Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/150-5.3.4-u-a+b-arctan-c-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1301]. This is test number [150].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1301)	0.00 (0)
Mathematica	98.31 (1279)	1.69 (22)
Maple	92.39 (1202)	7.61 (99)
Mupad	57.96 (754)	42.04 (547)
Sympy	44.20 (575)	55.80 (726)
Fricas	43.43 (565)	56.57 (736)
Giac	35.66 (464)	64.34 (837)
Maxima	32.51 (423)	67.49 (878)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

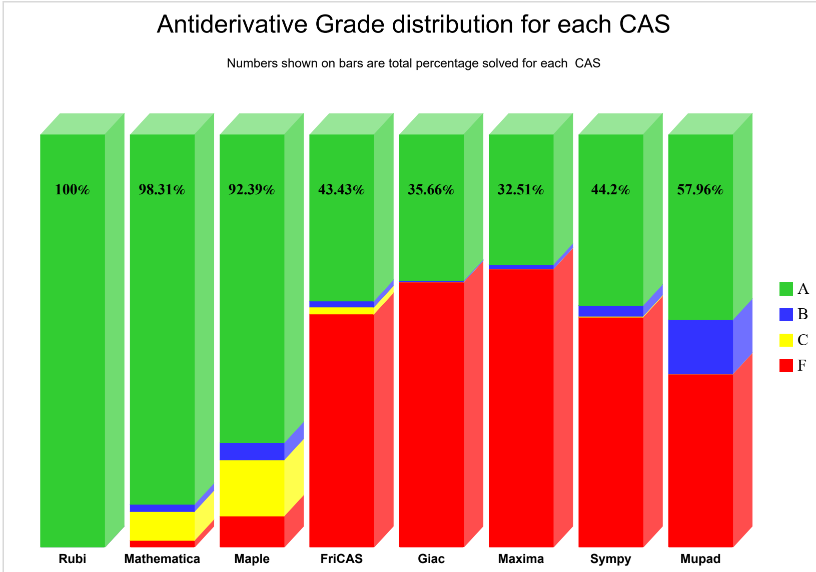
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

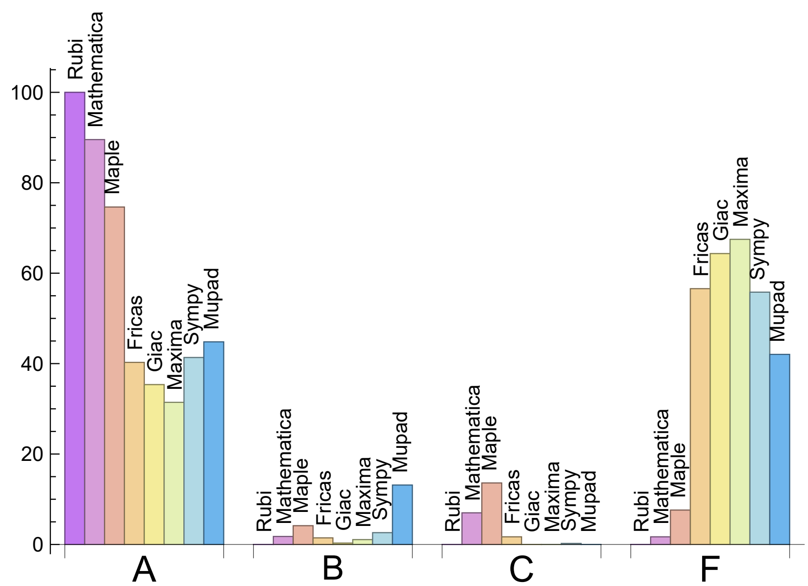
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	89.55	1.77	6.99	1.69
Maple	74.63	4.15	13.60	7.61
Mupad	N/A	13.14	0.00	42.04
Sympy	41.35	2.61	0.23	55.80
Fricas	40.28	1.46	1.69	56.57
Giac	35.36	0.31	0.00	64.34
Maxima	31.44	1.08	0.00	67.49

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	22	77.27 %	22.73 %	0.00 %
Maple	99	98.99 %	1.01 %	0.00 %
Fricas	736	52.85 %	0.00 %	47.15 %
Giac	837	66.79 %	10.87 %	22.34 %
Maxima	878	46.47 %	0.91 %	52.62 %
Sympy	726	64.46 %	25.21 %	10.33 %
Mupad	547	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

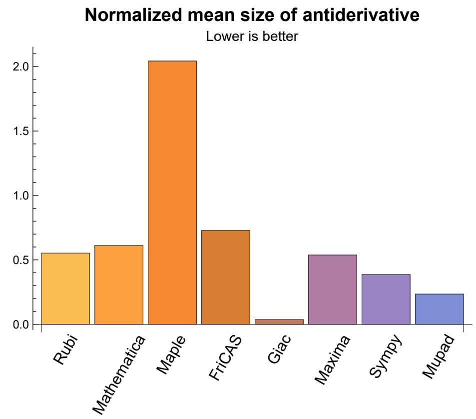
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

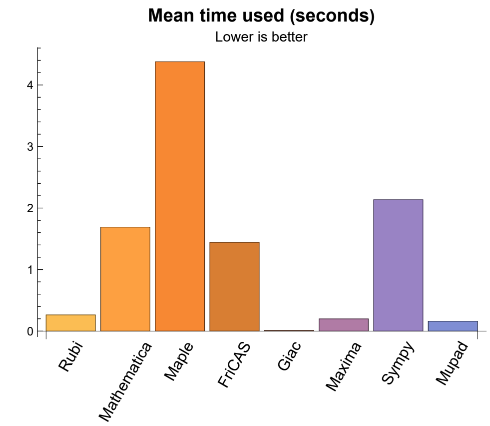
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	136.62	0.55	69.00	1.00
Mathematica	1.69	160.43	0.61	55.00	0.58
Maple	4.38	616.66	2.04	28.00	0.59
Maxima	0.20	71.98	0.54	0.00	0.00
Fricas	1.44	100.93	0.73	0.00	0.00
Sympy	2.13	51.80	0.39	0.00	0.00
Giac	0.01	1.60	0.04	0.00	0.00
Mupad	0.16	33.25	0.23	-1.00	-0.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{133, 148, 251, 252, 253, 254, 255, 256, 257, 355, 356, 357, 358, 359, 360, 361, 362, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 987, 988, 989, 990, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1057, 1058, 1059, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1113, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1299}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {217, 308, 315, 316, 318, 323, 324, 325, 327, 328, 332, 412, 413, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 451, 1162, 1163, 1170, 1171, 1229, 1261, 1263, 1265, 1267}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

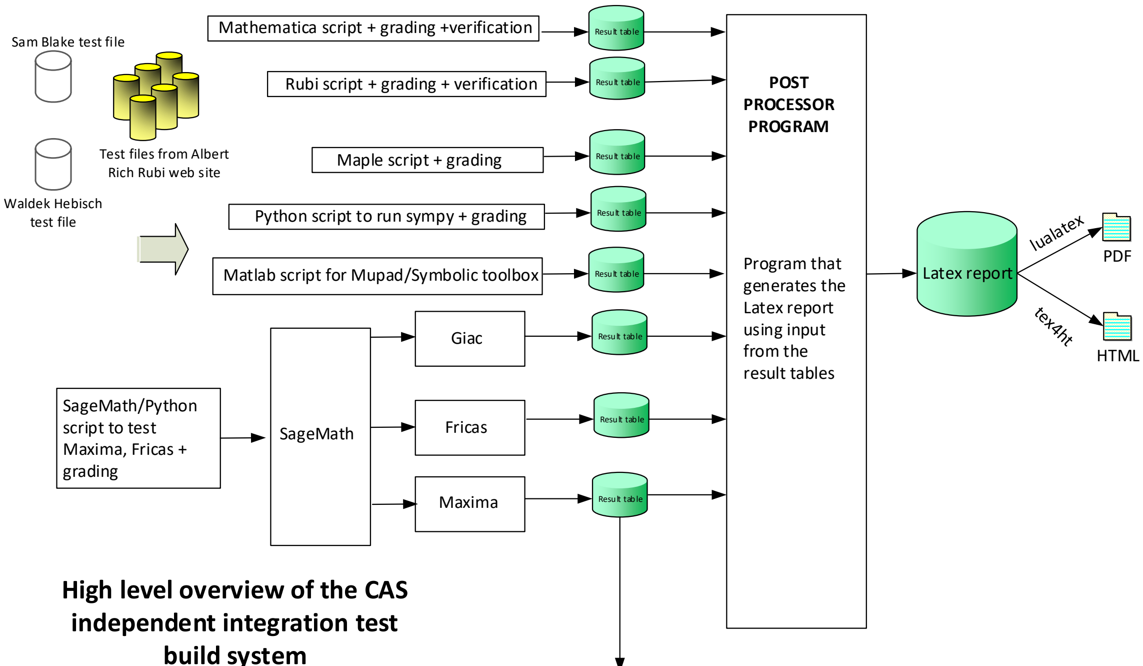
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427, 428, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 752, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 786, 787, 788, 789, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 825, 826, 827, 828, 829, 830, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 868, 869, 870, 871, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 933, 934, 935, 936, 937, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 963, 964, 965, 966, 967, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 993, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1061, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1123, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1244, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1270, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1292, 1293, 1294, 1295, 1296, 1299, 1300 }

B grade: { 130, 217, 287, 316, 323, 324, 325, 326, 392, 413, 421, 423, 429, 430, 431, 433, 1261, 1263, 1265, 1267, 1297, 1298, 1301 }

C grade: { 155, 180, 710, 715, 716, 717, 718, 745, 751, 753, 785, 791, 792, 793, 794, 824, 831, 832, 833, 867, 872, 873, 874, 875, 904, 909, 910, 911, 932, 938, 939, 940, 941, 962, 968, 969, 970, 992, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1120, 1122, 1124, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1229, 1230 }

F grade: { 141, 142, 143, 144, 145, 146, 147, 515, 1228, 1243, 1245, 1262, 1264, 1266, 1268, 1269, 1271, 1272, 1273, 1274, 1280, 1291 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 76, 77, 84, 85, 94, 118, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 179, 181, 182, 183, 184, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 198, 199, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 281, 282, 286, 292, 293, 294, 296, 298, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 369, 371, 373, 375, 377, 379, 381, 383, 385, 388, 391, 394, 397, 398, 399, 402, 404, 405, 406, 407, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 448, 449, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1150, 1159, 1167, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1290, 1299 }

B grade: { 28, 40, 45, 46, 47, 48, 49, 53, 67, 69, 70, 71, 74, 75, 78, 79, 83, 86, 87, 92, 93, 107, 114, 115, 119, 124, 125, 126, 176, 178, 180, 188, 190, 196, 263, 265, 284, 288, 290, 1120, 1149, 1156, 1162, 1163, 1166, 1170, 1171, 1248, 1250, 1252, 1255, 1257, 1259, 1270 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 341, 342, 349, 350, 351, 352, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 446, 447, 452, 453, 454, 455, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 643, 660, 661, 668, 669, 670, 671, 1151, 1152, 1153, 1154, 1155, 1157, 1158, 1160, 1161, 1164, 1165, 1168, 1169, 1172, 1251, 1253, 1258, 1260, 1262, 1264, 1266, 1269, 1271, 1275, 1276, 1277, 1278, 1279, 1280, 1286, 1287, 1288, 1289, 1291, 1297 }

F grade: { 334, 340, 348, 438, 439, 444, 445, 450, 451, 587, 745, 746, 751, 752, 753, 754, 824, 825, 831, 832, 833, 834, 904, 905, 909, 910, 911, 912, 962, 963, 967, 968, 969, 970, 1027, 1028, 1034, 1035, 1036, 1037, 1096, 1097, 1103, 1104, 1105, 1106, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1261, 1263, 1265, 1267, 1268, 1272, 1273, 1274, 1281, 1282, 1283, 1284, 1285, 1292, 1293, 1294, 1295, 1296, 1298, 1300, 1301 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 22, 24, 25, 29, 30, 31, 35, 36, 37, 42, 58, 59, 60, 61, 62, 66, 114, 115, 118, 125, 126, 133, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 207, 208, 216, 224, 226, 229, 231, 234, 235, 242, 243, 244, 245, 251, 252, 253, 254, 255, 256, 257, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 350, 352, 355, 356, 357, 358, 359, 360, 361, 362, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1148, 1149, 1150, 1159, 1166, 1167, 1177, 1178, 1179, 1181, 1189, 1191, 1204, 1205, 1207, 1213, 1215, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1299 }

B grade: { 13, 23, 28, 32, 33, 34, 40, 41, 63, 65, 67, 202, 210, 218 }

C grade: { }

F grade: { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 232, 233, 236, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 351, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 660, 661, 668, 669, 670, 671, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1120, 1133, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1180, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1206, 1208, 1209, 1210, 1211, 1212, 1214, 1216, 1217, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 41, 42, 47, 48, 49, 50, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 99, 107, 114, 115, 118, 124, 125, 126, 133, 148, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 251, 252, 253, 254, 255, 256, 257, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 362, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 694, 698, 702, 707, 713, 721, 725, 729, 733, 742, 749, 756, 762, 768, 774, 778, 783, 789, 797, 802, 807, 812, 821, 828, 837, 843, 849, 855, 859, 864, 870, 878, 883, 888, 893, 902, 907, 914, 918, 922, 926, 928, 930, 936, 944, 948, 952, 956, 960, 965, 972, 976, 980, 984, 986, 988, 998, 1007, 1011, 1015, 1019, 1024, 1033, 1042, 1046, 1050, 1054, 1056, 1058, 1067, 1076, 1080, 1084, 1088, 1093, 1102, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1142, 1144, 1146, 1148, 1149, 1150, 1159, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290, 1299 }

B grade: { 4, 28, 40, 130, 1166, 1167, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227 }

C grade: { 482, 483, 484, 489, 490, 491, 492, 493, 552, 553, 554, 559, 560, 561, 562, 628, 629, 630, 635, 636, 637, 638 }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 51, 52, 53, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 456, 457, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 660, 661, 668, 669, 670, 671, 687, 688, 689, 691, 692, 693, 695, 696, 697, 699, 700, 701, 703, 704, 705, 706, 708, 709, 710, 711, 712, 714, 715, 716, 717, 718, 719, 720, 722, 723, 724, 726, 727, 728, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 750, 751, 752, 753, 754, 755, 757, 758, 759, 760, 761, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 775, 776, 777, 779, 780, 781, 782, 784, 785, 786, 787, 788, 790, 791, 792, 793, 794, 795, 796, 798, 799, 800, 801, 803, 804, 805, 806, 808, 809, 810, 811, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 826, 827, 829, 830, 831, 832, 833, 834, 835, 836, 838, 839, 840, 841, 842, 844, 845, 846, 847, 848, 850, 851, 852, 853, 854, 856, 857, 858, 860, 861, 862, 863, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 879, 880, 881, 882, 884, 885, 886, 887, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 915, 916, 917, 919, 920, 921, 923, 924, 925, 927, 929, 931, 932, 933, 934, 935, 937, 938, 939, 940, 941, 942, 943, 945, 946, 947, 949, 950, 951, 953, 954, 955, 957, 958, 959, 961, 962, 963, 964, 966, 967, 968, 969, 970, 971, 973, 974, 975, 977, 978, 979, 981, 982, 983, 985, 987, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1043, 1044, 1045, 1047, 1048, 1049, 1051, 1052, 1053, 1055, 1057, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1081, 1082, 1083, 1085, 1086, 1087, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.1.6 Sympy

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 18, 19, 20, 21, 30, 31, 32, 42, 133, 148, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 179, 181, 186, 251, 252, 255, 256, 257, 258, 260, 266, 268, 274, 276, 355, 356, 357, 358, 360, 361, 362, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 606, 607, 608, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 682, 683, 684, 686, 687, 688, 689, 690, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 721, 722, 723, 724, 726, 727, 728, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 750, 755, 757, 758, 759, 760, 761, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 788, 790, 796, 798, 799, 800, 801, 806, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 826, 827, 835, 836, 838, 839, 840, 841, 842, 844, 845, 846, 847, 848, 851, 852, 853, 854, 856, 857, 858, 859, 860, 861, 862, 863, 865, 869, 871, 877, 897, 898, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 935, 937, 943, 944, 945, 946, 947, 949, 950, 951, 955, 956, 957, 958, 959, 961, 964, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 981, 982, 983, 984, 985, 986, 987, 989, 990, 994, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1012, 1013, 1014, 1020, 1021, 1022, 1023, 1025, 1026, 1029, 1030, 1043, 1044, 1045, 1047, 1048, 1049, 1051, 1052, 1053, 1055, 1056, 1057, 1059, 1063, 1064, 1065, 1066, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1089, 1090, 1091, 1094, 1095, 1111, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1140, 1142, 1144, 1146, 1148, 1150, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1231, 1235, 1236, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290, 1292, 1294, 1296, 1299 }

B grade: { 4, 7, 13, 17, 22, 23, 28, 29, 33, 34, 40, 41, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 177, 183, 189, 191, 192, 194, 197, 199, 1127, 1139, 1149, 1159 }

C grade: { 1281, 1283, 1285 }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 185, 187, 188, 190, 193, 195, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 528, 533, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 597, 604, 605, 609, 628, 629, 630, 635, 636, 637, 638, 643, 660, 661, 668, 669, 670, 671, 680, 681, 685, 694, 702, 709, 710, 711, 715, 716, 717, 718, 719, 725, 729, 730, 731, 732, 745, 746, 749, 751, 752, 753, 754, 756, 762, 768, 778, 785, 786, 787, 789, 791, 792, 793, 794, 795, 797, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 821, 824, 825, 828, 829, 830, 831, 832, 833, 834, 837, 843, 849, 850, 855, 864, 866, 867, 868, 870, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 932, 933, 934, 936, 938, 939, 940, 941, 942, 948, 952, 953, 954, 960, 962, 963, 965, 967, 968, 969, 970, 980, 988, 991, 992, 993, 998, 999, 1000, 1001, 1002, 1011, 1015, 1016, 1017, 1018, 1019, 1024, 1027, 1028, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1046, 1050, 1054, 1058, 1060, 1061, 1062, 1067, 1068, 1069, 1070, 1071, 1076, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1092, 1093, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1113, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1232, 1233, 1234, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1282, 1284, 1291, 1293, 1295, 1297, 1298, 1300, 1301 }

2.1.7 Giac

A grade: { 133, 148, 251, 252, 256, 257, 355, 356, 357, 358, 361, 362, 458, 459, 460, 461, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 513, 515, 521, 522, 523, 524, 525, 526, 527, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 584, 586, 588, 594, 596, 598, 599, 600, 601, 602, 603, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 663, 665, 667, 673, 674, 675, 676, 677, 678, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 722, 726, 730, 733, 735, 736, 737, 738, 739, 740, 741, 742, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 767, 768, 769, 770, 771, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 798, 803, 808, 812, 814, 815, 816, 817, 818, 819, 820, 821, 823, 826, 827, 828, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 849, 850, 851, 852, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 879, 884, 889, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 935, 936, 937, 943, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 957, 958, 959, 960, 961, 965, 966, 972, 973, 974, 975, 976, 978, 979, 980, 982, 983, 984, 985, 986, 987, 988, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1013, 1014, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1026, 1030, 1032, 1033, 1039, 1041, 1042, 1044, 1045, 1046, 1048, 1050, 1052, 1054, 1055, 1056, 1057, 1058, 1063, 1064, 1065, 1066, 1078, 1079, 1082, 1083, 1086, 1087, 1088, 1090, 1091, 1092, 1093, 1095, 1099, 1101, 1102, 1108, 1110, 1111, 1112, 1113, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1194, 1196, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1275, 1276 }

B grade: { 177, 1277, 1278, 1279 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 477, 482, 483, 484, 489, 490, 491, 492, 493, 508, 510, 511, 512, 514, 516, 517, 518, 519, 520, 528, 529, 530, 546, 552, 553, 554, 559, 560, 561, 562, 579, 581, 582, 583, 585, 587, 589, 590, 591, 592, 593, 595, 597, 604, 605, 606, 622, 628, 629, 630, 635, 636, 637, 638, 643, 658, 660, 661, 662, 664, 666, 668, 669, 670, 671, 672, 679, 680, 681, 682, 697, 709, 710, 711, 715, 716, 717, 718, 719, 721, 723, 724, 725, 727, 728, 729, 731, 732, 734, 743, 745, 746, 751, 752, 753, 754, 766, 772, 773, 785, 786, 787, 791, 792, 793, 794, 795, 797, 799, 800, 801, 802, 804, 805, 806, 807, 809, 810, 811, 813, 822, 824, 825, 829, 831, 832, 833, 834, 847, 848, 853, 854, 866, 867, 868, 872, 873, 874, 875, 876, 878, 880, 881, 882, 883, 885, 886, 887, 888, 890, 891, 892, 894, 904, 905, 909, 910, 911, 912, 932, 933, 934, 938, 939, 940, 941, 942, 944, 948, 952, 962, 963, 964, 967, 968, 969, 970, 971, 977, 981, 989, 990, 991, 992, 993, 999, 1000, 1001, 1002, 1007, 1011, 1012, 1015, 1016, 1025, 1027, 1028, 1029, 1031, 1034, 1035, 1036, 1037, 1038, 1040, 1043, 1047, 1049, 1051, 1053, 1059, 1060, 1061, 1062, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1084, 1085, 1089, 1094, 1096, 1097, 1098, 1100, 1103, 1104, 1105, 1106, 1107, 1109, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185,

1187, 1188, 1189, 1190, 1191, 1192, 1193, 1195, 1197, 1198, 1199, 1200, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301
}

2.1.8 Mupad

A grade: { 133, 148, 251, 252, 253, 254, 255, 256, 257, 355, 356, 357, 358, 359, 360, 361, 362, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486, 487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 931, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 987, 988, 989, 990, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1057, 1058, 1059, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1113, 1174, 1176, 1177, 1178, 1179, 1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205, 1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1242, 1244, 1246, 1299 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 391, 397, 398, 399, 404, 405, 406, 407, 477, 546, 622, 643, 702, 778, 859, 928, 986, 1056, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1159, 1166, 1167, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1281, 1286, 1287, 1288, 1289, 1290 }

C grade: { }

F grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 660, 661, 668, 669, 670, 671, 709, 710, 711, 715, 716, 717, 718, 719, 745, 746, 751, 752, 753, 754, 785, 786, 787, 791, 792, 793, 794, 795, 824, 825, 831, 832, 833, 834, 866, 867, 868, 872, 873, 874, 875, 876, 904, 905, 909, 910, 911, 912, 932, 933, 934, 938, 939, 940, 941, 942, 962, 963, 967, 968, 969, 970, 991, 992, 993, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1061, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	98	117	109	124	184	0	109
N.S.	1	1.00	0.84	1.00	0.93	1.06	1.57	0.00	0.93
time (sec)	N/A	0.074	0.050	0.122	0.482	4.120	1.716	0.000	0.739

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	107	99	113	167	0	99
N.S.	1	1.00	0.84	1.02	0.94	1.08	1.59	0.00	0.94
time (sec)	N/A	0.066	0.038	0.084	0.462	2.014	1.521	0.000	0.711

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	96	88	104	158	0	87
N.S.	1	1.00	0.84	1.05	0.97	1.14	1.74	0.00	0.96
time (sec)	N/A	0.055	0.035	0.103	0.470	1.588	1.415	0.000	0.382

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	79	73	89	128	0	73
N.S.	1	1.00	1.58	1.49	1.38	1.68	2.42	0.00	1.38
time (sec)	N/A	0.022	0.007	0.076	0.473	3.320	1.249	0.000	0.324

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	113	0	0	0	0	63
N.S.	1	1.00	1.00	1.49	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.059	0.006	0.059	0.000	0.000	0.000	0.000	0.618

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	127	0	0	0	0	93
N.S.	1	1.00	1.01	1.65	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.072	0.033	0.060	0.000	0.000	0.000	0.000	0.859

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	98	75	99	182	0	79
N.S.	1	1.00	1.17	1.51	1.15	1.52	2.80	0.00	1.22
time (sec)	N/A	0.040	0.037	0.128	0.469	2.483	1.827	0.000	0.541

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	89	108	87	109	197	0	176
N.S.	1	1.00	0.84	1.02	0.82	1.03	1.86	0.00	1.66
time (sec)	N/A	0.068	0.033	0.107	0.475	3.204	2.544	0.000	0.883

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	119	102	119	214	0	116
N.S.	1	1.00	0.81	0.96	0.82	0.96	1.73	0.00	0.94
time (sec)	N/A	0.068	0.049	0.138	0.484	2.191	4.019	0.000	0.580

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	124	171	185	172	270	0	152
N.S.	1	1.00	0.75	1.03	1.11	1.04	1.63	0.00	0.92
time (sec)	N/A	0.118	0.075	0.151	0.469	2.539	2.230	0.000	0.756

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	116	159	174	160	250	0	140
N.S.	1	1.00	0.76	1.05	1.14	1.05	1.64	0.00	0.92
time (sec)	N/A	0.104	0.065	0.129	0.472	1.836	1.988	0.000	0.740

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	101	146	155	148	240	0	125
N.S.	1	1.00	0.74	1.07	1.14	1.09	1.76	0.00	0.92
time (sec)	N/A	0.092	0.054	0.143	0.473	2.674	1.861	0.000	0.683

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	114	138	127	206	0	109
N.S.	1	1.00	0.69	1.37	1.66	1.53	2.48	0.00	1.31
time (sec)	N/A	0.033	0.027	0.077	0.488	2.401	1.540	0.000	0.399

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	103	177	142	0	0	0	131
N.S.	1	1.00	0.80	1.37	1.10	0.00	0.00	0.00	1.02
time (sec)	N/A	0.089	0.047	0.070	0.595	0.000	0.000	0.000	0.725

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	149	0	0	0	0	141
N.S.	1	1.00	0.89	1.67	0.00	0.00	0.00	0.00	1.58
time (sec)	N/A	0.101	0.048	0.069	0.000	0.000	0.000	0.000	0.603

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	137	206	0	0	0	0	161
N.S.	1	1.00	0.90	1.36	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.117	0.046	0.091	0.000	0.000	0.000	0.000	0.745

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	109	146	144	144	253	0	120
N.S.	1	1.00	1.25	1.68	1.66	1.66	2.91	0.00	1.38
time (sec)	N/A	0.057	0.040	0.122	0.463	3.278	5.182	0.000	0.641

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	119	161	152	155	275	0	142
N.S.	1	1.00	0.74	1.00	0.94	0.96	1.71	0.00	0.88
time (sec)	N/A	0.109	0.110	0.164	0.473	2.916	9.205	0.000	0.696

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	129	173	183	167	287	0	244
N.S.	1	1.00	0.75	1.01	1.07	0.98	1.68	0.00	1.43
time (sec)	N/A	0.115	0.048	0.131	0.466	1.440	15.021	0.000	0.919

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	154	210	261	202	328	0	186
N.S.	1	1.00	0.75	1.02	1.27	0.99	1.60	0.00	0.91
time (sec)	N/A	0.133	0.071	0.167	0.479	1.594	2.796	0.000	0.886

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	146	198	242	190	316	0	174
N.S.	1	1.00	0.76	1.04	1.27	0.99	1.65	0.00	0.91
time (sec)	N/A	0.121	0.061	0.133	0.476	2.087	2.464	0.000	0.828

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	132	185	222	177	296	0	160
N.S.	1	1.00	0.84	1.18	1.41	1.13	1.89	0.00	1.02
time (sec)	N/A	0.070	0.058	0.128	0.469	1.270	2.401	0.000	0.728

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	77	144	197	161	267	0	147
N.S.	1	1.00	0.77	1.44	1.97	1.61	2.67	0.00	1.47
time (sec)	N/A	0.037	0.028	0.084	0.472	1.458	2.118	0.000	0.693

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	139	220	184	0	0	0	196
N.S.	1	1.00	0.82	1.29	1.08	0.00	0.00	0.00	1.15
time (sec)	N/A	0.126	0.066	0.091	0.605	0.000	0.000	0.000	0.833

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	150	216	201	0	0	0	195
N.S.	1	1.00	0.93	1.33	1.24	0.00	0.00	0.00	1.20
time (sec)	N/A	0.120	0.065	0.084	0.602	0.000	0.000	0.000	0.721

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	164	228	0	0	0	0	205
N.S.	1	1.00	0.91	1.27	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.127	0.064	0.092	0.000	0.000	0.000	0.000	0.726

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	171	244	0	0	0	0	221
N.S.	1	1.00	0.90	1.29	0.00	0.00	0.00	0.00	1.17
time (sec)	N/A	0.147	0.057	0.090	0.000	0.000	0.000	0.000	0.973

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	141	187	202	174	311	0	154
N.S.	1	1.00	1.37	1.82	1.96	1.69	3.02	0.00	1.50
time (sec)	N/A	0.066	0.142	0.163	0.474	1.447	15.910	0.000	0.665

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	197	224	185	326	0	174
N.S.	1	1.00	0.99	1.31	1.49	1.23	2.17	0.00	1.16
time (sec)	N/A	0.077	0.054	0.138	0.467	2.332	28.597	0.000	0.951

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	161	212	248	198	347	0	192
N.S.	1	1.00	0.75	0.99	1.16	0.93	1.62	0.00	0.90
time (sec)	N/A	0.130	0.608	0.175	0.480	1.580	49.481	0.000	1.059

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	178	246	337	230	389	0	217
N.S.	1	1.00	0.75	1.03	1.42	0.97	1.63	0.00	0.91
time (sec)	N/A	0.155	0.076	0.172	0.470	1.701	3.476	0.000	2.589

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	170	234	318	218	367	0	205
N.S.	1	1.00	0.88	1.21	1.65	1.13	1.90	0.00	1.06
time (sec)	N/A	0.126	0.074	0.111	0.470	1.623	3.111	0.000	0.642

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	158	221	290	206	360	0	191
N.S.	1	1.00	0.89	1.24	1.63	1.16	2.02	0.00	1.07
time (sec)	N/A	0.083	0.071	0.134	0.475	1.841	2.812	0.000	0.793

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	172	264	188	316	0	175
N.S.	1	1.00	0.62	1.38	2.11	1.50	2.53	0.00	1.40
time (sec)	N/A	0.043	0.022	0.091	0.476	1.638	2.300	0.000	0.743

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	174	260	220	0	0	0	248
N.S.	1	1.00	0.86	1.28	1.08	0.00	0.00	0.00	1.22
time (sec)	N/A	0.149	0.072	0.092	0.600	0.000	0.000	0.000	0.986

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	181	257	240	0	0	0	253
N.S.	1	1.00	0.95	1.35	1.26	0.00	0.00	0.00	1.33
time (sec)	N/A	0.160	0.079	0.105	0.620	0.000	0.000	0.000	0.816

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	163	237	251	0	0	0	258
N.S.	1	1.00	0.94	1.37	1.45	0.00	0.00	0.00	1.49
time (sec)	N/A	0.148	0.073	0.097	0.630	0.000	0.000	0.000	0.892

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	262	0	0	0	0	261
N.S.	1	1.00	0.96	1.30	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.159	0.070	0.090	0.000	0.000	0.000	0.000	0.833

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	210	287	0	0	0	0	298
N.S.	1	1.00	0.93	1.26	0.00	0.00	0.00	0.00	1.31
time (sec)	N/A	0.171	0.064	0.106	0.000	0.000	0.000	0.000	0.943

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	167	223	275	202	366	0	186
N.S.	1	1.00	1.43	1.91	2.35	1.73	3.13	0.00	1.59
time (sec)	N/A	0.071	0.061	0.145	0.474	0.948	49.269	0.000	0.734

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	236	290	216	388	0	208
N.S.	1	1.00	1.06	1.40	1.73	1.29	2.31	0.00	1.24
time (sec)	N/A	0.081	0.985	0.197	0.489	1.385	91.683	0.000	0.897

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	187	248	329	228	398	0	317
N.S.	1	1.00	0.77	1.02	1.35	0.94	1.64	0.00	1.30
time (sec)	N/A	0.140	0.081	0.158	0.467	1.487	154.497	0.000	1.174

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	166	312	0	0	0	0	-1
N.S.	1	1.00	0.85	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.302	0.164	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	132	267	0	0	0	0	-1
N.S.	1	1.00	0.85	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.121	0.095	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	214	0	0	0	0	-1
N.S.	1	1.00	0.98	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.115	0.089	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	125	0	0	0	0	-1
N.S.	1	1.00	1.02	2.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.012	0.065	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	83	193	0	43	0	0	-1
N.S.	1	1.00	1.54	3.57	0.00	0.80	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.093	0.093	0.000	1.109	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	126	246	0	98	0	0	-1
N.S.	1	1.00	1.26	2.46	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.156	0.089	0.000	1.212	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	178	306	0	130	0	0	-1
N.S.	1	1.00	1.11	1.90	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.277	0.115	0.000	1.391	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	220	340	0	155	0	0	-1
N.S.	1	1.00	1.12	1.73	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.329	0.117	0.000	1.113	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	186	317	0	0	0	0	-1
N.S.	1	1.00	0.92	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.700	0.150	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	153	271	0	0	0	0	-1
N.S.	1	1.00	0.92	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.529	0.115	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	138	252	0	0	0	0	-1
N.S.	1	1.00	1.13	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.364	0.109	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	68	0	50	116	0	-1
N.S.	1	1.00	0.61	0.99	0.00	0.72	1.68	0.00	-0.01
time (sec)	N/A	0.035	0.027	0.091	0.000	1.700	0.965	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	251	0	127	0	0	-1
N.S.	1	1.00	0.92	1.67	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.477	0.110	0.000	1.413	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	175	329	0	182	0	0	-1
N.S.	1	1.00	0.90	1.70	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.791	0.124	0.000	1.243	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	219	345	0	221	0	0	-1
N.S.	1	1.00	0.90	1.41	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.198	1.062	0.135	0.000	1.664	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	235	364	358	0	0	0	-1
N.S.	1	1.00	0.92	1.42	1.40	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.654	0.160	0.357	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	216	321	334	0	0	0	-1
N.S.	1	1.00	0.96	1.43	1.48	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.519	0.142	0.341	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	187	299	291	0	0	0	-1
N.S.	1	1.00	1.06	1.70	1.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.400	0.125	0.324	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	63	109	70	85	189	0	-1
N.S.	1	1.00	0.72	1.24	0.80	0.97	2.15	0.00	-0.01
time (sec)	N/A	0.058	0.045	0.153	0.271	2.104	5.630	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	55	82	65	75	158	0	-1
N.S.	1	1.00	0.60	0.89	0.71	0.82	1.72	0.00	-0.01
time (sec)	N/A	0.040	0.036	0.096	0.283	1.254	1.822	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	198	327	406	201	0	0	-1
N.S.	1	1.00	1.02	1.68	2.08	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.611	0.132	0.333	1.963	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	241	380	0	263	0	0	-1
N.S.	1	1.00	0.96	1.52	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.212	1.027	0.142	0.000	1.794	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	274	431	590	311	0	0	-1
N.S.	1	1.00	0.90	1.41	1.93	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.238	1.319	0.152	0.360	2.066	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	79	83	93	168	0	-1
N.S.	1	1.00	0.73	0.79	0.83	0.93	1.68	0.00	-0.01
time (sec)	N/A	0.038	0.036	0.129	0.273	3.383	1.576	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	140	126	21	0	0	-1
N.S.	1	1.00	1.08	2.86	2.57	0.43	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.063	0.078	0.470	5.315	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	285	474	0	0	0	0	-1
N.S.	1	1.00	0.99	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.528	0.211	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	241	442	0	0	0	0	-1
N.S.	1	1.00	0.95	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.383	0.229	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	208	391	0	0	0	0	-1
N.S.	1	1.00	0.99	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.284	0.170	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	151	344	0	0	0	0	-1
N.S.	1	1.00	1.16	2.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.132	0.194	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	272	7034	0	0	0	0	-1
N.S.	1	1.00	1.26	32.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.263	2.566	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	289	5899	0	0	0	0	-1
N.S.	1	1.00	1.27	25.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.239	2.644	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	190	448	0	0	0	0	-1
N.S.	1	1.00	1.19	2.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.155	0.548	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	240	514	0	0	0	0	-1
N.S.	1	1.00	1.07	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.325	0.649	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	342	624	0	0	0	0	-1
N.S.	1	1.00	0.92	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.699	0.744	0.288	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	306	586	0	0	0	0	-1
N.S.	1	1.00	0.92	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	0.748	0.321	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	257	530	0	0	0	0	-1
N.S.	1	1.00	0.88	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.429	0.253	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	205	469	0	0	0	0	-1
N.S.	1	1.00	1.07	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.347	0.168	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	360	1542	0	0	0	0	-1
N.S.	1	1.00	1.20	5.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.380	4.228	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	378	11833	0	0	0	0	-1
N.S.	1	1.00	1.19	37.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.329	2.901	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	388	1565	0	0	0	0	-1
N.S.	1	1.00	1.15	4.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	0.507	6.798	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	253	619	0	0	0	0	-1
N.S.	1	1.00	0.95	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.413	0.588	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	408	720	0	0	0	0	-1
N.S.	1	1.00	0.93	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.004	1.163	0.312	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	369	682	0	0	0	0	-1
N.S.	1	1.00	0.92	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.847	0.885	0.342	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	325	626	0	0	0	0	-1
N.S.	1	1.00	1.06	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.746	0.229	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	267	553	0	0	0	0	-1
N.S.	1	1.00	1.18	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.479	0.202	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	465	1651	0	0	0	0	-1
N.S.	1	1.00	1.21	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	0.464	6.457	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	512	1702	0	0	0	0	-1
N.S.	1	1.00	1.27	4.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	0.323	8.267	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	500	1746	0	0	0	0	-1
N.S.	1	1.00	1.20	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	0.691	7.015	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	595	1726	0	0	0	0	-1
N.S.	1	1.00	1.39	4.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.640	0.431	10.272	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	322	701	0	0	0	0	-1
N.S.	1	1.00	1.10	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.510	0.707	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	363	760	0	0	0	0	-1
N.S.	1	1.00	0.95	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.827	1.041	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	401	797	0	0	0	0	-1
N.S.	1	1.00	0.78	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	1.006	0.872	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	421	1227	0	0	0	0	-1
N.S.	1	1.00	1.18	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	0.615	7.988	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	330	1113	0	0	0	0	-1
N.S.	1	1.00	1.19	4.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.346	4.636	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	239	4395	0	0	0	0	-1
N.S.	1	1.00	1.24	22.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.291	1.310	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	1003	0	0	0	0	-1
N.S.	1	1.00	0.97	10.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.024	2.501	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	173	1741	0	136	0	0	-1
N.S.	1	1.00	1.97	19.78	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.173	1.111	0.000	0.638	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	265	9130	0	0	0	0	-1
N.S.	1	1.00	1.42	49.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.536	1.588	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	372	2103	0	0	0	0	-1
N.S.	1	1.00	1.36	7.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	0.692	6.041	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	535	2256	0	0	0	0	-1
N.S.	1	1.00	1.47	6.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	0.691	10.141	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	502	1372	0	0	0	0	-1
N.S.	1	1.00	1.16	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	1.589	9.060	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	429	1274	0	0	0	0	-1
N.S.	1	1.00	1.18	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	1.205	5.536	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	362	4552	0	0	0	0	-1
N.S.	1	1.00	1.24	15.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.822	2.049	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	300	969	0	0	0	0	-1
N.S.	1	1.00	1.39	4.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.477	1.409	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	303	0	103	303	0	-1
N.S.	1	1.00	0.59	2.48	0.00	0.84	2.48	0.00	-0.01
time (sec)	N/A	0.092	0.087	0.296	0.000	0.955	5.639	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	299	1921	0	0	0	0	-1
N.S.	1	1.00	1.35	8.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	0.549	0.945	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	398	9303	0	0	0	0	-1
N.S.	1	1.00	1.30	30.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.557	1.611	1.886	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	491	2247	0	0	0	0	-1
N.S.	1	1.00	1.22	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	1.925	8.080	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	578	1475	0	0	0	0	-1
N.S.	1	1.00	1.25	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	1.595	6.382	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	507	4768	0	0	0	0	-1
N.S.	1	1.00	1.32	12.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	0.957	1.775	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	431	1164	0	0	0	0	-1
N.S.	1	1.00	1.42	3.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	0.723	1.609	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	117	401	141	165	502	0	-1
N.S.	1	1.00	0.66	2.25	0.79	0.93	2.82	0.00	-0.01
time (sec)	N/A	0.158	0.101	0.357	0.307	0.704	49.517	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	110	355	136	158	464	0	-1
N.S.	1	1.00	0.61	1.97	0.76	0.88	2.58	0.00	-0.01
time (sec)	N/A	0.138	0.080	0.310	0.300	0.771	40.483	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	435	2151	0	0	0	0	-1
N.S.	1	1.00	1.45	7.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	0.880	0.867	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	549	9532	0	0	0	0	-1
N.S.	1	1.00	1.40	24.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	2.238	2.120	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	155	345	184	206	552	0	-1
N.S.	1	1.00	0.75	1.67	0.89	1.00	2.67	0.00	-0.00
time (sec)	N/A	0.171	0.111	0.357	0.312	0.837	28.584	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	193	0	0	0	0	-1
N.S.	1	1.00	1.08	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.108	0.349	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	693	1875	0	0	0	0	-1
N.S.	1	1.00	1.81	4.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	1.005	13.181	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	528	1699	0	0	0	0	-1
N.S.	1	1.00	1.77	5.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.555	6.003	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	367	7165	0	0	0	0	-1
N.S.	1	1.00	1.67	32.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.258	1.692	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	133	1940	0	0	0	0	-1
N.S.	1	1.00	0.96	13.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.056	1.655	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	121	498	0	175	631	0	-1
N.S.	1	1.00	0.66	2.74	0.00	0.96	3.47	0.00	-0.01
time (sec)	N/A	0.161	0.093	0.727	0.000	1.261	18.393	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	183	656	233	265	954	0	-1
N.S.	1	1.00	0.68	2.42	0.86	0.98	3.52	0.00	-0.00
time (sec)	N/A	0.295	0.129	0.907	0.335	0.800	140.583	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	269	819	327	359	0	0	-1
N.S.	1	1.00	0.75	2.28	0.91	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.138	1.066	0.418	0.901	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	541	1586	0	0	0	0	-1
N.S.	1	1.00	1.32	3.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	0.628	9.622	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	393	5234	0	0	0	0	-1
N.S.	1	1.00	1.42	18.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.465	1.566	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	133	1940	0	0	0	0	-1
N.S.	1	1.00	0.96	13.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	268	3393	0	246	0	0	-1
N.S.	1	1.00	2.09	26.51	0.00	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.241	1.128	0.000	0.884	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	436	11097	0	0	0	0	-1
N.S.	1	1.00	1.66	42.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.908	1.612	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	634	2884	0	0	0	0	-1
N.S.	1	1.00	1.53	6.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.716	1.581	14.119	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.317	0.370	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	484	432	0	0	0	0	-1
N.S.	1	1.00	1.63	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	2.171	0.192	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	404	337	0	0	0	0	-1
N.S.	1	1.00	1.70	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	1.031	0.098	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	329	261	0	0	0	0	-1
N.S.	1	1.00	1.84	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.973	0.084	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	130	178	0	0	0	0	-1
N.S.	1	1.00	0.94	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	308	260	0	0	0	0	-1
N.S.	1	1.00	1.70	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.693	0.128	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	372	363	0	0	0	0	-1
N.S.	1	1.00	1.60	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.881	0.126	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	441	448	0	0	0	0	-1
N.S.	1	1.00	1.51	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	2.169	0.154	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	0	2208	0	0	0	0	-1
N.S.	1	1.00	0.00	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	156.363	35.816	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	0	1855	0	0	0	0	-1
N.S.	1	1.00	0.00	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	83.164	26.639	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	0	16245	0	0	0	0	-1
N.S.	1	1.00	0.00	50.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	148.844	14.026	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	0	1324	0	0	0	0	-1
N.S.	1	1.00	0.00	5.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.035	0.076	0.003	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	0	2363	0	0	0	0	-1
N.S.	1	1.00	0.00	6.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	148.179	2.247	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	0	40859	0	0	0	0	-1
N.S.	1	1.00	0.00	86.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	83.699	18.747	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	0	3004	0	0	0	0	-1
N.S.	1	1.00	0.00	5.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	165.043	33.840	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.374	0.252	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	60	64	57	65	0	57
N.S.	1	1.00	1.00	0.87	0.93	0.83	0.94	0.00	0.83
time (sec)	N/A	0.059	0.009	0.174	0.474	2.726	0.316	0.000	0.307

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	63	62	61	0	58
N.S.	1	1.00	1.00	0.95	0.95	0.94	0.92	0.00	0.88
time (sec)	N/A	0.068	0.021	0.148	0.272	2.101	0.248	0.000	0.230

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	54	50	44	54	0	48
N.S.	1	1.00	1.38	1.29	1.19	1.05	1.29	0.00	1.14
time (sec)	N/A	0.018	0.007	0.107	0.270	2.255	0.195	0.000	0.480

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	65	50	49	45	47	48	0	46
N.S.	1	1.30	1.00	0.98	0.90	0.94	0.96	0.00	0.92
time (sec)	N/A	0.016	0.011	0.075	0.259	2.316	0.169	0.000	0.164

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	90	66	0	0	0	57
N.S.	1	1.00	1.00	1.45	1.06	0.00	0.00	0.00	0.92
time (sec)	N/A	0.048	0.007	0.045	0.502	0.000	0.000	0.000	0.546

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	45	40	45	41	0	42
N.S.	1	1.00	1.00	1.12	1.00	1.12	1.02	0.00	1.05
time (sec)	N/A	0.040	0.007	0.103	0.246	2.254	0.304	0.000	0.163

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	96	95	0	0	0	71
N.S.	1	1.00	1.06	1.37	1.36	0.00	0.00	0.00	1.01
time (sec)	N/A	0.050	0.007	0.052	0.528	0.000	0.000	0.000	0.561

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	60	56	57	61	0	57
N.S.	1	1.00	0.92	0.95	0.89	0.90	0.97	0.00	0.90
time (sec)	N/A	0.059	0.018	0.125	0.263	1.492	0.370	0.000	0.146

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	68	88	98	91	104	0	89
N.S.	1	1.00	0.61	0.79	0.88	0.82	0.94	0.00	0.80
time (sec)	N/A	0.104	0.032	0.182	0.474	2.033	0.462	0.000	0.417

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	79	93	95	94	105	0	81
N.S.	1	1.00	0.75	0.88	0.90	0.89	0.99	0.00	0.76
time (sec)	N/A	0.122	0.028	0.132	0.269	5.533	0.410	0.000	0.536

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	85	62	77	92	0	71
N.S.	1	1.00	0.82	1.39	1.02	1.26	1.51	0.00	1.16
time (sec)	N/A	0.030	0.024	0.098	0.253	2.436	0.751	0.000	0.546

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	65	80	77	79	88	0	69
N.S.	1	1.00	0.56	0.68	0.66	0.68	0.75	0.00	0.59
time (sec)	N/A	0.033	0.019	0.100	0.251	2.308	0.253	0.000	0.198

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	74	119	104	0	0	0	105
N.S.	1	1.00	0.75	1.20	1.05	0.00	0.00	0.00	1.06
time (sec)	N/A	0.086	0.032	0.070	0.497	0.000	0.000	0.000	0.608

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	77	71	75	82	0	76
N.S.	1	1.00	0.77	0.95	0.88	0.93	1.01	0.00	0.94
time (sec)	N/A	0.085	0.023	0.099	0.259	2.051	0.483	0.000	0.212

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	117	120	0	0	0	110
N.S.	1	1.00	0.84	1.30	1.33	0.00	0.00	0.00	1.22
time (sec)	N/A	0.081	0.032	0.060	0.498	0.000	0.000	0.000	0.498

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	68	78	76	80	87	0	78
N.S.	1	1.00	0.80	0.92	0.89	0.94	1.02	0.00	0.92
time (sec)	N/A	0.084	0.025	0.129	0.253	2.266	0.452	0.000	0.466

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	76	112	120	113	138	0	111
N.S.	1	1.00	0.54	0.79	0.85	0.80	0.98	0.00	0.79
time (sec)	N/A	0.143	0.035	0.133	0.475	1.902	0.671	0.000	0.437

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	116	118	116	138	0	108
N.S.	1	1.00	0.70	0.85	0.87	0.85	1.01	0.00	0.79
time (sec)	N/A	0.163	0.030	0.142	0.289	2.593	0.588	0.000	0.438

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	107	73	99	124	0	100
N.S.	1	1.00	0.78	1.45	0.99	1.34	1.68	0.00	1.35
time (sec)	N/A	0.035	0.026	0.100	0.281	3.475	0.447	0.000	0.413

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	83	102	99	101	117	0	89
N.S.	1	1.00	0.52	0.63	0.61	0.63	0.73	0.00	0.55
time (sec)	N/A	0.054	0.022	0.091	0.251	3.571	0.412	0.000	0.246

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	94	143	127	0	0	0	156
N.S.	1	1.00	0.71	1.08	0.96	0.00	0.00	0.00	1.18
time (sec)	N/A	0.112	0.037	0.061	0.502	0.000	0.000	0.000	0.667

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	78	99	93	97	110	0	85
N.S.	1	1.00	0.72	0.92	0.86	0.90	1.02	0.00	0.79
time (sec)	N/A	0.115	0.026	0.124	0.262	3.074	0.651	0.000	0.563

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	109	148	155	0	0	0	152
N.S.	1	1.00	0.79	1.07	1.12	0.00	0.00	0.00	1.10
time (sec)	N/A	0.121	0.039	0.082	0.506	0.000	0.000	0.000	0.567

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	83	102	96	100	117	0	97
N.S.	1	1.00	0.72	0.88	0.83	0.86	1.01	0.00	0.84
time (sec)	N/A	0.112	0.026	0.129	0.254	3.547	0.628	0.000	0.536

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	56	76	74	54	70	0	73
N.S.	1	1.00	0.70	0.95	0.92	0.68	0.88	0.00	0.91
time (sec)	N/A	0.109	0.042	0.117	0.488	2.018	0.462	0.000	0.169

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	120	185	0	0	0	0	-1
N.S.	1	1.00	1.06	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.026	0.056	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	53	54	37	42	0	33
N.S.	1	1.00	1.00	1.08	1.10	0.76	0.86	0.00	0.67
time (sec)	N/A	0.052	0.023	0.090	0.473	3.033	0.304	0.000	0.157

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	159	0	0	0	0	-1
N.S.	1	1.00	1.07	2.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.006	0.073	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	36	35	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	2.25	2.19	0.88
time (sec)	N/A	0.012	0.004	0.064	0.471	4.990	1.453	0.406	0.385

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	103	221	0	0	0	0	-1
N.S.	1	1.00	1.61	3.45	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.020	0.059	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	62	53	43	42	0	48
N.S.	1	1.00	1.00	1.19	1.02	0.83	0.81	0.00	0.92
time (sec)	N/A	0.062	0.008	0.128	0.478	1.445	0.471	0.000	0.455

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	142	252	0	0	0	0	-1
N.S.	1	1.00	1.26	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.044	0.063	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	85	90	71	76	0	78
N.S.	1	1.00	1.00	0.97	1.02	0.81	0.86	0.00	0.89
time (sec)	N/A	0.116	0.014	0.070	0.495	1.855	0.688	0.000	0.469

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	90	220	0	0	0	0	-1
N.S.	1	1.00	0.57	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.154	0.070	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	86	114	81	223	0	94
N.S.	1	1.00	0.82	0.90	1.19	0.84	2.32	0.00	0.98
time (sec)	N/A	0.137	0.032	0.154	0.486	3.742	0.564	0.000	0.458

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	77	202	0	0	0	0	-1
N.S.	1	1.00	0.58	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.073	0.063	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	66	83	49	0	0	48
N.S.	1	1.00	0.73	1.03	1.30	0.77	0.00	0.00	0.75
time (sec)	N/A	0.047	0.023	0.128	0.470	3.431	0.000	0.000	0.401

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	39	53	59	40	82	0	40
N.S.	1	1.00	0.63	0.85	0.95	0.65	1.32	0.00	0.65
time (sec)	N/A	0.031	0.022	0.093	0.453	2.999	0.440	0.000	0.165

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	66	78	46	0	0	48
N.S.	1	1.00	0.72	1.08	1.28	0.75	0.00	0.00	0.79
time (sec)	N/A	0.019	0.018	0.119	0.466	1.708	0.000	0.000	0.420

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	72	262	0	0	0	0	-1
N.S.	1	1.00	0.62	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.102	0.072	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	94	95	119	97	274	0	91
N.S.	1	1.00	0.97	0.98	1.23	1.00	2.82	0.00	0.94
time (sec)	N/A	0.116	0.040	0.206	0.476	7.155	0.840	0.000	0.485

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	93	288	0	0	0	0	-1
N.S.	1	1.00	0.60	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.265	0.079	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	124	121	160	127	362	0	123
N.S.	1	1.00	0.91	0.89	1.18	0.93	2.66	0.00	0.90
time (sec)	N/A	0.284	0.046	0.085	0.493	1.726	1.178	0.000	0.545

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	58	84	108	69	209	0	62
N.S.	1	1.00	0.67	0.98	1.26	0.80	2.43	0.00	0.72
time (sec)	N/A	0.047	0.092	0.132	0.459	1.278	0.733	0.000	0.500

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	64	105	129	83	0	0	80
N.S.	1	1.00	0.58	0.95	1.16	0.75	0.00	0.00	0.72
time (sec)	N/A	0.058	0.027	0.161	0.474	1.406	0.000	0.000	0.468

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	55	68	86	69	209	0	103
N.S.	1	1.00	0.65	0.81	1.02	0.82	2.49	0.00	1.23
time (sec)	N/A	0.035	0.035	0.109	0.472	0.992	0.722	0.000	0.482

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	68	101	129	85	0	0	85
N.S.	1	1.00	0.65	0.96	1.23	0.81	0.00	0.00	0.81
time (sec)	N/A	0.034	0.021	0.152	0.472	1.108	0.000	0.000	0.479

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	90	292	0	0	0	0	-1
N.S.	1	1.00	0.57	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.136	0.082	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	118	136	181	149	604	0	133
N.S.	1	1.00	0.83	0.96	1.27	1.05	4.25	0.00	0.94
time (sec)	N/A	0.197	0.047	0.085	0.480	1.117	1.498	0.000	0.571

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	111	320	0	0	0	0	-1
N.S.	1	1.00	0.54	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	0.403	0.087	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	142	161	223	179	724	0	163
N.S.	1	1.00	0.78	0.88	1.22	0.98	3.96	0.00	0.89
time (sec)	N/A	0.507	0.061	0.248	0.493	1.146	2.137	0.000	0.594

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	105	176	127	94	0	0	-1
N.S.	1	1.00	0.66	1.10	0.79	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.085	1.466	0.328	1.245	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	278	199	0	0	0	0	-1
N.S.	1	1.00	0.93	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	2.100	0.496	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	156	260	77	0	0	-1
N.S.	1	1.00	1.00	1.81	3.02	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.075	0.360	0.542	0.858	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	141	178	0	0	0	0	-1
N.S.	1	1.00	0.58	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	0.466	0.222	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	164	151	0	0	0	0	-1
N.S.	1	1.00	0.72	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.164	0.277	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	163	221	0	0	0	0	-1
N.S.	1	1.00	0.67	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.351	0.260	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	165	169	0	0	0	0	-1
N.S.	1	1.00	0.69	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.830	0.366	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	153	73	84	0	0	-1
N.S.	1	1.00	1.25	1.82	0.87	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.084	0.501	0.348	6.806	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	119	199	214	118	0	0	-1
N.S.	1	1.00	0.55	0.92	0.99	0.54	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.118	1.356	0.467	4.508	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	576	221	0	0	0	0	-1
N.S.	1	1.00	1.61	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	4.236	0.497	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	101	179	406	98	0	0	-1
N.S.	1	1.00	0.93	1.64	3.72	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.118	0.217	0.587	4.970	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	351	201	0	0	0	0	-1
N.S.	1	1.00	1.18	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.104	1.866	0.245	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	233	174	0	0	0	0	-1
N.S.	1	1.00	0.83	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.181	0.268	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	218	240	0	0	0	0	-1
N.S.	1	1.00	0.73	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.681	0.280	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	301	180	0	0	0	0	-1
N.S.	1	1.00	0.99	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	1.202	0.376	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	263	245	0	0	0	0	-1
N.S.	1	1.00	0.85	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.407	0.488	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	129	225	338	154	0	0	-1
N.S.	1	1.00	0.45	0.78	1.17	0.53	0.00	0.00	-0.00
time (sec)	N/A	1.344	0.166	1.452	0.431	2.184	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	907	245	0	0	0	0	-1
N.S.	1	1.00	2.17	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.432	10.822	0.548	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	111	205	637	130	0	0	-1
N.S.	1	1.00	0.83	1.53	4.75	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.145	0.424	0.643	1.268	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	643	225	0	0	0	0	-1
N.S.	1	1.00	1.85	0.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	4.239	0.291	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	281	198	0	0	0	0	-1
N.S.	1	1.00	0.85	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	0.226	0.309	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	491	265	0	0	0	0	-1
N.S.	1	1.00	1.38	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	2.761	0.322	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	374	204	0	0	0	0	-1
N.S.	1	1.00	1.03	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.824	1.429	0.415	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	313	270	0	0	0	0	-1
N.S.	1	1.00	0.84	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.746	0.522	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	91	165	89	80	0	0	-1
N.S.	1	1.00	0.76	1.38	0.74	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.083	1.675	0.373	1.333	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	158	184	0	0	0	0	-1
N.S.	1	1.00	0.63	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	0.462	0.829	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	144	61	64	0	0	-1
N.S.	1	1.00	1.02	2.44	1.03	1.08	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.042	0.392	0.542	1.553	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	118	150	0	0	0	0	-1
N.S.	1	1.00	0.61	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.094	0.220	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	100	139	0	0	0	0	-1
N.S.	1	1.00	0.56	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.117	0.250	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	62	139	36	68	0	0	-1
N.S.	1	1.00	1.11	2.48	0.64	1.21	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.072	0.277	0.401	1.627	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	165	175	0	0	0	0	-1
N.S.	1	1.00	0.68	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.543	0.463	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	110	163	81	89	0	0	-1
N.S.	1	1.00	0.93	1.38	0.69	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.090	0.872	0.361	7.048	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	242	0	102	0	0	-1
N.S.	1	1.00	1.00	2.26	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.096	1.628	0.000	2.797	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	155	247	0	0	0	0	-1
N.S.	1	1.00	0.62	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.175	0.810	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	100	28	43	0	0	-1
N.S.	1	1.00	0.86	2.04	0.57	0.88	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.042	0.348	0.539	2.339	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	98	41	40	0	0	-1
N.S.	1	1.00	0.84	2.18	0.91	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.035	0.192	0.328	2.691	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	141	232	0	0	0	0	-1
N.S.	1	1.00	0.62	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.163	0.276	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	122	231	0	104	0	0	-1
N.S.	1	1.00	1.18	2.24	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.132	0.288	0.000	2.176	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	258	273	0	0	0	0	-1
N.S.	1	1.00	0.86	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.930	0.492	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	143	259	0	129	0	0	-1
N.S.	1	1.00	0.87	1.57	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.378	0.246	0.863	0.000	5.370	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	131	386	0	140	0	0	-1
N.S.	1	1.00	0.77	2.27	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.142	1.756	0.000	3.842	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	177	389	0	0	0	0	-1
N.S.	1	1.00	0.57	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.275	0.946	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	65	244	65	74	0	0	-1
N.S.	1	1.00	0.58	2.18	0.58	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.055	1.604	0.405	3.232	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	57	240	93	67	0	0	-1
N.S.	1	1.00	0.74	3.12	1.21	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.045	0.797	0.268	2.132	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	244	66	64	0	0	-1
N.S.	1	1.00	0.65	3.09	0.84	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.041	0.400	0.335	2.287	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	63	240	86	72	0	0	-1
N.S.	1	1.00	0.62	2.38	0.85	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.039	0.237	0.283	2.558	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	168	370	0	0	0	0	-1
N.S.	1	1.00	0.60	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.267	0.326	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	151	369	0	142	0	0	-1
N.S.	1	1.00	0.96	2.34	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.171	0.335	0.000	8.458	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	234	600	0	0	0	0	-1
N.S.	1	1.00	0.87	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.328	1.583	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	175	376	0	0	0	0	-1
N.S.	1	1.00	0.87	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.154	0.769	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	144	222	0	0	0	0	-1
N.S.	1	1.00	1.16	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.094	0.604	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.683	0.293	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.464	0.381	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	0.660	0.683	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.359	0.484	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.076	0.404	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	0.399	0.359	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.368	0.428	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	89	108	116	97	121	0	102
N.S.	1	1.00	0.72	0.87	0.94	0.78	0.98	0.00	0.82
time (sec)	N/A	0.303	0.031	0.213	0.521	6.026	0.408	0.000	0.573

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	104	227	0	0	0	0	-1
N.S.	1	1.00	0.67	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.471	0.387	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	64	88	87	74	94	0	83
N.S.	1	1.00	0.67	0.92	0.91	0.77	0.98	0.00	0.86
time (sec)	N/A	0.038	0.025	0.097	0.264	4.388	0.242	0.000	0.511

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	82	202	0	0	0	0	-1
N.S.	1	1.00	0.64	1.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.042	0.160	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	1055	0	0	0	0	-1
N.S.	1	1.00	1.05	6.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.033	6.434	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	240	0	0	0	0	-1
N.S.	1	1.00	1.09	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.111	0.132	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	208	1134	0	0	0	0	-1
N.S.	1	1.00	1.06	5.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.063	6.398	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	103	273	0	0	0	0	-1
N.S.	1	1.00	0.76	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.464	0.191	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	110	150	169	148	185	0	145
N.S.	1	1.00	0.58	0.79	0.88	0.77	0.97	0.00	0.76
time (sec)	N/A	0.557	0.044	0.279	0.488	3.287	0.585	0.000	0.545

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	133	271	0	0	0	0	-1
N.S.	1	1.00	0.59	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	0.889	0.355	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	84	133	111	123	158	0	135
N.S.	1	1.00	0.55	0.87	0.73	0.80	1.03	0.00	0.88
time (sec)	N/A	0.071	0.031	0.179	0.265	2.726	0.418	0.000	0.312

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	112	246	0	0	0	0	-1
N.S.	1	1.00	0.55	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.100	0.494	0.148	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	218	1185	0	0	0	0	-1
N.S.	1	1.00	0.93	5.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.214	7.823	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	167	287	0	0	0	0	-1
N.S.	1	1.00	0.81	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.250	0.218	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	226	1184	0	0	0	0	-1
N.S.	1	1.00	1.09	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.188	6.971	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	189	292	0	0	0	0	-1
N.S.	1	1.00	0.88	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.288	0.286	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	126	188	202	181	241	0	178
N.S.	1	1.00	0.52	0.78	0.84	0.75	1.00	0.00	0.74
time (sec)	N/A	0.841	0.045	0.215	0.512	2.929	0.886	0.000	0.513

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	157	308	0	0	0	0	-1
N.S.	1	1.00	0.57	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	1.659	0.613	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	100	169	133	156	207	0	156
N.S.	1	1.00	0.50	0.84	0.66	0.78	1.04	0.00	0.78
time (sec)	N/A	0.093	0.036	0.162	0.260	2.862	0.606	0.000	0.455

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	137	282	0	0	0	0	-1
N.S.	1	1.00	0.51	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.866	0.211	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	252	1405	0	0	0	0	-1
N.S.	1	1.00	0.88	4.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	0.375	12.039	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	202	323	0	0	0	0	-1
N.S.	1	1.00	0.80	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	0.510	0.278	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	333	1318	0	0	0	0	-1
N.S.	1	1.00	1.11	4.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	0.252	14.291	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	221	324	0	0	0	0	-1
N.S.	1	1.00	0.88	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.423	0.334	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	90	226	0	0	0	0	-1
N.S.	1	1.00	0.54	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.227	0.280	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	123	1492	0	0	0	0	-1
N.S.	1	1.00	0.73	8.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.083	8.569	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	69	189	0	0	0	0	-1
N.S.	1	1.00	0.70	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.128	0.221	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	756	0	0	0	0	-1
N.S.	1	1.00	1.08	7.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.008	1.188	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	0	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.019	0.004	0.077	0.470	2.257	0.000	0.000	0.165

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	243	1689	0	0	0	0	-1
N.S.	1	1.00	2.67	18.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.033	1.257	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	254	0	0	0	0	-1
N.S.	1	1.00	0.79	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.137	0.235	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	142	5037	0	0	0	0	-1
N.S.	1	1.00	0.80	28.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.231	11.478	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	120	295	0	0	0	0	-1
N.S.	1	1.00	0.72	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.280	0.415	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	117	855	0	0	0	0	-1
N.S.	1	1.00	0.61	4.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.106	0.997	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	68	93	151	69	0	0	96
N.S.	1	1.00	0.64	0.88	1.42	0.65	0.00	0.00	0.91
time (sec)	N/A	0.081	0.049	0.241	0.503	2.807	0.000	0.000	0.418

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	47	73	104	48	0	0	50
N.S.	1	1.00	0.52	0.80	1.14	0.53	0.00	0.00	0.55
time (sec)	N/A	0.053	0.026	0.194	0.473	4.400	0.000	0.000	0.420

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	65	93	146	67	0	0	101
N.S.	1	1.00	0.65	0.93	1.46	0.67	0.00	0.00	1.01
time (sec)	N/A	0.053	0.034	0.224	0.492	4.006	0.000	0.000	0.517

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	119	1795	0	0	0	0	-1
N.S.	1	1.00	0.70	10.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.101	1.121	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	109	315	0	0	0	0	-1
N.S.	1	1.00	0.62	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.220	0.208	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	183	4110	0	0	0	0	-1
N.S.	1	1.00	0.73	16.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	0.396	9.316	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	166	353	0	0	0	0	-1
N.S.	1	1.00	0.69	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	0.292	0.270	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	74	132	185	87	0	0	85
N.S.	1	1.00	0.53	0.94	1.32	0.62	0.00	0.00	0.61
time (sec)	N/A	0.143	0.042	0.284	0.480	3.959	0.000	0.000	0.573

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	95	149	232	114	0	0	150
N.S.	1	1.00	0.52	0.82	1.28	0.63	0.00	0.00	0.83
time (sec)	N/A	0.207	0.065	0.308	0.500	2.962	0.000	0.000	0.493

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	71	106	163	87	0	0	85
N.S.	1	1.00	0.51	0.77	1.18	0.63	0.00	0.00	0.62
time (sec)	N/A	0.071	0.030	0.247	0.480	3.859	0.000	0.000	0.510

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	98	143	232	113	0	0	157
N.S.	1	1.00	0.58	0.85	1.37	0.67	0.00	0.00	0.93
time (sec)	N/A	0.087	0.037	0.293	0.519	2.202	0.000	0.000	0.527

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	156	1840	0	0	0	0	-1
N.S.	1	1.00	0.66	7.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.136	1.494	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	139	372	0	0	0	0	-1
N.S.	1	1.00	0.56	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.268	0.263	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	226	1970	0	0	0	0	-1
N.S.	1	1.00	0.70	6.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.970	0.496	5.106	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	189	411	0	0	0	0	-1
N.S.	1	1.00	0.60	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.066	0.563	0.145	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	360	235	0	0	0	0	-1
N.S.	1	1.00	0.94	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	0.838	1.707	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	599	302	0	0	0	0	-1
N.S.	1	1.00	1.37	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	2.489	0.668	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	260	198	0	0	0	0	-1
N.S.	1	1.00	0.93	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.421	0.450	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	201	268	0	0	0	0	-1
N.S.	1	1.00	0.59	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.284	0.317	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	250	337	0	0	0	0	-1
N.S.	1	1.00	0.57	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	0.183	0.437	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	265	309	0	0	0	0	-1
N.S.	1	1.00	0.58	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.542	0.428	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	222	255	0	0	0	0	-1
N.S.	1	1.00	0.68	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	1.359	0.480	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	239	195	0	0	0	0	-1
N.S.	1	1.00	0.87	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	1.195	0.636	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	797	271	0	0	0	0	-1
N.S.	1	1.00	1.67	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.787	3.240	1.647	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	1101	338	0	0	0	0	-1
N.S.	1	1.00	2.07	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.219	2.276	0.697	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	601	237	0	0	0	0	-1
N.S.	1	1.00	1.80	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	2.857	0.247	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	811	304	0	0	0	0	-1
N.S.	1	1.00	1.85	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	1.417	0.349	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	496	365	0	0	0	0	-1
N.S.	1	1.00	0.94	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	2.123	0.461	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	376	356	0	0	0	0	-1
N.S.	1	1.00	0.68	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.647	0.743	0.429	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	455	412	0	0	0	0	-1
N.S.	1	1.00	0.80	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.101	2.131	0.556	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	453	343	0	0	0	0	-1
N.S.	1	1.00	0.78	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.798	5.197	0.712	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	1320	309	0	0	0	0	-1
N.S.	1	1.00	2.28	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.301	5.086	1.753	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1543	376	0	0	0	0	-1
N.S.	1	1.00	2.42	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.818	3.577	0.762	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	1039	275	0	0	0	0	-1
N.S.	1	1.00	2.68	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	6.605	0.530	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	1143	342	0	0	0	0	-1
N.S.	1	1.00	2.22	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	2.454	0.406	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	839	404	0	0	0	0	-1
N.S.	1	1.00	1.39	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.873	6.324	0.504	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	968	399	0	0	0	0	-1
N.S.	1	1.00	1.48	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.013	2.009	0.474	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	761	454	0	0	0	0	-1
N.S.	1	1.00	1.15	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.811	7.151	0.625	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	644	401	0	0	0	0	-1
N.S.	1	1.00	0.95	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.598	3.156	0.747	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	279	206	0	0	0	0	-1
N.S.	1	1.00	0.89	0.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.474	1.997	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	505	271	0	0	0	0	-1
N.S.	1	1.00	1.47	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.881	1.138	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	126	180	0	0	0	0	-1
N.S.	1	1.00	0.57	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.182	0.495	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	140	0	0	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.088	0.165	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	145	197	0	0	0	0	-1
N.S.	1	1.00	0.64	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.122	0.334	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	128	171	0	0	0	0	-1
N.S.	1	1.00	0.62	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.325	0.344	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	231	261	0	0	0	0	-1
N.S.	1	1.00	0.70	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	0.887	0.604	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	228	206	0	0	0	0	-1
N.S.	1	1.00	0.73	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	1.844	1.103	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	209	294	0	0	0	0	-1
N.S.	1	1.00	0.69	0.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.584	1.928	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	228	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.245	1.023	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	116	73	51	0	0	-1
N.S.	1	1.00	0.64	1.49	0.94	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.056	0.437	0.685	2.723	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	114	53	51	0	0	-1
N.S.	1	1.00	0.68	1.58	0.74	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.043	0.229	0.522	2.585	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	204	306	0	0	0	0	-1
N.S.	1	1.00	0.66	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.211	0.359	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	226	279	0	0	0	0	-1
N.S.	1	1.00	0.77	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.794	0.359	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	371	376	0	0	0	0	-1
N.S.	1	1.00	0.88	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.816	1.576	0.639	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	270	318	0	0	0	0	-1
N.S.	1	1.00	0.68	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.879	2.386	1.128	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	229	454	0	0	0	0	-1
N.S.	1	1.00	0.57	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	0.923	2.024	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	239	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	0.407	1.100	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	81	276	0	92	0	0	-1
N.S.	1	1.00	0.47	1.60	0.00	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.080	1.923	0.000	1.903	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	80	272	117	88	0	0	-1
N.S.	1	1.00	0.58	1.96	0.84	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.064	1.032	0.332	0.928	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	71	276	0	82	0	0	-1
N.S.	1	1.00	0.52	2.01	0.00	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.061	0.483	0.000	2.958	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	86	272	111	93	0	0	-1
N.S.	1	1.00	0.55	1.73	0.71	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.052	0.277	0.344	1.251	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	246	460	0	0	0	0	-1
N.S.	1	1.00	0.63	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	0.359	0.414	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	296	433	0	0	0	0	-1
N.S.	1	1.00	0.78	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	1.159	0.410	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	1.276	0.806	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	0.649	0.586	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	0.641	0.197	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.490	0.440	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.698	0.579	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.110	0.490	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.074	0.361	0.431	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.090	0.457	0.513	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	135	274	0	0	0	0	-1
N.S.	1	1.00	0.62	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.821	0.483	0.626	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	171	2466	0	0	0	0	-1
N.S.	1	1.00	0.81	11.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.630	0.419	13.182	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	101	240	0	0	0	0	-1
N.S.	1	1.00	0.63	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.051	0.913	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	144	1510	0	0	0	0	-1
N.S.	1	1.00	0.84	8.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.038	3.411	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	284	460	0	0	0	0	-1
N.S.	1	1.00	1.03	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.050	16.667	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	188	1765	0	0	0	0	-1
N.S.	1	1.00	1.11	10.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.132	37.765	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	337	525	0	0	0	0	-1
N.S.	1	1.00	1.09	1.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.189	29.948	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	177	5426	0	0	0	0	-1
N.S.	1	1.00	0.94	28.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.422	0.299	4.792	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	165	330	0	0	0	0	-1
N.S.	1	1.00	0.53	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.547	0.996	2.012	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	233	1256	0	0	0	0	-1
N.S.	1	1.00	0.73	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.269	0.824	99.169	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	131	299	0	0	0	0	-1
N.S.	1	1.00	0.54	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.563	3.254	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	195	2519	0	0	0	0	-1
N.S.	1	1.00	0.67	8.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.425	269.119	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	302	566	0	0	0	0	-1
N.S.	1	1.00	0.82	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.370	32.361	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	246	5077	0	0	0	0	-1
N.S.	1	1.00	0.87	17.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	0.270	137.654	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	302	622	0	0	0	0	-1
N.S.	1	1.00	0.76	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	0.386	29.614	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	289	5134	0	0	0	0	-1
N.S.	1	1.00	0.93	16.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	0.367	119.451	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	191	382	0	0	0	0	-1
N.S.	1	1.00	0.50	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.663	1.573	2.007	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	281	1576	0	0	0	0	-1
N.S.	1	1.00	0.72	4.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.142	1.412	75.766	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	157	349	0	0	0	0	-1
N.S.	1	1.00	0.51	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	1.001	2.723	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	243	1267	0	0	0	0	-1
N.S.	1	1.00	0.63	3.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.838	50.238	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	350	664	0	0	0	0	-1
N.S.	1	1.00	0.78	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.177	0.719	50.997	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	298	9696	0	0	0	0	-1
N.S.	1	1.00	0.84	27.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	0.525	248.309	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	464	763	0	0	0	0	-1
N.S.	1	1.00	0.92	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	0.548	73.456	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	331	7948	0	0	0	0	-1
N.S.	1	1.00	0.99	23.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.810	0.522	13.904	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	154	1536	0	0	0	0	-1
N.S.	1	1.00	0.71	7.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	0.190	44.526	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	162	259	0	0	0	0	-1
N.S.	1	1.00	0.62	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.220	73.761	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	785	0	0	0	0	-1
N.S.	1	1.00	0.72	6.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.152	20.483	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	149	789	0	0	0	0	-1
N.S.	1	1.00	1.08	5.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.009	20.964	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	0	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.019	0.004	0.618	0.461	13.787	0.000	0.000	0.127

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	354	1751	0	0	0	0	-1
N.S.	1	1.00	2.85	14.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.042	85.829	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	1722	0	0	0	0	-1
N.S.	1	1.00	0.89	14.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.123	40.050	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	441	0	0	0	0	-1
N.S.	1	1.00	0.72	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.287	82.510	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	180	5082	0	0	0	0	-1
N.S.	1	1.00	0.79	22.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	0.374	127.362	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	156	936	0	0	0	0	-1
N.S.	1	1.00	0.58	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.108	35.670	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	74	114	218	76	0	0	119
N.S.	1	1.00	0.55	0.84	1.61	0.56	0.00	0.00	0.88
time (sec)	N/A	0.116	0.033	0.769	0.516	3.253	0.000	0.000	0.446

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	68	101	174	69	0	0	114
N.S.	1	1.00	0.51	0.76	1.31	0.52	0.00	0.00	0.86
time (sec)	N/A	0.097	0.030	0.823	0.508	5.036	0.000	0.000	0.428

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	71	114	213	73	0	0	119
N.S.	1	1.00	0.55	0.88	1.65	0.57	0.00	0.00	0.92
time (sec)	N/A	0.079	0.023	0.767	0.518	3.591	0.000	0.000	0.430

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	156	1905	0	0	0	0	-1
N.S.	1	1.00	0.65	7.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.112	53.585	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	157	1849	0	0	0	0	-1
N.S.	1	1.00	0.67	7.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.235	26.325	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	243	531	0	0	0	0	-1
N.S.	1	1.00	0.65	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	0.460	45.957	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	243	4175	0	0	0	0	-1
N.S.	1	1.00	0.73	12.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	0.634	308.893	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	105	176	289	117	0	0	205
N.S.	1	1.00	0.50	0.83	1.36	0.55	0.00	0.00	0.97
time (sec)	N/A	0.206	0.163	1.000	0.508	4.815	0.000	0.000	0.587

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	111	197	334	130	0	0	188
N.S.	1	1.00	0.47	0.83	1.41	0.55	0.00	0.00	0.79
time (sec)	N/A	0.286	0.040	0.993	0.539	4.759	0.000	0.000	0.537

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	103	150	272	117	0	0	189
N.S.	1	1.00	0.50	0.72	1.31	0.56	0.00	0.00	0.91
time (sec)	N/A	0.138	0.062	0.991	0.509	2.918	0.000	0.000	0.531

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	114	193	335	132	0	0	199
N.S.	1	1.00	0.51	0.86	1.49	0.59	0.00	0.00	0.88
time (sec)	N/A	0.145	0.045	0.917	0.530	2.398	0.000	0.000	0.536

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	208	1959	0	0	0	0	-1
N.S.	1	1.00	0.63	5.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	0.161	75.354	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	232	1917	0	0	0	0	-1
N.S.	1	1.00	0.70	5.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	0.363	41.553	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	295	579	0	0	0	0	-1
N.S.	1	1.00	0.62	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.296	0.720	51.237	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	301	2062	0	0	0	0	-1
N.S.	1	1.00	0.70	4.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.537	0.815	43.813	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	594	417	0	0	0	0	-1
N.S.	1	1.00	1.14	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.701	4.662	7.313	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	747	747	1844	460	0	0	0	0	-1
N.S.	1	1.00	2.47	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.292	12.099	3.891	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	535	370	0	0	0	0	-1
N.S.	1	1.00	1.43	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	2.086	2.936	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	626	258	422	0	0	0	0	-1
N.S.	1	1.00	0.41	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.538	2.320	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	366	453	0	0	0	0	-1
N.S.	1	1.00	0.61	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	0.439	3.018	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	768	466	0	0	0	0	-1
N.S.	1	1.00	1.23	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	2.615	3.008	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	345	404	0	0	0	0	-1
N.S.	1	1.00	0.57	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	3.844	3.017	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	341	462	0	0	0	0	-1
N.S.	1	1.00	0.94	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	2.601	3.352	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	1296	469	0	0	0	0	-1
N.S.	1	1.00	1.99	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.918	7.724	7.877	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	882	882	4015	514	0	0	0	0	-1
N.S.	1	1.00	4.55	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.721	18.193	3.951	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	806	421	0	0	0	0	-1
N.S.	1	1.00	1.69	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	5.578	2.826	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	2105	466	0	0	0	0	-1
N.S.	1	1.00	2.77	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	12.544	2.211	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	726	726	854	511	0	0	0	0	-1
N.S.	1	1.00	1.18	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	1.659	2.726	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	901	901	1387	602	0	0	0	0	-1
N.S.	1	1.00	1.54	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	4.405	2.922	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	919	919	691	592	0	0	0	0	-1
N.S.	1	1.00	0.75	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.376	6.235	3.749	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	788	788	1508	557	0	0	0	0	-1
N.S.	1	1.00	1.91	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.290	8.954	4.479	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	798	798	1466	525	0	0	0	0	-1
N.S.	1	1.00	1.84	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	13.574	7.483	7.729	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1019	1019	6517	566	0	0	0	0	-1
N.S.	1	1.00	6.40	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	10.434	24.298	3.980	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	1699	477	0	0	0	0	-1
N.S.	1	1.00	3.03	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	4.267	2.956	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	870	870	4281	518	0	0	0	0	-1
N.S.	1	1.00	4.92	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	18.675	2.316	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	845	845	1400	562	0	0	0	0	-1
N.S.	1	1.00	1.66	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	5.737	2.937	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1027	1027	3267	655	0	0	0	0	-1
N.S.	1	1.00	3.18	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.428	14.283	2.990	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1043	1043	1460	660	0	0	0	0	-1
N.S.	1	1.00	1.40	0.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.421	9.353	3.846	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1061	1061	1771	699	0	0	0	0	-1
N.S.	1	1.00	1.67	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.338	9.462	4.527	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	552	380	0	0	0	0	-1
N.S.	1	1.00	1.35	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	2.282	8.595	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	812	430	0	0	0	0	-1
N.S.	1	1.00	1.30	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	4.256	7.145	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	168	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.188	2.368	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	190	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.104	0.938	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	208	261	0	0	0	0	-1
N.S.	1	1.00	0.64	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.157	2.549	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	174	230	0	0	0	0	-1
N.S.	1	1.00	0.67	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.235	2.444	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	345	410	0	0	0	0	-1
N.S.	1	1.00	0.58	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	2.630	3.281	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	343	487	0	0	0	0	-1
N.S.	1	1.00	0.87	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	4.095	4.832	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	308	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.576	4.967	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	639	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	1.235	5.640	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	61	134	98	62	0	0	-1
N.S.	1	1.00	0.57	1.25	0.92	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.066	2.493	0.769	3.759	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	56	132	99	58	0	0	-1
N.S.	1	1.00	0.56	1.32	0.99	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.049	1.559	0.692	2.675	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	295	388	0	0	0	0	-1
N.S.	1	1.00	0.67	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.308	2.289	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	301	356	0	0	0	0	-1
N.S.	1	1.00	0.80	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	1.048	2.137	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	367	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	1.746	10.270	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	691	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.715	1.705	5.069	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	104	312	0	113	0	0	-1
N.S.	1	1.00	0.44	1.32	0.00	0.48	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.101	7.628	0.000	9.423	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	95	308	0	106	0	0	-1
N.S.	1	1.00	0.48	1.55	0.00	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.080	5.589	0.000	4.977	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	91	312	0	103	0	0	-1
N.S.	1	1.00	0.46	1.57	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.070	2.482	0.000	6.950	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	104	308	0	111	0	0	-1
N.S.	1	1.00	0.48	1.43	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.063	1.548	0.000	4.111	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	347	560	0	0	0	0	-1
N.S.	1	1.00	0.63	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.648	0.537	2.273	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	399	528	0	0	0	0	-1
N.S.	1	1.00	0.81	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.651	1.663	2.167	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	1.321	3.247	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.678	2.815	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	0.640	0.664	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.516	1.711	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	0.718	2.180	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	0.111	2.188	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.394	2.153	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.462	2.171	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.304	15.569	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.282	14.792	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.457	27.672	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.401	32.192	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.305	25.158	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.556	57.400	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	0.383	59.746	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.323	45.912	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.528	107.757	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	1.110	3.811	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	0.390	3.116	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	15	12	8	0	12
N.S.	1	1.00	1.00	1.08	1.25	1.00	0.67	0.00	1.00
time (sec)	N/A	0.018	0.013	0.657	0.262	5.826	0.257	0.000	0.091

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.141	3.191	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	0.159	4.000	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	2.830	4.273	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	5.332	3.612	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	28	0	74	0	0	-1
N.S.	1	1.00	0.76	0.85	0.00	2.24	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.046	2.018	0.000	1.971	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	67	0	0	-1
N.S.	1	1.00	1.00	0.94	0.00	3.94	0.00	0.00	-0.06
time (sec)	N/A	0.052	0.033	1.999	0.000	1.267	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	28	0	70	0	0	-1
N.S.	1	1.00	0.70	0.85	0.00	2.12	0.00	0.00	-0.03
time (sec)	N/A	0.051	0.019	2.019	0.000	1.970	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.501	3.895	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	0.777	2.490	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	5.605	7.369	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	5.379	9.478	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	40	0	174	0	0	-1
N.S.	1	1.00	0.68	0.80	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.059	2.117	0.000	1.321	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	26	0	171	0	0	-1
N.S.	1	1.00	0.77	0.74	0.00	4.89	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.076	1.642	0.000	0.942	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	28	0	120	0	0	-1
N.S.	1	1.00	0.76	0.85	0.00	3.64	0.00	0.00	-0.03
time (sec)	N/A	0.082	0.036	1.783	0.000	1.819	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	26	0	171	0	0	-1
N.S.	1	1.00	0.77	0.74	0.00	4.89	0.00	0.00	-0.03
time (sec)	N/A	0.067	0.064	1.720	0.000	2.900	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	40	0	174	0	0	-1
N.S.	1	1.00	0.68	0.80	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.022	1.977	0.000	1.425	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.616	5.735	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	0.907	6.996	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	1.537	9.084	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.176	13.208	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	0.874	25.579	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	1.619	20.215	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	0.242	20.348	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	1.000	36.535	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	1.627	20.454	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.311	23.249	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	1.190	1.935	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.574	0.200	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.140	0.106	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.432	0.129	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.090	4.394	0.627	0.000	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.089	2.357	0.553	0.000	0.000	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	82	0	0	0	0	-1
N.S.	1	1.00	0.95	2.10	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.111	0.075	0.185	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	136	0	0	0	0	-1
N.S.	1	1.00	1.00	3.49	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.095	0.136	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	0.792	0.121	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.754	0.197	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	5.446	1.421	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.091	180.000	0.661	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	125	0	0	0	0	-1
N.S.	1	1.00	0.60	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.104	0.570	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	84	0	0	0	0	-1
N.S.	1	1.00	0.61	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.082	0.477	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	51	125	0	0	0	0	-1
N.S.	1	1.00	0.59	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.097	0.190	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	50	179	0	0	0	0	-1
N.S.	1	1.00	0.57	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.037	0.134	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	1.221	0.160	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	1.233	0.166	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	0.445	0.607	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.556	0.327	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.363	0.234	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.231	0.056	0.000	0.000	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.319	0.155	0.000	0.000	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.323	0.193	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.888	0.475	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	0.502	0.261	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	0.127	0.197	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.329	0.167	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.082	0.376	0.214	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	0.407	0.195	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.546	0.809	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.407	0.807	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.684	1.243	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.644	1.125	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.797	1.260	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.795	2.230	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.649	1.866	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.504	1.891	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.777	2.909	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	0.722	0.919	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	0.430	0.299	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.221	0.062	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	10	0	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.71	0.00	1.00
time (sec)	N/A	0.017	0.005	0.077	0.278	0.751	0.372	0.000	0.343

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.366	0.209	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.470	0.307	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.688	1.063	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.836	1.286	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	9.066	0.324	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	37	0	123	0	0	-1
N.S.	1	1.00	0.93	0.86	0.00	2.86	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.066	0.223	0.000	1.962	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	38	0	115	0	0	-1
N.S.	1	1.00	0.88	0.93	0.00	2.80	0.00	0.00	-0.02
time (sec)	N/A	0.157	0.054	0.243	0.000	2.807	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	37	0	112	0	0	-1
N.S.	1	1.00	0.83	0.90	0.00	2.73	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.051	0.119	0.000	2.058	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.979	0.242	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	1.835	0.299	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.351	2.189	1.279	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	2.126	1.288	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	58	0	292	0	0	-1
N.S.	1	1.00	0.97	0.67	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.360	0.079	0.271	0.000	1.298	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	37	0	196	0	0	-1
N.S.	1	1.00	0.88	0.55	0.00	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.122	0.234	0.000	1.260	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	60	0	286	0	0	-1
N.S.	1	1.00	1.23	0.98	0.00	4.69	0.00	0.00	-0.02
time (sec)	N/A	0.187	0.054	0.244	0.000	1.125	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	59	0	287	0	0	-1
N.S.	1	1.00	0.78	1.02	0.00	4.95	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.079	0.222	0.000	2.043	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.469	1.053	0.376	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.306	1.539	0.297	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.862	2.030	1.560	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.630	2.620	1.516	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	1.083	0.909	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.314	0.672	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	2.470	1.809	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	2.775	1.301	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.686	1.003	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.079	3.017	2.538	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	1.372	1.727	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.600	1.433	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	1.543	3.318	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	0.814	0.378	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.441	0.194	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.150	0.853	0.233	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	9.304	0.921	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	8.271	1.069	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	210	0	0	0	0	-1
N.S.	1	1.00	0.80	3.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.073	0.372	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	53	212	0	0	0	0	-1
N.S.	1	1.00	0.77	3.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.084	0.211	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.333	1.311	0.187	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	2.570	0.319	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.484	3.031	0.924	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.441	3.447	1.913	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.595	11.257	180.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.716	10.310	1.171	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	82	582	0	0	0	0	-1
N.S.	1	1.00	0.69	4.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.152	1.064	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	99	586	0	0	0	0	-1
N.S.	1	1.00	0.70	4.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.203	1.130	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	582	0	0	0	0	-1
N.S.	1	1.00	0.82	5.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	0.138	0.454	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	586	0	0	0	0	-1
N.S.	1	1.00	0.53	5.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.140	0.316	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.737	1.440	0.245	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.526	3.207	0.256	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.314	5.472	1.069	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	5.403	1.898	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.065	19.652	0.206	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.493	0.800	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.577	0.554	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.395	0.407	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.320	0.108	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.431	0.277	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.420	0.309	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	1.378	0.563	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	0.724	0.408	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	0.219	0.338	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	0.432	0.313	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.460	0.359	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.500	0.340	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.828	1.784	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.888	1.563	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	1.098	2.813	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.714	2.517	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.500	2.272	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.897	4.324	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.705	3.784	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.804	3.606	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.835	6.391	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.656	2.061	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	0.443	0.531	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.359	0.091	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.00	0.88
time (sec)	N/A	0.018	0.004	0.092	0.277	3.352	0.388	0.000	0.352

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.328	0.409	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.770	0.530	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.909	2.033	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	1.682	2.068	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	9.970	0.643	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	51	52	0	132	0	0	-1
N.S.	1	1.00	0.72	0.73	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.071	0.391	0.000	1.505	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	51	0	135	0	0	-1
N.S.	1	1.00	0.86	0.63	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.043	0.352	0.000	5.868	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	52	0	122	0	0	-1
N.S.	1	1.00	0.89	0.80	0.00	1.88	0.00	0.00	-0.02
time (sec)	N/A	0.174	0.051	0.178	0.000	4.199	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	1.242	0.265	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	1.807	0.288	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	1.432	2.388	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.376	4.811	2.433	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	72	90	0	328	0	0	-1
N.S.	1	1.00	0.41	0.51	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.439	0.173	0.418	0.000	2.540	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	60	52	0	215	0	0	-1
N.S.	1	1.00	0.50	0.43	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.408	0.084	0.418	0.000	3.248	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	98	88	0	318	0	0	-1
N.S.	1	1.00	0.87	0.78	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.323	0.137	0.362	0.000	5.841	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	89	0	297	0	0	-1
N.S.	1	1.00	1.10	1.10	0.00	3.67	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.077	0.371	0.000	5.753	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	1.799	0.535	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.516	2.670	0.513	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.891	3.485	3.013	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.979	6.843	2.550	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	14	14	0	0	14
N.S.	1	1.00	1.00	1.88	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.063	0.045	3.478	0.332	2.715	0.000	0.000	0.430

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	1.264	1.635	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.724	1.568	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.068	2.559	2.720	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	4.391	1.907	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.736	1.909	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	3.069	4.245	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	1.558	3.211	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.807	2.804	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.077	1.912	4.658	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	0.896	0.605	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.466	0.233	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	2.655	0.399	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	2.539	0.455	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	5.222	1.799	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	1.071	1.303	0.000	0.000	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.911	1.207	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	63	294	0	0	0	0	-1
N.S.	1	1.00	0.61	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.092	0.592	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	292	0	0	0	0	-1
N.S.	1	1.00	0.64	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.058	0.353	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.447	1.690	0.267	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	1.895	0.305	0.000	0.000	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	2.437	1.481	0.000	0.000	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.474	5.293	2.658	0.000	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.919	6.994	3.042	0.000	0.000	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.974	7.776	2.028	0.000	0.000	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	114	848	0	0	0	0	-1
N.S.	1	1.00	0.63	4.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.489	0.225	1.772	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	119	844	0	0	0	0	-1
N.S.	1	1.00	0.57	4.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.624	0.165	1.549	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	118	867	0	0	0	0	-1
N.S.	1	1.00	0.67	4.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.658	0.198	0.742	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	102	844	0	0	0	0	-1
N.S.	1	1.00	0.70	5.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.149	0.596	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.191	2.538	0.559	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.797	2.374	0.363	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.510	1.194	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.604	0.914	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.415	0.660	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.518	0.138	0.000	0.000	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.604	0.381	0.000	0.000	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.614	0.448	0.000	0.000	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.084	1.444	0.741	0.000	0.000	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	1.249	0.576	0.000	0.000	0.000	0.000	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	0.382	0.535	0.000	0.000	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.619	0.481	0.000	0.000	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	0.660	0.549	0.000	0.000	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	0.675	0.515	0.000	0.000	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.430	1.718	0.000	0.000	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.751	0.865	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	2.350	0.668	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	1.234	1.107	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.884	1.950	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	1.318	1.051	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.207	0.895	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.949	1.108	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.548	2.410	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	1.322	1.323	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.255	1.102	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.962	1.376	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.569	0.315	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	1.938	1.295	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.065	0.825	0.449	0.000	0.000	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.755	0.121	0.000	0.000	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	0	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.78	0.78
time (sec)	N/A	0.019	0.005	0.168	0.000	3.077	0.000	0.444	0.377

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.451	0.164	0.000	0.000	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	1.091	0.378	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.587	2.480	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.125	4.480	1.320	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	1.029	0.856	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	3.242	0.733	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	57	0	0	0	0	-1
N.S.	1	1.00	0.82	0.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.080	0.262	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	136	46	0	0	0	0	-1
N.S.	1	1.00	1.72	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.219	0.195	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	60	0	0	0	0	-1
N.S.	1	1.00	0.88	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.067	0.253	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	1.206	1.051	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.213	1.264	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	4.240	1.325	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	181	102	0	0	0	0	-1
N.S.	1	1.00	1.30	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.200	0.293	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	230	93	0	0	0	0	-1
N.S.	1	1.00	1.95	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.457	0.270	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	141	66	0	0	0	0	-1
N.S.	1	1.00	1.70	0.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.158	0.244	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	230	94	0	0	0	0	-1
N.S.	1	1.00	1.95	0.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.491	0.206	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	103	102	0	0	0	0	-1
N.S.	1	1.00	0.74	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.209	0.283	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	1.556	1.133	0.000	0.000	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.689	1.283	0.000	0.000	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.067	2.968	1.771	0.000	0.000	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	2.111	1.134	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.212	0.498	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	0.710	1.284	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	3.440	1.833	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	5.683	1.151	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.228	0.503	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.915	1.617	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	2.623	1.985	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	2.535	1.250	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.330	0.608	0.000	0.000	0.000	0.000	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.657	1.243	0.000	0.000	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	3.218	5.140	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	1.809	4.839	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.607	1.486	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.152	0.410	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.786	0.506	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.140	1.311	0.785	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	2.807	2.143	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	14.698	4.855	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.079	0.742	1.246	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	6.025	4.839	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	3.381	5.099	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	121	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.105	1.491	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.054	0.437	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.079	1.718	0.744	0.000	0.000	0.000	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	3.626	0.809	0.000	0.000	0.000	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.079	1.221	1.227	0.000	0.000	0.000	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	3.649	6.117	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	324	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.363	4.937	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.194	5.342	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	167	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.297	1.528	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	137	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.127	0.510	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	1.915	0.650	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.340	1.595	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	2.937	1.476	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.936	0.813	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	2.986	0.705	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	1.311	1.188	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.166	0.333	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	0.922	1.981	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	2.214	1.935	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.927	1.026	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	1.570	0.503	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	1.259	1.186	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	1.543	1.063	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.561	2.625	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	2.059	2.345	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.973	1.316	0.000	0.000	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.101	1.649	1.105	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	1.201	1.579	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.819	1.133	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.500	0.318	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	2.948	1.430	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.880	0.375	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.727	0.122	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	0	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.78	0.78
time (sec)	N/A	0.018	0.004	0.203	0.000	3.131	0.000	0.428	0.387

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.435	0.175	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.916	0.369	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.281	2.441	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.135	2.725	1.507	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.803	0.877	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	3.351	0.987	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	187	75	0	0	0	0	-1
N.S.	1	1.00	1.47	0.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.193	0.286	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	67	0	0	0	0	-1
N.S.	1	1.00	0.69	0.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.067	0.231	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	75	0	0	0	0	-1
N.S.	1	1.00	0.73	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.067	0.296	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.225	1.017	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	1.011	1.224	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	5.064	1.365	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	355	132	0	0	0	0	-1
N.S.	1	1.00	1.54	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.391	0.361	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	350	124	0	0	0	0	-1
N.S.	1	1.00	2.08	0.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.181	0.316	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	353	81	0	0	0	0	-1
N.S.	1	1.00	3.27	0.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.353	0.280	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	347	124	0	0	0	0	-1
N.S.	1	1.00	2.07	0.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.168	0.276	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	142	132	0	0	0	0	-1
N.S.	1	1.00	0.65	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.146	0.316	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	1.802	1.064	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.481	1.316	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	2.231	1.723	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	4.390	1.419	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.201	0.526	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	2.138	0.822	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	0.663	1.415	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	3.251	1.804	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.083	1.733	1.229	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	1.015	0.523	0.000	0.000	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	1.457	0.773	0.000	0.000	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.856	1.731	0.000	0.000	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	2.163	1.984	0.000	0.000	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	4.872	1.372	0.000	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.329	0.624	0.000	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	1.574	0.868	0.000	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.594	1.279	0.000	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	2.510	5.315	0.000	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.668	5.066	0.000	0.000	0.000	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.478	1.648	0.000	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.152	0.437	0.000	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.909	0.536	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.150	0.897	0.802	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	3.128	2.256	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.435	14.219	5.063	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.080	0.674	1.141	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	5.098	5.350	0.000	0.000	0.000	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	3.496	5.335	0.000	0.000	0.000	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	128	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.139	1.602	0.000	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	86	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.063	0.460	0.000	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	1.909	0.794	0.000	0.000	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	4.220	0.839	0.000	0.000	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	1.026	1.213	0.000	0.000	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.079	7.222	11.088	0.000	0.000	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	3.612	8.117	0.000	0.000	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	272	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	0.742	5.306	0.000	0.000	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	338	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.399	6.114	0.000	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	261	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.725	0.939	0.000	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	153	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.152	0.667	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	2.057	0.746	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	6.496	0.836	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	1.331	1.634	0.000	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	2.549	1.632	0.000	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.435	0.892	0.000	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	2.702	0.755	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	2.111	1.256	0.000	0.000	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	1.215	0.352	0.000	0.000	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.895	2.086	0.000	0.000	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.816	1.908	0.000	0.000	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.088	1.049	1.109	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	1.634	0.284	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.361	1.288	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	1.859	1.025	0.000	0.000	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.601	2.960	0.000	0.000	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	1.722	2.828	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	1.115	1.431	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.088	1.593	0.756	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	1.118	1.631	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	1.842	1.335	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.517	0.329	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.083	3.161	1.524	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.066	0.933	0.407	0.000	0.000	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.758	0.128	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	15	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.83	0.78	0.78
time (sec)	N/A	0.017	0.004	0.237	0.000	1.452	9.346	0.391	0.395

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.438	0.181	0.000	0.000	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.932	0.403	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.425	2.763	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.123	2.541	1.539	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.804	0.869	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	3.655	0.934	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	111	93	0	0	0	0	-1
N.S.	1	1.00	0.71	0.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.078	0.327	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	234	82	0	0	0	0	-1
N.S.	1	1.00	1.50	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.126	0.254	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	108	93	0	0	0	0	-1
N.S.	1	1.00	0.72	0.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.059	0.321	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	1.356	1.077	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	1.016	1.329	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	6.435	1.538	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	287	168	0	0	0	0	-1
N.S.	1	1.00	0.93	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.274	0.395	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	359	157	0	0	0	0	-1
N.S.	1	1.00	1.40	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.489	0.351	0.000	0.000	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	185	96	0	0	0	0	-1
N.S.	1	1.00	1.39	0.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.184	0.338	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	359	157	0	0	0	0	-1
N.S.	1	1.00	1.41	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.466	0.302	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	149	168	0	0	0	0	-1
N.S.	1	1.00	0.50	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.344	0.347	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	2.251	1.248	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.477	1.541	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	2.552	2.193	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	3.494	1.279	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.210	0.562	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	1.699	0.876	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	0.691	1.457	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	3.421	1.987	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	2.032	1.255	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	1.104	0.530	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	1.533	0.875	0.000	0.000	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.894	1.885	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	2.467	2.191	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	360	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	4.173	1.401	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.315	0.665	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	1.753	0.941	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.587	1.352	0.000	0.000	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	2.810	5.888	0.000	0.000	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	1.800	5.887	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.479	1.650	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.161	0.482	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.903	0.753	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.138	1.034	0.813	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	3.548	2.260	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	11.876	5.420	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	0.677	1.244	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	3.498	5.806	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	139	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.135	1.586	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	97	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.070	0.490	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	1.915	0.791	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	1.026	1.363	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	3.791	8.675	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	370	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.393	6.100	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	287	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	0.775	5.908	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	356	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.371	1.737	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	176	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.194	0.719	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	2.163	0.793	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	1.441	1.738	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.749	0.935	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.072	0.772	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.053	1.460	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.923	1.964	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	0.890	1.119	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.536	0.954	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.971	1.310	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.517	2.448	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	0.885	1.465	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.528	1.190	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.999	1.722	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.445	0.338	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.612	0.141	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	14	12	14	14
N.S.	1	1.00	1.00	0.94	0.00	0.88	0.75	0.88	0.88
time (sec)	N/A	0.019	0.004	0.131	0.000	2.964	0.765	0.438	0.343

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.184	0.179	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.906	0.743	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	3.125	0.436	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	122	38	0	0	0	0	-1
N.S.	1	1.00	2.60	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.093	0.201	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	0	0	0	0	-1
N.S.	1	1.00	1.00	0.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.041	0.138	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	38	0	0	0	0	-1
N.S.	1	1.00	0.91	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.035	0.204	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.959	1.171	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.115	1.418	0.000	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	3.254	1.157	0.000	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	230	59	0	0	0	0	-1
N.S.	1	1.00	2.58	0.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.208	0.269	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	131	47	0	0	0	0	-1
N.S.	1	1.00	1.85	0.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.081	0.203	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	229	48	0	0	0	0	-1
N.S.	1	1.00	3.95	0.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.191	0.214	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	133	46	0	0	0	0	-1
N.S.	1	1.00	1.87	0.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.084	0.187	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	59	0	0	0	0	-1
N.S.	1	1.00	0.81	0.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.056	0.260	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	1.140	1.214	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.065	0.600	1.426	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.373	1.504	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.203	0.725	0.000	0.000	0.000	0.000	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	2.371	0.881	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	0.676	1.385	0.000	0.000	0.000	0.000	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	2.237	1.378	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.255	0.575	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	2.049	0.865	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.904	1.464	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	1.673	1.428	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.304	0.593	0.000	0.000	0.000	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	2.072	0.924	0.000	0.000	0.000	0.000	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.607	1.311	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.757	1.490	0.000	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.156	0.591	0.000	0.000	0.000	0.000	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.708	0.753	0.000	0.000	0.000	0.000	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	0.699	1.331	0.000	0.000	0.000	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	3.360	6.670	0.000	0.000	0.000	0.000	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	97	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.093	1.468	0.000	0.000	0.000	0.000	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.037	0.624	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	1.311	0.758	0.000	0.000	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	1.132	1.291	0.000	0.000	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	3.488	8.431	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.150	6.241	0.000	0.000	0.000	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	159	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.144	7.189	0.000	0.000	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	156	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.156	1.478	0.000	0.000	0.000	0.000	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	159	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.156	0.721	0.000	0.000	0.000	0.000	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	1.815	0.793	0.000	0.000	0.000	0.000	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.424	1.718	0.000	0.000	0.000	0.000	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	1.134	0.950	0.000	0.000	0.000	0.000	0.000

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	1.074	0.802	0.000	0.000	0.000	0.000	0.000

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	1.433	1.385	0.000	0.000	0.000	0.000	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.957	2.263	0.000	0.000	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	1.191	1.171	0.000	0.000	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.117	0.957	0.000	0.000	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.467	1.388	0.000	0.000	0.000	0.000	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.562	3.217	0.000	0.000	0.000	0.000	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	1.237	1.578	0.000	0.000	0.000	0.000	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.129	1.242	0.000	0.000	0.000	0.000	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	1.543	1.666	0.000	0.000	0.000	0.000	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.518	0.453	0.000	0.000	0.000	0.000	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.784	0.141	0.000	0.000	0.000	0.000	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	14	14	14	14
N.S.	1	1.00	1.00	0.94	0.00	0.88	0.88	0.88	0.88
time (sec)	N/A	0.017	0.006	0.231	0.000	2.440	1.572	0.396	0.328

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.150	0.163	0.000	0.000	0.000	0.000	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.660	0.703	0.000	0.000	0.000	0.000	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	3.706	1.003	0.000	0.000	0.000	0.000	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	2.362	0.582	0.000	0.000	0.000	0.000	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	0	0	0	0	-1
N.S.	1	1.00	1.00	0.77	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.069	0.299	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	158	46	0	0	0	0	-1
N.S.	1	1.00	1.14	0.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.134	0.268	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	47	0	0	0	0	-1
N.S.	1	1.00	0.91	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.059	0.273	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	3.498	1.040	0.000	0.000	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	3.265	0.947	0.000	0.000	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	5.307	2.688	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	4.913	1.687	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.721	1.344	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	148	86	0	0	0	0	-1
N.S.	1	1.00	1.54	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.231	0.342	0.000	0.000	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	112	53	0	0	0	0	-1
N.S.	1	1.00	1.67	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.134	0.344	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	156	84	0	0	0	0	-1
N.S.	1	1.00	1.68	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.174	0.286	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	144	85	0	0	0	0	-1
N.S.	1	1.00	1.53	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.111	0.322	0.000	0.000	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	4.069	1.519	0.000	0.000	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	4.330	1.045	0.000	0.000	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	5.693	3.250	0.000	0.000	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	5.075	1.645	0.000	0.000	0.000	0.000	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.513	1.407	0.000	0.000	0.000	0.000	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	1.283	1.530	0.000	0.000	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.311	0.745	0.000	0.000	0.000	0.000	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	3.818	0.876	0.000	0.000	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	0.704	1.452	0.000	0.000	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	5.067	1.415	0.000	0.000	0.000	0.000	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.990	0.603	0.000	0.000	0.000	0.000	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	7.000	0.866	0.000	0.000	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	0.942	1.974	0.000	0.000	0.000	0.000	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	1.995	1.719	0.000	0.000	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.823	0.680	0.000	0.000	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	3.933	0.970	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.625	1.520	0.000	0.000	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	0.737	1.899	0.000	0.000	0.000	0.000	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.471	0.711	0.000	0.000	0.000	0.000	0.000

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	1.909	0.819	0.000	0.000	0.000	0.000	0.000

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.067	7.891	0.922	0.000	0.000	0.000	0.000	0.000

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	0.732	1.453	0.000	0.000	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	5.475	6.048	0.000	0.000	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	5.082	6.467	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	116	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.107	1.588	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.099	0.684	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	4.335	0.809	0.000	0.000	0.000	0.000	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	10.529	0.841	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	12.791	2.228	0.000	0.000	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	14.327	6.661	0.000	0.000	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.799	1.589	0.000	0.000	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	182	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	0.374	6.233	0.000	0.000	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	241	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.378	7.618	0.000	0.000	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	299	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.349	1.720	0.000	0.000	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	158	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.281	0.799	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	4.531	0.852	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	6.869	0.860	0.000	0.000	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	19.206	2.306	0.000	0.000	0.000	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	16.086	5.637	0.000	0.000	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	1.446	1.954	0.000	0.000	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.932	0.962	0.000	0.000	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	1.496	0.783	0.000	0.000	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	4.016	1.417	0.000	0.000	0.000	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.956	2.510	0.000	0.000	0.000	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	1.489	1.231	0.000	0.000	0.000	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.110	1.047	0.000	0.000	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	2.203	1.484	0.000	0.000	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.576	3.398	0.000	0.000	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	1.501	1.694	0.000	0.000	0.000	0.000	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.113	1.364	0.000	0.000	0.000	0.000	0.000

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	2.788	1.816	0.000	0.000	0.000	0.000	0.000

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.537	0.516	0.000	0.000	0.000	0.000	0.000

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	1.053	0.154	0.000	0.000	0.000	0.000	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	15	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.83	0.78	0.78
time (sec)	N/A	0.017	0.006	0.247	0.000	3.090	4.973	0.517	0.332

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.985	0.178	0.000	0.000	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.663	0.940	0.000	0.000	0.000	0.000	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	2.104	0.669	0.000	0.000	0.000	0.000	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	162	62	0	0	0	0	-1
N.S.	1	1.00	0.90	0.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.184	0.352	0.000	0.000	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	59	0	0	0	0	-1
N.S.	1	1.00	0.87	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.055	0.321	0.000	0.000	0.000	0.000	0.000

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	170	62	0	0	0	0	-1
N.S.	1	1.00	0.98	0.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.247	0.343	0.000	0.000	0.000	0.000	0.000

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	3.061	1.175	0.000	0.000	0.000	0.000	0.000

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	4.637	1.024	0.000	0.000	0.000	0.000	0.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	3.803	3.042	0.000	0.000	0.000	0.000	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	8.828	2.120	0.000	0.000	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.719	1.698	0.000	0.000	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	227	112	0	0	0	0	-1
N.S.	1	1.00	1.42	0.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.407	0.302	0.435	0.000	0.000	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	259	68	0	0	0	0	-1
N.S.	1	1.00	2.01	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.483	0.487	0.381	0.000	0.000	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	220	110	0	0	0	0	-1
N.S.	1	1.00	1.42	0.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.274	0.359	0.000	0.000	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	186	113	0	0	0	0	-1
N.S.	1	1.00	1.49	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.381	0.398	0.000	0.000	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	4.201	1.533	0.000	0.000	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	5.150	1.195	0.000	0.000	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	4.220	3.317	0.000	0.000	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.316	9.945	1.596	0.000	0.000	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.065	0.531	1.675	0.000	0.000	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.842	1.539	0.000	0.000	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	1.107	0.859	0.000	0.000	0.000	0.000	0.000

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	9.601	0.918	0.000	0.000	0.000	0.000	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.727	1.546	0.000	0.000	0.000	0.000	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	1.837	1.582	0.000	0.000	0.000	0.000	0.000

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	1.129	0.559	0.000	0.000	0.000	0.000	0.000

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	7.795	0.888	0.000	0.000	0.000	0.000	0.000

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	0.936	2.030	0.000	0.000	0.000	0.000	0.000

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	2.220	1.775	0.000	0.000	0.000	0.000	0.000

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	1.203	0.698	0.000	0.000	0.000	0.000	0.000

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	4.513	1.006	0.000	0.000	0.000	0.000	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.621	1.622	0.000	0.000	0.000	0.000	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	1.274	1.980	0.000	0.000	0.000	0.000	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.548	0.739	0.000	0.000	0.000	0.000	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	2.443	0.816	0.000	0.000	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	10.473	0.939	0.000	0.000	0.000	0.000	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	0.739	1.586	0.000	0.000	0.000	0.000	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.592	1.892	6.417	0.000	0.000	0.000	0.000	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	1.723	6.786	0.000	0.000	0.000	0.000	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	124	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.132	1.783	0.000	0.000	0.000	0.000	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	120	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.108	0.714	0.000	0.000	0.000	0.000	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	7.913	0.833	0.000	0.000	0.000	0.000	0.000

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	10.475	0.896	0.000	0.000	0.000	0.000	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	11.957	2.408	0.000	0.000	0.000	0.000	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	20.708	7.649	0.000	0.000	0.000	0.000	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	0.798	1.657	0.000	0.000	0.000	0.000	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	255	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.585	0.598	7.086	0.000	0.000	0.000	0.000	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	311	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.806	0.586	7.812	0.000	0.000	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	261	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	0.596	1.973	0.000	0.000	0.000	0.000	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	300	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.373	0.817	0.000	0.000	0.000	0.000	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	9.884	0.884	0.000	0.000	0.000	0.000	0.000

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.588	11.156	0.936	0.000	0.000	0.000	0.000	0.000

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.565	13.308	2.271	0.000	0.000	0.000	0.000	0.000

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	23.989	7.278	0.000	0.000	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.674	0.237	0.000	0.000	0.000	0.000	0.000

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	21	0	20	20
N.S.	1	1.00	1.00	1.05	0.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.021	0.006	0.175	0.000	3.090	0.000	0.399	0.361

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.066	0.501	2.809	0.000	0.000	0.000	0.000	0.000

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	127	119	110	109	138	0	105
N.S.	1	1.00	1.19	1.11	1.03	1.02	1.29	0.00	0.98
time (sec)	N/A	0.065	0.008	0.422	0.464	3.804	0.386	0.000	0.621

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	119	116	107	111	128	0	101
N.S.	1	1.00	1.27	1.23	1.14	1.18	1.36	0.00	1.07
time (sec)	N/A	0.087	0.020	0.263	0.262	2.085	0.338	0.000	0.573

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	103	100	90	90	114	0	85
N.S.	1	1.00	1.26	1.22	1.10	1.10	1.39	0.00	1.04
time (sec)	N/A	0.048	0.008	0.342	0.461	3.788	0.268	0.000	0.291

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	86	82	86	94	0	75
N.S.	1	1.00	1.25	1.26	1.21	1.26	1.38	0.00	1.10
time (sec)	N/A	0.052	0.011	0.054	0.257	2.899	0.202	0.000	0.516

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	83	117	108	0	0	0	88
N.S.	1	1.00	1.08	1.52	1.40	0.00	0.00	0.00	1.14
time (sec)	N/A	0.066	0.007	0.121	0.592	0.000	0.000	0.000	0.678

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	73	84	75	78	80	0	69
N.S.	1	1.00	1.28	1.47	1.32	1.37	1.40	0.00	1.21
time (sec)	N/A	0.055	0.007	0.105	0.252	2.049	0.356	0.000	0.226

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	144	0	0	0	0	91
N.S.	1	1.00	1.12	1.87	0.00	0.00	0.00	0.00	1.18
time (sec)	N/A	0.070	0.007	0.075	0.000	0.000	0.000	0.000	0.719

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	98	117	95	95	116	0	92
N.S.	1	1.00	1.18	1.41	1.14	1.14	1.40	0.00	1.11
time (sec)	N/A	0.074	0.027	0.085	0.267	5.131	0.431	0.000	0.553

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	107	82	80	99	0	162
N.S.	1	1.00	1.18	1.30	1.00	0.98	1.21	0.00	1.98
time (sec)	N/A	0.060	0.008	0.118	0.464	6.352	0.329	0.000	0.588

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	123	138	118	122	153	0	111
N.S.	1	1.00	1.12	1.25	1.07	1.11	1.39	0.00	1.01
time (sec)	N/A	0.093	0.034	0.097	0.257	1.866	0.606	0.000	0.236

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	127	105	103	122	0	130
N.S.	1	1.00	0.92	1.21	1.00	0.98	1.16	0.00	1.24
time (sec)	N/A	0.070	0.008	0.118	0.472	2.599	0.453	0.000	0.593

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	174	219	184	193	260	0	374
N.S.	1	1.00	0.94	1.18	0.99	1.04	1.41	0.00	2.02
time (sec)	N/A	0.121	0.382	0.369	0.465	2.492	0.579	0.000	0.536

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	162	208	181	184	245	0	191
N.S.	1	1.00	1.01	1.29	1.12	1.14	1.52	0.00	1.19
time (sec)	N/A	0.157	0.066	0.277	0.259	2.999	0.499	0.000	0.795

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	140	171	156	159	219	0	167
N.S.	1	1.00	1.22	1.49	1.36	1.38	1.90	0.00	1.45
time (sec)	N/A	0.071	0.196	0.330	0.467	2.633	0.409	0.000	0.458

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	130	161	147	149	194	0	150
N.S.	1	1.00	1.05	1.30	1.19	1.20	1.56	0.00	1.21
time (sec)	N/A	0.103	0.045	0.152	0.266	3.928	0.359	0.000	0.662

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	120	187	171	0	0	0	157
N.S.	1	1.00	0.88	1.36	1.25	0.00	0.00	0.00	1.15
time (sec)	N/A	0.117	0.071	0.202	0.611	0.000	0.000	0.000	0.713

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	114	159	130	141	165	0	135
N.S.	1	1.00	1.05	1.46	1.19	1.29	1.51	0.00	1.24
time (sec)	N/A	0.101	0.064	0.187	0.267	3.858	0.535	0.000	0.700

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	114	211	154	0	0	0	157
N.S.	1	1.00	0.89	1.65	1.20	0.00	0.00	0.00	1.23
time (sec)	N/A	0.105	0.078	0.171	0.596	0.000	0.000	0.000	0.683

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	119	166	135	149	180	0	142
N.S.	1	1.00	1.03	1.44	1.17	1.30	1.57	0.00	1.23
time (sec)	N/A	0.114	0.067	0.187	0.261	2.261	0.559	0.000	0.680

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	121	225	0	0	0	0	177
N.S.	1	1.00	0.87	1.62	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.130	0.068	0.175	0.000	0.000	0.000	0.000	0.762

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	153	213	166	173	235	0	179
N.S.	1	1.00	1.02	1.42	1.11	1.15	1.57	0.00	1.19
time (sec)	N/A	0.132	0.063	0.197	0.254	2.553	0.642	0.000	0.492

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	136	196	145	152	192	0	256
N.S.	1	1.00	1.23	1.77	1.31	1.37	1.73	0.00	2.31
time (sec)	N/A	0.102	0.445	0.251	0.482	2.740	0.484	0.000	0.812

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	185	249	197	209	289	0	232
N.S.	1	1.00	0.99	1.34	1.06	1.12	1.55	0.00	1.25
time (sec)	N/A	0.157	0.072	0.214	0.265	2.567	0.866	0.000	0.634

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	285	262	334	266	286	411	0	599
N.S.	1	1.19	1.09	1.39	1.11	1.19	1.71	0.00	2.50
time (sec)	N/A	0.319	3.370	0.310	0.472	2.197	0.864	0.000	0.621

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	236	315	263	269	389	0	296
N.S.	1	1.00	0.99	1.32	1.10	1.13	1.63	0.00	1.24
time (sec)	N/A	0.264	0.082	0.283	0.273	3.139	0.733	0.000	0.971

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	217	257	230	243	350	0	442
N.S.	1	1.00	1.37	1.63	1.46	1.54	2.22	0.00	2.80
time (sec)	N/A	0.099	1.838	0.574	0.472	3.135	0.604	0.000	0.561

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	192	249	220	222	306	0	238
N.S.	1	1.00	1.02	1.32	1.17	1.18	1.63	0.00	1.27
time (sec)	N/A	0.096	0.066	0.167	0.265	2.373	0.527	0.000	0.434

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	190	272	244	0	0	0	232
N.S.	1	1.00	0.83	1.19	1.07	0.00	0.00	0.00	1.02
time (sec)	N/A	0.152	0.106	0.171	0.607	0.000	0.000	0.000	0.782

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	169	237	195	202	258	0	236
N.S.	1	1.00	1.06	1.48	1.22	1.26	1.61	0.00	1.48
time (sec)	N/A	0.166	0.079	0.210	0.269	2.226	0.728	0.000	0.643

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	198	290	219	0	0	0	224
N.S.	1	1.00	0.99	1.45	1.10	0.00	0.00	0.00	1.12
time (sec)	N/A	0.138	0.093	0.207	0.611	0.000	0.000	0.000	0.730

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	166	243	191	214	272	0	203
N.S.	1	1.00	1.05	1.54	1.21	1.35	1.72	0.00	1.28
time (sec)	N/A	0.170	0.088	0.211	0.260	1.937	0.756	0.000	0.638

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	162	292	211	0	0	0	234
N.S.	1	1.00	0.81	1.46	1.06	0.00	0.00	0.00	1.17
time (sec)	N/A	0.140	0.108	0.164	0.595	0.000	0.000	0.000	0.709

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	184	260	206	228	289	0	194
N.S.	1	1.00	1.04	1.47	1.16	1.29	1.63	0.00	1.10
time (sec)	N/A	0.188	0.099	0.207	0.257	3.365	0.775	0.000	0.642

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	190	315	0	0	0	0	261
N.S.	1	1.00	0.83	1.38	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.152	0.099	0.180	0.000	0.000	0.000	0.000	0.842

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	229	324	245	260	362	0	236
N.S.	1	1.00	1.02	1.45	1.09	1.16	1.62	0.00	1.05
time (sec)	N/A	0.212	0.088	0.223	0.262	2.372	0.923	0.000	0.693

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	204	300	216	237	309	0	301
N.S.	1	1.00	1.34	1.97	1.42	1.56	2.03	0.00	1.98
time (sec)	N/A	0.141	3.009	0.431	0.477	3.377	0.656	0.000	0.633

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	212	254	226	237	314	0	233
N.S.	1	1.00	0.87	1.04	0.93	0.97	1.29	0.00	0.95
time (sec)	N/A	0.129	0.070	0.282	0.260	4.287	0.685	0.000	0.205

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	503	749	0	0	0	0	-1
N.S.	1	1.00	1.39	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.193	0.391	0.000	0.000	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	441	686	0	0	0	0	-1
N.S.	1	1.00	1.42	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.087	0.168	0.000	0.000	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	384	736	0	0	0	0	-1
N.S.	1	1.00	1.09	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	1.176	0.169	0.000	0.000	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	446	864	0	0	0	0	-1
N.S.	1	1.00	1.09	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	1.865	0.196	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	766	2437	0	0	0	0	-1
N.S.	1	1.00	1.38	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	2.349	0.870	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	461	879	0	0	0	0	-1
N.S.	1	1.00	0.89	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.174	0.565	0.000	0.000	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	468	2446	0	0	0	0	-1
N.S.	1	1.00	0.83	4.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	0.525	0.501	0.000	0.000	0.000	0.000	0.000

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	522	802	0	0	0	0	-1
N.S.	1	1.00	1.30	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	5.780	0.222	0.000	0.000	0.000	0.000	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	98	115	90	237	1054	0	696
N.S.	1	1.00	1.08	1.26	0.99	2.60	11.58	0.00	7.65
time (sec)	N/A	0.051	0.098	0.306	0.470	3.285	118.768	0.000	0.848

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	590	847	0	0	0	0	-1
N.S.	1	1.00	1.33	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	3.714	0.194	0.000	0.000	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	643	984	0	0	0	0	-1
N.S.	1	1.00	1.31	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	8.776	0.212	0.000	0.000	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1335	1335	877	2344	0	0	0	0	-1
N.S.	1	1.00	0.66	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.521	8.854	1.592	0.000	0.000	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	819	819	861	2320	0	0	0	0	-1
N.S.	1	1.00	1.05	2.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	8.909	0.911	0.000	0.000	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1382	1382	992	3849	0	0	0	0	-1
N.S.	1	1.00	0.72	2.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.235	12.531	0.543	0.000	0.000	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	589	1001	0	0	0	0	-1
N.S.	1	1.00	1.11	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.473	9.312	0.205	0.000	0.000	0.000	0.000	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	158	304	207	680	0	0	273
N.S.	1	1.00	1.22	2.34	1.59	5.23	0.00	0.00	2.10
time (sec)	N/A	0.135	2.325	0.377	0.471	2.342	0.000	0.000	3.299

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	222	183	626	0	0	201
N.S.	1	1.00	1.00	1.69	1.40	4.78	0.00	0.00	1.53
time (sec)	N/A	0.077	0.757	0.335	0.467	2.499	0.000	0.000	2.610

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	645	1041	0	0	0	0	-1
N.S.	1	1.00	1.12	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	9.469	0.214	0.000	0.000	0.000	0.000	0.000

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	723	1183	0	0	0	0	-1
N.S.	1	1.00	1.15	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	13.079	0.227	0.000	0.000	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	966	966	1744	3830	0	0	0	0	-1
N.S.	1	1.00	1.81	3.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.748	12.212	0.741	0.000	0.000	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	893	893	1745	4032	0	0	0	0	-1
N.S.	1	1.00	1.95	4.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	10.333	1.167	0.000	0.000	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1518	1518	2005	6658	0	0	0	0	-1
N.S.	1	1.00	1.32	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.053	12.777	0.842	0.000	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	391	0	0	617	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.374	0.141	0.000	7.925	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	11.175	0.134	0.000	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	279	0	0	472	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.396	0.128	0.000	5.723	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	4.997	0.984	0.000	0.000	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.114	6.847	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	7.102	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	8.135	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	288	0	0	926	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	6.76	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.455	0.132	0.000	3.832	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	10.095	0.137	0.000	0.000	0.000	0.000	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	413	0	0	1244	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	5.55	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.382	0.132	0.000	2.818	0.000	0.000	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	418	0	0	783	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	2.81	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.475	0.133	0.000	17.154	0.000	0.000	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	10.703	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	313	0	0	611	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	3.38	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.318	0.125	0.000	5.706	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.020	5.274	0.723	0.000	0.000	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	7.064	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	9.598	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	8.473	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	30.187	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	9.680	0.135	0.000	0.000	0.000	0.000	0.000

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	334	0	0	1227	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	6.89	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.343	0.132	0.000	2.578	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	470	0	0	971	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	2.81	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.619	0.136	0.000	90.010	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	11.559	0.129	0.000	0.000	0.000	0.000	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	353	0	0	781	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	3.35	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.380	0.127	0.000	25.067	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	5.179	0.834	0.000	0.000	0.000	0.000	0.000

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	6.962	0.131	0.000	0.000	0.000	0.000	0.000

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	9.379	0.125	0.000	0.000	0.000	0.000	0.000

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	8.627	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	9.474	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	377	0	0	476	0	0	-1
N.S.	1	1.00	2.14	0.00	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.345	0.137	0.000	2.641	0.000	0.000	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	10.388	0.138	0.000	0.000	0.000	0.000	0.000

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	251	0	0	347	0	0	-1
N.S.	1	1.00	2.44	0.00	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.285	0.137	0.000	3.701	0.000	0.000	0.000

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	2.609	0.889	0.000	0.000	0.000	0.000	0.000

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	4.002	0.133	0.000	0.000	0.000	0.000	0.000

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	247	0	0	688	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	6.88	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.298	0.134	0.000	2.746	0.000	0.000	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	9.003	0.136	0.000	0.000	0.000	0.000	0.000

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	372	0	0	936	0	0	-1
N.S.	1	1.00	2.08	0.00	0.00	5.23	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.376	0.139	0.000	2.720	0.000	0.000	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	321	0	0	681	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	4.97	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.445	0.137	0.000	3.280	0.000	0.000	0.000

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	16.508	0.134	0.000	0.000	0.000	0.000	0.000

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	210	0	0	401	0	0	-1
N.S.	1	1.00	2.96	0.00	0.00	5.65	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.259	0.134	0.000	2.391	0.000	0.000	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	202	0	0	409	0	0	-1
N.S.	1	1.00	2.89	0.00	0.00	5.84	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.193	0.888	0.000	2.620	0.000	0.000	0.000

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	8.419	0.131	0.000	0.000	0.000	0.000	0.000

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	306	0	0	1373	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	10.17	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.454	0.131	0.000	3.042	0.000	0.000	0.000

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	10.934	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	405	0	0	2008	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	8.06	0.00	0.00	-0.00
time (sec)	N/A	0.622	0.505	0.131	0.000	4.197	0.000	0.000	0.000

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	15.684	0.136	0.000	0.000	0.000	0.000	0.000

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	326	0	0	877	0	0	-1
N.S.	1	1.00	2.28	0.00	0.00	6.13	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.406	0.132	0.000	3.611	0.000	0.000	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	252	0	0	699	0	0	-1
N.S.	1	1.00	2.31	0.00	0.00	6.41	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.743	0.135	0.000	2.570	0.000	0.000	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	259	0	0	687	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	6.25	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.557	0.133	0.000	2.941	0.000	0.000	0.000

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	317	0	0	897	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	6.23	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.402	0.893	0.000	2.414	0.000	0.000	0.000

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	11.048	0.133	0.000	0.000	0.000	0.000	0.000

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	418	0	0	2774	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	10.12	0.00	0.00	-0.00
time (sec)	N/A	0.639	0.788	0.132	0.000	4.390	0.000	0.000	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	13.579	0.138	0.000	0.000	0.000	0.000	0.000

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	510	0	0	3608	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	8.53	0.00	0.00	-0.00
time (sec)	N/A	0.800	1.317	0.138	0.000	5.787	0.000	0.000	0.000

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	345	0	0	1280	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	6.15	0.00	0.00	-0.00
time (sec)	N/A	0.710	0.661	0.700	0.000	2.740	0.000	0.000	0.000

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	450	0	0	1986	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	6.78	0.00	0.00	-0.00
time (sec)	N/A	0.896	0.998	0.473	0.000	3.000	0.000	0.000	0.000

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	374	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.337	5.412	1.806	0.000	0.000	0.000	0.000	0.000

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	226	8408	0	0	0	0	0	-1
N.S.	1	0.98	36.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	18.932	0.913	0.000	0.000	0.000	0.000	0.000

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	733	0	0	0	0	0	-1
N.S.	1	1.00	6.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	3.274	0.585	0.000	0.000	0.000	0.000	0.000

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	1.685	0.786	0.000	0.000	0.000	0.000	0.000

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	3.871	0.824	0.000	0.000	0.000	0.000	0.000

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	2.560	0.447	0.000	0.000	0.000	0.000	0.000

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.072	0.441	0.000	0.000	0.000	0.000	0.000

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.068	0.455	0.000	0.000	0.000	0.000	0.000

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	2.809	0.450	0.000	0.000	0.000	0.000	0.000

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	3.651	0.430	0.000	0.000	0.000	0.000	0.000

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	4.894	0.423	0.000	0.000	0.000	0.000	0.000

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	2.324	1.467	0.000	0.000	0.000	0.000	0.000

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.303	1.243	0.000	0.000	0.000	0.000	0.000

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	166	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.450	1.283	0.000	0.000	0.000	0.000	0.000

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	2.526	1.231	0.000	0.000	0.000	0.000	0.000

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	3.330	1.297	0.000	0.000	0.000	0.000	0.000

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	2.685	1.220	0.000	0.000	0.000	0.000	0.000

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.026	3.724	1.253	0.000	0.000	0.000	0.000	0.000

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	2.157	1.254	0.000	0.000	0.000	0.000	0.000

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	240	333	310	286	398	0	338
N.S.	1	1.00	0.89	1.23	1.14	1.06	1.47	0.00	1.25
time (sec)	N/A	0.450	0.114	0.632	0.499	1.994	0.649	0.000	1.556

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	287	648	0	0	0	0	-1
N.S.	1	1.00	0.89	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.587	0.855	0.000	0.000	0.000	0.000	0.000

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	179	254	251	218	296	0	248
N.S.	1	1.00	0.90	1.28	1.26	1.10	1.49	0.00	1.25
time (sec)	N/A	0.290	0.082	0.415	0.497	2.456	0.425	0.000	1.003

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	208	550	0	0	0	0	-1
N.S.	1	1.00	0.90	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.293	0.308	0.000	0.000	0.000	0.000	0.000

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	263	1284	0	0	0	0	-1
N.S.	1	1.00	1.21	5.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.220	10.955	0.000	0.000	0.000	0.000	0.000

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	204	599	0	0	0	0	-1
N.S.	1	1.00	1.19	3.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.172	0.924	0.000	0.000	0.000	0.000	0.000

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	273	1390	0	0	0	0	-1
N.S.	1	1.00	1.24	6.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.212	9.837	0.000	0.000	0.000	0.000	0.000

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	502	414	624	516	508	758	0	929
N.S.	1	1.00	0.82	1.24	1.03	1.01	1.51	0.00	1.85
time (sec)	N/A	0.780	0.196	0.696	0.511	2.285	0.890	0.000	6.904

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	513	1137	0	0	0	0	-1
N.S.	1	1.00	0.88	1.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	1.176	0.983	0.000	0.000	0.000	0.000	0.000

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	317	474	433	395	575	0	780
N.S.	1	1.00	0.83	1.25	1.14	1.04	1.51	0.00	2.05
time (sec)	N/A	0.548	0.170	0.482	0.504	2.439	1.681	0.000	5.278

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	391	981	0	0	0	0	-1
N.S.	1	1.00	0.88	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	0.731	0.414	0.000	0.000	0.000	0.000	0.000

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	389	1549	0	0	0	0	-1
N.S.	1	1.00	1.10	4.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	0.429	8.567	0.000	0.000	0.000	0.000	0.000

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	349	1009	0	0	0	0	-1
N.S.	1	1.00	1.02	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.504	0.786	0.000	0.000	0.000	0.000	0.000

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	367	1597	0	0	0	0	-1
N.S.	1	1.00	1.15	4.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	0.350	7.382	0.000	0.000	0.000	0.000	0.000

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	1569	0	0	0	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	6.888	0.128	0.000	0.000	0.000	0.000	0.000

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	0	94172	0	0	0	0	-1
N.S.	1	1.00	0.00	169.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	180.002	73.259	0.000	0.000	0.000	0.000	0.000

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	1529	0	0	0	0	0	-1
N.S.	1	1.00	3.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	5.756	1.169	0.000	0.000	0.000	0.000	0.000

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	0	5665	0	0	0	0	-1
N.S.	1	1.00	0.00	12.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	180.001	9.077	0.000	0.000	0.000	0.000	0.000

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	637	637	1412	0	0	0	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.496	4.875	1.277	0.000	0.000	0.000	0.000	0.000

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	C	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	0	102882	0	0	0	0	-1
N.S.	1	1.00	0.00	186.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	180.003	39.353	0.000	0.000	0.000	0.000	0.000

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	1557	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	7.527	1.712	0.000	0.000	0.000	0.000	0.000

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	943	943	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.304	11.929	2.429	0.000	0.000	0.000	0.000	0.000

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1033	1033	0	6636	0	0	0	0	-1
N.S.	1	1.00	0.00	6.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.394	30.203	179.744	0.000	0.000	0.000	0.000	0.000

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	836	1209	0	0	0	0	-1
N.S.	1	1.00	1.83	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.803	6.498	5.197	0.000	0.000	0.000	0.000	0.000

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1039	1039	0	6570	0	0	0	0	-1
N.S.	1	1.00	0.00	6.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.985	17.180	179.982	0.000	0.000	0.000	0.000	0.000

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1087	1087	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.366	10.106	3.464	0.000	0.000	0.000	0.000	0.000

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.486	180.002	12.299	0.000	0.000	0.000	0.000	0.000

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1181	1181	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.495	20.110	75.355	0.000	0.000	0.000	0.000	0.000

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	79	1286	80	72	107	168	82
N.S.	1	1.00	0.71	11.59	0.72	0.65	0.96	1.51	0.74
time (sec)	N/A	0.305	0.020	4.895	0.475	3.124	0.911	0.390	0.481

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	997	62	49	83	124	69
N.S.	1	1.00	0.64	11.33	0.70	0.56	0.94	1.41	0.78
time (sec)	N/A	0.079	0.019	7.347	0.477	1.619	0.614	0.405	0.527

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	1078	65	55	78	135	65
N.S.	1	1.00	0.78	13.15	0.79	0.67	0.95	1.65	0.79
time (sec)	N/A	0.231	0.018	4.246	0.492	1.741	0.398	0.405	0.462

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	762	39	33	56	86	48
N.S.	1	1.00	0.78	15.55	0.80	0.67	1.14	1.76	0.98
time (sec)	N/A	0.035	0.017	3.104	0.547	2.266	0.264	0.385	0.471

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	697	42	33	39	92	39
N.S.	1	1.00	1.00	18.34	1.11	0.87	1.03	2.42	1.03
time (sec)	N/A	0.072	0.009	2.760	0.480	1.657	0.180	0.404	0.460

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	5237	0	0	0	0	-1
N.S.	1	1.00	0.00	27.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.543	13.265	0.000	0.000	0.000	0.000	0.000

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	58	0	37	0	36
N.S.	1	1.00	1.00	0.00	1.41	0.00	0.90	0.00	0.88
time (sec)	N/A	0.090	0.010	7.556	0.482	0.000	55.394	0.000	0.107

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	70	0	0	0	-1
N.S.	1	1.00	0.71	0.00	1.01	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.025	20.400	0.515	0.000	0.000	0.000	0.000

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	95	0	97	0	-1
N.S.	1	1.00	1.00	0.00	1.17	0.00	1.20	0.00	-0.01
time (sec)	N/A	0.139	0.013	6.705	0.486	0.000	18.525	0.000	0.000

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	89	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.87	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.030	7.874	0.524	0.000	0.000	0.000	0.000

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	0	115	0	134	0	-1
N.S.	1	1.00	1.00	0.00	1.01	0.00	1.18	0.00	-0.01
time (sec)	N/A	0.197	0.020	6.356	0.471	0.000	25.387	0.000	0.000

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	214	4941	259	224	338	0	276
N.S.	1	1.00	0.77	17.77	0.93	0.81	1.22	0.00	0.99
time (sec)	N/A	0.469	0.099	5.455	0.478	2.093	1.956	0.000	3.346

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	164	3897	227	179	279	0	297
N.S.	1	1.00	0.74	17.63	1.03	0.81	1.26	0.00	1.34
time (sec)	N/A	0.177	0.112	6.295	0.485	2.371	1.324	0.000	1.782

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	171	4146	215	177	258	0	212
N.S.	1	1.00	0.80	19.46	1.01	0.83	1.21	0.00	1.00
time (sec)	N/A	0.392	0.073	4.486	0.485	2.528	0.922	0.000	2.528

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	105	3074	152	120	202	0	227
N.S.	1	1.00	0.77	22.44	1.11	0.88	1.47	0.00	1.66
time (sec)	N/A	0.078	0.055	5.052	0.488	2.096	0.597	0.000	1.261

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	138	173	156	109	148	0	134
N.S.	1	1.00	1.38	1.73	1.56	1.09	1.48	0.00	1.34
time (sec)	N/A	0.130	0.018	1.988	0.493	2.503	0.392	0.000	0.986

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	0	6931	0	0	0	0	-1
N.S.	1	1.00	0.00	24.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.143	51.533	0.000	0.000	0.000	0.000	0.000

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	111	0	0	0	160	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	1.60	0.00	-0.01
time (sec)	N/A	0.157	0.071	15.905	0.000	0.000	95.403	0.000	0.000

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	189	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.075	74.662	0.000	0.000	0.000	0.000	0.000

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	181	0	0	0	428	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.26	0.00	-0.01
time (sec)	N/A	0.273	0.123	20.837	0.000	0.000	47.912	0.000	0.000

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	260	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.110	24.500	0.000	0.000	0.000	0.000	0.000

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	259	0	0	0	474	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	1.91	0.00	-0.00
time (sec)	N/A	0.401	0.229	25.481	0.000	0.000	43.591	0.000	0.000

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	1140	21442	0	0	0	0	-1
N.S.	1	1.00	2.03	38.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	5.784	17.071	0.000	0.000	0.000	0.000	0.000

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	656	656	1352	0	0	0	0	0	-1
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	2.889	14.230	0.000	0.000	0.000	0.000	0.000

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.145	4.718	0.000	0.000	0.000	0.000	0.000

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	552	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	0.717	7.079	0.000	0.000	0.000	0.000	0.000

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	1217	0	0	0	0	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	4.732	12.436	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [643] had the largest ratio of [38]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.00	21	0.333
2	A	7	7	1.00	21	0.333
3	A	7	7	1.00	19	0.368
4	A	4	3	1.00	18	0.167
5	A	8	5	1.00	21	0.238
6	A	10	8	1.00	21	0.381
7	A	4	4	1.00	21	0.190
8	A	4	4	1.00	21	0.190
9	A	4	4	1.00	21	0.190
10	A	7	7	1.00	23	0.304
11	A	7	7	1.00	23	0.304
12	A	7	7	1.00	21	0.333
13	A	4	3	1.00	20	0.150
14	A	11	8	1.00	23	0.348
15	A	13	10	1.00	23	0.435
16	A	13	10	1.00	23	0.435
17	A	4	4	1.00	23	0.174
18	A	4	4	1.00	23	0.174
19	A	4	4	1.00	23	0.174
20	A	7	7	1.00	23	0.304
21	A	7	7	1.00	23	0.304
22	A	4	4	1.00	21	0.190
23	A	4	3	1.00	20	0.150
24	A	15	10	1.00	23	0.435
25	A	16	12	1.00	23	0.522

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	16	12	1.00	23	0.522
27	A	17	11	1.00	23	0.478
28	A	4	4	1.00	23	0.174
29	A	4	5	1.00	23	0.217
30	A	4	4	1.00	23	0.174
31	A	7	7	1.00	23	0.304
32	A	4	4	1.00	23	0.174
33	A	4	4	1.00	21	0.190
34	A	4	3	1.00	20	0.150
35	A	19	11	1.00	23	0.478
36	A	20	13	1.00	23	0.565
37	A	19	13	1.00	23	0.565
38	A	20	13	1.00	23	0.565
39	A	21	11	1.00	23	0.478
40	A	4	4	1.00	23	0.174
41	A	4	5	1.00	23	0.217
42	A	4	4	1.00	23	0.174
43	A	16	11	1.00	23	0.478
44	A	11	9	1.00	23	0.391
45	A	7	6	1.00	21	0.286
46	A	3	3	1.00	20	0.150
47	A	2	2	1.00	23	0.087
48	A	8	8	1.00	23	0.348
49	A	12	10	1.00	23	0.435
50	A	17	11	1.00	23	0.478
51	A	16	12	1.00	23	0.522
52	A	13	10	1.00	23	0.435
53	A	10	8	1.00	21	0.381
54	A	5	4	1.00	20	0.200
55	A	13	10	1.00	23	0.435
56	A	18	15	1.00	23	0.652
57	A	21	16	1.00	23	0.696
58	A	21	12	1.00	23	0.522
59	A	18	10	1.00	23	0.435
60	A	15	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	21	0.238
62	A	5	4	1.00	20	0.200
63	A	18	10	1.00	23	0.435
64	A	23	15	1.00	23	0.652
65	A	26	16	1.00	23	0.696
66	A	5	4	1.00	19	0.210
67	A	3	3	1.00	20	0.150
68	A	27	15	1.00	23	0.652
69	A	22	14	1.00	23	0.609
70	A	17	12	1.00	21	0.571
71	A	9	7	1.00	20	0.350
72	A	13	11	1.00	23	0.478
73	A	12	10	1.00	23	0.435
74	A	14	11	1.00	23	0.478
75	A	18	13	1.00	23	0.565
76	A	43	15	1.00	25	0.600
77	A	36	15	1.00	25	0.600
78	A	28	14	1.00	23	0.609
79	A	12	10	1.00	22	0.454
80	A	19	14	1.00	25	0.560
81	A	17	15	1.00	25	0.600
82	A	20	15	1.00	25	0.600
83	A	16	14	1.00	25	0.560
84	A	62	15	1.00	25	0.600
85	A	52	15	1.00	25	0.600
86	A	38	14	1.00	23	0.609
87	A	16	12	1.00	22	0.546
88	A	28	16	1.00	25	0.640
89	A	23	17	1.00	25	0.680
90	A	25	20	1.00	25	0.800
91	A	28	17	1.00	25	0.680
92	A	20	15	1.00	25	0.600
93	A	24	16	1.00	25	0.640
94	A	31	15	1.00	25	0.600
95	A	26	14	1.00	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	16	12	1.00	25	0.480
97	A	9	9	1.00	23	0.391
98	A	3	4	1.00	22	0.182
99	A	3	4	1.00	25	0.160
100	A	8	8	1.00	25	0.320
101	A	17	13	1.00	25	0.520
102	A	26	15	1.00	25	0.600
103	A	33	18	1.00	25	0.720
104	A	24	17	1.00	25	0.680
105	A	18	14	1.00	25	0.560
106	A	13	10	1.00	23	0.435
107	A	8	6	1.00	22	0.273
108	A	19	12	1.00	25	0.480
109	A	23	16	1.00	25	0.640
110	A	31	21	1.00	25	0.840
111	A	37	17	1.00	25	0.680
112	A	31	14	1.00	25	0.560
113	A	26	10	1.00	25	0.400
114	A	13	7	1.00	23	0.304
115	A	13	6	1.00	22	0.273
116	A	32	12	1.00	25	0.480
117	A	36	16	1.00	25	0.640
118	A	18	6	1.00	21	0.286
119	A	4	5	1.00	22	0.227
120	A	26	15	1.00	22	0.682
121	A	17	13	1.00	22	0.591
122	A	11	10	1.00	20	0.500
123	A	4	5	1.00	22	0.227
124	A	11	6	1.00	22	0.273
125	A	24	6	1.00	22	0.273
126	A	42	6	1.00	22	0.273
127	A	19	12	1.00	25	0.480
128	A	10	8	1.00	23	0.348
129	A	4	5	1.00	22	0.227
130	A	4	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	10	9	1.00	25	0.360
132	A	18	11	1.00	25	0.440
133	A	0	0	0.00	0	0.000
134	A	16	12	1.00	19	0.632
135	A	12	10	1.00	19	0.526
136	A	9	7	1.00	17	0.412
137	A	4	4	1.00	16	0.250
138	A	9	7	1.00	19	0.368
139	A	14	12	1.00	19	0.632
140	A	17	14	1.00	19	0.737
141	A	23	13	1.00	21	0.619
142	A	14	11	1.00	21	0.524
143	A	8	7	1.00	19	0.368
144	A	1	1	1.00	18	0.056
145	A	9	7	1.00	21	0.333
146	A	13	11	1.00	21	0.524
147	A	21	16	1.00	21	0.762
148	A	0	0	0.00	0	0.000
149	A	9	4	1.00	18	0.222
150	A	9	4	1.00	18	0.222
151	A	2	1	1.00	16	0.062
152	A	3	3	1.30	15	0.200
153	A	7	6	1.00	18	0.333
154	A	8	8	1.00	18	0.444
155	A	7	6	1.00	18	0.333
156	A	10	7	1.00	18	0.389
157	A	14	4	1.00	20	0.200
158	A	14	4	1.00	20	0.200
159	A	3	2	1.00	18	0.111
160	A	4	3	1.00	17	0.176
161	A	12	7	1.00	20	0.350
162	A	13	9	1.00	20	0.450
163	A	11	7	1.00	20	0.350
164	A	13	9	1.00	20	0.450
165	A	18	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	18	4	1.00	20	0.200
167	A	3	2	1.00	18	0.111
168	A	5	3	1.00	17	0.176
169	A	16	7	1.00	20	0.350
170	A	17	9	1.00	20	0.450
171	A	15	8	1.00	20	0.400
172	A	17	10	1.00	20	0.500
173	A	9	7	1.00	20	0.350
174	A	8	8	1.00	20	0.400
175	A	4	4	1.00	20	0.200
176	A	4	4	1.00	18	0.222
177	A	1	1	1.00	17	0.059
178	A	3	3	1.00	20	0.150
179	A	7	7	1.00	20	0.350
180	A	7	7	1.00	20	0.350
181	A	12	8	1.00	20	0.400
182	A	17	12	1.00	20	0.600
183	A	7	6	1.00	20	0.300
184	A	8	8	1.00	20	0.400
185	A	2	2	1.00	20	0.100
186	A	3	3	1.00	18	0.167
187	A	2	2	1.00	17	0.118
188	A	7	7	1.00	20	0.350
189	A	10	10	1.00	20	0.500
190	A	15	11	1.00	20	0.550
191	A	23	11	1.00	20	0.550
192	A	4	3	1.00	20	0.150
193	A	3	3	1.00	20	0.150
194	A	4	3	1.00	18	0.167
195	A	3	3	1.00	17	0.176
196	A	12	7	1.00	20	0.350
197	A	14	11	1.00	20	0.550
198	A	28	11	1.00	20	0.550
199	A	38	12	1.00	20	0.600
200	A	12	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	7	1.00	22	0.318
202	A	4	4	1.00	20	0.200
203	A	3	3	1.00	19	0.158
204	A	5	5	1.00	22	0.227
205	A	7	7	1.00	22	0.318
206	A	6	5	1.00	22	0.227
207	A	5	5	1.00	22	0.227
208	A	31	7	1.00	22	0.318
209	A	21	8	1.00	22	0.364
210	A	5	4	1.00	20	0.200
211	A	4	3	1.00	19	0.158
212	A	10	8	1.00	22	0.364
213	A	11	8	1.00	22	0.364
214	A	12	8	1.00	22	0.364
215	A	13	8	1.00	22	0.364
216	A	76	7	1.00	22	0.318
217	A	51	8	1.00	22	0.364
218	A	6	4	1.00	20	0.200
219	A	5	3	1.00	19	0.158
220	A	16	8	1.00	22	0.364
221	A	16	8	1.00	22	0.364
222	A	23	10	1.00	22	0.454
223	A	25	9	1.00	22	0.409
224	A	7	5	1.00	22	0.227
225	A	4	4	1.00	22	0.182
226	A	3	3	1.00	20	0.150
227	A	2	2	1.00	19	0.105
228	A	2	2	1.00	22	0.091
229	A	4	4	1.00	22	0.182
230	A	4	4	1.00	22	0.182
231	A	9	6	1.00	22	0.273
232	A	6	5	1.00	22	0.227
233	A	3	3	1.00	22	0.136
234	A	2	2	1.00	20	0.100
235	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	5	5	1.00	22	0.227
237	A	6	6	1.00	22	0.273
238	A	10	7	1.00	22	0.318
239	A	16	8	1.00	22	0.364
240	A	10	6	1.00	22	0.273
241	A	8	7	1.00	22	0.318
242	A	3	3	1.00	22	0.136
243	A	4	3	1.00	22	0.136
244	A	3	3	1.00	20	0.150
245	A	2	2	1.00	19	0.105
246	A	9	6	1.00	22	0.273
247	A	9	7	1.00	22	0.318
248	A	10	3	1.00	20	0.150
249	A	8	3	1.00	20	0.150
250	A	5	3	1.00	18	0.167
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	0	0	0.00	0	0.000
257	A	0	0	0.00	0	0.000
258	A	26	8	1.00	20	0.400
259	A	24	10	1.00	20	0.500
260	A	4	4	1.00	18	0.222
261	A	7	7	1.00	17	0.412
262	A	12	10	1.00	20	0.500
263	A	10	10	1.00	20	0.500
264	A	15	12	1.00	20	0.600
265	A	13	8	1.00	20	0.400
266	A	47	8	1.00	22	0.364
267	A	44	10	1.00	22	0.454
268	A	5	4	1.00	20	0.200
269	A	9	7	1.00	19	0.368
270	A	23	12	1.00	22	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	20	13	1.00	22	0.591
272	A	21	15	1.00	22	0.682
273	A	19	13	1.00	22	0.591
274	A	72	8	1.00	22	0.364
275	A	68	10	1.00	22	0.454
276	A	6	4	1.00	20	0.200
277	A	12	8	1.00	19	0.421
278	A	38	12	1.00	22	0.546
279	A	34	14	1.00	22	0.636
280	A	31	16	1.00	22	0.727
281	A	28	15	1.00	22	0.682
282	A	17	10	1.00	22	0.454
283	A	10	9	1.00	22	0.409
284	A	7	7	1.00	22	0.318
285	A	4	5	1.00	20	0.250
286	A	1	1	1.00	19	0.053
287	A	4	5	1.00	22	0.227
288	A	6	6	1.00	22	0.273
289	A	13	11	1.00	22	0.500
290	A	15	8	1.00	22	0.364
291	A	8	9	1.00	22	0.409
292	A	4	4	1.00	22	0.182
293	A	3	3	1.00	20	0.150
294	A	4	4	1.00	19	0.210
295	A	8	9	1.00	22	0.409
296	A	11	11	1.00	22	0.500
297	A	22	15	1.00	22	0.682
298	A	27	13	1.00	22	0.591
299	A	4	4	1.00	22	0.182
300	A	13	6	1.00	22	0.273
301	A	4	4	1.00	20	0.200
302	A	8	5	1.00	19	0.263
303	A	13	10	1.00	22	0.454
304	A	20	12	1.00	22	0.546
305	A	36	16	1.00	22	0.727

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	48	14	1.00	22	0.636
307	A	26	8	1.00	24	0.333
308	A	35	12	1.00	24	0.500
309	A	4	4	1.00	22	0.182
310	A	12	9	1.00	21	0.429
311	A	13	10	1.00	24	0.417
312	A	13	10	1.00	24	0.417
313	A	24	12	1.00	24	0.500
314	A	7	6	1.00	24	0.250
315	A	75	8	1.00	24	0.333
316	A	92	12	1.00	24	0.500
317	A	5	4	1.00	22	0.182
318	A	16	10	1.00	21	0.476
319	A	18	11	1.00	24	0.458
320	A	26	13	1.00	24	0.542
321	A	38	15	1.00	24	0.625
322	A	21	13	1.00	24	0.542
323	A	203	8	1.00	24	0.333
324	A	238	12	1.00	24	0.500
325	A	6	4	1.00	22	0.182
326	A	21	10	1.00	21	0.476
327	A	24	11	1.00	24	0.458
328	A	43	14	1.00	24	0.583
329	A	57	16	1.00	24	0.667
330	A	48	16	1.00	24	0.667
331	A	8	5	1.00	24	0.208
332	A	13	10	1.00	24	0.417
333	A	3	3	1.00	22	0.136
334	A	9	6	1.00	21	0.286
335	A	9	6	1.00	24	0.250
336	A	3	3	1.00	24	0.125
337	A	14	11	1.00	24	0.458
338	A	8	5	1.00	24	0.208
339	A	6	5	1.00	24	0.208
340	A	12	9	1.00	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	2	2	1.00	22	0.091
342	A	2	2	1.00	21	0.095
343	A	12	9	1.00	24	0.375
344	A	6	6	1.00	24	0.250
345	A	27	14	1.00	24	0.583
346	A	15	8	1.00	24	0.333
347	A	13	8	1.00	24	0.333
348	A	17	12	1.00	24	0.500
349	A	6	5	1.00	24	0.208
350	A	4	4	1.00	24	0.167
351	A	3	3	1.00	22	0.136
352	A	5	4	1.00	21	0.190
353	A	16	10	1.00	24	0.417
354	A	12	8	1.00	24	0.333
355	A	0	0	0.00	0	0.000
356	A	0	0	0.00	0	0.000
357	A	0	0	0.00	0	0.000
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	52	12	1.00	20	0.600
364	A	34	12	1.00	20	0.600
365	A	8	8	1.00	18	0.444
366	A	8	8	1.00	17	0.471
367	A	17	14	1.00	20	0.700
368	A	11	11	1.00	20	0.550
369	A	16	12	1.00	20	0.600
370	A	20	12	1.00	20	0.600
371	A	106	12	1.00	22	0.546
372	A	73	12	1.00	22	0.546
373	A	10	8	1.00	20	0.400
374	A	12	9	1.00	19	0.474
375	A	36	16	1.00	22	0.727

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	23	13	1.00	22	0.591
377	A	25	18	1.00	22	0.818
378	A	26	16	1.00	22	0.727
379	A	184	12	1.00	22	0.546
380	A	132	12	1.00	22	0.546
381	A	13	9	1.00	20	0.450
382	A	17	9	1.00	19	0.474
383	A	69	17	1.00	22	0.773
384	A	45	15	1.00	22	0.682
385	A	43	20	1.00	22	0.909
386	A	37	18	1.00	22	0.818
387	A	19	9	1.00	22	0.409
388	A	14	11	1.00	22	0.500
389	A	7	7	1.00	22	0.318
390	A	5	6	1.00	20	0.300
391	A	1	1	1.00	19	0.053
392	A	5	6	1.00	22	0.273
393	A	7	7	1.00	22	0.318
394	A	13	9	1.00	22	0.409
395	A	22	11	1.00	22	0.500
396	A	11	11	1.00	22	0.500
397	A	4	4	1.00	22	0.182
398	A	5	4	1.00	20	0.200
399	A	4	3	1.00	19	0.158
400	A	11	11	1.00	22	0.500
401	A	12	11	1.00	22	0.500
402	A	25	14	1.00	22	0.636
403	A	35	15	1.00	22	0.682
404	A	9	7	1.00	22	0.318
405	A	13	6	1.00	22	0.273
406	A	9	5	1.00	20	0.250
407	A	8	5	1.00	19	0.263
408	A	21	12	1.00	22	0.546
409	A	21	13	1.00	22	0.591
410	A	47	15	1.00	22	0.682

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	57	17	1.00	22	0.773
412	A	71	12	1.00	24	0.500
413	A	40	12	1.00	24	0.500
414	A	13	10	1.00	22	0.454
415	A	14	9	1.00	21	0.429
416	A	22	12	1.00	24	0.500
417	A	22	12	1.00	24	0.500
418	A	27	11	1.00	24	0.458
419	A	25	12	1.00	24	0.500
420	A	200	12	1.00	24	0.500
421	A	108	14	1.00	24	0.583
422	A	17	11	1.00	22	0.500
423	A	18	10	1.00	21	0.476
424	A	36	15	1.00	24	0.625
425	A	37	14	1.00	24	0.583
426	A	50	15	1.00	24	0.625
427	A	48	16	1.00	24	0.667
428	A	547	12	1.00	24	0.500
429	A	293	14	1.00	24	0.583
430	A	22	11	1.00	22	0.500
431	A	23	10	1.00	21	0.476
432	A	54	16	1.00	24	0.667
433	A	56	15	1.00	24	0.625
434	A	87	18	1.00	24	0.750
435	A	86	18	1.00	24	0.750
436	A	24	10	1.00	24	0.417
437	A	15	10	1.00	24	0.417
438	A	10	7	1.00	22	0.318
439	A	11	7	1.00	21	0.333
440	A	11	7	1.00	24	0.292
441	A	10	7	1.00	24	0.292
442	A	15	10	1.00	24	0.417
443	A	25	11	1.00	24	0.458
444	A	14	10	1.00	24	0.417
445	A	14	10	1.00	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	3	1.00	22	0.136
447	A	2	2	1.00	21	0.095
448	A	15	11	1.00	24	0.458
449	A	13	10	1.00	24	0.417
450	A	22	12	1.00	24	0.500
451	A	22	15	1.00	24	0.625
452	A	7	5	1.00	24	0.208
453	A	7	6	1.00	24	0.250
454	A	6	5	1.00	22	0.227
455	A	5	4	1.00	21	0.190
456	A	22	13	1.00	24	0.542
457	A	19	12	1.00	24	0.500
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000
460	A	0	0	0.00	0	0.000
461	A	0	0	0.00	0	0.000
462	A	0	0	0.00	0	0.000
463	A	0	0	0.00	0	0.000
464	A	0	0	0.00	0	0.000
465	A	0	0	0.00	0	0.000
466	A	0	0	0.00	0	0.000
467	A	0	0	0.00	0	0.000
468	A	0	0	0.00	0	0.000
469	A	0	0	0.00	0	0.000
470	A	0	0	0.00	0	0.000
471	A	0	0	0.00	0	0.000
472	A	0	0	0.00	0	0.000
473	A	0	0	0.00	0	0.000
474	A	0	0	0.00	0	0.000
475	A	0	0	0.00	0	0.000
476	A	0	0	0.00	0	0.000
477	A	1	1	1.00	19	0.053
478	A	0	0	0.00	0	0.000
479	A	0	0	0.00	0	0.000
480	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	0	0	0.00	0	0.000
482	A	4	3	1.00	22	0.136
483	A	4	4	1.00	20	0.200
484	A	4	3	1.00	19	0.158
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	0	0	0.00	0	0.000
489	A	5	3	1.00	22	0.136
490	A	5	3	1.00	22	0.136
491	A	4	3	1.00	22	0.136
492	A	5	3	1.00	20	0.150
493	A	5	3	1.00	19	0.158
494	A	0	0	0.00	0	0.000
495	A	0	0	0.00	0	0.000
496	A	0	0	0.00	0	0.000
497	A	0	0	0.00	0	0.000
498	A	0	0	0.00	0	0.000
499	A	0	0	0.00	0	0.000
500	A	0	0	0.00	0	0.000
501	A	0	0	0.00	0	0.000
502	A	0	0	0.00	0	0.000
503	A	0	0	0.00	0	0.000
504	A	0	0	0.00	0	0.000
505	A	0	0	0.00	0	0.000
506	A	0	0	0.00	0	0.000
507	A	0	0	0.00	0	0.000
508	A	0	0	0.00	0	0.000
509	A	0	0	0.00	0	0.000
510	A	3	3	1.00	22	0.136
511	A	3	3	1.00	21	0.143
512	A	0	0	0.00	0	0.000
513	A	0	0	0.00	0	0.000
514	A	0	0	0.00	0	0.000
515	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	6	4	1.00	24	0.167
517	A	6	4	1.00	24	0.167
518	A	6	4	1.00	22	0.182
519	A	6	4	1.00	21	0.190
520	A	0	0	0.00	0	0.000
521	A	0	0	0.00	0	0.000
522	A	0	0	0.00	0	0.000
523	A	0	0	0.00	0	0.000
524	A	0	0	0.00	0	0.000
525	A	0	0	0.00	0	0.000
526	A	0	0	0.00	0	0.000
527	A	0	0	0.00	0	0.000
528	A	0	0	0.00	0	0.000
529	A	0	0	0.00	0	0.000
530	A	0	0	0.00	0	0.000
531	A	0	0	0.00	0	0.000
532	A	0	0	0.00	0	0.000
533	A	0	0	0.00	0	0.000
534	A	0	0	0.00	0	0.000
535	A	0	0	0.00	0	0.000
536	A	0	0	0.00	0	0.000
537	A	0	0	0.00	0	0.000
538	A	0	0	0.00	0	0.000
539	A	0	0	0.00	0	0.000
540	A	0	0	0.00	0	0.000
541	A	0	0	0.00	0	0.000
542	A	0	0	0.00	0	0.000
543	A	0	0	0.00	0	0.000
544	A	0	0	0.00	0	0.000
545	A	0	0	0.00	0	0.000
546	A	1	1	1.00	19	0.053
547	A	0	0	0.00	0	0.000
548	A	0	0	0.00	0	0.000
549	A	0	0	0.00	0	0.000
550	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	0	0	0.00	0	0.000
552	A	5	5	1.00	22	0.227
553	A	9	5	1.00	20	0.250
554	A	5	5	1.00	19	0.263
555	A	0	0	0.00	0	0.000
556	A	0	0	0.00	0	0.000
557	A	0	0	0.00	0	0.000
558	A	0	0	0.00	0	0.000
559	A	20	7	1.00	22	0.318
560	A	12	6	1.00	22	0.273
561	A	10	6	1.00	20	0.300
562	A	6	4	1.00	19	0.210
563	A	0	0	0.00	0	0.000
564	A	0	0	0.00	0	0.000
565	A	0	0	0.00	0	0.000
566	A	0	0	0.00	0	0.000
567	A	0	0	0.00	0	0.000
568	A	0	0	0.00	0	0.000
569	A	0	0	0.00	0	0.000
570	A	0	0	0.00	0	0.000
571	A	0	0	0.00	0	0.000
572	A	0	0	0.00	0	0.000
573	A	0	0	0.00	0	0.000
574	A	0	0	0.00	0	0.000
575	A	0	0	0.00	0	0.000
576	A	0	0	0.00	0	0.000
577	A	0	0	0.00	0	0.000
578	A	0	0	0.00	0	0.000
579	A	0	0	0.00	0	0.000
580	A	0	0	0.00	0	0.000
581	A	4	4	1.00	22	0.182
582	A	4	4	1.00	21	0.190
583	A	0	0	0.00	0	0.000
584	A	0	0	0.00	0	0.000
585	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	0	0	0.00	0	0.000
587	A	0	0	0.00	0	0.000
588	A	0	0	0.00	0	0.000
589	A	7	5	1.00	24	0.208
590	A	12	6	1.00	24	0.250
591	A	13	8	1.00	22	0.364
592	A	7	5	1.00	21	0.238
593	A	0	0	0.00	0	0.000
594	A	0	0	0.00	0	0.000
595	A	0	0	0.00	0	0.000
596	A	0	0	0.00	0	0.000
597	A	0	0	0.00	0	0.000
598	A	0	0	0.00	0	0.000
599	A	0	0	0.00	0	0.000
600	A	0	0	0.00	0	0.000
601	A	0	0	0.00	0	0.000
602	A	0	0	0.00	0	0.000
603	A	0	0	0.00	0	0.000
604	A	0	0	0.00	0	0.000
605	A	0	0	0.00	0	0.000
606	A	0	0	0.00	0	0.000
607	A	0	0	0.00	0	0.000
608	A	0	0	0.00	0	0.000
609	A	0	0	0.00	0	0.000
610	A	0	0	0.00	0	0.000
611	A	0	0	0.00	0	0.000
612	A	0	0	0.00	0	0.000
613	A	0	0	0.00	0	0.000
614	A	0	0	0.00	0	0.000
615	A	0	0	0.00	0	0.000
616	A	0	0	0.00	0	0.000
617	A	0	0	0.00	0	0.000
618	A	0	0	0.00	0	0.000
619	A	0	0	0.00	0	0.000
620	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	0	0	0.00	0	0.000
622	A	1	1	1.00	19	0.053
623	A	0	0	0.00	0	0.000
624	A	0	0	0.00	0	0.000
625	A	0	0	0.00	0	0.000
626	A	0	0	0.00	0	0.000
627	A	0	0	0.00	0	0.000
628	A	10	6	1.00	22	0.273
629	A	5	5	1.00	20	0.250
630	A	10	6	1.00	19	0.316
631	A	0	0	0.00	0	0.000
632	A	0	0	0.00	0	0.000
633	A	0	0	0.00	0	0.000
634	A	0	0	0.00	0	0.000
635	A	25	8	1.00	22	0.364
636	A	22	8	1.00	22	0.364
637	A	19	7	1.00	20	0.350
638	A	11	7	1.00	19	0.368
639	A	0	0	0.00	0	0.000
640	A	0	0	0.00	0	0.000
641	A	0	0	0.00	0	0.000
642	A	0	0	0.00	0	0.000
643	A	2	1	1.00	38	0.026
644	A	0	0	0.00	0	0.000
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000
648	A	0	0	0.00	0	0.000
649	A	0	0	0.00	0	0.000
650	A	0	0	0.00	0	0.000
651	A	0	0	0.00	0	0.000
652	A	0	0	0.00	0	0.000
653	A	0	0	0.00	0	0.000
654	A	0	0	0.00	0	0.000
655	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	0	0	0.00	0	0.000
657	A	0	0	0.00	0	0.000
658	A	0	0	0.00	0	0.000
659	A	0	0	0.00	0	0.000
660	A	5	5	1.00	22	0.227
661	A	5	5	1.00	21	0.238
662	A	0	0	0.00	0	0.000
663	A	0	0	0.00	0	0.000
664	A	0	0	0.00	0	0.000
665	A	0	0	0.00	0	0.000
666	A	0	0	0.00	0	0.000
667	A	0	0	0.00	0	0.000
668	A	13	7	1.00	24	0.292
669	A	20	11	1.00	24	0.458
670	A	20	7	1.00	22	0.318
671	A	14	9	1.00	21	0.429
672	A	0	0	0.00	0	0.000
673	A	0	0	0.00	0	0.000
674	A	0	0	0.00	0	0.000
675	A	0	0	0.00	0	0.000
676	A	0	0	0.00	0	0.000
677	A	0	0	0.00	0	0.000
678	A	0	0	0.00	0	0.000
679	A	0	0	0.00	0	0.000
680	A	0	0	0.00	0	0.000
681	A	0	0	0.00	0	0.000
682	A	0	0	0.00	0	0.000
683	A	0	0	0.00	0	0.000
684	A	0	0	0.00	0	0.000
685	A	0	0	0.00	0	0.000
686	A	0	0	0.00	0	0.000
687	A	0	0	0.00	0	0.000
688	A	0	0	0.00	0	0.000
689	A	0	0	0.00	0	0.000
690	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	0	0	0.00	0	0.000
692	A	0	0	0.00	0	0.000
693	A	0	0	0.00	0	0.000
694	A	0	0	0.00	0	0.000
695	A	0	0	0.00	0	0.000
696	A	0	0	0.00	0	0.000
697	A	0	0	0.00	0	0.000
698	A	0	0	0.00	0	0.000
699	A	0	0	0.00	0	0.000
700	A	0	0	0.00	0	0.000
701	A	0	0	0.00	0	0.000
702	A	1	1	1.00	21	0.048
703	A	0	0	0.00	0	0.000
704	A	0	0	0.00	0	0.000
705	A	0	0	0.00	0	0.000
706	A	0	0	0.00	0	0.000
707	A	0	0	0.00	0	0.000
708	A	0	0	0.00	0	0.000
709	A	6	6	1.00	24	0.250
710	A	6	5	1.00	22	0.227
711	A	6	6	1.00	21	0.286
712	A	0	0	0.00	0	0.000
713	A	0	0	0.00	0	0.000
714	A	0	0	0.00	0	0.000
715	A	9	5	1.00	24	0.208
716	A	8	5	1.00	24	0.208
717	A	6	5	1.00	24	0.208
718	A	8	5	1.00	22	0.227
719	A	9	5	1.00	21	0.238
720	A	0	0	0.00	0	0.000
721	A	0	0	0.00	0	0.000
722	A	0	0	0.00	0	0.000
723	A	0	0	0.00	0	0.000
724	A	0	0	0.00	0	0.000
725	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	0	0	0.00	0	0.000
727	A	0	0	0.00	0	0.000
728	A	0	0	0.00	0	0.000
729	A	0	0	0.00	0	0.000
730	A	0	0	0.00	0	0.000
731	A	0	0	0.00	0	0.000
732	A	0	0	0.00	0	0.000
733	A	0	0	0.00	0	0.000
734	A	0	0	0.00	0	0.000
735	A	0	0	0.00	0	0.000
736	A	0	0	0.00	0	0.000
737	A	0	0	0.00	0	0.000
738	A	0	0	0.00	0	0.000
739	A	0	0	0.00	0	0.000
740	A	0	0	0.00	0	0.000
741	A	0	0	0.00	0	0.000
742	A	0	0	0.00	0	0.000
743	A	0	0	0.00	0	0.000
744	A	0	0	0.00	0	0.000
745	A	5	5	1.00	24	0.208
746	A	5	5	1.00	23	0.217
747	A	0	0	0.00	0	0.000
748	A	0	0	0.00	0	0.000
749	A	0	0	0.00	0	0.000
750	A	0	0	0.00	0	0.000
751	A	10	6	1.00	26	0.231
752	A	9	6	1.00	26	0.231
753	A	9	6	1.00	24	0.250
754	A	10	6	1.00	23	0.261
755	A	0	0	0.00	0	0.000
756	A	0	0	0.00	0	0.000
757	A	0	0	0.00	0	0.000
758	A	0	0	0.00	0	0.000
759	A	0	0	0.00	0	0.000
760	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	0	0	0.00	0	0.000
762	A	0	0	0.00	0	0.000
763	A	0	0	0.00	0	0.000
764	A	0	0	0.00	0	0.000
765	A	0	0	0.00	0	0.000
766	A	0	0	0.00	0	0.000
767	A	0	0	0.00	0	0.000
768	A	0	0	0.00	0	0.000
769	A	0	0	0.00	0	0.000
770	A	0	0	0.00	0	0.000
771	A	0	0	0.00	0	0.000
772	A	0	0	0.00	0	0.000
773	A	0	0	0.00	0	0.000
774	A	0	0	0.00	0	0.000
775	A	0	0	0.00	0	0.000
776	A	0	0	0.00	0	0.000
777	A	0	0	0.00	0	0.000
778	A	1	1	1.00	21	0.048
779	A	0	0	0.00	0	0.000
780	A	0	0	0.00	0	0.000
781	A	0	0	0.00	0	0.000
782	A	0	0	0.00	0	0.000
783	A	0	0	0.00	0	0.000
784	A	0	0	0.00	0	0.000
785	A	7	6	1.00	24	0.250
786	A	7	7	1.00	22	0.318
787	A	7	6	1.00	21	0.286
788	A	0	0	0.00	0	0.000
789	A	0	0	0.00	0	0.000
790	A	0	0	0.00	0	0.000
791	A	15	8	1.00	24	0.333
792	A	10	6	1.00	24	0.250
793	A	7	5	1.00	24	0.208
794	A	10	6	1.00	22	0.273
795	A	15	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	0	0	0.00	0	0.000
797	A	0	0	0.00	0	0.000
798	A	0	0	0.00	0	0.000
799	A	0	0	0.00	0	0.000
800	A	0	0	0.00	0	0.000
801	A	0	0	0.00	0	0.000
802	A	0	0	0.00	0	0.000
803	A	0	0	0.00	0	0.000
804	A	0	0	0.00	0	0.000
805	A	0	0	0.00	0	0.000
806	A	0	0	0.00	0	0.000
807	A	0	0	0.00	0	0.000
808	A	0	0	0.00	0	0.000
809	A	0	0	0.00	0	0.000
810	A	0	0	0.00	0	0.000
811	A	0	0	0.00	0	0.000
812	A	0	0	0.00	0	0.000
813	A	0	0	0.00	0	0.000
814	A	0	0	0.00	0	0.000
815	A	0	0	0.00	0	0.000
816	A	0	0	0.00	0	0.000
817	A	0	0	0.00	0	0.000
818	A	0	0	0.00	0	0.000
819	A	0	0	0.00	0	0.000
820	A	0	0	0.00	0	0.000
821	A	0	0	0.00	0	0.000
822	A	0	0	0.00	0	0.000
823	A	0	0	0.00	0	0.000
824	A	6	6	1.00	24	0.250
825	A	5	5	1.00	23	0.217
826	A	0	0	0.00	0	0.000
827	A	0	0	0.00	0	0.000
828	A	0	0	0.00	0	0.000
829	A	0	0	0.00	0	0.000
830	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	15	10	1.00	26	0.385
832	A	11	7	1.00	26	0.269
833	A	11	7	1.00	24	0.292
834	A	14	7	1.00	23	0.304
835	A	0	0	0.00	0	0.000
836	A	0	0	0.00	0	0.000
837	A	0	0	0.00	0	0.000
838	A	0	0	0.00	0	0.000
839	A	0	0	0.00	0	0.000
840	A	0	0	0.00	0	0.000
841	A	0	0	0.00	0	0.000
842	A	0	0	0.00	0	0.000
843	A	0	0	0.00	0	0.000
844	A	0	0	0.00	0	0.000
845	A	0	0	0.00	0	0.000
846	A	0	0	0.00	0	0.000
847	A	0	0	0.00	0	0.000
848	A	0	0	0.00	0	0.000
849	A	0	0	0.00	0	0.000
850	A	0	0	0.00	0	0.000
851	A	0	0	0.00	0	0.000
852	A	0	0	0.00	0	0.000
853	A	0	0	0.00	0	0.000
854	A	0	0	0.00	0	0.000
855	A	0	0	0.00	0	0.000
856	A	0	0	0.00	0	0.000
857	A	0	0	0.00	0	0.000
858	A	0	0	0.00	0	0.000
859	A	1	1	1.00	21	0.048
860	A	0	0	0.00	0	0.000
861	A	0	0	0.00	0	0.000
862	A	0	0	0.00	0	0.000
863	A	0	0	0.00	0	0.000
864	A	0	0	0.00	0	0.000
865	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	8	8	1.00	24	0.333
867	A	8	6	1.00	22	0.273
868	A	8	7	1.00	21	0.333
869	A	0	0	0.00	0	0.000
870	A	0	0	0.00	0	0.000
871	A	0	0	0.00	0	0.000
872	A	18	11	1.00	24	0.458
873	A	16	9	1.00	24	0.375
874	A	8	5	1.00	24	0.208
875	A	16	7	1.00	22	0.318
876	A	18	11	1.00	21	0.524
877	A	0	0	0.00	0	0.000
878	A	0	0	0.00	0	0.000
879	A	0	0	0.00	0	0.000
880	A	0	0	0.00	0	0.000
881	A	0	0	0.00	0	0.000
882	A	0	0	0.00	0	0.000
883	A	0	0	0.00	0	0.000
884	A	0	0	0.00	0	0.000
885	A	0	0	0.00	0	0.000
886	A	0	0	0.00	0	0.000
887	A	0	0	0.00	0	0.000
888	A	0	0	0.00	0	0.000
889	A	0	0	0.00	0	0.000
890	A	0	0	0.00	0	0.000
891	A	0	0	0.00	0	0.000
892	A	0	0	0.00	0	0.000
893	A	0	0	0.00	0	0.000
894	A	0	0	0.00	0	0.000
895	A	0	0	0.00	0	0.000
896	A	0	0	0.00	0	0.000
897	A	0	0	0.00	0	0.000
898	A	0	0	0.00	0	0.000
899	A	0	0	0.00	0	0.000
900	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	0	0	0.00	0	0.000
902	A	0	0	0.00	0	0.000
903	A	0	0	0.00	0	0.000
904	A	6	6	1.00	24	0.250
905	A	6	6	1.00	23	0.261
906	A	0	0	0.00	0	0.000
907	A	0	0	0.00	0	0.000
908	A	0	0	0.00	0	0.000
909	A	17	11	1.00	26	0.423
910	A	16	11	1.00	26	0.423
911	A	15	8	1.00	24	0.333
912	A	17	8	1.00	23	0.348
913	A	0	0	0.00	0	0.000
914	A	0	0	0.00	0	0.000
915	A	0	0	0.00	0	0.000
916	A	0	0	0.00	0	0.000
917	A	0	0	0.00	0	0.000
918	A	0	0	0.00	0	0.000
919	A	0	0	0.00	0	0.000
920	A	0	0	0.00	0	0.000
921	A	0	0	0.00	0	0.000
922	A	0	0	0.00	0	0.000
923	A	0	0	0.00	0	0.000
924	A	0	0	0.00	0	0.000
925	A	0	0	0.00	0	0.000
926	A	0	0	0.00	0	0.000
927	A	0	0	0.00	0	0.000
928	A	1	1	1.00	21	0.048
929	A	0	0	0.00	0	0.000
930	A	0	0	0.00	0	0.000
931	A	0	0	0.00	0	0.000
932	A	5	4	1.00	24	0.167
933	A	5	5	1.00	22	0.227
934	A	5	4	1.00	21	0.190
935	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	0	0	0.00	0	0.000
937	A	0	0	0.00	0	0.000
938	A	7	4	1.00	24	0.167
939	A	7	4	1.00	24	0.167
940	A	5	4	1.00	24	0.167
941	A	7	4	1.00	22	0.182
942	A	7	4	1.00	21	0.190
943	A	0	0	0.00	0	0.000
944	A	0	0	0.00	0	0.000
945	A	0	0	0.00	0	0.000
946	A	0	0	0.00	0	0.000
947	A	0	0	0.00	0	0.000
948	A	0	0	0.00	0	0.000
949	A	0	0	0.00	0	0.000
950	A	0	0	0.00	0	0.000
951	A	0	0	0.00	0	0.000
952	A	0	0	0.00	0	0.000
953	A	0	0	0.00	0	0.000
954	A	0	0	0.00	0	0.000
955	A	0	0	0.00	0	0.000
956	A	0	0	0.00	0	0.000
957	A	0	0	0.00	0	0.000
958	A	0	0	0.00	0	0.000
959	A	0	0	0.00	0	0.000
960	A	0	0	0.00	0	0.000
961	A	0	0	0.00	0	0.000
962	A	4	4	1.00	24	0.167
963	A	4	4	1.00	23	0.174
964	A	0	0	0.00	0	0.000
965	A	0	0	0.00	0	0.000
966	A	0	0	0.00	0	0.000
967	A	8	5	1.00	26	0.192
968	A	8	5	1.00	26	0.192
969	A	8	5	1.00	24	0.208
970	A	8	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	0	0	0.00	0	0.000
972	A	0	0	0.00	0	0.000
973	A	0	0	0.00	0	0.000
974	A	0	0	0.00	0	0.000
975	A	0	0	0.00	0	0.000
976	A	0	0	0.00	0	0.000
977	A	0	0	0.00	0	0.000
978	A	0	0	0.00	0	0.000
979	A	0	0	0.00	0	0.000
980	A	0	0	0.00	0	0.000
981	A	0	0	0.00	0	0.000
982	A	0	0	0.00	0	0.000
983	A	0	0	0.00	0	0.000
984	A	0	0	0.00	0	0.000
985	A	0	0	0.00	0	0.000
986	A	1	1	1.00	21	0.048
987	A	0	0	0.00	0	0.000
988	A	0	0	0.00	0	0.000
989	A	0	0	0.00	0	0.000
990	A	0	0	0.00	0	0.000
991	A	6	6	1.00	24	0.250
992	A	7	6	1.00	22	0.273
993	A	6	6	1.00	21	0.286
994	A	0	0	0.00	0	0.000
995	A	0	0	0.00	0	0.000
996	A	0	0	0.00	0	0.000
997	A	0	0	0.00	0	0.000
998	A	0	0	0.00	0	0.000
999	A	13	6	1.00	24	0.250
1000	A	15	5	1.00	24	0.208
1001	A	13	7	1.00	22	0.318
1002	A	8	5	1.00	21	0.238
1003	A	0	0	0.00	0	0.000
1004	A	0	0	0.00	0	0.000
1005	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	0	0	0.00	0	0.000
1007	A	0	0	0.00	0	0.000
1008	A	0	0	0.00	0	0.000
1009	A	0	0	0.00	0	0.000
1010	A	0	0	0.00	0	0.000
1011	A	0	0	0.00	0	0.000
1012	A	0	0	0.00	0	0.000
1013	A	0	0	0.00	0	0.000
1014	A	0	0	0.00	0	0.000
1015	A	0	0	0.00	0	0.000
1016	A	0	0	0.00	0	0.000
1017	A	0	0	0.00	0	0.000
1018	A	0	0	0.00	0	0.000
1019	A	0	0	0.00	0	0.000
1020	A	0	0	0.00	0	0.000
1021	A	0	0	0.00	0	0.000
1022	A	0	0	0.00	0	0.000
1023	A	0	0	0.00	0	0.000
1024	A	0	0	0.00	0	0.000
1025	A	0	0	0.00	0	0.000
1026	A	0	0	0.00	0	0.000
1027	A	5	5	1.00	24	0.208
1028	A	5	5	1.00	23	0.217
1029	A	0	0	0.00	0	0.000
1030	A	0	0	0.00	0	0.000
1031	A	0	0	0.00	0	0.000
1032	A	0	0	0.00	0	0.000
1033	A	0	0	0.00	0	0.000
1034	A	9	6	1.00	26	0.231
1035	A	17	7	1.00	26	0.269
1036	A	17	9	1.00	24	0.375
1037	A	9	6	1.00	23	0.261
1038	A	0	0	0.00	0	0.000
1039	A	0	0	0.00	0	0.000
1040	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1041	A	0	0	0.00	0	0.000
1042	A	0	0	0.00	0	0.000
1043	A	0	0	0.00	0	0.000
1044	A	0	0	0.00	0	0.000
1045	A	0	0	0.00	0	0.000
1046	A	0	0	0.00	0	0.000
1047	A	0	0	0.00	0	0.000
1048	A	0	0	0.00	0	0.000
1049	A	0	0	0.00	0	0.000
1050	A	0	0	0.00	0	0.000
1051	A	0	0	0.00	0	0.000
1052	A	0	0	0.00	0	0.000
1053	A	0	0	0.00	0	0.000
1054	A	0	0	0.00	0	0.000
1055	A	0	0	0.00	0	0.000
1056	A	1	1	1.00	21	0.048
1057	A	0	0	0.00	0	0.000
1058	A	0	0	0.00	0	0.000
1059	A	0	0	0.00	0	0.000
1060	A	8	7	1.00	24	0.292
1061	A	6	6	1.00	22	0.273
1062	A	8	7	1.00	21	0.333
1063	A	0	0	0.00	0	0.000
1064	A	0	0	0.00	0	0.000
1065	A	0	0	0.00	0	0.000
1066	A	0	0	0.00	0	0.000
1067	A	0	0	0.00	0	0.000
1068	A	24	6	1.00	24	0.250
1069	A	27	7	1.00	24	0.292
1070	A	24	6	1.00	22	0.273
1071	A	14	8	1.00	21	0.381
1072	A	0	0	0.00	0	0.000
1073	A	0	0	0.00	0	0.000
1074	A	0	0	0.00	0	0.000
1075	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1076	A	0	0	0.00	0	0.000
1077	A	0	0	0.00	0	0.000
1078	A	0	0	0.00	0	0.000
1079	A	0	0	0.00	0	0.000
1080	A	0	0	0.00	0	0.000
1081	A	0	0	0.00	0	0.000
1082	A	0	0	0.00	0	0.000
1083	A	0	0	0.00	0	0.000
1084	A	0	0	0.00	0	0.000
1085	A	0	0	0.00	0	0.000
1086	A	0	0	0.00	0	0.000
1087	A	0	0	0.00	0	0.000
1088	A	0	0	0.00	0	0.000
1089	A	0	0	0.00	0	0.000
1090	A	0	0	0.00	0	0.000
1091	A	0	0	0.00	0	0.000
1092	A	0	0	0.00	0	0.000
1093	A	0	0	0.00	0	0.000
1094	A	0	0	0.00	0	0.000
1095	A	0	0	0.00	0	0.000
1096	A	6	6	1.00	24	0.250
1097	A	6	6	1.00	23	0.261
1098	A	0	0	0.00	0	0.000
1099	A	0	0	0.00	0	0.000
1100	A	0	0	0.00	0	0.000
1101	A	0	0	0.00	0	0.000
1102	A	0	0	0.00	0	0.000
1103	A	18	8	1.00	26	0.308
1104	A	27	10	1.00	26	0.385
1105	A	27	8	1.00	24	0.333
1106	A	18	10	1.00	23	0.435
1107	A	0	0	0.00	0	0.000
1108	A	0	0	0.00	0	0.000
1109	A	0	0	0.00	0	0.000
1110	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	0	0	0.00	0	0.000
1112	A	1	1	1.00	19	0.053
1113	A	0	0	0.00	0	0.000
1114	A	5	5	1.00	19	0.263
1115	A	4	4	1.00	19	0.210
1116	A	4	3	1.00	17	0.176
1117	A	5	4	1.00	16	0.250
1118	A	8	6	1.00	19	0.316
1119	A	4	4	1.00	19	0.210
1120	A	8	6	1.00	19	0.316
1121	A	5	5	1.00	19	0.263
1122	A	5	6	1.00	19	0.316
1123	A	5	5	1.00	19	0.263
1124	A	6	6	1.00	19	0.316
1125	A	4	5	1.00	21	0.238
1126	A	5	5	1.00	21	0.238
1127	A	4	3	1.00	19	0.158
1128	A	5	5	1.00	18	0.278
1129	A	12	7	1.00	21	0.333
1130	A	4	4	1.00	21	0.190
1131	A	11	7	1.00	21	0.333
1132	A	5	5	1.00	21	0.238
1133	A	12	6	1.00	21	0.286
1134	A	5	5	1.00	21	0.238
1135	A	5	5	1.00	21	0.238
1136	A	5	5	1.00	21	0.238
1137	A	8	7	1.19	21	0.333
1138	A	5	5	1.00	21	0.238
1139	A	4	3	1.00	19	0.158
1140	A	4	4	1.00	18	0.222
1141	A	16	7	1.00	21	0.333
1142	A	4	4	1.00	21	0.190
1143	A	15	8	1.00	21	0.381
1144	A	5	5	1.00	21	0.238
1145	A	15	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	5	5	1.00	21	0.238
1147	A	17	6	1.00	21	0.286
1148	A	5	5	1.00	21	0.238
1149	A	5	5	1.00	21	0.238
1150	A	4	4	1.00	14	0.286
1151	A	14	9	1.00	21	0.429
1152	A	10	5	1.00	19	0.263
1153	A	15	8	1.00	21	0.381
1154	A	19	12	1.00	21	0.571
1155	A	23	10	1.00	21	0.476
1156	A	19	7	1.00	18	0.389
1157	A	25	13	1.00	21	0.619
1158	A	16	9	1.00	21	0.429
1159	A	4	4	1.00	19	0.210
1160	A	19	11	1.00	21	0.524
1161	A	22	13	1.00	21	0.619
1162	A	45	14	1.00	21	0.667
1163	A	24	12	1.00	18	0.667
1164	A	50	17	1.00	21	0.810
1165	A	21	11	1.00	21	0.524
1166	A	6	6	1.00	21	0.286
1167	A	5	5	1.00	19	0.263
1168	A	24	13	1.00	21	0.619
1169	A	27	15	1.00	21	0.714
1170	A	49	15	1.00	21	0.714
1171	A	23	11	1.00	18	0.611
1172	A	73	19	1.00	21	0.905
1173	A	9	10	1.00	23	0.435
1174	A	0	0	0.00	0	0.000
1175	A	7	7	1.00	21	0.333
1176	A	0	0	0.00	0	0.000
1177	A	0	0	0.00	0	0.000
1178	A	0	0	0.00	0	0.000
1179	A	0	0	0.00	0	0.000
1180	A	9	8	1.00	23	0.348

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1181	A	0	0	0.00	0	0.000
1182	A	10	9	1.00	23	0.391
1183	A	10	10	1.00	23	0.435
1184	A	0	0	0.00	0	0.000
1185	A	8	8	1.00	21	0.381
1186	A	0	0	0.00	0	0.000
1187	A	0	0	0.00	0	0.000
1188	A	0	0	0.00	0	0.000
1189	A	0	0	0.00	0	0.000
1190	A	0	0	0.00	0	0.000
1191	A	0	0	0.00	0	0.000
1192	A	10	9	1.00	23	0.391
1193	A	11	10	1.00	23	0.435
1194	A	0	0	0.00	0	0.000
1195	A	9	8	1.00	21	0.381
1196	A	0	0	0.00	0	0.000
1197	A	0	0	0.00	0	0.000
1198	A	0	0	0.00	0	0.000
1199	A	0	0	0.00	0	0.000
1200	A	0	0	0.00	0	0.000
1201	A	8	10	1.00	23	0.435
1202	A	0	0	0.00	0	0.000
1203	A	6	6	1.00	21	0.286
1204	A	0	0	0.00	0	0.000
1205	A	0	0	0.00	0	0.000
1206	A	7	6	1.00	23	0.261
1207	A	0	0	0.00	0	0.000
1208	A	9	9	1.00	23	0.391
1209	A	7	9	1.00	23	0.391
1210	A	0	0	0.00	0	0.000
1211	A	3	3	1.00	21	0.143
1212	A	5	6	1.00	20	0.300
1213	A	0	0	0.00	0	0.000
1214	A	8	8	1.00	23	0.348
1215	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1216	A	14	10	1.00	23	0.435
1217	A	0	0	0.00	0	0.000
1218	A	6	7	1.00	23	0.304
1219	A	5	6	1.00	23	0.261
1220	A	4	4	1.00	21	0.190
1221	A	7	9	1.00	20	0.450
1222	A	0	0	0.00	0	0.000
1223	A	13	12	1.00	23	0.522
1224	A	0	0	0.00	0	0.000
1225	A	18	13	1.00	23	0.565
1226	A	8	9	1.00	16	0.562
1227	A	8	9	1.00	16	0.562
1228	A	4	4	0.99	21	0.190
1229	A	4	4	0.98	21	0.190
1230	A	3	4	1.00	19	0.210
1231	A	0	0	0.00	0	0.000
1232	A	0	0	0.00	0	0.000
1233	A	0	0	0.00	0	0.000
1234	A	0	0	0.00	0	0.000
1235	A	0	0	0.00	0	0.000
1236	A	0	0	0.00	0	0.000
1237	A	0	0	0.00	0	0.000
1238	A	0	0	0.00	0	0.000
1239	A	0	0	0.00	0	0.000
1240	A	0	0	0.00	0	0.000
1241	A	4	5	1.00	25	0.200
1242	A	0	0	0.00	0	0.000
1243	A	8	9	1.00	25	0.360
1244	A	0	0	0.00	0	0.000
1245	A	10	9	1.00	25	0.360
1246	A	0	0	0.00	0	0.000
1247	A	29	8	1.00	21	0.381
1248	A	25	10	1.00	21	0.476
1249	A	19	8	1.00	19	0.421
1250	A	16	10	1.00	18	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1251	A	14	10	1.00	21	0.476
1252	A	11	10	1.00	21	0.476
1253	A	16	12	1.00	21	0.571
1254	A	50	8	1.00	23	0.348
1255	A	44	10	1.00	23	0.435
1256	A	35	8	1.00	21	0.381
1257	A	30	11	1.00	20	0.550
1258	A	25	12	1.00	23	0.522
1259	A	20	13	1.00	23	0.565
1260	A	22	15	1.00	23	0.652
1261	A	11	7	1.00	23	0.304
1262	A	10	8	1.00	23	0.348
1263	A	4	2	1.00	21	0.095
1264	A	4	2	1.00	20	0.100
1265	A	12	7	1.00	23	0.304
1266	A	9	7	1.00	23	0.304
1267	A	21	13	1.00	23	0.565
1268	A	33	12	1.00	23	0.522
1269	A	38	12	1.00	23	0.522
1270	A	27	10	1.00	21	0.476
1271	A	32	11	1.00	20	0.550
1272	A	39	16	1.00	23	0.696
1273	A	42	15	1.00	23	0.652
1274	A	47	22	1.00	23	0.956
1275	A	24	14	1.00	12	1.167
1276	A	14	12	1.00	12	1.000
1277	A	19	12	1.00	12	1.000
1278	A	7	8	1.00	10	0.800
1279	A	8	8	1.00	9	0.889
1280	A	12	7	1.00	12	0.583
1281	A	8	12	1.00	12	1.000
1282	A	6	6	1.00	12	0.500
1283	A	18	15	1.00	12	1.250
1284	A	12	8	1.00	12	0.667
1285	A	26	15	1.00	12	1.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1286	A	26	15	1.00	26	0.577
1287	A	14	11	1.00	26	0.423
1288	A	21	15	1.00	26	0.577
1289	A	7	7	1.00	24	0.292
1290	A	9	8	1.00	23	0.348
1291	A	18	9	1.00	26	0.346
1292	A	6	6	1.00	26	0.231
1293	A	10	9	1.00	26	0.346
1294	A	15	14	1.00	26	0.538
1295	A	15	10	1.00	26	0.385
1296	A	24	16	1.00	26	0.615
1297	A	21	16	1.00	22	0.727
1298	A	28	12	1.00	21	0.571
1299	A	0	0	0.00	0	0.000
1300	A	28	14	1.00	24	0.583
1301	A	22	18	1.00	24	0.750

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(d+icdx)(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=117

$$\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4-\frac{bd\text{ArcTan}(cx)}{4c^4}+\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))+\frac{1}{5}icdx^5(a+b\text{ArcTan}(cx))-\frac{ibd\log(1}{10}$$

```
[Out] 1/4*b*d*x/c^3+1/10*I*b*d*x^2/c^2-1/12*b*d*x^3/c-1/20*I*b*d*x^4-1/4*b*d*arct
an(c*x)/c^4+1/4*d*x^4*(a+b*arctan(c*x))+1/5*I*c*d*x^5*(a+b*arctan(c*x))-1/1
0*I*b*d*ln(c^2*x^2+1)/c^4
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {45, 4992, 12, 815, 649, 209, 266}

$$\frac{1}{5}icdx^5(a+b\text{ArcTan}(cx))+\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))-\frac{bd\text{ArcTan}(cx)}{4c^4}+\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{ibd\log\left(c^2x^2+1\right)}{10c^4}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) + ((I/10)*b*d*x^2)/c^2 - (b*d*x^3)/(12*c) - (I/20)*b*d*x^4
- (b*d*ArcTan[c*x])/(4*c^4) + (d*x^4*(a + b*ArcTan[c*x]))/4 + (I/5)*c*d*x^5
*(a + b*ArcTan[c*x]) - ((I/10)*b*d*Log[1 + c^2*x^2])/c^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^4(5+\\ &= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\frac{x^4(5\\ &= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\Big(-\\ &= \frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\big(\\ &= \frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}icdx^5\big(\\ &= \frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4-\frac{bd\tan^{-1}(cx)}{4c^4}+\frac{1}{4}dx^4\big(a+b\tan^{-1}(\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.84

$$\frac{d(3ac^4x^4(5+4icx)+bcx(15+6icx-5c^2x^2-3ic^3x^3)+3b(-5+5c^4x^4+4ic^5x^5)\,\mathrm{ArcTan}(cx)-6ib\log\left(1+c^2x^2\right))}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

[Out] (d*(3*a*c^4*x^4*(5 + (4*I)*c*x) + b*c*x*(15 + (6*I)*c*x - 5*c^2*x^2 - (3*I)
*c^3*x^3) + 3*b*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5)*ArcTan[c*x] - (6*I)*b*Log[
1 + c^2*x^2]))/(60*c^4)

Maple [A]
time = 0.12, size = 117, normalized size = 1.00

method	result
derivativedivides	$\frac{da\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+\frac{idb\arctan(cx)c^5x^5}{5}+\frac{db\arctan(cx)c^4x^4}{4}+\frac{dbcx}{4}-\frac{idb\,c^4x^4}{20}-\frac{db\,c^3x^3}{12}+\frac{idb\,c^2x^2}{10}-\frac{idb\ln\left(c^2x^2+1\right)}{10}-\frac{db\,\mathrm{arcta}}{4}}{c^4}$
default	$\frac{da\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+\frac{idb\arctan(cx)c^5x^5}{5}+\frac{db\arctan(cx)c^4x^4}{4}+\frac{dbcx}{4}-\frac{idb\,c^4x^4}{20}-\frac{db\,c^3x^3}{12}+\frac{idb\,c^2x^2}{10}-\frac{idb\ln\left(c^2x^2+1\right)}{10}-\frac{db\,\mathrm{arcta}}{4}}{c^4}$
risch	$\frac{db\left(4c\,x^5-5ix^4\right)\ln(icx+1)}{40}-\frac{dcb\,x^5\ln(-icx+1)}{10}+\frac{idca\,x^5}{5}+\frac{idb\,x^4\ln(-icx+1)}{8}+\frac{da\,x^4}{4}-\frac{ibd\,x^4}{20}-\frac{bd\,x^3}{12c}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^4*(d*a*(1/5*I*c^5*x^5+1/4*c^4*x^4)+1/5*I*d*b*arctan(c*x)*c^5*x^5+1/4*d*
b*arctan(c*x)*c^4*x^4+1/4*d*b*c*x-1/20*I*d*b*c^4*x^4-1/12*d*b*c^3*x^3+1/10*
I*d*b*c^2*x^2-1/10*I*d*b*ln(c^2*x^2+1)-1/4*d*b*arctan(c*x))

Maxima [A]
time = 0.48, size = 109, normalized size = 0.93

$$\frac{1}{5}i\,acdx^5+\frac{1}{4}\,adx^4+\frac{1}{20}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")

[Out] 1/5*I*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2
*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d + 1/12*(3*x^4*arctan(c*x) - c*((
c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d

Fricas [A]
time = 4.12, size = 124, normalized size = 1.06

$$\frac{24i\,ac^5dx^5+6\,(5\,a-i\,b)c^4dx^4-10\,bc^3dx^3+12i\,bc^2dx^2+30\,bcdx-27i\,bd\log\left(\frac{cx+i}{c}\right)+3i\,bd\log\left(\frac{cx-i}{c}\right)-3\left(4\,bc^5dx^5-5i\,bc^4dx^4\right)\log\left(-\frac{cx+i}{cx-i}\right)}{120\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] 1/120*(24*I*a*c^5*d*x^5 + 6*(5*a - I*b)*c^4*d*x^4 - 10*b*c^3*d*x^3 + 12*I*b
*c^2*d*x^2 + 30*b*c*d*x - 27*I*b*d*log((c*x + I)/c) + 3*I*b*d*log((c*x - I)
/c) - 3*(4*b*c^5*d*x^5 - 5*I*b*c^4*d*x^4)*log(-(c*x + I)/(c*x - I)))/c^4

Sympy [A]
time = 1.72, size = 184, normalized size = 1.57

$$\frac{iacdx^5}{5}-\frac{bdx^3}{12c}+\frac{ibdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{bd\left(\frac{i\log\left(25bcdx-25ibd\right)}{40}-\frac{11i\log\left(25bcdx+25ibd\right)}{60}\right)}{c^4}+x^4\left(\frac{ad}{4}-\frac{ibd}{20}\right)+\left(\frac{bcdx^5}{10}-\frac{ibdx^4}{8}\right)\log\left(icx+1\right)+\frac{\left(-12bc^5dx^5+15ibc^4dx^4-5ibd\right)\log\left(-icx+1\right)}{120c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**5/5 - b*d*x**3/(12*c) + I*b*d*x**2/(10*c**2) + b*d*x/(4*c**3) +
b*d*(I*log(25*b*c*d*x - 25*I*b*d)/40 - 11*I*log(25*b*c*d*x + 25*I*b*d)/60)/
c**4 + x**4*(a*d/4 - I*b*d/20) + (b*c*d*x**5/10 - I*b*d*x**4/8)*log(I*c*x +
 1) + (-12*b*c**5*d*x**5 + 15*I*b*c**4*d*x**4 - 5*I*b*d)*log(-I*c*x + 1)/(1
20*c**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.74, size = 109, normalized size = 0.93

$$-\frac{\frac{d\,\left(15\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,6{}\mathrm{i}\right)}{60}-\frac{b\,c\,d\,x}{4}+\frac{b\,c^3\,d\,x^3}{12}-\frac{b\,c^2\,d\,x^2\,1{}\mathrm{i}}{10}}{c^4}+\frac{d\,\left(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}\left(c\,x\right)-b\,x^4\,3{}\mathrm{i}\right)}{60}+\frac{c\,d\,\left(a\,x^5\,12{}\mathrm{i}+b\,x^5\,\mathrm{atan}\left(c\,x\right)\,12{}\mathrm{i}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

```
[Out] (d*(15*a*x^4 - b*x^4*3i + 15*b*x^4*atan(c*x)))/60 - ((d*(15*b*atan(c*x) + b
*log(c^2*x^2 + 1)*6i))/60 - (b*c*d*x)/4 - (b*c^2*d*x^2*1i)/10 + (b*c^3*d*x^
3)/12)/c^4 + (c*d*(a*x^5*12i + b*x^5*atan(c*x)*12i))/60
```

3.2 $\int x^2(d+icdx)(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=105

$$\frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3-\frac{ibd\text{ArcTan}(cx)}{4c^3}+\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))+\frac{1}{4}icdx^4(a+b\text{ArcTan}(cx))+\frac{bd\log\left(1+c^2x^2\right)}{6c^3}$$

```
[Out] 1/4*I*b*d*x/c^2-1/6*b*d*x^2/c-1/12*I*b*d*x^3-1/4*I*b*d*arctan(c*x)/c^3+1/3*
d*x^3*(a+b*arctan(c*x))+1/4*I*c*d*x^4*(a+b*arctan(c*x))+1/6*b*d*ln(c^2*x^2+
1)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {45, 4992, 12, 815, 649, 209, 266}

$$\frac{1}{4}icdx^4(a+b\text{ArcTan}(cx))+\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))-\frac{ibd\text{ArcTan}(cx)}{4c^3}+\frac{ibdx}{4c^2}+\frac{bd\log\left(c^2x^2+1\right)}{6c^3}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((I/4)*b*d*x)/c^2 - (b*d*x^2)/(6*c) - (I/12)*b*d*x^3 - ((I/4)*b*d*ArcTan[c*
x])/c^3 + (d*x^3*(a + b*ArcTan[c*x]))/3 + (I/4)*c*d*x^4*(a + b*ArcTan[c*x])
 + (b*d*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^3(4+}{12(1+}\\ &= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^3(4}{1}\\ &= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\Big(-\\ &= \frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan\right.\\ &= \frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}icdx^4\left(a+b\tan\right.\\ &= \frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3-\frac{ibd\tan^{-1}(cx)}{4c^3}+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.84

$$\frac{d(ac^3x^3(4+3icx)+bcx(3i-2cx-ic^2x^2)+b(-3i+4c^3x^3+3ic^4x^4)\,\mathrm{ArcTan}(cx)+2b\log(1+c^2x^2))}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
[Out] (d*(a*c^3*x^3*(4 + (3*I)*c*x) + b*c*x*(3*I - 2*c*x - I*c^2*x^2) + b*(-3*I +
 4*c^3*x^3 + (3*I)*c^4*x^4)*ArcTan[c*x] + 2*b*Log[1 + c^2*x^2]))/(12*c^3)
```

Maple [A]
time = 0.08, size = 107, normalized size = 1.02

method	result
derivativedivides	$\frac{da\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+\frac{idb\arctan(cx)c^4x^4}{4}+\frac{db\arctan(cx)c^3x^3}{3}+\frac{idbcx}{4}-\frac{idb\,c^3x^3}{12}-\frac{db\,c^2x^2}{6}+\frac{bd\ln\left(c^2x^2+1\right)}{6}-\frac{idb\arctan(cx)}{4}}{c^3}$
default	$\frac{da\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+\frac{idb\arctan(cx)c^4x^4}{4}+\frac{db\arctan(cx)c^3x^3}{3}+\frac{idbcx}{4}-\frac{idb\,c^3x^3}{12}-\frac{db\,c^2x^2}{6}+\frac{bd\ln\left(c^2x^2+1\right)}{6}-\frac{idb\arctan(cx)}{4}}{c^3}$
risch	$\frac{db\left(3c\,x^4-4ix^3\right)\ln(icx+1)}{24}-\frac{dcb\,x^4\ln(-icx+1)}{8}+\frac{idca\,x^4}{4}+\frac{idb\,x^3\ln(-icx+1)}{6}-\frac{ibd\,x^3}{12}+\frac{da\,x^3}{3}-\frac{bd\,x^2}{6c}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^3*(d*a*(1/4*I*c^4*x^4+1/3*c^3*x^3)+1/4*I*d*b*arctan(c*x)*c^4*x^4+1/3*d*
b*arctan(c*x)*c^3*x^3+1/4*I*d*b*c*x-1/12*I*d*b*c^3*x^3-1/6*d*b*c^2*x^2+1/6*
b*d*ln(c^2*x^2+1)-1/4*I*d*b*arctan(c*x))
```

Maxima [A]
time = 0.46, size = 99, normalized size = 0.94

$$\frac{1}{4}i\,acdx^4+\frac{1}{3}\,adx^3+\frac{1}{12}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/4*I*a*c*d*x^4 + 1/3*a*d*x^3 + 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 -
 log(c^2*x^2 + 1)/c^4))*b*d
```

Fricas [A]
time = 2.01, size = 113, normalized size = 1.08

$$\frac{6i\,ac^4dx^4+2\,(4\,a-i\,b)c^3dx^3-4\,bc^2dx^2+6i\,bcdx+7\,bd\log\left(\frac{cx+i}{c}\right)+bd\log\left(\frac{cx-i}{c}\right)-\left(3\,bc^4dx^4-4i\,bc^3dx^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] 1/24*(6*I*a*c^4*d*x^4 + 2*(4*a - I*b)*c^3*d*x^3 - 4*b*c^2*d*x^2 + 6*I*b*c*d
*x + 7*b*d*log((c*x + I)/c) + b*d*log((c*x - I)/c) - (3*b*c^4*d*x^4 - 4*I*b
*c^3*d*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A]
time = 1.52, size = 167, normalized size = 1.59

$$\frac{iacdx^4}{4} - \frac{bdx^2}{6c} + \frac{ibdx}{4c^2} + \frac{bd\left(\frac{\log(11bcdx-11ibd)}{24} + \frac{9\log(11bcdx+11ibd)}{40}\right)}{c^3} + x^3\left(\frac{ad}{3} - \frac{ibd}{12}\right) + \left(\frac{bcdx^4}{8} - \frac{ibdx^3}{6}\right)\log(icx+1) + \frac{(-15bc^4dx^4 + 20ibc^3dx^3 + 8bd)\log(-icx+1)}{120c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**4/4 - b*d*x**2/(6*c) + I*b*d*x/(4*c**2) + b*d*(log(11*b*c*d*x -
11*I*b*d)/24 + 9*log(11*b*c*d*x + 11*I*b*d)/40)/c**3 + x**3*(a*d/3 - I*b*d/
12) + (b*c*d*x**4/8 - I*b*d*x**3/6)*log(I*c*x + 1) + (-15*b*c**4*d*x**4 + 2
0*I*b*c**3*d*x**3 + 8*b*d)*log(-I*c*x + 1)/(120*c**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.71, size = 99, normalized size = 0.94

$$-\frac{\frac{d\,(-2\,b\,\ln(c^2\,x^2+1)+b\,\mathrm{atan}(c\,x)\,3\mathrm{i})}{12} + \frac{b\,c^2\,d\,x^2}{6} - \frac{b\,c\,d\,x\,1\mathrm{i}}{4}}{c^3} + \frac{d\,(4\,a\,x^3 + 4\,b\,x^3\,\mathrm{atan}(c\,x) - b\,x^3\,1\mathrm{i})}{12} + \frac{c\,d\,(a\,x^4\,3\mathrm{i} + b\,x^4\,\mathrm{atan}(c\,x)\,3\mathrm{i})}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

```
[Out] (d*(4*a*x^3 - b*x^3*1i + 4*b*x^3*atan(c*x)))/12 - ((d*(b*atan(c*x)*3i - 2*b
*log(c^2*x^2 + 1)))/12 - (b*c*d*x*1i)/4 + (b*c^2*d*x^2)/6)/c^3 + (c*d*(a*x^
4*3i + b*x^4*atan(c*x)*3i))/12
```

3.3 $\int x(d+icdx)(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=91

$$-\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{bd\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))+\frac{1}{3}icdx^3(a+b\text{ArcTan}(cx))+\frac{ibd\log\left(1+c^2x^2\right)}{6c^2}$$

```
[Out] -1/2*b*d*x/c-1/6*I*b*d*x^2+1/2*b*d*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan(c*
x))+1/3*I*c*d*x^3*(a+b*arctan(c*x))+1/6*I*b*d*ln(c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {45, 4992, 12, 815, 649, 209, 266}

$$\frac{1}{3}icdx^3(a+b\text{ArcTan}(cx))+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))+\frac{bd\text{ArcTan}(cx)}{2c^2}+\frac{ibd\log\left(c^2x^2+1\right)}{6c^2}-\frac{bdx}{2c}-\frac{1}{6}ibdx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*d*x)/c - (I/6)*b*d*x^2 + (b*d*ArcTan[c*x])/(2*c^2) + (d*x^2*(a + b*
ArcTan[c*x]))/2 + (I/3)*c*d*x^3*(a + b*ArcTan[c*x]) + ((I/6)*b*d*Log[1 + c^
2*x^2])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{dx^2(3+2}{6+6c^2}\\
&= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bcd)\int\frac{x^2(3+2}{6+6c}\\
&= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)-(bcd)\int\left(\frac{1}{2c^2}+\right.\\
&= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)\\
&= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)\\
&= -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{bd\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}icdx^3(a
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.84

$$\frac{d\left(cx(b(-3-icx)+acx(3+2icx))+b(3+3c^2x^2+2ic^3x^3)\,\mathrm{ArcTan}(cx)+ib\log\left(1+c^2x^2\right)\right)}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]

[Out] (d*(c*x*(b*(-3 - I*c*x) + a*c*x*(3 + (2*I)*c*x)) + b*(3 + 3*c^2*x^2 + (2*I)*c^3*x^3)*ArcTan[c*x] + I*b*Log[1 + c^2*x^2]))/(6*c^2)

Maple [A]
time = 0.10, size = 96, normalized size = 1.05

method	result
derivativedivides	$\frac{da\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+\frac{idb\arctan(cx)c^3x^3}{3}+\frac{db\arctan(cx)c^2x^2}{2}-\frac{idb\,c^2x^2}{6}-\frac{dbcx}{2}+\frac{idb\ln\left(c^2x^2+1\right)}{6}+\frac{db\arctan(cx)}{2}}{c^2}$
default	$\frac{da\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+\frac{idb\arctan(cx)c^3x^3}{3}+\frac{db\arctan(cx)c^2x^2}{2}-\frac{idb\,c^2x^2}{6}-\frac{dbcx}{2}+\frac{idb\ln\left(c^2x^2+1\right)}{6}+\frac{db\arctan(cx)}{2}}{c^2}$
risch	$\frac{db(2c\,x^3-3ix^2)\ln(icx+1)}{12}-\frac{dcb\,x^3\ln(-icx+1)}{6}+\frac{idca\,x^3}{3}+\frac{idb\,x^2\ln(-icx+1)}{4}+\frac{da\,x^2}{2}-\frac{ibd\,x^2}{6}-\frac{bdx}{2c}+\frac{bc}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^2*(d*a*(1/3*I*c^3*x^3+1/2*c^2*x^2)+1/3*I*d*b*arctan(c*x)*c^3*x^3+1/2*d*b*arctan(c*x)*c^2*x^2-1/6*I*d*b*c^2*x^2-1/2*d*b*c*x+1/6*I*d*b*ln(c^2*x^2+1)+1/2*d*b*arctan(c*x))

Maxima [A]
time = 0.47, size = 88, normalized size = 0.97

$$\frac{1}{3}i\,acdx^3+\frac{1}{6}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")

[Out] 1/3*I*a*c*d*x^3 + 1/6*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d

Fricas [A]
time = 1.59, size = 104, normalized size = 1.14

$$\frac{4i\,ac^3dx^3+2\,(3\,a-i\,b)c^2dx^2-6\,bcdx+5i\,bd\log\left(\frac{cx+i}{c}\right)-i\,bd\log\left(\frac{cx-i}{c}\right)-\left(2\,bc^3dx^3-3i\,bc^2dx^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] 1/12*(4*I*a*c^3*d*x^3 + 2*(3*a - I*b)*c^2*d*x^2 - 6*b*c*d*x + 5*I*b*d*log((c*x + I)/c) - I*b*d*log((c*x - I)/c) - (2*b*c^3*d*x^3 - 3*I*b*c^2*d*x^2)*log(-(c*x + I)/(c*x - I)))/c^2

Sympy [A]
time = 1.42, size = 158, normalized size = 1.74

$$\frac{iacdx^3}{3} - \frac{bdx}{2c} + \frac{bd\left(-\frac{i\log(9bcdx-9ibd)}{12} + \frac{7i\log(9bcdx+9ibd)}{24}\right)}{c^2} + x^2\left(\frac{ad}{2} - \frac{ibd}{6}\right) + \left(\frac{bcdx^3}{6} - \frac{ibdx^2}{4}\right)\log(icx+1) + \frac{(-4bc^3dx^3 + 6ibc^2dx^2 + 3ibd)\log(-icx+1)}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**3/3 - b*d*x/(2*c) + b*d*(-I*log(9*b*c*d*x - 9*I*b*d)/12 + 7*I*lo
g(9*b*c*d*x + 9*I*b*d)/24)/c**2 + x**2*(a*d/2 - I*b*d/6) + (b*c*d*x**3/6 -
I*b*d*x**2/4)*log(I*c*x + 1) + (-4*b*c**3*d*x**3 + 6*I*b*c**2*d*x**2 + 3*I*
b*d)*log(-I*c*x + 1)/(24*c**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.38, size = 87, normalized size = 0.96

$$\frac{d\,(3\,a\,x^2 + 3\,b\,x^2\,\mathrm{atan}(c\,x) - b\,x^2\,1\mathrm{i})}{6} + \frac{\frac{d\,(3\,b\,\mathrm{atan}(c\,x) + b\,\ln(c^2\,x^2+1)\,1\mathrm{i})}{6} - \frac{b\,c\,d\,x}{2}}{c^2} + \frac{c\,d\,(a\,x^3\,2\mathrm{i} + b\,x^3\,\mathrm{atan}(c\,x)\,2\mathrm{i})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

```
[Out] (d*(3*a*x^2 - b*x^2*1i + 3*b*x^2*atan(c*x)))/6 + ((d*(3*b*atan(c*x) + b*log
(c^2*x^2 + 1)*1i))/6 - (b*c*d*x)/2)/c^2 + (c*d*(a*x^3*2i + b*x^3*atan(c*x)*
2i))/6
```

3.4 $\int (d+icdx)(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=53

$$-\frac{1}{2}ibdx-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))}{2c}-\frac{bd\log(i+cx)}{c}$$

```
[Out] -1/2*I*b*d*x-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))/c-b*d*ln(c*x+I)/c
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4972, 641, 45}

$$-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))}{2c}-\frac{bd\log(cx+i)}{c}-\frac{1}{2}ibdx$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-1/2*I)*b*d*x - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/c - (b*d*Log[I
 + c*x])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)\,dx = -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\frac{(d+icdx)^2}{1+c^2x^2}\,dx}{2d}$$
$$= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\frac{d+icdx}{\frac{1}{d}-\frac{icx}{d}}\,dx}{2d}$$
$$= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}+\frac{(ib)\int\left(-d^2+\frac{2id^2}{i+cx}\right)\,dx}{2d}$$
$$= -\frac{1}{2}ibdx-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)}{2c}-\frac{bd\log(i+cx)}{c}$$

Mathematica [A]
time = 0.01, size = 84, normalized size = 1.58

$$adx-\frac{1}{2}ibdx+\frac{1}{2}iacdx^2+\frac{ibd\text{ArcTan}(cx)}{2c}+bdx\text{ArcTan}(cx)+\frac{1}{2}ibcdx^2\text{ArcTan}(cx)-\frac{bd\log\left(1+c^2x^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] a*d*x - (I/2)*b*d*x + (I/2)*a*c*d*x^2 + ((I/2)*b*d*ArcTan[c*x])/c + b*d*x*A
rcTan[c*x] + (I/2)*b*c*d*x^2*ArcTan[c*x] - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Maple [A]
time = 0.08, size = 79, normalized size = 1.49

method	result
derivativedivides	$\frac{-ida\left(-\frac{1}{2}c^2x^2+icx\right)+\frac{idb\arctan(cx)c^2x^2}{2}+b\arctan(cx)dcx-\frac{bd\ln\left(c^2x^2+1\right)}{2}+\frac{idb\arctan(cx)}{2}-\frac{idbcx}{2}}{c}$
default	$\frac{-ida\left(-\frac{1}{2}c^2x^2+icx\right)+\frac{idb\arctan(cx)c^2x^2}{2}+b\arctan(cx)dcx-\frac{bd\ln\left(c^2x^2+1\right)}{2}+\frac{idb\arctan(cx)}{2}-\frac{idbcx}{2}}{c}$
risch	$\frac{db\left(c\,x^2-2ix\right)\ln(icx+1)}{4}-\frac{dcb\,x^2\ln(-icx+1)}{4}+\frac{iacd\,x^2}{2}+\frac{ibdx\ln(-icx+1)}{2}-\frac{ibdx}{2}+\frac{id\arctan(cx)b}{2c}+adx-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-I*d*a*(-1/2*c^2*x^2+I*c*x)+1/2*I*d*b*arctan(c*x)*c^2*x^2+b*arctan(c*x
)*d*c*x-1/2*b*d*ln(c^2*x^2+1)+1/2*I*d*b*arctan(c*x)-1/2*I*d*b*c*x)
```

Maxima [A]
time = 0.47, size = 73, normalized size = 1.38

$$\frac{1}{2}i\,acdx^2+\frac{1}{2}i\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bcd+adx+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*a*c*d*x^2 + 1/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c
*d + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(41) = 82$.
time = 3.32, size = 89, normalized size = 1.68

$$\frac{2i\,ac^2dx^2 + 2\,(2\,a - i\,b)cdx - 3\,bd\log\left(\frac{cx+i}{c}\right) - bd\log\left(\frac{cx-i}{c}\right) - (bc^2dx^2 - 2i\,bcdx)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*a*c^2*d*x^2 + 2*(2*a - I*b)*c*d*x - 3*b*d*log((c*x + I)/c) - b*d*l
og((c*x - I)/c) - (b*c^2*d*x^2 - 2*I*b*c*d*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(44) = 88$.
time = 1.25, size = 128, normalized size = 2.42

$$\frac{iacdx^2}{2} + \frac{bd\left(-\frac{\log(bcdx-ibd)}{4} - \frac{5\log(bcdx+ibd)}{12}\right)}{c} + x\left(ad - \frac{ibd}{2}\right) + \left(\frac{bcdx^2}{4} - \frac{ibdx}{2}\right)\log(icx+1) + \frac{(-3bc^2dx^2 + 6ibcdx - 4bd)\log(-icx+1)}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x)),x)
```

```
[Out] I*a*c*d*x**2/2 + b*d*(-log(b*c*d*x - I*b*d)/4 - 5*log(b*c*d*x + I*b*d)/12)/
c + x*(a*d - I*b*d/2) + (b*c*d*x**2/4 - I*b*d*x/2)*log(I*c*x + 1) + (-3*b*c
**2*d*x**2 + 6*I*b*c*d*x - 4*b*d)*log(-I*c*x + 1)/(12*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.32, size = 73, normalized size = 1.38

$$\frac{d\,(2\,a\,x + 2\,b\,x\,\mathrm{atan}(c\,x) - b\,x\,1\mathrm{i})}{2} + \frac{c\,d\,(a\,x^2\,1\mathrm{i} + b\,x^2\,\mathrm{atan}(c\,x)\,1\mathrm{i})}{2} + \frac{d\,(-b\,\ln(c^2\,x^2 + 1) + b\,\mathrm{atan}(c\,x)\,1\mathrm{i})}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + c*d*x*1i),x)
```

```
[Out] (d*(2*a*x - b*x*1i + 2*b*x*atan(c*x)))/2 + (c*d*(a*x^2*1i + b*x^2*atan(c*x)
*1i))/2 + (d*(b*atan(c*x)*1i - b*log(c^2*x^2 + 1)))/(2*c)
```

3.5 $\int \frac{(d+icdx)(a+b\mathbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=76

$$iacdx+ibcdx\text{ArcTan}(cx)+ad\log(x)-\frac{1}{2}ibd\log\left(1+c^2x^2\right)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)$$

```
[Out] I*a*c*d*x+I*b*c*d*x*arctan(c*x)+a*d*ln(x)-1/2*I*b*d*ln(c^2*x^2+1)+1/2*I*b*d
*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4996, 4930, 266, 4940, 2438}

$$iacdx+ad\log(x)+ibcdx\text{ArcTan}(cx)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+\frac{1}{2}ibd\text{Li}_2(-icx)-\frac{1}{2}ibd\text{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2]
 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x}\,dx &= \int \left(icd\left(a+b\tan^{-1}(cx)\right)+\frac{d\left(a+b\tan^{-1}(cx)\right)}{x}\right)dx\\ &= d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+(icd)\int \left(a+b\tan^{-1}(cx)\right)dx\\ &= iacdx+ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\int \frac{\log(1+icx)}{x}\\ &= iacdx+ibcdx\tan^{-1}(cx)+ad\log(x)+\frac{1}{2}ibd\text{Li}_2(-icx)-\frac{1}{2}ibd\text{Li}_2(icx)-\\ &= iacdx+ibcdx\tan^{-1}(cx)+ad\log(x)-\frac{1}{2}ibd\log\left(1+c^2x^2\right)+\frac{1}{2}ibd\text{Li}_2(-\end{aligned}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 1.00

$$iacdx+ibcdx\text{ArcTan}(cx)+ad\log(x)-\frac{1}{2}ibd\log\left(1+c^2x^2\right)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2]
 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Maple [A]
time = 0.06, size = 113, normalized size = 1.49

method	result
risch	$\frac{\ln(icx+1)xbcd}{2}+\frac{idb\,\text{dilog}(icx+1)}{2}-\frac{idb\ln(c^2x^2+1)}{2}+ibd-\frac{d\ln(-icx+1)xbc}{2}+iacdx-\frac{idb\,\text{dilog}(-icx+1)}{2}$
derivativedivides	$iacdx+\ln(cx)\,ad+ibcdx\arctan(cx)+db\ln(cx)\arctan(cx)+\frac{idb\ln(cx)\ln(icx+1)}{2}-\frac{idb\ln(c}{}$
default	$iacdx+\ln(cx)\,ad+ibcdx\arctan(cx)+db\ln(cx)\arctan(cx)+\frac{idb\ln(cx)\ln(icx+1)}{2}-\frac{idb\ln(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] I*a*c*d*x+ln(c*x)*a*d+I*b*c*d*x*arctan(c*x)+d*b*ln(c*x)*arctan(c*x)+1/2*I*d
*b*ln(c*x)*ln(1+I*c*x)-1/2*I*d*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d*b*dilog(1+I*c*
x)-1/2*I*d*b*dilog(1-I*c*x)-1/2*I*b*d*ln(c^2*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] I*a*c*d*x + 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d + b*d*integrat
e(arctan(c*x)/x, x) + a*d*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I)))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$id\left(\int ac\,dx + \int \left(-\frac{ia}{x}\right)\,dx + \int bc\operatorname{atan}(cx)\,dx + \int \left(-\frac{ib\operatorname{atan}(cx)}{x}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x,x)
```

```
[Out] I*d*(Integral(a*c, x) + Integral(-I*a/x, x) + Integral(b*c*atan(c*x), x) +
Integral(-I*b*atan(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.62, size = 63, normalized size = 0.83

$$-\frac{b\,d\,\left(\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}-c\,x\,\mathrm{atan}(c\,x)\,2\mathrm{i}\right)}{2}+a\,d\,\left(\ln\left(x\right)+c\,x\,1\mathrm{i}\right)-\frac{b\,d\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i))/x,x)
```

```
[Out] a*d*(log(x) + c*x*1i) - (b*d*(log(c^2*x^2 + 1)*1i - c*x*atan(c*x)*2i))/2 -
(b*d*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

3.6 $\int \frac{(d+icdx)(a+b\text{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{d(a+b\text{ArcTan}(cx))}{x}+iacd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1+c^2x^2\right)-\frac{1}{2}bcd\text{PolyLog}(2,-icx)+\frac{1}{2}bcd\text{PolyLog}$$

```
[Out] -d*(a+b*arctan(c*x))/x+I*a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(c^2*x^2+1)-1/
2*b*c*d*polylog(2,-I*c*x)+1/2*b*c*d*polylog(2,I*c*x)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {4996, 4946, 272, 36, 29, 31, 4940, 2438}

$$-\frac{d(a+b\text{ArcTan}(cx))}{x}+iacd\log(x)-\frac{1}{2}bcd\log\left(c^2x^2+1\right)-\frac{1}{2}bcd\text{Li}_2(-icx)+\frac{1}{2}bcd\text{Li}_2(icx)+bcd\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + I*a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
 + c^2*x^2])/2 - (b*c*d*PolyLog[2, (-I)*c*x])/2 + (b*c*d*PolyLog[2, I*c*x])
/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx &= \int \left(\frac{d(a+b\tan^{-1}(cx))}{x^2}+\frac{icd(a+b\tan^{-1}(cx))}{x}\right)dx \\
&= d\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx+(icd)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx \\
&= -\frac{d(a+b\tan^{-1}(cx))}{x}+iacd\log(x)-\frac{1}{2}(bcd)\int \frac{\log(1-icx)}{x}\,dx+\frac{1}{2}(bcd \\
&= -\frac{d(a+b\tan^{-1}(cx))}{x}+iacd\log(x)-\frac{1}{2}bcd\text{Li}_2(-icx)+\frac{1}{2}bcd\text{Li}_2(icx)+ \\
&= -\frac{d(a+b\tan^{-1}(cx))}{x}+iacd\log(x)-\frac{1}{2}bcd\text{Li}_2(-icx)+\frac{1}{2}bcd\text{Li}_2(icx)+ \\
&= -\frac{d(a+b\tan^{-1}(cx))}{x}+iacd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1+c^2x^2\right)-
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 1.01

$$-\frac{ad}{x}+iacd\log(x)+bcd\left(-\frac{\text{ArcTan}(cx)}{cx}+\log(cx)-\frac{1}{2}\log\left(1+c^2x^2\right)\right)-\frac{1}{2}bcd(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + I*a*c*d*Log[x] + b*c*d*(-(ArcTan[c*x]/(c*x)) + Log[c*x] - Log[
1 + c^2*x^2]/2) - (b*c*d*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]))/2
```

Maple [A]
time = 0.06, size = 127, normalized size = 1.65

method	result
risch	$-\frac{bdc\,\mathrm{dilog}(icx+1)}{2}+\frac{bdc\ln(icx)}{2}-\frac{bcd\ln(c^2x^2+1)}{2}+\frac{ibd\ln(icx+1)}{2x}+idca\ln(-icx)-\frac{da}{x}+\frac{dc\,\mathrm{dilog}(-i}{2}$
derivativedivides	$c\Big(-\frac{da}{cx}+ida\ln(cx)-\frac{db\arctan(cx)}{cx}+idb\arctan(cx)\ln(cx)-\frac{db\ln(cx)\ln(icx+1)}{2}+\frac{db\ln(cx)\ln(}{2}$
default	$c\Big(-\frac{da}{cx}+ida\ln(cx)-\frac{db\arctan(cx)}{cx}+idb\arctan(cx)\ln(cx)-\frac{db\ln(cx)\ln(icx+1)}{2}+\frac{db\ln(cx)\ln(}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-d*a/c/x+I*d*a*ln(c*x)-d*b*arctan(c*x)/c/x+I*d*b*arctan(c*x)*ln(c*x)-1/2
*d*b*ln(c*x)*ln(1+I*c*x)+1/2*d*b*ln(c*x)*ln(1-I*c*x)-1/2*d*b*dilog(1+I*c*x)
+1/2*d*b*dilog(1-I*c*x)-1/2*b*d*ln(c^2*x^2+1)+d*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] I*b*c*d*integrate(arctan(c*x)/x, x) + I*a*c*d*log(x) - 1/2*(c*(log(c^2*x^2
+ 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I)))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$id\left(\int\left(-\frac{ia}{x^2}\right)dx+\int\frac{ac}{x}\,dx+\int\left(-\frac{ib\operatorname{atan}(cx)}{x^2}\right)dx+\int\frac{bc\operatorname{atan}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] I*d*(Integral(-I*a/x**2, x) + Integral(a*c/x, x) + Integral(-I*b*atan(c*x)/
x**2, x) + Integral(b*c*atan(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.86, size = 93, normalized size = 1.21

$$\begin{cases} -\frac{a\,d}{x} & \text{if } c=0\\ \frac{b\,d\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+\frac{b\,c\,d\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2}+\frac{a\,d\,(-1+c\,x\,\ln(x)\,1\mathrm{i})}{x}-\frac{b\,d\,\mathrm{atan}(c\,x)}{x} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^2,x)
```

```
[Out] piecewise(c == 0, -(a*d)/x, c ~= 0, (b*d*(c^2*log(x) - (c^2*log(c^2*x^2 + 1
))/2))/c + (b*c*d*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 + (a*d*(c*x*
log(x)*1i - 1))/x - (b*d*atan(c*x))/x)
```

3.7 $\int \frac{(d+icdx)(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=65

$$-\frac{bcd}{2x}-\frac{d(1+icx)^2(a+b\text{ArcTan}(cx))}{2x^2}+ibc^2d\log(x)-ibc^2d\log(i+cx)$$

```
[Out] -1/2*b*c*d/x-1/2*d*(1+I*c*x)^2*(a+b*arctan(c*x))/x^2+I*b*c^2*d*ln(x)-I*b*c^
2*d*ln(c*x+I)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {37, 4992, 12, 78}

$$-\frac{d(1+icx)^2(a+b\text{ArcTan}(cx))}{2x^2}+ibc^2d\log(x)-ibc^2d\log(cx+i)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d)/x - (d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/(2*x^2) + I*b*c^2*d*
Log[x] - I*b*c^2*d*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)\,(a+b\tan^{-1}(cx))}{x^3}\,dx &= -\frac{d(1+icx)^2\,(a+b\tan^{-1}(cx))}{2x^2}-(bc)\int \frac{d(-i+cx)}{2x^2(i+cx)}\,dx\\ &= -\frac{d(1+icx)^2\,(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{2}(bcd)\int \frac{-i+cx}{x^2(i+cx)}\,dx\\ &= -\frac{d(1+icx)^2\,(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{2}(bcd)\int \left(-\frac{1}{x^2}-\frac{2ic}{x}+\frac{2ic^2}{i+cx}\right)\,dx\\ &= -\frac{bcd}{2x}-\frac{d(1+icx)^2\,(a+b\tan^{-1}(cx))}{2x^2}+ibc^2d\log(x)-ibc^2d\log(i+cx)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 1.17

$$-\frac{d(a+2iacx+bcx+b(1+2icx+c^2x^2)\,\text{ArcTan}(cx)-2ibc^2x^2\log(x)+ibc^2x^2\log\left(1+c^2x^2\right))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
[Out] -1/2*(d*(a + (2*I)*a*c*x + b*c*x + b*(1 + (2*I)*c*x + c^2*x^2)*ArcTan[c*x]
- (2*I)*b*c^2*x^2*Log[x] + I*b*c^2*x^2*Log[1 + c^2*x^2]))/x^2
```

Maple [A]
time = 0.13, size = 98, normalized size = 1.51

method	result
derivativedivides	$c^2\Big(da\big(-\frac{1}{2c^2x^2}-\frac{i}{cx}\big)-\frac{db\arctan(cx)}{2c^2x^2}-\frac{idb\arctan(cx)}{cx}-\frac{idb\ln(c^2x^2+1)}{2}-\frac{db\arctan(cx)}{2}-\frac{db}{2cx}+idb\ln$
default	$c^2\Big(da\big(-\frac{1}{2c^2x^2}-\frac{i}{cx}\big)-\frac{db\arctan(cx)}{2c^2x^2}-\frac{idb\arctan(cx)}{cx}-\frac{idb\ln(c^2x^2+1)}{2}-\frac{db\arctan(cx)}{2}-\frac{db}{2cx}+idb\ln$
risch	$-\frac{(2dbcx-ibd)\ln(icx+1)}{4x^2}+\frac{id(4bc^2\ln(-35cx)x^2-3bc^2\ln(-7cx-7i)x^2-bc^2\ln(5cx-5i)x^2-4acx-2ibcx\ln(-icx+1)-}{4x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d*a*(-1/2/c^2/x^2-I/c/x)-1/2*d*b*arctan(c*x)/c^2/x^2-I*d*b*arctan(c*x)
/c/x-1/2*I*b*d*ln(c^2*x^2+1)-1/2*d*b*arctan(c*x)-1/2*d*b/c/x+I*d*b*ln(c*x))
```

Maxima [A]
time = 0.47, size = 75, normalized size = 1.15

$$-\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bcd-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bd-\frac{i\,acd}{x}-\frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d - 1/2*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - I*a*c*d/x - 1/2*a*d/x^2
```

Fricas [A]
time = 2.48, size = 99, normalized size = 1.52

$$\frac{4i\,bc^2dx^2\log\left(x\right)-3i\,bc^2dx^2\log\left(\frac{cx+i}{c}\right)-i\,bc^2dx^2\log\left(\frac{cx-i}{c}\right)-2\left(2i\,a+b\right)cdx-2\,ad+\left(2\,bcdx-i\,bd\right)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*I*b*c^2*d*x^2*log(x) - 3*I*b*c^2*d*x^2*log((c*x + I)/c) - I*b*c^2*d*
x^2*log((c*x - I)/c) - 2*(2*I*a + b)*c*d*x - 2*a*d + (2*b*c*d*x - I*b*d)*lo
g(-(c*x + I)/(c*x - I)))/x^2
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(58) = 116.
time = 1.83, size = 182, normalized size = 2.80

$$ibc^2d\log\left(35b^2c^5d^2x\right)-\frac{ibc^2d\log\left(35b^2c^5d^2x-35ib^2c^4d^2\right)}{4}-\frac{3ibc^2d\log\left(35b^2c^5d^2x+35ib^2c^4d^2\right)}{4}+\frac{-ad+x(-2iacd-bcd)}{2x^2}+\frac{(-2bcdx+ibd)\log\left(icx+1\right)}{4x^2}+\frac{(2bcdx-ibd)\log\left(-icx+1\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] I*b*c**2*d*log(35*b**2*c**5*d**2*x) - I*b*c**2*d*log(35*b**2*c**5*d**2*x -
35*I*b**2*c**4*d**2)/4 - 3*I*b*c**2*d*log(35*b**2*c**5*d**2*x + 35*I*b**2*c
**4*d**2)/4 + (-a*d + x*(-2*I*a*c*d - b*c*d))/(2*x**2) + (-2*b*c*d*x + I*b*
d)*log(I*c*x + 1)/(4*x**2) + (2*b*c*d*x - I*b*d)*log(-I*c*x + 1)/(4*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 79, normalized size = 1.22

$$-\frac{\frac{d\,(a+b\,\mathrm{atan}(c\,x))}{2}+\frac{d\,x\,(a\,c\,2\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,2\mathrm{i})}{2}}{x^2}-\frac{d\,(b\,c^2\,\mathrm{atan}(c\,x)+b\,c^2\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}-b\,c^2\,\ln\left(x\right)\,2\mathrm{i})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^3,x)
```

```
[Out] - ((d*(a + b*atan(c*x)))/2 + (d*x*(a*c*2i + b*c + b*c*atan(c*x)*2i))/2)/x^2
 - (d*(b*c^2*atan(c*x) + b*c^2*log(c^2*x^2 + 1)*1i - b*c^2*log(x)*2i))/2
```

3.8 $\int \frac{(d+icdx)(a+b\mathrm{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{bcd}{6x^2}-\frac{ibc^2d}{2x}-\frac{d(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{icd(a+b\mathrm{ArcTan}(cx))}{2x^2}-\frac{1}{3}bc^3d\log(x)-\frac{1}{12}bc^3d\log(i-cx)+\frac{5}{12}bc^3d\log(i$$

```
[Out] -1/6*b*c*d/x^2-1/2*I*b*c^2*d/x-1/3*d*(a+b*arctan(c*x))/x^3-1/2*I*c*d*(a+b*a
rctan(c*x))/x^2-1/3*b*c^3*d*ln(x)-1/12*b*c^3*d*ln(I-c*x)+5/12*b*c^3*d*ln(c*
x+I)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {45, 4992, 12, 815}

$$-\frac{d(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{icd(a+b\mathrm{ArcTan}(cx))}{2x^2}-\frac{1}{3}bc^3d\log(x)-\frac{1}{12}bc^3d\log(-cx+i)+\frac{5}{12}bc^3d\log(cx+i)-\frac{ibc^2d}{2x}-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d)/x^2 - ((I/2)*b*c^2*d)/x - (d*(a + b*ArcTan[c*x]))/(3*x^3) - ((
I/2)*c*d*(a + b*ArcTan[c*x]))/x^2 - (b*c^3*d*Log[x])/3 - (b*c^3*d*Log[I - c
*x])/12 + (5*b*c^3*d*Log[I + c*x])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))}{2x^2}-(bc)\int\frac{d(-2-3icx)}{6x^3\,(1+c^2x^2)}\,dx\\ &= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{6}(bcd)\int\frac{-2-3icx}{x^3\,(1+c^2x^2)}\\ &= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{6}(bcd)\int\left(-\frac{2}{x^3}-\frac{3ic}{x^2}\right.\\ &= -\frac{bcd}{6x^2}-\frac{ibc^2d}{2x}-\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{3}bc^3d\log\end{aligned}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.84

$$\frac{d(-2a-3iacx-bcx-3ibc^2x^2-ib(-2i+3cx+3c^3x^3)\,\text{ArcTan}(cx)-2bc^3x^3\log(x)+bc^3x^3\log(1+c^2x^2))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (d*(-2*a - (3*I)*a*c*x - b*c*x - (3*I)*b*c^2*x^2 - I*b*(-2*I + 3*c*x + 3*c^
3*x^3)*ArcTan[c*x] - 2*b*c^3*x^3*Log[x] + b*c^3*x^3*Log[1 + c^2*x^2]))/(6*x
^3)
```

Maple [A]
time = 0.11, size = 108, normalized size = 1.02

method	result
derivativedivides	$c^3\Big(da\big(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\big)-\frac{idb\arctan(cx)}{2c^2x^2}-\frac{db\arctan(cx)}{3c^3x^3}+\frac{bd\ln(c^2x^2+1)}{6}-\frac{idb\arctan(cx)}{2}-\frac{idb}{2cx}-\frac{d}{6c^2}$
default	$c^3\Big(da\big(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\big)-\frac{idb\arctan(cx)}{2c^2x^2}-\frac{db\arctan(cx)}{3c^3x^3}+\frac{bd\ln(c^2x^2+1)}{6}-\frac{idb\arctan(cx)}{2}-\frac{idb}{2cx}-\frac{d}{6c^2}$
risch	$-\frac{(3dbcx-2ibd)\ln(icx+1)}{12x^3}-\frac{d(4c^3b\ln(-x)x^3-5c^3b\ln(-cx-i)x^3+c^3b\ln(cx-i)x^3+6ib\,c^2x^2+6iacx-3bcx\ln(-icx+1)+}{12x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d*a*(-1/2*I/c^2/x^2-1/3/c^3/x^3)-1/2*I*d*b*arctan(c*x)/c^2/x^2-1/3*d*b
*arctan(c*x)/c^3/x^3+1/6*b*d*ln(c^2*x^2+1)-1/2*I*d*b*arctan(c*x)-1/2*I*d*b/
c/x-1/6*d*b/c^2/x^2-1/3*d*b*ln(c*x))
```

Maxima [A]
time = 0.48, size = 87, normalized size = 0.82

$$-\frac{1}{2}i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd-\frac{i\,acd}{2\,x^2}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d + 1/6*((c^2*log(c^
2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d - 1/2*I*a*c*d
/x^2 - 1/3*a*d/x^3
```

Fricas [A]
time = 3.20, size = 109, normalized size = 1.03

$$-\frac{4\,bc^3dx^3\log\left(x\right)-5\,bc^3dx^3\log\left(\frac{cx+i}{c}\right)+bc^3dx^3\log\left(\frac{cx-i}{c}\right)+6i\,bc^2dx^2+2\,(3i\,a+b)cdx+4\,ad-(3\,bcdx-2i\,bd)\log\left(-\frac{cx+i}{cx-i}\right)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*c^3*d*x^3*log(x) - 5*b*c^3*d*x^3*log((c*x + I)/c) + b*c^3*d*x^3*
log((c*x - I)/c) + 6*I*b*c^2*d*x^2 + 2*(3*I*a + b)*c*d*x + 4*a*d - (3*b*c*d
*x - 2*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^3
```

Sympy [A]
time = 2.54, size = 197, normalized size = 1.86

$$-\frac{bc^3d\log\left(27b^2c^7d^2x\right)}{3}-\frac{bc^3d\log\left(27b^2c^7d^2x-27ib^2c^6d^2\right)}{12}+\frac{5bc^3d\log\left(27b^2c^7d^2x+27ib^2c^6d^2\right)}{12}+\frac{(-3bcdx+2ibd)\log\left(icx+1\right)}{12x^3}+\frac{(3bcdx-2ibd)\log\left(-icx+1\right)}{12x^3}+\frac{-2ad-3ibc^2dx^2+x(-3iacd-bcd)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] -b*c**3*d*log(27*b**2*c**7*d**2*x)/3 - b*c**3*d*log(27*b**2*c**7*d**2*x - 2
7*I*b**2*c**6*d**2)/12 + 5*b*c**3*d*log(27*b**2*c**7*d**2*x + 27*I*b**2*c**
6*d**2)/12 + (-3*b*c*d*x + 2*I*b*d)*log(I*c*x + 1)/(12*x**3) + (3*b*c*d*x -
 2*I*b*d)*log(-I*c*x + 1)/(12*x**3) + (-2*a*d - 3*I*b*c**2*d*x**2 + x*(-3*I
*a*c*d - b*c*d))/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.88, size = 176, normalized size = 1.66

$$\frac{b\,c^3\,d\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{\frac{a\,d}{3}-x^5\,\left(\frac{b\,c^5\,d}{6}+\frac{a\,c^5\,d\,1{}\mathrm{i}}{2}\right)+\frac{b\,d\,\mathrm{atan}\left(c\,x\right)}{3}+\frac{c\,d\,x\,\left(b+a\,3{}\mathrm{i}\right)}{6}+\frac{c^2\,d\,x^2\,\left(2\,a+b\,3{}\mathrm{i}\right)}{6}+\frac{b\,c^4\,d\,x^4\,1{}\mathrm{i}}{2}+\frac{b\,c^2\,d\,x^2\,\mathrm{atan}\left(c\,x\right)}{3}+\frac{b\,c^3\,d\,x^3\,\mathrm{atan}\left(c\,x\right)\,1{}\mathrm{i}}{2}+\frac{b\,c\,d\,x\,\mathrm{atan}\left(c\,x\right)\,1{}\mathrm{i}}{2}}{c^2\,x^5+x^3}-\frac{b\,c^3\,d\,\ln\left(x\right)}{3}-\frac{b\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,{\left(c^2\right)}^{3/2}\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^4,x)
```

```
[Out] (b*c^3*d*log(c^2*x^2 + 1))/6 - (b*d*atan((c^2*x)/(c^2)^(1/2))*(c^2)^(3/2)*1
i)/2 - ((a*d)/3 - x^5*((a*c^5*d*1i)/2 + (b*c^5*d)/6) + (b*d*atan(c*x))/3 +
(c*d*x*(a*3i + b))/6 + (c^2*d*x^2*(2*a + b*3i))/6 + (b*c^4*d*x^4*1i)/2 + (b
*c^2*d*x^2*atan(c*x))/3 + (b*c^3*d*x^3*atan(c*x)*1i)/2 + (b*c*d*x*atan(c*x)
*1i)/2)/(x^3 + c^2*x^5) - (b*c^3*d*log(x))/3
```

3.9 $\int \frac{(d+icdx)(a+b\mathbf{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=124

$$-\frac{bcd}{12x^3}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{d(a+b\text{ArcTan}(cx))}{4x^4}-\frac{icd(a+b\text{ArcTan}(cx))}{3x^3}-\frac{1}{3}ibc^4d\log(x)+\frac{1}{24}ibc^4d\log(i-cx)+\frac{7}{24}$$

```
[Out] -1/12*b*c*d/x^3-1/6*I*b*c^2*d/x^2+1/4*b*c^3*d/x-1/4*d*(a+b*arctan(c*x))/x^4
-1/3*I*c*d*(a+b*arctan(c*x))/x^3-1/3*I*b*c^4*d*ln(x)+1/24*I*b*c^4*d*ln(I-c*
x)+7/24*I*b*c^4*d*ln(c*x+I)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {45, 4992, 12, 815}

$$-\frac{d(a+b\text{ArcTan}(cx))}{4x^4}-\frac{icd(a+b\text{ArcTan}(cx))}{3x^3}-\frac{1}{3}ibc^4d\log(x)+\frac{1}{24}ibc^4d\log(-cx+i)+\frac{7}{24}ibc^4d\log(cx+i)+\frac{bc^3d}{4x}-\frac{ibc^2d}{6x^2}-\frac{bcd}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d)/x^3 - ((I/6)*b*c^2*d)/x^2 + (b*c^3*d)/(4*x) - (d*(a + b*ArcTa
n[c*x]))/(4*x^4) - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (I/3)*b*c^4*d*Log[
x] + (I/24)*b*c^4*d*Log[I - c*x] + ((7*I)/24)*b*c^4*d*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx = -\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{icd(a+b\tan^{-1}(cx))}{3x^3}-(bc)\int \frac{d(-3-4icx)}{12x^4\left(1+c^2x^2\right)}\,d$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{icd(a+b\tan^{-1}(cx))}{3x^3}-\frac{1}{12}(bcd)\int \frac{-3-4icx}{x^4\left(1+c^2x^2\right.}$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{icd(a+b\tan^{-1}(cx))}{3x^3}-\frac{1}{12}(bcd)\int \left(-\frac{3}{x^4}-\frac{4ic}{x^3}\right.$$
$$= -\frac{bcd}{12x^3}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{icd(a+b\tan^{-1}(cx))}{3x^3}-$$

Mathematica [A]
time = 0.05, size = 100, normalized size = 0.81

$$\frac{d\left(-3a-4iacx-bcx-2ibc^2x^2+3bc^3x^3+b\left(-3-4icx+3c^4x^4\right)\text{ArcTan}(cx)-4ibc^4x^4\log(x)+2ibc^4x^4\log\left(1+c^2x^2\right)\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d*(-3*a - (4*I)*a*c*x - b*c*x - (2*I)*b*c^2*x^2 + 3*b*c^3*x^3 + b*(-3 - (4
*I)*c*x + 3*c^4*x^4)*ArcTan[c*x] - (4*I)*b*c^4*x^4*Log[x] + (2*I)*b*c^4*x^4
*Log[1 + c^2*x^2]))/(12*x^4)
```

Maple [A]
time = 0.14, size = 119, normalized size = 0.96

method	result
derivativedivides	$c^4\left(da\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)-\frac{db\arctan(cx)}{4c^4x^4}-\frac{idb\arctan(cx)}{3c^3x^3}+\frac{idb\ln(c^2x^2+1)}{6}+\frac{db\arctan(cx)}{4}-\frac{idb}{6c^2x^2}-\right.$
default	$c^4\left(da\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)-\frac{db\arctan(cx)}{4c^4x^4}-\frac{idb\arctan(cx)}{3c^3x^3}+\frac{idb\ln(c^2x^2+1)}{6}+\frac{db\arctan(cx)}{4}-\frac{idb}{6c^2x^2}-\right.$
risch	$-\frac{(4dbcx-3ibd)\ln(icx+1)}{24x^4}-\frac{id\left(8c^4b\ln(-45cx)x^4-7c^4b\ln(-15cx-15i)x^4-c^4b\ln(9cx-9i)x^4+6ib\,c^3x^3+4b\,c^2x^2+8acx\right.}{24x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d*a*(-1/4/c^4/x^4-1/3*I/c^3/x^3)-1/4*d*b*arctan(c*x)/c^4/x^4-1/3*I*d*b
*arctan(c*x)/c^3/x^3+1/6*I*d*b*ln(c^2*x^2+1)+1/4*d*b*arctan(c*x)-1/6*I*d*b/
c^2/x^2-1/3*I*d*b*ln(c*x)-1/12*d*b/c^3/x^3+1/4*d*b/c/x)
```

Maxima [A]
time = 0.48, size = 102, normalized size = 0.82

$$\frac{1}{6}i\left(\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd+\frac{1}{12}\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd-\frac{iacd}{3x^3}-\frac{ad}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)
*b*c*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/
x^4)*b*d - 1/3*I*a*c*d/x^3 - 1/4*a*d/x^4
```

Fricas [A]
time = 2.19, size = 119, normalized size = 0.96

$$\frac{-8i\,bc^4dx^4\log\left(x\right)+7i\,bc^4dx^4\log\left(\frac{cx+i}{c}\right)+i\,bc^4dx^4\log\left(\frac{cx-i}{c}\right)+6\,bc^3dx^3-4i\,bc^2dx^2-2\left(4i\,a+b\right)cdx-6\,ad+\left(4\,bcdx-3i\,bd\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-8*I*b*c^4*d*x^4*log(x) + 7*I*b*c^4*d*x^4*log((c*x + I)/c) + I*b*c^4*
d*x^4*log((c*x - I)/c) + 6*b*c^3*d*x^3 - 4*I*b*c^2*d*x^2 - 2*(4*I*a + b)*c*
d*x - 6*a*d + (4*b*c*d*x - 3*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^4
```

Sympy [A]
time = 4.02, size = 214, normalized size = 1.73

$$-\frac{ibc^4d\log\left(135b^2c^9d^2x\right)}{3}+\frac{ibc^4d\log\left(135b^2c^9d^2x-135ib^2c^8d^2\right)}{24}+\frac{7ibc^4d\log\left(135b^2c^9d^2x+135ib^2c^8d^2\right)}{24}+\frac{\left(-4bcdx+3ibd\right)\log\left(icx+1\right)}{24x^4}+\frac{\left(4bcdx-3ibd\right)\log\left(-icx+1\right)}{24x^4}+\frac{-3ad+3bc^3dx^3-2ibc^2dx^2+x\left(-4iacd-bcd\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] -I*b*c**4*d*log(135*b**2*c**9*d**2*x)/3 + I*b*c**4*d*log(135*b**2*c**9*d**2
*x - 135*I*b**2*c**8*d**2)/24 + 7*I*b*c**4*d*log(135*b**2*c**9*d**2*x + 135
*I*b**2*c**8*d**2)/24 + (-4*b*c*d*x + 3*I*b*d)*log(I*c*x + 1)/(24*x**4) + (
4*b*c*d*x - 3*I*b*d)*log(-I*c*x + 1)/(24*x**4) + (-3*a*d + 3*b*c**3*d*x**3
- 2*I*b*c**2*d*x**2 + x*(-4*I*a*c*d - b*c*d))/(12*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.58, size = 116, normalized size = 0.94

$$\frac{d\left(\frac{3\,b\,c^7\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{{\left(c^2\right)}^{3/2}}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,2\mathrm{i}-b\,c^4\,\ln\left(x\right)\,4\mathrm{i}\right)}{12}-\frac{\frac{d\,\left(3\,a+3\,b\,\mathrm{atan}\left(c\,x\right)\right)}{12}+\frac{d\,x\,\left(a\,c\,4\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,4\mathrm{i}\right)}{12}-\frac{b\,c^3\,d\,x^3}{4}+\frac{b\,c^2\,d\,x^2\,1\mathrm{i}}{6}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^5,x)
```

```
[Out] (d*(b*c^4*log(c^2*x^2 + 1)*2i - b*c^4*log(x)*4i + (3*b*c^7*atan((c^2*x)/(c^
2)^(1/2)))/(c^2)^(3/2)))/12 - ((d*(3*a + 3*b*atan(c*x)))/12 + (d*x*(a*c*4i
+ b*c + b*c*atan(c*x)*4i))/12 + (b*c^2*d*x^2*1i)/6 - (b*c^3*d*x^3)/4)/x^4
```

3.10 $\int x^3(d+icdx)^2(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=166

$$\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5-\frac{5bd^2\text{ArcTan}(cx)}{12c^4}+\frac{1}{4}d^2x^4(a+b\text{ArcTan}(cx))+\frac{2}{5}icd^2x^5(a+b\text{Ar}$$

```
[Out] 5/12*b*d^2*x/c^3+1/5*I*b*d^2*x^2/c^2-5/36*b*d^2*x^3/c-1/10*I*b*d^2*x^4+1/30
*b*c*d^2*x^5-5/12*b*d^2*arctan(c*x)/c^4+1/4*d^2*x^4*(a+b*arctan(c*x))+2/5*I
*c*d^2*x^5*(a+b*arctan(c*x))-1/6*c^2*d^2*x^6*(a+b*arctan(c*x))-1/5*I*b*d^2*
ln(c^2*x^2+1)/c^4
```

Rubi [A]
time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.304$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$-\frac{1}{6}c^2d^2x^6(a+b\text{ArcTan}(cx))+\frac{2}{5}icd^2x^5(a+b\text{ArcTan}(cx))+\frac{1}{4}d^2x^4(a+b\text{ArcTan}(cx))-\frac{5bd^2\text{ArcTan}(cx)}{12c^4}+\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{ibd^2\log(c^2x^2+1)}{5c^4}+\frac{1}{30}bcd^2x^5-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (5*b*d^2*x)/(12*c^3) + ((I/5)*b*d^2*x^2)/c^2 - (5*b*d^2*x^3)/(36*c) - (I/10
)*b*d^2*x^4 + (b*c*d^2*x^5)/30 - (5*b*d^2*ArcTan[c*x])/(12*c^4) + (d^2*x^4*
(a + b*ArcTan[c*x]))/4 + ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x]) - (c^2*d^2
*x^6*(a + b*ArcTan[c*x]))/6 - ((I/5)*b*d^2*Log[1 + c^2*x^2])/c^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6(a+$$
$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6(a+$$
$$=\frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}c^2d^2x^6(a+$$
$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\,\mathrm{ta}$$
$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4(a+b\,\mathrm{ta}$$
$$=\frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5-\frac{5bd^2\tan^{-1}(cx)}{12c^4}$$

Mathematica [A]
time = 0.08, size = 124, normalized size = 0.75

$$\frac{d^2(3ac^4x^4(15+24icx-10c^2x^2)+bcx(75+36icx-25c^2x^2-18ic^3x^3+6c^4x^4)+3b(-25+15c^4x^4+24ic^5x^5-10c^6x^6)\,\mathrm{ArcTan}(cx)-36ib\log(1+c^2x^2))}{180c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(3*a*c^4*x^4*(15 + (24*I)*c*x - 10*c^2*x^2) + b*c*x*(75 + (36*I)*c*x -
 25*c^2*x^2 - (18*I)*c^3*x^3 + 6*c^4*x^4) + 3*b*(-25 + 15*c^4*x^4 + (24*I)*
c^5*x^5 - 10*c^6*x^6)*ArcTan[c*x] - (36*I)*b*Log[1 + c^2*x^2]))/(180*c^4)
```

Maple [A]
time = 0.15, size = 171, normalized size = 1.03

method	result
derivativedivides	$\frac{d^2 a\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)-\frac{d^2 b\arctan(cx)c^6x^6}{6}+\frac{2id^2 b\arctan(cx)c^5x^5}{5}+\frac{d^2 b\arctan(cx)c^4x^4}{4}+\frac{5bc\,d^2x}{12}+\frac{d^2b\,c^5x^5}{30}-\frac{id^2b\,c}{10}}{c^4}$
default	$\frac{d^2 a\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)-\frac{d^2 b\arctan(cx)c^6x^6}{6}+\frac{2id^2 b\arctan(cx)c^5x^5}{5}+\frac{d^2 b\arctan(cx)c^4x^4}{4}+\frac{5bc\,d^2x}{12}+\frac{d^2b\,c^5x^5}{30}-\frac{id^2b\,c}{10}}{c^4}$
risch	$\frac{id^2b(10c^2x^6-24icx^5-15x^4)\ln(icx+1)}{120}-\frac{d^2c^2a\,x^6}{6}-\frac{id^2c^2b\,x^6\ln(-icx+1)}{12}-\frac{d^2cb\,x^5\ln(-icx+1)}{5}+\frac{bc\,d^2x^5}{30}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^2*a*(-1/6*c^6*x^6+2/5*I*c^5*x^5+1/4*c^4*x^4)-1/6*d^2*b*arctan(c*x)
*c^6*x^6+2/5*I*d^2*b*arctan(c*x)*c^5*x^5+1/4*d^2*b*arctan(c*x)*c^4*x^4+5/12
*b*c*d^2*x+1/30*d^2*b*c^5*x^5-1/10*I*d^2*b*c^4*x^4-5/36*d^2*b*c^3*x^3+1/5*I
*d^2*b*c^2*x^2-1/5*I*d^2*b*ln(c^2*x^2+1)-5/12*b*d^2*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 185, normalized size = 1.11

$$-\frac{1}{6}\,ac^2d^2x^6+\frac{2}{5}i\,acd^2x^5+\frac{1}{4}\,ad^2x^4-\frac{1}{90}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^2d^2+\frac{1}{10}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd^2+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*a*c^2*d^2*x^6 + 2/5*I*a*c*d^2*x^5 + 1/4*a*d^2*x^4 - 1/90*(15*x^6*arcta
n(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2
*d^2 + 1/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2
 + 1)/c^6))*b*c*d^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*
arctan(c*x)/c^5))*b*d^2
```

Fricas [A]
time = 2.54, size = 172, normalized size = 1.04

$$-\frac{60\,ac^6d^2x^6+12\,(-12i\,a-b)c^5d^2x^5-18\,(5\,a-2i\,b)c^4d^2x^4+50\,bc^3d^2x^3-72i\,bc^2d^2x^2-150\,bcd^2x+147i\,bd^2\log\left(\frac{cx+i}{c}\right)-3i\,bd^2\log\left(\frac{cx-i}{c}\right)+3\,(10i\,bc^6d^2x^6+24\,bc^5d^2x^5-15i\,bc^4d^2x^4)\log\left(-\frac{cx+i}{cx-i}\right)}{360\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/360*(60*a*c^6*d^2*x^6 + 12*(-12*I*a - b)*c^5*d^2*x^5 - 18*(5*a - 2*I*b)*
c^4*d^2*x^4 + 50*b*c^3*d^2*x^3 - 72*I*b*c^2*d^2*x^2 - 150*b*c*d^2*x + 147*I
*b*d^2*log((c*x + I)/c) - 3*I*b*d^2*log((c*x - I)/c) + 3*(10*I*b*c^6*d^2*x^
6 + 24*b*c^5*d^2*x^5 - 15*I*b*c^4*d^2*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A]
time = 2.23, size = 270, normalized size = 1.63

$$-\frac{ac^2d^2x^6}{6}-\frac{5bd^2x^3}{36c}+\frac{ibd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}-\frac{bd^2\left(-\frac{i\log\left(291bcd^2x-291ibd^2\right)}{120}+\frac{71i\log\left(291bcd^2x+291ibd^2\right)}{210}\right)}{c^4}-x^5\left(-\frac{2iacd^2}{5}-\frac{bcd^2}{30}\right)-x^4\left(-\frac{ad^2}{4}+\frac{ibd^2}{10}\right)+\left(\frac{ibc^2d^2x^6}{12}+\frac{bcd^2x^5}{5}-\frac{ibd^2x^4}{8}\right)\log\left(icx+1\right)+\frac{\left(-70ibc^6d^2x^6-168bc^5d^2x^5+105ibc^4d^2x^4-59ibd^2\right)\log\left(-icx+1\right)}{840c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**6/6 - 5*b*d**2*x**3/(36*c) + I*b*d**2*x**2/(5*c**2) + 5*b*d
**2*x/(12*c**3) - b*d**2*(-I*log(291*b*c*d**2*x - 291*I*b*d**2)/120 + 71*I*
log(291*b*c*d**2*x + 291*I*b*d**2)/210)/c**4 - x**5*(-2*I*a*c*d**2/5 - b*c*
d**2/30) - x**4*(-a*d**2/4 + I*b*d**2/10) + (I*b*c**2*d**2*x**6/12 + b*c*d*
*2*x**5/5 - I*b*d**2*x**4/8)*log(I*c*x + 1) + (-70*I*b*c**6*d**2*x**6 - 168
*b*c**5*d**2*x**5 + 105*I*b*c**4*d**2*x**4 - 59*I*b*d**2)*log(-I*c*x + 1)/(
840*c**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.76, size = 152, normalized size = 0.92

$$-\frac{\frac{d^2\,\left(75\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,36{}\mathrm{i}\right)}{180}+\frac{5\,b\,c^3\,d^2\,x^3}{36}-\frac{5\,b\,c\,d^2\,x}{12}-\frac{b\,c^2\,d^2\,x^2\,1{}\mathrm{i}}{5}}{c^4}+\frac{d^2\,\left(45\,a\,x^4+45\,b\,x^4\,\mathrm{atan}\left(c\,x\right)-b\,x^4\,18{}\mathrm{i}\right)}{180}-\frac{c^2\,d^2\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atan}\left(c\,x\right)\right)}{180}+\frac{c\,d^2\,\left(a\,x^5\,72{}\mathrm{i}+6\,b\,x^5+b\,x^5\,\mathrm{atan}\left(c\,x\right)\,72{}\mathrm{i}\right)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

```
[Out] (d^2*(45*a*x^4 - b*x^4*18i + 45*b*x^4*atan(c*x)))/180 - ((d^2*(75*b*atan(c*
x) + b*log(c^2*x^2 + 1)*36i))/180 - (b*c^2*d^2*x^2*1i)/5 + (5*b*c^3*d^2*x^3
)/36 - (5*b*c*d^2*x)/12)/c^4 - (c^2*d^2*(30*a*x^6 + 30*b*x^6*atan(c*x)))/18
0 + (c*d^2*(a*x^5*72i + 6*b*x^5 + b*x^5*atan(c*x)*72i))/180
```

3.11 $\int x^2(d+icdx)^2(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=152

$$\frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4-\frac{ibd^2\text{ArcTan}(cx)}{2c^3}+\frac{1}{3}d^2x^3(a+b\text{ArcTan}(cx))+\frac{1}{2}icd^2x^4(a+b\text{ArcTan}(cx))$$

```
[Out] 1/2*I*b*d^2*x/c^2-4/15*b*d^2*x^2/c-1/6*I*b*d^2*x^3+1/20*b*c*d^2*x^4-1/2*I*b
*d^2*arctan(c*x)/c^3+1/3*d^2*x^3*(a+b*arctan(c*x))+1/2*I*c*d^2*x^4*(a+b*arc
tan(c*x))-1/5*c^2*d^2*x^5*(a+b*arctan(c*x))+4/15*b*d^2*ln(c^2*x^2+1)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$-\frac{1}{5}c^2d^2x^5(a+b\text{ArcTan}(cx))+\frac{1}{2}icd^2x^4(a+b\text{ArcTan}(cx))+\frac{1}{3}d^2x^3(a+b\text{ArcTan}(cx))-\frac{ibd^2\text{ArcTan}(cx)}{2c^3}+\frac{ibd^2x}{2c^2}+\frac{4bd^2\log(c^2x^2+1)}{15c^3}+\frac{1}{20}bcd^2x^4-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((I/2)*b*d^2*x)/c^2 - (4*b*d^2*x^2)/(15*c) - (I/6)*b*d^2*x^3 + (b*c*d^2*x^4
)/20 - ((I/2)*b*d^2*ArcTan[c*x])/c^3 + (d^2*x^3*(a + b*ArcTan[c*x]))/3 + (I
/2)*c*d^2*x^4*(a + b*ArcTan[c*x]) - (c^2*d^2*x^5*(a + b*ArcTan[c*x]))/5 + (
4*b*d^2*Log[1 + c^2*x^2])/(15*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx = \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+\right.$$
$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+\right.$$
$$= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}c^2d^2x^5\left(a+\right.$$
$$= \frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+$$
$$= \frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+$$
$$= \frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4-\frac{ibd^2\tan^{-1}(cx)}{2c^3}+\frac{1}{3}d^2x^3\left(a\right.$$

Mathematica [A]
time = 0.07, size = 116, normalized size = 0.76

$$\frac{d^2(2ac^3x^3(10+15icx-6c^2x^2)+bcx(30i-16cx-10ic^2x^2+3c^3x^3)+2b(-15i+10c^3x^3+15ic^4x^4-6c^5x^5)\,\mathrm{ArcTan}(cx)+16b\log(1+c^2x^2))}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(2*a*c^3*x^3*(10 + (15*I)*c*x - 6*c^2*x^2) + b*c*x*(30*I - 16*c*x - (1
0*I)*c^2*x^2 + 3*c^3*x^3) + 2*b*(-15*I + 10*c^3*x^3 + (15*I)*c^4*x^4 - 6*c^
5*x^5)*ArcTan[c*x] + 16*b*Log[1 + c^2*x^2]))/(60*c^3)
```

Maple [A]
time = 0.13, size = 159, normalized size = 1.05

method	result
derivativedivides	$\frac{d^2 a\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)-\frac{d^2 b\arctan(cx)c^5x^5}{5}+\frac{id^2 b\arctan(cx)c^4x^4}{2}+\frac{d^2 b\arctan(cx)c^3x^3}{3}+\frac{id^2 bcx}{2}+\frac{d^2 b\,c^4x^4}{20}-\frac{id^2 b\,c^3 x}{6}}{c^3}$
default	$\frac{d^2 a\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)-\frac{d^2 b\arctan(cx)c^5x^5}{5}+\frac{id^2 b\arctan(cx)c^4x^4}{2}+\frac{d^2 b\arctan(cx)c^3x^3}{3}+\frac{id^2 bcx}{2}+\frac{d^2 b\,c^4x^4}{20}-\frac{id^2 b\,c^3 x}{6}}{c^3}$
risch	$\frac{id^2 b\left(6c^2x^5-15ic\,x^4-10x^3\right)\ln(icx+1)}{60}-\frac{id^2c^2b\,x^5\ln(-icx+1)}{10}-\frac{d^2c^2a\,x^5}{5}-\frac{d^2cb\,x^4\ln(-icx+1)}{4}+\frac{id^2ca\,x^4}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^2*a*(-1/5*c^5*x^5+1/2*I*c^4*x^4+1/3*c^3*x^3)-1/5*d^2*b*arctan(c*x)
*c^5*x^5+1/2*I*d^2*b*arctan(c*x)*c^4*x^4+1/3*d^2*b*arctan(c*x)*c^3*x^3+1/2*
I*d^2*b*c*x+1/20*d^2*b*c^4*x^4-1/6*I*d^2*b*c^3*x^3-4/15*d^2*b*c^2*x^2+4/15*
b*ln(c^2*x^2+1)*d^2-1/2*I*d^2*b*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 174, normalized size = 1.14

$$-\frac{1}{5}\,ac^2d^2x^5+\frac{1}{2}i\,acd^2x^4-\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^2+\frac{1}{3}\,ad^2x^3+\frac{1}{6}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd^2+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*a*c^2*d^2*x^5 + 1/2*I*a*c*d^2*x^4 - 1/20*(4*x^5*arctan(c*x) - c*((c^2*
x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/6
*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^
2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2
```

Fricas [A]
time = 1.84, size = 160, normalized size = 1.05

$$-\frac{12\,ac^5d^2x^5+3\,(-10i\,a-b)c^4d^2x^4-10\,(2\,a-i\,b)c^3d^2x^3+16\,bc^2d^2x^2-30i\,bcd^2x-31\,bd^2\log\left(\frac{cx+i}{c}\right)-bd^2\log\left(\frac{cx-i}{c}\right)-(-6i\,bc^5d^2x^5-15\,bc^4d^2x^4+10i\,bc^3d^2x^3)\log\left(-\frac{cx+i}{cx-i}\right)}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*c^5*d^2*x^5 + 3*(-10*I*a - b)*c^4*d^2*x^4 - 10*(2*a - I*b)*c^3*
d^2*x^3 + 16*b*c^2*d^2*x^2 - 30*I*b*c*d^2*x - 31*b*d^2*log((c*x + I)/c) - b
*d^2*log((c*x - I)/c) - (-6*I*b*c^5*d^2*x^5 - 15*b*c^4*d^2*x^4 + 10*I*b*c^3
*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A]
time = 1.99, size = 250, normalized size = 1.64

$$-\frac{ac^2d^2x^5}{5}-\frac{4bd^2x^2}{15c}+\frac{ibd^2x}{2c^2}-\frac{bd^2\left(-\frac{\log\left(47bcd^2x-47ibd^2\right)}{60}-\frac{49\log\left(47bcd^2x+47ibd^2\right)}{120}\right)}{c^3}-x^4\left(-\frac{iacd^2}{2}-\frac{bcd^2}{20}\right)-x^3\left(-\frac{ad^2}{3}+\frac{ibd^2}{6}\right)+\left(\frac{ibc^2d^2x^5}{10}+\frac{bcd^2x^4}{4}-\frac{ibd^2x^3}{6}\right)\log\left(icx+1\right)+\frac{\left(-12ibc^5d^2x^5-30bc^4d^2x^4+20ibc^3d^2x^3+13bd^2\right)\log\left(-icx+1\right)}{120c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**5/5 - 4*b*d**2*x**2/(15*c) + I*b*d**2*x/(2*c**2) - b*d**2*(
-log(47*b*c*d**2*x - 47*I*b*d**2)/60 - 49*log(47*b*c*d**2*x + 47*I*b*d**2)/
120)/c**3 - x**4*(-I*a*c*d**2/2 - b*c*d**2/20) - x**3*(-a*d**2/3 + I*b*d**2
/6) + (I*b*c**2*d**2*x**5/10 + b*c*d**2*x**4/4 - I*b*d**2*x**3/6)*log(I*c*x
 + 1) + (-12*I*b*c**5*d**2*x**5 - 30*b*c**4*d**2*x**4 + 20*I*b*c**3*d**2*x*
*3 + 13*b*d**2)*log(-I*c*x + 1)/(120*c**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.74, size = 140, normalized size = 0.92

$$-\frac{\frac{d^2\,\left(-16\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}+\frac{4\,b\,c^2\,d^2\,x^2}{15}-\frac{b\,c\,d^2\,x\,1{}\mathrm{i}}{2}}{c^3}+\frac{d^2\,\left(20\,a\,x^3+20\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,10{}\mathrm{i}\right)}{60}-\frac{c^2\,d^2\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}\left(c\,x\right)\right)}{60}+\frac{c\,d^2\,\left(a\,x^4\,30{}\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

```
[Out] (d^2*(20*a*x^3 - b*x^3*10i + 20*b*x^3*atan(c*x)))/60 - ((d^2*(b*atan(c*x)*3
0i - 16*b*log(c^2*x^2 + 1)))/60 + (4*b*c^2*d^2*x^2)/15 - (b*c*d^2*x*1i)/2)/
c^3 - (c^2*d^2*(12*a*x^5 + 12*b*x^5*atan(c*x)))/60 + (c*d^2*(a*x^4*30i + 3*
b*x^4 + b*x^4*atan(c*x)*30i))/60
```

3.12 $\int x(d+icdx)^2(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=136

$$-\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{3bd^2\text{ArcTan}(cx)}{4c^2}+\frac{1}{2}d^2x^2(a+b\text{ArcTan}(cx))+\frac{2}{3}icd^2x^3(a+b\text{ArcTan}(cx))-\frac{1}{4}c^2d$$

```
[Out] -3/4*b*d^2*x/c-1/3*I*b*d^2*x^2+1/12*b*c*d^2*x^3+3/4*b*d^2*arctan(c*x)/c^2+1
/2*d^2*x^2*(a+b*arctan(c*x))+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))-1/4*c^2*d^2*
x^4*(a+b*arctan(c*x))+1/3*I*b*d^2*ln(c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$-\frac{1}{4}c^2d^2x^4(a+b\text{ArcTan}(cx))+\frac{2}{3}icd^2x^3(a+b\text{ArcTan}(cx))+\frac{1}{2}d^2x^2(a+b\text{ArcTan}(cx))+\frac{3bd^2\text{ArcTan}(cx)}{4c^2}+\frac{ibd^2\log(c^2x^2+1)}{3c^2}+\frac{1}{12}bcd^2x^3-\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-3*b*d^2*x)/(4*c) - (I/3)*b*d^2*x^2 + (b*c*d^2*x^3)/12 + (3*b*d^2*ArcTan[c
*x])/(4*c^2) + (d^2*x^2*(a + b*ArcTan[c*x]))/2 + ((2*I)/3)*c*d^2*x^3*(a + b
*ArcTan[c*x]) - (c^2*d^2*x^4*(a + b*ArcTan[c*x]))/4 + ((I/3)*b*d^2*Log[1 +
c^2*x^2])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+\right.\\
&= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+\right.\\
&= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^4\left(a+\right.\\
&= -\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\\
&= -\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}icd^2x^3\\
&= -\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{3bd^2\tan^{-1}(cx)}{4c^2}+\frac{1}{2}d^2x^2\left(a+b\tan\right.
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 0.74

$$\frac{d^2(cx(acx(6+8icx-3c^2x^2)+b(-9-4icx+c^2x^2))+b(9+6c^2x^2+8ic^3x^3-3c^4x^4)\,\mathrm{ArcTan}(cx)+4ib\log(1+c^2x^2))}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*(c*x*(a*c*x*(6 + (8*I)*c*x - 3*c^2*x^2) + b*(-9 - (4*I)*c*x + c^2*x^2)
) + b*(9 + 6*c^2*x^2 + (8*I)*c^3*x^3 - 3*c^4*x^4)*ArcTan[c*x] + (4*I)*b*Log
[1 + c^2*x^2]))/(12*c^2)
```

Maple [A]
time = 0.14, size = 146, normalized size = 1.07

method	result
derivativedivides	$\frac{d^2 a\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)-\frac{d^2 b\arctan(cx)c^4x^4}{4}+\frac{2id^2b\arctan(cx)c^3x^3}{3}+\frac{d^2b\arctan(cx)c^2x^2}{2}-\frac{3bc\,d^2x}{4}+\frac{d^2b\,c^3x^3}{12}-\frac{id^2b\,c}{3}}{c^2}$
default	$\frac{d^2 a\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)-\frac{d^2 b\arctan(cx)c^4x^4}{4}+\frac{2id^2b\arctan(cx)c^3x^3}{3}+\frac{d^2b\arctan(cx)c^2x^2}{2}-\frac{3bc\,d^2x}{4}+\frac{d^2b\,c^3x^3}{12}-\frac{id^2b\,c}{3}}{c^2}$
risch	$\frac{id^2b\left(3c^2x^4-8ic\,x^3-6x^2\right)\ln(icx+1)}{24}-\frac{id^2c^2b\,x^4\ln(-icx+1)}{8}-\frac{a\,c^2d^2x^4}{4}-\frac{d^2cb\,x^3\ln(-icx+1)}{3}+\frac{bc\,d^2x^3}{12}+2id$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^2*a*(-1/4*c^4*x^4+2/3*I*c^3*x^3+1/2*c^2*x^2)-1/4*d^2*b*arctan(c*x)
*c^4*x^4+2/3*I*d^2*b*arctan(c*x)*c^3*x^3+1/2*d^2*b*arctan(c*x)*c^2*x^2-3/4*
b*c*d^2*x+1/12*d^2*b*c^3*x^3-1/3*I*d^2*b*c^2*x^2+1/3*I*d^2*b*ln(c^2*x^2+1)+
3/4*b*d^2*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 155, normalized size = 1.14

$$-\frac{1}{4}\,ac^2d^2x^4+\frac{2}{3}i\,acd^2x^3-\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\arctan\left(cx\right)}{c^5}\right)\right)bc^2d^2+\frac{1}{3}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd^2+\frac{1}{2}\,ad^2x^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*a*c^2*d^2*x^4 + 2/3*I*a*c*d^2*x^3 - 1/12*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^2 + 1/3*I*(2*x^3*arctan(c*x) -
 c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^2 + 1/2*a*d^2*x^2 + 1/2*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^2
```

Fricas [A]
time = 2.67, size = 148, normalized size = 1.09

$$-\frac{6\,ac^4d^2x^4+2\left(-8i\,a-b\right)c^3d^2x^3-4\left(3\,a-2i\,b\right)c^2d^2x^2+18\,bcd^2x-17i\,bd^2\log\left(\frac{cx+i}{c}\right)+i\,bd^2\log\left(\frac{cx-i}{c}\right)-\left(-3i\,bc^4d^2x^4-8\,bc^3d^2x^3+6i\,bc^2d^2x^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] -1/24*(6*a*c^4*d^2*x^4 + 2*(-8*I*a - b)*c^3*d^2*x^3 - 4*(3*a - 2*I*b)*c^2*d^2*x^2 + 18*b*c*d^2*x - 17*I*b*d^2*log((c*x + I)/c) + I*b*d^2*log((c*x - I)/c) - (-3*I*b*c^4*d^2*x^4 - 8*b*c^3*d^2*x^3 + 6*I*b*c^2*d^2*x^2)*log(-(c*x + I)/(c*x - I)))/c^2

Sympy [A]
time = 1.86, size = 240, normalized size = 1.76

$$-\frac{ac^2d^2x^4}{4}-\frac{3bd^2x}{4c}-\frac{bd^2\left(\frac{i\log\left(67bcd^2x-67ibd^2\right)}{24}-\frac{31i\log\left(67bcd^2x+67ibd^2\right)}{60}\right)}{c^2}-x^3\left(-\frac{2iacd^2}{3}-\frac{bcd^2}{12}\right)-x^2\left(-\frac{ad^2}{2}+\frac{ibd^2}{3}\right)+\left(\frac{ibc^2d^2x^4}{8}+\frac{bcd^2x^3}{3}-\frac{ibd^2x^2}{4}\right)\log\left(icx+1\right)+\frac{\left(-15ibc^4d^2x^4-40bc^3d^2x^3+30ibc^2d^2x^2+23ibd^2\right)\log\left(-icx+1\right)}{120c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)

[Out] -a*c**2*d**2*x**4/4 - 3*b*d**2*x/(4*c) - b*d**2*(I*log(67*b*c*d**2*x - 67*I*b*d**2)/24 - 31*I*log(67*b*c*d**2*x + 67*I*b*d**2)/60)/c**2 - x**3*(-2*I*a*c*d**2/3 - b*c*d**2/12) - x**2*(-a*d**2/2 + I*b*d**2/3) + (I*b*c**2*d**2*x**4/8 + b*c*d**2*x**3/3 - I*b*d**2*x**2/4)*log(I*c*x + 1) + (-15*I*b*c**4*d**2*x**4 - 40*b*c**3*d**2*x**3 + 30*I*b*c**2*d**2*x**2 + 23*I*b*d**2)*log(-I*c*x + 1)/(120*c**2)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")

[Out] sage0*x

Mupad [B]
time = 0.68, size = 125, normalized size = 0.92

$$\frac{\frac{d^2\,\left(9\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,4{}\mathrm{i}\right)}{12}-\frac{3\,b\,c\,d^2\,x}{4}}{c^2}+\frac{d^2\,\left(6\,a\,x^2+6\,b\,x^2\,\mathrm{atan}\left(c\,x\right)-b\,x^2\,4{}\mathrm{i}\right)}{12}-\frac{c^2\,d^2\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}\left(c\,x\right)\right)}{12}+\frac{c\,d^2\,\left(a\,x^3\,8{}\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}\left(c\,x\right)\,8{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)

[Out] ((d^2*(9*b*atan(c*x) + b*log(c^2*x^2 + 1)*4i))/12 - (3*b*c*d^2*x)/4)/c^2 + (d^2*(6*a*x^2 - b*x^2*4i + 6*b*x^2*atan(c*x)))/12 - (c^2*d^2*(3*a*x^4 + 3*b*x^4*atan(c*x)))/12 + (c*d^2*(a*x^3*8i + b*x^3 + b*x^3*atan(c*x)*8i))/12

3.13 $\int (d+icdx)^2(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=83

$$-\frac{2}{3}ibd^2x-\frac{bd^2(1+icx)^2}{6c}-\frac{id^2(1+icx)^3(a+b\text{ArcTan}(cx))}{3c}-\frac{4bd^2\log(1-icx)}{3c}$$

```
[Out] -2/3*I*b*d^2*x-1/6*b*d^2*(1+I*c*x)^2/c-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan(c*
x))/c-4/3*b*d^2*ln(1-I*c*x)/c
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4972, 641, 45}

$$-\frac{id^2(1+icx)^3(a+b\text{ArcTan}(cx))}{3c}-\frac{bd^2(1+icx)^2}{6c}-\frac{4bd^2\log(1-icx)}{3c}-\frac{2}{3}ibd^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-2*I)/3)*b*d^2*x - (b*d^2*(1 + I*c*x)^2)/(6*c) - ((I/3)*d^2*(1 + I*c*x)^3
*(a + b*ArcTan[c*x]))/c - (4*b*d^2*Log[1 - I*c*x])/(3*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)\,dx = -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{(ib)\int\frac{(d+icdx)^3}{1+c^2x^2}\,dx}{3d}$$
$$= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{(ib)\int\frac{(d+icdx)^2}{\frac{1}{d}-\frac{icx}{d}}\,dx}{3d}$$
$$= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{(ib)\int\left(-2d^3+\frac{4d^2}{\frac{1}{d}-\frac{icx}{d}}-d^2(d+icd\right.}{3d}$$
$$= -\frac{2}{3}ibd^2x - \frac{bd^2(1+icx)^2}{6c} - \frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)}{3c} - \frac{4bd^2\log}{3}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.69

$$\frac{1}{3}d^2\left(\frac{1}{2}bx(-6i+cx)-\frac{(-i+cx)^3(a+b\text{ArcTan}(cx))}{c}-\frac{4b\log(i+cx)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^2*((b*x*(-6*I + c*x))/2 - ((-I + c*x)^3*(a + b*ArcTan[c*x]))/c - (4*b*Lo
g[I + c*x])/c))/3
```

Maple [A]
time = 0.08, size = 114, normalized size = 1.37

method	result
derivativedivides	$\frac{-\frac{id^2(icx+1)^3a}{3}-\frac{d^2b\arctan(cx)c^3x^3}{3}+id^2b\arctan(cx)c^2x^2+b\arctan(cx)d^2cx+id^2b\arctan(cx)-id^2bcx+\frac{d^2bc^2x^2}{6}-\frac{2b\ln\left(c^2x\right.}{3}}{c}$
default	$\frac{-\frac{id^2(icx+1)^3a}{3}-\frac{d^2b\arctan(cx)c^3x^3}{3}+id^2b\arctan(cx)c^2x^2+b\arctan(cx)d^2cx+id^2b\arctan(cx)-id^2bcx+\frac{d^2bc^2x^2}{6}-\frac{2b\ln\left(c^2x\right.}{3}}{c}$
risch	$\frac{id^2(cx-i)^3b\ln(icx+1)}{6c}-\frac{id^2c^2bx^3\ln(-icx+1)}{6}-\frac{d^2ac^2x^3}{3}+id^2acx^2-\frac{d^2cbx^2\ln(-icx+1)}{2}+\frac{ibd^2x\ln(-icx+}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*I*d^2*(1+I*c*x)^3*a-1/3*d^2*b*arctan(c*x)*c^3*x^3+I*d^2*b*arctan(
c*x)*c^2*x^2+b*arctan(c*x)*d^2*c*x+I*d^2*b*arctan(c*x)-I*d^2*b*c*x+1/6*d^2*
b*c^2*x^2-2/3*b*ln(c^2*x^2+1)*d^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.
time = 0.49, size = 138, normalized size = 1.66

$$-\frac{1}{3}ac^2d^2x^3-\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bc^2d^2+iacd^2x^2+i\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bcd^2+ad^2x+\frac{(2cx\arctan(cx)-\log(c^2x^2+1))bd^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/3*a*c^2*d^2*x^3 - 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*c^2*d^2 + I*a*c*d^2*x^2 + I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c
*x)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*
d^2/c
```

Fricas [A]
time = 2.40, size = 127, normalized size = 1.53

$$-\frac{2\,ac^3d^2x^3-(6i\,a+b)c^2d^2x^2-6\,(a-i\,b)cd^2x+7\,bd^2\log\left(\frac{cx+i}{c}\right)+bd^2\log\left(\frac{cx-i}{c}\right)-(-i\,bc^3d^2x^3-3\,bc^2d^2x^2+3i\,bcd^2x)\log\left(-\frac{cx+i}{cx-i}\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3*d^2*x^3 - (6*I*a + b)*c^2*d^2*x^2 - 6*(a - I*b)*c*d^2*x + 7*b
*d^2*log((c*x + I)/c) + b*d^2*log((c*x - I)/c) - (-I*b*c^3*d^2*x^3 - 3*b*c^
2*d^2*x^2 + 3*I*b*c*d^2*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.
time = 1.54, size = 206, normalized size = 2.48

$$-\frac{ac^2d^2x^3}{3}-\frac{bd^2\left(\frac{\log\left(13bcd^2x-13ibd^2\right)}{6}+\frac{17\log\left(13bcd^2x+13ibd^2\right)}{24}\right)}{c}-x^2\left(-iacd^2-\frac{bcd^2}{6}\right)-x\left(-ad^2+ibd^2\right)+\left(\frac{ibc^2d^2x^3}{6}+\frac{bcd^2x^2}{2}-\frac{ibd^2x}{2}\right)\log\left(icx+1\right)+\frac{\left(-4ibc^3d^2x^3-12bc^2d^2x^2+12ibcd^2x-11bd^2\right)\log\left(-icx+1\right)}{24c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

```
[Out] -a*c**2*d**2*x**3/3 - b*d**2*(log(13*b*c*d**2*x - 13*I*b*d**2)/6 + 17*log(1
3*b*c*d**2*x + 13*I*b*d**2)/24)/c - x**2*(-I*a*c*d**2 - b*c*d**2/6) - x*(-a
*d**2 + I*b*d**2) + (I*b*c**2*d**2*x**3/6 + b*c*d**2*x**2/2 - I*b*d**2*x/2)
*log(I*c*x + 1) + (-4*I*b*c**3*d**2*x**3 - 12*b*c**2*d**2*x**2 + 12*I*b*c*d
**2*x - 11*b*d**2)*log(-I*c*x + 1)/(24*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.40, size = 109, normalized size = 1.31

$$\frac{d^2 (6 a x + 6 b x \operatorname{atan}(c x) - b x 6\mathrm{i})}{6} - \frac{c^2 d^2 (2 a x^3 + 2 b x^3 \operatorname{atan}(c x))}{6} + \frac{d^2 (-4 b \ln (c^2 x^2 + 1) + b \operatorname{atan}(c x)\ 6\mathrm{i})}{6 c} + \frac{c d^2 (a x^2 6\mathrm{i} + b x^2 + b x^2 \operatorname{atan}(c x)\ 6\mathrm{i})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

```
[Out] (d^2*(6*a*x - b*x*6i + 6*b*x*atan(c*x)))/6 - (c^2*d^2*(2*a*x^3 + 2*b*x^3*at
an(c*x)))/6 + (d^2*(b*atan(c*x)*6i - 4*b*log(c^2*x^2 + 1)))/(6*c) + (c*d^2*
(a*x^2*6i + b*x^2 + b*x^2*atan(c*x)*6i))/6
```

3.14 $\int \frac{(d+icdx)^2(a+b\text{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=129

$$2iacd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\text{ArcTan}(cx)+2ibcd^2x\text{ArcTan}(cx)-\frac{1}{2}c^2d^2x^2(a+b\text{ArcTan}(cx))+ad^2\log(x)-ibd^2\log($$

```
[Out] 2*I*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctan(c*x)+2*I*b*c*d^2*x*arctan(c*x)
-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))+a*d^2*ln(x)-I*b*d^2*ln(c^2*x^2+1)+1/2*I*
b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4996, 4930, 266, 4940, 2438, 4946, 327, 209}

$$-\frac{1}{2}c^2d^2x^2(a+b\text{ArcTan}(cx))+2iacd^2x+ad^2\log(x)-\frac{1}{2}bd^2\text{ArcTan}(cx)+2ibcd^2x\text{ArcTan}(cx)-ibd^2\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^2\text{Li}_2(-icx)-\frac{1}{2}ibd^2\text{Li}_2(icx)+\frac{1}{2}bcd^2x$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (2*I)*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTan[c*x])/2 + (2*I)*b*c*d^2*x*A
rcTan[c*x] - (c^2*d^2*x^2*(a + b*ArcTan[c*x]))/2 + a*d^2*Log[x] - I*b*d^2*L
og[1 + c^2*x^2] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog[2,
 I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)}{x}\,dx &= \int \left(2icd^2\left(a+b\tan^{-1}(cx)\right)+\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x}-c^2d^2x\left(a+b\tan^{-1}\right.\right.\\
&= d^2\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(2icd^2\right)\int \left(a+b\tan^{-1}(cx)\right)\,dx-\left(c^2d^2\right)\int\\
&= 2iacd^2x-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+\frac{1}{2}\left(ibd^2\right)\int \frac{\log(1-}{x}\\
&= 2iacd^2x+\frac{1}{2}bcd^2x+2ibcd^2x\tan^{-1}(cx)-\frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)+\\
&= 2iacd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tan^{-1}(cx)+2ibcd^2x\tan^{-1}(cx)-\frac{1}{2}c^2d^2x^2(a
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 0.80

$$-\frac{1}{2}d^2\left(-4iacx-bcx+ac^2x^2+b\text{ArcTan}(cx)-4ibcx\text{ArcTan}(cx)+bc^2x^2\text{ArcTan}(cx)-2a\log(x)+2ib\log\left(1+c^2x^2\right)-ib\text{PolyLog}(2,-icx)+ib\text{PolyLog}(2,icx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -1/2*(d^2*((-4*I)*a*c*x - b*c*x + a*c^2*x^2 + b*ArcTan[c*x] - (4*I)*b*c*x*A
rcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - 2*a*Log[x] + (2*I)*b*Log[1 + c^2*x^2]
- I*b*PolyLog[2, (-I)*c*x] + I*b*PolyLog[2, I*c*x]))
```

Maple [A]
time = 0.07, size = 177, normalized size = 1.37

method	result
derivativedivides	$2iac\,d^2x-\frac{a\,c^2d^2x^2}{2}+d^2a\ln(cx)+2ibc\,d^2x\arctan(cx)-\frac{d^2b\arctan(cx)c^2x^2}{2}+d^2b\ln(cx)\,\text{ar}$
default	$2iac\,d^2x-\frac{a\,c^2d^2x^2}{2}+d^2a\ln(cx)+2ibc\,d^2x\arctan(cx)-\frac{d^2b\arctan(cx)c^2x^2}{2}+d^2b\ln(cx)\,\text{ar}$
risch	$-id^2b\ln\left(c^2x^2+1\right)-d^2b\ln(-icx+1)\,cx-\frac{id^2b\ln(-icx+1)x^2c^2}{4}-\frac{b\,d^2\arctan(cx)}{2}+2ib\,d^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*a*c*d^2*x-1/2*a*c^2*d^2*x^2+d^2*a*ln(c*x)+2*I*b*c*d^2*x*arctan(c*x)-1/2
*d^2*b*arctan(c*x)*c^2*x^2+d^2*b*ln(c*x)*arctan(c*x)+1/2*b*c*d^2*x-I*b*d^2*
ln(c^2*x^2+1)-1/2*b*d^2*arctan(c*x)+1/2*I*d^2*b*ln(c*x)*ln(1+I*c*x)-1/2*I*d
^2*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d^2*b*dilog(1+I*c*x)-1/2*I*d^2*b*dilog(1-I*c
*x)
```

Maxima [A]
time = 0.59, size = 142, normalized size = 1.10

$$-\frac{1}{2}ac^2d^2x^2+2i\,acd^2x+\frac{1}{2}bcd^2x-\frac{1}{4}\pi bd^2\log\left(c^2x^2+1\right)+bd^2\arctan(cx)\log(cx)+i\left(2\,cx\arctan(cx)-\log\left(c^2x^2+1\right)\right)bd^2-\frac{1}{2}i\,bd^2\text{Li}_2(i\,cx+1)+\frac{1}{2}i\,bd^2\text{Li}_2(-i\,cx+1)+ad^2\log(x)-\frac{1}{2}\left(bc^2d^2x^2+bd^2\right)\arctan(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*d^2*x^2 + 2*I*a*c*d^2*x + 1/2*b*c*d^2*x - 1/4*pi*b*d^2*log(c^2*x
^2 + 1) + b*d^2*arctan(c*x)*log(c*x) + I*(2*c*x*arctan(c*x) - log(c^2*x^2 +
 1))*b*d^2 - 1/2*I*b*d^2*dilog(I*c*x + 1) + 1/2*I*b*d^2*dilog(-I*c*x + 1) +
 a*d^2*log(x) - 1/2*(b*c^2*d^2*x^2 + b*d^2)*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^2\left(\int\left(-\frac{a}{x}\right)dx+\int(-2iac)\,dx+\int ac^2x\,dx+\int\left(-\frac{b\operatorname{atan}(cx)}{x}\right)dx+\int(-2ibc\operatorname{atan}(cx))\,dx+\int bc^2x\operatorname{atan}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x,x)
```

```
[Out] -d**2*(Integral(-a/x, x) + Integral(-2*I*a*c, x) + Integral(a*c**2*x, x) +
Integral(-b*atan(c*x)/x, x) + Integral(-2*I*b*c*atan(c*x), x) + Integral(b*
c**2*x*atan(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.73, size = 131, normalized size = 1.02

$$\begin{cases} a\,d^2\,\ln(x) & \text{if } c=0\\ \frac{b\,c\,d^2\,x}{2}+\frac{a\,d^2\,(2\,\ln(x)-c^2\,x^2+c\,x\,4\mathrm{i})}{2}-\frac{b\,d^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,d^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,d^2\,\ln(c^2\,x^2+1)\,1\mathrm{i}-b\,c^2\,d^2\,\mathrm{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)+b\,c\,d^2\,x\,\mathrm{atan}(c\,x)\,2\mathrm{i} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x,x)
```

```
[Out] piecewise(c == 0, a*d^2*log(x), c ~= 0, - b*d^2*log(c^2*x^2 + 1)*1i + (a*d^
2*(2*log(x) + c*x*4i - c^2*x^2))/2 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2 + (b*
d^2*dilog(c*x*1i + 1)*1i)/2 + (b*c*d^2*x)/2 - b*c^2*d^2*atan(c*x)*(1/(2*c^2
) + x^2/2) + b*c*d^2*x*atan(c*x)*2i)
```

3.15 $\int \frac{(d+icdx)^2(a+b\text{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=89

$$-ac^2d^2x-bc^2d^2x\text{ArcTan}(cx)-\frac{d^2(a+b\text{ArcTan}(cx))}{x}+2iacd^2\log(x)+bcd^2\log(x)-bcd^2\text{PolyLog}(2,-icx)+bc$$

```
[Out] -a*c^2*d^2*x-b*c^2*d^2*x*arctan(c*x)-d^2*(a+b*arctan(c*x))/x+2*I*a*c*d^2*ln
(x)+b*c*d^2*ln(x)-b*c*d^2*polylog(2,-I*c*x)+b*c*d^2*polylog(2,I*c*x)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4930, 266, 4946, 272, 36, 29, 31, 4940, 2438}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{x}-ac^2d^2x+2iacd^2\log(x)-bc^2d^2x\text{ArcTan}(cx)-bcd^2\text{Li}_2(-icx)+bcd^2\text{Li}_2(icx)+bcd^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -(a*c^2*d^2*x) - b*c^2*d^2*x*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/x + (2
*I)*a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, (-I)*c*x] + b*c*d^
2*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^2 (a+b\tan^{-1}(cx))}{x^2}\,dx = \int \left(-c^2d^2(a+b\tan^{-1}(cx)) + \frac{d^2(a+b\tan^{-1}(cx))}{x^2} + \frac{2icd^2(a+b\tan\right.$$
$$= d^2\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx + (2icd^2)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx - (c^2d^2)\int$$
$$= -ac^2d^2x - \frac{d^2(a+b\tan^{-1}(cx))}{x} + 2iacd^2\log(x) + (bcd^2)\int \frac{1}{x(1+c^2}$$
$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2(a+b\tan^{-1}(cx))}{x} + 2iacd^2\log(x) -$$
$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2(a+b\tan^{-1}(cx))}{x} + 2iacd^2\log(x) +$$
$$= -ac^2d^2x - bc^2d^2x\tan^{-1}(cx) - \frac{d^2(a+b\tan^{-1}(cx))}{x} + 2iacd^2\log(x) +$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.89

$$-\frac{d^2(a+ac^2x^2+b\text{ArcTan}(cx)+bc^2x^2\text{ArcTan}(cx)-2iacx\log(x)-bcx\log(cx)+bcx\text{PolyLog}(2,-icx)-bcx\text{PolyLog}(2,icx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d^2*(a + a*c^2*x^2 + b*ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - (2*I)*a*c*x
*Log[x] - b*c*x*Log[c*x] + b*c*x*PolyLog[2, (-I)*c*x] - b*c*x*PolyLog[2, I*
c*x]))/x)
```

Maple [A]
time = 0.07, size = 149, normalized size = 1.67

method	result
derivativedivides	$c\Big(-ac\,d^2x - \frac{d^2a}{cx} + 2id^2a\ln(cx) - b\arctan(cx)\,d^2cx - \frac{d^2b\arctan(cx)}{cx} + 2id^2b\arctan(cx)$
default	$c\Big(-ac\,d^2x - \frac{d^2a}{cx} + 2id^2a\ln(cx) - b\arctan(cx)\,d^2cx - \frac{d^2b\arctan(cx)}{cx} + 2id^2b\arctan(cx)$
risch	$-\frac{ic^2d^2b\ln(-icx+1)x}{2} + \frac{ib\,d^2\ln(icx+1)}{2x} - bc\,d^2 + c\,d^2b\,\text{dilog}(-icx+1) + \frac{c\,d^2b\ln(-icx)}{2} + 2ic\,d^2a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a*c*d^2*x-d^2*a/c/x+2*I*d^2*a*ln(c*x)-b*arctan(c*x)*d^2*c*x-d^2*b*arcta
n(c*x)/c/x+2*I*d^2*b*arctan(c*x)*ln(c*x)-d^2*b*ln(c*x)*ln(1+I*c*x)+d^2*b*ln
(c*x)*ln(1-I*c*x)+d^2*b*dilog(1-I*c*x)-d^2*b*dilog(1+I*c*x)+d^2*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -a*c^2*d^2*x - 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^2 + 2*I*b*c
*d^2*integrate(arctan(c*x)/x, x) + 2*I*a*c*d^2*log(x) - 1/2*(c*(log(c^2*x^2
 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^2 - a*d^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^{2}\left(\int ac^{2}\,dx+\int\left(-\frac{a}{x^{2}}\right)dx+\int bc^{2}\operatorname{atan}{\left(cx\right)}\,dx+\int\left(-\frac{b\operatorname{atan}{\left(cx\right)}}{x^{2}}\right)dx+\int\left(-\frac{2iac}{x}\right)dx+\int\left(-\frac{2ibc\operatorname{atan}{\left(cx\right)}}{x}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**2,x)
```

```
[Out] -d**2*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*atan(c*
x), x) + Integral(-b*atan(c*x)/x**2, x) + Integral(-2*I*a*c/x, x) + Integra
l(-2*I*b*c*atan(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.60, size = 141, normalized size = 1.58

$$\begin{cases} -\frac{a\,d^2}{x} & \text{if}\ \ c=0 \\ \frac{b\,d^2\,\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+b\,c\,d^2\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)+\frac{b\,c\,d^2\,\ln(c^2\,x^2+1)}{2}-\frac{a\,d^2\,(c^2\,x^2+1-c\,x\,\ln(x)\,2\mathrm{i})}{x}-\frac{b\,d^2\,\mathrm{atan}(c\,x)}{x}-b\,c^2\,d^2\,x\,\mathrm{atan}(c\,x) & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^2,x)
```

```
[Out] piecewise(c == 0, -(a*d^2)/x, c ~= 0, (b*d^2*(c^2*log(x) - (c^2*log(c^2*x^2
 + 1))/2))/c + b*c*d^2*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + (b*c*d^2
*log(c^2*x^2 + 1))/2 - (a*d^2*(c^2*x^2 - c*x*log(x)*2i + 1))/x - (b*d^2*ata
n(c*x))/x - b*c^2*d^2*x*atan(c*x))
```

3.16 $\int \frac{(d+icdx)^2(a+b\textbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{bcd^2}{2x}-\frac{1}{2}bc^2d^2\mathrm{ArcTan}(cx)-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{2x^2}-\frac{2icd^2(a+b\mathrm{ArcTan}(cx))}{x}-ac^2d^2\log(x)+2ibc^2d^2\log(x)-$$

```
[Out] -1/2*b*c*d^2/x-1/2*b*c^2*d^2*arctan(c*x)-1/2*d^2*(a+b*arctan(c*x))/x^2-2*I*
c*d^2*(a+b*arctan(c*x))/x-a*c^2*d^2*ln(x)+2*I*b*c^2*d^2*ln(x)-I*b*c^2*d^2*l
n(c^2*x^2+1)-1/2*I*b*c^2*d^2*polylog(2,-I*c*x)+1/2*I*b*c^2*d^2*polylog(2,I*
c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438}

$$-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{2x^2}-\frac{2icd^2(a+b\mathrm{ArcTan}(cx))}{x}-ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\mathrm{ArcTan}(cx)-\frac{1}{2}ibc^2d^2\mathrm{Li}_2(-icx)+\frac{1}{2}ibc^2d^2\mathrm{Li}_2(icx)-ibc^2d^2\log\left(c^2x^2+1\right)+2ibc^2d^2\log(x)-\frac{bcd^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2)/x - (b*c^2*d^2*ArcTan[c*x])/2 - (d^2*(a + b*ArcTan[c*x]))/(2
*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x]))/x - a*c^2*d^2*Log[x] + (2*I)*b*c^
2*d^2*Log[x] - I*b*c^2*d^2*Log[1 + c^2*x^2] - (I/2)*b*c^2*d^2*PolyLog[2, (-
I)*c*x] + (I/2)*b*c^2*d^2*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^2 (a+b\tan^{-1}(cx))}{x^3} dx = \int \left(\frac{d^2(a+b\tan^{-1}(cx))}{x^3} + \frac{2icd^2(a+b\tan^{-1}(cx))}{x^2} - \frac{c^2d^2(a+b\tan^{-1}(\right.$$

$$= d^2 \int \frac{a+b\tan^{-1}(cx)}{x^3} dx + (2icd^2) \int \frac{a+b\tan^{-1}(cx)}{x^2} dx - (c^2d^2) \int \frac{a}{}$$

$$= -\frac{d^2(a+b\tan^{-1}(cx))}{2x^2} - \frac{2icd^2(a+b\tan^{-1}(cx))}{x} - ac^2d^2\log(x) + \frac{1}{2}(bc$$

$$= -\frac{bcd^2}{2x} - \frac{d^2(a+b\tan^{-1}(cx))}{2x^2} - \frac{2icd^2(a+b\tan^{-1}(cx))}{x} - ac^2d^2\log(x)$$

$$= -\frac{bcd^2}{2x} - \frac{1}{2}bc^2d^2\tan^{-1}(cx) - \frac{d^2(a+b\tan^{-1}(cx))}{2x^2} - \frac{2icd^2(a+b\tan^{-1}(}{x}$$

$$= -\frac{bcd^2}{2x} - \frac{1}{2}bc^2d^2\tan^{-1}(cx) - \frac{d^2(a+b\tan^{-1}(cx))}{2x^2} - \frac{2icd^2(a+b\tan^{-1}(}{x}$$

Mathematica [A]
time = 0.05, size = 137, normalized size = 0.90

$$-\frac{d^2(a+4iacx+bcx+b\text{ArcTan}(cx)+4ibcx\text{ArcTan}(cx)+bc^2x^2\text{ArcTan}(cx)+2ac^2x^2\log(x)-4ibc^2x^2\log(cx)+2ibc^2x^2\log(1+c^2x^2)+ibc^2x^2\text{PolyLog}(2,-icx)-ibc^2x^2\text{PolyLog}(2,icx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(d^2*(a + (4*I)*a*c*x + b*c*x + b*ArcTan[c*x] + (4*I)*b*c*x*ArcTan[c*x
] + b*c^2*x^2*ArcTan[c*x] + 2*a*c^2*x^2*Log[x] - (4*I)*b*c^2*x^2*Log[c*x] +
 (2*I)*b*c^2*x^2*Log[1 + c^2*x^2] + I*b*c^2*x^2*PolyLog[2, (-I)*c*x] - I*b*
c^2*x^2*PolyLog[2, I*c*x]))/x^2
```

Maple [A]
time = 0.09, size = 206, normalized size = 1.36

method	result
derivativedivides	$c^2\left(-\frac{d^2a}{2c^2x^2} - \frac{2id^2a}{cx} - d^2a\ln(cx) - \frac{d^2b\arctan(cx)}{2c^2x^2} - \frac{2id^2b\arctan(cx)}{cx} - d^2b\ln(cx)\arctan(cx) - \right.$
default	$c^2\left(-\frac{d^2a}{2c^2x^2} - \frac{2id^2a}{cx} - d^2a\ln(cx) - \frac{d^2b\arctan(cx)}{2c^2x^2} - \frac{2id^2b\arctan(cx)}{cx} - d^2b\ln(cx)\arctan(cx) - \right.$
risch	$-\frac{2id^2ca}{x} - \frac{ib\,d^2c^2\,\text{dilog}(icx+1)}{2} + \frac{ib\,d^2\ln(icx+1)}{4x^2} - \frac{b\,c^2d^2\arctan(cx)}{2} + \frac{d^2cb\ln(-icx+1)}{x} - \frac{bc\,d^2}{2x} + \frac{3ib\,d^2c^2\ln}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*d^2*a/c^2/x^2-2*I*d^2*a/c/x-d^2*a*ln(c*x)-1/2*d^2*b*arctan(c*x)/c
^2/x^2-2*I*d^2*b*arctan(c*x)/c/x-d^2*b*ln(c*x)*arctan(c*x)-1/2*I*d^2*b*ln(c
```

```
*x)*ln(1+I*c*x)+1/2*I*d^2*b*ln(c*x)*ln(1-I*c*x)-1/2*I*d^2*b*dilog(1+I*c*x)+
1/2*I*d^2*b*dilog(1-I*c*x)-I*b*d^2*ln(c^2*x^2+1)-1/2*b*d^2*arctan(c*x)-1/2*
d^2*b/c/x+2*I*d^2*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -b*c^2*d^2*integrate(arctan(c*x)/x, x) - a*c^2*d^2*log(x) - I*(c*(log(c^2*x
^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^2 - 1/2*((c*arctan(c*x) + 1/x)
*c + arctan(c*x)/x^2)*b*d^2 - 2*I*a*c*d^2/x - 1/2*a*d^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x^
2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^{2}\left(\int\left(-\frac{a}{x^{3}}\right)dx+\int\frac{ac^{2}}{x}\,dx+\int\left(-\frac{b\operatorname{atan}{(cx)}}{x^{3}}\right)dx+\int\left(-\frac{2iac}{x^{2}}\right)dx+\int\frac{bc^{2}\operatorname{atan}{(cx)}}{x}\,dx+\int\left(-\frac{2ibc\operatorname{atan}{(cx)}}{x^{2}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**3,x)
```

```
[Out] -d**2*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*atan(c*x)
/x**3, x) + Integral(-2*I*a*c/x**2, x) + Integral(b*c**2*atan(c*x)/x, x) +
Integral(-2*I*b*c*atan(c*x)/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.74, size = 161, normalized size = 1.06

$$\begin{cases} -\frac{a\,d^2}{2\,x^2} & \text{if}\ \ c=0 \\ b\,d^2\,\left(c^2\,\ln\left(x\right)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)\,2\mathrm{i}+\frac{b\,c^2\,d^2\,\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}-\frac{b\,c^2\,d^2\,\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}-\frac{b\,d^2\,\left(c^3\,\mathrm{atan}\left(c\,x\right)+\frac{c^2}{x}\right)}{2\,c}-\frac{a\,d^2\,\left(2\,c^2\,x^2\,\ln\left(x\right)+1+c\,x\,4\mathrm{i}\right)}{2\,x^2}-\frac{b\,d^2\,\mathrm{atan}\left(c\,x\right)}{2\,x^2}-\frac{b\,c\,d^2\,\mathrm{atan}\left(c\,x\right)\,2\mathrm{i}}{x} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^3,x)
```

```
[Out] piecewise(c == 0, -(a*d^2)/(2*x^2), c ~= 0, b*d^2*(c^2*log(x) - (c^2*log(c^
2*x^2 + 1))/2)*2i + (b*c^2*d^2*dilog(- c*x*1i + 1)*1i)/2 - (b*c^2*d^2*dilog
(c*x*1i + 1)*1i)/2 - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (a*d^2*(c*x*4i
 + 2*c^2*x^2*log(x) + 1))/(2*x^2) - (b*d^2*atan(c*x))/(2*x^2) - (b*c*d^2*at
an(c*x)*2i)/x)
```

3.17 $\int \frac{(d+icdx)^2(a+b\text{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{bcd^2}{6x^2}-\frac{ibc^2d^2}{x}-\frac{d^2(1+icx)^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(i+cx)$$

```
[Out] -1/6*b*c*d^2/x^2-I*b*c^2*d^2/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c*x))/x^3-4/
3*b*c^3*d^2*ln(x)+4/3*b*c^3*d^2*ln(c*x+I)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {37, 4992, 12, 90}

$$-\frac{d^2(1+icx)^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(cx+i)-\frac{ibc^2d^2}{x}-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2)/x^2 - (I*b*c^2*d^2)/x - (d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x
]))/(3*x^3) - (4*b*c^3*d^2*Log[x])/3 + (4*b*c^3*d^2*Log[I + c*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^2\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= -\frac{d^2(1+icx)^3\,(a+b\tan^{-1}(cx))}{3x^3}-(bc)\int \frac{id^2(i-cx)^2}{3x^3(i+cx)}\,dx\\ &= -\frac{d^2(1+icx)^3\,(a+b\tan^{-1}(cx))}{3x^3}-\frac{1}{3}\left(ibcd^2\right)\int \frac{(i-cx)^2}{x^3(i+cx)}\,dx\\ &= -\frac{d^2(1+icx)^3\,(a+b\tan^{-1}(cx))}{3x^3}-\frac{1}{3}\left(ibcd^2\right)\int \left(\frac{i}{x^3}-\frac{3c}{x^2}-\frac{4ic^2}{x}+\frac{4}{i+}\right.\\ &= -\frac{bcd^2}{6x^2}-\frac{ibc^2d^2}{x}-\frac{d^2(1+icx)^3\,(a+b\tan^{-1}(cx))}{3x^3}-\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 109, normalized size = 1.25

$$\frac{d^2\left(-2a-6iacx-bcx+6ac^2x^2-6ibc^2x^2+2b(-1-3icx+3c^2x^2-3ic^3x^3)\operatorname{ArcTan}(cx)-8bc^3x^3\log(x)+4bc^3x^3\log\left(1+c^2x^2\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
[Out] (d^2*(-2*a - (6*I)*a*c*x - b*c*x + 6*a*c^2*x^2 - (6*I)*b*c^2*x^2 + 2*b*(-1
- (3*I)*c*x + 3*c^2*x^2 - (3*I)*c^3*x^3)*ArcTan[c*x] - 8*b*c^3*x^3*Log[x] +
 4*b*c^3*x^3*Log[1 + c^2*x^2]))/(6*x^3)
```

Maple [A]
time = 0.12, size = 146, normalized size = 1.68

method	result
derivativedivides	$c^3\Big(d^2a\big(-\frac{i}{c^2x^2}-\frac{1}{3c^3x^3}+\frac{1}{cx}\big)-\frac{id^2b\arctan(cx)}{c^2x^2}-\frac{d^2b\arctan(cx)}{3c^3x^3}+\frac{d^2b\arctan(cx)}{cx}+\frac{2b\ln(c^2x^2+1)d^2}{3}-$
default	$c^3\Big(d^2a\big(-\frac{i}{c^2x^2}-\frac{1}{3c^3x^3}+\frac{1}{cx}\big)-\frac{id^2b\arctan(cx)}{c^2x^2}-\frac{d^2b\arctan(cx)}{3c^3x^3}+\frac{d^2b\arctan(cx)}{cx}+\frac{2b\ln(c^2x^2+1)d^2}{3}-$
risch	$-\frac{id^2b(3c^2x^2-3icx-1)\ln(icx+1)}{6x^3}+\frac{d^2(c^3b\ln(cx-i)x^3-8c^3b\ln(-x)x^3+7c^3b\ln(-cx-i)x^3+3ib\,c^2x^2\ln(-icx+1)-6ib\,c}{6x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d^2*a*(-I/c^2/x^2-1/3/c^3/x^3+1/c/x)-I*d^2*b*arctan(c*x)/c^2/x^2-1/3*d
^2*b*arctan(c*x)/c^3/x^3+d^2*b*arctan(c*x)/c/x+2/3*b*ln(c^2*x^2+1)*d^2-I*d^
2*b*arctan(c*x)-I*d^2*b/c/x-1/6*d^2*b/c^2/x^2-4/3*d^2*b*ln(c*x))
```

Maxima [A]
time = 0.46, size = 144, normalized size = 1.66

$$\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^2d^2-i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd^2+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^2+\frac{ac^2d^2}{x}-\frac{iacd^2}{x^2}-\frac{ad^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^2 - I*((c*a
rctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^2 + 1/6*((c^2*log(c^2*x^2 + 1)
 - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^2 + a*c^2*d^2/x - I*a*c
*d^2/x^2 - 1/3*a*d^2/x^3
```

Fricas [A]
time = 3.28, size = 144, normalized size = 1.66

$$-\frac{8\,bc^3d^2x^3\log\left(x\right)-7\,bc^3d^2x^3\log\left(\frac{cx+i}{c}\right)-bc^3d^2x^3\log\left(\frac{cx-i}{c}\right)-6\left(a-i\,b\right)c^2d^2x^2-\left(-6i\,a-b\right)cd^2x+2\,ad^2-\left(3i\,bc^2d^2x^2+3\,bcd^2x-i\,bd^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(8*b*c^3*d^2*x^3*log(x) - 7*b*c^3*d^2*x^3*log((c*x + I)/c) - b*c^3*d^2
*x^3*log((c*x - I)/c) - 6*(a - I*b)*c^2*d^2*x^2 - (-6*I*a - b)*c*d^2*x + 2*
a*d^2 - (3*I*b*c^2*d^2*x^2 + 3*b*c*d^2*x - I*b*d^2)*log(-(c*x + I)/(c*x - I
)))/x^3
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(82) = 164$.
time = 5.18, size = 253, normalized size = 2.91

$$-\frac{4bc^3d^2\log\left(135b^2c^7d^4x\right)}{3}+\frac{bc^3d^2\log\left(135b^2c^7d^4x-135ib^2c^6d^4\right)}{6}+\frac{7bc^3d^2\log\left(135b^2c^7d^4x+135ib^2c^6d^4\right)}{6}-\frac{2ad^2+x^2\left(-6ac^2d^2+6ibc^2d^2\right)+x\left(6iacd^2+bcd^2\right)}{6x^3}+\frac{\left(-3ibc^2d^2x^2-3bcd^2x+ibd^2\right)\log\left(icx+1\right)}{6x^3}+\frac{\left(3ibc^2d^2x^2+3bcd^2x-ibd^2\right)\log\left(-icx+1\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**4,x)
```

```
[Out] -4*b*c**3*d**2*log(135*b**2*c**7*d**4*x)/3 + b*c**3*d**2*log(135*b**2*c**7*
d**4*x - 135*I*b**2*c**6*d**4)/6 + 7*b*c**3*d**2*log(135*b**2*c**7*d**4*x +
 135*I*b**2*c**6*d**4)/6 - (2*a*d**2 + x**2*(-6*a*c**2*d**2 + 6*I*b*c**2*d*
*2) + x*(6*I*a*c*d**2 + b*c*d**2))/(6*x**3) + (-3*I*b*c**2*d**2*x**2 - 3*b*
c*d**2*x + I*b*d**2)*log(I*c*x + 1)/(6*x**3) + (3*I*b*c**2*d**2*x**2 + 3*b*
c*d**2*x - I*b*d**2)*log(-I*c*x + 1)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.64, size = 120, normalized size = 1.38

$$-\frac{d^2\,\left(8\,b\,c^3\,\ln\left(x\right)-4\,b\,c^3\,\ln\left(c^2\,x^2+1\right)+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,6{}\mathrm{i}\right)}{6}-\frac{\frac{d^2\,\left(2\,a+2\,b\,\mathrm{atan}\left(c\,x\right)\right)}{6}+\frac{d^2\,x\,\left(a\,c\,6{}\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,6{}\mathrm{i}\right)}{6}-\frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,6{}\mathrm{i}\right)}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^4,x)
[Out] - (d^2*(b*c^3*atan(c*x)*6i - 4*b*c^3*log(c^2*x^2 + 1) + 8*b*c^3*log(x)))/6
- ((d^2*(2*a + 2*b*atan(c*x)))/6 + (d^2*x*(a*c*6i + b*c + b*c*atan(c*x)*6i)
)/6 - (d^2*x^2*(6*a*c^2 - b*c^2*6i + 6*b*c^2*atan(c*x)))/6)/x^3
```

3.18 $\int \frac{(d+icdx)^2(a+b\text{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=161

$$-\frac{bcd^2}{12x^3}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x}-\frac{d^2(a+b\text{ArcTan}(cx))}{4x^4}-\frac{2icd^2(a+b\text{ArcTan}(cx))}{3x^3}+\frac{c^2d^2(a+b\text{ArcTan}(cx))}{2x^2}-\frac{2}{3}ibc^4$$

```
[Out] -1/12*b*c*d^2/x^3-1/3*I*b*c^2*d^2/x^2+3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctan(c
*x))/x^4-2/3*I*c*d^2*(a+b*arctan(c*x))/x^3+1/2*c^2*d^2*(a+b*arctan(c*x))/x^
2-2/3*I*b*c^4*d^2*ln(x)-1/24*I*b*c^4*d^2*ln(I-c*x)+17/24*I*b*c^4*d^2*ln(c*x
+I)
```

Rubi [A]
time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {45, 4992, 12, 1816}

$$\frac{c^2d^2(a+b\text{ArcTan}(cx))}{2x^2}-\frac{d^2(a+b\text{ArcTan}(cx))}{4x^4}-\frac{2icd^2(a+b\text{ArcTan}(cx))}{3x^3}-\frac{2}{3}ibc^4d^2\log(x)-\frac{1}{24}ibc^4d^2\log(-cx+i)+\frac{17}{24}ibc^4d^2\log(cx+i)+\frac{3bc^3d^2}{4x}-\frac{ibc^2d^2}{3x^2}-\frac{bcd^2}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^2)/x^3 - ((I/3)*b*c^2*d^2)/x^2 + (3*b*c^3*d^2)/(4*x) - (d^2*(a
 + b*ArcTan[c*x]))/(4*x^4) - (((2*I)/3)*c*d^2*(a + b*ArcTan[c*x]))/x^3 + (c
^2*d^2*(a + b*ArcTan[c*x]))/(2*x^2) - ((2*I)/3)*b*c^4*d^2*Log[x] - (I/24)*b
*c^4*d^2*Log[I - c*x] + ((17*I)/24)*b*c^4*d^2*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)^2\,(a+b\tan^{-1}(cx))}{x^5}\,dx = -\frac{d^2(a+b\tan^{-1}(cx))}{4x^4} - \frac{2icd^2(a+b\tan^{-1}(cx))}{3x^3} + \frac{c^2d^2(a+b\tan^{-1}(cx}{2x^2}$$
$$= -\frac{d^2(a+b\tan^{-1}(cx))}{4x^4} - \frac{2icd^2(a+b\tan^{-1}(cx))}{3x^3} + \frac{c^2d^2(a+b\tan^{-1}(cx}{2x^2}$$
$$= -\frac{d^2(a+b\tan^{-1}(cx))}{4x^4} - \frac{2icd^2(a+b\tan^{-1}(cx))}{3x^3} + \frac{c^2d^2(a+b\tan^{-1}(cx}{2x^2}$$
$$= -\frac{bcd^2}{12x^3} - \frac{ibc^2d^2}{3x^2} + \frac{3bc^3d^2}{4x} - \frac{d^2(a+b\tan^{-1}(cx))}{4x^4} - \frac{2icd^2(a+b\tan^{-1}(}{3x^3}$$

Mathematica [A]
time = 0.11, size = 119, normalized size = 0.74

$$\frac{d^2(-3a-8iacx-bcx+6ac^2x^2-4ibc^2x^2+9bc^3x^3+b(-3-8icx+6c^2x^2+9c^4x^4)\operatorname{ArcTan}(cx)-8ibc^4x^4\log(x)+4ibc^4x^4\log(1+c^2x^2))}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^2*(-3*a - (8*I)*a*c*x - b*c*x + 6*a*c^2*x^2 - (4*I)*b*c^2*x^2 + 9*b*c^3*
x^3 + b*(-3 - (8*I)*c*x + 6*c^2*x^2 + 9*c^4*x^4)*ArcTan[c*x] - (8*I)*b*c^4*
x^4*Log[x] + (4*I)*b*c^4*x^4*Log[1 + c^2*x^2]))/(12*x^4)
```

Maple [A]
time = 0.16, size = 161, normalized size = 1.00

method	result
derivativedivides	$c^4\left(d^2a\left(\frac{1}{2c^2x^2}-\frac{2i}{3c^3x^3}-\frac{1}{4c^4x^4}\right)+\frac{d^2b\arctan(cx)}{2c^2x^2}-\frac{2id^2b\arctan(cx)}{3c^3x^3}-\frac{d^2b\arctan(cx)}{4c^4x^4}+\frac{id^2b\ln(c^2x^2+1)}{3}\right.$
default	$c^4\left(d^2a\left(\frac{1}{2c^2x^2}-\frac{2i}{3c^3x^3}-\frac{1}{4c^4x^4}\right)+\frac{d^2b\arctan(cx)}{2c^2x^2}-\frac{2id^2b\arctan(cx)}{3c^3x^3}-\frac{d^2b\arctan(cx)}{4c^4x^4}+\frac{id^2b\ln(c^2x^2+1)}{3}\right.$
risch	$-\frac{id^2b(6c^2x^2-8icx-3)\ln(icx+1)}{24x^4}+\frac{id^2\left(17bc^4\ln(-99cx-99i)x^4-bc^4\ln(45cx-45i)x^4-16bc^4\ln(-165cx)x^4+6bc^2x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d^2*a*(1/2/c^2/x^2-2/3*I/c^3/x^3-1/4/c^4/x^4)+1/2*d^2*b*arctan(c*x)/c^
2/x^2-2/3*I*d^2*b*arctan(c*x)/c^3/x^3-1/4*d^2*b*arctan(c*x)/c^4/x^4+1/3*I*d
^2*b*ln(c^2*x^2+1)+3/4*b*d^2*arctan(c*x)-1/3*I*d^2*b/c^2/x^2-2/3*I*d^2*b*ln
(c*x)-1/12*d^2*b/c^3/x^3+3/4*d^2*b/c/x)
```

Maxima [A]
time = 0.47, size = 152, normalized size = 0.94

$$\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^2d^2+\frac{1}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd^2+\frac{1}{12}\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd^2+\frac{ac^2d^2}{2x^2}-\frac{2i\,acd^2}{3x^3}-\frac{ad^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^2 + 1/3*I*((c^2*log
(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^2 + 1/12
*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^2 +
1/2*a*c^2*d^2/x^2 - 2/3*I*a*c*d^2/x^3 - 1/4*a*d^2/x^4
```

Fricas [A]
time = 2.92, size = 155, normalized size = 0.96

$$\frac{-16i\,bc^4d^2x^4\log\left(x\right)+17i\,bc^4d^2x^4\log\left(\frac{cx+i}{c}\right)-i\,bc^4d^2x^4\log\left(\frac{cx-i}{c}\right)+18\,bc^3d^2x^3+4\left(3\,a-2i\,b\right)c^2d^2x^2-2\left(8i\,a+b\right)cd^2x-6\,ad^2+\left(6i\,bc^2d^2x^2+8\,bcd^2x-3i\,bd^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-16*I*b*c^4*d^2*x^4*log(x) + 17*I*b*c^4*d^2*x^4*log((c*x + I)/c) - I*
b*c^4*d^2*x^4*log((c*x - I)/c) + 18*b*c^3*d^2*x^3 + 4*(3*a - 2*I*b)*c^2*d^2
*x^2 - 2*(8*I*a + b)*c*d^2*x - 6*a*d^2 + (6*I*b*c^2*d^2*x^2 + 8*b*c*d^2*x -
 3*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^4
```

Sympy [A]
time = 9.21, size = 275, normalized size = 1.71

$$-\frac{2ibc^4d^2\log\left(1485b^2c^9d^4x\right)}{3}-\frac{ibc^4d^2\log\left(1485b^2c^9d^4x-1485ib^2c^8d^4\right)}{24}+\frac{17ibc^4d^2\log\left(1485b^2c^9d^4x+1485ib^2c^8d^4\right)}{24}+\frac{\left(-6ibc^2d^2x^2-8bcd^2x+3ibd^2\right)\log\left(icx+1\right)}{24x^4}+\frac{\left(6ibc^2d^2x^2+8bcd^2x-3ibd^2\right)\log\left(-icx+1\right)}{24x^4}-\frac{3ad^2-9bc^3d^2x^3+x^2\left(-6ac^2d^2+4ibc^2d^2\right)+x\left(8iacd^2+bcd^2\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**5,x)
```

```
[Out] -2*I*b*c**4*d**2*log(1485*b**2*c**9*d**4*x)/3 - I*b*c**4*d**2*log(1485*b**2
*c**9*d**4*x - 1485*I*b**2*c**8*d**4)/24 + 17*I*b*c**4*d**2*log(1485*b**2*c
**9*d**4*x + 1485*I*b**2*c**8*d**4)/24 + (-6*I*b*c**2*d**2*x**2 - 8*b*c*d**
2*x + 3*I*b*d**2)*log(I*c*x + 1)/(24*x**4) + (6*I*b*c**2*d**2*x**2 + 8*b*c*
d**2*x - 3*I*b*d**2)*log(-I*c*x + 1)/(24*x**4) - (3*a*d**2 - 9*b*c**3*d**2*
x**3 + x**2*(-6*a*c**2*d**2 + 4*I*b*c**2*d**2) + x*(8*I*a*c*d**2 + b*c*d**2
))/(12*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.70, size = 142, normalized size = 0.88

$$\frac{d^2\left(9\,b\,c^3\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,4\mathrm{i}-b\,c^4\,\ln\left(x\right)\,8\mathrm{i}\right)}{12}-\frac{\frac{d^2\,\left(3\,a+3\,b\,\mathrm{atan}(c\,x)\right)}{12}+\frac{d^2\,x\,\left(a\,c\,8\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,8\mathrm{i}\right)}{12}-\frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,4\mathrm{i}\right)}{12}-\frac{3\,b\,c^3\,d^2\,x^3}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^5,x)
```

```
[Out] (d^2*(b*c^4*log(c^2*x^2 + 1)*4i - b*c^4*log(x)*8i + 9*b*c^3*atan(x*(c^2)^(1
/2))*(c^2)^(1/2)))/12 - ((d^2*(3*a + 3*b*atan(c*x)))/12 + (d^2*x*(a*c*8i +
b*c + b*c*atan(c*x)*8i))/12 - (d^2*x^2*(6*a*c^2 - b*c^2*4i + 6*b*c^2*atan(c
*x)))/12 - (3*b*c^3*d^2*x^3)/4)/x^4
```

3.19 $\int \frac{(d+icdx)^2(a+b\text{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=171

$$-\frac{bcd^2}{20x^4}-\frac{ibc^2d^2}{6x^3}+\frac{4bc^3d^2}{15x^2}+\frac{ibc^4d^2}{2x}-\frac{d^2(a+b\text{ArcTan}(cx))}{5x^5}-\frac{icd^2(a+b\text{ArcTan}(cx))}{2x^4}+\frac{c^2d^2(a+b\text{ArcTan}(cx))}{3x^3}+$$

```
[Out] -1/20*b*c*d^2/x^4-1/6*I*b*c^2*d^2/x^3+4/15*b*c^3*d^2/x^2+1/2*I*b*c^4*d^2/x-
1/5*d^2*(a+b*arctan(c*x))/x^5-1/2*I*c*d^2*(a+b*arctan(c*x))/x^4+1/3*c^2*d^2
*(a+b*arctan(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-1/60*b*c^5*d^2*ln(I-c*x)-31/60*
b*c^5*d^2*ln(c*x+I)
```

Rubi [A]
time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {45, 4992, 12, 1816}

$$\frac{c^2d^2(a+b\text{ArcTan}(cx))}{3x^3}-\frac{d^2(a+b\text{ArcTan}(cx))}{5x^5}-\frac{icd^2(a+b\text{ArcTan}(cx))}{2x^4}+\frac{8}{15}bc^5d^2\log(x)-\frac{1}{60}bc^5d^2\log(-cx+i)-\frac{31}{60}bc^5d^2\log(cx+i)+\frac{ibc^4d^2}{2x}+\frac{4bc^3d^2}{15x^2}-\frac{ibc^2d^2}{6x^3}-\frac{bcd^2}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^2)/x^4 - ((I/6)*b*c^2*d^2)/x^3 + (4*b*c^3*d^2)/(15*x^2) + ((I/
2)*b*c^4*d^2)/x - (d^2*(a + b*ArcTan[c*x]))/(5*x^5) - ((I/2)*c*d^2*(a + b*A
rcTan[c*x]))/x^4 + (c^2*d^2*(a + b*ArcTan[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log
[x])/15 - (b*c^5*d^2*Log[I - c*x])/60 - (31*b*c^5*d^2*Log[I + c*x])/60
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)^2\,(a+b\tan^{-1}(cx))}{x^6}\,dx = -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5} - \frac{icd^2(a+b\tan^{-1}(cx))}{2x^4} + \frac{c^2d^2(a+b\tan^{-1}(cx))}{3x^3}$$
$$= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5} - \frac{icd^2(a+b\tan^{-1}(cx))}{2x^4} + \frac{c^2d^2(a+b\tan^{-1}(cx))}{3x^3}$$
$$= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5} - \frac{icd^2(a+b\tan^{-1}(cx))}{2x^4} + \frac{c^2d^2(a+b\tan^{-1}(cx))}{3x^3}$$
$$= -\frac{bcd^2}{20x^4} - \frac{ibc^2d^2}{6x^3} + \frac{4bc^3d^2}{15x^2} + \frac{ibc^4d^2}{2x} - \frac{d^2(a+b\tan^{-1}(cx))}{5x^5} - \frac{icd^2(a+$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.75

$$\frac{d^2(-12a-30iacx-3bcx+20ac^2x^2-10ibc^2x^2+16bc^3x^3+30ibc^4x^4+2b(-6-15icx+10c^2x^2+15ic^5x^5)\operatorname{ArcTan}(cx)+32bc^5x^5\log(x)-16bc^5x^5\log(1+c^2x^2))}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (d^2*(-12*a - (30*I)*a*c*x - 3*b*c*x + 20*a*c^2*x^2 - (10*I)*b*c^2*x^2 + 16
*b*c^3*x^3 + (30*I)*b*c^4*x^4 + 2*b*(-6 - (15*I)*c*x + 10*c^2*x^2 + (15*I)*
c^5*x^5)*ArcTan[c*x] + 32*b*c^5*x^5*Log[x] - 16*b*c^5*x^5*Log[1 + c^2*x^2])
)/(60*x^5)
```

Maple [A]
time = 0.13, size = 173, normalized size = 1.01

method	result
derivativedivides	$c^5\left(d^2a\left(\frac{1}{3c^3x^3}-\frac{i}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{d^2b\arctan(cx)}{3c^3x^3}-\frac{id^2b\arctan(cx)}{2c^4x^4}-\frac{d^2b\arctan(cx)}{5c^5x^5}-\frac{4b\ln(c^2x^2+1)d^2}{15}\right.$
default	$c^5\left(d^2a\left(\frac{1}{3c^3x^3}-\frac{i}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{d^2b\arctan(cx)}{3c^3x^3}-\frac{id^2b\arctan(cx)}{2c^4x^4}-\frac{d^2b\arctan(cx)}{5c^5x^5}-\frac{4b\ln(c^2x^2+1)d^2}{15}\right.$
risch	$-\frac{id^2b(10c^2x^2-15icx-6)\ln(icx+1)}{60x^5}-\frac{d^2(c^5b\ln(cx-i)x^5+31c^5b\ln(-cx-i)x^5-32c^5b\ln(-x)x^5-30ib\,c^4x^4-10ib\,c^2x^2\ln$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)

[Out] c^5*(d^2*a*(1/3/c^3/x^3-1/2*I/c^4/x^4-1/5/c^5/x^5)+1/3*d^2*b*arctan(c*x)/c^3/x^3-1/2*I*d^2*b*arctan(c*x)/c^4/x^4-1/5*d^2*b*arctan(c*x)/c^5/x^5-4/15*b*ln(c^2*x^2+1)*d^2+1/2*I*d^2*b*arctan(c*x)-1/6*I*d^2*b/c^3/x^3+1/2*I*d^2*b/c/x-1/20*d^2*b/c^4/x^4+4/15*d^2*b/c^2/x^2+8/15*d^2*b*ln(c*x))

Maxima [A]
time = 0.47, size = 183, normalized size = 1.07

$$-\frac{1}{6}\left(\left(c^{2}\log\left(c^{2}x^{2}+1\right)-c^{2}\log\left(x^{2}\right)-\frac{1}{x^{2}}\right)c-\frac{2\arctan\left(cx\right)}{x^{3}}\right)bc^{2}d^{2}+\frac{1}{6}i\left(\left(3c^{3}\arctan\left(cx\right)+\frac{3c^{2}x^{2}-1}{x^{3}}\right)c-\frac{3\arctan\left(cx\right)}{x^{4}}\right)bcd^{2}-\frac{1}{20}\left(\left(2c^{4}\log\left(c^{2}x^{2}+1\right)-2c^{4}\log\left(x^{2}\right)-\frac{2c^{2}x^{2}-1}{x^{4}}\right)c+\frac{4\arctan\left(cx\right)}{x^{5}}\right)bd^{2}+\frac{ac^{2}d^{2}}{3x^{3}}-\frac{iacd^{2}}{2x^{4}}-\frac{ad^{2}}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")

[Out] -1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^2 + 1/6*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2 + 1/3*a*c^2*d^2/x^3 - 1/2*I*a*c*d^2/x^4 - 1/5*a*d^2/x^5

Fricas [A]
time = 1.44, size = 167, normalized size = 0.98

$$\frac{32bc^{5}d^{2}x^{5}\log\left(x\right)-31bc^{5}d^{2}x^{5}\log\left(\frac{cx+i}{c}\right)-bc^{5}d^{2}x^{5}\log\left(\frac{cx-i}{c}\right)+30i\,bc^{4}d^{2}x^{4}+16bc^{3}d^{2}x^{3}+10\left(2a-ib\right)c^{2}d^{2}x^{2}-3\left(10ia+b\right)cd^{2}x-12ad^{2}+\left(10i\,bc^{2}d^{2}x^{2}+15bcd^{2}x-6i\,bd^{2}\right)\log\left(-\frac{cx+i}{cx-i}\right)}{60x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")

[Out] 1/60*(32*b*c^5*d^2*x^5*log(x) - 31*b*c^5*d^2*x^5*log((c*x + I)/c) - b*c^5*d^2*x^5*log((c*x - I)/c) + 30*I*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a - I*b)*c^2*d^2*x^2 - 3*(10*I*a + b)*c*d^2*x - 12*a*d^2 + (10*I*b*c^2*d^2*x^2 + 15*b*c*d^2*x - 6*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^5

Sympy [A]
time = 15.02, size = 287, normalized size = 1.68

$$\frac{8bc^{5}d^{2}\log\left(10395b^{2}c^{11}d^{4}x\right)}{15}-\frac{bc^{5}d^{2}\log\left(10395b^{2}c^{11}d^{4}x-10395ib^{2}c^{10}d^{4}\right)}{60}-\frac{31bc^{5}d^{2}\log\left(10395b^{2}c^{11}d^{4}x+10395ib^{2}c^{10}d^{4}\right)}{60}+\frac{\left(-10ibc^{2}d^{2}x^{2}-15bcd^{2}x+6ibd^{2}\right)\log\left(icx+1\right)}{60x^{5}}+\frac{\left(10ibc^{2}d^{2}x^{2}+15bcd^{2}x-6ibd^{2}\right)\log\left(-icx+1\right)}{60x^{5}}-\frac{12ad^{2}-30ibc^{4}d^{2}x^{4}-16bc^{3}d^{2}x^{3}+x^{2}\left(-20ac^{2}d^{2}+10ibc^{2}d^{2}\right)+x\left(30iacd^{2}+3bcd^{2}\right)}{60x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**6,x)

[Out] 8*b*c**5*d**2*log(10395*b**2*c**11*d**4*x)/15 - b*c**5*d**2*log(10395*b**2*c**11*d**4*x - 10395*I*b**2*c**10*d**4)/60 - 31*b*c**5*d**2*log(10395*b**2*c**11*d**4*x + 10395*I*b**2*c**10*d**4)/60 + (-10*I*b*c**2*d**2*x**2 - 15*b*c*d**2*x + 6*I*b*d**2)*log(I*c*x + 1)/(60*x**5) + (10*I*b*c**2*d**2*x**2 + 15*b*c*d**2*x - 6*I*b*d**2)*log(-I*c*x + 1)/(60*x**5) - (12*a*d**2 - 30*I*

```
b*c**4*d**2*x**4 - 16*b*c**3*d**2*x**3 + x**2*(-20*a*c**2*d**2 + 10*I*b*c**
2*d**2) + x*(30*I*a*c*d**2 + 3*b*c*d**2))/(60*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.92, size = 244, normalized size = 1.43

$$\frac{8\,b\,c^5\,d^2\,\ln\left(x\right)}{15}-\frac{4\,b\,c^5\,d^2\,\ln\left(c^2\,x^2+1\right)}{15}-\frac{\frac{a\,d^2}{5}+\frac{b\,d^2\,\mathrm{atan}\left(c\,x\right)}{5}-\frac{4\,b\,c^5\,d^2\,x^5}{15}-\frac{b\,c^6\,d^2\,x^6\,1{}\mathrm{i}}{2}-\frac{c^4\,d^2\,x^4\,\left(a+b\,1{}\mathrm{i}\right)}{3}+\frac{c\,d^2\,x\,\left(b+a\,10{}\mathrm{i}\right)}{20}-\frac{c^2\,d^2\,x^2\,\left(4\,a-b\,5{}\mathrm{i}\right)}{30}+\frac{c^3\,d^2\,x^3\,\left(-13\,b+a\,30{}\mathrm{i}\right)}{60}-\frac{2\,b\,c^2\,d^2\,x^2\,\mathrm{atan}\left(c\,x\right)}{15}+\frac{b\,c^3\,d^2\,x^3\,\mathrm{atan}\left(c\,x\right)\,1{}\mathrm{i}}{2}-\frac{b\,c^4\,d^2\,x^4\,\mathrm{atan}\left(c\,x\right)}{3}+\frac{b\,c\,d^2\,x\,\mathrm{atan}\left(c\,x\right)\,1{}\mathrm{i}}{2}}{c^2\,x^7+x^5}+\frac{b\,c^8\,d^2\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,1{}\mathrm{i}}{2\,{\left(c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^6,x)
```

```
[Out] (8*b*c^5*d^2*log(x))/15 - (4*b*c^5*d^2*log(c^2*x^2 + 1))/15 - ((a*d^2)/5 +
(b*d^2*atan(c*x))/5 - (4*b*c^5*d^2*x^5)/15 - (b*c^6*d^2*x^6*1i)/2 - (c^4*d^
2*x^4*(a + b*1i))/3 + (c*d^2*x*(a*10i + b))/20 - (c^2*d^2*x^2*(4*a - b*5i))
/30 + (c^3*d^2*x^3*(a*30i - 13*b))/60 - (2*b*c^2*d^2*x^2*atan(c*x))/15 + (b
*c^3*d^2*x^3*atan(c*x)*1i)/2 - (b*c^4*d^2*x^4*atan(c*x))/3 + (b*c*d^2*x*ata
n(c*x)*1i)/2)/(x^5 + c^2*x^7) + (b*c^8*d^2*atan((c^2*x)/(c^2)^(1/2))*1i)/(2
*(c^2)^(3/2))
```

3.20 $\int x^3(d+icdx)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=205

$$\frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6-\frac{3bd^3\text{ArcTan}(cx)}{4c^4}+\frac{1}{4}d^3x^4(a+b\text{ArcTan}(cx))+\frac{3}{5}$$

```
[Out] 3/4*b*d^3*x/c^3+13/35*I*b*d^3*x^2/c^2-1/4*b*d^3*x^3/c-13/70*I*b*d^3*x^4+1/1
0*b*c*d^3*x^5+1/42*I*b*c^2*d^3*x^6-3/4*b*d^3*arctan(c*x)/c^4+1/4*d^3*x^4*(a
+b*arctan(c*x))+3/5*I*c*d^3*x^5*(a+b*arctan(c*x))-1/2*c^2*d^3*x^6*(a+b*arct
an(c*x))-1/7*I*c^3*d^3*x^7*(a+b*arctan(c*x))-13/35*I*b*d^3*ln(c^2*x^2+1)/c^
4
```

Rubi [A]
time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$-\frac{1}{7}ic^3d^3x^7(a+b\text{ArcTan}(cx))-\frac{1}{2}c^2d^3x^6(a+b\text{ArcTan}(cx))+\frac{3}{5}icd^3x^5(a+b\text{ArcTan}(cx))+\frac{1}{4}d^3x^4(a+b\text{ArcTan}(cx))-\frac{3bd^3\text{ArcTan}(cx)}{4c^4}+\frac{3bd^3x}{4c^3}+\frac{1}{42}ibc^2d^3x^6+\frac{13ibd^3x^2}{35c^2}-\frac{13ibd^3\log\left(c^2x^2+1\right)}{35c^4}+\frac{1}{10}bcd^3x^5-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (((13*I)/35)*b*d^3*x^2)/c^2 - (b*d^3*x^3)/(4*c) - ((1
3*I)/70)*b*d^3*x^4 + (b*c*d^3*x^5)/10 + (I/42)*b*c^2*d^3*x^6 - (3*b*d^3*Arc
Tan[c*x])/(4*c^4) + (d^3*x^4*(a + b*ArcTan[c*x]))/4 + ((3*I)/5)*c*d^3*x^5*(
a + b*ArcTan[c*x]) - (c^2*d^3*x^6*(a + b*ArcTan[c*x]))/2 - (I/7)*c^3*d^3*x^
7*(a + b*ArcTan[c*x]) - (((13*I)/35)*b*d^3*Log[1 + c^2*x^2])/c^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int x^3(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+\right.\\
&= \frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+\right.\\
&= \frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}c^2d^3x^6\left(a+\right.\\
&= \frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6+\\
&= \frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6+\\
&= \frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6-
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 154, normalized size = 0.75

$$\frac{d^3(3ac^4x^4(35+84icx-70c^2x^2-20ic^3x^3)+bcx(315+156icx-105c^2x^2-78ic^3x^3+42c^4x^4+10ic^5x^5)+3b(-105+35c^4x^4+84ic^5x^5-70c^6x^6-20ic^7x^7)\,\mathrm{ArcTan}(cx)-156ib\log(1+c^2x^2))}{420c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^3*(3*a*c^4*x^4*(35 + (84*I)*c*x - 70*c^2*x^2 - (20*I)*c^3*x^3) + b*c*x*(
315 + (156*I)*c*x - 105*c^2*x^2 - (78*I)*c^3*x^3 + 42*c^4*x^4 + (10*I)*c^5*
x^5) + 3*b*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7*x^
7)*ArcTan[c*x] - (156*I)*b*Log[1 + c^2*x^2]))/(420*c^4)
```

Maple [A]
time = 0.17, size = 210, normalized size = 1.02

method	result
derivativedivides	$\frac{d^3 a\left(-\frac{1}{7} i c^7 x^7-\frac{1}{2} c^6 x^6+\frac{3}{5} i c^5 x^5+\frac{1}{4} c^4 x^4\right)-\frac{i d^3 b \arctan(cx) c^7 x^7}{7}-\frac{d^3 b \arctan(cx) c^6 x^6}{2}+\frac{3 i d^3 b \arctan(cx) c^5 x^5}{5}+\frac{d^3 b \arctan(cx) c^4}{4}}{c^4}$
default	$\frac{d^3 a\left(-\frac{1}{7} i c^7 x^7-\frac{1}{2} c^6 x^6+\frac{3}{5} i c^5 x^5+\frac{1}{4} c^4 x^4\right)-\frac{i d^3 b \arctan(cx) c^7 x^7}{7}-\frac{d^3 b \arctan(cx) c^6 x^6}{2}+\frac{3 i d^3 b \arctan(cx) c^5 x^5}{5}+\frac{d^3 b \arctan(cx) c^4}{4}}{c^4}$
risch	$-\frac{d^3 b\left(20 c^3 x^7-70 i c^2 x^6-84 c x^5+35 i x^4\right) \ln(icx+1)}{280}+\frac{d^3 c^3 b x^7 \ln(-icx+1)}{14}+\frac{i b c^2 d^3 x^6}{42}-\frac{d^3 c^2 a x^6}{2}-\frac{13 i b d^3 x^4}{70}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^3*a*(-1/7*I*c^7*x^7-1/2*c^6*x^6+3/5*I*c^5*x^5+1/4*c^4*x^4)-1/7*I*d
^3*b*arctan(c*x)*c^7*x^7-1/2*d^3*b*arctan(c*x)*c^6*x^6+3/5*I*d^3*b*arctan(c
*x)*c^5*x^5+1/4*d^3*b*arctan(c*x)*c^4*x^4+3/4*b*c*d^3*x+1/42*I*d^3*b*c^6*x^
6+1/10*d^3*b*c^5*x^5-13/70*I*d^3*b*c^4*x^4-1/4*d^3*b*c^3*x^3+13/35*I*d^3*b*
c^2*x^2-13/35*I*d^3*b*ln(c^2*x^2+1)-3/4*b*d^3*arctan(c*x))
```

Maxima [A]
time = 0.48, size = 261, normalized size = 1.27

$$-\frac{1}{7} i a c^3 d^3 x^7-\frac{1}{2} a c^2 d^3 x^6+\frac{3}{5} i a c d^3 x^5-\frac{1}{84} i\left(12 x^7 \arctan(cx)-c\left(\frac{2 c^4 x^6-3 c^2 x^4+6 x^2}{c^6}-\frac{6 \log\left(c^2 x^2+1\right)}{c^8}\right)\right) b c^3 d^3+\frac{1}{4} a d^3 x^4-\frac{1}{30}\left(15 x^6 \arctan(cx)-c\left(\frac{3 c^4 x^5-5 c^2 x^3+15 x}{c^6}-\frac{15 \arctan(cx)}{c^7}\right)\right) b c^2 d^3+\frac{3}{20} i\left(4 x^5 \arctan(cx)-c\left(\frac{c^2 x^4-2 x^2}{c^4}+\frac{2 \log\left(c^2 x^2+1\right)}{c^6}\right)\right) b c d^3+\frac{1}{12}\left(3 x^4 \arctan(cx)-c\left(\frac{c^2 x^3-3 x}{c^4}+\frac{3 \arctan(cx)}{c^5}\right)\right) b d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*I*a*c^3*d^3*x^7 - 1/2*a*c^2*d^3*x^6 + 3/5*I*a*c*d^3*x^5 - 1/84*I*(12*x
^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1
)/c^8))*b*c^3*d^3 + 1/4*a*d^3*x^4 - 1/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^
5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^3 + 3/20*I*(4*x^5*
arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^3 +
 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d
^3
```

Fricas [A]
time = 1.59, size = 202, normalized size = 0.99

$$\frac{-120 i a c^7 d^3 x^7-20(21 a-i b) c^6 d^3 x^6-84(-6 i a-b) c^5 d^3 x^5+6(35 a-26 i b) c^4 d^3 x^4-210 b c^3 d^3 x^3+312 i b c^2 d^3 x^2+630 b c d^3 x-627 i b d^3 \log\left(\frac{cx+i}{c}\right)+3 i b d^3 \log\left(\frac{cx-i}{c}\right)+3\left(20 b c^7 d^3 x^7-70 i b c^6 d^3 x^6-84 b c^5 d^3 x^5+35 i b c^4 d^3 x^4\right) \log\left(-\frac{cx+i}{cx-i}\right)}{840 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(-120*I*a*c^7*d^3*x^7 - 20*(21*a - I*b)*c^6*d^3*x^6 - 84*(-6*I*a - b)
*c^5*d^3*x^5 + 6*(35*a - 26*I*b)*c^4*d^3*x^4 - 210*b*c^3*d^3*x^3 + 312*I*b*
c^2*d^3*x^2 + 630*b*c*d^3*x - 627*I*b*d^3*log((c*x + I)/c) + 3*I*b*d^3*log(
(c*x - I)/c) + 3*(20*b*c^7*d^3*x^7 - 70*I*b*c^6*d^3*x^6 - 84*b*c^5*d^3*x^5
+ 35*I*b*c^4*d^3*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A]
time = 2.80, size = 328, normalized size = 1.60

$$-\frac{iac^3d^3x^7}{7}-\frac{bd^3x^3}{4c}+\frac{13ibd^3x^2}{35c^2}+\frac{3bd^3x}{4c^3}-\frac{bd^3\left(-\frac{i\log\left(353bcd^3x-353ibd^3\right)}{280}+\frac{351i\log\left(353bcd^3x+353ibd^3\right)}{560}\right)}{c^4}-x^6\left(\frac{ac^2d^3}{2}-\frac{ibc^2d^3}{42}\right)-x^5\left(-\frac{3iacd^3}{5}-\frac{bcd^3}{10}\right)-x^4\left(-\frac{ad^3}{4}+\frac{13ibd^3}{70}\right)+\left(-\frac{bc^3d^3x^7}{14}+\frac{ibc^2d^3x^6}{4}+\frac{3bcd^3x^5}{10}-\frac{ibd^3x^4}{8}\right)\log\left(icx+1\right)+\frac{\left(40bc^7d^3x^7-140ibc^6d^3x^6-168bc^5d^3x^5+70ibc^4d^3x^4-67ibd^3\right)\log\left(-icx+1\right)}{560c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**7/7 - b*d**3*x**3/(4*c) + 13*I*b*d**3*x**2/(35*c**2) + 3*
b*d**3*x/(4*c**3) - b*d**3*(-I*log(353*b*c*d**3*x - 353*I*b*d**3)/280 + 351
*I*log(353*b*c*d**3*x + 353*I*b*d**3)/560)/c**4 - x**6*(a*c**2*d**3/2 - I*b
*c**2*d**3/42) - x**5*(-3*I*a*c*d**3/5 - b*c*d**3/10) - x**4*(-a*d**3/4 + 1
3*I*b*d**3/70) + (-b*c**3*d**3*x**7/14 + I*b*c**2*d**3*x**6/4 + 3*b*c*d**3*
x**5/10 - I*b*d**3*x**4/8)*log(I*c*x + 1) + (40*b*c**7*d**3*x**7 - 140*I*b*
c**6*d**3*x**6 - 168*b*c**5*d**3*x**5 + 70*I*b*c**4*d**3*x**4 - 67*I*b*d**3
)*log(-I*c*x + 1)/(560*c**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.89, size = 186, normalized size = 0.91

$$-\frac{\frac{d^3\left(315\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,156\mathrm{i}\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}-\frac{3\,b\,c\,d^3\,x}{4}-\frac{b\,c^2\,d^3\,x^2\,13\mathrm{i}}{35}}{c^4}+\frac{d^3\left(105\,a\,x^4+105\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,78\mathrm{i}\right)}{420}-\frac{c^3\,d^3\left(a\,x^7\,60\mathrm{i}+b\,x^7\,\mathrm{atan}(c\,x)\,60\mathrm{i}\right)}{420}+\frac{c\,d^3\left(a\,x^5\,252\mathrm{i}+42\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,252\mathrm{i}\right)}{420}-\frac{c^2\,d^3\left(210\,a\,x^6+210\,b\,x^6\,\mathrm{atan}(c\,x)-b\,x^6\,10\mathrm{i}\right)}{420}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

```
[Out] (d^3*(105*a*x^4 - b*x^4*78i + 105*b*x^4*atan(c*x)))/420 - ((d^3*(315*b*atan
(c*x) + b*log(c^2*x^2 + 1)*156i))/420 - (b*c^2*d^3*x^2*13i)/35 + (b*c^3*d^3
*x^3)/4 - (3*b*c*d^3*x)/4)/c^4 - (c^3*d^3*(a*x^7*60i + b*x^7*atan(c*x)*60i)
)/420 + (c*d^3*(a*x^5*252i + 42*b*x^5 + b*x^5*atan(c*x)*252i))/420 - (c^2*d
^3*(210*a*x^6 - b*x^6*10i + 210*b*x^6*atan(c*x)))/420
```

3.21 $\int x^2(d+icdx)^3(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=191

$$\frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5-\frac{11ibd^3\text{ArcTan}(cx)}{12c^3}+\frac{1}{3}d^3x^3(a+b\text{ArcTan}(cx))+\frac{3}{4}icd^3x$$

```
[Out] 11/12*I*b*d^3*x/c^2-7/15*b*d^3*x^2/c-11/36*I*b*d^3*x^3+3/20*b*c*d^3*x^4+1/3
0*I*b*c^2*d^3*x^5-11/12*I*b*d^3*arctan(c*x)/c^3+1/3*d^3*x^3*(a+b*arctan(c*x
))+3/4*I*c*d^3*x^4*(a+b*arctan(c*x))-3/5*c^2*d^3*x^5*(a+b*arctan(c*x))-1/6*
I*c^3*d^3*x^6*(a+b*arctan(c*x))+7/15*b*d^3*ln(c^2*x^2+1)/c^3
```

Rubi [A]
time = 0.12, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$-\frac{1}{6}ic^3d^3x^6(a+b\text{ArcTan}(cx))-\frac{3}{5}c^2d^3x^5(a+b\text{ArcTan}(cx))+\frac{3}{4}icd^3x^4(a+b\text{ArcTan}(cx))+\frac{1}{3}d^3x^3(a+b\text{ArcTan}(cx))-\frac{11ibd^3\text{ArcTan}(cx)}{12c^3}+\frac{1}{30}ibc^2d^3x^5+\frac{11ibd^3x}{12c^2}+\frac{7bd^3\log(c^2x^2+1)}{15c^3}+\frac{3}{20}bcd^3x^4-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (((11*I)/12)*b*d^3*x)/c^2 - (7*b*d^3*x^2)/(15*c) - ((11*I)/36)*b*d^3*x^3 +
(3*b*c*d^3*x^4)/20 + (I/30)*b*c^2*d^3*x^5 - (((11*I)/12)*b*d^3*ArcTan[c*x])
/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]))/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[
c*x]) - (3*c^2*d^3*x^5*(a + b*ArcTan[c*x]))/5 - (I/6)*c^3*d^3*x^6*(a + b*Ar
cTan[c*x]) + (7*b*d^3*Log[1 + c^2*x^2])/(15*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+\right.\\ &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+\right.\\ &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+\right.\\ &= \frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5+\frac{1}{3}d^3x^3(a\\ &= \frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5+\frac{1}{3}d^3x^3(a\\ &= \frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5-\frac{11ibd^3\text{ta}}{12}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 146, normalized size = 0.76

$$\frac{d^3(3ac^3x^3(20+45icx-36c^2x^2-10ic^3x^3)+bcx(165i-84cx-55ic^2x^2+27c^3x^3+6ic^4x^4)+3b(-55i+20c^3x^3+45ic^4x^4-36c^5x^5-10ic^6x^6)\,\text{ArcTan}(cx)+84b\log\left(1+c^2x^2\right))}{180c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^3*(3*a*c^3*x^3*(20 + (45*I)*c*x - 36*c^2*x^2 - (10*I)*c^3*x^3) + b*c*x*(
165*I - 84*c*x - (55*I)*c^2*x^2 + 27*c^3*x^3 + (6*I)*c^4*x^4) + 3*b*(-55*I
+ 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6)*ArcTan[c*x] +
84*b*Log[1 + c^2*x^2]))/(180*c^3)
```

Maple [A]
time = 0.13, size = 198, normalized size = 1.04

method	result
derivativedivides	$\frac{d^3 a\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)-\frac{id^3b\arctan(cx)c^6x^6}{6}-\frac{3d^3b\arctan(cx)c^5x^5}{5}+\frac{3id^3b\arctan(cx)c^4x^4}{4}+\frac{d^3b\arctan(cx)c^3}{3}}{c^3}$
default	$\frac{d^3 a\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)-\frac{id^3b\arctan(cx)c^6x^6}{6}-\frac{3d^3b\arctan(cx)c^5x^5}{5}+\frac{3id^3b\arctan(cx)c^4x^4}{4}+\frac{d^3b\arctan(cx)c^3}{3}}{c^3}$
risch	$-\frac{d^3b\left(10c^3x^6-36ic^2x^5-45c\,x^4+20ix^3\right)\ln(icx+1)}{120}-\frac{id^3c^3a\,x^6}{6}+\frac{d^3c^3b\,x^6\ln(-icx+1)}{12}+\frac{ib\,c^2d^3x^5}{30}+\frac{3id^3ca\,x^4}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d^3*a*(-1/6*I*c^6*x^6-3/5*c^5*x^5+3/4*I*c^4*x^4+1/3*c^3*x^3)-1/6*I*d
^3*b*arctan(c*x)*c^6*x^6-3/5*d^3*b*arctan(c*x)*c^5*x^5+3/4*I*d^3*b*arctan(c
*x)*c^4*x^4+1/3*d^3*b*arctan(c*x)*c^3*x^3+11/12*I*d^3*b*c*x+1/30*I*d^3*b*c^
5*x^5+3/20*d^3*b*c^4*x^4-11/36*I*d^3*b*c^3*x^3-7/15*d^3*b*c^2*x^2+7/15*b*ln
(c^2*x^2+1)*d^3-11/12*I*d^3*b*arctan(c*x))
```

Maxima [A]
time = 0.48, size = 242, normalized size = 1.27

$$-\frac{1}{6}i\,ac^3d^3x^6-\frac{3}{5}\,ac^2d^3x^5+\frac{3}{4}i\,acd^3x^4-\frac{1}{90}i\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^3d^3-\frac{3}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^3+\frac{1}{3}\,ad^3x^3+\frac{1}{4}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*I*a*c^3*d^3*x^6 - 3/5*a*c^2*d^3*x^5 + 3/4*I*a*c*d^3*x^4 - 1/90*I*(15*x
^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7
))*b*c^3*d^3 - 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/4*I*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3
```

Fricas [A]
time = 2.09, size = 190, normalized size = 0.99

$$\frac{-60i\,ac^6d^3x^6-12\left(18\,a-i\,b\right)c^5d^3x^5-54\left(-5i\,a-b\right)c^4d^3x^4+10\left(12\,a-11i\,b\right)c^3d^3x^3-168\,bc^2d^3x^2+330i\,bcd^3x+333\,bd^3\log\left(\frac{cx+i}{c}\right)+3\,bd^3\log\left(\frac{cx-i}{c}\right)+3\left(10\,bc^6d^3x^6-36i\,bc^5d^3x^5-45\,bc^4d^3x^4+20i\,bc^3d^3x^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{360\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(-60*I*a*c^6*d^3*x^6 - 12*(18*a - I*b)*c^5*d^3*x^5 - 54*(-5*I*a - b)*
c^4*d^3*x^4 + 10*(12*a - 11*I*b)*c^3*d^3*x^3 - 168*b*c^2*d^3*x^2 + 330*I*b*
c*d^3*x + 333*b*d^3*log((c*x + I)/c) + 3*b*d^3*log((c*x - I)/c) + 3*(10*b*c
^6*d^3*x^6 - 36*I*b*c^5*d^3*x^5 - 45*b*c^4*d^3*x^4 + 20*I*b*c^3*d^3*x^3)*lo
g(-(c*x + I)/(c*x - I)))/c^3
```

Sympy [A]
time = 2.46, size = 316, normalized size = 1.65

$$-\frac{iac^3d^3x^6}{6}-\frac{7bd^3x^2}{15c}+\frac{11ibd^3x}{12c^2}-\frac{bd^3\left(-\frac{\log\left(310bcd^3x-310ibd^3\right)}{120}-\frac{209\log\left(310bcd^3x+310ibd^3\right)}{280}\right)}{c^3}-x^5\cdot\left(\frac{3ac^2d^3}{5}-\frac{ibc^2d^3}{30}\right)-x^4\left(-\frac{3iacd^3}{4}-\frac{3bcd^3}{20}\right)-x^3\left(-\frac{ad^3}{3}+\frac{11ibd^3}{36}\right)+\left(-\frac{bc^3d^3x^6}{12}+\frac{3ibc^2d^3x^5}{10}+\frac{3bcd^3x^4}{8}-\frac{ibd^3x^3}{6}\right)\log\left(icx+1\right)+\frac{\left(70bc^6d^3x^6-252ibc^5d^3x^5-315bc^4d^3x^4+140ibc^3d^3x^3+150bd^3\right)\log\left(-icx+1\right)}{840c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**6/6 - 7*b*d**3*x**2/(15*c) + 11*I*b*d**3*x/(12*c**2) - b*
d**3*(-log(310*b*c*d**3*x - 310*I*b*d**3)/120 - 209*log(310*b*c*d**3*x + 31
0*I*b*d**3)/280)/c**3 - x**5*(3*a*c**2*d**3/5 - I*b*c**2*d**3/30) - x**4*(-
3*I*a*c*d**3/4 - 3*b*c*d**3/20) - x**3*(-a*d**3/3 + 11*I*b*d**3/36) + (-b*c
**3*d**3*x**6/12 + 3*I*b*c**2*d**3*x**5/10 + 3*b*c*d**3*x**4/8 - I*b*d**3*x
**3/6)*log(I*c*x + 1) + (70*b*c**6*d**3*x**6 - 252*I*b*c**5*d**3*x**5 - 315
*b*c**4*d**3*x**4 + 140*I*b*c**3*d**3*x**3 + 150*b*d**3)*log(-I*c*x + 1)/(8
40*c**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.83, size = 174, normalized size = 0.91

$$-\frac{\frac{d^3\,\left(-84\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,165{}\mathrm{i}\right)}{180}+\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{b\,c\,d^3\,x\,11{}\mathrm{i}}{12}}{c^3}+\frac{d^3\,\left(60\,a\,x^3+60\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,55{}\mathrm{i}\right)}{180}-\frac{c^3\,d^3\,\left(a\,x^6\,30{}\mathrm{i}+b\,x^6\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{180}+\frac{c\,d^3\,\left(a\,x^4\,135{}\mathrm{i}+27\,b\,x^4+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,135{}\mathrm{i}\right)}{180}-\frac{c^2\,d^3\,\left(108\,a\,x^5+108\,b\,x^5\,\mathrm{atan}\left(c\,x\right)-b\,x^5\,6{}\mathrm{i}\right)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

```
[Out] (d^3*(60*a*x^3 - b*x^3*55i + 60*b*x^3*atan(c*x)))/180 - ((d^3*(b*atan(c*x)*
165i - 84*b*log(c^2*x^2 + 1)))/180 + (7*b*c^2*d^3*x^2)/15 - (b*c*d^3*x*11i)
/12)/c^3 - (c^3*d^3*(a*x^6*30i + b*x^6*atan(c*x)*30i))/180 + (c*d^3*(a*x^4*
135i + 27*b*x^4 + b*x^4*atan(c*x)*135i))/180 - (c^2*d^3*(108*a*x^5 - b*x^5*
6i + 108*b*x^5*atan(c*x)))/180
```

3.22 $\int x(d+icdx)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=157

$$-\frac{3bd^3x}{5c}-\frac{3ibd^3(i-cx)^2}{20c^2}-\frac{bd^3(i-cx)^3}{20c^2}+\frac{ibd^3(i-cx)^4}{20c^2}+\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4c^2}-\frac{d^3(1+icx)^5(a+b}{5c^2}$$

```
[Out] -3/5*b*d^3*x/c-3/20*I*b*d^3*(I-c*x)^2/c^2-1/20*b*d^3*(I-c*x)^3/c^2+1/20*I*b
*d^3*(I-c*x)^4/c^2+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/c^2-1/5*d^3*(1+I*c
*x)^5*(a+b*arctan(c*x))/c^2+6/5*I*b*d^3*ln(c*x+I)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {45, 4992, 12, 78}

$$-\frac{d^3(1+icx)^5(a+b\text{ArcTan}(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4c^2}+\frac{ibd^3(-cx+i)^4}{20c^2}-\frac{bd^3(-cx+i)^3}{20c^2}-\frac{3ibd^3(-cx+i)^2}{20c^2}+\frac{6ibd^3\log(cx+i)}{5c^2}-\frac{3bd^3x}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-3*b*d^3*x)/(5*c) - (((3*I)/20)*b*d^3*(I - c*x)^2)/c^2 - (b*d^3*(I - c*x)^
3)/(20*c^2) + ((I/20)*b*d^3*(I - c*x)^4)/c^2 + (d^3*(1 + I*c*x)^4*(a + b*Ar
cTan[c*x]))/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) + (((
6*I)/5)*b*d^3*Log[I + c*x])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-(b$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{(b}{}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{(b}{}$$

$$=-\frac{3bd^3x}{5c}-\frac{3ibd^3(i-cx)^2}{20c^2}-\frac{bd^3(i-cx)^3}{20c^2}+\frac{ibd^3(i-cx)^4}{20c^2}+\frac{d^3(1+i}{}$$

Mathematica [A]
time = 0.06, size = 132, normalized size = 0.84

$$\frac{d^3(cx(b(-25-12icx+5c^2x^2+ic^3x^3)+acx(10+20icx-15c^2x^2-4ic^3x^3))+b(25+10c^2x^2+20ic^3x^3-15c^4x^4-4ic^5x^5)\operatorname{ArcTan}(cx)+12ib\log(1+c^2x^2))}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^3*(c*x*(b*(-25 - (12*I)*c*x + 5*c^2*x^2 + I*c^3*x^3) + a*c*x*(10 + (20*I
)*c*x - 15*c^2*x^2 - (4*I)*c^3*x^3)) + b*(25 + 10*c^2*x^2 + (20*I)*c^3*x^3
- 15*c^4*x^4 - (4*I)*c^5*x^5)*ArcTan[c*x] + (12*I)*b*Log[1 + c^2*x^2]))/(20
*c^2)
```

Maple [A]
time = 0.13, size = 185, normalized size = 1.18

method	result
derivativedivides	$\frac{d^3a\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)-\frac{id^3b\arctan(cx)c^5x^5}{5}-\frac{3d^3b\arctan(cx)c^4x^4}{4}+id^3b\arctan(cx)c^3x^3+\frac{d^3b\arctan(cx)c^2x^2}{2}}{c^2}$
default	$\frac{d^3a\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)-\frac{id^3b\arctan(cx)c^5x^5}{5}-\frac{3d^3b\arctan(cx)c^4x^4}{4}+id^3b\arctan(cx)c^3x^3+\frac{d^3b\arctan(cx)c^2x^2}{2}}{c^2}$
risch	$-\frac{d^3b(4c^3x^5-15ic^2x^4-20cx^3+10ix^2)\ln(icx+1)}{40}+\frac{d^3c^3bx^5\ln(-icx+1)}{10}-\frac{id^3c^3ax^5}{5}-\frac{3d^3c^2ax^4}{4}-\frac{3id^3c^2bx^4\ln}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^3*a*(-1/5*I*c^5*x^5-3/4*c^4*x^4+I*c^3*x^3+1/2*c^2*x^2)-1/5*I*d^3*b
*arctan(c*x)*c^5*x^5-3/4*d^3*b*arctan(c*x)*c^4*x^4+I*d^3*b*arctan(c*x)*c^3*
x^3+1/2*d^3*b*arctan(c*x)*c^2*x^2-5/4*b*c*d^3*x+1/20*I*d^3*b*c^4*x^4+1/4*d^
3*b*c^3*x^3-3/5*I*d^3*b*c^2*x^2+3/5*I*d^3*b*ln(c^2*x^2+1)+5/4*b*d^3*arctan(
c*x))
```

Maxima [A]
time = 0.47, size = 222, normalized size = 1.41

$$-\frac{1}{5} i a c^3 d^3 x^5 - \frac{3}{4} a c^2 d^3 x^4 - \frac{1}{20} i \left(4 x^5 \arctan(cx) - c\left(\frac{c^2 x^4 - 2 x^2}{c^4} + \frac{2 \log(c^2 x^2 + 1)}{c^6}\right)\right) b c^3 d^3 + i a c d^3 x^3 - \frac{1}{4}\left(3 x^4 \arctan(cx) - c\left(\frac{c^2 x^3 - 3x}{c^4} + \frac{3 \arctan(cx)}{c^5}\right)\right) b c^2 d^3 + \frac{1}{2} i \left(2 x^3 \arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2 x^2 + 1)}{c^4}\right)\right) b c d^3 + \frac{1}{2} a d^3 x^2 + \frac{1}{2}\left(x^2 \arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right) b d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*I*a*c^3*d^3*x^5 - 3/4*a*c^2*d^3*x^4 - 1/20*I*(4*x^5*arctan(c*x) - c*((
c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^3*d^3 + I*a*c*d^3*x^3 -
 1/4*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^
2*d^3 + 1/2*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*
d^3 + 1/2*a*d^3*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
*d^3
```

Fricas [A]
time = 1.27, size = 177, normalized size = 1.13

$$\frac{-8 i a c^5 d^3 x^5 - 2 (15 a - i b) c^4 d^3 x^4 - 10 (-4 i a - b) c^3 d^3 x^3 + 4 (5 a - 6 i b) c^2 d^3 x^2 - 50 b c d^3 x + 49 i b d^3 \log\left(\frac{cx+i}{c}\right) - i b d^3 \log\left(\frac{cx-i}{c}\right) + (4 b c^5 d^3 x^5 - 15 i b c^4 d^3 x^4 - 20 b c^3 d^3 x^3 + 10 i b c^2 d^3 x^2) \log\left(-\frac{cx+i}{cx-i}\right)}{40 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(-8*I*a*c^5*d^3*x^5 - 2*(15*a - I*b)*c^4*d^3*x^4 - 10*(-4*I*a - b)*c^3
*d^3*x^3 + 4*(5*a - 6*I*b)*c^2*d^3*x^2 - 50*b*c*d^3*x + 49*I*b*d^3*log((c*x
 + I)/c) - I*b*d^3*log((c*x - I)/c) + (4*b*c^5*d^3*x^5 - 15*I*b*c^4*d^3*x^4
 - 20*b*c^3*d^3*x^3 + 10*I*b*c^2*d^3*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(138) = 276.
time = 2.40, size = 296, normalized size = 1.89

$$-\frac{i a c^3 d^3 x^5}{5} - \frac{5 b d^3 x}{4c} - \frac{b d^3 \left(\frac{i \log(19 b c d^3 x - 19 i b d^3)}{40} - \frac{37 i \log(19 b c d^3 x + 19 i b d^3)}{40}\right)}{c^2} - x^4 \cdot \left(\frac{3 a c^2 d^3}{4} - \frac{i b c^2 d^3}{20}\right) - x^3 \left(-i a c d^3 - \frac{b c d^3}{4}\right) - x^2 \left(-\frac{a d^3}{2} + \frac{3 i b d^3}{5}\right) + \left(-\frac{b c^3 d^3 x^5}{10} + \frac{3 i b c^2 d^3 x^4}{8} + \frac{b c d^3 x^3}{2} - \frac{i b d^3 x^2}{4}\right) \log(icx + 1) + \frac{(4 b c^5 d^3 x^5 - 15 i b c^4 d^3 x^4 - 20 b c^3 d^3 x^3 + 10 i b c^2 d^3 x^2 + 12 i b d^3) \log(-icx + 1)}{40 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**5/5 - 5*b*d**3*x/(4*c) - b*d**3*(I*log(19*b*c*d**3*x - 19
*I*b*d**3)/40 - 37*I*log(19*b*c*d**3*x + 19*I*b*d**3)/40)/c**2 - x**4*(3*a*
c**2*d**3/4 - I*b*c**2*d**3/20) - x**3*(-I*a*c*d**3 - b*c*d**3/4) - x**2*(-
a*d**3/2 + 3*I*b*d**3/5) + (-b*c**3*d**3*x**5/10 + 3*I*b*c**2*d**3*x**4/8 +
 b*c*d**3*x**3/2 - I*b*d**3*x**2/4)*log(I*c*x + 1) + (4*b*c**5*d**3*x**5 -
15*I*b*c**4*d**3*x**4 - 20*b*c**3*d**3*x**3 + 10*I*b*c**2*d**3*x**2 + 12*I*
b*d**3)*log(-I*c*x + 1)/(40*c**2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.73, size = 160, normalized size = 1.02

$$\frac{\frac{d^3\,\left(25\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,12{}\mathrm{i}\right)}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2}+\frac{d^3\,\left(10\,a\,x^2+10\,b\,x^2\,\mathrm{atan}\left(c\,x\right)-b\,x^2\,12{}\mathrm{i}\right)}{20}-\frac{c^3\,d^3\,\left(a\,x^5\,4{}\mathrm{i}+b\,x^5\,\mathrm{atan}\left(c\,x\right)\,4{}\mathrm{i}\right)}{20}+\frac{c\,d^3\,\left(a\,x^3\,20{}\mathrm{i}+5\,b\,x^3+b\,x^3\,\mathrm{atan}\left(c\,x\right)\,20{}\mathrm{i}\right)}{20}-\frac{c^2\,d^3\,\left(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}\left(c\,x\right)-b\,x^4\,1{}\mathrm{i}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

```
[Out] ((d^3*(25*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i))/20 - (5*b*c*d^3*x)/4)/c^2
+ (d^3*(10*a*x^2 - b*x^2*12i + 10*b*x^2*atan(c*x)))/20 - (c^3*d^3*(a*x^5*4i
 + b*x^5*atan(c*x)*4i))/20 + (c*d^3*(a*x^3*20i + 5*b*x^3 + b*x^3*atan(c*x)*
20i))/20 - (c^2*d^3*(15*a*x^4 - b*x^4*1i + 15*b*x^4*atan(c*x)))/20
```

3.23 $\int (d+icdx)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=100

$$-ibd^3x-\frac{bd^3(1+icx)^2}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{id^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4c}-\frac{2bd^3\log(1-icx)}{c}$$

```
[Out] -I*b*d^3*x-1/4*b*d^3*(1+I*c*x)^2/c-1/12*b*d^3*(1+I*c*x)^3/c-1/4*I*d^3*(1+I*
c*x)^4*(a+b*arctan(c*x))/c-2*b*d^3*ln(1-I*c*x)/c
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4972, 641, 45}

$$-\frac{id^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{bd^3(1+icx)^2}{4c}-\frac{2bd^3\log(1-icx)}{c}-ibd^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-I)*b*d^3*x - (b*d^3*(1 + I*c*x)^2)/(4*c) - (b*d^3*(1 + I*c*x)^3)/(12*c) -
 ((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/c - (2*b*d^3*Log[1 - I*c*x])
/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right) dx = -\frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(cx)\right)}{4c} + \frac{(ib)\int \frac{(d+icdx)^4}{1+c^2x^2}\, dx}{4d}$$
$$= -\frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(cx)\right)}{4c} + \frac{(ib)\int \frac{(d+icdx)^3}{\frac{1}{d}-\frac{icx}{d}}\, dx}{4d}$$
$$= -\frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(cx)\right)}{4c} + \frac{(ib)\int \left(-4d^4+\frac{8d^3}{\frac{1}{d}-\frac{icx}{d}}-2d^3(d+ic\right.}{4d}$$
$$= -ibd^3x - \frac{bd^3(1+icx)^2}{4c} - \frac{bd^3(1+icx)^3}{12c} - \frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(\right.}{4c}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.77

$$-\frac{i(3(d+icdx)^4(a+b\text{ArcTan}(cx))-bd^4(4i-21cx-6ic^2x^2+c^3x^3+24i\log(i+cx)))}{12cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-1/12*I)*(3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]) - b*d^4*(4*I - 21*c*x - (
6*I)*c^2*x^2 + c^3*x^3 + (24*I)*Log[I + c*x])))/(c*d)
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{id^3(icx+1)^4a}{4}-\frac{id^3b\arctan(cx)c^4x^4}{4}-d^3b\arctan(cx)c^3x^3+\frac{3id^3b\arctan(cx)c^2x^2}{2}+b\arctan(cx)d^3cx+\frac{7id^3b\arctan(cx)}{4}-\frac{7id^3bc}{4}}{c}$
default	$\frac{-\frac{id^3(icx+1)^4a}{4}-\frac{id^3b\arctan(cx)c^4x^4}{4}-d^3b\arctan(cx)c^3x^3+\frac{3id^3b\arctan(cx)c^2x^2}{2}+b\arctan(cx)d^3cx+\frac{7id^3b\arctan(cx)}{4}-\frac{7id^3bc}{4}}{c}$
risch	$-\frac{d^3(cx-i)^4b\ln(icx+1)}{8c}-\frac{id^3ac^3x^4}{4}+\frac{d^3c^3bx^4\ln(-icx+1)}{8}-\frac{id^3c^2bx^3\ln(-icx+1)}{2}+\frac{id^3bc^2x^3}{12}-d^3ac^2x^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4*I*d^3*(1+I*c*x)^4*a-1/4*I*d^3*b*arctan(c*x)*c^4*x^4-d^3*b*arctan(
c*x)*c^3*x^3+3/2*I*d^3*b*arctan(c*x)*c^2*x^2+b*arctan(c*x)*d^3*c*x+7/4*I*d^
3*b*arctan(c*x)-7/4*I*d^3*b*c*x+1/12*I*d^3*b*c^3*x^3+1/2*d^3*b*c^2*x^2-b*ln
(c^2*x^2+1)*d^3)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(82) = 164.

time = 0.47, size = 197, normalized size = 1.97

$$-\frac{1}{4} i a c^3 d^3 x^4 - a c^2 d^3 x^3 - \frac{1}{12} i \left(3 x^4 \arctan(cx) - c\left(\frac{c^2 x^3 - 3x}{c^4} + \frac{3 \arctan(cx)}{c^5}\right)\right) b c^3 d^3 - \frac{1}{2} \left(2 x^3 \arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2 x^2 + 1)}{c^4}\right)\right) b c^2 d^3 + \frac{3}{2} i a c d^3 x^2 + \frac{3}{2} i \left(x^2 \arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right) b c d^3 + a d^3 x + \frac{(2 c x \arctan(cx) - \log(c^2 x^2 + 1)) b d^3}{2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*I*a*c^3*d^3*x^4 - a*c^2*d^3*x^3 - 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d^3 - 1/2*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + 3/2*I*a*c*d^3*x^2 + 3/2*I*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*
x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A]
time = 1.46, size = 161, normalized size = 1.61

$$\frac{-6 i a c^4 d^3 x^4 - 2 (12 a - i b) c^3 d^3 x^3 - 12 (-3 i a - b) c^2 d^3 x^2 + 6 (4 a - 7 i b) c d^3 x - 45 b d^3 \log\left(\frac{cx+i}{c}\right) - 3 b d^3 \log\left(\frac{cx-i}{c}\right) + 3 (b c^4 d^3 x^4 - 4 i b c^3 d^3 x^3 - 6 b c^2 d^3 x^2 + 4 i b c d^3 x) \log\left(-\frac{cx+i}{cx-i}\right)}{24 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-6*I*a*c^4*d^3*x^4 - 2*(12*a - I*b)*c^3*d^3*x^3 - 12*(-3*I*a - b)*c^2
*d^3*x^2 + 6*(4*a - 7*I*b)*c*d^3*x - 45*b*d^3*log((c*x + I)/c) - 3*b*d^3*lo
g((c*x - I)/c) + 3*(b*c^4*d^3*x^4 - 4*I*b*c^3*d^3*x^3 - 6*b*c^2*d^3*x^2 + 4
*I*b*c*d^3*x)*log(-(c*x + I)/(c*x - I)))/c
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.
time = 2.12, size = 267, normalized size = 2.67

$$-\frac{i a c^3 d^3 x^4}{4} - \frac{b d^3 \left(\frac{\log(22 b c d^3 x - 22 i b d^3)}{8} + \frac{49 \log(22 b c d^3 x + 22 i b d^3)}{40}\right)}{c} - x^3 \left(a c^2 d^3 - \frac{i b c^2 d^3}{12}\right) - x^2 \left(-\frac{3 i a c d^3}{2} - \frac{b c d^3}{2}\right) - x \left(-a d^3 + \frac{7 i b d^3}{4}\right) + \left(-\frac{b c^3 d^3 x^4}{8} + \frac{i b c^2 d^3 x^3}{2} + \frac{3 b c d^3 x^2}{4} - \frac{i b d^3 x}{2}\right) \log(i c x + 1) + \frac{(5 b c^4 d^3 x^4 - 20 i b c^3 d^3 x^3 - 30 b c^2 d^3 x^2 + 20 i b c d^3 x - 26 b d^3) \log(-i c x + 1)}{40 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

```
[Out] -I*a*c**3*d**3*x**4/4 - b*d**3*(log(22*b*c*d**3*x - 22*I*b*d**3)/8 + 49*log
(22*b*c*d**3*x + 22*I*b*d**3)/40)/c - x**3*(a*c**2*d**3 - I*b*c**2*d**3/12)
 - x**2*(-3*I*a*c*d**3/2 - b*c*d**3/2) - x*(-a*d**3 + 7*I*b*d**3/4) + (-b*c
**3*d**3*x**4/8 + I*b*c**2*d**3*x**3/2 + 3*b*c*d**3*x**2/4 - I*b*d**3*x/2)*
log(I*c*x + 1) + (5*b*c**4*d**3*x**4 - 20*I*b*c**3*d**3*x**3 - 30*b*c**2*d*
*3*x**2 + 20*I*b*c*d**3*x - 26*b*d**3)*log(-I*c*x + 1)/(40*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.69, size = 147, normalized size = 1.47

$$-\frac{d^3\,\left(a\,x\,12\mathrm{i}+21\,b\,x+b\,x\,\mathrm{atan}\left(c\,x\right)\,12\mathrm{i}\right)\,1\mathrm{i}}{12}-\frac{c^3\,d^3\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}\left(c\,x\right)\right)\,1\mathrm{i}}{12}+\frac{d^3\,\left(21\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,12\mathrm{i}\right)\,1\mathrm{i}}{12\,c}+\frac{c\,d^3\,\left(18\,a\,x^2+18\,b\,x^2\,\mathrm{atan}\left(c\,x\right)-b\,x^2\,6\mathrm{i}\right)\,1\mathrm{i}}{12}+\frac{c^2\,d^3\,\left(a\,x^3\,12\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}\left(c\,x\right)\,12\mathrm{i}\right)\,1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

```
[Out] (d^3*(21*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i)*1i)/(12*c) - (c^3*d^3*(3*a*x
^4 + 3*b*x^4*atan(c*x))*1i)/12 - (d^3*(a*x*12i + 21*b*x + b*x*atan(c*x)*12i
)*1i)/12 + (c*d^3*(18*a*x^2 - b*x^2*6i + 18*b*x^2*atan(c*x))*1i)/12 + (c^2*
d^3*(a*x^3*12i + b*x^3 + b*x^3*atan(c*x)*12i)*1i)/12
```

3.24 $\int \frac{(d+icdx)^3(a+b\textbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=170

$$3iacd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}ibc^2d^3x^2-\frac{3}{2}bd^3\text{ArcTan}(cx)+3ibcd^3x\text{ArcTan}(cx)-\frac{3}{2}c^2d^3x^2(a+b\text{ArcTan}(cx))-\frac{1}{3}ic^3d^3x^3($$

```
[Out] 3*I*a*c*d^3*x+3/2*b*c*d^3*x+1/6*I*b*c^2*d^3*x^2-3/2*b*d^3*arctan(c*x)+3*I*b
*c*d^3*x*arctan(c*x)-3/2*c^2*d^3*x^2*(a+b*arctan(c*x))-1/3*I*c^3*d^3*x^3*(a
+b*arctan(c*x))+a*d^3*ln(x)-5/3*I*b*d^3*ln(c^2*x^2+1)+1/2*I*b*d^3*polylog(2
,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4930, 266, 4940, 2438, 4946, 327, 209, 272, 45}

$$-\frac{1}{3}ic^3d^3x^3(a+b\text{ArcTan}(cx))-\frac{3}{2}c^2d^3x^2(a+b\text{ArcTan}(cx))+3iacd^3x+ad^3\log(x)-\frac{3}{2}bd^3\text{ArcTan}(cx)+3ibcd^3x\text{ArcTan}(cx)+\frac{1}{6}ibc^2d^3x^2-\frac{5}{3}ibd^3\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^3\text{Li}_2(-icx)-\frac{1}{2}ibd^3\text{Li}_2(icx)+\frac{3}{2}bcd^3x$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (3*I)*a*c*d^3*x + (3*b*c*d^3*x)/2 + (I/6)*b*c^2*d^3*x^2 - (3*b*d^3*ArcTan[c
*x])/2 + (3*I)*b*c*d^3*x*ArcTan[c*x] - (3*c^2*d^3*x^2*(a + b*ArcTan[c*x]))/
2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x]) + a*d^3*Log[x] - ((5*I)/3)*b*d^3*
Log[1 + c^2*x^2] + (I/2)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2
, I*c*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x}\,dx = \int \left(3icd^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3(a+b\tan^{-1}(cx))}{x}-3c^2d^3x\left(a+b\tan\right.\right.$$
$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(3icd^3\right)\int \left(a+b\tan^{-1}(cx)\right)\,dx-\left(3c^2d^3\right)$$
$$= 3iacd^3x-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}ic^3d^3x^3\left(a+b\tan^{-1}(cx)\right)+$$
$$= 3iacd^3x+\frac{3}{2}bcd^3x+3ibcd^3x\tan^{-1}(cx)-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-$$
$$= 3iacd^3x+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tan^{-1}(cx)+3ibcd^3x\tan^{-1}(cx)-\frac{3}{2}c^2d^3x^2($$
$$= 3iacd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}ibc^2d^3x^2-\frac{3}{2}bd^3\tan^{-1}(cx)+3ibcd^3x\tan^{-1}(cx$$

Mathematica [A]
time = 0.07, size = 139, normalized size = 0.82

$$-\frac{1}{6}id^3\left(-18acx+9ibcx-9iac^2x^2-bc^2x^2+2ac^3x^3-9ib\text{ArcTan}(cx)-18bcx\text{ArcTan}(cx)-9ibc^2x^2\text{ArcTan}(cx)+2bc^3x^3\text{ArcTan}(cx)+6ia\log(x)+10b\log\left(1+c^2x^2\right)-3b\text{PolyLog}(2,-icx)+3b\text{PolyLog}(2,icx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (-1/6*I)*d^3*(-18*a*c*x + (9*I)*b*c*x - (9*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*c
^3*x^3 - (9*I)*b*ArcTan[c*x] - 18*b*c*x*ArcTan[c*x] - (9*I)*b*c^2*x^2*ArcTa
n[c*x] + 2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*Log[x] + 10*b*Log[1 + c^2*x^2] -
 3*b*PolyLog[2, (-I)*c*x] + 3*b*PolyLog[2, I*c*x])
```

Maple [A]
time = 0.09, size = 220, normalized size = 1.29

method	result
derivativedivides	$\frac{id^3b\ln(cx)\ln(icx+1)}{2}-\frac{id^3b\,\text{dilog}(-icx+1)}{2}-\frac{3a\,c^2d^3x^2}{2}+d^3a\ln(cx)-\frac{id^3b\ln(cx)\ln(-icx+1)}{2}+3iac\,d^3$
default	$\frac{id^3b\ln(cx)\ln(icx+1)}{2}-\frac{id^3b\,\text{dilog}(-icx+1)}{2}-\frac{3a\,c^2d^3x^2}{2}+d^3a\ln(cx)-\frac{id^3b\ln(cx)\ln(-icx+1)}{2}+3iac\,d^3$
risch	$-\frac{d^3b\ln(icx+1)x^3c^3}{6}+3iac\,d^3x-\frac{3b\,d^3\arctan(cx)}{2}+\frac{3d^3b\ln(icx+1)xc}{2}-\frac{id^3b\,\text{dilog}(-icx+1)}{2}+\frac{3id^3b\ln(ic}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d^3*b*ln(c*x)*ln(1+I*c*x)-1/2*I*d^3*b*dilog(1-I*c*x)-3/2*a*c^2*d^3*x^
2+d^3*a*ln(c*x)-1/2*I*d^3*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d^3*b*dilog(1+I*c*x)-
3/2*d^3*b*arctan(c*x)*c^2*x^2+d^3*b*ln(c*x)*arctan(c*x)+3*I*a*c*d^3*x+1/6*I
*b*c^2*d^3*x^2-5/3*I*b*d^3*ln(c^2*x^2+1)-1/3*I*d^3*a*c^3*x^3+3/2*b*c*d^3*x+
3*I*b*c*d^3*x*arctan(c*x)-1/3*I*d^3*b*arctan(c*x)*c^3*x^3-3/2*b*d^3*arctan(
c*x)
```

Maxima [A]
time = 0.60, size = 184, normalized size = 1.08

$$-\frac{1}{3}i\,ac^3d^3x^3-\frac{3}{2}\,ac^2d^3x^2+\frac{1}{6}i\,bc^2d^3x^2+3i\,acd^3x+\frac{3}{2}\,bcd^3x-\frac{1}{12}\,(3\,\pi+2i)bd^3\log\left(c^2x^2+1\right)+bd^3\arctan\left(cx\right)\log\left(cx\right)+\frac{3}{2}i\,\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd^3-\frac{1}{2}i\,bd^3\mathrm{Li}_2(i\,cx+1)+\frac{1}{2}i\,bd^3\mathrm{Li}_2(-i\,cx+1)+ad^3\log\left(x\right)+\frac{1}{6}\left(-2i\,bc^3d^3x^3-9\,bc^2d^3x^2-9\,bd^3\right)\arctan\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*I*a*c^3*d^3*x^3 - 3/2*a*c^2*d^3*x^2 + 1/6*I*b*c^2*d^3*x^2 + 3*I*a*c*d^
3*x + 3/2*b*c*d^3*x - 1/12*(3*pi + 2*I)*b*d^3*log(c^2*x^2 + 1) + b*d^3*arct
an(c*x)*log(c*x) + 3/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3 - 1/2
*I*b*d^3*dilog(I*c*x + 1) + 1/2*I*b*d^3*dilog(-I*c*x + 1) + a*d^3*log(x) +
1/6*(-2*I*b*c^3*d^3*x^3 - 9*b*c^2*d^3*x^2 - 9*b*d^3)*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-id^3\left(\int\left(-3ac\right)\,dx+\int\frac{ia}{x}\,dx+\int ac^3x^2\,dx+\int\left(-3bc\operatorname{atan}\left(cx\right)\right)\,dx+\int\left(-3iac^2x\right)\,dx+\int\frac{ib\operatorname{atan}\left(cx\right)}{x}\,dx+\int bc^3x^2\operatorname{atan}\left(cx\right)dx+\int\left(-3ibc^2x\operatorname{atan}\left(cx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x,x)
```

```
[Out] -I*d**3*(Integral(-3*a*c, x) + Integral(I*a/x, x) + Integral(a*c**3*x**2, x
) + Integral(-3*b*c*atan(c*x), x) + Integral(-3*I*a*c**2*x, x) + Integral(I
*b*atan(c*x)/x, x) + Integral(b*c**3*x**2*atan(c*x), x) + Integral(-3*I*b*c
**2*x*atan(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.83, size = 196, normalized size = 1.15

$$\begin{cases} a\,d^3\,\ln\left(x\right) & \text{if}\ \ c=0\\ a\,d^3\,\ln\left(x\right)-\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)\,3\mathrm{i}}{2}-\frac{b\,d^3\,\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{b\,d^3\,\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}-\frac{3\,a\,c^2\,d^3\,x^2}{2}-\frac{a\,c^3\,d^3\,x^3\,1\mathrm{i}}{3}+a\,c\,d^3\,x\,3\mathrm{i}+\frac{3\,b\,c\,d^3\,x}{2}+\frac{b\,c^2\,d^3\,\left(\frac{x^2}{2}-\frac{\ln\left(c^2\,x^2+1\right)}{2\,c^2}\right)\,1\mathrm{i}}{3}-3\,b\,c^2\,d^3\,\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)-\frac{b\,c^3\,d^3\,x^3\,\mathrm{atan}(c\,x)\,1\mathrm{i}}{3}+b\,c\,d^3\,x\,\mathrm{atan}(c\,x)\,3\mathrm{i} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x,x)
```

```
[Out] piecewise(c == 0, a*d^3*log(x), c ~= 0, - (b*d^3*log(c^2*x^2 + 1)*3i)/2 + a
*d^3*log(x) - (b*d^3*dilog(- c*x*1i + 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)*1
i)/2 - (3*a*c^2*d^3*x^2)/2 - (a*c^3*d^3*x^3*1i)/3 + a*c*d^3*x*3i + (3*b*c*d
^3*x)/2 + (b*c^2*d^3*(x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*1i)/3 - 3*b*c^2*d^3
*atan(c*x)*(1/(2*c^2) + x^2/2) - (b*c^3*d^3*x^3*atan(c*x)*1i)/3 + b*c*d^3*x
*atan(c*x)*3i)
```

3.25 $\int \frac{(d+icdx)^3(a+b\textbf{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=162

$$-3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\text{ArcTan}(cx)-3bc^2d^3x\text{ArcTan}(cx)-\frac{d^3(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}ic^3d^3x^2(a+b\text{ArcTan}$$

```
[Out] -3*a*c^2*d^3*x+1/2*I*b*c^2*d^3*x-1/2*I*b*c*d^3*arctan(c*x)-3*b*c^2*d^3*x*ar
ctan(c*x)-d^3*(a+b*arctan(c*x))/x-1/2*I*c^3*d^3*x^2*(a+b*arctan(c*x))+3*I*a
*c*d^3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-I*c
*x)+3/2*b*c*d^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {4996, 4930, 266, 4946, 272, 36, 29, 31, 4940, 2438, 327, 209}

$$-\frac{1}{2}ic^3d^3x^2(a+b\text{ArcTan}(cx))-\frac{d^3(a+b\text{ArcTan}(cx))}{x}-3ac^2d^3x+3iacd^3\log(x)-3bc^2d^3x\text{ArcTan}(cx)-\frac{1}{2}ibcd^3\text{ArcTan}(cx)+bcd^3\log\left(c^2x^2+1\right)+\frac{1}{2}ibc^2d^3x-\frac{3}{2}bcd^3\text{Li}_2(-icx)+\frac{3}{2}bcd^3\text{Li}_2(icx)+bcd^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -3*a*c^2*d^3*x + (I/2)*b*c^2*d^3*x - (I/2)*b*c*d^3*ArcTan[c*x] - 3*b*c^2*d^
3*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/x - (I/2)*c^3*d^3*x^2*(a + b*Ar
cTan[c*x]) + (3*I)*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 + c^2*x^
2] - (3*b*c*d^3*PolyLog[2, (-I)*c*x])/2 + (3*b*c*d^3*PolyLog[2, I*c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx = \int \left(-3c^2d^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x^2}+\frac{3icd^3\left(a+b\tan\right.}{x}\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx+\left(3icd^3\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx-\left(3c^2d^3\right)\int$$

$$= -3ac^2d^3x-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}ic^3d^3x^2\left(a+b\tan^{-1}(cx)\right)+3iacd$$

$$= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}$$

$$= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\tan^{-1}(cx)-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3(a}{}$$

$$= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\tan^{-1}(cx)-3bc^2d^3x\tan^{-1}(cx)-\frac{d^3(a}{}$$

Mathematica [A]
time = 0.06, size = 150, normalized size = 0.93

$$\frac{d^3\left(-2a-6ac^2x^2+ibc^2x^2-iac^3x^3-2b\text{ArcTan}(cx)-ibcx\text{ArcTan}(cx)-6bc^2x^2\text{ArcTan}(cx)-ibc^3x^3\text{ArcTan}(cx)+6iacx\log(x)+2bcx\log(cx)+2bcx\log\left(1+c^2x^2\right)-3bcx\text{PolyLog}(2,-icx)+3bcx\text{PolyLog}(2,icx)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (d^3*(-2*a - 6*a*c^2*x^2 + I*b*c^2*x^2 - I*a*c^3*x^3 - 2*b*ArcTan[c*x] - I*
b*c*x*ArcTan[c*x] - 6*b*c^2*x^2*ArcTan[c*x] - I*b*c^3*x^3*ArcTan[c*x] + (6*
I)*a*c*x*Log[x] + 2*b*c*x*Log[c*x] + 2*b*c*x*Log[1 + c^2*x^2] - 3*b*c*x*Pol
yLog[2, (-I)*c*x] + 3*b*c*x*PolyLog[2, I*c*x]))/(2*x)
```

Maple [A]
time = 0.08, size = 216, normalized size = 1.33

method	result
derivativedivides	$c\Big(-3ac\,d^3x-\frac{id^3b\arctan(cx)c^2x^2}{2}-\frac{d^3a}{cx}-\frac{id^3b\arctan(cx)}{2}-3b\arctan(cx)\,d^3cx+3id^3a\ln(cx)$

default	$c\Big(-3ac\,d^3x - \frac{id^3b\arctan(cx)c^2x^2}{2} - \frac{d^3a}{cx} - \frac{id^3b\arctan(cx)}{2} - 3b\arctan(cx)\,d^3cx + 3id^3a\ln(cx)$
risch	$-\frac{b\,d^3c^3\ln(icx+1)x^2}{4} + \frac{3ib\,d^3c^2\ln(icx+1)x}{2} + bc\,d^3\ln(c^2x^2+1) - \frac{3id^3c^2b\ln(-icx+1)x}{2} - \frac{id^3b\ln(-icx+}{2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-3*a*c*d^3*x-1/2*I*d^3*b*arctan(c*x)*c^2*x^2-d^3*a/c/x-1/2*I*d^3*b*arcta
n(c*x)-3*b*arctan(c*x)*d^3*c*x+3*I*d^3*a*ln(c*x)-d^3*b*arctan(c*x)/c/x+3*I*
d^3*b*arctan(c*x)*ln(c*x)-3/2*d^3*b*ln(c*x)*ln(1+I*c*x)+3/2*d^3*b*ln(c*x)*l
n(1-I*c*x)-3/2*d^3*b*dilog(1+I*c*x)+3/2*d^3*b*dilog(1-I*c*x)-1/2*I*d^3*a*c^
2*x^2+b*ln(c^2*x^2+1)*d^3+1/2*I*d^3*b*c*x+d^3*b*ln(c*x))
```

Maxima [A]
time = 0.60, size = 201, normalized size = 1.24

$$-\frac{1}{2}i\,ac^3d^3x^2 - 3\,ac^2d^3x + \frac{1}{2}i\,bc^2d^3x - \frac{3}{4}i\,\pi bcd^3\log\left(c^2x^2+1\right) + 3i\,bcd^3\arctan\left(cx\right)\log\left(cx\right) - \frac{3}{2}\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bcd^3 + \frac{3}{2}\,bcd^3\mathrm{Li}_2(i\,cx+1) - \frac{3}{2}\,bcd^3\mathrm{Li}_2(-i\,cx+1) + 3i\,acd^3\log\left(x\right) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan\left(cx\right)}{x}\right)bd^3 - \frac{ad^3}{x} + \frac{1}{2}\left(-i\,bc^3d^3x^2 - i\,bcd^3\right)\arctan\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*I*a*c^3*d^3*x^2 - 3*a*c^2*d^3*x + 1/2*I*b*c^2*d^3*x - 3/4*I*pi*b*c*d^3
*log(c^2*x^2 + 1) + 3*I*b*c*d^3*arctan(c*x)*log(c*x) - 3/2*(2*c*x*arctan(c*
x) - log(c^2*x^2 + 1))*b*c*d^3 + 3/2*b*c*d^3*dilog(I*c*x + 1) - 3/2*b*c*d^3
*dilog(-I*c*x + 1) + 3*I*a*c*d^3*log(x) - 1/2*(c*(log(c^2*x^2 + 1) - log(x^
2)) + 2*arctan(c*x)/x)*b*d^3 - a*d^3/x + 1/2*(-I*b*c^3*d^3*x^2 - I*b*c*d^3)
*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-id^3\left(\int\left(-3iac^2\right)dx + \int\frac{ia}{x^2}\,dx + \int\left(-\frac{3ac}{x}\right)dx + \int ac^3x\,dx + \int\left(-3ibc^2\operatorname{atan}(cx)\right)dx + \int\frac{ib\operatorname{atan}(cx)}{x^2}\,dx + \int\left(-\frac{3bc\operatorname{atan}(cx)}{x}\right)dx + \int bc^3x\operatorname{atan}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**2,x)
```

```
[Out] -I*d**3*(Integral(-3*I*a*c**2, x) + Integral(I*a/x**2, x) + Integral(-3*a*c
/x, x) + Integral(a*c**3*x, x) + Integral(-3*I*b*c**2*atan(c*x), x) + Integ
ral(I*b*atan(c*x)/x**2, x) + Integral(-3*b*c*atan(c*x)/x, x) + Integral(b*c
**3*x*atan(c*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.72, size = 195, normalized size = 1.20

$$\begin{cases} -\frac{a\,d^3}{x} & \text{if } c=0\\ \frac{b\,d^3\,\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}-\frac{a\,c^3\,d^3\,x^2\,1\mathrm{i}}{2}-\frac{a\,d^3}{x}+\frac{3\,b\,c\,d^3\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)}{2}+\frac{3\,b\,c\,d^3\,\ln\left(c^2\,x^2+1\right)}{2}-3\,a\,c^2\,d^3\,x+\frac{b\,c^2\,d^3\,x\,1\mathrm{i}}{2}+a\,c\,d^3\,\ln\left(x\right)\,3\mathrm{i}-\frac{b\,d^3\,\mathrm{atan}(c\,x)}{x}-3\,b\,c^2\,d^3\,x\,\mathrm{atan}(c\,x)-b\,c^3\,d^3\,\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\,1\mathrm{i} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^2,x)
```

```
[Out] piecewise(c == 0, -(a*d^3)/x, c ~= 0, - (a*d^3)/x - (a*c^3*d^3*x^2*1i)/2 +
(b*d^3*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + (3*b*c*d^3*(dilog(- c*x
*1i + 1) - dilog(c*x*1i + 1)))/2 + (3*b*c*d^3*log(c^2*x^2 + 1))/2 - 3*a*c^2
*d^3*x + (b*c^2*d^3*x*1i)/2 + a*c*d^3*log(x)*3i - (b*d^3*atan(c*x))/x - 3*b
*c^2*d^3*x*atan(c*x) - b*c^3*d^3*atan(c*x)*(1/(2*c^2) + x^2/2)*1i)
```

3.26 $\int \frac{(d+icdx)^3(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=180

$$-\frac{bcd^3}{2x}-iac^3d^3x-\frac{1}{2}bc^2d^3\text{ArcTan}(cx)-ibc^3d^3x\text{ArcTan}(cx)-\frac{d^3(a+b\text{ArcTan}(cx))}{2x^2}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{x}$$

```
[Out] -1/2*b*c*d^3/x-I*a*c^3*d^3*x-1/2*b*c^2*d^3*arctan(c*x)-I*b*c^3*d^3*x*arctan
(c*x)-1/2*d^3*(a+b*arctan(c*x))/x^2-3*I*c*d^3*(a+b*arctan(c*x))/x-3*a*c^2*d
^3*ln(x)+3*I*b*c^2*d^3*ln(x)-I*b*c^2*d^3*ln(c^2*x^2+1)-3/2*I*b*c^2*d^3*poly
log(2,-I*c*x)+3/2*I*b*c^2*d^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$,
Rules used = {4996, 4930, 266, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{2x^2}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{x}-iac^3d^3x-3ac^2d^3\log(x)-ibc^3d^3x\text{ArcTan}(cx)-\frac{1}{2}bc^2d^3\text{ArcTan}(cx)-\frac{3}{2}ibc^2d^3\text{Li}_2(-icx)+\frac{3}{2}ibc^2d^3\text{Li}_2(icx)-ibc^2d^3\log\left(c^2x^2+1\right)+3ibc^2d^3\log(x)-\frac{bcd^3}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^3)/x - I*a*c^3*d^3*x - (b*c^2*d^3*ArcTan[c*x])/2 - I*b*c^3*d^3*
x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) - ((3*I)*c*d^3*(a + b*Arc
Tan[c*x]))/x - 3*a*c^2*d^3*Log[x] + (3*I)*b*c^2*d^3*Log[x] - I*b*c^2*d^3*Lo
g[1 + c^2*x^2] - ((3*I)/2)*b*c^2*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/2)*b*c^2
*d^3*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x^3}\,dx = \int \left(-ic^3d^3\left(a+b\tan^{-1}(cx)\right)+\frac{d^3(a+b\tan^{-1}(cx))}{x^3}+\frac{3icd^3(a+b\tan$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(3icd^3\right)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(3c^2d^3\right)\int$$

$$= -iac^3d^3x-\frac{d^3(a+b\tan^{-1}(cx))}{2x^2}-\frac{3icd^3(a+b\tan^{-1}(cx))}{x}-3ac^2d^3\,\mathrm{lo}$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3(a+b\tan^{-1}(cx))}{2x^2}-\frac{3icd^3}{}$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-\frac{1}{2}bc^2d^3\tan^{-1}(cx)-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3(a+b}{}$$

$$= -\frac{bcd^3}{2x}-iac^3d^3x-\frac{1}{2}bc^2d^3\tan^{-1}(cx)-ibc^3d^3x\tan^{-1}(cx)-\frac{d^3(a+b}{}$$

Mathematica [A]
time = 0.06, size = 164, normalized size = 0.91

$$-\frac{id^3(-ia+6acx-ibcx+2ac^3x^3-ib\mathrm{ArcTan}(cx)+6bcx\mathrm{ArcTan}(cx)-ibc^2x^2\mathrm{ArcTan}(cx)+2bc^3x^3\mathrm{ArcTan}(cx)-6iac^2x^2\log(x)-6bc^2x^2\log(cx)+2bc^2x^2\log(1+c^2x^2)+3bc^2x^2\mathrm{PolyLog}(2,-icx)-3bc^2x^2\mathrm{PolyLog}(2,icx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] ((-1/2*I)*d^3*((-I)*a + 6*a*c*x - I*b*c*x + 2*a*c^3*x^3 - I*b*ArcTan[c*x] +
 6*b*c*x*ArcTan[c*x] - I*b*c^2*x^2*ArcTan[c*x] + 2*b*c^3*x^3*ArcTan[c*x] -
(6*I)*a*c^2*x^2*Log[x] - 6*b*c^2*x^2*Log[c*x] + 2*b*c^2*x^2*Log[1 + c^2*x^2
] + 3*b*c^2*x^2*PolyLog[2, (-I)*c*x] - 3*b*c^2*x^2*PolyLog[2, I*c*x]))/x^2
```

Maple [A]
time = 0.09, size = 228, normalized size = 1.27

method	result
derivativedivides	$c^2\Big(-\frac{3id^3b\ln(cx)\ln(icx+1)}{2}-\frac{d^3a}{2c^2x^2}+\frac{3id^3b\ln(cx)\ln(-icx+1)}{2}-3d^3a\ln(cx)+\frac{3id^3b\,\mathrm{dilog}(-icx+1)}{2}-$

default	$c^2\left(-\frac{3id^3b\ln(cx)\ln(icx+1)}{2}-\frac{d^3a}{2c^2x^2}+\frac{3id^3b\ln(cx)\ln(-icx+1)}{2}-3d^3a\ln(cx)+\frac{3id^3b\,\mathrm{dilog}(-icx+1)}{2}-d^3\right.$
risch	$-\frac{d^3b\,c^3\ln(icx+1)x}{2}-\frac{id^3b\ln(-icx+1)}{4x^2}+\frac{5id^3b\,c^2\ln(icx)}{4}+\frac{7id^3c^2b\ln(-icx)}{4}-ia\,c^3d^3x-\frac{3d^3bc\ln(icx+1)}{2x}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-3/2*I*d^3*b*ln(c*x)*ln(1+I*c*x)-1/2*d^3*a/c^2/x^2+3/2*I*d^3*b*ln(c*x)
*ln(1-I*c*x)-3*d^3*a*ln(c*x)-3*I*d^3*b*arctan(c*x)/c/x-1/2*d^3*b*arctan(c*x
)/c^2/x^2-I*d^3*a*c*x-3*d^3*b*ln(c*x)*arctan(c*x)-3/2*I*d^3*b*dilog(1+I*c*x
)-3*I*d^3*a/c/x-I*d^3*b*ln(c^2*x^2+1)+3*I*d^3*b*ln(c*x)-I*d^3*b*arctan(c*x)
*c*x-1/2*b*d^3*arctan(c*x)-1/2*d^3*b/c/x+3/2*I*d^3*b*dilog(1-I*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -I*a*c^3*d^3*x - 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^2*d^3 - 3
*b*c^2*d^3*integrate(arctan(c*x)/x, x) - 3*a*c^2*d^3*log(x) - 3/2*I*(c*(log
(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^3 - 1/2*((c*arctan(c*x)
+ 1/x)*c + arctan(c*x)/x^2)*b*d^3 - 3*I*a*c*d^3/x - 1/2*a*d^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**3,x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.73, size = 205, normalized size = 1.14

$$\begin{cases} -\frac{a\,d^3}{2\,x^2} & \text{if}\ \ c=0 \\ b\,d^3\,\left(c^2\,\ln\left(x\right)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)\,3\mathrm{i}+\frac{b\,c^2\,d^3\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}}{2}+\frac{b\,c^2\,d^3\,\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)\,3\mathrm{i}}{2}-\frac{b\,c^2\,d^3\,\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\,3\mathrm{i}}{2}-\frac{b\,d^3\,\left(c^3\,\mathrm{atan}\left(c\,x\right)+\frac{c^2}{x}\right)}{2\,c}-\frac{a\,d^3\,\left(6\,c^2\,x^2\,\ln\left(x\right)+1+c\,x\,6\mathrm{i}+c^3\,x^3\,2\mathrm{i}\right)}{2\,x^2}-\frac{b\,d^3\,\mathrm{atan}\left(c\,x\right)}{2\,x^2}-\frac{b\,c\,d^3\,\mathrm{atan}\left(c\,x\right)\,3\mathrm{i}}{x}-b\,c^3\,d^3\,x\,\mathrm{atan}\left(c\,x\right)\,1\mathrm{i} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^3,x)
```

```
[Out] piecewise(c == 0, -(a*d^3)/(2*x^2), c ~= 0, b*d^3*(c^2*log(x) - (c^2*log(c^
2*x^2 + 1))/2)*3i + (b*c^2*d^3*log(c^2*x^2 + 1)*1i)/2 + (b*c^2*d^3*dilog(-
c*x*1i + 1)*3i)/2 - (b*c^2*d^3*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*(c^3*atan(c
*x) + c^2/x))/(2*c) - (a*d^3*(c*x*6i + c^3*x^3*2i + 6*c^2*x^2*log(x) + 1))/
(2*x^2) - (b*d^3*atan(c*x))/(2*x^2) - (b*c*d^3*atan(c*x)*3i)/x - b*c^3*d^3*
x*atan(c*x)*1i)
```

3.27 $\int \frac{(d+icdx)^3(a+b\textbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{bcd^3}{6x^2}-\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\text{ArcTan}(cx)-\frac{d^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{2x^2}+\frac{3c^2d^3(a+b\text{ArcTan}(cx))}{x}$$

```
[Out] -1/6*b*c*d^3/x^2-3/2*I*b*c^2*d^3/x-3/2*I*b*c^3*d^3*arctan(c*x)-1/3*d^3*(a+b
*arctan(c*x))/x^3-3/2*I*c*d^3*(a+b*arctan(c*x))/x^2+3*c^2*d^3*(a+b*arctan(c
*x))/x-I*a*c^3*d^3*ln(x)-10/3*b*c^3*d^3*ln(x)+5/3*b*c^3*d^3*ln(c^2*x^2+1)+1
/2*b*c^3*d^3*polylog(2,-I*c*x)-1/2*b*c^3*d^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4996, 4946, 272, 46, 331, 209, 36, 29, 31, 4940, 2438}

$$\frac{3c^2d^3(a+b\text{ArcTan}(cx))}{x}-\frac{d^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{2x^2}-iac^3d^3\log(x)-\frac{3}{2}ibc^3d^3\text{ArcTan}(cx)+\frac{1}{2}bc^3d^3\text{Li}_2(-icx)-\frac{1}{2}bc^3d^3\text{Li}_2(icx)-\frac{10}{3}bc^3d^3\log(x)-\frac{3ibc^2d^3}{2x}+\frac{5}{3}bc^3d^3\log\left(c^2x^2+1\right)-\frac{bcd^3}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^3)/x^2 - (((3*I)/2)*b*c^2*d^3)/x - ((3*I)/2)*b*c^3*d^3*ArcTan[c
*x] - (d^3*(a + b*ArcTan[c*x]))/(3*x^3) - (((3*I)/2)*c*d^3*(a + b*ArcTan[c*
x]))/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/x - I*a*c^3*d^3*Log[x] - (10*b*c
^3*d^3*Log[x])/3 + (5*b*c^3*d^3*Log[1 + c^2*x^2])/3 + (b*c^3*d^3*PolyLog[2,
 (-I)*c*x])/2 - (b*c^3*d^3*PolyLog[2, I*c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= \int \left(\frac{d^3(a+b\tan^{-1}(cx))}{x^4}+\frac{3icd^3(a+b\tan^{-1}(cx))}{x^3}-\frac{3c^2d^3(a+b\tan^{-1}\right.\\
&= d^3\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx+\left(3icd^3\right)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\left(3c^2d^3\right)\int\\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3icd^3(a+b\tan^{-1}(cx))}{2x^2}+\frac{3c^2d^3(a+b\tan^{-1}(c}{x}\\
&= -\frac{3ibc^2d^3}{2x}-\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3icd^3(a+b\tan^{-1}(cx))}{2x^2}+\frac{3c^2d^3(a+}{}\\
&= -\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\tan^{-1}(cx)-\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3icd^3(a+b\tan}{2x^2}\\
&= -\frac{bcd^3}{6x^2}-\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\tan^{-1}(cx)-\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3icd^3(}{}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 171, normalized size = 0.90

$$\frac{d^3(-2a-9iacx-bcx+18ac^2x^2-9ibc^2x^2-2b\text{ArcTan}(cx)-9ibcx\text{ArcTan}(cx)+18bc^2x^2\text{ArcTan}(cx)-9ibc^3x^3\text{ArcTan}(cx)-6iac^3x^3\log(x)-20bc^3x^3\log(cx)+10bc^3x^3\log(1+c^2x^2)+3bc^3x^3\text{PolyLog}(2,-icx)-3bc^3x^3\text{PolyLog}(2,icx))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (d^3*(-2*a - (9*I)*a*c*x - b*c*x + 18*a*c^2*x^2 - (9*I)*b*c^2*x^2 - 2*b*Arc
Tan[c*x] - (9*I)*b*c*x*ArcTan[c*x] + 18*b*c^2*x^2*ArcTan[c*x] - (9*I)*b*c^3
*x^3*ArcTan[c*x] - (6*I)*a*c^3*x^3*Log[x] - 20*b*c^3*x^3*Log[c*x] + 10*b*c^
3*x^3*Log[1 + c^2*x^2] + 3*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 3*b*c^3*x^3*Pol
yLog[2, I*c*x]))/(6*x^3)
```

Maple [A]
time = 0.09, size = 244, normalized size = 1.29

method	result
derivativedivides	$c^3\left(-\frac{3id^3b}{2cx}-\frac{d^3a}{3c^3x^3}+\frac{3d^3a}{cx}-\frac{3id^3b\arctan(cx)}{2}-id^3a\ln(cx)-\frac{d^3b\arctan(cx)}{3c^3x^3}+\frac{3d^3b\arctan(cx)}{cx}-id^3\right.$
default	$c^3\left(-\frac{3id^3b}{2cx}-\frac{d^3a}{3c^3x^3}+\frac{3d^3a}{cx}-\frac{3id^3b\arctan(cx)}{2}-id^3a\ln(cx)-\frac{d^3b\arctan(cx)}{3c^3x^3}+\frac{3d^3b\arctan(cx)}{cx}-id^3\right.$

risch	$\frac{d^3 b\, c^3 \operatorname{dilog}(icx+1)}{2} - \frac{11 d^3 b\, c^3 \ln(icx)}{12} + \frac{5 b\, c^3 d^3 \ln(c^2 x^2+1)}{3} - i d^3 c^3 a \ln(-icx) - \frac{3 i b\, c^2 d^3}{2x} - \frac{3 i d^3 b\, c^2 \ln(i}{2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-3/2*I*d^3*b/c/x-1/3*d^3*a/c^3/x^3+3*d^3*a/c/x-3/2*I*d^3*b*arctan(c*x)
-I*d^3*a*ln(c*x)-1/3*d^3*b*arctan(c*x)/c^3/x^3+3*d^3*b*arctan(c*x)/c/x-I*d^
3*b*arctan(c*x)*ln(c*x)+1/2*d^3*b*ln(c*x)*ln(1+I*c*x)-1/2*d^3*b*ln(c*x)*ln(
1-I*c*x)-1/2*d^3*b*dilog(1-I*c*x)+1/2*d^3*b*dilog(1+I*c*x)+5/3*b*ln(c^2*x^2
+1)*d^3-3/2*I*d^3*a/c^2/x^2-3/2*I*d^3*b*arctan(c*x)/c^2/x^2-1/6*d^3*b/c^2/x
^2-10/3*d^3*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -I*b*c^3*d^3*integrate(arctan(c*x)/x, x) - I*a*c^3*d^3*log(x) + 3/2*(c*(log
(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^3 - 3/2*I*((c*arctan(c
*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*
log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 3/2*I*a*c*
d^3/x^2 - 1/3*a*d^3/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d^
3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x +
 I)/(c*x - I)))/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.97, size = 221, normalized size = 1.17

$$\begin{cases} -\frac{a\,d^3}{3\,x^3} & \text{if}\ \ c=0\\ \frac{b\,c^3\,d^3\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6}-\frac{b\,c^3\,d^3\,\ln(x)}{3}-\frac{b\,c^3\,d^3\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)}{2}-3\,b\,c\,d^3\,\left(c^2\,\ln\left(x\right)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)-\frac{b\,c\,d^3}{6\,x^2}-\frac{a\,d^3\,\left(2-18\,c^2\,x^2+c\,x\,9\mathrm{i}+c^3\,x^3\,\ln(x)\,6\mathrm{i}\right)}{6\,x^3}-\frac{b\,d^3\,\mathrm{atan}(c\,x)}{3\,x^3}+\frac{3\,b\,c^2\,d^3\,\mathrm{atan}(c\,x)}{x}-\frac{b\,d^3\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)\,3\mathrm{i}}{2}-\frac{b\,c\,d^3\,\mathrm{atan}(c\,x)\,3\mathrm{i}}{2\,x^2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^4,x)
```

```
[Out] piecewise(c == 0, -(a*d^3)/(3*x^3), c ~= 0, - (b*d^3*(c^3*atan(c*x) + c^2/x
)*3i)/2 - (b*c^3*d^3*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 - (b*c^3*
d^3*log(x))/3 + (b*c^3*d^3*log(- (3*c^4)/2 - (3*c^6*x^2)/2))/6 - 3*b*c*d^3*
(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b*c*d^3)/(6*x^2) - (a*d^3*(c*x*9
i - 18*c^2*x^2 + c^3*x^3*log(x)*6i + 2))/(6*x^3) - (b*d^3*atan(c*x))/(3*x^3
) - (b*c*d^3*atan(c*x)*3i)/(2*x^2) + (3*b*c^2*d^3*atan(c*x))/x)
```

3.28 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=103

$$-\frac{bcd^3}{12x^3}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x}-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4x^4}-2ibc^4d^3\log(x)+2ibc^4d^3\log(i+cx)$$

```
[Out] -1/12*b*c*d^3/x^3-1/2*I*b*c^2*d^3/x^2+7/4*b*c^3*d^3/x-1/4*d^3*(1+I*c*x)^4*(
a+b*arctan(c*x))/x^4-2*I*b*c^4*d^3*ln(x)+2*I*b*c^4*d^3*ln(c*x+I)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {37, 4992, 12, 90}

$$-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{4x^4}-2ibc^4d^3\log(x)+2ibc^4d^3\log(cx+i)+\frac{7bc^3d^3}{4x}-\frac{ibc^2d^3}{2x^2}-\frac{bcd^3}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^3)/x^3 - ((I/2)*b*c^2*d^3)/x^2 + (7*b*c^3*d^3)/(4*x) - (d^3*(1
 + I*c*x)^4*(a + b*ArcTan[c*x]))/(4*x^4) - (2*I)*b*c^4*d^3*Log[x] + (2*I)*b
*c^4*d^3*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
```

```
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x^5}\,dx &= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{4x^4}-(bc)\int \frac{d^3(i-cx)^3}{4x^4(i+cx)}\,dx\\ &= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{4x^4}-\frac{1}{4}\left(bcd^3\right)\int \frac{(i-cx)^3}{x^4(i+cx)}\,dx\\ &= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{4x^4}-\frac{1}{4}\left(bcd^3\right)\int \left(-\frac{1}{x^4}-\frac{4ic}{x^3}+\frac{7c^2}{x^2}+\frac{8}{}\right.\\ &= -\frac{bcd^3}{12x^3}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x}-\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{4x^4}-2ibc^4d^3\,\mathrm{l}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 141, normalized size = 1.37

$$\frac{d^3(-3a-12iacx-bcx+18ac^2x^2-6ibc^2x^2+12iac^3x^3+21bc^3x^3+3b(-1-4icx+6c^2x^2+4ic^3x^3+7c^4x^4)\,\mathrm{ArcTan}(cx)-24ibc^4x^4\log(x)+12ibc^4x^4\log(1+c^2x^2))}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^3*(-3*a - (12*I)*a*c*x - b*c*x + 18*a*c^2*x^2 - (6*I)*b*c^2*x^2 + (12*I)
*a*c^3*x^3 + 21*b*c^3*x^3 + 3*b*(-1 - (4*I)*c*x + 6*c^2*x^2 + (4*I)*c^3*x^3
 + 7*c^4*x^4)*ArcTan[c*x] - (24*I)*b*c^4*x^4*Log[x] + (12*I)*b*c^4*x^4*Log[
1 + c^2*x^2]))/(12*x^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(90) = 180.
time = 0.16, size = 187, normalized size = 1.82

method	result
derivativedivides	$c^4\left(d^3a\left(\frac{3}{2c^2x^2}-\frac{i}{c^3x^3}+\frac{i}{cx}-\frac{1}{4c^4x^4}\right)+\frac{3d^3b\arctan(cx)}{2c^2x^2}-\frac{id^3b\arctan(cx)}{c^3x^3}+\frac{id^3b\arctan(cx)}{cx}-\frac{d^3b\arctan(}{4c^4x^4}\right.$
default	$c^4\left(d^3a\left(\frac{3}{2c^2x^2}-\frac{i}{c^3x^3}+\frac{i}{cx}-\frac{1}{4c^4x^4}\right)+\frac{3d^3b\arctan(cx)}{2c^2x^2}-\frac{id^3b\arctan(cx)}{c^3x^3}+\frac{id^3b\arctan(cx)}{cx}-\frac{d^3b\arctan(}{4c^4x^4}\right.$
risch	$\frac{(4d^3b\,c^3x^3-6id^3b\,c^2x^2-4bc\,d^3x+id^3b)\ln(icx+1)}{8x^4}-\frac{id^3(-45b\,c^4\ln(-217cx-217i)x^4-3b\,c^4\ln(119cx-119i)x^4+48b\,c^4\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(d^3*a*(3/2/c^2/x^2-I/c^3/x^3+I/c/x-1/4/c^4/x^4)+3/2*d^3*b*arctan(c*x)/
c^2/x^2-I*d^3*b*arctan(c*x)/c^3/x^3+I*d^3*b*arctan(c*x)/c/x-1/4*d^3*b*arcta
n(c*x)/c^4/x^4+I*d^3*b*ln(c^2*x^2+1)+7/4*b*d^3*arctan(c*x)-1/2*I*d^3*b/c^2/
x^2-2*I*d^3*b*ln(c*x)-1/12*d^3*b/c^3/x^3+7/4*d^3*b/c/x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(85) = 170$.
time = 0.47, size = 202, normalized size = 1.96

$$\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^3d^3+\frac{3}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^2d^3+\frac{1}{2}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd^3+\frac{i\,ac^3d^3}{x}+\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd^3+\frac{3\,ac^2d^3}{2\,x^2}-\frac{i\,acd^3}{x^3}-\frac{ad^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^3 + 3/2*(
(c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^3 + 1/2*I*((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^3 + I*a*c^3*d
^3/x + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^
4)*b*d^3 + 3/2*a*c^2*d^3/x^2 - I*a*c*d^3/x^3 - 1/4*a*d^3/x^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(85) = 170$.
time = 1.45, size = 174, normalized size = 1.69

$$\frac{-48i\,bc^4d^3x^4\log\left(x\right)+45i\,bc^4d^3x^4\log\left(\frac{cx+i}{c}\right)+3i\,bc^4d^3x^4\log\left(\frac{cx-i}{c}\right)-6\left(-4i\,a-7\,b\right)c^3d^3x^3+12\left(3\,a-i\,b\right)c^2d^3x^2-2\left(12i\,a+b\right)cd^3x-6\,ad^3-3\left(4\,bc^3d^3x^3-6i\,bc^2d^3x^2-4\,bcd^3x+i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-48*I*b*c^4*d^3*x^4*log(x) + 45*I*b*c^4*d^3*x^4*log((c*x + I)/c) + 3*
I*b*c^4*d^3*x^4*log((c*x - I)/c) - 6*(-4*I*a - 7*b)*c^3*d^3*x^3 + 12*(3*a -
 I*b)*c^2*d^3*x^2 - 2*(12*I*a + b)*c*d^3*x - 6*a*d^3 - 3*(4*b*c^3*d^3*x^3 -
 6*I*b*c^2*d^3*x^2 - 4*b*c*d^3*x + I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^4
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(99) = 198$.
time = 15.91, size = 311, normalized size = 3.02

$$-2ibc^4d^3\log\left(3689b^2c^9d^6x\right)+\frac{ibc^4d^3\log\left(3689b^2c^9d^6x-3689ib^2c^8d^6\right)}{8}+\frac{15ibc^4d^3\log\left(3689b^2c^9d^6x+3689ib^2c^8d^6\right)}{8}-\frac{3ad^3+x^3(-12iac^3d^3-21bc^3d^3)+x^2(-18ac^2d^3+6ibc^2d^3)+x(12iacd^3+bcd^3)}{12x^4}+\frac{(-4bc^3d^3x^3+6ibc^2d^3x^2+4bcd^3x-ibd^3)\log\left(-icx+1\right)}{8x^4}+\frac{(4bc^3d^3x^3-6ibc^2d^3x^2-4bcd^3x+ibd^3)\log\left(icx+1\right)}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**5,x)
```

```
[Out] -2*I*b*c**4*d**3*log(3689*b**2*c**9*d**6*x) + I*b*c**4*d**3*log(3689*b**2*c
**9*d**6*x - 3689*I*b**2*c**8*d**6)/8 + 15*I*b*c**4*d**3*log(3689*b**2*c**9
*d**6*x + 3689*I*b**2*c**8*d**6)/8 - (3*a*d**3 + x**3*(-12*I*a*c**3*d**3 -
```

```
21*b*c**3*d**3) + x**2*(-18*a*c**2*d**3 + 6*I*b*c**2*d**3) + x*(12*I*a*c*d*
*3 + b*c*d**3))/(12*x**4) + (-4*b*c**3*d**3*x**3 + 6*I*b*c**2*d**3*x**2 + 4
*b*c*d**3*x - I*b*d**3)*log(-I*c*x + 1)/(8*x**4) + (4*b*c**3*d**3*x**3 - 6*
I*b*c**2*d**3*x**2 - 4*b*c*d**3*x + I*b*d**3)*log(I*c*x + 1)/(8*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.66, size = 154, normalized size = 1.50

$$-\frac{\frac{d^3\,\left(3\,a+3\,b\,\mathrm{atan}\left(c\,x\right)\right)}{12}+\frac{d^3\,x\,\left(a\,c\,12{}\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,12{}\mathrm{i}\right)}{12}-\frac{d^3\,x^2\,\left(18\,a\,c^2+18\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,6{}\mathrm{i}\right)}{12}-\frac{d^3\,x^3\,\left(a\,c^3\,12{}\mathrm{i}+21\,b\,c^3+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,12{}\mathrm{i}\right)}{12}}{x^4}+\frac{d^3\,\left(21\,b\,c^4\,\mathrm{atan}\left(c\,x\right)+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,12{}\mathrm{i}-b\,c^4\,\ln\left(x\right)\,24{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^5,x)
```

```
[Out] (d^3*(21*b*c^4*atan(c*x) + b*c^4*log(c^2*x^2 + 1)*12i - b*c^4*log(x)*24i))/
12 - ((d^3*(3*a + 3*b*atan(c*x)))/12 + (d^3*x*(a*c*12i + b*c + b*c*atan(c*x
)*12i))/12 - (d^3*x^2*(18*a*c^2 - b*c^2*6i + 18*b*c^2*atan(c*x)))/12 - (d^3
*x^3*(a*c^3*12i + 21*b*c^3 + b*c^3*atan(c*x)*12i))/12)/x^4
```

3.29 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{bcd^3}{20x^4}-\frac{ibc^2d^3}{4x^3}+\frac{3bc^3d^3}{5x^2}+\frac{5ibc^4d^3}{4x}-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\text{ArcTan}(cx))}{20x^4}+\frac{6}{5}bc$$

```
[Out] -1/20*b*c*d^3/x^4-1/4*I*b*c^2*d^3/x^3+3/5*b*c^3*d^3/x^2+5/4*I*b*c^4*d^3/x-1
/5*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/x^5+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*arct
an(c*x))/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(c*x+I)
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {47, 37, 4992, 12, 153}

$$-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\text{ArcTan}(cx))}{20x^4}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(cx+i)+\frac{5ibc^4d^3}{4x}+\frac{3bc^3d^3}{5x^2}-\frac{ibc^2d^3}{4x^3}-\frac{bcd^3}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^3)/x^4 - ((I/4)*b*c^2*d^3)/x^3 + (3*b*c^3*d^3)/(5*x^2) + (((5*
I)/4)*b*c^4*d^3)/x - (d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/(5*x^5) + ((I/
20)*c*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^4 + (6*b*c^5*d^3*Log[x])/5 -
 (6*b*c^5*d^3*Log[I + c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x^6}\,dx &= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{5x^5}+\frac{icd^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{20x^4}-\\
&= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{5x^5}+\frac{icd^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{20x^4}-\\
&= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{5x^5}+\frac{icd^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{20x^4}-\\
&= -\frac{bcd^3}{20x^4}-\frac{ibc^2d^3}{4x^3}+\frac{3bc^3d^3}{5x^2}+\frac{5ibc^4d^3}{4x}-\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))}{5x^5}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 149, normalized size = 0.99

$$\frac{d^3(-4a-15iacx-bcx+20ac^2x^2-5ibc^2x^2+10iac^3x^3+12bc^3x^3+25ibc^4x^4+b(-4-15icx+20c^2x^2+10ic^3x^3+25ic^5x^5)\,\mathrm{ArcTan}(cx)+24bc^5x^5\log(x)-12bc^5x^5\log(1+c^2x^2))}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (d^3*(-4*a - (15*I)*a*c*x - b*c*x + 20*a*c^2*x^2 - (5*I)*b*c^2*x^2 + (10*I)
*a*c^3*x^3 + 12*b*c^3*x^3 + (25*I)*b*c^4*x^4 + b*(-4 - (15*I)*c*x + 20*c^2*
x^2 + (10*I)*c^3*x^3 + (25*I)*c^5*x^5)*ArcTan[c*x] + 24*b*c^5*x^5*Log[x] -
12*b*c^5*x^5*Log[1 + c^2*x^2]))/(20*x^5)
```

Maple [A]
time = 0.14, size = 197, normalized size = 1.31

method	result
derivativedivides	$c^5\left(d^3a\left(\frac{i}{2c^2x^2}+\frac{1}{c^3x^3}-\frac{3i}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{id^3b\arctan(cx)}{2c^2x^2}+\frac{d^3b\arctan(cx)}{c^3x^3}-\frac{3id^3b\arctan(cx)}{4c^4x^4}-\frac{d^3b\,\mathrm{a}}{5}\right.$
default	$c^5\left(d^3a\left(\frac{i}{2c^2x^2}+\frac{1}{c^3x^3}-\frac{3i}{4c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{id^3b\arctan(cx)}{2c^2x^2}+\frac{d^3b\arctan(cx)}{c^3x^3}-\frac{3id^3b\arctan(cx)}{4c^4x^4}-\frac{d^3b\,\mathrm{a}}{5}\right.$
risch	$\frac{(10d^3bc^3x^3-20id^3bc^2x^2-15bcd^3x+4id^3b)\ln(icx+1)}{40x^5}-\frac{d^3(49c^5b\ln(-cx-i)x^5-c^5b\ln(cx-i)x^5-48c^5b\ln(-x)x^5-50}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^3*a*(1/2*I/c^2/x^2+1/c^3/x^3-3/4*I/c^4/x^4-1/5/c^5/x^5)+1/2*I*d^3*b*
arctan(c*x)/c^2/x^2+d^3*b*arctan(c*x)/c^3/x^3-3/4*I*d^3*b*arctan(c*x)/c^4/x
^4-1/5*d^3*b*arctan(c*x)/c^5/x^5-3/5*b*ln(c^2*x^2+1)*d^3+5/4*I*d^3*b*arctan
(c*x)-1/4*I*d^3*b/c^3/x^3+5/4*I*d^3*b/c/x-1/20*d^3*b/c^4/x^4+3/5*d^3*b/c^2/
x^2+6/5*d^3*b*ln(c*x))
```

Maxima [A]
time = 0.47, size = 224, normalized size = 1.49

$$\frac{1}{2}i\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^3d^3-\frac{1}{2}\left(\left(c^2\log(c^2x^2+1)-c^2\log(x^2)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^2d^3+\frac{1}{4}i\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bcd^3-\frac{1}{20}\left(\left(2c^4\log(c^2x^2+1)-2c^4\log(x^2)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bd^3+\frac{iac^3d^3}{2x^2}+\frac{ac^2d^3}{x^3}-\frac{3iacd^3}{4x^4}-\frac{ad^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^3 - 1/2*((c^2*log
(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^3 + 1/
4*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c*d
^3 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*
c + 4*arctan(c*x)/x^5)*b*d^3 + 1/2*I*a*c^3*d^3/x^2 + a*c^2*d^3/x^3 - 3/4*I*
a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

Fricas [A]
time = 2.33, size = 185, normalized size = 1.23

$$\frac{48\,bc^5d^3x^5\log(x)-49\,bc^5d^3x^5\log\left(\frac{cx+i}{c}\right)+bc^5d^3x^5\log\left(\frac{cx-i}{c}\right)+50i\,bc^4d^3x^4-4\,(-5i\,a-6\,b)c^3d^3x^3+10\,(4\,a-i\,b)c^2d^3x^2-2\,(15i\,a+b)cd^3x-8\,ad^3-(10\,bc^3d^3x^3-20i\,bc^2d^3x^2-15\,bcd^3x+4i\,bd^3)\log\left(-\frac{cx+i}{cx-i}\right)}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(48*b*c^5*d^3*x^5*log(x) - 49*b*c^5*d^3*x^5*log((c*x + I)/c) + b*c^5*d
^3*x^5*log((c*x - I)/c) + 50*I*b*c^4*d^3*x^4 - 4*(-5*I*a - 6*b)*c^3*d^3*x^3
 + 10*(4*a - I*b)*c^2*d^3*x^2 - 2*(15*I*a + b)*c*d^3*x - 8*a*d^3 - (10*b*c^
3*d^3*x^3 - 20*I*b*c^2*d^3*x^2 - 15*b*c*d^3*x + 4*I*b*d^3)*log(-(c*x + I)/(
c*x - I)))/x^5
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.
time = 28.60, size = 326, normalized size = 2.17

$$\frac{6bc^5d^3\log(113975b^2c^{11}d^6x)}{5}+\frac{bc^5d^3\log(113975b^2c^{11}d^6x-113975ib^2c^{10}d^6)}{40}-\frac{49bc^5d^3\log(113975b^2c^{11}d^6x+113975ib^2c^{10}d^6)}{40}+\frac{(-10bc^3d^3x^3+20ibc^2d^3x^2+15bcd^3x-4ibd^3)\log(-icx+1)}{40x^5}+\frac{(10bc^3d^3x^3-20ibc^2d^3x^2-15bcd^3x+4ibd^3)\log(icx+1)}{40x^5}-\frac{4ad^3-25ibc^4d^3x^4+x^3(-10iac^3d^3-12bc^3d^3)+x^2(-20ac^2d^3+5ibc^2d^3)+x(15iacd^3+bcd^3)}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**6,x)
```

```
[Out] 6*b*c**5*d**3*log(113975*b**2*c**11*d**6*x)/5 + b*c**5*d**3*log(113975*b**2
*c**11*d**6*x - 113975*I*b**2*c**10*d**6)/40 - 49*b*c**5*d**3*log(113975*b*
*2*c**11*d**6*x + 113975*I*b**2*c**10*d**6)/40 + (-10*b*c**3*d**3*x**3 + 20
*I*b*c**2*d**3*x**2 + 15*b*c*d**3*x - 4*I*b*d**3)*log(-I*c*x + 1)/(40*x**5)
 + (10*b*c**3*d**3*x**3 - 20*I*b*c**2*d**3*x**2 - 15*b*c*d**3*x + 4*I*b*d**
3)*log(I*c*x + 1)/(40*x**5) - (4*a*d**3 - 25*I*b*c**4*d**3*x**4 + x**3*(-10
*I*a*c**3*d**3 - 12*b*c**3*d**3) + x**2*(-20*a*c**2*d**3 + 5*I*b*c**2*d**3)
 + x*(15*I*a*c*d**3 + b*c*d**3))/(20*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.95, size = 174, normalized size = 1.16

$$\frac{d^3\left(24\,b\,c^5\,\ln(x)-12\,b\,c^5\,\ln(c^2\,x^2+1)+b\,c^4\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}\,25\mathrm{i}\right)}{20}+\frac{-\frac{d^3\,(4\,a+4\,b\,\mathrm{atan}(c\,x))}{20}-\frac{d^3\,x\,(a\,c\,15\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,15\mathrm{i})}{20}+\frac{d^3\,x^3\,(a\,c^3\,10\mathrm{i}+12\,b\,c^3+b\,c^3\,\mathrm{atan}(c\,x)\,10\mathrm{i})}{20}+\frac{d^3\,x^2\,(20\,a\,c^2+20\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,5\mathrm{i})}{20}+\frac{b\,c^4\,d^3\,x^4\,5\mathrm{i}}{4}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^6,x)
```

```
[Out] (d^3*(24*b*c^5*log(x) - 12*b*c^5*log(c^2*x^2 + 1) + b*c^4*atan(x*(c^2)^(1/2
))*(c^2)^(1/2)*25i))/20 + ((d^3*x^3*(a*c^3*10i + 12*b*c^3 + b*c^3*atan(c*x)
*10i))/20 - (d^3*x*(a*c*15i + b*c + b*c*atan(c*x)*15i))/20 - (d^3*(4*a + 4*
b*atan(c*x)))/20 + (d^3*x^2*(20*a*c^2 - b*c^2*5i + 20*b*c^2*atan(c*x)))/20
+ (b*c^4*d^3*x^4*5i)/4)/x^5
```

3.30 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))}{x^7}\,dx$

Optimal. Leaf size=214

$$-\frac{bcd^3}{30x^5}-\frac{3ibc^2d^3}{20x^4}+\frac{11bc^3d^3}{36x^3}+\frac{7ibc^4d^3}{15x^2}-\frac{11bc^5d^3}{12x}-\frac{d^3(a+b\text{ArcTan}(cx))}{6x^6}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{5x^5}+\frac{3c^2d^3(a+$$

```
[Out] -1/30*b*c*d^3/x^5-3/20*I*b*c^2*d^3/x^4+11/36*b*c^3*d^3/x^3+7/15*I*b*c^4*d^3
/x^2-11/12*b*c^5*d^3/x-1/6*d^3*(a+b*arctan(c*x))/x^6-3/5*I*c*d^3*(a+b*arcta
n(c*x))/x^5+3/4*c^2*d^3*(a+b*arctan(c*x))/x^4+1/3*I*c^3*d^3*(a+b*arctan(c*x
))/x^3+14/15*I*b*c^6*d^3*ln(x)-1/120*I*b*c^6*d^3*ln(I-c*x)-37/40*I*b*c^6*d^
3*ln(c*x+I)
```

Rubi [A]
time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {45, 4992, 12, 1816}

$$\frac{ic^3d^3(a+b\text{ArcTan}(cx))}{3x^3}+\frac{3c^2d^3(a+b\text{ArcTan}(cx))}{4x^4}-\frac{d^3(a+b\text{ArcTan}(cx))}{6x^6}-\frac{3icd^3(a+b\text{ArcTan}(cx))}{5x^5}+\frac{14}{15}ibc^6d^3\log(x)-\frac{1}{120}ibc^6d^3\log(-cx+i)-\frac{37}{40}ibc^6d^3\log(cx+i)-\frac{11bc^5d^3}{12x}+\frac{7ibc^4d^3}{15x^2}+\frac{11bc^3d^3}{36x^3}-\frac{3ibc^2d^3}{20x^4}-\frac{bcd^3}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^3)/x^5 - (((3*I)/20)*b*c^2*d^3)/x^4 + (11*b*c^3*d^3)/(36*x^3)
+ (((7*I)/15)*b*c^4*d^3)/x^2 - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTan[c
*x]))/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]))/x^5 + (3*c^2*d^3*(a +
 b*ArcTan[c*x]))/(4*x^4) + ((I/3)*c^3*d^3*(a + b*ArcTan[c*x]))/x^3 + ((14*I
)/15)*b*c^6*d^3*Log[x] - (I/120)*b*c^6*d^3*Log[I - c*x] - ((37*I)/40)*b*c^6
*d^3*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))}{x^7}\,dx = -\frac{d^3(a+b\tan^{-1}(cx))}{6x^6} - \frac{3icd^3(a+b\tan^{-1}(cx))}{5x^5} + \frac{3c^2d^3(a+b\tan^{-1}(c}{4x^4}$$

$$= -\frac{d^3(a+b\tan^{-1}(cx))}{6x^6} - \frac{3icd^3(a+b\tan^{-1}(cx))}{5x^5} + \frac{3c^2d^3(a+b\tan^{-1}(c}{4x^4}$$

$$= -\frac{d^3(a+b\tan^{-1}(cx))}{6x^6} - \frac{3icd^3(a+b\tan^{-1}(cx))}{5x^5} + \frac{3c^2d^3(a+b\tan^{-1}(c}{4x^4}$$

$$= -\frac{bcd^3}{30x^5} - \frac{3ibc^2d^3}{20x^4} + \frac{11bc^3d^3}{36x^3} + \frac{7ibc^4d^3}{15x^2} - \frac{11bc^5d^3}{12x} - \frac{d^3(a+b\tan^{-1}(cx}{6x^6}$$

Mathematica [A]

time = 0.61, size = 161, normalized size = 0.75

$$\frac{d^3(-30a-108iacx-6bcx+135ac^2x^2-27ibc^2x^2+60iac^3x^3+55bc^3x^3+84ibc^4x^4-165bc^5x^5-3b(10+36icx-45c^2x^2-20ic^3x^3+55c^6x^6)\,\mathrm{ArcTan}(cx)+168ibc^6x^6\log(x)-84ibc^6x^6\log(1+c^2x^2))}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] (d^3*(-30*a - (108*I)*a*c*x - 6*b*c*x + 135*a*c^2*x^2 - (27*I)*b*c^2*x^2 +
(60*I)*a*c^3*x^3 + 55*b*c^3*x^3 + (84*I)*b*c^4*x^4 - 165*b*c^5*x^5 - 3*b*(1
0 + (36*I)*c*x - 45*c^2*x^2 - (20*I)*c^3*x^3 + 55*c^6*x^6)*ArcTan[c*x] + (1
68*I)*b*c^6*x^6*Log[x] - (84*I)*b*c^6*x^6*Log[1 + c^2*x^2]))/(180*x^6)
```

Maple [A]

time = 0.18, size = 212, normalized size = 0.99

method	result
derivativedivides	$c^6\Big(d^3a\big(-\frac{1}{6c^6x^6}+\frac{i}{3c^3x^3}+\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}\big)-\frac{d^3b\arctan(cx)}{6c^6x^6}+\frac{id^3b\arctan(cx)}{3c^3x^3}+\frac{3d^3b\arctan(cx)}{4c^4x^4}-\frac{3id^3}{}$
default	$c^6\Big(d^3a\big(-\frac{1}{6c^6x^6}+\frac{i}{3c^3x^3}+\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}\big)-\frac{d^3b\arctan(cx)}{6c^6x^6}+\frac{id^3b\arctan(cx)}{3c^3x^3}+\frac{3d^3b\arctan(cx)}{4c^4x^4}-\frac{3id^3}{}$

risch	$\frac{(20d^3b\,c^3x^3-45id^3b\,c^2x^2-36bc\,d^3x+10id^3b)\ln(icx+1)}{120x^6}-\frac{id^3(3c^6b\ln(6215cx-6215i)x^6+333c^6b\ln(-12265cx-12265$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(d^3*a*(-1/6/c^6/x^6+1/3*I/c^3/x^3+3/4/c^4/x^4-3/5*I/c^5/x^5)-1/6*d^3*b
*arctan(c*x)/c^6/x^6+1/3*I*d^3*b*arctan(c*x)/c^3/x^3+3/4*d^3*b*arctan(c*x)/
c^4/x^4-3/5*I*d^3*b*arctan(c*x)/c^5/x^5-7/15*I*d^3*b*ln(c^2*x^2+1)-11/12*b*
d^3*arctan(c*x)+14/15*I*d^3*b*ln(c*x)-3/20*I*d^3*b/c^4/x^4+7/15*I*d^3*b/c^2
/x^2-1/30*d^3*b/c^5/x^5+11/36*d^3*b/c^3/x^3-11/12*d^3*b/c/x)
```

Maxima [A]
time = 0.48, size = 248, normalized size = 1.16

$$-\frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^3d^3-\frac{1}{4}\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^2d^3-\frac{3}{20}i\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bcd^3-\frac{1}{90}\left(\left(15c^5\arctan\left(cx\right)+\frac{15c^4x^4-5c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bd^3+\frac{i\,ac^3d^3}{3x^3}+\frac{3\,ac^2d^3}{4x^4}-\frac{3i\,acd^3}{5x^5}-\frac{ad^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3
)*b*c^3*d^3 - 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c
*x)/x^4)*b*c^2*d^3 - 3/20*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*
c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^3 - 1/90*((15*c^5*arctan(c*x
) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^3 + 1/3*I
*a*c^3*d^3/x^3 + 3/4*a*c^2*d^3/x^4 - 3/5*I*a*c*d^3/x^5 - 1/6*a*d^3/x^6
```

Fricas [A]
time = 1.58, size = 198, normalized size = 0.93

$$\frac{336i\,bc^6d^3x^6\log\left(x\right)-333i\,bc^6d^3x^6\log\left(\frac{cx+i}{c}\right)-3i\,bc^6d^3x^6\log\left(\frac{cx-i}{c}\right)-330\,bc^5d^3x^5+168i\,bc^4d^3x^4-10\left(-12i\,a-11\,b\right)c^3d^3x^3+54\left(5\,a-i\,b\right)c^2d^3x^2-12\left(18i\,a+b\right)cd^3x-60\,ad^3-3\left(20\,bc^3d^3x^3-45i\,bc^2d^3x^2-36\,bcd^3x+10i\,bd^3\right)\log\left(-\frac{cx+i}{cx-i}\right)}{360\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/360*(336*I*b*c^6*d^3*x^6*log(x) - 333*I*b*c^6*d^3*x^6*log((c*x + I)/c) -
3*I*b*c^6*d^3*x^6*log((c*x - I)/c) - 330*b*c^5*d^3*x^5 + 168*I*b*c^4*d^3*x^
4 - 10*(-12*I*a - 11*b)*c^3*d^3*x^3 + 54*(5*a - I*b)*c^2*d^3*x^2 - 12*(18*I
*a + b)*c*d^3*x - 60*a*d^3 - 3*(20*b*c^3*d^3*x^3 - 45*I*b*c^2*d^3*x^2 - 36*
b*c*d^3*x + 10*I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^6
```

Sympy [A]
time = 49.48, size = 347, normalized size = 1.62

$$\frac{14ibc^6d^3\log\left(1385945b^2c^{13}d^6x\right)}{15}-\frac{ibc^6d^3\log\left(1385945b^2c^{13}d^6x-1385945ib^2c^{12}d^6\right)}{120}-\frac{37ibc^6d^3\log\left(1385945b^2c^{13}d^6x+1385945ib^2c^{12}d^6\right)}{40}+\frac{(-20bc^3d^3x^3+45ibc^2d^3x^2+36bcd^3x-10ibd^3)\log\left(-icx+1\right)}{120x^6}+\frac{(20bc^3d^3x^3-45ibc^2d^3x^2-36bcd^3x+10ibd^3)\log\left(icx+1\right)}{120x^6}-\frac{30ad^3+165bc^5d^3x^5-84ibc^4d^3x^4+x^3(-60iac^3d^3-55bc^3d^3)+x^2(-135ac^2d^3+27ibc^2d^3)+x(108iacd^3+6bcd^3)}{180x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**7,x)
```

```
[Out] 14*I*b*c**6*d**3*log(1385945*b**2*c**13*d**6*x)/15 - I*b*c**6*d**3*log(1385
945*b**2*c**13*d**6*x - 1385945*I*b**2*c**12*d**6)/120 - 37*I*b*c**6*d**3*l
og(1385945*b**2*c**13*d**6*x + 1385945*I*b**2*c**12*d**6)/40 + (-20*b*c**3*
d**3*x**3 + 45*I*b*c**2*d**3*x**2 + 36*b*c*d**3*x - 10*I*b*d**3)*log(-I*c*x
 + 1)/(120*x**6) + (20*b*c**3*d**3*x**3 - 45*I*b*c**2*d**3*x**2 - 36*b*c*d*
*3*x + 10*I*b*d**3)*log(I*c*x + 1)/(120*x**6) - (30*a*d**3 + 165*b*c**5*d**
3*x**5 - 84*I*b*c**4*d**3*x**4 + x**3*(-60*I*a*c**3*d**3 - 55*b*c**3*d**3)
+ x**2*(-135*a*c**2*d**3 + 27*I*b*c**2*d**3) + x*(108*I*a*c*d**3 + 6*b*c*d*
*3))/(180*x**6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.06, size = 192, normalized size = 0.90

$$-\frac{\frac{d^3\,\left(30\,a+30\,b\,\mathrm{atan}\left(c\,x\right)\right)}{180}+\frac{d^3\,x\,\left(a\,c\,108{}\mathrm{i}+6\,b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,108{}\mathrm{i}\right)}{180}-\frac{d^3\,x^3\,\left(a\,c^3\,60{}\mathrm{i}+55\,b\,c^3+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,60{}\mathrm{i}\right)}{180}-\frac{d^3\,x^2\,\left(135\,a\,c^2+135\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,27{}\mathrm{i}\right)}{180}+\frac{11\,b\,c^5\,d^3\,x^5}{12}-\frac{b\,c^4\,d^3\,x^4\,7{}\mathrm{i}}{15}}{x^6}-\frac{d^3\,\left(\frac{165\,b\,c^9\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{{\left(c^2\right)}^{3/2}}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,84{}\mathrm{i}-b\,c^6\,\ln\left(x\right)\,168{}\mathrm{i}\right)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^7,x)
```

```
[Out] - ((d^3*(30*a + 30*b*atan(c*x)))/180 + (d^3*x*(a*c*108i + 6*b*c + b*c*atan(
c*x)*108i))/180 - (d^3*x^3*(a*c^3*60i + 55*b*c^3 + b*c^3*atan(c*x)*60i))/18
0 - (d^3*x^2*(135*a*c^2 - b*c^2*27i + 135*b*c^2*atan(c*x)))/180 - (b*c^4*d^
3*x^4*7i)/15 + (11*b*c^5*d^3*x^5)/12)/x^6 - (d^3*(b*c^6*log(c^2*x^2 + 1)*84
i - b*c^6*log(x)*168i + (165*b*c^9*atan((c^2*x)/(c^2)^(1/2)))/(c^2)^(3/2)))
/180
```

3.31 $\int x^3(d+icdx)^4(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=238

$$\frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4x^7-\frac{11bd^4\text{ArcTan}(cx)}{8c^4}+\frac{1}{4}d^4x^4(a+$$

```
[Out] 11/8*b*d^4*x/c^3+24/35*I*b*d^4*x^2/c^2-11/24*b*d^4*x^3/c-12/35*I*b*d^4*x^4+
9/40*b*c*d^4*x^5+2/21*I*b*c^2*d^4*x^6-1/56*b*c^3*d^4*x^7-11/8*b*d^4*arctan(
c*x)/c^4+1/4*d^4*x^4*(a+b*arctan(c*x))+4/5*I*c*d^4*x^5*(a+b*arctan(c*x))-c^
2*d^4*x^6*(a+b*arctan(c*x))-4/7*I*c^3*d^4*x^7*(a+b*arctan(c*x))+1/8*c^4*d^4
*x^8*(a+b*arctan(c*x))-24/35*I*b*d^4*ln(c^2*x^2+1)/c^4
```

Rubi [A]

time = 0.15, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {45, 4992, 12, 1816, 649, 209, 266}

$$\frac{1}{8}c^4d^4x^8(a+b\text{ArcTan}(cx))-\frac{4}{7}ic^3d^4x^7(a+b\text{ArcTan}(cx))-c^2d^4x^6(a+b\text{ArcTan}(cx))+\frac{4}{5}icd^4x^5(a+b\text{ArcTan}(cx))+\frac{1}{4}d^4x^4(a+b\text{ArcTan}(cx))-\frac{11bd^4\text{ArcTan}(cx)}{8c^4}-\frac{1}{56}bc^3d^4x^7+\frac{11bd^4x}{8c^3}+\frac{2}{21}ibc^2d^4x^6+\frac{24ibd^4x^2}{35c^2}-\frac{24ibd^4\log\left(c^2x^2+1\right)}{35c^4}+\frac{9}{40}bcd^4x^5-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
- ((12*I)/35)*b*d^4*x^4 + (9*b*c*d^4*x^5)/40 + ((2*I)/21)*b*c^2*d^4*x^6 - (
b*c^3*d^4*x^7)/56 - (11*b*d^4*ArcTan[c*x])/(8*c^4) + (d^4*x^4*(a + b*ArcTan
[c*x]))/4 + ((4*I)/5)*c*d^4*x^5*(a + b*ArcTan[c*x]) - c^2*d^4*x^6*(a + b*Ar
cTan[c*x]) - ((4*I)/7)*c^3*d^4*x^7*(a + b*ArcTan[c*x]) + (c^4*d^4*x^8*(a +
b*ArcTan[c*x]))/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^3(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)\,dx = \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\right.$$

$$= \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\right.$$

$$= \frac{1}{4}d^4x^4\left(a+b\tan^{-1}(cx)\right)+\frac{4}{5}icd^4x^5\left(a+b\tan^{-1}(cx)\right)-c^2d^4x^6\left(a+b\right.$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x$$

$$= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x$$

Mathematica [A]
time = 0.08, size = 178, normalized size = 0.75

$$\frac{d^4(3ac^4x^4(70+224icx-280c^2x^2-160ic^3x^3+35c^4x^4)+bcx(1155+576icx-385c^2x^2-288ic^3x^3+189c^4x^4+80ic^5x^5-15c^6x^6)+3b(-385+70c^4x^4+224ic^5x^5-280c^6x^6-160ic^7x^7+35c^8x^8)\operatorname{ArcTan}(cx)-576ib\log(1+c^2x^2))}{840c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^4*(3*a*c^4*x^4*(70 + (224*I)*c*x - 280*c^2*x^2 - (160*I)*c^3*x^3 + 35*c^
4*x^4) + b*c*x*(1155 + (576*I)*c*x - 385*c^2*x^2 - (288*I)*c^3*x^3 + 189*c^
4*x^4 + (80*I)*c^5*x^5 - 15*c^6*x^6) + 3*b*(-385 + 70*c^4*x^4 + (224*I)*c^5
*x^5 - 280*c^6*x^6 - (160*I)*c^7*x^7 + 35*c^8*x^8)*ArcTan[c*x] - (576*I)*b*
Log[1 + c^2*x^2]))/(840*c^4)
```

Maple [A]
time = 0.17, size = 246, normalized size = 1.03

method	result
derivativedivides	$d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^4b\arctan(cx)c^8x^8}{8}-\frac{4id^4b\arctan(cx)c^7x^7}{7}-d^4b\arctan(cx)c^6x^6+\frac{4id^4b\operatorname{arcta}}{5}$
default	$d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+\frac{d^4b\arctan(cx)c^8x^8}{8}-\frac{4id^4b\arctan(cx)c^7x^7}{7}-d^4b\arctan(cx)c^6x^6+\frac{4id^4b\operatorname{arcta}}{5}$
risch	$-\frac{id^4c^2b\,x^6\ln(-icx+1)}{2}+\frac{d^4c^4a\,x^8}{8}+\frac{4id^4ca\,x^5}{5}+\frac{2d^4c^3b\,x^7\ln(-icx+1)}{7}-\frac{b\,c^3d^4x^7}{56}+\frac{2ib\,c^2d^4x^6}{21}-d^4c^2a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(d^4*a*(1/8*c^8*x^8-4/7*I*c^7*x^7-c^6*x^6+4/5*I*c^5*x^5+1/4*c^4*x^4)+
1/8*d^4*b*arctan(c*x)*c^8*x^8-4/7*I*d^4*b*arctan(c*x)*c^7*x^7-d^4*b*arctan(
c*x)*c^6*x^6+4/5*I*d^4*b*arctan(c*x)*c^5*x^5+1/4*d^4*b*arctan(c*x)*c^4*x^4+
11/8*b*c*d^4*x-1/56*d^4*b*c^7*x^7+2/21*I*d^4*b*c^6*x^6+9/40*d^4*b*c^5*x^5-1
2/35*I*d^4*b*c^4*x^4-11/24*b*c^3*d^4*x^3+24/35*I*d^4*b*c^2*x^2-24/35*I*d^4*
b*ln(c^2*x^2+1)-11/8*b*d^4*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 337, normalized size = 1.42

$$\frac{1}{8}ac^4d^4x^8-\frac{4}{7}i\,ac^3d^4x^7-ac^2d^4x^6+\frac{4}{5}i\,acd^4x^5+\frac{1}{840}\left(105\,x^8\arctan(cx)-c\left(\frac{15\,c^6x^7-21\,c^4x^5+35\,c^2x^3-105\,x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\right)bc^4d^4-\frac{1}{21}i\left(12\,x^7\arctan(cx)-c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6}-\frac{6\log(c^2x^2+1)}{c^8}\right)\right)bc^3d^4+\frac{1}{4}ad^4x^4-\frac{1}{15}\left(15\,x^6\arctan(cx)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)bc^2d^4+\frac{1}{5}i\left(4\,x^5\arctan(cx)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)bcd^4+\frac{1}{12}\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^4*x^8 - 4/7*I*a*c^3*d^4*x^7 - a*c^2*d^4*x^6 + 4/5*I*a*c*d^4*x^5
 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 -
```

```
105*x)/c^8 + 105*arctan(c*x)/c^9))*b*c^4*d^4 - 1/21*I*(12*x^7*arctan(c*x) -
 c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*c^3*d^
4 + 1/4*a*d^4*x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 +
15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^4 + 1/5*I*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^4 + 1/12*(3*x^4*arct
an(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^4
```

Fricas [A]
time = 1.70, size = 230, normalized size = 0.97

$$\frac{210\,ac^8d^4x^8 - 30\,(32i\,a+b)c^7d^4x^7 - 80\,(21\,a-2i\,b)c^6d^4x^6 - 42\,(-32i\,a-9\,b)c^5d^4x^5 + 12\,(35\,a-48i\,b)c^4d^4x^4 - 770\,bc^3d^4x^3 + 1152i\,bc^2d^4x^2 + 2310\,bcd^4x - 2307i\,bd^4\log\left(\frac{cx+i}{c}\right) + 3i\,bd^4\log\left(\frac{cx-i}{c}\right) - 3\,(-35i\,bc^8d^4x^8 - 160\,bc^7d^4x^7 + 280i\,bc^6d^4x^6 + 224\,bc^5d^4x^5 - 70i\,bc^4d^4x^4)\log\left(-\frac{cx+i}{cx-i}\right)}{1680\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(210*a*c^8*d^4*x^8 - 30*(32*I*a + b)*c^7*d^4*x^7 - 80*(21*a - 2*I*b)
*c^6*d^4*x^6 - 42*(-32*I*a - 9*b)*c^5*d^4*x^5 + 12*(35*a - 48*I*b)*c^4*d^4*
x^4 - 770*b*c^3*d^4*x^3 + 1152*I*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 2307*I*b*
d^4*log((c*x + I)/c) + 3*I*b*d^4*log((c*x - I)/c) - 3*(-35*I*b*c^8*d^4*x^8
- 160*b*c^7*d^4*x^7 + 280*I*b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 - 70*I*b*c^4*
d^4*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

Sympy [A]
time = 3.48, size = 389, normalized size = 1.63

$$\frac{ac^4d^4x^8}{8} - \frac{11bd^4x^3}{24c} + \frac{24ibd^4x^2}{35c^2} + \frac{11bd^4x}{8c^3} + \frac{bd^4\left(\frac{i\log\left(5893bcd^4x-5893ibd^4\right)}{560} - \frac{1471i\log\left(5893bcd^4x+5893ibd^4\right)}{1260}\right)}{c^4} + x^7\left(-\frac{4iac^3d^4}{7} - \frac{bc^3d^4}{56}\right) + x^6\left(-ac^2d^4 + \frac{2ibc^2d^4}{21}\right) + x^5\cdot\left(\frac{4iacd^4}{5} + \frac{9bcd^4}{40}\right) + x^4\left(\frac{ad^4}{4} - \frac{12ibd^4}{35}\right) + \left(-\frac{ibc^4d^4x^8}{16} - \frac{2bc^3d^4x^7}{7} + \frac{ibc^2d^4x^6}{2} + \frac{2bcd^4x^5}{5} - \frac{ibd^4x^4}{8}\right)\log\left(icx+1\right) + \frac{\left(315ibc^8d^4x^8 + 1440bc^7d^4x^7 - 2520ibc^6d^4x^6 - 2016bc^5d^4x^5 + 630ibc^4d^4x^4 - 1037ibd^4\right)\log\left(-icx+1\right)}{5040c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**8/8 - 11*b*d**4*x**3/(24*c) + 24*I*b*d**4*x**2/(35*c**2) + 1
1*b*d**4*x/(8*c**3) + b*d**4*(I*log(5893*b*c*d**4*x - 5893*I*b*d**4)/560 -
1471*I*log(5893*b*c*d**4*x + 5893*I*b*d**4)/1260)/c**4 + x**7*(-4*I*a*c**3*
d**4/7 - b*c**3*d**4/56) + x**6*(-a*c**2*d**4 + 2*I*b*c**2*d**4/21) + x**5*
(4*I*a*c*d**4/5 + 9*b*c*d**4/40) + x**4*(a*d**4/4 - 12*I*b*d**4/35) + (-I*b
*c**4*d**4*x**8/16 - 2*b*c**3*d**4*x**7/7 + I*b*c**2*d**4*x**6/2 + 2*b*c*d*
*4*x**5/5 - I*b*d**4*x**4/8)*log(I*c*x + 1) + (315*I*b*c**8*d**4*x**8 + 144
0*b*c**7*d**4*x**7 - 2520*I*b*c**6*d**4*x**6 - 2016*b*c**5*d**4*x**5 + 630*
I*b*c**4*d**4*x**4 - 1037*I*b*d**4)*log(-I*c*x + 1)/(5040*c**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.59, size = 217, normalized size = 0.91

$$\frac{c^4\,d^4\,\left(105\,a\,x^8+105\,b\,x^8\,\mathrm{atan}\left(c\,x\right)\right)}{840}+\frac{d^4\,\left(210\,a\,x^4+210\,b\,x^4\,\mathrm{atan}\left(c\,x\right)-b\,x^4\,288\mathrm{i}\right)}{840}-\frac{\frac{d^4\,\left(1155\,b\,\mathrm{atan}\left(c\,x\right)+b\,\ln\left(c^2\,x^2+1\right)\,576\mathrm{i}\right)}{840}+\frac{11\,b\,c^3\,d^4\,x^3}{24}-\frac{11\,b\,c\,d^4\,x}{8}-\frac{b\,c^2\,d^4\,x^2\,24\mathrm{i}}{35}}{c^4}+\frac{c\,d^4\,\left(a\,x^5\,672\mathrm{i}+189\,b\,x^5+b\,x^5\,\mathrm{atan}\left(c\,x\right)\,672\mathrm{i}\right)}{840}-\frac{c^3\,d^4\,\left(a\,x^7\,480\mathrm{i}+15\,b\,x^7+b\,x^7\,\mathrm{atan}\left(c\,x\right)\,480\mathrm{i}\right)}{840}-\frac{c^2\,d^4\,\left(840\,a\,x^6+840\,b\,x^6\,\mathrm{atan}\left(c\,x\right)-b\,x^6\,80\mathrm{i}\right)}{840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

```
[Out] (d^4*(210*a*x^4 - b*x^4*288i + 210*b*x^4*atan(c*x)))/840 - ((d^4*(1155*b*at
an(c*x) + b*log(c^2*x^2 + 1)*576i))/840 - (b*c^2*d^4*x^2*24i)/35 + (11*b*c^
3*d^4*x^3)/24 - (11*b*c*d^4*x)/8)/c^4 + (c^4*d^4*(105*a*x^8 + 105*b*x^8*ata
n(c*x)))/840 + (c*d^4*(a*x^5*672i + 189*b*x^5 + b*x^5*atan(c*x)*672i))/840
- (c^3*d^4*(a*x^7*480i + 15*b*x^7 + b*x^7*atan(c*x)*480i))/840 - (c^2*d^4*(
840*a*x^6 - b*x^6*80i + 840*b*x^6*atan(c*x)))/840
```

3.32 $\int x^2(d+icdx)^4(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=193

$$\frac{5ibd^4x}{3c^2}-\frac{88bd^4x^2}{105c}-\frac{5}{9}ibd^4x^3+\frac{47}{140}bcd^4x^4+\frac{2}{15}ibc^2d^4x^5-\frac{1}{42}bc^3d^4x^6+\frac{id^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c^3}-\frac{id^4(1+$$

```
[Out] 5/3*I*b*d^4*x/c^2-88/105*b*d^4*x^2/c-5/9*I*b*d^4*x^3+47/140*b*c*d^4*x^4+2/1
5*I*b*c^2*d^4*x^5-1/42*b*c^3*d^4*x^6+1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*x)
)/c^3-1/3*I*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^3+1/7*I*d^4*(1+I*c*x)^7*(a+
b*arctan(c*x))/c^3+176/105*b*d^4*ln(c*x+I)/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {45, 4992, 12, 907}

$$\frac{id^4(1+icx)^7(a+b\text{ArcTan}(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\text{ArcTan}(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c^3}-\frac{1}{42}bc^3d^4x^6+\frac{176bd^4\log(cx+i)}{105c^3}+\frac{2}{15}ibc^2d^4x^5+\frac{5ibd^4x}{3c^2}+\frac{47}{140}bcd^4x^4-\frac{88bd^4x^2}{105c}-\frac{5}{9}ibd^4x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) - ((5*I)/9)*b*d^4*x^3 + (4
7*b*c*d^4*x^4)/140 + ((2*I)/15)*b*c^2*d^4*x^5 - (b*c^3*d^4*x^6)/42 + ((I/5)
*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c^3 - ((I/3)*d^4*(1 + I*c*x)^6*(a +
 b*ArcTan[c*x]))/c^3 + ((I/7)*d^4*(1 + I*c*x)^7*(a + b*ArcTan[c*x]))/c^3 +
(176*b*d^4*Log[I + c*x])/(105*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int x^2(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3}$$
$$=\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3}$$
$$=\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{id^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{3c^3}$$
$$=\frac{5ibd^4x}{3c^2}-\frac{88bd^4x^2}{105c}-\frac{5}{9}ibd^4x^3+\frac{47}{140}bcd^4x^4+\frac{2}{15}ibc^2d^4x^5-\frac{1}{42}bc^3d$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.88

$$\frac{d^4\left(12ac^3x^3\left(35+105icx-126c^2x^2-70ic^3x^3+15c^4x^4\right)+bcx\left(2100i-1056cx-700ic^2x^2+423c^3x^3+168ic^4x^4-30c^5x^5\right)+12b\left(-175i+35c^3x^3+105ic^4x^4-126c^5x^5-70ic^6x^6+15c^7x^7\right)\text{ArcTan}(cx)+1056b\log\left(1+c^2x^2\right)\right)}{1260c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^4*(12*a*c^3*x^3*(35 + (105*I)*c*x - 126*c^2*x^2 - (70*I)*c^3*x^3 + 15*c^
4*x^4) + b*c*x*(2100*I - 1056*c*x - (700*I)*c^2*x^2 + 423*c^3*x^3 + (168*I)
*c^4*x^4 - 30*c^5*x^5) + 12*b*(-175*I + 35*c^3*x^3 + (105*I)*c^4*x^4 - 126*
c^5*x^5 - (70*I)*c^6*x^6 + 15*c^7*x^7)*ArcTan[c*x] + 1056*b*Log[1 + c^2*x^2
]))/(1260*c^3)
```

Maple [A]
time = 0.11, size = 234, normalized size = 1.21

method	result
derivativedivides	$\frac{d^4a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+\frac{d^4b\arctan(cx)c^7x^7}{7}-\frac{2id^4b\arctan(cx)c^6x^6}{3}-\frac{6d^4b\arctan(cx)c^5x^5}{5}+id^4b\arctan$

default	$\frac{d^4a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+\frac{d^4b\arctan(cx)c^7x^7}{7}-\frac{2id^4b\arctan(cx)c^6x^6}{3}-\frac{6d^4b\arctan(cx)c^5x^5}{5}+id^4b\arctan(}{}$
risch	$-\frac{3id^4c^2b\,x^5\ln(-icx+1)}{5}+\frac{id^4c^4b\,x^7\ln(-icx+1)}{14}+\frac{d^4c^4a\,x^7}{7}-\frac{id^4b\left(15c^4x^7-70ic^3x^6-126c^2x^5+105ic\,x^4+35x^3\right)\ln}{210}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^3*(d^4*a*(1/7*c^7*x^7-2/3*I*c^6*x^6-6/5*c^5*x^5+I*c^4*x^4+1/3*c^3*x^3)+1/7*d^4*b*arctan(c*x)*c^7*x^7-2/3*I*d^4*b*arctan(c*x)*c^6*x^6-6/5*d^4*b*arctan(c*x)*c^5*x^5+I*d^4*b*arctan(c*x)*c^4*x^4+1/3*d^4*b*arctan(c*x)*c^3*x^3+5/3*I*d^4*b*c*x-1/42*d^4*b*c^6*x^6+2/15*I*d^4*b*c^5*x^5+47/140*d^4*b*c^4*x^4-5/9*I*d^4*b*c^3*x^3-88/105*d^4*b*c^2*x^2+88/105*b*ln(c^2*x^2+1)*d^4-5/3*I*d^4*b*arctan(c*x))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(153) = 306$.
time = 0.47, size = 318, normalized size = 1.65

$$\frac{1}{7}\,ac^4d^4x^7-\frac{2}{3}i\,ac^3d^4x^6-\frac{6}{5}\,ac^2d^4x^5+\frac{1}{84}\left(12\,x^7\arctan\left(cx\right)-c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6}-\frac{6\,\log\left(c^2x^2+1\right)}{c^8}\right)\right)bc^4d^4+i\,acd^4x^4-\frac{2}{45}i\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^3d^4-\frac{3}{10}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^4+\frac{1}{3}\,ad^4x^3+\frac{1}{3}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd^4+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")

[Out] 1/7*a*c^4*d^4*x^7 - 2/3*I*a*c^3*d^4*x^6 - 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*c^4*d^4 + I*a*c*d^4*x^4 - 2/45*I*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^3*d^4 - 3/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^4 + 1/3*a*d^4*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^4 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^4

Fricas [A]
time = 1.62, size = 218, normalized size = 1.13

$$\frac{180\,ac^7d^4x^7-30\,(28i\,a+b)c^6d^4x^6-168\,(9\,a-i\,b)c^5d^4x^5-9\,(-140i\,a-47\,b)c^4d^4x^4+140\,(3\,a-5i\,b)c^3d^4x^3-1056\,bc^2d^4x^2+2100i\,bcd^4x+2106\,bd^4\log\left(\frac{cx+i}{c}\right)+6\,bd^4\log\left(\frac{cx-i}{c}\right)-6\,(-15i\,bc^7d^4x^7-70\,bc^6d^4x^6+126i\,bc^5d^4x^5+105\,bc^4d^4x^4-35i\,bc^3d^4x^3)\log\left(-\frac{cx+i}{cx-i}\right)}{1260\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] 1/1260*(180*a*c^7*d^4*x^7 - 30*(28*I*a + b)*c^6*d^4*x^6 - 168*(9*a - I*b)*c^5*d^4*x^5 - 9*(-140*I*a - 47*b)*c^4*d^4*x^4 + 140*(3*a - 5*I*b)*c^3*d^4*x^3 - 1056*b*c^2*d^4*x^2 + 2100*I*b*c*d^4*x + 2106*b*d^4*log((c*x + I)/c) + 6*b*d^4*log((c*x - I)/c) - 6*(-15*I*b*c^7*d^4*x^7 - 70*b*c^6*d^4*x^6 + 126*I

```
*b*c^5*d^4*x^5 + 105*b*c^4*d^4*x^4 - 35*I*b*c^3*d^4*x^3)*log(-(c*x + I)/(c*
x - I)))/c^3
```

Sympy [A]
time = 3.11, size = 367, normalized size = 1.90

$$\frac{ac^4d^4x^7}{7} - \frac{88bd^4x^2}{105c} + \frac{5ibd^4x}{3c^2} + \frac{bd^4\left(\frac{\log\left(2299bcd^4x-2299ibd^4\right)}{210} + \frac{769\log\left(2299bcd^4x+2299ibd^4\right)}{560}\right)}{c^3} + x^6\left(-\frac{2iac^3d^4}{3} - \frac{bc^3d^4}{42}\right) + x^5\left(-\frac{6ac^2d^4}{5} + \frac{2ibc^2d^4}{15}\right) + x^4\left(iacd^4 + \frac{47bcd^4}{140}\right) + x^3\left(\frac{ad^4}{3} - \frac{5ibd^4}{9}\right) + \left(-\frac{ibc^4d^4x^7}{14} - \frac{bc^3d^4x^6}{3} + \frac{3ibc^2d^4x^5}{5} + \frac{bcd^4x^4}{2} - \frac{ibd^4x^3}{6}\right)\log\left(icx+1\right) + \frac{\left(120ibc^7d^4x^7 + 560bc^6d^4x^6 - 1008ibc^5d^4x^5 - 840bc^4d^4x^4 + 280ibc^3d^4x^3 + 501bd^4\right)\log\left(-icx+1\right)}{1680c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**7/7 - 88*b*d**4*x**2/(105*c) + 5*I*b*d**4*x/(3*c**2) + b*d**
4*(log(2299*b*c*d**4*x - 2299*I*b*d**4)/210 + 769*log(2299*b*c*d**4*x + 229
9*I*b*d**4)/560)/c**3 + x**6*(-2*I*a*c**3*d**4/3 - b*c**3*d**4/42) + x**5*(
-6*a*c**2*d**4/5 + 2*I*b*c**2*d**4/15) + x**4*(I*a*c*d**4 + 47*b*c*d**4/140
) + x**3*(a*d**4/3 - 5*I*b*d**4/9) + (-I*b*c**4*d**4*x**7/14 - b*c**3*d**4*
x**6/3 + 3*I*b*c**2*d**4*x**5/5 + b*c*d**4*x**4/2 - I*b*d**4*x**3/6)*log(I*
c*x + 1) + (120*I*b*c**7*d**4*x**7 + 560*b*c**6*d**4*x**6 - 1008*I*b*c**5*d
**4*x**5 - 840*b*c**4*d**4*x**4 + 280*I*b*c**3*d**4*x**3 + 501*b*d**4)*log(
-I*c*x + 1)/(1680*c**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.64, size = 205, normalized size = 1.06

$$\frac{c^4\,d^4\,\left(180\,a\,x^7+180\,b\,x^7\,\mathrm{atan}(c\,x)\right)}{1260}+\frac{d^4\,\left(420\,a\,x^3+420\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,700\mathrm{i}\right)}{1260}-\frac{\frac{d^4\,\left(-1056\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\,2100\mathrm{i}\right)}{1260}+\frac{88\,b\,c^2\,d^4\,x^2}{105}-\frac{b\,c\,d^4\,x\,5\mathrm{i}}{3}}{c^3}+\frac{c\,d^4\,\left(a\,x^4\,1260\mathrm{i}+423\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,1260\mathrm{i}\right)}{1260}-\frac{c^3\,d^4\,\left(a\,x^6\,840\mathrm{i}+30\,b\,x^6+b\,x^6\,\mathrm{atan}(c\,x)\,840\mathrm{i}\right)}{1260}-\frac{c^2\,d^4\,\left(1512\,a\,x^5+1512\,b\,x^5\,\mathrm{atan}(c\,x)-b\,x^5\,168\mathrm{i}\right)}{1260}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

```
[Out] (d^4*(420*a*x^3 - b*x^3*700i + 420*b*x^3*atan(c*x)))/1260 - ((d^4*(b*atan(c
*x)*2100i - 1056*b*log(c^2*x^2 + 1)))/1260 + (88*b*c^2*d^4*x^2)/105 - (b*c*
d^4*x*5i)/3)/c^3 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atan(c*x)))/1260 + (c*d^
4*(a*x^4*1260i + 423*b*x^4 + b*x^4*atan(c*x)*1260i))/1260 - (c^3*d^4*(a*x^6
*840i + 30*b*x^6 + b*x^6*atan(c*x)*840i))/1260 - (c^2*d^4*(1512*a*x^5 - b*x
^5*168i + 1512*b*x^5*atan(c*x)))/1260
```

3.33 $\int x(d+icdx)^4(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=178

$$-\frac{16bd^4x}{15c}-\frac{4ibd^4(i-cx)^2}{15c^2}-\frac{4bd^4(i-cx)^3}{45c^2}+\frac{ibd^4(i-cx)^4}{30c^2}+\frac{bd^4(i-cx)^5}{30c^2}+\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c^2}-d^4$$

```
[Out] -16/15*b*d^4*x/c-4/15*I*b*d^4*(I-c*x)^2/c^2-4/45*b*d^4*(I-c*x)^3/c^2+1/30*I
*b*d^4*(I-c*x)^4/c^2+1/30*b*d^4*(I-c*x)^5/c^2+1/5*d^4*(1+I*c*x)^5*(a+b*arct
an(c*x))/c^2-1/6*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^2+32/15*I*b*d^4*ln(c*x
+I)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {45, 4992, 12, 78}

$$-\frac{d^4(1+icx)^6(a+b\text{ArcTan}(cx))}{6c^2}+\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c^2}+\frac{bd^4(-cx+i)^5}{30c^2}+\frac{ibd^4(-cx+i)^4}{30c^2}-\frac{4bd^4(-cx+i)^3}{45c^2}-\frac{4ibd^4(-cx+i)^2}{15c^2}+\frac{32ibd^4\log(cx+i)}{15c^2}-\frac{16bd^4x}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-16*b*d^4*x)/(15*c) - (((4*I)/15)*b*d^4*(I - c*x)^2)/c^2 - (4*b*d^4*(I - c
*x)^3)/(45*c^2) + ((I/30)*b*d^4*(I - c*x)^4)/c^2 + (b*d^4*(I - c*x)^5)/(30*
c^2) + (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) - (d^4*(1 + I*c*x)^6
*(a + b*ArcTan[c*x]))/(6*c^2) + (((32*I)/15)*b*d^4*Log[I + c*x])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int x(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-(\\ &= \frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-\\ &= \frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c^2}-\frac{d^4(1+icx)^6\left(a+b\tan^{-1}(cx)\right)}{6c^2}-\\ &= -\frac{16bd^4x}{15c}-\frac{4ibd^4(i-cx)^2}{15c^2}-\frac{4bd^4(i-cx)^3}{45c^2}+\frac{ibd^4(i-cx)^4}{30c^2}+\frac{bd^4(i}{3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 158, normalized size = 0.89

$$\frac{d^4(cx(b(-195-96icx+50c^2x^2+18ic^3x^3-3c^4x^4)+3acx(15+40icx-45c^2x^2-24ic^3x^3+5c^4x^4))+3b(65+15c^2x^2+40ic^3x^3-45c^4x^4-24ic^5x^5+5c^6x^6)\,\text{ArcTan}(cx)+96ib\log\left(1+c^2x^2\right))}{90c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^4*(c*x*(b*(-195 - (96*I)*c*x + 50*c^2*x^2 + (18*I)*c^3*x^3 - 3*c^4*x^4)
+ 3*a*c*x*(15 + (40*I)*c*x - 45*c^2*x^2 - (24*I)*c^3*x^3 + 5*c^4*x^4)) + 3*
b*(65 + 15*c^2*x^2 + (40*I)*c^3*x^3 - 45*c^4*x^4 - (24*I)*c^5*x^5 + 5*c^6*x
^6)*ArcTan[c*x] + (96*I)*b*Log[1 + c^2*x^2]))/(90*c^2)
```

Maple [A]
time = 0.13, size = 221, normalized size = 1.24

method	result
derivativedivides	$d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+\frac{d^4b\arctan(cx)c^6x^6}{6}-\frac{4id^4b\arctan(cx)c^5x^5}{5}-\frac{3d^4b\arctan(cx)c^4x^4}{2}+\frac{4id^4b\,\text{arct}}{}$

default	$\frac{d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+\frac{d^4b\arctan(cx)c^6x^6}{6}-\frac{4id^4b\arctan(cx)c^5x^5}{5}-\frac{3d^4b\arctan(cx)c^4x^4}{2}+\frac{4id^4b\arctan}{3}}{}$
risch	$\frac{id^4c^4b\,x^6\ln(-icx+1)}{12}+\frac{a\,c^4d^4x^6}{6}+\frac{4id^4ca\,x^3}{3}+\frac{2d^4c^3b\,x^5\ln(-icx+1)}{5}-\frac{c^3b\,d^4x^5}{30}-\frac{3id^4c^2b\,x^4\ln(-icx+1)}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d^4*a*(1/6*c^6*x^6-4/5*I*c^5*x^5-3/2*c^4*x^4+4/3*I*c^3*x^3+1/2*c^2*x
^2)+1/6*d^4*b*arctan(c*x)*c^6*x^6-4/5*I*d^4*b*arctan(c*x)*c^5*x^5-3/2*d^4*b
*arctan(c*x)*c^4*x^4+4/3*I*d^4*b*arctan(c*x)*c^3*x^3+1/2*d^4*b*arctan(c*x)*
c^2*x^2-13/6*b*c*d^4*x-1/30*d^4*b*c^5*x^5+1/5*I*d^4*b*c^4*x^4+5/9*b*c^3*d^4
*x^3-16/15*I*d^4*b*c^2*x^2+16/15*I*d^4*b*ln(c^2*x^2+1)+13/6*b*d^4*arctan(c*
x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(138) = 276$.
time = 0.47, size = 290, normalized size = 1.63

$$\frac{1}{6}ac^4d^4x^6-\frac{4}{5}iac^3d^4x^5-\frac{3}{2}ac^2d^4x^4+\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)bc^4d^4-\frac{1}{5}i\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)bc^3d^4+\frac{4}{3}iacd^4x^3-\frac{1}{2}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bc^2d^4+\frac{2}{3}i\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bcd^4+\frac{1}{2}ad^4x^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^4*x^6 - 4/5*I*a*c^3*d^4*x^5 - 3/2*a*c^2*d^4*x^4 + 1/90*(15*x^6*
arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*
b*c^4*d^4 - 1/5*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2
*x^2 + 1)/c^6))*b*c^3*d^4 + 4/3*I*a*c*d^4*x^3 - 1/2*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^4 + 2/3*I*(2*x^3*arctan(
c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/2*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^4
```

Fricas [A]
time = 1.84, size = 206, normalized size = 1.16

$$\frac{30\,ac^6d^4x^6-6\,(24i\,a+b)c^5d^4x^5-18\,(15\,a-2i\,b)c^4d^4x^4-20\,(-12i\,a-5\,b)c^3d^4x^3+6\,(15\,a-32i\,b)c^2d^4x^2-390\,bcd^4x+387i\,bd^4\log\left(\frac{cx+i}{c}\right)-3i\,bd^4\log\left(\frac{cx-i}{c}\right)-3\,(-5i\,bc^6d^4x^6-24\,bc^5d^4x^5+45i\,bc^4d^4x^4+40\,bc^3d^4x^3-15i\,bc^2d^4x^2)\log\left(-\frac{cx+i}{cx-i}\right)}{180\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*d^4*x^6 - 6*(24*I*a + b)*c^5*d^4*x^5 - 18*(15*a - 2*I*b)*c^
4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 6*(15*a - 32*I*b)*c^2*d^4*x^2
- 390*b*c*d^4*x + 387*I*b*d^4*log((c*x + I)/c) - 3*I*b*d^4*log((c*x - I)/c)
 - 3*(-5*I*b*c^6*d^4*x^6 - 24*b*c^5*d^4*x^5 + 45*I*b*c^4*d^4*x^4 + 40*b*c^3
*d^4*x^3 - 15*I*b*c^2*d^4*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(156) = 312$.
time = 2.81, size = 360, normalized size = 2.02

$$\frac{ac^4d^4x^6}{6} - \frac{13bd^4x}{6c} + \frac{bd^4\left(-\frac{i\log\left(709bcd^4x-709ibd^4\right)}{60} + \frac{117i\log\left(709bcd^4x+709ibd^4\right)}{70}\right)}{c^2} + x^5\left(-\frac{4iac^3d^4}{5} - \frac{bc^3d^4}{30}\right) + x^4\left(-\frac{3ac^2d^4}{2} + \frac{ibc^2d^4}{5}\right) + x^3\cdot\left(\frac{4iacd^4}{3} + \frac{5bcd^4}{9}\right) + x^2\left(\frac{ad^4}{2} - \frac{16ibd^4}{15}\right) + \left(-\frac{ibc^4d^4x^6}{12} - \frac{2bc^3d^4x^5}{5} + \frac{3ibc^2d^4x^4}{4} + \frac{2bcd^4x^3}{3} - \frac{ibd^4x^2}{4}\right)\log\left(icx+1\right) + \frac{\left(35ibc^6d^4x^6 + 168bc^5d^4x^5 - 315ibc^4d^4x^4 - 280bc^3d^4x^3 + 105ibc^2d^4x^2 + 201ibd^4\right)\log\left(-icx+1\right)}{420c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**6/6 - 13*b*d**4*x/(6*c) + b*d**4*(-I*log(709*b*c*d**4*x - 70
9*I*b*d**4)/60 + 117*I*log(709*b*c*d**4*x + 709*I*b*d**4)/70)/c**2 + x**5*(
-4*I*a*c**3*d**4/5 - b*c**3*d**4/30) + x**4*(-3*a*c**2*d**4/2 + I*b*c**2*d*
*4/5) + x**3*(4*I*a*c*d**4/3 + 5*b*c*d**4/9) + x**2*(a*d**4/2 - 16*I*b*d**4
/15) + (-I*b*c**4*d**4*x**6/12 - 2*b*c**3*d**4*x**5/5 + 3*I*b*c**2*d**4*x**
4/4 + 2*b*c*d**4*x**3/3 - I*b*d**4*x**2/4)*log(I*c*x + 1) + (35*I*b*c**6*d*
*4*x**6 + 168*b*c**5*d**4*x**5 - 315*I*b*c**4*d**4*x**4 - 280*b*c**3*d**4*x
**3 + 105*I*b*c**2*d**4*x**2 + 201*I*b*d**4)*log(-I*c*x + 1)/(420*c**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.79, size = 191, normalized size = 1.07

$$\frac{\frac{d^4\,\left(195\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,96\mathrm{i}\right)}{90} - \frac{13\,b\,c\,d^4\,x}{6}}{c^2} + \frac{d^4\,\left(45\,a\,x^2 + 45\,b\,x^2\,\mathrm{atan}(c\,x) - b\,x^2\,96\mathrm{i}\right)}{90} + \frac{c^4\,d^4\,\left(15\,a\,x^6 + 15\,b\,x^6\,\mathrm{atan}(c\,x)\right)}{90} + \frac{c\,d^4\,\left(a\,x^3\,120\mathrm{i} + 50\,b\,x^3 + b\,x^3\,\mathrm{atan}(c\,x)\,120\mathrm{i}\right)}{90} - \frac{c^3\,d^4\,\left(a\,x^5\,72\mathrm{i} + 3\,b\,x^5 + b\,x^5\,\mathrm{atan}(c\,x)\,72\mathrm{i}\right)}{90} - \frac{c^2\,d^4\,\left(135\,a\,x^4 + 135\,b\,x^4\,\mathrm{atan}(c\,x) - b\,x^4\,18\mathrm{i}\right)}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

```
[Out] ((d^4*(195*b*atan(c*x) + b*log(c^2*x^2 + 1)*96i))/90 - (13*b*c*d^4*x)/6)/c^
2 + (d^4*(45*a*x^2 - b*x^2*96i + 45*b*x^2*atan(c*x)))/90 + (c^4*d^4*(15*a*x
^6 + 15*b*x^6*atan(c*x)))/90 + (c*d^4*(a*x^3*120i + 50*b*x^3 + b*x^3*atan(c
*x)*120i))/90 - (c^3*d^4*(a*x^5*72i + 3*b*x^5 + b*x^5*atan(c*x)*72i))/90 -
(c^2*d^4*(135*a*x^4 - b*x^4*18i + 135*b*x^4*atan(c*x)))/90
```

3.34 $\int (d+icdx)^4(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=125

$$-\frac{8}{5}ibd^4x-\frac{2bd^4(1+icx)^2}{5c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{bd^4(1+icx)^4}{20c}-\frac{id^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c}-\frac{16bd^4\log(1-i}{5c}$$

```
[Out] -8/5*I*b*d^4*x-2/5*b*d^4*(1+I*c*x)^2/c-2/15*b*d^4*(1+I*c*x)^3/c-1/20*b*d^4*
(1+I*c*x)^4/c-1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/c-16/5*b*d^4*ln(1-I*c
*x)/c
```

Rubi [A]
time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4972, 641, 45}

$$-\frac{id^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5c}-\frac{bd^4(1+icx)^4}{20c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{2bd^4(1+icx)^2}{5c}-\frac{16bd^4\log(1-icx)}{5c}-\frac{8}{5}ibd^4x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((-8*I)/5)*b*d^4*x - (2*b*d^4*(1 + I*c*x)^2)/(5*c) - (2*b*d^4*(1 + I*c*x)^3
)/(15*c) - (b*d^4*(1 + I*c*x)^4)/(20*c) - ((I/5)*d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/c - (16*b*d^4*Log[1 - I*c*x])/(5*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int (d+icdx)^4 \left(a+b\tan^{-1}(cx)\right)\,dx = -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\frac{(d+icdx)^5}{1+c^2x^2}\,dx}{5d}$$
$$= -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\frac{(d+icdx)^4}{\frac{1}{d}-\frac{icx}{d}}\,dx}{5d}$$
$$= -\frac{id^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{5c}+\frac{(ib)\int\left(-8d^5+\frac{16d^4}{\frac{1}{d}-\frac{icx}{d}}-4d^4(d+$$
$$= -\frac{8}{5}ibd^4x-\frac{2bd^4(1+icx)^2}{5c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{bd^4(1+icx)^4}{20c}-\frac{id^4(1$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.62

$$\frac{d^4(12(-i+cx)^5(a+b\text{ArcTan}(cx))-b(35+180icx-66c^2x^2-20ic^3x^3+3c^4x^4+192\log(i+cx)))}{60c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (d^4*(12*(-I + c*x)^5*(a + b*ArcTan[c*x]) - b*(35 + (180*I)*c*x - 66*c^2*x^
2 - (20*I)*c^3*x^3 + 3*c^4*x^4 + 192*Log[I + c*x])))/(60*c)
```

Maple [A]
time = 0.09, size = 172, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{id^4(icx+1)^5a}{5}+\frac{d^4b\arctan(cx)c^5x^5}{5}-id^4b\arctan(cx)c^4x^4-2d^4b\arctan(cx)c^3x^3+2id^4b\arctan(cx)c^2x^2+b\arctan(cx)d^4c}{c}$
default	$\frac{-\frac{id^4(icx+1)^5a}{5}+\frac{d^4b\arctan(cx)c^5x^5}{5}-id^4b\arctan(cx)c^4x^4-2d^4b\arctan(cx)c^3x^3+2id^4b\arctan(cx)c^2x^2+b\arctan(cx)d^4c}{c}$
risch	$-id^4ac^3x^4+2id^4acx^2+\frac{d^4ac^4x^5}{5}-3id^4bx+\frac{d^4c^3bx^4\ln(-icx+1)}{2}+\frac{id^4bc^2x^3}{3}-\frac{d^4bc^3x^4}{20}+\frac{id^4c^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/5*I*d^4*(1+I*c*x)^5*a+1/5*d^4*b*arctan(c*x)*c^5*x^5-I*d^4*b*arctan(
c*x)*c^4*x^4-2*d^4*b*arctan(c*x)*c^3*x^3+2*I*d^4*b*arctan(c*x)*c^2*x^2+b*ar
ctan(c*x)*d^4*c*x+3*I*d^4*b*arctan(c*x)-3*I*d^4*b*c*x-1/20*d^4*b*c^4*x^4+1/
3*I*d^4*b*c^3*x^3+11/10*d^4*b*c^2*x^2-8/5*b*ln(c^2*x^2+1)*d^4)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(99) = 198.
time = 0.48, size = 264, normalized size = 2.11

$$\frac{1}{5}ac^4d^4x^5 - i\,ac^3d^4x^4 + \frac{1}{20}\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^4d^4 - 2\,ac^2d^4x^3 - \frac{1}{3}i\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^3d^4 - \left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bc^2d^4 + 2i\,acd^4x^2 + 2i\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bcd^4 + ad^4x + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^4}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^4*x^5 - I*a*c^3*d^4*x^4 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4
 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^4*d^4 - 2*a*c^2*d^4*x^3 - 1/3*
I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d
^4 - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^4 + 2
*I*a*c*d^4*x^2 + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^
4 + a*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

Fricas [**A**]
time = 1.64, size = 188, normalized size = 1.50

$$\frac{12\,ac^5d^4x^5 - 3\,(20i\,a + b)c^4d^4x^4 - 20\,(6\,a - i\,b)c^3d^4x^3 - 6\,(-20i\,a - 11\,b)c^2d^4x^2 + 60\,(a - 3i\,b)cd^4x - 186\,bd^4\log\left(\frac{cx+i}{c}\right) - 6\,bd^4\log\left(\frac{cx-i}{c}\right) - 6\,(-i\,bc^5d^4x^5 - 5\,bc^4d^4x^4 + 10i\,bc^3d^4x^3 + 10\,bc^2d^4x^2 - 5i\,bcd^4x)\log\left(-\frac{cx+i}{cx-i}\right)}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^4*x^5 - 3*(20*I*a + b)*c^4*d^4*x^4 - 20*(6*a - I*b)*c^3*d^
4*x^3 - 6*(-20*I*a - 11*b)*c^2*d^4*x^2 + 60*(a - 3*I*b)*c*d^4*x - 186*b*d^4
*log((c*x + I)/c) - 6*b*d^4*log((c*x - I)/c) - 6*(-I*b*c^5*d^4*x^5 - 5*b*c^
4*d^4*x^4 + 10*I*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 - 5*I*b*c*d^4*x)*log(-(c*
x + I)/(c*x - I)))/c
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(110) = 220.
time = 2.30, size = 316, normalized size = 2.53

$$\frac{ac^4d^4x^5}{5} + \frac{bd^4\left(-\frac{\log\left(41bcd^4x - 41ibd^4\right)}{10} - \frac{43\log\left(41bcd^4x + 41ibd^4\right)}{20}\right)}{c} + x^4\left(-iac^3d^4 - \frac{bc^3d^4}{20}\right) + x^3\left(-2ac^2d^4 + \frac{ibc^2d^4}{3}\right) + x^2\cdot\left(2iacd^4 + \frac{11bcd^4}{10}\right) + x\left(ad^4 - 3ibd^4\right) + \left(-\frac{ibc^4d^4x^5}{10} - \frac{bc^3d^4x^4}{2} + ibc^2d^4x^3 + bcd^4x^2 - \frac{ibd^4x}{2}\right)\log\left(icx + 1\right) + \frac{\left(2ibc^5d^4x^5 + 10bc^4d^4x^4 - 20ibc^3d^4x^3 - 20bc^2d^4x^2 + 10ibcd^4x - 19bd^4\right)\log\left(-icx + 1\right)}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

```
[Out] a*c**4*d**4*x**5/5 + b*d**4*(-log(41*b*c*d**4*x - 41*I*b*d**4)/10 - 43*log(
41*b*c*d**4*x + 41*I*b*d**4)/20)/c + x**4*(-I*a*c**3*d**4 - b*c**3*d**4/20)
 + x**3*(-2*a*c**2*d**4 + I*b*c**2*d**4/3) + x**2*(2*I*a*c*d**4 + 11*b*c*d*
*4/10) + x*(a*d**4 - 3*I*b*d**4) + (-I*b*c**4*d**4*x**5/10 - b*c**3*d**4*x*
*4/2 + I*b*c**2*d**4*x**3 + b*c*d**4*x**2 - I*b*d**4*x/2)*log(I*c*x + 1) +
(2*I*b*c**5*d**4*x**5 + 10*b*c**4*d**4*x**4 - 20*I*b*c**3*d**4*x**3 - 20*b*
c**2*d**4*x**2 + 10*I*b*c*d**4*x - 19*b*d**4)*log(-I*c*x + 1)/(20*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.74, size = 175, normalized size = 1.40

$$\frac{d^4\,\left(60\,a\,x+60\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,180\mathrm{i}\right)}{60}+\frac{c^4\,d^4\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}(c\,x)\right)}{60}+\frac{d^4\,\left(-96\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\;180\mathrm{i}\right)}{60\,c}+\frac{c\,d^4\,\left(a\,x^2\,120\mathrm{i}+66\,b\,x^2+b\,x^2\,\mathrm{atan}(c\,x)\;120\mathrm{i}\right)}{60}-\frac{c^3\,d^4\,\left(a\,x^4\,60\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\;60\mathrm{i}\right)}{60}-\frac{c^2\,d^4\,\left(120\,a\,x^3+120\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,20\mathrm{i}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

```
[Out] (d^4*(60*a*x - b*x*180i + 60*b*x*atan(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*b
*x^5*atan(c*x)))/60 + (d^4*(b*atan(c*x)*180i - 96*b*log(c^2*x^2 + 1)))/(60*
c) + (c*d^4*(a*x^2*120i + 66*b*x^2 + b*x^2*atan(c*x)*120i))/60 - (c^3*d^4*(
a*x^4*60i + 3*b*x^4 + b*x^4*atan(c*x)*60i))/60 - (c^2*d^4*(120*a*x^3 - b*x^
3*20i + 120*b*x^3*atan(c*x)))/60
```

3.35 $\int \frac{(d+icdx)^4(a+b\mathbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=203

$$4iacd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}ibc^2d^4x^2-\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\text{ArcTan}(cx)+4ibcd^4x\text{ArcTan}(cx)-3c^2d^4x^2(a+b\text{ArcTan}(cx)$$

```
[Out] 4*I*a*c*d^4*x+13/4*b*c*d^4*x+2/3*I*b*c^2*d^4*x^2-1/12*b*c^3*d^4*x^3-13/4*b*
d^4*arctan(c*x)+4*I*b*c*d^4*x*arctan(c*x)-3*c^2*d^4*x^2*(a+b*arctan(c*x))-4
/3*I*c^3*d^4*x^3*(a+b*arctan(c*x))+1/4*c^4*d^4*x^4*(a+b*arctan(c*x))+a*d^4*
ln(x)-8/3*I*b*d^4*ln(c^2*x^2+1)+1/2*I*b*d^4*polylog(2,-I*c*x)-1/2*I*b*d^4*p
olylog(2,I*c*x)
```

Rubi [A]
time = 0.15, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4996, 4930, 266, 4940, 2438, 4946, 327, 209, 272, 45, 308}

$$\frac{1}{4}c^4d^4x^4(a+b\text{ArcTan}(cx))-\frac{4}{3}ic^3d^4x^3(a+b\text{ArcTan}(cx))-3c^2d^4x^2(a+b\text{ArcTan}(cx))+4iacd^4x+ad^4\log(x)-\frac{13}{4}bd^4\text{ArcTan}(cx)+4ibcd^4x\text{ArcTan}(cx)-\frac{1}{12}bc^3d^4x^3+\frac{2}{3}ibc^2d^4x^2-\frac{8}{3}ibd^4\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^4\text{Li}_2(-icx)-\frac{1}{2}ibd^4\text{Li}_2(icx)+\frac{13}{4}bcd^4x$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (4*I)*a*c*d^4*x + (13*b*c*d^4*x)/4 + ((2*I)/3)*b*c^2*d^4*x^2 - (b*c^3*d^4*x
^3)/12 - (13*b*d^4*ArcTan[c*x])/4 + (4*I)*b*c*d^4*x*ArcTan[c*x] - 3*c^2*d^4
*x^2*(a + b*ArcTan[c*x]) - ((4*I)/3)*c^3*d^4*x^3*(a + b*ArcTan[c*x]) + (c^4
*d^4*x^4*(a + b*ArcTan[c*x]))/4 + a*d^4*Log[x] - ((8*I)/3)*b*d^4*Log[1 + c^
2*x^2] + (I/2)*b*d^4*PolyLog[2, (-I)*c*x] - (I/2)*b*d^4*PolyLog[2, I*c*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x}\,dx=\int\left(4icd^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4\left(a+b\tan^{-1}(cx)\right)}{x}-6c^2d^4x\left(a+b\tan\right.\right.$$

$$=d^4\int\frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(4icd^4\right)\int\left(a+b\tan^{-1}(cx)\right)\,dx-\left(6c^2d^4\right)$$

$$=4iacd^4x-3c^2d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}ic^3d^4x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}$$

$$=4iacd^4x+3bcd^4x+4ibcd^4x\tan^{-1}(cx)-3c^2d^4x^2\left(a+b\tan^{-1}(cx)\right)-\frac{4}{3}$$

$$=4iacd^4x+\frac{13}{4}bcd^4x-\frac{1}{12}bc^3d^4x^3-3bd^4\tan^{-1}(cx)+4ibcd^4x\tan^{-1}(cx)$$

$$=4iacd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}ibc^2d^4x^2-\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tan^{-1}(cx)+4ib$$

Mathematica [A]
time = 0.07, size = 174, normalized size = 0.86

$$\frac{1}{12}d^4\left(48iacx+39bcx-36ac^2x^2+8ibc^2x^2-16iac^3x^3-bc^3x^3+3ac^4x^4-39b\text{ArcTan}(cx)+48ibcx\text{ArcTan}(cx)-36bc^2x^2\text{ArcTan}(cx)-16ibc^3x^3\text{ArcTan}(cx)+3bc^4x^4\text{ArcTan}(cx)+12a\log(x)-32ib\log\left(1+c^2x^2\right)+6ib\text{PolyLog}(2,-icx)-6ib\text{PolyLog}(2,icx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (d^4*((48*I)*a*c*x + 39*b*c*x - 36*a*c^2*x^2 + (8*I)*b*c^2*x^2 - (16*I)*a*c
^3*x^3 - b*c^3*x^3 + 3*a*c^4*x^4 - 39*b*ArcTan[c*x] + (48*I)*b*c*x*ArcTan[c
*x] - 36*b*c^2*x^2*ArcTan[c*x] - (16*I)*b*c^3*x^3*ArcTan[c*x] + 3*b*c^4*x^4
*ArcTan[c*x] + 12*a*Log[x] - (32*I)*b*Log[1 + c^2*x^2] + (6*I)*b*PolyLog[2,
 (-I)*c*x] - (6*I)*b*PolyLog[2, I*c*x]))/12
```

Maple [A]
time = 0.09, size = 260, normalized size = 1.28

method	result
derivativedivides	$4ibc\,d^4x\arctan(cx)+\frac{a\,c^4d^4x^4}{4}+\frac{id^4b\,\mathrm{dilog}(icx+1)}{2}-3a\,c^2d^4x^2+d^4a\ln(cx)-\frac{8id^4b\ln(c^2x^2+1)}{3}$

default	$4ibc\,d^4x\arctan(cx)+\frac{a\,c^4d^4x^4}{4}+\frac{id^4b\,\mathrm{dilog}(icx+1)}{2}-3a\,c^2d^4x^2+d^4a\ln(cx)-\frac{8id^4b\ln(c^2x^2+1)}{3}$
risch	$\frac{a\,c^4d^4x^4}{4}-3a\,c^2d^4x^2+\frac{3ib\,d^4\ln(icx+1)x^2c^2}{2}+\frac{13bc\,d^4x}{4}-\frac{b\,c^3d^4x^3}{12}+\frac{id^4b\,\mathrm{dilog}(icx+1)}{2}+\frac{58ib\,d^4}{9}-8id$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*I*b*c*d^4*x*arctan(c*x)+1/4*a*c^4*d^4*x^4-8/3*I*b*d^4*ln(c^2*x^2+1)-3*a*c
^2*d^4*x^2+d^4*a*ln(c*x)+1/2*I*d^4*b*dilog(1+I*c*x)+1/4*d^4*b*arctan(c*x)*c
^4*x^4-4/3*I*d^4*a*c^3*x^3-3*d^4*b*arctan(c*x)*c^2*x^2+d^4*b*ln(c*x)*arctan
(c*x)-1/2*I*d^4*b*dilog(1-I*c*x)+2/3*I*b*c^2*d^4*x^2-1/2*I*d^4*b*ln(c*x)*ln
(1-I*c*x)+1/2*I*d^4*b*ln(c*x)*ln(1+I*c*x)+13/4*b*c*d^4*x-1/12*b*c^3*d^4*x^3
-4/3*I*d^4*b*arctan(c*x)*c^3*x^3+4*I*a*c*d^4*x-13/4*b*d^4*arctan(c*x)
```

Maxima [A]
time = 0.60, size = 220, normalized size = 1.08

$$\frac{1}{4}ac^4d^4x^4-\frac{4}{3}i\,ac^3d^4x^3-\frac{1}{12}bc^3d^4x^3-3\,ac^2d^4x^2+\frac{2}{3}i\,bc^2d^4x^2+4i\,acd^4x+\frac{13}{4}bcd^4x-\frac{1}{12}(3\pi+8i)bd^4\log(c^2x^2+1)+bd^4\arctan(cx)\log(cx)+2i\left(2\,cx\arctan(cx)-\log(c^2x^2+1)\right)bd^4-\frac{1}{2}i\,bd^4\mathrm{Li}_2(i\,cx+1)+\frac{1}{2}i\,bd^4\mathrm{Li}_2(-i\,cx+1)+ad^4\log(x)+\frac{1}{12}\left(3\,bc^4d^4x^4-16i\,bc^3d^4x^3-36\,bc^2d^4x^2-39\,bd^4\right)\arctan(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^4*x^4 - 4/3*I*a*c^3*d^4*x^3 - 1/12*b*c^3*d^4*x^3 - 3*a*c^2*d^4*
x^2 + 2/3*I*b*c^2*d^4*x^2 + 4*I*a*c*d^4*x + 13/4*b*c*d^4*x - 1/12*(3*pi + 8
*I)*b*d^4*log(c^2*x^2 + 1) + b*d^4*arctan(c*x)*log(c*x) + 2*I*(2*c*x*arctan
(c*x) - log(c^2*x^2 + 1))*b*d^4 - 1/2*I*b*d^4*dilog(I*c*x + 1) + 1/2*I*b*d^
4*dilog(-I*c*x + 1) + a*d^4*log(x) + 1/12*(3*b*c^4*d^4*x^4 - 16*I*b*c^3*d^4
*x^3 - 36*b*c^2*d^4*x^2 - 39*b*d^4)*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.99, size = 248, normalized size = 1.22

$$\begin{cases} a\,d^4\,\ln\left(x\right) & \text{if}\ \ c=0\\ a\,d^4\,\ln\left(x\right)-b\,d^4\,\ln\left(c^2\,x^2+1\right)\,2{}\mathrm{i}-\frac{b\,d^4\,\left(3\,\mathrm{atan}\left(c\,x\right)-3\,c\,x+c^3\,x^3\right)}{12}-\frac{b\,d^4\,\mathrm{Li}_2\left(1-c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\frac{b\,d^4\,\mathrm{Li}_2\left(1+c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}-3\,a\,c^2\,d^4\,x^2-\frac{a\,c^3\,d^4\,x^3\,4{}\mathrm{i}}{3}+\frac{a\,c^4\,d^4\,x^4}{4}+a\,c\,d^4\,x\,4{}\mathrm{i}+3\,b\,c\,d^4\,x+\frac{b\,c^2\,d^4\,\left(\frac{x^2}{2}-\frac{\ln\left(c^2\,x^2+1\right)}{2\,c^2}\right)\,4{}\mathrm{i}}{3}-6\,b\,c^2\,d^4\,\mathrm{atan}\left(c\,x\right)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)-\frac{b\,c^3\,d^4\,x^3\,\mathrm{atan}\left(c\,x\right)\,4{}\mathrm{i}}{3}+\frac{b\,c^4\,d^4\,x^4\,\mathrm{atan}\left(c\,x\right)}{4}+b\,c\,d^4\,x\,\mathrm{atan}\left(c\,x\right)\,4{}\mathrm{i} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x,x)
```

```
[Out] piecewise(c == 0, a*d^4*log(x), c ~= 0, - (b*d^4*(3*atan(c*x) - 3*c*x + c^3
*x^3))/12 - b*d^4*log(c^2*x^2 + 1)*2i + a*d^4*log(x) - (b*d^4*dilog(- c*x*1
i + 1)*1i)/2 + (b*d^4*dilog(c*x*1i + 1)*1i)/2 - 3*a*c^2*d^4*x^2 - (a*c^3*d^
4*x^3*4i)/3 + (a*c^4*d^4*x^4)/4 + a*c*d^4*x*4i + 3*b*c*d^4*x + (b*c^2*d^4*(
x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*4i)/3 - 6*b*c^2*d^4*atan(c*x)*(1/(2*c^2)
+ x^2/2) - (b*c^3*d^4*x^3*atan(c*x)*4i)/3 + (b*c^4*d^4*x^4*atan(c*x))/4 + b
*c*d^4*x*atan(c*x)*4i)
```

3.36 $\int \frac{(d+icdx)^4(a+b\mathbf{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=190

$$-6ac^2d^4x+2ibc^2d^4x-\frac{1}{6}bc^3d^4x^2-2ibcd^4\text{ArcTan}(cx)-6bc^2d^4x\text{ArcTan}(cx)-\frac{d^4(a+b\text{ArcTan}(cx))}{x}-2ic^3d^4x^2($$

```
[Out] -6*a*c^2*d^4*x+2*I*b*c^2*d^4*x-1/6*b*c^3*d^4*x^2-2*I*b*c*d^4*arctan(c*x)-6*
b*c^2*d^4*x*arctan(c*x)-d^4*(a+b*arctan(c*x))/x-2*I*c^3*d^4*x^2*(a+b*arctan
(c*x))+1/3*c^4*d^4*x^3*(a+b*arctan(c*x))+4*I*a*c*d^4*ln(x)+b*c*d^4*ln(x)+8/
3*b*c*d^4*ln(c^2*x^2+1)-2*b*c*d^4*polylog(2,-I*c*x)+2*b*c*d^4*polylog(2,I*c
*x)
```

Rubi [A]
time = 0.16, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4996, 4930, 266, 4946, 272, 36, 29, 31, 4940, 2438, 327, 209, 45}

$$\frac{1}{3}c^4d^4x^3(a+b\text{ArcTan}(cx))-2ic^3d^4x^2(a+b\text{ArcTan}(cx))-\frac{d^4(a+b\text{ArcTan}(cx))}{x}-6ac^2d^4x+4iacd^4\log(x)-6bc^2d^4x\text{ArcTan}(cx)-2ibcd^4\text{ArcTan}(cx)-\frac{1}{6}bc^3d^4x^2+\frac{8}{3}bcd^4\log\left(c^2x^2+1\right)+2ibc^2d^4x-2bcd^4\text{Li}_2(-icx)+2bcd^4\text{Li}_2(icx)+bcd^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -6*a*c^2*d^4*x + (2*I)*b*c^2*d^4*x - (b*c^3*d^4*x^2)/6 - (2*I)*b*c*d^4*ArcT
an[c*x] - 6*b*c^2*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/x - (2*I)*c
^3*d^4*x^2*(a + b*ArcTan[c*x]) + (c^4*d^4*x^3*(a + b*ArcTan[c*x]))/3 + (4*I
)*a*c*d^4*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 + c^2*x^2])/3 - 2*b*c*
d^4*PolyLog[2, (-I)*c*x] + 2*b*c*d^4*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^2}\,dx &= \int \left(-6c^2d^4\big(a+b\tan^{-1}(cx)\big)+\frac{d^4(a+b\tan^{-1}(cx))}{x^2}+\frac{4icd^4(a+b\,\text{ta}}{x}\right. \\
&= d^4\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx+\left(4icd^4\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx-\left(6c^2d^4\right)\int \\
&= -6ac^2d^4x-\frac{d^4(a+b\tan^{-1}(cx))}{x}-2ic^3d^4x^2\big(a+b\tan^{-1}(cx)\big)+\frac{1}{3}c^4d \\
&= -6ac^2d^4x+2ibc^2d^4x-6bc^2d^4x\tan^{-1}(cx)-\frac{d^4(a+b\tan^{-1}(cx))}{x}-2 \\
&= -6ac^2d^4x+2ibc^2d^4x-2ibcd^4\tan^{-1}(cx)-6bc^2d^4x\tan^{-1}(cx)-\frac{d^4(a}{} \\
&= -6ac^2d^4x+2ibc^2d^4x-\frac{1}{6}bc^3d^4x^2-2ibcd^4\tan^{-1}(cx)-6bc^2d^4x\tan^{-1}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 181, normalized size = 0.95

$$\frac{d^4(-6a-36ac^2x^2+12ibc^2x^2-12iac^3x^3-bc^3x^3+2ac^4x^4-6b\text{ArcTan}(cx)-12ibcx\text{ArcTan}(cx)-36bc^2x^2\text{ArcTan}(cx)-12ibc^3x^3\text{ArcTan}(cx)+2bc^4x^4\text{ArcTan}(cx)+24iacx\log(x)+6bcx\log(cx)+16bcx\log(1+c^2x^2)-12bcx\text{PolyLog}(2,-icx)+12bcx\text{PolyLog}(2,icx))}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
[Out] (d^4*(-6*a - 36*a*c^2*x^2 + (12*I)*b*c^2*x^2 - (12*I)*a*c^3*x^3 - b*c^3*x^3
 + 2*a*c^4*x^4 - 6*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] - 36*b*c^2*x^2*
```

```
ArcTan[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 2*b*c^4*x^4*ArcTan[c*x] + (24*
I)*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 16*b*c*x*Log[1 + c^2*x^2] - 12*b*c*x*P
olyLog[2, (-I)*c*x] + 12*b*c*x*PolyLog[2, I*c*x]))/(6*x)
```

Maple [A]
time = 0.10, size = 257, normalized size = 1.35

method	result
derivativedivides	$c\left(-6ac\,d^4x+\frac{a\,c^3d^4x^3}{3}-2id^4b\arctan\left(cx\right)-\frac{d^4a}{cx}+2id^4bcx-6b\arctan\left(cx\right)d^4cx+\frac{d^4b\,\text{arct}}{}\right.$
default	$c\left(-6ac\,d^4x+\frac{a\,c^3d^4x^3}{3}-2id^4b\arctan\left(cx\right)-\frac{d^4a}{cx}+2id^4bcx-6b\arctan\left(cx\right)d^4cx+\frac{d^4b\,\text{arct}}{}\right.$
risch	$2d^4cb\,\text{dilog}\left(-icx+1\right)+\frac{d^4cb\ln\left(-icx\right)}{2}+\frac{8bc\,d^4\ln\left(c^2x^2+1\right)}{3}+d^4c^3b\ln\left(-icx+1\right)x^2-b\,d^4c^3\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-6*a*c*d^4*x+1/3*a*c^3*d^4*x^3-2*I*d^4*b*arctan(c*x)-d^4*a/c/x+2*I*d^4*b
*c*x-6*b*arctan(c*x)*d^4*c*x+1/3*d^4*b*arctan(c*x)*c^3*x^3-2*I*d^4*a*c^2*x^
2-d^4*b*arctan(c*x)/c/x-2*I*d^4*b*arctan(c*x)*c^2*x^2-2*d^4*b*ln(c*x)*ln(1+
I*c*x)+2*d^4*b*ln(c*x)*ln(1-I*c*x)-2*d^4*b*dilog(1+I*c*x)+2*d^4*b*dilog(1-I
*c*x)+4*I*d^4*a*ln(c*x)-1/6*d^4*b*c^2*x^2+8/3*b*ln(c^2*x^2+1)*d^4+4*I*d^4*b
*arctan(c*x)*ln(c*x)+d^4*b*ln(c*x))
```

Maxima [A]
time = 0.62, size = 240, normalized size = 1.26

$$\frac{1}{3}\,ac^4d^4x^3-2i\,ac^3d^4x^2-\frac{1}{6}\,bc^3d^4x^2-6\,ac^2d^4x+2i\,bc^2d^4x-\frac{1}{6}\,(6i\,\pi-1)bcd^4\log\left(c^2x^2+1\right)+4i\,bcd^4\arctan\left(cx\right)\log\left(cx\right)-3\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bcd^4+2\,bcd^4\text{Li}_2(i\,cx+1)-2\,bcd^4\text{Li}_2(-i\,cx+1)+4i\,acd^4\log\left(x\right)-\frac{1}{2}\left(c(\log\left(c^2x^2+1\right)-\log\left(x^2\right))+\frac{2\,\arctan\left(cx\right)}{x}\right)bd^4-\frac{ad^4}{x}+\frac{1}{3}\left(bc^4d^4x^3-6i\,bc^3d^4x^2-6i\,bcd^4\right)\arctan\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^4*x^3 - 2*I*a*c^3*d^4*x^2 - 1/6*b*c^3*d^4*x^2 - 6*a*c^2*d^4*x +
 2*I*b*c^2*d^4*x - 1/6*(6*I*pi - 1)*b*c*d^4*log(c^2*x^2 + 1) + 4*I*b*c*d^4*
arctan(c*x)*log(c*x) - 3*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^4 + 2
*b*c*d^4*dilog(I*c*x + 1) - 2*b*c*d^4*dilog(-I*c*x + 1) + 4*I*a*c*d^4*log(x
) - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^4 - a*d^4/x
 + 1/3*(b*c^4*d^4*x^3 - 6*I*b*c^3*d^4*x^2 - 6*I*b*c*d^4)*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.82, size = 253, normalized size = 1.33

$$\begin{cases} -\frac{a\,d^4}{x} & \text{if}\ \ c=0\\ \frac{a\,c^4\,d^4\,x^3}{3}-\frac{a\,d^4}{x}+\frac{b\,d^4\,\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+2\,b\,c\,d^4\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)+3\,b\,c\,d^4\,\ln\left(c^2\,x^2+1\right)-6\,a\,c^2\,d^4\,x-\frac{b\,c^3\,d^4\,\left(\frac{x^2}{2}-\frac{\ln\left(c^2\,x^2+1\right)}{2\,c^2}\right)}{3}-\frac{b\,d^4\,\mathrm{atan}(c\,x)}{x}-6\,b\,c^2\,d^4\,x\,\mathrm{atan}(c\,x)+\frac{b\,c^4\,d^4\,x^3\,\mathrm{atan}(c\,x)}{3}-a\,c^3\,d^4\,x^2\,2\mathrm{i}+b\,c^2\,d^4\,x\,2\mathrm{i}+a\,c\,d^4\,\ln\left(x\right)\,4\mathrm{i}-b\,c^3\,d^4\,\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\,4\mathrm{i} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^2,x)
```

```
[Out] piecewise(c == 0, -(a*d^4)/x, c ~= 0, - (a*d^4)/x - a*c^3*d^4*x^2*2i + (a*c
^4*d^4*x^3)/3 + (b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + 2*b*c*d
^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + 3*b*c*d^4*log(c^2*x^2 + 1) -
 6*a*c^2*d^4*x + b*c^2*d^4*x*2i - (b*c^3*d^4*(x^2/2 - log(c^2*x^2 + 1)/(2*c
^2)))/3 + a*c*d^4*log(x)*4i - (b*d^4*atan(c*x))/x - 6*b*c^2*d^4*x*atan(c*x)
 - b*c^3*d^4*atan(c*x)*(1/(2*c^2) + x^2/2)*4i + (b*c^4*d^4*x^3*atan(c*x))/3
)
```

3.37 $\int \frac{(d+icdx)^4(a+b\text{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=173

$$-\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\text{ArcTan}(cx)-\frac{d^4(a+b\text{ArcTan}(cx))}{2x^2}-\frac{4icd^4(a+b\text{ArcTan}(cx))}{x}+\frac{1}{2}c^4d^4x^2$$

```
[Out] -1/2*b*c*d^4/x-4*I*a*c^3*d^4*x-1/2*b*c^3*d^4*x-4*I*b*c^3*d^4*x*arctan(c*x)-
1/2*d^4*(a+b*arctan(c*x))/x^2-4*I*c*d^4*(a+b*arctan(c*x))/x+1/2*c^4*d^4*x^2
*(a+b*arctan(c*x))-6*a*c^2*d^4*ln(x)+4*I*b*c^2*d^4*ln(x)-3*I*b*c^2*d^4*poly
log(2,-I*c*x)+3*I*b*c^2*d^4*polylog(2,I*c*x)
```

Rubi [A]

time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4996, 4930, 266, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438, 327}

$$\frac{1}{2}c^4d^4x^2(a+b\text{ArcTan}(cx))-\frac{d^4(a+b\text{ArcTan}(cx))}{2x^2}-\frac{4icd^4(a+b\text{ArcTan}(cx))}{x}-4iac^3d^4x-6ac^2d^4\log(x)-4ibc^3d^4x\text{ArcTan}(cx)-\frac{1}{2}bc^3d^4x-3ibc^2d^4\text{Li}_2(-icx)+3ibc^2d^4\text{Li}_2(icx)+4ibc^2d^4\log(x)-\frac{bcd^4}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^4)/x - (4*I)*a*c^3*d^4*x - (b*c^3*d^4*x)/2 - (4*I)*b*c^3*d^4*x*
ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(2*x^2) - ((4*I)*c*d^4*(a + b*ArcTa
n[c*x]))/x + (c^4*d^4*x^2*(a + b*ArcTan[c*x]))/2 - 6*a*c^2*d^4*Log[x] + (4*
I)*b*c^2*d^4*Log[x] - (3*I)*b*c^2*d^4*PolyLog[2, (-I)*c*x] + (3*I)*b*c^2*d^
4*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^3}\,dx &= \int \left(-4ic^3d^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4(a+b\tan^{-1}(cx))}{x^3}+\frac{4icd^4(a+b\tan\right.\\
&= d^4\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(4icd^4\right)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(6c^2d^4\right)\int\\
&= -4iac^3d^4x-\frac{d^4(a+b\tan^{-1}(cx))}{2x^2}-\frac{4icd^4(a+b\tan^{-1}(cx))}{x}+\frac{1}{2}c^4d^4x^2\\
&= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4(a+b\tan^{-1}(c}{2x^2}\\
&= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4(a+b\tan^{-1}(c}{2x^2}\\
&= -\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\tan^{-1}(cx)-\frac{d^4(a+b\tan^{-1}(c}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 163, normalized size = 0.94

$$\frac{d^4(-a-8iacx-bcx-8iac^3x^3-bc^3x^3+ac^4x^4-b\text{ArcTan}(cx)-8ibcx\text{ArcTan}(cx)-8ibc^3x^3\text{ArcTan}(cx)+bc^4x^4\text{ArcTan}(cx)-12ac^2x^2\log(x)+8ibc^2x^2\log(cx)-6ibc^2x^2\text{PolyLog}(2,-icx)+6ibc^2x^2\text{PolyLog}(2,icx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (d^4*(-a - (8*I)*a*c*x - b*c*x - (8*I)*a*c^3*x^3 - b*c^3*x^3 + a*c^4*x^4 -
b*ArcTan[c*x] - (8*I)*b*c*x*ArcTan[c*x] - (8*I)*b*c^3*x^3*ArcTan[c*x] + b*c
^4*x^4*ArcTan[c*x] - 12*a*c^2*x^2*Log[x] + (8*I)*b*c^2*x^2*Log[c*x] - (6*I)
```

```
*b*c^2*x^2*PolyLog[2, (-I)*c*x] + (6*I)*b*c^2*x^2*PolyLog[2, I*c*x]))/(2*x^
2)
```

Maple [A]
time = 0.10, size = 237, normalized size = 1.37

method	result
derivativedivides	$c^2\left(-4ibc\,d^4x\arctan\left(cx\right)+\frac{a\,c^2d^4x^2}{2}-\frac{d^4a}{2c^2x^2}-3id^4b\,\mathrm{dilog}\left(icx+1\right)-6d^4a\ln\left(cx\right)-3id^4\right.$
default	$c^2\left(-4ibc\,d^4x\arctan\left(cx\right)+\frac{a\,c^2d^4x^2}{2}-\frac{d^4a}{2c^2x^2}-3id^4b\,\mathrm{dilog}\left(icx+1\right)-6d^4a\ln\left(cx\right)-3id^4\right.$
risch	$-\frac{d^4a}{2x^2}+\frac{9id^4c^2b\ln(-icx)}{4}-3ib\,d^4c^2\,\mathrm{dilog}\left(icx+1\right)+\frac{id^4c^4b\ln(-icx+1)x^2}{4}-6d^4c^2a\ln\left(-icx\right)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-3*I*d^4*b*dilog(1+I*c*x)+1/2*a*c^2*d^4*x^2-1/2*d^4*a/c^2/x^2-4*I*d^4*
b*arctan(c*x)*c*x-6*d^4*a*ln(c*x)+3*I*d^4*b*dilog(1-I*c*x)+1/2*d^4*b*arctan
(c*x)*c^2*x^2-1/2*d^4*b*arctan(c*x)/c^2/x^2-3*I*d^4*b*ln(c*x)*ln(1+I*c*x)-6
*d^4*b*ln(c*x)*arctan(c*x)-1/2*b*c*d^4*x-1/2*d^4*b/c/x+3*I*d^4*b*ln(c*x)*ln
(1-I*c*x)-4*I*d^4*b*arctan(c*x)/c/x+4*I*d^4*b*ln(c*x)-4*I*d^4*a/c/x-4*I*d^4
*a*c*x)
```

Maxima [A]
time = 0.63, size = 251, normalized size = 1.45

$$\frac{1}{2}\,ac^4d^4x^2-4i\,ac^3d^4x-\frac{1}{2}\,bc^3d^4x+\frac{3}{2}\,\pi bc^2d^4\log\left(c^2x^2+1\right)-6\,bc^2d^4\arctan\left(cx\right)\log\left(cx\right)-2i\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bc^2d^4+3i\,bc^2d^4\mathrm{Li}_2(i\,cx+1)-3i\,bc^2d^4\mathrm{Li}_2(-i\,cx+1)-6\,ac^2d^4\log\left(x\right)-2i\left(c(\log\left(c^2x^2+1\right)-\log\left(x^2\right))+\frac{2\arctan\left(cx\right)}{x}\right)bcd^4-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bd^4-\frac{4i\,acd^4}{x}-\frac{ad^4}{2\,x^2}+\frac{1}{2}\left(bc^4d^4x^2+bc^2d^4\right)\arctan\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^4*x^2 - 4*I*a*c^3*d^4*x - 1/2*b*c^3*d^4*x + 3/2*pi*b*c^2*d^4*lo
g(c^2*x^2 + 1) - 6*b*c^2*d^4*arctan(c*x)*log(c*x) - 2*I*(2*c*x*arctan(c*x)
- log(c^2*x^2 + 1))*b*c^2*d^4 + 3*I*b*c^2*d^4*dilog(I*c*x + 1) - 3*I*b*c^2*
d^4*dilog(-I*c*x + 1) - 6*a*c^2*d^4*log(x) - 2*I*(c*(log(c^2*x^2 + 1) - log
(x^2)) + 2*arctan(c*x)/x)*b*c*d^4 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c
*x)/x^2)*b*d^4 - 4*I*a*c*d^4/x - 1/2*a*d^4/x^2 + 1/2*(b*c^4*d^4*x^2 + b*c^2
*d^4)*arctan(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.89, size = 258, normalized size = 1.49

$$\begin{cases} -\frac{a\,d^4}{2\,x^2} & \text{if}\ \ c=0\\ \frac{a\,c^4\,d^4\,x^2}{2}-\frac{\frac{a\,d^4}{2}+a\,c\,d^4\,x\,4\mathrm{i}}{x^2}-6\,a\,c^2\,d^4\,\ln\left(x\right)-\frac{b\,d^4\,\left(c^3\,\mathrm{atan}\left(c\,x\right)+\frac{c^2}{x}\right)}{2\,c}-\frac{b\,c^3\,d^4\,x}{2}-\frac{b\,d^4\,\mathrm{atan}\left(c\,x\right)}{2\,x^2}+b\,c^4\,d^4\,\mathrm{atan}\left(c\,x\right)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)+b\,d^4\,\left(c^2\,\ln\left(x\right)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)\,4\mathrm{i}+b\,c^2\,d^4\,\ln\left(c^2\,x^2+1\right)\,2\mathrm{i}+b\,c^2\,d^4\,\mathrm{Li_2}\left(1-c\,x\,1\mathrm{i}\right)\,3\mathrm{i}-b\,c^2\,d^4\,\mathrm{Li_2}\left(1+c\,x\,1\mathrm{i}\right)\,3\mathrm{i}-a\,c^3\,d^4\,x\,4\mathrm{i}-\frac{b\,c\,d^4\,\mathrm{atan}\left(c\,x\right)\,4\mathrm{i}}{x}-b\,c^3\,d^4\,x\,\mathrm{atan}\left(c\,x\right)\,4\mathrm{i} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^3,x)
```

```
[Out] piecewise(c == 0, -(a*d^4)/(2*x^2), c ~= 0, - ((a*d^4)/2 + a*c*d^4*x*4i)/x^
2 + b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2)*4i + b*c^2*d^4*log(c^2*x^
2 + 1)*2i + (a*c^4*d^4*x^2)/2 - 6*a*c^2*d^4*log(x) + b*c^2*d^4*dilog(- c*x*
1i + 1)*3i - b*c^2*d^4*dilog(c*x*1i + 1)*3i - (b*d^4*(c^3*atan(c*x) + c^2/x
))/(2*c) - a*c^3*d^4*x*4i - (b*c^3*d^4*x)/2 - (b*d^4*atan(c*x))/(2*x^2) - (
b*c*d^4*atan(c*x)*4i)/x - b*c^3*d^4*x*atan(c*x)*4i + b*c^4*d^4*atan(c*x)*(1
/(2*c^2) + x^2/2))
```

3.38 $\int \frac{(d+icdx)^4(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=201

$$-\frac{bcd^4}{6x^2}-\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\mathrm{ArcTan}(cx)+bc^4d^4x\mathrm{ArcTan}(cx)-\frac{d^4(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{2icd^4(a+b\mathrm{ArcT}}{x^2}$$

```
[Out] -1/6*b*c*d^4/x^2-2*I*b*c^2*d^4/x+a*c^4*d^4*x-2*I*b*c^3*d^4*arctan(c*x)+b*c^
4*d^4*x*arctan(c*x)-1/3*d^4*(a+b*arctan(c*x))/x^3-2*I*c*d^4*(a+b*arctan(c*x
))/x^2+6*c^2*d^4*(a+b*arctan(c*x))/x-4*I*a*c^3*d^4*ln(x)-19/3*b*c^3*d^4*ln(
x)+8/3*b*c^3*d^4*ln(c^2*x^2+1)+2*b*c^3*d^4*polylog(2,-I*c*x)-2*b*c^3*d^4*po
lylog(2,I*c*x)
```

Rubi [A]
time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4996, 4930, 266, 4946, 272, 46, 331, 209, 36, 29, 31, 4940, 2438}

$$\frac{6c^2d^4(a+b\mathrm{ArcTan}(cx))}{x}-\frac{d^4(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{2icd^4(a+b\mathrm{ArcTan}(cx))}{x^2}+ac^4d^4x-4iac^3d^4\log(x)+bc^4d^4x\mathrm{ArcTan}(cx)-2ibc^3d^4\mathrm{ArcTan}(cx)+2bc^3d^4\mathrm{Li}_2(-icx)-2bc^3d^4\mathrm{Li}_2(icx)-\frac{19}{3}bc^3d^4\log(x)-\frac{2ibc^2d^4}{x}+\frac{8}{3}bc^3d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^4)/x^2 - ((2*I)*b*c^2*d^4)/x + a*c^4*d^4*x - (2*I)*b*c^3*d^4*Ar
cTan[c*x] + b*c^4*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (
(2*I)*c*d^4*(a + b*ArcTan[c*x]))/x^2 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/x -
(4*I)*a*c^3*d^4*Log[x] - (19*b*c^3*d^4*Log[x])/3 + (8*b*c^3*d^4*Log[1 + c^2
*x^2])/3 + 2*b*c^3*d^4*PolyLog[2, (-I)*c*x] - 2*b*c^3*d^4*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= \int \left(c^4d^4\left(a+b\tan^{-1}(cx)\right)+\frac{d^4(a+b\tan^{-1}(cx))}{x^4}+\frac{4icd^4(a+b\tan^{-1}\right.\\ &= d^4\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx+\left(4icd^4\right)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\left(6c^2d^4\right)\int\\ &= ac^4d^4x-\frac{d^4(a+b\tan^{-1}(cx))}{3x^3}-\frac{2icd^4(a+b\tan^{-1}(cx))}{x^2}+\frac{6c^2d^4(a+}{}\\ &= -\frac{2ibc^2d^4}{x}+ac^4d^4x+bc^4d^4x\tan^{-1}(cx)-\frac{d^4(a+b\tan^{-1}(cx))}{3x^3}-\frac{2icd}{}\\ &= -\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\tan^{-1}(cx)+bc^4d^4x\tan^{-1}(cx)-\frac{d^4(a+}{}\\ &= -\frac{bcd^4}{6x^2}-\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\tan^{-1}(cx)+bc^4d^4x\tan^{-1}(cx)-\end{aligned}$$

Mathematica [A]
time = 0.07, size = 193, normalized size = 0.96

$$\frac{d^4(-2a-12iacx-bcx+36ac^2x^2-12ibc^2x^2+6ac^4x^4-2b\text{ArcTan}(cx)-12ibcx\text{ArcTan}(cx)+36bc^2x^2\text{ArcTan}(cx)-12ibc^3x^3\text{ArcTan}(cx)+6bc^4x^4\text{ArcTan}(cx)-24iac^3x^3\log(x)-38bc^3x^3\log(cx)+16bc^3x^3\log(1+c^2x^2)+12bc^3x^3\text{PolyLog}(2,-icx)-12bc^3x^3\text{PolyLog}(2,icx))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
[Out] (d^4*(-2*a - (12*I)*a*c*x - b*c*x + 36*a*c^2*x^2 - (12*I)*b*c^2*x^2 + 6*a*c
^4*x^4 - 2*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan[c
```

```
*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 6*b*c^4*x^4*ArcTan[c*x] - (24*I)*a*c^3
*x^3*Log[x] - 38*b*c^3*x^3*Log[c*x] + 16*b*c^3*x^3*Log[1 + c^2*x^2] + 12*b*
c^3*x^3*PolyLog[2, (-I)*c*x] - 12*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3)
```

Maple [A]
time = 0.09, size = 262, normalized size = 1.30

method	result
derivativedivides	$c^3\Big(ac\,d^4x - 2id^4b\arctan(cx) - \frac{d^4a}{3c^3x^3} + \frac{6d^4a}{cx} - \frac{2id^4b\arctan(cx)}{c^2x^2} + b\arctan(cx)\,d^4cx - \frac{2id^4a}{c^2x^2}$
default	$c^3\Big(ac\,d^4x - 2id^4b\arctan(cx) - \frac{d^4a}{3c^3x^3} + \frac{6d^4a}{cx} - \frac{2id^4b\arctan(cx)}{c^2x^2} + b\arctan(cx)\,d^4cx - \frac{2id^4a}{c^2x^2}$
risch	$-\frac{d^4a}{3x^3} + \frac{8b\,c^3d^4\ln(c^2x^2+1)}{3} + \frac{ib\,d^4\ln(icx+1)}{6x^3} - \frac{id^4b\ln(-icx+1)}{6x^3} + a\,c^4d^4x - 2d^4c^3b\,\mathrm{dilog}(-icx+1)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a*c*d^4*x-2*I*d^4*b*arctan(c*x)-1/3*d^4*a/c^3/x^3+6*d^4*a/c/x-2*I*d^4*
b*arctan(c*x)/c^2/x^2+b*arctan(c*x)*d^4*c*x-2*I*d^4*a/c^2/x^2-1/3*d^4*b*arc
tan(c*x)/c^3/x^3+6*d^4*b*arctan(c*x)/c/x-2*I*d^4*b/c/x+2*d^4*b*ln(c*x)*ln(1
+I*c*x)-2*d^4*b*ln(c*x)*ln(1-I*c*x)+2*d^4*b*dilog(1+I*c*x)-2*d^4*b*dilog(1-
I*c*x)+8/3*b*ln(c^2*x^2+1)*d^4-4*I*d^4*a*ln(c*x)-1/6*d^4*b/c^2/x^2-4*I*d^4*
b*arctan(c*x)*ln(c*x)-19/3*d^4*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^3*d^4 - 4*I*b*
c^3*d^4*integrate(arctan(c*x)/x, x) - 4*I*a*c^3*d^4*log(x) + 3*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^4 - 2*I*((c*arctan(c*x) + 1
/x)*c + arctan(c*x)/x^2)*b*c*d^4 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2
) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^4 + 6*a*c^2*d^4/x - 2*I*a*c*d^4/x^2 -
 1/3*a*d^4/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.83, size = 261, normalized size = 1.30

$$\begin{cases} -\frac{a\,d^4}{3\,x^3} & \text{if } c=0\\ \frac{b\,c^3\,d^4\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6}-\frac{b\,c^3\,d^4\,\ln\left(c^2\,x^2+1\right)}{2}-\frac{b\,c^3\,d^4\,\ln(x)}{3}-2\,b\,c^3\,d^4\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)-6\,b\,c\,d^4\,\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)-\frac{b\,c\,d^4}{6\,x^2}-\frac{a\,d^4\,\left(1-3\,c^4\,x^4-18\,c^2\,x^2+c\,x\,6\mathrm{i}+c^3\,x^3\,\ln(x)\,12\mathrm{i}\right)}{3\,x^3}-\frac{b\,d^4\,\mathrm{atan}(c\,x)}{3\,x^3}+b\,c^4\,d^4\,x\,\mathrm{atan}(c\,x)+\frac{6\,b\,c^2\,d^4\,\mathrm{atan}(c\,x)}{x}-b\,d^4\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)\,2\mathrm{i}-\frac{b\,c\,d^4\,\mathrm{atan}(c\,x)\,2\mathrm{i}}{x^2} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^4,x)
```

```
[Out] piecewise(c == 0, -(a*d^4)/(3*x^3), c ~= 0, - b*d^4*(c^3*atan(c*x) + c^2/x)
*2i - 2*b*c^3*d^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) - (b*c^3*d^4*lo
g(c^2*x^2 + 1))/2 - (b*c^3*d^4*log(x))/3 + (b*c^3*d^4*log(- (3*c^4)/2 - (3*
c^6*x^2)/2))/6 - 6*b*c*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b*c*d
^4)/(6*x^2) - (a*d^4*(c*x*6i - 18*c^2*x^2 - 3*c^4*x^4 + c^3*x^3*log(x)*12i
+ 1))/(3*x^3) - (b*d^4*atan(c*x))/(3*x^3) - (b*c*d^4*atan(c*x)*2i)/x^2 + b*
c^4*d^4*x*atan(c*x) + (6*b*c^2*d^4*atan(c*x))/x)
```

3.39 $\int \frac{(d+icdx)^4(a+b\text{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=227

$$-\frac{bcd^4}{12x^3}-\frac{2ibc^2d^4}{3x^2}+\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\text{ArcTan}(cx)-\frac{d^4(a+b\text{ArcTan}(cx))}{4x^4}-\frac{4icd^4(a+b\text{ArcTan}(cx))}{3x^3}+\frac{3c^2d^4(a-$$

```
[Out] -1/12*b*c*d^4/x^3-2/3*I*b*c^2*d^4/x^2+13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arcta
n(c*x)-1/4*d^4*(a+b*arctan(c*x))/x^4-4/3*I*c*d^4*(a+b*arctan(c*x))/x^3+3*c^
2*d^4*(a+b*arctan(c*x))/x^2+4*I*c^3*d^4*(a+b*arctan(c*x))/x+a*c^4*d^4*ln(x)
-16/3*I*b*c^4*d^4*ln(x)+8/3*I*b*c^4*d^4*ln(c^2*x^2+1)+1/2*I*b*c^4*d^4*polyl
og(2,-I*c*x)-1/2*I*b*c^4*d^4*polylog(2,I*c*x)
```

Rubi [A]
time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4996, 4946, 331, 209, 272, 46, 36, 29, 31, 4940, 2438}

$$\frac{4ic^3d^4(a+b\text{ArcTan}(cx))}{x}+\frac{3c^2d^4(a+b\text{ArcTan}(cx))}{x^2}-\frac{d^4(a+b\text{ArcTan}(cx))}{4x^4}-\frac{4icd^4(a+b\text{ArcTan}(cx))}{3x^3}+ac^4d^4\log(x)+\frac{13}{4}bc^4d^4\text{ArcTan}(cx)+\frac{1}{2}ibc^4d^4\text{Li}_2(-icx)-\frac{1}{2}ibc^4d^4\text{Li}_2(icx)-\frac{16}{3}ibc^4d^4\log(x)+\frac{13bc^3d^4}{4x}-\frac{2ibc^2d^4}{3x^2}+\frac{8}{3}ibc^4d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^4)/x^3 - (((2*I)/3)*b*c^2*d^4)/x^2 + (13*b*c^3*d^4)/(4*x) + (1
3*b*c^4*d^4*ArcTan[c*x])/4 - (d^4*(a + b*ArcTan[c*x]))/(4*x^4) - (((4*I)/3)
*c*d^4*(a + b*ArcTan[c*x]))/x^3 + (3*c^2*d^4*(a + b*ArcTan[c*x]))/x^2 + ((4
*I)*c^3*d^4*(a + b*ArcTan[c*x]))/x + a*c^4*d^4*Log[x] - ((16*I)/3)*b*c^4*d^
4*Log[x] + ((8*I)/3)*b*c^4*d^4*Log[1 + c^2*x^2] + (I/2)*b*c^4*d^4*PolyLog[2
, (-I)*c*x] - (I/2)*b*c^4*d^4*PolyLog[2, I*c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^5}\,dx = \int \left(\frac{d^4(a+b\tan^{-1}(cx))}{x^5} + \frac{4icd^4(a+b\tan^{-1}(cx))}{x^4} - \frac{6c^2d^4(a+b\tan^{-1}}{x^3}\right.$$
$$= d^4\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx + \left(4icd^4\right)\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx - \left(6c^2d^4\right)\int$$
$$= -\frac{d^4(a+b\tan^{-1}(cx))}{4x^4} - \frac{4icd^4(a+b\tan^{-1}(cx))}{3x^3} + \frac{3c^2d^4(a+b\tan^{-1}(c}{x^2}$$
$$= -\frac{bcd^4}{12x^3} + \frac{3bc^3d^4}{x} - \frac{d^4(a+b\tan^{-1}(cx))}{4x^4} - \frac{4icd^4(a+b\tan^{-1}(cx))}{3x^3} + \frac{3c}{}$$
$$= -\frac{bcd^4}{12x^3} + \frac{13bc^3d^4}{4x} + 3bc^4d^4\tan^{-1}(cx) - \frac{d^4(a+b\tan^{-1}(cx))}{4x^4} - \frac{4icd^4(a}{}$$
$$= -\frac{bcd^4}{12x^3} - \frac{2ibc^2d^4}{3x^2} + \frac{13bc^3d^4}{4x} + \frac{13}{4}bc^4d^4\tan^{-1}(cx) - \frac{d^4(a+b\tan^{-1}(cx}{4x^4}$$

Mathematica [A]
time = 0.06, size = 210, normalized size = 0.93

$$\frac{d^4\left(-3a-16iacx-bcx+36ac^2x^2-8ibc^2x^2+48iac^3x^3+39bc^3x^3-3b\text{ArcTan}(cx)-16ibcx\text{ArcTan}(cx)+36bc^2x^2\text{ArcTan}(cx)+48ibc^3x^3\text{ArcTan}(cx)+39bc^4x^4\text{ArcTan}(cx)+12ac^4x^4\log(x)-64ibc^4x^4\log(cx)+32ibc^4x^4\log\left(1+c^2x^2\right)+6ibc^4x^4\text{PolyLog}(2,-icx)-6ibc^4x^4\text{PolyLog}(2,icx)\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (d^4*(-3*a - (16*I)*a*c*x - b*c*x + 36*a*c^2*x^2 - (8*I)*b*c^2*x^2 + (48*I)
*a*c^3*x^3 + 39*b*c^3*x^3 - 3*b*ArcTan[c*x] - (16*I)*b*c*x*ArcTan[c*x] + 36
*b*c^2*x^2*ArcTan[c*x] + (48*I)*b*c^3*x^3*ArcTan[c*x] + 39*b*c^4*x^4*ArcTan
[c*x] + 12*a*c^4*x^4*Log[x] - (64*I)*b*c^4*x^4*Log[c*x] + (32*I)*b*c^4*x^4*
Log[1 + c^2*x^2] + (6*I)*b*c^4*x^4*PolyLog[2, (-I)*c*x] - (6*I)*b*c^4*x^4*P
olyLog[2, I*c*x]))/(12*x^4)
```

Maple [A]
time = 0.11, size = 287, normalized size = 1.26

method	result
derivativedivides	$c^4\left(\frac{3d^4a}{c^2x^2} - \frac{4id^4b\arctan(cx)}{3c^3x^3} + \frac{id^4b\ln(cx)\ln(icx+1)}{2} + d^4a\ln(cx) - \frac{d^4a}{4c^4x^4} + \frac{3d^4b\arctan(cx)}{c^2x^2} + \frac{8id^4b\ln(c^2}{3}\right.$

default	$c^4\left(\frac{3d^4a}{c^2x^2}-\frac{4id^4b\arctan(cx)}{3c^3x^3}+\frac{id^4b\ln(cx)\ln(icx+1)}{2}+d^4a\ln(cx)-\frac{d^4a}{4c^4x^4}+\frac{3d^4b\arctan(cx)}{c^2x^2}+\frac{8id^4b\ln(}{3}\right.$
risch	$\frac{2b\,d^4c^3\ln(icx+1)}{x}-\frac{2b\,d^4c\ln(icx+1)}{3x^3}+\frac{2d^4cb\ln(-icx+1)}{3x^3}-\frac{2d^4c^3b\ln(-icx+1)}{x}-\frac{3ib\,d^4c^2\ln(icx+1)}{2x^2}-\frac{25ib\,d^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(3*d^4*a/c^2/x^2-4/3*I*d^4*b*arctan(c*x)/c^3/x^3+1/2*I*d^4*b*dilog(1+I*
c*x)+d^4*a*ln(c*x)-1/4*d^4*a/c^4/x^4+3*d^4*b*arctan(c*x)/c^2/x^2-1/2*I*d^4*
b*dilog(1-I*c*x)+1/2*I*d^4*b*ln(c*x)*ln(1+I*c*x)+d^4*b*ln(c*x)*arctan(c*x)-
1/4*d^4*b*arctan(c*x)/c^4/x^4+8/3*I*d^4*b*ln(c^2*x^2+1)+13/4*b*d^4*arctan(c
*x)-1/2*I*d^4*b*ln(c*x)*ln(1-I*c*x)-2/3*I*d^4*b/c^2/x^2-1/12*d^4*b/c^3/x^3+
13/4*d^4*b/c/x+4*I*d^4*b*arctan(c*x)/c/x-4/3*I*d^4*a/c^3/x^3-16/3*I*d^4*b*l
n(c*x)+4*I*d^4*a/c/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] b*c^4*d^4*integrate(arctan(c*x)/x, x) + a*c^4*d^4*log(x) + 2*I*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^4 + 3*((c*arctan(c*x) + 1/x
)*c + arctan(c*x)/x^2)*b*c^2*d^4 + 2/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x
^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^4 + 4*I*a*c^3*d^4/x + 1/12*((3*c^
3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^4 + 3*a*c^2
*d^4/x^2 - 4/3*I*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I*
a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*x^
2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.94, size = 298, normalized size = 1.31

$$\begin{cases} -\frac{a\,d^4}{4\,x^4} & \text{if}\ \ c=0\\ 3\,b\,c\,d^4\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)-\frac{b\,d^4\,\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c}-\frac{b\,c^4\,d^4\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,c^4\,d^4\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,c^2\,d^4\,\left(c^2\,\ln(x)-\frac{c^2\,\ln(c^2\,x^2+1)}{2}\right)\,4\mathrm{i}+\frac{a\,d^4\,(36\,c^2\,x^2+12\,c^4\,x^4\,\ln(x)-3-c\,x\,16\mathrm{i}+c^3\,x^3\,48\mathrm{i})}{12\,x^4}-\frac{b\,d^4\,\mathrm{atan}(c\,x)}{4\,x^4}-\frac{b\,d^4\,\left(c^4\,\ln(x)-\frac{c^4\,\ln\left(-\frac{c^4\,(3\,c^2\,x^2+1)}{2}-c^4\right)}{2}+\frac{c^2}{2\,x^2}\right)\,4\mathrm{i}}{3}-\frac{b\,c\,d^4\,\mathrm{atan}(c\,x)\,4\mathrm{i}}{3\,x^3}+\frac{3\,b\,c^2\,d^4\,\mathrm{atan}(c\,x)}{x^2}+\frac{b\,c^3\,d^4\,\mathrm{atan}(c\,x)\,4\mathrm{i}}{x} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^5,x)
```

```
[Out] piecewise(c == 0, -(a*d^4)/(4*x^4), c ~= 0, - (b*d^4*(c^4*log(x) - (c^4*log
(- (c^4*(3*c^2*x^2 + 1))/2 - c^4))/2 + c^2/(2*x^2))*4i)/3 - (b*d^4*((c^2/3
- c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - (b*c^4*d^4*dilog(- c*x*1i + 1)*1i)
/2 + (b*c^4*d^4*dilog(c*x*1i + 1)*1i)/2 - b*c^2*d^4*(c^2*log(x) - (c^2*log(
c^2*x^2 + 1))/2)*4i + (a*d^4*(- c*x*16i + 36*c^2*x^2 + c^3*x^3*48i + 12*c^4
*x^4*log(x) - 3))/(12*x^4) - (b*d^4*atan(c*x))/(4*x^4) + 3*b*c*d^4*(c^3*ata
n(c*x) + c^2/x) - (b*c*d^4*atan(c*x)*4i)/(3*x^3) + (3*b*c^2*d^4*atan(c*x))/
x^2 + (b*c^3*d^4*atan(c*x)*4i)/x)
```

3.40 $\int \frac{(d+icdx)^4(a+b\text{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=117

$$-\frac{bcd^4}{20x^4}-\frac{ibc^2d^4}{3x^3}+\frac{11bc^3d^4}{10x^2}+\frac{3ibc^4d^4}{x}-\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5x^5}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(i+cx)$$

```
[Out] -1/20*b*c*d^4/x^4-1/3*I*b*c^2*d^4/x^3+11/10*b*c^3*d^4/x^2+3*I*b*c^4*d^4/x-1
/5*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*d^
4*ln(c*x+I)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {37, 4992, 12, 90}

$$-\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{5x^5}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(cx+i)+\frac{3ibc^4d^4}{x}+\frac{11bc^3d^4}{10x^2}-\frac{ibc^2d^4}{3x^3}-\frac{bcd^4}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^4)/x^4 - ((I/3)*b*c^2*d^4)/x^3 + (11*b*c^3*d^4)/(10*x^2) + ((3
*I)*b*c^4*d^4)/x - (d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*x^5) + (16*b*
c^5*d^4*Log[x])/5 - (16*b*c^5*d^4*Log[I + c*x])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^6}\,dx &= -\frac{d^4(1+icx)^5\,(a+b\tan^{-1}(cx))}{5x^5}-(bc)\int -\frac{id^4(i-cx)^4}{5x^5(i+cx)}\,dx \\
&= -\frac{d^4(1+icx)^5\,(a+b\tan^{-1}(cx))}{5x^5}+\frac{1}{5}\left(ibcd^4\right)\int \frac{(i-cx)^4}{x^5(i+cx)}\,dx \\
&= -\frac{d^4(1+icx)^5\,(a+b\tan^{-1}(cx))}{5x^5}+\frac{1}{5}\left(ibcd^4\right)\int \left(-\frac{i}{x^5}+\frac{5c}{x^4}+\frac{11ic^2}{x^3}-\right. \\
&= -\frac{bcd^4}{20x^4}-\frac{ibc^2d^4}{3x^3}+\frac{11bc^3d^4}{10x^2}+\frac{3ibc^4d^4}{x}-\frac{d^4(1+icx)^5\,(a+b\tan^{-1}(cx)}{5x^5}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 167, normalized size = 1.43

$$\frac{d^4(-12a-60iacx-3bcx+120ac^2x^2-20ibc^2x^2+120iac^3x^3+66bc^3x^3-60ac^4x^4+180ibc^4x^4+12b(-1-5icx+10c^2x^2+10ic^3x^3-5c^4x^4+15ic^5x^5)\,\mathrm{ArcTan}(cx)+192bc^5x^5\log(x)-96bc^5x^5\log\left(1+c^2x^2\right))}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (d^4*(-12*a - (60*I)*a*c*x - 3*b*c*x + 120*a*c^2*x^2 - (20*I)*b*c^2*x^2 + (
120*I)*a*c^3*x^3 + 66*b*c^3*x^3 - 60*a*c^4*x^4 + (180*I)*b*c^4*x^4 + 12*b*(
-1 - (5*I)*c*x + 10*c^2*x^2 + (10*I)*c^3*x^3 - 5*c^4*x^4 + (15*I)*c^5*x^5)*
ArcTan[c*x] + 192*b*c^5*x^5*Log[x] - 96*b*c^5*x^5*Log[1 + c^2*x^2]))/(60*x^
5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(101) = 202.
time = 0.14, size = 223, normalized size = 1.91

method	result
derivativedivides	$c^5\left(d^4a\left(\frac{2i}{c^2x^2}+\frac{2}{c^3x^3}-\frac{1}{cx}-\frac{i}{c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{2id^4b\arctan(cx)}{c^2x^2}+\frac{2d^4b\arctan(cx)}{c^3x^3}-\frac{d^4b\arctan(cx)}{cx}-\right.$ id
default	$c^5\left(d^4a\left(\frac{2i}{c^2x^2}+\frac{2}{c^3x^3}-\frac{1}{cx}-\frac{i}{c^4x^4}-\frac{1}{5c^5x^5}\right)+\frac{2id^4b\arctan(cx)}{c^2x^2}+\frac{2d^4b\arctan(cx)}{c^3x^3}-\frac{d^4b\arctan(cx)}{cx}-\right.$ id
risch	$\frac{id^4b(5c^4x^4-10ic^3x^3-10c^2x^2+5icx+1)\ln(icx+1)}{10x^5}-\frac{d^4(186c^5b\ln(-cx-i)x^5-192c^5b\ln(-x)x^5+6c^5b\ln(cx-i)x^5+60ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(d^4*a*(2*I/c^2/x^2+2/c^3/x^3-1/c/x-I/c^4/x^4-1/5/c^5/x^5)+2*I*d^4*b*ar
ctan(c*x)/c^2/x^2+2*d^4*b*arctan(c*x)/c^3/x^3-d^4*b*arctan(c*x)/c/x-I*d^4*b
*arctan(c*x)/c^4/x^4-1/5*d^4*b*arctan(c*x)/c^5/x^5-8/5*b*ln(c^2*x^2+1)*d^4+
3*I*d^4*b*arctan(c*x)-1/3*I*d^4*b/c^3/x^3+3*I*d^4*b/c/x-1/20*d^4*b/c^4/x^4+
11/10*d^4*b/c^2/x^2+16/5*d^4*b*ln(c*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(97) = 194$.
time = 0.47, size = 275, normalized size = 2.35

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^4d^4+2i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^3d^4-\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^2d^4-\frac{ac^4d^4}{x}+\frac{1}{3}i\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bcd^4-\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^4+\frac{2i\,ac^3d^4}{x^2}+\frac{2\,ac^2d^4}{x^3}-\frac{i\,acd^4}{x^4}-\frac{ad^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^4*d^4 + 2*I*((
c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^4 - ((c^2*log(c^2*x^2 + 1
) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^4 - a*c^4*d^4/x +
1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c
*d^4 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
)*c + 4*arctan(c*x)/x^5)*b*d^4 + 2*I*a*c^3*d^4/x^2 + 2*a*c^2*d^4/x^3 - I*a*
c*d^4/x^4 - 1/5*a*d^4/x^5
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.
time = 0.95, size = 202, normalized size = 1.73

$$\frac{192\,bc^5d^4x^5\log\left(x\right)-186\,bc^5d^4x^5\log\left(\frac{cx+i}{c}\right)-6\,bc^5d^4x^5\log\left(\frac{cx-i}{c}\right)-60\,(a-3i\,b)c^4d^4x^4-6\,(-20i\,a-11\,b)c^3d^4x^3+20\,(6\,a-i\,b)c^2d^4x^2-3\,(20i\,a+b)cd^4x-12\,ad^4-6\,(5i\,bc^4d^4x^4+10\,bc^3d^4x^3-10i\,bc^2d^4x^2-5\,bcd^4x+i\,bd^4)\log\left(-\frac{cx+i}{cx-i}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(192*b*c^5*d^4*x^5*log(x) - 186*b*c^5*d^4*x^5*log((c*x + I)/c) - 6*b*c
^5*d^4*x^5*log((c*x - I)/c) - 60*(a - 3*I*b)*c^4*d^4*x^4 - 6*(-20*I*a - 11*
b)*c^3*d^4*x^3 + 20*(6*a - I*b)*c^2*d^4*x^2 - 3*(20*I*a + b)*c*d^4*x - 12*a
*d^4 - 6*(5*I*b*c^4*d^4*x^4 + 10*b*c^3*d^4*x^3 - 10*I*b*c^2*d^4*x^2 - 5*b*c
*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(114) = 228$.
time = 49.27, size = 366, normalized size = 3.13

$$\frac{16bc^5d^4\log\left(10395b^2c^{11}d^8x\right)}{5}-\frac{bc^5d^4\log\left(10395b^2c^{11}d^8x-10395ib^2c^{10}d^8\right)}{10}-\frac{31bc^5d^4\log\left(10395b^2c^{11}d^8x+10395ib^2c^{10}d^8\right)}{10}+\frac{-12ad^4+x^4(-60ac^4d^4+180ibc^4d^4)+x^3\cdot(120iac^3d^4+66bc^3d^4)+x^2\cdot(120ac^2d^4-20ibc^2d^4)+x(-60iacd^4-3bcd^4)}{60x^5}+\frac{(-5ibc^4d^4x^4-10bc^3d^4x^3+10ibc^2d^4x^2+5bcd^4x-ibd^4)\log\left(-icx+1\right)}{10x^5}+\frac{(5ibc^4d^4x^4+10bc^3d^4x^3-10ibc^2d^4x^2-5bcd^4x+ibd^4)\log\left(icx+1\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**6,x)
```

```
[Out] 16*b*c**5*d**4*log(10395*b**2*c**11*d**8*x)/5 - b*c**5*d**4*log(10395*b**2*
c**11*d**8*x - 10395*I*b**2*c**10*d**8)/10 - 31*b*c**5*d**4*log(10395*b**2*
c**11*d**8*x + 10395*I*b**2*c**10*d**8)/10 + (-12*a*d**4 + x**4*(-60*a*c**4
*d**4 + 180*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 66*b*c**3*d**4) + x*
*2*(120*a*c**2*d**4 - 20*I*b*c**2*d**4) + x*(-60*I*a*c*d**4 - 3*b*c*d**4))/
(60*x**5) + (-5*I*b*c**4*d**4*x**4 - 10*b*c**3*d**4*x**3 + 10*I*b*c**2*d**4
*x**2 + 5*b*c*d**4*x - I*b*d**4)*log(-I*c*x + 1)/(10*x**5) + (5*I*b*c**4*d*
*4*x**4 + 10*b*c**3*d**4*x**3 - 10*I*b*c**2*d**4*x**2 - 5*b*c*d**4*x + I*b*
d**4)*log(I*c*x + 1)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.73, size = 186, normalized size = 1.59

$$\frac{d^4\,\left(192\,b\,c^5\,\ln\left(x\right)-96\,b\,c^5\,\ln\left(c^2\,x^2+1\right)+b\,c^5\,\mathrm{atan}\left(c\,x\right)\,180\mathrm{i}\right)}{60}-\frac{\frac{d^4\,\left(12\,a+12\,b\,\mathrm{atan}\left(c\,x\right)\right)}{60}+\frac{d^4\,x\,\left(a\,c\,60\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,60\mathrm{i}\right)}{60}-\frac{d^4\,x^2\,\left(120\,a\,c^2+120\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,20\mathrm{i}\right)}{60}+\frac{d^4\,x^4\,\left(60\,a\,c^4+60\,b\,c^4\,\mathrm{atan}\left(c\,x\right)-b\,c^4\,180\mathrm{i}\right)}{60}-\frac{d^4\,x^3\,\left(a\,c^3\,120\mathrm{i}+66\,b\,c^3+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,120\mathrm{i}\right)}{60}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^6,x)
```

```
[Out] (d^4*(b*c^5*atan(c*x)*180i - 96*b*c^5*log(c^2*x^2 + 1) + 192*b*c^5*log(x)))
/60 - ((d^4*(12*a + 12*b*atan(c*x)))/60 + (d^4*x*(a*c*60i + 3*b*c + b*c*ata
n(c*x)*60i))/60 - (d^4*x^2*(120*a*c^2 - b*c^2*20i + 120*b*c^2*atan(c*x)))/6
0 + (d^4*x^4*(60*a*c^4 - b*c^4*180i + 60*b*c^4*atan(c*x)))/60 - (d^4*x^3*(a
*c^3*120i + 66*b*c^3 + b*c^3*atan(c*x)*120i))/60)/x^5
```

3.41 $\int \frac{(d+icdx)^4(a+b\text{ArcTan}(cx))}{x^7}\,dx$

Optimal. Leaf size=168

$$-\frac{bcd^4}{30x^5}-\frac{ibc^2d^4}{5x^4}+\frac{5bc^3d^4}{9x^3}+\frac{16ibc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\text{ArcTan}(cx))}{30x^5}$$

```
[Out] -1/30*b*c*d^4/x^5-1/5*I*b*c^2*d^4/x^4+5/9*b*c^3*d^4/x^3+16/15*I*b*c^4*d^4/x
^2-13/6*b*c^5*d^4/x-1/6*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^6+1/30*I*c*d^4*
(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+32/15*I*b*c^6*d^4*ln(x)-32/15*I*b*c^6*d^4
*ln(c*x+I)
```

Rubi [A]
time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {47, 37, 4992, 12, 153}

$$-\frac{d^4(1+icx)^5(a+b\text{ArcTan}(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\text{ArcTan}(cx))}{30x^5}+\frac{32}{15}ibc^6d^4\log(x)-\frac{32}{15}ibc^6d^4\log(cx+i)-\frac{13bc^5d^4}{6x}+\frac{16ibc^4d^4}{15x^2}+\frac{5bc^3d^4}{9x^3}-\frac{ibc^2d^4}{5x^4}-\frac{bcd^4}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^4)/x^5 - ((I/5)*b*c^2*d^4)/x^4 + (5*b*c^3*d^4)/(9*x^3) + (((16
*I)/15)*b*c^4*d^4)/x^2 - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + I*c*x)^5*(a + b*A
rcTan[c*x]))/(6*x^6) + ((I/30)*c*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^5
 + ((32*I)/15)*b*c^6*d^4*Log[x] - ((32*I)/15)*b*c^6*d^4*Log[I + c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{x^7}\,dx &= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\\ &= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\\ &= -\frac{d^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{6x^6}+\frac{icd^4(1+icx)^5\left(a+b\tan^{-1}(cx)\right)}{30x^5}-\\ &= -\frac{bcd^4}{30x^5}-\frac{ibc^2d^4}{5x^4}+\frac{5bc^3d^4}{9x^3}+\frac{16ibc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+icx)^5\left(a+b\right.}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.98, size = 178, normalized size = 1.06

$$-\frac{d^4\left(15a+72iacx+3bcx-135ac^2x^2+18ibc^2x^2-120iac^3x^3-50bc^3x^3+45ac^4x^4-96ibc^4x^4+195bc^5x^5+3b\left(5+24icx-45c^2x^2-40ic^3x^3+15c^4x^4+65c^6x^6\right)\mathrm{ArcTan}(cx)-192ibc^6x^6\log(x)+96ibc^6x^6\log\left(1+c^2x^2\right)\right)}{90x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/90*(d^4*(15*a + (72*I)*a*c*x + 3*b*c*x - 135*a*c^2*x^2 + (18*I)*b*c^2*x^
2 - (120*I)*a*c^3*x^3 - 50*b*c^3*x^3 + 45*a*c^4*x^4 - (96*I)*b*c^4*x^4 + 19
5*b*c^5*x^5 + 3*b*(5 + (24*I)*c*x - 45*c^2*x^2 - (40*I)*c^3*x^3 + 15*c^4*x^
4 + 65*c^6*x^6)*ArcTan[c*x] - (192*I)*b*c^6*x^6*Log[x] + (96*I)*b*c^6*x^6*L
og[1 + c^2*x^2]))/x^6
```

Maple [A]
time = 0.20, size = 236, normalized size = 1.40

method	result
derivativedivides	$c^{6}\left(d^{4}a\left(-\frac{1}{2c^{2}x^{2}}-\frac{1}{6c^{6}x^{6}}+\frac{4i}{3c^{3}x^{3}}+\frac{3}{2c^{4}x^{4}}-\frac{4i}{5c^{5}x^{5}}\right)-\frac{d^{4}b\arctan(cx)}{2c^{2}x^{2}}-\frac{d^{4}b\arctan(cx)}{6c^{6}x^{6}}+\frac{4id^{4}b\arctan(\right.$
default	$c^{6}\left(d^{4}a\left(-\frac{1}{2c^{2}x^{2}}-\frac{1}{6c^{6}x^{6}}+\frac{4i}{3c^{3}x^{3}}+\frac{3}{2c^{4}x^{4}}-\frac{4i}{5c^{5}x^{5}}\right)-\frac{d^{4}b\arctan(cx)}{2c^{2}x^{2}}-\frac{d^{4}b\arctan(cx)}{6c^{6}x^{6}}+\frac{4id^{4}b\arctan(\right.$
risch	$\frac{id^{4}b(15c^{4}x^{4}-40ic^{3}x^{3}-45c^{2}x^{2}+24icx+5)\ln(icx+1)}{60x^{6}}-\frac{id^{4}(387b\,c^{6}\ln(-16705cx-16705i)x^{6}-3b\,c^{6}\ln(8255cx-8255i)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(d^4*a*(-1/2/c^2/x^2-1/6/c^6/x^6+4/3*I/c^3/x^3+3/2/c^4/x^4-4/5*I/c^5/x^
5)-1/2*d^4*b*arctan(c*x)/c^2/x^2-1/6*d^4*b*arctan(c*x)/c^6/x^6+4/3*I*d^4*b*
arctan(c*x)/c^3/x^3+3/2*d^4*b*arctan(c*x)/c^4/x^4-4/5*I*d^4*b*arctan(c*x)/c
^5/x^5-16/15*I*d^4*b*ln(c^2*x^2+1)-13/6*b*d^4*arctan(c*x)+32/15*I*d^4*b*ln(
c*x)-1/5*I*d^4*b/c^4/x^4+16/15*I*d^4*b/c^2/x^2-1/30*d^4*b/c^5/x^5+5/9*d^4*b
/c^3/x^3-13/6*d^4*b/c/x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(134) = 268.
time = 0.49, size = 290, normalized size = 1.73

$$-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^{2}}\right)bc^{4}d^{4}-\frac{2}{3}i\left(\left(c^{2}\log(c^{2}x^{2}+1)-c^{2}\log(x^{2})-\frac{1}{x^{2}}\right)c-\frac{2\arctan(cx)}{x^{3}}\right)bc^{3}d^{4}-\frac{1}{2}\left(\left(3c^{3}\arctan(cx)+\frac{3c^{2}x^{2}-1}{x^{3}}\right)c-\frac{3\arctan(cx)}{x^{4}}\right)bc^{2}d^{4}-\frac{1}{5}i\left(\left(2c^{4}\log(c^{2}x^{2}+1)-2c^{4}\log(x^{2})-\frac{2c^{2}x^{2}-1}{x^{4}}\right)c+\frac{4\arctan(cx)}{x^{5}}\right)bcd^{4}-\frac{ac^{4}d^{4}}{2x^{2}}-\frac{1}{90}\left(\left(15c^{5}\arctan(cx)+\frac{15c^{4}x^{4}-5c^{2}x^{2}+3}{x^{5}}\right)c+\frac{15\arctan(cx)}{x^{6}}\right)bd^{4}+\frac{4i\,ac^{3}d^{4}}{3x^{3}}+\frac{3ac^{2}d^{4}}{2x^{4}}-\frac{4i\,acd^{4}}{5x^{5}}-\frac{ad^{4}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^4*d^4 - 2/3*I*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^3*d^4 - 1
/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c^2*
d^4 - 1/5*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
)*c + 4*arctan(c*x)/x^5)*b*c*d^4 - 1/2*a*c^4*d^4/x^2 - 1/90*((15*c^5*arctan
(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^4 + 4
/3*I*a*c^3*d^4/x^3 + 3/2*a*c^2*d^4/x^4 - 4/5*I*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

Fricas [A]
time = 1.39, size = 216, normalized size = 1.29

$$\frac{384i\,bc^{6}d^{4}x^{6}\log(x)-387i\,bc^{6}d^{4}x^{6}\log\left(\frac{cx+i}{c}\right)+3i\,bc^{6}d^{4}x^{6}\log\left(\frac{cx-i}{c}\right)-390\,bc^{5}d^{4}x^{5}-6\,(15\,a-32i\,b)c^{4}d^{4}x^{4}-20\,(-12i\,a-5\,b)c^{3}d^{4}x^{3}+18\,(15\,a-2i\,b)c^{2}d^{4}x^{2}-6\,(24i\,a+b)cd^{4}x-30\,ad^{4}-3\,(15i\,bc^{4}d^{4}x^{4}+40\,bc^{3}d^{4}x^{3}-45i\,bc^{2}d^{4}x^{2}-24\,bcd^{4}x+5i\,bd^{4})\log\left(-\frac{cx+i}{cx-i}\right)}{180\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/180*(384*I*b*c^6*d^4*x^6*log(x) - 387*I*b*c^6*d^4*x^6*log((c*x + I)/c) +
3*I*b*c^6*d^4*x^6*log((c*x - I)/c) - 390*b*c^5*d^4*x^5 - 6*(15*a - 32*I*b)*
c^4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 18*(15*a - 2*I*b)*c^2*d^4*x^
2 - 6*(24*I*a + b)*c*d^4*x - 30*a*d^4 - 3*(15*I*b*c^4*d^4*x^4 + 40*b*c^3*d^
4*x^3 - 45*I*b*c^2*d^4*x^2 - 24*b*c*d^4*x + 5*I*b*d^4)*log(-(c*x + I)/(c*x
- I)))/x^6
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(163) = 326$.
time = 91.68, size = 388, normalized size = 2.31

$$\frac{32ibc^6d^4\log\left(2121535b^2c^{13}d^8x\right)}{15} + \frac{ibc^6d^4\log\left(2121535b^2c^{13}d^8x - 2121535ib^2c^{12}d^8\right)}{60} - \frac{43ibc^6d^4\log\left(2121535b^2c^{13}d^8x + 2121535ib^2c^{12}d^8\right)}{20} + \frac{\left(-15ibc^4d^4x^4 - 40bc^3d^4x^3 + 45ibc^2d^4x^2 + 24bcd^4x - 5ibd^4\right)\log\left(-icx+1\right)}{60x^6} + \frac{\left(15ibc^4d^4x^4 + 40bc^3d^4x^3 - 45ibc^2d^4x^2 - 24bcd^4x + 5ibd^4\right)\log\left(icx+1\right)}{60x^6} + \frac{-15ad^4 - 195bc^5d^4x^5 + x^4\left(-45ac^4d^4 + 96ibc^4d^4\right) + x^3\cdot\left(120iac^3d^4 + 50bc^3d^4\right) + x^2\cdot\left(135ac^2d^4 - 18ibc^2d^4\right) + x\left(-72iacd^4 - 3bcd^4\right)}{90x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**7,x)
```

```
[Out] 32*I*b*c**6*d**4*log(2121535*b**2*c**13*d**8*x)/15 + I*b*c**6*d**4*log(2121
535*b**2*c**13*d**8*x - 2121535*I*b**2*c**12*d**8)/60 - 43*I*b*c**6*d**4*lo
g(2121535*b**2*c**13*d**8*x + 2121535*I*b**2*c**12*d**8)/20 + (-15*I*b*c**4
*d**4*x**4 - 40*b*c**3*d**4*x**3 + 45*I*b*c**2*d**4*x**2 + 24*b*c*d**4*x -
5*I*b*d**4)*log(-I*c*x + 1)/(60*x**6) + (15*I*b*c**4*d**4*x**4 + 40*b*c**3*
d**4*x**3 - 45*I*b*c**2*d**4*x**2 - 24*b*c*d**4*x + 5*I*b*d**4)*log(I*c*x +
 1)/(60*x**6) + (-15*a*d**4 - 195*b*c**5*d**4*x**5 + x**4*(-45*a*c**4*d**4
+ 96*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 50*b*c**3*d**4) + x**2*(135
*a*c**2*d**4 - 18*I*b*c**2*d**4) + x*(-72*I*a*c*d**4 - 3*b*c*d**4))/(90*x**
6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.90, size = 208, normalized size = 1.24

$$-\frac{d^4\left(195\,b\,c^5\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,96\mathrm{i}-b\,c^6\,\ln\left(x\right)\,192\mathrm{i}\right)}{90}-\frac{\frac{d^4\,\left(15\,a+15\,b\,\mathrm{atan}\left(c\,x\right)\right)}{90}+\frac{d^4\,x\,\left(a\,c\,72\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,72\mathrm{i}\right)}{90}+\frac{d^4\,x^4\,\left(45\,a\,c^4+45\,b\,c^4\,\mathrm{atan}\left(c\,x\right)-b\,c^4\,96\mathrm{i}\right)}{90}-\frac{d^4\,x^2\,\left(135\,a\,c^2+135\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,18\mathrm{i}\right)}{90}-\frac{d^4\,x^3\,\left(a\,c^3\,120\mathrm{i}+50\,b\,c^3+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,120\mathrm{i}\right)}{90}+\frac{13\,b\,c^5\,d^4\,x^5}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^7,x)
```

```
[Out] - (d^4*(b*c^6*log(c^2*x^2 + 1)*96i - b*c^6*log(x)*192i + 195*b*c^5*atan(x*(
c^2)^(1/2))*(c^2)^(1/2)))/90 - ((d^4*(15*a + 15*b*atan(c*x)))/90 + (d^4*x*(
```

```
a*c*72i + 3*b*c + b*c*atan(c*x)*72i))/90 + (d^4*x^4*(45*a*c^4 - b*c^4*96i +
 45*b*c^4*atan(c*x)))/90 - (d^4*x^2*(135*a*c^2 - b*c^2*18i + 135*b*c^2*atan
(c*x)))/90 - (d^4*x^3*(a*c^3*120i + 50*b*c^3 + b*c^3*atan(c*x)*120i))/90 +
(13*b*c^5*d^4*x^5)/6)/x^6
```

3.42 $\int \frac{(d+icdx)^4(a+b\textbf{ArcTan}(cx))}{x^8}\,dx$

Optimal. Leaf size=243

$$-\frac{bcd^4}{42x^6}-\frac{2ibc^2d^4}{15x^5}+\frac{47bc^3d^4}{140x^4}+\frac{5ibc^4d^4}{9x^3}-\frac{88bc^5d^4}{105x^2}-\frac{5ibc^6d^4}{3x}-\frac{d^4(a+b\text{ArcTan}(cx))}{7x^7}-\frac{2icd^4(a+b\text{ArcTan}(cx))}{3x^6}+6c$$

```
[Out] -1/42*b*c*d^4/x^6-2/15*I*b*c^2*d^4/x^5+47/140*b*c^3*d^4/x^4+5/9*I*b*c^4*d^4
/x^3-88/105*b*c^5*d^4/x^2-5/3*I*b*c^6*d^4/x-1/7*d^4*(a+b*arctan(c*x))/x^7-2
/3*I*c*d^4*(a+b*arctan(c*x))/x^6+6/5*c^2*d^4*(a+b*arctan(c*x))/x^5+I*c^3*d^
4*(a+b*arctan(c*x))/x^4-1/3*c^4*d^4*(a+b*arctan(c*x))/x^3-176/105*b*c^7*d^4
*ln(x)+1/210*b*c^7*d^4*ln(I-c*x)+117/70*b*c^7*d^4*ln(c*x+I)
```

Rubi [A]
time = 0.14, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {45, 4992, 12, 1816}

$$-\frac{c^4d^4(a+b\text{ArcTan}(cx))}{3x^3}+\frac{ic^3d^4(a+b\text{ArcTan}(cx))}{x^4}+\frac{6c^2d^4(a+b\text{ArcTan}(cx))}{5x^5}-\frac{d^4(a+b\text{ArcTan}(cx))}{7x^7}-\frac{2icd^4(a+b\text{ArcTan}(cx))}{3x^6}-\frac{176}{105}bc^7d^4\log(x)+\frac{1}{210}bc^7d^4\log(-cx+i)+\frac{117}{70}bc^7d^4\log(cx+i)-\frac{5ibc^6d^4}{3x}-\frac{88bc^5d^4}{105x^2}+\frac{5ibc^4d^4}{9x^3}+\frac{47bc^3d^4}{140x^4}-\frac{2ibc^2d^4}{15x^5}-\frac{bcd^4}{42x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d^4)/x^6 - (((2*I)/15)*b*c^2*d^4)/x^5 + (47*b*c^3*d^4)/(140*x^4)
 + (((5*I)/9)*b*c^4*d^4)/x^3 - (88*b*c^5*d^4)/(105*x^2) - (((5*I)/3)*b*c^6*
d^4)/x - (d^4*(a + b*ArcTan[c*x]))/(7*x^7) - (((2*I)/3)*c*d^4*(a + b*ArcTan
[c*x]))/x^6 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/(5*x^5) + (I*c^3*d^4*(a + b*A
rcTan[c*x]))/x^4 - (c^4*d^4*(a + b*ArcTan[c*x]))/(3*x^3) - (176*b*c^7*d^4*L
og[x])/105 + (b*c^7*d^4*Log[I - c*x])/210 + (117*b*c^7*d^4*Log[I + c*x])/70
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\int \frac{(d+icdx)^4\,(a+b\tan^{-1}(cx))}{x^8}\,dx = -\frac{d^4(a+b\tan^{-1}(cx))}{7x^7} - \frac{2icd^4(a+b\tan^{-1}(cx))}{3x^6} + \frac{6c^2d^4(a+b\tan^{-1}(\ldots}{5x^5}$$

$$= -\frac{d^4(a+b\tan^{-1}(cx))}{7x^7} - \frac{2icd^4(a+b\tan^{-1}(cx))}{3x^6} + \frac{6c^2d^4(a+b\tan^{-1}(\ldots}{5x^5}$$

$$= -\frac{d^4(a+b\tan^{-1}(cx))}{7x^7} - \frac{2icd^4(a+b\tan^{-1}(cx))}{3x^6} + \frac{6c^2d^4(a+b\tan^{-1}(\ldots}{5x^5}$$

$$= -\frac{bcd^4}{42x^6} - \frac{2ibc^2d^4}{15x^5} + \frac{47bc^3d^4}{140x^4} + \frac{5ibc^4d^4}{9x^3} - \frac{88bc^5d^4}{105x^2} - \frac{5ibc^6d^4}{3x} - \frac{d^4(a\ldots}{}$$

Mathematica [A]
time = 0.08, size = 187, normalized size = 0.77

$$-\frac{d^4(180a+840iacx+30bcx-1512ac^2x^2+168ibc^2x^2-1260iac^3x^3-423bc^3x^3+420ac^4x^4-700ibc^4x^4+1056bc^5x^5+2100ibc^6x^6+12b(15+70icx-126c^2x^2-105ic^3x^3+35c^4x^4+175ic^7x^7)\,\mathrm{ArcTan}(cx)+2112bc^7x^7\log(x)-1056bc^7x^7\log(1+c^2x^2))}{1260x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/1260*(d^4*(180*a + (840*I)*a*c*x + 30*b*c*x - 1512*a*c^2*x^2 + (168*I)*b
*c^2*x^2 - (1260*I)*a*c^3*x^3 - 423*b*c^3*x^3 + 420*a*c^4*x^4 - (700*I)*b*c
^4*x^4 + 1056*b*c^5*x^5 + (2100*I)*b*c^6*x^6 + 12*b*(15 + (70*I)*c*x - 126*
c^2*x^2 - (105*I)*c^3*x^3 + 35*c^4*x^4 + (175*I)*c^7*x^7)*ArcTan[c*x] + 211
2*b*c^7*x^7*Log[x] - 1056*b*c^7*x^7*Log[1 + c^2*x^2]))/x^7
```

Maple [A]
time = 0.16, size = 248, normalized size = 1.02

method	result
derivativedivides	$c^7\Big(d^4a\big(-\frac{2i}{3c^6x^6}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}+\frac{i}{c^4x^4}+\frac{6}{5c^5x^5}\big)-\frac{2id^4b\arctan(cx)}{3c^6x^6}-\frac{d^4b\arctan(cx)}{3c^3x^3}-\frac{d^4b\arctan(c\ldots}{7c^7x^7}$

default	$c^7\left(d^4a\left(-\frac{2i}{3c^6x^6}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}+\frac{i}{c^4x^4}+\frac{6}{5c^5x^5}\right)-\frac{2id^4b\arctan(cx)}{3c^6x^6}-\frac{d^4b\arctan(cx)}{3c^3x^3}-\frac{d^4b\arctan(cx)}{7c^7x^7}\right.$
risch	$\frac{id^4b(35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15)\ln(icx+1)}{210x^7}+\frac{d^4(6c^7b\ln(cx-i)x^7+2106c^7b\ln(-cx-i)x^7-2112c^7b\ln(-x)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)

[Out] c^7*(d^4*a*(-2/3*I/c^6/x^6-1/3/c^3/x^3-1/7/c^7/x^7+I/c^4/x^4+6/5/c^5/x^5)-2/3*I*d^4*b*arctan(c*x)/c^6/x^6-1/3*d^4*b*arctan(c*x)/c^3/x^3-1/7*d^4*b*arctan(c*x)/c^7/x^7+I*d^4*b*arctan(c*x)/c^4/x^4+6/5*d^4*b*arctan(c*x)/c^5/x^5+88/105*b*ln(c^2*x^2+1)*d^4-5/3*I*d^4*b*arctan(c*x)-5/3*I*d^4*b/c/x-2/15*I*d^4*b/c^5/x^5+5/9*I*d^4*b/c^3/x^3-1/42*d^4*b/c^6/x^6+47/140*d^4*b/c^4/x^4-88/105*d^4*b/c^2/x^2-176/105*d^4*b*ln(c*x))

Maxima [A]
time = 0.47, size = 329, normalized size = 1.35

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^4d^4-\frac{1}{3}i\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^3d^4+\frac{3}{10}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bc^2d^4-\frac{2}{45}i\left(\left(15c^5\arctan\left(cx\right)+\frac{15c^4x^4-5c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bcd^4+\frac{1}{84}\left(\left(6c^6\log\left(c^2x^2+1\right)-6c^6\log\left(x^2\right)-\frac{6c^4x^4-3c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^4-\frac{ac^4d^4}{3x^3}+\frac{iac^3d^4}{x^4}+\frac{6ac^2d^4}{5x^5}-\frac{2iacd^4}{3x^6}-\frac{ad^4}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")

[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^4*d^4 - 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c^3*d^4 + 3/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c^2*d^4 - 2/45*I*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*c*d^4 + 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 + I*a*c^3*d^4/x^4 + 6/5*a*c^2*d^4/x^5 - 2/3*I*a*c*d^4/x^6 - 1/7*a*d^4/x^7

Fricas [A]
time = 1.49, size = 228, normalized size = 0.94

$$-\frac{2112\,bc^7d^4x^7\log\left(x\right)-2106\,bc^7d^4x^7\log\left(\frac{cx+i}{c}\right)-6\,bc^7d^4x^7\log\left(\frac{cx-i}{c}\right)+2100i\,bc^6d^4x^6+1056\,bc^5d^4x^5+140\left(3\,a-5i\,b\right)c^4d^4x^4+9\left(-140i\,a-47\,b\right)c^3d^4x^3-168\left(9\,a-i\,b\right)c^2d^4x^2+30\left(28i\,a+b\right)cd^4x+180\,ad^4+6\left(35i\,bc^4d^4x^4+105\,bc^3d^4x^3-126i\,bc^2d^4x^2-70\,bcd^4x+15i\,bd^4\right)\log\left(-\frac{cx+i}{cx-i}\right)}{1260\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")

[Out] -1/1260*(2112*b*c^7*d^4*x^7*log(x) - 2106*b*c^7*d^4*x^7*log((c*x + I)/c) - 6*b*c^7*d^4*x^7*log((c*x - I)/c) + 2100*I*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 140*(3*a - 5*I*b)*c^4*d^4*x^4 + 9*(-140*I*a - 47*b)*c^3*d^4*x^3 - 168*(9*a - I*b)*c^2*d^4*x^2 + 30*(28*I*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*I*b*c^4*d^4*x^4 + 105*b*c^3*d^4*x^3 - 126*I*b*c^2*d^4*x^2 - 70*b*c*d^4*x + 15*I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^7

Sympy [A]
time = 154.50, size = 398, normalized size = 1.64

$$-\frac{176bc^7d^4\log\left(43427825b^2c^{15}d^8x\right)}{105}+\frac{bc^7d^4\log\left(43427825b^2c^{15}d^8x-43427825ib^2c^{14}d^8\right)}{210}+\frac{117bc^7d^4\log\left(43427825b^2c^{15}d^8x+43427825ib^2c^{14}d^8\right)}{70}+\frac{\left(-35ibc^4d^4x^4-105bc^3d^4x^3+126ibc^2d^4x^2+70bcd^4x-15ibd^4\right)\log\left(-icx+1\right)}{210x^7}+\frac{\left(35ibc^4d^4x^4+105bc^3d^4x^3-126ibc^2d^4x^2-70bcd^4x+15ibd^4\right)\log\left(icx+1\right)}{210x^7}+\frac{-180ad^4-2100ibc^6d^4x^6-1056bc^5d^4x^5+x^4(-420ac^4d^4+700ibc^4d^4)+x^3\cdot(1260iac^3d^4+423bc^3d^4)+x^2\cdot(1512ac^2d^4-168ibc^2d^4)+x(-840iacd^4-30bcd^4)}{1260x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**8,x)
```

```
[Out] -176*b*c**7*d**4*log(43427825*b**2*c**15*d**8*x)/105 + b*c**7*d**4*log(4342
7825*b**2*c**15*d**8*x - 43427825*I*b**2*c**14*d**8)/210 + 117*b*c**7*d**4*
log(43427825*b**2*c**15*d**8*x + 43427825*I*b**2*c**14*d**8)/70 + (-35*I*b*
c**4*d**4*x**4 - 105*b*c**3*d**4*x**3 + 126*I*b*c**2*d**4*x**2 + 70*b*c*d**
4*x - 15*I*b*d**4)*log(-I*c*x + 1)/(210*x**7) + (35*I*b*c**4*d**4*x**4 + 10
5*b*c**3*d**4*x**3 - 126*I*b*c**2*d**4*x**2 - 70*b*c*d**4*x + 15*I*b*d**4)*
log(I*c*x + 1)/(210*x**7) + (-180*a*d**4 - 2100*I*b*c**6*d**4*x**6 - 1056*b
*c**5*d**4*x**5 + x**4*(-420*a*c**4*d**4 + 700*I*b*c**4*d**4) + x**3*(1260*
I*a*c**3*d**4 + 423*b*c**3*d**4) + x**2*(1512*a*c**2*d**4 - 168*I*b*c**2*d*
*4) + x*(-840*I*a*c*d**4 - 30*b*c*d**4))/(1260*x**7)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.17, size = 317, normalized size = 1.30

$$\frac{88\,b\,c^7\,d^4\,\ln\left(c^2\,x^2+1\right)}{105}-\frac{\frac{a\,d^4}{7}+\frac{b\,d^4\,\mathrm{atan}(c\,x)}{7}+\frac{88\,b\,c^7\,d^4\,x^7}{105}+\frac{b\,c^8\,d^4\,x^8\,5\mathrm{i}}{3}+\frac{c\,d^4\,x\,(b+a\,28\mathrm{i})}{42}+\frac{c^6\,d^4\,x^6\,(3\,a+b\,10\mathrm{i})}{9}-\frac{c^4\,d^4\,x^4\,(39\,a+b\,19\mathrm{i})}{45}-\frac{c^2\,d^4\,x^2\,(111\,a-b\,14\mathrm{i})}{105}-\frac{c^3\,d^4\,x^3\,(131\,b+a\,140\mathrm{i})}{420}-c^5\,d^4\,x^5\,\left(-\frac{211\,b}{420}+a\,1\mathrm{i}\right)-\frac{37\,b\,c^2\,d^4\,x^2\,\mathrm{atan}(c\,x)}{35}-\frac{b\,c^3\,d^4\,x^3\,\mathrm{atan}(c\,x)\,1\mathrm{i}}{3}-\frac{13\,b\,c^4\,d^4\,x^4\,\mathrm{atan}(c\,x)}{15}-b\,c^5\,d^4\,x^5\,\mathrm{atan}(c\,x)\,1\mathrm{i}+\frac{b\,c^6\,d^4\,x^6\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c\,d^4\,x\,\mathrm{atan}(c\,x)\,2\mathrm{i}}{3}}{c^2\,x^9+x^7}-\frac{176\,b\,c^7\,d^4\,\ln\left(x\right)}{105}-\frac{b\,c^{10}\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,5\mathrm{i}}{3\,{\left(c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^8,x)
```

```
[Out] (88*b*c^7*d^4*log(c^2*x^2 + 1))/105 - ((a*d^4)/7 + (b*d^4*atan(c*x))/7 + (8
8*b*c^7*d^4*x^7)/105 + (b*c^8*d^4*x^8*5i)/3 + (c*d^4*x*(a*28i + b))/42 + (c
^6*d^4*x^6*(3*a + b*10i))/9 - (c^4*d^4*x^4*(39*a + b*19i))/45 - (c^2*d^4*x^
2*(111*a - b*14i))/105 - (c^3*d^4*x^3*(a*140i + 131*b))/420 - c^5*d^4*x^5*(
a*1i - (211*b)/420) - (37*b*c^2*d^4*x^2*atan(c*x))/35 - (b*c^3*d^4*x^3*atan
(c*x)*1i)/3 - (13*b*c^4*d^4*x^4*atan(c*x))/15 - b*c^5*d^4*x^5*atan(c*x)*1i
+ (b*c^6*d^4*x^6*atan(c*x))/3 + (b*c*d^4*x*atan(c*x)*2i)/3)/(x^7 + c^2*x^9)
 - (176*b*c^7*d^4*log(x))/105 - (b*c^10*d^4*atan((c^2*x)/(c^2)^(1/2))*5i)/(
3*(c^2)^(3/2))
```

3.43 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))}{d+icdx}\,dx$

Optimal. Leaf size=196

$$\frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}+\frac{b\text{ArcTan}(cx)}{2c^4d}+\frac{ibx\text{ArcTan}(cx)}{c^3d}+\frac{x^2(a+b\text{ArcTan}(cx))}{2c^2d}-\frac{ix^3(a+b\text{ArcTan}(cx))}{3cd}+\frac{(a+b\text{Arc}}{}$$

```
[Out] I*a*x/c^3/d-1/2*b*x/c^3/d+1/6*I*b*x^2/c^2/d+1/2*b*arctan(c*x)/c^4/d+I*b*x*a
rctan(c*x)/c^3/d+1/2*x^2*(a+b*arctan(c*x))/c^2/d-1/3*I*x^3*(a+b*arctan(c*x)
)/c/d+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d-2/3*I*b*ln(c^2*x^2+1)/c^4/d+1
/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d
```

Rubi [A]
time = 0.22, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4986, 4946, 272, 45, 327, 209, 4930, 266, 4964, 2449, 2352}

$$\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4d}+\frac{x^2(a+b\text{ArcTan}(cx))}{2c^2d}-\frac{ix^3(a+b\text{ArcTan}(cx))}{3cd}+\frac{iax}{c^3d}+\frac{b\text{ArcTan}(cx)}{2c^4d}+\frac{ibx\text{ArcTan}(cx)}{c^3d}+\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^4d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}-\frac{2ib\log\left(c^2x^2+1\right)}{3c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] (I*a*x)/(c^3*d) - (b*x)/(2*c^3*d) + ((I/6)*b*x^2)/(c^2*d) + (b*ArcTan[c*x])
/(2*c^4*d) + (I*b*x*ArcTan[c*x])/(c^3*d) + (x^2*(a + b*ArcTan[c*x]))/(2*c^2
*d) - ((I/3)*x^3*(a + b*ArcTan[c*x]))/(c*d) + ((a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/(c^4*d) - (((2*I)/3)*b*Log[1 + c^2*x^2])/(c^4*d) + ((I/2)*b*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{d+icdx}\,dx = \frac{i\int \frac{x^2(a+b\tan^{-1}(cx))}{d+icdx}\,dx}{c} - \frac{i\int x^2(a+b\tan^{-1}(cx))\,dx}{cd}$$

$$= -\frac{ix^3(a+b\tan^{-1}(cx))}{3cd} - \frac{\int \frac{x(a+b\tan^{-1}(cx))}{d+icdx}\,dx}{c^2} + \frac{(ib)\int \frac{x^3}{1+c^2x^2}\,dx}{3d} + \frac{\int x(a+b\tan}{c^2d}$$

$$= \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d} - \frac{ix^3(a+b\tan^{-1}(cx))}{3cd} - \frac{i\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c^3} + \frac{(ib)\mathrm{Subst}(\int}{}$$

$$= \frac{iax}{c^3d} - \frac{bx}{2c^3d} + \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d} - \frac{ix^3(a+b\tan^{-1}(cx))}{3cd} + \frac{(a+b\tan^{-1}(cx))\,\mathrm{l}}{c^4d}$$

$$= \frac{iax}{c^3d} - \frac{bx}{2c^3d} + \frac{ibx^2}{6c^2d} + \frac{b\tan^{-1}(cx)}{2c^4d} + \frac{ibx\tan^{-1}(cx)}{c^3d} + \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d} - \frac{ix^3}{}$$

$$= \frac{iax}{c^3d} - \frac{bx}{2c^3d} + \frac{ibx^2}{6c^2d} + \frac{b\tan^{-1}(cx)}{2c^4d} + \frac{ibx\tan^{-1}(cx)}{c^3d} + \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d} - \frac{ix^3}{}$$

Mathematica [A]
time = 0.30, size = 166, normalized size = 0.85

$$-\frac{i\left(-b-6acx-3ibcx+3iac^2x^2-bc^2x^2+2ac^3x^3+6b\mathrm{ArcTan}(cx)^2+\mathrm{ArcTan}(cx)\left(6a+b(3i-6cx+3ic^2x^2+2c^3x^3)+6ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-3ia\log\left(1+c^2x^2\right)+4b\log\left(1+c^2x^2\right)+3b\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{6c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-1/6*I)*(-b - 6*a*c*x - (3*I)*b*c*x + (3*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*c
^3*x^3 + 6*b*ArcTan[c*x]^2 + ArcTan[c*x]*(6*a + b*(3*I - 6*c*x + (3*I)*c^2*
x^2 + 2*c^3*x^3) + (6*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (3*I)*a*Log[1
+ c^2*x^2] + 4*b*Log[1 + c^2*x^2] + 3*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])])
)/(c^4*d)
```

Maple [A]
time = 0.16, size = 312, normalized size = 1.59

method	result
derivativedivides	$-\frac{11ib\ln(c^2x^2+1)}{24d}-\frac{ia\arctan(cx)}{d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ib\ln(cx-i)^2}{4d}-\frac{5ib\ln(c^4x^4+10c^2x^2+9)}{48d}+\frac{iacx}{d}+\frac{b\arctan(cx)c^2x^2}{2d}-$
default	$-\frac{11ib\ln(c^2x^2+1)}{24d}-\frac{ia\arctan(cx)}{d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ib\ln(cx-i)^2}{4d}-\frac{5ib\ln(c^4x^4+10c^2x^2+9)}{48d}+\frac{iacx}{d}+\frac{b\arctan(cx)c^2x^2}{2d}-$
risch	$\frac{ib\ln(icx+1)^2}{4c^4d}+\frac{b\arctan(cx)}{2c^4d}-\frac{b\left(\frac{1}{3}c^2x^3+\frac{1}{2}ic\,x^2-x\right)\ln(icx+1)}{2c^3d}+\frac{\ln(-icx+1)x^3b}{6dc}-\frac{ia\arctan(cx)}{d\,c^4}-\frac{\ln(-icx+1}{2d\,c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*I*b/d*dilog(-1/2*I*(c*x+I))-11/24*I*b/d*ln(c^2*x^2+1)+1/2*a/d*c^
2*x^2-1/2*a/d*ln(c^2*x^2+1)-I*a/d*arctan(c*x)-1/4*I*b/d*ln(c*x-I)^2-5/48*I*
b/d*ln(c^4*x^4+10*c^2*x^2+9)+1/2*b/d*arctan(c*x)*c^2*x^2-b/d*arctan(c*x)*ln
(c*x-I)+I*a/d*c*x+1/6*I*b/d*c^2*x^2+2/3*I*b/d-1/2*b/d*c*x-1/3*I*b/d*arctan(
c*x)*c^3*x^3+1/2*I*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I*b/d*arctan(c*x)*c*x+1
1/12*b/d*arctan(c*x)-1/3*I*a/d*c^3*x^3+5/24*b/d*arctan(1/2*c*x)-5/24*b/d*ar
ctan(1/6*c^3*x^3+7/6*c*x)-5/12*b/d*arctan(1/2*c*x-1/2*I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*a*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*d))
 - 1/72*(432*I*c^8*d*integrate(1/12*x^4*arctan(c*x)/(c^5*d*x^2 + c^3*d), x)
 + 216*c^8*d*integrate(1/12*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) -
432*c^7*d*integrate(1/12*x^3*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) + 216*I*c^
7*d*integrate(1/12*x^3*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 432*c^5*d
*integrate(1/12*x*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) - 216*I*c^5*d*integra
te(1/12*x*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 4*c^3*x^3 - 216*c^4*d*
integrate(1/12*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 3*I*c^2*x^2 - 30*
c*x - 6*(-2*I*c^3*x^3 + 3*c^2*x^2 + 6*I*c*x - 5)*arctan(c*x) + 18*I*arctan(
c*x)^2 - 3*(2*c^3*x^3 + 3*I*c^2*x^2 - 6*c*x + I)*log(c^2*x^2 + 1) + 9*I*log
(c^2*x^2 + 1)^2 + 18*I*log(12*c^5*d*x^2 + 12*c^3*d))*b/(c^4*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c*d*x - I*d), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{6ib\log{(icx+1)}}{c^2x^2+1}\,dx + \int \frac{12ac^4x^4}{c^2x^2+1}\,dx + \int \frac{6bcx}{c^2x^2+1}\,dx + \int \frac{bc^3x^3}{c^2x^2+1}\,dx + \int \frac{12iac^3x^3}{c^2x^2+1}\,dx + \int \frac{3ibc^2x^2}{c^2x^2+1}\,dx + \int \left(-\frac{2ibc^4x^4}{c^2x^2+1}\right)dx + \int \left(-\frac{6bcx\log{(icx+1)}}{c^2x^2+1}\right)dx + \int \frac{6bc^3x^3\log{(icx+1)}}{c^2x^2+1}\,dx + \int \left(-\frac{6ibc^4x^4\log{(icx+1)}}{c^2x^2+1}\right)dx\right)}{12c^3d} + \frac{(2bc^3x^3 + 3ibc^2x^2 - 6bcx - 6ib\log{(icx+1)})\log{(-icx+1)}}{12c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(6*I*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(12*a*c**4*
x**4/(c**2*x**2 + 1), x) + Integral(6*b*c*x/(c**2*x**2 + 1), x) + Integral(
b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(12*I*a*c**3*x**3/(c**2*x**2 + 1)
, x) + Integral(3*I*b*c**2*x**2/(c**2*x**2 + 1), x) + Integral(-2*I*b*c**4*
x**4/(c**2*x**2 + 1), x) + Integral(-6*b*c*x*log(I*c*x + 1)/(c**2*x**2 + 1)
, x) + Integral(6*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral
(-6*I*b*c**4*x**4*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(12*c**3*d) + (2*b*c*
*3*x**3 + 3*I*b*c**2*x**2 - 6*b*c*x - 6*I*b*log(I*c*x + 1))*log(-I*c*x + 1)
/(12*c**4*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.44 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{d+icdx}\,dx$

Optimal. Leaf size=156

$$\frac{ax}{c^2d}+\frac{ibx}{2c^2d}-\frac{ib\mathrm{ArcTan}(cx)}{2c^3d}+\frac{bx\mathrm{ArcTan}(cx)}{c^2d}-\frac{ix^2(a+b\mathrm{ArcTan}(cx))}{2cd}-\frac{i(a+b\mathrm{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d}-\frac{b\log}{}$$

```
[Out] a*x/c^2/d+1/2*I*b*x/c^2/d-1/2*I*b*arctan(c*x)/c^3/d+b*x*arctan(c*x)/c^2/d-1
/2*I*x^2*(a+b*arctan(c*x))/c/d-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d-1/
2*b*ln(c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(1+I*c*x))/c^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4986, 4946, 327, 209, 4930, 266, 4964, 2449, 2352}

$$-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\mathrm{ArcTan}(cx))}{c^3d}-\frac{ix^2(a+b\mathrm{ArcTan}(cx))}{2cd}+\frac{ax}{c^2d}-\frac{ib\mathrm{ArcTan}(cx)}{2c^3d}+\frac{bx\mathrm{ArcTan}(cx)}{c^2d}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^3d}+\frac{ibx}{2c^2d}-\frac{b\log\left(c^2x^2+1\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] (a*x)/(c^2*d) + ((I/2)*b*x)/(c^2*d) - ((I/2)*b*ArcTan[c*x])/(c^3*d) + (b*x*
ArcTan[c*x])/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c*d) - (I*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) - (b*Log[1 + c^2*x^2])/(2*c^3*d) + (
b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{d+icdx}\,dx = \frac{i\int \frac{x(a+b\tan^{-1}(cx))}{d+icdx}\,dx}{c} - \frac{i\int x(a+b\tan^{-1}(cx))\,dx}{cd}$$
$$= -\frac{ix^2(a+b\tan^{-1}(cx))}{2cd} - \frac{\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c^2} + \frac{(ib)\int \frac{x^2}{1+c^2x^2}\,dx}{2d} + \frac{\int (a+b\tan^{-1}}{c^2d}$$
$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))}{2cd} - \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{(ib)\int \frac{}{1}}{2c}$$
$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ib\tan^{-1}(cx)}{2c^3d} + \frac{bx\tan^{-1}(cx)}{c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))}{2cd} - \frac{i(a+b\,\mathrm{t}}{}$$
$$= \frac{ax}{c^2d} + \frac{ibx}{2c^2d} - \frac{ib\tan^{-1}(cx)}{2c^3d} + \frac{bx\tan^{-1}(cx)}{c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))}{2cd} - \frac{i(a+b\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.12, size = 132, normalized size = 0.85

$$-\frac{-2acx - ibcx + iac^2x^2 + 2b\text{ArcTan}(cx)^2 + i\text{ArcTan}(cx)\left(-2ia + b + 2ibcx + bc^2x^2 + 2b\log\left(1 + e^{2i\text{ArcTan}(cx)}\right)\right) - ia\log\left(1 + c^2x^2\right) + b\log\left(1 + c^2x^2\right) + b\text{PolyLog}\left(2, -e^{2i\text{ArcTan}(cx)}\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] -1/2*(-2*a*c*x - I*b*c*x + I*a*c^2*x^2 + 2*b*ArcTan[c*x]^2 + I*ArcTan[c*x]*
((-2*I)*a + b + (2*I)*b*c*x + b*c^2*x^2 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x])
]) - I*a*Log[1 + c^2*x^2] + b*Log[1 + c^2*x^2] + b*PolyLog[2, -E^((2*I)*Arc
Tan[c*x])])/(c^3*d)
```

Maple [A]
time = 0.10, size = 267, normalized size = 1.71

method	result
derivativedivides	$\frac{\frac{acx}{d} + \frac{ib\arctan(cx)\ln(cx-i)}{d} + \frac{ibcx}{2d} - \frac{a\arctan(cx)}{d} + \frac{b\arctan(cx)cx}{d} + \frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4d} + \frac{ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{8d} - \frac{b\ln(cx-i)^2}{4d} + \frac{b}{}}{}$
default	$\frac{\frac{acx}{d} + \frac{ib\arctan(cx)\ln(cx-i)}{d} + \frac{ibcx}{2d} - \frac{a\arctan(cx)}{d} + \frac{b\arctan(cx)cx}{d} + \frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4d} + \frac{ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{8d} - \frac{b\ln(cx-i)^2}{4d} + \frac{b}{}}{}$
risch	$\frac{b\ln(icx+1)^2}{4c^3d} - \frac{b\left(\frac{1}{2}cx^2+ix\right)\ln(icx+1)}{2c^2d} - \frac{ix^2a}{2dc} + \frac{ax}{c^2d} + \frac{ia}{2d\,c^3} - \frac{ib\arctan(cx)}{2c^3d} - \frac{a\arctan(cx)}{d\,c^3} + \frac{\ln(-icx+1)}{4dc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/d*c*x+I*b/d*arctan(c*x)*ln(c*x-I)+1/2*I*b/d*c*x-a/d*arctan(c*x)+b/
d*arctan(c*x)*c*x+1/4*I*b/d*arctan(1/2*c*x-1/2*I)+1/8*I*b/d*arctan(1/6*c^3*
```

```
x^3+7/6*c*x)-1/4*b/d*ln(c*x-I)^2+1/2*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*b
/d*dilog(-1/2*I*(c*x+I))+1/2*b/d-1/8*I*b/d*arctan(1/2*c*x)-3/8*b/d*ln(c^2*x
^2+1)+1/2*I*a/d*ln(c^2*x^2+1)-1/16*b/d*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I*b/d*a
rctan(c*x)*c^2*x^2-1/2*I*a/d*c^2*x^2-3/4*I*b/d*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*a*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/8*(32*I*c
^6*d*integrate(1/8*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*c^6*d*integ
rate(1/8*x^3*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*c^5*d*integrate(
1/8*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^5*d*integrate(1/8*x^2*
log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*I*c^4*d*integrate(1/8*x*arcta
n(c*x)/(c^4*d*x^2 + c^2*d), x) - 16*c^4*d*integrate(1/8*x*log(c^2*x^2 + 1)/
(c^4*d*x^2 + c^2*d), x) + 16*I*c^3*d*integrate(1/8*log(c^2*x^2 + 1)/(c^4*d*
x^2 + c^2*d), x) + c^2*x^2 + 2*I*c*x - 2*(-I*c^2*x^2 + 2*c*x + I)*arctan(c*
x) + 2*arctan(c*x)^2 - (c^2*x^2 + 2*I*c*x + 1)*log(c^2*x^2 + 1) + log(c^2*x
^2 + 1)^2 + 2*log(8*c^4*d*x^2 + 8*c^2*d))*b/(c^3*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c*d*x - I*d), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{2b\log\left(icx+1\right)}{c^2x^2+1}\,dx + \int \frac{4ac^3x^3}{c^2x^2+1}\,dx + \int \frac{bc^2x^2}{c^2x^2+1}\,dx + \int \frac{4iac^2x^2}{c^2x^2+1}\,dx + \int \left(-\frac{2ibcx}{c^2x^2+1}\right)dx + \int \left(-\frac{ibc^3x^3}{c^2x^2+1}\right)dx + \int \frac{2bc^2x^2\log\left(icx+1\right)}{c^2x^2+1}\,dx + \int \frac{2ibcx\log\left(icx+1\right)}{c^2x^2+1}\,dx + \int \left(-\frac{2ibc^3x^3\log\left(icx+1\right)}{c^2x^2+1}\right)dx\right)}{4c^2d} + \frac{\left(bc^2x^2 + 2ibcx - 2b\log\left(icx+1\right)\right)\log\left(-icx+1\right)}{4c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(2*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(4*a*c**3*x**
3/(c**2*x**2 + 1), x) + Integral(b*c**2*x**2/(c**2*x**2 + 1), x) + Integral
(4*I*a*c**2*x**2/(c**2*x**2 + 1), x) + Integral(-2*I*b*c*x/(c**2*x**2 + 1),
```

```
x) + Integral(-I*b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(2*b*c**2*x**2*
log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*I*b*c*x*log(I*c*x + 1)/(c**
2*x**2 + 1), x) + Integral(-2*I*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2 + 1),
 x))/(4*c**2*d) + (b*c**2*x**2 + 2*I*b*c*x - 2*b*log(I*c*x + 1))*log(-I*c*x
 + 1)/(4*c**3*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
[Out] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.45 $\int \frac{x(a+b\mathbf{ArcTan}(cx))}{d+icdx} dx$

Optimal. Leaf size=110

$$-\frac{iax}{cd}-\frac{ibx\text{ArcTan}(cx)}{cd}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{ib\log\left(1+c^2x^2\right)}{2c^2d}-\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}$$

```
[Out] -I*a*x/c/d-I*b*x*arctan(c*x)/c/d-(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d+1/
2*I*b*ln(c^2*x^2+1)/c^2/d-1/2*I*b*polylog(2,1-2/(1+I*c*x))/c^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4986, 4930, 266, 4964, 2449, 2352}

$$-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^2d}-\frac{iax}{cd}-\frac{ibx\text{ArcTan}(cx)}{cd}-\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^2d}+\frac{ib\log\left(c^2x^2+1\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-I)*a*x)/(c*d) - (I*b*x*ArcTan[c*x])/(c*d) - ((a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/(c^2*d) + ((I/2)*b*Log[1 + c^2*x^2])/(c^2*d) - ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/(c^2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+b\tan^{-1}(cx))}{d+icdx}\,dx &= \frac{i\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx}{c} - \frac{i\int (a+b\tan^{-1}(cx))\,dx}{cd}\\
&= -\frac{iax}{cd} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{(ib)\int \tan^{-1}(cx)\,dx}{cd} + \frac{b\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{cd}\\
&= -\frac{iax}{cd} - \frac{ibx\tan^{-1}(cx)}{cd} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{(ib)\int \frac{x}{1+c^2x^2}\,dx}{d} - \frac{(ib)}{}\\
&= -\frac{iax}{cd} - \frac{ibx\tan^{-1}(cx)}{cd} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{ib\log(1+c^2x^2)}{2c^2d} - \frac{ib\mathrm{L}}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 108, normalized size = 0.98

$$\frac{-2iacx + 2ib\mathrm{ArcTan}(cx)^2 + 2i\mathrm{ArcTan}(cx)\left(a - bcx + ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right) + a\log(1+c^2x^2) + ib\log(1+c^2x^2) + ib\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

```
[Out] ((-2*I)*a*c*x + (2*I)*b*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]*(a - b*c*x + I*b*
Log[1 + E^((2*I)*ArcTan[c*x])]) + a*Log[1 + c^2*x^2] + I*b*Log[1 + c^2*x^2]
 + I*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(100) = 200$.

time = 0.09, size = 214, normalized size = 1.95

method	result
risch	$-\frac{ib\ln(icx+1)^2}{4c^2d} - \frac{xb\ln(icx+1)}{2cd} + \frac{ib\ln(c^2x^2+1)}{2c^2d} + \frac{\ln(-icx+1)xb}{2dc} - \frac{ib\ln(\frac{1}{2}+\frac{icx}{2})\ln(\frac{1}{2}-\frac{icx}{2})}{2d\,c^2} + \frac{ib\ln(\frac{1}{2}+\frac{icx}{2})\ln}{2d\,c^2}$
derivativedivides	$\frac{-\frac{iacx}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{ib\ln(cx-i)^2}{4d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{ib\,\mathrm{dilog}}{}}{c^2}$
default	$\frac{-\frac{iacx}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{ib\ln(cx-i)^2}{4d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{ib\,\mathrm{dilog}}{}}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-I*a/d*c*x+1/2*a/d*ln(c^2*x^2+1)+I*a/d*arctan(c*x)-I*b/d*arctan(c*x)
*c*x+b/d*arctan(c*x)*ln(c*x-I)+1/4*I*b/d*ln(c*x-I)^2-1/2*I*b/d*ln(c*x-I)*ln
(-1/2*I*(c*x+I))-1/2*I*b/d*dilog(-1/2*I*(c*x+I))+1/8*I*b/d*ln(c^8*x^8+12*c^
6*x^6+30*c^4*x^4+28*c^2*x^2+9)-1/4*b/d*arctan(1/12*c^3*x^3+13/12*c*x)-1/4*b
/d*arctan(1/4*c*x)+1/2*b/d*arctan(1/2*c*x-1/2*I))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) - 1/8*(8*I*c^4*d*integrate(1/2*x^2*
arctan(c*x)/(c^3*d*x^2 + c*d), x) + 4*c^4*d*integrate(1/2*x^2*log(c^2*x^2 +
 1)/(c^3*d*x^2 + c*d), x) - 16*c^3*d*integrate(1/2*x*arctan(c*x)/(c^3*d*x^2
 + c*d), x) + 8*I*c^3*d*integrate(1/2*x*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d),
 x) + 4*c^2*d*integrate(1/2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 2*c*x*
log(c^2*x^2 + 1) + 4*c*x - 4*(-I*c*x + 1)*arctan(c*x) - 2*I*arctan(c*x)^2 -
 I*log(c^2*x^2 + 1)^2 - 2*I*log(2*c^3*d*x^2 + 2*c*d))*b/(c^2*d)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*x*log(-(c*x + I)/(c*x - I)) - 2*I*a*x)/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\left(-\frac{ib\log\left(icx+1\right)}{c^2x^2+1}\right)dx+\int\frac{2ac^2x^2}{c^2x^2+1}\,dx+\int\left(-\frac{bcx}{c^2x^2+1}\right)dx+\int\frac{2iacx}{c^2x^2+1}\,dx+\int\left(-\frac{ibc^2x^2}{c^2x^2+1}\right)dx+\int\frac{2bcx\log\left(icx+1\right)}{c^2x^2+1}\,dx+\int\left(-\frac{ibc^2x^2\log\left(icx+1\right)}{c^2x^2+1}\right)dx\right)}{2cd}+\frac{\left(bcx+ib\log\left(icx+1\right)\right)\log\left(-icx+1\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(-I*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*a*c**2*x*
*2/(c**2*x**2 + 1), x) + Integral(-b*c*x/(c**2*x**2 + 1), x) + Integral(2*I
*a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c**2*x**2/(c**2*x**2 + 1), x) +
Integral(2*b*c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-I*b*c**2*x*
*2*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(2*c*d) + (b*c*x + I*b*log(I*c*x + 1
))*log(-I*c*x + 1)/(2*c**2*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

```
[Out] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.46 $\int \frac{a+b\mathbf{ArcTan}(cx)}{d+icdx}\,dx$

Optimal. Leaf size=59

$$\frac{i(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}$$

[Out] I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/d-1/2*b*polylog(2,1-2/(1+I*c*x))/c/d

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4964, 2449, 2352}

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{cd}-\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2cd}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]

[Out] (I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*d) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d)

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+icdx}\,dx = \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(ib)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$
$$= \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{cd}$$
$$= \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.02

$$\frac{2i(a+b\text{ArcTan}(cx))\log\left(\frac{2d}{d+icdx}\right) - b\text{PolyLog}\left(2, \frac{i+cx}{-i+cx}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

```
[Out] ((2*I)*(a + b*ArcTan[c*x])*Log[(2*d)/(d + I*c*d*x)] - b*PolyLog[2, (I + c*x
)/(-I + c*x)])/(2*c*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(54) = 108$.
time = 0.06, size = 125, normalized size = 2.12

method	result
risch	$-\frac{b\ln(icx+1)^2}{4cd} - \frac{ia\ln(c^2x^2+1)}{2cd} + \frac{\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)b}{2cd} - \frac{\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)b}{2cd} + \frac{a\arctan(cx)}{cd} - \mathrm{dilog}($
derivativedivides	$\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d} + \frac{a\arctan(cx)}{d} - \frac{ib\ln(icx+1)\arctan(cx)}{d} + \frac{b\ln(icx+1)^2}{4d} + \frac{b\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)}{2d} - \frac{b\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln(icx+1)}{2d} + \cdots}{c}$
default	$\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d} + \frac{a\arctan(cx)}{d} - \frac{ib\ln(icx+1)\arctan(cx)}{d} + \frac{b\ln(icx+1)^2}{4d} + \frac{b\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)}{2d} - \frac{b\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln(icx+1)}{2d} + \cdots}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*I*a/d*ln(c^2*x^2+1)+a/d*arctan(c*x)-I*b/d*ln(1+I*c*x)*arctan(c*x)
+1/4*b/d*ln(1+I*c*x)^2+1/2*b/d*ln(1/2-1/2*I*c*x)*ln(1/2+1/2*I*c*x)-1/2*b/d*
ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+1/2*b/d*dilog(1/2+1/2*I*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/8*(8*I*c^2*d*integrate(x*arctan(c*x)/(c^2*d*x^2 + d), x) + 4*c^2*d*integ
rate(x*log(c^2*x^2 + 1)/(c^2*d*x^2 + d), x) - 4*arctan(c*x)^2 - log(c^2*x^2
 + 1)^2)*b/(c*d) - I*a*log(I*c*d*x + d)/(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x - I*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\log\left(-icx+1\right)\log\left(icx+1\right)}{2cd}-\frac{i\left(\int\frac{ia}{c^2x^2+1}\,dx+\int\frac{acx}{c^2x^2+1}\,dx+\int\left(-\frac{ibcx\log\left(icx+1\right)}{c^2x^2+1}\right)\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x),x)
```

```
[Out] b*log(-I*c*x + 1)*log(I*c*x + 1)/(2*c*d) - I*(Integral(I*a/(c**2*x**2 + 1),
 x) + Integral(a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c*x*log(I*c*x + 1)
/(c**2*x**2 + 1), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{atan}(c\,x)}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))/(d + c*d*x*1i), x)
```

3.47 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+icdx)}\,dx$

Optimal. Leaf size=54

$$\frac{(a+b\text{ArcTan}(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

```
[Out] (a+b*arctan(c*x))*ln(2-2/(1+I*c*x))/d+1/2*I*b*polylog(2,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4988, 2497}

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + ((I/2)*b*PolyLog[2, -1 + 2
/(1 + I*c*x)])/d
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx&=\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d}-\frac{(bc)\int\frac{\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}\\&=\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 83, normalized size = 1.54

$$-\frac{i\left(2b\mathrm{ArcTan}(cx)^2+2\mathrm{ArcTan}(cx)\left(a+ib\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)\right)+ia(2\log(x)-\log\left(1+c^2x^2\right))+b\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

```
[Out] ((-1/2*I)*(2*b*ArcTan[c*x]^2 + 2*ArcTan[c*x]*(a + I*b*Log[1 - E^((2*I)*ArcT
an[c*x])]) + I*a*(2*Log[x] - Log[1 + c^2*x^2]) + b*PolyLog[2, E^((2*I)*ArcT
an[c*x])]))/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(49) = 98.
time = 0.09, size = 193, normalized size = 3.57

method	result
risch	$\frac{ib\,\mathrm{dilog}(icx+1)}{2d}+\frac{ib\ln(icx+1)^2}{4d}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d}+\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d}+\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d}-\frac{ib}{}$
derivativedivides	$-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{a\ln(cx)}{d}-\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{b\ln(cx)\arctan(cx)}{d}+\frac{ib\ln(cx)\ln(icx+1)}{2d}$
default	$-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{a\ln(cx)}{d}-\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{b\ln(cx)\arctan(cx)}{d}+\frac{ib\ln(cx)\ln(icx+1)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*ln(c^2*x^2+1)-I*a/d*arctan(c*x)+a/d*ln(c*x)-b/d*arctan(c*x)*ln(c*x
-I)+b/d*ln(c*x)*arctan(c*x)+1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d*ln(c*x)
*ln(1-I*c*x)+1/2*I*b/d*dilog(1+I*c*x)-1/2*I*b/d*dilog(1-I*c*x)-1/4*I*b/d*ln
(c*x-I)^2+1/2*I*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*I*b/d*dilog(-1/2*I*(c*
x+I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*(I*arctan(c*x)^2/d - 2*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x))
- a*(log(I*c*x + 1)/d - log(x)/d)
```

Fricas [A]
time = 1.11, size = 43, normalized size = 0.80

$$\frac{-i\,b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,a\log(x)-2\,a\log\left(\frac{cx-i}{c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*dilog((c*x + I)/(c*x - I) + 1) + 2*a*log(x) - 2*a*log((c*x - I)/c
))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a}{cx^2-ix}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{cx^2-ix}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a/(c*x**2 - I*x), x) + Integral(b*atan(c*x)/(c*x**2 - I*x), x)
)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)), x)
```

3.48 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^2(d+icdx)}\,dx$

Optimal. Leaf size=100

$$-\frac{a+b\text{ArcTan}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log\left(1+c^2x^2\right)}{2d}-\frac{ic(a+b\text{ArcTan}(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{bc\text{PolyLog}(2,-1+}{2d}$$

```
[Out] (-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(c^2*x^2+1)/d-I*c*(a+b*arctan(
c*x))*ln(2-2/(1+I*c*x))/d+1/2*b*c*polylog(2,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4990, 4946, 272, 36, 29, 31, 4988, 2497}

$$-\frac{a+b\text{ArcTan}(cx)}{dx}-\frac{ic\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{bc\log\left(c^2x^2+1\right)}{2d}+\frac{bc\text{Li}_2\left(\frac{2}{icx+1}-1\right)}{2d}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 + c^2*x^2])/(2*d
) - (I*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + (b*c*PolyLog[2, -1
 + 2/(1 + I*c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx\right)+\frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx}-\frac{ic(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{(bc)\int \frac{1}{x(1+c^2x^2)}\,dx}{d}+\frac{(ib$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx}-\frac{ic(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{bc\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d}+\frac{(b$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx}-\frac{ic(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{bc\text{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d}+\frac{(b$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log\left(1+c^2x^2\right)}{2d}-\frac{ic(a+b\tan^{-1}(cx))\log\left(2-\frac{}{1+}\right.}{d}$$

Mathematica [A]
time = 0.16, size = 126, normalized size = 1.26

$$\frac{-2a-2bcx\text{ArcTan}(cx)^2+\text{ArcTan}(cx)\left(-2(b+acx)-2ibcx\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)-2iacx\log(x)+2bcx\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+iacx\log\left(1+c^2x^2\right)-bcx\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

```
[Out] (-2*a - 2*b*c*x*ArcTan[c*x]^2 + ArcTan[c*x]*(-2*(b + a*c*x) - (2*I)*b*c*x*L
og[1 - E^((2*I)*ArcTan[c*x])]) - (2*I)*a*c*x*Log[x] + 2*b*c*x*Log[(c*x)/Sqr
t[1 + c^2*x^2]] + I*a*c*x*Log[1 + c^2*x^2] - b*c*x*PolyLog[2, E^((2*I)*ArcT
an[c*x])])/(2*d*x)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(95) = 190$.
time = 0.09, size = 246, normalized size = 2.46

method	result
risch	$\frac{cb\,\text{dilog}(icx+1)}{2d}+\frac{cb\ln(icx)}{2d}-\frac{bc\ln(c^2x^2+1)}{2d}+\frac{ib\ln(icx+1)}{2dx}-\frac{ica\ln(-icx)}{d}+\frac{cb\ln(icx+1)^2}{4d}+\frac{ica\ln(c^2x^2+1)}{2d}$
derivativedivides	$c\left(\frac{ib\arctan(cx)\ln(cx-i)}{d}-\frac{a\arctan(cx)}{d}-\frac{a}{dcx}-\frac{ia\ln(cx)}{d}-\frac{ib\arctan(cx)\ln(cx)}{d}-\frac{b\arctan(cx)}{dcx}+\frac{ia\ln(c^2x^2}{2d}\right.$
default	$c\left(\frac{ib\arctan(cx)\ln(cx-i)}{d}-\frac{a\arctan(cx)}{d}-\frac{a}{dcx}-\frac{ia\ln(cx)}{d}-\frac{ib\arctan(cx)\ln(cx)}{d}-\frac{b\arctan(cx)}{dcx}+\frac{ia\ln(c^2x^2}{2d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c*(I*b/d*arctan(c*x)*ln(c*x-I)-a/d*arctan(c*x)-a/d/c/x-I*a/d*ln(c*x)-I*b/d*
arctan(c*x)*ln(c*x)-b/d*arctan(c*x)/c/x+1/2*I*a/d*ln(c^2*x^2+1)+1/2*b/d*ln(
c*x)*ln(1+I*c*x)-1/2*b/d*ln(c*x)*ln(1-I*c*x)+1/2*b/d*dilog(1+I*c*x)-1/2*b/d
*dilog(1-I*c*x)-1/4*b/d*ln(c*x-I)^2+1/2*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/
2*b/d*dilog(-1/2*I*(c*x+I))-1/2*b/d*ln(c^2*x^2+1)+b/d*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] (-I*c*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x) + integrate(arctan(c*x)/(
c^2*d*x^4 + d*x^2), x))*b + a*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x
))
```

Fricas [A]
time = 1.21, size = 98, normalized size = 0.98

$$-\frac{bcx\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,(i\,a-b)cx\log\left(x\right)+bcx\log\left(\frac{cx+i}{c}\right)-(2i\,a-b)cx\log\left(\frac{cx-i}{c}\right)+i\,b\log\left(-\frac{cx+i}{cx-i}\right)+2\,a}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x*dilog((c*x + I)/(c*x - I) + 1) + 2*(I*a - b)*c*x*log(x) + b*c*x
*log((c*x + I)/c) - (2*I*a - b)*c*x*log((c*x - I)/c) + I*b*log(-(c*x + I)/(
c*x - I)) + 2*a)/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\frac{a}{cx^3-ix^2}\,dx+\int\frac{b\operatorname{atan}\left(cx\right)}{cx^3-ix^2}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a/(c*x**3 - I*x**2), x) + Integral(b*atan(c*x)/(c*x**3 - I*x**
2), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)), x)
```

3.49 $\int \frac{a+b\textbf{ArcTan}(cx)}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=161

$$-\frac{bc}{2dx}-\frac{bc^2\text{ArcTan}(cx)}{2d}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}+\frac{ic(a+b\text{ArcTan}(cx))}{dx}-\frac{ibc^2\log(x)}{d}+\frac{ibc^2\log\left(1+c^2x^2\right)}{2d}-\frac{c^2(a+b$$

```
[Out] -1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2+I*c*(a+b*
arctan(c*x))/d/x-I*b*c^2*ln(x)/d+1/2*I*b*c^2*ln(c^2*x^2+1)/d-c^2*(a+b*arcta
n(c*x))*ln(2-2/(1+I*c*x))/d-1/2*I*b*c^2*polylog(2,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4990, 4946, 331, 209, 272, 36, 29, 31, 4988, 2497}

$$-\frac{c^2\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}+\frac{ic(a+b\text{ArcTan}(cx))}{dx}-\frac{bc^2\text{ArcTan}(cx)}{2d}-\frac{ibc^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)}{2d}+\frac{ibc^2\log\left(c^2x^2+1\right)}{2d}-\frac{ibc^2\log(x)}{d}-\frac{bc}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

```
[Out] -1/2*(b*c)/(d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2
) + (I*c*(a + b*ArcTan[c*x]))/(d*x) - (I*b*c^2*Log[x])/d + ((I/2)*b*c^2*Log
[1 + c^2*x^2])/d - (c^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - ((I
/2)*b*c^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx\right)+\frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2}-c^2\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx-\frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d}+\frac{(bc)\int \frac{1}{x^2(1+c^2}}{2d}$$

$$= -\frac{bc}{2dx}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic(a+b\tan^{-1}(cx))}{dx}-\frac{c^2(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1+icx}\right.}{d}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic(a+b\tan^{-1}(cx))}{dx}-\frac{c^2(a+b\tan^{-1}(cx}{}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic(a+b\tan^{-1}(cx))}{dx}-\frac{c^2(a+b\tan^{-1}(cx}{}$$

$$= -\frac{bc}{2dx}-\frac{bc^2\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{2dx^2}+\frac{ic(a+b\tan^{-1}(cx))}{dx}-\frac{ibc^2\log(x)}{d}+\frac{ibc}{}$$

Mathematica [A]
time = 0.28, size = 178, normalized size = 1.11

$$-\frac{a-2iacx+bcx-2ibc^2x^2\text{ArcTan}(cx)^2+\text{ArcTan}(cx)\left(b-2ibcx-2iac^2x^2+bc^2x^2+2bc^2x^2\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+2ac^2x^2\log(x)+2ibc^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-ac^2x^2\log\left(1+c^2x^2\right)-ibc^2x^2\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

```
[Out] -1/2*(a - (2*I)*a*c*x + b*c*x - (2*I)*b*c^2*x^2*ArcTan[c*x]^2 + ArcTan[c*x]
*(b - (2*I)*b*c*x - (2*I)*a*c^2*x^2 + b*c^2*x^2 + 2*b*c^2*x^2*Log[1 - E^((2
*I)*ArcTan[c*x])]) + 2*a*c^2*x^2*Log[x] + (2*I)*b*c^2*x^2*Log[(c*x)/Sqrt[1
+ c^2*x^2]] - a*c^2*x^2*Log[1 + c^2*x^2] - I*b*c^2*x^2*PolyLog[2, E^((2*I)*
ArcTan[c*x])])/(d*x^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(148) = 296.
time = 0.12, size = 306, normalized size = 1.90

method	result
derivativedivides	$c^2\left(\frac{a\ln(c^2x^2+1)}{2d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{a}{2d\,c^2x^2}-\frac{ib\ln(cx)}{d}-\frac{a\ln(cx)}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}-\frac{b\,\text{ar}}{2}\right.$
default	$c^2\left(\frac{a\ln(c^2x^2+1)}{2d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{a}{2d\,c^2x^2}-\frac{ib\ln(cx)}{d}-\frac{a\ln(cx)}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}-\frac{b\,\text{ar}}{2}\right.$
risch	$\frac{ic^2b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d}+\frac{ib\,c^2\ln(c^2x^2+1)}{2d}+\frac{ic^2a\arctan(cx)}{d}+\frac{bc\ln(icx+1)}{2dx}-\frac{bc}{2dx}-\frac{ib\,c^2\,\text{dilog}(icx+1)}{2d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/d*ln(c^2*x^2+1)-1/2*I*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*a/d/c
^2/x^2-I*b/d*ln(c*x)-a/d*ln(c*x)+b/d*arctan(c*x)*ln(c*x-I)-1/2*b/d*arctan(c
*x)/c^2/x^2+I*a/d/c/x-b/d*ln(c*x)*arctan(c*x)-1/2*I*b/d*dilog(-1/2*I*(c*x+I
))-1/2*b/d*arctan(c*x)+1/2*I*b/d*dilog(1-I*c*x)-1/2*b/d/c/x+1/2*I*b/d*ln(c^
2*x^2+1)+I*b/d*arctan(c*x)/c/x+1/4*I*b/d*ln(c*x-I)^2+1/2*I*b/d*ln(c*x)*ln(1
-I*c*x)-1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d*dilog(1+I*c*x)+I*a/d*arctan
(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a + (
-I*c*integrate(arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + integrate(arctan(c*x)/
(c^2*d*x^5 + d*x^3), x))*b
```

Fricas [A]
time = 1.39, size = 130, normalized size = 0.81

$$\frac{2i\,bc^2x^2\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(a+i\,b)c^2x^2\log\left(x\right)+i\,bc^2x^2\log\left(\frac{cx+i}{c}\right)+(4\,a+3i\,b)c^2x^2\log\left(\frac{cx-i}{c}\right)-2\,(-2i\,a+b)cx-(2\,bcx+i\,b)\log\left(-\frac{cx+i}{cx-i}\right)-2\,a}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b*c^2*x^2*dilog((c*x + I)/(c*x - I) + 1) - 4*(a + I*b)*c^2*x^2*log
(x) + I*b*c^2*x^2*log((c*x + I)/c) + (4*a + 3*I*b)*c^2*x^2*log((c*x - I)/c)
 - 2*(-2*I*a + b)*c*x - (2*b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) - 2*a)/(d
*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a}{cx^4-ix^3}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a/(c*x**4 - I*x**3), x) + Integral(b*atan(c*x)/(c*x**4 - I*x**
3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)), x)
```

3.50 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^4(d+icdx)}\,dx$

Optimal. Leaf size=197

$$-\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\text{ArcTan}(cx)}{2d}-\frac{a+b\text{ArcTan}(cx)}{3dx^3}+\frac{ic(a+b\text{ArcTan}(cx))}{2dx^2}+\frac{c^2(a+b\text{ArcTan}(cx))}{dx}-\frac{4bc^3\log(}{3d}$$

```
[Out] -1/6*b*c/d/x^2+1/2*I*b*c^2/d/x+1/2*I*b*c^3*arctan(c*x)/d+1/3*(-a-b*arctan(c
*x))/d/x^3+1/2*I*c*(a+b*arctan(c*x))/d/x^2+c^2*(a+b*arctan(c*x))/d/x-4/3*b*
c^3*ln(x)/d+2/3*b*c^3*ln(c^2*x^2+1)/d+I*c^3*(a+b*arctan(c*x))*ln(2-2/(1+I*c
*x))/d-1/2*b*c^3*polylog(2,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.25, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4990, 4946, 272, 46, 331, 209, 36, 29, 31, 4988, 2497}

$$\frac{ic^3\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{c^2(a+b\text{ArcTan}(cx))}{dx}-\frac{a+b\text{ArcTan}(cx)}{3dx^3}+\frac{ic(a+b\text{ArcTan}(cx))}{2dx^2}+\frac{ibc^3\text{ArcTan}(cx)}{2d}-\frac{bc^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)}{2d}-\frac{4bc^3\log(x)}{3d}+\frac{ibc^2}{2dx}+\frac{2bc^3\log\left(c^2x^2+1\right)}{3d}-\frac{bc}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -1/6*(b*c)/(d*x^2) + ((I/2)*b*c^2)/(d*x) + ((I/2)*b*c^3*ArcTan[c*x])/d - (a
 + b*ArcTan[c*x])/(3*d*x^3) + ((I/2)*c*(a + b*ArcTan[c*x]))/(d*x^2) + (c^2*
(a + b*ArcTan[c*x]))/(d*x) - (4*b*c^3*Log[x])/(3*d) + (2*b*c^3*Log[1 + c^2*
x^2])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d - (b*c^3
*PolyLog[2, -1 + 2/(1 + I*c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^4(d+icdx)}\,dx &= -\left((ic)\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)}\,dx\right)+\frac{\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx}{d}\\
&= -\frac{a+b\tan^{-1}(cx)}{3dx^3}-c^2\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)}\,dx-\frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d}+\frac{(bc)\int \frac{1}{x^3(1+}}{3d}\\
&= -\frac{a+b\tan^{-1}(cx)}{3dx^3}+\frac{ic(a+b\tan^{-1}(cx))}{2dx^2}+\left(ic^3\right)\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)}\,dx+\frac{(bc)\text{Subst}}{}\\
&= \frac{ibc^2}{2dx}-\frac{a+b\tan^{-1}(cx)}{3dx^3}+\frac{ic(a+b\tan^{-1}(cx))}{2dx^2}+\frac{c^2(a+b\tan^{-1}(cx))}{dx}+\frac{ic^3(a+b\,\text{ta}}{}\\
&= -\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{3dx^3}+\frac{ic(a+b\tan^{-1}(cx))}{2dx^2}+\frac{c^2(a+}{}\\
&= -\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{3dx^3}+\frac{ic(a+b\tan^{-1}(cx))}{2dx^2}+\frac{c^2(a+}{}\\
&= -\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\tan^{-1}(cx)}{2d}-\frac{a+b\tan^{-1}(cx)}{3dx^3}+\frac{ic(a+b\tan^{-1}(cx))}{2dx^2}+\frac{c^2(a+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.33, size = 220, normalized size = 1.12

$$
\frac{-2a+3iacx-bcx+6ac^2x^2+3ibc^2x^2-bc^3x^3+6bc^3x^3\text{ArcTan}(cx)^2+\text{ArcTan}(cx)\left(6ac^3x^3+b(-2+3icx+6c^2x^2+3ic^3x^3)+6ibc^3x^3\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+6iac^3x^3\log(x)-8bc^3x^3\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-3iac^3x^3\log\left(1+c^2x^2\right)+3bc^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)}{6dx^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

```
[Out] (-2*a + (3*I)*a*c*x - b*c*x + 6*a*c^2*x^2 + (3*I)*b*c^2*x^2 - b*c^3*x^3 + 6
*b*c^3*x^3*ArcTan[c*x]^2 + ArcTan[c*x]*(6*a*c^3*x^3 + b*(-2 + (3*I)*c*x + 6
*c^2*x^2 + (3*I)*c^3*x^3) + (6*I)*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])])
 + (6*I)*a*c^3*x^3*Log[x] - 8*b*c^3*x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (3*I
)*a*c^3*x^3*Log[1 + c^2*x^2] + 3*b*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])
])/(6*d*x^3)
```

Maple [A]
time = 0.12, size = 340, normalized size = 1.73

method	result
derivativedivides	$c^3\left(-\frac{ib\arctan(cx)\ln(cx-i)}{d}+\frac{a\arctan(cx)}{d}-\frac{a}{3d\,c^3x^3}+\frac{ib}{2dcx}+\frac{ib\arctan(cx)\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}\right.$
default	$c^3\left(-\frac{ib\arctan(cx)\ln(cx-i)}{d}+\frac{a\arctan(cx)}{d}-\frac{a}{3d\,c^3x^3}+\frac{ib}{2dcx}+\frac{ib\arctan(cx)\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}\right.$
risch	$\frac{ica}{2d\,x^2}+\frac{bc\ln(icx+1)}{4d\,x^2}+\frac{c^2a}{dx}+\frac{ib\,c^3\arctan(cx)}{2d}-\frac{cb\ln(-icx+1)}{4d\,x^2}-\frac{c^3b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d}+\frac{c^3b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-I*b/d*arctan(c*x)*ln(c*x-I)+a/d*arctan(c*x)-1/3*a/d/c^3/x^3+1/2*I*b/d
/c/x+I*b/d*arctan(c*x)*ln(c*x)+a/d/c/x-1/2*I*a/d*ln(c^2*x^2+1)-1/3*b/d*arct
an(c*x)/c^3/x^3+I*a/d*ln(c*x)+1/2*I*a/d/c^2/x^2+b/d*arctan(c*x)/c/x+1/4*b/d
*ln(c*x-I)^2-1/2*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*b/d*dilog(-1/2*I*(c*x
+I))-1/2*b/d*ln(c*x)*ln(1+I*c*x)+1/2*b/d*ln(c*x)*ln(1-I*c*x)-1/2*b/d*dilog(
1+I*c*x)+1/2*b/d*dilog(1-I*c*x)+2/3*b/d*ln(c^2*x^2+1)+1/2*I*b/d*arctan(c*x)
/c^2/x^2+1/2*I*b/d*arctan(c*x)-1/6*b/d/c^2/x^2-4/3*b/d*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
2)/(d*x^3))*a + (-I*c*integrate(arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + integ
rate(arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*b
```

Fricas [A]
time = 1.11, size = 155, normalized size = 0.79

$$\frac{6\,bc^3x^3\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(-3i\,a+4\,b)c^3x^3\log\left(x\right)+5\,bc^3x^3\log\left(\frac{cx+i}{c}\right)+(-12i\,a+11\,b)c^3x^3\log\left(\frac{cx-i}{c}\right)+6\,(2\,a+i\,b)c^2x^2-2\,(-3i\,a+b)cx+(6i\,bc^2x^2-3\,bcx-2i\,b)\log\left(-\frac{cx+i}{cx-i}\right)-4\,a}{12\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*c^3*x^3*dilog((c*x + I)/(c*x - I) + 1) - 4*(-3*I*a + 4*b)*c^3*x^3
*log(x) + 5*b*c^3*x^3*log((c*x + I)/c) + (-12*I*a + 11*b)*c^3*x^3*log((c*x
```

```
- I)/c) + 6*(2*a + I*b)*c^2*x^2 - 2*(-3*I*a + b)*c*x + (6*I*b*c^2*x^2 - 3*b
*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) - 4*a)/(d*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(d+I*c*d*x),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)),x)
[Out] int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)), x)
```

3.51 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))}{(d+icdx)^2} dx$

Optimal. Leaf size=203

$$-\frac{2iax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(i-cx)}-\frac{b\text{ArcTan}(cx)}{c^4d^2}-\frac{2ibx\text{ArcTan}(cx)}{c^3d^2}-\frac{x^2(a+b\text{ArcTan}(cx))}{2c^2d^2}+\frac{i(a+b\text{ArcTan}(cx))}{c^4d^2(i-cx)}$$

```
[Out] -2*I*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(I-c*x)-b*arctan(c*x)/c^4/d^
2-2*I*b*x*arctan(c*x)/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))/c^2/d^2+I*(a+b*arct
an(c*x))/c^4/d^2/(I-c*x)-3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^2+I*b*ln
(c^2*x^2+1)/c^4/d^2-3/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$,
Rules used = {4996, 4930, 266, 4946, 327, 209, 4972, 641, 46, 4964, 2449, 2352}

$$\frac{i(a+b\text{ArcTan}(cx))}{c^4d^2(-cx+i)}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4d^2}-\frac{x^2(a+b\text{ArcTan}(cx))}{2c^2d^2}-\frac{2iax}{c^3d^2}-\frac{b\text{ArcTan}(cx)}{c^4d^2}-\frac{2ibx\text{ArcTan}(cx)}{c^3d^2}-\frac{3ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(-cx+i)}+\frac{bx}{2c^3d^2}+\frac{ib\log\left(c^2x^2+1\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] ((-2*I)*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(I - c*x)) - (b*A
rcTan[c*x])/(c^4*d^2) - ((2*I)*b*x*ArcTan[c*x])/(c^3*d^2) - (x^2*(a + b*Arc
Tan[c*x]))/(2*c^2*d^2) + (I*(a + b*ArcTan[c*x]))/(c^4*d^2*(I - c*x)) - (3*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^2) + (I*b*Log[1 + c^2*x^2])/(
c^4*d^2) - (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+icdx)^2}\,dx = \int \left(-\frac{2i(a+b\tan^{-1}(cx))}{c^3d^2} - \frac{x(a+b\tan^{-1}(cx))}{c^2d^2} + \frac{i(a+b\tan^{-1}(cx))}{c^3d^2(-i+cx)^2} + \frac{3(a+b}{c^3d^2(}\right.$$

$$= \frac{i\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^3d^2} - \frac{(2i)\int (a+b\tan^{-1}(cx))\,dx}{c^3d^2} + \frac{3\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^3d^2} - \frac{\int x(a+}{}$$

$$= -\frac{2iax}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d^2} + \frac{i(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} - \frac{3(a+b\tan^{-1}(cx))\log\left(\frac{}{1+}\right.}{c^4d^2}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d^2} + \frac{i(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} -$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} - \frac{b\tan^{-1}(cx)}{2c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))}{2c^2d^2} + \frac{i(a+}{c^4}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(i-cx)} - \frac{b\tan^{-1}(cx)}{2c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2(a+b\tan}{2c^2d}$$

$$= -\frac{2iax}{c^3d^2} + \frac{bx}{2c^3d^2} + \frac{b}{2c^4d^2(i-cx)} - \frac{b\tan^{-1}(cx)}{c^4d^2} - \frac{2ibx\tan^{-1}(cx)}{c^3d^2} - \frac{x^2(a+b\tan}{2c^2d}$$

Mathematica [A]
time = 0.70, size = 186, normalized size = 0.92

$$-\frac{8iacx+2ac^2x^2+\frac{4ia}{-i+cx}-12ia\text{ArcTan}(cx)-6a\log\left(1+c^2x^2\right)+b\left(-2cx-12i\text{ArcTan}(cx)^2+i\cos(2\text{ArcTan}(cx))-4i\log\left(1+c^2x^2\right)-6i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+2\text{ArcTan}(cx)\left(1+4icx+c^2x^2-\cos(2\text{ArcTan}(cx))+6\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+i\sin(2\text{ArcTan}(cx))\right)+\sin(2\text{ArcTan}(cx))\right)}{4c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -1/4*((8*I)*a*c*x + 2*a*c^2*x^2 + ((4*I)*a)/(-I + c*x) - (12*I)*a*ArcTan[c*
x] - 6*a*Log[1 + c^2*x^2] + b*(-2*c*x - (12*I)*ArcTan[c*x]^2 + I*Cos[2*ArcT
an[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])
] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x]] + 6*Log[1 +
 E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*x]]))/(c^4
*d^2)
```

Maple [A]
time = 0.15, size = 317, normalized size = 1.56

method	result
derivativedivides	$\frac{-\frac{2ib\arctan(cx)cx}{d^2}-\frac{a\,c^2x^2}{2d^2}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia}{d^2(cx-i)}+\frac{3ib\ln\left(c^2x^2+1\right)}{4d^2}+\frac{3ia\arctan(cx)}{d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{3b\arctan(cx)\ln(c}{d^2}}{}$
default	$\frac{-\frac{2ib\arctan(cx)cx}{d^2}-\frac{a\,c^2x^2}{2d^2}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia}{d^2(cx-i)}+\frac{3ib\ln\left(c^2x^2+1\right)}{4d^2}+\frac{3ia\arctan(cx)}{d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{3b\arctan(cx)\ln(c}{d^2}}{}$
risch	$-\frac{ib\ln(-icx+1)}{4d^2c^4(-icx-1)}+\left(\frac{ib\left(\frac{1}{2}c\,x^2+2ix\right)}{2c^3d^2}-\frac{b}{2c^4d^2(cx-i)}\right)\ln\left(icx+1\right)+\frac{9ib\ln\left(c^2x^2+1\right)}{8c^4d^2}+\frac{\ln(-icx+1)xb}{d^2c^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-2*I*b/d^2*arctan(c*x)*c*x-1/2*a/d^2*c^2*x^2+3/2*a/d^2*ln(c^2*x^2+1)
-3/2*I*b/d^2*dilog(-1/2*I*(c*x+I))-I*a/d^2/(c*x-I)+3/4*I*b/d^2*ln(c^2*x^2+1
)-1/2*b/d^2*arctan(c*x)*c^2*x^2+3*b/d^2*arctan(c*x)*ln(c*x-I)+3*I*a/d^2*arc
tan(c*x)-1/2*I*b/d^2+1/8*I*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-I*b/d^2*arctan(c*
x)/(c*x-I)+1/2*b/d^2*c*x-3/2*I*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)-1/2*b/d^2
/(c*x-I)-2*I*a/d^2*c*x-3/2*b*arctan(c*x)/d^2+3/4*I*b/d^2*ln(c*x-I)^2-1/4*b/
d^2*arctan(1/2*c*x)+1/4*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2*b/d^2*arctan(
1/2*c*x-1/2*I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(c*x
 - I)/(c^4*d^2)) + 1/8*(I*c^3*x^3 - 5*c^2*x^2 - 2*c*x*(arctan2(1, c*x) - 3*
I) - 12*(-I*c*x - 1)*arctan(c*x)^2 - 3*(-I*c*x - 1)*log(c^2*x^2 + 1)^2 - 3*
(c^5*d^2*x - I*c^4*d^2)*((c*(x/(c^7*d^2*x^2 + c^5*d^2) + arctan(c*x)/(c^6*d
^2)) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*d^2))*c + 16*integrate(1/8*log(c^2*
```

```
x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) + 3*(-I*c^5*d^2*x - c
^4*d^2)*(c*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d^2*x^2 + c
^5*d^2)) + 32*integrate(1/8*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*
d^2), x)) + 6*(c^6*d^2*x - I*c^5*d^2)*(c*(x/(c^7*d^2*x^2 + c^5*d^2) + arcta
n(c*x)/(c^6*d^2)) - 16*c*integrate(1/8*x^2*log(c^2*x^2 + 1)/(c^7*d^2*x^4 +
2*c^5*d^2*x^2 + c^3*d^2), x) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*d^2)) - 6*(
I*c^6*d^2*x + c^5*d^2)*(32*c*integrate(1/8*x^2*arctan(c*x)/(c^7*d^2*x^4 + 2
*c^5*d^2*x^2 + c^3*d^2), x) - c^2/(c^9*d^2*x^2 + c^7*d^2) - log(c^2*x^2 + 1
)/(c^7*d^2*x^2 + c^5*d^2)) - 2*(c^3*x^3 + 3*I*c^2*x^2 + 4*c*x + 2*I)*arctan
(c*x) - 16*(c^9*d^2*x - I*c^8*d^2)*integrate(1/8*(2*c*x^5*arctan(c*x) + x^4
*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(-I*c^9
*d^2*x - c^8*d^2)*integrate(1/8*(c*x^5*log(c^2*x^2 + 1) - 2*x^4*arctan(c*x)
)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(I*c^8*d^2*x + c^7*d^2)*
integrate(1/8*(2*c*x^4*arctan(c*x) + x^3*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2
*c^5*d^2*x^2 + c^3*d^2), x) - 16*(c^8*d^2*x - I*c^7*d^2)*integrate(1/8*(c*x
^4*log(c^2*x^2 + 1) - 2*x^3*arctan(c*x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3
*d^2), x) + 48*(c^7*d^2*x - I*c^6*d^2)*integrate(1/8*(2*c*x^3*arctan(c*x) +
 x^2*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 48*(I*
c^7*d^2*x + c^6*d^2)*integrate(1/8*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c
*x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + (-I*c^3*x^3 + 3*c^2*x^2
- I*c*x + 5)*log(c^2*x^2 + 1) + 2*I*arctan2(1, c*x))*b/(c^5*d^2*x - I*c^4*d
^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-I*b*x^3*log(-(c*x + I)/(c*x - I)) - 2*a*x^3)/(c^2*d^2*x^2 -
2*I*c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.52 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{(d+icdx)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{ax}{c^2d^2}-\frac{ib}{2c^3d^2(i-cx)}+\frac{ib\text{ArcTan}(cx)}{2c^3d^2}-\frac{bx\text{ArcTan}(cx)}{c^2d^2}+\frac{a+b\text{ArcTan}(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}$$

```
[Out] -a*x/c^2/d^2-1/2*I*b/c^3/d^2/(I-c*x)+1/2*I*b*arctan(c*x)/c^3/d^2-b*x*arctan
(c*x)/c^2/d^2+(a+b*arctan(c*x))/c^3/d^2/(I-c*x)+2*I*(a+b*arctan(c*x))*ln(2/
(1+I*c*x))/c^3/d^2+1/2*b*ln(c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(1+I*c*x))/c
^3/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4930, 266, 4972, 641, 46, 209, 4964, 2449, 2352}

$$\frac{a+b\text{ArcTan}(cx)}{c^3d^2(-cx+i)}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^3d^2}-\frac{ax}{c^2d^2}+\frac{ib\text{ArcTan}(cx)}{2c^3d^2}-\frac{bx\text{ArcTan}(cx)}{c^2d^2}-\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{ib}{2c^3d^2(-cx+i)}+\frac{b\log\left(c^2x^2+1\right)}{2c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -((a*x)/(c^2*d^2)) - ((I/2)*b)/(c^3*d^2*(I - c*x)) + ((I/2)*b*ArcTan[c*x])/
(c^3*d^2) - (b*x*ArcTan[c*x])/(c^2*d^2) + (a + b*ArcTan[c*x])/(c^3*d^2*(I -
 c*x)) + ((2*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + (b*Log[
1 + c^2*x^2])/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+icdx)^2}\,dx &= \int \left(-\frac{a+b\tan^{-1}(cx)}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^2d^2(-i+cx)^2}-\frac{2i(a+b\tan^{-1}(cx))}{c^2d^2(-i+cx)}\right)dx \\
&= -\frac{(2i)\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^2d^2}-\frac{\int \left(a+b\tan^{-1}(cx)\right)dx}{c^2d^2}+\frac{\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^2d^2} \\
&= -\frac{ax}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}-\frac{(2ib)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,d}{c^2d^2} \\
&= -\frac{ax}{c^2d^2}-\frac{bx\tan^{-1}(cx)}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}-\frac{(2}{} \\
&= -\frac{ax}{c^2d^2}-\frac{bx\tan^{-1}(cx)}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}+\frac{b}{} \\
&= -\frac{ax}{c^2d^2}-\frac{ib}{2c^3d^2(i-cx)}-\frac{bx\tan^{-1}(cx)}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\tan^{-1}(cx)}{c^3d^2} \\
&= -\frac{ax}{c^2d^2}-\frac{ib}{2c^3d^2(i-cx)}+\frac{ib\tan^{-1}(cx)}{2c^3d^2}-\frac{bx\tan^{-1}(cx)}{c^2d^2}+\frac{a+b\tan^{-1}(cx)}{c^3d^2(i-cx)}+\frac{2i(}{}
\end{aligned}
$$

Mathematica [A]

time = 0.53, size = 153, normalized size = 0.92

$$
-\frac{4acx+\frac{4a}{-i+cx}-8a\text{ArcTan}(cx)+4ia\log\left(1+c^2x^2\right)+b\left(-8\text{ArcTan}(cx)^2+\cos(2\text{ArcTan}(cx))-2\log\left(1+c^2x^2\right)-4\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)-i\sin(2\text{ArcTan}(cx))+2\text{ArcTan}(cx)\left(2cx+i\cos(2\text{ArcTan}(cx))-4i\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+\sin(2\text{ArcTan}(cx))\right)\right)}{4c^3d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -1/4*(4*a*c*x + (4*a)/(-I + c*x) - 8*a*ArcTan[c*x] + (4*I)*a*Log[1 + c^2*x^
2] + b*(-8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*
x + I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*Arc
Tan[c*x]])))/(c^3*d^2)
```

Maple [A]

time = 0.12, size = 271, normalized size = 1.62

method	result
derivativedivides	$\frac{-\frac{acx}{d^2}+\frac{2a\arctan(cx)}{d^2}+\frac{3ib\arctan(cx)}{4d^2}-\frac{a}{d^2(cx-i)}-\frac{b\arctan(cx)cx}{d^2}-\frac{2ib\arctan(cx)\ln(cx-i)}{d^2}-\frac{b\arctan(cx)}{d^2(cx-i)}+\frac{b\ln(cx-i)^2}{2d^2}-\frac{b\ln(cx-i)}{}}{}$

default	$\frac{-\frac{acx}{d^2}+\frac{2a\arctan(cx)}{d^2}+\frac{3ib\arctan(cx)}{4d^2}-\frac{a}{d^2(cx-i)}-\frac{b\arctan(cx)cx}{d^2}-\frac{2ib\arctan(cx)\ln(cx-i)}{d^2}-\frac{b\arctan(cx)}{d^2(cx-i)}+\frac{b\ln(cx-i)^2}{2d^2}-\frac{b\ln(cx-}{}}{}$
risch	$-\frac{b\ln(icx+1)^2}{2c^3d^2}+\left(\frac{ibx}{2c^2d^2}+\frac{ib}{2c^3d^2(cx-i)}\right)\ln(icx+1)-\frac{ia\ln(c^2x^2+1)}{d^2c^3}+\frac{5b\ln(c^2x^2+1)}{8c^3d^2}-\frac{i\ln(-icx+1)}{2d^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a/d^2*c*x+2*a/d^2*arctan(c*x)+3/4*I*b/d^2*arctan(c*x)-a/d^2/(c*x-I)
-b/d^2*arctan(c*x)*c*x-2*I*b/d^2*arctan(c*x)*ln(c*x-I)-b/d^2*arctan(c*x)/(c
*x-I)+1/2*b/d^2*ln(c*x-I)^2-b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-b/d^2*dilog(
-1/2*I*(c*x+I))-1/4*I*b/d^2*arctan(1/2*c*x-1/2*I)+3/8*b/d^2*ln(c^2*x^2+1)-I
*a/d^2*ln(c^2*x^2+1)+1/16*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)+1/2*I*b/d^2/(c*x-I
)+1/8*I*b/d^2*arctan(1/2*c*x)-1/8*I*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -a*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2)) +
 1/4*(2*I*c^2*x^2 + 4*(c*x - I)*arctan(c*x)^2 + (c*x - I)*log(c^2*x^2 + 1)^
2 - (-I*c^4*d^2*x - c^3*d^2)*((c*(x/(c^6*d^2*x^2 + c^4*d^2) + arctan(c*x)/(
c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2))*c + 8*integrate(1/4*log(
c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)) - (c^4*d^2*x - I*
c^3*d^2)*(c*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d^2*x^2 +
c^4*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2
*d^2), x)) + 2*(-I*c^5*d^2*x - c^4*d^2)*(c*(x/(c^6*d^2*x^2 + c^4*d^2) + arc
tan(c*x)/(c^5*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^6*d^2*x^4 +
 2*c^4*d^2*x^2 + c^2*d^2), x) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2)) - 2*
(c^5*d^2*x - I*c^4*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^6*d^2*x^4 +
2*c^4*d^2*x^2 + c^2*d^2), x) - c^2/(c^8*d^2*x^2 + c^6*d^2) - log(c^2*x^2 +
1)/(c^6*d^2*x^2 + c^4*d^2)) + 2*c*x - 2*(c^2*x^2 - I*c*x + 1)*arctan(c*x) -
 4*(c^7*d^2*x - I*c^6*d^2)*integrate(1/4*(2*c*x^4*arctan(c*x) + x^3*log(c^2
*x^2 + 1))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 4*(-I*c^7*d^2*x -
c^6*d^2)*integrate(1/4*(c*x^4*log(c^2*x^2 + 1) - 2*x^3*arctan(c*x))/(c^6*d^
2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 12*(I*c^6*d^2*x + c^5*d^2)*integrate
(1/4*(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^6*d^2*x^4 + 2*c^4*d^2*
x^2 + c^2*d^2), x) - 12*(c^6*d^2*x - I*c^5*d^2)*integrate(1/4*(c*x^3*log(c^
2*x^2 + 1) - 2*x^2*arctan(c*x))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)
 + (-I*c^2*x^2 - 2*I)*log(c^2*x^2 + 1))*b/(c^4*d^2*x - I*c^3*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
[Out] integral(1/2*(-I*b*x^2*log(-(c*x + I)/(c*x - I)) - 2*a*x^2)/(c^2*d^2*x^2 -
2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
[Out] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.53 $\int \frac{x(a+b\text{ArcTan}(cx))}{(d+icdx)^2} dx$

Optimal. Leaf size=122

$$-\frac{b}{2c^2d^2(i-cx)}+\frac{b\text{ArcTan}(cx)}{2c^2d^2}-\frac{i(a+b\text{ArcTan}(cx))}{c^2d^2(i-cx)}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d^2}$$

```
[Out] -1/2*b/c^2/d^2/(I-c*x)+1/2*b*arctan(c*x)/c^2/d^2-I*(a+b*arctan(c*x))/c^2/d^
2/(I-c*x)+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d^2+1/2*I*b*polylog(2,1-2/(
1+I*c*x))/c^2/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {4996, 4972, 641, 46, 209, 4964, 2449, 2352}

$$-\frac{i(a+b\text{ArcTan}(cx))}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^2d^2}+\frac{b\text{ArcTan}(cx)}{2c^2d^2}+\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^2d^2}-\frac{b}{2c^2d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] -1/2*b/(c^2*d^2*(I - c*x)) + (b*ArcTan[c*x])/(2*c^2*d^2) - (I*(a + b*ArcTan
[c*x]))/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2
*d^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{(d+icdx)^2}\,dx = \int \left(-\frac{i(a+b\tan^{-1}(cx))}{cd^2(-i+cx)^2} - \frac{a+b\tan^{-1}(cx)}{cd^2(-i+cx)}\right) dx$$

$$= -\frac{i\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{cd^2} - \frac{\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{cd^2}$$

$$= -\frac{i(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2} - \frac{(ib)\int \frac{1}{(-i+cx)(1+c^2x^2)}\,dx}{cd^2}$$

$$= -\frac{i(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2} + \frac{(ib)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x\right.}{c^2d^2}$$

$$= -\frac{i(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2} + \frac{ib\text{Li}_2\left(1-\frac{2}{1+icx}\right)}{2c^2d^2} - \frac{(ib)}{}$$

$$= -\frac{b}{2c^2d^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2} + \frac{ib\text{Li}_2(1}{2c}$$

$$= -\frac{b}{2c^2d^2(i-cx)} + \frac{b\tan^{-1}(cx)}{2c^2d^2} - \frac{i(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{}{1+}\right.}{c^2d^2}$$

Mathematica [A]
time = 0.36, size = 138, normalized size = 1.13

$$\frac{\frac{4ia}{-i+cx} - 4ib\text{ArcTan}(cx)^2 + ib\cos(2\text{ArcTan}(cx)) - 2a\log(1+c^2x^2) - 2ib\text{PolyLog}\left(2, -e^{2i\text{ArcTan}(cx)}\right) + b\sin(2\text{ArcTan}(cx)) + \text{ArcTan}(cx)\left(-4ia - 2b\cos(2\text{ArcTan}(cx)) + 4b\log\left(1+e^{2i\text{ArcTan}(cx)}\right) + 2ib\sin(2\text{ArcTan}(cx))\right)}{4c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

```
[Out] (((4*I)*a)/(-I + c*x) - (4*I)*b*ArcTan[c*x]^2 + I*b*Cos[2*ArcTan[c*x]] - 2*
a*Log[1 + c^2*x^2] - (2*I)*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + b*Sin[2*A
rcTan[c*x]] + ArcTan[c*x]*((-4*I)*a - 2*b*Cos[2*ArcTan[c*x]] + 4*b*Log[1 +
E^((2*I)*ArcTan[c*x])] + (2*I)*b*Sin[2*ArcTan[c*x]]))/(4*c^2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(110) = 220.
time = 0.11, size = 252, normalized size = 2.07

method	result
derivativedivides	$\frac{-\frac{a\ln(c^2x^2+1)}{2d^2} - \frac{ia\arctan(cx)}{d^2} + \frac{ia}{d^2(cx-i)} - \frac{b\arctan(cx)\ln(cx-i)}{d^2} + \frac{ib\arctan(cx)}{d^2(cx-i)} + \frac{b}{2d^2(cx-i)} - \frac{ib\ln(c^2x^2+1)}{8d^2} + \frac{b\arctan(cx)}{4d^2} + \frac{ib}{}}{}$
default	$\frac{-\frac{a\ln(c^2x^2+1)}{2d^2} - \frac{ia\arctan(cx)}{d^2} + \frac{ia}{d^2(cx-i)} - \frac{b\arctan(cx)\ln(cx-i)}{d^2} + \frac{ib\arctan(cx)}{d^2(cx-i)} + \frac{b}{2d^2(cx-i)} - \frac{ib\ln(c^2x^2+1)}{8d^2} + \frac{b\arctan(cx)}{4d^2} + \frac{ib}{}}{}$

risch	$\frac{ib\ln(icx+1)^2}{4c^2d^2}+\frac{b\ln(icx+1)}{2c^2d^2(cx-i)}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^2c^2}+\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^2c^2}+\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^2c^2}-\frac{ib\ln(}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a/d^2*ln(c^2*x^2+1)-I*a/d^2*arctan(c*x)+I*a/d^2/(c*x-I)-b/d^2*a
rctan(c*x)*ln(c*x-I)+I*b/d^2*arctan(c*x)/(c*x-I)+1/2*b/d^2/(c*x-I)-1/8*I*b/
d^2*ln(c^2*x^2+1)+1/4*b*arctan(c*x)/d^2+1/16*I*b/d^2*ln(c^4*x^4+10*c^2*x^2+
9)-1/8*b/d^2*arctan(1/2*c*x)+1/8*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/4*b/d^
2*arctan(1/2*c*x-1/2*I)-1/4*I*b/d^2*ln(c*x-I)^2+1/2*I*b/d^2*ln(c*x-I)*ln(-1
/2*I*(c*x+I))+1/2*I*b/d^2*dilog(-1/2*I*(c*x+I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] a*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/8*(4*(I*c*x + 1)
*arctan(c*x)^2 + 4*c*x*arctan2(1, c*x) - (-I*c*x - 1)*log(c^2*x^2 + 1)^2 -
(c^3*d^2*x - I*c^2*d^2)*((c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x)/(c^4*d
^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*c + 8*integrate(1/4*log(c^2*x
^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x)) - (I*c^3*d^2*x + c^2*d^2
)*(c*(c^2/(c^7*d^2*x^2 + c^5*d^2) + log(c^2*x^2 + 1)/(c^5*d^2*x^2 + c^3*d^2
)) + 16*integrate(1/4*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x)
) + (c^4*d^2*x - I*c^3*d^2)*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x)/(c^
4*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x
^2 + c*d^2), x) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2)) + (-I*c^4*d^2*x -
c^3*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 +
 c*d^2), x) - c^2/(c^7*d^2*x^2 + c^5*d^2) - log(c^2*x^2 + 1)/(c^5*d^2*x^2 +
 c^3*d^2)) + 16*(c^5*d^2*x - I*c^4*d^2)*integrate(1/4*(2*c*x^3*arctan(c*x)
+ x^2*log(c^2*x^2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 16*(-I*
c^5*d^2*x - c^4*d^2)*integrate(1/4*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c
*x))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 4*I*arctan(c*x) - 4*I*arct
an2(1, c*x) + 2*log(c^2*x^2 + 1))*b/(c^3*d^2*x - I*c^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/2*(-I*b*x*log(-(c*x + I)/(c*x - I)) - 2*a*x)/(c^2*d^2*x^2 - 2*I*
c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

```
[Out] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.54 $\int \frac{a+b\text{ArcTan}(cx)}{(d+icdx)^2}\,dx$

Optimal. Leaf size=69

$$\frac{ib}{2cd^2(i-cx)}-\frac{ib\text{ArcTan}(cx)}{2cd^2}+\frac{i(a+b\text{ArcTan}(cx))}{cd^2(1+icx)}$$

```
[Out] 1/2*I*b/c/d^2/(I-c*x)-1/2*I*b*arctan(c*x)/c/d^2+I*(a+b*arctan(c*x))/c/d^2/(
1+I*c*x)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4972, 641, 46, 209}

$$\frac{i(a+b\text{ArcTan}(cx))}{cd^2(1+icx)}-\frac{ib\text{ArcTan}(cx)}{2cd^2}+\frac{ib}{2cd^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

```
[Out] ((I/2)*b)/(c*d^2*(I - c*x)) - ((I/2)*b*ArcTan[c*x])/(c*d^2) + (I*(a + b*Arc
Tan[c*x]))/(c*d^2*(1 + I*c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^2}\,dx &= \frac{i(a+b\tan^{-1}(cx))}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{(d+icdx)(1+c^2x^2)}\,dx}{d}\\ &= \frac{i(a+b\tan^{-1}(cx))}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{\left(\frac{1}{d}-\frac{icx}{d}\right)(d+icdx)^2}\,dx}{d}\\ &= \frac{i(a+b\tan^{-1}(cx))}{cd^2(1+icx)}-\frac{(ib)\int \left(-\frac{1}{2d(-i+cx)^2}+\frac{1}{2d(1+c^2x^2)}\right)dx}{d}\\ &= \frac{ib}{2cd^2(i-cx)}+\frac{i(a+b\tan^{-1}(cx))}{cd^2(1+icx)}-\frac{(ib)\int \frac{1}{1+c^2x^2}\,dx}{2d^2}\\ &= \frac{ib}{2cd^2(i-cx)}-\frac{ib\tan^{-1}(cx)}{2cd^2}+\frac{i(a+b\tan^{-1}(cx))}{cd^2(1+icx)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.61

$$\frac{2a-ib+(b-ibcx)\text{ArcTan}(cx)}{2cd^2(-i+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
[Out] (2*a - I*b + (b - I*b*c*x)*ArcTan[c*x])/(2*c*d^2*(-I + c*x))
```

Maple [A]
time = 0.09, size = 68, normalized size = 0.99

method	result	size
derivativedivides	$\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$	68
default	$\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$	68
risch	$-\frac{ib\ln(icx+1)}{2c\,d^2(cx-i)}+\frac{2ib\ln(-icx+1)+\ln(-cx-i)bcx-\ln(cx-i)bcx-i\ln(-cx-i)b+i\ln(cx-i)b+4a-2ib}{4d^2(cx-i)c}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*a/d^2/(1+I*c*x)+I*b/d^2/(1+I*c*x)*arctan(c*x)-1/2*I*b/d^2*arctan(c*x
)-1/2*I*b/d^2/(c*x-I))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 1.70, size = 50, normalized size = 0.72

$$\frac{(bcx + i\,b)\log\left(-\frac{cx+i}{cx-i}\right) + 4\,a - 2i\,b}{4\,(c^2d^2x - i\,cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) + 4*a - 2*I*b)/(c^2*d^2*x - I*
c*d^2)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(51) = 102$.
time = 0.97, size = 116, normalized size = 1.68

$$\frac{ib\log\left(-icx+1\right)}{2c^2d^2x-2icd^2} - \frac{ib\log\left(icx+1\right)}{2c^2d^2x-2icd^2} - \frac{b\left(\frac{\log\left(bx-\frac{ib}{c}\right)}{4} - \frac{\log\left(bx+\frac{ib}{c}\right)}{4}\right)}{cd^2} - \frac{-2a+ib}{2c^2d^2x-2icd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

```
[Out] I*b*log(-I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) - I*b*log(I*c*x + 1)/(2*c*
*2*d**2*x - 2*I*c*d**2) - b*(log(b*x - I*b/c)/4 - log(b*x + I*b/c)/4)/(c*d*
*2) - (-2*a + I*b)/(2*c**2*d**2*x - 2*I*c*d**2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + c*d*x*1i)^2,x)
```

```
[Out] int((a + b*atan(c*x))/(d + c*d*x*1i)^2, x)
```

3.55 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+icdx)^2}\,dx$

Optimal. Leaf size=150

$$\frac{b}{2d^2(i-cx)}-\frac{b\text{ArcTan}(cx)}{2d^2}+\frac{i(a+b\text{ArcTan}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{ib\text{PolyLog}(2,-}{2d^2}$$

```
[Out] 1/2*b/d^2/(I-c*x)-1/2*b*arctan(c*x)/d^2+I*(a+b*arctan(c*x))/d^2/(I-c*x)+a*l
n(x)/d^2+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+1/2*I*b*polylog(2,-I*c*x)/d^
2-1/2*I*b*polylog(2,I*c*x)/d^2+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4940, 2438, 4972, 641, 46, 209, 4964, 2449, 2352}

$$\frac{i(a+b\text{ArcTan}(cx))}{d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\text{ArcTan}(cx)}{2d^2}+\frac{ib\text{Li}_2(-icx)}{2d^2}-\frac{ib\text{Li}_2(icx)}{2d^2}+\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2d^2}+\frac{b}{2d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

```
[Out] b/(2*d^2*(I - c*x)) - (b*ArcTan[c*x])/(2*d^2) + (I*(a + b*ArcTan[c*x]))/(d^
2*(I - c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^
2 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 +
((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x}+\frac{ic(a+b\tan^{-1}(cx))}{d^2(-i+cx)^2}-\frac{c(a+b\tan^{-1}(cx))}{d^2(-i+cx)}\right)dx$$

$$=\frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2}+\frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2}-\frac{c\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^2}$$

$$=\frac{i(a+b\tan^{-1}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^2}-$$

$$=\frac{i(a+b\tan^{-1}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{ib\text{Li}_2(-icx)}{2d^2}-\frac{ib\text{Li}}{2}$$

$$=\frac{i(a+b\tan^{-1}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{ib\text{Li}_2(-icx)}{2d^2}-\frac{ib\text{Li}}{2}$$

$$=\frac{b}{2d^2(i-cx)}+\frac{i(a+b\tan^{-1}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{ib\text{Li}_2}{2}$$

$$=\frac{b}{2d^2(i-cx)}-\frac{b\tan^{-1}(cx)}{2d^2}+\frac{i(a+b\tan^{-1}(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\tan^{-1}(cx))\log(}{d^2}$$

Mathematica [A]
time = 0.48, size = 138, normalized size = 0.92

$$-\frac{\frac{4ia}{-i+cx}+4ia\text{ArcTan}(cx)-4a\log(x)+2a\log\left(1+c^2x^2\right)+b\left(4i\text{ArcTan}(cx)^2+i\cos(2\text{ArcTan}(cx))+2i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)-2\text{ArcTan}(cx)\left(\cos(2\text{ArcTan}(cx))+2\log\left(1-e^{2i\text{ArcTan}(cx)}\right)-i\sin(2\text{ArcTan}(cx))\right)+\sin(2\text{ArcTan}(cx))\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

```
[Out] -1/4*(((4*I)*a)/(-I + c*x) + (4*I)*a*ArcTan[c*x] - 4*a*Log[x] + 2*a*Log[1 +
 c^2*x^2] + b*((4*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c*x]] + (2*I)*PolyLog[2
, E^((2*I)*ArcTan[c*x])] - 2*ArcTan[c*x]*(Cos[2*ArcTan[c*x]] + 2*Log[1 - E^
((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*x]]))/d^2
```

Maple [A]
time = 0.11, size = 251, normalized size = 1.67

method	result
derivativedivides	$-\frac{ib\,\text{dilog}(-icx+1)}{2d^2}-\frac{a\ln(c^2x^2+1)}{2d^2}+\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^2}+\frac{a\ln(cx)}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}$
default	$-\frac{ib\,\text{dilog}(-icx+1)}{2d^2}-\frac{a\ln(c^2x^2+1)}{2d^2}+\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^2}+\frac{a\ln(cx)}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}$
risch	$-\frac{ib\,\text{dilog}(-icx+1)}{2d^2}+\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^2}+\frac{ib\,\text{dilog}(icx+1)}{2d^2}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^2}-\frac{ib\ln(icx+1)}{2d^2(icx+1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*b/d^2*dilog(1-I*c*x)-1/2*a/d^2*ln(c^2*x^2+1)+1/2*I*b/d^2*ln(c*x-I)*l
n(-1/2*I*(c*x+I))+a/d^2*ln(c*x)-I*a/d^2/(c*x-I)-b/d^2*arctan(c*x)*ln(c*x-I)
+b/d^2*ln(c*x)*arctan(c*x)-1/2*I*b/d^2*ln(c*x)*ln(1-I*c*x)+1/2*I*b/d^2*ln(c
*x)*ln(1+I*c*x)-I*b/d^2*arctan(c*x)/(c*x-I)+1/2*I*b/d^2*dilog(-1/2*I*(c*x+I
))-1/2*b/d^2/(c*x-I)-1/2*b*arctan(c*x)/d^2+1/2*I*b/d^2*dilog(1+I*c*x)-I*a/d
^2*arctan(c*x)-1/4*I*b/d^2*ln(c*x-I)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x) - int
egrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x))
*b + a*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2)
```

Fricas [**A**]
time = 1.41, size = 127, normalized size = 0.85

$$-\frac{2\,(i\,bcx+b)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(acx-i\,a)\log\left(x\right)-2\,b\log\left(-\frac{cx+i}{cx-i}\right)-(-i\,bcx-b)\log\left(\frac{cx+i}{c}\right)+((4\,a-i\,b)cx-4i\,a-b)\log\left(\frac{cx-i}{c}\right)+4i\,a+2\,b}{4\,(cd^2x-i\,d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(I*b*c*x + b)*dilog((c*x + I)/(c*x - I) + 1) - 4*(a*c*x - I*a)*log(
x) - 2*b*log(-(c*x + I)/(c*x - I)) - (-I*b*c*x - b)*log((c*x + I)/c) + ((4*
a - I*b)*c*x - 4*I*a - b)*log((c*x - I)/c) + 4*I*a + 2*b)/(c*d^2*x - I*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(d+c\,d\,x\,1\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2), x)
```

3.56 $\int \frac{a+b\text{ArcTan}(cx)}{x^2(d+icdx)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{ibc}{2d^2(i-cx)}+\frac{ibc\text{ArcTan}(cx)}{2d^2}-\frac{a+b\text{ArcTan}(cx)}{d^2x}+\frac{c(a+b\text{ArcTan}(cx))}{d^2(i-cx)}-\frac{2iac\log(x)}{d^2}+\frac{bc\log(x)}{d^2}-\frac{2ic(a+b$$

```
[Out] -1/2*I*b*c/d^2/(I-c*x)+1/2*I*b*c*arctan(c*x)/d^2+(-a-b*arctan(c*x))/d^2/x+c
*(a+b*arctan(c*x))/d^2/(I-c*x)-2*I*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*I*c*(a+b*a
rctan(c*x))*ln(2/(1+I*c*x))/d^2-1/2*b*c*ln(c^2*x^2+1)/d^2+b*c*polylog(2,-I*
c*x)/d^2-b*c*polylog(2,I*c*x)/d^2+b*c*polylog(2,1-2/(1+I*c*x))/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {4996, 4946, 272, 36, 29, 31, 4940, 2438, 4972, 641, 46, 209, 4964, 2449, 2352}

$$\frac{c(a+b\text{ArcTan}(cx))}{d^2(-cx+i)}-\frac{a+b\text{ArcTan}(cx)}{d^2x}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{2iac\log(x)}{d^2}+\frac{ibc\text{ArcTan}(cx)}{2d^2}-\frac{bc\log\left(c^2x^2+1\right)}{2d^2}+\frac{bc\text{Li}_2(-icx)}{d^2}-\frac{bc\text{Li}_2(icx)}{d^2}+\frac{bc\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{d^2}-\frac{ibc}{2d^2(-cx+i)}+\frac{bc\log(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] ((-1/2*I)*b*c)/(d^2*(I - c*x)) + ((I/2)*b*c*ArcTan[c*x])/d^2 - (a + b*ArcTa
n[c*x])/(d^2*x) + (c*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) - ((2*I)*a*c*Log[
x])/d^2 + (b*c*Log[x])/d^2 - ((2*I)*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)
])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, (-I)*c*x])/d^2 -
(b*c*PolyLog[2, I*c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^2} - \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} + \frac{c^2(a+b\tan^{-1}(cx))}{d^2(-i+cx)^2} + \frac{2ic^2(a+b\tan}{d^2(-i+c}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(2ic)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{(2ic^2)\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^2} + \frac{c^2\int \frac{a+b\tan^{-1}(c}{(-i+cx)^2}}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic(a+b\tan^{-1}(cx))\log\left(\frac{}{1+}\right.}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic(a+b\tan^{-1}(cx))\log\left(\frac{}{1+}\right.}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} - \frac{2ic(a+b\tan^{-1}(cx))\log\left(\frac{}{1+}\right.}{d^2}$$

$$= -\frac{ibc}{2d^2(i-cx)} - \frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2iac\log(x)}{d^2} + \frac{bc\log(x)}{d^2} -$$

$$= -\frac{ibc}{2d^2(i-cx)} + \frac{ibc\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{d^2x} + \frac{c(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2iac\log(x}{d^2}$$

Mathematica [A]
time = 0.79, size = 175, normalized size = 0.90

$$-\frac{\frac{4a}{x} + \frac{4ac}{-i+cx} + 8ac\text{ArcTan}(cx) + 8iac\log(x) - 4iac\log(1+c^2x^2) + bc\left(8\text{ArcTan}(cx)^2 + \cos(2\text{ArcTan}(cx)) - 4\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) + 4\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right) - i\sin(2\text{ArcTan}(cx)) + \text{ArcTan}(cx)\left(\frac{4}{cx} + 2i\cos(2\text{ArcTan}(cx)) + 8i\log\left(1-e^{2i\text{ArcTan}(cx)}\right) + 2\sin(2\text{ArcTan}(cx))\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] -1/4*((4*a)/x + (4*a*c)/(-I + c*x) + 8*a*c*ArcTan[c*x] + (8*I)*a*c*Log[x] -
 (4*I)*a*c*Log[1 + c^2*x^2] + b*c*(8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 4
*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Sin
[2*ArcTan[c*x]] + ArcTan[c*x]*(4/(c*x) + (2*I)*Cos[2*ArcTan[c*x]] + (8*I)*L
og[1 - E^((2*I)*ArcTan[c*x])] + 2*Sin[2*ArcTan[c*x]])))/d^2
```

Maple [A]
time = 0.12, size = 329, normalized size = 1.70

method	result
derivativedivides	$c\left(-\frac{a}{d^2(cx-i)} - \frac{2a\arctan(cx)}{d^2} - \frac{2ia\ln(cx)}{d^2} - \frac{a}{d^2cx} + \frac{ib\arctan(cx)}{2d^2} - \frac{b\arctan(cx)}{d^2(cx-i)} + \frac{2ib\arctan(cx)\ln(cx-i)}{d^2}\right.$
default	$c\left(-\frac{a}{d^2(cx-i)} - \frac{2a\arctan(cx)}{d^2} - \frac{2ia\ln(cx)}{d^2} - \frac{a}{d^2cx} + \frac{ib\arctan(cx)}{2d^2} - \frac{b\arctan(cx)}{d^2(cx-i)} + \frac{2ib\arctan(cx)\ln(cx-i)}{d^2}\right.$

risch	$-\frac{3bc\ln(c^2x^2+1)}{8d^2}+\frac{ibc\arctan(cx)}{4d^2}-\frac{ib\ln(-icx+1)}{2d^2x}+\frac{ica}{d^2(-icx-1)}-\frac{2ica\ln(-icx)}{d^2}+\frac{bc\,\mathrm{dilog}(icx+1)}{d^2}+\frac{cb\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/d^2/(c*x-I)-2*a/d^2*arctan(c*x)-2*I*a/d^2*ln(c*x)-a/d^2/c/x+1/2*I*b/d
^2*arctan(c*x)-b/d^2*arctan(c*x)/(c*x-I)+2*I*b/d^2*arctan(c*x)*ln(c*x-I)-b/
d^2*arctan(c*x)/c/x+I*a/d^2*ln(c^2*x^2+1)+b/d^2*ln(-I*(-c*x+I))*ln(c*x)-b/d
^2*ln(-I*(-c*x+I))*ln(-I*c*x)-b/d^2*dilog(-I*c*x)-b/d^2*dilog(-I*(c*x+I))-b
/d^2*ln(c*x)*ln(-I*(c*x+I))-1/2*b/d^2*ln(c*x-I)^2+b/d^2*dilog(-1/2*I*(c*x+I
))+b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2*I*b/d^2*arctan(c*x)*ln(c*x)-1/2*b/d
^2*ln(c^2*x^2+1)+1/2*I*b/d^2/(c*x-I)+b/d^2*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - i
ntegrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2),
 x))*b - a*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/d^2
 + 1/(d^2*x))
```

Fricas [A]
time = 1.24, size = 182, normalized size = 0.94

$$-\frac{2\,(4\,a-i\,b)cx+4\,(bc^2x^2-i\,bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+4\,((2i\,a-b)c^2x^2+(2\,a+i\,b)cx)\log\left(x\right)+2\,(2i\,bcx+b)\log\left(-\frac{cx+i}{cx-i}\right)+3\,(bc^2x^2-i\,bcx)\log\left(\frac{cx+i}{c}\right)-((8i\,a-b)c^2x^2+(8\,a+i\,b)cx)\log\left(\frac{cx-i}{c}\right)-4i\,a}{4\,(cd^2x^2-i\,d^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(4*a - I*b)*c*x + 4*(b*c^2*x^2 - I*b*c*x)*dilog((c*x + I)/(c*x - I)
 + 1) + 4*((2*I*a - b)*c^2*x^2 + (2*a + I*b)*c*x)*log(x) + 2*(2*I*b*c*x + b
)*log(-(c*x + I)/(c*x - I)) + 3*(b*c^2*x^2 - I*b*c*x)*log((c*x + I)/c) - ((
8*I*a - b)*c^2*x^2 + (8*a + I*b)*c*x)*log((c*x - I)/c) - 4*I*a)/(c*d^2*x^2
- I*d^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(d+c\,d\,x\,1\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2), x)
```

3.57 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^3(d+icdx)^2}\,dx$

Optimal. Leaf size=244

$$-\frac{bc}{2d^2x}-\frac{bc^2}{2d^2(i-cx)}-\frac{a+b\text{ArcTan}(cx)}{2d^2x^2}+\frac{2ic(a+b\text{ArcTan}(cx))}{d^2x}-\frac{ic^2(a+b\text{ArcTan}(cx))}{d^2(i-cx)}-\frac{3ac^2\log(x)}{d^2}-\frac{2ibc}{}$$

```
[Out] -1/2*b*c/d^2/x-1/2*b*c^2/d^2/(I-c*x)+1/2*(-a-b*arctan(c*x))/d^2/x^2+2*I*c*(
a+b*arctan(c*x))/d^2/x-I*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-3*a*c^2*ln(x)/d^
2-2*I*b*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+I*b*c^2*l
n(c^2*x^2+1)/d^2-3/2*I*b*c^2*polylog(2,-I*c*x)/d^2+3/2*I*b*c^2*polylog(2,I*
c*x)/d^2-3/2*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^2
```

Rubi [A]
time = 0.20, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {4996, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438, 4972, 641, 46, 4964, 2449, 2352}

$$-\frac{ic^2(a+b\text{ArcTan}(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{a+b\text{ArcTan}(cx)}{2d^2x^2}+\frac{2ic(a+b\text{ArcTan}(cx))}{d^2x}-\frac{3ac^2\log(x)}{d^2}-\frac{3ibc^2\text{Li}_2(-icx)}{2d^2}+\frac{3ibc^2\text{Li}_2(icx)}{2d^2}-\frac{3ibc^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2d^2}+\frac{ibc^2\log\left(c^2x^2+1\right)}{d^2}-\frac{bc^2}{2d^2(-cx+i)}-\frac{2ibc^2\log(x)}{d^2}-\frac{bc}{2d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*x) - (b*c^2)/(2*d^2*(I - c*x)) - (a + b*ArcTan[c*x])/(2*d^2
*x^2) + ((2*I)*c*(a + b*ArcTan[c*x]))/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x]))
/(d^2*(I - c*x)) - (3*a*c^2*Log[x])/d^2 - ((2*I)*b*c^2*Log[x])/d^2 - (3*c^2
*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^2 + (I*b*c^2*Log[1 + c^2*x^2])/d
^2 - (((3*I)/2)*b*c^2*PolyLog[2, (-I)*c*x])/d^2 + (((3*I)/2)*b*c^2*PolyLog[
2, I*c*x])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^3} - \frac{2ic(a+b\tan^{-1}(cx))}{d^2x^2} - \frac{3c^2(a+b\tan^{-1}(cx))}{d^2x} - \frac{ic^3(a+b\tan}{d^2(-i+c}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(3c^2)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} - \frac{(ic^3)\int \frac{a+b\tan^{-}}{(-i+cx}}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} - \frac{ic^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{3ac^2\log(x)}{d^2} - \frac{3}{}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} - \frac{ic^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{3ac^2\log}{d^2}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} - \frac{ic^2(a+b\tan^{-1}}{d^2(i-cx)}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2}{2d^2(i-cx)} - \frac{bc^2\tan^{-1}(cx)}{2d^2} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} -$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2}{2d^2(i-cx)} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))}{d^2x} - \frac{ic^2(a+b\tan^{-1}(c}{d^2(i-cx)}$$

Mathematica [A]
time = 1.06, size = 219, normalized size = 0.90

$$\frac{-\frac{2a}{x^2} + \frac{8iac}{x} + \frac{4iac^2}{-i+cx} + 12iac^2\text{ArcTan}(cx) - 12ac^2\log(x) + 6ac^2\log(1+c^2x^2) + ibc^2\left(\frac{2i}{cx} + 12\text{ArcTan}(cx)^2 + \cos(2\text{ArcTan}(cx)) - 8\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) + 6\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right) - i\sin(2\text{ArcTan}(cx)) + 2\text{ArcTan}(cx)\left(i + \frac{i}{c^2x^2} + \frac{4}{cx} + i\cos(2\text{ArcTan}(cx)) + 6i\log\left(1 - e^{2i\text{ArcTan}(cx)}\right) + \sin(2\text{ArcTan}(cx))\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] ((-2*a)/x^2 + ((8*I)*a*c)/x + ((4*I)*a*c^2)/(-I + c*x) + (12*I)*a*c^2*ArcTa
n[c*x] - 12*a*c^2*Log[x] + 6*a*c^2*Log[1 + c^2*x^2] + I*b*c^2*(((2*I)/(c*x)
+ 12*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 8*Log[(c*x)/Sqrt[1 + c^2*x^2]] +
6*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*
(I + I/(c^2*x^2) + 4/(c*x) + I*Cos[2*ArcTan[c*x]] + (6*I)*Log[1 - E^((2*I)*
ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(4*d^2)
```

Maple [A]
time = 0.14, size = 345, normalized size = 1.41

method	result
derivativedivides	$c^2\left(\frac{ia}{d^2(cx-i)} + \frac{3a\ln(c^2x^2+1)}{2d^2} + \frac{2ib\arctan(cx)}{d^2cx} - \frac{a}{2d^2c^2x^2} + \frac{3ib\ln(cx)\ln(-icx+1)}{2d^2} - \frac{3a\ln(cx)}{d^2} - \frac{3ib\,\mathrm{dilog}(ic}{2d^2}\right.$

default	$c^2\left(\frac{ia}{d^2(cx-i)}+\frac{3a\ln(c^2x^2+1)}{2d^2}+\frac{2ib\arctan(cx)}{d^2cx}-\frac{a}{2d^2c^2x^2}+\frac{3ib\ln(cx)\ln(-icx+1)}{2d^2}-\frac{3a\ln(cx)}{d^2}-\frac{3ib\,\mathrm{dilog}(}{2d}\right.$
risch	$\frac{ib\,c^2\ln(icx+1)}{2d^2(icx+1)}-\frac{bc}{2d^2x}-\frac{b\,c^2\arctan(cx)}{4d^2}+\frac{3c^2a\ln(c^2x^2+1)}{2d^2}+\frac{c^2a}{d^2(-icx-1)}-\frac{3c^2a\ln(-icx)}{d^2}+\frac{3ic^2b\ln(\frac{1}{2}+\frac{i}{}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(I*a/d^2/(c*x-I)+3/2*a/d^2*ln(c^2*x^2+1)+2*I*b/d^2*arctan(c*x)/c/x-1/2*
a/d^2/c^2/x^2+3/2*I*b/d^2*ln(c*x)*ln(1-I*c*x)-3*a/d^2*ln(c*x)-3/2*I*b/d^2*d
ilog(-1/2*I*(c*x+I))+3*b/d^2*arctan(c*x)*ln(c*x-I)-1/2*b/d^2*arctan(c*x)/c^
2/x^2-3/2*I*b/d^2*dilog(1+I*c*x)-3*b/d^2*ln(c*x)*arctan(c*x)+I*b/d^2*ln(c^2
*x^2+1)+3*I*a/d^2*arctan(c*x)-3/2*I*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)+3/2*
I*b/d^2*dilog(1-I*c*x)+I*b/d^2*arctan(c*x)/(c*x-I)-3/2*I*b/d^2*ln(c*x)*ln(1
+I*c*x)+2*I*a/d^2/c/x-2*I*b/d^2*ln(c*x)+1/2*b/d^2/(c*x-I)-1/2*b/d^2/c/x+3/4
*I*b/d^2*ln(c*x-I)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] (-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) -
 integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3
), x))*b - 1/2*a*(-2*I*c^2/(c*d^2*x - I*d^2) - 6*c^2*log(c*x - I)/d^2 + 6*c
^2*log(x)/d^2 - (4*I*c*x - 1)/(d^2*x^2))
```

Fricas [A]
time = 1.66, size = 221, normalized size = 0.91

$$\frac{12i\,ac^2x^2+2\,(3\,a+i\,b)cx-6\,(-i\,bc^3x^3-bc^2x^2)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,((3\,a+2i\,b)c^3x^3+(-3i\,a+2\,b)c^2x^2)\log\left(x\right)-(6\,bc^2x^2-3i\,bcx+b)\log\left(-\frac{cx+i}{cx-i}\right)-4\,(-i\,bc^3x^3-bc^2x^2)\log\left(\frac{cx+i}{c}\right)+4\,((3\,a+i\,b)c^3x^3-(3i\,a-b)c^2x^2)\log\left(\frac{cx-i}{c}\right)+2i\,a}{4\,(cd^2x^3-i\,d^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(12*I*a*c^2*x^2 + 2*(3*a + I*b)*c*x - 6*(-I*b*c^3*x^3 - b*c^2*x^2)*dilo
g((c*x + I)/(c*x - I) + 1) - 4*((3*a + 2*I*b)*c^3*x^3 + (-3*I*a + 2*b)*c^2*
x^2)*log(x) - (6*b*c^2*x^2 - 3*I*b*c*x + b)*log(-(c*x + I)/(c*x - I)) - 4*(
-I*b*c^3*x^3 - b*c^2*x^2)*log((c*x + I)/c) + 4*((3*a + I*b)*c^3*x^3 - (3*I*
a - b)*c^2*x^2)*log((c*x - I)/c) + 2*I*a)/(c*d^2*x^3 - I*d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^5-2icx^4-x^3}\,dx+\int \frac{b\operatorname{atan}{(cx)}}{c^2x^5-2icx^4-x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**2,x)
```

```
[Out] -(Integral(a/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b*atan(c*x)/(c*
*2*x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2), x)
```

3.58 $\int \frac{x^4(a+b\text{ArcTan}(cx))}{(d+icdx)^3}\,dx$

Optimal. Leaf size=256

$$-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}+\frac{19ib\text{ArcTan}(cx)}{8c^5d^3}-\frac{3bx\text{ArcTan}(cx)}{c^4d^3}+\frac{ix^2(a+b\text{ArcTan}(cx))}{2c^3d^3}$$

```
[Out] -3*a*x/c^4/d^3-1/2*I*b*x/c^4/d^3-1/8*b/c^5/d^3/(I-c*x)^2-15/8*I*b/c^5/d^3/(
I-c*x)+19/8*I*b*arctan(c*x)/c^5/d^3-3*b*x*arctan(c*x)/c^4/d^3+1/2*I*x^2*(a+
b*arctan(c*x))/c^3/d^3-1/2*I*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2+4*(a+b*arc
tan(c*x))/c^5/d^3/(I-c*x)+6*I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^3+3/2
*b*ln(c^2*x^2+1)/c^5/d^3-3*b*polylog(2,1-2/(1+I*c*x))/c^5/d^3
```

Rubi [A]
time = 0.21, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {4996, 4930, 266, 4946, 327, 209, 4972, 641, 46, 4964, 2449, 2352}

$$\frac{4(a+b\text{ArcTan}(cx))}{c^5d^3(-cx+i)}-\frac{i(a+b\text{ArcTan}(cx))}{2c^5d^3(-cx+i)^2}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^5d^3}+\frac{ix^2(a+b\text{ArcTan}(cx))}{2c^3d^3}-\frac{3ax}{c^4d^3}+\frac{19ib\text{ArcTan}(cx)}{8c^5d^3}-\frac{3bx\text{ArcTan}(cx)}{c^4d^3}-\frac{3b\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{15ib}{8c^5d^3(-cx+i)}-\frac{b}{8c^5d^3(-cx+i)^2}-\frac{ibx}{2c^4d^3}+\frac{3b\log\left(c^2x^2+1\right)}{2c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (-3*a*x)/(c^4*d^3) - ((I/2)*b*x)/(c^4*d^3) - b/(8*c^5*d^3*(I - c*x)^2) - ((
(15*I)/8)*b)/(c^5*d^3*(I - c*x)) + (((19*I)/8)*b*ArcTan[c*x])/(c^5*d^3) - (
3*b*x*ArcTan[c*x])/(c^4*d^3) + ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c^3*d^3) -
((I/2)*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)^2) + (4*(a + b*ArcTan[c*x]))
/(c^5*d^3*(I - c*x)) + ((6*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^5*
d^3) + (3*b*Log[1 + c^2*x^2])/(2*c^5*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+icdx)^3}\,dx &= \int \left(-\frac{3(a+b\tan^{-1}(cx))}{c^4d^3}+\frac{ix(a+b\tan^{-1}(cx))}{c^3d^3}+\frac{i(a+b\tan^{-1}(cx))}{c^4d^3(-i+cx)^3}+\frac{4(a+}{c^4d^3}\right.\\
&= \frac{i\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{c^4d^3}-\frac{(6i)\int\frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^4d^3}-\frac{3\int\left(a+b\tan^{-1}(cx)\right)dx}{c^4d^3}+\frac{4\int\frac{a+}{}}{}\\
&= -\frac{3ax}{c^4d^3}+\frac{ix^2(a+b\tan^{-1}(cx))}{2c^3d^3}-\frac{i(a+b\tan^{-1}(cx))}{2c^5d^3(i-cx)^2}+\frac{4(a+b\tan^{-1}(cx))}{c^5d^3(i-cx)}+\frac{6i}{}\\
&= -\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2(a+b\tan^{-1}(cx))}{2c^3d^3}-\frac{i(a+b\tan^{-1}(cx))}{2c^5d^3(i-cx)^2}\\
&= -\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}+\frac{ib\tan^{-1}(cx)}{2c^5d^3}-\frac{3bx\tan^{-1}(cx)}{c^4d^3}+\frac{ix^2(a+b\tan^{-1}(cx))}{2c^3d^3}-\frac{i(a}{2c}\\
&= -\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}+\frac{ib\tan^{-1}(cx)}{2c^5d^3}-\frac{3bx\tan^{-1}(}{c^4d^3}\\
&= -\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}+\frac{19ib\tan^{-1}(cx)}{8c^5d^3}-\frac{3bx\tan}{c^4d}
\end{aligned}$$

Mathematica [A]
time = 0.65, size = 235, normalized size = 0.92

$$\frac{-96acx+16iac^2x^2-\frac{16ia}{(-i+cx)^2}-\frac{128a}{-i+cx}+192a\text{ArcTan}(cx)-96ia\log\left(1+c^2x^2\right)+b\left(-16icx+192\text{ArcTan}(cx)^2-28\cos(2\text{ArcTan}(cx))+\cos(4\text{ArcTan}(cx))+48\log\left(1+c^2x^2\right)+96\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+28i\sin(2\text{ArcTan}(cx))+4i\text{ArcTan}(cx)\left(4+24icx+4c^2x^2-14\cos(2\text{ArcTan}(cx))+\cos(4\text{ArcTan}(cx))+48\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+14i\sin(2\text{ArcTan}(cx))-i\sin(4\text{ArcTan}(cx))\right)-i\sin(4\text{ArcTan}(cx))\right)}{32c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (-96*a*c*x + (16*I)*a*c^2*x^2 - ((16*I)*a)/(-I + c*x)^2 - (128*a)/(-I + c*x
) + 192*a*ArcTan[c*x] - (96*I)*a*Log[1 + c^2*x^2] + b*((-16*I)*c*x + 192*Ar
cTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + c^2*x
^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan[c*x]] + (
4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*x]] + Cos[
4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2*ArcTan[c*
x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]))/(32*c^5*d^3)
```

Maple [A]
time = 0.16, size = 364, normalized size = 1.42

method	result
derivativedivides	$-\frac{3acx}{d^3}-\frac{ibcx}{2d^3}+\frac{6a\arctan(cx)}{d^3}+\frac{ia\,c^2x^2}{2d^3}-\frac{6ib\arctan(cx)\ln(cx-i)}{d^3}-\frac{4a}{d^3(cx-i)}-\frac{3b\arctan(cx)cx}{d^3}+\frac{ib\arctan(cx)c^2x^2}{2d^3}-\frac{5ib\arctan\left(\frac{cx}{2}-\right.}{16d^3}$
default	$-\frac{3acx}{d^3}-\frac{ibcx}{2d^3}+\frac{6a\arctan(cx)}{d^3}+\frac{ia\,c^2x^2}{2d^3}-\frac{6ib\arctan(cx)\ln(cx-i)}{d^3}-\frac{4a}{d^3(cx-i)}-\frac{3b\arctan(cx)cx}{d^3}+\frac{ib\arctan(cx)c^2x^2}{2d^3}-\frac{5ib\arctan\left(\frac{cx}{2}-\right.}{16d^3}$
risch	$-\frac{3ax}{c^4d^3}+\frac{63b\ln(c^2x^2+1)}{32c^5d^3}+\frac{6a\arctan(cx)}{d^3c^5}-\frac{3i\ln(-icx+1)xb}{2d^3c^4}-\frac{b\ln(-icx+1)x^2}{16d^3c^3(-icx-1)^2}+\frac{ib\ln(-icx+1)x}{d^3c^4(-icx-1)}+\frac{ib\ln(-}{8d^3c^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-3*a/d^3*c*x-1/2*I*b/d^3*c*x+6*a/d^3*arctan(c*x)+1/2*I*a/d^3*c^2*x^2
-6*I*b/d^3*arctan(c*x)*ln(c*x-I)-4*a/d^3/(c*x-I)-3*b/d^3*arctan(c*x)*c*x+1/
2*I*b/d^3*arctan(c*x)*c^2*x^2-5/16*I*b/d^3*arctan(1/2*c*x-1/2*I)+43/16*I*b/
d^3*arctan(c*x)-4*b/d^3*arctan(c*x)/(c*x-I)-1/2*b/d^3+5/32*I*b/d^3*arctan(1
/2*c*x)-1/2*I*b/d^3*arctan(c*x)/(c*x-I)^2-1/8*b/d^3/(c*x-I)^2+43/32*b/d^3*l
n(c^2*x^2+1)-3*I*a/d^3*ln(c^2*x^2+1)+5/64*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-5/
32*I*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-1/2*I*a/d^3/(c*x-I)^2+15/8*I*b/d^3/(
c*x-I)+3/2*b/d^3*ln(c*x-I)^2-3*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3*b/d^3*d
ilog(-1/2*I*(c*x+I)))
```

Maxima [A]
time = 0.36, size = 358, normalized size = 1.40

$$\frac{8i\,ac^4x^4-8\,(4\,a+i\,b)c^3x^3+(b(5i\,\arctan(1,cx)-16)+88i\,a)c^2x^2+2\,(b(5\,\arctan(1,cx)+19i)-8\,a)cx+24\,(bc^2x^2-2i\,bcx-b)\arctan(cx)^2+6\,(bc^2x^2-2i\,bcx-b)\log(c^2x^2+1)^2-24\,(i\,bc^2x^2+2\,bcx-i\,b)\arctan(cx)\log\left(\frac{1}{4}\,c^2x^2+\frac{1}{4}\right)+b(-5i\,\arctan(1,cx)+28)+(8i\,bc^4x^4-32\,bc^3x^3+(96\,a+131i\,b)c^2x^2-2\,(96i\,a-35\,b)cx-96\,a+13i\,b)\arctan(cx)-48\,(bc^2x^2-2i\,bcx-b)\mathrm{Li}_2\left(\frac{1}{2}i\,cx+\frac{1}{2}\right)-12\,(2\,(2i\,a-b)c^2x^2+4\,(2\,a+i\,b)cx+(bc^2x^2-2i\,bcx-b)\log\left(\frac{1}{4}\,c^2x^2+\frac{1}{4}\right)-4i\,a+2\,b)\log(c^2x^2+1)+56i\,a}{16\,(c^7d^3x^2-2i\,c^6d^3x-c^5d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(8*I*a*c^4*x^4 - 8*(4*a + I*b)*c^3*x^3 + (b*(5*I*arctan2(1, c*x) - 16)
 + 88*I*a)*c^2*x^2 + 2*(b*(5*arctan2(1, c*x) + 19*I) - 8*a)*c*x + 24*(b*c^2
*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + 6*(b*c^2*x^2 - 2*I*b*c*x - b)*log(c^2
*x^2 + 1)^2 - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*log(1/4*c^2*x^2
+ 1/4) + b*(-5*I*arctan2(1, c*x) + 28) + (8*I*b*c^4*x^4 - 32*b*c^3*x^3 + (9
6*a + 131*I*b)*c^2*x^2 - 2*(96*I*a - 35*b)*c*x - 96*a + 13*I*b)*arctan(c*x)
 - 48*(b*c^2*x^2 - 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 12*(2*(2*I*a - b
)*c^2*x^2 + 4*(2*a + I*b)*c*x + (b*c^2*x^2 - 2*I*b*c*x - b)*log(1/4*c^2*x^2
 + 1/4) - 4*I*a + 2*b)*log(c^2*x^2 + 1) + 56*I*a)/(c^7*d^3*x^2 - 2*I*c^6*d^
3*x - c^5*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(b*x^4*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^4)/(c^3*d^3*x^3 -
3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
[Out] int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.59 $\int \frac{x^3(a+b\text{ArcTan}(cx))}{(d+icdx)^3}\,dx$

Optimal. Leaf size=225

$$\frac{iax}{c^3d^3}+\frac{ib}{8c^4d^3(i-cx)^2}-\frac{11b}{8c^4d^3(i-cx)}+\frac{11b\text{ArcTan}(cx)}{8c^4d^3}+\frac{ibx\text{ArcTan}(cx)}{c^3d^3}-\frac{a+b\text{ArcTan}(cx)}{2c^4d^3(i-cx)^2}-\frac{3i(a+b\text{ArcTa}}{c^4d^3(i-c}$$

```
[Out] I*a*x/c^3/d^3+1/8*I*b/c^4/d^3/(I-c*x)^2-11/8*b/c^4/d^3/(I-c*x)+11/8*b*arcta
n(c*x)/c^4/d^3+I*b*x*arctan(c*x)/c^3/d^3+1/2*(-a-b*arctan(c*x))/c^4/d^3/(I-
c*x)^2-3*I*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)+3*(a+b*arctan(c*x))*ln(2/(1+I*
c*x))/c^4/d^3-1/2*I*b*ln(c^2*x^2+1)/c^4/d^3+3/2*I*b*polylog(2,1-2/(1+I*c*x)
)/c^4/d^3
```

Rubi [A]
time = 0.18, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4930, 266, 4972, 641, 46, 209, 4964, 2449, 2352}

$$-\frac{3i(a+b\text{ArcTan}(cx))}{c^4d^3(-cx+i)}-\frac{a+b\text{ArcTan}(cx)}{2c^4d^3(-cx+i)^2}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4d^3}+\frac{iax}{c^3d^3}+\frac{11b\text{ArcTan}(cx)}{8c^4d^3}+\frac{ibx\text{ArcTan}(cx)}{c^3d^3}+\frac{3ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(-cx+i)}+\frac{ib}{8c^4d^3(-cx+i)^2}-\frac{ib\log\left(c^2x^2+1\right)}{2c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (I*a*x)/(c^3*d^3) + ((I/8)*b)/(c^4*d^3*(I - c*x)^2) - (11*b)/(8*c^4*d^3*(I
- c*x)) + (11*b*ArcTan[c*x])/(8*c^4*d^3) + (I*b*x*ArcTan[c*x])/(c^3*d^3) -
(a + b*ArcTan[c*x])/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x]))/(
c^4*d^3*(I - c*x)) + (3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^3) -
 ((I/2)*b*Log[1 + c^2*x^2])/(c^4*d^3) + (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^4*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+icdx)^3}\,dx = \int \left(\frac{i(a+b\tan^{-1}(cx))}{c^3d^3} + \frac{a+b\tan^{-1}(cx)}{c^3d^3(-i+cx)^3} - \frac{3i(a+b\tan^{-1}(cx))}{c^3d^3(-i+cx)^2} - \frac{3(a+b\tan\right.$$

$$= \frac{i\int (a+b\tan^{-1}(cx))\,dx}{c^3d^3} - \frac{(3i)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^3d^3} + \frac{\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{c^3d^3} - \frac{3\int \frac{a+b}{}}{c}$$

$$= \frac{iax}{c^3d^3} - \frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2} - \frac{3i(a+b\tan^{-1}(cx))}{c^4d^3(i-cx)} + \frac{3(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^3}$$

$$= \frac{iax}{c^3d^3} + \frac{ibx\tan^{-1}(cx)}{c^3d^3} - \frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2} - \frac{3i(a+b\tan^{-1}(cx))}{c^4d^3(i-cx)} + \frac{3(a+b\tan^{-}}{}$$

$$= \frac{iax}{c^3d^3} + \frac{ibx\tan^{-1}(cx)}{c^3d^3} - \frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2} - \frac{3i(a+b\tan^{-1}(cx))}{c^4d^3(i-cx)} + \frac{3(a+b\tan^{-}}{}$$

$$= \frac{iax}{c^3d^3} + \frac{ib}{8c^4d^3(i-cx)^2} - \frac{11b}{8c^4d^3(i-cx)} + \frac{ibx\tan^{-1}(cx)}{c^3d^3} - \frac{a+b\tan^{-1}(cx)}{2c^4d^3(i-cx)^2} - \frac{3}{}$$

$$= \frac{iax}{c^3d^3} + \frac{ib}{8c^4d^3(i-cx)^2} - \frac{11b}{8c^4d^3(i-cx)} + \frac{11b\tan^{-1}(cx)}{8c^4d^3} + \frac{ibx\tan^{-1}(cx)}{c^3d^3} - \frac{a-}{2c}$$

Mathematica [A]
time = 0.52, size = 216, normalized size = 0.96

$$\frac{32iacx - \frac{16a}{(-i+cx)^2} + \frac{96ia}{-i+cx} - 96ia\mathrm{ArcTan}(cx) - 48a\log(1+c^2x^2) + ib\left(-96\mathrm{ArcTan}(cx)^2 + 20\cos(2\mathrm{ArcTan}(cx)) - \cos(4\mathrm{ArcTan}(cx)) - 16\log(1+c^2x^2) - 48\mathrm{PolyLog}(2,-e^{2i\mathrm{ArcTan}(cx)}) - 20i\sin(2\mathrm{ArcTan}(cx)) + 4\mathrm{ArcTan}(cx)\left(8cx + 10i\cos(2\mathrm{ArcTan}(cx)) - i\cos(4\mathrm{ArcTan}(cx)) - 24i\log(1+e^{2i\mathrm{ArcTan}(cx)}) + 10\sin(2\mathrm{ArcTan}(cx)) - \sin(4\mathrm{ArcTan}(cx))\right) + i\sin(4\mathrm{ArcTan}(cx))\right)}{32c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] ((32*I)*a*c*x - (16*a)/(-I + c*x)^2 + ((96*I)*a)/(-I + c*x) - (96*I)*a*ArcT
an[c*x] - 48*a*Log[1 + c^2*x^2] + I*b*(-96*ArcTan[c*x]^2 + 20*Cos[2*ArcTan[
c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*PolyLog[2, -E^((2*I)*
ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]*(8*c*x + (10*I)*C
os[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 + E^((2*I)*ArcTan[c
*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]])
)/(32*c^4*d^3)
```

Maple [A]
time = 0.14, size = 321, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{3ib\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}-\frac{19ib\ln\left(c^2x^2+1\right)}{32d^3}-\frac{a}{2d^3(cx-i)^2}+\frac{3ib\arctan(cx)}{d^3(cx-i)}+\frac{ib}{8d^3(cx-i)^2}-\frac{3b\arctan(cx)\ln(cx-i)}{d^3}-}{}$
default	$\frac{\frac{3ib\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}-\frac{19ib\ln\left(c^2x^2+1\right)}{32d^3}-\frac{a}{2d^3(cx-i)^2}+\frac{3ib\arctan(cx)}{d^3(cx-i)}+\frac{ib}{8d^3(cx-i)^2}-\frac{3b\arctan(cx)\ln(cx-i)}{d^3}-}{}$
risch	$-\frac{3b}{2c^4d^3(-cx+i)}+\frac{11b\arctan(cx)}{16c^4d^3}-\frac{27ib\ln(c^2x^2+1)}{32c^4d^3}+\frac{3ib\ln(-icx+1)}{16d^3c^4(-icx-1)^2}+\frac{3b\ln(-icx+1)x}{4d^3c^3(-icx-1)}-\frac{3ia\arctan(cx)}{d^3c^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(3/64*I*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-3/2*a/d^3*ln(c^2*x^2+1)-19/32*
I*b/d^3*ln(c^2*x^2+1)-1/2*a/d^3/(c*x-I)^2+3*I*b/d^3*arctan(c*x)/(c*x-I)+1/8
*I*b/d^3/(c*x-I)^2-3*b/d^3*arctan(c*x)*ln(c*x-I)-1/2*b/d^3*arctan(c*x)/(c*x
-I)^2+3*I*a/d^3/(c*x-I)-3/4*I*b/d^3*ln(c*x-I)^2+I*b/d^3*arctan(c*x)*c*x+19/
16*b*arctan(c*x)/d^3+11/8*b/d^3/(c*x-I)+3/2*I*b/d^3*ln(-1/2*I*(c*x+I))*ln(c
*x-I)-3/32*b/d^3*arctan(1/2*c*x)+3/32*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)+3/1
6*b/d^3*arctan(1/2*c*x-1/2*I)-3*I*a/d^3*arctan(c*x)+3/2*I*b/d^3*dilog(-1/2*
I*(c*x+I))+I*a/d^3*c*x)
```

Maxima [A]
time = 0.34, size = 334, normalized size = 1.48

$$-\frac{-16i\,ac^3x^3-32\,ac^2x^2-2\,(16i\,a+11\,b)cx-12\,(-i\,bc^2x^2-2\,bcx+i\,b)\arctan(cx)^2-3\,(-i\,bc^2x^2-2\,bcx+i\,b)\log(c^2x^2+1)^2+12\,(bc^2x^2-2i\,bcx-b)\arctan(cx)\log\left(\frac{1}{4}c^2x^2+\frac{1}{4}\right)+(-16i\,bc^3x^3-3\,(-16i\,a+17\,b)c^2x^2+6\,(16\,a+i\,b)cx-48i\,a-21\,b)\arctan(cx)+3\,(bc^2x^2-2i\,bcx-b)\arctan(cx,-1)-24\,(i\,bc^2x^2+2\,bcx-i\,b)\mathrm{Li}_2\left(\frac{1}{2}i\,cx+\frac{1}{2}\right)+2\,(4\,(3\,a+i\,b)c^2x^2-8\,(3i\,a-b)cx-3\,(i\,bc^2x^2+2\,bcx-i\,b)\log\left(\frac{1}{4}c^2x^2+\frac{1}{4}\right)-12\,a-4i\,b)\log(c^2x^2+1)-40\,a+20i\,b}{16\,(c^6d^3x^2-2i\,c^5d^3x-c^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(-16*I*a*c^3*x^3 - 32*a*c^2*x^2 - 2*(16*I*a + 11*b)*c*x - 12*(-I*b*c^
2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^2 - 3*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*log
(c^2*x^2 + 1)^2 + 12*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(1/4*c^2*x^
2 + 1/4) + (-16*I*b*c^3*x^3 - 3*(-16*I*a + 17*b)*c^2*x^2 + 6*(16*a + I*b)*c
*x - 48*I*a - 21*b)*arctan(c*x) + 3*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x
, -1) - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 2*(4*(3*a
 + I*b)*c^2*x^2 - 8*(3*I*a - b)*c*x - 3*(I*b*c^2*x^2 + 2*b*c*x - I*b)*log(1
/4*c^2*x^2 + 1/4) - 12*a - 4*I*b)*log(c^2*x^2 + 1) - 40*a + 20*I*b)/(c^6*d^
3*x^2 - 2*I*c^5*d^3*x - c^4*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c^3*d^3*x^3 -
3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.60 $\int \frac{x^2(a+b\text{ArcTan}(cx))}{(d+icdx)^3}\,dx$

Optimal. Leaf size=176

$$\frac{b}{8c^3d^3(i-cx)^2}+\frac{7ib}{8c^3d^3(i-cx)}-\frac{7ib\text{ArcTan}(cx)}{8c^3d^3}+\frac{i(a+b\text{ArcTan}(cx))}{2c^3d^3(i-cx)^2}-\frac{2(a+b\text{ArcTan}(cx))}{c^3d^3(i-cx)}-\frac{i(a+b\text{ArcTan}}{c^3}$$

```
[Out] 1/8*b/c^3/d^3/(I-c*x)^2+7/8*I*b/c^3/d^3/(I-c*x)-7/8*I*b*arctan(c*x)/c^3/d^3
+1/2*I*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))/c^3/d^3/(I-c
*x)-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d^3+1/2*b*polylog(2,1-2/(1+I*c*
x))/c^3/d^3
```

Rubi [A]

time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4996, 4972, 641, 46, 209, 4964, 2449, 2352}

$$-\frac{2(a+b\text{ArcTan}(cx))}{c^3d^3(-cx+i)}+\frac{i(a+b\text{ArcTan}(cx))}{2c^3d^3(-cx+i)^2}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^3d^3}-\frac{7ib\text{ArcTan}(cx)}{8c^3d^3}+\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{7ib}{8c^3d^3(-cx+i)}+\frac{b}{8c^3d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] b/(8*c^3*d^3*(I - c*x)^2) + (((7*I)/8)*b)/(c^3*d^3*(I - c*x)) - (((7*I)/8)*
b*ArcTan[c*x])/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)^2
) - (2*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (I*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+icdx)^3}\,dx = \int \left(-\frac{i(a+b\tan^{-1}(cx))}{c^2d^3(-i+cx)^3} - \frac{2(a+b\tan^{-1}(cx))}{c^2d^3(-i+cx)^2} + \frac{i(a+b\tan^{-1}(cx))}{c^2d^3(-i+cx)}\right)dx$$

$$= -\frac{i\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{c^2d^3} + \frac{i\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{c^2d^3} - \frac{2\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{c^2d^3}$$

$$= \frac{i(a+b\tan^{-1}(cx))}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))}{c^3d^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^3} - \frac{(ib)}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))}{c^3d^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^3} + \frac{b\mathrm{Su}}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))}{c^3d^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^3} + \frac{b\mathrm{Li}_2}{}$$

$$= \frac{b}{8c^3d^3(i-cx)^2} + \frac{7ib}{8c^3d^3(i-cx)} + \frac{i(a+b\tan^{-1}(cx))}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))}{c^3d^3(i-cx)} - \frac{i(}{}$$

$$= \frac{b}{8c^3d^3(i-cx)^2} + \frac{7ib}{8c^3d^3(i-cx)} - \frac{7ib\tan^{-1}(cx)}{8c^3d^3} + \frac{i(a+b\tan^{-1}(cx))}{2c^3d^3(i-cx)^2} - \frac{2(a+b}{c^3d^3}$$

Mathematica [A]
time = 0.40, size = 187, normalized size = 1.06

$$\frac{\frac{16ia}{(-i+cx)^2} + \frac{64a}{-i+cx} - 32a\mathrm{ArcTan}(cx) + 16ia\log\left(1+c^2x^2\right) - b\left(32\mathrm{ArcTan}(cx)^2 - 12\cos(2\mathrm{ArcTan}(cx)) + \cos(4\mathrm{ArcTan}(cx)) + 16\mathrm{PolyLog}\left(2, -e^{2i\mathrm{ArcTan}(cx)}\right) + 12i\sin(2\mathrm{ArcTan}(cx)) - i\sin(4\mathrm{ArcTan}(cx)) + 4\mathrm{ArcTan}(cx)\left(-6i\cos(2\mathrm{ArcTan}(cx)) + i\cos(4\mathrm{ArcTan}(cx)) + 8i\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) - 6\sin(2\mathrm{ArcTan}(cx)) + \sin(4\mathrm{ArcTan}(cx))\right)\right)}{32c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (((16*I)*a)/(-I + c*x)^2 + (64*a)/(-I + c*x) - 32*a*ArcTan[c*x] + (16*I)*a*
Log[1 + c^2*x^2] - b*(32*ArcTan[c*x]^2 - 12*Cos[2*ArcTan[c*x]] + Cos[4*ArcT
an[c*x]] + 16*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (12*I)*Sin[2*ArcTan[c*x]
] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*((-6*I)*Cos[2*ArcTan[c*x]] + I*Cos
[4*ArcTan[c*x]] + (8*I)*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*Sin[2*ArcTan[c*x
]] + Sin[4*ArcTan[c*x]])))/(32*c^3*d^3)
```

Maple [A]
time = 0.12, size = 299, normalized size = 1.70

method	result
derivativedivides	$\frac{ia}{2d^3(cx-i)^2} - \frac{a\arctan(cx)}{d^3} + \frac{7ib\arctan\left(\frac{cx}{2}\right)}{32d^3} + \frac{2a}{d^3(cx-i)} + \frac{ia\ln\left(c^2x^2+1\right)}{2d^3} - \frac{7ib\arctan(cx)}{16d^3} + \frac{2b\arctan(cx)}{d^3(cx-i)} - \frac{7ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{32d^3}$

default	$\frac{\frac{ia}{2d^3(cx-i)^2}-\frac{a\arctan(cx)}{d^3}+\frac{7ib\arctan\left(\frac{cx}{2}\right)}{32d^3}+\frac{2a}{d^3(cx-i)}+\frac{ia\ln\left(c^2x^2+1\right)}{2d^3}-\frac{7ib\arctan(cx)}{16d^3}+\frac{2b\arctan(cx)}{d^3(cx-i)}-\frac{7ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right.}{32d^3}}{}$
risch	$\frac{b\ln(icx+1)^2}{4c^3d^3}+\frac{\left(-\frac{ibx}{c^2}-\frac{3b}{4c^3}\right)\ln(icx+1)}{d^3(cx-i)^2}-\frac{ib\ln(-icx+1)x}{8d^3c^2(-icx-1)^2}-\frac{7ib\arctan(cx)}{16c^3d^3}+\frac{b}{8c^3d^3(-cx+i)^2}-\frac{2ia}{d^3c^3(-icx-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/2*I*a/d^3/(c*x-I)^2-a/d^3*arctan(c*x)+7/32*I*b/d^3*arctan(1/2*c*x)
+2*a/d^3/(c*x-I)+1/2*I*a/d^3*ln(c^2*x^2+1)-7/16*I*b/d^3*arctan(c*x)+2*b/d^3
*arctan(c*x)/(c*x-I)-7/32*I*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)+1/8*b/d^3/(c*
x-I)^2-7/32*b/d^3*ln(c^2*x^2+1)-7/16*I*b/d^3*arctan(1/2*c*x-1/2*I)+7/64*b/d
^3*ln(c^4*x^4+10*c^2*x^2+9)-7/8*I*b/d^3/(c*x-I)+I*b/d^3*arctan(c*x)*ln(c*x-
I)+1/2*I*b/d^3*arctan(c*x)/(c*x-I)^2-1/4*b/d^3*ln(c*x-I)^2+1/2*b/d^3*ln(c*x
-I)*ln(-1/2*I*(c*x+I))+1/2*b/d^3*dilog(-1/2*I*(c*x+I)))
```

Maxima [A]
time = 0.32, size = 291, normalized size = 1.65

$$-\frac{-7i\,bc^2x^2\arctan\left(1,cx\right)-2\left(7\,b(\arctan\left(1,cx\right)-i)+16\,a\right)cx+4\left(bc^2x^2-2i\,bcx-b\right)\arctan\left(cx\right)^2+\left(bc^2x^2-2i\,bcx-b\right)\log\left(c^2x^2+1\right)^2-4\left(i\,bc^2x^2+2\,bcx-i\,b\right)\arctan\left(cx\right)\log\left(\frac{1}{4}\,c^2x^2+\frac{1}{4}\right)+b(7i\arctan\left(1,cx\right)+12)+\left((16\,a+7i\,b)c^2x^2-2\left(16i\,a+9\,b\right)cx-16\,a+17i\,b\right)\arctan\left(cx\right)-8\left(bc^2x^2-2i\,bcx-b\right)\mathrm{Li}_2\left(\frac{1}{2}i\,cx+\frac{1}{2}\right)-2\left(4i\,ac^2x^2+8\,acx+\left(bc^2x^2-2i\,bcx-b\right)\log\left(\frac{1}{4}\,c^2x^2+\frac{1}{4}\right)-4i\,a\right)\log\left(c^2x^2+1\right)+24i\,a}{16\left(c^5d^3x^2-2i\,c^4d^3x-c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(-7*I*b*c^2*x^2*arctan2(1, c*x) - 2*(7*b*(arctan2(1, c*x) - I) + 16*a
)*c*x + 4*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + (b*c^2*x^2 - 2*I*b*c*
x - b)*log(c^2*x^2 + 1)^2 - 4*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*log
(1/4*c^2*x^2 + 1/4) + b*(7*I*arctan2(1, c*x) + 12) + ((16*a + 7*I*b)*c^2*x^
2 - 2*(16*I*a + 9*b)*c*x - 16*a + 17*I*b)*arctan(c*x) - 8*(b*c^2*x^2 - 2*I*
b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 2*(4*I*a*c^2*x^2 + 8*a*c*x + (b*c^2*x^2
 - 2*I*b*c*x - b)*log(1/4*c^2*x^2 + 1/4) - 4*I*a)*log(c^2*x^2 + 1) + 24*I*a
)/(c^5*d^3*x^2 - 2*I*c^4*d^3*x - c^3*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c^3*d^3*x^3 -
3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.61 $\int \frac{x(a+b\text{ArcTan}(cx))}{(d+icdx)^3}\,dx$

Optimal. Leaf size=88

$$-\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{b\text{ArcTan}(cx)}{8c^2d^3}+\frac{x^2(a+b\text{ArcTan}(cx))}{2d^3(1+icx)^2}$$

```
[Out] -1/8*I*b/c^2/d^3/(I-c*x)^2+3/8*b/c^2/d^3/(I-c*x)+1/8*b*arctan(c*x)/c^2/d^3+
1/2*x^2*(a+b*arctan(c*x))/d^3/(1+I*c*x)^2
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {37, 4992, 12, 90, 209}

$$\frac{x^2(a+b\text{ArcTan}(cx))}{2d^3(1+icx)^2}+\frac{b\text{ArcTan}(cx)}{8c^2d^3}+\frac{3b}{8c^2d^3(-cx+i)}-\frac{ib}{8c^2d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] ((-1/8*I)*b)/(c^2*d^3*(I - c*x)^2) + (3*b)/(8*c^2*d^3*(I - c*x)) + (b*ArcTa
n[c*x])/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x]))/(2*d^3*(1 + I*c*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4992

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
+ b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2), x
], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2*m
] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+b\tan^{-1}(cx))}{(d+icdx)^3}\,dx &= \frac{x^2(a+b\tan^{-1}(cx))}{2d^3(1+icx)^2}-(bc)\int \frac{x^2}{2d^3(i-cx)^3(i+cx)}\,dx\\
&= \frac{x^2(a+b\tan^{-1}(cx))}{2d^3(1+icx)^2}-\frac{(bc)\int \frac{x^2}{(i-cx)^3(i+cx)}\,dx}{2d^3}\\
&= \frac{x^2(a+b\tan^{-1}(cx))}{2d^3(1+icx)^2}-\frac{(bc)\int \left(-\frac{i}{2c^2(-i+cx)^3}-\frac{3}{4c^2(-i+cx)^2}-\frac{1}{4c^2(1+c^2x^2)}\right)dx}{2d^3}\\
&= -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{x^2(a+b\tan^{-1}(cx))}{2d^3(1+icx)^2}+\frac{b\int \frac{1}{1+c^2x^2}\,dx}{8cd^3}\\
&= -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{b\tan^{-1}(cx)}{8c^2d^3}+\frac{x^2(a+b\tan^{-1}(cx))}{2d^3(1+icx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.72

$$\frac{b(2i-3cx)+a(-4-8icx)-b(1+2icx+3c^2x^2)\,\text{ArcTan}(cx)}{8c^2d^3(-i+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

```
[Out] (b*(2*I - 3*c*x) + a*(-4 - (8*I)*c*x) - b*(1 + (2*I)*c*x + 3*c^2*x^2)*ArcTa
n[c*x])/(8*c^2*d^3*(-I + c*x)^2)
```

Maple [A]
time = 0.15, size = 109, normalized size = 1.24

method	result

derivativedivides	$\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$
default	$\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$
risch	$-\frac{b(2cx-i)\ln(icx+1)}{4c^2d^3(cx-i)^2}-\frac{i(-3\ln(-cx+i)b\,c^2x^2+3\ln(cx+i)b\,c^2x^2+6i\ln(-cx+i)bcx-6i\ln(cx+i)bcx+8ibcx\ln(-icx+}{16c^2d^3(cx-i)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a/d^3*(1/2/(c*x-I)^2-I/(c*x-I))+1/2*b/d^3*arctan(c*x)/(c*x-I)^2-I*b/
d^3*arctan(c*x)/(c*x-I)-3/8*b*arctan(c*x)/d^3-1/8*I*b/d^3/(c*x-I)^2-3/8*b/d
^3/(c*x-I))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 0.80

$$-\frac{(8i\,a+3\,b)cx+(3\,bc^2x^2+2i\,bcx+b)\arctan{(cx)}+4\,a-2i\,b}{8\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((8*I*a + 3*b)*c*x + (3*b*c^2*x^2 + 2*I*b*c*x + b)*arctan(c*x) + 4*a -
 2*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

Fricas [A]
time = 2.10, size = 85, normalized size = 0.97

$$-\frac{2\,(8i\,a+3\,b)cx-(-3i\,bc^2x^2+2\,bcx-i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+8\,a-4i\,b}{16\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(8*I*a + 3*b)*c*x - (-3*I*b*c^2*x^2 + 2*b*c*x - I*b)*log(-(c*x + I
)/(c*x - I)) + 8*a - 4*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(75) = 150.
time = 5.63, size = 189, normalized size = 2.15

$$\frac{b\left(\frac{3i\log\left(x-\frac{i}{c}\right)}{16}-\frac{3i\log\left(x+\frac{i}{c}\right)}{16}\right)}{c^2d^3}+\frac{(-2bcx+ib)\log\left(icx+1\right)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{(2bcx-ib)\log\left(-icx+1\right)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{-4a+2ib+x(-8iac-3bc)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] b*(3*I*log(x - I/c)/16 - 3*I*log(x + I/c)/16)/(c**2*d**3) + (-2*b*c*x + I*b
)*log(I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*d**3) + (2*b*
c*x - I*b)*log(-I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*d**
3) + (-4*a + 2*I*b + x*(-8*I*a*c - 3*b*c))/(8*c**4*d**3*x**2 - 16*I*c**3*d*
*3*x - 8*c**2*d**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{atan}(c\,x)\right)}{{\left(d+c\,d\,x\,1\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

```
[Out] int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.62 $\int \frac{a+b\text{ArcTan}(cx)}{(d+icdx)^3}\,dx$

Optimal. Leaf size=92

$$-\frac{b}{8cd^3(i-cx)^2}+\frac{ib}{8cd^3(i-cx)}-\frac{ib\text{ArcTan}(cx)}{8cd^3}+\frac{i(a+b\text{ArcTan}(cx))}{2cd^3(1+icx)^2}$$

```
[Out] -1/8*b/c/d^3/(I-c*x)^2+1/8*I*b/c/d^3/(I-c*x)-1/8*I*b*arctan(c*x)/c/d^3+1/2*
I*(a+b*arctan(c*x))/c/d^3/(1+I*c*x)^2
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4972, 641, 46, 209}

$$\frac{i(a+b\text{ArcTan}(cx))}{2cd^3(1+icx)^2}-\frac{ib\text{ArcTan}(cx)}{8cd^3}+\frac{ib}{8cd^3(-cx+i)}-\frac{b}{8cd^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

```
[Out] -1/8*b/(c*d^3*(I - c*x)^2) + ((I/8)*b)/(c*d^3*(I - c*x)) - ((I/8)*b*ArcTan[
c*x])/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c*d^3*(1 + I*c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(d+icdx)^3}\,dx &= \frac{i(a+b\tan^{-1}(cx))}{2cd^3(1+icx)^2} - \frac{(ib)\int \frac{1}{(d+icdx)^2(1+c^2x^2)}\,dx}{2d} \\
&= \frac{i(a+b\tan^{-1}(cx))}{2cd^3(1+icx)^2} - \frac{(ib)\int \frac{1}{\left(\frac{1}{d}-\frac{icx}{d}\right)(d+icdx)^3}\,dx}{2d} \\
&= \frac{i(a+b\tan^{-1}(cx))}{2cd^3(1+icx)^2} - \frac{(ib)\int \left(\frac{i}{2d^2(-i+cx)^3} - \frac{1}{4d^2(-i+cx)^2} + \frac{1}{4d^2(1+c^2x^2)}\right)dx}{2d} \\
&= -\frac{b}{8cd^3(i-cx)^2} + \frac{ib}{8cd^3(i-cx)} + \frac{i(a+b\tan^{-1}(cx))}{2cd^3(1+icx)^2} - \frac{(ib)\int \frac{1}{1+c^2x^2}\,dx}{8d^3} \\
&= -\frac{b}{8cd^3(i-cx)^2} + \frac{ib}{8cd^3(i-cx)} - \frac{ib\tan^{-1}(cx)}{8cd^3} + \frac{i(a+b\tan^{-1}(cx))}{2cd^3(1+icx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.60

$$-\frac{i(4a+b(-2i+cx)+b(3-2icx+c^2x^2)\,\mathrm{ArcTan}(cx))}{8cd^3(-i+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
[Out] ((-1/8*I)*(4*a + b*(-2*I + c*x) + b*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[c*x]))
/(c*d^3*(-I + c*x)^2)
```

Maple [A]
time = 0.10, size = 82, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$
default	$\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$
risch	$-\frac{b\ln(icx+1)}{4c\,d^3(cx-i)^2}+\frac{4b\ln(-icx+1)-\ln(-cx+i)b\,c^2x^2+\ln(cx+i)b\,c^2x^2+2i\ln(-cx+i)bcx-2i\ln(cx+i)bcx+b\ln(-cx+i)-b}{16d^3(cx-i)^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*I*a/d^3/(1+I*c*x)^2+1/2*I*b/d^3/(1+I*c*x)^2*arctan(c*x)-1/8*I*b/d^
3*arctan(c*x)-1/8*b/d^3/(c*x-I)^2-1/8*I*b/d^3/(c*x-I))
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.71

$$-\frac{i\,bcx+\left(i\,bc^2x^2+2\,bcx+3i\,b\right)\arctan\left(cx\right)+4i\,a+2\,b}{8\,\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(I*b*c*x + (I*b*c^2*x^2 + 2*b*c*x + 3*I*b)*arctan(c*x) + 4*I*a + 2*b)/
(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Fricas [A]
time = 1.25, size = 75, normalized size = 0.82

$$\frac{-2i\,bcx+\left(bc^2x^2-2i\,bcx+3\,b\right)\log\left(-\frac{cx+i}{cx-i}\right)-8i\,a-4\,b}{16\,\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(-2*I*b*c*x + (b*c^2*x^2 - 2*I*b*c*x + 3*b)*log(-(c*x + I)/(c*x - I))
- 8*I*a - 4*b)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(70) = 140.
time = 1.82, size = 158, normalized size = 1.72

$$\frac{b\log\left(-icx+1\right)}{4c^3d^3x^2-8ic^2d^3x-4cd^3}-\frac{b\log\left(icx+1\right)}{4c^3d^3x^2-8ic^2d^3x-4cd^3}+\frac{b\left(-\frac{\log\left(bx-\frac{ib}{c}\right)}{16}+\frac{\log\left(bx+\frac{ib}{c}\right)}{16}\right)}{cd^3}+\frac{-4ia-ibcx-2b}{8c^3d^3x^2-16ic^2d^3x-8cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

```
[Out] b*log(-I*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) - b*log(I
*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) + b*(-log(b*x - I
*b/c)/16 + log(b*x + I*b/c)/16)/(c*d**3) + (-4*I*a - I*b*c*x - 2*b)/(8*c**3
*d**3*x**2 - 16*I*c**2*d**3*x - 8*c*d**3)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + c*d*x*1i)^3,x)
```

```
[Out] int((a + b*atan(c*x))/(d + c*d*x*1i)^3, x)
```

3.63 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+icdx)^3} \, dx$

Optimal. Leaf size=195

$$\frac{ib}{8d^3(i-cx)^2}+\frac{5b}{8d^3(i-cx)}-\frac{5b\text{ArcTan}(cx)}{8d^3}-\frac{a+b\text{ArcTan}(cx)}{2d^3(i-cx)^2}+\frac{i(a+b\text{ArcTan}(cx))}{d^3(i-cx)}+\frac{a\log(x)}{d^3}+\frac{(a+b\text{ArcT}}{}$$

```
[Out] 1/8*I*b/d^3/(I-c*x)^2+5/8*b/d^3/(I-c*x)-5/8*b*arctan(c*x)/d^3+1/2*(-a-b*arc
tan(c*x))/d^3/(I-c*x)^2+I*(a+b*arctan(c*x))/d^3/(I-c*x)+a*ln(x)/d^3+(a+b*ar
ctan(c*x))*ln(2/(1+I*c*x))/d^3+1/2*I*b*polylog(2,-I*c*x)/d^3-1/2*I*b*polylo
g(2,I*c*x)/d^3+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^3
```

Rubi [A]
time = 0.19, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4940, 2438, 4972, 641, 46, 209, 4964, 2449, 2352}

$$\frac{i(a+b\text{ArcTan}(cx))}{d^3(-cx+i)}-\frac{a+b\text{ArcTan}(cx)}{2d^3(-cx+i)^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{a\log(x)}{d^3}-\frac{5b\text{ArcTan}(cx)}{8d^3}+\frac{ib\text{Li}_2(-icx)}{2d^3}-\frac{ib\text{Li}_2(icx)}{2d^3}+\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2d^3}+\frac{5b}{8d^3(-cx+i)}+\frac{ib}{8d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

```
[Out] ((I/8)*b)/(d^3*(I - c*x)^2) + (5*b)/(8*d^3*(I - c*x)) - (5*b*ArcTan[c*x])/(
8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x]))/
(d^3*(I - c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/d^3 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])/d^3
 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{c(a+b\tan^{-1}(cx))}{d^3(-i+cx)^3} + \frac{ic(a+b\tan^{-1}(cx))}{d^3(-i+cx)^2} - \frac{c(a+b\tan^{-1}(c}{d^3(-i+cx)}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} + \frac{(ic)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^3} + \frac{c\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{d^3} - \frac{c\int \frac{a+b\tan^{-1}(cx)}{-i+cx}\,dx}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))}{d^3(i-cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))}{d^3(i-cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))}{d^3(i-cx)} + \frac{a\log(x)}{d^3} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}$$

$$= \frac{ib}{8d^3(i-cx)^2} + \frac{5b}{8d^3(i-cx)} - \frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))}{d^3(i-cx)} + \frac{a\log(x)}{d^3} +$$

$$= \frac{ib}{8d^3(i-cx)^2} + \frac{5b}{8d^3(i-cx)} - \frac{5b\tan^{-1}(cx)}{8d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx)}{d^3(i-cx)}$$

Mathematica [A]
time = 0.61, size = 198, normalized size = 1.02

$$-\frac{\frac{16a}{(-i+cx)^2} + \frac{32ia}{-i+cx} + 32ib\text{ArcTan}(cx)^2 + 12ib\cos(2\text{ArcTan}(cx)) + ib\cos(4\text{ArcTan}(cx)) - 32a\log(x) + 16a\log\left(1+c^2x^2\right) + 16ib\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right) + 12b\sin(2\text{ArcTan}(cx)) + b\sin(4\text{ArcTan}(cx)) + 4i\text{ArcTan}(cx)\left(8a + 6ib\cos(2\text{ArcTan}(cx)) + ib\cos(4\text{ArcTan}(cx)) + 8ib\log\left(1-e^{2i\text{ArcTan}(cx)}\right) + 6b\sin(2\text{ArcTan}(cx)) + b\sin(4\text{ArcTan}(cx))\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

```
[Out] -1/32*((16*a)/(-I + c*x)^2 + ((32*I)*a)/(-I + c*x) + (32*I)*b*ArcTan[c*x]^2
 + (12*I)*b*Cos[2*ArcTan[c*x]] + I*b*Cos[4*ArcTan[c*x]] - 32*a*Log[x] + 16*
a*Log[1 + c^2*x^2] + (16*I)*b*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 12*b*Sin[
2*ArcTan[c*x]] + b*Sin[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*(8*a + (6*I)*b*Co
s[2*ArcTan[c*x]] + I*b*Cos[4*ArcTan[c*x]] + (8*I)*b*Log[1 - E^((2*I)*ArcTan
[c*x])] + 6*b*Sin[2*ArcTan[c*x]] + b*Sin[4*ArcTan[c*x]]))/d^3
```

Maple [A]
time = 0.13, size = 327, normalized size = 1.68

method	result
derivativedivides	$-\frac{a}{2d^3(cx-i)^2} - \frac{ib\,\mathrm{dilog}(-icx)}{2d^3} - \frac{a\ln(c^2x^2+1)}{2d^3} - \frac{ib\arctan(cx)}{d^3(cx-i)} + \frac{a\ln(cx)}{d^3} - \frac{b\arctan(cx)}{2d^3(cx-i)^2} + \frac{ib}{8d^3(cx-i)^2} -$
default	$-\frac{a}{2d^3(cx-i)^2} - \frac{ib\,\mathrm{dilog}(-icx)}{2d^3} - \frac{a\ln(c^2x^2+1)}{2d^3} - \frac{ib\arctan(cx)}{d^3(cx-i)} + \frac{a\ln(cx)}{d^3} - \frac{b\arctan(cx)}{2d^3(cx-i)^2} + \frac{ib}{8d^3(cx-i)^2} -$

risch	$\frac{ib}{8d^3(-icx-1)} - \frac{ib\ln(icx+1)}{2d^3(icx+1)} - \frac{5b\arctan(cx)}{16d^3} + \frac{ib\ln(-icx+1)x^2c^2}{16d^3(-icx-1)^2} + \frac{5ib\ln(c^2x^2+1)}{32d^3} - \frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a/d^3/(c*x-I)^2-1/2*I*b/d^3*dilog(-I*(c*x+I))-1/2*a/d^3*ln(c^2*x^2+1)-I*b/d^3*arctan(c*x)/(c*x-I)+a/d^3*ln(c*x)-1/2*b/d^3*arctan(c*x)/(c*x-I)^2+1/8*I*b/d^3/(c*x-I)^2-b/d^3*arctan(c*x)*ln(c*x-I)+b/d^3*ln(c*x)*arctan(c*x)-I*a/d^3/(c*x-I)-5/8*b*arctan(c*x)/d^3-5/8*b/d^3/(c*x-I)-1/4*I*b/d^3*ln(c*x-I)^2+1/2*I*b/d^3*ln(c*x)*ln(-I*(-c*x+I))+1/2*I*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*b/d^3*ln(c*x)*ln(-I*(c*x+I))-I*a/d^3*arctan(c*x)+1/2*I*b/d^3*dilog(-1/2*I*(c*x+I))-1/2*I*b/d^3*dilog(-I*c*x)-1/2*I*b/d^3*ln(-I*c*x)*ln(-I*(-c*x+I))

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(151) = 302.
time = 0.33, size = 406, normalized size = 2.08

$$\frac{2(-8ia-5b)cx+4(-ibc^2x^2-2bcx+ib)\arctan(cx)^2-(ibc^2x^2+2bcx-ib)\log(c^2x^2+1)^2-4(bc^2x^2-2ibcx-b)\arctan(cx)\log\left(\frac{1}{4}c^2x^2+\frac{1}{4}\right)+16(bc^2x^2-2ibcx-b)\arctan(cx)\log(cx)-((16ia+5b)c^2x^2+2(16a+3ib)cx-16ia+19b)\arctan(cx)+5(bc^2x^2-2ibcx-b)\arctan(cx,-1)+8(-ibc^2x^2-2bcx+ib)\mathrm{Li}_2(icx+1)+8(ibc^2x^2+2bcx-ib)\mathrm{Li}_2\left(\frac{1}{2}icx+\frac{1}{2}\right)+8(ibc^2x^2+2bcx-ib)\mathrm{Li}_2(-icx+1)-2\left(2(\pi b+2a)c^2x^2-4(i\pi b+2ia)cx-2\pi b-(ibc^2x^2+2bcx-ib)\log\left(\frac{1}{4}c^2x^2+\frac{1}{4}\right)-4a\right)\log(c^2x^2+1)+16(ac^2x^2-2iacx-a)\log(x)-24a+12ib}{16(c^2d^3x^2-2icd^3x-d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="maxima")

[Out] 1/16*(2*(-8*I*a - 5*b)*c*x + 4*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^2 - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(c^2*x^2 + 1)^2 - 4*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 16*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(c*x) - ((16*I*a + 5*b)*c^2*x^2 + 2*(16*a + 3*I*b)*c*x - 16*I*a + 19*b)*arctan(c*x) + 5*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x, -1) + 8*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*dilog(I*c*x + 1) + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog(-I*c*x + 1) - 2*(2*(pi*b + 2*a)*c^2*x^2 - 4*(I*pi*b + 2*I*a)*c*x - 2*pi*b - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(1/4*c^2*x^2 + 1/4) - 4*a)*log(c^2*x^2 + 1) + 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 24*a + 12*I*b)/(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)

Fricas [**A**]
time = 1.96, size = 201, normalized size = 1.03

$$-\frac{2(8ia+5b)cx+8(ibc^2x^2+2bcx-ib)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-16(ac^2x^2-2iacx-a)\log(x)-4(2bcx-3ib)\log\left(-\frac{cx+i}{cx-i}\right)+5(ibc^2x^2+2bcx-ib)\log\left(\frac{cx+i}{c}\right)+((16a-5ib)c^2x^2+2(-16ia-5b)cx-16a+5ib)\log\left(\frac{cx-i}{c}\right)+24a-12ib}{16(c^2d^3x^2-2icd^3x-d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="fricas")

[Out] -1/16*(2*(8*I*a + 5*b)*c*x + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog((c*x + I)/(c*x - I) + 1) - 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 4*(2*b*c*x - 3*I

```
*b)*log(-(c*x + I)/(c*x - I)) + 5*(I*b*c^2*x^2 + 2*b*c*x - I*b)*log((c*x +
I)/c) + ((16*a - 5*I*b)*c^2*x^2 + 2*(-16*I*a - 5*b)*c*x - 16*a + 5*I*b)*log
((c*x - I)/c) + 24*a - 12*I*b)/(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(d+c\,d\,x\,1\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3), x)
```

3.64 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^2(d+icdx)^3}\,dx$

Optimal. Leaf size=250

$$\frac{bc}{8d^3(i-cx)^2}-\frac{9ibc}{8d^3(i-cx)}+\frac{9ibc\text{ArcTan}(cx)}{8d^3}-\frac{a+b\text{ArcTan}(cx)}{d^3x}+\frac{ic(a+b\text{ArcTan}(cx))}{2d^3(i-cx)^2}+\frac{2c(a+b\text{ArcTan}(cx)}{d^3(i-cx)}$$

```
[Out] 1/8*b*c/d^3/(I-c*x)^2-9/8*I*b*c/d^3/(I-c*x)+9/8*I*b*c*arctan(c*x)/d^3+(-a-b
*arctan(c*x))/d^3/x+1/2*I*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2+2*c*(a+b*arctan
(c*x))/d^3/(I-c*x)-3*I*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*I*c*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/d^3-1/2*b*c*ln(c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-I*c*x)/d^3
-3/2*b*c*polylog(2,I*c*x)/d^3+3/2*b*c*polylog(2,1-2/(1+I*c*x))/d^3
```

Rubi [A]
time = 0.21, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {4996, 4946, 272, 36, 29, 31, 4940, 2438, 4972, 641, 46, 209, 4964, 2449, 2352}

$$\frac{2c(a+b\text{ArcTan}(cx))}{d^3(-cx+i)}+\frac{ic(a+b\text{ArcTan}(cx))}{2d^3(-cx+i)^2}-\frac{a+b\text{ArcTan}(cx)}{d^3x}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{3iac\log(x)}{d^3}+\frac{9ibc\text{ArcTan}(cx)}{8d^3}-\frac{bc\log\left(c^2x^2+1\right)}{2d^3}+\frac{3bc\text{Li}_2(-icx)}{2d^3}-\frac{3bc\text{Li}_2(icx)}{2d^3}+\frac{3bc\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2d^3}-\frac{9ibc}{8d^3(-cx+i)}+\frac{bc}{8d^3(-cx+i)^2}+\frac{bc\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] (b*c)/(8*d^3*(I - c*x)^2) - (((9*I)/8)*b*c)/(d^3*(I - c*x)) + (((9*I)/8)*b*
c*ArcTan[c*x])/d^3 - (a + b*ArcTan[c*x])/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c
*x]))/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - ((3*I
)*a*c*Log[x])/d^3 + (b*c*Log[x])/d^3 - ((3*I)*c*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/d^3 - (b*c*Log[1 + c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, (-I)*
c*x])/(2*d^3) - (3*b*c*PolyLog[2, I*c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^2} - \frac{3ic(a+b\tan^{-1}(cx))}{d^3x} - \frac{ic^2(a+b\tan^{-1}(cx))}{d^3(-i+cx)^3} + \frac{2c^2(a+b\,\mathrm{ta}}{d^3(-i+}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(3ic)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{(ic^2)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{d^3} + \frac{(3ic^2)\int \frac{a+b\,\mathrm{ta}}{-i}}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} + \frac{2c(a+b\tan^{-1}(cx))}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3i}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} + \frac{2c(a+b\tan^{-1}(cx))}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3i}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} + \frac{2c(a+b\tan^{-1}(cx))}{d^3(i-cx)} - \frac{3iac\log(x)}{d^3} - \frac{3i}{}$$

$$= \frac{bc}{8d^3(i-cx)^2} - \frac{9ibc}{8d^3(i-cx)} - \frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} + \frac{2c(a+b\,\mathrm{ta}}{d^3(i-}$$

$$= \frac{bc}{8d^3(i-cx)^2} - \frac{9ibc}{8d^3(i-cx)} + \frac{9ibc\tan^{-1}(cx)}{8d^3} - \frac{a+b\tan^{-1}(cx)}{d^3x} + \frac{ic(a+b\tan^{-1}(c}{2d^3(i-cx)^2}$$

Mathematica [A]
time = 1.03, size = 241, normalized size = 0.96

$$-\frac{\frac{32a}{x} - \frac{16iac}{(-i+cx)^2} + \frac{64ac}{-i+cx} + 96ac\,\mathrm{ArcTan}(cx) + 96bc\,\mathrm{ArcTan}(cx)^2 + 96iac\log(x) - 48iac\log(1+c^2x^2) + 48bc\,\mathrm{PolyLog}(2, e^{2i\mathrm{ArcTan}(cx)}) + bc\left(20\cos(2\mathrm{ArcTan}(cx)) + \cos(4\mathrm{ArcTan}(cx)) - 32\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) - 20i\sin(2\mathrm{ArcTan}(cx)) - i\sin(4\mathrm{ArcTan}(cx))\right) + \frac{4b\,\mathrm{ArcTan}(cx)\left(8+10icx\cos(2\mathrm{ArcTan}(cx)) + icx\cos(4\mathrm{ArcTan}(cx)) + 24icx\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right) + 10cx\sin(2\mathrm{ArcTan}(cx)) + cx\sin(4\mathrm{ArcTan}(cx))\right)}{x}}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] -1/32*((32*a)/x - ((16*I)*a*c)/(-I + c*x)^2 + (64*a*c)/(-I + c*x) + 96*a*c*
ArcTan[c*x] + 96*b*c*ArcTan[c*x]^2 + (96*I)*a*c*Log[x] - (48*I)*a*c*Log[1 +
 c^2*x^2] + 48*b*c*PolyLog[2, E^((2*I)*ArcTan[c*x])] + b*c*(20*Cos[2*ArcTan
[c*x]] + Cos[4*ArcTan[c*x]] - 32*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (20*I)*Sin[
2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + (4*b*ArcTan[c*x]*(8 + (10*I)*c*x*C
os[2*ArcTan[c*x]] + I*c*x*Cos[4*ArcTan[c*x]] + (24*I)*c*x*Log[1 - E^((2*I)*
ArcTan[c*x])] + 10*c*x*Sin[2*ArcTan[c*x]] + c*x*Sin[4*ArcTan[c*x]]))/x)/d^3
```

Maple [A]
time = 0.14, size = 380, normalized size = 1.52

method	result
derivativedivides	$c\left(\frac{ia}{2d^3(cx-i)^2} - \frac{3ib\arctan(cx)\ln(cx)}{d^3} - \frac{3a\arctan(cx)}{d^3} - \frac{2a}{d^3(cx-i)} - \frac{a}{d^3cx} + \frac{3ia\ln(c^2x^2+1)}{2d^3} + \frac{9ib\arctan(c}{8d^3}\right.$

default	$c\left(\frac{ia}{2d^3(cx-i)^2}-\frac{3ib\arctan(cx)\ln(cx)}{d^3}-\frac{3a\arctan(cx)}{d^3}-\frac{2a}{d^3(cx-i)}-\frac{a}{d^3cx}+\frac{3ia\ln(c^2x^2+1)}{2d^3}+\frac{9ib\arctan(cx)}{8d^3}\right.$
risch	$\frac{3ica\ln(c^2x^2+1)}{2d^3}-\frac{cb\ln(icx+1)}{d^3(icx+1)}-\frac{cb\ln(icx+1)}{4d^3(icx+1)^2}+\frac{3cb\ln(\frac{1}{2}+\frac{icx}{2})\ln(\frac{1}{2}-\frac{icx}{2})}{2d^3}-\frac{3cb\ln(\frac{1}{2}+\frac{icx}{2})\ln(-icx+1)}{2d^3}-\frac{cb}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*I*a/d^3/(c*x-I)^2-3*I*b/d^3*arctan(c*x)*ln(c*x)-3*a/d^3*arctan(c*x)-
2*a/d^3/(c*x-I)-a/d^3/c/x+3/2*I*a/d^3*ln(c^2*x^2+1)+9/8*I*b/d^3*arctan(c*x)
-3*I*a/d^3*ln(c*x)-2*b/d^3*arctan(c*x)/(c*x-I)-b/d^3*arctan(c*x)/c/x+9/8*I*
b/d^3/(c*x-I)+3*I*b/d^3*arctan(c*x)*ln(c*x-I)+1/8*b/d^3/(c*x-I)^2-1/2*b/d^3
*ln(c^2*x^2+1)+1/2*I*b/d^3*arctan(c*x)/(c*x-I)^2+b/d^3*ln(c*x)+3/2*b/d^3*ln
(-I*(-c*x+I))*ln(c*x)-3/2*b/d^3*ln(-I*(-c*x+I))*ln(-I*c*x)-3/2*b/d^3*dilog(
-I*c*x)-3/2*b/d^3*dilog(-I*(c*x+I))-3/2*b/d^3*ln(c*x)*ln(-I*(c*x+I))-3/4*b/
d^3*ln(c*x-I)^2+3/2*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*b/d^3*dilog(-1/2
*I*(c*x+I)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.79, size = 263, normalized size = 1.05

$$-\frac{6\,(8\,a-3i\,b)c^2x^2+4\,(-18i\,a-5\,b)cx+24\,(bc^3x^3-2i\,bc^2x^2-bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+16\,((3i\,a-b)c^3x^3+2\,(3\,a+i\,b)c^2x^2+(-3i\,a+b)cx)\log\left(x\right)+4\,(6i\,bc^2x^2+9\,bcx-2i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+17\,(bc^3x^3-2i\,bc^2x^2-bcx)\log\left(\frac{cx+i}{c}\right)-((48i\,a+b)c^3x^3+2\,(48\,a-i\,b)c^2x^2+(-48i\,a-b)cx)\log\left(\frac{cx-i}{c}\right)-16\,a}{16\,(c^2d^3x^3-2i\,cd^3x^2-d^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(8*a - 3*I*b)*c^2*x^2 + 4*(-18*I*a - 5*b)*c*x + 24*(b*c^3*x^3 - 2*
I*b*c^2*x^2 - b*c*x)*dilog((c*x + I)/(c*x - I) + 1) + 16*((3*I*a - b)*c^3*x
^3 + 2*(3*a + I*b)*c^2*x^2 + (-3*I*a + b)*c*x)*log(x) + 4*(6*I*b*c^2*x^2 +
9*b*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) + 17*(b*c^3*x^3 - 2*I*b*c^2*x^2
- b*c*x)*log((c*x + I)/c) - ((48*I*a + b)*c^3*x^3 + 2*(48*a - I*b)*c^2*x^2
+ (-48*I*a - b)*c*x)*log((c*x - I)/c) - 16*a)/(c^2*d^3*x^3 - 2*I*c*d^3*x^2
- d^3*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3), x)
```

3.65 $\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+icdx)^3}\,dx$

Optimal. Leaf size=306

$$-\frac{bc}{2d^3x}-\frac{ibc^2}{8d^3(i-cx)^2}-\frac{13bc^2}{8d^3(i-cx)}+\frac{9bc^2\text{ArcTan}(cx)}{8d^3}-\frac{a+b\text{ArcTan}(cx)}{2d^3x^2}+\frac{3ic(a+b\text{ArcTan}(cx))}{d^3x}+\frac{c^2(a+bA}{2d^3(i}$$

```
[Out] -1/2*b*c/d^3/x-1/8*I*b*c^2/d^3/(I-c*x)^2-13/8*b*c^2/d^3/(I-c*x)+9/8*b*c^2*a
rctan(c*x)/d^3+1/2*(-a-b*arctan(c*x))/d^3/x^2+3*I*c*(a+b*arctan(c*x))/d^3/x
+1/2*c^2*(a+b*arctan(c*x))/d^3/(I-c*x)^2-3*I*c^2*(a+b*arctan(c*x))/d^3/(I-c
*x)-6*a*c^2*ln(x)/d^3-3*I*b*c^2*ln(x)/d^3-6*c^2*(a+b*arctan(c*x))*ln(2/(1+I
*c*x))/d^3+3/2*I*b*c^2*ln(c^2*x^2+1)/d^3-3*I*b*c^2*polylog(2,-I*c*x)/d^3+3*
I*b*c^2*polylog(2,I*c*x)/d^3-3*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^3
```

Rubi [A]
time = 0.24, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {4996, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438, 4972, 641, 46, 4964, 2449, 2352}

$$-\frac{3ic^2(a+b\text{ArcTan}(cx))}{d^3(-cx+i)}+\frac{c^2(a+b\text{ArcTan}(cx))}{2d^3(-cx+i)^2}-\frac{6c^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{a+b\text{ArcTan}(cx)}{2d^3x^2}+\frac{3ic(a+b\text{ArcTan}(cx))}{d^3x}-\frac{6ac^2\log(x)}{d^3}+\frac{9bc^2\text{ArcTan}(cx)}{8d^3}-\frac{3ibc^2\text{Li}_2(-icx)}{d^3}+\frac{3ibc^2\text{Li}_2(icx)}{d^3}-\frac{3ibc^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{d^3}+\frac{3ibc^2\log\left(c^2x^2+1\right)}{2d^3}-\frac{13bc^2}{8d^3(-cx+i)}-\frac{ibc^2}{8d^3(-cx+i)^2}-\frac{3ibc^2\log(x)}{d^3}-\frac{bc}{2d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

```
[Out] -1/2*(b*c)/(d^3*x) - ((I/8)*b*c^2)/(d^3*(I - c*x)^2) - (13*b*c^2)/(8*d^3*(I
 - c*x)) + (9*b*c^2*ArcTan[c*x])/(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*x^2)
+ ((3*I)*c*(a + b*ArcTan[c*x]))/(d^3*x) + (c^2*(a + b*ArcTan[c*x]))/(2*d^3*
(I - c*x)^2) - ((3*I)*c^2*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - (6*a*c^2*L
og[x])/d^3 - ((3*I)*b*c^2*Log[x])/d^3 - (6*c^2*(a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/d^3 + (((3*I)/2)*b*c^2*Log[1 + c^2*x^2])/d^3 - ((3*I)*b*c^2*Pol
yLog[2, (-I)*c*x])/d^3 + ((3*I)*b*c^2*PolyLog[2, I*c*x])/d^3 - ((3*I)*b*c^2
*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+icdx)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^3} - \frac{3ic(a+b\tan^{-1}(cx))}{d^3x^2} - \frac{6c^2(a+b\tan^{-1}(cx))}{d^3x} - \frac{c^3(a+b\tan\cdots}{d^3(-i+\cdots}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3ic)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(6c^2)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{(3ic^3)\int \frac{a+b\tan\cdots}{(-i\cdots}}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic(a+b\tan^{-1}(cx))}{d^3x} + \frac{c^2(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} - \frac{3ic^2(a+b\tan^{-}\cdots}{d^3(i-cx\cdots}$$

$$= -\frac{bc}{2d^3x} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic(a+b\tan^{-1}(cx))}{d^3x} + \frac{c^2(a+b\tan^{-1}(cx))}{2d^3(i-cx)^2} - \frac{3ic^2(a\cdots}{d^3\cdots}$$

$$= -\frac{bc}{2d^3x} - \frac{bc^2\tan^{-1}(cx)}{2d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic(a+b\tan^{-1}(cx))}{d^3x} + \frac{c^2(a+b\tan^{-}\cdots}{2d^3(i-c\cdots}$$

$$= -\frac{bc}{2d^3x} - \frac{ibc^2}{8d^3(i-cx)^2} - \frac{13bc^2}{8d^3(i-cx)} - \frac{bc^2\tan^{-1}(cx)}{2d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic(a\cdots}{\cdots}$$

$$= -\frac{bc}{2d^3x} - \frac{ibc^2}{8d^3(i-cx)^2} - \frac{13bc^2}{8d^3(i-cx)} + \frac{9bc^2\tan^{-1}(cx)}{8d^3} - \frac{a+b\tan^{-1}(cx)}{2d^3x^2} + \frac{3ic(a\cdots}{\cdots}$$

Mathematica [A]
time = 1.32, size = 274, normalized size = 0.90

$$\frac{-\frac{16a}{x^2} + \frac{96iac}{x} + \frac{16ac^2}{(-i+cx)^2} + \frac{96iac^2}{-i+cx} + 192iac^2\text{ArcTan}(cx) - 192ac^2\log(x) + 96ac^2\log\left(1+c^2x^2\right) + ibc^2\left(\frac{16i}{cx} + 192\text{ArcTan}(cx)^2 + 28\cos(2\text{ArcTan}(cx)) + \cos(4\text{ArcTan}(cx)) - 96\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) + 96\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right) - 28i\sin(2\text{ArcTan}(cx)) - i\sin(4\text{ArcTan}(cx)) + 4\text{ArcTan}(cx)\left(4i + \frac{4i}{c^2x^2} + \frac{24}{cx} + 14i\cos(2\text{ArcTan}(cx)) + i\cos(4\text{ArcTan}(cx)) + 48i\log\left(1-e^{2i\text{ArcTan}(cx)}\right) + 14\sin(2\text{ArcTan}(cx)) + \sin(4\text{ArcTan}(cx))\right)\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

```
[Out] ((-16*a)/x^2 + ((96*I)*a*c)/x + (16*a*c^2)/(-I + c*x)^2 + ((96*I)*a*c^2)/(-
I + c*x) + (192*I)*a*c^2*ArcTan[c*x] - 192*a*c^2*Log[x] + 96*a*c^2*Log[1 +
c^2*x^2] + I*b*c^2*((16*I)/(c*x) + 192*ArcTan[c*x]^2 + 28*Cos[2*ArcTan[c*x]
] + Cos[4*ArcTan[c*x]] - 96*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 96*PolyLog[2, E^
((2*I)*ArcTan[c*x])] - (28*I)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4
*ArcTan[c*x]*(4*I + (4*I)/(c^2*x^2) + 24/(c*x) + (14*I)*Cos[2*ArcTan[c*x]]
 + I*Cos[4*ArcTan[c*x]] + (48*I)*Log[1 - E^((2*I)*ArcTan[c*x])] + 14*Sin[2*A
rcTan[c*x]] + Sin[4*ArcTan[c*x]])))/(32*d^3)
```

Maple [A]
time = 0.15, size = 431, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3*a/d^3*ln(c^2*x^2+1)+1/2*a/d^3/(c*x-I)^2+3*I*b/d^3*arctan(c*x)/c/x+9/
8*b*arctan(c*x)/d^3+3*I*a/d^3/(c*x-I)+6*b/d^3*arctan(c*x)*ln(c*x-I)+1/2*b/d
^3*arctan(c*x)/(c*x-I)^2+3*I*b/d^3*arctan(c*x)/(c*x-I)-1/8*I*b/d^3/(c*x-I)^
2-6*a/d^3*ln(c*x)+13/8*b/d^3/(c*x-I)+3*I*a/d^3/c/x-1/2*b/d^3*arctan(c*x)/c^
2/x^2+3*I*b/d^3*ln(c*x)*ln(-I*(c*x+I))+3*I*b/d^3*ln(-I*c*x)*ln(-I*(-c*x+I))
-3*I*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3*I*b/d^3*ln(c*x)*ln(-I*(-c*x+I))+6
*I*a/d^3*arctan(c*x)-1/2*b/d^3/c/x-1/2*a/d^3/c^2/x^2+3/2*I*b/d^3*ln(c^2*x^2
+1)-3*I*b/d^3*ln(c*x)-3*I*b/d^3*dilog(-1/2*I*(c*x+I))+3/2*I*b/d^3*ln(c*x-I)
^2+3*I*b/d^3*dilog(-I*(c*x+I))+3*I*b/d^3*dilog(-I*c*x)-6*b/d^3*ln(c*x)*arct
an(c*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(253) = 506$.
time = 0.36, size = 590, normalized size = 1.93

$$-\frac{33\,bc^4x^4\arctan\left(1, cx\right) + 6\left(b\left(-11i\arctan\left(1, cx\right) - 3\right) - 16i\,a\right)c^3x^3 - 3\left(b\left(11\arctan\left(1, cx\right) - 4i\right) + 48\,a\right)c^2x^2 + 8\left(4i\,a - b\right)cx + 24\left(-i\,bc^4x^4 - 2\,bc^3x^3 + i\,bc^2x^2\right)\arctan\left(cx\right)^2 + 6\left(-i\,bc^4x^4 - 2\,bc^3x^3 + i\,bc^2x^2\right)\log\left(c^2x^2 + 1\right)^2 - 24\left(bc^4x^4 - 2i\,bc^3x^3 - bc^2x^2\right)\arctan\left(cx\right)\log\left(\frac{1}{4}c^2x^2 + \frac{1}{4}\right) + 96\left(bc^4x^4 - 2i\,bc^3x^3 - bc^2x^2\right)\arctan\left(cx\right)\log\left(cx\right) + \left(3\left(-32i\,a + 5\,b\right)c^4x^4 - 6\left(32\,a + 21i\,b\right)c^3x^3 + 3\left(32i\,a - 53\,b\right)c^2x^2 + 32i\,bcx - 8\,b\right)\arctan\left(cx\right) + 48\left(-i\,bc^4x^4 - 2\,bc^3x^3 + i\,bc^2x^2\right){\rm Li}_2\left(i\,cx + 1\right) + 48\left(i\,bc^4x^4 + 2\,bc^3x^3 - i\,bc^2x^2\right){\rm Li}_2\left(\frac{1}{2}i\,cx + \frac{1}{2}\right) + 48\left(i\,bc^4x^4 + 2\,bc^3x^3 - i\,bc^2x^2\right){\rm Li}_2\left(-i\,cx + 1\right) - 12\left(2\left(\left(\pi + i\right)b + 2\,a\right)c^4x^4 - 4\left(\left(i\pi - 1\right)b + 2i\,a\right)c^3x^3 - 2\left(\left(\pi + i\right)b + 2\,a\right)c^2x^2 - \left(i\,bc^4x^4 + 2\,bc^3x^3 - i\,bc^2x^2\right)\log\left(\frac{1}{4}c^2x^2 + \frac{1}{4}\right)\right)\log\left(c^2x^2 + 1\right) + 48\left(\left(2\,a + i\,b\right)c^4x^4 + 2\left(-2i\,a + b\right)c^3x^3 - \left(2\,a + i\,b\right)c^2x^2\right)\log\left(x\right) - 8\,a}{16\left(c^2d^3x^4 - 2i\,cd^3x^3 - d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(33*b*c^4*x^4*arctan2(1, c*x) + 6*(b*(-11*I*arctan2(1, c*x) - 3) - 16
*I*a)*c^3*x^3 - 3*(b*(11*arctan2(1, c*x) - 4*I) + 48*a)*c^2*x^2 + 8*(4*I*a
- b)*c*x + 24*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*arctan(c*x)^2 + 6*
(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log(c^2*x^2 + 1)^2 - 24*(b*c^4*x
^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 96*(b*
c^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(c*x) + (3*(-32*I*a + 5
*b)*c^4*x^4 - 6*(32*a + 21*I*b)*c^3*x^3 + 3*(32*I*a - 53*b)*c^2*x^2 + 32*I*
b*c*x - 8*b)*arctan(c*x) + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*di
log(I*c*x + 1) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilog(1/2*I*c
*x + 1/2) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilog(-I*c*x + 1)
- 12*(2*((pi + I)*b + 2*a)*c^4*x^4 - 4*((I*pi - 1)*b + 2*I*a)*c^3*x^3 - 2*(
(pi + I)*b + 2*a)*c^2*x^2 - (I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*log(1
/4*c^2*x^2 + 1/4))*log(c^2*x^2 + 1) + 48*((2*a + I*b)*c^4*x^4 + 2*(-2*I*a +
 b)*c^3*x^3 - (2*a + I*b)*c^2*x^2)*log(x) - 8*a)/(c^2*d^3*x^4 - 2*I*c*d^3*x
^3 - d^3*x^2)
```

Fricas [A]
time = 2.07, size = 311, normalized size = 1.02

$$-\frac{6\left(-16i\,a - 3\,b\right)c^3x^3 - 12\left(12\,a - i\,b\right)c^2x^2 + 8\left(4i\,a - b\right)cx + 48\left(-i\,bc^4x^4 - 2\,bc^3x^3 + i\,bc^2x^2\right){\rm Li}_2\left(\frac{cx + i}{cx - i} + 1\right) + 48\left(\left(2\,a + i\,b\right)c^4x^4 + 2\left(-2i\,a + b\right)c^3x^3 - \left(2\,a + i\,b\right)c^2x^2\right)\log\left(x\right) + 4\left(12\,bc^3x^3 - 18i\,bc^2x^2 - 4\,bcx - i\,b\right)\log\left(-\frac{cx + i}{cx - i}\right) + 33\left(-i\,bc^4x^4 - 2\,bc^3x^3 + i\,bc^2x^2\right)\log\left(\frac{cx + i}{c}\right) - 3\left(\left(32\,a + 5i\,b\right)c^4x^4 - 2\left(32i\,a - 5\,b\right)c^3x^3 - \left(32\,a + 5i\,b\right)c^2x^2\right)\log\left(\frac{cx - i}{c}\right) - 8\,a}{16\left(c^2d^3x^4 - 2i\,cd^3x^3 - d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(-16*I*a - 3*b)*c^3*x^3 - 12*(12*a - I*b)*c^2*x^2 + 8*(4*I*a - b)*
c*x + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*dilog((c*x + I)/(c*x -
```

```
I) + 1) + 48*((2*a + I*b)*c^4*x^4 + 2*(-2*I*a + b)*c^3*x^3 - (2*a + I*b)*c^
2*x^2)*log(x) + 4*(12*b*c^3*x^3 - 18*I*b*c^2*x^2 - 4*b*c*x - I*b)*log(-(c*x
 + I)/(c*x - I)) + 33*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log((c*x +
 I)/c) - 3*((32*a + 5*I*b)*c^4*x^4 - 2*(32*I*a - 5*b)*c^3*x^3 - (32*a + 5*I
*b)*c^2*x^2)*log((c*x - I)/c) - 8*a)/(c^2*d^3*x^4 - 2*I*c*d^3*x^3 - d^3*x^2
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3),x)
[Out] int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3), x)
```

3.66 $\int \frac{a+b\mathbf{ArcTan}(cx)}{(1+icx)^4} dx$

Optimal. Leaf size=100

$$-\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)}-\frac{ib\text{ArcTan}(cx)}{24c}+\frac{i(a+b\text{ArcTan}(cx))}{3c(1+icx)^3}$$

```
[Out] -1/18*I*b/c/(I-c*x)^3-1/24*b/c/(I-c*x)^2+1/24*I*b/c/(I-c*x)-1/24*I*b*arctan
(c*x)/c+1/3*I*(a+b*arctan(c*x))/c/(1+I*c*x)^3
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {4972, 641, 46, 209}

$$\frac{i(a+b\text{ArcTan}(cx))}{3c(1+icx)^3}-\frac{ib\text{ArcTan}(cx)}{24c}+\frac{ib}{24c(-cx+i)}-\frac{b}{24c(-cx+i)^2}-\frac{ib}{18c(-cx+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

```
[Out] ((-1/18*I)*b)/(c*(I - c*x)^3) - b/(24*c*(I - c*x)^2) + ((I/24)*b)/(c*(I - c
*x)) - ((I/24)*b*ArcTan[c*x])/c + ((I/3)*(a + b*ArcTan[c*x]))/(c*(1 + I*c*x
)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(1+icx)^4}\,dx &= \frac{i(a+b\tan^{-1}(cx))}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \frac{1}{(1+icx)^3\,(1+c^2x^2)}\,dx\\
&= \frac{i(a+b\tan^{-1}(cx))}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \frac{1}{(1-icx)(1+icx)^4}\,dx\\
&= \frac{i(a+b\tan^{-1}(cx))}{3c(1+icx)^3}-\frac{1}{3}(ib)\int \left(\frac{1}{2(-i+cx)^4}+\frac{i}{4(-i+cx)^3}-\frac{1}{8(-i+cx)^2}+\frac{}{8\,(1}\right.\\
&= -\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)}+\frac{i(a+b\tan^{-1}(cx))}{3c(1+icx)^3}-\frac{1}{24}(ib)\int \frac{}{1+}\\
&= -\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)}-\frac{ib\tan^{-1}(cx)}{24c}+\frac{i(a+b\tan^{-1}(cx))}{3c(1+icx)^3}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 0.73

$$
\frac{-24a+b(10i-9cx-3ic^2x^2)+3b(-7+3icx-3c^2x^2-ic^3x^3)\,\text{ArcTan}(cx)}{72c(-i+cx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
[Out] (-24*a + b*(10*I - 9*c*x - (3*I)*c^2*x^2) + 3*b*(-7 + (3*I)*c*x - 3*c^2*x^2
 - I*c^3*x^3)*ArcTan[c*x])/(72*c*(-I + c*x)^3)
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{ia}{3(icx+1)^3}+\frac{ib\arctan(cx)}{3(icx+1)^3}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^2}+\frac{ib}{18(cx-i)^3}-\frac{ib}{24(cx-i)}}{c}$
default	$\frac{\frac{ia}{3(icx+1)^3}+\frac{ib\arctan(cx)}{3(icx+1)^3}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^2}+\frac{ib}{18(cx-i)^3}-\frac{ib}{24(cx-i)}}{c}$
risch	$\frac{ib\ln(icx+1)}{6c(cx-i)^3}-\frac{24ib\ln(-icx+1)-3c^3b\ln(-cx-i)x^3+3c^3b\ln(cx-i)x^3+9i\ln(-cx-i)bc^2x^2-9i\ln(cx-i)bc^2x^2+9\ln(-}{144(cx-i)^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*I*a/(1+I*c*x)^3+1/3*I*b/(1+I*c*x)^3*arctan(c*x)-1/24*I*b*arctan(c*
x)-1/24*b/(c*x-I)^2+1/18*I*b/(c*x-I)^3-1/24*I*b/(c*x-I))
```

Maxima [A]
time = 0.27, size = 83, normalized size = 0.83

$$-\frac{3i\,bc^2x^2+9\,bcx-3\left(-i\,bc^3x^3-3\,bc^2x^2+3i\,bcx-7\,b\right)\arctan\left(cx\right)+24\,a-10i\,b}{72\left(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="maxima")
```

```
[Out] -1/72*(3*I*b*c^2*x^2 + 9*b*c*x - 3*(-I*b*c^3*x^3 - 3*b*c^2*x^2 + 3*I*b*c*x
- 7*b)*arctan(c*x) + 24*a - 10*I*b)/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

Fricas [A]
time = 3.38, size = 93, normalized size = 0.93

$$\frac{-6i\,bc^2x^2-18\,bcx+3\left(bc^3x^3-3i\,bc^2x^2-3\,bcx-7i\,b\right)\log\left(-\frac{cx+i}{cx-i}\right)-48\,a+20i\,b}{144\left(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="fricas")
```

```
[Out] 1/144*(-6*I*b*c^2*x^2 - 18*b*c*x + 3*(b*c^3*x^3 - 3*I*b*c^2*x^2 - 3*b*c*x -
 7*I*b)*log(-(c*x + I)/(c*x - I)) - 48*a + 20*I*b)/(c^4*x^3 - 3*I*c^3*x^2 -
 3*c^2*x + I*c)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.
time = 1.58, size = 168, normalized size = 1.68

$$-\frac{ib\log\left(-icx+1\right)}{6c^4x^3-18ic^3x^2-18c^2x+6ic}+\frac{ib\log\left(icx+1\right)}{6c^4x^3-18ic^3x^2-18c^2x+6ic}+\frac{b\left(-\frac{\log\left(bx-\frac{ib}{c}\right)}{48}+\frac{\log\left(bx+\frac{ib}{c}\right)}{48}\right)}{c}+\frac{-24a-3ibc^2x^2-9bcx+10ib}{72c^4x^3-216ic^3x^2-216c^2x+72ic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(1+I*c*x)**4,x)
```

```
[Out] -I*b*log(-I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) + I
*b*log(I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) + b*(-
```

```
log(b*x - I*b/c)/48 + log(b*x + I*b/c)/48)/c + (-24*a - 3*I*b*c**2*x**2 - 9
*b*c*x + 10*I*b)/(72*c**4*x**3 - 216*I*c**3*x**2 - 216*c**2*x + 72*I*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{{(1+c\,x\,1\mathrm{i})}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(c*x*1i + 1)^4,x)
```

```
[Out] int((a + b*atan(c*x))/(c*x*1i + 1)^4, x)
```

3.67 $\int \frac{\text{ArcTan}(ax)}{cx+iacx^2} dx$

Optimal. Leaf size=49

$$\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}+\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)}{2c}$$

```
[Out] arctan(a*x)*ln(2-2/(1+I*a*x))/c+1/2*I*polylog(2,-1+2/(1+I*a*x))/c
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1607, 4988, 2497}

$$\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}+\frac{i\text{Li}_2\left(\frac{2}{iax+1}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

```
[Out] (ArcTan[a*x]*Log[2 - 2/(1 + I*a*x)])/c + ((I/2)*PolyLog[2, -1 + 2/(1 + I*a*
x)])/c
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{cx + iacx^2}\,dx = \int \frac{\tan^{-1}(ax)}{x(c + iacx)}\,dx$$
$$= \frac{\tan^{-1}(ax)\log\left(2 - \frac{2}{1+iax}\right)}{c} - \frac{a\int \frac{\log\left(2-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{c}$$
$$= \frac{\tan^{-1}(ax)\log\left(2 - \frac{2}{1+iax}\right)}{c} + \frac{i\mathrm{Li}_2\left(-1 + \frac{2}{1+iax}\right)}{2c}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.08

$$-\frac{i\left(\mathrm{ArcTan}(ax)\left(\mathrm{ArcTan}(ax) + i\log\left(1 - e^{2i\mathrm{ArcTan}(ax)}\right)\right) + \frac{1}{2}\mathrm{PolyLog}\left(2, e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

```
[Out] ((-I)*(ArcTan[a*x]*(ArcTan[a*x] + I*Log[1 - E^((2*I)*ArcTan[a*x])]) + PolyL
og[2, E^((2*I)*ArcTan[a*x])]/2))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(44) = 88.
time = 0.08, size = 140, normalized size = 2.86

method	result
risch	$\frac{i\,\mathrm{dilog}(iax+1)}{2c} + \frac{i\ln(iax+1)^2}{4c} + \frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c} - \frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{2c} - \frac{i\,\mathrm{dilog}(-iax+1)}{2c} +$
derivativedivides	$\frac{\frac{a\arctan(ax)\ln(ax)}{c} - \frac{a\arctan(ax)\ln(ax-i)}{c} - \frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(-iax+1)}{2} - \frac{i\,\mathrm{dilog}(iax+1)}{2} + \frac{i\,\mathrm{dilog}(-iax+1)}{2} + \right.}{c}}{a}$
default	$\frac{\frac{a\arctan(ax)\ln(ax)}{c} - \frac{a\arctan(ax)\ln(ax-i)}{c} - \frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(-iax+1)}{2} - \frac{i\,\mathrm{dilog}(iax+1)}{2} + \frac{i\,\mathrm{dilog}(-iax+1)}{2} + \right.}{c}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(c*x+I*a*c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a/c*arctan(a*x)*ln(a*x)-a/c*arctan(a*x)*ln(a*x-I)-a/c*(-1/2*I*ln(a*x)*
ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*I*dilog(1-I*
a*x)+1/4*I*ln(a*x-I)^2-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*I*dilog(-1/2*
I*(I+a*x))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(40) = 80.
time = 0.47, size = 126, normalized size = 2.57

$$\frac{1}{4}a\left(-\frac{i\log\left(iax+1\right)^2}{ac}+\frac{2i\left(\log\left(iax+1\right)\log\left(-\frac{1}{2}iax+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}iax+\frac{1}{2}\right)\right)}{ac}+\frac{2i\left(\log\left(iax+1\right)\log\left(x\right)+\text{Li}_2(-iax)\right)}{ac}-\frac{2i\left(\log\left(-iax+1\right)\log\left(x\right)+\text{Li}_2(iax)\right)}{ac}\right)-\left(\frac{\log\left(iax+1\right)}{c}-\frac{\log\left(x\right)}{c}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(-I*log(I*a*x + 1)^2/(a*c) + 2*I*(log(I*a*x + 1)*log(-1/2*I*a*x + 1/2
) + dilog(1/2*I*a*x + 1/2))/(a*c) + 2*I*(log(I*a*x + 1)*log(x) + dilog(-I*a
*x))/(a*c) - 2*I*(log(-I*a*x + 1)*log(x) + dilog(I*a*x))/(a*c)) - (log(I*a*
x + 1)/c - log(x)/c)*arctan(a*x)
```

Fricas [A]
time = 5.31, size = 21, normalized size = 0.43

$$-\frac{i\,\text{Li}_2\left(\frac{ax+i}{ax-i}+1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*I*dilog((a*x + I)/(a*x - I) + 1)/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{atan}\left(ax\right)}{ax^2-ix}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(c*x+I*a*c*x**2),x)
```

```
[Out] -I*Integral(atan(a*x)/(a*x**2 - I*x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)}{1\mathrm{i}\,a\,c\,x^2+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c*x + a*c*x^2*1i),x)
```

```
[Out] int(atan(a*x)/(c*x + a*c*x^2*1i), x)
```

3.68 $\int x^3(d+icdx)(a+b\text{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=287

$$\frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}+\frac{3ib^2d\text{ArcTan}(cx)}{10c^4}+\frac{b^2dx\text{ArcTan}(cx)}{2c^3}+\frac{ibdx^2(a+b\text{ArcTan}(cx))}{5c^2}-\frac{bdx^3(a+bA}{6}$$

```
[Out] 1/2*a*b*d*x/c^3-3/10*I*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*I*b^2*d*x^3/c+3/
10*I*b^2*d*arctan(c*x)/c^4+1/2*b^2*d*x*arctan(c*x)/c^3+1/5*I*b*d*x^2*(a+b*a
rctan(c*x))/c^2-1/6*b*d*x^3*(a+b*arctan(c*x))/c-1/10*I*b*d*x^4*(a+b*arctan(
c*x))-9/20*d*(a+b*arctan(c*x))^2/c^4+1/4*d*x^4*(a+b*arctan(c*x))^2+1/5*I*c*
d*x^5*(a+b*arctan(c*x))^2+2/5*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4-1
/3*b^2*d*ln(c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(1+I*c*x))/c^4
```

Rubi [A]

time = 0.43, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {4996, 4946, 5036, 272, 45, 4930, 266, 5004, 308, 209, 327, 5040, 4964, 2449, 2352}

$$-\frac{9d(a+b\text{ArcTan}(cx))^2}{20c^4}+\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{5c^4}+\frac{ibdx^2(a+b\text{ArcTan}(cx))}{5c^2}+\frac{1}{5}icdx^5(a+b\text{ArcTan}(cx))^2+\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))^2-\frac{1}{10}ibdx^4(a+b\text{ArcTan}(cx))-\frac{bdx^3(a+b\text{ArcTan}(cx))}{6c}+\frac{abdx}{2c^3}+\frac{3ib^2d\text{ArcTan}(cx)}{10c^4}+\frac{b^2dx\text{ArcTan}(cx)}{2c^3}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}-\frac{b^2d\log\left(c^2x^2+1\right)}{3c^4}+\frac{ib^2dx^3}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^3) - (((3*I)/10)*b^2*d*x)/c^3 + (b^2*d*x^2)/(12*c^2) + ((I/3
0)*b^2*d*x^3)/c + (((3*I)/10)*b^2*d*ArcTan[c*x])/c^4 + (b^2*d*x*ArcTan[c*x]
)/(2*c^3) + ((I/5)*b*d*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d*x^3*(a + b*ArcTa
n[c*x]))/(6*c) - (I/10)*b*d*x^4*(a + b*ArcTan[c*x]) - (9*d*(a + b*ArcTan[c*
x])^2)/(20*c^4) + (d*x^4*(a + b*ArcTan[c*x])^2)/4 + (I/5)*c*d*x^5*(a + b*Ar
cTan[c*x])^2 + (((2*I)/5)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 -
 (b^2*d*Log[1 + c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(
5*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(dx^3\left(a+b\tan^{-1}(cx)\right)^2+icdx^4\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx \\
&= d\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int \\
&= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}icdx^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}(2ibd)\int \\
&= -\frac{bdx^3(a+b\tan^{-1}(cx))}{6c}-\frac{1}{10}ibdx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+\right. \\
&= \frac{abdx}{2c^3}+\frac{ibdx^2(a+b\tan^{-1}(cx))}{5c^2}-\frac{bdx^3(a+b\tan^{-1}(cx))}{6c}-\frac{1}{10}ibdx \\
&= \frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{ib^2dx^3}{30c}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}+\frac{ibdx^2(a+b\tan^{-1}(cx}{5c^2} \\
&= \frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}+\frac{3ib^2d\tan^{-1}(cx)}{10c^4}+\frac{b^2dx\tan^{-1}}{2c^3} \\
&= \frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}+\frac{3ib^2d\tan^{-1}(cx)}{10c^4}+\frac{b^2dx\tan^{-1}}{2c^3}
\end{aligned}$$

Mathematica [A]
time = 0.53, size = 285, normalized size = 0.99

$$\frac{d\left(18iab+5b^2+30abcx-18ib^2cx+12iabc^2x^2+5b^2c^2x^2-10abc^3x^3+2ib^2c^3x^3+15a^2c^4x^4-6iabc^4x^4+12ia^2c^5x^5+3b^2(-1+5c^4x^4+4ic^5x^5)\,\mathrm{ArcTan}(cx)^2+2b\,\mathrm{ArcTan}(cx)\left(b(9i+15cx+6ic^2x^2-5c^3x^3-3ic^4x^4)+3a(-5+5c^4x^4+4ic^5x^5)+12ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-12iab\log\left(1+c^2x^2\right)-20b^2\log\left(1+c^2x^2\right)+12b^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d*((18*I)*a*b + 5*b^2 + 30*a*b*c*x - (18*I)*b^2*c*x + (12*I)*a*b*c^2*x^2 +
 5*b^2*c^2*x^2 - 10*a*b*c^3*x^3 + (2*I)*b^2*c^3*x^3 + 15*a^2*c^4*x^4 - (6*I
)*a*b*c^4*x^4 + (12*I)*a^2*c^5*x^5 + 3*b^2*(-1 + 5*c^4*x^4 + (4*I)*c^5*x^5)
*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(9*I + 15*c*x + (6*I)*c^2*x^2 - 5*c^3*x
^3 - (3*I)*c^4*x^4) + 3*a*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5) + (12*I)*b*Log[1
 + E^((2*I)*ArcTan[c*x])]) - (12*I)*a*b*Log[1 + c^2*x^2] - 20*b^2*Log[1 + c
^2*x^2] + 12*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^4)
```

Maple [A]
time = 0.21, size = 474, normalized size = 1.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/30*I*d*b^2*c^3*x^3-3/10*I*d*b^2*c*x+d*a^2*(1/5*I*c^5*x^5+1/4*c^4*x
^4)+1/5*I*d*a*b*c^2*x^2+1/2*d*a*b*arctan(c*x)*c^4*x^4+1/5*I*d*b^2*arctan(c*
x)*c^2*x^2+1/5*I*d*b^2*arctan(c*x)^2*c^5*x^5-1/10*I*d*b^2*arctan(c*x)*c^4*x
^4-1/10*I*d*a*b*c^4*x^4+1/10*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/10*d*b^2*ln(c*
x-I)*ln(-1/2*I*(c*x+I))-1/10*d*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/10*d*b^2*ln(c*
x+I)*ln(1/2*I*(c*x-I))+3/10*I*d*b^2*arctan(c*x)+1/12*d*b^2*c^2*x^2-1/2*d*a*
b*arctan(c*x)-1/5*I*d*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/4*d*b^2*arctan(c*x)^2
*c^4*x^4+1/2*d*b^2*arctan(c*x)*c*x-1/6*d*b^2*arctan(c*x)*c^3*x^3-1/5*I*d*a*
b*ln(c^2*x^2+1)-1/6*d*a*b*c^3*x^3+1/2*d*a*b*c*x-1/4*d*b^2*arctan(c*x)^2-1/2
0*d*b^2*ln(c*x-I)^2-1/10*d*b^2*dilog(-1/2*I*(c*x+I))+1/20*d*b^2*ln(c*x+I)^2
+1/10*d*b^2*dilog(1/2*I*(c*x-I))-1/3*d*b^2*ln(c^2*x^2+1)+2/5*I*d*a*b*arctan
(c*x)*c^5*x^5)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*I*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctan(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*I*(4
*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*
c*d + 1/80*I*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 + 80*integrate(1
/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(c^2*x^6 + x^4)*
arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^
2*c*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5
))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x)
- (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(-4*I*b^2*c*d*x^5 - 5*b^2*d*x^4)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/20*(20*I*a^2*c^3*d*x^6 + 20*a^2*c^2*d*x^5 + 20*I*a^2*c*d*x^4 + 20*a^2*d
*x^3 - (20*a*b*c^3*d*x^6 + 4*(-5*I*a*b - b^2)*c^2*d*x^5 + 5*(4*a*b + I*b^2)
*c*d*x^4 - 20*I*a*b*d*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

```
[Out] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.69 $\int x^2(d+icdx)(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=255

$$\frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{ib^2dx^2}{12c}-\frac{b^2d\text{ArcTan}(cx)}{3c^3}+\frac{ib^2dx\text{ArcTan}(cx)}{2c^2}-\frac{bdx^2(a+b\text{ArcTan}(cx))}{3c}-\frac{1}{6}ibdx^3(a+b\text{ArcTan}(cx$$

```
[Out] 1/2*I*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*I*b^2*d*x^2/c-1/3*b^2*d*arctan(c*x)/
c^3+1/2*I*b^2*d*x*arctan(c*x)/c^2-1/3*b*d*x^2*(a+b*arctan(c*x))/c-1/6*I*b*d
*x^3*(a+b*arctan(c*x))-7/12*I*d*(a+b*arctan(c*x))^2/c^3+1/3*d*x^3*(a+b*arct
an(c*x))^2+1/4*I*c*d*x^4*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*ln(2
/(1+I*c*x))/c^3-1/3*I*b^2*d*ln(c^2*x^2+1)/c^3-1/3*I*b^2*d*polylog(2,1-2/(1+
I*c*x))/c^3
```

Rubi [A]
time = 0.36, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {4996, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 272, 45, 4930, 266, 5004}

$$-\frac{7id(a+b\text{ArcTan}(cx))^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}+\frac{1}{4}icdx^4(a+b\text{ArcTan}(cx))^2+\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))^2-\frac{1}{6}ibdx^3(a+b\text{ArcTan}(cx))-\frac{bdx^2(a+b\text{ArcTan}(cx))}{3c}+\frac{iabdx}{2c^2}-\frac{b^2d\text{ArcTan}(cx)}{3c^3}+\frac{ib^2dx\text{ArcTan}(cx)}{2c^2}-\frac{ib^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}-\frac{ib^2d\log\left(c^2x^2+1\right)}{3c^3}+\frac{ib^2dx^2}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((I/2)*a*b*d*x)/c^2 + (b^2*d*x)/(3*c^2) + ((I/12)*b^2*d*x^2)/c - (b^2*d*Arc
Tan[c*x])/(3*c^3) + ((I/2)*b^2*d*x*ArcTan[c*x])/c^2 - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) - (I/6)*b*d*x^3*(a + b*ArcTan[c*x]) - (((7*I)/12)*d*(a + b*A
rcTan[c*x])^2)/c^3 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/4)*c*d*x^4*(a + b
*ArcTan[c*x])^2 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) -
((I/3)*b^2*d*Log[1 + c^2*x^2])/c^3 - ((I/3)*b^2*d*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(dx^2\left(a+b\tan^{-1}(cx)\right)^2+icdx^3\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx$$

$$=d\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bcd)\int$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}icdx^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(ibd)\int x$$

$$=-\frac{bdx^2(a+b\tan^{-1}(cx))}{3c}-\frac{1}{6}ibdx^3\left(a+b\tan^{-1}(cx)\right)-\frac{id(a+b\tan}{3c^3}$$

$$=\frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{bdx^2(a+b\tan^{-1}(cx))}{3c}-\frac{1}{6}ibdx^3\left(a+b\tan^{-1}(cx)\right)$$

$$=\frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{ib^2dx\tan^{-1}(cx)}{2c^2}-\frac{bdx^2(a+b\tan}{3c}$$

$$=\frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{ib^2dx^2}{12c}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{ib^2dx\tan^{-1}(cx)}{2c^2}-\frac{bdx}{}$$

Mathematica [A]
time = 0.38, size = 241, normalized size = 0.95

$$\frac{id\left(b^2+6abcx-4ib^2cx+4iabc^2x^2+b^2c^2x^2-4ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(1-4ic^3x^3+3c^4x^4)\operatorname{ArcTan}(cx)^2+2b\operatorname{ArcTan}(cx)\left(b(2i+3cx+2ic^2x^2-c^3x^3)+a(-3-4ic^3x^3+3c^4x^4)+4ib\log\left(1+e^{2i\operatorname{ArcTan}(cx)}\right)\right)-4iab\log\left(1+c^2x^2\right)-4b^2\log\left(1+c^2x^2\right)+4b^2\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx)}\right)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((I/12)*d*(b^2 + 6*a*b*c*x - (4*I)*b^2*c*x + (4*I)*a*b*c^2*x^2 + b^2*c^2*x^
2 - (4*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(1 - (4*I)*c^3*
x^3 + 3*c^4*x^4)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(2*I + 3*c*x + (2*I)*c^
2*x^2 - c^3*x^3) + a*(-3 - (4*I)*c^3*x^3 + 3*c^4*x^4) + (4*I)*b*Log[1 + E^(
(2*I)*ArcTan[c*x])]) - (4*I)*a*b*Log[1 + c^2*x^2] - 4*b^2*Log[1 + c^2*x^2]
+ 4*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(219) = 438.
time = 0.23, size = 442, normalized size = 1.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(d*a^2*(1/4*I*c^4*x^4+1/3*c^3*x^3)+1/2*I*d*a*b*arctan(c*x)*c^4*x^4+2/
3*d*a*b*arctan(c*x)*c^3*x^3+1/2*I*d*a*b*c*x-1/6*I*d*a*b*c^3*x^3-1/6*I*d*b^2
*arctan(c*x)*c^3*x^3+1/2*I*d*b^2*arctan(c*x)*c*x+1/4*I*d*b^2*arctan(c*x)^2*
c^4*x^4+1/3*d*b^2*c*x+1/12*I*d*b^2*ln(c*x+I)^2-1/4*I*d*b^2*arctan(c*x)^2-1/
3*I*d*b^2*ln(c^2*x^2+1)-1/6*I*d*b^2*dilog(-1/2*I*(c*x+I))-1/12*I*d*b^2*ln(c
*x-I)^2+1/6*I*d*b^2*dilog(1/2*I*(c*x-I))-1/6*I*d*b^2*ln(c*x+I)*ln(c^2*x^2+1
)+1/6*I*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/6*I*d*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+
I))+1/6*I*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/3*d*a*b*c^2*x^2+1/12*I*d*b^2*
c^2*x^2+1/3*d*b^2*arctan(c*x)^2*c^3*x^3-1/3*d*b^2*arctan(c*x)*c^2*x^2-1/2*I
*d*a*b*arctan(c*x)-1/3*d*b^2*arctan(c*x)+1/3*b^2*ln(c^2*x^2+1)*arctan(c*x)*
d+1/3*a*b*d*ln(c^2*x^2+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3
- 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d + 1/3*(2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d - 1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*
arctan(c*x)^2 - 1/48*(3*b^2*c*d*x^4 - 4*I*b^2*d*x^3)*arctan(c*x)*log(c^2*x^
2 + 1) + 1/192*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + I*inte
grate(-1/48*(14*b^2*c^2*d*x^4*arctan(c*x) - 36*(b^2*c^3*d*x^5 + b^2*c*d*x^3
)*arctan(c*x)^2 - 3*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*log(c^2*x^2 + 1)^2 - (3*b
^2*c^3*d*x^5 - 4*b^2*c*d*x^3 - 12*(b^2*c^2*d*x^4 + b^2*d*x^2)*arctan(c*x))*
log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + integrate(1/48*(36*(b^2*c^2*d*x^4 + b
^2*d*x^2)*arctan(c*x)^2 + 3*(b^2*c^2*d*x^4 + b^2*d*x^2)*log(c^2*x^2 + 1)^2
+ 2*(3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3)*arctan(c*x) + (7*b^2*c^2*d*x^4 + 12*(
b^2*c^3*d*x^5 + b^2*c*d*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1),
x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/12*(12*I*a^2*c^3*d*x^5 + 12*a^2*c^2*d*x^4 + 12*I*a^2*c*d*x^3 + 12*a^2*d
*x^2 - (12*a*b*c^3*d*x^5 + 3*(-4*I*a*b - b^2)*c^2*d*x^4 + 4*(3*a*b + I*b^2)
*c*d*x^3 - 12*I*a*b*d*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.70 $\int x(d+icdx)(a+b\textbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=211

$$-\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\text{ArcTan}(cx)}{3c^2}-\frac{b^2dx\text{ArcTan}(cx)}{c}-\frac{1}{3}ibdx^2(a+b\text{ArcTan}(cx))+\frac{5d(a+b\text{ArcTan}(cx))^2}{6c^2}+\frac{1}{2}dx$$

```
[Out] -a*b*d*x/c+1/3*I*b^2*d*x/c-1/3*I*b^2*d*arctan(c*x)/c^2-b^2*d*x*arctan(c*x)/
c-1/3*I*b*d*x^2*(a+b*arctan(c*x))+5/6*d*(a+b*arctan(c*x))^2/c^2+1/2*d*x^2*(
a+b*arctan(c*x))^2+1/3*I*c*d*x^3*(a+b*arctan(c*x))^2-2/3*I*b*d*(a+b*arctan(
c*x))*ln(2/(1+I*c*x))/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2+1/3*b^2*d*polylog(2,1
-2/(1+I*c*x))/c^2
```

Rubi [A]
time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4996, 4946, 5036, 4930, 266, 5004, 327, 209, 5040, 4964, 2449, 2352}

$$\frac{5d(a+b\text{ArcTan}(cx))^2}{6c^2}-\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^2}+\frac{1}{3}icdx^3(a+b\text{ArcTan}(cx))^2+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))^2-\frac{1}{3}ibdx^2(a+b\text{ArcTan}(cx))-\frac{abdx}{c}-\frac{ib^2d\text{ArcTan}(cx)}{3c^2}-\frac{b^2dx\text{ArcTan}(cx)}{c}+\frac{b^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^2}+\frac{b^2d\log\left(c^2x^2+1\right)}{2c^2}+\frac{ib^2dx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*d*x)/c) + ((I/3)*b^2*d*x)/c - ((I/3)*b^2*d*ArcTan[c*x])/c^2 - (b^2*d
*x*ArcTan[c*x])/c - (I/3)*b*d*x^2*(a + b*ArcTan[c*x]) + (5*d*(a + b*ArcTan[
c*x])^2)/(6*c^2) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (I/3)*c*d*x^3*(a + b*A
rcTan[c*x])^2 - (((2*I)/3)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2
 + (b^2*d*Log[1 + c^2*x^2])/(2*c^2) + (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(3*c^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(dx\left(a+b\tan^{-1}(cx)\right)^2+icdx^2\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx \\
&= d\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+(icd)\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2-(bcd)\int \frac{x^2(a
\end{aligned}$$

$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}icdx^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2ibd)\int x($$

$$= -\frac{abdx}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+\frac{5d\left(a+b\tan^{-1}(cx)\right)^2}{6c^2}+\frac{1}{2}dx^2(a$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2\left(a+b\tan^{-1}(cx)\right)+\frac{5d(a$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\tan^{-1}(cx)}{3c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2(a+b\tan$$

$$= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\tan^{-1}(cx)}{3c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{1}{3}ibdx^2(a+b\tan$$

Mathematica [A]

time = 0.28, size = 208, normalized size = 0.99

$$\frac{d\left(-6abcx+2ib^2cx+3a^2c^2x^2-2iabc^2x^2+2ia^2c^3x^3+b^2(1+3c^2x^2+2ic^3x^3)\,\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(-ib(1-3icx+c^2x^2)+a(3+3c^2x^2+2ic^3x^3)-2ib\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)+2iab\log\left(1+c^2x^2\right)+3b^2\log\left(1+c^2x^2\right)-2b^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d*(-6*a*b*c*x + (2*I)*b^2*c*x + 3*a^2*c^2*x^2 - (2*I)*a*b*c^2*x^2 + (2*I)*
a^2*c^3*x^3 + b^2*(1 + 3*c^2*x^2 + (2*I)*c^3*x^3)*ArcTan[c*x]^2 + 2*b*ArcTa
n[c*x]*((-I)*b*(1 - (3*I)*c*x + c^2*x^2) + a*(3 + 3*c^2*x^2 + (2*I)*c^3*x^3
) - (2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] +
3*b^2*Log[1 + c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(6*c^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(186) = 372$.
time = 0.17, size = 391, normalized size = 1.85

method	result
derivativedivides	$\frac{d a^2\left(\frac{1}{3} i c^3 x^3+\frac{1}{2} c^2 x^2\right)-\frac{i d b^2 \arctan(cx) c^2 x^2}{3}+\frac{d b^2 \arctan(cx)^2 c^2 x^2}{2}+\frac{i d b^2 c x}{3}+\frac{2 i d a b \arctan(cx) c^3 x^3}{3}+\frac{d b^2 \arctan(cx)^2}{2}-d b^2 \mathrm{ar}}{}$
default	$\frac{d a^2\left(\frac{1}{3} i c^3 x^3+\frac{1}{2} c^2 x^2\right)-\frac{i d b^2 \arctan(cx) c^2 x^2}{3}+\frac{d b^2 \arctan(cx)^2 c^2 x^2}{2}+\frac{i d b^2 c x}{3}+\frac{2 i d a b \arctan(cx) c^3 x^3}{3}+\frac{d b^2 \arctan(cx)^2}{2}-d b^2 \mathrm{ar}}{}$
risch	$\frac{i d b a \ln(c^2x^2+1)}{3c^2}+\frac{d b a \arctan(cx)}{c^2}+\frac{d x^2 a^2}{2}+\frac{73 b^2 d \ln(c^2x^2+1)}{144 c^2}+\left(\frac{i d b^2 (2 c x^3-3 i x^2) \ln(-icx+1)}{12}+\frac{db(4a c^3}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(d*a^2*(1/3*I*c^3*x^3+1/2*c^2*x^2)-1/3*I*d*b^2*arctan(c*x)*c^2*x^2+1/
2*d*b^2*arctan(c*x)^2*c^2*x^2+1/3*I*d*b^2*c*x+2/3*I*d*a*b*arctan(c*x)*c^3*x
^3+1/2*d*b^2*arctan(c*x)^2-d*b^2*arctan(c*x)*c*x-1/6*d*b^2*ln(c*x-I)*ln(c^2
*x^2+1)+1/12*d*b^2*ln(c*x-I)^2+1/6*d*b^2*dilog(-1/2*I*(c*x+I))+1/6*d*b^2*ln
(c*x-I)*ln(-1/2*I*(c*x+I))+1/6*d*b^2*ln(c*x+I)*ln(c^2*x^2+1)-1/12*d*b^2*ln(
c*x+I)^2-1/6*d*b^2*dilog(1/2*I*(c*x-I))-1/6*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I
))+1/3*I*d*a*b*ln(c^2*x^2+1)+1/2*d*b^2*ln(c^2*x^2+1)-1/3*I*d*a*b*c^2*x^2+1/
3*I*d*b^2*ln(c^2*x^2+1)*arctan(c*x)+d*a*b*arctan(c*x)*c^2*x^2-1/3*I*d*b^2*a
rctan(c*x)-d*a*b*c*x+1/3*I*d*b^2*arctan(c*x)^2*c^3*x^3+d*a*b*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctan(c*x)^2 + 1/3*I*(2*x^3*arctan(c*x)
- c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d + 1/48*I*(4*x^3*arctan(c*x)^2
```

```
 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/48*(4*c^2*x^4*log(c^2*x^2 + 1) -
 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*arctan(c*x)^2 + 3*(c^2*x^4 + x^2)
*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2*c*d + 1/2*a^2*d*x^2 + (x^2*arct
an(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/2*(2*c*(x/c^2 - arctan(c*x
)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*b^2*c*d*x^3 - 3*b^2*d*x^2)*log(-(c*x + I)/(c*x - I))^2 + integra
l(1/6*(6*I*a^2*c^3*d*x^4 + 6*a^2*c^2*d*x^3 + 6*I*a^2*c*d*x^2 + 6*a^2*d*x -
(6*a*b*c^3*d*x^4 + 2*(-3*I*a*b - b^2)*c^2*d*x^3 + 3*(2*a*b + I*b^2)*c*d*x^2
 - 6*I*a*b*d*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

```
[Out] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.71 $\int (d+icdx)(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=130

$$-iabdx-ib^2dx\text{ArcTan}(cx)-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))^2}{2c}+\frac{2bd(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{c}+\frac{ib^2d\log\left(\right.}{2}$$

```
[Out] -I*a*b*d*x-I*b^2*d*x*arctan(c*x)-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))^2/c+
2*b*d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+1/2*I*b^2*d*ln(c^2*x^2+1)/c-I*b^2
*d*polylog(2,1-2/(1-I*c*x))/c
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4974, 4930, 266, 1600, 4964, 2449, 2352}

$$-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))^2}{2c}+\frac{2bd\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{c}-iabdx-ib^2dx\text{ArcTan}(cx)+\frac{ib^2d\log\left(c^2x^2+1\right)}{2c}-\frac{ib^2d\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-I)*a*b*d*x - I*b^2*d*x*ArcTan[c*x] - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan
[c*x])^2)/c + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + ((I/2)*b^2
*d*Log[1 + c^2*x^2])/c - (I*b^2*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/c
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^2\,dx &= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{(ib)\int\left(-d^2(a+b\tan^{-1}(cx))-\frac{2i(\cdots}{\cdots}\right.}{d}\cdots\\
&= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{(2b)\int\frac{(id^2-cd^2x)(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{d}-\cdots\\
&= -iabdx-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{(2b)\int\frac{a+b\tan^{-1}(cx)}{-\frac{i}{d^2}-\frac{cx}{d^2}}\,dx}{d}-(i\cdots\\
&= -iabdx-ib^2dx\tan^{-1}(cx)-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{2bd(a+\cdots}{\cdots}\\
&= -iabdx-ib^2dx\tan^{-1}(cx)-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{2bd(a+\cdots}{\cdots}\\
&= -iabdx-ib^2dx\tan^{-1}(cx)-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{2bd(a+\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 151, normalized size = 1.16

$$\frac{id\left(-2ia^2cx-2abcx+a^2c^2x^2+b^2(-i+cx)^2\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(a-2iacx-bcx+ac^2x^2-2ib\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)+2iab\log\left(1+c^2x^2\right)+b^2\log\left(1+c^2x^2\right)-2b^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((I/2)*d*((-2*I)*a^2*c*x - 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-I + c*x)^2*ArcTa
n[c*x]^2 + 2*b*ArcTan[c*x]*(a - (2*I)*a*c*x - b*c*x + a*c^2*x^2 - (2*I)*b*L
og[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] + b^2*Log[1 + c
^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(118) = 236.
time = 0.19, size = 344, normalized size = 2.65

method	result
derivativedivides	$\frac{-id\,b^2\arctan(cx)cx+idab\arctan(cx)+b^2\arctan(cx)^2dcx-b^2\ln\left(c^2x^2+1\right)\arctan(cx)d-id\,a^2\left(-\frac{1}{2}c^2x^2+icx\right)+\frac{id\,b^2\ln\left(c^2}{2}}{}$
default	$\frac{-id\,b^2\arctan(cx)cx+idab\arctan(cx)+b^2\arctan(cx)^2dcx-b^2\ln\left(c^2x^2+1\right)\arctan(cx)d-id\,a^2\left(-\frac{1}{2}c^2x^2+icx\right)+\frac{id\,b^2\ln\left(c^2}{2}}{}$
risch	$a^2dx+\frac{ib^2\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)d}{c}+\frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)d}{c}-\frac{3i\ln(-icx+1)^2b^2d}{8c}-\frac{abd\ln\left(c^2x^2+1\right)}{c}-\frac{\ln(-icx+}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-I*d*b^2*arctan(c*x)*c*x+I*d*a*b*arctan(c*x)+b^2*arctan(c*x)^2*d*c*x-b
^2*ln(c^2*x^2+1)*arctan(c*x)*d-I*d*a^2*(-1/2*c^2*x^2+I*c*x)-1/2*I*b^2*d*dil
og(1/2*I*(c*x-I))+1/2*I*d*b^2*ln(c^2*x^2+1)-1/2*I*b^2*d*ln(c*x+I)*ln(1/2*I*
(c*x-I))+1/2*I*b^2*d*dilog(-1/2*I*(c*x+I))-1/4*I*b^2*d*ln(c*x+I)^2-I*d*a*b*
c*x+1/2*I*b^2*d*ln(c*x+I)*ln(c^2*x^2+1)+1/4*I*b^2*d*ln(c*x-I)^2-1/2*I*b^2*d
*ln(c*x-I)*ln(c^2*x^2+1)+1/2*I*b^2*d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*I*d*b
^2*arctan(c*x)^2+1/2*I*d*b^2*arctan(c*x)^2*c^2*x^2+2*a*b*arctan(c*x)*d*c*x+
I*d*a*b*arctan(c*x)*c^2*x^2-a*b*d*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) + 1/2*I*a
^2*c*d*x^2 + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x
) + b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 6*b
```

```
^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + I*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d + 1/4*b^2*d*arctan(c*x)^3/c
 + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x
) - 8*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d*x + b^
2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x
) - log(c^2*x^2 + 1))*a*b*d/c - 1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*arctan(c*x
)^2 - 1/8*(b^2*c*d*x^2 - 2*I*b^2*d*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/32*(
-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2 + I*integrate(-1/16*(12*b^2*
c^2*d*x^2*arctan(c*x) - 12*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 - (b^2
*c^3*d*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(b^2*c^3*d*x^3 - 2*b^2*c*d*x
 - 2*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/2
*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*x + 2*a^2*d - (2*a*b*c^
3*d*x^3 - (2*I*a*b + b^2)*c^2*d*x^2 + 2*(a*b + I*b^2)*c*d*x - 2*I*a*b*d)*lo
g(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{id\left(\int\left(-\frac{4ia^2}{c^2x^2+1}\right)dx+\int\left(-\frac{ib^2\log(icx+1)}{c^2x^2+1}\right)dx+\int\left(-\frac{4ab\log(icx+1)}{c^2x^2+1}\right)dx+\int\frac{4a^2cx}{c^2x^2+1}dx+\int\frac{4a^2c^3x^3}{c^2x^2+1}dx+\int\frac{2b^2cx}{c^2x^2+1}dx+\int\left(-\frac{4ia^2c^2x^2}{c^2x^2+1}\right)dx+\int\frac{2ib^2c^2x^2}{c^2x^2+1}dx+\int\left(-\frac{6abc^2x^2}{c^2x^2+1}\right)dx+\int\frac{5b^2cx\log(icx+1)}{c^2x^2+1}dx+\int\frac{4iabcx}{c^2x^2+1}dx+\int\left(-\frac{2iabc^3x^3}{c^2x^2+1}\right)dx+\int\frac{2ib^2c^2x^2\log(icx+1)}{c^2x^2+1}dx+\int\left(-\frac{4abc^2x^2\log(icx+1)}{c^2x^2+1}\right)dx+\int\left(-\frac{4iabcx\log(icx+1)}{c^2x^2+1}\right)dx+\int\left(-\frac{4iabc^3x^3\log(icx+1)}{c^2x^2+1}\right)dx\right)}{4}+\left(-\frac{ib^2cdx^2}{8}-\frac{b^2dx}{4}\right)\log(icx+1)^2+\frac{(-ib^2c^2dx^2-2b^2cdx-3ib^2d)\log(-icx+1)^2}{8c}+\frac{(-2abc^2dx^2+4iabcdx+ib^2c^2dx^2\log(icx+1)+2b^2cdx\log(icx+1)+2b^2cdx-ib^2d\log(icx+1))\log(-icx+1)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

```
[Out] I*d*(Integral(-4*I*a**2/(c**2*x**2 + 1), x) + Integral(-I*b**2*log(I*c*x +
1)/(c**2*x**2 + 1), x) + Integral(-4*a*b*log(I*c*x + 1)/(c**2*x**2 + 1), x)
 + Integral(4*a**2*c*x/(c**2*x**2 + 1), x) + Integral(4*a**2*c**3*x**3/(c**
2*x**2 + 1), x) + Integral(2*b**2*c*x/(c**2*x**2 + 1), x) + Integral(-4*I*a
**2*c**2*x**2/(c**2*x**2 + 1), x) + Integral(2*I*b**2*c**2*x**2/(c**2*x**2
+ 1), x) + Integral(-6*a*b*c**2*x**2/(c**2*x**2 + 1), x) + Integral(5*b**2*
c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(4*I*a*b*c*x/(c**2*x**2 +
1), x) + Integral(-2*I*a*b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(2*I*b**
2*c**2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-4*a*b*c**2*x**2*
log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-4*I*a*b*c*x*log(I*c*x + 1)/(
c**2*x**2 + 1), x) + Integral(-4*I*a*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2
+ 1), x))/4 + (-I*b**2*c*d*x**2/8 - b**2*d*x/4)*log(I*c*x + 1)**2 + (-I*b**
```

```
2*c**2*d*x**2 - 2*b**2*c*d*x - 3*I*b**2*d)*log(-I*c*x + 1)**2/(8*c) + (-2*a
*b*c**2*d*x**2 + 4*I*a*b*c*d*x + I*b**2*c**2*d*x**2*log(I*c*x + 1) + 2*b**2
*c*d*x*log(I*c*x + 1) + 2*b**2*c*d*x - I*b**2*d*log(I*c*x + 1))*log(-I*c*x
+ 1)/(4*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.72 $\int \frac{(d+icdx)(a+b\text{ArcTan}(cx))^2}{x}\,dx$

Optimal. Leaf size=216

$$-d(a+b\text{ArcTan}(cx))^2+icdx(a+b\text{ArcTan}(cx))^2+2d(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+2ibd(a+b\text{ArcT}$$

```
[Out] -d*(a+b*arctan(c*x))^2+I*c*d*x*(a+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2*
arctanh(-1+2/(1+I*c*x))+2*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-b^2*d*pol
ylog(2,1-2/(1+I*c*x))-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*
d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(1+I*
c*x))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.30, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$,
Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4942, 5108, 5004, 5114, 6745}

$$-ibd\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibd\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))-d(a+b\text{ArcTan}(cx))^2+icdx(a+b\text{ArcTan}(cx))^2+2ibd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+2d\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+b^2(-d)\text{Li}_2\left(1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] -(d*(a + b*ArcTan[c*x])^2) + I*c*d*x*(a + b*ArcTan[c*x])^2 + 2*d*(a + b*Arc
Tan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (2*I)*b*d*(a + b*ArcTan[c*x])*Log[
2/(1 + I*c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*Poly
Log[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx &= \int \left(icd\left(a+b\tan^{-1}(cx)\right)^2+\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)dx\\
&= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+(icd)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx\\
&= icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)\\
&= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)\\
&= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)\\
&= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)\\
&= -d\left(a+b\tan^{-1}(cx)\right)^2+icdx\left(a+b\tan^{-1}(cx)\right)^2+2d\left(a+b\tan^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 272, normalized size = 1.26

$$d\left(ia^2cx+a^2\log(cx)+iab\left(2cx\text{ArcTan}(cx)-\log\left(1+c^2x^2\right)\right)+b^2\left(\text{ArcTan}(cx)\left((1+icx)\text{ArcTan}(cx)+2i\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)+iab(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+b^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\text{ArcTan}(cx)^3+\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)-\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+i\text{ArcTan}(cx)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] d*(I*a^2*c*x + a^2*Log[c*x] + I*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2])
+ b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*ArcTan
[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + I*a*b*(PolyLog[2, (-I)*c*x
```

```
] - PolyLog[2, I*c*x]) + b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I
)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcT
an[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*
x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.57, size = 7034, normalized size = 32.56

method	result	size
derivativedivides	Expression too large to display	7034
default	Expression too large to display	7034

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*I*b^2*c*d*x*arctan(c*x)^2 + 12*I*b^2*c^3*d*integrate(1/16*x^3*arctan(c*
x)^2/(c^2*x^3 + x), x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 +
1)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^
3 + x), x) + 4*I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x
), x) - 1/4*b^2*c*d*x*arctan(c*x)*log(c^2*x^2 + 1) - 1/16*I*b^2*c*d*x*log(c
^2*x^2 + 1)^2 + 1/4*I*b^2*d*arctan(c*x)^3 + 12*b^2*c^2*d*integrate(1/16*x^2
*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*c^2*d*integrate(1/16*x^2*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*a
rctan(c*x)/(c^2*x^3 + x), x) - 8*I*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)
/(c^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + I*a^2*c*d*x + 4*b^2*c*
d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c
*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 1/16*b^2*d*log(c
^2*x^2 + 1)^2 + I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d + 12*b^2*d*i
ntegrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*d*integrate(1/16*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + b^2*d*integrate(1/16*log(c^2
*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integrate(1/16*arctan(c*x)/(c^2*x^
3 + x), x) + a^2*d*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$id\left(\int a^2c\,dx + \int\left(-\frac{ia^2}{x}\right)dx + \int b^2c\,\mathrm{atan}^2\left(cx\right)dx + \int\left(-\frac{ib^2\,\mathrm{atan}^2\left(cx\right)}{x}\right)dx + \int 2abc\,\mathrm{atan}\left(cx\right)dx + \int\left(-\frac{2iab\,\mathrm{atan}\left(cx\right)}{x}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x,x)
```

```
[Out] I*d*(Integral(a**2*c, x) + Integral(-I*a**2/x, x) + Integral(b**2*c*atan(c*
x)**2, x) + Integral(-I*b**2*atan(c*x)**2/x, x) + Integral(2*a*b*c*atan(c*x
), x) + Integral(-2*I*a*b*atan(c*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}(c\,x)\right)}^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x, x)
```

3.73 $\int \frac{(d+icdx)(a+b\text{ArcTan}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=228

$$-icd(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{x}+2icd(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+2bcd(a+b\text{A}$$

```
[Out] -I*c*d*(a+b*arctan(c*x))^2-d*(a+b*arctan(c*x))^2/x-2*I*c*d*(a+b*arctan(c*x)
)^2*arctanh(-1+2/(1+I*c*x))+2*b*c*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I*b
^2*c*d*polylog(2,-1+2/(1-I*c*x))+b*c*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I
*c*x))-b*c*d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*I*b^2*c*d*poly
log(3,1-2/(1+I*c*x))+1/2*I*b^2*c*d*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.34, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745}

$$bcd\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))-bcd\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))-icd(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+2icd\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-ib^2cd\text{Li}_2\left(\frac{2}{1-icx}-1\right)-\frac{1}{2}ib^2cd\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}ib^2cd\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/x + (2*I)*c*d*(a
 + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + 2*b*c*d*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + b*c*d
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - b*c*d*(a + b*ArcTan[c*
x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (I/2)*b^2*c*d*PolyLog[3, 1 - 2/(1 + I*
c*x)] + (I/2)*b^2*c*d*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^2}+\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx+(icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2icd\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2icd\left(a+b\tan^{-1}(cx\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2icd\left(a+b\tan^{-1}(cx\right.$$

$$= -icd\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x}+2icd\left(a+b\tan^{-1}(cx\right.$$

Mathematica [A]
time = 0.24, size = 289, normalized size = 1.27

$$\frac{id\left(ia^2+a^2cx\log(x)+iab(2\text{ArcTan}(cx)+cx(-2\log(cx)+\log(1+c^2x^2)))+ib^2\left(\text{ArcTan}(cx)^2-2cx\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)+icx\left(\text{ArcTan}(cx)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)\right)+iabcx(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+\frac{1}{24}b^2cx\left(-i\pi^3+16i\text{ArcTan}(cx)^3+24\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)-24\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+24i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+24i\text{ArcTan}(cx)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)-12\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (I*d*(I*a^2 + a^2*c*x*Log[x] + I*a*b*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Lo
g[1 + c^2*x^2])) + I*b^2*(ArcTan[c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I
)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + I*a*b*c*x*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c*x*((-I)*P
i^3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x]
)] - 24*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*P
olyLog[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)
```

```
*ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^(
(2*I)*ArcTan[c*x])]))/24))/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.64, size = 5899, normalized size = 25.87

method	result	size
derivativedivides	Expression too large to display	5899
default	Expression too large to display	5899

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] I*a^2*c*d*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*
d - a^2*d/x - 1/96*(24*b^2*d*arctan(c*x)^2 + 24*I*b^2*d*arctan(c*x)*log(c^2
*x^2 + 1) - 6*b^2*d*log(c^2*x^2 + 1)^2 - 24*(b^2*c*d*arctan(c*x)^3 + 16*b^2
*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x)
+ 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 1
6*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 16*b^
2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 3
2*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*b^2*d*integ
rate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integrate(1/16*log(c^
2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x - I*(1152*b^2*c^3*d*integrate(1/16*x^3*
arctan(c*x)^2/(c^2*x^4 + x^2), x) + 3072*a*b*c^3*d*integrate(1/16*x^3*arcta
n(c*x)/(c^2*x^4 + x^2), x) + b^2*c*d*log(c^2*x^2 + 1)^3 + 24*b^2*c*d*arctan
(c*x)^2 - 384*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^
2*x^4 + x^2), x) + 1152*b^2*c*d*integrate(1/16*x*arctan(c*x)^2/(c^2*x^4 + x
^2), x) + 96*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x
) + 3072*a*b*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 384*b^2
*c*d*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 384*b^2*d*inte
grate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x^2, x
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$id\left(\int\left(-\frac{ia^2}{x^2}\right)dx+\int\frac{a^2c}{x}\,dx+\int\left(-\frac{ib^2\operatorname{atan}^2(cx)}{x^2}\right)dx+\int\frac{b^2c\operatorname{atan}^2(cx)}{x}\,dx+\int\left(-\frac{2iab\operatorname{atan}(cx)}{x^2}\right)dx+\int\frac{2abc\operatorname{atan}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] I*d*(Integral(-I*a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(-I*b**2*a
tan(c*x)**2/x**2, x) + Integral(b**2*c*atan(c*x)**2/x, x) + Integral(-2*I*a
*b*atan(c*x)/x**2, x) + Integral(2*a*b*c*atan(c*x)/x, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\operatorname{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2, x)
```

3.74 $\int \frac{(d+icdx)(a+b\text{ArcTan}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=159

$$-\frac{bcd(a+b\text{ArcTan}(cx))}{x}+\frac{1}{2}c^2d(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{icd(a+b\text{ArcTan}(cx))^2}{x}+b^2c^2d\log$$

[Out] -b*c*d*(a+b*arctan(c*x))/x+1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan(c*x))^2/x^2-I*c*d*(a+b*arctan(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*ln(c^2*x^2+1)+2*I*b*c^2*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+b^2*c^2*d*polylog(2,-1+2/(1-I*c*x))

Rubi [A]
time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4996, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497}

$$\frac{1}{2}c^2d(a+b\text{ArcTan}(cx))^2+2ibc^2d\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{d(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{icd(a+b\text{ArcTan}(cx))^2}{x}-\frac{bcd(a+b\text{ArcTan}(cx))}{x}+b^2c^2d\text{Li}_2\left(\frac{2}{1-icx}-1\right)-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)$$

Antiderivative was successfully verified.

[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]

[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) + (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a + b*ArcTan[c*x])^2)/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x])^2)/x + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 + (2*I)*b*c^2*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + b^2*c^2*d*PolyLog[2, -1 + 2/(1 - I*c*x)]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx &= \int \left(\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{x^3}+\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\right)dx\\
&= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx+(icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx\\
&= -\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}+(bcd)\int \frac{a+b\tan^{-1}(c}{x^2\left(1+c^2x\right.}\\
&= c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{icd\left(a+b\tan^{-1}(cx)\right)^2}{x}\\
&= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right.}{2x^2}\\
&= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right.}{2x^2}\\
&= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right.}{2x^2}\\
&= -\frac{bcd\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\tan^{-1}(cx)\right)^2-\frac{d\left(a+b\tan^{-1}(cx)\right.}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 190, normalized size = 1.19

$$-\frac{d\left(a^2+2ia^2cx+2abcx-b^2(-i+cx)^2\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(a+2iacx+bcx+ac^2x^2-2ibc^2x^2\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)-4iabc^2x^2\log(cx)-2b^2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+2iabc^2x^2\log\left(1+c^2x^2\right)-2b^2c^2x^2\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d*(a^2 + (2*I)*a^2*c*x + 2*a*b*c*x - b^2*(-I + c*x)^2*ArcTan[c*x]^2 +
 2*b*ArcTan[c*x]*(a + (2*I)*a*c*x + b*c*x + a*c^2*x^2 - (2*I)*b*c^2*x^2*Log
[1 - E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*c^2*x^2*Log[c*x] - 2*b^2*c^2*x^2*L
og[(c*x)/Sqrt[1 + c^2*x^2]] + (2*I)*a*b*c^2*x^2*Log[1 + c^2*x^2] - 2*b^2*c^
2*x^2*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(149) = 298.
time = 0.55, size = 448, normalized size = 2.82

method	result
derivativedivides	$c^2\left(-id\,b^2\arctan\left(cx\right)\ln\left(c^2x^2+1\right)-idab\ln\left(c^2x^2+1\right)-\frac{id\,b^2\arctan(cx)^2}{cx}+\frac{d\,b^2\ln(cx-i)\ln(c^2}{2}\right.$
default	$c^2\left(-id\,b^2\arctan\left(cx\right)\ln\left(c^2x^2+1\right)-idab\ln\left(c^2x^2+1\right)-\frac{id\,b^2\arctan(cx)^2}{cx}+\frac{d\,b^2\ln(cx-i)\ln(c^2}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(d*a^2*(-1/2/c^2/x^2-I/c/x)-2*I*d*a*b*arctan(c*x)/c/x-d*a*b*arctan(c*x)
/c^2/x^2-I*d*b^2*arctan(c*x)^2/c/x+1/2*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/2*d*
b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*d*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/2*d*b^
2*ln(c*x+I)*ln(1/2*I*(c*x-I))-d*a*b*arctan(c*x)-d*a*b/c/x-1/2*d*b^2*arctan(
c*x)^2/c^2/x^2-d*b^2*arctan(c*x)/c/x+2*I*d*b^2*arctan(c*x)*ln(c*x)-I*d*b^2*
ln(c^2*x^2+1)*arctan(c*x)+2*I*d*a*b*ln(c*x)-1/2*d*b^2*arctan(c*x)^2-1/4*d*b
^2*ln(c*x-I)^2-1/2*d*b^2*dilog(-1/2*I*(c*x+I))+1/4*d*b^2*ln(c*x+I)^2+1/2*d*
b^2*dilog(1/2*I*(c*x-I))-1/2*d*b^2*ln(c^2*x^2+1)-d*b^2*ln(c*x)*ln(1+I*c*x)+
d*b^2*ln(c*x)*ln(1-I*c*x)+d*b^2*ln(c*x)-d*b^2*dilog(1+I*c*x)+d*b^2*dilog(1-
I*c*x)-I*d*a*b*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c*d - ((c*arctan
(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + 1/2*((arctan(c*x)^2 - log(c^2*x^2
 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2*d + 1/16
*I*(4*(c*arctan(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x
^4 + x^2), x) - 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 32*c*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*integrate(
1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^2 + 1)^
2/(c^2*x^4 + x^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2*c*d/x
- I*a^2*c*d/x - 1/2*b^2*d*arctan(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^2*integral(1/2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*
x + 2*a^2*d - (2*a*b*c^3*d*x^3 + 2*(-I*a*b + b^2)*c^2*d*x^2 + (2*a*b - I*b^
2)*c*d*x - 2*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^5 + x^3), x) + (2*I
*b^2*c*d*x + b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3, x)
```

3.75 $\int \frac{(d+icdx)(a+b\textbf{ArcTan}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=224

$$-\frac{b^2c^2d}{3x}-\frac{1}{3}b^2c^3d\text{ArcTan}(cx)-\frac{bcd(a+b\text{ArcTan}(cx))}{3x^2}-\frac{ibc^2d(a+b\text{ArcTan}(cx))}{x}-\frac{1}{6}ic^3d(a+b\text{ArcTan}(cx))^2-$$

```
[Out] -1/3*b^2*c^2*d/x-1/3*b^2*c^3*d*arctan(c*x)-1/3*b*c*d*(a+b*arctan(c*x))/x^2-
I*b*c^2*d*(a+b*arctan(c*x))/x-1/6*I*c^3*d*(a+b*arctan(c*x))^2-1/3*d*(a+b*ar
ctan(c*x))^2/x^3-1/2*I*c*d*(a+b*arctan(c*x))^2/x^2+I*b^2*c^3*d*ln(x)-1/2*I*
b^2*c^3*d*ln(c^2*x^2+1)-2/3*b*c^3*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+1/3
*I*b^2*c^3*d*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.32, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4996, 4946, 5038, 331, 209, 5044, 4988, 2497, 272, 36, 29, 31, 5004}

$$-\frac{1}{6}ic^3d(a+b\text{ArcTan}(cx))^2-\frac{2}{3}bc^3d\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{ibc^2d(a+b\text{ArcTan}(cx))}{x}-\frac{d(a+b\text{ArcTan}(cx))^2}{3x^3}-\frac{icd(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{bcd(a+b\text{ArcTan}(cx))}{3x^2}-\frac{1}{3}b^2c^3d\text{ArcTan}(cx)+\frac{1}{3}ib^2c^3d\text{Li}_2\left(\frac{2}{1-icx}-1\right)+ib^2c^3d\log(x)-\frac{b^2c^2d}{3x}-\frac{1}{2}ib^2c^3d\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d)/x - (b^2*c^3*d*ArcTan[c*x])/3 - (b*c*d*(a + b*ArcTan[c*x])
)/(3*x^2) - (I*b*c^2*d*(a + b*ArcTan[c*x]))/x - (I/6)*c^3*d*(a + b*ArcTan[c
*x])^2 - (d*(a + b*ArcTan[c*x])^2)/(3*x^3) - ((I/2)*c*d*(a + b*ArcTan[c*x])
^2)/x^2 + I*b^2*c^3*d*Log[x] - (I/2)*b^2*c^3*d*Log[1 + c^2*x^2] - (2*b*c^3*
d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*d*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+icdx)\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d(a+b\tan^{-1}(cx))^2}{x^4}+\frac{icd(a+b\tan^{-1}(cx))^2}{x^3}\right)dx$$

$$= d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx+(icd)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$$

$$= -\frac{d(a+b\tan^{-1}(cx))^2}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b\tan}{x^3\,(1+}$$

$$= -\frac{d(a+b\tan^{-1}(cx))^2}{3x^3}-\frac{icd(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b\tan}{x^3}$$

$$= -\frac{bcd(a+b\tan^{-1}(cx))}{3x^2}-\frac{ibc^2d(a+b\tan^{-1}(cx))}{x}-\frac{1}{6}ic^3d\left(a+b\tan^{-1}\right.$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd(a+b\tan^{-1}(cx))}{3x^2}-\frac{ibc^2d(a+b\tan^{-1}(cx))}{x}-\frac{1}{6}ic^3d(a$$

$$= -\frac{b^2c^2d}{3x}-\frac{1}{3}b^2c^3d\tan^{-1}(cx)-\frac{bcd(a+b\tan^{-1}(cx))}{3x^2}-\frac{ibc^2d(a+b\tan}{x}$$

$$= -\frac{b^2c^2d}{3x}-\frac{1}{3}b^2c^3d\tan^{-1}(cx)-\frac{bcd(a+b\tan^{-1}(cx))}{3x^2}-\frac{ibc^2d(a+b\tan}{x}$$

Mathematica [A]
time = 0.32, size = 240, normalized size = 1.07

$$\frac{d\left(-2a^2-3ia^2cx-2abcx-6iabc^2x^2-2b^2c^2x^2-ib^2(-2i+3cx+c^3x^3)\operatorname{ArcTan}(cx)^2-2b\operatorname{ArcTan}(cx)\left(bcx(1+3icx+c^2x^2)+a(2+3icx+3ic^3x^3)+2bc^3x^3\log\left(1-e^{2i\operatorname{ArcTan}(cx)}\right)\right)-4abc^3x^3\log(cx)+6ib^2c^3x^3\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+2abc^3x^3\log\left(1+c^2x^2\right)+2ib^2c^3x^3\operatorname{PolyLog}\left(2,e^{2i\operatorname{ArcTan}(cx)}\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d*(-2*a^2 - (3*I)*a^2*c*x - 2*a*b*c*x - (6*I)*a*b*c^2*x^2 - 2*b^2*c^2*x^2
- I*b^2*(-2*I + 3*c*x + c^3*x^3)*ArcTan[c*x]^2 - 2*b*ArcTan[c*x]*(b*c*x*(1
+ (3*I)*c*x + c^2*x^2) + a*(2 + (3*I)*c*x + (3*I)*c^3*x^3) + 2*b*c^3*x^3*Lo
g[1 - E^((2*I)*ArcTan[c*x])]) - 4*a*b*c^3*x^3*Log[c*x] + (6*I)*b^2*c^3*x^3*
Log[(c*x)/Sqrt[1 + c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 + c^2*x^2] + (2*I)*b^2*c
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(6*x^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(198) = 396.
time = 0.65, size = 514, normalized size = 2.29

method	result
derivativedivides	$c^3\left(-idab\arctan(cx)-\frac{idb^2\arctan(cx)^2}{2}-\frac{2dab\arctan(cx)}{3c^3x^3}-\frac{ib^2d\ln(cx+i)\ln(c^2x^2+1)}{6}+\frac{ib^2d\ln(cx+i)\ln}{6}\right.$
default	$c^3\left(-idab\arctan(cx)-\frac{idb^2\arctan(cx)^2}{2}-\frac{2dab\arctan(cx)}{3c^3x^3}-\frac{ib^2d\ln(cx+i)\ln(c^2x^2+1)}{6}+\frac{ib^2d\ln(cx+i)\ln}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(d*a^2*(-1/2*I/c^2/x^2-1/3/c^3/x^3)-I*d*b^2*arctan(c*x)/c/x-1/2*I*d*b^2
*arctan(c*x)^2/c^2/x^2-2/3*d*a*b*arctan(c*x)/c^3/x^3-I*d*a*b/c/x+1/12*I*d*b
^2*ln(c*x+I)^2-1/6*I*d*b^2*dilog(-1/2*I*(c*x+I))-1/12*I*d*b^2*ln(c*x-I)^2+1
/6*I*d*b^2*dilog(1/2*I*(c*x-I))-1/3*I*d*b^2*dilog(1+I*c*x)-2/3*d*b^2*ln(c*x
)*arctan(c*x)+I*d*b^2*ln(c*x)-1/3*d*b^2/c/x-1/2*I*d*b^2*arctan(c*x)^2-1/2*I
*d*b^2*ln(c^2*x^2+1)+1/3*I*d*b^2*dilog(1-I*c*x)-1/6*I*d*b^2*ln(c*x+I)*ln(c^
2*x^2+1)+1/6*I*d*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/6*I*d*b^2*ln(c*x-I)*ln(-1/2*
I*(c*x+I))+1/6*I*d*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/3*d*b^2*arctan(c*x)+1/
3*b^2*ln(c^2*x^2+1)*arctan(c*x)*d+1/3*a*b*d*ln(c^2*x^2+1)-I*d*a*b*arctan(c*
x)/c^2/x^2-I*d*a*b*arctan(c*x)+1/3*I*d*b^2*ln(c*x)*ln(1-I*c*x)-1/3*I*d*b^2*
ln(c*x)*ln(1+I*c*x)-1/3*d*a*b/c^2/x^2-1/3*d*b^2*arctan(c*x)^2/c^3/x^3-1/3*d
*b^2*arctan(c*x)/c^2/x^2-2/3*d*a*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d + 1/3*((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d - 1/2*I*a^2*c
*d/x^2 - 1/3*a^2*d/x^3 + 1/96*(96*I*x^3*integrate(1/48*(20*b^2*c^2*d*x^2*ar
ctan(c*x) + 36*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 + 3*(b^2*c^3*d*x^3
 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x + 6*(b^
2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) + 9
6*x^3*integrate(1/48*(36*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x)^2 + 3*(b^2*c^2
*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 - 4*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x)*arct
an(c*x) - 2*(5*b^2*c^2*d*x^2 - 6*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x))*l
og(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) - 4*(3*I*b^2*c*d*x + 2*b^2*d)*arctan(c
*x)^2 + 4*(3*b^2*c*d*x - 2*I*b^2*d)*arctan(c*x)*log(c^2*x^2 + 1) + (3*I*b^2
*c*d*x + 2*b^2*d)*log(c^2*x^2 + 1)^2)/x^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(1/6*(6*I*a^2*c^3*d*x^3 + 6*a^2*c^2*d*x^2 + 6*I*a^2*c*
d*x + 6*a^2*d - (6*a*b*c^3*d*x^3 + 3*(-2*I*a*b + b^2)*c^2*d*x^2 + 2*(3*a*b
- I*b^2)*c*d*x - 6*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^6 + x^4), x)
+ (3*I*b^2*c*d*x + 2*b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4, x)
```

3.76 $\int x^3(d+icdx)^2(a+b\textbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=373

$$\frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\text{ArcTan}(cx)}{5c^4}+\frac{5b^2d^2x\text{ArcTan}(cx)}{6c^3}+\frac{2ibd^2x^2(a+b\text{A}}{5c^2}$$

```
[Out] 5/6*a*b*d^2*x/c^3+4/5*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+31/180*
b^2*d^2*x^2/c^2+2/5*I*b*d^2*x^2*(a+b*arctan(c*x))/c^2-1/60*b^2*d^2*x^4+2/5*
I*c*d^2*x^5*(a+b*arctan(c*x))^2+5/6*b^2*d^2*x*arctan(c*x)/c^3-3/5*I*b^2*d^2
*x/c^3-5/18*b*d^2*x^3*(a+b*arctan(c*x))/c+1/15*I*b^2*d^2*x^3/c+1/15*b*c*d^2
*x^5*(a+b*arctan(c*x))-49/60*d^2*(a+b*arctan(c*x))^2/c^4+1/4*d^2*x^4*(a+b*a
rctan(c*x))^2+3/5*I*b^2*d^2*arctan(c*x)/c^4-1/6*c^2*d^2*x^6*(a+b*arctan(c*x
))^2-1/5*I*b*d^2*x^4*(a+b*arctan(c*x))-53/90*b^2*d^2*ln(c^2*x^2+1)/c^4-2/5*
b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^4
```

Rubi [A]
time = 0.70, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4996, 4946, 5036, 272, 45, 4930, 266, 5004, 308, 209, 327, 5040, 4964, 2449, 2352}

$$-\frac{49d^2(a+b\text{ArcTan}(cx))^2}{60c^4}+\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{5c^4}-\frac{1}{6}c^2d^2x^6(a+b\text{ArcTan}(cx))^2+\frac{2ibd^2x^2(a+b\text{ArcTan}(cx))}{5c^2}+\frac{2}{5}icd^2x^5(a+b\text{ArcTan}(cx))^2+\frac{1}{15}bcd^2x^5(a+b\text{ArcTan}(cx))+\frac{1}{4}d^2x^4(a+b\text{ArcTan}(cx))^2-\frac{1}{5}ibd^2x^4(a+b\text{ArcTan}(cx))-\frac{5bd^2x^3(a+b\text{ArcTan}(cx))}{18c}+\frac{5abd^2x}{6c^3}+\frac{3ib^2d^2\text{ArcTan}(cx)}{5c^4}+\frac{5b^2d^2x\text{ArcTan}(cx)}{6c^3}-\frac{2b^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}-\frac{53b^2d^2\log\left(c^2x^2+1\right)}{90c^4}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (5*a*b*d^2*x)/(6*c^3) - (((3*I)/5)*b^2*d^2*x)/c^3 + (31*b^2*d^2*x^2)/(180*c
^2) + ((I/15)*b^2*d^2*x^3)/c - (b^2*d^2*x^4)/60 + (((3*I)/5)*b^2*d^2*ArcTan
[c*x])/c^4 + (5*b^2*d^2*x*ArcTan[c*x])/(6*c^3) + (((2*I)/5)*b*d^2*x^2*(a +
b*ArcTan[c*x]))/c^2 - (5*b*d^2*x^3*(a + b*ArcTan[c*x]))/(18*c) - (I/5)*b*d^
2*x^4*(a + b*ArcTan[c*x]) + (b*c*d^2*x^5*(a + b*ArcTan[c*x]))/15 - (49*d^2*
(a + b*ArcTan[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + ((2*I
)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^6*(a + b*ArcTan[c*x])^2)/
6 + (((4*I)/5)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (53*b^2*
d^2*Log[1 + c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/
(5*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^5\right($$

$$= d^2\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(2icd^2\right)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{6}c^2d^2x^6($$

$$= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}icd^2x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{6}c^2d^2x^6($$

$$= -\frac{bd^2x^3(a+b\tan^{-1}(cx))}{6c}-\frac{1}{5}ibd^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{15}bcd^2x^5($$

$$= \frac{abd^2x}{2c^3}+\frac{2ibd^2x^2(a+b\tan^{-1}(cx))}{5c^2}-\frac{5bd^2x^3(a+b\tan^{-1}(cx))}{18c}-\frac{1}{5}i$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{ib^2d^2x^3}{15c}+\frac{b^2d^2x\tan^{-1}(cx)}{2c^3}+\frac{2ibd^2x^2(a+b\tan}{5c^2}$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{7b^2d^2x^2}{60c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\tan^{-}}{5c^4}$$

$$= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\tan}{5c^4}$$

Mathematica [A]
time = 0.74, size = 342, normalized size = 0.92

$$\frac{d^2\left(108iab+34b^2+150abcx-108ib^2cx+72iabc^2x^2+31b^2c^2x^2-50abc^3x^3+12ib^2c^3x^3+45a^2c^4x^4-36iabc^4x^4-3b^2c^4x^4+72ia^2c^5x^5+12abc^5x^5-30a^2c^6x^6-3b^2(1-15c^4x^4-24ic^5x^5+10c^6x^6)\,\text{ArcTan}(cx)^2+2b\,\text{ArcTan}(cx)\left(b(54i+75cx+36ic^2x^2-25c^3x^3-18ic^4x^4+6c^5x^5)+a(-75+45c^4x^4+72ic^5x^5-30c^6x^6)+72ib\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)-72iab\log\left(1+c^2x^2\right)-106b^2\log\left(1+c^2x^2\right)+72b^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{180c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^2*((108*I)*a*b + 34*b^2 + 150*a*b*c*x - (108*I)*b^2*c*x + (72*I)*a*b*c^2
*x^2 + 31*b^2*c^2*x^2 - 50*a*b*c^3*x^3 + (12*I)*b^2*c^3*x^3 + 45*a^2*c^4*x^
4 - (36*I)*a*b*c^4*x^4 - 3*b^2*c^4*x^4 + (72*I)*a^2*c^5*x^5 + 12*a*b*c^5*x^
5 - 30*a^2*c^6*x^6 - 3*b^2*(1 - 15*c^4*x^4 - (24*I)*c^5*x^5 + 10*c^6*x^6)*A
rcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(54*I + 75*c*x + (36*I)*c^2*x^2 - 25*c^3*
x^3 - (18*I)*c^4*x^4 + 6*c^5*x^5) + a*(-75 + 45*c^4*x^4 + (72*I)*c^5*x^5 -
30*c^6*x^6) + (72*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (72*I)*a*b*Log[1 +
 c^2*x^2] - 106*b^2*Log[1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x])]))/(180*c^4)
```

Maple [A]
time = 0.29, size = 624, normalized size = 1.67 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(4/5*I*d^2*a*b*arctan(c*x)*c^5*x^5+5/6*a*b*c*d^2*x+5/6*b^2*c*d^2*x*ar
ctan(c*x)-1/3*d^2*a*b*arctan(c*x)*c^6*x^6+1/2*d^2*a*b*arctan(c*x)*c^4*x^4-1
/5*I*d^2*a*b*c^4*x^4+2/5*I*d^2*a*b*c^2*x^2-1/5*I*d^2*b^2*arctan(c*x)*c^4*x^
4+2/5*I*d^2*b^2*arctan(c*x)^2*c^5*x^5+2/5*I*d^2*b^2*arctan(c*x)*c^2*x^2+1/1
5*d^2*b^2*arctan(c*x)*c^5*x^5-5/18*d^2*b^2*arctan(c*x)*c^3*x^3+1/15*I*d^2*b
^2*c^3*x^3-3/5*I*d^2*b^2*c*x-1/6*d^2*b^2*arctan(c*x)^2*c^6*x^6+1/4*d^2*b^2*
arctan(c*x)^2*c^4*x^4+1/15*d^2*a*b*c^5*x^5-5/18*d^2*a*b*c^3*x^3-2/5*I*d^2*a
*b*ln(c^2*x^2+1)-2/5*I*d^2*b^2*ln(c^2*x^2+1)*arctan(c*x)+3/5*I*d^2*b^2*arct
an(c*x)-1/60*d^2*b^2*c^4*x^4+31/180*d^2*b^2*c^2*x^2-5/6*d^2*a*b*arctan(c*x)
+1/5*d^2*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/5*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1
)-1/5*d^2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/5*d^2*b^2*ln(c*x-I)*ln(c^2*x^2
+1)+d^2*a^2*(-1/6*c^6*x^6+2/5*I*c^5*x^5+1/4*c^4*x^4)-5/12*d^2*b^2*arctan(c*
x)^2+1/10*d^2*b^2*ln(c*x+I)^2-1/10*d^2*b^2*ln(c*x-I)^2+1/5*d^2*b^2*dilog(1/
2*I*(c*x-I))-1/5*d^2*b^2*dilog(-1/2*I*(c*x+I))-53/90*b^2*d^2*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^2*c^2*d^2*x^6 + 2/5*I*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctan(c*x)^2
+ 1/4*a^2*d^2*x^4 - 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 +
15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b*c^2*d^2 + 1/5*I*(4*x^5*arctan(c*x) - c
*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^2 + 1/6*(3*x^4*a
rctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^2 - 1/12*(2
*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arc
tan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 - 1/120*(5*b^2*c^2*d^2*x^6 -
12*I*b^2*c*d^2*x^5)*arctan(c*x)^2 + 1/120*(-5*I*b^2*c^2*d^2*x^6 - 12*b^2*c*
d^2*x^5)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(5*b^2*c^2*d^2*x^6 - 12*I*b^2
*c*d^2*x^5)*log(c^2*x^2 + 1)^2 - integrate(-1/240*(68*b^2*c^3*d^2*x^6*arcta
n(c*x) - 180*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x)^2 - 15*(b^2*c^
4*d^2*x^7 + b^2*c^2*d^2*x^5)*log(c^2*x^2 + 1)^2 - 2*(5*b^2*c^4*d^2*x^7 - 12
*b^2*c^2*d^2*x^5 - 60*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan(c*x))*log(c^
2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1/120*(180*(b^2*c^3*d^2*x^6 + b
^2*c*d^2*x^4)*arctan(c*x)^2 + 15*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*log(c^2*
x^2 + 1)^2 + 2*(5*b^2*c^4*d^2*x^7 - 12*b^2*c^2*d^2*x^5)*arctan(c*x) + (17*b
^2*c^3*d^2*x^6 + 30*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x))*log(c^
2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(10*b^2*c^2*d^2*x^6 - 24*I*b^2*c*d^2*x^5 - 15*b^2*d^2*x^4)*log(-(c*x
+ I)/(c*x - I))^2 + integral(-1/60*(60*a^2*c^4*d^2*x^7 - 120*I*a^2*c^3*d^2*
x^6 - 120*I*a^2*c*d^2*x^4 - 60*a^2*d^2*x^3 - (-60*I*a*b*c^4*d^2*x^7 - 10*(1
2*a*b - I*b^2)*c^3*d^2*x^6 + 24*b^2*c^2*d^2*x^5 - 15*(8*a*b + I*b^2)*c*d^2*
x^4 + 60*I*a*b*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

```
[Out] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.77 $\int x^2(d+icdx)^2(a+b\textbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=333

$$\frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\text{ArcTan}(cx)}{30c^3}+\frac{ib^2d^2x\text{ArcTan}(cx)}{c^2}-\frac{8bd^2x^2(a+b\text{ArcTan}(cx))}{15c}$$

```
[Out] -1/3*I*b*d^2*x^3*(a+b*arctan(c*x))+19/30*b^2*d^2*x/c^2+I*a*b*d^2*x/c^2-1/30
*b^2*d^2*x^3-19/30*b^2*d^2*arctan(c*x)/c^3+1/6*I*b^2*d^2*x^2/c-8/15*b*d^2*x
^2*(a+b*arctan(c*x))/c-31/30*I*d^2*(a+b*arctan(c*x))^2/c^3+1/10*b*c*d^2*x^4
*(a+b*arctan(c*x))-2/3*I*b^2*d^2*ln(c^2*x^2+1)/c^3+1/3*d^2*x^3*(a+b*arctan(
c*x))^2-8/15*I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^3-1/5*c^2*d^2*x^5*(a+b*ar
ctan(c*x))^2-16/15*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+I*b^2*d^2*x*
arctan(c*x)/c^2+1/2*I*c*d^2*x^4*(a+b*arctan(c*x))^2
```

Rubi [A]
time = 0.62, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4996, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 272, 45, 4930, 266, 5004, 308}

$$-\frac{31id^2(a+b\text{ArcTan}(cx))^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{15c^3}-\frac{1}{5}c^2d^2x^5(a+b\text{ArcTan}(cx))^2+\frac{1}{2}icd^2x^4(a+b\text{ArcTan}(cx))^2+\frac{1}{10}bcd^2x^4(a+b\text{ArcTan}(cx))+\frac{1}{3}d^2x^3(a+b\text{ArcTan}(cx))^2-\frac{1}{3}ibd^2x^3(a+b\text{ArcTan}(cx))-\frac{8bd^2x^2(a+b\text{ArcTan}(cx))}{15c}+\frac{iabd^2x}{c^2}-\frac{19b^2d^2\text{ArcTan}(cx)}{30c^3}+\frac{ib^2d^2x\text{ArcTan}(cx)}{c^2}-\frac{8ib^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{15c^3}+\frac{19b^2d^2x}{30c^2}-\frac{2ib^2d^2\log\left(c^2x^2+1\right)}{3c^3}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (I*a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + ((I/6)*b^2*d^2*x^2)/c - (b^2*
d^2*x^3)/30 - (19*b^2*d^2*ArcTan[c*x])/(30*c^3) + (I*b^2*d^2*x*ArcTan[c*x])
/c^2 - (8*b*d^2*x^2*(a + b*ArcTan[c*x]))/(15*c) - (I/3)*b*d^2*x^3*(a + b*Ar
cTan[c*x]) + (b*c*d^2*x^4*(a + b*ArcTan[c*x]))/10 - (((31*I)/30)*d^2*(a + b
*ArcTan[c*x])^2)/c^3 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/2)*c*d^2*x^4*
(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^5*(a + b*ArcTan[c*x])^2)/5 - (16*b*d^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((2*I)/3)*b^2*d^2*Log[1
+ c^2*x^2])/c^3 - (((8*I)/15)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^4\left(\right.\right. \\
&= d^2\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(2icd^2\right)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}c^2d^2x^5\left(\right. \\
&= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}icd^2x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}c^2d^2x^5\left(\right. \\
&= -\frac{bd^2x^2(a+b\tan^{-1}(cx))}{3c}-\frac{1}{3}ibd^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{10}bcd^2x^4\left(\right. \\
&= \frac{iabd^2x}{c^2}+\frac{b^2d^2x}{3c^2}-\frac{8bd^2x^2(a+b\tan^{-1}(cx))}{15c}-\frac{1}{3}ibd^2x^3\left(a+b\tan^{-1}\right. \\
&= \frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}-\frac{1}{30}b^2d^2x^3-\frac{b^2d^2\tan^{-1}(cx)}{3c^3}+\frac{ib^2d^2x\tan^{-1}(cx}{c^2} \\
&= \frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tan^{-1}(cx)}{30c^3}+\frac{ib^2}{} \\
&= \frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tan^{-1}(cx)}{30c^3}+\frac{ib^2}{}
\end{aligned}
$$

Mathematica [A]
time = 0.75, size = 306, normalized size = 0.92

$$
-\frac{d^2\left(9ab-5ib^2-30iabcx-19b^2cx+16abc^2x^2-5ib^2c^2x^2-10a^2c^3x^3+10iabc^3x^3+b^2c^3x^3-15ia^2c^4x^4-3abc^4x^4+6a^2c^5x^5+b^2(-i+cx)^3\left(-1+3icx+6c^2x^2\right)\mathrm{ArcTan}(cx)^2+b\mathrm{ArcTan}(cx)\left(b(19-30icx+16c^2x^2+10ic^3x^3-3c^4x^4)+2a(15i-10c^3x^3-15ic^4x^4+6c^5x^5)+32b\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-16ab\log\left(1+c^2x^2\right)+20ib^2\log\left(1+c^2x^2\right)-16ib^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{30c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -1/30*(d^2*(9*a*b - (5*I)*b^2 - (30*I)*a*b*c*x - 19*b^2*c*x + 16*a*b*c^2*x^
2 - (5*I)*b^2*c^2*x^2 - 10*a^2*c^3*x^3 + (10*I)*a*b*c^3*x^3 + b^2*c^3*x^3 -
 (15*I)*a^2*c^4*x^4 - 3*a*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-I + c*x)^3*(-1
+ (3*I)*c*x + 6*c^2*x^2)*ArcTan[c*x]^2 + b*ArcTan[c*x]*(b*(19 - (30*I)*c*x
+ 16*c^2*x^2 + (10*I)*c^3*x^3 - 3*c^4*x^4) + 2*a*(15*I - 10*c^3*x^3 - (15*I
)*c^4*x^4 + 6*c^5*x^5) + 32*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 16*a*b*Log[
1 + c^2*x^2] + (20*I)*b^2*Log[1 + c^2*x^2] - (16*I)*b^2*PolyLog[2, -E^((2*I
)*ArcTan[c*x])]))/c^3
```

Maple [A]
time = 0.32, size = 586, normalized size = 1.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(I*d^2*a*b*arctan(c*x)*c^4*x^4+I*d^2*a*b*c*x-1/3*I*d^2*a*b*c^3*x^3-2/
5*d^2*a*b*arctan(c*x)*c^5*x^5+2/3*d^2*a*b*arctan(c*x)*c^3*x^3+I*d^2*b^2*arc
tan(c*x)*c*x+1/2*I*d^2*b^2*arctan(c*x)^2*c^4*x^4-1/3*I*d^2*b^2*arctan(c*x)*
c^3*x^3+1/10*d^2*a*b*c^4*x^4+4/15*I*d^2*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/6*I*d
^2*b^2*c^2*x^2-1/5*d^2*b^2*arctan(c*x)^2*c^5*x^5+1/3*d^2*b^2*arctan(c*x)^2*
c^3*x^3+1/10*d^2*b^2*arctan(c*x)*c^4*x^4-8/15*d^2*b^2*arctan(c*x)*c^2*x^2-4
/15*I*d^2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+4/15*I*d^2*b^2*ln(c*x+I)*ln(1/2*
I*(c*x-I))-4/15*I*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)-I*d^2*a*b*arctan(c*x)-8/1
5*d^2*a*b*c^2*x^2-2/3*I*d^2*b^2*ln(c^2*x^2+1)-4/15*I*d^2*b^2*dilog(-1/2*I*(
c*x+I))-2/15*I*d^2*b^2*ln(c*x-I)^2+4/15*I*d^2*b^2*dilog(1/2*I*(c*x-I))+2/15
*I*d^2*b^2*ln(c*x+I)^2-1/2*I*d^2*b^2*arctan(c*x)^2-1/30*d^2*b^2*c^3*x^3+19/
30*d^2*b^2*c*x+d^2*a^2*(-1/5*c^5*x^5+1/2*I*c^4*x^4+1/3*c^3*x^3)-19/30*d^2*b
^2*arctan(c*x)+8/15*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^2+8/15*a*b*ln(c^2*x^2+1
)*d^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/5*a^2*c^2*d^2*x^5 + 1/2*I*a^2*c*d^2*x^4 - 1/10*(4*x^5*arctan(c*x) - c*((
c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x
^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)
)*a*b*c*d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b*d^2 - 1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*a
rctan(c*x)^2 + 1/120*(-6*I*b^2*c^2*d^2*x^5 - 15*b^2*c*d^2*x^4 + 10*I*b^2*d^
2*x^3)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c
*d^2*x^4 - 10*b^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - integrate(1/240*(180*(b^2*c
^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x)^2 + 15*(b^2*c^4*d^2*x^6 - b^2*d^2*x^2
)*log(c^2*x^2 + 1)^2 - 4*(21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3)*arctan(c*x
) + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^2*d^2*x^4 - 60*(b^2*c^3*d^2*x^5 + b^2*c
*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1/
120*(180*(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*arctan(c*x)^2 + 15*(b^2*c^3*d^2*
x^5 + b^2*c*d^2*x^3)*log(c^2*x^2 + 1)^2 + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^2
*d^2*x^4)*arctan(c*x) + (21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3 + 30*(b^2*c^
4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(-(c*x +
 I)/(c*x - I))^2 + integral(-1/30*(30*a^2*c^4*d^2*x^6 - 60*I*a^2*c^3*d^2*x^
5 - 60*I*a^2*c*d^2*x^3 - 30*a^2*d^2*x^2 - (-30*I*a*b*c^4*d^2*x^6 - 6*(10*a*
b - I*b^2)*c^3*d^2*x^5 + 15*b^2*c^2*d^2*x^4 - 10*(6*a*b + I*b^2)*c*d^2*x^3
+ 30*I*a*b*d^2*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.78 $\int x(d+icdx)^2(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=293

$$-\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2-\frac{2ib^2d^2\text{ArcTan}(cx)}{3c^2}-\frac{3b^2d^2x\text{ArcTan}(cx)}{2c}-\frac{2}{3}ibd^2x^2(a+b\text{ArcTan}(cx))+\frac{1}{6}bcd^2x$$

```
[Out] -3/2*a*b*d^2*x/c+2/3*I*b^2*d^2*x/c-1/12*b^2*d^2*x^2-2/3*I*b^2*d^2*arctan(c*
x)/c^2-3/2*b^2*d^2*x*arctan(c*x)/c-2/3*I*b*d^2*x^2*(a+b*arctan(c*x))+1/6*b*
c*d^2*x^3*(a+b*arctan(c*x))+17/12*d^2*(a+b*arctan(c*x))^2/c^2+1/2*d^2*x^2*(
a+b*arctan(c*x))^2+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))^2-1/4*c^2*d^2*x^4*(a+b
*arctan(c*x))^2-4/3*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2+5/6*b^2*d
^2*ln(c^2*x^2+1)/c^2+2/3*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^2
```

Rubi [A]
time = 0.45, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {4996, 4946, 5036, 4930, 266, 5004, 327, 209, 5040, 4964, 2449, 2352, 272, 45}

$$-\frac{1}{4}c^2d^2x^4(a+b\text{ArcTan}(cx))^2+\frac{17d^2(a+b\text{ArcTan}(cx))^2}{12c^2}-\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^2}+\frac{2}{3}icd^2x^3(a+b\text{ArcTan}(cx))^2+\frac{1}{6}bcd^2x^3(a+b\text{ArcTan}(cx))+\frac{1}{2}d^2x^2(a+b\text{ArcTan}(cx))^2-\frac{2}{3}ibd^2x^2(a+b\text{ArcTan}(cx))-\frac{3abd^2x}{2c}-\frac{2ib^2d^2\text{ArcTan}(cx)}{3c^2}-\frac{3b^2d^2x\text{ArcTan}(cx)}{2c}+\frac{2b^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^2}+\frac{5b^2d^2\log\left(c^2x^2+1\right)}{6c^2}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-3*a*b*d^2*x)/(2*c) + (((2*I)/3)*b^2*d^2*x)/c - (b^2*d^2*x^2)/12 - (((2*I)
/3)*b^2*d^2*ArcTan[c*x])/c^2 - (3*b^2*d^2*x*ArcTan[c*x])/(2*c) - ((2*I)/3)*
b*d^2*x^2*(a + b*ArcTan[c*x]) + (b*c*d^2*x^3*(a + b*ArcTan[c*x]))/6 + (17*d
^2*(a + b*ArcTan[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + ((
2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^4*(a + b*ArcTan[c*x])^
2)/4 - (((4*I)/3)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (5*b^
2*d^2*Log[1 + c^2*x^2])/(6*c^2) + (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(3*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int\left(d^2x\left(a+b\tan^{-1}(cx)\right)^2+2icd^2x^2\left(a+b\tan^{-1}(cx)\right)^2-c^2d^2x^3(a\right. \\
&= d^2\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(2icd^2\right)\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{4}c^2d^2x^4(a \\
&= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}icd^2x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{4}c^2d^2x^4(a \\
&= -\frac{abd^2x}{c}-\frac{2}{3}ibd^2x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a+b\tan^{-1}(cx)\right)+ \\
&= -\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{b^2d^2x\tan^{-1}(cx)}{c}-\frac{2}{3}ibd^2x^2\left(a+b\tan^{-1}(cx)\right) \\
&= -\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{2ib^2d^2\tan^{-1}(cx)}{3c^2}-\frac{3b^2d^2x\tan^{-1}(cx)}{2c}-\frac{2}{3}ibd \\
&= -\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2-\frac{2ib^2d^2\tan^{-1}(cx)}{3c^2}-\frac{3b^2d^2x\tan^{-1}}{2c}
\end{aligned}$$

Mathematica [A]
time = 0.43, size = 257, normalized size = 0.88

$$-\frac{d^2\left(b^2+18abcx-8ib^2cx-6a^2c^2x^2+8iabc^2x^2+b^2c^2x^2-8ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(-i+cx)^3(i+3cx)\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(b(4i+9cx+4ic^2x^2-c^3x^3)+a(-9-6c^2x^2-8ic^3x^3+3c^4x^4)+8ib\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)-8iab\log\left(1+c^2x^2\right)-10b^2\log\left(1+c^2x^2\right)+8b^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -1/12*(d^2*(b^2 + 18*a*b*c*x - (8*I)*b^2*c*x - 6*a^2*c^2*x^2 + (8*I)*a*b*c^
2*x^2 + b^2*c^2*x^2 - (8*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b
^2*(-I + c*x)^3*(I + 3*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(4*I + 9*c*x
 + (4*I)*c^2*x^2 - c^3*x^3) + a*(-9 - 6*c^2*x^2 - (8*I)*c^3*x^3 + 3*c^4*x^4
) + (8*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (8*I)*a*b*Log[1 + c^2*x^2] -
10*b^2*Log[1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(258) = 516.
time = 0.25, size = 530, normalized size = 1.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-3/2*a*b*c*d^2*x-3/2*b^2*c*d^2*x*arctan(c*x)-1/2*d^2*a*b*arctan(c*x)
*c^4*x^4+d^2*a*b*arctan(c*x)*c^2*x^2-2/3*I*d^2*a*b*c^2*x^2+2/3*I*d^2*b^2*ar
ctan(c*x)^2*c^3*x^3-2/3*I*d^2*b^2*arctan(c*x)*c^2*x^2+4/3*I*d^2*a*b*arctan(
c*x)*c^3*x^3+1/6*d^2*b^2*arctan(c*x)*c^3*x^3-1/4*d^2*b^2*arctan(c*x)^2*c^4*
x^4+1/6*d^2*a*b*c^3*x^3+1/2*d^2*b^2*arctan(c*x)^2*c^2*x^2+2/3*I*d^2*b^2*c*x
+2/3*I*d^2*b^2*ln(c^2*x^2+1)*arctan(c*x)+2/3*I*d^2*a*b*ln(c^2*x^2+1)-2/3*I*
d^2*b^2*arctan(c*x)-1/12*d^2*b^2*c^2*x^2+3/2*d^2*a*b*arctan(c*x)-1/3*d^2*b^
2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/3*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/3*d^2*b
^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/3*d^2*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/4*d^2
*b^2*arctan(c*x)^2-1/6*d^2*b^2*ln(c*x+I)^2+1/6*d^2*b^2*ln(c*x-I)^2-1/3*d^2*
b^2*dilog(1/2*I*(c*x-I))+1/3*d^2*b^2*dilog(-1/2*I*(c*x+I))+d^2*a^2*(-1/4*c^
4*x^4+2/3*I*c^3*x^3+1/2*c^2*x^2)+5/6*b^2*d^2*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*c^2*d^2*x^4 + 2/3*I*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctan(c*x)^2
- 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b
*c^2*d^2 + 2/3*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
*b*c*d^2 + 1/2*a^2*d^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3)
)*a*b*d^2 - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2
 - log(c^2*x^2 + 1))/c^2)*b^2*d^2 - 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2
*x^3)*arctan(c*x)^2 + 1/48*(-3*I*b^2*c^2*d^2*x^4 - 8*b^2*c*d^2*x^3)*arctan(
c*x)*log(c^2*x^2 + 1) + 1/192*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3)*log(c
^2*x^2 + 1)^2 - integrate(-1/48*(22*b^2*c^3*d^2*x^4*arctan(c*x) - 36*(b^2*c
^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x)^2 - 3*(b^2*c^4*d^2*x^5 + b^2*c^2*
d^2*x^3)*log(c^2*x^2 + 1)^2 - (3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2*x^3 - 24*(
b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 +
1), x) + I*integrate(1/48*(72*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*x)
^2 + 6*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*log(c^2*x^2 + 1)^2 + 2*(3*b^2*c^4*
d^2*x^5 - 8*b^2*c^2*d^2*x^3)*arctan(c*x) + (11*b^2*c^3*d^2*x^4 + 12*(b^2*c^
4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3 - 6*b^2*d^2*x^2)*log(-(c*x + I)
/(c*x - I))^2 + integral(-1/12*(12*a^2*c^4*d^2*x^5 - 24*I*a^2*c^3*d^2*x^4 -
 24*I*a^2*c*d^2*x^2 - 12*a^2*d^2*x - (-12*I*a*b*c^4*d^2*x^5 - 3*(8*a*b - I*
b^2)*c^3*d^2*x^4 + 8*b^2*c^2*d^2*x^3 - 6*(4*a*b + I*b^2)*c*d^2*x^2 + 12*I*a
*b*d^2*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

```
[Out] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.79 $\int (d+icdx)^2(a+b\textbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=192

$$-2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\text{ArcTan}(cx)}{3c}-2ib^2d^2x\text{ArcTan}(cx)+\frac{1}{3}bcd^2x^2(a+b\text{ArcTan}(cx))-\frac{id^2(1+icx)^3(a+b}{3c}$$

```
[Out] -2*I*a*b*d^2*x-1/3*b^2*d^2*x+1/3*b^2*d^2*arctan(c*x)/c-2*I*b^2*d^2*x*arctan
(c*x)+1/3*b*c*d^2*x^2*(a+b*arctan(c*x))-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan(c
*x))^2/c+8/3*b*d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+I*b^2*d^2*ln(c^2*x^2
+1)/c-4/3*I*b^2*d^2*polylog(2,1-2/(1-I*c*x))/c
```

Rubi [A]

time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4974, 4930, 266, 4946, 327, 209, 1600, 4964, 2449, 2352}

$$\frac{1}{3}bcd^2x^2(a+b\text{ArcTan}(cx))-\frac{id^2(1+icx)^3(a+b\text{ArcTan}(cx))^2}{3c}+\frac{8bd^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{3c}-2iabd^2x+\frac{b^2d^2\text{ArcTan}(cx)}{3c}-2ib^2d^2x\text{ArcTan}(cx)+\frac{ib^2d^2\log(c^2x^2+1)}{c}-\frac{4ib^2d^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{3c}-\frac{1}{3}b^2d^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-2*I)*a*b*d^2*x - (b^2*d^2*x)/3 + (b^2*d^2*ArcTan[c*x])/(3*c) - (2*I)*b^2*
d^2*x*ArcTan[c*x] + (b*c*d^2*x^2*(a + b*ArcTan[c*x]))/3 - ((I/3)*d^2*(1 + I
*c*x)^3*(a + b*ArcTan[c*x])^2)/c + (8*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/(3*c) + (I*b^2*d^2*Log[1 + c^2*x^2])/c - (((4*I)/3)*b^2*d^2*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/c
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int (d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{(2ib)\int\left(-3d^3(a+b\tan^{-1}(cx))\right.}{} \\
&= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3c}+\frac{(8b)\int\frac{\left(id^3-cd^3x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{3d} \\
&= -2iabd^2x+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)-\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(\right.}{3c} \\
&= -2iabd^2x-\frac{1}{3}b^2d^2x-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x^2\left(a+b\tan^{-1}(cx)\right. \\
&= -2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\tan^{-1}(cx)}{3c}-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x \\
&= -2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\tan^{-1}(cx)}{3c}-2ib^2d^2x\tan^{-1}(cx)+\frac{1}{3}bcd^2x
\end{aligned}
$$

Mathematica [**A**]
time = 0.35, size = 205, normalized size = 1.07

$$
-\frac{d^2\left(-3a^2cx+6iabcx+b^2cx-3ia^2c^2x^2-abc^2x^2+a^2c^3x^3+b^2(-i+cx)^3\mathrm{ArcTan}(cx)^2-b\mathrm{ArcTan}(cx)\left(b(1-6icx+c^2x^2)+a(6i+6cx+6ic^2x^2-2c^3x^3)+8b\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)+4ab\log\left(1+c^2x^2\right)-3ib^2\log\left(1+c^2x^2\right)+4ib^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{3c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -1/3*(d^2*(-3*a^2*c*x + (6*I)*a*b*c*x + b^2*c*x - (3*I)*a^2*c^2*x^2 - a*b*c
^2*x^2 + a^2*c^3*x^3 + b^2*(-I + c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*(1
 - (6*I)*c*x + c^2*x^2) + a*(6*I + 6*c*x + (6*I)*c^2*x^2 - 2*c^3*x^3) + 8*b
*Log[1 + E^((2*I)*ArcTan[c*x])]) + 4*a*b*Log[1 + c^2*x^2] - (3*I)*b^2*Log[1
 + c^2*x^2] + (4*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(172) = 344.
time = 0.17, size = 469, normalized size = 2.44 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*d^2*b^2*arctan(c*x)^2*c^2*x^2-2*I*d^2*b^2*arctan(c*x)*c*x-2*I*d^2*a*
b*c*x-2/3*d^2*a*b*arctan(c*x)*c^3*x^3+2*a*b*arctan(c*x)*d^2*c*x+2*I*d^2*a*b
*arctan(c*x)*c^2*x^2-1/3*d^2*b^2*arctan(c*x)^2*c^3*x^3+1/3*d^2*b^2*arctan(c
*x)*c^2*x^2+1/3*d^2*a*b*c^2*x^2-1/3*d^2*b^2*c*x-1/3*I*d^2*b^2*ln(c*x+I)^2+I
```

```
*d^2*b^2*arctan(c*x)^2+I*d^2*b^2*ln(c^2*x^2+1)+2/3*I*d^2*b^2*dilog(-1/2*I*(
c*x+I))+1/3*I*d^2*b^2*ln(c*x-I)^2-2/3*I*d^2*b^2*dilog(1/2*I*(c*x-I))-1/3*I*
d^2*(1+I*c*x)^3*a^2+2*I*d^2*a*b*arctan(c*x)-2/3*I*d^2*b^2*ln(c*x-I)*ln(c^2*
x^2+1)+2/3*I*d^2*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2/3*I*d^2*b^2*ln(c*x+I)*l
n(1/2*I*(c*x-I))+2/3*I*d^2*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/3*d^2*b^2*arctan(c
*x)-4/3*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^2-4/3*a*b*ln(c^2*x^2+1)*d^2+b^2*arc
tan(c*x)^2*d^2*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c^2*d^2*x^3 - 36*b^2*c^4*d^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2
*x^2 + 1), x) - 3*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^
2 + 1), x) - 4*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1
), x) + 24*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^2 + 1), x) + 32*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1)
, x) - 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c^2
*d^2 + I*a^2*c*d^2*x^2 + 24*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)
/(c^2*x^2 + 1), x) + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*
b*c*d^2 + 1/4*b^2*d^2*arctan(c*x)^3/c + 24*b^2*c*d^2*integrate(1/48*x*arcta
n(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 24*b^2*c*d^2*integrate(1/48*x*a
rctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^2*x + 3*b^2*d^2*integrate(1/48*log(c^2
*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*
d^2/c - 1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*x)*arctan(c*x
)^2 + 1/12*(-I*b^2*c^2*d^2*x^3 - 3*b^2*c*d^2*x^2 + 3*I*b^2*d^2*x)*arctan(c*
x)*log(c^2*x^2 + 1) + 1/48*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2
*x)*log(c^2*x^2 + 1)^2 + I*integrate(1/24*(36*(b^2*c^3*d^2*x^3 + b^2*c*d^2*
x)*arctan(c*x)^2 + 3*(b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*log(c^2*x^2 + 1)^2 + 4
*(b^2*c^4*d^2*x^4 - 6*b^2*c^2*d^2*x^2)*arctan(c*x) + 2*(4*b^2*c^3*d^2*x^3 -
 3*b^2*c*d^2*x + 3*(b^2*c^4*d^2*x^4 - b^2*d^2)*arctan(c*x))*log(c^2*x^2 + 1
))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*x)*log(-(c*x + I)/(c*
x - I))^2 + integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*a^
2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - (6*a*b - I*b^2)*c^3*d^2*x^3
 + 3*b^2*c^2*d^2*x^2 - 3*(2*a*b + I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c*x +
 I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.80 $\int \frac{(d+icdx)^2(a+b\mathbf{ArcTan}(cx))^2}{x}\,dx$

Optimal. Leaf size=300

$$abcd^2x+b^2cd^2x\text{ArcTan}(cx)-\frac{5}{2}d^2(a+b\text{ArcTan}(cx))^2+2icd^2x(a+b\text{ArcTan}(cx))^2-\frac{1}{2}c^2d^2x^2(a+b\text{ArcTan}(cx))^2+$$

```
[Out] a*b*c*d^2*x+b^2*c*d^2*x*arctan(c*x)-5/2*d^2*(a+b*arctan(c*x))^2+2*I*c*d^2*x
*(a+b*arctan(c*x))^2-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))^2-2*d^2*(a+b*arctan(
c*x))^2*arctanh(-1+2/(1+I*c*x))+4*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))
-1/2*b^2*d^2*ln(c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(1+I*c*x))-I*b*d^2*(a+b*
arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d^2*(a+b*arctan(c*x))*polylog(2,-
1+2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^2*polylog(3,-
1+2/(1+I*c*x))
```

Rubi [A]
time = 0.41, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 266}

$$-\frac{1}{2}c^2d^2x^2(a+b\text{ArcTan}(cx))^2-ibd^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibd^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+2icd^2x(a+b\text{ArcTan}(cx))^2-\frac{5}{2}d^2(a+b\text{ArcTan}(cx))^2+4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+2d^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+abcd^2x+b^2cd^2x\text{ArcTan}(cx)-\frac{1}{2}b^2d^2\log\left(c^2x^2+1\right)-2b^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTan[c*x] - (5*d^2*(a + b*ArcTan[c*x])^2)/2 + (
2*I)*c*d^2*x*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^2*(a + b*ArcTan[c*x])^2)/2
+ 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (4*I)*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*d^2*Log[1 + c^2*x^2])/2 - 2*b^2*d
^2*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^2*PolyLog[3, -1 +
2/(1 + I*c*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^2 \left(a+b\tan^{-1}(cx)\right)^2}{x} dx = \int \left(2icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x} - c^2d^2x\left(a+b\,\mathrm{t}\right.\right.$$

$$= d^2 \int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} dx + \left(2icd^2\right) \int \left(a+b\tan^{-1}(cx)\right)^2 dx - \left(c^2\right.$$

$$= 2icd^2x\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{2}c^2d^2x^2\left(a+b\tan^{-1}(cx)\right)^2 + 2d^2\left(a+b\,\mathrm{t}\right.$$

$$= -2d^2\left(a+b\tan^{-1}(cx)\right)^2 + 2icd^2x\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{2}c^2d^2x^2\left(a+\right.$$

$$= abcd^2x - \frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2 + 2icd^2x\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{2}c^2d$$

$$= abcd^2x + b^2cd^2x\tan^{-1}(cx) - \frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2 + 2icd^2x\left(a+b\right.$$

$$= abcd^2x + b^2cd^2x\tan^{-1}(cx) - \frac{5}{2}d^2\left(a+b\tan^{-1}(cx)\right)^2 + 2icd^2x\left(a+b\right.$$

Mathematica [A]
time = 0.38, size = 360, normalized size = 1.20

$$\frac{1}{2}d^2\left(4ia^2cx - a^2c^2x^2 + 2b^2cx\mathrm{ArcTan}(cx) - b^2(1+c^2x^2)\,\mathrm{ArcTan}(cx)^2 - 2ab\left(-cx+(1+c^2x^2)\,\mathrm{ArcTan}(cx)\right) + 2a^2\log(cx) + 4iab\left(2cx\mathrm{ArcTan}(cx) - \log\left(1+c^2x^2\right)\right) - b^2\log\left(1+c^2x^2\right) + 4b^2\left(\mathrm{ArcTan}(cx)\left((1+icx)\mathrm{ArcTan}(cx) + 2i\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right) + \mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right) + 2iab(\mathrm{PolyLog}(2,-icx) - \mathrm{PolyLog}(2,icx)) + 2b^2\left(-\frac{i\pi^3}{24} + \frac{2}{3}i\mathrm{ArcTan}(cx)^3 + \mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right) - \mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) + i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right) + i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right) - \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (d^2*((4*I)*a^2*c*x - a^2*c^2*x^2 + 2*b^2*c*x*ArcTan[c*x] - b^2*(1 + c^2*x^
2)*ArcTan[c*x]^2 - 2*a*b*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]) + 2*a^2*Log[c
*x] + (4*I)*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - b^2*Log[1 + c^2*x^
2] + 4*b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*A
rcTan[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*(PolyLog[2,
 (-I)*c*x] - PolyLog[2, I*c*x]) + 2*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[
c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x]
)] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I
)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.23, size = 1542, normalized size = 5.14

method	result	size

derivativedivides	Expression too large to display	1542
default	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(
c*x)^2+1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x
^2+1)+1))*arctan(c*x)^2+a*b*c*d^2*x+b^2*c*d^2*x*arctan(c*x)-1/2*I*d^2*b^2*P
i*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*arctan(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+4*I*d^2*a*b*arctan(c*x)*c*x-d^2*a*b*arct
an(c*x)*c^2*x^2+d^2*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*d^2*b^2*polylog(3,(
1+I*c*x)/(c^2*x^2+1)^(1/2))+2*d^2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2
))+4*d^2*b^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*d^2*b^2*polylog(3,-
(1+I*c*x)^2/(c^2*x^2+1))+4*d^2*b^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2
*I*d^2*a^2*c*x+I*d^2*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/
2*I*d^2*b^2*Pi*arctan(c*x)^2-2*I*d^2*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c
^2*x^2+1)^(1/2))+4*I*d^2*b^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+4*I*d^2*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^2*b^2*a
rctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*d^2*a*b*dilog(1+I*c*x)
+2*d^2*a*b*ln(c*x)*arctan(c*x)-I*d^2*a*b*dilog(1-I*c*x)-2*I*d^2*a*b*ln(c^2*
x^2+1)-1/2*d^2*b^2*arctan(c*x)^2*c^2*x^2+d^2*a^2*ln(c*x)+d^2*b^2*arctan(c*x
)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+d^2*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/
(c^2*x^2+1)^(1/2))+d^2*b^2*ln(c*x)*arctan(c*x)^2-d^2*b^2*arctan(c*x)^2*ln((
1+I*c*x)^2/(c^2*x^2+1)-1)-I*d^2*b^2*arctan(c*x)-d^2*a*b*arctan(c*x)+3/2*d^2
*b^2*arctan(c*x)^2-1/2*d^2*a^2*c^2*x^2+2*I*d^2*b^2*arctan(c*x)^2*c*x+I*d^2*
a*b*ln(c*x)*ln(1+I*c*x)-I*d^2*a*b*ln(c*x)*ln(1-I*c*x)+1/2*I*d^2*b^2*Pi*csgn
(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1
/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^3*arctan(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -12*b^2*c^4*d^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*I*b^
2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x)
- b^2*c^4*d^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 2*I
*b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 32*a*b*c^4*d
^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 2*b^2*c^4*d^2*integra
te(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 1/2*a^2*c^2*d^2*x^2 + 12*I
*b^2*c^3*d^2*integrate(1/8*x^3*arctan(c*x)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*
d^2*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b
^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 20*b^2*
c^3*d^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 5*I*b^2*c^3*d^2*
integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 1/2*I*b^2*d^2*arctan
(c*x)^3 - 8*I*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^3 + x), x) +
 2*I*a^2*c*d^2*x + 8*b^2*c*d^2*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1
)/(c^2*x^3 + x), x) + I*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2 + 1)^2/(c^2*x
^3 + x), x) + 1/8*b^2*d^2*log(c^2*x^2 + 1)^2 + 2*I*(2*c*x*arctan(c*x) - log
(c^2*x^2 + 1))*a*b*d^2 + 12*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 +
 x), x) - 2*I*b^2*d^2*integrate(1/8*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 +
 x), x) + b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*
a*b*d^2*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + a^2*d^2*log(x) - 1/8
*(b^2*c^2*d^2*x^2 - 4*I*b^2*c*d^2*x)*arctan(c*x)^2 + 1/8*(-I*b^2*c^2*d^2*x^
2 - 4*b^2*c*d^2*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/32*(b^2*c^2*d^2*x^2 - 4
*I*b^2*c*d^2*x)*log(c^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d
^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*b*
c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-d^2\left(\int\left(-\frac{a^2}{x}\right)dx+\int(-2ia^2c)\,dx+\int a^2c^2x\,dx+\int\left(-\frac{b^2\operatorname{atan}^2(cx)}{x}\right)dx+\int\left(-2ib^2c\operatorname{atan}^2(cx)\right)dx+\int\left(-\frac{2ab\operatorname{atan}(cx)}{x}\right)dx+\int b^2c^2x\operatorname{atan}^2(cx)\,dx+\int\left(-4iabc\operatorname{atan}(cx)\right)dx+\int 2abc^2x\operatorname{atan}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x,x)
```

```
[Out] -d**2*(Integral(-a**2/x, x) + Integral(-2*I*a**2*c, x) + Integral(a**2*c**2
*x, x) + Integral(-b**2*atan(c*x)**2/x, x) + Integral(-2*I*b**2*c*atan(c*x)
```

```
**2, x) + Integral(-2*a*b*atan(c*x)/x, x) + Integral(b**2*c**2*x*atan(c*x)*
*2, x) + Integral(-4*I*a*b*c*atan(c*x), x) + Integral(2*a*b*c**2*x*atan(c*x
), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x, x)
```

3.81 $\int \frac{(d+icdx)^2(a+b\textbf{ArcTan}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=317

$$-2icd^2(a+b\text{ArcTan}(cx))^2-\frac{d^2(a+b\text{ArcTan}(cx))^2}{x}-c^2d^2x(a+b\text{ArcTan}(cx))^2+4icd^2(a+b\text{ArcTan}(cx))^2\tanh$$

```
[Out] -2*I*c*d^2*(a+b*arctan(c*x))^2-d^2*(a+b*arctan(c*x))^2/x-c^2*d^2*x*(a+b*arc
tan(c*x))^2-4*I*c*d^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-2*b*c*d^2
*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+2*b*c*d^2*(a+b*arctan(c*x))*ln(2-2/(1-I*
c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,1-2/(1+I*
c*x))+2*b*c*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-2*b*c*d^2*(a+b*a
rctan(c*x))*polylog(2,-1+2/(1+I*c*x))-I*b^2*c*d^2*polylog(3,1-2/(1+I*c*x))+
I*b^2*c*d^2*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]

time = 0.46, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745}

$$-c^2d^2x(a+b\text{ArcTan}(cx))^2+2bcd^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))-2bcd^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))-2icd^2(a+b\text{ArcTan}(cx))^2-\frac{d^2(a+b\text{ArcTan}(cx))^2}{x}-2bcd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+4icd^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-ib^2cd^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)-ib^2cd^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)-ib^2cd^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)+ib^2cd^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-2*I)*c*d^2*(a + b*ArcTan[c*x])^2 - (d^2*(a + b*ArcTan[c*x])^2)/x - c^2*d^
2*x*(a + b*ArcTan[c*x])^2 + (4*I)*c*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2
/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + 2*b*c*d^
2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 +
2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + 2*b*c*d^2*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x
])*PolyLog[2, -1 + 2/(1 + I*c*x)] - I*b^2*c*d^2*PolyLog[3, 1 - 2/(1 + I*c*x
)] + I*b^2*c*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(-c^2d^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}+\frac{2icd^2\left(a+b\tan\right.}{x}\right.$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx+\left(2icd^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx-\left(c^2d\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-1}(cx)\right)^2+4icd^2\left(a+b\tan^{-}\right.$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-}\right.$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-}\right.$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-}\right.$$

$$= -2icd^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x}-c^2d^2x\left(a+b\tan^{-}\right.$$

Mathematica [A]
time = 0.33, size = 378, normalized size = 1.19

$$-\frac{d^2\left(12a^2-b^2c\pi^3x+12a^2c^2x^2+24ab\mathrm{ArcTan}(cx)+24abc^2x^2\mathrm{ArcTan}(cx)+12b^2\mathrm{ArcTan}(cx)^2+12b^2c^2x^2\mathrm{ArcTan}(cx)^2+16b^2cx\mathrm{ArcTan}(cx)^3-24ib^2cx\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)-24b^2cx\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+24b^2cx\mathrm{ArcTan}(cx)\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)+24ib^2cx\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)-24ia^2cx\log(cx)-24abcx\log(cx)+24b^2cx\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)+12b^2cx(-i+2\mathrm{ArcTan}(cx))\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)+12ib^2cx\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)+24abcx\mathrm{PolyLog}(2,-icx)-24abcx\mathrm{PolyLog}(2,icx)-12ib^2cx\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)+12ib^2cx\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{12x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] -1/12*(d^2*(12*a^2 - b^2*c*Pi^3*x + 12*a^2*c^2*x^2 + 24*a*b*ArcTan[c*x] + 2
4*a*b*c^2*x^2*ArcTan[c*x] + 12*b^2*ArcTan[c*x]^2 + 12*b^2*c^2*x^2*ArcTan[c*
x]^2 + 16*b^2*c*x*ArcTan[c*x]^3 - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((
-2*I)*ArcTan[c*x])] - 24*b^2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])]
 + 24*b^2*c*x*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*b^2*c*x*A
rcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c*x*Log[c*x] - 24*
a*b*c*x*Log[c*x] + 24*b^2*c*x*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])
] + 12*b^2*c*x*(-I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (1
2*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 24*a*b*c*x*PolyLog[2, (-I)
*c*x] - 24*a*b*c*x*PolyLog[2, I*c*x] - (12*I)*b^2*c*x*PolyLog[3, E^((-2*I)*
ArcTan[c*x])] + (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.90, size = 11833, normalized size = 37.33

method	result	size
derivativedivides	Expression too large to display	11833
default	Expression too large to display	11833

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -a^2*c^2*d^2*x - (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c*d^2 + 2*I*a^2
*c*d^2*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d^2
 - a^2*d^2/x + 1/16*(8*b^2*c^2*d^2*x^2 - 2*b^2*c^2*d^2*x*integrate(4*arctan
(c*x)^2 + log(c^2*x^2 + 1)^2, x) + 4*I*b^2*c^2*d^2*x*integrate(-1/4*(8*(c^2
*x^2 + 1)*c*x*arctan(c*x)^2 - 2*(c^2*x^2 + 1)*c*x*log(c^2*x^2 + 1)^2 + 8*(c
^2*x^2 + 1)*arctan(c*x)*log(c^2*x^2 + 1) - (4*(c^2*x^2 + 1)^(3/2)*arctan(c*
x)*cos(2*arctan(c*x))*log(c^2*x^2 + 1) + 4*sqrt(c^2*x^2 + 1)*arctan(c*x)*lo
g(c^2*x^2 + 1) + (4*(c^2*x^2 + 1)^(3/2)*arctan(c*x)^2 - (c^2*x^2 + 1)^(3/2)
*log(c^2*x^2 + 1)^2)*sin(2*arctan(c*x)))*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^
3*cos(2*arctan(c*x))^2 + (c^2*x^2 + 1)^3*sin(2*arctan(c*x))^2 + c^2*x^2 + 2
*(c^2*x^2 + 1)^2*cos(2*arctan(c*x)) + 4*(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)^
(3/2)*c*x*sin(2*arctan(c*x)) + (c^2*x^2 + 1)^(3/2)*cos(2*arctan(c*x)) + sqr
t(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1) + 1), x) - 4*b^2*c^2*d^2*x*integrate(1/4*
(8*(c^2*x^2 + 1)*c*x*arctan(c*x)*log(c^2*x^2 + 1) - 8*(c^2*x^2 + 1)*arctan(
c*x)^2 + 2*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 + 1)^(3/2)*arctan
(c*x)*log(c^2*x^2 + 1)*sin(2*arctan(c*x)) - 4*sqrt(c^2*x^2 + 1)*arctan(c*x)
^2 + sqrt(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 + 1)^(3/2)*arctan(c
*x)^2 - (c^2*x^2 + 1)^(3/2)*log(c^2*x^2 + 1)^2)*cos(2*arctan(c*x)))*sqrt(c^
2*x^2 + 1))/((c^2*x^2 + 1)^3*cos(2*arctan(c*x))^2 + (c^2*x^2 + 1)^3*sin(2*a
rctan(c*x))^2 + c^2*x^2 + 2*(c^2*x^2 + 1)^2*cos(2*arctan(c*x)) + 4*(c^2*x^2
 + 1)^2 - 4*((c^2*x^2 + 1)^(3/2)*c*x*sin(2*arctan(c*x)) + (c^2*x^2 + 1)^(3/
2)*cos(2*arctan(c*x)) + sqrt(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1) + 1), x) - 4*b
^2*c*d^2*x*(I*gamma(3, -log(I*c*x + 1)) - 2*I) + 8*b^2*c*d^2*x*integrate(ar
ctan(c*x)*log(c^2*x^2 + 1)/x, x) + 4*I*b^2*c*d^2*x*integrate((4*arctan(c*x)
^2 + log(c^2*x^2 + 1)^2)/x, x) - 2*I*b^2*c*d^2*x*integrate(-(4*arctan(c*x)^
2 - log(c^2*x^2 + 1)^2)/x, x) + 4*I*b^2*c*d^2*x*integrate(log(c^2*x^2 + 1)/
x, x) - 8*(-8*I*a*b - b^2)*c*d^2*x*integrate(arctan(c*x)/x, x) + 2*b^2*d^2*
```

```
x*integrate((4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)/x^2, x) - 4*(b^2*c^2*d^2
*x^2 + b^2*d^2)*arctan(c*x)^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*log(c^2*x^2 + 1
)^2 - 8*(-I*b^2*c^2*d^2*x^2 + b^2*c*d^2*x)*arctan(c*x) - 4*(b^2*c^2*d^2*x^2
 + I*b^2*c*d^2*x + (I*b^2*c^2*d^2*x^2 + I*b^2*d^2)*arctan(c*x))*log(c^2*x^2
 + 1))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d
^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*b*
c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**2,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2, x)
```

3.82 $\int \frac{(d+icdx)^2(a+b\mathbf{ArcTan}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=337

$$-\frac{bcd^2(a+b\text{ArcTan}(cx))}{x}+\frac{3}{2}c^2d^2(a+b\text{ArcTan}(cx))^2-\frac{d^2(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{2icd^2(a+b\text{ArcTan}(cx))^2}{x}-2c$$

```
[Out] -b*c*d^2*(a+b*arctan(c*x))/x+3/2*c^2*d^2*(a+b*arctan(c*x))^2-1/2*d^2*(a+b*a
rctan(c*x))^2/x^2-2*I*c*d^2*(a+b*arctan(c*x))^2/x+2*c^2*d^2*(a+b*arctan(c*x
))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^2+1
)+4*I*b*c^2*d^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+2*b^2*c^2*d^2*polylog(2
,-1+2/(1-I*c*x))+I*b*c^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-I*b
*c^2*d^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+1/2*b^2*c^2*d^2*polylo
g(3,1-2/(1+I*c*x))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.47, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4996, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497, 4942, 5108, 5114, 6745}

$$ibc^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))-ibc^2d^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+\frac{3}{2}c^2d^2(a+b\text{ArcTan}(cx))^2+4ibc^2d^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-2c^2d^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-\frac{d^2(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{2icd^2(a+b\text{ArcTan}(cx))^2}{x}-\frac{bcd^2(a+b\text{ArcTan}(cx))}{x}+2b^2c^2d^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)+\frac{1}{2}b^2c^2d^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2c^2d^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^2*(a + b*ArcTan[c*x]))/x) + (3*c^2*d^2*(a + b*ArcTan[c*x])^2)/2 -
(d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x])^2)/x
 - 2*c^2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2
*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (4*I)*b*c^2*d^2*(a + b*ArcTan[
c*x])*Log[2 - 2/(1 - I*c*x)] + 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)]
 + I*b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*c^2*
d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (b^2*c^2*d^2*PolyL
og[3, 1 - 2/(1 + I*c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/
2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^3} + \frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2} - \frac{c^2d^2(a+b\tan}{x}\right.$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx + \left(2icd^2\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx - \left(c^2d\right.$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{2icd^2\left(a+b\tan^{-1}(cx)\right)^2}{x} - 2c^2d^2\left(a+b\tan^{-}\right.$$

$$= 2c^2d^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{2icd^2\left(a+b\tan^{-1}(\right.}{x}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d^2\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^2\left(a+b\tan^{-1}(\right.}{2x^2}$$

Mathematica [**A**]
time = 0.51, size = 388, normalized size = 1.15

$$-\frac{d^2\left(a^2+4ia^2cx+2ab(\mathrm{ArcTan}(cx)+cx(1+cx\mathrm{ArcTan}(cx)))+2a^2c^2x^2\log(x)+b^2\left(2cx\mathrm{ArcTan}(cx)+(1+c^2x^2)\mathrm{ArcTan}(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+4iabcx(2\mathrm{ArcTan}(cx)+cx(-2\log(cx)+\log(1+c^2x^2)))+4ib^2cx\left(\mathrm{ArcTan}(cx)^2-2cx\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+icx\left(\mathrm{ArcTan}(cx)^2+\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)\right)\right)+2iabc^2x^2(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+\frac{1}{12}b^2c^2x^2\left(-i\pi^3+16i\mathrm{ArcTan}(cx)^3+24\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)-24\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)+24i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)+24i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)+12\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)-12\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(d^2*(a^2 + (4*I)*a^2*c*x + 2*a*b*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c
*x])) + 2*a^2*c^2*x^2*Log[x] + b^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTa
n[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (4*I)*a*b*c*x*(2*ArcTa
n[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])) + (4*I)*b^2*c*x*(ArcTan[c*x]
^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^
2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])) + (2*I)*a*b*c^2*x^2*(PolyLog[2, (-I
)*c*x] - PolyLog[2, I*c*x]) + (b^2*c^2*x^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^
3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log
[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTa
n[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyL
og[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/12)
)/x^2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.80, size = 1565, normalized size = 4.64

method	result	size
derivativedivides	Expression too large to display	1565
default	Expression too large to display	1565

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^3*arctan(c*x)^2-1/2*d^2*a^2/c^2/x^2-1/2*I*d^2*b^2*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-2*I*d^2*a^
2/c/x-1/2*d^2*b^2*arctan(c*x)^2/c^2/x^2-d^2*b^2*arctan(c*x)/c/x-d^2*a*b/c/x
-I*d^2*a*b*dilog(1+I*c*x)+4*I*d^2*a*b*ln(c*x)+I*d^2*a*b*dilog(1-I*c*x)-1/2*
I*d^2*b^2*Pi*arctan(c*x)^2+2*I*d^2*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2
*x^2+1)^(1/2))+4*I*d^2*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*
I*d^2*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*d^2*b^2*arc
tan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*d^2*b^2*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+
1))*arctan(c*x)^2+1/2*I*d^2*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)^2-2*d^2*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))
-2*d^2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*d^2*b^2*polylog(3,-(
1+I*c*x)^2/(c^2*x^2+1))-2*d^2*a*b*ln(c*x)*arctan(c*x)-2*I*d^2*a*b*ln(c^2*x^
2+1)-d^2*a^2*ln(c*x)-1/2*I*d^2*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*d^2*b^2*Pi*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-d^2*a*b*arctan
(c*x)/c^2/x^2-2*I*d^2*b^2*arctan(c*x)^2/c/x+I*d^2*a*b*ln(c*x)*ln(1-I*c*x)-I
*d^2*a*b*ln(c*x)*ln(1+I*c*x)-d^2*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+
1)^(1/2))-d^2*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-d^2*b^2*l
n(c*x)*arctan(c*x)^2+d^2*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-I*
d^2*b^2*arctan(c*x)-d^2*a*b*arctan(c*x)+3/2*d^2*b^2*arctan(c*x)^2+d^2*b^2*l
n(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+d^2*b^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-
4*d^2*b^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+4*d^2*b^2*dilog(1+(1+I*c*x)/(c
^2*x^2+1)^(1/2))-4*I*d^2*a*b*arctan(c*x)/c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")

[Out] -a^2*c^2*d^2*log(x) - 2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c*d^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 - 2*I*a^2*c*d^2/x - 1/2*a^2*d^2/x^2 + 1/96*(48*I*(b^2*c^2*d^2*arctan(c*x)^3 + 4*b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 2*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 8*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 20*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 24*b^2*c*d^2*integrate(1/8*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 4*b^2*d^2*integrate(1/8*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 - (1152*b^2*c^4*d^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^4*d^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + b^2*c^2*d^2*log(c^2*x^2 + 1)^3 + 48*b^2*c^2*d^2*arctan(c*x)^2 - 768*b^2*c^3*d^2*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*d^2*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 960*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 768*b^2*c*d^2*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 384*b^2*c*d^2*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) - 1152*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 96*b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x))*x^2 + 12*(-4*I*b^2*c*d^2*x - b^2*d^2)*arctan(c*x)^2 + 12*(4*b^2*c*d^2*x - I*b^2*d^2)*arctan(c*x)*log(c^2*x^2 + 1) - 3*(-4*I*b^2*c*d^2*x - b^2*d^2)*log(c^2*x^2 + 1)^2)/x^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")

[Out] integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3, x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**3,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3, x)
```

3.83 $\int \frac{(d+icdx)^2(a+b\mathbf{ArcTan}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=267

$$-\frac{b^2c^2d^2}{3x}-\frac{1}{3}b^2c^3d^2\text{ArcTan}(cx)-\frac{bcd^2(a+b\text{ArcTan}(cx))}{3x^2}-\frac{2ibc^2d^2(a+b\text{ArcTan}(cx))}{x}-\frac{d^2(1+icx)^3(a+b\text{ArcT}}{3x^3}$$

```
[Out] -1/3*b^2*c^2*d^2/x-1/3*b^2*c^3*d^2*arctan(c*x)-1/3*b*c*d^2*(a+b*arctan(c*x)
)/x^2-2*I*b*c^2*d^2*(a+b*arctan(c*x))/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c*x
))^2/x^3-8/3*a*b*c^3*d^2*ln(x)+2*I*b^2*c^3*d^2*ln(x)-8/3*b*c^3*d^2*(a+b*arc
tan(c*x))*ln(2/(1-I*c*x))-I*b^2*c^3*d^2*ln(c^2*x^2+1)-4/3*I*b^2*c^3*d^2*pol
ylog(2,-I*c*x)+4/3*I*b^2*c^3*d^2*polylog(2,I*c*x)+4/3*I*b^2*c^3*d^2*polylog
(2,1-2/(1-I*c*x))
```

Rubi [A]
time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$,
Rules used = {37, 4994, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438, 4964, 2449, 2352}

$$-\frac{8}{3}bc^3d^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{2ibc^2d^2(a+b\text{ArcTan}(cx))}{x}-\frac{d^2(1+icx)^3(a+b\text{ArcTan}(cx))^2}{3x^3}-\frac{bcd^2(a+b\text{ArcTan}(cx))}{3x^2}-\frac{8}{3}abc^3d^2\log(x)-\frac{1}{3}b^2c^3d^2\text{ArcTan}(cx)-\frac{4}{3}ib^2c^3d^2\text{Li}_2(-icx)+\frac{4}{3}ib^2c^3d^2\text{Li}_2(icx)+\frac{4}{3}ib^2c^3d^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)+2ib^2c^3d^2\log(x)-\frac{b^2c^2d^2}{3x}-ib^2c^3d^2\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^2)/x - (b^2*c^3*d^2*ArcTan[c*x])/3 - (b*c*d^2*(a + b*ArcTan
[c*x]))/(3*x^2) - ((2*I)*b*c^2*d^2*(a + b*ArcTan[c*x]))/x - (d^2*(1 + I*c*x
)^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (8*a*b*c^3*d^2*Log[x])/3 + (2*I)*b^2*c
^3*d^2*Log[x] - (8*b*c^3*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/3 - I*
b^2*c^3*d^2*Log[1 + c^2*x^2] - ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, (-I)*c*x] +
 ((4*I)/3)*b^2*c^3*d^2*PolyLog[2, I*c*x] + ((4*I)/3)*b^2*c^3*d^2*PolyLog[2,
 1 - 2/(1 - I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-(2bc)\int\left(-\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}\right.\\
&= -\frac{d^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bcd^2\right)\int\frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\\
&= -\frac{bcd^2(a+b\tan^{-1}(cx))}{3x^2}-\frac{2ibc^2d^2(a+b\tan^{-1}(cx))}{x}-\frac{d^2(1+icx)^3\,(a}{3}\\
&= -\frac{b^2c^2d^2}{3x}-\frac{bcd^2(a+b\tan^{-1}(cx))}{3x^2}-\frac{2ibc^2d^2(a+b\tan^{-1}(cx))}{x}-\frac{d^2(1}{}\\
&= -\frac{b^2c^2d^2}{3x}-\frac{1}{3}b^2c^3d^2\tan^{-1}(cx)-\frac{bcd^2(a+b\tan^{-1}(cx))}{3x^2}-\frac{2ibc^2d^2(a+}{}\\
&= -\frac{b^2c^2d^2}{3x}-\frac{1}{3}b^2c^3d^2\tan^{-1}(cx)-\frac{bcd^2(a+b\tan^{-1}(cx))}{3x^2}-\frac{2ibc^2d^2(a+}{}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 253, normalized size = 0.95

$$\frac{d^2\left(-a^2-3ia^2cx-abcx+3a^2c^2x^2-6iabc^2x^2-b^2c^2x^2+b^2(-1-icx)^3\text{ArcTan}(cx)^2-b\text{ArcTan}(cx)\left(bcx(1+6icx+c^2x^2)+a(2+6icx-6c^2x^2+6ic^3x^3)+8bc^3x^3\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)-8abc^3x^3\log(cx)+6ib^2c^3x^3\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+4abc^3x^3\log\left(1+c^2x^2\right)+4ib^2c^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - (3*I)*a^2*c*x - a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a*b*c^2*x^2 -
b^2*c^2*x^2 + b^2*(-1 - I*c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*c*x*(1 +
(6*I)*c*x + c^2*x^2) + a*(2 + (6*I)*c*x - 6*c^2*x^2 + (6*I)*c^3*x^3) + 8*b*
c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + (6*I)*b^
2*c^3*x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 + c^2*x^2] + (
4*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(3*x^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(238) = 476$.
time = 0.59, size = 619, normalized size = 2.32

method	result
derivativedivides	$c^3\left(\frac{2d^2ab\arctan(cx)}{cx}-\frac{2d^2ab\arctan(cx)}{3c^3x^3}+\frac{4id^2b^2\,\text{dilog}(-icx+1)}{3}-\frac{d^2ab}{3c^2x^2}-id^2b^2\ln\left(c^2x^2+1\right)-id^2b\right.$
default	$c^3\left(\frac{2d^2ab\arctan(cx)}{cx}-\frac{2d^2ab\arctan(cx)}{3c^3x^3}+\frac{4id^2b^2\,\text{dilog}(-icx+1)}{3}-\frac{d^2ab}{3c^2x^2}-id^2b^2\ln\left(c^2x^2+1\right)-id^2b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(2*d^2*a*b*arctan(c*x)/c/x-2*I*d^2*b^2*arctan(c*x)/c/x-I*d^2*b^2*arctan
(c*x)^2/c^2/x^2-2/3*d^2*a*b*arctan(c*x)/c^3/x^3-2*I*d^2*a*b/c/x-2/3*I*d^2*b
^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+4/3*I*d^2*b^2*ln(c*x)*ln(1-I*c*x)+2/3*I*d^2
*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-4/3*I*d^2*b^2*ln(c*x)*ln(1+I*c*x)-2/3*I*d^
2*b^2*ln(c*x+I)*ln(c^2*x^2+1)+2/3*I*d^2*b^2*ln(c*x-I)*ln(c^2*x^2+1)-2*I*d^2
*a*b*arctan(c*x)-1/3*d^2*a*b/c^2/x^2+d^2*a^2*(-I/c^2/x^2-1/3/c^3/x^3+1/c/x)
-1/3*d^2*b^2/c/x+1/3*I*d^2*b^2*ln(c*x+I)^2-8/3*d^2*b^2*ln(c*x)*arctan(c*x)-
I*d^2*b^2*arctan(c*x)^2-I*d^2*b^2*ln(c^2*x^2+1)+2*I*d^2*b^2*ln(c*x)-2/3*I*d
^2*b^2*dilog(-1/2*I*(c*x+I))+4/3*I*d^2*b^2*dilog(1-I*c*x)-1/3*I*d^2*b^2*ln(
c*x-I)^2-4/3*I*d^2*b^2*dilog(1+I*c*x)+2/3*I*d^2*b^2*dilog(1/2*I*(c*x-I))-1/
3*d^2*b^2*arctan(c*x)-2*I*d^2*a*b*arctan(c*x)/c^2/x^2+4/3*b^2*arctan(c*x)*l
n(c^2*x^2+1)*d^2+4/3*a*b*ln(c^2*x^2+1)*d^2-1/3*d^2*b^2*arctan(c*x)^2/c^3/x^
3-1/3*d^2*b^2*arctan(c*x)/c^2/x^2+d^2*b^2*arctan(c*x)^2/c/x-8/3*d^2*a*b*ln(
c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^2*d^2 - 2*I*((c*a
rctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^2 + 1/3*((c^2*log(c^2*x^2 +
1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d^2 + a^2*c^2*d^2/x -
 I*a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/48*(12*(b^2*c^3*d^2*arctan(c*x)^3 +
12*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) -
48*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 96
*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4
), x) + 192*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x^4), x)
+ 64*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) -
96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4),
 x) - 32*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^6 + x^4), x) - 144*b
^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*d^2*integr
ate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 + 12*I*(b^2*c^3*d^2*ar
ctan(c*x)^2 - 24*b^2*c^4*d^2*integrate(1/24*x^4*arctan(c*x)*log(c^2*x^2 + 1
)/(c^2*x^6 + x^4), x) - 144*b^2*c^3*d^2*integrate(1/24*x^3*arctan(c*x)^2/(c
^2*x^6 + x^4), x) - 12*b^2*c^3*d^2*integrate(1/24*x^3*log(c^2*x^2 + 1)^2/(c
^2*x^6 + x^4), x) + 48*b^2*c^3*d^2*integrate(1/24*x^3*log(c^2*x^2 + 1)/(c^2
*x^6 + x^4), x) - 64*b^2*c^2*d^2*integrate(1/24*x^2*arctan(c*x)/(c^2*x^6 +
x^4), x) - 144*b^2*c*d^2*integrate(1/24*x*arctan(c*x)^2/(c^2*x^6 + x^4), x)
 - 12*b^2*c*d^2*integrate(1/24*x*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) - 8
*b^2*c*d^2*integrate(1/24*x*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 24*b^2*d
^2*integrate(1/24*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x))*x^3 - 4
*(3*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d^2*x - b^2*d^2)*arctan(c*x)^2 - 4*(3*I*b^2
*c^2*d^2*x^2 + 3*b^2*c*d^2*x - I*b^2*d^2)*arctan(c*x)*log(c^2*x^2 + 1) + (3
*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d^2*x - b^2*d^2)*log(c^2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*x^3*integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*a
^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - 3*(2*a*b + I*b^2)*c^3*d^2*
x^3 - 3*b^2*c^2*d^2*x^2 - (6*a*b - I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c*x
+ I)/(c*x - I)))/(c^2*x^6 + x^4), x) - (3*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d^2*x
 - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4, x)
```

3.84 $\int x^3(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=438

$$\frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5+\frac{122ib^2d^3\text{ArcTan}(cx)}{105c^4}+\frac{3b^2d^3x\text{ArcTan}(cx}{2c^3}$$

```
[Out] 3/2*a*b*d^3*x/c^3-1/7*I*c^3*d^3*x^7*(a+b*arctan(c*x))^2+7/20*b^2*d^3*x^2/c^
2+26/35*I*b*d^3*x^2*(a+b*arctan(c*x))/c^2-1/20*b^2*d^3*x^4+52/35*I*b*d^3*(a
+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4-122/105*I*b^2*d^3*x/c^3+3/2*b^2*d^3*x*a
rctan(c*x)/c^3+44/315*I*b^2*d^3*x^3/c-1/2*b*d^3*x^3*(a+b*arctan(c*x))/c-13/
35*I*b*d^3*x^4*(a+b*arctan(c*x))+1/5*b*c*d^3*x^5*(a+b*arctan(c*x))+3/5*I*c*
d^3*x^5*(a+b*arctan(c*x))^2-209/140*d^3*(a+b*arctan(c*x))^2/c^4+1/4*d^3*x^4
*(a+b*arctan(c*x))^2-1/105*I*b^2*c*d^3*x^5-1/2*c^2*d^3*x^6*(a+b*arctan(c*x)
)^2+122/105*I*b^2*d^3*arctan(c*x)/c^4+1/21*I*b*c^2*d^3*x^6*(a+b*arctan(c*x)
)-11/10*b^2*d^3*ln(c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(1+I*c*x))/c^
4
```

Rubi [A]
time = 1.00, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4996, 4946, 5036, 272, 45, 4930, 266, 5004, 308, 209, 327, 5040, 4964, 2449, 2352}

$$-\frac{209d^3(a+b\text{ArcTan}(cx))^2}{140c^4}+\frac{52ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{35c^4}-\frac{1}{7}ic^3d^3x^7(a+b\text{ArcTan}(cx))^2-\frac{1}{2}c^2d^3x^6(a+b\text{ArcTan}(cx))^2+\frac{1}{21}ibc^2d^3x^6(a+b\text{ArcTan}(cx))+\frac{26ibd^3x^2(a+b\text{ArcTan}(cx))}{35c^2}+\frac{3}{5}icd^3x^5(a+b\text{ArcTan}(cx))^2+\frac{1}{5}bcd^3x^5(a+b\text{ArcTan}(cx))+\frac{1}{4}d^3x^4(a+b\text{ArcTan}(cx))^2-\frac{13}{35}ibd^3x^4(a+b\text{ArcTan}(cx))-\frac{bd^3x^3(a+b\text{ArcTan}(cx))}{2c}+\frac{3abd^3x}{2c^3}+\frac{122ib^2d^3\text{ArcTan}(cx)}{105c^4}+\frac{3b^2d^3x\text{ArcTan}(cx)}{2c^3}-\frac{26b^2d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{35c^4}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}-\frac{11b^2d^3\log\left(c^2x^2+1\right)}{10c^4}-\frac{1}{105}ib^2cd^3x^5+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (3*a*b*d^3*x)/(2*c^3) - (((122*I)/105)*b^2*d^3*x)/c^3 + (7*b^2*d^3*x^2)/(20
*c^2) + (((44*I)/315)*b^2*d^3*x^3)/c - (b^2*d^3*x^4)/20 - (I/105)*b^2*c*d^3
*x^5 + (((122*I)/105)*b^2*d^3*ArcTan[c*x])/c^4 + (3*b^2*d^3*x*ArcTan[c*x])/
(2*c^3) + (((26*I)/35)*b*d^3*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d^3*x^3*(a +
 b*ArcTan[c*x]))/(2*c) - ((13*I)/35)*b*d^3*x^4*(a + b*ArcTan[c*x]) + (b*c*d
^3*x^5*(a + b*ArcTan[c*x]))/5 + (I/21)*b*c^2*d^3*x^6*(a + b*ArcTan[c*x]) -
(209*d^3*(a + b*ArcTan[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTan[c*x])^2)
/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^3*x^6*(a + b*ArcTan
[c*x])^2)/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*x])^2 + (((52*I)/35)*b*d^3*
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (11*b^2*d^3*Log[1 + c^2*x^2])
/(10*c^4) - (26*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(35*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int\Big(d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-3c^2d^3x$$

$$= d^3\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(3icd^3\right)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,d$$

$$= \frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6$$

$$= \frac{1}{4}d^3x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{5}icd^3x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2d^3x^6$$

$$= -\frac{bd^3x^3(a+b\tan^{-1}(cx))}{6c}-\frac{3}{10}ibd^3x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}bcd^3x^5$$

$$= \frac{abd^3x}{2c^3}+\frac{3ibd^3x^2(a+b\tan^{-1}(cx))}{5c^2}-\frac{bd^3x^3(a+b\tan^{-1}(cx))}{2c}-\frac{13}{35}$$

$$= \frac{3abd^3x}{2c^3}-\frac{199ib^2d^3x}{210c^3}+\frac{73ib^2d^3x^3}{630c}-\frac{1}{105}ib^2cd^3x^5+\frac{b^2d^3x\tan^{-1}(c}{2c^3}$$

$$= \frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{11b^2d^3x^2}{60c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}$$

$$= \frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib$$

$$= \frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib$$

Mathematica [A]
time = 1.16, size = 408, normalized size = 0.93

$$\frac{d^3\left(1464iab+504b^2+1890abcx-1464ib^2cx+936iabc^2x^2+441b^2c^2x^2-630abc^3x^3+176ib^2c^3x^3+315a^2c^4x^4-468iabc^4x^4-63b^2c^4x^4+756ia^2c^5x^5+252abc^5x^5-12ib^2c^5x^5-630a^2c^6x^6+60iabc^6x^6-180ia^2c^7x^7+9b^2(-i+cx)^4\left(-1+4icx+10c^2x^2-20ic^3x^3\right)\mathrm{ArcTan}(cx)^2+6b\,\mathrm{ArcTan}(cx)\left(b(244i+315cx+156ic^2x^2-105c^3x^3-78ic^4x^4+42c^5x^5+10ic^6x^6)+3a(-105+35c^4x^4+84ic^5x^5-70c^6x^6-20ic^7x^7)+312ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-936iab\log\left(1+c^2x^2\right)-1386b^2\log\left(1+c^2x^2\right)+936b^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{1260c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*((1464*I)*a*b + 504*b^2 + 1890*a*b*c*x - (1464*I)*b^2*c*x + (936*I)*a*
b*c^2*x^2 + 441*b^2*c^2*x^2 - 630*a*b*c^3*x^3 + (176*I)*b^2*c^3*x^3 + 315*a
^2*c^4*x^4 - (468*I)*a*b*c^4*x^4 - 63*b^2*c^4*x^4 + (756*I)*a^2*c^5*x^5 + 2
52*a*b*c^5*x^5 - (12*I)*b^2*c^5*x^5 - 630*a^2*c^6*x^6 + (60*I)*a*b*c^6*x^6
- (180*I)*a^2*c^7*x^7 + 9*b^2*(-I + c*x)^4*(-1 + (4*I)*c*x + 10*c^2*x^2 - (
20*I)*c^3*x^3)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*(244*I + 315*c*x + (156*I
)*c^2*x^2 - 105*c^3*x^3 - (78*I)*c^4*x^4 + 42*c^5*x^5 + (10*I)*c^6*x^6) + 3
```

```
*a*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7*x^7) + (31
2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (936*I)*a*b*Log[1 + c^2*x^2] - 138
6*b^2*Log[1 + c^2*x^2] + 936*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(1260
*c^4)
```

Maple [A]
time = 0.31, size = 720, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/21*I*d^3*a*b*c^6*x^6+3/2*b^2*c*d^3*x*arctan(c*x)+44/315*I*d^3*b^2*
c^3*x^3-1/105*I*d^3*b^2*c^5*x^5-122/105*I*d^3*b^2*c*x+1/5*d^3*a*b*c^5*x^5-1
/2*d^3*a*b*c^3*x^3-1/2*d^3*b^2*arctan(c*x)^2*c^6*x^6+1/4*d^3*b^2*arctan(c*x
)^2*c^4*x^4+1/5*d^3*b^2*arctan(c*x)*c^5*x^5-1/2*d^3*b^2*arctan(c*x)*c^3*x^3
-26/35*I*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)-26/35*I*d^3*a*b*ln(c^2*x^2+1)-13
/35*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+122/105*I*d^3*b^2*arctan(c*x)-1/20
*d^3*b^2*c^4*x^4+7/20*d^3*b^2*c^2*x^2-3/2*d^3*a*b*arctan(c*x)-13/35*d^3*b^2
*ln(c*x+I)*ln(c^2*x^2+1)+13/35*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+13/35*d^
3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/2*d^3*a*b*arctan(c*x)*c^4*x^4+3/5*I*d^3*b^2
*arctan(c*x)^2*c^5*x^5-13/35*I*d^3*b^2*arctan(c*x)*c^4*x^4+26/35*I*d^3*b^2*
arctan(c*x)*c^2*x^2-1/7*I*d^3*b^2*arctan(c*x)^2*c^7*x^7+1/21*I*d^3*b^2*arct
an(c*x)*c^6*x^6-13/35*I*d^3*a*b*c^4*x^4+26/35*I*d^3*a*b*c^2*x^2-d^3*a*b*arc
tan(c*x)*c^6*x^6+d^3*a^2*(-1/7*I*c^7*x^7-1/2*c^6*x^6+3/5*I*c^5*x^5+1/4*c^4*
x^4)+13/35*d^3*b^2*dilog(1/2*I*(c*x-I))-13/35*d^3*b^2*dilog(-1/2*I*(c*x+I))
-3/4*d^3*b^2*arctan(c*x)^2+13/70*d^3*b^2*ln(c*x+I)^2-13/70*d^3*b^2*ln(c*x-I
)^2-2/7*I*d^3*a*b*arctan(c*x)*c^7*x^7+6/5*I*d^3*a*b*arctan(c*x)*c^5*x^5+3/2
*a*b*c*d^3*x-11/10*b^2*d^3*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/7*I*a^2*c^3*d^3*x^7 - 1/2*a^2*c^2*d^3*x^6 + 3/5*I*a^2*c*d^3*x^5 + 1/4*b^
2*d^3*x^4*arctan(c*x)^2 - 1/42*I*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^
2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4
 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*ar
ctan(c*x)/c^7))*a*b*c^2*d^3 + 3/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x
^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^3 + 1/6*(3*x^4*arctan(c*x) - c*(
(c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^3 - 1/12*(2*c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*l
og(c^2*x^2 + 1))/c^4)*b^2*d^3 + 1/280*(-10*I*b^2*c^3*d^3*x^7 - 35*b^2*c^2*d
```

```
^3*x^6 + 42*I*b^2*c*d^3*x^5)*arctan(c*x)^2 + 1/280*(10*b^2*c^3*d^3*x^7 - 35
*I*b^2*c^2*d^3*x^6 - 42*b^2*c*d^3*x^5)*arctan(c*x)*log(c^2*x^2 + 1) - 1/112
0*(-10*I*b^2*c^3*d^3*x^7 - 35*b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*x^5)*log(c^2
*x^2 + 1)^2 - I*integrate(1/560*(420*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*x^6 -
 3*b^2*c*d^3*x^4)*arctan(c*x)^2 + 35*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*x^6 -
 3*b^2*c*d^3*x^4)*log(c^2*x^2 + 1)^2 - 12*(15*b^2*c^4*d^3*x^7 - 14*b^2*c^2*
d^3*x^5)*arctan(c*x) + 2*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6 - 210*(b^
2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 +
1), x) - integrate(1/560*(1260*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arctan(c
*x)^2 + 105*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*log(c^2*x^2 + 1)^2 + 4*(10*
b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6)*arctan(c*x) + 2*(45*b^2*c^4*d^3*x^7 -
 42*b^2*c^2*d^3*x^5 + 70*(b^2*c^5*d^3*x^8 - 2*b^2*c^3*d^3*x^6 - 3*b^2*c*d^3
*x^4)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/560*(20*I*b^2*c^3*d^3*x^7 + 70*b^2*c^2*d^3*x^6 - 84*I*b^2*c*d^3*x^5 - 35*
b^2*d^3*x^4)*log(-(c*x + I)/(c*x - I))^2 + integral(1/140*(-140*I*a^2*c^5*d
^3*x^8 - 420*a^2*c^4*d^3*x^7 + 280*I*a^2*c^3*d^3*x^6 - 280*a^2*c^2*d^3*x^5
+ 420*I*a^2*c*d^3*x^4 + 140*a^2*d^3*x^3 + (140*a*b*c^5*d^3*x^8 - 20*(21*I*a
*b + b^2)*c^4*d^3*x^7 - 70*(4*a*b - I*b^2)*c^3*d^3*x^6 - 28*(10*I*a*b - 3*b
^2)*c^2*d^3*x^5 - 35*(12*a*b + I*b^2)*c*d^3*x^4 + 140*I*a*b*d^3*x^3)*log(-(
c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

```
[Out] int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.85 $\int x^2(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=402

$$\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2d^3\text{ArcTan}(cx)}{30c^3}+\frac{11ib^2d^3x\text{ArcTan}(cx)}{6c^2}-\frac{14bd^3x}{}$$

```
[Out] 11/6*I*a*b*d^3*x/c^2+37/30*b^2*d^3*x/c^2+1/15*I*b*c^2*d^3*x^5*(a+b*arctan(c
*x))-1/10*b^2*d^3*x^3-37/20*I*d^3*(a+b*arctan(c*x))^2/c^3-37/30*b^2*d^3*arc
tan(c*x)/c^3-14/15*I*b^2*d^3*polylog(2,1-2/(1+I*c*x))/c^3-14/15*b*d^3*x^2*(
a+b*arctan(c*x))/c-1/6*I*c^3*d^3*x^6*(a+b*arctan(c*x))^2+3/10*b*c*d^3*x^4*(
a+b*arctan(c*x))-113/90*I*b^2*d^3*ln(c^2*x^2+1)/c^3+11/6*I*b^2*d^3*x*arctan
(c*x)/c^2+1/3*d^3*x^3*(a+b*arctan(c*x))^2-11/18*I*b*d^3*x^3*(a+b*arctan(c*x
))-3/5*c^2*d^3*x^5*(a+b*arctan(c*x))^2+3/4*I*c*d^3*x^4*(a+b*arctan(c*x))^2-
28/15*b*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+61/180*I*b^2*d^3*x^2/c-1/
60*I*b^2*c*d^3*x^4
```

Rubi [A]
time = 0.85, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4996, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 272, 45, 4930, 266, 5004, 308}

$$-\frac{1}{6}ic^3d^3x^6(a+b\text{ArcTan}(cx))^2-\frac{37id^3(a+b\text{ArcTan}(cx))^2}{20c^3}-\frac{28bd^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{15c^3}-\frac{3}{5}c^2d^3x^5(a+b\text{ArcTan}(cx))^2+\frac{1}{15}ibc^2d^3x^5(a+b\text{ArcTan}(cx))+\frac{3}{4}icd^3x^4(a+b\text{ArcTan}(cx))^2+\frac{3}{10}bcd^3x^4(a+b\text{ArcTan}(cx))+\frac{1}{3}d^3x^3(a+b\text{ArcTan}(cx))^2-\frac{11}{18}ibd^3x^3(a+b\text{ArcTan}(cx))-\frac{14bd^3x^2(a+b\text{ArcTan}(cx))}{15c}+\frac{11iabd^3x}{6c^2}-\frac{37b^2d^3\text{ArcTan}(cx)}{30c^3}+\frac{11ib^2d^3x\text{ArcTan}(cx)}{6c^2}-\frac{14ib^2d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{15c^3}+\frac{37b^2d^3x}{30c^2}-\frac{113ib^2d^3\log\left(c^2x^2+1\right)}{90c^3}-\frac{1}{60}ib^2cd^3x^4+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (((11*I)/6)*a*b*d^3*x)/c^2 + (37*b^2*d^3*x)/(30*c^2) + (((61*I)/180)*b^2*d^
3*x^2)/c - (b^2*d^3*x^3)/10 - (I/60)*b^2*c*d^3*x^4 - (37*b^2*d^3*ArcTan[c*x
])/(30*c^3) + (((11*I)/6)*b^2*d^3*x*ArcTan[c*x])/c^2 - (14*b*d^3*x^2*(a + b
*ArcTan[c*x]))/(15*c) - ((11*I)/18)*b*d^3*x^3*(a + b*ArcTan[c*x]) + (3*b*c*
d^3*x^4*(a + b*ArcTan[c*x]))/10 + (I/15)*b*c^2*d^3*x^5*(a + b*ArcTan[c*x])
- (((37*I)/20)*d^3*(a + b*ArcTan[c*x])^2)/c^3 + (d^3*x^3*(a + b*ArcTan[c*x]
)^2)/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^5*(a + b*
ArcTan[c*x])^2)/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[c*x])^2 - (28*b*d^3*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((113*I)/90)*b^2*d^3*Log[1
 + c^2*x^2])/c^3 - (((14*I)/15)*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\Big(d^3x^2\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x^3\left(a+b\tan^{-1}(cx)\right)^2-3c^2d^3x^4$$

$$=d^3\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+\left(3icd^3\right)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5($$

$$=\frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{3}{4}icd^3x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{5}c^2d^3x^5($$

$$=-\frac{bd^3x^2(a+b\tan^{-1}(cx))}{3c}-\frac{1}{2}ibd^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{10}bcd^3x^4($$

$$=\frac{3iabd^3x}{2c^2}+\frac{b^2d^3x}{3c^2}-\frac{14bd^3x^2(a+b\tan^{-1}(cx))}{15c}-\frac{11}{18}ibd^3x^3\left(a+b\tan\right.$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}-\frac{1}{10}b^2d^3x^3-\frac{b^2d^3\tan^{-1}(cx)}{3c^3}+\frac{3ib^2d^3x\tan^{-1}}{2c^2}$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{17ib^2d^3x^2}{60c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2}{}$$

$$=\frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2}{}$$

Mathematica [A]
time = 0.89, size = 369, normalized size = 0.92

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*(-162*a*b + (64*I)*b^2 + (330*I)*a*b*c*x + 222*b^2*c*x - 168*a*b*c^2*x
^2 + (61*I)*b^2*c^2*x^2 + 60*a^2*c^3*x^3 - (110*I)*a*b*c^3*x^3 - 18*b^2*c^3
*x^3 + (135*I)*a^2*c^4*x^4 + 54*a*b*c^4*x^4 - (3*I)*b^2*c^4*x^4 - 108*a^2*c
^5*x^5 + (12*I)*a*b*c^5*x^5 - (30*I)*a^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(I +
4*c*x - (10*I)*c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(-111 + (165*I)*
c*x - 84*c^2*x^2 - (55*I)*c^3*x^3 + 27*c^4*x^4 + (6*I)*c^5*x^5) + 3*a*(-55*
I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6) - 168*b*Log[
1 + E^((2*I)*ArcTan[c*x])]) + 168*a*b*Log[1 + c^2*x^2] - (226*I)*b^2*Log[1
+ c^2*x^2] + (168*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(180*c^3)
```

Maple [A]
time = 0.34, size = 682, normalized size = 1.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^3*(-1/3*I*d^3*a*b*arctan(c*x)*c^6*x^6+3/2*I*d^3*a*b*arctan(c*x)*c^4*x^4 +3/4*I*d^3*b^2*arctan(c*x)^2*c^4*x^4-11/18*I*d^3*b^2*arctan(c*x)*c^3*x^3-1/6*I*d^3*b^2*arctan(c*x)^2*c^6*x^6+1/15*I*d^3*b^2*arctan(c*x)*c^5*x^5-6/5*d^3*a*b*arctan(c*x)*c^5*x^5+2/3*d^3*a*b*arctan(c*x)*c^3*x^3+11/6*I*d^3*a*b*c*x+1/15*I*d^3*a*b*c^5*x^5-11/18*I*d^3*a*b*c^3*x^3+11/6*I*d^3*b^2*arctan(c*x)*c*x+3/10*d^3*a*b*c^4*x^4-1/10*d^3*b^2*c^3*x^3+37/30*d^3*b^2*c*x-7/15*I*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)+7/15*I*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+7/15*I*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-14/15*d^3*b^2*arctan(c*x)*c^2*x^2+3/10*d^3*b^2*arctan(c*x)*c^4*x^4-3/5*d^3*b^2*arctan(c*x)^2*c^5*x^5-7/15*I*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+61/180*I*d^3*b^2*c^2*x^2+1/3*d^3*b^2*arctan(c*x)^2*c^3*x^3-14/15*d^3*a*b*c^2*x^2-1/60*I*d^3*b^2*c^4*x^4-11/6*I*d^3*a*b*arctan(c*x)-113/90*I*d^3*b^2*ln(c^2*x^2+1)+7/30*I*d^3*b^2*ln(c*x+I)^2+7/15*I*d^3*b^2*dilog(1/2*I*(c*x-I))-7/30*I*d^3*b^2*ln(c*x-I)^2-7/15*I*d^3*b^2*dilog(-1/2*I*(c*x+I))-11/12*I*d^3*b^2*arctan(c*x)^2+14/15*a*b*ln(c^2*x^2+1)*d^3+14/15*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^3+d^3*a^2*(-1/6*I*c^6*x^6-3/5*c^5*x^5+3/4*I*c^4*x^4+1/3*c^3*x^3)-37/30*d^3*b^2*arctan(c*x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] -1/6*I*a^2*c^3*d^3*x^6 - 3/5*a^2*c^2*d^3*x^5 + 3/4*I*a^2*c*d^3*x^4 - 1/45*I*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b*c^3*d^3 - 3/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3 + 1/2*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^3 + 1/240*(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^2*d^3*x^3)*arctan(c*x)^2 + 1/240*(10*b^2*c^3*d^3*x^6 - 36*I*b^2*c^2*d^3*x^5 - 45*b^2*c*d^3*x^4 + 20*I*b^2*d^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/960*(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^2*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/240*(180*(b^2*c^5*d^3*x^7 - 2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^7 - 2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - 2*(46*b^2*c^4*d^3*x^6 - 65*b^2*c^2*d^3*x^4)*arctan(c*x) + (10*b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 + 20*b^2*c*d^3*x^3 - 60*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/240*(180*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*arctan(c*x)^2 + 15*(3*b^2

```
*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*log(c^2*x^2 + 1)^2 + 2*(10*
b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 + 20*b^2*c*d^3*x^3)*arctan(c*x) + (46*
b^2*c^4*d^3*x^6 - 65*b^2*c^2*d^3*x^4 + 60*(b^2*c^5*d^3*x^7 - 2*b^2*c^3*d^3*
x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] 1/240*(10*I*b^2*c^3*d^3*x^6 + 36*b^2*c^2*d^3*x^5 - 45*I*b^2*c*d^3*x^4 - 20*
b^2*d^3*x^3)*log(-(c*x + I)/(c*x - I))^2 + integral(1/60*(-60*I*a^2*c^5*d^3
*x^7 - 180*a^2*c^4*d^3*x^6 + 120*I*a^2*c^3*d^3*x^5 - 120*a^2*c^2*d^3*x^4 +
180*I*a^2*c*d^3*x^3 + 60*a^2*d^3*x^2 + (60*a*b*c^5*d^3*x^7 - 10*(18*I*a*b +
 b^2)*c^4*d^3*x^6 - 12*(10*a*b - 3*I*b^2)*c^3*d^3*x^5 - 15*(8*I*a*b - 3*b^2
)*c^2*d^3*x^4 - 20*(9*a*b + I*b^2)*c*d^3*x^3 + 60*I*a*b*d^3*x^2)*log(-(c*x
+ I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
[Out] int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.86 $\int x(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=307

$$-\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2-\frac{1}{30}ib^2cd^3x^3-\frac{13ib^2d^3\text{ArcTan}(cx)}{10c^2}-\frac{5b^2d^3x\text{ArcTan}(cx)}{2c}-\frac{6}{5}ibd^3x^2(a+b\text{ArcTa}$$

```
[Out] -5/2*a*b*d^3*x/c+13/10*I*b^2*d^3*x/c-1/4*b^2*d^3*x^2-1/30*I*b^2*c*d^3*x^3-1
3/10*I*b^2*d^3*arctan(c*x)/c^2-5/2*b^2*d^3*x*arctan(c*x)/c-6/5*I*b*d^3*x^2*
(a+b*arctan(c*x))+1/2*b*c*d^3*x^3*(a+b*arctan(c*x))+1/10*I*b*c^2*d^3*x^4*(a
+b*arctan(c*x))+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c^2-1/5*d^3*(1+I*c*
x)^5*(a+b*arctan(c*x))^2/c^2-12/5*I*b*d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))
/c^2+3/2*b^2*d^3*ln(c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(1-I*c*x))/c^2
```

Rubi [A]
time = 0.45, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {4996, 4974, 4930, 266, 4946, 327, 209, 272, 45, 1600, 4964, 2449, 2352, 308}

$$\frac{1}{10}ibc^2d^3x^4(a+b\text{ArcTan}(cx))-\frac{d^3(1+icx)^5(a+b\text{ArcTan}(cx))^2}{5c^2}+\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))^2}{4c^2}-\frac{12ibd^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{5c^2}+\frac{1}{2}bcd^3x^3(a+b\text{ArcTan}(cx))-\frac{6}{5}ibd^3x^2(a+b\text{ArcTan}(cx))-\frac{5abd^3x}{2c}-\frac{13ib^2d^3\text{ArcTan}(cx)}{10c^2}-\frac{5b^2d^3x\text{ArcTan}(cx)}{2c}-\frac{6b^2d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{5c^2}+\frac{3b^2d^3\log\left(c^2x^2+1\right)}{2c^2}-\frac{1}{30}ib^2cd^3x^3+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-5*a*b*d^3*x)/(2*c) + (((13*I)/10)*b^2*d^3*x)/c - (b^2*d^3*x^2)/4 - (I/30)
*b^2*c*d^3*x^3 - (((13*I)/10)*b^2*d^3*ArcTan[c*x])/c^2 - (5*b^2*d^3*x*ArcTa
n[c*x])/(2*c) - ((6*I)/5)*b*d^3*x^2*(a + b*ArcTan[c*x]) + (b*c*d^3*x^3*(a +
 b*ArcTan[c*x]))/2 + (I/10)*b*c^2*d^3*x^4*(a + b*ArcTan[c*x]) + (d^3*(1 + I
*c*x)^4*(a + b*ArcTan[c*x])^2)/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*ArcTan[c
*x])^2)/(5*c^2) - (((12*I)/5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])
/c^2 + (3*b^2*d^3*Log[1 + c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(
1 - I*c*x)])/(5*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int x(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(\frac{i(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{c}-\frac{i(d+icdx)^4\left(a+b\tan^{-1}(cx)\right)}{cd}\right.$$

$$=\frac{i\int(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2\,dx}{c}-\frac{i\int(d+icdx)^4\left(a+b\tan^{-1}(\right.}{cd}$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}+$$

$$=\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c^2}-\frac{d^3(1+icx)^5\left(a+b\tan^{-1}(cx)\right)^2}{5c^2}-$$

$$=-\frac{5abd^3x}{2c}-\frac{6}{5}ibd^3x^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3\left(a+b\tan^{-1}(cx)\right)-$$

$$=-\frac{5abd^3x}{2c}+\frac{6ib^2d^3x}{5c}-\frac{5b^2d^3x\tan^{-1}(cx)}{2c}-\frac{6}{5}ibd^3x^2\left(a+b\tan^{-1}(cx)\right.$$

$$=-\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{30}ib^2cd^3x^3-\frac{6ib^2d^3\tan^{-1}(cx)}{5c^2}-\frac{5b^2d^3x\tan}{2c}$$

$$=-\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2-\frac{1}{30}ib^2cd^3x^3-\frac{13ib^2d^3\tan^{-1}(cx)}{10c^2}$$

Mathematica [**A**]
time = 0.75, size = 325, normalized size = 1.06

$$\frac{d^3\left(-18iab-15b^2-150abcx+78ib^2cx+30a^2c^2x^2-72iabc^2x^2-15b^2c^2x^2+60ia^2c^3x^3+30abc^3x^3-2ib^2c^3x^3-45a^2c^4x^4+6iabc^4x^4-12ia^2c^5x^5+3b^2(1-4icx)(-i+cx)^4\mathrm{ArcTan}(cx)^2+6b\mathrm{ArcTan}(cx)\left(b(-13i-25cx-12ic^2x^2+5c^3x^3+ic^4x^4)+a(25+10c^2x^2+20ic^3x^3-15c^4x^4-4ic^5x^5)-24ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)+72iab\log\left(1+c^2x^2\right)+90b^2\log\left(1+c^2x^2\right)-72b^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{60c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (d^3*((-18*I)*a*b - 15*b^2 - 150*a*b*c*x + (78*I)*b^2*c*x + 30*a^2*c^2*x^2
- (72*I)*a*b*c^2*x^2 - 15*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 30*a*b*c^3*x^3
 - (2*I)*b^2*c^3*x^3 - 45*a^2*c^4*x^4 + (6*I)*a*b*c^4*x^4 - (12*I)*a^2*c^5*
x^5 + 3*b^2*(1 - (4*I)*c*x)*(-I + c*x)^4*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b
*(-13*I - 25*c*x - (12*I)*c^2*x^2 + 5*c^3*x^3 + I*c^4*x^4) + a*(25 + 10*c^2
*x^2 + (20*I)*c^3*x^3 - 15*c^4*x^4 - (4*I)*c^5*x^5) - (24*I)*b*Log[1 + E^((
2*I)*ArcTan[c*x])]) + (72*I)*a*b*Log[1 + c^2*x^2] + 90*b^2*Log[1 + c^2*x^2]
 - 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(269) = 538.
time = 0.23, size = 626, normalized size = 2.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-2/5*I*d^3*a*b*arctan(c*x)*c^5*x^5+2*I*d^3*a*b*arctan(c*x)*c^3*x^3-5
/2*b^2*c*d^3*x*arctan(c*x)+1/2*d^3*a*b*c^3*x^3-3/4*d^3*b^2*arctan(c*x)^2*c^
4*x^4+1/2*d^3*b^2*arctan(c*x)*c^3*x^3-13/10*I*d^3*b^2*arctan(c*x)+6/5*I*d^3
*b^2*ln(c^2*x^2+1)*arctan(c*x)+1/2*d^3*b^2*arctan(c*x)^2*c^2*x^2-1/30*I*d^3
*b^2*c^3*x^3+13/10*I*d^3*b^2*c*x+6/5*I*d^3*a*b*ln(c^2*x^2+1)+3/5*d^3*b^2*ln
(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*d^3*b^2*c^2*x^2+5/2*d^3*a*b*arctan(c*x)+3/5*
d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)-3/5*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-3/5
*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)-3/2*d^3*a*b*arctan(c*x)*c^4*x^4+1/10*I*d^3
*a*b*c^4*x^4-6/5*I*d^3*a*b*c^2*x^2+d^3*a*b*arctan(c*x)*c^2*x^2+I*d^3*b^2*ar
ctan(c*x)^2*c^3*x^3+1/10*I*d^3*b^2*arctan(c*x)*c^4*x^4-1/5*I*d^3*b^2*arctan
(c*x)^2*c^5*x^5-6/5*I*d^3*b^2*arctan(c*x)*c^2*x^2-3/5*d^3*b^2*dilog(1/2*I*(
c*x-I))+3/5*d^3*b^2*dilog(-1/2*I*(c*x+I))+5/4*d^3*b^2*arctan(c*x)^2-3/10*d^
3*b^2*ln(c*x+I)^2+3/10*d^3*b^2*ln(c*x-I)^2+d^3*a^2*(-1/5*I*c^5*x^5-3/4*c^4*
x^4+I*c^3*x^3+1/2*c^2*x^2)-5/2*a*b*c*d^3*x+3/2*b^2*d^3*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/5*I*a^2*c^3*d^3*x^5 - 3/4*a^2*c^2*d^3*x^4 - 1/10*I*(4*x^5*arctan(c*x) -
c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^3*d^3 + I*a^2*c*d
^3*x^3 + 1/2*b^2*d^3*x^2*arctan(c*x)^2 - 1/2*(3*x^4*arctan(c*x) - c*((c^2*x
^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^2*d^3 + I*(2*x^3*arctan(c*x) - c*
(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^3*x^2 + (x^2*arctan
(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^3 - 1/2*(2*c*(x/c^2 - arctan(c*x
)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^3 + 1/80
*(-4*I*b^2*c^3*d^3*x^5 - 15*b^2*c^2*d^3*x^4 + 20*I*b^2*c*d^3*x^3)*arctan(c*
x)^2 + 1/80*(4*b^2*c^3*d^3*x^5 - 15*I*b^2*c^2*d^3*x^4 - 20*b^2*c*d^3*x^3)*a
rctan(c*x)*log(c^2*x^2 + 1) - 1/320*(-4*I*b^2*c^3*d^3*x^5 - 15*b^2*c^2*d^3*
x^4 + 20*I*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/80*(60*(b^2*c^
5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x)^2 + 5*(b^2*c^5
*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*log(c^2*x^2 + 1)^2 - 2*(19*
b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3)*arctan(c*x) + (4*b^2*c^5*d^3*x^6 - 35
*b^2*c^3*d^3*x^4 - 60*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*arctan(c*x))*log(
c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/80*(180*(b^2*c^4*d^3*x^5 + b^
2*c^2*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*log(c
^2*x^2 + 1)^2 + 2*(4*b^2*c^5*d^3*x^6 - 35*b^2*c^3*d^3*x^4)*arctan(c*x) + (1
9*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3 + 20*(b^2*c^5*d^3*x^6 - 2*b^2*c^3*d^
3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(4*I*b^2*c^3*d^3*x^5 + 15*b^2*c^2*d^3*x^4 - 20*I*b^2*c*d^3*x^3 - 10*b^
2*d^3*x^2)*log(-(c*x + I)/(c*x - I))^2 + integral(1/20*(-20*I*a^2*c^5*d^3*x
^6 - 60*a^2*c^4*d^3*x^5 + 40*I*a^2*c^3*d^3*x^4 - 40*a^2*c^2*d^3*x^3 + 60*I*
a^2*c*d^3*x^2 + 20*a^2*d^3*x + (20*a*b*c^5*d^3*x^6 - 4*(15*I*a*b + b^2)*c^4
*d^3*x^5 - 5*(8*a*b - 3*I*b^2)*c^3*d^3*x^4 - 20*(2*I*a*b - b^2)*c^2*d^3*x^3
 - 10*(6*a*b + I*b^2)*c*d^3*x^2 + 20*I*a*b*d^3*x)*log(-(c*x + I)/(c*x - I))
)/(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

```
[Out] int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.87 $\int (d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=226

$$-\frac{7}{2}iabd^3x-b^2d^3x-\frac{1}{12}ib^2cd^3x^2+\frac{b^2d^3\text{ArcTan}(cx)}{c}-\frac{7}{2}ib^2d^3x\text{ArcTan}(cx)+bcd^3x^2(a+b\text{ArcTan}(cx))+\frac{1}{6}ibc^2d^3x$$

```
[Out] -7/2*I*a*b*d^3*x-b^2*d^3*x-1/12*I*b^2*c*d^3*x^2+b^2*d^3*arctan(c*x)/c-7/2*I
*b^2*d^3*x*arctan(c*x)+b*c*d^3*x^2*(a+b*arctan(c*x))+1/6*I*b*c^2*d^3*x^3*(a
+b*arctan(c*x))-1/4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c+4*b*d^3*(a+b*ar
ctan(c*x))*ln(2/(1-I*c*x))/c+11/6*I*b^2*d^3*ln(c^2*x^2+1)/c-2*I*b^2*d^3*pol
ylog(2,1-2/(1-I*c*x))/c
```

Rubi [A]
time = 0.16, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4974, 4930, 266, 4946, 327, 209, 272, 45, 1600, 4964, 2449, 2352}

$$\frac{1}{6}ibc^2d^3x^3(a+b\text{ArcTan}(cx))+bcd^3x^2(a+b\text{ArcTan}(cx))-\frac{id^3(1+icx)^4(a+b\text{ArcTan}(cx))^2}{4c}+\frac{4bd^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{c}-\frac{7}{2}iabd^3x+\frac{b^2d^3\text{ArcTan}(cx)}{c}-\frac{7}{2}ib^2d^3x\text{ArcTan}(cx)+\frac{11ib^2d^3\log\left(c^2x^2+1\right)}{6c}-\frac{2ib^2d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{c}-\frac{1}{12}ib^2cd^3x^2-b^2d^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] ((-7*I)/2)*a*b*d^3*x - b^2*d^3*x - (I/12)*b^2*c*d^3*x^2 + (b^2*d^3*ArcTan[c
*x])/c - ((7*I)/2)*b^2*d^3*x*ArcTan[c*x] + b*c*d^3*x^2*(a + b*ArcTan[c*x])
+ (I/6)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x]) - ((I/4)*d^3*(1 + I*c*x)^4*(a + b
*ArcTan[c*x])^2)/c + (4*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c + (
((11*I)/6)*b^2*d^3*Log[1 + c^2*x^2])/c - ((2*I)*b^2*d^3*PolyLog[2, 1 - 2/(1
 - I*c*x)])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rubi steps

$$\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2 \, dx = -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{(ib)\int \left(-7d^4(a+b\tan^{-1}(cx))\right.}{}$$

$$= -\frac{id^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{(4b)\int \frac{(id^4-cd^4x)(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{d}$$

$$= -\frac{7}{2}iabd^3x + bcd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{1}{6}ibc^2d^3x^3\left(a+b\tan^{-1}(cx)\right)$$

$$= -\frac{7}{2}iabd^3x - b^2d^3x - \frac{7}{2}ib^2d^3x\tan^{-1}(cx) + bcd^3x^2\left(a+b\tan^{-1}(cx)\right) -$$

$$= -\frac{7}{2}iabd^3x - b^2d^3x + \frac{b^2d^3\tan^{-1}(cx)}{c} - \frac{7}{2}ib^2d^3x\tan^{-1}(cx) + bcd^3x^2($$

$$= -\frac{7}{2}iabd^3x - b^2d^3x - \frac{1}{12}ib^2cd^3x^2 + \frac{b^2d^3\tan^{-1}(cx)}{c} - \frac{7}{2}ib^2d^3x\tan^{-1}($$

Mathematica [A]
time = 0.48, size = 267, normalized size = 1.18

$$-\frac{id^3\left(b^2+12ia^2cx+42abcx-12ib^2cx-18a^2c^2x^2+12iabc^2x^2+b^2c^2x^2-12ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+3b^2(-i+cx)^4\mathrm{ArcTan}(cx)^2+2b\mathrm{ArcTan}(cx)\left(b(6i+21cx+6ic^2x^2-c^3x^3)+3a(-7+4icx-6c^2x^2-4ic^3x^3+c^4x^4)+24ib\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-24iab\log\left(1+c^2x^2\right)-22b^2\log\left(1+c^2x^2\right)+24b^2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
[Out] ((-1/12*I)*d^3*(b^2 + (12*I)*a^2*c*x + 42*a*b*c*x - (12*I)*b^2*c*x - 18*a^2
*c^2*x^2 + (12*I)*a*b*c^2*x^2 + b^2*c^2*x^2 - (12*I)*a^2*c^3*x^3 - 2*a*b*c^
```

```
3*x^3 + 3*a^2*c^4*x^4 + 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*
(b*(6*I + 21*c*x + (6*I)*c^2*x^2 - c^3*x^3) + 3*a*(-7 + (4*I)*c*x - 6*c^2*x
^2 - (4*I)*c^3*x^3 + c^4*x^4) + (24*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) -
(24*I)*a*b*Log[1 + c^2*x^2] - 22*b^2*Log[1 + c^2*x^2] + 24*b^2*PolyLog[2, -
E^((2*I)*ArcTan[c*x])]))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(204) = 408.
time = 0.20, size = 553, normalized size = 2.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*I*d^3*a*b*arctan(c*x)*c^4*x^4+3*I*d^3*a*b*arctan(c*x)*c^2*x^2+2*a
*b*arctan(c*x)*d^3*c*x-2*d^3*a*b*arctan(c*x)*c^3*x^3-7/2*I*d^3*b^2*arctan(c
*x)*c*x+3/2*I*d^3*b^2*arctan(c*x)^2*c^2*x^2+1/6*I*d^3*b^2*arctan(c*x)*c^3*x
^3-1/4*I*d^3*b^2*arctan(c*x)^2*c^4*x^4+1/6*I*d^3*a*b*c^3*x^3-7/2*I*d^3*a*b*
c*x-d^3*b^2*c*x+d^3*b^2*arctan(c*x)*c^2*x^2-d^3*b^2*arctan(c*x)^2*c^3*x^3+d
^3*a*b*c^2*x^2+b^2*arctan(c*x)^2*d^3*c*x-I*d^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-
I))-I*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+7/2*I*d^3*a*b*arctan(c*x)-1/12*I*d^3*
b^2*c^2*x^2+I*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I*d^3*b^2*ln(c*x+I)*ln(c
^2*x^2+1)-2*a*b*ln(c^2*x^2+1)*d^3-2*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^3-1/4*I
*d^3*(1+I*c*x)^4*a^2+I*d^3*b^2*dilog(-1/2*I*(c*x+I))+7/4*I*d^3*b^2*arctan(c
*x)^2+11/6*I*d^3*b^2*ln(c^2*x^2+1)+1/2*I*d^3*b^2*ln(c*x-I)^2-I*d^3*b^2*dilo
g(1/2*I*(c*x-I))-1/2*I*d^3*b^2*ln(c*x+I)^2+d^3*b^2*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*I*a^2*c^3*d^3*x^4 - 4*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(c
^2*x^2 + 1)/(c^2*x^2 + 1), x) - 2*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x
)/(c^2*x^2 + 1), x) - a^2*c^2*d^3*x^3 - 36*b^2*c^4*d^3*integrate(1/16*x^4*a
rctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - 5*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2*x^2
+ 1)/(c^2*x^2 + 1), x) - 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4
+ 3*arctan(c*x)/c^5))*a*b*c^3*d^3 + 8*b^2*c^3*d^3*integrate(1/16*x^3*arctan
(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 20*b^2*c^3*d^3*integrate(1/16*x^
3*arctan(c*x)/(c^2*x^2 + 1), x) - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2
*x^2 + 1)/c^4))*a*b*c^2*d^3 + 3/2*I*a^2*c*d^3*x^2 - 24*b^2*c^2*d^3*integrat
e(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) - 2*b^2*c^2*d^3*integrate(1/16*x
^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 10*b^2*c^2*d^3*integrate(1/16*x^2
```

```
*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b*c*d^3 + 1/4*b^2*d^3*arctan(c*x)^3/c + 12*b^2*c*d^3*integ
rate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 8*b^2*c*d^3*in
tegrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^3*x + b^2*d^3*integrat
e(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*
x^2 + 1))*a*b*d^3/c + 1/16*(-I*b^2*c^3*d^3*x^4 - 4*b^2*c^2*d^3*x^3 + 6*I*b^
2*c*d^3*x^2 + 4*b^2*d^3*x)*arctan(c*x)^2 + 1/16*(b^2*c^3*d^3*x^4 - 4*I*b^2*
c^2*d^3*x^3 - 6*b^2*c*d^3*x^2 + 4*I*b^2*d^3*x)*arctan(c*x)*log(c^2*x^2 + 1)
 - 1/64*(-I*b^2*c^3*d^3*x^4 - 4*b^2*c^2*d^3*x^3 + 6*I*b^2*c*d^3*x^2 + 4*b^2
*d^3*x)*log(c^2*x^2 + 1)^2 - I*integrate(1/16*(12*(b^2*c^5*d^3*x^5 - 2*b^2*
c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 + (b^2*c^5*d^3*x^5 - 2*b^2*c^3*d
^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^2 - 10*(b^2*c^4*d^3*x^4 - 2*b^2*c^
2*d^3*x^2)*arctan(c*x) + (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 + 4*b^2*c*d^
3*x - 4*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*arctan(c*x))*log(
c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(I*b^2*c^3*d^3*x^4 + 4*b^2*c^2*d^3*x^3 - 6*I*b^2*c*d^3*x^2 - 4*b^2*d^3
*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a
^2*c^4*d^3*x^4 + 8*I*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (4*a*b*c^5*d^3*x^5 + (-12*I*a*b - b^2)*c^4*d^3*x^4 - 4*(2*a*
b - I*b^2)*c^3*d^3*x^3 - 2*(4*I*a*b - 3*b^2)*c^2*d^3*x^2 - 4*(3*a*b + I*b^2
)*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.88 $\int \frac{(d+icdx)^3(a+b\textbf{ArcTan}(cx))^2}{x} dx$

Optimal. Leaf size=385

$$3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\text{ArcTan}(cx)+3b^2cd^3x\text{ArcTan}(cx)+\frac{1}{3}ibc^2d^3x^2(a+b\text{ArcTan}(cx))-\frac{29}{6}d^3(a+b\text{ArcT}$$

```
[Out] 3*a*b*c*d^3*x+1/3*I*b*c^2*d^3*x^2*(a+b*arctan(c*x))+I*b*d^3*(a+b*arctan(c*x
))*polylog(2,-1+2/(1+I*c*x))+3*b^2*c*d^3*x*arctan(c*x)-1/3*I*c^3*d^3*x^3*(a
+b*arctan(c*x))^2-29/6*d^3*(a+b*arctan(c*x))^2-I*b*d^3*(a+b*arctan(c*x))*po
lylog(2,1-2/(1+I*c*x))-3/2*c^2*d^3*x^2*(a+b*arctan(c*x))^2-1/3*I*b^2*c*d^3*
x-2*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+1/3*I*b^2*d^3*arctan(c*
x)-3/2*b^2*d^3*ln(c^2*x^2+1)-10/3*b^2*d^3*polylog(2,1-2/(1+I*c*x))+3*I*c*d^
3*x*(a+b*arctan(c*x))^2+20/3*I*b*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-1/2*
b^2*d^3*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^3*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.55, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 266, 327, 209}

$$-\frac{1}{3}ic^3d^3x^3(a+b\text{ArcTan}(cx))^2-\frac{3}{2}c^2d^3x^2(a+b\text{ArcTan}(cx))^2+\frac{1}{3}ibc^2d^3x^2(a+b\text{ArcTan}(cx))-ibd^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibd^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+3icd^3x(a+b\text{ArcTan}(cx))^2-\frac{29}{6}d^3(a+b\text{ArcTan}(cx))^2+\frac{20}{3}ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+2d^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+3abcd^3x+\frac{1}{3}ib^2d^3\text{ArcTan}(cx)+3b^2cd^3x\text{ArcTan}(cx)-\frac{3}{2}b^2d^3\log\left(c^2x^2+1\right)-\frac{10}{3}b^2d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^3\text{Li}_3\left(\frac{2}{icx+1}-1\right)-\frac{1}{3}ib^2cd^3x$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] 3*a*b*c*d^3*x - (I/3)*b^2*c*d^3*x + (I/3)*b^2*d^3*ArcTan[c*x] + 3*b^2*c*d^3
*x*ArcTan[c*x] + (I/3)*b*c^2*d^3*x^2*(a + b*ArcTan[c*x]) - (29*d^3*(a + b*A
rcTan[c*x])^2)/6 + (3*I)*c*d^3*x*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^2*(a
+ b*ArcTan[c*x])^2)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x])^2 + 2*d^3*(a
+ b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + ((20*I)/3)*b*d^3*(a + b*Arc
Tan[c*x])*Log[2/(1 + I*c*x)] - (3*b^2*d^3*Log[1 + c^2*x^2])/2 - (10*b^2*d^3
*PolyLog[2, 1 - 2/(1 + I*c*x)])/3 - I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 + I*c*x)] + I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c
*x)] - (b^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^3*PolyLog[3, -1 +
 2/(1 + I*c*x)])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = \int \left(3icd^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}-3c^2d^3x\left(a+b\,\mathrm{t}\right.\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx+\left(3icd^3\right)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx-\left(3c^2\right.$$

$$= 3icd^3x\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{2}c^2d^3x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}ic^3d^3x^3\left(a\right.$$

$$= -3d^3\left(a+b\tan^{-1}(cx)\right)^2+3icd^3x\left(a+b\tan^{-1}(cx)\right)^2-\frac{3}{2}c^2d^3x^2\left(a+b\right.$$

$$= 3abcd^3x+\frac{1}{3}ibc^2d^3x^2\left(a+b\tan^{-1}(cx)\right)-\frac{29}{6}d^3\left(a+b\tan^{-1}(cx)\right)^2+3a$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d^3x^2\left(a+b\tan^{-1}(cx)\right.$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\tan^{-1}(cx)+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d$$

$$= 3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\tan^{-1}(cx)+3b^2cd^3x\tan^{-1}(cx)+\frac{1}{3}ibc^2d$$

Mathematica [A]
time = 0.46, size = 465, normalized size = 1.21

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (-1/24*I)*d^3*(b^2*Pi^3 - 72*a^2*c*x + (72*I)*a*b*c*x + 8*b^2*c*x - (36*I)*
a^2*c^2*x^2 - 8*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - (72*I)*a*b*ArcTan[c*x] - 8*b^
2*ArcTan[c*x] - 144*a*b*c*x*ArcTan[c*x] + (72*I)*b^2*c*x*ArcTan[c*x] - (72*
I)*a*b*c^2*x^2*ArcTan[c*x] - 8*b^2*c^2*x^2*ArcTan[c*x] + 16*a*b*c^3*x^3*Arc
Tan[c*x] + (44*I)*b^2*ArcTan[c*x]^2 - 72*b^2*c*x*ArcTan[c*x]^2 - (36*I)*b^2
```

```
*c^2*x^2*ArcTan[c*x]^2 + 8*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*ArcTan[c*x]^3
 + (24*I)*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 160*b^2*ArcTa
n[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*ArcTan[c*x]^2*Log[1 + E^
((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] + 80*a*b*Log[1 + c^2*x^2] - (36*
I)*b^2*Log[1 + c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*
x])] - 8*b^2*(-10*I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 2
4*a*b*PolyLog[2, (-I)*c*x] + 24*a*b*PolyLog[2, I*c*x] + (12*I)*b^2*PolyLog[
3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.46, size = 1651, normalized size = 4.29

method	result	size
derivativedivides	Expression too large to display	1651
default	Expression too large to display	1651

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^3*arctan(c*x)^2+3*b^2*c*d^3*x*arctan(c*x)-1/3*I*b^2*c*d^3*x-1/2*I*d^3*b
^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arcta
n(c*x)^2+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*arctan(c*x)^2+3*I*d^3*b^2*arctan(c*x)^2*c*x-1/3*I*d^3*b^2*
arctan(c*x)^2*c^3*x^3+1/3*I*d^3*b^2*arctan(c*x)*c^2*x^2+1/3*I*d^3*a*b*c^2*x
^2+I*d^3*a*b*ln(c*x)*ln(1+I*c*x)-I*d^3*a*b*ln(c*x)*ln(1-I*c*x)-3/2*d^3*b^2*
arctan(c*x)^2*c^2*x^2-3*d^3*a*b*arctan(c*x)-3*d^3*a*b*arctan(c*x)*c^2*x^2+1
1/6*d^3*b^2*arctan(c*x)^2+1/3*d^3*b^2-1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+6*I*d^3*a*b*arctan(c*x)*c*x-
2/3*I*d^3*a*b*arctan(c*x)*c^3*x^3+3*a*b*c*d^3*x-1/2*I*d^3*b^2*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^2+1/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3*I*d^3*a^2*c*x-1/3*I*d^3*a^2*c^3*x^3
+20/3*I*d^3*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^3*b^2
*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*b^2*Pi*arctan
(c*x)^2-2*I*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*d
^3*a*b*ln(c*x)*arctan(c*x)+I*d^3*a*b*dilog(1+I*c*x)-I*d^3*a*b*dilog(1-I*c*x
)-10/3*I*d^3*a*b*ln(c^2*x^2+1)+I*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))+d^3*a^2*ln(c*x)+20/3*d^3*b^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^
```

```
(1/2))+2*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*d^3*b^2*polylo
g(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*d^3*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/
2))+3*d^3*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+20/3*d^3*b^2*dilog(1+I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))+20/3*I*d^3*b^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-d^3*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+d^3*b^2*ln(c*x
)*arctan(c*x)^2+d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+d^3
*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-8/3*I*d^3*b^2*arctan(c
*x)-3/2*d^3*a^2*c^2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*I*a^2*c^3*d^3*x^3 - 36*I*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)^2/
(c^2*x^3 + x), x) - 12*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)*log(c^2*x
^2 + 1)/(c^2*x^3 + x), x) - 3*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2
+ 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/
(c^2*x^3 + x), x) - 8*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/(c^2*x^3 +
 x), x) - 4*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2 + 1)/(c^2*x^3 + x)
, x) - 108*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) +
 36*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3
+ x), x) - 9*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x
), x) - 288*a*b*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x) +
44*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 22*b^2*
c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 3/2*a^2*c^2
*d^3*x^2 + 72*I*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)^2/(c^2*x^3 + x),
 x) + 24*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x
^3 + x), x) + 6*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)^2/(c^2*x^
3 + x), x) - 96*I*a*b*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^3 + x),
 x) + 108*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 54
*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 3/4*
I*b^2*d^3*arctan(c*x)^3 - 72*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)^2/(
c^2*x^3 + x), x) + 24*I*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*x^3 + x), x) - 192*a*b*c^2*d^3*integrate(1/48*x^2*arctan(c*x)
/(c^2*x^3 + x), x) - 72*I*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)/(c^2*x
^3 + x), x) - 1/48*b^2*d^3*log(c^2*x^2 + 1)^3 + 3*I*a^2*c*d^3*x + 36*b^2*c*
d^3*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 9*I*b
^2*c*d^3*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 3/16*b^2*d
^3*log(c^2*x^2 + 1)^2 + 3*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^3
+ 36*b^2*d^3*integrate(1/48*arctan(c*x)^2/(c^2*x^3 + x), x) - 12*I*b^2*d^3*
integrate(1/48*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 3*b^2*d^3*i
```

```
ntegrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 96*a*b*d^3*integrate(1
/48*arctan(c*x)/(c^2*x^3 + x), x) + a^2*d^3*log(x) + 1/24*(-2*I*b^2*c^3*d^3
*x^3 - 9*b^2*c^2*d^3*x^2 + 18*I*b^2*c*d^3*x)*arctan(c*x)^2 + 1/24*(2*b^2*c^
3*d^3*x^3 - 9*I*b^2*c^2*d^3*x^2 - 18*b^2*c*d^3*x)*arctan(c*x)*log(c^2*x^2 +
 1) - 1/96*(-2*I*b^2*c^3*d^3*x^3 - 9*b^2*c^2*d^3*x^2 + 18*I*b^2*c*d^3*x)*lo
g(c^2*x^2 + 1)^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^3*x
^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x, x)
```

3.89 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))^2}{x^2} \, dx$

Optimal. Leaf size=402

$$iabc^2d^3x+ib^2c^2d^3x\text{ArcTan}(cx)-\frac{9}{2}icd^3(a+b\text{ArcTan}(cx))^2-\frac{d^3(a+b\text{ArcTan}(cx))^2}{x}-3c^2d^3x(a+b\text{ArcTan}(cx))^2$$

```
[Out] I*b^2*c^2*d^3*x*arctan(c*x)-1/2*I*b^2*c*d^3*ln(c^2*x^2+1)+I*a*b*c^2*d^3*x-d
^3*(a+b*arctan(c*x))^2/x-3*c^2*d^3*x*(a+b*arctan(c*x))^2-6*I*c*d^3*(a+b*arc
tan(c*x))^2*arctanh(-1+2/(1+I*c*x))-9/2*I*c*d^3*(a+b*arctan(c*x))^2-6*b*c*d
^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-3*I*b^2*c*d^3*polylog(2,1-2/(1+I*c*x))
+2*b*c*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d^3*polylog(2,-1+2/(
1-I*c*x))+3/2*I*b^2*c*d^3*polylog(3,-1+2/(1+I*c*x))+3*b*c*d^3*(a+b*arctan(c
*x))*polylog(2,1-2/(1+I*c*x))-3*b*c*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1
+I*c*x))-3/2*I*b^2*c*d^3*polylog(3,1-2/(1+I*c*x))-1/2*I*c^3*d^3*x^2*(a+b*ar
ctan(c*x))^2
```

Rubi [A]
time = 0.54, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745, 5036, 266}

$$-\frac{1}{2}ic^3d^3x^2(a+b\text{ArcTan}(cx))^2-3c^2d^3x(a+b\text{ArcTan}(cx))^2+3bcd^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))-3bcd^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))-\frac{9}{2}icd^3(a+b\text{ArcTan}(cx))^2-\frac{d^3(a+b\text{ArcTan}(cx))^2}{x}-6bcd^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+2bcd^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+6icd^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+iabc^2d^3x+ib^2c^2d^3x\text{ArcTan}(cx)-\frac{1}{2}ib^2cd^3\log\left(c^2x^2+1\right)-ib^2cd^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)-3ib^2cd^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)-\frac{3}{2}ib^2cd^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{3}{2}ib^2cd^3\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] I*a*b*c^2*d^3*x + I*b^2*c^2*d^3*x*ArcTan[c*x] - ((9*I)/2)*c*d^3*(a + b*ArcT
an[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/x - 3*c^2*d^3*x*(a + b*ArcTan[c*x]
)^2 - (I/2)*c^3*d^3*x^2*(a + b*ArcTan[c*x])^2 + (6*I)*c*d^3*(a + b*ArcTan[c
*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*d^3*(a + b*ArcTan[c*x])*Log[2/(1
+ I*c*x)] - (I/2)*b^2*c*d^3*Log[1 + c^2*x^2] + 2*b*c*d^3*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)] - (3*
I)*b^2*c*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + 3*b*c*d^3*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 + I*c*x)] - 3*b*c*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - ((3*I)/2)*b^2*c*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] + ((3
*I)/2)*b^2*c*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
```

```
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(-3c^2d^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x^2}+\frac{3icd^3(a+b\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx+\left(3icd^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx-(3c$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}-3c^2d^3x\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}ic^3d^3x^2(a+$$

$$= -4icd^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}-3c^2d^3x(a+b\tan$$

$$= iabc^2d^3x-\frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x}-3c^2d^3$$

$$= iabc^2d^3x+ib^2c^2d^3x\tan^{-1}(cx)-\frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^3(a+b}{}$$

$$= iabc^2d^3x+ib^2c^2d^3x\tan^{-1}(cx)-\frac{9}{2}icd^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^3(a+b}{}$$

Mathematica [A]
time = 0.32, size = 512, normalized size = 1.27

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (d^3*(-8*a^2 + b^2*c*Pi^3*x - 24*a^2*c^2*x^2 + (8*I)*a*b*c^2*x^2 - (4*I)*a^
2*c^3*x^3 - 16*a*b*ArcTan[c*x] - (8*I)*a*b*c*x*ArcTan[c*x] - 48*a*b*c^2*x^2
*ArcTan[c*x] + (8*I)*b^2*c^2*x^2*ArcTan[c*x] - (8*I)*a*b*c^3*x^3*ArcTan[c*x
] - 8*b^2*ArcTan[c*x]^2 + (12*I)*b^2*c*x*ArcTan[c*x]^2 - 24*b^2*c^2*x^2*Arc
Tan[c*x]^2 - (4*I)*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*c*x*ArcTan[c*x]^3 + (
24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 16*b^2*c*x*Ar
cTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] - 48*b^2*c*x*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTa
n[c*x])] + (24*I)*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 16*a*b*c*x*Log[1 +
 c^2*x^2] - (4*I)*b^2*c*x*Log[1 + c^2*x^2] - 24*b^2*c*x*ArcTan[c*x]*PolyLog
[2, E^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*(-I + ArcTan[c*x])*PolyLog[2, -E^(
(2*I)*ArcTan[c*x])] - (8*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 24*
a*b*c*x*PolyLog[2, (-I)*c*x] + 24*a*b*c*x*PolyLog[2, I*c*x] + (12*I)*b^2*c*
x*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*
ArcTan[c*x])]))/(8*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.27, size = 1702, normalized size = 4.23

method	result	size
derivativedivides	Expression too large to display	1702
default	Expression too large to display	1702

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-I*d^3*a*b*arctan(c*x)*c^2*x^2-3/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-6*a*b*arctan(c*x)*d^3*c*x-
3*b^2*arctan(c*x)^2*d^3*c*x+2*a*b*ln(c^2*x^2+1)*d^3+d^3*b^2*arctan(c*x)+3/2
*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*d^3*b^2*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+6*I*d^3*a*b*arctan(c*x)*ln(c*
x)+I*d^3*b^2*arctan(c*x)*c*x-1/2*I*d^3*b^2*arctan(c*x)^2*c^2*x^2-2*d^3*a*b*
arctan(c*x)/c/x+I*d^3*a*b*c*x-3*a^2*d^3*c*x-a^2*d^3/c/x+3*I*a^2*d^3*ln(c*x)
-3*d^3*a*b*dilog(1+I*c*x)+3*d^3*a*b*dilog(1-I*c*x)+2*d^3*a*b*ln(c*x)+2*d^3*
b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*d^3*b^2*arctan(c*x)*pol
ylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))-3*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^
2+1))+I*d^3*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-6*d^3*b^2*arctan(c*x)*ln(1+I*
```

```
(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*d^3*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))-3/2*d^3*b^2*Pi*arctan(c*x)^2-2*I*d^3*b^2*dilog(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))+6*I*d^3*b^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*d^3*b
^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*d^3*b^2*dilog((1+I*c*x)/(c^2*
x^2+1)^(1/2))-3/2*I*d^3*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+6*I*d^3*b^2
*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*d^3*b^2*polylog(3,(1+I*c*x)/(c
^2*x^2+1)^(1/2))+3/2*I*d^3*b^2*arctan(c*x)^2-1/2*I*a^2*d^3*c^2*x^2+3/2*d^3*
b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arct
an(c*x)^2-3/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3*I*d^3*b^2*arctan(c*x)^2*l
n(c*x)+3*I*d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*d^3*
b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+3*I*d^3*b^2*arctan(c*x)^2*l
n(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-d^3*b^2*arctan(c*x)^2/c/x-3*d^3*a*b*ln(c*x
)*ln(1+I*c*x)+3*d^3*a*b*ln(c*x)*ln(1-I*c*x)-I*d^3*a*b*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*I*a^2*c^3*d^3*x^2 - 3*a^2*c^2*d^3*x - 3*(2*c*x*arctan(c*x) - log(c^2*x
^2 + 1))*a*b*c*d^3 + 3*I*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)
) + 2*arctan(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/96*(12*(-I*b^2*c^3*d^3*x^3 - 6
*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*arctan(c*x)^2 + 12*(b^2*c^3*d^3*x^3 - 6*I*b^2
*c^2*d^3*x^2 - 2*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) - 3*(-I*b^2*c^3*d^
3*x^3 - 6*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*log(c^2*x^2 + 1)^2 - 2*I*(576*b^2*c^
5*d^3*integrate(1/16*x^5*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 48*b^2*c^5*d^3
*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 1536*a*b*c^5*d
^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^4 + x^2), x) + 96*b^2*c^5*d^3*inte
grate(1/16*x^5*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 576*b^2*c^4*d^3*integ
rate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 1344*b^2*c
^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^4 + x^2), x) - 1152*b^2*c^3*d^
3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 3072*a*b*c^3*d^3*i
ntegrate(1/16*x^3*arctan(c*x)/(c^2*x^4 + x^2), x) - b^2*c*d^3*log(c^2*x^2 +
 1)^3 - 12*b^2*c*d^3*arctan(c*x)^2 - 384*b^2*c^2*d^3*integrate(1/16*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 9*b^2*c*d^3*log(c^2*x^2 + 1
)^2 - 1728*b^2*c*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 1
44*b^2*c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 4608
*a*b*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) - 192*b^2*c*d^3
*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 192*b^2*d^3*integr
ate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x))*x - 48*(8*b^2*c^
```

```
5*d^3*integrate(1/16*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) +
 8*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^4 + x^2), x) + 72*b^2*
c^4*d^3*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 6*b^2*c^4*d^
3*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 28*b^2*c^4*d^
3*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + b^2*c*d^3*arcta
n(c*x)^3 - 16*b^2*c^3*d^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(
c^2*x^4 + x^2), x) - 48*b^2*c^3*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^4
 + x^2), x) + 4*b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4
+ x^2), x) + 8*b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x
^2), x) - 24*b^2*c*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x
^4 + x^2), x) - 16*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2),
x) - 24*b^2*d^3*integrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 2*b^2*d^
3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^3*x
^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2, x)
```

3.90 $\int \frac{(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=416

$$-\frac{bcd^3(a+b\text{ArcTan}(cx))}{x}+\frac{7}{2}c^2d^3(a+b\text{ArcTan}(cx))^2-\frac{d^3(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{3icd^3(a+b\text{ArcTan}(cx))^2}{x}-ic^3d$$

[Out] -b*c*d^3*(a+b*arctan(c*x))/x+7/2*c^2*d^3*(a+b*arctan(c*x))^2-1/2*d^3*(a+b*arctan(c*x))^2/x^2-I*c^3*d^3*x*(a+b*arctan(c*x))^2-3*I*b*c^2*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+6*c^2*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^3*ln(x)-3*I*c*d^3*(a+b*arctan(c*x))^2/x-1/2*b^2*c^2*d^3*ln(c^2*x^2+1)+6*I*b*c^2*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+3*b^2*c^2*d^3*polylog(2,-1+2/(1-I*c*x))+b^2*c^2*d^3*polylog(2,1-2/(1+I*c*x))-2*I*b*c^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+3*I*b*c^2*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+3/2*b^2*c^2*d^3*polylog(3,1-2/(1+I*c*x))-3/2*b^2*c^2*d^3*polylog(3,-1+2/(1+I*c*x))

Rubi [A]
time = 0.54, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497, 4942, 5108, 5114, 6745}

$$-ic^3d^3x(a+b\text{ArcTan}(cx))^2+3ibc^2d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))-3ibc^2d^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+\frac{7}{2}c^2d^3(a+b\text{ArcTan}(cx))^2-2ibc^2d^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))+6ibc^2d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-6c^2d^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-\frac{d^3(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{3icd^3(a+b\text{ArcTan}(cx))^2}{x}-\frac{bcd^3(a+b\text{ArcTan}(cx))}{x}+3b^2c^2d^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)+b^2c^2d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)+\frac{3}{2}b^2c^2d^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)-\frac{3}{2}b^2c^2d^3\text{Li}_3\left(\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^3\log\left(c^2x^2+1\right)+b^2c^2d^3\log(x)$$

Antiderivative was successfully verified.

[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]

[Out] -((b*c*d^3*(a + b*ArcTan[c*x]))/x) + (7*c^2*d^3*(a + b*ArcTan[c*x])^2)/2 - (d^3*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((3*I)*c*d^3*(a + b*ArcTan[c*x])^2)/x - I*c^3*d^3*x*(a + b*ArcTan[c*x])^2 - 6*c^2*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^3*Log[x] - (2*I)*b*c^2*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*c^2*d^3*Log[1 + c^2*x^2])/2 + (6*I)*b*c^2*d^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + 3*b^2*c^2*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)] + b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + (3*I)*b*c^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - (3*I)*b*c^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)])/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(-ic^3d^3\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x^3} + \frac{3icd^3(a+b\,t}{x^2}\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx + \left(3icd^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx - \left(3c^2\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x} - ic^3d^3x\left(a+b\tan\right.$$

$$= 4c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} - \frac{3icd^3\left(a+b\tan^{-1}(\right.}{x}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{x} + \frac{7}{2}c^2d^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tan^{-1}(\right.}{2x^2}$$

<u>Mathematica</u> [A]
time = 0.69, size = 500, normalized size = 1.20

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-(a^2/x^2) - ((6*I)*a^2*c)/x - (2*I)*a^2*c^3*x - (2*a*b*(ArcTan[c*x]
+ c*x*(1 + c*x*ArcTan[c*x])))/x^2 - 6*a^2*c^2*Log[x] - (b^2*(2*c*x*ArcTan[c
*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]])
)/x^2 - (2*I)*a*b*c^2*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - ((6*I)*a*b*c
*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])))/x - (2*I)*b^2*c^2*
(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) -
I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (6*b^2*c*(ArcTan[c*x]*((-I + c*x)*A
rcTan[c*x] + (2*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])]) + c*x*PolyLog[2, E^(
(2*I)*ArcTan[c*x])]))/x - (6*I)*a*b*c^2*(PolyLog[2, (-I)*c*x] - PolyLog[2,
I*c*x]) + 6*b^2*c^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c*x]^3 - ArcTan[c*x]^2*
Log[1 - E^((-2*I)*ArcTan[c*x])] + ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x
])] - I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - I*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 + Pol
yLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.02, size = 1746, normalized size = 4.20

method	result	size
derivativedivides	Expression too large to display	1746
default	Expression too large to display	1746

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
[Out] c^2*(d^3*b^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-3/2*I*d^3*b^2*Pi*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-d^3*a*b*arc
tan(c*x)+3/2*d^3*b^2*arctan(c*x)^2-6*d^3*b^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1
/2))+d^3*b^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*d^3*b^2*dilog(1+(1+I*c*x)/
(c^2*x^2+1)^(1/2))-3*I*d^3*a*b*ln(c*x)*ln(1+I*c*x)+3*I*d^3*a*b*ln(c*x)*ln(1
-I*c*x)+3/2*I*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*d^3*b^2*Pi*csgn(((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-d^3*a
*b*arctan(c*x)/c^2/x^2-I*d^3*b^2*arctan(c*x)^2*c*x-3*I*d^3*b^2*arctan(c*x)^
2/c/x-2*I*d^3*a*b*arctan(c*x)*c*x-6*I*d^3*a*b*arctan(c*x)/c/x-3/2*I*d^3*b^2
*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*
d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*d^3*b^2*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*d^3*b^2*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*a^2*d^3*c*x-3*I*a^2*d^3/c/x-d
^3*a*b/c/x-3*I*d^3*a*b*dilog(1+I*c*x)+3*I*d^3*a*b*dilog(1-I*c*x)-2*I*d^3*a*
b*ln(c^2*x^2+1)+6*I*d^3*a*b*ln(c*x)-1/2*d^3*b^2*arctan(c*x)^2/c^2/x^2-d^3*b
^2*arctan(c*x)/c/x+6*I*d^3*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-2*I*d^3*b^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*d^3*b^2*a
rctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*d^3*b^2*Pi*arctan(c*x)
^2-2*I*d^3*b^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*d^3*b^2*
arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*d^3*b^2*arctan(c*x)*
polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*a^2*d^3/c^2/x^2-I*d^3*b^2*arcta
n(c*x)-6*d^3*a*b*ln(c*x)*arctan(c*x)-3*d^3*a^2*ln(c*x)-2*d^3*b^2*dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/
2))+3/2*d^3*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-6*d^3*b^2*polylog(3,(1+
I*c*x)/(c^2*x^2+1)^(1/2))-2*d^3*b^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
3*d^3*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3*d^3*b^2*ln(c*x)*arc
tan(c*x)^2-3*d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*d^3*
b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -I*a^2*c^3*d^3*x - I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c^2*d^3 - 3
*a^2*c^2*d^3*log(x) - 3*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/
x)*a*b*c*d^3 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^3 - 3*I*a^
2*c*d^3/x - 1/2*a^2*d^3/x^2 - 1/32*(16*I*(24*b^2*c^5*d^3*integrate(1/16*x^5
*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c^5*d^3*integrate(1/16*x^5*log(c
^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 8*b^2*c^5*d^3*integrate(1/16*x^5*log(c^
2*x^2 + 1)/(c^2*x^5 + x^3), x) - b^2*c^2*d^3*arctan(c*x)^3 - 24*b^2*c^4*d^3
*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 16*b
^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) - 4*b^2*c^3*d
^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 24*b^2*c^3*d
^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 16*b^2*c^2*d^3
*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 56*b
^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) - 72*b^2*c*d^
3*integrate(1/16*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 6*b^2*c*d^3*integrat
e(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 4*b^2*c*d^3*integrate(1/1
6*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 8*b^2*d^3*integrate(1/16*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 + (128*b^2*c^5*d^3*integrate
(1/16*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 256*b^2*c^5*d^
3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^4*d^3*int
egrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^4*d^3*integr
ate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + b^2*c^2*d^3*log(c^2*x^2 + 1)
^3 + 24*b^2*c^2*d^3*arctan(c*x)^2 - 256*b^2*c^3*d^3*integrate(1/16*x^3*arct
an(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 2*b^2*c^2*d^3*log(c^2*x^2 +
1)^2 + 768*b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x)
 + 64*b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x)
 + 3072*a*b*c^2*d^3*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 44
8*b^2*c^2*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 384
*b^2*c*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x
) - 128*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) - 384*b^
2*d^3*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 32*b^2*d^3*integra
te(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x))*x^2 - 4*(-2*I*b^2*c^3*d^3*x
^3 - 6*I*b^2*c*d^3*x - b^2*d^3)*arctan(c*x)^2 - 4*(2*b^2*c^3*d^3*x^3 + 6*b^
2*c*d^3*x - I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) + (-2*I*b^2*c^3*d^3*x^3
 - 6*I*b^2*c*d^3*x - b^2*d^3)*log(c^2*x^2 + 1)^2)/x^2
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^3*x
^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3, x)
```

3.91 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=429

$$-\frac{b^2c^2d^3}{3x}-\frac{1}{3}b^2c^3d^3\text{ArcTan}(cx)-\frac{bcd^3(a+b\text{ArcTan}(cx))}{3x^2}-\frac{3ibc^2d^3(a+b\text{ArcTan}(cx))}{x}+\frac{11}{6}ic^3d^3(a+b\text{ArcTan}(cx$$

```
[Out] -1/3*b^2*c^2*d^3/x-1/3*b^2*c^3*d^3*arctan(c*x)-1/3*b*c*d^3*(a+b*arctan(c*x)
)/x^2+10/3*I*b^2*c^3*d^3*polylog(2,-1+2/(1-I*c*x))-3/2*I*c*d^3*(a+b*arctan(
c*x))^2/x^2-1/3*d^3*(a+b*arctan(c*x))^2/x^3-3*I*b*c^2*d^3*(a+b*arctan(c*x))
/x+3*c^2*d^3*(a+b*arctan(c*x))^2/x+11/6*I*c^3*d^3*(a+b*arctan(c*x))^2+3*I*b
^2*c^3*d^3*ln(x)+2*I*c^3*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-20
/3*b*c^3*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+1/2*I*b^2*c^3*d^3*polylog(
3,1-2/(1+I*c*x))-b*c^3*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+b*c^3
*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*I*b^2*c^3*d^3*polylog(
3,-1+2/(1+I*c*x))-3/2*I*b^2*c^3*d^3*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.64, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {4996, 4946, 5038, 331, 209, 5044, 4988, 2497, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745}

$$-bc^3d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+bc^3d^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+\frac{11}{6}ic^3d^3(a+b\text{ArcTan}(cx))^2-\frac{20}{3}bc^3d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-2ic^3d^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+\frac{3c^2d^3(a+b\text{ArcTan}(cx))^2}{x}-\frac{3ibc^2d^3(a+b\text{ArcTan}(cx))}{x}-\frac{d^3(a+b\text{ArcTan}(cx))^2}{3x^3}-\frac{3icd^3(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{bcd^3(a+b\text{ArcTan}(cx))}{3x^2}-\frac{1}{3}b^2c^3d^3\text{ArcTan}(cx)+\frac{10}{3}ib^2c^3d^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)+\frac{1}{2}ib^2c^3d^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)-\frac{1}{2}ib^2c^3d^3\text{Li}_3\left(\frac{2}{icx+1}-1\right)+3ib^2c^3d^3\log(x)-\frac{b^2c^2d^3}{3x}-\frac{3}{2}ib^2c^3d^3\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(b^2*c^2*d^3)/x - (b^2*c^3*d^3*ArcTan[c*x])/3 - (b*c*d^3*(a + b*ArcTan
[c*x]))/(3*x^2) - ((3*I)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x + ((11*I)/6)*c^3*
d^3*(a + b*ArcTan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (((3*I)/2
)*c*d^3*(a + b*ArcTan[c*x])^2)/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/x -
(2*I)*c^3*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (3*I)*b^2*
c^3*d^3*Log[x] - ((3*I)/2)*b^2*c^3*d^3*Log[1 + c^2*x^2] - (20*b*c^3*d^3*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + ((10*I)/3)*b^2*c^3*d^3*PolyLog
[2, -1 + 2/(1 - I*c*x)] - b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
 + I*c*x)] + b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] +
 (I/2)*b^2*c^3*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] - (I/2)*b^2*c^3*d^3*PolyLo
g[3, -1 + 2/(1 + I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{x^4}+\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{x^3}-\frac{3c^2d^3(a+b\,\mathrm{t}}{x^2}\right.$$

$$= d^3\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx+\left(3icd^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx-\left(3c\right.$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{3c^2d^3(a+b\tan}{x}$$

$$= 3ic^3d^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{3icd^3(a+b\tan^{-}}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{3x^2}-\frac{3ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{11}{6}ic^3d^3(a+b$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{3x^2}-\frac{3ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{11}{6}ic$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{1}{3}b^2c^3d^3\tan^{-1}(cx)-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{3x^2}-\frac{3ibc^2d^3(a-}{}$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{1}{3}b^2c^3d^3\tan^{-1}(cx)-\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{3x^2}-\frac{3ibc^2d^3(a-}{}$$

Mathematica [A]
time = 0.43, size = 595, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-8*a^2 - (36*I)*a^2*c*x - 8*a*b*c*x + 72*a^2*c^2*x^2 - (72*I)*a*b*c^2
*x^2 - 8*b^2*c^2*x^2 - b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTan[c*x] - (72*I)*a*b*c
*x*ArcTan[c*x] - 8*b^2*c*x*ArcTan[c*x] + 144*a*b*c^2*x^2*ArcTan[c*x] - (72*
I)*b^2*c^2*x^2*ArcTan[c*x] - (72*I)*a*b*c^3*x^3*ArcTan[c*x] - 8*b^2*c^3*x^3
*ArcTan[c*x] - 8*b^2*ArcTan[c*x]^2 - (36*I)*b^2*c*x*ArcTan[c*x]^2 + 72*b^2*
c^2*x^2*ArcTan[c*x]^2 + (44*I)*b^2*c^3*x^3*ArcTan[c*x]^2 + 16*b^2*c^3*x^3*A
rcTan[c*x]^3 - (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*
x])] - 160*b^2*c^3*x^3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (24*I)*
b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c^3*x
^3*Log[x] - 160*a*b*c^3*x^3*Log[c*x] + (72*I)*b^2*c^3*x^3*Log[(c*x)/Sqrt[1
+ c^2*x^2]] + 80*a*b*c^3*x^3*Log[1 + c^2*x^2] + 24*b^2*c^3*x^3*ArcTan[c*x]*
PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*b^2*c^3*x^3*ArcTan[c*x]*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] + (80*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x
])] + 24*a*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 24*a*b*c^3*x^3*PolyLog[2, I*c*x
] - (12*I)*b^2*c^3*x^3*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (12*I)*b^2*c^3*
x^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(24*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.27, size = 1726, normalized size = 4.02

method	result	size
derivativedivides	Expression too large to display	1726
default	Expression too large to display	1726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(1/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^2-I*a^2*d^3*ln(c*x)+3*I*d^3*b^2*ln((1+I*c*x)/(c^2*x^2+1)^
(1/2)-1)+3*I*d^3*b^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^3*b^2*polylog(
3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*I*d^3*b^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)
^(1/2))-2*I*d^3*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*b^2*p
olylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+11/6*I*d^3*b^2*arctan(c*x)^2-20/3*I*d^3*
b^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+10/3*a*b*ln(c^2*x^2+1)*d^3+8/3*d^3*b
^2*arctan(c*x)-2*I*d^3*a*b*arctan(c*x)*ln(c*x)-2/3*d^3*a*b*arctan(c*x)/c^3/
x^3-3*I*d^3*a*b/c/x-3/2*I*d^3*b^2*arctan(c*x)^2/c^2/x^2-3*I*d^3*b^2*arctan(
c*x)/c/x-1/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*d^3*b^
2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*d^3*b^2*Pi*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*d^3*b^2*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+6*d^3*a*b*arctan(
```

```
c*x)/c/x+3*a^2*d^3/c/x+d^3*a*b*dilog(1+I*c*x)-d^3*a*b*dilog(1-I*c*x)-20/3*d
^3*a*b*ln(c*x)-20/3*d^3*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2
*d^3*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*d^3*b^2*arcta
n(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+d^3*b^2*arctan(c*x)*polylog(
2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*d^3*b^2*Pi*arctan(c*x)^2-3*I*d^3*a*b*arctan
(c*x)/c^2/x^2-1/3*a^2*d^3/c^3/x^3-1/3*d^3*b^2*arctan(c*x)/c^2/x^2-1/3*d^3*b
^2*arctan(c*x)^2/c^3/x^3-3*I*d^3*a*b*arctan(c*x)+I*d^3*b^2*arctan(c*x)^2*ln
((1+I*c*x)^2/(c^2*x^2+1)-1)+1/3*I*d^3*b^2/(1+I*c*x+(c^2*x^2+1)^(1/2))*(c^2*
x^2+1)^(1/2)-I*d^3*b^2*arctan(c*x)^2*ln(c*x)-1/3*I*d^3*b^2/(I*c*x-(c^2*x^2+
1)^(1/2)+1)*(c^2*x^2+1)^(1/2)-I*d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x
^2+1)^(1/2))-I*d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*
d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2+1/2*d^3*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*d^3*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3*d^3*b^2*arctan(c*x)^2
/c/x+d^3*a*b*ln(c*x)*ln(1+I*c*x)-d^3*a*b*ln(c*x)*ln(1-I*c*x)-3/2*I*a^2*d^3/
c^2/x^2-1/3*d^3*a*b/c^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -I*a^2*c^3*d^3*log(x) + 3*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/
x)*a*b*c^2*d^3 - 3*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^3
+ 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)
*a*b*d^3 + 3*a^2*c^2*d^3/x - 3/2*I*a^2*c*d^3/x^2 - 1/3*a^2*d^3/x^3 - 1/96*(
24*(3*b^2*c^3*d^3*arctan(c*x)^3 + 48*b^2*c^5*d^3*integrate(1/48*x^5*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 36*b^2*c^4*d^3*integrate(1/48*x
^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) - 144*b^2*c^4*d^3*integrate(1/48*
x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 96*b^2*c^3*d^3*integrate(1/48*x^
3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 432*b^2*c^3*d^3*integr
ate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x^4), x) + 288*b^2*c^2*d^3*integrate(1/
48*x^2*arctan(c*x)^2/(c^2*x^6 + x^4), x) + 24*b^2*c^2*d^3*integrate(1/48*x^
2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 88*b^2*c^2*d^3*integrate(1/48*x^
2*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 144*b^2*c*d^3*integrate(1/48*x*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 32*b^2*c*d^3*integrate(1/48
*x*arctan(c*x)/(c^2*x^6 + x^4), x) - 144*b^2*d^3*integrate(1/48*arctan(c*x)
^2/(c^2*x^6 + x^4), x) - 12*b^2*d^3*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*
x^6 + x^4), x))*x^3 + I*(3456*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)^2/
(c^2*x^6 + x^4), x) + 9216*a*b*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/(c^2*
x^6 + x^4), x) + b^2*c^3*d^3*log(c^2*x^2 + 1)^3 + 72*b^2*c^3*d^3*arctan(c*x
```

```
)^2 - 3456*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^6 + x^4), x) - 6912*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)^2/(c^2*x^
6 + x^4), x) - 576*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)^2/(c^2*x
^6 + x^4), x) + 9216*a*b*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 +
x^4), x) + 5184*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x^6 +
x^4), x) - 2304*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)*log(c^2*x^2 + 1)
/(c^2*x^6 + x^4), x) - 4224*b^2*c^2*d^3*integrate(1/48*x^2*arctan(c*x)/(c^2
*x^6 + x^4), x) - 10368*b^2*c*d^3*integrate(1/48*x*arctan(c*x)^2/(c^2*x^6 +
 x^4), x) - 864*b^2*c*d^3*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^
4), x) - 384*b^2*c*d^3*integrate(1/48*x*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x
) + 1152*b^2*d^3*integrate(1/48*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4
), x))*x^3 - 4*(18*b^2*c^2*d^3*x^2 - 9*I*b^2*c*d^3*x - 2*b^2*d^3)*arctan(c*
x)^2 - 4*(18*I*b^2*c^2*d^3*x^2 + 9*b^2*c*d^3*x - 2*I*b^2*d^3)*arctan(c*x)*l
og(c^2*x^2 + 1) + (18*b^2*c^2*d^3*x^2 - 9*I*b^2*c*d^3*x - 2*b^2*d^3)*log(c^
2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
+ 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x - b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^3*x
^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4, x)
```

3.92 $\int \frac{(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2}{x^5}\,dx$

Optimal. Leaf size=293

$$-\frac{b^2c^2d^3}{12x^2}-\frac{ib^2c^3d^3}{x}-ib^2c^4d^3\text{ArcTan}(cx)-\frac{bcd^3(a+b\text{ArcTan}(cx))}{6x^3}-\frac{ibc^2d^3(a+b\text{ArcTan}(cx))}{x^2}+\frac{7bc^3d^3(a+b\text{Arc}}{2x}$$

```
[Out] -1/12*b^2*c^2*d^3/x^2-I*b^2*c^3*d^3/x-I*b^2*c^4*d^3*arctan(c*x)-1/6*b*c*d^3
*(a+b*arctan(c*x))/x^3-I*b*c^2*d^3*(a+b*arctan(c*x))/x^2+7/2*b*c^3*d^3*(a+b
*arctan(c*x))/x-1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^4-4*I*a*b*c^4*d^3
*ln(x)-11/3*b^2*c^4*d^3*ln(x)-4*I*b*c^4*d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x
))+11/6*b^2*c^4*d^3*ln(c^2*x^2+1)+2*b^2*c^4*d^3*polylog(2,-I*c*x)-2*b^2*c^4
*d^3*polylog(2,I*c*x)-2*b^2*c^4*d^3*polylog(2,1-2/(1-I*c*x))
```

Rubi [A]
time = 0.25, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {37, 4994, 4946, 272, 46, 331, 209, 36, 29, 31, 4940, 2438, 4964, 2449, 2352}

$$-4ibc^4d^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+\frac{7bc^3d^3(a+b\text{ArcTan}(cx))}{2x}-\frac{ibc^2d^3(a+b\text{ArcTan}(cx))}{x^2}-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))^2}{4x^4}-\frac{bcd^3(a+b\text{ArcTan}(cx))}{6x^3}-4iabc^4d^3\log(x)-ib^2c^4d^3\text{ArcTan}(cx)+2b^2c^4d^3\text{Li}_2(-icx)-2b^2c^4d^3\text{Li}_2(icx)-2b^2c^4d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)-\frac{11}{3}b^2c^4d^3\log(x)-\frac{ib^2c^3d^3}{x}-\frac{b^2c^2d^3}{12x^2}+\frac{11}{6}b^2c^4d^3\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

```
[Out] -1/12*(b^2*c^2*d^3)/x^2 - (I*b^2*c^3*d^3)/x - I*b^2*c^4*d^3*ArcTan[c*x] - (
b*c*d^3*(a + b*ArcTan[c*x]))/(6*x^3) - (I*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^
2 + (7*b*c^3*d^3*(a + b*ArcTan[c*x]))/(2*x) - (d^3*(1 + I*c*x)^4*(a + b*Arc
Tan[c*x])^2)/(4*x^4) - (4*I)*a*b*c^4*d^3*Log[x] - (11*b^2*c^4*d^3*Log[x])/3
 - (4*I)*b*c^4*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] + (11*b^2*c^4*d^3
*Log[1 + c^2*x^2])/6 + 2*b^2*c^4*d^3*PolyLog[2, (-I)*c*x] - 2*b^2*c^4*d^3*P
olyLog[2, I*c*x] - 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^3\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} - (2bc)\int\left(-\frac{d^3(a+b\tan^{-1}(cx))}{4x^4}\right.$$

$$= -\frac{d^3(1+icx)^4\left(a+b\tan^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bcd^3\right)\int\frac{a+b\tan^{-1}(cx)}{x^4}\,dx +$$

$$= -\frac{bcd^3(a+b\tan^{-1}(cx))}{6x^3} - \frac{ibc^2d^3(a+b\tan^{-1}(cx))}{x^2} + \frac{7bc^3d^3(a+b\,\text{ta}}{2x}$$

$$= -\frac{ib^2c^3d^3}{x} - \frac{bcd^3(a+b\tan^{-1}(cx))}{6x^3} - \frac{ibc^2d^3(a+b\tan^{-1}(cx))}{x^2} + \frac{7bc^3}{}$$

$$= -\frac{ib^2c^3d^3}{x} - ib^2c^4d^3\tan^{-1}(cx) - \frac{bcd^3(a+b\tan^{-1}(cx))}{6x^3} - \frac{ibc^2d^3(a+}{}$$

$$= -\frac{b^2c^2d^3}{12x^2} - \frac{ib^2c^3d^3}{x} - ib^2c^4d^3\tan^{-1}(cx) - \frac{bcd^3(a+b\tan^{-1}(cx))}{6x^3} - $$

Mathematica [A]
time = 0.51, size = 322, normalized size = 1.10

$$\frac{d^3\left(-3a^2-12ia^2cx-2abcx+18a^2c^2x^2-12iabc^2x^2-b^2c^2x^2+12ia^2c^3x^3+42abc^3x^3-12ib^2c^3x^3-b^2c^4x^4-3b^2(-i+cx)^4\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(bcx(-1-6icx+21c^2x^2-6ic^3x^3)+3a(-1-4icx+6c^2x^2+4ic^3x^3+7c^4x^4)-24ibc^4x^4\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)-48iabc^4x^4\log(cx)-44b^2c^4x^4\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+24iabc^4x^4\log\left(1+c^2x^2\right)-24b^2c^4x^4\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

```
[Out] (d^3*(-3*a^2 - (12*I)*a^2*c*x - 2*a*b*c*x + 18*a^2*c^2*x^2 - (12*I)*a*b*c^2
*x^2 - b^2*c^2*x^2 + (12*I)*a^2*c^3*x^3 + 42*a*b*c^3*x^3 - (12*I)*b^2*c^3*x
^3 - b^2*c^4*x^4 - 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*
x*(-1 - (6*I)*c*x + 21*c^2*x^2 - (6*I)*c^3*x^3) + 3*a*(-1 - (4*I)*c*x + 6*c
^2*x^2 + (4*I)*c^3*x^3 + 7*c^4*x^4) - (24*I)*b*c^4*x^4*Log[1 - E^((2*I)*Arc
Tan[c*x])]) - (48*I)*a*b*c^4*x^4*Log[c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1
 + c^2*x^2]] + (24*I)*a*b*c^4*x^4*Log[1 + c^2*x^2] - 24*b^2*c^4*x^4*PolyLog
[2, E^((2*I)*ArcTan[c*x])]))/(12*x^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(271) = 542.
time = 0.71, size = 701, normalized size = 2.39

method	result
derivativedivides	$c^4\left(-\frac{d^3ab\arctan(cx)}{2c^4x^4}+\frac{2id^3ab\arctan(cx)}{cx}-\frac{2id^3ab\arctan(cx)}{c^3x^3}+2id^3b^2\arctan(cx)\ln\left(c^2x^2+1\right)+\right.$
default	$c^4\left(-\frac{d^3ab\arctan(cx)}{2c^4x^4}+\frac{2id^3ab\arctan(cx)}{cx}-\frac{2id^3ab\arctan(cx)}{c^3x^3}+2id^3b^2\arctan(cx)\ln\left(c^2x^2+1\right)+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(2*I*d^3*a*b*ln(c^2*x^2+1)-1/2*d^3*a*b*arctan(c*x)/c^4/x^4-2*I*d^3*a*b*
arctan(c*x)/c^3/x^3+2*I*d^3*a*b*arctan(c*x)/c/x-I*d^3*a*b/c^2/x^2+I*d^3*b^2
*arctan(c*x)^2/c/x-I*d^3*b^2*arctan(c*x)/c^2/x^2-I*d^3*b^2*arctan(c*x)^2/c^
3/x^3-4*I*d^3*a*b*ln(c*x)+2*I*d^3*b^2*arctan(c*x)*ln(c^2*x^2+1)-4*I*d^3*b^2
*arctan(c*x)*ln(c*x)-1/6*d^3*a*b/c^3/x^3-I*d^3*b^2/c/x-1/6*d^3*b^2*arctan(c
*x)/c^3/x^3-1/4*d^3*b^2*arctan(c*x)^2/c^4/x^4-1/12*d^3*b^2/c^2/x^2-2*d^3*b^
2*ln(c*x)*ln(1-I*c*x)+2*d^3*b^2*ln(c*x)*ln(1+I*c*x)+d^3*b^2*ln(c*x-I)*ln(-1
/2*I*(c*x+I))+7/2*d^3*a*b*arctan(c*x)+d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)-d^3*b
^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+a^2*d^3*(3/2
/c^2/x^2-I/c^3/x^3+I/c/x-1/4/c^4/x^4)-11/3*d^3*b^2*ln(c*x)+2*d^3*b^2*dilog(
1+I*c*x)-2*d^3*b^2*dilog(1-I*c*x)-d^3*b^2*dilog(1/2*I*(c*x-I))+d^3*b^2*dilo
g(-1/2*I*(c*x+I))+7/4*d^3*b^2*arctan(c*x)^2-1/2*d^3*b^2*ln(c*x+I)^2+1/2*d^3
*b^2*ln(c*x-I)^2+3*d^3*a*b*arctan(c*x)/c^2/x^2+11/6*b^2*d^3*ln(c^2*x^2+1)+7
/2*d^3*a*b/c/x+3/2*d^3*b^2*arctan(c*x)^2/c^2/x^2+7/2*d^3*b^2*arctan(c*x)/c/
x-I*d^3*b^2*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^3*d^3 + 3*((c*a
rctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^2*d^3 + I*((c^2*log(c^2*x^2 +
1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c*d^3 + I*a^2*c^3*d^3
/x + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*
a*b*d^3 + 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) -
 (3*c^2*x^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) +
 1)*c^2/x^2)*b^2*d^3 + 3/2*a^2*c^2*d^3/x^2 - I*a^2*c*d^3/x^3 - 1/4*b^2*d^3*
arctan(c*x)^2/x^4 - 1/4*a^2*d^3/x^4 - 1/32*(8*I*(b^2*c^4*d^3*arctan(c*x)^3
+ 4*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) -
 16*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 4
8*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) + 80*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^6 + x^4), x)
- 96*b^2*c^3*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 8*b
^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 40*b
^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 48*b^2
*c^2*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x)
```

```
- 32*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^6 + x^4), x) - 144*b^2
*c*d^3*integrate(1/16*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*c*d^3*inte
grate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 - 8*(b^2*c^4*d^3*arc
tan(c*x)^2 - 16*b^2*c^5*d^3*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)
/(c^2*x^6 + x^4), x) - 144*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)^2/(c^
2*x^6 + x^4), x) - 12*b^2*c^4*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^
2*x^6 + x^4), x) + 40*b^2*c^4*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*
x^6 + x^4), x) + 32*b^2*c^3*d^3*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2
+ 1)/(c^2*x^6 + x^4), x) - 80*b^2*c^3*d^3*integrate(1/16*x^2*arctan(c*x)/(c
^2*x^6 + x^4), x) - 144*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x^6
 + x^4), x) - 12*b^2*c^2*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^6 +
 x^4), x) - 16*b^2*c^2*d^3*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^6 + x^4
), x) + 48*b^2*c*d^3*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 +
 x^4), x))*x^3 - 4*(2*I*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x - 2*I*b^2*c*d^3)*
arctan(c*x)^2 + 4*(2*b^2*c^3*d^3*x^2 - 3*I*b^2*c^2*d^3*x - 2*b^2*c*d^3)*arc
tan(c*x)*log(c^2*x^2 + 1) + (2*I*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x - 2*I*b^
2*c*d^3)*log(c^2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^4*integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a^2*c^4*d^3*x^4 + 8*I*
a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x + 4*a^2*d^3 + (4*a*b
*c^5*d^3*x^5 - 4*(3*I*a*b - b^2)*c^4*d^3*x^4 - 2*(4*a*b + 3*I*b^2)*c^3*d^3*
x^3 - 4*(2*I*a*b + b^2)*c^2*d^3*x^2 - (12*a*b - I*b^2)*c*d^3*x + 4*I*a*b*d^
3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^7 + x^5), x) + (-4*I*b^2*c^3*d^3*x^3 -
 6*b^2*c^2*d^3*x^2 + 4*I*b^2*c*d^3*x + b^2*d^3)*log(-(c*x + I)/(c*x - I))^2
)/x^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5, x)
```

3.93 $\int \frac{(d+icdx)^3(a+b\mathbf{ArcTan}(cx))^2}{x^6}\,dx$

Optimal. Leaf size=384

$$-\frac{b^2c^2d^3}{30x^3}-\frac{ib^2c^3d^3}{4x^2}+\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\text{ArcTan}(cx)-\frac{bcd^3(a+b\text{ArcTan}(cx))}{10x^4}-\frac{ibc^2d^3(a+b\text{ArcTan}(cx))}{2x^3}+$$

```
[Out] -1/30*b^2*c^2*d^3/x^3-1/4*I*b^2*c^3*d^3/x^2+13/10*b^2*c^4*d^3/x+13/10*b^2*c
^5*d^3*arctan(c*x)-1/10*b*c*d^3*(a+b*arctan(c*x))/x^4-6/5*I*b^2*c^5*d^3*pol
ylog(2,I*c*x)+6/5*b*c^3*d^3*(a+b*arctan(c*x))/x^2-3*I*b^2*c^5*d^3*ln(x)-1/5
*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^5+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*arct
an(c*x))^2/x^4+12/5*a*b*c^5*d^3*ln(x)+5/2*I*b*c^4*d^3*(a+b*arctan(c*x))/x+1
2/5*b*c^5*d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))-1/2*I*b*c^2*d^3*(a+b*arctan
(c*x))/x^3-6/5*I*b^2*c^5*d^3*polylog(2,1-2/(1-I*c*x))+3/2*I*b^2*c^5*d^3*ln(
c^2*x^2+1)+6/5*I*b^2*c^5*d^3*polylog(2,-I*c*x)
```

Rubi [A]
time = 0.28, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {47, 37, 4994, 4946, 331, 209, 272, 46, 36, 29, 31, 4940, 2438, 4964, 2449, 2352}

$$\frac{12}{5}bc^5d^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+\frac{5ibc^4d^3(a+b\text{ArcTan}(cx))}{2x}+\frac{6bc^3d^3(a+b\text{ArcTan}(cx))}{5x^2}-\frac{ibc^2d^3(a+b\text{ArcTan}(cx))}{2x^3}-\frac{d^3(1+icx)^4(a+b\text{ArcTan}(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4(a+b\text{ArcTan}(cx))^2}{20x^4}-\frac{bcd^3(a+b\text{ArcTan}(cx))}{10x^4}+\frac{12}{5}abc^5d^3\log(x)+\frac{13}{10}b^2c^5d^3\text{ArcTan}(cx)+\frac{6}{5}ib^2c^5d^3\text{Li}_2(-icx)-\frac{6}{5}ib^2c^5d^3\text{Li}_2(icx)-\frac{6}{5}ib^2c^5d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)-3ib^2c^5d^3\log(x)+\frac{13b^2c^4d^3}{10x}-\frac{ib^2c^3d^3}{4x^2}-\frac{b^2c^2d^3}{30x^3}+\frac{3}{2}ib^2c^5d^3\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

```
[Out] -1/30*(b^2*c^2*d^3)/x^3 - ((I/4)*b^2*c^3*d^3)/x^2 + (13*b^2*c^4*d^3)/(10*x)
 + (13*b^2*c^5*d^3*ArcTan[c*x])/10 - (b*c*d^3*(a + b*ArcTan[c*x]))/(10*x^4)
 - ((I/2)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x^3 + (6*b*c^3*d^3*(a + b*ArcTan[c
*x]))/(5*x^2) + (((5*I)/2)*b*c^4*d^3*(a + b*ArcTan[c*x]))/x - (d^3*(1 + I*c
*x)^4*(a + b*ArcTan[c*x])^2)/(5*x^5) + ((I/20)*c*d^3*(1 + I*c*x)^4*(a + b*A
rcTan[c*x])^2)/x^4 + (12*a*b*c^5*d^3*Log[x])/5 - (3*I)*b^2*c^5*d^3*Log[x] +
 (12*b*c^5*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/5 + ((3*I)/2)*b^2*c^
5*d^3*Log[1 + c^2*x^2] + ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, (-I)*c*x] - ((6*I
)/5)*b^2*c^5*d^3*PolyLog[2, I*c*x] - ((6*I)/5)*b^2*c^5*d^3*PolyLog[2, 1 - 2
/(1 - I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
-1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+icdx)^3\,(a+b\tan^{-1}(cx))^2}{x^6}\,dx &= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4\,(a+b\tan^{-1}(cx))^2}{20x^4} \\
&= -\frac{d^3(1+icx)^4\,(a+b\tan^{-1}(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4\,(a+b\tan^{-1}(cx))^2}{20x^4} \\
&= -\frac{bcd^3(a+b\tan^{-1}(cx))}{10x^4}-\frac{ibc^2d^3(a+b\tan^{-1}(cx))}{2x^3}+\frac{6bc^3d^3(a+b\tan}{5x^2} \\
&= -\frac{b^2c^2d^3}{30x^3}+\frac{6b^2c^4d^3}{5x}-\frac{bcd^3(a+b\tan^{-1}(cx))}{10x^4}-\frac{ibc^2d^3(a+b\tan^{-1}(cx)}{2x^3} \\
&= -\frac{b^2c^2d^3}{30x^3}+\frac{13b^2c^4d^3}{10x}+\frac{6}{5}b^2c^5d^3\tan^{-1}(cx)-\frac{bcd^3(a+b\tan^{-1}(cx))}{10x^4}- \\
&= -\frac{b^2c^2d^3}{30x^3}-\frac{ib^2c^3d^3}{4x^2}+\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\tan^{-1}(cx)-\frac{bcd^3(a+b\tan}{10x^4}
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 363, normalized size = 0.95

$$\frac{d^3\left(-12a^2-45ia^2cx-6abcx+60a^2c^2x^2-30iabc^2x^2-2b^2c^2x^2+30ia^2c^3x^3+72abc^3x^3-15ib^2c^3x^3+150iabc^4x^4+78b^2c^4x^4-15ib^2c^5x^5+3ib^2(-i+cx)^4(4i+cx)\text{ArcTan}(cx)^2+6b\text{ArcTan}(cx)\left(bcx(-1-5icx+12c^2x^2+25ic^3x^3+13c^4x^4)+a(-4-15icx+20c^2x^2+10ic^3x^3+25ic^5x^5)+24bc^5x^5\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+144abc^5x^5\log(cx)-180ib^2c^5x^5\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-72abc^5x^5\log\left(1+c^2x^2\right)-72ib^2c^5x^5\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

```
[Out] (d^3*(-12*a^2 - (45*I)*a^2*c*x - 6*a*b*c*x + 60*a^2*c^2*x^2 - (30*I)*a*b*c^
2*x^2 - 2*b^2*c^2*x^2 + (30*I)*a^2*c^3*x^3 + 72*a*b*c^3*x^3 - (15*I)*b^2*c^
3*x^3 + (150*I)*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - (15*I)*b^2*c^5*x^5 + (3*I)*b
^2*(-I + c*x)^4*(4*I + c*x)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*c*x*(-1 - (5
*I)*c*x + 12*c^2*x^2 + (25*I)*c^3*x^3 + 13*c^4*x^4) + a*(-4 - (15*I)*c*x +
20*c^2*x^2 + (10*I)*c^3*x^3 + (25*I)*c^5*x^5) + 24*b*c^5*x^5*Log[1 - E^((2*
I)*ArcTan[c*x])]) + 144*a*b*c^5*x^5*Log[c*x] - (180*I)*b^2*c^5*x^5*Log[(c*x
)/Sqrt[1 + c^2*x^2]] - 72*a*b*c^5*x^5*Log[1 + c^2*x^2] - (72*I)*b^2*c^5*x^5
*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(60*x^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 759 vs. 2(337) = 674.
time = 1.04, size = 760, normalized size = 1.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/2*I*d^3*a*b/c^3/x^3+5/2*I*d^3*a*b/c/x-2/5*d^3*a*b*arctan(c*x)/c^5/x
^5+1/2*I*d^3*b^2*arctan(c*x)^2/c^2/x^2-3/4*I*d^3*b^2*arctan(c*x)^2/c^4/x^4-
1/2*I*d^3*b^2*arctan(c*x)/c^3/x^3+5/2*I*d^3*b^2*arctan(c*x)/c/x+3/10*I*d^3*
b^2*ln(c*x-I)^2+13/10*d^3*b^2/c/x-1/30*d^3*b^2/c^3/x^3+3/5*I*d^3*b^2*dilog(
-1/2*I*(c*x+I))+6/5*I*d^3*b^2*dilog(1+I*c*x)-6/5*I*d^3*b^2*dilog(1-I*c*x)+5
/4*I*d^3*b^2*arctan(c*x)^2-3*I*d^3*b^2*ln(c*x)+12/5*d^3*b^2*ln(c*x)*arctan(
c*x)+3/2*I*d^3*b^2*ln(c^2*x^2+1)-3/10*I*d^3*b^2*ln(c*x+I)^2-3/5*I*d^3*b^2*d
ilog(1/2*I*(c*x-I))+a^2*d^3*(1/2*I/c^2/x^2+1/c^3/x^3-3/4*I/c^4/x^4-1/5/c^5/
x^5)-6/5*a*b*ln(c^2*x^2+1)*d^3-6/5*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^3+13/10*
d^3*b^2*arctan(c*x)-3/2*I*d^3*a*b*arctan(c*x)/c^4/x^4+I*d^3*a*b*arctan(c*x)
/c^2/x^2+2*d^3*a*b*arctan(c*x)/c^3/x^3+6/5*I*d^3*b^2*ln(c*x)*ln(1+I*c*x)-3/
5*I*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/5*I*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+
I))+3/5*I*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)-3/5*I*d^3*b^2*ln(c*x+I)*ln(1/2*I*
(c*x-I))-1/10*d^3*a*b/c^4/x^4-1/4*I*d^3*b^2/c^2/x^2-1/10*d^3*b^2*arctan(c*x
)/c^4/x^4-1/5*d^3*b^2*arctan(c*x)^2/c^5/x^5+5/2*I*d^3*a*b*arctan(c*x)-6/5*I
*d^3*b^2*ln(c*x)*ln(1-I*c*x)+12/5*d^3*a*b*ln(c*x)+6/5*d^3*b^2*arctan(c*x)/c
^2/x^2+d^3*b^2*arctan(c*x)^2/c^3/x^3+6/5*d^3*a*b/c^2/x^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="maxima")
```

```
[Out] I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^3*d^3 - ((c^2*log(c^2*x
^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c^2*d^3 + 1/2*I*
((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b*c*d^3
 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c
+ 4*arctan(c*x)/x^5)*a*b*d^3 + 1/2*I*a^2*c^3*d^3/x^2 + a^2*c^2*d^3/x^3 - 3/
4*I*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/320*(320*I*x^5*integrate(1/80*(60*(
b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 + 5*(b^2
*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^2 + 2*(3
0*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2)*arctan(c*x) - (10*b^2*c^5*d^3*x^5 -
 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x + 20*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3
*x^2 - b^2*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) + 320*x^
5*integrate(1/80*(60*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*arct
an(c*x)^2 + 5*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3)*log(c^2*x^2
 + 1)^2 - 2*(10*b^2*c^5*d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x)*arcta
n(c*x) - (30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2 - 20*(b^2*c^5*d^3*x^5 - 2
*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 +
 x^6), x) - 4*(10*I*b^2*c^3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 - 15*I*b^2*c*d^3*x
 - 4*b^2*d^3)*arctan(c*x)^2 + 4*(10*b^2*c^3*d^3*x^3 - 20*I*b^2*c^2*d^3*x^2
```

```
- 15*b^2*c*d^3*x + 4*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) + (10*I*b^2*c^
3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 - 15*I*b^2*c*d^3*x - 4*b^2*d^3)*log(c^2*x^2
+ 1)^2)/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="fricas")
[Out] 1/80*(80*x^5*integral(1/20*(-20*I*a^2*c^5*d^3*x^5 - 60*a^2*c^4*d^3*x^4 + 40
*I*a^2*c^3*d^3*x^3 - 40*a^2*c^2*d^3*x^2 + 60*I*a^2*c*d^3*x + 20*a^2*d^3 + (
20*a*b*c^5*d^3*x^5 - 10*(6*I*a*b - b^2)*c^4*d^3*x^4 - 20*(2*a*b + I*b^2)*c^
3*d^3*x^3 - 5*(8*I*a*b + 3*b^2)*c^2*d^3*x^2 - 4*(15*a*b - I*b^2)*c*d^3*x +
20*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^8 + x^6), x) + (-10*I*b^2*c
^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 + 15*I*b^2*c*d^3*x + 4*b^2*d^3)*log(-(c*x +
 I)/(c*x - I))^2)/x^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**6,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6, x)
```

3.94 $\int \frac{(d+icdx)^3(a+b\text{ArcTan}(cx))^2}{x^7}\,dx$

Optimal. Leaf size=513

$$-\frac{b^2c^2d^3}{60x^4}-\frac{ib^2c^3d^3}{10x^3}+\frac{61b^2c^4d^3}{180x^2}+\frac{37ib^2c^5d^3}{30x}+\frac{37}{30}ib^2c^6d^3\text{ArcTan}(cx)-\frac{bcd^3(a+b\text{ArcTan}(cx))}{15x^5}-\frac{3ibc^2d^3(a+b\text{Arc}}{10x^4}$$

```
[Out] -1/60*b^2*c^2*d^3/x^4+28/15*I*a*b*c^6*d^3*ln(x)+61/180*b^2*c^4*d^3/x^2-1/10
*I*b^2*c^3*d^3/x^3-3/5*I*c*d^3*(a+b*arctan(c*x))^2/x^5-1/15*b*c*d^3*(a+b*ar
ctan(c*x))/x^5+1/3*I*c^3*d^3*(a+b*arctan(c*x))^2/x^3+11/18*b*c^3*d^3*(a+b*a
rctan(c*x))/x^3+37/30*I*b^2*c^5*d^3/x-11/6*b*c^5*d^3*(a+b*arctan(c*x))/x-1/
6*d^3*(a+b*arctan(c*x))^2/x^6+37/20*I*b*c^6*d^3*(a+b*arctan(c*x))*ln(2/(1-I
*c*x))+3/4*c^2*d^3*(a+b*arctan(c*x))^2/x^4+37/30*I*b^2*c^6*d^3*arctan(c*x)+
1/60*I*b*c^6*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+113/45*b^2*c^6*d^3*ln(x)
-3/10*I*b*c^2*d^3*(a+b*arctan(c*x))/x^4+14/15*I*b*c^4*d^3*(a+b*arctan(c*x))
/x^2-113/90*b^2*c^6*d^3*ln(c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,-I*c*x)+1
4/15*b^2*c^6*d^3*polylog(2,I*c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2/(1-I*c*x)
)-1/120*b^2*c^6*d^3*polylog(2,1-2/(1+I*c*x))
```

Rubi [A]
time = 0.37, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {45, 4994, 4946, 272, 46, 331, 209, 36, 29, 31, 4940, 2438, 4964, 2449, 2352}

$$\frac{37}{20}ibc^6d^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+\frac{1}{60}ibc^6d^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))-\frac{11bc^5d^3(a+b\text{ArcTan}(cx))}{6x}+\frac{14ibc^4d^3(a+b\text{ArcTan}(cx))}{15x^2}+\frac{ic^3d^3(a+b\text{ArcTan}(cx))^2}{3x^3}+\frac{11bc^3d^3(a+b\text{ArcTan}(cx))}{18x^3}+\frac{3c^2d^3(a+b\text{ArcTan}(cx))^2}{4x^4}-\frac{3ibc^2d^3(a+b\text{ArcTan}(cx))}{10x^4}-\frac{d^3(a+b\text{ArcTan}(cx))^2}{6x^6}-\frac{3icd^3(a+b\text{ArcTan}(cx))^2}{5x^5}-\frac{bcd^3(a+b\text{ArcTan}(cx))}{15x^5}+\frac{28}{15}iabc^6d^3\log(x)+\frac{37}{30}ib^2c^6d^3\text{ArcTan}(cx)-\frac{14}{15}b^2c^6d^3\text{Li}_2(-icx)+\frac{14}{15}b^2c^6d^3\text{Li}_2(icx)+\frac{37}{40}b^2c^6d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)-\frac{1}{120}b^2c^6d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)+\frac{113}{45}b^2c^6d^3\log(x)+\frac{37ib^2c^5d^3}{30x}+\frac{61b^2c^4d^3}{180x^2}-\frac{ib^2c^3d^3}{10x^3}-\frac{b^2c^2d^3}{60x^4}-\frac{113}{90}b^2c^6d^3\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

```
[Out] -1/60*(b^2*c^2*d^3)/x^4 - ((I/10)*b^2*c^3*d^3)/x^3 + (61*b^2*c^4*d^3)/(180*
x^2) + (((37*I)/30)*b^2*c^5*d^3)/x + ((37*I)/30)*b^2*c^6*d^3*ArcTan[c*x] -
(b*c*d^3*(a + b*ArcTan[c*x]))/(15*x^5) - (((3*I)/10)*b*c^2*d^3*(a + b*ArcTa
n[c*x]))/x^4 + (11*b*c^3*d^3*(a + b*ArcTan[c*x]))/(18*x^3) + (((14*I)/15)*b
*c^4*d^3*(a + b*ArcTan[c*x]))/x^2 - (11*b*c^5*d^3*(a + b*ArcTan[c*x]))/(6*x
) - (d^3*(a + b*ArcTan[c*x])^2)/(6*x^6) - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*
x])^2)/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/(4*x^4) + ((I/3)*c^3*d^3*(a
+ b*ArcTan[c*x])^2)/x^3 + ((28*I)/15)*a*b*c^6*d^3*Log[x] + (113*b^2*c^6*d^3
*Log[x])/45 + ((37*I)/20)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
+ (I/60)*b*c^6*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (113*b^2*c^6*d^
3*Log[1 + c^2*x^2])/90 - (14*b^2*c^6*d^3*PolyLog[2, (-I)*c*x])/15 + (14*b^2
*c^6*d^3*PolyLog[2, I*c*x])/15 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - I*c*
x)])/40 - (b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/120
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^2}{x^7}\,dx = -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{6x^6} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{5x^5} + \frac{3c^2d^3\left(a+b\tan^{-1}\right.}{4x^4}$$

$$= -\frac{d^3\left(a+b\tan^{-1}(cx)\right)^2}{6x^6} - \frac{3icd^3\left(a+b\tan^{-1}(cx)\right)^2}{5x^5} + \frac{3c^2d^3\left(a+b\tan^{-1}\right.}{4x^4}$$

$$= -\frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5} - \frac{3ibc^2d^3\left(a+b\tan^{-1}(cx)\right)}{10x^4} + \frac{11bc^3d^3\left(a+b\,t\right.}{18x^3}$$

$$= -\frac{ib^2c^3d^3}{10x^3} + \frac{14ib^2c^5d^3}{15x} - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5} - \frac{3ibc^2d^3\left(a+b\tan^{-1}\right.}{10x^4}$$

$$= -\frac{ib^2c^3d^3}{10x^3} + \frac{37ib^2c^5d^3}{30x} + \frac{14}{15}ib^2c^6d^3\tan^{-1}(cx) - \frac{bcd^3\left(a+b\tan^{-1}(cx)\right)}{15x^5}$$

$$= -\frac{b^2c^2d^3}{60x^4} - \frac{ib^2c^3d^3}{10x^3} + \frac{61b^2c^4d^3}{180x^2} + \frac{37ib^2c^5d^3}{30x} + \frac{37}{30}ib^2c^6d^3\tan^{-1}(cx) -$$

Mathematica [A]
time = 1.01, size = 401, normalized size = 0.78

$$\frac{d^3\left(-30a^2-108ia^2cx-12abcx+135a^2c^2x^2-54iabc^2x^2-3b^2c^2x^2+60ia^2c^3x^3+110abc^3x^3-18ib^2c^3x^3+168iabc^4x^4+61b^2c^4x^4-330abc^5x^5+222ib^2c^5x^5+64b^2c^6x^6+3b^2(-i+cx)^4\left(-10+4icx+c^2x^2\right)\mathrm{ArcTan}(cx)^2+2b\mathrm{ArcTan}(cx)\left(bcx(-6-27icx+55c^2x^2+84ic^3x^3-165c^4x^4+111ic^5x^5)-3a(10+36icx-45c^2x^2-20ic^3x^3+55c^6x^6)+168ibc^6x^6\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)\right)+336iabc^6x^6\log(cx)+452b^2c^6x^6\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-168iabc^6x^6\log\left(1+c^2x^2\right)+168b^2c^6x^6\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

```
[Out] (d^3*(-30*a^2 - (108*I)*a^2*c*x - 12*a*b*c*x + 135*a^2*c^2*x^2 - (54*I)*a*b
*c^2*x^2 - 3*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 110*a*b*c^3*x^3 - (18*I)*b^
2*c^3*x^3 + (168*I)*a*b*c^4*x^4 + 61*b^2*c^4*x^4 - 330*a*b*c^5*x^5 + (222*I
)*b^2*c^5*x^5 + 64*b^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(-10 + (4*I)*c*x + c^2*
x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*x*(-6 - (27*I)*c*x + 55*c^2*x^2 +
 (84*I)*c^3*x^3 - 165*c^4*x^4 + (111*I)*c^5*x^5) - 3*a*(10 + (36*I)*c*x - 4
5*c^2*x^2 - (20*I)*c^3*x^3 + 55*c^6*x^6) + (168*I)*b*c^6*x^6*Log[1 - E^((2*
I)*ArcTan[c*x])]) + (336*I)*a*b*c^6*x^6*Log[c*x] + 452*b^2*c^6*x^6*Log[(c*x
)/Sqrt[1 + c^2*x^2]] - (168*I)*a*b*c^6*x^6*Log[1 + c^2*x^2] + 168*b^2*c^6*x
^6*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(180*x^6)
```

Maple [A]
time = 0.87, size = 797, normalized size = 1.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(3/2*d^3*a*b*arctan(c*x)/c^4/x^4+1/3*I*d^3*b^2*arctan(c*x)^2/c^3/x^3-3/
5*I*d^3*b^2*arctan(c*x)^2/c^5/x^5-3/10*I*d^3*b^2*arctan(c*x)/c^4/x^4+14/15*
```

```
I*d^3*b^2*arctan(c*x)/c^2/x^2-3/10*I*d^3*a*b/c^4/x^4+14/15*I*d^3*a*b/c^2/x^
2-1/3*d^3*a*b*arctan(c*x)/c^6/x^6+11/18*d^3*a*b/c^3/x^3+11/18*d^3*b^2*arcta
n(c*x)/c^3/x^3+3/4*d^3*b^2*arctan(c*x)^2/c^4/x^4+37/30*I*d^3*b^2*arctan(c*x
)-1/60*d^3*b^2/c^4/x^4+61/180*d^3*b^2/c^2/x^2+14/15*d^3*b^2*ln(c*x)*ln(1-I*
c*x)-14/15*d^3*b^2*ln(c*x)*ln(1+I*c*x)+a^2*d^3*(-1/6/c^6/x^6+1/3*I/c^3/x^3+
3/4/c^4/x^4-3/5*I/c^5/x^5)-7/15*d^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-11/6*d
^3*a*b*arctan(c*x)-7/15*d^3*b^2*ln(c*x+I)*ln(c^2*x^2+1)+7/15*d^3*b^2*ln(c*x
+I)*ln(1/2*I*(c*x-I))+7/15*d^3*b^2*ln(c*x-I)*ln(c^2*x^2+1)+113/45*d^3*b^2*l
n(c*x)-14/15*d^3*b^2*dilog(1+I*c*x)+14/15*d^3*b^2*dilog(1-I*c*x)+7/15*d^3*b
^2*dilog(1/2*I*(c*x-I))-7/15*d^3*b^2*dilog(-1/2*I*(c*x+I))-11/12*d^3*b^2*ar
ctan(c*x)^2+7/30*d^3*b^2*ln(c*x+I)^2-7/30*d^3*b^2*ln(c*x-I)^2+2/3*I*d^3*a*b
*arctan(c*x)/c^3/x^3-6/5*I*d^3*a*b*arctan(c*x)/c^5/x^5-14/15*I*d^3*b^2*arct
an(c*x)*ln(c^2*x^2+1)-1/15*d^3*a*b/c^5/x^5-1/10*I*d^3*b^2/c^3/x^3+37/30*I*d
^3*b^2/c/x-1/15*d^3*b^2*arctan(c*x)/c^5/x^5-1/6*d^3*b^2*arctan(c*x)^2/c^6/x
^6+28/15*I*d^3*b^2*arctan(c*x)*ln(c*x)-14/15*I*d^3*a*b*ln(c^2*x^2+1)+28/15*
I*d^3*a*b*ln(c*x)-113/90*b^2*d^3*ln(c^2*x^2+1)-11/6*d^3*a*b/c/x-11/6*d^3*b^
2*arctan(c*x)/c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3
)*a*b*c^3*d^3 - 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*a*b*c^2*d^3 - 3/10*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) -
 (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*a*b*c*d^3 - 1/45*((15*c^5*arct
an(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*a*b*d^3
 - 1/180*(4*(15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c*arcta
n(c*x) - (30*c^4*x^4*arctan(c*x)^2 - 46*c^4*x^4*log(c^2*x^2 + 1) + 92*c^4*x
^4*log(x) + 16*c^2*x^2 - 3)*c^2/x^4)*b^2*d^3 + 1/3*I*a^2*c^3*d^3/x^3 + 3/4*
a^2*c^2*d^3/x^4 - 3/5*I*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctan(c*x)^2/x^6 - 1/6
*a^2*d^3/x^6 - 1/960*(960*I*x^5*integrate(1/240*(180*(b^2*c^5*d^3*x^4 - 2*b
^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^4 - 2*b^2*c
^3*d^3*x^2 - 3*b^2*c*d^3)*log(c^2*x^2 + 1)^2 + 2*(65*b^2*c^4*d^3*x^3 - 36*b
^2*c^2*d^3*x)*arctan(c*x) - (20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2 + 180*
(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 +
 x^6), x) + 960*x^5*integrate(1/240*(540*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*
arctan(c*x)^2 + 45*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*log(c^2*x^2 + 1)^2 - 2
*(20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2)*arctan(c*x) - (65*b^2*c^4*d^3*x^
3 - 36*b^2*c^2*d^3*x - 60*(b^2*c^5*d^3*x^4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^
3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) - 4*(20*I*b^2*c^3*d^3
```

```
*x^2 + 45*b^2*c^2*d^3*x - 36*I*b^2*c*d^3)*arctan(c*x)^2 + 4*(20*b^2*c^3*d^3
*x^2 - 45*I*b^2*c^2*d^3*x - 36*b^2*c*d^3)*arctan(c*x)*log(c^2*x^2 + 1) + (2
0*I*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x - 36*I*b^2*c*d^3)*log(c^2*x^2 + 1)^2
)/x^5
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="fricas")
[Out] 1/240*(240*x^6*integral(1/60*(-60*I*a^2*c^5*d^3*x^5 - 180*a^2*c^4*d^3*x^4 +
 120*I*a^2*c^3*d^3*x^3 - 120*a^2*c^2*d^3*x^2 + 180*I*a^2*c*d^3*x + 60*a^2*d
^3 + (60*a*b*c^5*d^3*x^5 - 20*(9*I*a*b - b^2)*c^4*d^3*x^4 - 15*(8*a*b + 3*I
*b^2)*c^3*d^3*x^3 - 12*(10*I*a*b + 3*b^2)*c^2*d^3*x^2 - 10*(18*a*b - I*b^2)
*c*d^3*x + 60*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^9 + x^7), x) + (
-20*I*b^2*c^3*d^3*x^3 - 45*b^2*c^2*d^3*x^2 + 36*I*b^2*c*d^3*x + 10*b^2*d^3)
*log(-(c*x + I)/(c*x - I))^2)/x^6
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**7,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7,x)
[Out] int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7, x)
```

3.95 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))^2}{d+icdx}\,dx$

Optimal. Leaf size=356

$$-\frac{abx}{c^3d}-\frac{ib^2x}{3c^3d}+\frac{ib^2\text{ArcTan}(cx)}{3c^4d}-\frac{b^2x\text{ArcTan}(cx)}{c^3d}+\frac{ibx^2(a+b\text{ArcTan}(cx))}{3c^2d}-\frac{5(a+b\text{ArcTan}(cx))^2}{6c^4d}+\frac{ix(a+bA$$

```
[Out] -a*b*x/c^3/d-1/3*I*b^2*x/c^3/d+1/3*I*b^2*arctan(c*x)/c^4/d-b^2*x*arctan(c*x
)/c^3/d+1/3*I*b*x^2*(a+b*arctan(c*x))/c^2/d-5/6*(a+b*arctan(c*x))^2/c^4/d+I
*x*(a+b*arctan(c*x))^2/c^3/d+1/2*x^2*(a+b*arctan(c*x))^2/c^2/d-1/3*I*x^3*(a
+b*arctan(c*x))^2/c/d+8/3*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d+(a+b*
arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d+1/2*b^2*ln(c^2*x^2+1)/c^4/d-4/3*b^2*po
lylog(2,1-2/(1+I*c*x))/c^4/d+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))
/c^4/d+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^4/d
```

Rubi [A]

time = 0.61, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4986, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 266, 5004, 5114, 6745}

$$\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^4d}-\frac{5(a+b\text{ArcTan}(cx))^2}{6c^4d}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^4d}+\frac{8ib\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^4d}+\frac{ix(a+b\text{ArcTan}(cx))^2}{c^3d}+\frac{x^2(a+b\text{ArcTan}(cx))^2}{2c^2d}+\frac{ibx^2(a+b\text{ArcTan}(cx))}{3c^2d}-\frac{ix^3(a+b\text{ArcTan}(cx))^2}{3cd}-\frac{abx}{c^3d}+\frac{ib^2\text{ArcTan}(cx)}{3c^4d}-\frac{b^2x\text{ArcTan}(cx)}{c^3d}-\frac{4b^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^4d}+\frac{b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^4d}-\frac{ib^2x}{3c^3d}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] -((a*b*x)/(c^3*d)) - ((I/3)*b^2*x)/(c^3*d) + ((I/3)*b^2*ArcTan[c*x])/(c^4*d
) - (b^2*x*ArcTan[c*x])/(c^3*d) + ((I/3)*b*x^2*(a + b*ArcTan[c*x]))/(c^2*d)
 - (5*(a + b*ArcTan[c*x])^2)/(6*c^4*d) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d
) + (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d) - ((I/3)*x^3*(a + b*ArcTan[c*x])^
2)/(c*d) + (((8*I)/3)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d) + (
(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d) + (b^2*Log[1 + c^2*x^2])/
(2*c^4*d) - (4*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(3*c^4*d) + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d) + (b^2*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(2*c^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
```

```
, 0] && GtQ[m, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx = \frac{i\int \frac{x^2(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c} - \frac{i\int x^2(a+b\tan^{-1}(cx))^2\,dx}{cd}$$

$$= -\frac{ix^3(a+b\tan^{-1}(cx))^2}{3cd} - \frac{\int \frac{x(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c^2} + \frac{(2ib)\int \frac{x^3(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{3d} + \int$$

$$= \frac{x^2(a+b\tan^{-1}(cx))^2}{2c^2d} - \frac{ix^3(a+b\tan^{-1}(cx))^2}{3cd} - \frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c^3} + \frac{i\int (a-$$

$$= \frac{ibx^2(a+b\tan^{-1}(cx))}{3c^2d} - \frac{(a+b\tan^{-1}(cx))^2}{3c^4d} + \frac{ix(a+b\tan^{-1}(cx))^2}{c^3d} + \frac{x^2(a+b$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ibx^2(a+b\tan^{-1}(cx))}{3c^2d} - \frac{5(a+b\tan^{-1}(cx))^2}{6c^4d} + \frac{ix(a+b\tan^{-1}}{c^3d}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2(a+b\tan^{-1}(cx))}{3c^2d} - \frac{5(a}{}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2(a+b\tan^{-1}(cx))}{3c^2d} - \frac{5(a}{}$$

$$= -\frac{abx}{c^3d} - \frac{ib^2x}{3c^3d} + \frac{ib^2\tan^{-1}(cx)}{3c^4d} - \frac{b^2x\tan^{-1}(cx)}{c^3d} + \frac{ibx^2(a+b\tan^{-1}(cx))}{3c^2d} - \frac{5(a}{}$$

Mathematica [A]
time = 0.61, size = 421, normalized size = 1.18

$$\frac{ia^2x}{c^3d} + \frac{a^2x^2}{2c^2d} - \frac{ia^2x^3}{3cd} - \frac{ia^2\text{ArcTan}(cx)}{c^4d} - \frac{a^2\log(1+c^2x^2)}{2c^4d} - \frac{iab\left(-3icx - 8cx\text{ArcTan}(cx) + 6\text{ArcTan}(cx)^2 + (1+c^2x^2)(-1+3i\text{ArcTan}(cx)+2cx\text{ArcTan}(cx)) + 6i\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right) - 8\log\left(\frac{1}{\sqrt{1+c^2x^2}}\right) + 3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{3c^4d} - \frac{ib^2\left(2cx - 6icx\text{ArcTan}(cx) - 2(1+c^2x^2)\text{ArcTan}(cx) + 8i\text{ArcTan}(cx)^2 - 8cx\text{ArcTan}(cx)^2 + 3i(1+c^2x^2)\text{ArcTan}(cx)^2 + 2cx(1+c^2x^2)\text{ArcTan}(cx)^2 + 4\text{ArcTan}(cx)^3 - 16\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right) + 6i\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right) - 6i\log\left(\frac{1}{\sqrt{1+c^2x^2}}\right) + (8i+6\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right) + 3i\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)}{6c^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] (I*a^2*x)/(c^3*d) + (a^2*x^2)/(2*c^2*d) - ((I/3)*a^2*x^3)/(c*d) - (I*a^2*Ar
cTan[c*x])/(c^4*d) - (a^2*Log[1 + c^2*x^2])/(2*c^4*d) - ((I/3)*a*b*((-3*I)*
c*x - 8*c*x*ArcTan[c*x] + 6*ArcTan[c*x]^2 + (1 + c^2*x^2)*(-1 + (3*I)*ArcTa
n[c*x] + 2*c*x*ArcTan[c*x]) + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x
])] - 8*Log[1/Sqrt[1 + c^2*x^2]] + 3*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(
c^4*d) - ((I/6)*b^2*(2*c*x - (6*I)*c*x*ArcTan[c*x] - 2*(1 + c^2*x^2)*ArcTan
[c*x] + (8*I)*ArcTan[c*x]^2 - 8*c*x*ArcTan[c*x]^2 + (3*I)*(1 + c^2*x^2)*Arc
Tan[c*x]^2 + 2*c*x*(1 + c^2*x^2)*ArcTan[c*x]^2 + 4*ArcTan[c*x]^3 - 16*ArcTa
n[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (6*I)*ArcTan[c*x]^2*Log[1 + E^((2*I
)*ArcTan[c*x])] - (6*I)*Log[1/Sqrt[1 + c^2*x^2]] + (8*I + 6*ArcTan[c*x])*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])]
))/(c^4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.99, size = 1227, normalized size = 3.45

method	result	size
derivativedivides	Expression too large to display	1227
default	Expression too large to display	1227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*a^2/d*ln(c^2*x^2+1)+I*b^2/d*arctan(c*x)^2*c*x+11/6*b^2/d*arctan
(c*x)^2-b^2/d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+8/3*b^2/d*dilog(1-I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))+8/3*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b^2/
d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*I*a*b/d*arctan(c*x)*c*x-2/3*I*a*b/d
*arctan(c*x)*c^3*x^3+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*
I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/3*b^2/d-1/3*I*b^2/d*arctan(c*x
)^2*c^3*x^3+I*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/3*I*b^2/d*arctan(c*x)*c^
2*x^2-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^3*arctan(c*x)^2-I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*arctan(c*x)^2+a*b/d*arctan(c*x)*c^2*x^2+1/3*I*a*b/d*c^2*x^2
+I*b^2/d*Pi*arctan(c*x)^2+8/3*I*b^2/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))-I*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+8/3*I*b^
2/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*a^2/d*c*x-1/3*I*a^2/d
*c^3*x^3-2*a*b/d*arctan(c*x)*ln(c*x-I)+I*a*b/d*dilog(-1/2*I*(c*x+I))-11/12*
I*a*b/d*ln(c^2*x^2+1)-5/24*I*a*b/d*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I*a*b/d*ln(
c*x-I)^2-b^2/d*arctan(c*x)*c*x+1/2*b^2/d*arctan(c*x)^2*c^2*x^2-1/3*I*b^2/d*
c*x-a*b/d*c*x+1/2*a^2/d*c^2*x^2+4/3*I*a*b/d-b^2/d*arctan(c*x)^2*ln(c*x-I)+b
^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2/3*I*b^2/d*arctan(c*x)^
3+4/3*I*b^2/d*arctan(c*x)+11/6*a*b/d*arctan(c*x)+5/12*a*b/d*arctan(1/2*c*x)
-5/12*a*b/d*arctan(1/6*c^3*x^3+7/6*c*x)-5/6*a*b/d*arctan(1/2*c*x-1/2*I)-I*a
^2/d*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*a^2*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*d
)) - 1/96*(16*I*(216*b^2*c^4*integrate(1/48*x^4*arctan(c*x)^2/(c^5*d*x^2 +
c^3*d), x) + 18*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^5*d*x^2 +
c^3*d), x) + 576*a*b*c^4*integrate(1/48*x^4*arctan(c*x)/(c^5*d*x^2 + c^3*d)
, x) + 24*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d),
x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*d*x^2
+ c^3*d), x) + 24*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^5*d*x^2 + c^3*d
), x) - 36*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d),
 x) + 144*b^2*c*integrate(1/48*x*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) - 36*b
^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^5*d*x^2 + c^3*d), x) - b^2*arctan(c
*x)^3/(c^4*d))*c^4*d - 96*c^4*d*integrate(1/48*(12*(3*b^2*c^2*x^3 - 2*b^2*x
)*arctan(c*x)^2 + 3*(b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1)^2 - 4*(2*b^2*c
^3*x^4 - 24*a*b*c^2*x^3 - 3*b^2*c*x^2)*arctan(c*x) - 2*(6*b^2*c^3*x^4*arcta
n(c*x) - b^2*c^2*x^3 - 6*b^2*x)*log(c^2*x^2 + 1))/(c^4*d*x^2 + c^2*d), x) +
 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 4*(-2*I*b^2*c^3*x^3 +
3*b^2*c^2*x^2 + 6*I*b^2*c*x)*arctan(c*x)^2 + (-2*I*b^2*c^3*x^3 + 3*b^2*c^2*
x^2 + 6*I*b^2*c*x + 6*I*b^2*arctan(c*x))*log(c^2*x^2 + 1)^2 - 4*(3*b^2*arct
an(c*x)^2 + (2*b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 - 6*b^2*c*x)*arctan(c*x))*log(
c^2*x^2 + 1))/(c^4*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^3*log(-(c*x +
 I)/(c*x - I)) - 4*I*a^2*x^3)/(c*d*x - I*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

```
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.96 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))^2}{d+icdx}\,dx$

Optimal. Leaf size=277

$$\frac{iabx}{c^2d}+\frac{ib^2x\text{ArcTan}(cx)}{c^2d}+\frac{i(a+b\text{ArcTan}(cx))^2}{2c^3d}+\frac{x(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{ix^2(a+b\text{ArcTan}(cx))^2}{2cd}+\frac{2b(a+b\text{Arc}}{}$$

```
[Out] I*a*b*x/c^2/d+I*b^2*x*arctan(c*x)/c^2/d+1/2*I*(a+b*arctan(c*x))^2/c^3/d+x*(
a+b*arctan(c*x))^2/c^2/d-1/2*I*x^2*(a+b*arctan(c*x))^2/c/d+2*b*(a+b*arctan(
c*x))*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d-1/2
*I*b^2*ln(c^2*x^2+1)/c^3/d+I*b^2*polylog(2,1-2/(1+I*c*x))/c^3/d+b*(a+b*arct
an(c*x))*polylog(2,1-2/(1+I*c*x))/c^3/d-1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/
c^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {4986, 4946, 5036, 4930, 266, 5004, 5040, 4964, 2449, 2352, 5114, 6745}

$$\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^3d}+\frac{i(a+b\text{ArcTan}(cx))^2}{2c^3d}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^3d}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^3d}+\frac{x(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{ix^2(a+b\text{ArcTan}(cx))^2}{2cd}+\frac{iabx}{c^2d}+\frac{ib^2x\text{ArcTan}(cx)}{c^2d}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^3d}-\frac{ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^3d}-\frac{ib^2\log\left(c^2x^2+1\right)}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] (I*a*b*x)/(c^2*d) + (I*b^2*x*ArcTan[c*x])/(c^2*d) + ((I/2)*(a + b*ArcTan[c*
x])^2)/(c^3*d) + (x*(a + b*ArcTan[c*x])^2)/(c^2*d) - ((I/2)*x^2*(a + b*ArcT
an[c*x])^2)/(c*d) + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) -
(I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*Log[1 + c
^2*x^2])/(c^3*d) + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) - ((I/2)*b^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(c^3*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx = \frac{i\int \frac{x(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c} - \frac{i\int x(a+b\tan^{-1}(cx))^2\,dx}{cd}$$

$$= -\frac{ix^2(a+b\tan^{-1}(cx))^2}{2cd} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c^2} + \frac{(ib)\int \frac{x^2(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{d} + \int($$

$$= \frac{x(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))^2}{2cd} - \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d}$$

$$= \frac{iabx}{c^2d} + \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{x(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))^2}{2cd} - i$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{x(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix^2(a+}{}$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{x(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix^2(a+}{}$$

$$= \frac{iabx}{c^2d} + \frac{ib^2x\tan^{-1}(cx)}{c^2d} + \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{x(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix^2(a+}{}$$

Mathematica [A]
time = 0.35, size = 330, normalized size = 1.19

$$-\frac{i\left(6ia^2cx-6abcx+3a^2c^2x^2-6ia^2\text{ArcTan}(cx)+6ab\text{ArcTan}(cx)+12iabcx\text{ArcTan}(cx)-6b^2cx\text{ArcTan}(cx)+6abc^2x^2\text{ArcTan}(cx)-12iab\text{ArcTan}(cx)^2+9b^2\text{ArcTan}(cx)^2+6ib^2cx\text{ArcTan}(cx)^2+3b^2c^2x^2\text{ArcTan}(cx)^2-4ib^2\text{ArcTan}(cx)^3+12ab\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+12ib^2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+6b^2\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-3a^2\log\left(1+c^2x^2\right)-6iab\log\left(1+c^2x^2\right)+3b^2\log\left(1+c^2x^2\right)+6b(-ia+b-ib\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+3b^2\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)}{6c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] ((-1/6*I)*((6*I)*a^2*c*x - 6*a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a^2*ArcTan[c*x
] + 6*a*b*ArcTan[c*x] + (12*I)*a*b*c*x*ArcTan[c*x] - 6*b^2*c*x*ArcTan[c*x]
+ 6*a*b*c^2*x^2*ArcTan[c*x] - (12*I)*a*b*ArcTan[c*x]^2 + 9*b^2*ArcTan[c*x]^
2 + (6*I)*b^2*c*x*ArcTan[c*x]^2 + 3*b^2*c^2*x^2*ArcTan[c*x]^2 - (4*I)*b^2*A
rcTan[c*x]^3 + 12*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b
^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b^2*ArcTan[c*x]^2*Log[1 +
 E^((2*I)*ArcTan[c*x])] - 3*a^2*Log[1 + c^2*x^2] - (6*I)*a*b*Log[1 + c^2*x^
2] + 3*b^2*Log[1 + c^2*x^2] + 6*b*((-I)*a + b - I*b*ArcTan[c*x])*PolyLog[2,
 -E^((2*I)*ArcTan[c*x])] + 3*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^3*
d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.64, size = 1113, normalized size = 4.02

method	result	size
derivativedivides	Expression too large to display	1113
default	Expression too large to display	1113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a^2/d*arctan(c*x)-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+b*a/d+1
/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csg
n(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+2*b*a/d*arctan(c*x)*c*x+I*b*
a/d*c*x+2*I*b*a/d*arctan(c*x)*ln(c*x-I)+I*b^2/d*arctan(c*x)*c*x-1/2*I*b^2/d
*arctan(c*x)^2*c^2*x^2-I*b*a/d*arctan(c*x)*c^2*x^2-1/2*b^2/d*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+b^2/d*arctan(c*x)-2/3*b
^2/d*arctan(c*x)^3+a^2/d*c*x-1/2*b*a/d*ln(c*x-I)^2+b*a/d*dilog(-1/2*I*(c*x+
I))-3/4*b*a/d*ln(c^2*x^2+1)-1/8*b*a/d*ln(c^4*x^4+10*c^2*x^2+9)+1/2*I*a^2/d*
ln(c^2*x^2+1)+2*b^2/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2
/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*Pi*arctan(c*x)^2+I
*b^2/d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)
^2/(c^2*x^2+1))-3/2*I*b^2/d*arctan(c*x)^2-1/2*I*b^2/d*polylog(3,-(1+I*c*x)^
2/(c^2*x^2+1))+I*b^2/d*arctan(c*x)^2*ln(c*x-I)-b^2/d*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*b^2/d*Pi*csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2/d*
arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2*I*b^2/d*dilog(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-2*I*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b*a/d*
ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*a^2/d*c^2*x^2+b^2/d*arctan(c*x)^2*c*x-3/
```

```
2*I*b*a/d*arctan(c*x)-1/4*I*b*a/d*arctan(1/2*c*x)+1/4*I*b*a/d*arctan(1/6*c^
3*x^3+7/6*c*x)+1/2*I*b*a/d*arctan(1/2*c*x-1/2*I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/96*(16*
(24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^
2*d), x) + 24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d), x
) - 72*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 6
*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 19
2*a*b*c^2*integrate(1/16*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12*b^2*c
^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) + 48*b^2*c*i
ntegrate(1/16*x*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12*b^2*integrate(1/16
*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - b^2*arctan(c*x)^3/(c^3*d))*c^
3*d + 24*b^2*arctan(c*x)^3 + 96*I*c^3*d*integrate(1/16*(4*(3*b^2*c^2*x^3 -
2*b^2*x)*arctan(c*x)^2 + (b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1)^2 + 4*(8*
a*b*c^2*x^3 + b^2*c*x^2)*arctan(c*x) + 2*(b^2*c^2*x^3 + 2*b^2*c*x^2*arctan(
c*x) + 2*b^2*x)*log(c^2*x^2 + 1))/(c^3*d*x^2 + c*d), x) + 3*I*b^2*log(c^2*x
^2 + 1)^3 - 12*(-I*b^2*c^2*x^2 + 2*b^2*c*x)*arctan(c*x)^2 + 3*(-I*b^2*c^2*x
^2 + 2*b^2*c*x + 2*b^2*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*(-I*b^2*arctan(
c*x)^2 + (b^2*c^2*x^2 + 2*I*b^2*c*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^3*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^2*log(-(c*x +
 I)/(c*x - I)) - 4*I*a^2*x^2)/(c*d*x - I*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

```
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.97 $\int \frac{x(a+b\text{ArcTan}(cx))^2}{d+icdx}\,dx$

Optimal. Leaf size=192

$$\frac{(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{ix(a+b\text{ArcTan}(cx))^2}{cd}-\frac{2ib(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1+}\right.}{c^2d}$$

```
[Out] (a+b*arctan(c*x))^2/c^2/d-I*x*(a+b*arctan(c*x))^2/c/d-2*I*b*(a+b*arctan(c*x
))*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d+b^2*poly
log(2,1-2/(1+I*c*x))/c^2/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c
^2/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4986, 4930, 5040, 4964, 2449, 2352, 5004, 5114, 6745}

$$-\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^2d}+\frac{(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{2ib\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^2d}-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{ix(a+b\text{ArcTan}(cx))^2}{cd}+\frac{b^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^2d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] (a + b*ArcTan[c*x])^2/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^2)/(c*d) - ((2*I)*
b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^2*
Log[2/(1 + I*c*x)])/(c^2*d) + (b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) -
 (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d) - (b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx = \frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx}{c} - \frac{i\int (a+b\tan^{-1}(cx))^2\,dx}{cd}$$

$$= -\frac{ix(a+b\tan^{-1}(cx))^2}{cd} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{(2ib)\int \frac{x(a+b\tan^{-1}(cx))}{1+c^2x^2}}{d}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^2}{cd} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{ib}{}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^2}{cd} - \frac{2ib(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} -$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^2}{cd} - \frac{2ib(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} -$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^2}{cd} - \frac{2ib(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} -$$

Mathematica [A]
time = 0.29, size = 239, normalized size = 1.24

$$-\frac{i\left(6a^2cx - 6a^2\text{ArcTan}(cx) + 12abcx\text{ArcTan}(cx) - 12ab\text{ArcTan}(cx)^2 - 6ib^2\text{ArcTan}(cx)^2 + 6b^2cx\text{ArcTan}(cx)^2 - 4b^2\text{ArcTan}(cx)^3 - 12iab\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right) + 12b^2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right) - 6ib^2\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right) + 3ia^2\log\left(1+c^2x^2\right) - 6ab\log\left(1+c^2x^2\right) - 6b(a+ib+b\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right) - 3ib^2\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)}{6c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

```
[Out] ((-1/6*I)*(6*a^2*c*x - 6*a^2*ArcTan[c*x] + 12*a*b*c*x*ArcTan[c*x] - 12*a*b*
ArcTan[c*x]^2 - (6*I)*b^2*ArcTan[c*x]^2 + 6*b^2*c*x*ArcTan[c*x]^2 - 4*b^2*A
rcTan[c*x]^3 - (12*I)*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b
^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^2*ArcTan[c*x]^2*Log
[1 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*Log[1 + c^2*x^2] - 6*a*b*Log[1 + c^
2*x^2] - 6*b*(a + I*b + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] -
 (3*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.31, size = 4395, normalized size = 22.89

method	result	size
derivativedivides	Expression too large to display	4395
default	Expression too large to display	4395

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

[Out] 1/c^2*(1/2*a^2/d*ln(c^2*x^2+1)+I*a^2/d*arctan(c*x)-1/2*b*a/d*arctan(1/12*c^3*x^3+13/12*c*x)-1/2*b*a/d*arctan(1/4*c*x)-2*I*b*a/d*arctan(c*x)*c*x+1/4*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*b^2/d*arctan(c*x)^2*c*x-b^2/d*arctan(c*x)^2-b^2/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)

```
)+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-I*a^2/d*c*x-1/2*b^2/d*polylog(2,-(1+I
*c*x)^2/(c^2*x^2+1))+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln((1+I*
c*x)^2/(c^2*x^2+1)+1)-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln((1+I*c*x)^
2/(c^2*x^2+1)+1)-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1-I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2*b^2/d*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*
arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I*b^2/d*Pi*csgn((1+I*c*
x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/
2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a^2*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/96*(-24*I*b^2*c*x*arctan(c*x)
^2 + 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 16*I*(72*b^2*c^2*i
ntegrate(1/16*x^2*arctan(c*x)^2/(c^3*d*x^2 + c*d), x) + 6*b^2*c^2*integrate
(1/16*x^2*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 192*a*b*c^2*integrate(
1/16*x^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 24*b^2*c^2*integrate(1/16*x^2*
log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 24*b^2*c*integrate(1/16*x*arctan(c
*x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 48*b^2*c*integrate(1/16*x*arct
an(c*x)/(c^3*d*x^2 + c*d), x) + 12*b^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c
^3*d*x^2 + c*d), x) + b^2*arctan(c*x)^3/(c^2*d))*c^2*d + 96*c^2*d*integrate
(1/16*(20*b^2*x*arctan(c*x)^2 + 3*b^2*x*log(c^2*x^2 + 1)^2 - 8*(b^2*c*x^2 -
 4*a*b*x)*arctan(c*x) - 4*(b^2*c*x^2*arctan(c*x) + b^2*x)*log(c^2*x^2 + 1))
/(c^2*d*x^2 + d), x) + 6*(I*b^2*c*x + I*b^2*arctan(c*x))*log(c^2*x^2 + 1)^2
 + 12*(2*b^2*c*x*arctan(c*x) - b^2*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^2*d)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*x*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x*log(-(c*x + I)/
(c*x - I)) - 4*I*a^2*x)/(c*d*x - I*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

```
[Out] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.98 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{d+icdx}\,dx$

Optimal. Leaf size=98

$$\frac{i(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{cd}+\frac{ib^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd}$$

```
[Out] I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c/d-b*(a+b*arctan(c*x))*polylog(2,1-2
/(1+I*c*x))/c/d+1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c/d
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4964, 5004, 5114, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{cd}+\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{cd}+\frac{ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c*d) - (b*(a + b*ArcTan[c*x])
*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*d) + ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*
c*x)])/(c*d)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{d+icdx}\,dx = \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(2ib)\int\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b(a+b\tan^{-1}(cx))\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{cd} + \frac{b^2\int\frac{\operatorname{Li}_2(1}{1+}}{c}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b(a+b\tan^{-1}(cx))\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{cd} + \frac{ib^2\operatorname{Li}_3(1-}{2cd}$$

Mathematica [A]
time = 0.02, size = 95, normalized size = 0.97

$$\frac{i\left(2(a+b\text{ArcTan}(cx))^2\log\left(\frac{2d}{d+icdx}\right)+2ib(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)+b^2\text{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)\right)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

```
[Out] ((I/2)*(2*(a + b*ArcTan[c*x])^2*Log[(2*d)/(d + I*c*d*x)] + (2*I)*b*(a + b*A
rcTan[c*x])*PolyLog[2, (I + c*x)/(-I + c*x)] + b^2*PolyLog[3, (I + c*x)/(-I
 + c*x)]))/(c*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.50, size = 1003, normalized size = 10.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-2*I*b*a/d*ln(1+I*c*x)*arctan(c*x)+a^2/d*arctan(c*x)+1/2*I*b^2/d*polyl
og(3,-(1+I*c*x)^2/(c^2*x^2+1))-I*b^2/d*ln(1+I*c*x)*arctan(c*x)^2-1/2*b^2/d*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^
2+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*arctan(c*x)^2+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+
1))*arctan(c*x)^2+1/2*b^2/d*arctan(c*x)^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(
```

```
(1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^2+1/2*b^2/d*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3-1/2*b^2/d*arctan(c*x)^2*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))-1/2*b^2/d*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*b^2/d*Pi*arctan(c*x)^2+b^2/d*arct
an(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*a^2/d*ln(c^2*x^2+1)+2/3*b
^2/d*arctan(c*x)^3+I*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-b*
a/d*ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+b*a/d*ln(1/2-1/2*I*c*x)*ln(1/2+1/2*I*c*x)
+b*a/d*dilog(1/2+1/2*I*c*x)+1/2*b*a/d*ln(1+I*c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^2*log(I*c*d*x + d)/(c*d) + 1/96*(24*b^2*arctan(c*x)^3 + 12*I*b^2*arcta
n(c*x)^2*log(c^2*x^2 + 1) + 6*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 + 3*I*b^2*
log(c^2*x^2 + 1)^3 - 8*(48*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 +
 1)/(c^2*d*x^2 + d), x) - b^2*arctan(c*x)^3/(c*d) + 12*b^2*integrate(1/16*l
og(c^2*x^2 + 1)^2/(c^2*d*x^2 + d), x) - 12*a*b*arctan(c*x)^2/(c*d))*c*d - 9
6*I*c*d*integrate(1/16*(20*b^2*c*x*arctan(c*x)^2 + 3*b^2*c*x*log(c^2*x^2 +
1)^2 + 32*a*b*c*x*arctan(c*x) + 4*b^2*arctan(c*x)*log(c^2*x^2 + 1))/(c^2*d*
x^2 + d), x))/(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*x
 - I)) - 4*I*a^2)/(c*d*x - I*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2}{d + c\,d\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))^2/(d + c*d*x*1i), x)
```

3.99 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{x(d+icdx)}\,dx$

Optimal. Leaf size=88

$$\frac{(a+b\text{ArcTan}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}+\frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}$$

```
[Out] (a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1+
2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4988, 5004, 5114, 6745}

$$\frac{ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}+\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}+\frac{b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcTan[c*x])
*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
2*d)
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+icdx)}\,dx = \frac{(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{(2bc)\int \frac{(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\tan^{-1}(cx))\operatorname{Li}_2\left(-1+\frac{2}{1+icx}\right)}{d} - \frac{(ib^2}{}$$

$$= \frac{(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\tan^{-1}(cx))\operatorname{Li}_2\left(-1+\frac{2}{1+icx}\right)}{d} + \frac{b^2\mathrm{Li}}{}$$

Mathematica [A]
time = 0.17, size = 173, normalized size = 1.97

$$-\frac{i\left(b^2\pi^3+24a^2\mathrm{ArcTan}(cx)+48ab\mathrm{ArcTan}(cx)^2+24ib^2\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)+48iab\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+24ia^2\log(cx)-12ia^2\log\left(1+c^2x^2\right)-24b^2\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)+24ab\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)+12ib^2\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

```
[Out] ((-1/24*I)*(b^2*Pi^3 + 24*a^2*ArcTan[c*x] + 48*a*b*ArcTan[c*x]^2 + (24*I)*b
^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (48*I)*a*b*ArcTan[c*x]*L
og[1 - E^((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] - (12*I)*a^2*Log[1 + c^
2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*a*b*Pol
yLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*b^2*PolyLog[3, E^((-2*I)*ArcTan[c*x
])]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.11, size = 1741, normalized size = 19.78

method	result	size
derivativedivides	Expression too large to display	1741
default	Expression too large to display	1741

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*ln(c*x)-1/2*a^2/d*ln(c^2*x^2+1)-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
```

```
+1))*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-
1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*ar
ctan(c*x)^2+b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*arc
tan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*arctan(c*x)^2*ln(c*x)+2*
b*a/d*arctan(c*x)*ln(c*x)+I*b*a/d*dilog(1+I*c*x)-I*b*a/d*dilog(1-I*c*x)-2*I
*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d*arctan
(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^2/d*Pi*arctan(c*x)^2+1
/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*
arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d
*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c
*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1
/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*arctan(c*x)^2+I*b*a/d*ln(c*x)*ln(1+I*c*x)-I*b*a/d*ln(c*x)*ln(1-I*c*x)-b^
2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+2*b^2/d*polylog(3,(1+I*c*x)
/(c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*a*b/d
*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-2*a*b/d*arctan(c*x)*l
n(c*x-I)+I*a*b/d*dilog(-1/2*I*(c*x+I))-1/2*I*a*b/d*ln(c*x-I)^2-b^2/d*arctan
(c*x)^2*ln(c*x-I)+b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2/3*I
*b^2/d*arctan(c*x)^3-I*a^2/d*arctan(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -a^2*(log(I*c*x + 1)/d - log(x)/d) + 1/96*(-24*I*b^2*arctan(c*x)^3 + 12*b^2
*arctan(c*x)^2*log(c^2*x^2 + 1) - 6*I*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 +
3*b^2*log(c^2*x^2 + 1)^3 - 2*(384*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/
(c^2*d*x^3 + d*x), x) + 192*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2
+ 1)/(c^2*d*x^3 + d*x), x) + b^2*log(c^2*x^2 + 1)^3/d - 576*b^2*integrate(1
```

```
/16*arctan(c*x)^2/(c^2*d*x^3 + d*x), x) - 48*b^2*integrate(1/16*log(c^2*x^2
 + 1)^2/(c^2*d*x^3 + d*x), x) - 1536*a*b*integrate(1/16*arctan(c*x)/(c^2*d*
x^3 + d*x), x))*d - 8*I*(b^2*arctan(c*x)^3/d - 12*b^2*c*integrate(1/16*x*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) + 12*a*b*arctan(c*x)^2/d + 48*b^2*in
tegrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x))*d)/d
```

Fricas [A]
time = 0.64, size = 136, normalized size = 1.55

$$-\frac{b^2\log\left(\frac{2\,cx}{cx-i}\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+2\,b^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)+4i\,ab\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,a^2\log\left(x\right)+4\,a^2\log\left(\frac{cx-i}{c}\right)-2\,b^2\mathrm{polylog}\left(3,-\frac{cx+i}{cx-i}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*b^2*dilog(-2
*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*I*a*b*dilog((c*x + I)/(c*
x - I) + 1) - 4*a^2*log(x) + 4*a^2*log((c*x - I)/c) - 2*b^2*polylog(3, -(c*
x + I)/(c*x - I)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\frac{a^2}{cx^2-ix}\,dx+\int\frac{b^2\,\mathrm{atan}^2\,(cx)}{cx^2-ix}\,dx+\int\frac{2ab\,\mathrm{atan}\,(cx)}{cx^2-ix}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**2/(c*x**2 - I*x), x) + Integral(b**2*atan(c*x)**2/(c*x**2 -
 I*x), x) + Integral(2*a*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)), x)
```

3.100 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2(d+icdx)}\,dx$

Optimal. Leaf size=186

$$-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}+\frac{2bc(a+b\text{ArcTan}(cx))\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}$$

```
[Out] -I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*b*c*(a+b*arctan(c*x))*
ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d-I*b^2*c*pol
ylog(2,-1+2/(1-I*c*x))/d+b*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d-
1/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d
```

Rubi [A]

time = 0.29, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4990, 4946, 5044, 4988, 2497, 5004, 5114, 6745}

$$\frac{bc\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{ic\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d}-\frac{ib^2c\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (2*b*c*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^2*Log
[2 - 2/(1 + I*c*x)])/d - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (b*c*
(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - ((I/2)*b^2*c*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/d
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
```

```
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+icdx)}\,dx\right) + \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{ic(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x(1+c^2x^2)}}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{ic(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{2bc(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1-icx}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{2bc(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1-icx}\right.}{d}$$

Mathematica [A]
time = 0.54, size = 265, normalized size = 1.42

$$-\frac{\frac{2a^2}{x} + 2a^2c\text{ArcTan}(cx) + 2ia^2c\log(x) - ia^2c\log(1+c^2x^2) + 2abc\left(2\left(\text{ArcTan}(cx)^2 + \text{ArcTan}(cx)\left(\frac{1}{cx} + i\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right) - \log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right) + \text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right)\right) + 2ib^2c\left(-\frac{i\pi^3}{24} + \text{ArcTan}(cx)^2 - \frac{i\text{ArcTan}(cx)^2}{cx} + \text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right) + 2i\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right) + i\text{ArcTan}(cx)\text{PolyLog}\left(2, e^{-2i\text{ArcTan}(cx)}\right) + \text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3, e^{-2i\text{ArcTan}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -1/2*((2*a^2)/x + 2*a^2*c*ArcTan[c*x] + (2*I)*a^2*c*Log[x] - I*a^2*c*Log[1
 + c^2*x^2] + 2*a*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1 - E
^((2*I)*ArcTan[c*x])]) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^((2*I
)*ArcTan[c*x])]) + (2*I)*b^2*c*((-1/24*I)*Pi^3 + ArcTan[c*x]^2 - (I*ArcTan[
c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2*I)*ArcTa
n[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*
ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*Ar
cTan[c*x])]/2))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.59, size = 9130, normalized size = 49.09

method	result	size
derivativedivides	Expression too large to display	9130
default	Expression too large to display	9130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a^2*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/96*(24*b^2*c*x*arct
an(c*x)^3 + 3*I*b^2*c*x*log(c^2*x^2 + 1)^3 + 24*b^2*arctan(c*x)^2 - 2*I*(38
4*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + b^2*c*
log(c^2*x^2 + 1)^3/d + 12*b^2*c*arctan(c*x)^2/d - 576*b^2*c*integrate(1/16*
x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 48*b^2*c*integrate(1/16*x*log(c^2
*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 1536*a*b*c*integrate(1/16*x*arctan(c*
x)/(c^2*d*x^4 + d*x^2), x) + 192*b^2*c*integrate(1/16*x*log(c^2*x^2 + 1)/(c
^2*d*x^4 + d*x^2), x) - 192*b^2*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)
/(c^2*d*x^4 + d*x^2), x))*d*x - 16*(b^2*c*arctan(c*x)^3/d + 12*b^2*c^2*inte
grate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 24*b^2*c^2*inte
grate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 24*b^2*c*integrat
e(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 48*b^2*c*in
tegrate(1/16*x*arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + 72*b^2*integrate(1/16*
arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 6*b^2*integrate(1/16*log(c^2*x^2 +
1)^2/(c^2*d*x^4 + d*x^2), x) + 192*a*b*integrate(1/16*arctan(c*x)/(c^2*d*x^
4 + d*x^2), x))*d*x + 6*(b^2*c*x*arctan(c*x) - b^2)*log(c^2*x^2 + 1)^2 + 12
*(I*b^2*c*x*arctan(c*x)^2 + 2*I*b^2*arctan(c*x))*log(c^2*x^2 + 1))/(d*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/4*(I*b^2*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*I*b^2*c
*x*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 2*I*b^2*c*x*poly
log(3, -(c*x + I)/(c*x - I)) + b^2*log(-(c*x + I)/(c*x - I))^2 + 4*d*x*inte
gral((-I*a^2*c*x + a^2 + ((a*b + I*b^2)*c*x + I*a*b)*log(-(c*x + I)/(c*x -
I)))/(c^2*d*x^4 + d*x^2), x))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a^2}{cx^3-ix^2}\,dx+\int \frac{b^2\operatorname{atan}^2{(cx)}}{cx^3-ix^2}\,dx+\int \frac{2ab\operatorname{atan}{(cx)}}{cx^3-ix^2}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**2/(c*x**3 - I*x**2), x) + Integral(b**2*atan(c*x)**2/(c*x**
3 - I*x**2), x) + Integral(2*a*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)), x)
```

3.101 $\int \frac{(a+b\text{ArcTan}(cx))^2}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=273

$$-\frac{bc(a+b\text{ArcTan}(cx))}{dx}-\frac{3c^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{(a+b\text{ArcTan}(cx))^2}{2dx^2}+\frac{ic(a+b\text{ArcTan}(cx))^2}{dx}+\frac{b^2c^2\log(x)}{d}$$

```
[Out] -b*c*(a+b*arctan(c*x))/d/x-3/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c*
x))^2/d/x^2+I*c*(a+b*arctan(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(c^2*
x^2+1)/d-2*I*b*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan(c*
x))^2*ln(2-2/(1+I*c*x))/d-b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d-I*b*c^2*(a+b*
arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d-1/2*b^2*c^2*polylog(3,-1+2/(1+I*c*
x))/d
```

Rubi [A]
time = 0.44, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {4990, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497, 5114, 6745}

$$-\frac{ibc^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}-\frac{3c^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{2ibc^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{c^2\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{2dx^2}+\frac{ic(a+b\text{ArcTan}(cx))^2}{dx}-\frac{bc(a+b\text{ArcTan}(cx))}{dx}-\frac{b^2c^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d}-\frac{b^2c^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d}+\frac{b^2c^2\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (3*c^2*(a + b*ArcTan[c*x])^2)/(2*d) -
(a + b*ArcTan[c*x])^2/(2*d*x^2) + (I*c*(a + b*ArcTan[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - ((2*I)*b*c^2*(a + b*ArcT
an[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(
1 + I*c*x)])/d - (b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (I*b*c^2*(a +
 b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (b^2*c^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2(d+icdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} - c^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(d+icdx)}\,dx - \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d} + \frac{(b}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+icx}\right.}{d}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic(a+b\,\mathrm{ta}}{d}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic(a+b\,\mathrm{ta}}{d}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic(a+b\,\mathrm{ta}}{d}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2dx^2} + \frac{ic(a+b\,\mathrm{ta}}{d}$$

Mathematica [A]
time = 0.69, size = 372, normalized size = 1.36

$$\frac{-\frac{a^2}{x^2}+\frac{2ia^2c}{x}+2ia^2c^2\mathrm{ArcTan}(cx)-2a^2c^2\log(x)+a^2c^2\log\left(1+c^2x^2\right)+\frac{2iab\left(2c^2x^2\mathrm{ArcTan}(cx)^2+\mathrm{ArcTan}(cx)\left(i+2cx+ic^2x^2+2ic^2x^2\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)\right)+cx\left(i-2cx\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+c^2x^2\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{x^2}+2b^2c^2\left(\frac{i\pi^3}{24}-\frac{\mathrm{ArcTan}(cx)}{cx}-\frac{3}{2}\mathrm{ArcTan}(cx)^2-\frac{\mathrm{ArcTan}(cx)^2}{2c^2x^2}+\frac{i\mathrm{ArcTan}(cx)^2}{cx}-\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)-2i\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

```
[Out] (-(a^2/x^2) + ((2*I)*a^2*c)/x + (2*I)*a^2*c^2*ArcTan[c*x] - 2*a^2*c^2*Log[x
] + a^2*c^2*Log[1 + c^2*x^2] + ((2*I)*a*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcTan
[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]
) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2, E^((2
*I)*ArcTan[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x) - (3*A
rcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c*x) - Arc
Tan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Log[1 - E^((
2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*x]*PolyLog[2
, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])] - PolyLog[3,
E^((-2*I)*ArcTan[c*x])]/2))/(2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.04, size = 2103, normalized size = 7.70

method	result	size
derivativedivides	Expression too large to display	2103
default	Expression too large to display	2103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a^2/d*ln(c*x)+1/2*a^2/d*ln(c^2*x^2+1)-1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*a^2/d*arcta
n(c*x)-1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-b^2/d*arctan(c*x)^2*ln(1-
(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)
^(1/2))-b^2/d*arctan(c*x)^2*ln(c*x)-2*b*a/d*arctan(c*x)*ln(c*x)-3/2*b^2/d*a
rctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b*a/d*ln(c*x)*ln(1-I*c
*x)-I*b*a/d*ln(c*x)*ln(1+I*c*x)-b*a/d*arctan(c*x)/c^2/x^2+I*b^2/d*arctan(c*
x)^2/c/x-I*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+b^2/d*ln(1+(1+I*c*x)/(c^2*x^2
+1)^(1/2))-2*b^2/d*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*ln((1+I*c*x)/
(c^2*x^2+1)^(1/2)-1)+2*b^2/d*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b^2/d*arc
tan(c*x)-1/2*a^2/d/c^2/x^2+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan
(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d
*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*
b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*arctan(c*x)^2+1/2*I*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1))*arctan(c*x)^2+2*I*b*a/d*arctan(c*x)/c/x+2/3*I*b^2/d*arctan(c*x)^3+b^2/
d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-2*b^2/d*polylog(3,(1+I*c*x)/(
c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-b*a/d/c/x
+1/2*I*b*a/d*ln(c*x-I)^2-I*a*b/d*dilog(-1/2*I*(c*x+I))-b^2/d*arctan(c*x)/c/
x-1/2*b^2/d*arctan(c*x)^2/c^2/x^2+I*b*a/d*ln(c^2*x^2+1)+I*b*a/d*dilog(1-I*c
*x)-2*I*b*a/d*ln(c*x)-I*b*a/d*dilog(1+I*c*x)+I*a^2/d/c/x+2*I*b^2/d*arctan(c
*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*b^2/d*Pi*arctan(c*x)^2-2*I
```

```
*b^2/d*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2/d*arctan(c*x)*
polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*a*b/d*arctan(c*x)*ln(c*x-I)+b^2/d
*arctan(c*x)^2*ln(c*x-I)-b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1)
)-a*b/d*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^2 -
 1/96*(-24*I*b^2*c^2*x^2*arctan(c*x)^3 + 3*b^2*c^2*x^2*log(c^2*x^2 + 1)^3 -
 2*(384*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) +
b^2*c^2*log(c^2*x^2 + 1)^3/d + 12*b^2*c^2*arctan(c*x)^2/d + 96*b^2*c^2*inte
grate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 192*b^2*c*integra
te(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 192*b^2*c*
integrate(1/16*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 576*b^2*integrate(1/
16*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 48*b^2*integrate(1/16*log(c^2*x^
2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 1536*a*b*integrate(1/16*arctan(c*x)/(c^2
*d*x^5 + d*x^3), x))*d*x^2 + 16*I*(b^2*c^2*arctan(c*x)^3/d + 12*b^2*c^3*int
egrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) - 24*b^2*c^3*int
egrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b^2*c^2*integ
rate(1/16*x^2*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 72*b^2*c*integrate(1/16
*x*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 6*b^2*c*integrate(1/16*x*log(c^2
*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 192*a*b*c*integrate(1/16*x*arctan(c*x
)/(c^2*d*x^5 + d*x^3), x) - 12*b^2*c*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2
*d*x^5 + d*x^3), x) + 24*b^2*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*d*x^5 + d*x^3), x))*d*x^2 + 12*(-2*I*b^2*c*x + b^2)*arctan(c*x)^2 - 3*(2
*I*b^2*c^2*x^2*arctan(c*x) - 2*I*b^2*c*x + b^2)*log(c^2*x^2 + 1)^2 + 12*(b^
2*c^2*x^2*arctan(c*x)^2 + (2*b^2*c*x + I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)
)/(d*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^2*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 4*b^2
*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 4*b^2*c^2*
x^2*polylog(3, -(c*x + I)/(c*x - I)) + 8*d*x^2*integral(1/2*(-2*I*a^2*c*x +
```

```
 2*a^2 + (2*b^2*c^2*x^2 + (2*a*b + I*b^2)*c*x + 2*I*a*b)*log(-(c*x + I)/(c*
x - I)))/(c^2*d*x^5 + d*x^3), x) + (-2*I*b^2*c*x + b^2)*log(-(c*x + I)/(c*x
 - I))^2)/(d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a^2}{cx^4-ix^3}\,dx+\int \frac{b^2\operatorname{atan}^2{(cx)}}{cx^4-ix^3}\,dx+\int \frac{2ab\operatorname{atan}{(cx)}}{cx^4-ix^3}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**2/(c*x**4 - I*x**3), x) + Integral(b**2*atan(c*x)**2/(c*x**
4 - I*x**3), x) + Integral(2*a*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x^3\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)), x)
```

3.102 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^4(d+icdx)} dx$

Optimal. Leaf size=365

$$-\frac{b^2c^2}{3dx}-\frac{b^2c^3\text{ArcTan}(cx)}{3d}-\frac{bc(a+b\text{ArcTan}(cx))}{3dx^2}+\frac{ibc^2(a+b\text{ArcTan}(cx))}{dx}+\frac{11ic^3(a+b\text{ArcTan}(cx))^2}{6d}-\frac{(a+bA}{}$$

```
[Out] -1/3*b^2*c^2/d/x-1/3*b^2*c^3*arctan(c*x)/d-1/3*b*c*(a+b*arctan(c*x))/d/x^2+
I*b*c^2*(a+b*arctan(c*x))/d/x+11/6*I*c^3*(a+b*arctan(c*x))^2/d-1/3*(a+b*arc
tan(c*x))^2/d/x^3+1/2*I*c*(a+b*arctan(c*x))^2/d/x^2+c^2*(a+b*arctan(c*x))^2
/d/x-I*b^2*c^3*ln(x)/d+1/2*I*b^2*c^3*ln(c^2*x^2+1)/d-8/3*b*c^3*(a+b*arctan(
c*x))*ln(2-2/(1-I*c*x))/d+I*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d+4/3
*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))/d-b*c^3*(a+b*arctan(c*x))*polylog(2,-1
+2/(1+I*c*x))/d+1/2*I*b^2*c^3*polylog(3,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.69, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4990, 4946, 5038, 331, 209, 5044, 4988, 2497, 272, 36, 29, 31, 5004, 5114, 6745}

$$-\frac{bc^3\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}+\frac{11ic^3(a+b\text{ArcTan}(cx))^2}{6d}-\frac{8bc^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{3d}+\frac{ic^3\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}+\frac{c^2(a+b\text{ArcTan}(cx))^2}{dx}+\frac{ibc^2(a+b\text{ArcTan}(cx))}{dx}-\frac{(a+b\text{ArcTan}(cx))^2}{3dx^3}+\frac{ic(a+b\text{ArcTan}(cx))^2}{2dx^2}-\frac{bc(a+b\text{ArcTan}(cx))}{3dx^2}-\frac{b^2c^3\text{ArcTan}(cx)}{3d}+\frac{4ib^2c^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{3d}+\frac{ib^2c^3\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}-\frac{ib^2c^3\log(x)}{d}-\frac{b^2c^2}{3dx}+\frac{ib^2c^3\log\left(c^2x^2+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -1/3*(b^2*c^2)/(d*x) - (b^2*c^3*ArcTan[c*x])/(3*d) - (b*c*(a + b*ArcTan[c*x
]))/(3*d*x^2) + (I*b*c^2*(a + b*ArcTan[c*x]))/(d*x) + (((11*I)/6)*c^3*(a +
b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(3*d*x^3) + ((I/2)*c*(a + b*Arc
Tan[c*x])^2)/(d*x^2) + (c^2*(a + b*ArcTan[c*x])^2)/(d*x) - (I*b^2*c^3*Log[x
])/d + ((I/2)*b^2*c^3*Log[1 + c^2*x^2])/d - (8*b*c^3*(a + b*ArcTan[c*x])*Lo
g[2 - 2/(1 - I*c*x)])/(3*d) + (I*c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I
*c*x)])/d + (((4*I)/3)*b^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (b*c^3*(
a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b^2*c^3*PolyL
og[3, -1 + 2/(1 + I*c*x)])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^4(d+icdx)}\,dx = -\left((ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+icdx)}\,dx\right) + \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^4}\,dx}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{3dx^3} - c^2\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+icdx)}\,dx - \frac{(ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx}{d} + \cdots$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{3dx^3} + \frac{ic(a+b\tan^{-1}(cx))^2}{2dx^2} + (ic^3)\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+icdx)}\,dx + \cdots$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{3dx^2} + \frac{ic^3(a+b\tan^{-1}(cx))^2}{3d} - \frac{(a+b\tan^{-1}(cx))^2}{3dx^3} + \frac{ic(a+b\,t\cdots}{2\cdots}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc(a+b\tan^{-1}(cx))}{3dx^2} + \frac{ibc^2(a+b\tan^{-1}(cx))}{dx} + \frac{11ic^3(a+b\tan^{-1}(cx))^2}{6d}\cdots$$

$$= -\frac{b^2c^2}{3dx} - \frac{b^2c^3\tan^{-1}(cx)}{3d} - \frac{bc(a+b\tan^{-1}(cx))}{3dx^2} + \frac{ibc^2(a+b\tan^{-1}(cx))}{dx} + \frac{11ic^3(\cdots}{\cdots}$$

$$= -\frac{b^2c^2}{3dx} - \frac{b^2c^3\tan^{-1}(cx)}{3d} - \frac{bc(a+b\tan^{-1}(cx))}{3dx^2} + \frac{ibc^2(a+b\tan^{-1}(cx))}{dx} + \frac{11ic^3(\cdots}{\cdots}$$

$$= -\frac{b^2c^2}{3dx} - \frac{b^2c^3\tan^{-1}(cx)}{3d} - \frac{bc(a+b\tan^{-1}(cx))}{3dx^2} + \frac{ibc^2(a+b\tan^{-1}(cx))}{dx} + \frac{11ic^3(\cdots}{\cdots}$$

Mathematica [A]
time = 0.69, size = 535, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

```
[Out] -1/3*a^2/(d*x^3) + ((I/2)*a^2*c)/(d*x^2) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTa
n[c*x])/d + (I*a^2*c^3*Log[x])/d - ((I/2)*a^2*c^3*Log[1 + c^2*x^2])/d - ((2
*I)*a*b*c^3*(-1/2*1/(c*x) - ((I/6)*(1 + c^2*x^2))/(c^2*x^2) + (((4*I)/3)*Ar
cTan[c*x])/(c*x) - ((I/3)*(1 + c^2*x^2)*ArcTan[c*x])/(c^3*x^3) - ((1 + c^2*
x^2)*ArcTan[c*x])/(2*c^2*x^2) + (I/2)*ArcTan[c*x]^2 - ArcTan[c*x]*Log[1 - E
^((2*I)*ArcTan[c*x])] - ((4*I)/3)*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (I/2)*(Arc
Tan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])))/d + (b^2*c^3*(Pi^3 - 8/(c
*x) + ((24*I)*ArcTan[c*x])/(c*x) - (8*(1 + c^2*x^2)*ArcTan[c*x])/(c^2*x^2)
+ (32*I)*ArcTan[c*x]^2 + (32*ArcTan[c*x]^2)/(c*x) - (8*(1 + c^2*x^2)*ArcTan
[c*x]^2)/(c^3*x^3) + ((12*I)*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*x^2) + (24*I
)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 64*ArcTan[c*x]*Log[1 - E^
((2*I)*ArcTan[c*x])] - (24*I)*Log[(c*x)/Sqrt[1 + c^2*x^2]] - 24*ArcTan[c*x]
```

```
*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (32*I)*PolyLog[2, E^((2*I)*ArcTan[c*x
])] + (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*x])]))/(24*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.14, size = 2256, normalized size = 6.18

method	result	size
derivativedivides	Expression too large to display	2256
default	Expression too large to display	2256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a^2/d*arctan(c*x)+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b*a/d*ar
ctan(c*x)-1/3*b^2/d*arctan(c*x)^2/c^3/x^3-1/3*b^2/d*arctan(c*x)/c^2/x^2+1/2
*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*b^2/d*Pi*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+b^2/d*arctan(c*x)^2/c/x+1/2*b^2/d*Pi*
csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)
^2-1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^3*arctan(c*x)^2-1/2*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+2*b*a/d*arctan(c*x)/c/x-1/2*b^2/d*Pi*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)
)*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*
x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-b*a/d*ln(c*x)*ln(1+I*c*x)+b*a/d*ln(c*x)*
ln(1-I*c*x)-4/3*b^2/d*arctan(c*x)+2/3*b^2/d*arctan(c*x)^3-1/3*a^2/d/c^3/x^3
+I*a^2/d*ln(c*x)-I*b^2/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-b*a
/d*dilog(1+I*c*x)+b*a/d*dilog(1-I*c*x)+a^2/d/c/x-8/3*b*a/d*ln(c*x)+2*b^2/d*
arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b^2/d*arctan(c*x)/c/x+
1/2*I*b^2/d*arctan(c*x)^2/c^2/x^2+2*I*b*a/d*arctan(c*x)*ln(c*x)-2*I*b*a/d*a
rctan(c*x)*ln(c*x-I)-2/3*b*a/d*arctan(c*x)/c^3/x^3+I*b*a/d/c/x+2*b^2/d*arct
an(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-8/3*b^2/d*arctan(c*x)*ln(1+
(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b*a/d*arctan(c*x)/c^2/x^2+I*b^2/d*arctan(c*x
)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*a^2/d*ln(c^2*x^2+1)+1/2*b*a/d*ln(
c*x-I)^2-b*a/d*dilog(-1/2*I*(c*x+I))+4/3*b*a/d*ln(c^2*x^2+1)-3/2*b^2/d*Pi*a
rctan(c*x)^2+11/6*I*b^2/d*arctan(c*x)^2+2*I*b^2/d*polylog(3,-(1+I*c*x)/(c^2
```

```
*x^2+1)^(1/2))-I*b^2/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+8/3*I*b^2/d*dilog(
1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1
/2))-8/3*I*b^2/d*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*Pi*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*b^2/d*Pi*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-b*a
/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+I*b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+I*b^2/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*
a^2/d/c^2/x^2+I*b^2/d*arctan(c*x)^2*ln(c*x)+1/3*I*b^2/d/(1+I*c*x+(c^2*x^2+1
)^(1/2))*(c^2*x^2+1)^(1/2)-I*b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)
-1)-I*b^2/d*arctan(c*x)^2*ln(c*x-I)-1/3*I*b^2/d/(I*c*x-(c^2*x^2+1)^(1/2)+1)
*(c^2*x^2+1)^(1/2)-1/3*b*a/d/c^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
2)/(d*x^3))*a^2 + 1/96*(24*b^2*c^3*x^3*arctan(c*x)^3 + 3*I*b^2*c^3*x^3*log(
c^2*x^2 + 1)^3 - 2*I*(1152*b^2*c^5*integrate(1/48*x^5*arctan(c*x)^2/(c^2*d*
x^6 + d*x^4), x) + b^2*c^3*log(c^2*x^2 + 1)^3/d + 12*b^2*c^3*arctan(c*x)^2/
d + 288*b^2*c^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x)
 + 192*b^2*c^2*integrate(1/48*x^2*arctan(c*x)/(c^2*d*x^6 + d*x^4), x) + 172
8*b^2*c*integrate(1/48*x*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x) + 144*b^2*c*
integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) + 4608*a*b*c*in
tegrate(1/48*x*arctan(c*x)/(c^2*d*x^6 + d*x^4), x) - 192*b^2*c*integrate(1/
48*x*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x) + 576*b^2*integrate(1/48*arct
an(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x))*d*x^3 - 16*(b^2*c^3*arcta
n(c*x)^3/d + 36*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*d*x^6 +
d*x^4), x) - 72*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*
x^4), x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^2*d*x^6 + d*x^4), x
) - 12*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x)
+ 72*b^2*c*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4
), x) - 48*b^2*c*integrate(1/48*x*arctan(c*x)/(c^2*d*x^6 + d*x^4), x) - 216
*b^2*integrate(1/48*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x) - 18*b^2*integrat
e(1/48*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) - 576*a*b*integrate(1/48*
arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*d*x^3 + 4*(6*b^2*c^2*x^2 + 3*I*b^2*c*x
 - 2*b^2)*arctan(c*x)^2 + (6*b^2*c^3*x^3*arctan(c*x) - 6*b^2*c^2*x^2 - 3*I*
b^2*c*x + 2*b^2)*log(c^2*x^2 + 1)^2 + 4*(3*I*b^2*c^3*x^3*arctan(c*x)^2 + (6
*I*b^2*c^2*x^2 - 3*b^2*c*x - 2*I*b^2)*arctan(c*x))*log(c^2*x^2 + 1))/(d*x^3
)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/24*(-6*I*b^2*c^3*x^3*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 - 1
2*I*b^2*c^3*x^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 12*
I*b^2*c^3*x^3*polylog(3, -(c*x + I)/(c*x - I)) + 24*d*x^3*integral(1/6*(-6*
I*a^2*c*x + 6*a^2 + (-6*I*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2*(3*a*b + I*b^2)*c
*x + 6*I*a*b)*log(-(c*x + I)/(c*x - I)))/(c^2*d*x^6 + d*x^4), x) - (6*b^2*c
^2*x^2 + 3*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2)/(d*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**4/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^4\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)), x)
```

3.103 $\int \frac{x^4(a+b\text{ArcTan}(cx))^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=433

$$\frac{2iabx}{c^4d^2}-\frac{b^2x}{3c^4d^2}+\frac{b^2}{2c^5d^2(i-cx)}-\frac{b^2\text{ArcTan}(cx)}{6c^5d^2}+\frac{2ib^2x\text{ArcTan}(cx)}{c^4d^2}+\frac{bx^2(a+b\text{ArcTan}(cx))}{3c^3d^2}+\frac{ib(a+b\text{ArcTan}}{c^5d^2(i-cx}$$

```
[Out] I*b*(a+b*arctan(c*x))/c^5/d^2/(I-c*x)-1/3*b^2*x/c^4/d^2+1/2*b^2/c^5/d^2/(I-
c*x)-1/6*b^2*arctan(c*x)/c^5/d^2+2*I*b^2*x*arctan(c*x)/c^4/d^2+1/3*b*x^2*(a
+b*arctan(c*x))/c^3/d^2+10/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5/d^2-2*I*b^2
*polylog(3,1-2/(1+I*c*x))/c^5/d^2+3*x*(a+b*arctan(c*x))^2/c^4/d^2-I*b^2*ln(
c^2*x^2+1)/c^5/d^2-1/3*x^3*(a+b*arctan(c*x))^2/c^2/d^2-(a+b*arctan(c*x))^2/
c^5/d^2/(I-c*x)+20/3*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^2+2*I*a*b*x/
c^4/d^2-I*x^2*(a+b*arctan(c*x))^2/c^3/d^2-4*I*(a+b*arctan(c*x))^2*ln(2/(1+I
*c*x))/c^5/d^2+4*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^5/d^2+11/6*
I*(a+b*arctan(c*x))^2/c^5/d^2
```

Rubi [A]
time = 0.60, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 18, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 327, 209, 4974, 4972, 641, 46, 5114, 6745}

$$\frac{4b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^5d^2}-\frac{(a+b\text{ArcTan}(cx))^2}{c^5d^2(-cx+i)}+\frac{11i(a+b\text{ArcTan}(cx))^2}{6c^5d^2}+\frac{ib(a+b\text{ArcTan}(cx))}{c^5d^2(-cx+i)}-\frac{4i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^5d^2}+\frac{20b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^5d^2}+\frac{3x(a+b\text{ArcTan}(cx))^2}{c^4d^2}-\frac{ix^2(a+b\text{ArcTan}(cx))^2}{c^3d^2}+\frac{bx^2(a+b\text{ArcTan}(cx))}{3c^3d^2}-\frac{x^3(a+b\text{ArcTan}(cx))^2}{3c^2d^2}+\frac{2iabx}{c^4d^2}-\frac{b^2\text{ArcTan}(cx)}{6c^5d^2}+\frac{2ib^2x\text{ArcTan}(cx)}{c^4d^2}+\frac{10ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^5d^2}-\frac{2ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{c^5d^2}+\frac{b^2}{2c^5d^2(-cx+i)}-\frac{b^2x}{3c^4d^2}-\frac{ib^2\log\left(c^2x^2+1\right)}{c^5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] ((2*I)*a*b*x)/(c^4*d^2) - (b^2*x)/(3*c^4*d^2) + b^2/(2*c^5*d^2*(I - c*x)) -
 (b^2*ArcTan[c*x])/(6*c^5*d^2) + ((2*I)*b^2*x*ArcTan[c*x])/(c^4*d^2) + (b*x
^2*(a + b*ArcTan[c*x]))/(3*c^3*d^2) + (I*b*(a + b*ArcTan[c*x]))/(c^5*d^2*(I
 - c*x)) + (((11*I)/6)*(a + b*ArcTan[c*x])^2)/(c^5*d^2) + (3*x*(a + b*ArcTa
n[c*x])^2)/(c^4*d^2) - (I*x^2*(a + b*ArcTan[c*x])^2)/(c^3*d^2) - (x^3*(a +
b*ArcTan[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTan[c*x])^2/(c^5*d^2*(I - c*x)) +
 (20*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^5*d^2) - ((4*I)*(a + b*
ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^5*d^2) - (I*b^2*Log[1 + c^2*x^2])/(c^
5*d^2) + (((10*I)/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) + (4*b*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) - ((2*I)*b^2*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4(a+b\tan^{-1}(cx))^2}{(d+icdx)^2}\,dx = \int \left(\frac{3(a+b\tan^{-1}(cx))^2}{c^4d^2} - \frac{2ix(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))^2}{c^2d^2} - \right.$$

$$= \frac{(4i)\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{c^4d^2} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{c^4d^2} + \frac{3\int (a+b\tan^{-1}(cx))^2\,dx}{c^4d^2} -$$

$$= \frac{3x(a+b\tan^{-1}(cx))^2}{c^4d^2} - \frac{ix^2(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x^3(a+b\tan^{-1}(cx))^2}{3c^2d^2} - \frac{(a+}{c}$$

$$= \frac{3i(a+b\tan^{-1}(cx))^2}{c^5d^2} + \frac{3x(a+b\tan^{-1}(cx))^2}{c^4d^2} - \frac{ix^2(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x^3(a}{}$$

$$= \frac{2iabx}{c^4d^2} + \frac{bx^2(a+b\tan^{-1}(cx))}{3c^3d^2} + \frac{ib(a+b\tan^{-1}(cx))}{c^5d^2(i-cx)} + \frac{11i(a+b\tan^{-1}(cx))^2}{6c^5d^2}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2(a+b\tan^{-1}(cx))}{3c^3d^2} + \frac{ib(a+b\tan^{-1}(cx}{c^5d^2(i-cx)}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2\tan^{-1}(cx)}{3c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2(a+b\tan^{-1}(cx))}{3c^3d^2} + \frac{i}{}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2}{2c^5d^2(i-cx)} + \frac{b^2\tan^{-1}(cx)}{3c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2(a+}{3}$$

$$= \frac{2iabx}{c^4d^2} - \frac{b^2x}{3c^4d^2} + \frac{b^2}{2c^5d^2(i-cx)} - \frac{b^2\tan^{-1}(cx)}{6c^5d^2} + \frac{2ib^2x\tan^{-1}(cx)}{c^4d^2} + \frac{bx^2(a+}{3}$$

Mathematica [A]
time = 1.59, size = 502, normalized size = 1.16

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -1/12*(-36*a^2*c*x + (12*I)*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(-I + c*
x) + 48*a^2*ArcTan[c*x] - (24*I)*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2 - (12*I)*
c*x - 2*c^2*x^2 + 48*ArcTan[c*x]^2 - 3*Cos[2*ArcTan[c*x]] + 20*Log[1 + c^2*
x^2] + 24*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(6*I - 18*c*x
+ (6*I)*c^2*x^2 + 2*c^3*x^3 - (3*I)*Cos[2*ArcTan[c*x]] + (24*I)*Log[1 + E^(
(2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]]) + (3*I)*Sin[2*ArcTan[c*x]]) + b
^2*(4*c*x - 4*ArcTan[c*x] - (24*I)*c*x*ArcTan[c*x] - 4*c^2*x^2*ArcTan[c*x]
+ (52*I)*ArcTan[c*x]^2 - 36*c*x*ArcTan[c*x]^2 + (12*I)*c^2*x^2*ArcTan[c*x]^
```

```
2 + 4*c^3*x^3*ArcTan[c*x]^2 + 32*ArcTan[c*x]^3 + (3*I)*Cos[2*ArcTan[c*x]] -
 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]]
- 80*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (48*I)*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] + (12*I)*Log[1 + c^2*x^2] + 8*(5*I + 6*ArcTan[c*
x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (24*I)*PolyLog[3, -E^((2*I)*ArcTan
[c*x])] + 3*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 6*A
rcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(c^5*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.06, size = 1372, normalized size = 3.17

method	result	size
derivativedivides	Expression too large to display	1372
default	Expression too large to display	1372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(8*I*b*a/d^2*arctan(c*x)*ln(c*x-I)+7/3*b*a/d^2-1/3*I*b^2/d^2-1/2*b^2/
d^2*arctan(c*x)/(c*x-I)*c*x-I*b^2/d^2*arctan(c*x)^2*c^2*x^2+2*I*b^2/d^2*arc
tan(c*x)*c*x+2*I*b^2/d^2/(8*c*x-8*I)*c*x-2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan
(c*x)^2+2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+6*b*a/d^2*arctan(c*
x)*c*x-2/3*b*a/d^2*arctan(c*x)*c^3*x^3+2*I*b*a/d^2*c*x-11/24*b*a/d^2*ln(c^4
*x^4+10*c^2*x^2+9)+20/3*b^2/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))+20/3*b^2/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*b^2/d^2
*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/3*b^2/d^2*c*x+3*a^2/d^2*
c*x-1/3*a^2/d^2*c^3*x^3+2*I*a^2/d^2*ln(c^2*x^2+1)-2*I*b^2/d^2*polylog(3,-(1
+I*c*x)^2/(c^2*x^2+1))-29/6*I*b^2/d^2*arctan(c*x)^2-20/3*I*b^2/d^2*dilog(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2/d^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-20
/3*I*b^2/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*b*a/d^2*ln(c*x-I)^2+4
*b*a/d^2*dilog(-1/2*I*(c*x+I))-29/12*b*a/d^2*ln(c^2*x^2+1)+3*b^2/d^2*arctan
(c*x)^2*c*x-1/3*b^2/d^2*arctan(c*x)^2*c^3*x^3+1/3*b^2/d^2*arctan(c*x)*c^2*x
^2+1/3*b*a/d^2*c^2*x^2-I*a^2/d^2*c^2*x^2-2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-4*b^2/d^2*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-4*I*b^2/d^
2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+4*I*b^2/d^2*arctan(c*x)^2*l
n(c*x-I)-1/2*I*b^2/d^2*arctan(c*x)/(c*x-I)+2*b*a/d^2*arctan(c*x)/(c*x-I)+4*
b*a/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+11/12*I*b*a/d^2*arctan(1/6*c^3*x^3+7/6
*c*x)+11/6*I*b*a/d^2*arctan(1/2*c*x-1/2*I)-11/12*I*b*a/d^2*arctan(1/2*c*x)+
4*b^2/d^2*Pi*arctan(c*x)^2+b^2/d^2*arctan(c*x)^2/(c*x-I)+7/3*b^2/d^2*arctan
(c*x)-2*b^2/d^2/(8*c*x-8*I)-8/3*b^2/d^2*arctan(c*x)^3+a^2/d^2/(c*x-I)-4*a^2
/d^2*arctan(c*x)-I*b*a/d^2/(c*x-I)-29/6*I*b*a/d^2*arctan(c*x)-2*I*b*a/d^2*a
rctan(c*x)*c^2*x^2-2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x
```

```
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3/(c^6*d^2*x - I*c^5*d^2) - (c^2*x^3 + 3*I*c*x^2 - 9*x)/(c^4*d^2)
+ 12*I*log(c*x - I)/(c^5*d^2)) - 1/48*(48*(b^2*c*x - I*b^2)*arctan(c*x)^3 -
 6*(-I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 4*(b^2*c^4*x^4 + 2*I*b^2*c^3*x^3
 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2)*arctan(c*x)^2 - (b^2*c^4*x^4 + 2*I*
b^2*c^3*x^3 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2 - 12*(b^2*c*x - I*b^2)*ar
ctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(c^6*d^2*x - I*c^5*d^2)*(288*b^2*c^6*inte
grate(1/48*x^6*arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) +
24*b^2*c^6*integrate(1/48*x^6*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x
^2 + c^4*d^2), x) + 768*a*b*c^6*integrate(1/48*x^6*arctan(c*x)/(c^8*d^2*x^4
 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 32*b^2*c^6*integrate(1/48*x^6*log(c^2*x^2
 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 192*b^2*c^5*integrate(1
/48*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2
), x) + 128*b^2*c^5*integrate(1/48*x^5*arctan(c*x)/(c^8*d^2*x^4 + 2*c^6*d^2
*x^2 + c^4*d^2), x) - 288*b^2*c^4*integrate(1/48*x^4*arctan(c*x)^2/(c^8*d^2
*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*b^2*c^4*integrate(1/48*x^4*log(c^2
*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 768*a*b*c^4*integ
rate(1/48*x^4*arctan(c*x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 160
*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 +
 c^4*d^2), x) + 704*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^8*d^2*x^4 + 2
*c^6*d^2*x^2 + c^4*d^2), x) - 768*b^2*c^2*integrate(1/48*x^2*arctan(c*x)^2/
(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 192*b^2*c^2*integrate(1/48*x^
2*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 288*b^2*
c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*
d^2), x) + 3*(c*(x/(c^8*d^2*x^2 + c^6*d^2) + arctan(c*x)/(c^7*d^2)) - 2*arc
tan(c*x)/(c^8*d^2*x^2 + c^6*d^2))*b^2*c - 768*b^2*integrate(1/48*arctan(c*x
)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 192*b^2*integrate(1/48*lo
g(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 96*b^2*integ
rate(1/48*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x)) + 6
*(-I*c^6*d^2*x - c^5*d^2)*(48*b^2*c^6*integrate(1/24*x^6*arctan(c*x)*log(c^
2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 32*b^2*c^6*integra
te(1/24*x^6*arctan(c*x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 288*b
^2*c^5*integrate(1/24*x^5*arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*
d^2), x) - 24*b^2*c^5*integrate(1/24*x^5*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 +
2*c^6*d^2*x^2 + c^4*d^2), x) - 768*a*b*c^5*integrate(1/24*x^5*arctan(c*x)/(
```

```
c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 32*b^2*c^5*integrate(1/24*x^5*
log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 48*b^2*c^4*i
ntegrate(1/24*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2
 + c^4*d^2), x) - 160*b^2*c^4*integrate(1/24*x^4*arctan(c*x)/(c^8*d^2*x^4 +
 2*c^6*d^2*x^2 + c^4*d^2), x) - 4*b^2*c^3*(c^2/(c^12*d^2*x^2 + c^10*d^2) +
log(c^2*x^2 + 1)/(c^10*d^2*x^2 + c^8*d^2)) + 384*b^2*c^3*integrate(1/24*x^3
*arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 96*b^2*c^3*int
egrate(1/24*x^3*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2),
 x) - 176*b^2*c^3*integrate(1/24*x^3*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*
d^2*x^2 + c^4*d^2), x) - 288*b^2*c^2*integrate(1/24*x^2*arctan(c*x)/(c^8*d^
2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 3*b^2*c*(c^2/(c^10*d^2*x^2 + c^8*d^2
) + log(c^2*x^2 + 1)/(c^8*d^2*x^2 + c^6*d^2)) + 384*b^2*c*integrate(1/24*x*
arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 2*b^2*c*log(c^2
*x^2 + 1)^2/(c^8*d^2*x^2 + c^6*d^2) - 96*b^2*integrate(1/24*arctan(c*x)/(c^
8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x)) - 4*(6*(-I*b^2*c*x - b^2)*arctan(
c*x)^2 + (-I*b^2*c^4*x^4 + 2*b^2*c^3*x^3 + 6*I*b^2*c^2*x^2 + 9*b^2*c*x + 3*
I*b^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^6*d^2*x - I*c^5*d^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^4*log(-(c*x +
 I)/(c*x - I)) - 4*a^2*x^4)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(a+b \operatorname{atan}(c x)\right)^2}{\left(d+c d x 1 \mathrm{i}\right)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
[Out] int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.104 $\int \frac{x^3(a+b\text{ArcTan}(cx))^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=364

$$\frac{abx}{c^3d^2}-\frac{ib^2}{2c^4d^2(i-cx)}+\frac{ib^2\text{ArcTan}(cx)}{2c^4d^2}+\frac{b^2x\text{ArcTan}(cx)}{c^3d^2}+\frac{b(a+b\text{ArcTan}(cx))}{c^4d^2(i-cx)}+\frac{(a+b\text{ArcTan}(cx))^2}{c^4d^2}-\frac{2ix(a+$$

```
[Out] a*b*x/c^3/d^2-1/2*I*b^2/c^4/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/c^4/d^2+b^2*x
*arctan(c*x)/c^3/d^2+b*(a+b*arctan(c*x))/c^4/d^2/(I-c*x)+(a+b*arctan(c*x))^
2/c^4/d^2-2*I*x*(a+b*arctan(c*x))^2/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))^2/c^2
/d^2+I*(a+b*arctan(c*x))^2/c^4/d^2/(I-c*x)-4*I*b*(a+b*arctan(c*x))*ln(2/(1+
I*c*x))/c^4/d^2-3*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^2-1/2*b^2*ln(c^
2*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3*I*b*(a+b*arctan(c
*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3/2*b^2*polylog(3,1-2/(1+I*c*x))/c^4/
d^2
```

Rubi [A]

time = 0.44, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 4974, 4972, 641, 46, 209, 5114, 6745}

$$-\frac{3ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^4d^2}+\frac{b(a+b\text{ArcTan}(cx))}{c^4d^2(-cx+i)}+\frac{i(a+b\text{ArcTan}(cx))^2}{c^4d^2(-cx+i)}+\frac{(a+b\text{ArcTan}(cx))^2}{c^4d^2}-\frac{4ib\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4d^2}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^4d^2}-\frac{2ix(a+b\text{ArcTan}(cx))^2}{c^3d^2}-\frac{x^2(a+b\text{ArcTan}(cx))^2}{2c^2d^2}+\frac{abx}{c^3d^2}+\frac{ib^2\text{ArcTan}(cx)}{2c^4d^2}+\frac{b^2x\text{ArcTan}(cx)}{c^3d^2}+\frac{2b^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^4d^2}-\frac{3b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^4d^2}-\frac{ib^2}{2c^4d^2(-cx+i)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] (a*b*x)/(c^3*d^2) - ((I/2)*b^2)/(c^4*d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x
])/(c^4*d^2) + (b^2*x*ArcTan[c*x])/(c^3*d^2) + (b*(a + b*ArcTan[c*x]))/(c^4
*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(c^4*d^2) - ((2*I)*x*(a + b*ArcTan[
c*x])^2)/(c^3*d^2) - (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d^2) + (I*(a + b*Ar
cTan[c*x])^2)/(c^4*d^2*(I - c*x)) - ((4*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c^4*d^2) - (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^2
) - (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 + I*c*x
)])/(c^4*d^2) - ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])
/(c^4*d^2) - (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^4*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))^2}{(d+icdx)^2}\,dx = \int \left(-\frac{2i(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{c^3d^2(-i+cx)^2} + \right.$$

$$= \frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{c^3d^2} - \frac{(2i)\int (a+b\tan^{-1}(cx))^2\,dx}{c^3d^2} + \frac{3\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{c^3d^2} -$$

$$= -\frac{2ix(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))^2}{2c^2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{c^4d^2(i-cx)} -$$

$$= \frac{2(a+b\tan^{-1}(cx))^2}{c^4d^2} - \frac{2ix(a+b\tan^{-1}(cx))^2}{c^3d^2} - \frac{x^2(a+b\tan^{-1}(cx))^2}{2c^2d^2} +$$

$$= \frac{abx}{c^3d^2} + \frac{b(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{c^4d^2} - \frac{2ix(a+b\tan^{-1}(cx))^2}{c^3d^2} -$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{c^4d^2} -$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{c^4d^2} -$$

$$= \frac{abx}{c^3d^2} - \frac{ib^2}{2c^4d^2(i-cx)} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)} +$$

$$= \frac{abx}{c^3d^2} - \frac{ib^2}{2c^4d^2(i-cx)} + \frac{ib^2\tan^{-1}(cx)}{2c^4d^2} + \frac{b^2x\tan^{-1}(cx)}{c^3d^2} + \frac{b(a+b\tan^{-1}(cx))}{c^4d^2(i-cx)}$$

Mathematica [A]
time = 1.21, size = 429, normalized size = 1.18

$$-\frac{8ia^2cx+2a^2c^2x^2+\frac{4a^2}{-i+cx}-12ia^2\text{ArcTan}(cx)-6a^2\log(1+c^2x^2)+2ab\left(-2cx-12i\text{ArcTan}(cx)^2+i\cos(2\text{ArcTan}(cx))-4i\log(1+c^2x^2)-6i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+2\text{ArcTan}(cx)\left(1+4icx+c^2x^2-\cos(2\text{ArcTan}(cx))+6\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+i\sin(2\text{ArcTan}(cx))\right)+\sin(2\text{ArcTan}(cx))\right)+b^2\left(-4cx\text{ArcTan}(cx)+10\text{ArcTan}(cx)^2+8icx\text{ArcTan}(cx)^2+2c^2x^2\text{ArcTan}(cx)^2-8i\text{ArcTan}(cx)^3+\cos(2\text{ArcTan}(cx))+2i\text{ArcTan}(cx)\cos(2\text{ArcTan}(cx))-2\text{ArcTan}(cx)^2\cos(2\text{ArcTan}(cx))+16i\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+12\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+2\log(1+c^2x^2)+4(2-3i\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+6\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)-i\sin(2\text{ArcTan}(cx))+2\text{ArcTan}(cx)\sin(2\text{ArcTan}(cx))+2i\text{ArcTan}(cx)^2\sin(2\text{ArcTan}(cx))\right)}{4c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -1/4*((8*I)*a^2*c*x + 2*a^2*c^2*x^2 + ((4*I)*a^2)/(-I + c*x) - (12*I)*a^2*A
rcTan[c*x] - 6*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2*c*x - (12*I)*ArcTan[c*x]^2
+ I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I
)*ArcTan[c*x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x
]] + 6*Log[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTa
n[c*x]]) + b^2*(-4*c*x*ArcTan[c*x] + 10*ArcTan[c*x]^2 + (8*I)*c*x*ArcTan[c*
x]^2 + 2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*ArcTan[c*x]^3 + Cos[2*ArcTan[c*x]] +
 (2*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 2*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]]
+ (16*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] + 2*Log[1 + c^2*x^2] + 4*(2 - (3*I)*ArcTan[c*x])
*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[c*x])]
- I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (2*I)*ArcTan[c*
x]^2*Sin[2*ArcTan[c*x]]))/(c^4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.54, size = 1274, normalized size = 3.50

method	result	size
derivativedivides	Expression too large to display	1274
default	Expression too large to display	1274

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-b*a/d^2*arctan(c*x)*c^2*x^2+3/2*a^2/d^2*ln(c^2*x^2+1)-3*b^2/d^2*arc
tan(c*x)^2-3/2*b^2/d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+b^2/d^2*ln((1+I*
c*x)^2/(c^2*x^2+1)+1)-4*b^2/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*b^
2/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b*a/d^2*arctan(c*x)*ln(c*x-I
)+3/2*I*b*a/d^2*ln(c^2*x^2+1)-3*I*b*a/d^2*dilog(-1/2*I*(c*x+I))+3/2*I*b*a/d
^2*ln(c*x-I)^2+1/4*I*b*a/d^2*ln(c^4*x^4+10*c^2*x^2+9)-3/2*I*b^2/d^2*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^
2-4*I*b*a/d^2*arctan(c*x)*c*x+I*b^2/d^2*arctan(c*x)/(2*c*x-2*I)*c*x-I*b*a/d
^2-1/2*a^2/d^2*c^2*x^2+3*I*a^2/d^2*arctan(c*x)-I*a^2/d^2/(c*x-I)-3*b^2/d^2*
arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-b^2/d^2*arctan(c*x)/(2*c*x-2*
I)+3*b^2/d^2*arctan(c*x)^2*ln(c*x-I)-3*I*b*a/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-
I)-2*I*b*a/d^2*arctan(c*x)/(c*x-I)+3*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*b^2/d^2*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-2*I*b^2/d^
2*arctan(c*x)^2*c*x+3/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+
```

```
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*arctan(c*x)^2-I*b^2/d^2*arctan(c*x)+2*I*b^2/d^2*arctan(c*x)^3+1/
4*I*b^2/d^2/(c*x-I)-b*a/d^2/(c*x-I)-3*b*a/d^2*arctan(c*x)-1/2*b*a/d^2*arcta
n(1/2*c*x)+1/2*b*a/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+b*a/d^2*arctan(1/2*c*x-1
/2*I)-2*I*a^2/d^2*c*x-1/2*b^2/d^2*arctan(c*x)^2*c^2*x^2+1/4*b^2/d^2/(c*x-I)
*c*x+b^2/d^2*arctan(c*x)*c*x-4*I*b^2/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+3*I*b^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3
*I*b^2/d^2*Pi*arctan(c*x)^2-4*I*b^2/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))-I*b^2/d^2*arctan(c*x)^2/(c*x-I)+b*a/d^2*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(c
*x - I)/(c^4*d^2)) + 1/32*(24*(I*b^2*c*x + b^2)*arctan(c*x)^3 - 3*(b^2*c*x
- I*b^2)*log(c^2*x^2 + 1)^3 - 4*(b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c*x
+ 2*I*b^2)*arctan(c*x)^2 + (b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c*x + 2*I
*b^2 + 6*(I*b^2*c*x + b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 2*(c^5*d^2*x -
 I*c^4*d^2)*(192*b^2*c^5*integrate(1/16*x^5*arctan(c*x)^2/(c^7*d^2*x^4 + 2*
c^5*d^2*x^2 + c^3*d^2), x) + 16*b^2*c^5*integrate(1/16*x^5*log(c^2*x^2 + 1)
^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 512*a*b*c^5*integrate(1/16
*x^5*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 32*b^2*c^5*i
ntegrate(1/16*x^5*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2),
 x) + 128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^7*d^2*
x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 192*b^2*c^4*integrate(1/16*x^4*arctan(
c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 6*b^2*c^3*(c^2/(c^11*d^2
*x^2 + c^9*d^2) + log(c^2*x^2 + 1)/(c^9*d^2*x^2 + c^7*d^2)) - 576*b^2*c^3*i
ntegrate(1/16*x^3*arctan(c*x)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)
 - 112*b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^7*d^2*x^4 + 2*c^5*d
^2*x^2 + c^3*d^2), x) - 512*a*b*c^3*integrate(1/16*x^3*arctan(c*x)/(c^7*d^2
*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 160*b^2*c^3*integrate(1/16*x^3*log(c^
2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 320*b^2*c^2*integr
ate(1/16*x^2*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 4*b^
2*c*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d^2*x^2 + c^5*d^2)
) - 384*b^2*c*integrate(1/16*x*arctan(c*x)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 +
 c^3*d^2), x) + 3*b^2*c*log(c^2*x^2 + 1)^2/(c^7*d^2*x^2 + c^5*d^2) + 128*b^
2*integrate(1/16*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) -
 8*(-I*c^5*d^2*x - c^4*d^2)*(8*b^2*c^5*integrate(1/8*x^5*arctan(c*x)*log(c^
2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 8*b^2*c^5*integrat
e(1/8*x^5*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 48*b^2*
```

```
c^4*integrate(1/8*x^4*arctan(c*x)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)
, x) - 4*b^2*c^4*integrate(1/8*x^4*log(c^2*x^2 + 1)^2/(c^7*d^2*x^4 + 2*c^5*
d^2*x^2 + c^3*d^2), x) - 128*a*b*c^4*integrate(1/8*x^4*arctan(c*x)/(c^7*d^2
*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 12*b^2*c^4*integrate(1/8*x^4*log(c^2*
x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 8*b^2*c^3*integrate(
1/8*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2
), x) + 40*b^2*c^3*integrate(1/8*x^3*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x
^2 + c^3*d^2), x) - 48*b^2*c^2*integrate(1/8*x^2*arctan(c*x)^2/(c^7*d^2*x^4
 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 12*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2
+ 1)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 20*b^2*c^2*integrate(1
/8*x^2*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + (c*(x
/(c^7*d^2*x^2 + c^5*d^2) + arctan(c*x)/(c^6*d^2)) - 2*arctan(c*x)/(c^7*d^2*
x^2 + c^5*d^2))*b^2*c - 48*b^2*integrate(1/8*arctan(c*x)^2/(c^7*d^2*x^4 + 2
*c^5*d^2*x^2 + c^3*d^2), x) - 12*b^2*integrate(1/8*log(c^2*x^2 + 1)^2/(c^7*
d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 8*b^2*integrate(1/8*log(c^2*x^2 +
1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) - 4*(3*(b^2*c*x - I*b^2)*ar
ctan(c*x)^2 - (-I*b^2*c^3*x^3 + 3*b^2*c^2*x^2 - 4*I*b^2*c*x + 2*b^2)*arctan
(c*x))*log(c^2*x^2 + 1))/(c^5*d^2*x - I*c^4*d^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^3*log(-(c*x +
 I)/(c*x - I)) - 4*a^2*x^3)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

```
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.105 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{b^2}{2c^3d^2(i-cx)}+\frac{b^2\text{ArcTan}(cx)}{2c^3d^2}-\frac{ib(a+b\text{ArcTan}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\text{ArcTan}(cx))^2}{2c^3d^2}-\frac{x(a+b\text{ArcTan}(cx))^2}{c^2d^2}+\frac{(a+}{c^3}$$

```
[Out] -1/2*b^2/c^3/d^2/(I-c*x)+1/2*b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arctan(c*x))/
c^3/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c^3/d^2-x*(a+b*arctan(c*x))^2/c^2
/d^2+(a+b*arctan(c*x))^2/c^3/d^2/(I-c*x)-2*b*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c^3/d^2+2*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d^2-I*b^2*polylog(2
,1-2/(1+I*c*x))/c^3/d^2-2*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3/
d^2+I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^2
```

Rubi [A]
time = 0.37, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4974, 4972, 641, 46, 209, 5004, 5114, 6745}

$$-\frac{2b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^3d^2}-\frac{ib(a+b\text{ArcTan}(cx))}{c^3d^2(-cx+i)}+\frac{(a+b\text{ArcTan}(cx))^2}{c^3d^2(-cx+i)}-\frac{i(a+b\text{ArcTan}(cx))^2}{2c^3d^2}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^3d^2}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^3d^2}-\frac{x(a+b\text{ArcTan}(cx))^2}{c^2d^2}+\frac{b^2\text{ArcTan}(cx)}{2c^3d^2}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^3d^2}+\frac{ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{b^2}{2c^3d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -1/2*b^2/(c^3*d^2*(I - c*x)) + (b^2*ArcTan[c*x])/(2*c^3*d^2) - (I*b*(a + b*
ArcTan[c*x]))/(c^3*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^2)
 - (x*(a + b*ArcTan[c*x])^2)/(c^2*d^2) + (a + b*ArcTan[c*x])^2/(c^3*d^2*(I
- c*x)) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + ((2*I)*(
a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^2) - (I*b^2*PolyLog[2, 1 -
2/(1 + I*c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^3*d^2) + (I*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{(d+icdx)^2}\,dx = \int \left(-\frac{(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\tan^{-1}(cx))^2}{c^2d^2(-i+cx)^2}-\frac{2i(a+b\tan^{-1}(cx))^2}{c^2d^2(-i+cx)}\right)dx$$

$$= -\frac{(2i)\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}dx}{c^2d^2}-\frac{\int (a+b\tan^{-1}(cx))^2\,dx}{c^2d^2}+\frac{\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}dx}{c^2d^2}$$

$$= -\frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\tan^{-1}(cx))^2}{c^3d^2(i-cx)}+\frac{2i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^2}$$

$$= -\frac{i(a+b\tan^{-1}(cx))^2}{c^3d^2}-\frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\tan^{-1}(cx))^2}{c^3d^2(i-cx)}+\frac{2i(a+b$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^2}-\frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\,t}{c^3d^2}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^2}-\frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\,t}{c^3d^2}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^2}-\frac{x(a+b\tan^{-1}(cx))^2}{c^2d^2}+\frac{(a+b\,t}{c^3d^2}$$

$$= -\frac{b^2}{2c^3d^2(i-cx)}-\frac{ib(a+b\tan^{-1}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^2}-\frac{x(a+b\tan^{-1}}{c^2d^2}$$

$$= -\frac{b^2}{2c^3d^2(i-cx)}+\frac{b^2\tan^{-1}(cx)}{2c^3d^2}-\frac{ib(a+b\tan^{-1}(cx))}{c^3d^2(i-cx)}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^2}$$

Mathematica [A]
time = 0.82, size = 362, normalized size = 1.24

$$-\frac{12a^2cx+\frac{12a^2}{-i+cx}-24a^2\text{ArcTan}(cx)+12ia^2\log(1+c^2x^2)+b^2\left(-12i\text{ArcTan}(cx)^2+12cx\text{ArcTan}(cx)^2-16\text{ArcTan}(cx)^3-3i\cos(2\text{ArcTan}(cx))+6\text{ArcTan}(cx)\cos(2\text{ArcTan}(cx))+6i\text{ArcTan}(cx)^2\cos(2\text{ArcTan}(cx))+24\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-24i\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-12(i+2\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)-12i\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)-3\sin(2\text{ArcTan}(cx))-6i\text{ArcTan}(cx)\sin(2\text{ArcTan}(cx))+6\text{ArcTan}(cx)^2\sin(2\text{ArcTan}(cx))\right)+6ab\left(-8\text{ArcTan}(cx)^2+\cos(2\text{ArcTan}(cx))-2\log(1+c^2x^2)-4\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)-i\sin(2\text{ArcTan}(cx))+2\text{ArcTan}(cx)\left(2cx+i\cos(2\text{ArcTan}(cx))-4i\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+\sin(2\text{ArcTan}(cx))\right)\right)}{12c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] -1/12*(12*a^2*c*x + (12*a^2)/(-I + c*x) - 24*a^2*ArcTan[c*x] + (12*I)*a^2*L
og[1 + c^2*x^2] + b^2*((-12*I)*ArcTan[c*x]^2 + 12*c*x*ArcTan[c*x]^2 - 16*Ar
cTan[c*x]^3 - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]] +
 (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]*Log[1 + E^((2*I)*A
rcTan[c*x])] - (24*I)*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 12*(I
+ 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (12*I)*PolyLog[3, -E^
((2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTa
n[c*x]] + 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*(-8*ArcTan[c*x]^2 + C
```

```
os[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*PolyLog[2, -E^((2*I)*ArcTan[c*x]
)] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*x + I*Cos[2*ArcTan[c*x]] - (
4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(c^3*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.05, size = 4552, normalized size = 15.59

method	result	size
derivativedivides	Expression too large to display	4552
default	Expression too large to display	4552

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-4*I*b*a/d^2*arctan(c*x)*ln(c*x-I)+1/2*b^2/d^2*arctan(c*x)/(c*x-I)*c
*x+b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*
arctan(c*x)^2-2*b*a/d^2*arctan(c*x)*c*x+1/2*I*b^2/d^2*arctan(c*x)/(c*x-I)+2
*I*b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2*b^2/d^2*Pi*csgn(
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))+b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2/d^2*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*polylog(2,-(1+I*c*x)^2/(
c^2*x^2+1))-2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d^2*Pi*csgn((1+I*c*x)^
2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*
b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arct
an(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d^2*Pi*csgn(
(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(
1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(
1+I*c*x)^2/(c^2*x^2+1))-1/2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog(2,-(1+I
*c*x)^2/(c^2*x^2+1))+1/8*b*a/d^2*ln(c^4*x^4+10*c^2*x^2+9)-I*b^2/d^2*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
```

```
2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1-I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))+I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1))*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-I*b^2/d^2*Pi*csgn((1+I*c*x)^
2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))+b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b
^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2/d^2
*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d^2*Pi*arctan(c
*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*b^2/d^2*Pi*dilog(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+2*b^2/d^2*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/d^2*Pi*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*b^2/d^2*arctan(c*x)*ln((1+I*c*x)^2/(
c^2*x^2+1)+1)+I*b^2/d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*b^2/d^2*a
rctan(c*x)^2+3/4*I*b^2/d^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2/d^
2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2/d^2*dilog(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))+1/2*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2
+1)+1))*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2
/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/d^2*arctan(c*x
)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d^2*arctan(c*x)*polylog(2,-(1+I
*c*x)^2/(c^2*x^2+1))-a^2/d^2*c*x+b*a/d^2*ln(c*x-I)^2-2*b*a/d^2*dilog(-1/2*I
*(c*x+I))+3/4*b*a/d^2*ln(c^2*x^2+1)-b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^2/
d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b*a/d^2*arctan(c*x)+1/4*I*b*a/d^2*arct
an(1/2*c*x)-1/4*I*b*a/d^2*arctan(1/6*c^3*x^3+7/6*c*x)-1/2*I*b*a/d^2*arctan(
1/2*c*x-1/2*I)-b^2/d^2*arctan(c*x)^2*c*x+b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+2*b^2/d^2*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-2*b*a/d^2*ar
ctan(c*x)/(c*x-I)-2*b*a/d^2*ln(c*x-I)*ln(-1/2*I...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2))
 + 1/16*(8*(b^2*c*x - I*b^2)*arctan(c*x)^3 - (-I*b^2*c*x - b^2)*log(c^2*x^2
 + 1)^3 - 4*(b^2*c^2*x^2 - I*b^2*c*x + b^2)*arctan(c*x)^2 + (b^2*c^2*x^2 -
I*b^2*c*x + b^2 + 2*(b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 2*(
c^4*d^2*x - I*c^3*d^2)*(96*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^6*d^
2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 8*b^2*c^4*integrate(1/16*x^4*log(c^2
*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 256*a*b*c^4*integ
rate(1/16*x^4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 32*
b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 +
c^2*d^2), x) + 64*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(
c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 64*b^2*c^3*integrate(1/16*x^3*
arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 32*b^2*c^2*integr
ate(1/16*x^2*arctan(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 24
*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2
 + c^2*d^2), x) - 256*a*b*c^2*integrate(1/16*x^2*arctan(c*x)/(c^6*d^2*x^4 +
 2*c^4*d^2*x^2 + c^2*d^2), x) + 64*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 +
 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - (c*(x/(c^6*d^2*x^2 + c^4*
d^2) + arctan(c*x)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2))*b^2*
c + 128*b^2*integrate(1/16*arctan(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2
*d^2), x) + 32*b^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d
^2*x^2 + c^2*d^2), x) + 32*b^2*integrate(1/16*log(c^2*x^2 + 1)/(c^6*d^2*x^4
 + 2*c^4*d^2*x^2 + c^2*d^2), x)) - 2*(-I*c^4*d^2*x - c^3*d^2)*(16*b^2*c^4*i
ntegrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2
+ c^2*d^2), x) + 32*b^2*c^4*integrate(1/8*x^4*arctan(c*x)/(c^6*d^2*x^4 + 2*
c^4*d^2*x^2 + c^2*d^2), x) + 2*b^2*c^3*(c^2/(c^10*d^2*x^2 + c^8*d^2) + log(
c^2*x^2 + 1)/(c^8*d^2*x^2 + c^6*d^2)) - 160*b^2*c^3*integrate(1/8*x^3*arcta
n(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 24*b^2*c^3*integrate
(1/8*x^3*log(c^2*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 2
56*a*b*c^3*integrate(1/8*x^3*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2
*d^2), x) + 16*b^2*c^3*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*
c^4*d^2*x^2 + c^2*d^2), x) - 16*b^2*c^2*integrate(1/8*x^2*arctan(c*x)*log(c
^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 64*b^2*c^2*integr
ate(1/8*x^2*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - b^2*c
*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d^2*x^2 + c^4*d^2)) -
 64*b^2*c*integrate(1/8*x*arctan(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*
d^2), x) + b^2*c*log(c^2*x^2 + 1)^2/(c^6*d^2*x^2 + c^4*d^2) + 32*b^2*integr
ate(1/8*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)) + 4*((I*b^
2*c*x + b^2)*arctan(c*x)^2 + (-I*b^2*c^2*x^2 - b^2*c*x - I*b^2)*arctan(c*x)
)*log(c^2*x^2 + 1))/(c^4*d^2*x - I*c^3*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^2*log(-(c*x +
 I)/(c*x - I)) - 4*a^2*x^2)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

```
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.106 $\int \frac{x(a+b\mathbf{ArcTan}(cx))^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=216

$$\frac{ib^2}{2c^2d^2(i-cx)}-\frac{ib^2\text{ArcTan}(cx)}{2c^2d^2}-\frac{b(a+b\text{ArcTan}(cx))}{c^2d^2(i-cx)}+\frac{(a+b\text{ArcTan}(cx))^2}{2c^2d^2}-\frac{i(a+b\text{ArcTan}(cx))^2}{c^2d^2(i-cx)}+\frac{(a+b\text{Ar}}{}$$

```
[Out] 1/2*I*b^2/c^2/d^2/(I-c*x)-1/2*I*b^2*arctan(c*x)/c^2/d^2-b*(a+b*arctan(c*x))
/c^2/d^2/(I-c*x)+1/2*(a+b*arctan(c*x))^2/c^2/d^2-I*(a+b*arctan(c*x))^2/c^2/
d^2/(I-c*x)+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d^2+I*b*(a+b*arctan(c*x
))*polylog(2,1-2/(1+I*c*x))/c^2/d^2+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^2/d^
2
```

Rubi [A]
time = 0.26, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4996, 4974, 4972, 641, 46, 209, 5004, 4964, 5114, 6745}

$$\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^2d^2}-\frac{b(a+b\text{ArcTan}(cx))}{c^2d^2(-cx+i)}-\frac{i(a+b\text{ArcTan}(cx))^2}{c^2d^2(-cx+i)}+\frac{(a+b\text{ArcTan}(cx))^2}{2c^2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^2d^2}-\frac{ib^2\text{ArcTan}(cx)}{2c^2d^2}+\frac{b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^2d^2}+\frac{ib^2}{2c^2d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] ((I/2)*b^2)/(c^2*d^2*(I - c*x)) - ((I/2)*b^2*ArcTan[c*x])/(c^2*d^2) - (b*(a
 + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(2*c^2*d^2)
- (I*(a + b*ArcTan[c*x])^2)/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])^2*Lo
g[2/(1 + I*c*x)])/(c^2*d^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{(d+icdx)^2}\,dx = \int \left(-\frac{i(a+b\tan^{-1}(cx))^2}{cd^2(-i+cx)^2} - \frac{(a+b\tan^{-1}(cx))^2}{cd^2(-i+cx)}\right)dx$$

$$= -\frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{cd^2} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{cd^2}$$

$$= -\frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2} - \frac{(2ib)\int\left(-\frac{i(a+b\tan^{-1}(cx)}{2(-i+cx)^2}\right.}{c}$$

$$= -\frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2} + \frac{ib(a+b\tan^{-1}(cx))\,\mathrm{Li}_2}{c^2d^2}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{2c^2d^2} - \frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}}{}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{2c^2d^2} - \frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}}{}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{2c^2d^2} - \frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}}{}$$

$$= \frac{ib^2}{2c^2d^2(i-cx)} - \frac{b(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{2c^2d^2} - \frac{i(a+b\tan^{-1}(cx))^2}{c^2d^2(i-cx)}$$

$$= \frac{ib^2}{2c^2d^2(i-cx)} - \frac{ib^2\tan^{-1}(cx)}{2c^2d^2} - \frac{b(a+b\tan^{-1}(cx))}{c^2d^2(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{2c^2d^2} - \frac{i(a}{}$$

Mathematica [A]
time = 0.48, size = 300, normalized size = 1.39

$$\frac{\frac{12ia^2}{-i+cx} - 12ia^2\mathrm{ArcTan}(cx) - 6a^2\log(1+c^2x^2) - 6iab\left(4\mathrm{ArcTan}(cx)^2 - \cos(2\mathrm{ArcTan}(cx)) + 2\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right) - 2i\mathrm{ArcTan}(cx)\left(\cos(2\mathrm{ArcTan}(cx)) - 2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) - i\sin(2\mathrm{ArcTan}(cx))\right) + i\sin(2\mathrm{ArcTan}(cx))\right) + b^2\left(-8i\mathrm{ArcTan}(cx)^3 + 3\cos(2\mathrm{ArcTan}(cx)) + 6i\mathrm{ArcTan}(cx)\cos(2\mathrm{ArcTan}(cx)) - 6\mathrm{ArcTan}(cx)^2\cos(2\mathrm{ArcTan}(cx)) + 12\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) - 12i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right) + 6\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right) - 3i\sin(2\mathrm{ArcTan}(cx)) + 6\mathrm{ArcTan}(cx)\sin(2\mathrm{ArcTan}(cx)) + 6i\mathrm{ArcTan}(cx)^2\sin(2\mathrm{ArcTan}(cx))\right)}{12c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

```
[Out] (((12*I)*a^2)/(-I + c*x) - (12*I)*a^2*ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2]
- (6*I)*a*b*(4*ArcTan[c*x]^2 - Cos[2*ArcTan[c*x]] + 2*PolyLog[2, -E^((2*I)*
ArcTan[c*x])] - (2*I)*ArcTan[c*x]*(Cos[2*ArcTan[c*x]] - 2*Log[1 + E^((2*I)*
```

```
ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + I*Sin[2*ArcTan[c*x]]) + b^2*((-8*I)
*ArcTan[c*x]^3 + 3*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]
] - 6*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Log[1 + E^((2*I)*
ArcTan[c*x])] - (12*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*P
olyLog[3, -E^((2*I)*ArcTan[c*x])] - (3*I)*Sin[2*ArcTan[c*x]] + 6*ArcTan[c*x
]*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(12*c^2*d^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.41, size = 969, normalized size = 4.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(I*b*a/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*a^2/d^2*ln(c^2*x^2+1)+1/2
*b^2/d^2*arctan(c*x)^2+1/2*b^2/d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-2*b*
a/d^2*arctan(c*x)*ln(c*x-I)-2*I*b^2/d^2*arctan(c*x)/(4*c*x-4*I)*c*x-1/2*I*b
^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(
(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*arctan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+2*I*b*a/d^2*arctan(c*x)/(c
*x-I)+b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-b^2/d^2*arctan(
c*x)^2*ln(c*x-I)+2*b^2/d^2*arctan(c*x)/(4*c*x-4*I)-2/3*I*b^2/d^2*arctan(c*x
)^3-1/4*I*b^2/d^2/(c*x-I)+I*a^2/d^2/(c*x-I)-I*a^2/d^2*arctan(c*x)+b*a/d^2/(
c*x-I)+1/2*b*a/d^2*arctan(c*x)-1/4*b*a/d^2*arctan(1/2*c*x)+1/4*b*a/d^2*arct
an(1/6*c^3*x^3+7/6*c*x)+1/2*b*a/d^2*arctan(1/2*c*x-1/2*I)+I*b*a/d^2*dilog(-
1/2*I*(c*x+I))-1/4*I*b*a/d^2*ln(c^2*x^2+1)+1/8*I*b*a/d^2*ln(c^4*x^4+10*c^2*
x^2+9)-1/2*I*b*a/d^2*ln(c*x-I)^2+I*b^2/d^2*arctan(c*x)^2/(c*x-I)+I*b^2/d^2*
Pi*arctan(c*x)^2-I*b^2/d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-
1/4*b^2/d^2/(c*x-I)*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] a^2*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/32*(-8*I*b^2*a
rctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*log(c
^2*x^2 + 1)^3 - 2*(-I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1
```

```
)^2 - (2*b^2*c^3*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d^2*x
^2 + c^5*d^2)) - 640*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^5*d^2*x^4
+ 2*c^3*d^2*x^2 + c*d^2), x) - 96*b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 +
1)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 1024*a*b*c^3*integrate(1/1
6*x^3*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 256*b^2*c^2*i
ntegrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2
 + c*d^2), x) + 256*b^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^5*d^2*x^4 + 2
*c^3*d^2*x^2 + c*d^2), x) + 16*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x)/
(c^4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*a*b*c + 128*b^2*c*integ
rate(1/16*x*arctan(c*x)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + b^2*c
*log(c^2*x^2 + 1)^2/(c^5*d^2*x^2 + c^3*d^2) + 256*b^2*integrate(1/16*arctan
(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x))*(c^3*d^2*x - I*c^2*d^2) -
32*(I*c^3*d^2*x + c^2*d^2)*integrate(-1/8*(32*a*b*c^2*x^2*arctan(c*x) - b^2
*log(c^2*x^2 + 1)^2 + 4*(2*b^2*c^2*x^2 - b^2)*arctan(c*x)^2 - 2*(b^2*c^2*x^
2 + b^2 + (b^2*c^3*x^3 - b^2*c*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^5*d^2*x
^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 4*(2*b^2*arctan(c*x) - (b^2*c*x - I*b^2)*
arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^3*d^2*x - I*c^2*d^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(b^2*x*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x*log(-(c*x + I)/
(c*x - I)) - 4*a^2*x)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
[Out] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.107 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{(d+icdx)^2}\,dx$

Optimal. Leaf size=122

$$\frac{b^2}{2cd^2(i-cx)}-\frac{b^2\text{ArcTan}(cx)}{2cd^2}+\frac{ib(a+b\text{ArcTan}(cx))}{cd^2(i-cx)}-\frac{i(a+b\text{ArcTan}(cx))^2}{2cd^2}+\frac{i(a+b\text{ArcTan}(cx))^2}{cd^2(1+icx)}$$

```
[Out] 1/2*b^2/c/d^2/(I-c*x)-1/2*b^2*arctan(c*x)/c/d^2+I*b*(a+b*arctan(c*x))/c/d^2
/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c/d^2+I*(a+b*arctan(c*x))^2/c/d^2/(1+I*c
*x)
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{ib(a+b\text{ArcTan}(cx))}{cd^2(-cx+i)}+\frac{i(a+b\text{ArcTan}(cx))^2}{cd^2(1+icx)}-\frac{i(a+b\text{ArcTan}(cx))^2}{2cd^2}-\frac{b^2\text{ArcTan}(cx)}{2cd^2}+\frac{b^2}{2cd^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

```
[Out] b^2/(2*c*d^2*(I - c*x)) - (b^2*ArcTan[c*x])/(2*c*d^2) + (I*b*(a + b*ArcTan[
c*x]))/(c*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^2) + (I*(a +
b*ArcTan[c*x])^2)/(c*d^2*(1 + I*c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx))^2}{(d+icdx)^2}\,dx &= \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} - \frac{(2ib)\int\left(-\frac{a+b\tan^{-1}(cx)}{2d(-i+cx)^2}+\frac{a+b\tan^{-1}(cx)}{2d(1+c^2x^2)}\right)dx}{d} \\
&= \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} + \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{d^2} - \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{d^2} \\
&= \frac{ib(a+b\tan^{-1}(cx))}{cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} + \cdots \\
&= \frac{ib(a+b\tan^{-1}(cx))}{cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} + \cdots \\
&= \frac{ib(a+b\tan^{-1}(cx))}{cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} + \cdots \\
&= \frac{b^2}{2cd^2(i-cx)} + \frac{ib(a+b\tan^{-1}(cx))}{cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^2}{cd^2(1+icx)} \cdots \\
&= \frac{b^2}{2cd^2(i-cx)} - \frac{b^2\tan^{-1}(cx)}{2cd^2} + \frac{ib(a+b\tan^{-1}(cx))}{cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{2cd^2} + \cdots
\end{aligned}
$$

Mathematica [A]

time = 0.09, size = 72, normalized size = 0.59

$$-\frac{-2a^2+2iab+b^2+b(2ia+b)(i+cx)\text{ArcTan}(cx)+b^2(-1+icx)\text{ArcTan}(cx)^2}{2cd^2(-i+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

```
[Out] -1/2*(-2*a^2 + (2*I)*a*b + b^2 + b*((2*I)*a + b)*(I + c*x)*ArcTan[c*x] + b^
2*(-1 + I*c*x)*ArcTan[c*x]^2)/(c*d^2*(-I + c*x))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(110) = 220.
time = 0.30, size = 303, normalized size = 2.48

method	result
derivativedivides	$\frac{ia^2}{d^2(icx+1)}+\frac{ib^2\arctan(cx)^2}{d^2(icx+1)}-\frac{b^2\arctan(cx)\ln(cx-i)}{2d^2}-\frac{ib^2\arctan(cx)}{d^2(cx-i)}+\frac{b^2\arctan(cx)\ln(cx+i)}{2d^2}-\frac{ib^2\ln(cx-i)^2}{8d^2}+\frac{ib^2\ln(cx-i)\ln\left(-\frac{i(c}{}\right.}{4d^2}$
default	$\frac{ia^2}{d^2(icx+1)}+\frac{ib^2\arctan(cx)^2}{d^2(icx+1)}-\frac{b^2\arctan(cx)\ln(cx-i)}{2d^2}-\frac{ib^2\arctan(cx)}{d^2(cx-i)}+\frac{b^2\arctan(cx)\ln(cx+i)}{2d^2}-\frac{ib^2\ln(cx-i)^2}{8d^2}+\frac{ib^2\ln(cx-i)\ln\left(-\frac{i(c}{}\right.}{4d^2}$
risch	$\frac{ib^2(cx+i)\ln(icx+1)^2}{8d^2(cx-i)c}-\frac{ib(bcx\ln(-icx+1)+ib\ln(-icx+1)-2ib+4a)\ln(icx+1)}{4d^2(cx-i)c}+\frac{8i\ln(-icx+1)ab+4b^2\ln(-icx+1)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*a^2/d^2/(1+I*c*x)+I*b^2/d^2/(1+I*c*x)*arctan(c*x)^2-1/2*b^2/d^2*arct
an(c*x)*ln(c*x-I)-I*b^2/d^2*arctan(c*x)/(c*x-I)+1/2*b^2/d^2*arctan(c*x)*ln(
c*x+I)-1/8*I*b^2/d^2*ln(c*x-I)^2+1/4*I*b^2/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))
-1/2*b^2/d^2/(c*x-I)-1/2*b^2/d^2*arctan(c*x)+1/4*I*b^2/d^2*ln(-1/2*I*(-c*x+
I))*ln(c*x+I)-1/4*I*b^2/d^2*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I))-1/8*I*b^
2/d^2*ln(c*x+I)^2+2*I*b*a/d^2/(1+I*c*x)*arctan(c*x)-I*b*a/d^2*arctan(c*x)-I
*b*a/d^2/(c*x-I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.95, size = 103, normalized size = 0.84

$$\frac{\left(i\,b^2cx-b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+8\,a^2-8i\,ab-4\,b^2+2\left(\left(2\,ab-i\,b^2\right)cx+2i\,ab+b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{8\left(c^2d^2x-i\,cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*b^2*c*x - b^2)*log(-(c*x + I)/(c*x - I))^2 + 8*a^2 - 8*I*a*b - 4*b^
2 + 2*((2*a*b - I*b^2)*c*x + 2*I*a*b + b^2)*log(-(c*x + I)/(c*x - I)))/(c^2
*d^2*x - I*c*d^2)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(94) = 188$.
time = 5.64, size = 303, normalized size = 2.48

$$-\frac{b(2a-ib)\log\left(-\frac{ib(2a-ib)}{c}+x(2ab-ib^2)\right)}{4cd^2}+\frac{b(2a-ib)\log\left(\frac{ib(2a-ib)}{c}+x(2ab-ib^2)\right)}{4cd^2}+\frac{(-2iab-b^2)\log(icx+1)}{2c^2d^2x-2icd^2}+\frac{(ib^2cx-b^2)\log(-icx+1)^2}{8c^2d^2x-8icd^2}+\frac{(ib^2cx-b^2)\log(icx+1)^2}{8c^2d^2x-8icd^2}+\frac{(4iab-ib^2cx\log(icx+1)+b^2\log(icx+1)+2b^2)\log(-icx+1)}{4c^2d^2x-4icd^2}-\frac{-2a^2+2iab+b^2}{2c^2d^2x-2icd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

```
[Out] -b*(2*a - I*b)*log(-I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + b*
(2*a - I*b)*log(I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + (-2*I*
a*b - b**2)*log(I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) + (I*b**2*c*x - b**
2)*log(-I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (I*b**2*c*x - b**2)*lo
g(I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (4*I*a*b - I*b**2*c*x*log(I*
c*x + 1) + b**2*log(I*c*x + 1) + 2*b**2)*log(-I*c*x + 1)/(4*c**2*d**2*x - 4
*I*c*d**2) - (-2*a**2 + 2*I*a*b + b**2)/(2*c**2*d**2*x - 2*I*c*d**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{\left(d+c\,d\,x\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2,x)
```

```
[Out] int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2, x)
```

3.108 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x(d+icdx)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{ib^2}{2d^2(i-cx)}+\frac{ib^2\mathrm{ArcTan}(cx)}{2d^2}+\frac{b(a+b\mathrm{ArcTan}(cx))}{d^2(i-cx)}-\frac{(a+b\mathrm{ArcTan}(cx))^2}{2d^2}+\frac{i(a+b\mathrm{ArcTan}(cx))^2}{d^2(i-cx)}+\frac{2(a+b\mathrm{A}}{}$$

```
[Out] -1/2*I*b^2/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/d^2+b*(a+b*arctan(c*x))/d^2/(I
-c*x)-1/2*(a+b*arctan(c*x))^2/d^2+I*(a+b*arctan(c*x))^2/d^2/(I-c*x)-2*(a+b*
arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/(1+I*c*
x))/d^2+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2+1/2*b^2*polylog
(3,-1+2/(1+I*c*x))/d^2
```

Rubi [A]
time = 0.45, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$,
Rules used = {4996, 4942, 5108, 5004, 5114, 6745, 4974, 4972, 641, 46, 209, 4964}

$$\frac{ib\mathrm{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\mathrm{ArcTan}(cx))}{d^2}+\frac{b(a+b\mathrm{ArcTan}(cx))}{d^2(-cx+i)}+\frac{i(a+b\mathrm{ArcTan}(cx))^2}{d^2(-cx+i)}-\frac{(a+b\mathrm{ArcTan}(cx))^2}{2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\mathrm{ArcTan}(cx))^2}{d^2}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\mathrm{ArcTan}(cx))^2}{d^2}+\frac{ib^2\mathrm{ArcTan}(cx)}{2d^2}+\frac{b^2\mathrm{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^2}-\frac{ib^2}{2d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

```
[Out] ((-1/2*I)*b^2)/(d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x])/d^2 + (b*(a + b*Ar
cTan[c*x]))/(d^2*(I - c*x)) - (a + b*ArcTan[c*x])^2/(2*d^2) + (I*(a + b*Arc
Tan[c*x])^2)/(d^2*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 +
I*c*x)])/d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (I*b*(a + b
*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + (b^2*PolyLog[3, -1 + 2/
(1 + I*c*x)])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+icdx)^2}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{ic(a+b\tan^{-1}(cx))^2}{d^2(-i+cx)^2} - \frac{c(a+b\tan^{-1}(cx))^2}{d^2(-i+cx)}\right)dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^2} + \frac{(ic)\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{d^2} - \frac{c\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{d^2}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b\tan^{-1}(cx}{d^2}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b\tan^{-1}(cx}{d^2}$$

$$= \frac{b(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} + \frac{2(a+b\tan^{-1}(}{}$$

$$= \frac{b(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} + \frac{2(a+b\tan^{-1}(}{}$$

$$= \frac{b(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} + \frac{2(a+b\tan^{-1}(}{}$$

$$= -\frac{ib^2}{2d^2(i-cx)} + \frac{b(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^2(i-cx)}$$

$$= -\frac{ib^2}{2d^2(i-cx)} + \frac{ib^2\tan^{-1}(cx)}{2d^2} + \frac{b(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{i(a}{}$$

<u>Mathematica</u> [A]
time = 0.55, size = 299, normalized size = 1.35

$$\frac{-\frac{24ia^2}{-i+cx} - 24ia^2\text{ArcTan}(cx) + 24a^2\log(cx) - 12a^2\log(1+c^2x^2) - 12ab\left(4i\text{ArcTan}(cx)^2 + i\cos(2\text{ArcTan}(cx)) + 2i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right) - 2\text{ArcTan}(cx)\left(\cos(2\text{ArcTan}(cx)) + 2\log\left(1-e^{2i\text{ArcTan}(cx)}\right) - i\sin(2\text{ArcTan}(cx))\right) + \sin(2\text{ArcTan}(cx))\right) + b^2\left(-i\pi^3 - 6\cos(2\text{ArcTan}(cx)) - 12i\text{ArcTan}(cx)\cos(2\text{ArcTan}(cx)) + 12\text{ArcTan}(cx)^2\cos(2\text{ArcTan}(cx)) + 24\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right) + 24i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right) + 12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right) + 6i\sin(2\text{ArcTan}(cx)) - 12\text{ArcTan}(cx)\sin(2\text{ArcTan}(cx)) - 12i\text{ArcTan}(cx)^2\sin(2\text{ArcTan}(cx))\right)}{24d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

```
[Out] (((-24*I)*a^2)/(-I + c*x) - (24*I)*a^2*ArcTan[c*x] + 24*a^2*Log[c*x] - 12*a
^2*Log[1 + c^2*x^2] - 12*a*b*((4*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c*x]] +
(2*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 2*ArcTan[c*x]*(Cos[2*ArcTan[c*x]]
 + 2*Log[1 - E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[
c*x]]) + b^2*((-I)*Pi^3 - 6*Cos[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Cos[2*A
rcTan[c*x]] + 12*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Log[1
- E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[
c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (6*I)*Sin[2*ArcTan[c*x]] -
```

```
12*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]
]))/(24*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 1921, normalized size = 8.69

method	result	size
derivativedivides	Expression too large to display	1921
default	Expression too large to display	1921

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*b*a/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*a^2/d^2*ln(c
^2*x^2+1)-1/2*b^2/d^2*arctan(c*x)^2+a^2/d^2*ln(c*x)-2*b*a/d^2*arctan(c*x)*l
n(c*x-I)-1/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+1/2*I*b^2/d^2*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^
2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2/d^2*Pi*csgn
(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I
*b*a/d^2*ln(c*x)*ln(1+I*c*x)-I*b*a/d^2*ln(c*x)*ln(1-I*c*x)-1/2*I*b^2/d^2*Pi
*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2
/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d
^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*
x)^2-I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*arctan(c*x)^2+3/2*I*b^2/d^2*Pi*arctan(c*x)^2-2*I*b^2/d^2*arctan(c*x)*pol
ylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d^2*arctan(c*x)*polylog(2,(1+I
*c*x)/(c^2*x^2+1)^(1/2))+2*b*a/d^2*arctan(c*x)*ln(c*x)+I*b*a/d^2*dilog(1+I*
c*x)-I*b*a/d^2*dilog(1-I*c*x)+1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+b^2/d^2*arctan(c*x)^2*ln(1+(
1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1
)^(1/2))-b^2/d^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2/d^2*arctan
(c*x)^2*ln(c*x)-I*a^2/d^2/(c*x-I)+b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/
(c^2*x^2+1))-b^2/d^2*arctan(c*x)^2*ln(c*x-I)+2*I*b^2/d^2*arctan(c*x)/(4*c*x
-4*I)*c*x-1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-2*b^2/d
^2*arctan(c*x)/(4*c*x-4*I)-2/3*I*b^2/d^2*arctan(c*x)^3-2*I*b*a/d^2*arctan(c
*x)/(c*x-I)-1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^2-1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x
)^2+1/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
```

```
^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^2+2*b^2/d^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d^2*polyl
og(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*a^2/d^2*arctan(c*x)+1/4*I*b^2/d^2/(c*x
-I)-b*a/d^2/(c*x-I)-b*a/d^2*arctan(c*x)+I*b*a/d^2*dilog(-1/2*I*(c*x+I))-1/2
*I*b*a/d^2*ln(c*x-I)^2+1/4*b^2/d^2/(c*x-I)*c*x-I*b^2/d^2*arctan(c*x)^2/(c*x
-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] a^2*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2) - 1/32*(8*I*b^2*
arctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*log(
c^2*x^2 + 1)^3 - 2*(I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1
)^2 - (6*b^2*c^4*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d^2*x
^2 + c^4*d^2)) - 256*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2*x^5
+ 2*c^2*d^2*x^3 + d^2*x), x) - 64*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 +
1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c^3*integrate(1/16
*x^3*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 16*(c*(x/(c^4*
d^2*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d^2*x^2 +
c^2*d^2))*a*b*c^2 - 640*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4*d^2*x
^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 3*b^2*c^2*log(c^2*x^2 + 1)^2/(c^4*d^2*x^2
 + c^2*d^2) - 256*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*
d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c*integrate(1/16*x*arctan(c*
x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 384*b^2*integrate(1/16*arcta
n(c*x)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 32*b^2*integrate(1/16*
log(c^2*x^2 + 1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 1024*a*b*int
egrate(1/16*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x))*(c*d^2*x
 - I*d^2) - 32*(I*c*d^2*x + d^2)*integrate(1/8*(b^2*c^3*x^3*log(c^2*x^2 + 1
)^2 - 32*a*b*c*x*arctan(c*x) + 4*(b^2*c^3*x^3 - 2*b^2*c*x)*arctan(c*x)^2 -
2*(b^2*c^3*x^3 + b^2*c*x - (b^2*c^2*x^2 - b^2)*arctan(c*x))*log(c^2*x^2 + 1
))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 4*(2*b^2*arctan(c*x) + (b^2*
c*x - I*b^2)*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c*d^2*x - I*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 - (b^2*c*x - I*b^2)*log(2*c*x/(c*x -
 I))*log(-(c*x + I)/(c*x - I))^2 - 2*(b^2*c*x - I*b^2)*dilog(-2*c*x/(c*x -
I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*(c*d^2*x - I*d^2)*integral(-(a^2*c*x
+ I*a^2 - ((-I*a*b - b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^
4 - I*c^2*d^2*x^3 + c*d^2*x^2 - I*d^2*x), x) + 2*(b^2*c*x - I*b^2)*polylog(
3, -(c*x + I)/(c*x - I)))/(c*d^2*x - I*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2), x)
```

3.109 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2(d+icdx)^2}\,dx$

Optimal. Leaf size=306

$$-\frac{b^2c}{2d^2(i-cx)}+\frac{b^2c\text{ArcTan}(cx)}{2d^2}-\frac{ibc(a+b\text{ArcTan}(cx))}{d^2(i-cx)}-\frac{ic(a+b\text{ArcTan}(cx))^2}{2d^2}-\frac{(a+b\text{ArcTan}(cx))^2}{d^2x}+\frac{c(a+$$

```
[Out] -1/2*b^2*c/d^2/(I-c*x)+1/2*b^2*c*arctan(c*x)/d^2-I*b*c*(a+b*arctan(c*x))/d^
2/(I-c*x)-1/2*I*c*(a+b*arctan(c*x))^2/d^2-(a+b*arctan(c*x))^2/d^2/x+c*(a+b*
arctan(c*x))^2/d^2/(I-c*x)+4*I*c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x)
)/d^2-2*I*c*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^2+2*b*c*(a+b*arctan(c*x))
*ln(2-2/(1-I*c*x))/d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d^2+2*b*c*(a+b*arc
tan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d
^2
```

Rubi [A]
time = 0.56, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {4996, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745, 4974, 4972, 641, 46, 209, 4964}

$$\frac{2bc\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{ibc(a+b\text{ArcTan}(cx))}{d^2(-cx+i)}-\frac{(a+b\text{ArcTan}(cx))^2}{d^2x}+\frac{c(a+b\text{ArcTan}(cx))^2}{d^2(-cx+i)}-\frac{ic(a+b\text{ArcTan}(cx))^2}{2d^2}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{4ic\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}+\frac{b^2c\text{ArcTan}(cx)}{2d^2}-\frac{ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d^2}-\frac{ib^2c\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{d^2}-\frac{b^2c}{2d^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] -1/2*(b^2*c)/(d^2*(I - c*x)) + (b^2*c*ArcTan[c*x])/(2*d^2) - (I*b*c*(a + b*
ArcTan[c*x]))/(d^2*(I - c*x)) - ((I/2)*c*(a + b*ArcTan[c*x])^2)/d^2 - (a +
b*ArcTan[c*x])^2/(d^2*x) + (c*(a + b*ArcTan[c*x])^2)/(d^2*(I - c*x)) - ((4*
I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - ((2*I)*c*(a +
b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2
 - 2/(1 - I*c*x)])/d^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 + (2*
b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b^2*c*Poly
Log[3, -1 + 2/(1 + I*c*x)])/d^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+icdx)^2}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^2x^2} - \frac{2ic(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c^2(a+b\tan^{-1}(cx))^2}{d^2(-i+cx)^2} + \frac{2ic^2(a}{d}\right.$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d^2} - \frac{(2ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^2} + \frac{(2ic^2)\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{d^2} + \frac{c^2\int}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} - \frac{4ic(a+b\tan^{-1}(cx))^2\tanh^{-1}(1-}{d^2}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} - \frac{4ic(a+b\,\mathrm{ta}}{}$$

$$= -\frac{ibc(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{ic(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c(a+b\,\mathrm{tan}}{d^2(i-}$$

$$= -\frac{ibc(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{ic(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c(a+b\,\mathrm{tan}}{d^2(i-}$$

$$= -\frac{ibc(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{ic(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{c(a+b\,\mathrm{tan}}{d^2(i-}$$

$$= -\frac{b^2c}{2d^2(i-cx)} - \frac{ibc(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{ic(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))}{d^2x}$$

$$= -\frac{b^2c}{2d^2(i-cx)} + \frac{b^2c\tan^{-1}(cx)}{2d^2} - \frac{ibc(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{ic(a+b\tan^{-1}(cx))^2}{2d^2} - ($$

Mathematica [A]
time = 1.61, size = 398, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

```
[Out] -1/12*((12*a^2)/x + (12*a^2*c)/(-I + c*x) + 24*a^2*c*ArcTan[c*x] + (24*I)*a
^2*c*Log[c*x] - (12*I)*a^2*c*Log[1 + c^2*x^2] + b^2*c*(Pi^3 + (12*I)*ArcTan
[c*x]^2 + (12*ArcTan[c*x]^2)/(c*x) - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*
x]*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + (24*I)*Arc
Tan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*Log[1 - E^((2*I
)*ArcTan[c*x])] - 24*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (12*I
)*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*
x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 6*ArcT
an[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*c*(8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x
]] - 4*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*PolyLog[2, E^((2*I)*ArcTan[c*x])] -
 I*Sin[2*ArcTan[c*x]] + ArcTan[c*x]*(4/(c*x) + (2*I)*Cos[2*ArcTan[c*x]] + (
8*I)*Log[1 - E^((2*I)*ArcTan[c*x])] + 2*Sin[2*ArcTan[c*x]])))/d^2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.89, size = 9303, normalized size = 30.40

method	result	size
derivativedivides	Expression too large to display	9303
default	Expression too large to display	9303

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/d^2 + 1/(
d^2*x)) - 1/16*(8*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x)^3 - (-I*b^2*c^2*x^2
 - b^2*c*x)*log(c^2*x^2 + 1)^3 + 4*(2*b^2*c*x - I*b^2)*arctan(c*x)^2 - (2*b
^2*c*x - I*b^2 - 2*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x))*log(c^2*x^2 + 1)^
2 - 2*(128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c^2*d^
2*x^4 + d^2*x^2), x) + 32*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^
4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^4*integrate(1/16*x^4*lo
g(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + (c*(x/(c^4*d^2
*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d^2*x^2 + c^2
*d^2))*b^2*c^3 + 32*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4*d^2*x^6 +
 2*c^2*d^2*x^4 + d^2*x^2), x) + 24*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 +
```

```
1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 256*a*b*c^2*integrate(1
/16*x^2*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^
2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^
2), x) - 64*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d^2*x^
6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 64*b^2*c*integrate(1/16*x*arctan(c*x)/(c
^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 96*b^2*integrate(1/16*arctan(c*
x)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 8*b^2*integrate(1/16*log
(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 256*a*b*integ
rate(1/16*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x))*(c*d^2*x
^2 - I*d^2*x) - 2*(2*b^2*c^5*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1
)/(c^6*d^2*x^2 + c^4*d^2)) - 64*b^2*c^5*integrate(1/8*x^5*arctan(c*x)^2/(c^
4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 16*b^2*c^5*integrate(1/8*x^5*log
(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^4*in
tegrate(1/8*x^4*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + b
^2*c^3*(c^2/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2*x^2 + c^2*d
^2)) - 64*b^2*c^3*integrate(1/8*x^3*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c^2*d^2*
x^4 + d^2*x^2), x) + b^2*c^3*log(c^2*x^2 + 1)^2/(c^4*d^2*x^2 + c^2*d^2) - 1
6*b^2*c^2*integrate(1/8*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^2*integrate(1/8*x^2*arctan(c*x)/(c^4*d
^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 96*b^2*c*integrate(1/8*x*arctan(c*x
)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 8*b^2*c*integrate(1/8*x*l
og(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 256*a*b*c*i
ntegrate(1/8*x*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 16
*b^2*c*integrate(1/8*x*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*
x^2), x) + 16*b^2*integrate(1/8*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d^2*x^6 +
 2*c^2*d^2*x^4 + d^2*x^2), x))*(-I*c*d^2*x^2 - d^2*x) + 4*((I*b^2*c^2*x^2 +
 b^2*c*x)*arctan(c*x)^2 + (2*I*b^2*c*x + b^2)*arctan(c*x))*log(c^2*x^2 + 1)
)/(c*d^2*x^2 - I*d^2*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(-I*b^2*c^2*x^2 - b^2*c*x)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x
 - I))^2 + 4*(-I*b^2*c^2*x^2 - b^2*c*x)*dilog(-2*c*x/(c*x - I) + 1)*log(-(c
*x + I)/(c*x - I)) - (2*b^2*c*x - I*b^2)*log(-(c*x + I)/(c*x - I))^2 - 4*(c
*d^2*x^2 - I*d^2*x)*integral(-(a^2*c*x + I*a^2 - (2*I*b^2*c^2*x^2 + (-I*a*b
 + b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^5 - I*c^2*d^2*x^4
+ c*d^2*x^3 - I*d^2*x^2), x) + 4*(I*b^2*c^2*x^2 + b^2*c*x)*polylog(3, -(c*x
 + I)/(c*x - I)))/(c*d^2*x^2 - I*d^2*x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2), x)
```

3.110 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^3(d+icdx)^2}\,dx$

Optimal. Leaf size=403

$$\frac{ib^2c^2}{2d^2(i-cx)}-\frac{ib^2c^2\text{ArcTan}(cx)}{2d^2}-\frac{bc(a+b\text{ArcTan}(cx))}{d^2x}-\frac{bc^2(a+b\text{ArcTan}(cx))}{d^2(i-cx)}-\frac{2c^2(a+b\text{ArcTan}(cx))^2}{d^2}-(a$$

```
[Out] -1/2*I*b^2*c^2*arctan(c*x)/d^2+1/2*I*b^2*c^2/d^2/(I-c*x)-b*c*(a+b*arctan(c*
x))/d^2/x-b*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-2*c^2*(a+b*arctan(c*x))^2/d^2
-1/2*(a+b*arctan(c*x))^2/d^2/x^2-4*I*b*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*
x))/d^2-I*c^2*(a+b*arctan(c*x))^2/d^2/(I-c*x)+6*c^2*(a+b*arctan(c*x))^2*arc
tanh(-1+2/(1+I*c*x))/d^2+b^2*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))^2*ln(2/(
1+I*c*x))/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+2*I*c*(a+b*arctan(c*x))^2/d^2/x
-2*b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d^2-3*I*b*c^2*(a+b*arctan(c*x))*polylo
g(2,-1+2/(1+I*c*x))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(1+I*c*x))/d^2
```

Rubi [A]
time = 0.67, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 21, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {4996, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497, 4942, 5108, 5114, 6745, 4974, 4972, 641, 46, 209, 4964}

$$-\frac{3ibc^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{ic^2(a+b\text{ArcTan}(cx))^2}{d^2(-cx+i)}-\frac{2c^2(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{bc^2(a+b\text{ArcTan}(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{4ibc^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{6c^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{(a+b\text{ArcTan}(cx))^2}{2d^2x^2}+\frac{2ic(a+b\text{ArcTan}(cx))^2}{d^2x}-\frac{bc(a+b\text{ArcTan}(cx))}{d^2x}-\frac{ib^2c^2\text{ArcTan}(cx)}{2d^2}-\frac{2b^2c^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d^2}-\frac{3b^2c^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^2}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d^2}+\frac{ib^2c^2}{2d^2(-cx+i)}+\frac{b^2c^2\log(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] ((I/2)*b^2*c^2)/(d^2*(I - c*x)) - ((I/2)*b^2*c^2*ArcTan[c*x])/d^2 - (b*c*(a
 + b*ArcTan[c*x]))/(d^2*x) - (b*c^2*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) -
(2*c^2*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d^2*x^2) + ((2
*I)*c*(a + b*ArcTan[c*x])^2)/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x])^2)/(d^2*(
I - c*x)) - (6*c^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 +
(b^2*c^2*Log[x])/d^2 - (3*c^2*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2
 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) - ((4*I)*b*c^2*(a + b*ArcTan[c*x])*Lo
g[2 - 2/(1 - I*c*x)])/d^2 - (2*b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2
 - ((3*I)*b*c^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (3
*b^2*c^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+icdx)^2}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^2x^3} - \frac{2ic(a+b\tan^{-1}(cx))^2}{d^2x^2} - \frac{3c^2(a+b\tan^{-1}(cx))^2}{d^2x} - \frac{ic^3(a}{d}\right.$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx}{d^2} - \frac{(2ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d^2} - \frac{(3c^2)\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^2} - \frac{(ic^3)}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))^2}{d^2x} - \frac{ic^2(a+b\tan^{-1}(cx))^2}{d^2(i-cx)} - \frac{6c^2(a+}{}$$

$$= -\frac{2c^2(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} + \frac{2ic(a+b\tan^{-1}(cx))^2}{d^2x} - \frac{ic^2(a+}{d^2(}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{bc^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2c^2(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\,\mathrm{ta}}{2d}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{bc^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2c^2(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\,\mathrm{ta}}{2d}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{bc^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2c^2(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\,\mathrm{ta}}{2d}$$

$$= \frac{ib^2c^2}{2d^2(i-cx)} - \frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{bc^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2c^2(a+b\tan^{-1}(cx))}{d^2}$$

$$= \frac{ib^2c^2}{2d^2(i-cx)} - \frac{ib^2c^2\tan^{-1}(cx)}{2d^2} - \frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{bc^2(a+b\tan^{-1}(cx))}{d^2(i-cx)} - \frac{2}{}$$

Mathematica [A]
time = 1.92, size = 491, normalized size = 1.22

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

```
[Out] ((-4*a^2)/x^2 + ((16*I)*a^2*c)/x + ((8*I)*a^2*c^2)/(-I + c*x) + (24*I)*a^2*
c^2*ArcTan[c*x] - 24*a^2*c^2*Log[x] + 12*a^2*c^2*Log[1 + c^2*x^2] - b^2*c^2
*((-I)*Pi^3 + (8*ArcTan[c*x])/(c*x) + 20*ArcTan[c*x]^2 + (4*ArcTan[c*x]^2)/
(c^2*x^2) - ((16*I)*ArcTan[c*x]^2)/(c*x) - 2*Cos[2*ArcTan[c*x]] - (4*I)*Arc
Tan[c*x]*Cos[2*ArcTan[c*x]] + 4*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTa
n[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (32*I)*ArcTan[c*x]*Log[1 - E^((2
*I)*ArcTan[c*x])] - 8*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (24*I)*ArcTan[c*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c*x])] + 16*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 12
```

```
*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (2*I)*Sin[2*ArcTan[c*x]] - 4*ArcTan[c
*x]*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + (4*I)*a*
b*c^2*((2*I)/(c*x) + 12*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 8*Log[(c*x)/Sq
rt[1 + c^2*x^2]] + 6*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x
]] + 2*ArcTan[c*x]*(I + I/(c^2*x^2) + 4/(c*x) + I*Cos[2*ArcTan[c*x]] + (6*I
)*Log[1 - E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(8*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.08, size = 2247, normalized size = 5.58

method	result	size
derivativedivides	Expression too large to display	2247
default	Expression too large to display	2247

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3/2*a^2/d^2*ln(c^2*x^2+1)-2*b^2/d^2*arctan(c*x)^2-3*a^2/d^2*ln(c*x)-3/
2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*
x)^2+4*I*b*a/d^2*arctan(c*x)/c/x-I*b^2/d^2*arctan(c*x)/(2*c*x-2*I)*c*x+6*b*
a/d^2*arctan(c*x)*ln(c*x-I)-3*I*b*a/d^2*dilog(-1/2*I*(c*x+I))+3/2*I*b*a/d^2
*ln(c*x-I)^2-3/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*b^2/
d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+2*I*b*a/d^2*arctan(c*x)/(c*x-I)-6*b*a/d
^2*arctan(c*x)*ln(c*x)+b^2/d^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^2*ln
((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-4*b^2/d^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/
2))-3/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2-3*b^2/d^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/
2))-3*b^2/d^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*b^2/d^2*arc
tan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3*b^2/d^2*arctan(c*x)^2*ln(c*x)+3*
I*a^2/d^2*arctan(c*x)-3*b^2/d^2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1
))+b^2/d^2*arctan(c*x)/(2*c*x-2*I)+3*b^2/d^2*arctan(c*x)^2*ln(c*x-I)-1/4*I*
b^2/d^2/(c*x-I)-3*I*b*a/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)+3*I*b^2/d^2*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*
I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*ar
ctan(c*x)^2-6*b^2/d^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*b^2/d^2*poly
log(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*a^2/d^2/(c*x-I)+2*I*b*a/d^2*ln(c^2*x^
2+1)+3*I*b*a/d^2*dilog(1-I*c*x)-3*I*b*a/d^2*dilog(1+I*c*x)-4*I*b*a/d^2*ln(c
*x)-1/2*b^2/d^2*arctan(c*x)^2/c^2/x^2-b^2/d^2*arctan(c*x)/c/x-4*I*b^2/d^2*a
rctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-9/2*I*b^2/d^2*Pi*arctan(c*x)^2
+6*I*b^2/d^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*b^2/d^2
*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*a^2/d^2/c/x-b*a/d^
```

```
2/c/x+3/2*I*b^2/d^2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^2+3/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^2*arctan(c*x)^2+3/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)
^2+3/2*I*b^2/d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*a^2/d^2/c^
2/x^2-I*b^2/d^2*arctan(c*x)+2*I*b^2/d^2*arctan(c*x)^3+b*a/d^2/(c*x-I)+I*b^2
/d^2*arctan(c*x)^2/(c*x-I)+4*b^2/d^2*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4
*b^2/d^2/(c*x-I)*c*x-b*a/d^2*arctan(c*x)/c^2/x^2+3*I*b*a/d^2*ln(c*x)*ln(1-I
*c*x)-3*I*b*a/d^2*ln(c*x)*ln(1+I*c*x)+3/2*I*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*b^2/d^2*Pi*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*
x)^2-3/2*I*b^2/d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^3*arctan(c*x)^2+2*I*b^2/d^2*arctan(c*x)^2/c/x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c
*x - I))^2 + 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*dilog(-2*c*x/(c*x - I) + 1)*l
og(-(c*x + I)/(c*x - I)) + (-6*I*b^2*c^2*x^2 - 3*b^2*c*x - I*b^2)*log(-(c*x
 + I)/(c*x - I))^2 + 8*(c*d^2*x^3 - I*d^2*x^2)*integral(-1/2*(2*a^2*c*x + 2
*I*a^2 - (6*b^2*c^3*x^3 - 3*I*b^2*c^2*x^2 + (-2*I*a*b + b^2)*c*x + 2*a*b)*l
og(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^6 - I*c^2*d^2*x^5 + c*d^2*x^4 - I*d^2*
x^3), x) - 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*polylog(3, -(c*x + I)/(c*x - I)
))/(c*d^2*x^3 - I*d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a^2}{c^2x^5-2icx^4-x^3}\,dx+\int \frac{b^2\,\mathrm{atan}^2\,(cx)}{c^2x^5-2icx^4-x^3}\,dx+\int \frac{2ab\,\mathrm{atan}\,(cx)}{c^2x^5-2icx^4-x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x)**2,x)
```

```
[Out] -(Integral(a**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b**2*atan(c*
x)**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(2*a*b*atan(c*x)/(c**2*
x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x^3\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2), x)
```

3.111 $\int \frac{x^4(a+b\text{ArcTan}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=462

$$-\frac{iabx}{c^4d^3}+\frac{ib^2}{16c^5d^3(i-cx)^2}-\frac{29b^2}{16c^5d^3(i-cx)}+\frac{29b^2\text{ArcTan}(cx)}{16c^5d^3}-\frac{ib^2x\text{ArcTan}(cx)}{c^4d^3}-\frac{b(a+b\text{ArcTan}(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a$$

```
[Out] -I*a*b*x/c^4/d^3+1/2*I*x^2*(a+b*arctan(c*x))^2/c^3/d^3-29/16*b^2/c^5/d^3/(I
-c*x)+29/16*b^2*arctan(c*x)/c^5/d^3-3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5/d^
3-1/4*b*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2-15/4*I*b*(a+b*arctan(c*x))/c^5/
d^3/(I-c*x)+1/16*I*b^2/c^5/d^3/(I-c*x)^2-3*x*(a+b*arctan(c*x))^2/c^4/d^3-1/
2*I*(a+b*arctan(c*x))^2/c^5/d^3/(I-c*x)^2+6*I*(a+b*arctan(c*x))^2*ln(2/(1+I
*c*x))/c^5/d^3+4*(a+b*arctan(c*x))^2/c^5/d^3/(I-c*x)-6*b*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^5/d^3-I*b^2*x*arctan(c*x)/c^4/d^3+3*I*b^2*polylog(3,1-2/(
1+I*c*x))/c^5/d^3+1/2*I*b^2*ln(c^2*x^2+1)/c^5/d^3-6*b*(a+b*arctan(c*x))*pol
ylog(2,1-2/(1+I*c*x))/c^5/d^3-5/8*I*(a+b*arctan(c*x))^2/c^5/d^3
```

Rubi [A]
time = 0.60, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 4974, 4972, 641, 46, 209, 5114, 6745}

$$-\frac{6b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^5d^3}-\frac{15ib(a+b\text{ArcTan}(cx))}{4c^5d^3(-cx+i)}-\frac{b(a+b\text{ArcTan}(cx))}{4c^5d^3(-cx+i)^2}+\frac{4(a+b\text{ArcTan}(cx))^2}{c^5d^3(-cx+i)}-\frac{i(a+b\text{ArcTan}(cx))^2}{2c^5d^3(-cx+i)^2}-\frac{5i(a+b\text{ArcTan}(cx))^2}{8c^5d^3}-\frac{6b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^5d^3}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^5d^3}-\frac{3x(a+b\text{ArcTan}(cx))^2}{c^4d^3}+\frac{ix^2(a+b\text{ArcTan}(cx))^2}{2c^3d^3}-\frac{iabx}{c^4d^3}+\frac{29b^2\text{ArcTan}(cx)}{16c^5d^3}-\frac{ib^2x\text{ArcTan}(cx)}{c^4d^3}-\frac{3ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^5d^3}+\frac{3ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{29b^2}{16c^5d^3(-cx+i)}+\frac{ib^2}{16c^5d^3(-cx+i)^2}+\frac{ib^2\log\left(c^2x^2+1\right)}{2c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] ((-I)*a*b*x)/(c^4*d^3) + ((I/16)*b^2)/(c^5*d^3*(I - c*x)^2) - (29*b^2)/(16*
c^5*d^3*(I - c*x)) + (29*b^2*ArcTan[c*x])/(16*c^5*d^3) - (I*b^2*x*ArcTan[c*
x])/(c^4*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c^5*d^3*(I - c*x)^2) - (((15*I)/
4)*b*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)) - (((5*I)/8)*(a + b*ArcTan[c*
x])^2)/(c^5*d^3) - (3*x*(a + b*ArcTan[c*x])^2)/(c^4*d^3) + ((I/2)*x^2*(a +
b*ArcTan[c*x])^2)/(c^3*d^3) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c
*x)^2) + (4*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c*x)) - (6*b*(a + b*ArcTan
[c*x])*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((6*I)*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/(c^5*d^3) + ((I/2)*b^2*Log[1 + c^2*x^2])/(c^5*d^3) - ((3*I)*b^
2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) - (6*b*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) + ((3*I)*b^2*PolyLog[3, 1 - 2/(1 + I*c*
x)])/(c^5*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx = \int \left(-\frac{3(a+b\tan^{-1}(cx))^2}{c^4d^3}+\frac{ix(a+b\tan^{-1}(cx))^2}{c^3d^3}+\frac{i(a+b\tan^{-1}(cx))^2}{c^4d^3(-i+cx)^3}+\frac{4(\cdots\right.$$

$$=\frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^3}\,dx}{c^4d^3}-\frac{(6i)\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{c^4d^3}-\frac{3\int (a+b\tan^{-1}(cx))^2\,dx}{c^4d^3}+\cdots$$

$$=-\frac{3x(a+b\tan^{-1}(cx))^2}{c^4d^3}+\frac{ix^2(a+b\tan^{-1}(cx))^2}{2c^3d^3}-\frac{i(a+b\tan^{-1}(cx))^2}{2c^5d^3(i-cx)^2}+\frac{4(a\cdots}{\cdots}$$

$$=-\frac{3i(a+b\tan^{-1}(cx))^2}{c^5d^3}-\frac{3x(a+b\tan^{-1}(cx))^2}{c^4d^3}+\frac{ix^2(a+b\tan^{-1}(cx))^2}{2c^3d^3}-\frac{i(a\cdots}{2\cdots}$$

$$=-\frac{iabx}{c^4d^3}-\frac{b(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)}-\frac{5i(a+b\tan^{-1}(cx))^2}{8c^5d^3}\cdots$$

$$=-\frac{iabx}{c^4d^3}-\frac{ib^2x\tan^{-1}(cx)}{c^4d^3}-\frac{b(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)}-\frac{5i(\cdots}{\cdots}$$

$$=-\frac{iabx}{c^4d^3}-\frac{ib^2x\tan^{-1}(cx)}{c^4d^3}-\frac{b(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a+b\tan^{-1}(cx))}{4c^5d^3(i-cx)}-\frac{5i(\cdots}{\cdots}$$

$$=-\frac{iabx}{c^4d^3}+\frac{ib^2}{16c^5d^3(i-cx)^2}-\frac{29b^2}{16c^5d^3(i-cx)}-\frac{ib^2x\tan^{-1}(cx)}{c^4d^3}-\frac{b(a+b\tan^{-1}\cdots}{4c^5d^3(i-c\cdots}$$

$$=-\frac{iabx}{c^4d^3}+\frac{ib^2}{16c^5d^3(i-cx)^2}-\frac{29b^2}{16c^5d^3(i-cx)}+\frac{29b^2\tan^{-1}(cx)}{16c^5d^3}-\frac{ib^2x\tan^{-1}(c\cdots}{c^4d^3}$$

Mathematica [A]
time = 1.60, size = 578, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (-48*a^2*c*x + (8*I)*a^2*c^2*x^2 - ((8*I)*a^2)/(-I + c*x)^2 - (64*a^2)/(-I
+ c*x) + 96*a^2*ArcTan[c*x] - (48*I)*a^2*Log[1 + c^2*x^2] + a*b*((-16*I)*c*
x + 192*ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log
[1 + c^2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan
[c*x]] + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*
x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2
*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]) + (16*I)*b^2*
(-(c*x*ArcTan[c*x]) + 3*ArcTan[c*x]^2 + (3*I)*c*x*ArcTan[c*x]^2 + ((1 + c^2
*x^2)*ArcTan[c*x]^2)/2 - (4*I)*ArcTan[c*x]^3 - (7*(-1 - (2*I)*ArcTan[c*x] +
 2*ArcTan[c*x]^2)*Cos[2*ArcTan[c*x]])/8 - Cos[4*ArcTan[c*x]]/64 - (I/16)*Ar
cTan[c*x]*Cos[4*ArcTan[c*x]] + (ArcTan[c*x]^2*Cos[4*ArcTan[c*x]])/8 + (6*I)
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*ArcTan[c*x]^2*Log[1 + E^((2
*I)*ArcTan[c*x])] + Log[1 + c^2*x^2]/2 + (3 - (6*I)*ArcTan[c*x])*PolyLog[2,
 -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - ((7*I)/8)
*Sin[2*ArcTan[c*x]] + (7*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/4 + ((7*I)/4)*ArcT
an[c*x]^2*Sin[2*ArcTan[c*x]] + (I/64)*Sin[4*ArcTan[c*x]] - (ArcTan[c*x]*Sin
[4*ArcTan[c*x]])/16 - (I/8)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(16*c^5*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.38, size = 1475, normalized size = 3.19

method	result	size
derivativedivides	Expression too large to display	1475
default	Expression too large to display	1475

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-6*b*a/d^3*arctan(c*x)*c*x-b^2/d^3*arctan(c*x)+7*b^2/d^3/(8*c*x-8*I)
+4*b^2/d^3*arctan(c*x)^3-3*a^2/d^3*c*x-1/4*b*a/d^3/(c*x-I)^2+43/16*b*a/d^3*
ln(c^2*x^2+1)+5/32*b*a/d^3*ln(c^4*x^4+10*c^2*x^2+9)+3*b*a/d^3*ln(c*x-I)^2+6
*a^2/d^3*arctan(c*x)-4*a^2/d^3/(c*x-I)+3*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x
^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1
+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+I*b*a/d^3*arctan(c*x)*c^2*x^2+1/8*I
*b^2/d^3*arctan(c*x)/(c*x-I)^2*c*x-I*b^2/d^3*arctan(c*x)*c*x-1/64*I*b^2/d^3
/(c*x-I)^2*c^2*x^2-7*I*b^2/d^3/(8*c*x-8*I)*c*x+1/2*I*b^2/d^3*arctan(c*x)^2*
c^2*x^2-6*b*a/d^3*dilog(-1/2*I*(c*x+I))-3*I*a^2/d^3*ln(c^2*x^2+1)-1/2*I*a^2
```

```
/d^3/(c*x-I)^2-6*b^2/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*
b^2/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*b^2/d^3*Pi*arctan
(c*x)^2+6*b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-4*b^2/d^3
*arctan(c*x)^2/(c*x-I)-1/16*b^2/d^3*arctan(c*x)/(c*x-I)^2+6*I*b^2/d^3*dilog
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^2/d^3*polylog(3,-(1+I*c*x)^2/(c^2*x
^2+1))-I*b^2/d^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/64*I*b^2/d^3/(c*x-I)^2+6*I
*b^2/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+43/8*I*b^2/d^3*arctan(c*x)^
2-b*a/d^3-I*b*a/d^3*c*x-12*I*b*a/d^3*arctan(c*x)*ln(c*x-I)-I*b*a/d^3*arctan
(c*x)/(c*x-I)^2+3*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3*b^2/d^3*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*arctan(c*x)^2+7/4*b^2/d^3*arctan(c*x)/(c*x-I)*c*x+1/16*b^2
/d^3*arctan(c*x)/(c*x-I)^2*c^2*x^2-8*b*a/d^3*arctan(c*x)/(c*x-I)-6*b*a/d^3*
ln(c*x-I)*ln(-1/2*I*(c*x+I))-5/16*I*b*a/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-5/8
*I*b*a/d^3*arctan(1/2*c*x-1/2*I)+5/16*I*b*a/d^3*arctan(1/2*c*x)+15/4*I*b*a/
d^3/(c*x-I)+43/8*I*b*a/d^3*arctan(c*x)+1/2*I*a^2/d^3*c^2*x^2-3*b^2/d^3*arct
an(c*x)^2*c*x+1/32*b^2/d^3/(c*x-I)^2*c*x+3*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+6*b^2/d^3*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+6*I*b^2/d^
3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-6*I*b^2/d^3*arctan(c*x)^2*l
n(c*x-I)-1/2*I*b^2/d^3*arctan(c*x)^2/(c*x-I)^2+7/4*I*b^2/d^3*arctan(c*x)/(c
*x-I))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(64*I*a^2*c^4*x^4 - 256*a^2*c^3*x^3 - 32*a^2*c^2*x^2*(15*arctan2(1, c
*x) - 22*I) + 64*a^2*c*x*(15*I*arctan2(1, c*x) - 2) + 192*(b^2*c^2*x^2 - 2*
I*b^2*c*x - b^2)*arctan(c*x)^3 + 24*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*log
(c^2*x^2 + 1)^3 + 32*a^2*(15*arctan2(1, c*x) + 14*I) + 16*(I*b^2*c^4*x^4 -
4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^2)*arctan(c*x)^2 - 4*(
I*b^2*c^4*x^4 - 4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^2 - 12
*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 36*(I*
b^2*c^8*d^3*x^2 + 2*b^2*c^7*d^3*x - I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7)*c^2/(c
^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 +
1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*
log(c^2*x^2 + 1)^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) - c^2*(c^2/(c^
14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*log(c^2*x^2 + 1)/(c^12*d^3*x^4
+ 2*c^10*d^3*x^2 + c^8*d^3)) - 2048*c^2*integrate(1/64*x^3*arctan(c*x)^2/(c
^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) - 2*log(c^2*x^2
```

+ 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2048*integrate(1/64*x*arc
tan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x)) +
12*(-I*b^2*c^10*d^3*x^2 - 2*b^2*c^9*d^3*x + I*b^2*c^8*d^3)*(((8*c^2*x^2 + 7
)*c^2/(c^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c^2*x^2 + 3)*log(c^
2*x^2 + 1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c^2 + 2048*c^2*integ
rate(1/64*x^5*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 +
 c^4*d^3), x) + 512*c^2*integrate(1/64*x^5*log(c^2*x^2 + 1)^2/(c^10*d^3*x^6
 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c^2
*x^2 + 1)^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) - 2048*integrate(1/64
*x^3*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3)
, x)) - 72*(-I*b^2*c^9*d^3*x^2 - 2*b^2*c^8*d^3*x + I*b^2*c^7*d^3)*(((8*c^2*
x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3) + 2*(4*c^2*x^2 + 3)
*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3))*c^2 + 2*(2*c^2
*x^2 + 1)*log(c^2*x^2 + 1)^2/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) - 102
4*integrate(1/32*x^3*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3
*x^2 + c^3*d^3), x)) - 18*(I*b^2*c^8*d^3*x^2 + 2*b^2*c^7*d^3*x - I*b^2*c^6*
d^3)*(((4*c^2*x^2 + 3)*c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(
2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*
c^2 - 1024*c*integrate(1/32*x^2*arctan(c*x)/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 +
 3*c^6*d^3*x^2 + c^4*d^3), x) - c^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^
3) - 2*log(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)) + 9*(b^2*
c^8*d^3*x^2 - 2*I*b^2*c^7*d^3*x - b^2*c^6*d^3)*((c*((5*c^2*x^3 + 3*x)/(c^12
*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 5*arctan(c*x)/(c^9*d^3)) - 8*(2*c^2*
x^2 + 1)*arctan(c*x)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^2 - c*((3
*c^2*x^3 + 5*x)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 3*arctan(c*x)/(c
^7*d^3)) - 512*c*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^10*d^3*x^6 + 3*c^8*
d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + 8*arctan(c*x)/(c^10*d^3*x^4 + 2*c^
8*d^3*x^2 + c^6*d^3)) - 24*(I*b^2*c^7*d^3*x^2 + 2*b^2*c^6*d^3*x - I*b^2*c^5
d^3)(c^2*(c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*log(c^2*x^2 +
 1)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) + 2*log(c^2*x^2 + 1)^2/(c^9*d
^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3) - 1024*integrate(1/32*x*arctan(c*x)^2/(c^
9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x)) + 15*(b^2*c^10*d^
3*x^2 - 2*I*b^2*c^9*d^3*x - b^2*c^8*d^3)*(512*c^2*integrate(1/16*x^5*arctan
(c*x)/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) - c*((5*
c^2*x^3 + 3*x)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 5*arctan(c*x)/(c
^9*d^3)) - 512*c*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^10*d^3*x^6 + 3*c^8*
d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + 8*(2*c^2*x^2 + 1)*arctan(c*x)/(c^1
2*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3)) + 30*(I*b^2*c^10*d^3*x^2 + 2*b^2*c^9
*d^3*x - I*b^2*c^8*d^3)*(256*c^2*integrate(1/32*x^5*log(c^2*x^2 + 1)/(c^10*
d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + (4*c^2*x^2 + 3)*c^
2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 1024*c*integrate(1/32*x^4*ar
ctan(c*x)/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + 2*
(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))
 - 36*(I*b^2*c^9*d^3*x^2 + 2*b^2*c^8*d^3*x - I*b^2*c^7*d^3)*(128*c^2*integr
ate(1/16*x^4*arctan(c*x)/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^

```
4*d^3), x) + ((4*c^2*x^2 + 3)*c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3
) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8
*d^3))*c - 128*integrate(1/16*x^2*arctan(c*x)/(c^10*d^3*x^6 + 3*c^8*d^3*x^4
 + 3*c^6*d^3*x^2 + c^4*d^3), x)) + 18*(b^2*c^9*d^3*x^2 - 2*I*b^2*c^8*d^3*x
- b^2*c^7*d^3)*(256*c^2*integrate(1/32*x^4*log(c^2*x^2 + 1)/(c^10*d^3*x^6 +
 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) + (c*((5*c^2*x^3 + 3*x)/(c^12
*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 5*arctan(c*x)/(c^9*d^3)) - 8*(2*c^2*
x^2 + 1)*arctan(c*x)/(c^12*d^3*x^4 + 2*c^10*d^3...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
[Out] integral(1/4*(-I*b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^4*log(-(c*x
+ I)/(c*x - I)) + 4*I*a^2*x^4)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x +
 I*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
[Out] int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.112 $\int \frac{x^3(a+b\textbf{ArcTan}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=383

$$\frac{b^2}{16c^4d^3(i-cx)^2}+\frac{21ib^2}{16c^4d^3(i-cx)}-\frac{21ib^2\text{ArcTan}(cx)}{16c^4d^3}+\frac{ib(a+b\text{ArcTan}(cx))}{4c^4d^3(i-cx)^2}-\frac{11b(a+b\text{ArcTan}(cx))}{4c^4d^3(i-cx)}+\frac{3(a+b$$

```
[Out] 1/16*b^2/c^4/d^3/(I-c*x)^2+21/16*I*b^2/c^4/d^3/(I-c*x)-21/16*I*b^2*arctan(c
*x)/c^4/d^3+1/4*I*b*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)^2-11/4*b*(a+b*arctan(
c*x))/c^4/d^3/(I-c*x)+3/8*(a+b*arctan(c*x))^2/c^4/d^3+I*x*(a+b*arctan(c*x))
^2/c^3/d^3-1/2*(a+b*arctan(c*x))^2/c^4/d^3/(I-c*x)^2-3*I*(a+b*arctan(c*x))^
2/c^4/d^3/(I-c*x)+2*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^3+3*(a+b*ar
ctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^3-b^2*polylog(2,1-2/(1+I*c*x))/c^4/d^3+3
*I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^3+3/2*b^2*polylog(3,1
-2/(1+I*c*x))/c^4/d^3
```

Rubi [A]
time = 0.49, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4974, 4972, 641, 46, 209, 5004, 5114, 6745}

$$\frac{3ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^4d^3}-\frac{11b(a+b\text{ArcTan}(cx))}{4c^4d^3(-cx+i)}+\frac{ib(a+b\text{ArcTan}(cx))}{4c^4d^3(-cx+i)^2}-\frac{3i(a+b\text{ArcTan}(cx))^2}{c^4d^3(-cx+i)}-\frac{(a+b\text{ArcTan}(cx))^2}{2c^4d^3(-cx+i)^2}+\frac{3(a+b\text{ArcTan}(cx))^2}{8c^4d^3}+\frac{2ib\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4d^3}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^4d^3}+\frac{ix(a+b\text{ArcTan}(cx))^2}{c^3d^3}-\frac{21ib^2\text{ArcTan}(cx)}{16c^4d^3}-\frac{b^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^4d^3}+\frac{3b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^4d^3}+\frac{21ib^2}{16c^4d^3(-cx+i)}+\frac{b^2}{16c^4d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] b^2/(16*c^4*d^3*(I - c*x)^2) + (((21*I)/16)*b^2)/(c^4*d^3*(I - c*x)) - (((2
1*I)/16)*b^2*ArcTan[c*x])/(c^4*d^3) + ((I/4)*b*(a + b*ArcTan[c*x]))/(c^4*d^
3*(I - c*x)^2) - (11*b*(a + b*ArcTan[c*x]))/(4*c^4*d^3*(I - c*x)) + (3*(a +
 b*ArcTan[c*x])^2)/(8*c^4*d^3) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d^3) - (a
 + b*ArcTan[c*x])^2/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x])^2)
/(c^4*d^3*(I - c*x)) + ((2*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^
4*d^3) + (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^3) - (b^2*Poly
Log[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(
2*c^4*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx = \int \left(\frac{i(a+b\tan^{-1}(cx))^2}{c^3d^3} + \frac{(a+b\tan^{-1}(cx))^2}{c^3d^3(-i+cx)^3} - \frac{3i(a+b\tan^{-1}(cx))^2}{c^3d^3(-i+cx)^2} - \frac{3(a+}{c^3}\right.$$

$$= \frac{i\int(a+b\tan^{-1}(cx))^2\,dx}{c^3d^3} - \frac{(3i)\int\frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{c^3d^3} + \frac{\int\frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^3}\,dx}{c^3d^3} - \frac{3}{}$$

$$= \frac{ix(a+b\tan^{-1}(cx))^2}{c^3d^3} - \frac{(a+b\tan^{-1}(cx))^2}{2c^4d^3(i-cx)^2} - \frac{3i(a+b\tan^{-1}(cx))^2}{c^4d^3(i-cx)} + \frac{3(a+b\mathrm{t}}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{c^4d^3} + \frac{ix(a+b\tan^{-1}(cx))^2}{c^3d^3} - \frac{(a+b\tan^{-1}(cx))^2}{2c^4d^3(i-cx)^2} - \frac{3i(a+b}{c^4d^3}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)^2} - \frac{11b(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)} + \frac{3(a+b\tan^{-1}(cx))^2}{8c^4d^3} + \frac{ix(a+b}{}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)^2} - \frac{11b(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)} + \frac{3(a+b\tan^{-1}(cx))^2}{8c^4d^3} + \frac{ix(a+b}{}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)^2} - \frac{11b(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)} + \frac{3(a+b\tan^{-1}(cx))^2}{8c^4d^3} + \frac{ix(a+b}{}$$

$$= \frac{b^2}{16c^4d^3(i-cx)^2} + \frac{21ib^2}{16c^4d^3(i-cx)} + \frac{ib(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)^2} - \frac{11b(a+b\tan^{-1}(cx}{4c^4d^3(i-cx)}$$

$$= \frac{b^2}{16c^4d^3(i-cx)^2} + \frac{21ib^2}{16c^4d^3(i-cx)} - \frac{21ib^2\tan^{-1}(cx)}{16c^4d^3} + \frac{ib(a+b\tan^{-1}(cx))}{4c^4d^3(i-cx)^2} -$$

Mathematica [A]
time = 0.96, size = 507, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] ((64*I)*a^2*c*x - (32*a^2)/(-I + c*x)^2 + ((192*I)*a^2)/(-I + c*x) - (192*I
)*a^2*ArcTan[c*x] - 96*a^2*Log[1 + c^2*x^2] + (4*I)*a*b*(-96*ArcTan[c*x]^2
+ 20*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*Pol
yLog[2, -E^((2*I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]
*(8*c*x + (10*I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 +
 E^((2*I)*ArcTan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*S
in[4*ArcTan[c*x]]) + I*b^2*((-64*I)*ArcTan[c*x]^2 + 64*c*x*ArcTan[c*x]^2 -
128*ArcTan[c*x]^3 - (40*I)*Cos[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Cos[2*ArcTan
```

```
[c*x]] + (80*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + I*Cos[4*ArcTan[c*x]] - 4
*ArcTan[c*x]*Cos[4*ArcTan[c*x]] - (8*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] +
128*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (192*I)*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] - 64*(I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*Ar
cTan[c*x])] - (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - 40*Sin[2*ArcTan[c
*x]] - (80*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]^2*Sin[2*ArcTa
n[c*x]] + Sin[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] - 8*Arc
Tan[c*x]^2*Sin[4*ArcTan[c*x]]))/(64*c^4*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.78, size = 4768, normalized size = 12.45

method	result	size
derivativedivides	Expression too large to display	4768
default	Expression too large to display	4768

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(3*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*b^2/d^3*Pi*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*
csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))+3/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*ar
ctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-3/4*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*polylog
(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))
*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^2/d^3*Pi
*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*dilog(1-I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1))*arctan(c*x)^2+3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))-3/4*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3*b^2/d^3*Pi*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((
```

1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+3/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*b^2/d^3*Pi*csgn((1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))-3/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*b*a/d^
3*arctan(c*x)/(c*x-I)+3*I*b*a/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)-1/32*I*b^2/d
^3/(c*x-I)^2*c*x+3*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^2/d^3*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-3/4*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^3*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*b^2/d^3*Pi*
csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1+I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^2/d^
3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2-3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^3*arctan(c*x)^2-3/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/8*b^2/d^3*arcta
n(c*x)/(c*x-I)^2*c*x+I*b^2/d^3*arctan(c*x)^2*c*x+3/2*I*b^2/d^3*Pi*csgn((1+I
*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+
1))^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*b^2/d^3*Pi*arctan(c*x)*ln(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*b^2/d^3*Pi*arctan(c*x)*ln((1+I*c*x)^2/(c^2
*x^2+1)+1)+3*b^2/d^3*Pi*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I
*b^2/d^3*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^2/d^3*Pi*polylog
(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/8*I*b^2/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))-3/8*I*b^2/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+1/16*I*b^2/d^3*arctan(c*x)/(c*x-I)^2-3*I*b^2/d^3*arctan(c*x)*polylog(2,-
(1+I*c*x)^2/(c^2*x^2+1))+3*I*b^2/d^3*Pi*arctan(c*x)^2-3*I*b^2/d^3*Pi*dilog(
1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^2/d^3*arctan(c*x)^2/(c*x-I)+19/8*I*b
^2/d^3*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+I*a^2/d^3*c*x-6*b*a/d^3*ar
ctan(c*x)*ln(c*x-I)-b*a/d^3*arctan(c*x)/(c*x-I)^2+1/4*I*b*a/d^3/(c*x-I)^2-1
9/16*I*b*a/d^3*ln(c^2*x^2+1)+3/32*I*b*a/d^3*ln(c^4*x^4+10*c^2*x^2+9)-3/2*I*
b*a/d^3*ln(c*x-I)^2+3*I*b*a/d^3*dilog(-1/2*I*(c*x+I))-1/64*b^2/d^3/(c*x-I)^
2*c^2*x^2-5/8*b^2/d^3/(c*x-I)*c*x-3*I*a^2/d^3*arctan(c*x)+3*I*a^2/d^3/(c*x-
I)+19/8*b*a/d^3*arctan(c*x)+11/4*b*a/d^3/(c*x-I)-3/16*b*a/d^3*arctan(1/2*c*
x)+3/16*b*a/d^3*arctan(1/6*c^3*x^3+7/6*c*x)+3/8*b*a/d^3*arctan(1/2*c*x-1/2*
I)+5*b^2/d^3*arctan(c*x)/(4*c*x-4*I)-3*b^2/d^3*arctan(c*x)^2*ln(c*x-I)-1/2*
b^2/d^3*arctan(c*x)^2/(c*x-I)^2+3*b^2/d^3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/
(c^2*x^2+1))-5/8*I*b^2/d^3/(c*x-I)-2*I*b^2/d^3*arctan(c*x)^3+19/8*b^2/d^3*a
rctan(c*x)^2+3/2*b^2/d^3*polylog(3,-(1+I*c*x)^2...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")

[Out] 1/128*(128*I*a^2*c^3*x^3 + 32*a^2*c^2*x^2*(3*I*arctan2(1, c*x) + 8) + 64*a^2*c*x*(3*arctan2(1, c*x) + 4*I) + 96*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)*arctan(c*x)^3 + 12*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 32*a^2*(-3*I*arctan2(1, c*x) + 10) + 16*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2 + 4*I*b^2*c*x + 5*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2 + 4*I*b^2*c*x + 5*b^2 - 6*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 18*(b^2*c^7*d^3*x^2 - 2*I*b^2*c^6*d^3*x - b^2*c^5*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*log(c^2*x^2 + 1)^2/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) - c^2*(c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*log(c^2*x^2 + 1)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) - 4096*c^2*integrate(1/128*x^3*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3) + 4096*integrate(1/128*x*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x)) - 8*(b^2*c^9*d^3*x^2 - 2*I*b^2*c^8*d^3*x - b^2*c^7*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3))*c^2 + 4096*c^2*integrate(1/128*x^5*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 1024*c^2*integrate(1/128*x^5*log(c^2*x^2 + 1)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)^2/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) - 4096*integrate(1/128*x^3*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x)) + 36*(b^2*c^8*d^3*x^2 - 2*I*b^2*c^7*d^3*x - b^2*c^6*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^2 + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - 2048*integrate(1/64*x^3*arctan(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) - 18*(b^2*c^7*d^3*x^2 - 2*I*b^2*c^6*d^3*x - b^2*c^5*d^3)*(((4*c^2*x^2 + 3)*c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3))*c^2 - 2048*c*integrate(1/64*x^2*arctan(c*x)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) - c^2/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) - 2*log(c^2*x^2 + 1)/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3)) - 9*(I*b^2*c^7*d^3*x^2 + 2*b^2*c^6*d^3*x - I*b^2*c^5*d^3)*((c*((5*c^2*x^3 + 3*x)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) + 5*arctan(c*x)/(c^8*d^3)) - 8*(2*c^2*x^2 + 1)*arctan(c*x)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3))*c^2 - c*((3*c^2*x^3 + 5*x)/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3) + 3*arctan(c*x)/(c^6*d^3)) - 1024*c*integrate(1/32*x^2*log(c^2*x^2 + 1)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 8*arctan(c*x)/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3)) - 12*(b^2*c^6*d^3*x^2 - 2*I*b^2*c^5*d^3*x - b^2*c^4*d^3

```
)*(c^2*(c^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*log(c^2*x^2 + 1)/
(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)) + 2*log(c^2*x^2 + 1)^2/(c^8*d^3*x
^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 2048*integrate(1/64*x*arctan(c*x)^2/(c^8*d^
3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) - 4*(a*b*c^9*d^3*x^2
- 2*I*a*b*c^8*d^3*x - a*b*c^7*d^3)*(1024*c^2*integrate(1/32*x^5*arctan(c*x)
/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) - c*((5*c^2*x^
3 + 3*x)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) + 5*arctan(c*x)/(c^8*d^3)
) + 1024*c*integrate(1/32*x^4*log(c^2*x^2 + 1)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4
 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 8*(2*c^2*x^2 + 1)*arctan(c*x)/(c^11*d^3*x
^4 + 2*c^9*d^3*x^2 + c^7*d^3)) - 2*((2*a*b + 3*I*b^2)*c^9*d^3*x^2 + 2*(-2*I
*a*b + 3*b^2)*c^8*d^3*x - (2*a*b + 3*I*b^2)*c^7*d^3)*(1024*c^2*integrate(1/
32*x^5*arctan(c*x)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3),
 x) - c*((5*c^2*x^3 + 3*x)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3) + 5*arc
tan(c*x)/(c^8*d^3)) - 1024*c*integrate(1/32*x^4*log(c^2*x^2 + 1)/(c^9*d^3*x
^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 8*(2*c^2*x^2 + 1)*arcta
n(c*x)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) + 4*((-2*I*a*b + 3*b^2)*c^
9*d^3*x^2 - 2*(2*a*b + 3*I*b^2)*c^8*d^3*x + (2*I*a*b - 3*b^2)*c^7*d^3)*(512
*c^2*integrate(1/64*x^5*log(c^2*x^2 + 1)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c
^5*d^3*x^2 + c^3*d^3), x) + (4*c^2*x^2 + 3)*c^2/(c^13*d^3*x^4 + 2*c^11*d^3*
x^2 + c^9*d^3) + 2048*c*integrate(1/64*x^4*arctan(c*x)/(c^9*d^3*x^6 + 3*c^7
*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1
)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) + 8*(I*a*b*c^9*d^3*x^2 + 2*a*b*
c^8*d^3*x - I*a*b*c^7*d^3)*(512*c^2*integrate(1/64*x^5*log(c^2*x^2 + 1)/(c^
9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + (4*c^2*x^2 + 3)*
c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) - 2048*c*integrate(1/64*x^4*a
rctan(c*x)/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^3*log(-(c*x
+ I)/(c*x - I)) + 4*I*a^2*x^3)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x +
 I*d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(a + b \operatorname{atan}(c x)\right)^2}{\left(d + c d x 1\mathrm{i}\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

```
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.113 $\int \frac{x^2(a+b\text{ArcTan}(cx))^2}{(d+icdx)^3} dx$

Optimal. Leaf size=304

$$-\frac{ib^2}{16c^3d^3(i-cx)^2}+\frac{13b^2}{16c^3d^3(i-cx)}-\frac{13b^2\text{ArcTan}(cx)}{16c^3d^3}+\frac{b(a+b\text{ArcTan}(cx))}{4c^3d^3(i-cx)^2}+\frac{7ib(a+b\text{ArcTan}(cx))}{4c^3d^3(i-cx)}-\frac{7i(a+}{}$$

```
[Out] -1/16*I*b^2/c^3/d^3/(I-c*x)^2+13/16*b^2/c^3/d^3/(I-c*x)-13/16*b^2*arctan(c*
x)/c^3/d^3+1/4*b*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2+7/4*I*b*(a+b*arctan(c*
x))/c^3/d^3/(I-c*x)-7/8*I*(a+b*arctan(c*x))^2/c^3/d^3+1/2*I*(a+b*arctan(c*x
))^2/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))^2/c^3/d^3/(I-c*x)-I*(a+b*arctan(
c*x))^2*ln(2/(1+I*c*x))/c^3/d^3+b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x)
)/c^3/d^3-1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^3
```

Rubi [A]

time = 0.40, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4996, 4974, 4972, 641, 46, 209, 5004, 4964, 5114, 6745}

$$\frac{b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^3d^3}+\frac{7ib(a+b\text{ArcTan}(cx))}{4c^3d^3(-cx+i)}+\frac{b(a+b\text{ArcTan}(cx))}{4c^3d^3(-cx+i)^2}-\frac{2(a+b\text{ArcTan}(cx))^2}{c^3d^3(-cx+i)}+\frac{i(a+b\text{ArcTan}(cx))^2}{2c^3d^3(-cx+i)^2}-\frac{7i(a+b\text{ArcTan}(cx))^2}{8c^3d^3}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^3d^3}-\frac{13b^2\text{ArcTan}(cx)}{16c^3d^3}-\frac{ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{13b^2}{16c^3d^3(-cx+i)}-\frac{ib^2}{16c^3d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] ((-1/16*I)*b^2)/(c^3*d^3*(I - c*x)^2) + (13*b^2)/(16*c^3*d^3*(I - c*x)) - (
13*b^2*ArcTan[c*x])/(16*c^3*d^3) + (b*(a + b*ArcTan[c*x]))/(4*c^3*d^3*(I -
c*x)^2) + (((7*I)/4)*b*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (((7*I)/8
)*(a + b*ArcTan[c*x])^2)/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^3
*(I - c*x)^2) - (2*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(I - c*x)) - (I*(a + b*A
rcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*(a + b*ArcTan[c*x])*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/(c^3*d^3) - ((I/2)*b^2*PolyLog[3, 1 - 2/(1 + I*c*x
)])/(c^3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx = \int \left(-\frac{i(a+b\tan^{-1}(cx))^2}{c^2d^3(-i+cx)^3} - \frac{2(a+b\tan^{-1}(cx))^2}{c^2d^3(-i+cx)^2} + \frac{i(a+b\tan^{-1}(cx))^2}{c^2d^3(-i+cx)}\right)dx$$

$$= -\frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^3}\,dx}{c^2d^3} + \frac{i\int \frac{(a+b\tan^{-1}(cx))^2}{-i+cx}\,dx}{c^2d^3} - \frac{2\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{c^2d^3}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))^2}{c^3d^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3} -$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{2c^3d^3(i-cx)^2} - \frac{2(a+b\tan^{-1}(cx))^2}{c^3d^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3} +$$

$$= \frac{b(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)^2} + \frac{7ib(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)} - \frac{7i(a+b\tan^{-1}(cx))^2}{8c^3d^3} + \frac{i(a+b\,\mathrm{t}}{2c^3d^3(}$$

$$= \frac{b(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)^2} + \frac{7ib(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)} - \frac{7i(a+b\tan^{-1}(cx))^2}{8c^3d^3} + \frac{i(a+b\,\mathrm{t}}{2c^3d^3(}$$

$$= \frac{b(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)^2} + \frac{7ib(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)} - \frac{7i(a+b\tan^{-1}(cx))^2}{8c^3d^3} + \frac{i(a+b\,\mathrm{t}}{2c^3d^3(}$$

$$= -\frac{ib^2}{16c^3d^3(i-cx)^2} + \frac{13b^2}{16c^3d^3(i-cx)} + \frac{b(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)^2} + \frac{7ib(a+b\tan^{-1}(c}{4c^3d^3(i-cx)}$$

$$= -\frac{ib^2}{16c^3d^3(i-cx)^2} + \frac{13b^2}{16c^3d^3(i-cx)} - \frac{13b^2\tan^{-1}(cx)}{16c^3d^3} + \frac{b(a+b\tan^{-1}(cx))}{4c^3d^3(i-cx)^2} +$$

Mathematica [A]
time = 0.72, size = 431, normalized size = 1.42

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (((96*I)*a^2)/(-I + c*x)^2 + (384*a^2)/(-I + c*x) - 192*a^2*ArcTan[c*x] + (
96*I)*a^2*Log[1 + c^2*x^2] - b^2*(128*ArcTan[c*x]^3 + (72*I)*Cos[2*ArcTan[c
*x]] - 144*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2*Cos[2*Arc
Tan[c*x]] - (3*I)*Cos[4*ArcTan[c*x]] + 12*ArcTan[c*x]*Cos[4*ArcTan[c*x]] +
(24*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + (192*I)*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] + 192*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] +
(96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] + 72*Sin[2*ArcTan[c*x]] + (144*I)
*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]] - 3*
Sin[4*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] + 24*ArcTan[c*x]
^2*Sin[4*ArcTan[c*x]]) - 12*a*b*(32*ArcTan[c*x]^2 - 12*Cos[2*ArcTan[c*x]] +
 Cos[4*ArcTan[c*x]] + 16*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (12*I)*Sin[2*
ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*((-6*I)*Cos[2*ArcTan[c*
x]] + I*Cos[4*ArcTan[c*x]] + (8*I)*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*Sin[2
*ArcTan[c*x]] + Sin[4*ArcTan[c*x]])))/(192*c^3*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.61, size = 1164, normalized size = 3.83

method	result	size
derivativedivides	Expression too large to display	1164
default	Expression too large to display	1164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/64*I*b^2/d^3/(c*x-I)^2*c^2*x^2-3*b^2/d^3/(8*c*x-8*I)-2/3*b^2/d^3*a
rctan(c*x)^3+1/4*b*a/d^3/(c*x-I)^2-7/16*b*a/d^3*ln(c^2*x^2+1)+7/32*b*a/d^3*
ln(c^4*x^4+10*c^2*x^2+9)-1/2*b*a/d^3*ln(c*x-I)^2-a^2/d^3*arctan(c*x)+2*a^2/
d^3/(c*x-I)-1/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))
*arctan(c*x)^2-1/8*I*b^2/d^3*arctan(c*x)/(c*x-I)^2*c*x+3*I*b^2/d^3/(8*c*x-8
*I)*c*x+I*b*a/d^3*arctan(c*x)/(c*x-I)^2+2*I*b*a/d^3*arctan(c*x)*ln(c*x-I)+I
*b^2/d^3*arctan(c*x)^2*ln(c*x-I)+1/2*I*b^2/d^3*arctan(c*x)^2/(c*x-I)^2-3/4*
I*b^2/d^3*arctan(c*x)/(c*x-I)-I*b^2/d^3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c
^2*x^2+1))-7/4*I*b*a/d^3/(c*x-I)-7/8*I*b*a/d^3*arctan(c*x)+7/16*I*b*a/d^3*a
rctan(1/2*c*x)-7/16*I*b*a/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-7/8*I*b*a/d^3*arc
tan(1/2*c*x-1/2*I)-7/8*I*b^2/d^3*arctan(c*x)^2-1/64*I*b^2/d^3/(c*x-I)^2-1/2
*I*b^2/d^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*a^2/d^3*ln(c^2*x^2+1)+
1/2*I*a^2/d^3/(c*x-I)^2+b*a/d^3*dilog(-1/2*I*(c*x+I))+b^2/d^3*Pi*arctan(c*x
)^2-b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2/d^3*arcta
n(c*x)^2/(c*x-I)+1/16*b^2/d^3*arctan(c*x)/(c*x-I)^2-1/2*b^2/d^3*Pi*csgn((1+
I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^2+1/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/4*
b^2/d^3*arctan(c*x)/(c*x-I)*c*x-1/16*b^2/d^3*arctan(c*x)/(c*x-I)^2*c^2*x^2+
```

```
4*b*a/d^3*arctan(c*x)/(c*x-I)+b*a/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/32*b^2
/d^3/(c*x-I)^2*c*x-1/2*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^3*arctan(c*x)^2-b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(144*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(9*I*arctan2(1, c*x) -
8) - 32*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x)^3 + 4*(-I*b^2*c^2*x^2
 - 2*b^2*c*x + I*b^2)*log(c^2*x^2 + 1)^3 - 48*a^2*(3*arctan2(1, c*x) + 4*I)
 + 16*(4*b^2*c*x - 3*I*b^2)*arctan(c*x)^2 - 4*(4*b^2*c*x - 3*I*b^2 + 2*(b^2
*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(I*b^2*c^
6*d^3*x^2 + 2*b^2*c^5*d^3*x - I*b^2*c^4*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^14*d^
3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^
12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*log(c^2
*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - c^2*(c^2/(c^12*d^3*x
^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*log(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c^8*d
^3*x^2 + c^6*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c^8*d^3*x^6
+ 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^8*
d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 512*integrate(1/16*x*arctan(c*x)^2/(c^
8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) - 4*(-I*b^2*c^8*d
^3*x^2 - 2*b^2*c^7*d^3*x + I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^14*d^3*x
^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^12*
d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^2 + 512*c^2*integrate(1/16*x^5*arcta
n(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 128*
c^2*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*
c^4*d^3*x^2 + c^2*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)^2/(c^10*d^3
*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - 512*integrate(1/16*x^3*arctan(c*x)^2/(c^8
*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) + 8*(-I*b^2*c^7*d^
3*x^2 - 2*b^2*c^6*d^3*x + I*b^2*c^5*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^13*d^3*x^
4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^11*d^
3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3))*c^2 + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)^
2/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3) - 256*integrate(1/8*x^3*arctan(c*
x)^2/(c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3), x)) + 14*(I*b^2
*c^6*d^3*x^2 + 2*b^2*c^5*d^3*x - I*b^2*c^4*d^3)*(((4*c^2*x^2 + 3)*c^2/(c^12
*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(
c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3))*c^2 - 256*c*integrate(1/8*x^2*arct
an(c*x)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) - c^2/(
c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 +
```

```
 2*c^6*d^3*x^2 + c^4*d^3)) - 7*(b^2*c^6*d^3*x^2 - 2*I*b^2*c^5*d^3*x - b^2*c
^4*d^3)*((c*((5*c^2*x^3 + 3*x)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 5
*arctan(c*x)/(c^7*d^3)) - 8*(2*c^2*x^2 + 1)*arctan(c*x)/(c^10*d^3*x^4 + 2*c
^8*d^3*x^2 + c^6*d^3))*c^2 - c*((3*c^2*x^3 + 5*x)/(c^8*d^3*x^4 + 2*c^6*d^3*
x^2 + c^4*d^3) + 3*arctan(c*x)/(c^5*d^3)) - 128*c*integrate(1/4*x^2*log(c^2
*x^2 + 1)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 8*a
rctan(c*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)) + 4*(I*b^2*c^5*d^3*x^2
+ 2*b^2*c^4*d^3*x - I*b^2*c^3*d^3)*(c^2*(c^2/(c^11*d^3*x^4 + 2*c^9*d^3*x^2
+ c^7*d^3) + 2*log(c^2*x^2 + 1)/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^3)) +
2*log(c^2*x^2 + 1)^2/(c^7*d^3*x^4 + 2*c^5*d^3*x^2 + c^3*d^3) - 256*integrat
e(1/8*x*arctan(c*x)^2/(c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)
, x)) - 4*(-I*a*b*c^8*d^3*x^2 - 2*a*b*c^7*d^3*x + I*a*b*c^6*d^3)*(128*c^2*i
ntegrate(1/4*x^5*arctan(c*x)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 +
 c^2*d^3), x) - c*((5*c^2*x^3 + 3*x)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^
3) + 5*arctan(c*x)/(c^7*d^3)) + 128*c*integrate(1/4*x^4*log(c^2*x^2 + 1)/(c
^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 8*(2*c^2*x^2 +
1)*arctan(c*x)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)) + 4*(I*a*b*c^8*d^3
*x^2 + 2*a*b*c^7*d^3*x - I*a*b*c^6*d^3)*(128*c^2*integrate(1/4*x^5*arctan(c
*x)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) - c*((5*c^2
*x^3 + 3*x)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 5*arctan(c*x)/(c^7*d
^3)) - 128*c*integrate(1/4*x^4*log(c^2*x^2 + 1)/(c^8*d^3*x^6 + 3*c^6*d^3*x^
4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 8*(2*c^2*x^2 + 1)*arctan(c*x)/(c^10*d^3*
x^4 + 2*c^8*d^3*x^2 + c^6*d^3)) - 8*(a*b*c^8*d^3*x^2 - 2*I*a*b*c^7*d^3*x -
a*b*c^6*d^3)*(64*c^2*integrate(1/8*x^5*log(c^2*x^2 + 1)/(c^8*d^3*x^6 + 3*c^
6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + (4*c^2*x^2 + 3)*c^2/(c^12*d^3*x^
4 + 2*c^10*d^3*x^2 + c^8*d^3) + 256*c*integrate(1/8*x^4*arctan(c*x)/(c^8*d^
3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 2*(2*c^2*x^2 + 1)*lo
g(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)) + 8*(a*b*c^8*d^3*x
^2 - 2*I*a*b*c^7*d^3*x - a*b*c^6*d^3)*(64*c^2*integrate(1/8*x^5*log(c^2*x^2
 + 1)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + (4*c^2*
x^2 + 3)*c^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) - 256*c*integrate(1/
8*x^4*arctan(c*x)/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3),
x) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6
*d^3)) - 16*((a*b + I*b^2)*c^7*d^3*x^2 - 2*(I*a*b - b^2)*c^6*d^3*x - (a*b +
 I*b^2)*c^5*d^3)*(32*c^2*integrate(1/4*x^4*arct...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/4*(-I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^2*log(-(c*x
```

```
+ I)/(c*x - I)) + 4*I*a^2*x^2)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x +
 I*d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.114 $\int \frac{x(a+b\text{ArcTan}(cx))^2}{(d+icdx)^3} dx$

Optimal. Leaf size=178

$$-\frac{b^2}{16c^2d^3(i-cx)^2}-\frac{5ib^2}{16c^2d^3(i-cx)}+\frac{5ib^2\text{ArcTan}(cx)}{16c^2d^3}-\frac{ib(a+b\text{ArcTan}(cx))}{4c^2d^3(i-cx)^2}+\frac{3b(a+b\text{ArcTan}(cx))}{4c^2d^3(i-cx)}+\frac{(a+b\text{A}}{8}$$

```
[Out] -1/16*b^2/c^2/d^3/(I-c*x)^2-5/16*I*b^2/c^2/d^3/(I-c*x)+5/16*I*b^2*arctan(c*
x)/c^2/d^3-1/4*I*b*(a+b*arctan(c*x))/c^2/d^3/(I-c*x)^2+3/4*b*(a+b*arctan(c*
x))/c^2/d^3/(I-c*x)+1/8*(a+b*arctan(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctan(c*x
))^2/d^3/(1+I*c*x)^2
```

Rubi [A]
time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {37, 4994, 4972, 641, 46, 209, 5004}

$$\frac{3b(a+b\text{ArcTan}(cx))}{4c^2d^3(-cx+i)}-\frac{ib(a+b\text{ArcTan}(cx))}{4c^2d^3(-cx+i)^2}+\frac{(a+b\text{ArcTan}(cx))^2}{8c^2d^3}+\frac{x^2(a+b\text{ArcTan}(cx))^2}{2d^3(1+icx)^2}+\frac{5ib^2\text{ArcTan}(cx)}{16c^2d^3}-\frac{5ib^2}{16c^2d^3(-cx+i)}-\frac{b^2}{16c^2d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] -1/16*b^2/(c^2*d^3*(I - c*x)^2) - (((5*I)/16)*b^2)/(c^2*d^3*(I - c*x)) + ((
(5*I)/16)*b^2*ArcTan[c*x])/(c^2*d^3) - ((I/4)*b*(a + b*ArcTan[c*x]))/(c^2*d
^3*(I - c*x)^2) + (3*b*(a + b*ArcTan[c*x]))/(4*c^2*d^3*(I - c*x)) + (a + b*
ArcTan[c*x])^2/(8*c^2*d^3) + (x^2*(a + b*ArcTan[c*x])^2)/(2*d^3*(1 + I*c*x)
^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4994

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dis
t[(a + b*ArcTan[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*Arc
Tan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
 q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[m,
 -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx = \frac{x^2(a+b\tan^{-1}(cx))^2}{2d^3(1+icx)^2} - (2bc)\int \left(-\frac{i(a+b\tan^{-1}(cx))}{4c^2d^3(-i+cx)^3} - \frac{3(a+b\tan^{-1}(cx))}{8c^2d^3(-i+cx)^2} - \right.$$

$$= \frac{x^2(a+b\tan^{-1}(cx))^2}{2d^3(1+icx)^2} + \frac{(ib)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^3}\,dx}{2cd^3} + \frac{b\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{4cd^3} + \frac{(3b)\int \frac{a+b\tan^{-1}(cx)}{(-i}}{4cd^3}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{8c^2d^3} + \frac{x^2(a+b\tan}{2d^3(1+}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{8c^2d^3} + \frac{x^2(a+b\tan}{2d^3(1+}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)} + \frac{(a+b\tan^{-1}(cx))^2}{8c^2d^3} + \frac{x^2(a+b\tan}{2d^3(1+}$$

$$= -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} - \frac{ib(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\tan^{-1}(cx)}{4c^2d^3(i-cx)}$$

$$= -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} + \frac{5ib^2\tan^{-1}(cx)}{16c^2d^3} - \frac{ib(a+b\tan^{-1}(cx))}{4c^2d^3(i-cx)^2} + \frac{3}{}$$

Mathematica [A]
time = 0.10, size = 117, normalized size = 0.66

$$\frac{4ab(2i-3cx)+b^2(4+5icx)+a^2(-8-16icx)+b(i+cx)(a(4i-12cx)+b(3+5icx))\text{ArcTan}(cx)-2b^2(1+2icx+3c^2x^2)\text{ArcTan}(cx)^2}{16c^2d^3(-i+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

```
[Out] (4*a*b*(2*I - 3*c*x) + b^2*(4 + (5*I)*c*x) + a^2*(-8 - (16*I)*c*x) + b*(I +
 c*x)*(a*(4*I - 12*c*x) + b*(3 + (5*I)*c*x))*ArcTan[c*x] - 2*b^2*(1 + (2*I)
*c*x + 3*c^2*x^2)*ArcTan[c*x]^2)/(16*c^2*d^3*(-I + c*x)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(156) = 312.
time = 0.36, size = 401, normalized size = 2.25

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{5ib^2\arctan(cx)}{16d^3}+\frac{b^2\arctan(cx)^2}{2d^3(cx-i)^2}+\frac{5ib^2}{16d^3(cx-i)}-\frac{2iba\arctan(cx)}{d^3(cx-i)}-\frac{3b^2\arctan(cx)}{4d^3(cx-i)}-\frac{iba}{4d^3(cx-i)^2}-\frac{ib^2\arctan}{4d^3(cx-}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{5ib^2\arctan(cx)}{16d^3}+\frac{b^2\arctan(cx)^2}{2d^3(cx-i)^2}+\frac{5ib^2}{16d^3(cx-i)}-\frac{2iba\arctan(cx)}{d^3(cx-i)}-\frac{3b^2\arctan(cx)}{4d^3(cx-i)}-\frac{iba}{4d^3(cx-i)^2}-\frac{ib^2\arctan}{4d^3(cx-}}{}$

risch	$\frac{(3b^2c^2x^2+2ib^2cx+b^2)\ln(icx+1)^2}{32c^2d^3(cx-i)^2} - \frac{(2i\ln(-icx+1)b^2cx+b^2\ln(-icx+1)+3\ln(-icx+1)b^2c^2x^2-6ib^2cx+16abcx-8ib}{16c^2d^3(cx-i)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a^2/d^3*(1/2/(c*x-I)^2-I/(c*x-I))+5/16*I*b^2/d^3*arctan(c*x)+1/2*b^2
/d^3*arctan(c*x)^2/(c*x-I)^2+5/16*I*b^2/d^3/(c*x-I)-2*I*b*a/d^3*arctan(c*x)
/(c*x-I)-3/4*b^2/d^3*arctan(c*x)/(c*x-I)-1/4*I*b*a/d^3/(c*x-I)^2-1/4*I*b^2/
d^3*arctan(c*x)/(c*x-I)^2-1/16*b^2/d^3/(c*x-I)^2+3/8*I*b^2/d^3*arctan(c*x)*
ln(c*x-I)-3/32*b^2/d^3*ln(c*x-I)^2+3/16*b^2/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I)
)-3/32*b^2/d^3*ln(c*x+I)^2-3/16*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+
I))+3/16*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(c*x+I)-3/8*I*b^2/d^3*arctan(c*x)*ln
(c*x+I)+b*a/d^3*arctan(c*x)/(c*x-I)^2-I*b^2/d^3*arctan(c*x)^2/(c*x-I)-3/4*b
*a/d^3*arctan(c*x)-3/4*b*a/d^3/(c*x-I))
```

Maxima [A]
time = 0.31, size = 141, normalized size = 0.79

$$-\frac{(16i\,a^2+12\,ab-5i\,b^2)cx+2\,(3\,b^2c^2x^2+2i\,b^2cx+b^2)\arctan(cx)^2+8\,a^2-8i\,ab-4\,b^2+((12\,ab-5i\,b^2)c^2x^2-2\,(-4i\,ab-b^2)cx+4\,ab-3i\,b^2)\arctan(cx)}{16\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((16*I*a^2 + 12*a*b - 5*I*b^2)*c*x + 2*(3*b^2*c^2*x^2 + 2*I*b^2*c*x +
 b^2)*arctan(c*x)^2 + 8*a^2 - 8*I*a*b - 4*b^2 + ((12*a*b - 5*I*b^2)*c^2*x^2
 - 2*(-4*I*a*b - b^2)*c*x + 4*a*b - 3*I*b^2)*arctan(c*x))/(c^4*d^3*x^2 - 2*
I*c^3*d^3*x - c^2*d^3)
```

Fricas [A]
time = 0.70, size = 165, normalized size = 0.93

$$-\frac{2\,(16i\,a^2+12\,ab-5i\,b^2)cx-(3\,b^2c^2x^2+2i\,b^2cx+b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16\,a^2-16i\,ab-8\,b^2-((-12i\,ab-5\,b^2)c^2x^2+2\,(4\,ab-i\,b^2)cx-4i\,ab-3\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)}{32\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(16*I*a^2 + 12*a*b - 5*I*b^2)*c*x - (3*b^2*c^2*x^2 + 2*I*b^2*c*x +
 b^2)*log(-(c*x + I)/(c*x - I))^2 + 16*a^2 - 16*I*a*b - 8*b^2 - ((-12*I*a*b
 - 5*b^2)*c^2*x^2 + 2*(4*a*b - I*b^2)*c*x - 4*I*a*b - 3*b^2)*log(-(c*x + I)
/(c*x - I)))/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(156) = 312.
time = 49.52, size = 502, normalized size = 2.82

$$\frac{ib(12a-5ib)\log\left(-\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3}-\frac{ib(12a-5ib)\log\left(\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3}+\frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(-icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3}+\frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3}+\frac{-8a^2+8iab+4b^2+x(-16ia^2c-12abc+5ib^2c)}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3}+\frac{(16abcx-8iab-3b^2c^2x^2\log(icx+1)-2ib^2cx\log(icx+1)-6ib^2cx-b^2\log(icx+1)-4b^2)\log(-icx+1)}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3}+\frac{(-8abcx+4iab+3ib^2cx+2b^2)\log(icx+1)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] I*b*(12*a - 5*I*b)*log(-I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I*b**2))/(32*c
**2*d**3) - I*b*(12*a - 5*I*b)*log(I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I*b
**2))/(32*c**2*d**3) + (3*b**2*c**2*x**2 + 2*I*b**2*c*x + b**2)*log(-I*c*x
+ 1)**2/(32*c**4*d**3*x**2 - 64*I*c**3*d**3*x - 32*c**2*d**3) + (3*b**2*c**
2*x**2 + 2*I*b**2*c*x + b**2)*log(I*c*x + 1)**2/(32*c**4*d**3*x**2 - 64*I*c
**3*d**3*x - 32*c**2*d**3) + (-8*a**2 + 8*I*a*b + 4*b**2 + x*(-16*I*a**2*c
- 12*a*b*c + 5*I*b**2*c))/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16*c**2*d
**3) + (16*a*b*c*x - 8*I*a*b - 3*b**2*c**2*x**2*log(I*c*x + 1) - 2*I*b**2*c
*x*log(I*c*x + 1) - 6*I*b**2*c*x - b**2*log(I*c*x + 1) - 4*b**2)*log(-I*c*x
 + 1)/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16*c**2*d**3) + (-8*a*b*c*x +
 4*I*a*b + 3*I*b**2*c*x + 2*b**2)*log(I*c*x + 1)/(8*c**4*d**3*x**2 - 16*I*c
**3*d**3*x - 8*c**2*d**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

```
[Out] int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.115 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{(d+icdx)^3}\,dx$

Optimal. Leaf size=180

$$\frac{ib^2}{16cd^3(i-cx)^2}+\frac{3b^2}{16cd^3(i-cx)}-\frac{3b^2\text{ArcTan}(cx)}{16cd^3}-\frac{b(a+b\text{ArcTan}(cx))}{4cd^3(i-cx)^2}+\frac{ib(a+b\text{ArcTan}(cx))}{4cd^3(i-cx)}-\frac{i(a+b\text{ArcT}}{8cd}$$

```
[Out] 1/16*I*b^2/c/d^3/(I-c*x)^2+3/16*b^2/c/d^3/(I-c*x)-3/16*b^2*arctan(c*x)/c/d^
3-1/4*b*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+1/4*I*b*(a+b*arctan(c*x))/c/d^3/(
I-c*x)-1/8*I*(a+b*arctan(c*x))^2/c/d^3+1/2*I*(a+b*arctan(c*x))^2/c/d^3/(1+I
*c*x)^2
```

Rubi [A]

time = 0.14, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{ib(a+b\text{ArcTan}(cx))}{4cd^3(-cx+i)}-\frac{b(a+b\text{ArcTan}(cx))}{4cd^3(-cx+i)^2}+\frac{i(a+b\text{ArcTan}(cx))^2}{2cd^3(1+icx)^2}-\frac{i(a+b\text{ArcTan}(cx))^2}{8cd^3}-\frac{3b^2\text{ArcTan}(cx)}{16cd^3}+\frac{3b^2}{16cd^3(-cx+i)}+\frac{ib^2}{16cd^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

```
[Out] ((I/16)*b^2)/(c*d^3*(I - c*x)^2) + (3*b^2)/(16*c*d^3*(I - c*x)) - (3*b^2*Ar
cTan[c*x])/(16*c*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c*d^3*(I - c*x)^2) + ((I
/4)*b*(a + b*ArcTan[c*x]))/(c*d^3*(I - c*x)) - ((I/8)*(a + b*ArcTan[c*x])^2
)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^3*(1 + I*c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{(d+icdx)^3}\,dx = \frac{i(a+b\tan^{-1}(cx))^2}{2cd^3(1+icx)^2} - \frac{(ib)\int\left(\frac{i(a+b\tan^{-1}(cx))}{2d^2(-i+cx)^3} - \frac{a+b\tan^{-1}(cx)}{4d^2(-i+cx)^2} + \frac{a+b\tan^{-1}(cx)}{4d^2(1+c^2x^2)}\right)dx}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{2cd^3(1+icx)^2} + \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx}{4d^3} - \frac{(ib)\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{4d^3} + \frac{b\int\frac{a+b\tan^{-1}}{(-i+cx}}{2d^3}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{4cd^3(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))}{4cd^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{8cd^3} + \frac{i(a+b\tan^{-1}}{2cd^3(1+i}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{4cd^3(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))}{4cd^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{8cd^3} + \frac{i(a+b\tan^{-1}}{2cd^3(1+i}$$

$$= -\frac{b(a+b\tan^{-1}(cx))}{4cd^3(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))}{4cd^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^2}{8cd^3} + \frac{i(a+b\tan^{-1}}{2cd^3(1+i}$$

$$= \frac{ib^2}{16cd^3(i-cx)^2} + \frac{3b^2}{16cd^3(i-cx)} - \frac{b(a+b\tan^{-1}(cx))}{4cd^3(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))}{4cd^3(i-cx)} - \frac{i(}{}$$

$$= \frac{ib^2}{16cd^3(i-cx)^2} + \frac{3b^2}{16cd^3(i-cx)} - \frac{3b^2\tan^{-1}(cx)}{16cd^3} - \frac{b(a+b\tan^{-1}(cx))}{4cd^3(i-cx)^2} + \frac{ib(a+b}{4cd^3}$$

Mathematica [A]
time = 0.08, size = 110, normalized size = 0.61

$$-\frac{i(8a^2+b^2(-4-3icx)+4ab(-2i+cx)+b(i+cx)(b(-5-3icx)+4a(-3i+cx))\text{ArcTan}(cx)+2b^2(3-2icx+c^2x^2)\,\text{ArcTan}(cx)^2)}{16cd^3(-i+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

```
[Out] ((-1/16*I)*(8*a^2 + b^2*(-4 - (3*I)*c*x) + 4*a*b*(-2*I + c*x) + b*(I + c*x)
*(b*(-5 - (3*I)*c*x) + 4*a*(-3*I + c*x))*ArcTan[c*x] + 2*b^2*(3 - (2*I)*c*x
 + c^2*x^2)*ArcTan[c*x]^2))/(c*d^3*(-I + c*x)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(157) = 314.
time = 0.31, size = 355, normalized size = 1.97

method	result
derivativedivides	$\frac{\frac{iba\arctan(cx)}{d^3(icx+1)^2}-\frac{ib^2\arctan(cx)}{4d^3(cx-i)}-\frac{b^2\arctan(cx)\ln(cx-i)}{8d^3}-\frac{b^2\arctan(cx)}{4d^3(cx-i)^2}+\frac{ib^2}{16d^3(cx-i)^2}+\frac{b^2\arctan(cx)\ln(cx+i)}{8d^3}-\frac{ib^2\ln(cx-i)^2}{32d^3}-\cdots}{}$
default	$\frac{\frac{iba\arctan(cx)}{d^3(icx+1)^2}-\frac{ib^2\arctan(cx)}{4d^3(cx-i)}-\frac{b^2\arctan(cx)\ln(cx-i)}{8d^3}-\frac{b^2\arctan(cx)}{4d^3(cx-i)^2}+\frac{ib^2}{16d^3(cx-i)^2}+\frac{b^2\arctan(cx)\ln(cx+i)}{8d^3}-\frac{ib^2\ln(cx-i)^2}{32d^3}-\cdots}{}$
risch	$\frac{ib^2(c^2x^2-2icx+3)\ln(icx+1)^2}{32d^3(cx-i)^2c}-\frac{(3ib^2\ln(-icx+1)+i\ln(-icx+1)b^2c^2x^2+2\ln(-icx+1)b^2cx+2b^2cx-4ib^2+8ba)\ln(ic\cdots}{16d^3(cx-i)^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*b*a/d^3/(1+I*c*x)^2*arctan(c*x)-1/4*I*b^2/d^3*arctan(c*x)/(c*x-I)-1/
8*b^2/d^3*arctan(c*x)*ln(c*x-I)-1/4*b^2/d^3*arctan(c*x)/(c*x-I)^2+1/16*I*b^
2/d^3/(c*x-I)^2+1/8*b^2/d^3*arctan(c*x)*ln(c*x+I)-1/32*I*b^2/d^3*ln(c*x-I)^
2-1/4*I*b*a/d^3/(c*x-I)-1/16*I*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(-1/2*I*(c*x+I
))-3/16*b^2/d^3/(c*x-I)+1/16*I*b^2/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3/16*b^
2/d^3*arctan(c*x)-1/4*I*b*a/d^3*arctan(c*x)+1/2*I*a^2/d^3/(1+I*c*x)^2+1/16*
I*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(c*x+I)+1/2*I*b^2/d^3/(1+I*c*x)^2*arctan(c*
x)^2-1/4*b*a/d^3/(c*x-I)^2-1/32*I*b^2/d^3*ln(c*x+I)^2)
```

Maxima [A]
time = 0.30, size = 136, normalized size = 0.76

$$-\frac{(4i\,ab+3\,b^2)cx-2\,(-i\,b^2c^2x^2-2\,b^2cx-3i\,b^2)\arctan(cx)^2+8i\,a^2+8\,ab-4i\,b^2+((4i\,ab+3\,b^2)c^2x^2+2\,(4\,ab-i\,b^2)cx+12i\,ab+5\,b^2)\arctan(cx)}{16\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((4*I*a*b + 3*b^2)*c*x - 2*(-I*b^2*c^2*x^2 - 2*b^2*c*x - 3*I*b^2)*arc
tan(c*x)^2 + 8*I*a^2 + 8*a*b - 4*I*b^2 + ((4*I*a*b + 3*b^2)*c^2*x^2 + 2*(4*
a*b - I*b^2)*c*x + 12*I*a*b + 5*b^2)*arctan(c*x))/(c^3*d^3*x^2 - 2*I*c^2*d^
3*x - c*d^3)
```

Fricas [A]
time = 0.77, size = 158, normalized size = 0.88

$$-\frac{2\,(4i\,ab+3\,b^2)cx-(i\,b^2c^2x^2+2\,b^2cx+3i\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16i\,a^2+16\,ab-8i\,b^2-((4\,ab-3i\,b^2)c^2x^2-2\,(4i\,ab+b^2)cx+12\,ab-5i\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)}{32\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(4*I*a*b + 3*b^2)*c*x - (I*b^2*c^2*x^2 + 2*b^2*c*x + 3*I*b^2)*log(
-(c*x + I)/(c*x - I))^2 + 16*I*a^2 + 16*a*b - 8*I*b^2 - ((4*a*b - 3*I*b^2)*
c^2*x^2 - 2*(4*I*a*b + b^2)*c*x + 12*a*b - 5*I*b^2)*log(-(c*x + I)/(c*x - I
)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(144) = 288$.
time = 40.48, size = 464, normalized size = 2.58

$$-\frac{b(4a-3ib)\log\left(-\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3}+\frac{b(4a-3ib)\log\left(\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3}+\frac{(-4ab-b^2cx+2ib^2)\log\left(icx+1\right)}{8c^3d^3x^2-16ic^2d^3x-8cd^3}+\frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log\left(-icx+1\right)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3}+\frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log\left(icx+1\right)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3}+\frac{-8ia^2-8ab+4ib^2+x(-4iabc-3b^2c)}{16c^3d^3x^2-32ic^2d^3x-16cd^3}+\frac{(8ab-ib^2c^2x^2\log\left(icx+1\right)-2b^2cx\log\left(icx+1\right)+2b^2cx-3ib^2\log\left(icx+1\right)-4ib^2)\log\left(-icx+1\right)}{16c^3d^3x^2-32ic^2d^3x-16cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

```
[Out] -b*(4*a - 3*I*b)*log(-I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*c*d**
3) + b*(4*a - 3*I*b)*log(I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*c*
d**3) + (-4*a*b - b**2*c*x + 2*I*b**2)*log(I*c*x + 1)/(8*c**3*d**3*x**2 - 1
6*I*c**2*d**3*x - 8*c*d**3) + (I*b**2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2)*lo
g(-I*c*x + 1)**2/(32*c**3*d**3*x**2 - 64*I*c**2*d**3*x - 32*c*d**3) + (I*b*
*2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2)*log(I*c*x + 1)**2/(32*c**3*d**3*x**2
- 64*I*c**2*d**3*x - 32*c*d**3) + (-8*I*a**2 - 8*a*b + 4*I*b**2 + x*(-4*I*a
*b*c - 3*b**2*c))/(16*c**3*d**3*x**2 - 32*I*c**2*d**3*x - 16*c*d**3) + (8*a
*b - I*b**2*c**2*x**2*log(I*c*x + 1) - 2*b**2*c*x*log(I*c*x + 1) + 2*b**2*c
*x - 3*I*b**2*log(I*c*x + 1) - 4*I*b**2)*log(-I*c*x + 1)/(16*c**3*d**3*x**2
 - 32*I*c**2*d**3*x - 16*c*d**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3,x)
[Out] int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3, x)
```

3.116 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{x(d+icdx)^3}\,dx$

Optimal. Leaf size=299

$$\frac{b^2}{16d^3(i-cx)^2}-\frac{11ib^2}{16d^3(i-cx)}+\frac{11ib^2\text{ArcTan}(cx)}{16d^3}+\frac{ib(a+b\text{ArcTan}(cx))}{4d^3(i-cx)^2}+\frac{5b(a+b\text{ArcTan}(cx))}{4d^3(i-cx)}-\frac{5(a+b\text{ArcT}}{8d^3}$$

```
[Out] 1/16*b^2/d^3/(I-c*x)^2-11/16*I*b^2/d^3/(I-c*x)+11/16*I*b^2*arctan(c*x)/d^3+
1/4*I*b*(a+b*arctan(c*x))/d^3/(I-c*x)^2+5/4*b*(a+b*arctan(c*x))/d^3/(I-c*x)
-5/8*(a+b*arctan(c*x))^2/d^3-1/2*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+I*(a+b*a
rctan(c*x))^2/d^3/(I-c*x)-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3
+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^3+I*b*(a+b*arctan(c*x))*polylog(2,-1
+2/(1+I*c*x))/d^3+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d^3
```

Rubi [A]
time = 0.57, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {4996, 4942, 5108, 5004, 5114, 6745, 4974, 4972, 641, 46, 209, 4964}

$$\frac{ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{5b(a+b\text{ArcTan}(cx))}{4d^3(-cx+i)}+\frac{ib(a+b\text{ArcTan}(cx))}{4d^3(-cx+i)^2}+\frac{i(a+b\text{ArcTan}(cx))^2}{d^3(-cx+i)}-\frac{(a+b\text{ArcTan}(cx))^2}{2d^3(-cx+i)^2}-\frac{5(a+b\text{ArcTan}(cx))^2}{8d^3}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}+\frac{11ib^2\text{ArcTan}(cx)}{16d^3}+\frac{b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^3}-\frac{11ib^2}{16d^3(-cx+i)}+\frac{b^2}{16d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

```
[Out] b^2/(16*d^3*(I - c*x)^2) - (((11*I)/16)*b^2)/(d^3*(I - c*x)) + (((11*I)/16)
*b^2*ArcTan[c*x])/d^3 + ((I/4)*b*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)^2) + (
5*b*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)) - (5*(a + b*ArcTan[c*x])^2)/(8*d
^3) - (a + b*ArcTan[c*x])^2/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x])^2)
/(d^3*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3
 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
)/(2*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+icdx)^3}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^3x} + \frac{c(a+b\tan^{-1}(cx))^2}{d^3(-i+cx)^3} + \frac{ic(a+b\tan^{-1}(cx))^2}{d^3(-i+cx)^2} - \frac{c(a+b}{d^3(}\right.$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^3} + \frac{(ic)\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{d^3} + \frac{c\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^3}\,dx}{d^3} - \frac{c\int \frac{(a+b}{}}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^3(i-cx)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{}{1}\right.}{d^3}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2d^3(i-cx)^2} + \frac{i(a+b\tan^{-1}(cx))^2}{d^3(i-cx)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{}{1}\right.}{d^3}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} + \frac{5b(a+b\tan^{-1}(cx))}{4d^3(i-cx)} - \frac{5(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\tan^{-}}{2d^3(i-}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} + \frac{5b(a+b\tan^{-1}(cx))}{4d^3(i-cx)} - \frac{5(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\tan^{-}}{2d^3(i-}$$

$$= \frac{ib(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} + \frac{5b(a+b\tan^{-1}(cx))}{4d^3(i-cx)} - \frac{5(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\tan^{-}}{2d^3(i-}$$

$$= \frac{b^2}{16d^3(i-cx)^2} - \frac{11ib^2}{16d^3(i-cx)} + \frac{ib(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} + \frac{5b(a+b\tan^{-1}(cx))}{4d^3(i-cx)} - \frac{5}{}$$

$$= \frac{b^2}{16d^3(i-cx)^2} - \frac{11ib^2}{16d^3(i-cx)} + \frac{11ib^2\tan^{-1}(cx)}{16d^3} + \frac{ib(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} + \frac{5b(a-}{4}$$

Mathematica [A]
time = 0.88, size = 435, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

```
[Out] ((-96*a^2)/(-I + c*x)^2 - ((192*I)*a^2)/(-I + c*x) - (192*I)*a^2*ArcTan[c*x
] + 192*a^2*Log[c*x] - 96*a^2*Log[1 + c^2*x^2] + (12*I)*a*b*(-32*ArcTan[c*x
]^2 - 12*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*PolyLog[2, E^((2*I)*A
rcTan[c*x])] + (12*I)*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]*(6*Cos[2*ArcTa
n[c*x]] + Cos[4*ArcTan[c*x]] + 8*Log[1 - E^((2*I)*ArcTan[c*x])] - (6*I)*Sin
[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]]) + b^2*((-8*
I)*Pi^3 - 72*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] +
144*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] - 3*Cos[4*ArcTan[c*x]] - (12*I)*ArcTan
```

```
[c*x]*Cos[4*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + 192*ArcTan
[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*ArcTan[c*x]*PolyLog[2, E^
((-2*I)*ArcTan[c*x])] + 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (72*I)*Sin[
2*ArcTan[c*x]] - 144*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2
*Sin[2*ArcTan[c*x]] + (3*I)*Sin[4*ArcTan[c*x]] - 12*ArcTan[c*x]*Sin[4*ArcTa
n[c*x]] - (24*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(192*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 2151, normalized size = 7.19

method	result	size
derivativedivides	Expression too large to display	2151
default	Expression too large to display	2151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*b*a/d^3*arctan(c*x)/(c*x-I)-1/32*I*b^2/d^3/(c*x-I)^2*c*x+1/8*b^2/d^3*a
rctan(c*x)/(c*x-I)^2*c*x-2*I*b^2/d^3*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*
x^2+1)^(1/2))+3/2*I*b^2/d^3*Pi*arctan(c*x)^2-2*I*b^2/d^3*arctan(c*x)*polylo
g(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b*a/d^3*arctan(c*x)*ln(c*x)+I*b*a/d^3*di
log(-1/2*I*(c*x+I))-I*b*a/d^3*dilog(-I*c*x)-I*b*a/d^3*dilog(-I*(c*x+I))-1/2
*I*b*a/d^3*ln(c*x-I)^2+1/16*I*b^2/d^3*arctan(c*x)/(c*x-I)^2-2*b*a/d^3*arcta
n(c*x)*ln(c*x-I)-b*a/d^3*arctan(c*x)/(c*x-I)^2+1/4*I*b*a/d^3/(c*x-I)^2-1/64
*b^2/d^3/(c*x-I)^2*c^2*x^2+3/8*b^2/d^3/(c*x-I)*c*x-5/4*b*a/d^3*arctan(c*x)-
5/4*b*a/d^3/(c*x-I)-3*b^2/d^3*arctan(c*x)/(4*c*x-4*I)-b^2/d^3*arctan(c*x)^2
*ln(c*x-I)-1/2*b^2/d^3*arctan(c*x)^2/(c*x-I)^2+b^2/d^3*arctan(c*x)^2*ln(2*I
*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2/d^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^2/d^3*Pi*csgn((1+I*c*
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^2/d^3
*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^2-I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*arctan(c*x)^2+3*I*b^2/d^3*arctan(c*x)/(4*c*x-4*I)*c*x-5/8*b^2/d^3*arct
an(c*x)^2+1/64*b^2/d^3/(c*x-I)^2-1/2*a^2/d^3*ln(c^2*x^2+1)-1/2*a^2/d^3/(c*x
-I)^2+2*b^2/d^3*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d^3*polylog(3,
-(1+I*c*x)/(c^2*x^2+1)^(1/2))+a^2/d^3*ln(c*x)-I*b^2/d^3*arctan(c*x)^2/(c*x-
I)-1/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan
(c*x)^2+1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*arctan(c*x)^2+b^2/d^3*arctan(c*x)^2*ln(c*x)+b^2/d^3*arctan(c*x)^
2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d^3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c
^2*x^2+1)^(1/2))-b^2/d^3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-I*a^2/
d^3/(c*x-I)-I*a^2/d^3*arctan(c*x)+3/8*I*b^2/d^3/(c*x-I)-2/3*I*b^2/d^3*arcta
n(c*x)^3-1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
```

```
c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^
2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
+1/2*I*b^2/d^3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d^3*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d^3*P
i*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2-1/16*I*b^2/d^3*arctan(c*x)/(c*x-I)^2*c^2*x^2+1/2*I*b^2/d^3*Pi
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c
*x)^2+I*b*a/d^3*ln(c*x)*ln(-I*(-c*x+I))+I*b*a/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+
I))-I*b*a/d^3*ln(-I*c*x)*ln(-I*(-c*x+I))-I*b*a/d^3*ln(c*x)*ln(-I*(c*x+I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/128*(-16*I*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(arctan2(1, c*x) - 4
*I) + 32*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x)^3 - 4*(b^2*c^2*x^2
 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 16*a^2*(I*arctan2(1, c*x) + 12)
+ 16*(2*I*b^2*c*x + 3*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c*x + 3*b^2 - 2*(I*b^
2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(b^2*c^4
*d^3*x^2 - 2*I*b^2*c^3*d^3*x - b^2*c^2*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^12*d^3
*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^10
*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*log(c^2*x^
2 + 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - c^2*(c^2/(c^10*d^3*x^4 +
 2*c^8*d^3*x^2 + c^6*d^3) + 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2
 + c^4*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c^6*d^3*x^6 + 3*c^
4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^6*d^3*x^4 +
2*c^4*d^3*x^2 + c^2*d^3) + 512*integrate(1/16*x*arctan(c*x)^2/(c^6*d^3*x^6
+ 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)) - 2*(b^2*c^2*d^3*x^2 - 2*I*b^2*
c*d^3*x - b^2*d^3)*(c^4*(c^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2*l
og(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)) - 512*c^2*integrat
e(1/16*x^2*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3
*x), x) + 2*c^2*log(c^2*x^2 + 1)^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
+ 512*integrate(1/16*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3
*x^3 + d^3*x), x) + 128*integrate(1/16*log(c^2*x^2 + 1)^2/(c^6*d^3*x^7 + 3*
c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)) - 12*(b^2*c^5*d^3*x^2 - 2*I*b^2*c
^4*d^3*x - b^2*c^3*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^
```

```
2 + c^8*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c^8*d^3
*x^2 + c^6*d^3))*c^3 + 2*(2*c^2*x^2 + 1)*c*log(c^2*x^2 + 1)^2/(c^8*d^3*x^4
+ 2*c^6*d^3*x^2 + c^4*d^3) - 256*c*integrate(1/8*x^3*arctan(c*x)^2/(c^6*d^3
*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)) + 2*(b^2*c^6*d^3*x^2 - 2*I
*b^2*c^5*d^3*x - b^2*c^4*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^12*d^3*x^4 + 2*c^10*
d^3*x^2 + c^8*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^10*d^3*x^4 + 2*c
^8*d^3*x^2 + c^6*d^3))*c^2 + 512*c^2*integrate(1/16*x^5*arctan(c*x)^2/(c^6*
d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x) + 128*c^2*integrate(1/16
*x^5*log(c^2*x^2 + 1)^2/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3)
, x) + 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 +
c^4*d^3) - 512*integrate(1/16*x^3*arctan(c*x)^2/(c^6*d^3*x^6 + 3*c^4*d^3*x^
4 + 3*c^2*d^3*x^2 + d^3), x)) - 10*(b^2*c^4*d^3*x^2 - 2*I*b^2*c^3*d^3*x - b
^2*c^2*d^3)*(((4*c^2*x^2 + 3)*c^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
+ 2*(2*c^2*x^2 + 1)*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3
))*c^2 - 256*c*integrate(1/8*x^2*arctan(c*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 +
 3*c^2*d^3*x^2 + d^3), x) - c^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 2
*log(c^2*x^2 + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)) - 5*(I*b^2*c^4*d
^3*x^2 + 2*b^2*c^3*d^3*x - I*b^2*c^2*d^3)*((c*((5*c^2*x^3 + 3*x)/(c^8*d^3*x
^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 5*arctan(c*x)/(c^5*d^3)) - 8*(2*c^2*x^2 + 1
)*arctan(c*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3))*c^2 - c*((3*c^2*x^3
+ 5*x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + 3*arctan(c*x)/(c^3*d^3)) -
 128*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*
c^2*d^3*x^2 + d^3), x) + 8*arctan(c*x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d
^3)) + 4*(a*b*c^2*d^3*x^2 - 2*I*a*b*c*d^3*x - a*b*d^3)*((c*((3*c^2*x^3 + 5*
x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + 3*arctan(c*x)/(c^3*d^3)) - 8*a
rctan(c*x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3))*c^2 + 128*c*integrate(1
/4*x*log(c^2*x^2 + 1)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x)
, x) - 128*integrate(1/4*arctan(c*x)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d
^3*x^3 + d^3*x), x)) + 4*(a*b*c^2*d^3*x^2 - 2*I*a*b*c*d^3*x - a*b*d^3)*((c*
((3*c^2*x^3 + 5*x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + 3*arctan(c*x)/
(c^3*d^3)) - 8*arctan(c*x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3))*c^2 - 1
28*c*integrate(1/4*x*log(c^2*x^2 + 1)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*
d^3*x^3 + d^3*x), x) - 128*integrate(1/4*arctan(c*x)/(c^6*d^3*x^7 + 3*c^4*d
^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)) - 8*(-I*a*b*c^2*d^3*x^2 - 2*a*b*c*d^3*
x + I*a*b*d^3)*(c^2*(c^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 2*log(c^
2*x^2 + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)) + 256*c*integrate(1/8*x
*arctan(c*x)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x) + 64
*integrate(1/8*log(c^2*x^2 + 1)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^
3 + d^3*x), x)) - 8*(I*a*b*c^2*d^3*x^2 + 2*a*b*c*d^3*x - I*a*b*d^3)*(c^2*(c
^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 2*log(c^2*x^2 + 1)/(c^6*d^3*x^
4 + 2*c^4*d^3*x^2 + c^2*d^3)) - 256*c*integrate(1/8*x*arctan(c*x)/(c^6*d^3*
x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x) + 64*integrate(1/8*log(c^2
*x^2 + 1)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)) + 8*(b
^2*c^5*d^3*x^2 - 2*I*b^2*c^4*d^3*x - b^2*c^3*d^3)*(32*c^2*integrate(1/4*x^4
*arctan(c*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(2*c*x/(c*x - I))*log(-(c*x +
I)/(c*x - I))^2 + 4*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*dilog(-2*c*x/(c*x - I
) + 1)*log(-(c*x + I)/(c*x - I)) - (2*I*b^2*c*x + 3*b^2)*log(-(c*x + I)/(c*
x - I))^2 - 8*(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)*integral(1/2*(2*I*a^2*c*x -
 2*a^2 - (2*b^2*c^2*x^2 + (2*a*b - 3*I*b^2)*c*x + 2*I*a*b)*log(-(c*x + I)/(
c*x - I)))/(c^4*d^3*x^5 - 2*I*c^3*d^3*x^4 - 2*I*c*d^3*x^2 - d^3*x), x) - 4*
(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*polylog(3, -(c*x + I)/(c*x - I)))/(c^2*d^
3*x^2 - 2*I*c*d^3*x - d^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**3,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded while
calling a Python object
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3), x)
```

3.117 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2(d+icdx)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{ib^2c}{16d^3(i-cx)^2}-\frac{19b^2c}{16d^3(i-cx)}+\frac{19b^2c\text{ArcTan}(cx)}{16d^3}+\frac{bc(a+b\text{ArcTan}(cx))}{4d^3(i-cx)^2}-\frac{9ibc(a+b\text{ArcTan}(cx))}{4d^3(i-cx)}+\frac{ic(a+b$$

```
[Out] -1/16*I*b^2*c/d^3/(I-c*x)^2-19/16*b^2*c/d^3/(I-c*x)+19/16*b^2*c*arctan(c*x)
/d^3+1/4*b*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2-9/4*I*b*c*(a+b*arctan(c*x))/d^
3/(I-c*x)+1/8*I*c*(a+b*arctan(c*x))^2/d^3-(a+b*arctan(c*x))^2/d^3/x+1/2*I*c
*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+2*c*(a+b*arctan(c*x))^2/d^3/(I-c*x)+6*I*
c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3-3*I*c*(a+b*arctan(c*x))^2
*ln(2/(1+I*c*x))/d^3+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^3-I*b^2*c*
polylog(2,-1+2/(1-I*c*x))/d^3+3*b*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c
*x))/d^3-3/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d^3
```

Rubi [A]
time = 0.68, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {4996, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745, 4974, 4972, 641, 46, 209, 4964}

$$\frac{3bc\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{9ibc(a+b\text{ArcTan}(cx))}{4d^3(-cx+i)}+\frac{bc(a+b\text{ArcTan}(cx))}{4d^3(-cx+i)^2}-\frac{(a+b\text{ArcTan}(cx))^2}{d^3x}+\frac{2c(a+b\text{ArcTan}(cx))^2}{d^3(-cx+i)}+\frac{ic(a+b\text{ArcTan}(cx))^2}{2d^3(-cx+i)^2}+\frac{ic(a+b\text{ArcTan}(cx))^2}{8d^3}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}-\frac{6ic\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}+\frac{19b^2c\text{ArcTan}(cx)}{16d^3}-\frac{ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d^3}-\frac{3ib^2c\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^3}-\frac{19b^2c}{16d^3(-cx+i)}-\frac{ib^2c}{16d^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] ((-1/16*I)*b^2*c)/(d^3*(I - c*x)^2) - (19*b^2*c)/(16*d^3*(I - c*x)) + (19*b
^2*c*ArcTan[c*x])/(16*d^3) + (b*c*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)^2)
- (((9*I)/4)*b*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) + ((I/8)*c*(a + b*Arc
Tan[c*x])^2)/d^3 - (a + b*ArcTan[c*x])^2/(d^3*x) + ((I/2)*c*(a + b*ArcTan[c
*x])^2)/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x])^2)/(d^3*(I - c*x)) - (
(6*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 - ((3*I)*c*(a
 + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (2*b*c*(a + b*ArcTan[c*x])*Lo
g[2 - 2/(1 - I*c*x)])/d^3 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^3 +
(3*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 - (((3*I)/2)
*b^2*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
```

```
c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+icdx)^3}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^3x^2} - \frac{3ic(a+b\tan^{-1}(cx))^2}{d^3x} - \frac{ic^2(a+b\tan^{-1}(cx))^2}{d^3(-i+cx)^3} + \frac{2c^2(\cdots}{\cdots}\right.$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d^3} - \frac{(3ic)\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^3} - \frac{(ic^2)\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^3}\,dx}{d^3} + \frac{(3i\cdots}{\cdots}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))^2}{2d^3(i-cx)^2} + \frac{2c(a+b\tan^{-1}(cx))^2}{d^3(i-cx)} - \frac{6ic(a+b\cdots}{\cdots}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d^3} - \frac{(a+b\tan^{-1}(cx))^2}{d^3x} + \frac{ic(a+b\tan^{-1}(cx))^2}{2d^3(i-cx)^2} + \frac{2c(a+b\cdots}{d^3(i\cdots}$$

$$= \frac{bc(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} - \frac{9ibc(a+b\tan^{-1}(cx))}{4d^3(i-cx)} + \frac{ic(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\,\mathrm{ta}\cdots}{d\cdots}$$

$$= \frac{bc(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} - \frac{9ibc(a+b\tan^{-1}(cx))}{4d^3(i-cx)} + \frac{ic(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\,\mathrm{ta}\cdots}{d\cdots}$$

$$= \frac{bc(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} - \frac{9ibc(a+b\tan^{-1}(cx))}{4d^3(i-cx)} + \frac{ic(a+b\tan^{-1}(cx))^2}{8d^3} - \frac{(a+b\,\mathrm{ta}\cdots}{d\cdots}$$

$$= -\frac{ib^2c}{16d^3(i-cx)^2} - \frac{19b^2c}{16d^3(i-cx)} + \frac{bc(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} - \frac{9ibc(a+b\tan^{-1}(cx))}{4d^3(i-cx)}$$

$$= -\frac{ib^2c}{16d^3(i-cx)^2} - \frac{19b^2c}{16d^3(i-cx)} + \frac{19b^2c\tan^{-1}(cx)}{16d^3} + \frac{bc(a+b\tan^{-1}(cx))}{4d^3(i-cx)^2} - \frac{9ib\cdots}{\cdots}$$

Mathematica [A]
time = 2.24, size = 549, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

```
[Out] -1/64*((64*a^2)/x - ((32*I)*a^2*c)/(-I + c*x)^2 + (128*a^2*c)/(-I + c*x) +
192*a^2*c*ArcTan[c*x] + (192*I)*a^2*c*Log[x] - (96*I)*a^2*c*Log[1 + c^2*x^2
] - I*b^2*c*((8*I)*Pi^3 - 64*ArcTan[c*x]^2 + ((64*I)*ArcTan[c*x]^2)/(c*x) +
 40*Cos[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 80*ArcTan[
c*x]^2*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Cos[4*Ar
cTan[c*x]] - 8*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] - 192*ArcTan[c*x]^2*Log[1 -
 E^((-2*I)*ArcTan[c*x])] - (128*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x]
)] - (192*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 64*PolyLog[2,
 E^((2*I)*ArcTan[c*x])] - 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (40*I)*Si
n[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]^2
*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*Sin[4*ArcTan[c*x
]] + (8*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]) + (4*a*b*(96*c*x*ArcTan[c*x]^2
 + 48*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + c*x*(20*Cos[2*ArcTan[c*x]] +
Cos[4*ArcTan[c*x]] - 32*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (20*I)*Sin[2*ArcTan[
c*x]] - I*Sin[4*ArcTan[c*x]]) + 4*ArcTan[c*x]*(8 + (10*I)*c*x*Cos[2*ArcTan[
c*x]] + I*c*x*Cos[4*ArcTan[c*x]] + (24*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])
] + 10*c*x*Sin[2*ArcTan[c*x]] + c*x*Sin[4*ArcTan[c*x]])))/x)/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.12, size = 9532, normalized size = 24.38

method	result	size
derivativedivides	Expression too large to display	9532
default	Expression too large to display	9532

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c*x)*log(2*c*x/(c*x - I))*l
og(-(c*x + I)/(c*x - I))^2 + 12*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c*x
)*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - (6*b^2*c^2*x^2 -
9*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2 - 8*(c^2*d^3*x^3 - 2*I*c*d
^3*x^2 - d^3*x)*integral(1/2*(2*I*a^2*c*x - 2*a^2 + (6*I*b^2*c^3*x^3 + 9*b^
2*c^2*x^2 - 2*(a*b + I*b^2)*c*x - 2*I*a*b)*log(-(c*x + I)/(c*x - I)))/(c^4*
d^3*x^6 - 2*I*c^3*d^3*x^5 - 2*I*c*d^3*x^3 - d^3*x^2), x) + 12*(I*b^2*c^3*x^
3 + 2*b^2*c^2*x^2 - I*b^2*c*x)*polylog(3, -(c*x + I)/(c*x - I)))/(c^2*d^3*x
^3 - 2*I*c*d^3*x^2 - d^3*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3), x)
```

3.118 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{(1+icx)^4}\,dx$

Optimal. Leaf size=207

$$-\frac{b^2}{54c(i-cx)^3}+\frac{5ib^2}{144c(i-cx)^2}+\frac{11b^2}{144c(i-cx)}-\frac{11b^2\text{ArcTan}(cx)}{144c}-\frac{ib(a+b\text{ArcTan}(cx))}{9c(i-cx)^3}-\frac{b(a+b\text{ArcTan}(cx))}{12c(i-cx)^2}$$

```
[Out] -1/54*b^2/c/(I-c*x)^3+5/144*I*b^2/c/(I-c*x)^2+11/144*b^2/c/(I-c*x)-11/144*b
^2*arctan(c*x)/c-1/9*I*b*(a+b*arctan(c*x))/c/(I-c*x)^3-1/12*b*(a+b*arctan(c
*x))/c/(I-c*x)^2+1/12*I*b*(a+b*arctan(c*x))/c/(I-c*x)-1/24*I*(a+b*arctan(c*
x))^2/c+1/3*I*(a+b*arctan(c*x))^2/c/(1+I*c*x)^3
```

Rubi [A]
time = 0.17, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{ib(a+b\text{ArcTan}(cx))}{12c(-cx+i)}-\frac{b(a+b\text{ArcTan}(cx))}{12c(-cx+i)^2}-\frac{ib(a+b\text{ArcTan}(cx))}{9c(-cx+i)^3}-\frac{i(a+b\text{ArcTan}(cx))^2}{24c}+\frac{i(a+b\text{ArcTan}(cx))^2}{3c(1+icx)^3}-\frac{11b^2\text{ArcTan}(cx)}{144c}+\frac{11b^2}{144c(-cx+i)}+\frac{5ib^2}{144c(-cx+i)^2}-\frac{b^2}{54c(-cx+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

```
[Out] -1/54*b^2/(c*(I - c*x)^3) + (((5*I)/144)*b^2)/(c*(I - c*x)^2) + (11*b^2)/(1
44*c*(I - c*x)) - (11*b^2*ArcTan[c*x])/(144*c) - ((I/9)*b*(a + b*ArcTan[c*x
]))/(c*(I - c*x)^3) - (b*(a + b*ArcTan[c*x]))/(12*c*(I - c*x)^2) + ((I/12)*
b*(a + b*ArcTan[c*x]))/(c*(I - c*x)) - ((I/24)*(a + b*ArcTan[c*x])^2)/c + (
(I/3)*(a + b*ArcTan[c*x])^2)/(c*(1 + I*c*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(1+icx)^4}\,dx = \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3} - \frac{1}{3}(2ib)\int \left(\frac{a+b\tan^{-1}(cx)}{2(-i+cx)^4} + \frac{i\left(a+b\tan^{-1}(cx)\right)}{4(-i+cx)^3} - \frac{a+b}{8(-}\right.$$

$$= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c(1+icx)^3} + \frac{1}{12}(ib)\int \frac{a+b\tan^{-1}(cx)}{(-i+cx)^2}\,dx - \frac{1}{12}(ib)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i(a+b\tan}{24c}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i(a+b\tan}{24c}$$

$$= -\frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)}{12c(i-cx)} - \frac{i(a+b\tan}{24c}$$

$$= -\frac{b^2}{54c(i-cx)^3} + \frac{5ib^2}{144c(i-cx)^2} + \frac{11b^2}{144c(i-cx)} - \frac{ib\left(a+b\tan^{-1}(cx)\right)}{9c(i-cx)^3} - \frac{b(a+b}{12c}$$

$$= -\frac{b^2}{54c(i-cx)^3} + \frac{5ib^2}{144c(i-cx)^2} + \frac{11b^2}{144c(i-cx)} - \frac{11b^2\tan^{-1}(cx)}{144c} - \frac{ib(a+b\tan}{9c(i-}$$

Mathematica [A]
time = 0.11, size = 155, normalized size = 0.75

$$-\frac{144a^2+12ab(-10i+9cx+3ic^2x^2)+b^2(-56-81icx+33c^2x^2)+3b(i+cx)\left(12a(-7i+4cx+ic^2x^2)+b(-29-32icx+11c^2x^2)\right)\mathrm{ArcTan}(cx)+18b^2(7-3icx+3c^2x^2+ic^3x^3)\,\mathrm{ArcTan}(cx)^2}{432c(-i+cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

```
[Out] -1/432*(144*a^2 + 12*a*b*(-10*I + 9*c*x + (3*I)*c^2*x^2) + b^2*(-56 - (81*I
)*c*x + 33*c^2*x^2) + 3*b*(I + c*x)*(12*a*(-7*I + 4*c*x + I*c^2*x^2) + b*(-
29 - (32*I)*c*x + 11*c^2*x^2))*ArcTan[c*x] + 18*b^2*(7 - (3*I)*c*x + 3*c^2*
x^2 + I*c^3*x^3)*ArcTan[c*x]^2)/(c*(-I + c*x)^3)
```

Maple [A]
time = 0.36, size = 345, normalized size = 1.67

method	result
derivativedivides	$\frac{\frac{2iba\arctan(cx)}{3(icx+1)^3}+\frac{ia^2}{3(icx+1)^3}-\frac{b^2\arctan(cx)\ln(cx-i)}{24}-\frac{b^2\arctan(cx)}{12(cx-i)^2}-\frac{ib^2\ln\left(-\frac{i(-cx+i)}{2}\right)\ln\left(-\frac{i(cx+i)}{2}\right)}{48}+\frac{ib^2\ln\left(-\frac{i(-cx+i)}{2}\right)\ln(c}{48}}{}$
default	$\frac{\frac{2iba\arctan(cx)}{3(icx+1)^3}+\frac{ia^2}{3(icx+1)^3}-\frac{b^2\arctan(cx)\ln(cx-i)}{24}-\frac{b^2\arctan(cx)}{12(cx-i)^2}-\frac{ib^2\ln\left(-\frac{i(-cx+i)}{2}\right)\ln\left(-\frac{i(cx+i)}{2}\right)}{48}+\frac{ib^2\ln\left(-\frac{i(-cx+i)}{2}\right)\ln(c}{48}}{}$
risch	$\frac{ib^2(c^3x^3-3ic^2x^2-3cx-7i)\ln(icx+1)^2}{96(cx-i)^3c}+\frac{ib(-3bc^3x^3\ln(-icx+1)+9ibc^2x^2\ln(-icx+1)+6ibc^2x^2+9bcx\ln(-icx+1)+2}{144(cx-i)^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(2/3*I*b*a/(1+I*c*x)^3*arctan(c*x)+1/3*I*a^2/(1+I*c*x)^3-1/24*b^2*arcta
n(c*x)*ln(c*x-I)-1/12*b^2*arctan(c*x)/(c*x-I)^2-1/48*I*b^2*ln(-1/2*I*(-c*x+
I))*ln(-1/2*I*(c*x+I))+1/48*I*b^2*ln(-1/2*I*(-c*x+I))*ln(c*x+I)+1/24*b^2*ar
ctan(c*x)*ln(c*x+I)+1/54*b^2/(c*x-I)^3-11/144*b^2/(c*x-I)+1/3*I*b^2/(1+I*c*
x)^3*arctan(c*x)^2-11/144*b^2*arctan(c*x)-1/12*I*b*a/(c*x-I)-1/12*I*b^2*arc
tan(c*x)/(c*x-I)-1/96*I*b^2*ln(c*x-I)^2+5/144*I*b^2/(c*x-I)^2-1/96*I*b^2*ln
(c*x+I)^2-1/12*I*b*a*arctan(c*x)+1/9*I*b^2*arctan(c*x)/(c*x-I)^3-1/12*b*a/(
c*x-I)^2+1/9*I*b*a/(c*x-I)^3+1/48*I*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I)))
```

Maxima [A]
time = 0.31, size = 184, normalized size = 0.89

$$\frac{3\left(-12i\,ab-11\,b^2\right)c^2x^2-27\left(4\,ab-3i\,b^2\right)cx+18\left(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2\right)\arctan(cx)^2-144\,a^2+120i\,ab+56\,b^2+3\left(\left(-12i\,ab-11\,b^2\right)c^3x^3-3\left(12\,ab-7i\,b^2\right)c^2x^2+3\left(12i\,ab-b^2\right)cx-84\,ab+29i\,b^2\right)\arctan(cx)}{432\left(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="maxima")
```

```
[Out] 1/432*(3*(-12*I*a*b - 11*b^2)*c^2*x^2 - 27*(4*a*b - 3*I*b^2)*c*x + 18*(-I*b
^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*arctan(c*x)^2 - 144*a^2 +
 120*I*a*b + 56*b^2 + 3*((-12*I*a*b - 11*b^2)*c^3*x^3 - 3*(12*a*b - 7*I*b^2
)*c^2*x^2 + 3*(12*I*a*b - b^2)*c*x - 84*a*b + 29*I*b^2)*arctan(c*x))/(c^4*x
^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

Fricas [A]
time = 0.84, size = 206, normalized size = 1.00

$$-\frac{6\,(12i\,ab+11\,b^2)c^2x^2+54\,(4\,ab-3i\,b^2)cx+9\,(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+288\,a^2-240i\,ab-112\,b^2-3\,((12\,ab-11i\,b^2)c^3x^3-3\,(12i\,ab+7\,b^2)c^2x^2-3\,(12\,ab+i\,b^2)cx-84i\,ab-29\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)}{864\,(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="fricas")
```

```
[Out] -1/864*(6*(12*I*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b - 3*I*b^2)*c*x + 9*(-I*b^
2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*log(-(c*x + I)/(c*x - I))^
2 + 288*a^2 - 240*I*a*b - 112*b^2 - 3*((12*a*b - 11*I*b^2)*c^3*x^3 - 3*(12*
I*a*b + 7*b^2)*c^2*x^2 - 3*(12*a*b + I*b^2)*c*x - 84*I*a*b - 29*b^2)*log(-(
c*x + I)/(c*x - I)))/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(158) = 316.
time = 28.58, size = 552, normalized size = 2.67

$$-\frac{b(12a-11ib)\log\left(-\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c}+\frac{b(12a-11ib)\log\left(\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c}+\frac{-144a^2+120iab+56b^2+x^2(-36iabc^2-33b^2c^2)+x(-108abc+81ib^2c)}{432c^4x^3-1296ic^3x^2-1296c^2x+432ic}+\frac{(-48iab-3ib^2c^3x^3\log(icx+1)-9b^2c^2x^2\log(icx+1)+6b^2c^2x^2+9ib^2cx\log(icx+1)-18ib^2cx-21b^2\log(icx+1)-20b^2)\log(-icx+1)}{144c^4x^3-432ic^3x^2-432c^2x+144ic}+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(-icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic}+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic}+\frac{(24iab-3b^2c^2x^2+9ib^2cx+10b^2)\log(icx+1)}{72c^4x^3-216ic^3x^2-216c^2x+72ic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(1+I*c*x)**4,x)
```

```
[Out] -b*(12*a - 11*I*b)*log(-I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**2))/(28
8*c) + b*(12*a - 11*I*b)*log(I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**2)
)/(288*c) + (-144*a**2 + 120*I*a*b + 56*b**2 + x**2*(-36*I*a*b*c**2 - 33*b*
*2*c**2) + x*(-108*a*b*c + 81*I*b**2*c))/(432*c**4*x**3 - 1296*I*c**3*x**2
- 1296*c**2*x + 432*I*c) + (-48*I*a*b - 3*I*b**2*c**3*x**3*log(I*c*x + 1) -
 9*b**2*c**2*x**2*log(I*c*x + 1) + 6*b**2*c**2*x**2 + 9*I*b**2*c*x*log(I*c*
x + 1) - 18*I*b**2*c*x - 21*b**2*log(I*c*x + 1) - 20*b**2)*log(-I*c*x + 1)/
(144*c**4*x**3 - 432*I*c**3*x**2 - 432*c**2*x + 144*I*c) + (I*b**2*c**3*x**
3 + 3*b**2*c**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(-I*c*x + 1)**2/(96*c**4*x
**3 - 288*I*c**3*x**2 - 288*c**2*x + 96*I*c) + (I*b**2*c**3*x**3 + 3*b**2*c
**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(I*c*x + 1)**2/(96*c**4*x**3 - 288*I*c
**3*x**2 - 288*c**2*x + 96*I*c) + (24*I*a*b - 3*b**2*c**2*x**2 + 9*I*b**2*c
*x + 10*b**2)*log(I*c*x + 1)/(72*c**4*x**3 - 216*I*c**3*x**2 - 216*c**2*x +
 72*I*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(1+c\,x\,1\mathrm{i})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(c*x*1i + 1)^4,x)
```

```
[Out] int((a + b*atan(c*x))^2/(c*x*1i + 1)^4, x)
```

3.119 $\int \frac{\text{ArcTan}(ax)^2}{cx-iacx^2} \, dx$

Optimal. Leaf size=76

$$\frac{\text{ArcTan}(ax)^2 \log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}+\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

```
[Out] arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c
+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1607, 4988, 5004, 5112, 6745}

$$-\frac{i\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}+\frac{\text{ArcTan}(ax)^2 \log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

```
[Out] (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*ArcTan[a*x]*PolyLog[2, -1 + 2
/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^2}{cx-iacx^2}\,dx &= \int \frac{\tan^{-1}(ax)^2}{x(c-iacx)}\,dx\\ &= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{(2a)\int \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{(ia)\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{\mathrm{Li}_3\left(-1+\frac{2}{1-iax}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 1.08

$$\frac{-i\pi^3+16i\mathrm{ArcTan}(ax)^3+24\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right)+24i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right)+12\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

```
[Out] ((-I)*Pi^3 + (16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcT
an[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*Poly
Log[3, E^((-2*I)*ArcTan[a*x])])/(24*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(70) = 140.
time = 0.35, size = 193, normalized size = 2.54

method	result

derivativedivides	$\frac{\frac{a \arctan(ax)^2 \ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{2ia \arctan(ax) \operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{2a \operatorname{polylog}\left(3,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+}{a}$
default	$\frac{\frac{a \arctan(ax)^2 \ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{2ia \arctan(ax) \operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{2a \operatorname{polylog}\left(3,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(c*x-I*a*c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a/c*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*arctan(a*x
)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))+a/c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*ar
ctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,(1+I*a*x)/
(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="maxima")
```

```
[Out] 1/96*(8*I*arctan(a*x)^3 - 12*arctan(a*x)^2*log(a^2*x^2 + 1) - 6*I*arctan(a*
x)*log(a^2*x^2 + 1)^2 + log(a^2*x^2 + 1)^3 + 24*I*(arctan(a*x)^3/c + 4*a*in
tegrate(1/16*x*log(a^2*x^2 + 1)^2/(a^2*c*x^3 + c*x), x) - 16*integrate(1/16
*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^3 + c*x), x))*c + 96*c*integrate(1/1
6*(4*a*x*arctan(a*x)*log(a^2*x^2 + 1) + 12*arctan(a*x)^2 + log(a^2*x^2 + 1)
^2)/(a^2*c*x^3 + c*x), x))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="fricas")
```

```
[Out] integral(-1/4*I*log(-(a*x + I)/(a*x - I))^2/(a*c*x^2 + I*c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \int \frac{\operatorname{atan}^{2}(ax)}{ax^{2}+ix}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(c*x-I*a*c*x**2),x)
```

```
[Out] I*Integral(atan(a*x)**2/(a*x**2 + I*x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^2}{c\,x-a\,c\,x^2\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c*x - a*c*x^2*1i),x)
```

```
[Out] int(atan(a*x)^2/(c*x - a*c*x^2*1i), x)
```

3.120 $\int (d+icdx)^3(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=382

$$-3ab^2d^3x+\frac{1}{4}ib^3d^3x-\frac{ib^3d^3\text{ArcTan}(cx)}{4c}-3b^3d^3x\text{ArcTan}(cx)-\frac{1}{4}ib^2cd^3x^2(a+b\text{ArcTan}(cx))+\frac{7bd^3(a+b\text{ArcTa}}{c}$$

```
[Out] -3*a*b^2*d^3*x-1/4*I*b^2*c*d^3*x^2*(a+b*arctan(c*x))+1/4*I*b*c^2*d^3*x^3*(a
+b*arctan(c*x))^2-3*b^3*d^3*x*arctan(c*x)+1/4*I*b^3*d^3*x+7*b*d^3*(a+b*arct
an(c*x))^2/c-11*I*b^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+3/2*b*c*d^3*x
^2*(a+b*arctan(c*x))^2-1/4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^3/c-21/4*I*b
*d^3*x*(a+b*arctan(c*x))^2+6*b*d^3*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-1/
4*I*b^3*d^3*arctan(c*x)/c+3/2*b^3*d^3*ln(c^2*x^2+1)/c-6*I*b^2*d^3*(a+b*arct
an(c*x))*polylog(2,1-2/(1-I*c*x))/c+11/2*b^3*d^3*polylog(2,1-2/(1+I*c*x))/c
+3*b^3*d^3*polylog(3,1-2/(1-I*c*x))/c
```

Rubi [A]
time = 0.51, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 327, 209, 1600, 5112, 6745}

$$-\frac{6ib^2d^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{c}-\frac{1}{4}ib^2cd^3x^2(a+b\text{ArcTan}(cx))-\frac{11ib^2d^3\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{1}{4}ibc^2d^3x^3(a+b\text{ArcTan}(cx))^2+\frac{3}{2}bcd^3x^2(a+b\text{ArcTan}(cx))^2-\frac{id^3(1+icx)^4(a+b\text{ArcTan}(cx))^3}{4c}+\frac{7bd^3(a+b\text{ArcTan}(cx))^2}{c}-\frac{21}{4}ibd^3x(a+b\text{ArcTan}(cx))^2+\frac{6bd^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{c}-3ab^2d^3x-\frac{ib^3d^3\text{ArcTan}(cx)}{4c}-3b^3d^3x\text{ArcTan}(cx)+\frac{3b^3d^3\log\left(c^2x^2+1\right)}{2c}+\frac{11b^3d^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c}+\frac{3b^3d^3\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{c}+\frac{1}{4}ib^3d^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -3*a*b^2*d^3*x + (I/4)*b^3*d^3*x - ((I/4)*b^3*d^3*ArcTan[c*x])/c - 3*b^3*d^
3*x*ArcTan[c*x] - (I/4)*b^2*c*d^3*x^2*(a + b*ArcTan[c*x]) + (7*b*d^3*(a + b
*ArcTan[c*x])^2)/c - ((21*I)/4)*b*d^3*x*(a + b*ArcTan[c*x])^2 + (3*b*c*d^3*
x^2*(a + b*ArcTan[c*x])^2)/2 + (I/4)*b*c^2*d^3*x^3*(a + b*ArcTan[c*x])^2 -
((I/4)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^3)/c + (6*b*d^3*(a + b*ArcTan[
c*x])^2*Log[2/(1 - I*c*x)])/c - ((11*I)*b^2*d^3*(a + b*ArcTan[c*x])*Log[2/(
1 + I*c*x)])/c + (3*b^3*d^3*Log[1 + c^2*x^2])/(2*c) - ((6*I)*b^2*d^3*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (11*b^3*d^3*PolyLog[2, 1 -
 2/(1 + I*c*x)])/(2*c) + (3*b^3*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)^3 \left(a+b\tan^{-1}(cx)\right)^3 \, dx = -\frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(cx)\right)^3}{4c} + \frac{(3ib)\int \left(-7d^4(a+b\tan^{-1}(cx))^2\right.}{}$$

$$= -\frac{id^3(1+icx)^4 \left(a+b\tan^{-1}(cx)\right)^3}{4c} + \frac{(6b)\int \frac{(id^4-cd^4x)(a+b\tan^{-1}(cx))^2}{1+c^2x^2}\, dx}{d}$$

$$= -\frac{21}{4} ibd^3x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{3}{2}bcd^3x^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}ibc^2d^3x$$

$$= \frac{21bd^3(a+b\tan^{-1}(cx))^2}{4c} - \frac{21}{4}ibd^3x\left(a+b\tan^{-1}(cx)\right)^2 + \frac{3}{2}bcd^3x^2(a+$$

$$= -3ab^2d^3x - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan^{-1}(cx)\right) + \frac{7bd^3(a+b\tan^{-1}(cx))^2}{c} -$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3x^2\left(a+b\tan^{-1}(cx)\right.$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - \frac{ib^3d^3\tan^{-1}(cx)}{4c} - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3$$

$$= -3ab^2d^3x + \frac{1}{4}ib^3d^3x - \frac{ib^3d^3\tan^{-1}(cx)}{4c} - 3b^3d^3x\tan^{-1}(cx) - \frac{1}{4}ib^2cd^3$$

Mathematica [A]
time = 1.01, size = 693, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] ((-1/4*I)*d^3*(a*b^2 + (4*I)*a^3*c*x + 21*a^2*b*c*x - (12*I)*a*b^2*c*x - b^
3*c*x - 6*a^3*c^2*x^2 + (6*I)*a^2*b*c^2*x^2 + a*b^2*c^2*x^2 - (4*I)*a^3*c^3
*x^3 - a^2*b*c^3*x^3 + a^3*c^4*x^4 - 21*a^2*b*ArcTan[c*x] + (12*I)*a*b^2*Ar
cTan[c*x] + b^3*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] + 42*a*b^2*c*x*A
rcTan[c*x] - (12*I)*b^3*c*x*ArcTan[c*x] - 18*a^2*b*c^2*x^2*ArcTan[c*x] + (1
2*I)*a*b^2*c^2*x^2*ArcTan[c*x] + b^3*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*c^3
*x^3*ArcTan[c*x] - 2*a*b^2*c^3*x^3*ArcTan[c*x] + 3*a^2*b*c^4*x^4*ArcTan[c*x
] + 3*a*b^2*ArcTan[c*x]^2 - (16*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*Arc
Tan[c*x]^2 + 21*b^3*c*x*ArcTan[c*x]^2 - 18*a*b^2*c^2*x^2*ArcTan[c*x]^2 + (6
*I)*b^3*c^2*x^2*ArcTan[c*x]^2 - (12*I)*a*b^2*c^3*x^3*ArcTan[c*x]^2 - b^3*c^
3*x^3*ArcTan[c*x]^2 + 3*a*b^2*c^4*x^4*ArcTan[c*x]^2 + b^3*ArcTan[c*x]^3 + (
4*I)*b^3*c*x*ArcTan[c*x]^3 - 6*b^3*c^2*x^2*ArcTan[c*x]^3 - (4*I)*b^3*c^3*x^
```

```
3*ArcTan[c*x]^3 + b^3*c^4*x^4*ArcTan[c*x]^3 + (48*I)*a*b^2*ArcTan[c*x]*Log[
1 + E^((2*I)*ArcTan[c*x])] + 44*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x
])] + (24*I)*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*a^2*
b*Log[1 + c^2*x^2] - 22*a*b^2*Log[1 + c^2*x^2] + (6*I)*b^3*Log[1 + c^2*x^2]
 + 2*b^2*(12*a - (11*I)*b + 12*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c
*x])] + (12*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.18, size = 1875, normalized size = 4.91

method	result	size
derivativedivides	Expression too large to display	1875
default	Expression too large to display	1875

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-3*d^3*a^2*b*arctan(c*x)*c^3*x^3-1/4*I*d^3*(1+I*c*x)^4*a^3-1/4*d^3*b^3
+d^3*b^3*arctan(c*x)^3*c*x-d^3*b^3*arctan(c*x)^3*c^3*x^3-3*d^3*b^3*arctan(c
*x)*c*x+3/2*d^3*b^3*arctan(c*x)^2*c^2*x^2+1/4*I*d^3*b^3*c*x-6*d^3*a*b^2*arc
tan(c*x)*ln(c^2*x^2+1)+21/4*I*d^3*a*b^2*arctan(c*x)^2+11/2*I*d^3*a*b^2*ln(c
^2*x^2+1)+3*I*d^3*a*b^2*dilog(-1/2*I*(c*x+I))-3/4*I*d^3*a^2*b*arctan(c*x)*c
^4*x^4+9/2*I*d^3*a^2*b*arctan(c*x)*c^2*x^2+3/2*I*d^3*b^3*Pi*csgn(I/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1)^2)^2*arctan(c*x)^2-3/2*I*d^3*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+3*I*d^3*b^3*Pi*csgn(I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2
-3/2*I*d^3*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*
arctan(c*x)^2+3*d^3*a^2*b*arctan(c*x)*c*x-21/4*I*d^3*a^2*b*c*x+1/4*I*d^3*a^
2*b*c^3*x^3+3*I*d^3*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3*I*d^3*a*b^2*ln(c*x
+I)*ln(1/2*I*(c*x-I))+3*I*d^3*a*b^2*ln(c*x+I)*ln(c^2*x^2+1)+3/2*I*d^3*b^3*P
i*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2-3/2*I*d^3*b^3*Pi*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2-3/2*I*d^3*b^3*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2-3*I*d^3*a
*b^2*ln(c*x-I)*ln(c^2*x^2+1)-3*d^3*a*b^2*arctan(c*x)^2*c^3*x^3+3*d^3*a*b^2*
arctan(c*x)^2*c*x+3*d^3*a*b^2*arctan(c*x)*c^2*x^2-1/4*I*d^3*a*b^2*c^2*x^2-1
/4*I*d^3*b^3*arctan(c*x)*c^2*x^2-1/4*I*d^3*b^3*arctan(c*x)^3*c^4*x^4+3/2*I*
d^3*b^3*arctan(c*x)^3*c^2*x^2+1/4*I*d^3*b^3*arctan(c*x)^2*c^3*x^3-21/4*I*d^
3*b^3*arctan(c*x)^2*c*x+3/2*I*d^3*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2
-3*I*d^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)^2*arctan(c*x)^2+3/2*I*d^3*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2+3*d^3*b^3*po
lylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-4*d^3*b^3*arctan(c*x)^2-3*d^3*b^3*ln((1+I
```

```
*c*x)^2/(c^2*x^2+1)+1)-11*d^3*b^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-11
*d^3*b^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*a*b^2*arctan(c*x)
*c^3*x^3+9/2*I*d^3*a*b^2*arctan(c*x)^2*c^2*x^2-21/2*I*d^3*a*b^2*arctan(c*x)
*c*x-3/4*I*d^3*a*b^2*arctan(c*x)^2*c^4*x^4+6*d^3*b^3*arctan(c*x)^2*ln((1+I*
c*x)/(c^2*x^2+1)^(1/2))+6*d^3*b^3*ln(2)*arctan(c*x)^2-3*d^3*b^3*arctan(c*x)
^2*ln(c^2*x^2+1)+11/4*I*d^3*b^3*arctan(c*x)-1/4*I*d^3*b^3*arctan(c*x)^3+3*d
^3*a*b^2*arctan(c*x)-3*d^3*a^2*b*ln(c^2*x^2+1)-6*I*d^3*b^3*arctan(c*x)*poly
log(2,-(1+I*c*x)^2/(c^2*x^2+1))-11*I*d^3*b^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))-11*I*d^3*b^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+3/2*I*d^3*a*b^2*ln(c*x-I)^2-3*I*d^3*a*b^2*dilog(1/2*I*(c*x-I))-3/2*I*d^
3*a*b^2*ln(c*x+I)^2-3*d^3*a*b^2*c*x+3/2*d^3*a^2*b*c^2*x^2+21/4*I*d^3*a^2*b*
arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*a^3*c^3*d^3*x^4 - 24*b^3*c^5*d^3*integrate(1/128*x^5*arctan(c*x)^2*l
og(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 2*b^3*c^5*d^3*integrate(1/128*x^5*log(c
^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c^5*d^3*integrate(1/128*x^5*arctan
(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c^5*d^3*integrate(1/128*x^5*log(c^2*x^2 +
 1)^2/(c^2*x^2 + 1), x) - a^3*c^2*d^3*x^3 - 336*b^3*c^4*d^3*integrate(1/128
*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) - 36*b^3*c^4*d^3*integrate(1/128*x^4*a
rctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 1152*a*b^2*c^4*d^3*integr
ate(1/128*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 60*b^3*c^4*d^3*integrate(1/
128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 1/4*I*(3*x^4*arcta
n(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*c^3*d^3 + 48*b^
3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) - 4*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x
) + 120*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3
0*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 3/
2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*c^2*d^3 +
3/2*I*a^3*c*d^3*x^2 + 7/32*b^3*d^3*arctan(c*x)^4/c - 224*b^3*c^2*d^3*integr
ate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) - 24*b^3*c^2*d^3*integrate(1/
128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 768*a*b^2*c^2*d^
3*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 120*b^3*c^2*d^3*int
egrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 9/2*I*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*c*d^3 + a*b^2*d^3*arctan
(c*x)^3/c + 72*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(
c^2*x^2 + 1), x) - 6*b^3*c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^3/(c^2*x^
2 + 1), x) - 48*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x)
```

```
 + 12*b^3*c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + a^
3*d^3*x + 12*b^3*d^3*integrate(1/128*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^
2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^3/c - 1/32*
(I*b^3*c^3*d^3*x^4 + 4*b^3*c^2*d^3*x^3 - 6*I*b^3*c*d^3*x^2 - 4*b^3*d^3*x)*a
rctan(c*x)^3 + 3/64*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*x^
2 + 4*I*b^3*d^3*x)*arctan(c*x)^2*log(c^2*x^2 + 1) - 3/128*(-I*b^3*c^3*d^3*x
^4 - 4*b^3*c^2*d^3*x^3 + 6*I*b^3*c*d^3*x^2 + 4*b^3*d^3*x)*arctan(c*x)*log(c
^2*x^2 + 1)^2 - 1/256*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*
x^2 + 4*I*b^3*d^3*x)*log(c^2*x^2 + 1)^3 - I*integrate(1/128*(112*(b^3*c^5*d
^3*x^5 - 2*b^3*c^3*d^3*x^3 - 3*b^3*c*d^3*x)*arctan(c*x)^3 + 2*(3*b^3*c^4*d^
3*x^4 + 2*b^3*c^2*d^3*x^2 - b^3*d^3)*log(c^2*x^2 + 1)^3 + 12*(32*a*b^2*c^5*
d^3*x^5 - 5*b^3*c^4*d^3*x^4 - 64*a*b^2*c^3*d^3*x^3 + 10*b^3*c^2*d^3*x^2 - 9
6*a*b^2*c*d^3*x)*arctan(c*x)^2 + 3*(5*b^3*c^4*d^3*x^4 - 10*b^3*c^2*d^3*x^2
+ 4*(b^3*c^5*d^3*x^5 - 2*b^3*c^3*d^3*x^3 - 3*b^3*c*d^3*x)*arctan(c*x))*log(
c^2*x^2 + 1)^2 - 12*(2*(3*b^3*c^4*d^3*x^4 + 2*b^3*c^2*d^3*x^2 - b^3*d^3)*ar
ctan(c*x)^2 - (b^3*c^5*d^3*x^5 - 10*b^3*c^3*d^3*x^3 + 4*b^3*c*d^3*x)*arctan
(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*x^2 + 4*I*b^3*d^
3*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/16*(-16*I*a^3*c^5*d^3*x^5 - 4
8*a^3*c^4*d^3*x^4 + 32*I*a^3*c^3*d^3*x^3 - 32*a^3*c^2*d^3*x^2 + 48*I*a^3*c*
d^3*x + 16*a^3*d^3 - 3*(-4*I*a*b^2*c^5*d^3*x^5 - (12*a*b^2 - I*b^3)*c^4*d^3
*x^4 + 4*(2*I*a*b^2 + b^3)*c^3*d^3*x^3 - 2*(4*a*b^2 + 3*I*b^3)*c^2*d^3*x^2
+ 4*a*b^2*d^3 + 4*(3*I*a*b^2 - b^3)*c*d^3*x)*log(-(c*x + I)/(c*x - I))^2 +
24*(a^2*b*c^5*d^3*x^5 - 3*I*a^2*b*c^4*d^3*x^4 - 2*a^2*b*c^3*d^3*x^3 - 2*I*a
^2*b*c^2*d^3*x^2 - 3*a^2*b*c*d^3*x + I*a^2*b*d^3)*log(-(c*x + I)/(c*x - I))
)/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3,x)
```

```
[Out] int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3, x)
```

3.121 $\int (d+icdx)^2(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=298

$$-ab^2d^2x-b^3d^2x\text{ArcTan}(cx)+\frac{7bd^2(a+b\text{ArcTan}(cx))^2}{2c}-3ibd^2x(a+b\text{ArcTan}(cx))^2+\frac{1}{2}bcd^2x^2(a+b\text{ArcTan}(cx$$

```
[Out] -a*b^2*d^2*x-b^3*d^2*x*arctan(c*x)+7/2*b*d^2*(a+b*arctan(c*x))^2/c-3*I*b*d^
2*x*(a+b*arctan(c*x))^2+1/2*b*c*d^2*x^2*(a+b*arctan(c*x))^2-1/3*I*d^2*(1+I*
c*x)^3*(a+b*arctan(c*x))^3/c+4*b*d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-
6*I*b^2*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+1/2*b^3*d^2*ln(c^2*x^2+1)/c
-4*I*b^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/c+3*b^3*d^2*polylog
(2,1-2/(1+I*c*x))/c+2*b^3*d^2*polylog(3,1-2/(1-I*c*x))/c
```

Rubi [A]
time = 0.35, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 1600, 5112, 6745}

$$-\frac{4ib^2d^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{c}-\frac{6ib^2d^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{1}{2}bcd^2x^2(a+b\text{ArcTan}(cx))^2+\frac{7bd^2(a+b\text{ArcTan}(cx))^2}{2c}-3ibd^2x(a+b\text{ArcTan}(cx))^2-\frac{id^2(1+icx)^3(a+b\text{ArcTan}(cx))^3}{3c}+\frac{4bd^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{c}-ab^2d^2x+b^3\left(-d^2\right)x\text{ArcTan}(cx)+\frac{b^3d^2\log\left(c^2x^2+1\right)}{2c}+\frac{3b^3d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}+\frac{2b^3d^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -(a*b^2*d^2*x) - b^3*d^2*x*ArcTan[c*x] + (7*b*d^2*(a + b*ArcTan[c*x])^2)/(2
*c) - (3*I)*b*d^2*x*(a + b*ArcTan[c*x])^2 + (b*c*d^2*x^2*(a + b*ArcTan[c*x]
)^2)/2 - ((I/3)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^3)/c + (4*b*d^2*(a +
b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - ((6*I)*b^2*d^2*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/c + (b^3*d^2*Log[1 + c^2*x^2])/(2*c) - ((4*I)*b^2*d^2*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c + (3*b^3*d^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c + (2*b^3*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/c
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
```

```
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)^2\left(a+b\tan^{-1}(cx)\right)^3\,dx = -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^3}{3c}+\frac{(ib)\int\left(-3d^3\left(a+b\tan^{-1}(cx)\right)^2\right.}{}$$

$$= -\frac{id^2(1+icx)^3\left(a+b\tan^{-1}(cx)\right)^3}{3c}+\frac{(4b)\int\frac{\left(id^3-cd^3x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{d}$$

$$= -3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}bcd^2x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{id^2(1+}{}$$

$$= \frac{3bd^2\left(a+b\tan^{-1}(cx)\right)^2}{c}-3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}bcd^2x^2(a+b$$

$$= -ab^2d^2x+\frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-3ibd^2x\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}b$$

$$= -ab^2d^2x-b^3d^2x\tan^{-1}(cx)+\frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-3ibd^2x(a+$$

$$= -ab^2d^2x-b^3d^2x\tan^{-1}(cx)+\frac{7bd^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-3ibd^2x(a+$$

Mathematica [A]
time = 0.56, size = 528, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -1/6*(d^2*(-6*a^3*c*x + (18*I)*a^2*b*c*x + 6*a*b^2*c*x - (6*I)*a^3*c^2*x^2
- 3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 - (18*I)*a^2*b*ArcTan[c*x] - 6*a*b^2*ArcT
an[c*x] - 18*a^2*b*c*x*ArcTan[c*x] + (36*I)*a*b^2*c*x*ArcTan[c*x] + 6*b^3*c
*x*ArcTan[c*x] - (18*I)*a^2*b*c^2*x^2*ArcTan[c*x] - 6*a*b^2*c^2*x^2*ArcTan[
c*x] + 6*a^2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*b^2*ArcTan[c*x]^2 + 15*b^3*Arc
Tan[c*x]^2 - 18*a*b^2*c*x*ArcTan[c*x]^2 + (18*I)*b^3*c*x*ArcTan[c*x]^2 - (1
8*I)*a*b^2*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^2*x^2*ArcTan[c*x]^2 + 6*a*b^2*c^
3*x^3*ArcTan[c*x]^2 + (2*I)*b^3*ArcTan[c*x]^3 - 6*b^3*c*x*ArcTan[c*x]^3 - (
6*I)*b^3*c^2*x^2*ArcTan[c*x]^3 + 2*b^3*c^3*x^3*ArcTan[c*x]^3 - 48*a*b^2*Arc
Tan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (36*I)*b^3*ArcTan[c*x]*Log[1 + E^
((2*I)*ArcTan[c*x])] - 24*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])]
+ 12*a^2*b*Log[1 + c^2*x^2] - (18*I)*a*b^2*Log[1 + c^2*x^2] - 3*b^3*Log[1 +
 c^2*x^2] + 6*b^2*((4*I)*a + 3*b + (4*I)*b*ArcTan[c*x])*PolyLog[2, -E^((2*I
)*ArcTan[c*x])] - 12*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.00, size = 1699, normalized size = 5.70

method	result	size
derivativedivides	Expression too large to display	1699
default	Expression too large to display	1699

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*d^2*b^3*arctan(c*x)^3*c^2*x^2+3*I*d^2*a^2*b*arctan(c*x)*c^2*x^2+I*d^
2*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+I*d^2*b^3*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)^2*arctan(c*x)^2+I*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-I*d^2*b^3*Pi*csgn(I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+2*
I*d^2*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))^2*arctan(c*x)^2-2*I*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+3*I*d^2*a*b^2*arctan(c*
x)^2*c^2*x^2-6*I*d^2*a*b^2*arctan(c*x)*c*x-2*a^2*b*ln(c^2*x^2+1)*d^2+4*b^3*
d^2*ln(2)*arctan(c*x)^2+4*b^3*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*arctan(c*
```

```
x)^2-2*b^3*arctan(c*x)^2*ln(c^2*x^2+1)*d^2+2*b^3*polylog(3,-(1+I*c*x)^2/(c^
2*x^2+1))*d^2-I*d^2*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)*arctan(c*x)^2-3*I*d^2*b^3*arctan(c*x)^2*c*x-d^2*a*b^2*arctan(c*x)^
2*c^3*x^3+d^2*a*b^2*arctan(c*x)*c^2*x^2-2*I*d^2*a*b^2*ln(c*x-I)*ln(c^2*x^2+
1)+2*I*d^2*a*b^2*ln(c*x+I)*ln(c^2*x^2+1)+2*I*d^2*a*b^2*ln(c*x-I)*ln(-1/2*I*
(c*x+I))-2*I*d^2*a*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-d^2*a^2*b*arctan(c*x)*c^
3*x^3-3*I*d^2*a^2*b*c*x+I*d^2*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^
3*arctan(c*x)^2-I*d^2*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^
2-I*d^2*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2
)^3*arctan(c*x)^2+3*a*b^2*arctan(c*x)^2*d^2*c*x+3*a^2*b*arctan(c*x)*d^2*c*x
-6*I*d^2*b^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*d^2*b^3*ar
ctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I*d^2*b^3*arctan(c*x)*polyl
og(2,-(1+I*c*x)^2/(c^2*x^2+1))+I*d^2*a*b^2*ln(c*x-I)^2-I*d^2*a*b^2*ln(c*x+I
)^2+3*I*d^2*a*b^2*arctan(c*x)^2+1/2*d^2*a^2*b*c^2*x^2+3*I*d^2*a^2*b*arctan(
c*x)-d^2*b^3*arctan(c*x)*c*x-1/3*d^2*b^3*arctan(c*x)^3*c^3*x^3+1/2*d^2*b^3*
arctan(c*x)^2*c^2*x^2+b^3*arctan(c*x)^3*d^2*c*x-4*a*b^2*arctan(c*x)*ln(c^2*
x^2+1)*d^2-1/3*I*d^2*(1+I*c*x)^3*a^3+d^2*a*b^2*arctan(c*x)+I*d^2*b^3*arctan
(c*x)-1/3*I*d^2*b^3*arctan(c*x)^3-6*d^2*b^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-5/2*d^2*b^3*arctan(c*x)^2-6*d^2*b^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-d^2*b^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+3*I*d^2*a*b^2*ln(c^2*x^2+1)-2
*I*d^2*a*b^2*dilog(1/2*I*(c*x-I))+2*I*d^2*a*b^2*dilog(-1/2*I*(c*x+I))-d^2*a
*b^2*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/3*a^3*c^2*d^2*x^3 - 28*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^3/(c^2
*x^2 + 1), x) - 3*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^2 +
1)^2/(c^2*x^2 + 1), x) - 96*a*b^2*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^2/
(c^2*x^2 + 1), x) - 4*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*x)^2*
log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c^3*d^2*integrate(1/32*x^3*log(c^2
*x^2 + 1)^3/(c^2*x^2 + 1), x) + 16*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*
x)^2/(c^2*x^2 + 1), x) - 4*b^3*c^3*d^2*integrate(1/32*x^3*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) - 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1
)/c^4))*a^2*b*c^2*d^2 + I*a^3*c*d^2*x^2 + 7/32*b^3*d^2*arctan(c*x)^4/c + 24
*b^3*c^2*d^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*c*d^2 + a*b
^2*d^2*arctan(c*x)^3/c + 12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2*log(c^
```

```
2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^
3/(c^2*x^2 + 1), x) - 12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 3*b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + a^3*d^2*x + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^
2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^2/c - 1
/24*(b^3*c^2*d^2*x^3 - 3*I*b^3*c*d^2*x^2 - 3*b^3*d^2*x)*arctan(c*x)^3 + 1/1
6*(-I*b^3*c^2*d^2*x^3 - 3*b^3*c*d^2*x^2 + 3*I*b^3*d^2*x)*arctan(c*x)^2*log(
c^2*x^2 + 1) + 1/32*(b^3*c^2*d^2*x^3 - 3*I*b^3*c*d^2*x^2 - 3*b^3*d^2*x)*arc
tan(c*x)*log(c^2*x^2 + 1)^2 - 1/192*(-I*b^3*c^2*d^2*x^3 - 3*b^3*c*d^2*x^2 +
 3*I*b^3*d^2*x)*log(c^2*x^2 + 1)^3 - I*integrate(-1/64*(112*(b^3*c^3*d^2*x^
3 + b^3*c*d^2*x)*arctan(c*x)^3 - (b^3*c^4*d^2*x^4 - b^3*d^2)*log(c^2*x^2 +
1)^3 + 8*(b^3*c^4*d^2*x^4 + 48*a*b^2*c^3*d^2*x^3 - 6*b^3*c^2*d^2*x^2 + 48*a
*b^2*c*d^2*x)*arctan(c*x)^2 - 2*(b^3*c^4*d^2*x^4 - 6*b^3*c^2*d^2*x^2 - 6*(b
^3*c^3*d^2*x^3 + b^3*c*d^2*x)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 4*(3*(b^3*c
^4*d^2*x^4 - b^3*d^2)*arctan(c*x)^2 + 2*(4*b^3*c^3*d^2*x^3 - 3*b^3*c*d^2*x)
*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(I*b^3*c^2*d^2*x^3 + 3*b^3*c*d^2*x^2 - 3*I*b^3*d^2*x)*log(-(c*x + I)/(
c*x - I))^3 + integral(-1/4*(4*a^3*c^4*d^2*x^4 - 8*I*a^3*c^3*d^2*x^3 - 8*I*
a^3*c*d^2*x - 4*a^3*d^2 - (3*a*b^2*c^4*d^2*x^4 + 3*I*b^3*c^2*d^2*x^2 + (-6*
I*a*b^2 - b^3)*c^3*d^2*x^3 - 3*a*b^2*d^2 - 3*(2*I*a*b^2 - b^3)*c*d^2*x)*log
(-(c*x + I)/(c*x - I))^2 + 6*(I*a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^3*d^2*x^3 + 2
*a^2*b*c*d^2*x - I*a^2*b*d^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2,x)
```

```
[Out] int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2, x)
```

3.122 $\int (d+icdx)(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=220

$$\frac{3bd(a+b\text{ArcTan}(cx))^2}{2c}-\frac{3}{2}ibdx(a+b\text{ArcTan}(cx))^2-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))^3}{2c}+\frac{3bd(a+b\text{ArcTan}(cx))}{c}$$

```
[Out] 3/2*b*d*(a+b*arctan(c*x))^2/c-3/2*I*b*d*x*(a+b*arctan(c*x))^2-1/2*I*d*(1+I*
c*x)^2*(a+b*arctan(c*x))^3/c+3*b*d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-3*
I*b^2*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c-3*I*b^2*d*(a+b*arctan(c*x))*pol
ylog(2,1-2/(1-I*c*x))/c+3/2*b^3*d*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*d*poly
log(3,1-2/(1-I*c*x))/c
```

Rubi [A]
time = 0.25, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4974, 4930, 5040, 4964, 2449, 2352, 1600, 5004, 5112, 6745}

$$-\frac{3ib^2d\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{c}-\frac{3ib^2d\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{3bd(a+b\text{ArcTan}(cx))^2}{2c}-\frac{3}{2}ibdx(a+b\text{ArcTan}(cx))^2-\frac{id(1+icx)^2(a+b\text{ArcTan}(cx))^3}{2c}+\frac{3bd\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{c}+\frac{3b^3d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c}+\frac{3b^3d\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b*d*(a + b*ArcTan[c*x])^2)/(2*c) - ((3*I)/2)*b*d*x*(a + b*ArcTan[c*x])^2
 - ((I/2)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^3)/c + (3*b*d*(a + b*ArcTan[c
*x])^2*Log[2/(1 - I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*Log[2/(1 +
I*c*x)])/c - ((3*I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)]
)/c + (3*b^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c) + (3*b^3*d*PolyLog[3, 1
 - 2/(1 - I*c*x)])/(2*c)
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+icdx)\left(a+b\tan^{-1}(cx)\right)^3\,dx = -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3ib)\int\left(-d^2\left(a+b\tan^{-1}(cx)\right)^2-\right.}{2d}$$

$$= -\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3b)\int\frac{\left(id^2-cd^2x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{d}-$$

$$= -\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2\left(a+b\tan^{-1}(cx)\right)^3}{2c}+\frac{(3b)\int}{}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2(a+}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2(a+}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2(a+}{2c}$$

$$= \frac{3bd\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{3}{2}ibdx\left(a+b\tan^{-1}(cx)\right)^2-\frac{id(1+icx)^2(a+}{2c}$$

Mathematica [A]
time = 0.26, size = 367, normalized size = 1.67

$$\frac{id\left(-2ia^3cx-3a^2bcx+a^3c^2x^2+3a^2b\text{ArcTan}(cx)-6ia^2bcx\text{ArcTan}(cx)-6ab^2cx\text{ArcTan}(cx)+3a^2bc^2x^2\text{ArcTan}(cx)-3ab^2\text{ArcTan}(cx)^2+3ib^3\text{ArcTan}(cx)^2-6iab^2cx\text{ArcTan}(cx)^2-3b^3cx\text{ArcTan}(cx)^2+3ab^2c^2x^2\text{ArcTan}(cx)^2-b^3\text{ArcTan}(cx)^3-2ib^3cx\text{ArcTan}(cx)^3+b^3c^2x^2\text{ArcTan}(cx)^3-12iab^2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-6b^3\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-6ib^3\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+3ia^2b\log\left(1+c^2x^2\right)+3ab^2\log\left(1+c^2x^2\right)-3b^2(2a-ib+2b\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)-3ib^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] ((I/2)*d*((-2*I)*a^3*c*x - 3*a^2*b*c*x + a^3*c^2*x^2 + 3*a^2*b*ArcTan[c*x]
- (6*I)*a^2*b*c*x*ArcTan[c*x] - 6*a*b^2*c*x*ArcTan[c*x] + 3*a^2*b*c^2*x^2*A
rcTan[c*x] - 3*a*b^2*ArcTan[c*x]^2 + (3*I)*b^3*ArcTan[c*x]^2 - (6*I)*a*b^2*
c*x*ArcTan[c*x]^2 - 3*b^3*c*x*ArcTan[c*x]^2 + 3*a*b^2*c^2*x^2*ArcTan[c*x]^2
 - b^3*ArcTan[c*x]^3 - (2*I)*b^3*c*x*ArcTan[c*x]^3 + b^3*c^2*x^2*ArcTan[c*x
]^3 - (12*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*b^3*ArcTa
n[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^3*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] + (3*I)*a^2*b*Log[1 + c^2*x^2] + 3*a*b^2*Log[1 + c^2*x^
2] - 3*b^2*(2*a - I*b + 2*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
 - (3*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.69, size = 7165, normalized size = 32.57

method	result	size
derivativedivides	Expression too large to display	7165
default	Expression too large to display	7165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1
), x) - b^3*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) +
 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^3*c^
3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1/2*I*a^3*c*d
*x^2 + 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*d*integrate(1/64*x^2*arctan(
c*x)^3/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2*arctan
(c*x)^2/(c^2*x^2 + 1), x) + 36*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log
(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arcta
n(c*x)/c^3))*a^2*b*c*d + a*b^2*d*arctan(c*x)^3/c + 12*b^3*c*d*integrate(1/6
4*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d*integrate(1/
64*x*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrate(1/64*x*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) + a^3*d*x + 6*b^3*d*integrate(1/64*arctan(c*x)*log(c^2*
x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a
^2*b*d/c + 1/16*(I*b^3*c*d*x^2 + 2*b^3*d*x)*arctan(c*x)^3 - 3/32*(b^3*c*d*x
^2 - 2*I*b^3*d*x)*arctan(c*x)^2*log(c^2*x^2 + 1) + 3/64*(-I*b^3*c*d*x^2 - 2
*b^3*d*x)*arctan(c*x)*log(c^2*x^2 + 1)^2 + 1/128*(b^3*c*d*x^2 - 2*I*b^3*d*x
)*log(c^2*x^2 + 1)^3 + I*integrate(1/64*(56*(b^3*c^3*d*x^3 + b^3*c*d*x)*arc
tan(c*x)^3 + (b^3*c^2*d*x^2 + b^3*d)*log(c^2*x^2 + 1)^3 + 12*(16*a*b^2*c^3*
d*x^3 - 3*b^3*c^2*d*x^2 + 16*a*b^2*c*d*x)*arctan(c*x)^2 + 3*(3*b^3*c^2*d*x^
2 + 2*(b^3*c^3*d*x^3 + b^3*c*d*x)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*((b^
3*c^2*d*x^2 + b^3*d)*arctan(c*x)^2 - (b^3*c^3*d*x^3 - 2*b^3*c*d*x)*arctan(c
*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(b^3*c*d*x^2 - 2*I*b^3*d*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/8
*(8*I*a^3*c^3*d*x^3 + 8*a^3*c^2*d*x^2 + 8*I*a^3*c*d*x + 8*a^3*d - 3*(2*I*a*
b^2*c^3*d*x^3 + (2*a*b^2 - I*b^3)*c^2*d*x^2 + 2*a*b^2*d + 2*(I*a*b^2 - b^3)
*c*d*x)*log(-(c*x + I)/(c*x - I))^2 - 12*(a^2*b*c^3*d*x^3 - I*a^2*b*c^2*d*x
^2 + a^2*b*c*d*x - I*a^2*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*atan(c*x))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{atan}(c\,x))^3\,(d + c\,d\,x\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3*(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))^3*(d + c*d*x*1i), x)
```

3.123 $\int \frac{(a+b\text{ArcTan}(cx))^3}{d+icdx}\,dx$

Optimal. Leaf size=139

$$\frac{i(a+b\text{ArcTan}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{3b(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}+\frac{3ib^2(a+b\text{ArcTan}(cx))\text{Poly}}{2cd}$$

```
[Out] I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylog
(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))/
c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4964, 5004, 5114, 5118, 6745}

$$\frac{3ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{2cd}-\frac{3b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))^2}{2cd}+\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{cd}+\frac{3b^3\text{Li}_4\left(1-\frac{2}{icx+1}\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx = \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(3ib)\int\frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{(3b^2)\int \frac{(a}{}}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a+}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a+}{}$$

Mathematica [A]
time = 0.06, size = 133, normalized size = 0.96

$$\frac{i\left(4(a+b\text{ArcTan}(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)+b\text{PolyLog}\left(4,\frac{i+cx}{-i+cx}\right)\right)\right)\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
[Out] ((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTan[
c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*x)]
))))/(c*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.66, size = 1940, normalized size = 13.96

method	result	size
derivativedivides	Expression too large to display	1940
default	Expression too large to display	1940

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^3/d*arctan(c*x)+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+3/2*a*b^2/
d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*Pi+3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*Pi+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*Pi-3/4*b^3/d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^3/d*arctan(c*x)^4
-1/2*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*Pi-I*b^3/d*ln(1+I*c*x)*arctan(c*x)^3+3/2*I*b^3/d*arctan(c*x)*po
lylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a*b^2/d*Pi*arctan(c*x)^2+3*a*b^2/d*ar
ctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a^2*b/d*ln(1/2-1/2*I*c*x)
*ln(1/2+1/2*I*c*x)-3/2*a^2*b/d*ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+3/2*I*a*b^2/d*
polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+I*b^3/d*arctan(c*x)^3*ln(2*I*(1+I*c*x)^
2/(c^2*x^2+1))-1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^3*Pi+1/2*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi+1/2*b^3/d*Pi*arctan(c*x)^3+3/2*b^3
/d*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a*b^2/d*arctan(c*x
)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+3/2*a^2*b/d*dilog(1
/2+1/2*I*c*x)+3/4*a^2*b/d*ln(1+I*c*x)^2+2*a*b^2/d*arctan(c*x)^3-1/2*b^3/d*a
rctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*Pi-3*I*a^2*b/d*ln(1+I*c*x)*arctan(c*x)-3/2*a*b^2/
d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3
*Pi+3/2*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3*I*a*b^2/d*ln(1+I*c*x)*arctan(c*x)^2+3
*I*a*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*a^3/d*ln(c^2
```

```
*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arct
an(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3*l
og(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16*x*
log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) + 12*a
^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*arctan
(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arctan(c*
x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 + 1) + b
^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

3.124 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{(d+icdx)^2}\,dx$

Optimal. Leaf size=182

$$-\frac{3ib^3}{4cd^2(i-cx)}+\frac{3ib^3\text{ArcTan}(cx)}{4cd^2}+\frac{3b^2(a+b\text{ArcTan}(cx))}{2cd^2(i-cx)}-\frac{3b(a+b\text{ArcTan}(cx))^2}{4cd^2}+\frac{3ib(a+b\text{ArcTan}(cx))^2}{2cd^2(i-cx)}-$$

```
[Out] -3/4*I*b^3/c/d^2/(I-c*x)+3/4*I*b^3*arctan(c*x)/c/d^2+3/2*b^2*(a+b*arctan(c*
x))/c/d^2/(I-c*x)-3/4*b*(a+b*arctan(c*x))^2/c/d^2+3/2*I*b*(a+b*arctan(c*x))
^2/c/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^3/c/d^2+I*(a+b*arctan(c*x))^3/c/d^
2/(1+I*c*x)
```

Rubi [A]
time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{3b^2(a+b\text{ArcTan}(cx))}{2cd^2(-cx+i)}+\frac{3ib(a+b\text{ArcTan}(cx))^2}{2cd^2(-cx+i)}-\frac{3b(a+b\text{ArcTan}(cx))^2}{4cd^2}+\frac{i(a+b\text{ArcTan}(cx))^3}{cd^2(1+icx)}-\frac{i(a+b\text{ArcTan}(cx))^3}{2cd^2}+\frac{3ib^3\text{ArcTan}(cx)}{4cd^2}-\frac{3ib^3}{4cd^2(-cx+i)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

```
[Out] (((-3*I)/4)*b^3)/(c*d^2*(I - c*x)) + (((3*I)/4)*b^3*ArcTan[c*x])/(c*d^2) +
(3*b^2*(a + b*ArcTan[c*x]))/(2*c*d^2*(I - c*x)) - (3*b*(a + b*ArcTan[c*x])^
2)/(4*c*d^2) + (((3*I)/2)*b*(a + b*ArcTan[c*x])^2)/(c*d^2*(I - c*x)) - ((I/
2)*(a + b*ArcTan[c*x])^3)/(c*d^2) + (I*(a + b*ArcTan[c*x])^3)/(c*d^2*(1 + I
*c*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{(d+icdx)^2}\,dx = \frac{i(a+b\tan^{-1}(cx))^3}{cd^2(1+icx)} - \frac{(3ib)\int \left(-\frac{(a+b\tan^{-1}(cx))^2}{2d(-i+cx)^2} + \frac{(a+b\tan^{-1}(cx))^2}{2d(1+c^2x^2)}\right)dx}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3}{cd^2(1+icx)} + \frac{(3ib)\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}dx}{2d^2} - \frac{(3ib)\int \frac{(a+b\tan^{-1}(cx))^2}{1+c^2x^2}dx}{2d^2}$$

$$= \frac{3ib(a+b\tan^{-1}(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^3}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^3}{cd^2(1+icx)} + \frac{(3ib^2)\int \Big(-}{}$$

$$= \frac{3ib(a+b\tan^{-1}(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^3}{2cd^2} + \frac{i(a+b\tan^{-1}(cx))^3}{cd^2(1+icx)} + \frac{(3b^2)\int \frac{a+b\,\mathrm{t}}{(-}}{2d}$$

$$= \frac{3b^2(a+b\tan^{-1}(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\tan^{-1}(cx))^2}{4cd^2} + \frac{3ib(a+b\tan^{-1}(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\,\mathrm{ta}}{2}$$

$$= \frac{3b^2(a+b\tan^{-1}(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\tan^{-1}(cx))^2}{4cd^2} + \frac{3ib(a+b\tan^{-1}(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\,\mathrm{ta}}{2}$$

$$= \frac{3b^2(a+b\tan^{-1}(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\tan^{-1}(cx))^2}{4cd^2} + \frac{3ib(a+b\tan^{-1}(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\,\mathrm{ta}}{2}$$

$$= -\frac{3ib^3}{4cd^2(i-cx)} + \frac{3b^2(a+b\tan^{-1}(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\tan^{-1}(cx))^2}{4cd^2} + \frac{3ib(a+b\tan^{-1}(}{2cd^2(i-cx}$$

$$= -\frac{3ib^3}{4cd^2(i-cx)} + \frac{3ib^3\tan^{-1}(cx)}{4cd^2} + \frac{3b^2(a+b\tan^{-1}(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\tan^{-1}(cx))^2}{4cd^2}$$

Mathematica [A]
time = 0.09, size = 121, normalized size = 0.66

$$\frac{4a^3-6ia^2b-6ab^2+3ib^3+3ib(-2a^2+2iab+b^2)(i+cx)\mathrm{ArcTan}(cx)-3b^2(2ia+b)(i+cx)\mathrm{ArcTan}(cx)^2+2b^3(1-icx)\mathrm{ArcTan}(cx)^3}{4cd^2(-i+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

```
[Out] (4*a^3 - (6*I)*a^2*b - 6*a*b^2 + (3*I)*b^3 + (3*I)*b*(-2*a^2 + (2*I)*a*b +
b^2)*(I + c*x)*ArcTan[c*x] - 3*b^2*((2*I)*a + b)*(I + c*x)*ArcTan[c*x]^2 +
2*b^3*(1 - I*c*x)*ArcTan[c*x]^3)/(4*c*d^2*(-I + c*x))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(161) = 322.
time = 0.73, size = 498, normalized size = 2.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(3*I*a^2*b/d^2/(1+I*c*x)*arctan(c*x)+3*I*a*b^2/d^2/(1+I*c*x)*arctan(c*x
)^2-1/2*b^3/d^2/(c*x-I)*arctan(c*x)^3+3/4*I*b^3/d^2/(c*x-I)-3/4*b^3/d^2/(c*
x-I)*arctan(c*x)^2*c*x+3/4*I*b^3/d^2/(c*x-I)*arctan(c*x)*c*x-3/4*b^3/d^2/(c
*x-I)*arctan(c*x)+3/4*I*a*b^2/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/4*I*a*b^2/
d^2*ln(-1/2*I*(-c*x+I))*ln(c*x+I)-1/2*I*b^3/d^2/(c*x-I)*arctan(c*x)^3*c*x-3
/2*a*b^2/d^2*arctan(c*x)*ln(c*x-I)-3/4*I*a*b^2/d^2*ln(-1/2*I*(-c*x+I))*ln(-
1/2*I*(c*x+I))+3/2*a*b^2/d^2*arctan(c*x)*ln(c*x+I)-3*I*a*b^2/d^2*arctan(c*x
)/(c*x-I)-3/4*I*b^3/d^2/(c*x-I)*arctan(c*x)^2-3/2*a*b^2/d^2/(c*x-I)-3/2*a*b
^2/d^2*arctan(c*x)+I*b^3/d^2/(1+I*c*x)*arctan(c*x)^3-3/2*I*a^2*b/d^2/(c*x-I
)-3/8*I*a*b^2/d^2*ln(c*x-I)^2+I*a^3/d^2/(1+I*c*x)-3/2*I*a^2*b/d^2*arctan(c*
x)-3/8*I*a*b^2/d^2*ln(c*x+I)^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 1.26, size = 175, normalized size = 0.96

$$-\frac{(b^3cx+ib^3)\log\left(-\frac{cx+i}{cx-i}\right)^3-16a^3+24ia^2b+24ab^2-12ib^3+3\left(2ab^2-ib^3+(-2iab^2-b^3)cx\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+6\left(-2ia^2b-2ab^2+ib^3-(2a^2b-2iab^2-b^3)cx\right)\log\left(-\frac{cx+i}{cx-i}\right)}{16\left(c^2d^2x-icd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*((b^3*c*x + I*b^3)*log(-(c*x + I)/(c*x - I))^3 - 16*a^3 + 24*I*a^2*b
+ 24*a*b^2 - 12*I*b^3 + 3*(2*a*b^2 - I*b^3 + (-2*I*a*b^2 - b^3)*c*x)*log(-(
c*x + I)/(c*x - I))^2 + 6*(-2*I*a^2*b - 2*a*b^2 + I*b^3 - (2*a^2*b - 2*I*a*
b^2 - b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^2*d^2*x - I*c*d^2)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(151) = 302.
time = 18.39, size = 631, normalized size = 3.47

$$\frac{3ib(a(1-i)-b)(a(1-i)-ib)\log\left(-\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2}-\frac{3ib(a(1-i)-b)(a(1-i)-ib)\log\left(\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2}+\frac{(-b^3cx-ib^3)\log(-icx+1)^3}{16c^2d^2x-16icd^2}+\frac{(b^3cx+ib^3)\log(icx+1)^3}{16c^2d^2x-16icd^2}+\frac{(6iab^2cx-6ab^2+3b^3cx+3ib^3)\log(icx+1)^2}{16c^2d^2x-16icd^2}+\frac{(6iab^2cx-6ab^2+3b^3cx\log(icx+1)+3b^3cx+3ib^3\log(icx+1)+3ib^3)\log(-icx+1)^2}{16c^2d^2x-16icd^2}+\frac{(24ia^2b-12iab^2cx\log(icx+1)+12ab^2\log(icx+1)+24ab^2-3b^3cx\log(icx+1)^2-6b^3cx\log(icx+1)-3ib^3\log(icx+1)^2-6ib^3\log(icx+1)-12ib^3)\log(-icx+1)}{16c^2d^2x-16icd^2}+\frac{(-6ia^2b-6ab^2+3ib^3)\log(icx+1)}{4c^2d^2x-4icd^2}-\frac{-4a^3+6ia^2b+6ab^2-3ib^3}{4c^2d^2x-4icd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**2,x)
```

```
[Out] 3*I*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)*log(-3*b*(a*(1 - I) - b)*(a*(1 - I)
 - I*b)/c + x*(6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) - 3*I*b*(a*(1 -
```

```
I) - b)*(a*(1 - I) - I*b)*log(3*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)/c + x*(
6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) + (-b**3*c*x - I*b**3)*log(-I*c
*x + 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (b**3*c*x + I*b**3)*log(I*c*x +
 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (6*I*a*b**2*c*x - 6*a*b**2 + 3*b**3
*c*x + 3*I*b**3)*log(I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*c*d**2) + (6*I*a*
b**2*c*x - 6*a*b**2 + 3*b**3*c*x*log(I*c*x + 1) + 3*b**3*c*x + 3*I*b**3*log
(I*c*x + 1) + 3*I*b**3)*log(-I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*c*d**2) +
 (24*I*a**2*b - 12*I*a*b**2*c*x*log(I*c*x + 1) + 12*a*b**2*log(I*c*x + 1) +
 24*a*b**2 - 3*b**3*c*x*log(I*c*x + 1)**2 - 6*b**3*c*x*log(I*c*x + 1) - 3*I
*b**3*log(I*c*x + 1)**2 - 6*I*b**3*log(I*c*x + 1) - 12*I*b**3)*log(-I*c*x +
 1)/(16*c**2*d**2*x - 16*I*c*d**2) + (-6*I*a**2*b - 6*a*b**2 + 3*I*b**3)*lo
g(I*c*x + 1)/(4*c**2*d**2*x - 4*I*c*d**2) - (-4*a**3 + 6*I*a**2*b + 6*a*b**
2 - 3*I*b**3)/(4*c**2*d**2*x - 4*I*c*d**2)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^3}{(d + c\,d\,x\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2,x)
```

```
[Out] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2, x)
```

3.125 $\int \frac{(a+b\text{ArcTan}(cx))^3}{(d+icdx)^3}\,dx$

Optimal. Leaf size=271

$$\frac{3b^3}{64cd^3(i-cx)^2}-\frac{21ib^3}{64cd^3(i-cx)}+\frac{21ib^3\text{ArcTan}(cx)}{64cd^3}+\frac{3ib^2(a+b\text{ArcTan}(cx))}{16cd^3(i-cx)^2}+\frac{9b^2(a+b\text{ArcTan}(cx))}{16cd^3(i-cx)}-\frac{9b(a-}{}$$

```
[Out] 3/64*b^3/c/d^3/(I-c*x)^2-21/64*I*b^3/c/d^3/(I-c*x)+21/64*I*b^3*arctan(c*x)/
c/d^3+3/16*I*b^2*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+9/16*b^2*(a+b*arctan(c*x
))/c/d^3/(I-c*x)-9/32*b*(a+b*arctan(c*x))^2/c/d^3-3/8*b*(a+b*arctan(c*x))^2
/c/d^3/(I-c*x)^2+3/8*I*b*(a+b*arctan(c*x))^2/c/d^3/(I-c*x)-1/8*I*(a+b*arcta
n(c*x))^3/c/d^3+1/2*I*(a+b*arctan(c*x))^3/c/d^3/(1+I*c*x)^2
```

Rubi [A]
time = 0.30, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{9b^2(a+b\text{ArcTan}(cx))}{16cd^3(-cx+i)}+\frac{3ib^2(a+b\text{ArcTan}(cx))}{16cd^3(-cx+i)^2}+\frac{3ib(a+b\text{ArcTan}(cx))^2}{8cd^3(-cx+i)}-\frac{3b(a+b\text{ArcTan}(cx))^2}{8cd^3(-cx+i)^2}-\frac{9b(a+b\text{ArcTan}(cx))^2}{32cd^3}+\frac{i(a+b\text{ArcTan}(cx))^3}{2cd^3(1+icx)^2}-\frac{i(a+b\text{ArcTan}(cx))^3}{8cd^3}+\frac{21ib^3\text{ArcTan}(cx)}{64cd^3}-\frac{21ib^3}{64cd^3(-cx+i)}+\frac{3b^3}{64cd^3(-cx+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

```
[Out] (3*b^3)/(64*c*d^3*(I - c*x)^2) - (((21*I)/64)*b^3)/(c*d^3*(I - c*x)) + (((2
1*I)/64)*b^3*ArcTan[c*x])/(c*d^3) + (((3*I)/16)*b^2*(a + b*ArcTan[c*x]))/(c
*d^3*(I - c*x)^2) + (9*b^2*(a + b*ArcTan[c*x]))/(16*c*d^3*(I - c*x)) - (9*b
*(a + b*ArcTan[c*x])^2)/(32*c*d^3) - (3*b*(a + b*ArcTan[c*x])^2)/(8*c*d^3*(
I - c*x)^2) + (((3*I)/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^3*(I - c*x)) - ((I/8
)*(a + b*ArcTan[c*x])^3)/(c*d^3) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c*d^3*(1
+ I*c*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{(d+icdx)^3}\,dx = \frac{i(a+b\tan^{-1}(cx))^3}{2cd^3(1+icx)^2} - \frac{(3ib)\int\left(\frac{i(a+b\tan^{-1}(cx))^2}{2d^2(-i+cx)^3} - \frac{(a+b\tan^{-1}(cx))^2}{4d^2(-i+cx)^2} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2(1+c^2x^2)}\right)}{2d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3}{2cd^3(1+icx)^2} + \frac{(3ib)\int\frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}dx}{8d^3} - \frac{(3ib)\int\frac{(a+b\tan^{-1}(cx))^2}{1+c^2x^2}dx}{8d^3} + \frac{(3b}{}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{8cd^3(i-cx)^2} + \frac{3ib(a+b\tan^{-1}(cx))^2}{8cd^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^3}{8cd^3} + \frac{i(a+b}{2cd^3}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{8cd^3(i-cx)^2} + \frac{3ib(a+b\tan^{-1}(cx))^2}{8cd^3(i-cx)} - \frac{i(a+b\tan^{-1}(cx))^3}{8cd^3} + \frac{i(a+b}{2cd^3}$$

$$= \frac{3ib^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)^2} + \frac{9b^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)} - \frac{9b(a+b\tan^{-1}(cx))^2}{32cd^3} - \frac{3b(a+}{8cd}$$

$$= \frac{3ib^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)^2} + \frac{9b^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)} - \frac{9b(a+b\tan^{-1}(cx))^2}{32cd^3} - \frac{3b(a+}{8cd}$$

$$= \frac{3ib^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)^2} + \frac{9b^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)} - \frac{9b(a+b\tan^{-1}(cx))^2}{32cd^3} - \frac{3b(a+}{8cd}$$

$$= \frac{3b^3}{64cd^3(i-cx)^2} - \frac{21ib^3}{64cd^3(i-cx)} + \frac{3ib^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)^2} + \frac{9b^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)}$$

$$= \frac{3b^3}{64cd^3(i-cx)^2} - \frac{21ib^3}{64cd^3(i-cx)} + \frac{21ib^3\tan^{-1}(cx)}{64cd^3} + \frac{3ib^2(a+b\tan^{-1}(cx))}{16cd^3(i-cx)^2} + \frac{9}{}$$

Mathematica [A]
time = 0.13, size = 183, normalized size = 0.68

$$-\frac{i(32a^3+3b^3(8i-7cx)+12ab^2(-4-3icx)+24a^2b(-2i+cx)+3b(i+cx)\left(b^2(9i-7cx)+4ab(-5-3icx)+8a^2(-3i+cx)\right)\mathrm{ArcTan}(cx)+6b^2(i+cx)(b(-5-3icx)+4a(-3i+cx))\mathrm{ArcTan}(cx)^2+8b^3(3-2icx+c^2x^2)\,\mathrm{ArcTan}(cx)^3)}{64cd^3(-i+cx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

```
[Out] ((-1/64*I)*(32*a^3 + 3*b^3*(8*I - 7*c*x) + 12*a*b^2*(-4 - (3*I)*c*x) + 24*a
^2*b*(-2*I + c*x) + 3*b*(I + c*x)*(b^2*(9*I - 7*c*x) + 4*a*b*(-5 - (3*I)*c*
x) + 8*a^2*(-3*I + c*x))*ArcTan[c*x] + 6*b^2*(I + c*x)*(b*(-5 - (3*I)*c*x)
+ 4*a*(-3*I + c*x))*ArcTan[c*x]^2 + 8*b^3*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[
c*x]^3))/(c*d^3*(-I + c*x)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(238) = 476.
time = 0.91, size = 656, normalized size = 2.42

method	result
derivativedivides	$\frac{-\frac{9b^3\arctan(cx)^2c^2x^2}{32d^3(cx-i)^2}+\frac{3b^3\arctan(cx)cx}{32d^3(cx-i)^2}+\frac{21ib^3cx}{64d^3(cx-i)^2}+\frac{3ia^2b\arctan(cx)}{2d^3(icx+1)^2}+\frac{3iab^2\arctan(cx)^2}{2d^3(icx+1)^2}-\frac{3iab^2\arctan(cx)}{4d^3(cx-i)}-\frac{3iab^2\ln\left(-\frac{i(-}{}\right.}{}}{}$
default	$\frac{-\frac{9b^3\arctan(cx)^2c^2x^2}{32d^3(cx-i)^2}+\frac{3b^3\arctan(cx)cx}{32d^3(cx-i)^2}+\frac{21ib^3cx}{64d^3(cx-i)^2}+\frac{3ia^2b\arctan(cx)}{2d^3(icx+1)^2}+\frac{3iab^2\arctan(cx)^2}{2d^3(icx+1)^2}-\frac{3iab^2\arctan(cx)}{4d^3(cx-i)}-\frac{3iab^2\ln\left(-\frac{i(-}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(3/8*b^3/d^3/(c*x-I)^2-1/4*b^3/d^3/(c*x-I)^2*arctan(c*x)^3*c*x-3/8*I*a^
2*b/d^3*arctan(c*x)-3/8*I*a^2*b/d^3/(c*x-I)+1/2*I*b^3/d^3/(1+I*c*x)^2*arcta
n(c*x)^3+1/8*I*b^3/d^3/(c*x-I)^2*arctan(c*x)^3+27/64*I*b^3/d^3/(c*x-I)^2*ar
ctan(c*x)-3/8*a*b^2/d^3*arctan(c*x)*ln(c*x-I)-3/4*a*b^2/d^3/(c*x-I)^2*arcta
n(c*x)+3/8*a*b^2/d^3*arctan(c*x)*ln(c*x+I)-3/32*I*a*b^2/d^3*ln(c*x+I)^2+3/1
6*I*a*b^2/d^3/(c*x-I)^2-3/32*I*a*b^2/d^3*ln(c*x-I)^2-1/8*I*b^3/d^3/(c*x-I)^
2*arctan(c*x)^3*c^2*x^2+3/16*I*b^3/d^3/(c*x-I)^2*arctan(c*x)^2*c*x+21/64*I*
b^3/d^3/(c*x-I)^2*c^2*x^2*arctan(c*x)+1/2*I*a^3/d^3/(1+I*c*x)^2-3/8*a^2*b/d
^3/(c*x-I)^2-15/32*b^3/d^3/(c*x-I)^2*arctan(c*x)^2-9/16*a*b^2/d^3/(c*x-I)-9
/16*a*b^2/d^3*arctan(c*x)-9/32*b^3/d^3/(c*x-I)^2*arctan(c*x)^2*c^2*x^2+3/32
*b^3/d^3/(c*x-I)^2*arctan(c*x)*c*x+21/64*I*b^3/d^3/(c*x-I)^2*c*x+3/2*I*a^2*
b/d^3/(1+I*c*x)^2*arctan(c*x)+3/2*I*a*b^2/d^3/(1+I*c*x)^2*arctan(c*x)^2-3/4
*I*a*b^2/d^3*arctan(c*x)/(c*x-I)-3/16*I*a*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(-1
/2*I*(c*x+I))+3/16*I*a*b^2/d^3*ln(-1/2*I*(-c*x+I))*ln(c*x+I)+3/16*I*a*b^2/d
^3*ln(c*x-I)*ln(-1/2*I*(c*x+I)))
```

Maxima [A]

time = 0.33, size = 233, normalized size = 0.86

$$\frac{8\left(-i\,b^3c^2x^2-2\,b^3cx-3i\,b^3\right)\arctan\left(cx\right)^3-32i\,a^3-48\,a^2b+48i\,ab^2+24\,b^3+3\left(-8i\,a^2b-12\,ab^2+7i\,b^3\right)cx+6\left(\left(-4i\,ab^2-3\,b^3\right)c^2x^2-12i\,ab^2-5\,b^3-2\left(4\,ab^2-i\,b^3\right)cx\right)\arctan\left(cx\right)^2+3\left(\left(-8i\,a^2b-12\,ab^2+7i\,b^3\right)c^2x^2-24i\,a^2b-20\,ab^2+9i\,b^3-2\left(8\,a^2b-4i\,ab^2-b^3\right)cx\right)\arctan\left(cx\right)}{64\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(8*(-I*b^3*c^2*x^2 - 2*b^3*c*x - 3*I*b^3)*arctan(c*x)^3 - 32*I*a^3 - 4
8*a^2*b + 48*I*a*b^2 + 24*b^3 + 3*(-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c*x + 6
*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a*b^2 - I*b^3)*c
*x)*arctan(c*x)^2 + 3*((-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c^2*x^2 - 24*I*a^2
*b - 20*a*b^2 + 9*I*b^3 - 2*(8*a^2*b - 4*I*a*b^2 - b^3)*c*x)*arctan(c*x))/(
c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Fricas [A]

time = 0.80, size = 265, normalized size = 0.98

$$-\frac{2\left(b^3c^2x^2-2i\,b^3cx+3\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+64i\,a^3+96\,a^2b-96i\,ab^2-48\,b^3+6\left(8i\,a^2b+12\,ab^2-7i\,b^3\right)cx+3\left(\left(-4i\,ab^2-3\,b^3\right)c^2x^2-12i\,ab^2-5\,b^3-2\left(4\,ab^2-i\,b^3\right)cx\right)\log\left(-\frac{cx+i}{cx-i}\right)^2-3\left(\left(8\,a^2b-12i\,ab^2-7\,b^3\right)c^2x^2+24\,a^2b-20i\,ab^2-9\,b^3-2\left(8i\,a^2b+4\,ab^2-i\,b^3\right)cx\right)\log\left(-\frac{cx+i}{cx-i}\right)}{128\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(2*(b^3*c^2*x^2 - 2*I*b^3*c*x + 3*b^3)*log(-(c*x + I)/(c*x - I))^3 +
 64*I*a^3 + 96*a^2*b - 96*I*a*b^2 - 48*b^3 + 6*(8*I*a^2*b + 12*a*b^2 - 7*I*
b^3)*c*x + 3*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a*b^
2 - I*b^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 - 3*((8*a^2*b - 12*I*a*b^2 - 7*
b^3)*c^2*x^2 + 24*a^2*b - 20*I*a*b^2 - 9*b^3 - 2*(8*I*a^2*b + 4*a*b^2 - I*b
^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(230) = 460$.
time = 140.58, size = 954, normalized size = 3.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**3,x)
```

```
[Out] -3*b*(8*a**2 - 12*I*a*b - 7*b**2)*log(-3*I*b*(8*a**2 - 12*I*a*b - 7*b**2)/c
 + x*(24*a**2*b - 36*I*a*b**2 - 21*b**3))/(128*c*d**3) + 3*b*(8*a**2 - 12*I
*a*b - 7*b**2)*log(3*I*b*(8*a**2 - 12*I*a*b - 7*b**2)/c + x*(24*a**2*b - 36
*I*a*b**2 - 21*b**3))/(128*c*d**3) + (-b**3*c**2*x**2 + 2*I*b**3*c*x - 3*b*
*3)*log(-I*c*x + 1)**3/(64*c**3*d**3*x**2 - 128*I*c**2*d**3*x - 64*c*d**3)
+ (b**3*c**2*x**2 - 2*I*b**3*c*x + 3*b**3)*log(I*c*x + 1)**3/(64*c**3*d**3*
x**2 - 128*I*c**2*d**3*x - 64*c*d**3) + (12*I*a*b**2*c**2*x**2 + 24*a*b**2*
c*x + 36*I*a*b**2 + 9*b**3*c**2*x**2 - 6*I*b**3*c*x + 15*b**3)*log(I*c*x +
1)**2/(128*c**3*d**3*x**2 - 256*I*c**2*d**3*x - 128*c*d**3) + (12*I*a*b**2*
c**2*x**2 + 24*a*b**2*c*x + 36*I*a*b**2 + 6*b**3*c**2*x**2*log(I*c*x + 1) +
 9*b**3*c**2*x**2 - 12*I*b**3*c*x*log(I*c*x + 1) - 6*I*b**3*c*x + 18*b**3*l
og(I*c*x + 1) + 15*b**3)*log(-I*c*x + 1)**2/(128*c**3*d**3*x**2 - 256*I*c**
2*d**3*x - 128*c*d**3) + (-32*I*a**3 - 48*a**2*b + 48*I*a*b**2 + 24*b**3 +
x*(-24*I*a**2*b*c - 36*a*b**2*c + 21*I*b**3*c))/(64*c**3*d**3*x**2 - 128*I*
c**2*d**3*x - 64*c*d**3) + (48*a**2*b - 12*I*a*b**2*c**2*x**2*log(I*c*x + 1
) - 24*a*b**2*c*x*log(I*c*x + 1) + 24*a*b**2*c*x - 36*I*a*b**2*log(I*c*x +
1) - 48*I*a*b**2 - 3*b**3*c**2*x**2*log(I*c*x + 1)**2 - 9*b**3*c**2*x**2*lo
g(I*c*x + 1) + 6*I*b**3*c*x*log(I*c*x + 1)**2 + 6*I*b**3*c*x*log(I*c*x + 1)
 - 18*I*b**3*c*x - 9*b**3*log(I*c*x + 1)**2 - 15*b**3*log(I*c*x + 1) - 24*b
**3)*log(-I*c*x + 1)/(64*c**3*d**3*x**2 - 128*I*c**2*d**3*x - 64*c*d**3) +
(-24*a**2*b - 12*a*b**2*c*x + 24*I*a*b**2 + 9*I*b**3*c*x + 12*b**3)*log(I*c
*x + 1)/(32*c**3*d**3*x**2 - 64*I*c**2*d**3*x - 32*c*d**3)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^3}{(d + c\,d\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3,x)
```

```
[Out] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3, x)
```

3.126 $\int \frac{(a+b\textbf{ArcTan}(cx))^3}{(d+icdx)^4}\,dx$

Optimal. Leaf size=360

$$\frac{ib^3}{108cd^4(i-cx)^3}+\frac{19b^3}{576cd^4(i-cx)^2}-\frac{85ib^3}{576cd^4(i-cx)}+\frac{85ib^3\text{ArcTan}(cx)}{576cd^4}-\frac{b^2(a+b\text{ArcTan}(cx))}{18cd^4(i-cx)^3}+\frac{5ib^2(a+b\text{A}}{48cd^4(i}$$

```
[Out] 1/108*I*b^3/c/d^4/(I-c*x)^3+19/576*b^3/c/d^4/(I-c*x)^2-85/576*I*b^3/c/d^4/(
I-c*x)+85/576*I*b^3*arctan(c*x)/c/d^4-1/18*b^2*(a+b*arctan(c*x))/c/d^4/(I-c
*x)^3+5/48*I*b^2*(a+b*arctan(c*x))/c/d^4/(I-c*x)^2+11/48*b^2*(a+b*arctan(c*
x))/c/d^4/(I-c*x)-11/96*b*(a+b*arctan(c*x))^2/c/d^4-1/6*I*b*(a+b*arctan(c*x
))^2/c/d^4/(I-c*x)^3-1/8*b*(a+b*arctan(c*x))^2/c/d^4/(I-c*x)^2+1/8*I*b*(a+b
*arctan(c*x))^2/c/d^4/(I-c*x)-1/24*I*(a+b*arctan(c*x))^3/c/d^4+1/3*I*(a+b*a
rctan(c*x))^3/c/d^4/(1+I*c*x)^3
```

Rubi [A]
time = 0.50, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4974, 4972, 641, 46, 209, 5004}

$$\frac{11b^2(a+b\text{ArcTan}(cx))}{48cd^4(-cx+i)}+\frac{5ib^2(a+b\text{ArcTan}(cx))}{48cd^4(-cx+i)^2}-\frac{b^2(a+b\text{ArcTan}(cx))}{18cd^4(-cx+i)^3}+\frac{ib(a+b\text{ArcTan}(cx))^2}{8cd^4(-cx+i)}-\frac{b(a+b\text{ArcTan}(cx))^2}{8cd^4(-cx+i)^2}-\frac{ib(a+b\text{ArcTan}(cx))^2}{6cd^4(-cx+i)^3}-\frac{11b(a+b\text{ArcTan}(cx))^2}{96cd^4}+\frac{i(a+b\text{ArcTan}(cx))^3}{3cd^4(1+icx)^3}-\frac{i(a+b\text{ArcTan}(cx))^3}{24cd^4}+\frac{85ib^3\text{ArcTan}(cx)}{576cd^4}-\frac{85ib^3}{576cd^4(-cx+i)}+\frac{19b^3}{576cd^4(-cx+i)^2}+\frac{ib^3}{108cd^4(-cx+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

```
[Out] ((I/108)*b^3)/(c*d^4*(I - c*x)^3) + (19*b^3)/(576*c*d^4*(I - c*x)^2) - (((8
5*I)/576)*b^3)/(c*d^4*(I - c*x)) + (((85*I)/576)*b^3*ArcTan[c*x])/(c*d^4) -
 (b^2*(a + b*ArcTan[c*x]))/(18*c*d^4*(I - c*x)^3) + (((5*I)/48)*b^2*(a + b*
ArcTan[c*x]))/(c*d^4*(I - c*x)^2) + (11*b^2*(a + b*ArcTan[c*x]))/(48*c*d^4*
(I - c*x)) - (11*b*(a + b*ArcTan[c*x])^2)/(96*c*d^4) - ((I/6)*b*(a + b*ArcT
an[c*x])^2)/(c*d^4*(I - c*x)^3) - (b*(a + b*ArcTan[c*x])^2)/(8*c*d^4*(I - c
*x)^2) + ((I/8)*b*(a + b*ArcTan[c*x])^2)/(c*d^4*(I - c*x)) - ((I/24)*(a + b
*ArcTan[c*x])^3)/(c*d^4) + ((I/3)*(a + b*ArcTan[c*x])^3)/(c*d^4*(1 + I*c*x)
^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{(d+icdx)^4}\,dx = \frac{i(a+b\tan^{-1}(cx))^3}{3cd^4(1+icx)^3} - \frac{(ib)\int \left(\frac{(a+b\tan^{-1}(cx))^2}{2d^3(-i+cx)^4} + \frac{i(a+b\tan^{-1}(cx))^2}{4d^3(-i+cx)^3} - \frac{(a+b\tan^{-1}(cx))^2}{8d^3(-i+cx)^2} + \right.}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3}{3cd^4(1+icx)^3} + \frac{(ib)\int \frac{(a+b\tan^{-1}(cx))^2}{(-i+cx)^2}\,dx}{8d^4} - \frac{(ib)\int \frac{(a+b\tan^{-1}(cx))^2}{1+c^2x^2}\,dx}{8d^4} - \frac{(ib)\int}{}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))^2}{6cd^4(i-cx)^3} - \frac{b(a+b\tan^{-1}(cx))^2}{8cd^4(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))^2}{8cd^4(i-cx)} - \frac{i(a+b\,\mathrm{t}}{2}$$

$$= -\frac{ib(a+b\tan^{-1}(cx))^2}{6cd^4(i-cx)^3} - \frac{b(a+b\tan^{-1}(cx))^2}{8cd^4(i-cx)^2} + \frac{ib(a+b\tan^{-1}(cx))^2}{8cd^4(i-cx)} - \frac{i(a+b\,\mathrm{t}}{2}$$

$$= -\frac{b^2(a+b\tan^{-1}(cx))}{18cd^4(i-cx)^3} + \frac{5ib^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)^2} + \frac{11b^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)} - \frac{11b(a}{}$$

$$= -\frac{b^2(a+b\tan^{-1}(cx))}{18cd^4(i-cx)^3} + \frac{5ib^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)^2} + \frac{11b^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)} - \frac{11b(a}{}$$

$$= -\frac{b^2(a+b\tan^{-1}(cx))}{18cd^4(i-cx)^3} + \frac{5ib^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)^2} + \frac{11b^2(a+b\tan^{-1}(cx))}{48cd^4(i-cx)} - \frac{11b(a}{}$$

$$= \frac{ib^3}{108cd^4(i-cx)^3} + \frac{19b^3}{576cd^4(i-cx)^2} - \frac{85ib^3}{576cd^4(i-cx)} - \frac{b^2(a+b\tan^{-1}(cx))}{18cd^4(i-cx)^3} + \frac{5}{}$$

$$= \frac{ib^3}{108cd^4(i-cx)^3} + \frac{19b^3}{576cd^4(i-cx)^2} - \frac{85ib^3}{576cd^4(i-cx)} + \frac{85ib^3\tan^{-1}(cx)}{576cd^4} - \frac{b^2(a}{18}$$

Mathematica [A]
time = 0.14, size = 269, normalized size = 0.75

$$\frac{-576a^3+12ab^2(56+81icx-33c^2x^2)+b^3(-328i+567cx+255ic^2x^2)-72ia^2b(-10-9icx+3c^2x^2)+3b(i+cx)(12ab(29+32icx-11c^2x^2)+b^2(-139i+208cx+85ic^2x^2)-72ia^2(-7-4icx+c^2x^2))\mathrm{ArcTan}(cx)-18ib^2(i+cx)(b(29i-32cx-11ic^2x^2)+12a(-7-4icx+c^2x^2))\mathrm{ArcTan}(cx)^2-72ib^3(-7i-3cx-3ic^2x^2+c^3x^3)\mathrm{ArcTan}(cx)^3}{1728cd^4(-i+cx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

```
[Out] (-576*a^3 + 12*a*b^2*(56 + (81*I)*c*x - 33*c^2*x^2) + b^3*(-328*I + 567*c*x
 + (255*I)*c^2*x^2) - (72*I)*a^2*b*(-10 - (9*I)*c*x + 3*c^2*x^2) + 3*b*(I +
 c*x)*(12*a*b*(29 + (32*I)*c*x - 11*c^2*x^2) + b^2*(-139*I + 208*c*x + (85*
I)*c^2*x^2) - (72*I)*a^2*(-7 - (4*I)*c*x + c^2*x^2))*ArcTan[c*x] - (18*I)*b
^2*(I + c*x)*(b*(29*I - 32*c*x - (11*I)*c^2*x^2) + 12*a*(-7 - (4*I)*c*x + c
^2*x^2))*ArcTan[c*x]^2 - (72*I)*b^3*(-7*I - 3*c*x - (3*I)*c^2*x^2 + c^3*x^3
)*ArcTan[c*x]^3)/(1728*c*d^4*(-I + c*x)^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(316) = 632.
time = 1.07, size = 819, normalized size = 2.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(I*a*b^2/d^4/(1+I*c*x)^3*arctan(c*x)^2+139/576*b^3/d^4/(c*x-I)^3*arctan
(c*x)-41/216*I*b^3/d^4/(c*x-I)^3-1/8*a^2*b/d^4/(c*x-I)^2-1/24*I*b^3/d^4/(c*
x-I)^3*arctan(c*x)^3*c^3*x^3+1/8*I*b^3/d^4/(c*x-I)^3*arctan(c*x)^3*c*x+7/32
*I*b^3/d^4/(c*x-I)^3*arctan(c*x)^2*c^2*x^2+85/576*I*b^3/d^4/(c*x-I)^3*arcta
n(c*x)*c^3*x^3+23/192*I*b^3/d^4/(c*x-I)^3*arctan(c*x)*c*x-1/4*I*a*b^2/d^4*a
rctan(c*x)/(c*x-I)+1/16*I*a*b^2/d^4*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/16*I*a*b
^2/d^4*ln(-1/2*I*(-c*x+I))*ln(c*x+I)-1/16*I*a*b^2/d^4*ln(-1/2*I*(-c*x+I))*l
n(-1/2*I*(c*x+I))+I*a^2*b/d^4/(1+I*c*x)^3*arctan(c*x)-1/8*b^3/d^4/(c*x-I)^3
*arctan(c*x)^3*c^2*x^2-11/96*b^3/d^4/(c*x-I)^3*arctan(c*x)^2*c^3*x^3-1/32*b
^3/d^4/(c*x-I)^3*arctan(c*x)^2*c*x+41/192*b^3/d^4/(c*x-I)^3*c^2*x^2*arctan(
c*x)+85/576*I*b^3/d^4/(c*x-I)^3*c^2*x^2+1/3*I*a*b^2/d^4*arctan(c*x)/(c*x-I)
^3+1/18*a*b^2/d^4/(c*x-I)^3-11/48*a*b^2/d^4/(c*x-I)-11/48*a*b^2/d^4*arctan(
c*x)+1/3*I*a^3/d^4/(1+I*c*x)^3+1/24*b^3/d^4/(c*x-I)^3*arctan(c*x)^3+21/64*b
^3/d^4/(c*x-I)^3*c*x-1/8*a*b^2/d^4*arctan(c*x)*ln(c*x-I)-1/4*a*b^2/d^4*arct
an(c*x)/(c*x-I)^2+1/8*a*b^2/d^4*arctan(c*x)*ln(c*x+I)+5/48*I*a*b^2/d^4/(c*x
-I)^2-1/32*I*a*b^2/d^4*ln(c*x-I)^2-1/32*I*a*b^2/d^4*ln(c*x+I)^2-1/8*I*a^2*b
/d^4*arctan(c*x)+1/6*I*a^2*b/d^4/(c*x-I)^3-1/8*I*a^2*b/d^4/(c*x-I)+1/3*I*b^
3/d^4/(1+I*c*x)^3*arctan(c*x)^3+29/96*I*b^3/d^4/(c*x-I)^3*arctan(c*x)^2)
```

Maxima [A]
time = 0.42, size = 327, normalized size = 0.91

$$\frac{3\,(-72i\,a^2b-132\,ab^2+85i\,b^3)c^2x^2+72\,(-i\,b^3c^3x^3-3\,b^3c^2x^2+3i\,b^3cx-7\,b^3)\arctan\left(cx\right)^3-576\,a^3+720i\,a^2b+672\,ab^2-328i\,b^3-81\,(8\,a^2b-12i\,ab^2-7\,b^3)cx+18\,((-12i\,ab^2-11\,b^3)c^3x^3-3\,(12\,ab^2-7i\,b^3)c^2x^2-84\,ab^2+29i\,b^3+3\,(12i\,ab^2-b^3)cx)\arctan\left(cx\right)^2+3\,((-72i\,a^2b-132\,ab^2+85i\,b^3)c^3x^3-3\,(72\,a^2b-84i\,ab^2-41\,b^3)c^2x^2-504\,a^2b+348i\,ab^2+139\,b^3+3\,(72i\,a^2b-12\,ab^2+23i\,b^3)cx)\arctan\left(cx\right)}{1728\,(c^4d^4x^3-3i\,c^3d^4x^2-3\,c^2d^4x+i\,cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="maxima")
```

```
[Out] 1/1728*(3*(-72*I*a^2*b - 132*a*b^2 + 85*I*b^3)*c^2*x^2 + 72*(-I*b^3*c^3*x^3
 - 3*b^3*c^2*x^2 + 3*I*b^3*c*x - 7*b^3)*arctan(c*x)^3 - 576*a^3 + 720*I*a^2
*b + 672*a*b^2 - 328*I*b^3 - 81*(8*a^2*b - 12*I*a*b^2 - 7*b^3)*c*x + 18*((-
12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c^2*x^2 - 84*a*b^2 +
29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*arctan(c*x)^2 + 3*((-72*I*a^2*b - 132*
a*b^2 + 85*I*b^3)*c^3*x^3 - 3*(72*a^2*b - 84*I*a*b^2 - 41*b^3)*c^2*x^2 - 50
4*a^2*b + 348*I*a*b^2 + 139*b^3 + 3*(72*I*a^2*b - 12*a*b^2 + 23*I*b^3)*c*x)
*arctan(c*x))/(c^4*d^4*x^3 - 3*I*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^4)
```

Fricas [A]
time = 0.90, size = 359, normalized size = 1.00

$$-\frac{6\,(72i\,a^2b+132\,ab^2-85i\,b^3)c^2x^2+18\,(b^3c^3x^3-3i\,b^3c^2x^2-3\,b^3cx-7i\,b^3)\log\left(-\frac{cx+i}{cx-i}\right)^3+1152\,a^3-1440i\,a^2b-1344\,ab^2+656i\,b^3+162\,(8\,a^2b-12i\,ab^2-7\,b^3)cx+9\,((-12i\,ab^2-11\,b^3)c^3x^3-3\,(12\,ab^2-7i\,b^3)c^2x^2-84\,ab^2+29i\,b^3+3\,(12i\,ab^2-b^3)cx)\log\left(-\frac{cx+i}{cx-i}\right)^2-3\,((72\,a^2b-132i\,ab^2-85\,b^3)c^3x^3-3\,(72i\,a^2b+84\,ab^2-41i\,b^3)c^2x^2-504i\,a^2b-348\,ab^2+139i\,b^3-3\,(72\,a^2b+12i\,ab^2+23\,b^3)cx)\log\left(-\frac{cx+i}{cx-i}\right)}{3456\,(c^4d^4x^3-3i\,c^3d^4x^2-3\,c^2d^4x+i\,cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="fricas")
```

```
[Out] -1/3456*(6*(72*I*a^2*b + 132*a*b^2 - 85*I*b^3)*c^2*x^2 + 18*(b^3*c^3*x^3 -
3*I*b^3*c^2*x^2 - 3*b^3*c*x - 7*I*b^3)*log(-(c*x + I)/(c*x - I))^3 + 1152*a
^3 - 1440*I*a^2*b - 1344*a*b^2 + 656*I*b^3 + 162*(8*a^2*b - 12*I*a*b^2 - 7*
b^3)*c*x + 9*((-12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c^2*x
^2 - 84*a*b^2 + 29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*log(-(c*x + I)/(c*x -
I))^2 - 3*((72*a^2*b - 132*I*a*b^2 - 85*b^3)*c^3*x^3 - 3*(72*I*a^2*b + 84*a
*b^2 - 41*I*b^3)*c^2*x^2 - 504*I*a^2*b - 348*a*b^2 + 139*I*b^3 - 3*(72*a^2*
b + 12*I*a*b^2 + 23*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^4*d^4*x^3 - 3*I
*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**4,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4,x)
```

```
[Out] int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4, x)
```

3.127 $\int \frac{x^2(a+b\text{ArcTan}(cx))^3}{d+icdx}\,dx$

Optimal. Leaf size=410

$$-\frac{3b(a+b\text{ArcTan}(cx))^2}{2c^3d}+\frac{3ibx(a+b\text{ArcTan}(cx))^2}{2c^2d}+\frac{i(a+b\text{ArcTan}(cx))^3}{2c^3d}+\frac{x(a+b\text{ArcTan}(cx))^3}{c^2d}-\frac{ix^2(a+b\text{ArcTan}(cx))^3}{\ldots}$$

```
[Out] -3/2*b*(a+b*arctan(c*x))^2/c^3/d+3/2*I*b*x*(a+b*arctan(c*x))^2/c^2/d+1/2*I*
(a+b*arctan(c*x))^3/c^3/d+x*(a+b*arctan(c*x))^3/c^2/d-1/2*I*x^2*(a+b*arctan
(c*x))^3/c/d+3*I*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d+3*b*(a+b*arcta
n(c*x))^2*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c^3/d
-3/2*b^3*polylog(2,1-2/(1+I*c*x))/c^3/d+3*I*b^2*(a+b*arctan(c*x))*polylog(2
,1-2/(1+I*c*x))/c^3/d+3/2*b*(a+b*arctan(c*x))^2*polylog(2,1-2/(1+I*c*x))/c^
3/d+3/2*b^3*polylog(3,1-2/(1+I*c*x))/c^3/d-3/2*I*b^2*(a+b*arctan(c*x))*poly
log(3,1-2/(1+I*c*x))/c^3/d-3/4*b^3*polylog(4,1-2/(1+I*c*x))/c^3/d
```

Rubi [A]
time = 0.62, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {4986, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004, 5114, 6745, 5118}

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^3d}-\frac{3ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{2c^3d}+\frac{3ib^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^3d}+\frac{3b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))^2}{2c^3d}-\frac{3b(a+b\text{ArcTan}(cx))^2}{2c^3d}+\frac{i(a+b\text{ArcTan}(cx))^3}{2c^3d}+\frac{3b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^3d}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{c^3d}+\frac{3ibx(a+b\text{ArcTan}(cx))^2}{2c^2d}+\frac{x(a+b\text{ArcTan}(cx))^3}{c^2d}-\frac{ix^2(a+b\text{ArcTan}(cx))^3}{2cd}-\frac{3b^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^3d}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^3d}-\frac{3b^3\text{Li}_4\left(1-\frac{2}{icx+1}\right)}{4c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] (-3*b*(a + b*ArcTan[c*x])^2)/(2*c^3*d) + (((3*I)/2)*b*x*(a + b*ArcTan[c*x])
^2)/(c^2*d) + ((I/2)*(a + b*ArcTan[c*x])^3)/(c^3*d) + (x*(a + b*ArcTan[c*x]
)^3)/(c^2*d) - ((I/2)*x^2*(a + b*ArcTan[c*x])^3)/(c*d) + ((3*I)*b^2*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x])^2*Log[2
/(1 + I*c*x)])/(c^3*d) - (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c^3*
d) - (3*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + ((3*I)*b^2*(a + b*Ar
cTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d) + (3*b*(a + b*ArcTan[c*x]
)^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*d) + (3*b^3*PolyLog[3, 1 - 2/(1 +
 I*c*x)])/(2*c^3*d) - (((3*I)/2)*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(
1 + I*c*x)])/(c^3*d) - (3*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(4*c^3*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx = \frac{i\int \frac{x(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx}{c} - \frac{i\int x(a+b\tan^{-1}(cx))^3\,dx}{cd}$$

$$= -\frac{ix^2(a+b\tan^{-1}(cx))^3}{2cd} - \frac{\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx}{c^2} + \frac{(3ib)\int \frac{x^2(a+b\tan^{-1}(cx))^2}{1+c^2x^2}\,dx}{2d} +$$

$$= \frac{x(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix^2(a+b\tan^{-1}(cx))^3}{2cd} - \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^3d}$$

$$= \frac{3ibx(a+b\tan^{-1}(cx))^2}{2c^2d} + \frac{i(a+b\tan^{-1}(cx))^3}{2c^3d} + \frac{x(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix^2(a}{}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{3ibx(a+b\tan^{-1}(cx))^2}{2c^2d} + \frac{i(a+b\tan^{-1}(cx))^3}{2c^3d} + \frac{x(a}{}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{3ibx(a+b\tan^{-1}(cx))^2}{2c^2d} + \frac{i(a+b\tan^{-1}(cx))^3}{2c^3d} + \frac{x(a}{}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{3ibx(a+b\tan^{-1}(cx))^2}{2c^2d} + \frac{i(a+b\tan^{-1}(cx))^3}{2c^3d} + \frac{x(a}{}$$

$$= -\frac{3b(a+b\tan^{-1}(cx))^2}{2c^3d} + \frac{3ibx(a+b\tan^{-1}(cx))^2}{2c^2d} + \frac{i(a+b\tan^{-1}(cx))^3}{2c^3d} + \frac{x(a}{}$$

Mathematica [A]
time = 0.63, size = 541, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] ((-1/4*I)*((4*I)*a^3*c*x - 6*a^2*b*c*x + 2*a^3*c^2*x^2 - (4*I)*a^3*ArcTan[c
*x] + 6*a^2*b*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] - 12*a*b^2*c*x*Arc
Tan[c*x] + 6*a^2*b*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*ArcTan[c*x]^2 + 18*a*
b^2*ArcTan[c*x]^2 + (6*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*ArcTan[c*x]^
2 - 6*b^3*c*x*ArcTan[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*a*b^2*A
rcTan[c*x]^3 + 6*b^3*ArcTan[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 + 2*b^3*c^
2*x^2*ArcTan[c*x]^3 - (2*I)*b^3*ArcTan[c*x]^4 + 12*a^2*b*ArcTan[c*x]*Log[1
+ E^((2*I)*ArcTan[c*x])] + (24*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan
[c*x])] - 12*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*a*b^2*ArcT
an[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*ArcTan[c*x]^2*Log[1 +
 E^((2*I)*ArcTan[c*x])] + 4*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*ArcTan[c*x])
] - 2*a^3*Log[1 + c^2*x^2] - (6*I)*a^2*b*Log[1 + c^2*x^2] + 6*a*b^2*Log[1 +
```

```
c^2*x^2] - (6*I)*b*(a + I*b + b*ArcTan[c*x])^2*PolyLog[2, -E^((2*I)*ArcTan
[c*x])] + 6*b^2*(a + I*b + b*ArcTan[c*x])*PolyLog[3, -E^((2*I)*ArcTan[c*x])
] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.62, size = 1586, normalized size = 3.87

method	result	size
derivativedivides	Expression too large to display	1586
default	Expression too large to display	1586

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a^3/d*arctan(c*x)+3*a*b^2/d*arctan(c*x)^2*c*x+6*a*b^2/d*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*a*b^2/d*arctan(c*x)*ln(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-9/2*I*a*b^2/d*arctan(c*x)^2-3/2*a*b^2/d*arctan(c*x)^2*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2
/(c^2*x^2+1))*Pi+3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+3/4*b^3/
d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*b^3/d*arctan(c*x)^4+3*a*b^2/d*Pi*
arctan(c*x)^2-3*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2
*b^3/d*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*a*b^2/d*arctan
(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi-2*a*b^2/d*arcta
n(c*x)^3-3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^3*Pi+3/2*b^3/d*arctan(c*x)^2+3/2*b^3/d*polylog(2,-(1+I*c*
x)^2/(c^2*x^2+1))+3/2*b^3/d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*a^3/d
*ln(c^2*x^2+1)-3/4*a^2*b/d*ln(c*x-I)^2+3/2*a^2*b/d*dilog(-1/2*I*(c*x+I))-9/
8*a^2*b/d*ln(c^2*x^2+1)-3/16*a^2*b/d*ln(c^4*x^4+10*c^2*x^2+9)+3*b^3/d*arcta
n(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*I*b^3/d*arctan(c*x)^3+a^3/d*c*x+
3*a*b^2/d*arctan(c*x)-3/2*I*a*b^2/d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-6*I
*a*b^2/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*a*b^2/d*ln((1+I*c*x)^2/
(c^2*x^2+1)+1)-6*I*a*b^2/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*a^2*b
/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-9/4*I*a^2*b/d*arctan(c*x)-3/8*I*a^2*b/d*arc
tan(1/2*c*x)+3/8*I*a^2*b/d*arctan(1/6*c^3*x^3+7/6*c*x)+3/4*I*a^2*b/d*arctan
(1/2*c*x-1/2*I)-1/2*I*a^3/d*c^2*x^2-1/2*I*b^3/d*arctan(c*x)^3*c^2*x^2+3/2*I
*b^3/d*arctan(c*x)^2*c*x-3*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3*I*a*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*
c*x)^2/(c^2*x^2+1))+3*I*a*b^2/d*arctan(c*x)^2*ln(c*x-I)+3*I*a^2*b/d*arctan(
c*x)*ln(c*x-I)+3/2*a^2*b/d+3*a^2*b/d*arctan(c*x)*c*x+3/2*I*a^2*b/d*c*x+3*I*
a*b^2/d*arctan(c*x)*c*x-3/2*I*a*b^2/d*arctan(c*x)^2*c^2*x^2-3/2*I*a^2*b/d*a
rctan(c*x)*c^2*x^2-I*b^3/d*arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3/2*
I*b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3*I*b^3/d*arctan(c*
```

```
x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3*I*b^3/d*arctan(c*x)*polylog(2,-(1+I*c*x)
^2/(c^2*x^2+1))+b^3/d*arctan(c*x)^3*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/128*(16
*b^3*arctan(c*x)^4 - b^3*log(c^2*x^2 + 1)^4 + 4*(384*b^3*c^3*integrate(1/64
*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*b^3*c^3*in
tegrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^4*d*x^2 + c^2*d), x) + 384*b^3*c^3*i
ntegrate(1/64*x^3*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 96*b^3*c^3*integr
ate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 1792*b^3*c^2*inte
grate(1/64*x^2*arctan(c*x)^3/(c^4*d*x^2 + c^2*d), x) - 192*b^3*c^2*integrat
e(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 6144*a*
b^2*c^2*integrate(1/64*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 384*b^3*
c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x)
 - 6144*a^2*b*c^2*integrate(1/64*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) +
384*b^3*c*integrate(1/64*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*
d), x) + 96*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^3/(c^4*d*x^2 + c^2*d),
x) + 768*b^3*c*integrate(1/64*x*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 192
*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 192*b^
3*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 3
*b^3*arctan(c*x)^4/(c^3*d))*c^3*d - 128*I*c^3*d*integrate(-1/64*(192*a^2*b*
c^3*x^3*arctan(c*x) + 8*(7*b^3*c^3*x^3 - 3*b^3*c*x)*arctan(c*x)^3 - (b^3*c^
2*x^2 + 3*b^3)*log(c^2*x^2 + 1)^3 + 12*(16*a*b^2*c^3*x^3 + b^3*c^2*x^2)*arc
tan(c*x)^2 - 3*(b^3*c^2*x^2 - 2*(b^3*c^3*x^3 - b^3*c*x)*arctan(c*x))*log(c^
2*x^2 + 1)^2 + 12*((b^3*c^2*x^2 - b^3)*arctan(c*x)^2 + (b^3*c^3*x^3 + 2*b^3
*c*x)*arctan(c*x))*log(c^2*x^2 + 1))/(c^4*d*x^2 + c^2*d), x) + 8*(I*b^3*c^2
*x^2 - 2*b^3*c*x)*arctan(c*x)^3 + 6*(-I*b^3*c^2*x^2 + 2*b^3*c*x)*arctan(c*x
)*log(c^2*x^2 + 1)^2 + (b^3*c^2*x^2 + 2*I*b^3*c*x + 4*I*b^3*arctan(c*x))*lo
g(c^2*x^2 + 1)^3 + 4*(4*I*b^3*arctan(c*x)^3 - 3*(b^3*c^2*x^2 + 2*I*b^3*c*x)
*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^3*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*x^2*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x^2*log(-(c*
x + I)/(c*x - I))^2 - 12*a^2*b*x^2*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x^2)
/(c*d*x - I*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

```
[Out] int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

3.128 $\int \frac{x(a+b\text{ArcTan}(cx))^3}{d+icdx} dx$

Optimal. Leaf size=277

$$\frac{(a+b\text{ArcTan}(cx))^3}{c^2d}-\frac{ix(a+b\text{ArcTan}(cx))^3}{cd}-\frac{3ib(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d}-\frac{(a+b\text{ArcTan}(cx))^3\log(}{c^2d}$$

```
[Out] (a+b*arctan(c*x))^3/c^2/d-I*x*(a+b*arctan(c*x))^3/c/d-3*I*b*(a+b*arctan(c*x
))^2*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c^2/d+3*b^2*
(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b*(a+b*arctan(c*x))^
2*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b^3*polylog(3,1-2/(1+I*c*x))/c^2/d-3
/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))/c^2/d+3/4*I*b^3*polylog(4
,1-2/(1+I*c*x))/c^2/d
```

Rubi [A]
time = 0.35, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4986, 4930, 5040, 4964, 5004, 5114, 6745, 5118}

$$\frac{3b^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^2d}-\frac{3b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{2c^2d}-\frac{3ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))^2}{2c^2d}+\frac{(a+b\text{ArcTan}(cx))^3}{c^2d}-\frac{3ib\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^2d}-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{c^2d}-\frac{ix(a+b\text{ArcTan}(cx))^3}{cd}-\frac{3ib^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^2d}+\frac{3ib^3\text{Li}_4\left(1-\frac{2}{icx+1}\right)}{4c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] (a + b*ArcTan[c*x])^3/(c^2*d) - (I*x*(a + b*ArcTan[c*x])^3)/(c*d) - ((3*I)*
b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^2*d) - ((a + b*ArcTan[c*x])^
3*Log[2/(1 + I*c*x)])/(c^2*d) + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 -
2/(1 + I*c*x)])/(c^2*d) - (((3*I)/2)*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^
2*d) - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d)
+ (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)])/(c^2*d)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4986

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p,
 x], x] - Dist[d*(f/e), Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2
, 0] && GtQ[m, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx &= \frac{i\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx}{c} - \frac{i\int (a+b\tan^{-1}(cx))^3\,dx}{cd} \\
&= -\frac{ix(a+b\tan^{-1}(cx))^3}{cd} - \frac{(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{(3ib)\int \frac{x(a+b\tan^{-1}(cx)}{1+c^2x^2}}{d} \\
&= \frac{(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^3}{cd} - \frac{(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^2d} - \\
&= \frac{(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^3}{cd} - \frac{3ib(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d} \\
&= \frac{(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^3}{cd} - \frac{3ib(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d} \\
&= \frac{(a+b\tan^{-1}(cx))^3}{c^2d} - \frac{ix(a+b\tan^{-1}(cx))^3}{cd} - \frac{3ib(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 393, normalized size = 1.42

$$
-\frac{i\left(4a^3cx - 4a^3\mathrm{ArcTan}(cx) + 12a^2bcx\mathrm{ArcTan}(cx) - 12a^2b\mathrm{ArcTan}(cx)^2 - 12iab^2\mathrm{ArcTan}(cx)^2 + 12ab^2cx\mathrm{ArcTan}(cx)^2 - 8ab^2\mathrm{ArcTan}(cx)^3 - 4ib^3\mathrm{ArcTan}(cx)^3 + 4b^3cx\mathrm{ArcTan}(cx)^3 - 2b^3\mathrm{ArcTan}(cx)^4 - 12ia^2b\mathrm{ArcTan}(cx)\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) + 24ab^2\mathrm{ArcTan}(cx)\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) - 12iab^2\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) + 12b^3\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) - 4ib^3\mathrm{ArcTan}(cx)^3\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right) + 2ia^3\log\left(1+c^2x^2\right) - 6a^2b\log\left(1+c^2x^2\right) - 6b\left(a(a+2ib) + 2(a+ib)b\mathrm{ArcTan}(cx) + b^2\mathrm{ArcTan}(cx)^2\right)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right) + 6b^2(-ia+b-ib\mathrm{ArcTan}(cx))\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right) + 3b^3\mathrm{PolyLog}\left(4,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{4c^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

```
[Out] ((-1/4*I)*(4*a^3*c*x - 4*a^3*ArcTan[c*x] + 12*a^2*b*c*x*ArcTan[c*x] - 12*a^
2*b*ArcTan[c*x]^2 - (12*I)*a*b^2*ArcTan[c*x]^2 + 12*a*b^2*c*x*ArcTan[c*x]^2
 - 8*a*b^2*ArcTan[c*x]^3 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3*c*x*ArcTan[c*x]^
3 - 2*b^3*ArcTan[c*x]^4 - (12*I)*a^2*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[
c*x])] + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*a*b^2
*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b^3*ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] - (4*I)*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*ArcTan[
c*x])] + (2*I)*a^3*Log[1 + c^2*x^2] - 6*a^2*b*Log[1 + c^2*x^2] - 6*b*(a*(a
+ (2*I)*b) + 2*(a + I*b)*b*ArcTan[c*x] + b^2*ArcTan[c*x]^2)*PolyLog[2, -E^(
(2*I)*ArcTan[c*x])] + 6*b^2*((-I)*a + b - I*b*ArcTan[c*x])*PolyLog[3, -E^((
2*I)*ArcTan[c*x])] + 3*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.57, size = 5234, normalized size = 18.90

method	result	size

derivativedivides	Expression too large to display	5234
default	Expression too large to display	5234

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a^3*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/128*(-16*I*b^3*c*x*arctan(c*x
)^3 + 12*I*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 16*I*b^3*arctan(c*x)^4
- I*b^3*log(c^2*x^2 + 1)^4 - 4*I*(896*b^3*c^2*integrate(1/32*x^2*arctan(c*x
)^3/(c^3*d*x^2 + c*d), x) + 96*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c
^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 3072*a*b^2*c^2*integrate(1/32*x^2*arc
tan(c*x)^2/(c^3*d*x^2 + c*d), x) + 384*b^3*c^2*integrate(1/32*x^2*arctan(c*
x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 3072*a^2*b*c^2*integrate(1/32*x
^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) - 64*b^3*c*integrate(1/32*x*log(c^2*x^
2 + 1)^3/(c^3*d*x^2 + c*d), x) - 384*b^3*c*integrate(1/32*x*arctan(c*x)^2/(
c^3*d*x^2 + c*d), x) + 96*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^3*d*
x^2 + c*d), x) + 3*b^3*arctan(c*x)^4/(c^2*d) + 96*b^3*integrate(1/32*arctan
(c*x)*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x))*c^2*d + 128*c^2*d*integrate
(1/64*(80*b^3*c*x*arctan(c*x)^3 + 192*a^2*b*c*x*arctan(c*x) + (b^3*c^2*x^2
+ 3*b^3)*log(c^2*x^2 + 1)^3 - 24*(b^3*c^2*x^2 - 8*a*b^2*c*x)*arctan(c*x)^2
+ 6*(b^3*c^2*x^2 + 2*b^3*c*x*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*(2*b^3*c*
x*arctan(c*x) + (b^3*c^2*x^2 - b^3)*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^3*d
*x^2 + c*d), x) - 2*(b^3*c*x + 2*b^3*arctan(c*x))*log(c^2*x^2 + 1)^3 + 8*(3
*b^3*c*x*arctan(c*x)^2 - 2*b^3*arctan(c*x)^3)*log(c^2*x^2 + 1))/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*x*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x*log(-(c*x +
I)/(c*x - I))^2 - 12*a^2*b*x*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x)/(c*d*x
- I*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

```
[Out] int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

3.129 $\int \frac{(a+b\textbf{ArcTan}(cx))^3}{d+icdx} dx$

Optimal. Leaf size=139

$$\frac{i(a+b\text{ArcTan}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{3b(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}+\frac{3ib^2(a+b\text{ArcTan}(cx))\text{PolyI}}{2cd}$$

```
[Out] I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylog
(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))/
c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

Rubi [A]
time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4964, 5004, 5114, 5118, 6745}

$$\frac{3ib^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{2cd}-\frac{3b\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))^2}{2cd}+\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{cd}+\frac{3b^3\text{Li}_4\left(1-\frac{2}{icx+1}\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - (3*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c*d) + (((3*I)/2)*b^2*(a + b*ArcTan[
c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c*d) + (3*b^3*PolyLog[4, 1 - 2/(1 + I
*c*x)])/(4*c*d)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{d+icdx}\,dx = \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{(3ib)\int \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{(3b^2)\int$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a-$$

$$= \frac{i(a+b\tan^{-1}(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\tan^{-1}(cx))^2\operatorname{Li}_2\left(1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a-$$

Mathematica [A]
time = 0.07, size = 133, normalized size = 0.96

$$\frac{i\left(4(a+b\text{ArcTan}(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)+b\text{PolyLog}\left(4,\frac{i+cx}{-i+cx}\right)\right)\right)\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

```
[Out] ((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTan[
c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*x)]
))))/(c*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.00, size = 1940, normalized size = 13.96

method	result	size
derivativedivides	Expression too large to display	1940
default	Expression too large to display	1940

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^3/d*arctan(c*x)+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi+3/2*a*b^2/
d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
*csgn((1+I*c*x)^2/(c^2*x^2+1))*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1))*Pi+3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*Pi+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*Pi-3/4*b^3/d*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^3/d*arctan(c*x)^4
-1/2*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*Pi-I*b^3/d*ln(1+I*c*x)*arctan(c*x)^3+3/2*I*b^3/d*arctan(c*x)*po
lylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a*b^2/d*Pi*arctan(c*x)^2+3*a*b^2/d*ar
ctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a^2*b/d*ln(1/2-1/2*I*c*x)
*ln(1/2+1/2*I*c*x)-3/2*a^2*b/d*ln(1/2-1/2*I*c*x)*ln(1+I*c*x)+3/2*I*a*b^2/d*
polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+I*b^3/d*arctan(c*x)^3*ln(2*I*(1+I*c*x)^
2/(c^2*x^2+1))-1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^3*Pi+1/2*b^3/d*arctan(c*x)^3*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi+1/2*b^3/d*Pi*arctan(c*x)^3+3/2*b^3
/d*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*a*b^2/d*arctan(c*x
)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+3/2*a^2*b/d*dilog(1
/2+1/2*I*c*x)+3/4*a^2*b/d*ln(1+I*c*x)^2+2*a*b^2/d*arctan(c*x)^3-1/2*b^3/d*a
rctan(c*x)^3*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi+1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)
^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-1/2*b^3/d*arctan(c*x)^3*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*Pi-3*I*a^2*b/d*ln(1+I*c*x)*arctan(c*x)-3/2*a*b^2/
d*arctan(c*x)^2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3
*Pi+3/2*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^3*Pi-3/2*a*b^2/d*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi-3*I*a*b^2/d*ln(1+I*c*x)*arctan(c*x)^2+3
*I*a*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*a^3/d*ln(c^2
```

```
*x^2+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arct
an(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3*l
og(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16*x*
log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) + 12*a
^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*arctan
(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arctan(c*
x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 + 1) + b
^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/(
c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

```
[Out] int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

3.130 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{x(d+icdx)}\,dx$

Optimal. Leaf size=128

$$\frac{(a+b\text{ArcTan}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{3ib(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}+\frac{3b^2(a+b\text{ArcTan}(cx)}{}$$

```
[Out] (a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d+3/2*I*b*(a+b*arctan(c*x))^2*polylog
(2,-1+2/(1+I*c*x))/d+3/2*b^2*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/d-
3/4*I*b^3*polylog(4,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.17, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4988, 5004, 5114, 5118, 6745}

$$\frac{3b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{2d}+\frac{3ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))^2}{2d}+\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{d}-\frac{3ib^3\text{Li}_4\left(\frac{2}{icx+1}-1\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

```
[Out] ((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d + (((3*I)/2)*b*(a + b*ArcT
an[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (3*b^2*(a + b*ArcTan[c*x])*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1
+ I*c*x)])/d
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{x(d+icdx)}\,dx = \frac{(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{(3bc)\int \frac{(a+b\tan^{-1}(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$
$$= \frac{(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib(a+b\tan^{-1}(cx))^2\,\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} - (3i$$
$$= \frac{(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib(a+b\tan^{-1}(cx))^2\,\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + 3b^2$$
$$= \frac{(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib(a+b\tan^{-1}(cx))^2\,\mathrm{Li}_2\left(-1+\frac{2}{1+icx}\right)}{2d} + 3b^2$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. 2(128) = 256.
time = 0.24, size = 268, normalized size = 2.09

$$-\frac{i\left(8ab^2\pi^3+b^3\pi^4+64a^3\mathrm{ArcTan}(cx)+192a^2b\mathrm{ArcTan}(cx)^2+192iab^2\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)+64ib^3\mathrm{ArcTan}(cx)^3\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)+192ia^2b\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+64ia^3\log(cx)-32ia^3\log\left(1+c^2x^2\right)-96b^2\mathrm{ArcTan}(cx)(2a+b\mathrm{ArcTan}(cx))\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)+96a^2b\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)+96iab^2\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)+96ib^3\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)+48b^3\mathrm{PolyLog}\left(4,e^{-2i\mathrm{ArcTan}(cx)}\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

```
[Out] ((-1/64*I)*(8*a*b^2*Pi^3 + b^3*Pi^4 + 64*a^3*ArcTan[c*x] + 192*a^2*b*ArcTan
[c*x]^2 + (192*I)*a*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (64
*I)*b^3*ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*a^2*b*ArcTa
n[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (64*I)*a^3*Log[c*x] - (32*I)*a^3*Lo
g[1 + c^2*x^2] - 96*b^2*ArcTan[c*x]*(2*a + b*ArcTan[c*x])*PolyLog[2, E^((-2
```

```
*I)*ArcTan[c*x])] + 96*a^2*b*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (96*I)*a*b
^2*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (96*I)*b^3*ArcTan[c*x]*PolyLog[3, E
^((-2*I)*ArcTan[c*x])] + 48*b^3*PolyLog[4, E^((-2*I)*ArcTan[c*x])]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 3393, normalized size = 26.51

method	result	size
derivativedivides	Expression too large to display	3393
default	Expression too large to display	3393

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3/d*ln(c^2*x^2+1)+1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*a*b^2/d
*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+a^3/d*ln
(c*x)-1/2*I*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3+1/2*I*b^3/d*Pi*arctan(c*x
)^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))-1/2*I*b^3/d*Pi*arcta
n(c*x)^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^3-1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))-1/2*I*b^
3/d*Pi*arctan(c*x)^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-I
*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arcta
n(c*x)^3-1/2*I*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))^3*arctan(c*x)^3-1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn(((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-6*I*a*b^2/d*arctan(c*x)*polylog(2,
(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))+9/2*I*a*b^2/d*Pi*arctan(c*x)^2+3/2*I*a^2*b/d*ln(c*x)*ln(1
+I*c*x)-3/2*I*a^2*b/d*ln(c*x)*ln(1-I*c*x)+3/2*I*a^2*b/d*ln(-1/2*I*(c*x+I))*
ln(c*x-I)+1/2*I*b^3/d*Pi*arctan(c*x)^3*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^3-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2
*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan
(c*x)^2+3/2*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*a*b^2/d*P
```

```
i*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3*I*a*b^2/d*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*a*b^2/
d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan
(c*x)^2-3/2*I*a*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3/2*I*b^3/d*Pi*arctan(c*x
)^3-3*I*b^3/d*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3
/d*arctan(c*x)^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2/d*arctan(c*
x)^2*ln(c*x)-3*a*b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+3*a*b^2/
d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2/d*arctan(c*x)^2*l
n(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*a*b^2/d*arctan(c*x)^3+3*a^2*b/d*arctan
(c*x)*ln(c*x)+3/2*I*a^2*b/d*dilog(1+I*c*x)-3/2*I*a^2*b/d*dilog(1-I*c*x)-3/4
*I*a^2*b/d*ln(c*x-I)^2+3/2*I*a^2*b/d*dilog(-1/2*I*(c*x+I))+1/2*I*b^3/d*Pi*a
rctan(c*x)^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))-1
/2*I*b^3/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^
3-I*a^3/d*arctan(c*x)+6*a*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*a*
b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*b^3/d*polylog(4,(1+I*c*x)
/(c^2*x^2+1)^(1/2))+6*I*b^3/d*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I
*b^3/d*arctan(c*x)^4+b^3/d*arctan(c*x)^3*ln(c*x)+b^3/d*arctan(c*x)^3*ln(1+(
1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3/d*arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^
(1/2))+6*b^3/d*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3/d*a
rctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^3/d*arctan(c*x)^3*ln((
1+I*c*x)^2/(c^2*x^2+1)-1)-b^3/d*ln(c*x-I)*arctan(c*x)^3+b^3/d*arctan(c*x)^3
*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-3*a^2*b/d*ln(c*x-I)*arctan(c*x)+3*a*b^2/d*
arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-3*a*b^2/d*ln(c*x-I)*arctan(c*
x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] -a^3*(log(I*c*x + 1)/d - log(x)/d) + 1/512*(-64*I*b^3*arctan(c*x)^4 + 64*b^
3*arctan(c*x)^3*log(c^2*x^2 + 1) + 16*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 +
```

```
4*I*b^3*log(c^2*x^2 + 1)^4 - I*(64*b^3*arctan(c*x)^4/d + 6144*b^3*c^2*integ
rate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x) + 3*b^3*
log(c^2*x^2 + 1)^4/d + 512*a*b^2*arctan(c*x)^3/d + 768*a^2*b*arctan(c*x)^2/
d + 6144*b^3*integrate(1/64*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x
), x) - 512*b^3*integrate(1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^3 + d*x), x))*d
- 512*d*integrate(1/32*(12*b^3*c*x*arctan(c*x)^2*log(c^2*x^2 + 1) + b^3*c*x
*log(c^2*x^2 + 1)^3 - 96*a*b^2*arctan(c*x)^2 - 96*a^2*b*arctan(c*x) + 4*(3*
b^3*c^2*x^2 - 7*b^3)*arctan(c*x)^3 + 3*(b^3*c^2*x^2 - b^3)*arctan(c*x)*log(
c^2*x^2 + 1)^2)/(c^2*d*x^3 + d*x), x))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(109) = 218$.
time = 0.88, size = 246, normalized size = 1.92

$$\frac{-3i\,b^3\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,ab^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right) - 12i\,a^2b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right) + 8\,a^3\log\left(x\right) - 8\,a^3\log\left(\frac{cx-i}{c}\right) - 6i\,b^3\mathrm{polylog}\left(4,-\frac{cx+i}{cx-i}\right) + \left(-i\,b^3\log\left(-\frac{cx+i}{cx-i}\right)^3 - 6\,ab^2\log\left(-\frac{cx+i}{cx-i}\right)^2\right)\log\left(\frac{2\,cx}{cx-i}\right) - 6\left(-i\,b^3\log\left(-\frac{cx+i}{cx-i}\right) - 2\,ab^2\right)\mathrm{polylog}\left(3,-\frac{cx+i}{cx-i}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*b^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 12*
a*b^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 12*I*a^2*b*di
log((c*x + I)/(c*x - I) + 1) + 8*a^3*log(x) - 8*a^3*log((c*x - I)/c) - 6*I*
b^3*polylog(4, -(c*x + I)/(c*x - I)) + (-I*b^3*log(-(c*x + I)/(c*x - I))^3
- 6*a*b^2*log(-(c*x + I)/(c*x - I))^2)*log(2*c*x/(c*x - I)) - 6*(-I*b^3*log
(-(c*x + I)/(c*x - I)) - 2*a*b^2)*polylog(3, -(c*x + I)/(c*x - I)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a^3}{cx^2-ix}\,dx + \int \frac{b^3\operatorname{atan}^3(cx)}{cx^2-ix}\,dx + \int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^2-ix}\,dx + \int \frac{3a^2b\operatorname{atan}(cx)}{cx^2-ix}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**3/(c*x**2 - I*x), x) + Integral(b**3*atan(c*x)**3/(c*x**2 -
 I*x), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**2 - I*x), x) + Integral(3*
a**2*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)),x)
[Out] int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)), x)
```

3.131 $\int \frac{(a+b\text{ArcTan}(cx))^3}{x^2(d+icdx)} dx$

Optimal. Leaf size=263

$$-\frac{ic(a+b\text{ArcTan}(cx))^3}{d}-\frac{(a+b\text{ArcTan}(cx))^3}{dx}+\frac{3bc(a+b\text{ArcTan}(cx))^2\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic(a+b\text{ArcTan}(cx$$

```
[Out] -I*c*(a+b*arctan(c*x))^3/d-(a+b*arctan(c*x))^3/d/x+3*b*c*(a+b*arctan(c*x))^
2*ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d-3*I*b^2*c
*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))/d+3/2*b*c*(a+b*arctan(c*x))^2*
polylog(2,-1+2/(1+I*c*x))/d+3/2*b^3*c*polylog(3,-1+2/(1-I*c*x))/d-3/2*I*b^2
*c*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/d-3/4*b^3*c*polylog(4,-1+2/(
1+I*c*x))/d
```

Rubi [A]
time = 0.43, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4990, 4946, 5044, 4988, 5004, 5112, 6745, 5114, 5118}

$$-\frac{3ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)(a+b\text{ArcTan}(cx))}{d}-\frac{3ib^2c\text{Li}_3\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{2d}+\frac{3bc\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))^2}{2d}-\frac{ic(a+b\text{ArcTan}(cx))^3}{d}-\frac{(a+b\text{ArcTan}(cx))^3}{dx}+\frac{3bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{ic\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{d}+\frac{3b^3c\text{Li}_3\left(\frac{2}{1-icx}-1\right)}{2d}-\frac{3b^3c\text{Li}_4\left(\frac{2}{icx+1}-1\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^3)/d - (a + b*ArcTan[c*x])^3/(d*x) + (3*b*c*(a
+ b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)])/d - (I*c*(a + b*ArcTan[c*x])^3*L
og[2 - 2/(1 + I*c*x)])/d - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 +
 2/(1 - I*c*x)])/d + (3*b*c*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*
c*x)])/(2*d) + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/(2*d) - (((3*I)/2)*
b^2*c*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/d - (3*b^3*c*Poly
Log[4, -1 + 2/(1 + I*c*x)])/(4*d)
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^3}{x^2(d+icdx)}\,dx = -\left((ic)\int \frac{(a+b\tan^{-1}(cx))^3}{x(d+icdx)}\,dx\right) + \frac{\int \frac{(a+b\tan^{-1}(cx))^3}{x^2}\,dx}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^3}{dx} - \frac{ic(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{(3bc)\int \frac{(a+b\tan^{-1}(c}{x(1+c^2x^2}}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^3}{d} - \frac{(a+b\tan^{-1}(cx))^3}{dx} - \frac{ic(a+b\tan^{-1}(cx))^3\log\left(2-\frac{2}{1+ic}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^3}{d} - \frac{(a+b\tan^{-1}(cx))^3}{dx} + \frac{3bc(a+b\tan^{-1}(cx))^2\log\left(2-\frac{}{1-}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^3}{d} - \frac{(a+b\tan^{-1}(cx))^3}{dx} + \frac{3bc(a+b\tan^{-1}(cx))^2\log\left(2-\frac{}{1-}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^3}{d} - \frac{(a+b\tan^{-1}(cx))^3}{dx} + \frac{3bc(a+b\tan^{-1}(cx))^2\log\left(2-\frac{}{1-}\right.}{d}$$

Mathematica [A]
time = 0.91, size = 436, normalized size = 1.66

$$-\frac{\frac{2a^3}{x}+2a^3c\text{ArcTan}(cx)+2ia^3c\log(x)-ia^3c\log\left(1+c^2x^2\right)+3a^2bc\left(2\left(\text{ArcTan}(cx)^2+\text{ArcTan}(cx)\left(\frac{1}{cx}+i\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)-\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)+6iab^2c\left(-\frac{i\pi^3}{24}+\text{ArcTan}(cx)^2-\frac{i\text{ArcTan}(cx)^2}{cx}+\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)+2i\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)+i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)+\frac{1}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)\right)+2ib^3c\left(\frac{\pi^3}{8}-\frac{i\pi^4}{64}-\text{ArcTan}(cx)^3-\frac{i\text{ArcTan}(cx)^3}{cx}+3i\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)+\text{ArcTan}(cx)^3\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)+\frac{3}{2}i\text{ArcTan}(cx)(2i+\text{ArcTan}(cx))\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+\frac{3}{2}(i+\text{ArcTan}(cx))\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)-\frac{3}{4}i\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(cx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

```
[Out] -1/2*((2*a^3)/x + 2*a^3*c*ArcTan[c*x] + (2*I)*a^3*c*Log[x] - I*a^3*c*Log[1
+ c^2*x^2] + 3*a^2*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1 -
 E^((2*I)*ArcTan[c*x])]) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^((2
*I)*ArcTan[c*x])]) + (6*I)*a*b^2*c*((-1/24*I)*Pi^3 + ArcTan[c*x]^2 - (I*Arc
Tan[c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2*I)*A
rcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2
*I)*ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I
)*ArcTan[c*x])]/2) + (2*I)*b^3*c*(Pi^3/8 - (I/64)*Pi^4 - ArcTan[c*x]^3 - (I
*ArcTan[c*x]^3)/(c*x) + (3*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])]
 + ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + ((3*I)/2)*ArcTan[c*x]*(2
```

```
*I + ArcTan[c*x])*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (3*(I + ArcTan[c*x])
*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2 - ((3*I)/4)*PolyLog[4, E^((-2*I)*Arc
Tan[c*x])]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.61, size = 11097, normalized size = 42.19

method	result	size
derivativedivides	Expression too large to display	11097
default	Expression too large to display	11097

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] a^3*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/512*(64*b^3*c*x*arc
tan(c*x)^4 - 4*b^3*c*x*log(c^2*x^2 + 1)^4 + 64*b^3*arctan(c*x)^3 - 48*b^3*a
rctan(c*x)*log(c^2*x^2 + 1)^2 - 8*(-2*I*b^3*c*x*arctan(c*x) + I*b^3)*log(c^
2*x^2 + 1)^3 - (48*b^3*c*arctan(c*x)^4/d - 6144*b^3*c^3*integrate(1/64*x^3*
arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 3*b^3*c*log(c^2*x^
2 + 1)^4/d + 3072*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2
/(c^2*d*x^4 + d*x^2), x) - 12288*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log
(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 6144*b^3*c*integrate(1/64*x*arctan(
c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 512*b^3*c*integrate(1/64*
x*log(c^2*x^2 + 1)^3/(c^2*d*x^4 + d*x^2), x) + 12288*b^3*c*integrate(1/64*x
*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 3072*b^3*c*integrate(1/64*x*log(c^
2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 28672*b^3*integrate(1/64*arctan(c*x)
^3/(c^2*d*x^4 + d*x^2), x) + 3072*b^3*integrate(1/64*arctan(c*x)*log(c^2*x^
2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 98304*a*b^2*integrate(1/64*arctan(c*x)^2
/(c^2*d*x^4 + d*x^2), x) + 98304*a^2*b*integrate(1/64*arctan(c*x)/(c^2*d*x^
4 + d*x^2), x))*d*x - 64*I*(192*b^3*c^3*integrate(1/64*x^3*arctan(c*x)^3/(c
^2*d*x^4 + d*x^2), x) + 48*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*log(c^2*x
^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + b^3*c*arctan(c*x)^3/d + 96*b^3*c^2*inte
grate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 24*
b^3*c^2*integrate(1/64*x^2*log(c^2*x^2 + 1)^3/(c^2*d*x^4 + d*x^2), x) - 48*
b^3*c^2*integrate(1/64*x^2*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 448
```

```
*b^3*c*integrate(1/64*x*arctan(c*x)^3/(c^2*d*x^4 + d*x^2), x) - 48*b^3*c*in
tegrate(1/64*x*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 153
6*a*b^2*c*integrate(1/64*x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 192*b^3*
c*integrate(1/64*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 1
536*a^2*b*c*integrate(1/64*x*arctan(c*x)/(c^2*d*x^4 + d*x^2), x) - 96*b^3*i
ntegrate(1/64*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 8*b^
3*integrate(1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^4 + d*x^2), x))*d*x - 32*(-2*I
*b^3*c*x*arctan(c*x)^3 - 3*I*b^3*arctan(c*x)^2)*log(c^2*x^2 + 1))/(d*x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] -1/8*(b^3*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 3*b^3*c*x*
dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 6*b^3*c*x*log(-(c
*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I)) - I*b^3*log(-(c*x + I)/
(c*x - I))^3 + 6*b^3*c*x*polylog(4, -(c*x + I)/(c*x - I)) - 8*d*x*integral(
1/4*(-4*I*a^3*c*x + 4*a^3 - 3*(a*b^2 + (-I*a*b^2 + b^3)*c*x)*log(-(c*x + I)
/(c*x - I))^2 + 6*(a^2*b*c*x + I*a^2*b)*log(-(c*x + I)/(c*x - I)))/(c^2*d*x
^4 + d*x^2), x))/(d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a^{3}}{cx^{3}-ix^{2}}\, dx + \int \frac{b^{3}\operatorname{atan}^{3}{\left(cx \right)}}{cx^{3}-ix^{2}}\, dx + \int \frac{3ab^{2}\operatorname{atan}^{2}{\left(cx \right)}}{cx^{3}-ix^{2}}\, dx + \int \frac{3a^{2}b\operatorname{atan}{\left(cx \right)}}{cx^{3}-ix^{2}}\, dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**2/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**3/(c*x**3 - I*x**2), x) + Integral(b**3*atan(c*x)**3/(c*x**
3 - I*x**2), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**3 - I*x**2), x) + In
tegral(3*a**2*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^2\,\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)),x)
[Out] int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)), x)
```

3.132 $\int \frac{(a+b\textbf{ArcTan}(cx))^3}{x^3(d+icdx)}\,dx$

Optimal. Leaf size=414

$$-\frac{3ibc^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{3bc(a+b\text{ArcTan}(cx))^2}{2dx}-\frac{3c^2(a+b\text{ArcTan}(cx))^3}{2d}-\frac{(a+b\text{ArcTan}(cx))^3}{2dx^2}+\frac{ic(a\ -$$

```
[Out] -3/2*I*b*c^2*(a+b*arctan(c*x))^2/d-3/2*b*c*(a+b*arctan(c*x))^2/d/x-3/2*c^2*
(a+b*arctan(c*x))^3/d-1/2*(a+b*arctan(c*x))^3/d/x^2+I*c*(a+b*arctan(c*x))^3
/d/x+3*b^2*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-3*I*b*c^2*(a+b*arctan(
c*x))^2*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d-3/2
*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))/d-3*b^2*c^2*(a+b*arctan(c*x))*polylog(
2,-1+2/(1-I*c*x))/d-3/2*I*b*c^2*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*c*x
))/d-3/2*I*b^3*c^2*polylog(3,-1+2/(1-I*c*x))/d-3/2*b^2*c^2*(a+b*arctan(c*x)
)*polylog(3,-1+2/(1+I*c*x))/d+3/4*I*b^3*c^2*polylog(4,-1+2/(1+I*c*x))/d
```

Rubi [A]
time = 0.72, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4990, 4946, 5038, 5044, 4988, 2497, 5004, 5112, 6745, 5114, 5118}

$$-\frac{3b^2c^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)(a+b\text{ArcTan}(cx))}{d}-\frac{3b^2c^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{2d}+\frac{3b^2c^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{3ibc^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))^2}{2d}-\frac{3ibc^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{3c^2(a+b\text{ArcTan}(cx))^3}{2d}-\frac{3ibc^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{c^2\log\left(2-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3}{d}-\frac{(a+b\text{ArcTan}(cx))^3}{2dx^2}-\frac{3bc(a+b\text{ArcTan}(cx))^2}{2dx}+\frac{ic(a+b\text{ArcTan}(cx))^3}{dx}-\frac{3ib^3c^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{2d}-\frac{3ib^3c^2\text{Li}_3\left(\frac{2}{1-icx}-1\right)}{2d}+\frac{3ib^3c^2\text{Li}_4\left(\frac{2}{icx+1}-1\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

```
[Out] (((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2)/d - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*d*x) - (3*c^2*(a + b*ArcTan[c*x])^3)/(2*d) - (a + b*ArcTan[c*x])^3/(2*d*
x^2) + (I*c*(a + b*ArcTan[c*x])^3)/(d*x) + (3*b^2*c^2*(a + b*ArcTan[c*x])*L
og[2 - 2/(1 - I*c*x)])/d - ((3*I)*b*c^2*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1
 - I*c*x)])/d - (c^2*(a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d - (((3*
I)/2)*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/d - (((3*I)/2)*b*c^2*(a + b*ArcTan[c*
x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/d - (((3*I)/2)*b^3*c^2*PolyLog[3, -1
+ 2/(1 - I*c*x)])/d - (3*b^2*c^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 +
 I*c*x)])/(2*d) + (((3*I)/4)*b^3*c^2*PolyLog[4, -1 + 2/(1 + I*c*x)])/d
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4990

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x]
 - Dist[e/(d*f), Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0] &&
 LtQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3(d+icdx)}\,dx = -\left((ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2(d+icdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} - c^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x(d+icdx)}\,dx - \frac{(ic)\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx}{d} + \frac{(3}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} + \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{dx} - \frac{c^2\left(a+b\tan^{-1}(cx)\right)^3\log\left(2-\frac{2}{1+icx}\right.}{d}$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{2dx^2} + \frac{ic(a+b}{}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{(a}{}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{(a}{}$$

$$= -\frac{3ibc^2\left(a+b\tan^{-1}(cx)\right)^2}{2d} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2dx} - \frac{3c^2\left(a+b\tan^{-1}(cx)\right)^3}{2d} - \frac{(a}{}$$

Mathematica [A]
time = 1.58, size = 634, normalized size = 1.53

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

```
[Out] (-(a^3/x^2) + ((2*I)*a^3*c)/x + (2*I)*a^3*c^2*ArcTan[c*x] - 2*a^3*c^2*Log[x
] + a^3*c^2*Log[1 + c^2*x^2] + ((3*I)*a^2*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcT
an[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x]
)]) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2, E^(
(2*I)*ArcTan[c*x])]))/x^2 + 6*a*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x) -
(3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c*x) -
 ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Log[1 -
E^((2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*x]*PolyL
og[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])] - PolyLog
[3, E^((-2*I)*ArcTan[c*x])]/2) + 2*b^3*c^2*(-1/8*Pi^3 + (I/64)*Pi^4 - ((3*I
)/2)*ArcTan[c*x]^2 - (3*ArcTan[c*x]^2)/(2*c*x) + ArcTan[c*x]^3 + (I*ArcTan[
c*x]^3)/(c*x) - ((1 + c^2*x^2)*ArcTan[c*x]^3)/(2*c^2*x^2) - (3*I)*ArcTan[c*
x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcT
an[c*x])] + 3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (3*(2 - I*ArcTan
```

```
[c*x])*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])])/2 - ((3*I)/2)*PolyLo
g[2, E^((2*I)*ArcTan[c*x])] - (3*(I + ArcTan[c*x])*PolyLog[3, E^((-2*I)*Arc
Tan[c*x])])/2 + ((3*I)/4)*PolyLog[4, E^((-2*I)*ArcTan[c*x])]))/(2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.12, size = 2884, normalized size = 6.97

method	result	size
derivativedivides	Expression too large to display	2884
default	Expression too large to display	2884

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a^3/d*ln(c^2*x^2+1)-3/2*I*a^2*b/d*ln(c*x)*ln(1+I*c*x)+3/2*I*a^2*b/
d*ln(c*x)*ln(1-I*c*x)-3/2*a^2*b/d*arctan(c*x)/c^2/x^2-3/2*a*b^2/d*arctan(c*
x)^2/c^2/x^2+6*I*a*b^2/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))
-6*I*a*b^2/d*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-9/2*I*a*b^2/d*Pi
*arctan(c*x)^2+6*I*a*b^2/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/
2))+I*b^3/d*arctan(c*x)^3/c/x-3*a*b^2/d*arctan(c*x)/c/x-a^3/d*ln(c*x)-3/2*I
*a^2*b/d*ln(-1/2*I*(c*x+I))*ln(c*x-I)+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2+3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x
^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arc
tan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3/2*I*a*b^2/
d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*b^3/d*arctan(c*x)^3+3/2*I*a*b^2/d*Pi*
csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3*a*b^2/d*ar
ctan(c*x)^2*ln(c*x)+3*a*b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3
*a*b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*a*b^2/d*arctan(c
*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*a^2*b/d*arctan(c*x)*ln(c*x)+3/2*I
*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2+3/2*I*a^2*b/d*ln(c^2*x^2+1)-3*I*a^2*b/d*ln(c*x)-3/2*I*a^2*b/d*dilog(1+I
*c*x)+3/2*I*a^2*b/d*dilog(1-I*c*x)-6*a*b^2/d*polylog(3,(1+I*c*x)/(c^2*x^2+1
)^(1/2))-6*a*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^3/d*arctan(c*x
)^3*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^3/d*arctan(c*x)^3*ln(1-(1+I*c*x)/(c
^2*x^2+1)^(1/2))-6*b^3/d*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))
-6*b^3/d*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2/d*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^
2+3*I*a*b^2/d*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
```

```
2*arctan(c*x)^2+I*a^3/d*arctan(c*x)+1/2*I*b^3/d*arctan(c*x)^4-9/2*a*b^2/d*a
rctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*a^3/d/c^2/x^2+3*b^3/d*arctan(c*x)*ln(1+(1
+I*c*x)/(c^2*x^2+1)^(1/2))+3*b^3/d*arctan(c*x)*ln(1-(1+I*c*x)/(c^2*x^2+1)^(
1/2))-6*b^3/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*b^3/d*a
rctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3/d*polylog(2,(1+I*
c*x)/(c^2*x^2+1)^(1/2))-6*I*b^3/d*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*
I*b^3/d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3/d*polylog(2,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))-3/2*I*b^3/d*arctan(c*x)^2+I*a^3/d/c/x-6*I*b^3/d*poly
log(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*b^3/d*polylog(4,-(1+I*c*x)/(c^2*x^2+
1)^(1/2))-6*a*b^2/d*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*a*b^2/d*dilog((1
+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+3*a*
b^2/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*a^2*b/d*arctan(c*x)+3*I*a*b^2/d
*arctan(c*x)^2/c/x-3/2*I*a*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*a*b^2/d*Pi*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+3/2*I*a*b^2/d*
Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c
*x)^2+3*I*a^2*b/d*arctan(c*x)/c/x-3*I*a*b^2/d*arctan(c*x)+3*I*b^3/d*arctan(
c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^3/d*arctan(c*x)^2*poly
log(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(
c^2*x^2+1)^(1/2))-3*I*b^3/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))
-1/2*b^3/d*arctan(c*x)^3/c^2/x^2-3/2*b^3/d*arctan(c*x)^2/c/x-3/2*a^2*b/d/c/
x+3*a^2*b/d*ln(c*x-I)*arctan(c*x)+3/4*I*a^2*b/d*ln(c*x-I)^2-3/2*I*a^2*b/d*d
ilog(-1/2*I*(c*x+I))+2*I*a*b^2/d*arctan(c*x)^3-3*a*b^2/d*arctan(c*x)^2*ln(2
*I*(1+I*c*x)^2/(c^2*x^2+1))+3*a*b^2/d*ln(c*x-I)*arctan(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^3 -
 1/512*(-64*I*b^3*c^2*x^2*arctan(c*x)^4 + 4*I*b^3*c^2*x^2*log(c^2*x^2 + 1)^
4 + I*(48*b^3*c^2*arctan(c*x)^4/d - 6144*b^3*c^4*integrate(1/64*x^4*arctan(
c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 3*b^3*c^2*log(c^2*x^2 + 1
)^4/d + 3072*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2
*d*x^5 + d*x^3), x) - 12288*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*c^2*integrate(1/64*x^2*arctan(c
*x)^2/(c^2*d*x^5 + d*x^3), x) - 1536*b^3*c^2*integrate(1/64*x^2*log(c^2*x^2
 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 28672*b^3*c*integrate(1/64*x*arctan(c*x)^
3/(c^2*d*x^5 + d*x^3), x) + 3072*b^3*c*integrate(1/64*x*arctan(c*x)*log(c^2
```

```
*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 98304*a*b^2*c*integrate(1/64*x*arctan
(c*x)^2/(c^2*d*x^5 + d*x^3), x) - 6144*b^3*c*integrate(1/64*x*arctan(c*x)*l
og(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 98304*a^2*b*c*integrate(1/64*x*ar
ctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*integrate(1/64*arctan(c*x)^2*l
og(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 512*b^3*integrate(1/64*log(c^2*x^
2 + 1)^3/(c^2*d*x^5 + d*x^3), x))*d*x^2 - 64*(192*b^3*c^4*integrate(1/64*x^
4*arctan(c*x)^3/(c^2*d*x^5 + d*x^3), x) + 48*b^3*c^4*integrate(1/64*x^4*arc
tan(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + b^3*c^2*arctan(c*x)^3
/d + 96*b^3*c^3*integrate(1/64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^
5 + d*x^3), x) + 24*b^3*c^3*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^2*d*x^
5 + d*x^3), x) - 48*b^3*c^3*integrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^2*d*x^
5 + d*x^3), x) + 96*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)
/(c^2*d*x^5 + d*x^3), x) - 96*b^3*c*integrate(1/64*x*arctan(c*x)^2*log(c^2*
x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 8*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1
)^3/(c^2*d*x^5 + d*x^3), x) + 96*b^3*c*integrate(1/64*x*arctan(c*x)^2/(c^2*
d*x^5 + d*x^3), x) - 24*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^2/(c^2*d*x^
5 + d*x^3), x) + 448*b^3*integrate(1/64*arctan(c*x)^3/(c^2*d*x^5 + d*x^3),
x) + 48*b^3*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^
3), x) + 1536*a*b^2*integrate(1/64*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) +
1536*a^2*b*integrate(1/64*arctan(c*x)/(c^2*d*x^5 + d*x^3), x))*d*x^2 + 32*(
-2*I*b^3*c*x + b^3)*arctan(c*x)^3 + 24*(2*I*b^3*c*x - b^3)*arctan(c*x)*log(
c^2*x^2 + 1)^2 + 4*(4*b^3*c^2*x^2*arctan(c*x) - 2*b^3*c*x - I*b^3)*log(c^2*
x^2 + 1)^3 + 16*(4*b^3*c^2*x^2*arctan(c*x)^3 + 3*(2*b^3*c*x + I*b^3)*arctan
(c*x)^2)*log(c^2*x^2 + 1))/(d*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

```
[Out] 1/16*(2*I*b^3*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 6*
I*b^3*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 12*
I*b^3*c^2*x^2*log(-(c*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I)) +
12*I*b^3*c^2*x^2*polylog(4, -(c*x + I)/(c*x - I)) + 16*d*x^2*integral(1/8*(
-8*I*a^3*c*x + 8*a^3 - 3*(-2*I*b^3*c^2*x^2 + 2*a*b^2 + (-2*I*a*b^2 + b^3)*c
*x)*log(-(c*x + I)/(c*x - I))^2 + 12*(a^2*b*c*x + I*a^2*b)*log(-(c*x + I)/(
c*x - I)))/(c^2*d*x^5 + d*x^3), x) + (2*b^3*c*x + I*b^3)*log(-(c*x + I)/(c*
x - I))^3)/(d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{a^3}{cx^4 - ix^3}\,dx + \int \frac{b^3\operatorname{atan}^3(cx)}{cx^4 - ix^3}\,dx + \int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^4 - ix^3}\,dx + \int \frac{3a^2b\operatorname{atan}(cx)}{cx^4 - ix^3}\,dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**3/(d+I*c*d*x),x)
```

```
[Out] -I*(Integral(a**3/(c*x**4 - I*x**3), x) + Integral(b**3*atan(c*x)**3/(c*x**
4 - I*x**3), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**4 - I*x**3), x) + In
tegral(3*a**2*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^3\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)),x)
```

```
[Out] int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)), x)
```

3.133 $\int \frac{1}{(d+icdx)(a+b\textbf{ArcTan}(cx))}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+icdx)(a+b\text{ArcTan}(cx))},x\right)$$

[Out] Unintegrable(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+icdx)(a+b\text{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]

[Out] Defer[Int][1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]

Rubi steps

$$\int \frac{1}{(d+icdx)\,(a+b\tan^{-1}(cx))}\,dx=\int \frac{1}{(d+icdx)\,(a+b\tan^{-1}(cx))}\,dx$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+icdx)(a+b\text{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]

[Out] Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(icdx+d)\,(a+b\arctan(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

```
[Out] int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-2/(-2*I*a*c*d*x - 2*a*d + (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x -
 I))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{acx-ia+bcx\operatorname{atan}(cx)-ib\operatorname{atan}(cx)}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*atan(c*x)),x)
```

```
[Out] -I*Integral(1/(a*c*x - I*a + b*c*x*atan(c*x) - I*b*atan(c*x)), x)/d
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{atan}\,(c\,x))\,\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*atan(c*x))*(d + c*d*x*1i)),x)
[Out] int(1/((a + b*atan(c*x))*(d + c*d*x*1i)), x)
```

3.134 $\int \frac{x^3(a+b\textbf{ArcTan}(cx))}{d+ex}\,dx$

Optimal. Leaf size=297

$$\frac{ad^2x}{e^3}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}-\frac{bd\text{ArcTan}(cx)}{2c^2e^2}+\frac{bd^2x\text{ArcTan}(cx)}{e^3}-\frac{dx^2(a+b\text{ArcTan}(cx))}{2e^2}+\frac{x^3(a+b\text{ArcTan}(cx))}{3e}+\frac{d^3(a+$$

```
[Out] a*d^2*x/e^3+1/2*b*d*x/c/e^2-1/6*b*x^2/c/e-1/2*b*d*arctan(c*x)/c^2/e^2+b*d^2
*x*arctan(c*x)/e^3-1/2*d*x^2*(a+b*arctan(c*x))/e^2+1/3*x^3*(a+b*arctan(c*x)
)/e+d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^4-d^3*(a+b*arctan(c*x))*ln(2*c*
(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4-1/2*b*d^2*ln(c^2*x^2+1)/c/e^3+1/6*b*ln(c^2
*x^2+1)/c^3/e-1/2*I*b*d^3*polylog(2,1-2/(1-I*c*x))/e^4+1/2*I*b*d^3*polylog(
2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

Rubi [A]
time = 0.20, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {4996, 4930, 266, 4946, 327, 209, 272, 45, 4966, 2449, 2352, 2497}

$$\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^4}-\frac{d^3(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}-\frac{dx^2(a+b\text{ArcTan}(cx))}{2e^2}+\frac{x^3(a+b\text{ArcTan}(cx))}{3e}+\frac{ad^2x}{e^3}-\frac{bd\text{ArcTan}(cx)}{2c^2e^2}+\frac{bd^2x\text{ArcTan}(cx)}{e^3}-\frac{bd^2\log\left(c^2x^2+1\right)}{2ce^3}+\frac{b\log\left(c^2x^2+1\right)}{6c^3e}-\frac{ibd^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^4}+\frac{ibd^3\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (a*d^2*x)/e^3 + (b*d*x)/(2*c*e^2) - (b*x^2)/(6*c*e) - (b*d*ArcTan[c*x])/(2*
c^2*e^2) + (b*d^2*x*ArcTan[c*x])/e^3 - (d*x^2*(a + b*ArcTan[c*x]))/(2*e^2)
+ (x^3*(a + b*ArcTan[c*x]))/(3*e) + (d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c
*x)])/e^4 - (d^3*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/e^4 - (b*d^2*Log[1 + c^2*x^2])/(2*c*e^3) + (b*Log[1 + c^2*x^2])/(
6*c^3*e) - ((I/2)*b*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + ((I/2)*b*d^3*P
olyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{d+ex}\,dx = \int \left(\frac{d^2(a+b\tan^{-1}(cx))}{e^3} - \frac{dx(a+b\tan^{-1}(cx))}{e^2} + \frac{x^2(a+b\tan^{-1}(cx))}{e} - \frac{d^3(a+\cdots}{e^{3}\cdots}\right.$$

$$= \frac{d^2\int (a+b\tan^{-1}(cx))\,dx}{e^3} - \frac{d^3\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{e^3} - \frac{d\int x(a+b\tan^{-1}(cx))\,dx}{e^2} + \cdots$$

$$= \frac{ad^2x}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))}{2e^2} + \frac{x^3(a+b\tan^{-1}(cx))}{3e} + \frac{d^3(a+b\tan^{-1}(cx))\log(\cdots}{e^4}$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))}{2e^2} + \frac{x^3(a+b\tan^{-1}(cx))}{3e} + \cdots$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} - \frac{bd\tan^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))}{2e^2} + \frac{x^3(a+\cdots}{\cdots}$$

$$= \frac{ad^2x}{e^3} + \frac{bdx}{2ce^2} - \frac{bx^2}{6ce} - \frac{bd\tan^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tan^{-1}(cx)}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))}{2e^2} + \cdots$$

Mathematica [A]
time = 2.17, size = 484, normalized size = 1.63

$$-\frac{\frac{be^3}{c^3} - 6ad^2ex - \frac{3bde^2x}{c} + 3ade^2x^2 + \frac{be^3x^2}{c} - 2ae^3x^3 + \frac{3bde^2\text{ArcTan}(cx)}{c^2} + 3ibd^3\pi\text{ArcTan}(cx) - 6bd^2ex\text{ArcTan}(cx) + 3bde^2x^2\text{ArcTan}(cx) - 2be^3x^3\text{ArcTan}(cx) - 6ibd^3\text{ArcTan}\left(\frac{cd}{e}\right)\text{ArcTan}(cx) + 3ibd^3\text{ArcTan}(cx)^2 + \frac{3bd^2e\text{ArcTan}(cx)^2}{c} - \frac{3bd^2\sqrt{1+\frac{c^2d^2}{e^2}}\,e\,e^{i\text{ArcTan}\left(\frac{cd}{e}\right)}\text{ArcTan}(cx)^2}{c} + 3bd^3\pi\log\left(1+e^{-2i\text{ArcTan}(cx)}\right) - 6bd^3\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right) + 6bd^3\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right) + 6bd^3\text{ArcTan}(cx)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right) + 6ad^3\log(d+ex) + \frac{3bd^2e\log(1+c^2x^2)}{c} - \frac{be^3\log(1+c^2x^2)}{c^3} + \frac{3}{2}bd^3\pi\log(1+c^2x^2) - 6bd^3\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)\right) + 3ibd^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right) - 3ibd^3\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] -1/6*((b*e^3)/c^3 - 6*a*d^2*e*x - (3*b*d*e^2*x)/c + 3*a*d*e^2*x^2 + (b*e^3*
x^2)/c - 2*a*e^3*x^3 + (3*b*d*e^2*ArcTan[c*x])/c^2 + (3*I)*b*d^3*Pi*ArcTan[
c*x] - 6*b*d^2*e*x*ArcTan[c*x] + 3*b*d*e^2*x^2*ArcTan[c*x] - 2*b*e^3*x^3*Ar
cTan[c*x] - (6*I)*b*d^3*ArcTan[(c*d)/e]*ArcTan[c*x] + (3*I)*b*d^3*ArcTan[c*
x]^2 + (3*b*d^2*e*ArcTan[c*x]^2)/c - (3*b*d^2*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(
I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/c + 3*b*d^3*Pi*Log[1 + E^((-2*I)*ArcTan[c
*x])] - 6*b*d^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b*d^3*ArcTan
[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*b*d^3*ArcT
an[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*a*d^3*Log[d
+ e*x] + (3*b*d^2*e*Log[1 + c^2*x^2])/c - (b*e^3*Log[1 + c^2*x^2])/c^3 + (3
*b*d^3*Pi*Log[1 + c^2*x^2])/2 - 6*b*d^3*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d
)/e] + ArcTan[c*x]]] + (3*I)*b*d^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (3*
I)*b*d^3*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/e^4
```

Maple [A]

time = 0.19, size = 432, normalized size = 1.45

method	result
derivativedivides	$\frac{a c^4 d^2 x}{e^3}-\frac{a c^4 d x^2}{2e^2}+\frac{a c^4 x^3}{3e}-\frac{a c^4 d^3 \ln(cex+cd)}{e^4}+\frac{b c^4 \arctan(cx) d^2 x}{e^3}-\frac{b c^4 \arctan(cx) d x^2}{2e^2}+\frac{b c^4 \arctan(cx) x^3}{3e}-\frac{b c^4 \arctan(cx) d^3 \mathrm{l}}{e^4}$
default	$\frac{a c^4 d^2 x}{e^3}-\frac{a c^4 d x^2}{2e^2}+\frac{a c^4 x^3}{3e}-\frac{a c^4 d^3 \ln(cex+cd)}{e^4}+\frac{b c^4 \arctan(cx) d^2 x}{e^3}-\frac{b c^4 \arctan(cx) d x^2}{2e^2}+\frac{b c^4 \arctan(cx) x^3}{3e}-\frac{b c^4 \arctan(cx) d^3 \mathrm{l}}{e^4}$
risch	$-\frac{ad}{2c^2e^2}-\frac{a d^3 \ln(icd-(-icx+1)e+e)}{e^4}+\frac{ib d^3 \operatorname{dilog}\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2e^4}-\frac{ib d^3 \ln(-icx+1) \ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^4/e^3*d^2*x-1/2*a*c^4/e^2*d*x^2+1/3*a*c^4/e*x^3-a*c^4*d^3/e^4*ln
(c*e*x+c*d)+b*c^4*arctan(c*x)/e^3*d^2*x-1/2*b*c^4*arctan(c*x)/e^2*d*x^2+1/3
*b*c^4*arctan(c*x)/e*x^3-b*c^4*arctan(c*x)*d^3/e^4*ln(c*e*x+c*d)-1/2*b*c^3/
e^3*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)*d^2-1/2*b*c^2/e^2*arcta
n(c*x)*d+1/6*b*c/e*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)+2/3*b*c^
3*d^2/e^3+1/2*b*c^3*d/e^2*x-1/6*b*c^3/e*x^2+1/2*I*b*c^4/e^4*d^3*dilog((I*e+
c*e*x)/(I*e-c*d))+1/2*I*b*c^4/e^4*d^3*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d
))-1/2*I*b*c^4/e^4*d^3*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))-1/2*I*b*c^4/
e^4*d^3*dilog((I*e-c*e*x)/(c*d+I*e)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*(6*d^3*e^(-4)*log(x*e + d) - (2*x^3*e^2 - 3*d*x^2*e + 6*d^2*x)*e^(-3))
*a + 2*b*integrate(1/2*x^3*arctan(c*x)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{atan}\,(cx))}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a + b\,\mathrm{atan}(c\,x)\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x),x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x), x)
```

3.135 $\int \frac{x^2(a+b\text{ArcTan}(cx))}{d+ex}\,dx$

Optimal. Leaf size=237

$$-\frac{adx}{e^2}-\frac{bx}{2ce}+\frac{b\text{ArcTan}(cx)}{2c^2e}-\frac{bdx\text{ArcTan}(cx)}{e^2}+\frac{x^2(a+b\text{ArcTan}(cx))}{2e}-\frac{d^2(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e^3}+\frac{d^2(\cdots}{\cdots}$$

```
[Out] -a*d*x/e^2-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e-b*d*x*arctan(c*x)/e^2+1/2*x^
2*(a+b*arctan(c*x))/e-d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^3+d^2*(a+b*ar
ctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3+1/2*b*d*ln(c^2*x^2+1)/c/
e^2+1/2*I*b*d^2*polylog(2,1-2/(1-I*c*x))/e^3-1/2*I*b*d^2*polylog(2,1-2*c*(e
*x+d)/(c*d+I*e)/(1-I*c*x))/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {4996, 4930, 266, 4946, 327, 209, 4966, 2449, 2352, 2497}

$$-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^3}+\frac{d^2(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}+\frac{x^2(a+b\text{ArcTan}(cx))}{2e}-\frac{adx}{e^2}+\frac{b\text{ArcTan}(cx)}{2c^2e}-\frac{bdx\text{ArcTan}(cx)}{e^2}+\frac{bd\log\left(c^2x^2+1\right)}{2ce^2}+\frac{ibd^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{bx}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] -((a*d*x)/e^2) - (b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) - (b*d*x*ArcTan[
c*x])/e^2 + (x^2*(a + b*ArcTan[c*x]))/(2*e) - (d^2*(a + b*ArcTan[c*x])*Log[
2/(1 - I*c*x)])/e^3 + (d^2*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d +
I*e)*(1 - I*c*x))])/e^3 + (b*d*Log[1 + c^2*x^2])/(2*c*e^2) + ((I/2)*b*d^2*P
olyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - ((I/2)*b*d^2*PolyLog[2, 1 - (2*c*(d + e
*x))/((c*d + I*e)*(1 - I*c*x))])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{d+ex}\,dx = \int \left(-\frac{d(a+b\tan^{-1}(cx))}{e^2}+\frac{x(a+b\tan^{-1}(cx))}{e}+\frac{d^2(a+b\tan^{-1}(cx))}{e^2(d+ex)}\right)\,dx$$

$$= -\frac{d\int (a+b\tan^{-1}(cx))\,dx}{e^2}+\frac{d^2\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{e^2}+\frac{\int x(a+b\tan^{-1}(cx))\,dx}{e}$$

$$= -\frac{adx}{e^2}+\frac{x^2(a+b\tan^{-1}(cx))}{2e}-\frac{d^2(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^3}+\frac{d^2(a+b\tan$$

$$= -\frac{adx}{e^2}-\frac{bx}{2ce}-\frac{bdx\tan^{-1}(cx)}{e^2}+\frac{x^2(a+b\tan^{-1}(cx))}{2e}-\frac{d^2(a+b\tan^{-1}(cx))\log}{e^3}$$

$$= -\frac{adx}{e^2}-\frac{bx}{2ce}+\frac{b\tan^{-1}(cx)}{2c^2e}-\frac{bdx\tan^{-1}(cx)}{e^2}+\frac{x^2(a+b\tan^{-1}(cx))}{2e}-\frac{d^2(a+}{}$$

Mathematica [A]
time = 1.03, size = 404, normalized size = 1.70

$$\frac{-2adex-\frac{be^2x}{c}+ae^2x^2+\frac{be^2\text{ArcTan}(cx)}{c^2}+ibd^2\pi\text{ArcTan}(cx)-2bdex\text{ArcTan}(cx)+be^2x^2\text{ArcTan}(cx)-2ibd^2\text{ArcTan}\left(\frac{cd}{e}\right)\text{ArcTan}(cx)+ibd^2\text{ArcTan}(cx)^2+\frac{bde\text{ArcTan}(cx)^2}{c}-\frac{bd\sqrt{1+\frac{c^2d^2}{e^2}}\,ee^{i\text{ArcTan}\left(\frac{cd}{e}\right)}\text{ArcTan}(cx)^2}{c}+bd^2\pi\log\left(1+e^{-2i\text{ArcTan}(cx)}\right)-2bd^2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+2bd^2\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)+2bd^2\text{ArcTan}(cx)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)+2ad^2\log(d+ex)+\frac{bde\log(1+c^2x^2)}{c}+\frac{1}{2}bd^2\pi\log(1+c^2x^2)-2bd^2\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)\right)+ibd^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)-ibd^2\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (-2*a*d*e*x - (b*e^2*x)/c + a*e^2*x^2 + (b*e^2*ArcTan[c*x])/c^2 + I*b*d^2*P
i*ArcTan[c*x] - 2*b*d*e*x*ArcTan[c*x] + b*e^2*x^2*ArcTan[c*x] - (2*I)*b*d^2
*ArcTan[(c*d)/e]*ArcTan[c*x] + I*b*d^2*ArcTan[c*x]^2 + (b*d*e*ArcTan[c*x]^2
)/c - (b*d*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/c
 + b*d^2*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*b*d^2*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] + 2*b*d^2*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c
*d)/e] + ArcTan[c*x]))] + 2*b*d^2*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d
)/e] + ArcTan[c*x]))] + 2*a*d^2*Log[d + e*x] + (b*d*e*Log[1 + c^2*x^2])/c +
 (b*d^2*Pi*Log[1 + c^2*x^2])/2 - 2*b*d^2*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*
d)/e] + ArcTan[c*x]]] + I*b*d^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - I*b*d^
2*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/(2*e^3)
```

Maple [A]
time = 0.10, size = 337, normalized size = 1.42

method	result
derivativedivides	$-\frac{a\,c^3 dx}{e^2}+\frac{a\,c^3 x^2}{2e}+\frac{a\,c^3 d^2 \ln(cex+cd)}{e^3}-\frac{b\,c^3 \arctan(cx) dx}{e^2}+\frac{b\,c^3 \arctan(cx) x^2}{2e}+\frac{b\,c^3 \arctan(cx) d^2 \ln(cex+cd)}{e^3}+\frac{ib\,c^3 d^2 \ln(cex+cd) \ln(}{2e^3}$
default	$-\frac{a\,c^3 dx}{e^2}+\frac{a\,c^3 x^2}{2e}+\frac{a\,c^3 d^2 \ln(cex+cd)}{e^3}-\frac{b\,c^3 \arctan(cx) dx}{e^2}+\frac{b\,c^3 \arctan(cx) x^2}{2e}+\frac{b\,c^3 \arctan(cx) d^2 \ln(cex+cd)}{e^3}+\frac{ib\,c^3 d^2 \ln(cex+cd) \ln(}{2e^3}$
risch	$\frac{a\,x^2}{2e}+\frac{ib\ln(icx+1)xd}{2e^2}-\frac{ib\ln(-icx+1)dx}{2e^2}+\frac{bd\ln(c^2x^2+1)}{2c\,e^2}+\frac{ib\,d^2\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e^3}-\frac{ib\,d^2\ln(icx+1)\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-a*c^3/e^2*d*x+1/2*a*c^3/e*x^2+a*c^3*d^2/e^3*ln(c*e*x+c*d)-b*c^3*arc
tan(c*x)/e^2*d*x+1/2*b*c^3*arctan(c*x)/e*x^2+b*c^3*arctan(c*x)*d^2/e^3*ln(c
*e*x+c*d)+1/2*I*b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))-1/2*I
*b*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))+1/2*I*b*c^3/e^3*d^2*
dilog((I*e-c*e*x)/(c*d+I*e))-1/2*I*b*c^3/e^3*d^2*dilog((I*e+c*e*x)/(I*e-c*d
))+1/2*b*c^2/e^2*d*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)+1/2*b*c/
e*arctan(c*x)-1/2*b*c^2*d/e^2-1/2*b*c^2/e*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^2*e^(-3)*log(x*e + d) + (x^2*e - 2*d*x)*e^(-2))*a + 2*b*integrate(
1/2*x^2*arctan(c*x)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x), x)
```

3.136 $\int \frac{x(a+b\text{ArcTan}(cx))}{d+ex}\,dx$

Optimal. Leaf size=179

$$\frac{ax}{e}+\frac{bx\text{ArcTan}(cx)}{e}+\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}-\frac{b\log(1+c^2x$$

```
[Out] a*x/e+b*x*arctan(c*x)/e+d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^2-d*(a+b*arct
an(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-1/2*b*ln(c^2*x^2+1)/c/e-1/
2*I*b*d*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*polylog(2,1-2*c*(e*x+d)/(c*d
+I*e)/(1-I*c*x))/e^2
```

Rubi [A]
time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4996, 4930, 266, 4966, 2449, 2352, 2497}

$$\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^2}-\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{ax}{e}+\frac{bx\text{ArcTan}(cx)}{e}-\frac{b\log(c^2x^2+1)}{2ce}-\frac{ibd\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
e^2 - (d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/e^2 - (b*Log[1 + c^2*x^2])/(2*c*e) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 - I*c
*x)])/e^2 + ((I/2)*b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c
*x))])/e^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{d+ex}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{e}-\frac{d(a+b\tan^{-1}(cx))}{e(d+ex)}\right)dx$$

$$= \frac{\int (a+b\tan^{-1}(cx))\,dx}{e}-\frac{d\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{e}$$

$$= \frac{ax}{e}+\frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}-$$

$$= \frac{ax}{e}+\frac{bx\tan^{-1}(cx)}{e}+\frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(a+b\tan^{-1}(cx))\log\left(\right.}{e^2}$$

$$= \frac{ax}{e}+\frac{bx\tan^{-1}(cx)}{e}+\frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(a+b\tan^{-1}(cx))\log\left(\right.}{e^2}$$

Mathematica [A]

time = 0.97, size = 329, normalized size = 1.84

$$\frac{2aex-2ad\log(d+ex)+\frac{b\left(-icd\pi\text{ArcTan}(cx)+2cex\text{ArcTan}(cx)+2icd\text{ArcTan}\left(\frac{cd}{e}\right)\text{ArcTan}(cx)-icd\text{ArcTan}(cx)^2-e\text{ArcTan}(cx)^2+\sqrt{1+\frac{c^2d^2}{e^2}}ee^{i\text{ArcTan}\left(\frac{cd}{e}\right)}\text{ArcTan}(cx)^2-cd\pi\log\left(1+e^{-2i\text{ArcTan}(cx)}\right)+2cd\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-2cd\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)-2cd\text{ArcTan}(cx)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)-e\log(1+c^2x^2)-\frac{1}{2}cd\pi\log(1+c^2x^2)+2cd\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)\right)-icd\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+icd\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)\right)}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

```
[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTan[c*x] + 2*c*e*x*ArcTa
n[c*x] + (2*I)*c*d*ArcTan[(c*d)/e]*ArcTan[c*x] - I*c*d*ArcTan[c*x]^2 - e*Ar
cTan[c*x]^2 + Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2
 - c*d*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] + 2*c*d*ArcTan[c*x]*Log[1 + E^((2
*I)*ArcTan[c*x])] - 2*c*d*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e]
 + ArcTan[c*x]))] - 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + A
rcTan[c*x]))] - e*Log[1 + c^2*x^2] - (c*d*Pi*Log[1 + c^2*x^2])/2 + 2*c*d*Ar
cTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] - I*c*d*PolyLog[2, -E
^((2*I)*ArcTan[c*x])] + I*c*d*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan
[c*x]))]))/c)/(2*e^2)
```

Maple [A]

time = 0.08, size = 261, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+\frac{b\,c^2\arctan(cx)x}{e}-\frac{b\,c^2\arctan(cx)d\ln(cex+cd)}{e^2}-\frac{bc\ln\left(c^2d^2-2cd(cex+cd)+e^2+(cex+cd)^2\right)}{2e}-\frac{ib\,c^2d\ln(cex+}{}}{c^2}$
default	$\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+\frac{b\,c^2\arctan(cx)x}{e}-\frac{b\,c^2\arctan(cx)d\ln(cex+cd)}{e^2}-\frac{bc\ln\left(c^2d^2-2cd(cex+cd)+e^2+(cex+cd)^2\right)}{2e}-\frac{ib\,c^2d\ln(cex+}{}}{c^2}$
risch	$\frac{ib\ln(-icx+1)x}{2e}-\frac{b\ln(c^2x^2+1)}{2ce}+\frac{b}{ce}-\frac{ibd\,\text{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e^2}-\frac{ibd\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(a*c^2/e*x-a*c^2*d/e^2*ln(c*e*x+c*d)+b*c^2*arctan(c*x)/e*x-b*c^2*arct
an(c*x)*d/e^2*ln(c*e*x+c*d)-1/2*b*c/e*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e
*x+c*d)^2)-1/2*I*b*c^2/e^2*d*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))+1/2*I*
b*c^2/e^2*d*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))-1/2*I*b*c^2/e^2*d*dilog
((I*e-c*e*x)/(c*d+I*e))+1/2*I*b*c^2/e^2*d*dilog((I*e+c*e*x)/(I*e-c*d)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -(d*e^(-2)*log(x*e + d) - x*e^(-1))*a + 2*b*integrate(1/2*x*arctan(c*x)/(x*
e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctan(c*x) + a*x)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{atan}\,(cx))}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\mathrm{atan}(c\,x)\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x),x)
```

```
[Out] int((x*(a + b*atan(c*x)))/(d + e*x), x)
```

3.137 $\int \frac{a+b\mathbf{ArcTan}(cx)}{d+ex}\,dx$

Optimal. Leaf size=138

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}-\frac{ib\text{PolyLo}}{}$$

```
[Out] -(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+
I*e)/(1-I*c*x))/e+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*polylog(2,1-2*
c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4966, 2449, 2352, 2497}

$$\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e}-\frac{ib\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx = -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{(bc)\int \frac{\log(}{1-}}{e}$$

$$= -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} - \frac{ib\mathrm{Li}_2\left(1-\right.}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\mathrm{Li}_2(1-}{2e}$$

Mathematica [A]

time = 0.04, size = 130, normalized size = 0.94

$$\frac{a\log(d+ex)}{e} + \frac{ib\left(\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right) - \log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right) - \mathrm{PolyLog}\left(2,-\frac{e(-i+cx)}{cd+ie}\right) + \mathrm{PolyLog}\left(2,-\frac{e(i+cx)}{cd-ie}\right)\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + ((I/2)*b*(Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)
] - Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] - PolyLog[2, -((e*(-I + c
*x))/(c*d + I*e))] + PolyLog[2, -((e*(I + c*x))/(c*d - I*e))]))/e
```

Maple [A]

time = 0.00, size = 178, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+\frac{bc\ln(cex+cd)\arctan(cx)}{e}+\frac{ibc\ln(cex+cd)\ln\left(\frac{-cex+ie}{cd+ie}\right)}{2e}-\frac{ibc\ln(cex+cd)\ln\left(\frac{cex+ie}{-cd+ie}\right)}{2e}+\frac{ibc\,\mathrm{dilog}\left(\frac{-cex+ie}{cd+ie}\right)}{2e}-\frac{ibc}{}}{c}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+\frac{bc\ln(cex+cd)\arctan(cx)}{e}+\frac{ibc\ln(cex+cd)\ln\left(\frac{-cex+ie}{cd+ie}\right)}{2e}-\frac{ibc\ln(cex+cd)\ln\left(\frac{cex+ie}{-cd+ie}\right)}{2e}+\frac{ibc\,\mathrm{dilog}\left(\frac{-cex+ie}{cd+ie}\right)}{2e}-\frac{ibc}{}}{c}$
risch	$\frac{ib\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{a\ln(icd-(-icx+1)e+e)}{e}-\frac{ib\,\mathrm{dilog}\left(\frac{icd+(}{}\right)}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c*ln(c*e*x+c*d)/e+b*c*ln(c*e*x+c*d)/e*arctan(c*x)+1/2*I*b*c*ln(c*e*x
+c*d)/e*ln((I*e-c*e*x)/(c*d+I*e))-1/2*I*b*c*ln(c*e*x+c*d)/e*ln((I*e+c*e*x)/
(I*e-c*d))+1/2*I*b*c/e*dilog((I*e-c*e*x)/(c*d+I*e))-1/2*I*b*c/e*dilog((I*e+
c*e*x)/(I*e-c*d)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + 2*b*integrate(1/2*arctan(c*x)/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x), x)
```

3.138 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex)}\,dx$

Optimal. Leaf size=181

$$\frac{a\log(x)}{d}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}+\frac{ib\text{PolyLog}(2,-icx)}{2d}-\frac{ib\text{Po}}{}$$

```
[Out] a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-(a+b*arctan(c*x))*ln(2*c*(e*x
+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,I*
c*x)/d-1/2*I*b*polylog(2,1-2/(1-I*c*x))/d+1/2*I*b*polylog(2,1-2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/d
```

Rubi [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{ib\text{Li}_2(-icx)}{2d}-\frac{ib\text{Li}_2(icx)}{2d}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + ((I/2)*b*PolyLog[
2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2
/(1 - I*c*x)])/d + ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
- I*c*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x(d+ex)}\,dx &= \int \left(\frac{a+b\tan^{-1}(cx)}{dx}-\frac{e(a+b\tan^{-1}(cx))}{d(d+ex)}\right)dx\\
&= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d}-\frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d}\\
&= \frac{a\log(x)}{d}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}+\\
&= \frac{a\log(x)}{d}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}+\\
&= \frac{a\log(x)}{d}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}+
\end{aligned}$$

Mathematica [A]

time = 0.69, size = 308, normalized size = 1.70

$$\frac{2ad\log(x)-2ad\log(d+ex)+\frac{b\left(-icd\pi\text{ArcTan}(cx)+2icd\text{ArcTan}\left(\frac{cd}{e}\right)\text{ArcTan}(cx)-icd\text{ArcTan}(cx)^2-e\text{ArcTan}(cx)^2+\sqrt{1+\frac{c^2d^2}{e^2}}\,ee^{i\text{ArcTan}\left(\frac{cd}{e}\right)}\text{ArcTan}(cx)^2-cd\pi\log\left(1+e^{-2i\text{ArcTan}(cx)}\right)+2cd\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)-2cd\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)-2cd\text{ArcTan}(cx)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)-\frac{1}{2}cd\pi\log(1+c^2x^2)+2cd\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)\right)-icd\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)+icd\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)\right)}{c}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

```
[Out] (2*a*d*Log[x] - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTan[c*x] + (2*I)*c*
d*ArcTan[(c*d)/e]*ArcTan[c*x] - I*c*d*ArcTan[c*x]^2 - e*ArcTan[c*x]^2 + Sqr
t[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2 - c*d*Pi*Log[1 +
 E^((-2*I)*ArcTan[c*x])] + 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])]
 - 2*c*d*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]
 - 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] - (
c*d*Pi*Log[1 + c^2*x^2])/2 + 2*c*d*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e]
+ ArcTan[c*x]]] - I*c*d*PolyLog[2, E^((2*I)*ArcTan[c*x])] + I*c*d*PolyLog[2
, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]))/c)/(2*d^2)
```

Maple [A]

time = 0.13, size = 260, normalized size = 1.44

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d}-\frac{ib\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}-\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}+\frac{a\ln(-icx)}{d}-\frac{a\ln(i}{}$
derivativedivides	$-\frac{a\ln(cex+cd)}{d}+\frac{a\ln(cx)}{d}-\frac{b\arctan(cx)\ln(cex+cd)}{d}+\frac{b\ln(cx)\arctan(cx)}{d}+\frac{ib\ln(cx)\ln(icx+1)}{2d}-\frac{ib\ln(cx)\ln(-}{2d}$
default	$-\frac{a\ln(cex+cd)}{d}+\frac{a\ln(cx)}{d}-\frac{b\arctan(cx)\ln(cex+cd)}{d}+\frac{b\ln(cx)\arctan(cx)}{d}+\frac{ib\ln(cx)\ln(icx+1)}{2d}-\frac{ib\ln(cx)\ln(-}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d*ln(c*e*x+c*d)+a/d*ln(c*x)-b*arctan(c*x)/d*ln(c*e*x+c*d)+b/d*ln(c*x)*ar
ctan(c*x)+1/2*I*b/d*ln(c*x)*ln(1+I*c*x)-1/2*I*b/d*ln(c*x)*ln(1-I*c*x)+1/2*I
*b/d*dilog(1+I*c*x)-1/2*I*b/d*dilog(1-I*c*x)-1/2*I*b/d*ln(c*e*x+c*d)*ln((I*
e-c*e*x)/(c*d+I*e))+1/2*I*b/d*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))-1/2*I
*b/d*dilog((I*e-c*e*x)/(c*d+I*e))+1/2*I*b/d*dilog((I*e+c*e*x)/(I*e-c*d))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(x*e + d)/d - log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(x^2*e + d*x
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}(cx)}{x(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atan}(c\,x)}{x\,(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + e*x)), x)
```

3.139 $\int \frac{a+b\text{ArcTan}(cx)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=232

$$-\frac{a+b\text{ArcTan}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{ae\log(x)}{d^2}-\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+}{(cd+ie)(}\right.}{d^2}$$

```
[Out] (-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctan(c*x))*ln(2/(
1-I*c*x))/d^2+e*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2-1
/2*b*c*ln(c^2*x^2+1)/d-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog(2,
I*c*x)/d^2+1/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/2*I*b*e*polylog(2,1-2*c
*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
```

Rubi [A]

time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {4996, 4946, 272, 36, 29, 31, 4940, 2438, 4966, 2449, 2352, 2497}

$$-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{a+b\text{ArcTan}(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log\left(c^2x^2+1\right)}{2d}-\frac{ibe\text{Li}_2(-icx)}{2d^2}+\frac{ibe\text{Li}_2(icx)}{2d^2}+\frac{ibe\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(d
 + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d) - (
(I/2)*b*e*PolyLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + (
(I/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b*e*PolyLog[2, 1 - (2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx^2} - \frac{e(a+b\tan^{-1}(cx))}{d^2x} + \frac{e^2(a+b\tan^{-1}(cx))}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d^2}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+b\tan^{-1}(cx))}{\ldots}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+b\tan^{-1}(cx))}{\ldots}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+b\tan^{-1}(cx))}{\ldots}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+\ldots}{\ldots}$$

Mathematica [A]
time = 0.88, size = 372, normalized size = 1.60

$$\frac{-\frac{2ad^2}{x} + ibde\pi\text{ArcTan}(cx) - \frac{2bd^2\text{ArcTan}(cx)}{x} - 2ibde\text{ArcTan}\left(\frac{cd}{e}\right)\text{ArcTan}(cx) + ibde\text{ArcTan}(cx)^2 + \frac{be^2\text{ArcTan}(cx)^2}{c} - \frac{b\sqrt{1+\frac{c^2d^2}{e^2}}\,e^2e^{i\text{ArcTan}\left(\frac{cd}{e}\right)}\text{ArcTan}(cx)^2}{c} + bde\pi\log\left(1+e^{-2i\text{ArcTan}(cx)}\right) - 2bde\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right) + 2bde\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right) + 2bde\text{ArcTan}(cx)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right) - 2ade\log(x) + 2ade\log(d+ex) + 2bcd^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right) + \frac{1}{2}bde\pi\log\left(1+c^2x^2\right) - 2bde\text{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)\right) + ibde\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right) - ibde\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{cd}{e}\right)+\text{ArcTan}(cx)\right)}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] ((-2*a*d^2)/x + I*b*d*e*Pi*ArcTan[c*x] - (2*b*d^2*ArcTan[c*x])/x - (2*I)*b*
d*e*ArcTan[(c*d)/e]*ArcTan[c*x] + I*b*d*e*ArcTan[c*x]^2 + (b*e^2*ArcTan[c*x
]^2)/c - (b*Sqrt[1 + (c^2*d^2)/e^2]*e^2*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2
)/c + b*d*e*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*b*d*e*ArcTan[c*x]*Log[1
```

```
- E^((2*I)*ArcTan[c*x])] + 2*b*d*e*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan
[(c*d)/e] + ArcTan[c*x]))] + 2*b*d*e*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(
c*d)/e] + ArcTan[c*x]))] - 2*a*d*e*Log[x] + 2*a*d*e*Log[d + e*x] + 2*b*c*d^
2*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (b*d*e*Pi*Log[1 + c^2*x^2])/2 - 2*b*d*e*Ar
cTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] + I*b*d*e*PolyLog[2,
E^((2*I)*ArcTan[c*x])] - I*b*d*e*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + Arc
Tan[c*x]))])/(2*d^3)
```

Maple [A]
time = 0.13, size = 363, normalized size = 1.56

method	result
risch	$-\frac{ibe\ln(icx+1)\ln\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2d^2}+\frac{ib\,\mathrm{dilog}(-icx+1)e}{2d^2}+\frac{ibe\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^2}+\frac{cb\ln(-icx)}{2d}$
derivativedivides	$c\left(\frac{ae\ln(cex+cd)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{b\arctan(cx)e\ln(cex+cd)}{c\,d^2}-\frac{b\arctan(cx)}{dcx}-\frac{b\arctan(cx)e\ln(cx)}{c\,d^2}+\right.$
default	$c\left(\frac{ae\ln(cex+cd)}{c\,d^2}-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{b\arctan(cx)e\ln(cex+cd)}{c\,d^2}-\frac{b\arctan(cx)}{dcx}-\frac{b\arctan(cx)e\ln(cx)}{c\,d^2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c/d^2*e*ln(c*e*x+c*d)-a/d/c/x-a/c/d^2*e*ln(c*x)+b/c*arctan(c*x)/d^2*e*
ln(c*e*x+c*d)-b/d*arctan(c*x)/c/x-b/c*arctan(c*x)/d^2*e*ln(c*x)-1/2*I*b/c/d
^2*e*ln(c*x)*ln(1+I*c*x)-1/2*I*b/c/d^2*e*dilog((I*e+c*e*x)/(I*e-c*d))+1/2*I
*b/c/d^2*e*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))+1/2*I*b/c/d^2*e*dilog(1-
I*c*x)-1/2*b/d*ln(c^2*x^2+1)+b/d*ln(c*x)+1/2*I*b/c/d^2*e*dilog((I*e-c*e*x)/
(c*d+I*e))-1/2*I*b/c/d^2*e*dilog(1+I*c*x)-1/2*I*b/c/d^2*e*ln(c*e*x+c*d)*ln(
(I*e+c*e*x)/(I*e-c*d))+1/2*I*b/c/d^2*e*ln(c*x)*ln(1-I*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 2*b*integrate(1/2*arctan(
c*x)/(x^3*e + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^3*e + d*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x)), x)
```

3.140 $\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=293

$$-\frac{bc}{2dx}-\frac{bc^2\text{ArcTan}(cx)}{2d}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}+\frac{e(a+b\text{ArcTan}(cx))}{d^2x}-\frac{bce\log(x)}{d^2}+\frac{ae^2\log(x)}{d^3}+\frac{e^2(a+b\text{ArcTan}}{d}$$

```
[Out] -1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2+e*(a+b*ar
ctan(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctan(c*x))*ln(2
/(1-I*c*x))/d^3-e^2*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d
^3+1/2*b*c*e*ln(c^2*x^2+1)/d^2+1/2*I*b*e^2*polylog(2,-I*c*x)/d^3-1/2*I*b*e^
2*polylog(2,I*c*x)/d^3-1/2*I*b*e^2*polylog(2,1-2/(1-I*c*x))/d^3+1/2*I*b*e^2
*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

Rubi [A]
time = 0.20, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.737$, Rules used = {4996, 4946, 331, 209, 272, 36, 29, 31, 4940, 2438, 4966, 2449, 2352, 2497}

$$\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{e^2(a+b\text{ArcTan}(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{e(a+b\text{ArcTan}(cx))}{d^2x}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}+\frac{ae^2\log(x)}{d^3}-\frac{bc^2\text{ArcTan}(cx)}{2d}+\frac{bce\log\left(c^2x^2+1\right)}{2d^2}+\frac{ibe^2\text{Li}_2(-icx)}{2d^3}-\frac{ibe^2\text{Li}_2(icx)}{2d^3}-\frac{ibe^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ibe^2\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}-\frac{bce\log(x)}{d^2}-\frac{bc}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(b*c)/(d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2
) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/d
^3 + (e^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcTan[
c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3 + (b*c*e*Log[1 +
c^2*x^2])/(2*d^2) + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*P
olyLog[2, I*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + (
(I/2)*b*e^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx^3} - \frac{e(a+b\tan^{-1}(cx))}{d^2x^2} + \frac{e^2(a+b\tan^{-1}(cx))}{d^3x} - \frac{e^3(a+b\tan^{-1}(c\cdots}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{e^3\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-ic\cdots}\right.}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2(a+b\tan^{-1}(cx))\,\mathrm{l}\cdots}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2(a\cdots}{}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))}{d^2x} - \frac{bce\log(x)}{d^2} + \frac{ae^2\,\mathrm{l}\cdots}{\cdots}$$

Mathematica [A]
time = 2.17, size = 441, normalized size = 1.51

$$-\frac{\frac{ad^3}{x^2}-\frac{2ad^2e}{x}-2ade^2\log(x)+2ade^2\log(d+ex)+\frac{b\left(\frac{c^2d^3}{x}+c^3d^3\mathrm{ArcTan}(cx)+icde^2\pi\mathrm{ArcTan}(cx)+\frac{cd^3\mathrm{ArcTan}(cx)}{x^2}-\frac{2cd^2e\mathrm{ArcTan}(cx)}{x}-2icde^2\mathrm{ArcTan}\left(\frac{cd}{e}\right)\mathrm{ArcTan}(cx)+icde^2\mathrm{ArcTan}(cx)^2+e^3\mathrm{ArcTan}(cx)^2-\sqrt{1+\frac{c^2d^2}{e^2}}\,e^3e^{i\mathrm{ArcTan}\left(\frac{cd}{e}\right)}\mathrm{ArcTan}(cx)^2+cde^2\pi\log\left(1+e^{-2i\mathrm{ArcTan}(cx)}\right)-2cde^2\mathrm{ArcTan}(cx)\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)+2cde^2\mathrm{ArcTan}\left(\frac{cd}{e}\right)\log\left(1-e^{2i\left(\mathrm{ArcTan}\left(\frac{cd}{e}\right)+\mathrm{ArcTan}(cx)\right)}\right)+2cde^2\mathrm{ArcTan}(cx)\log\left(1-e^{2i\left(\mathrm{ArcTan}\left(\frac{cd}{e}\right)+\mathrm{ArcTan}(cx)\right)}\right)+2c^2d^2e\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)+\frac{1}{2}cde^2\pi\log\left(1+c^2x^2\right)-2cde^2\mathrm{ArcTan}\left(\frac{cd}{e}\right)\log\left(\sin\left(\mathrm{ArcTan}\left(\frac{cd}{e}\right)+\mathrm{ArcTan}(cx)\right)\right)+icde^2\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)-icde^2\mathrm{PolyLog}\left(2,e^{2i\left(\mathrm{ArcTan}\left(\frac{cd}{e}\right)+\mathrm{ArcTan}(cx)\right)}\right)\right)}{c}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((a*d^3)/x^2 - (2*a*d^2*e)/x - 2*a*d*e^2*Log[x] + 2*a*d*e^2*Log[d + e*
x] + (b*((c^2*d^3)/x + c^3*d^3*ArcTan[c*x] + I*c*d*e^2*Pi*ArcTan[c*x] + (c*
d^3*ArcTan[c*x])/x^2 - (2*c*d^2*e*ArcTan[c*x])/x - (2*I)*c*d*e^2*ArcTan[(c*
d)/e]*ArcTan[c*x] + I*c*d*e^2*ArcTan[c*x]^2 + e^3*ArcTan[c*x]^2 - Sqrt[1 +
(c^2*d^2)/e^2]*e^3*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2 + c*d*e^2*Pi*Log[1 +
 E^((-2*I)*ArcTan[c*x])] - 2*c*d*e^2*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*
x])] + 2*c*d*e^2*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan
[c*x]))] + 2*c*d*e^2*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan
[c*x]))] + 2*c^2*d^2*e*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (c*d*e^2*Pi*Log[1 + c
^2*x^2])/2 - 2*c*d*e^2*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x
]]] + I*c*d*e^2*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*c*d*e^2*PolyLog[2, E^
((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]))/c)/d^4
```

Maple [A]
time = 0.15, size = 448, normalized size = 1.53

method	result

derivativedivides	$c^2\left(-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{b\arctan(cx)e^2\ln(cex+cd)}{c^2d^3}-\frac{b\arctan(cx)}{2d\,c^2x^2}+\frac{b\arctan\right.$
default	$c^2\left(-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{b\arctan(cx)e^2\ln(cex+cd)}{c^2d^3}-\frac{b\arctan(cx)}{2d\,c^2x^2}+\frac{b\arctan\right.$
risch	$-\frac{ib\,e^2\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^3}-\frac{ib\,e^2\operatorname{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^3}-\frac{ibe\ln(icx+1)}{2d^2x}-\frac{ib\operatorname{dilog}(-icx+1)}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-a/c^2/d^3*e^2*ln(c*e*x+c*d)-1/2*a/d/c^2/x^2+a/c^2/d^3*e^2*ln(c*x)+a/c
^2/d^2*e/x-b/c^2*arctan(c*x)/d^3*e^2*ln(c*e*x+c*d)-1/2*b/d*arctan(c*x)/c^2/
x^2+b/c^2*arctan(c*x)/d^3*e^2*ln(c*x)+b/c^2*arctan(c*x)/d^2*e/x+1/2*I*b/c^2
/d^3*e^2*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))+1/2*I*b/c^2/d^3*e^2*dilog(
1+I*c*x)+1/2*I*b/c^2/d^3*e^2*dilog((I*e+c*e*x)/(I*e-c*d))-1/2*I*b/c^2/d^3*e
^2*dilog((I*e-c*e*x)/(c*d+I*e))+1/2*I*b/c^2/d^3*e^2*ln(c*x)*ln(1+I*c*x)-1/2
*I*b/c^2/d^3*e^2*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))-1/2*I*b/c^2/d^3*e^
2*dilog(1-I*c*x)-1/2*I*b/c^2/d^3*e^2*ln(c*x)*ln(1-I*c*x)+1/2*b/c/d^2*e*ln(c
^2*x^2+1)-1/2*b/d*arctan(c*x)-1/2*b/d/c/x-b/c/d^2*e*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*e^2*log(x*e + d)/d^3 - 2*e^2*log(x)/d^3 - (2*x*e - d)/(d^2*x^2))
+ 2*b*integrate(1/2*arctan(c*x)/(x^4*e + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^4*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{atan}(cx)}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**3*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x)), x)
```

3.141 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=598

$$\frac{abdx}{ce^2}+\frac{b^2x}{3c^2e}-\frac{b^2\text{ArcTan}(cx)}{3c^3e}+\frac{b^2dx\text{ArcTan}(cx)}{ce^2}-\frac{bx^2(a+b\text{ArcTan}(cx))}{3ce}+\frac{id^2(a+b\text{ArcTan}(cx))^2}{ce^3}-\frac{d(a+b\text{A}}{2}$$

```
[Out] a*b*d*x/c/e^2+1/3*b^2*x/c^2/e-1/3*b^2*arctan(c*x)/c^3/e+b^2*d*x*arctan(c*x)
/c/e^2-1/3*b*x^2*(a+b*arctan(c*x))/c/e+I*b*d^3*(a+b*arctan(c*x))*polylog(2,
1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4-1/2*d*(a+b*arctan(c*x))^2/c^2/e^2-1/
3*I*(a+b*arctan(c*x))^2/c^3/e+d^2*x*(a+b*arctan(c*x))^2/e^3-1/2*d*x^2*(a+b*
arctan(c*x))^2/e^2+1/3*x^3*(a+b*arctan(c*x))^2/e+d^3*(a+b*arctan(c*x))^2*ln
(2/(1-I*c*x))/e^4+2*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e^3-2/3*b*(a+
b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/e-d^3*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)
/(c*d+I*e)/(1-I*c*x))/e^4-1/2*b^2*d*ln(c^2*x^2+1)/c^2/e^2+I*b^2*d^2*polylog
(2,1-2/(1+I*c*x))/c/e^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3/e+I*d^2*(a+b
*arctan(c*x))^2/c/e^3-I*b*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e^
4+1/2*b^2*d^3*polylog(3,1-2/(1-I*c*x))/e^4-1/2*b^2*d^3*polylog(3,1-2*c*(e*x
+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

Rubi [A]
time = 0.46, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 327, 209, 4968}

$$-\frac{i(a+b\text{ArcTan}(cx))^2}{3c^3e}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3e}-\frac{d(a+b\text{ArcTan}(cx))^2}{2c^2e^2}-\frac{ibd^3\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^4}+\frac{ibd^3(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4}+\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e^4}-\frac{d^3(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}+\frac{d^2x(a+b\text{ArcTan}(cx))^2}{e^3}+\frac{id^2(a+b\text{ArcTan}(cx))^2}{ce^3}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{ce^3}-\frac{dx^2(a+b\text{ArcTan}(cx))^2}{2e^2}+\frac{x^3(a+b\text{ArcTan}(cx))^2}{3e}-\frac{bx^2(a+b\text{ArcTan}(cx))}{3ce}+\frac{abdx}{ce^2}-\frac{b^2\text{ArcTan}(cx)}{3c^3e}+\frac{b^2dx\text{ArcTan}(cx)}{ce^2}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3e}-\frac{b^2d\log\left(c^2x^2+1\right)}{2c^2e^2}+\frac{b^2x}{3c^2e}+\frac{b^2d^3\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e^4}-\frac{b^2d^3\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{ib^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] (a*b*d*x)/(c*e^2) + (b^2*x)/(3*c^2*e) - (b^2*ArcTan[c*x])/(3*c^3*e) + (b^2*
d*x*ArcTan[c*x])/(c*e^2) - (b*x^2*(a + b*ArcTan[c*x]))/(3*c*e) + (I*d^2*(a
+ b*ArcTan[c*x])^2)/(c*e^3) - (d*(a + b*ArcTan[c*x])^2)/(2*c^2*e^2) - ((I/3
)*(a + b*ArcTan[c*x])^2)/(c^3*e) + (d^2*x*(a + b*ArcTan[c*x])^2)/e^3 - (d*x
^2*(a + b*ArcTan[c*x])^2)/(2*e^2) + (x^3*(a + b*ArcTan[c*x])^2)/(3*e) + (d^
3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^4 + (2*b*d^2*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/(c*e^3) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(3*c^3*e) - (d^3*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^4 - (b^2*d*Log[1 + c^2*x^2])/(2*c^2*e^2) - (I*b*d^3*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + (I*b^2*d^2*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c*e^3) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3
*e) + (I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^4 + (b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^4) -
(b^2*d^3*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))^2}{d+ex}\,dx = \int \left(\frac{d^2(a+b\tan^{-1}(cx))^2}{e^3} - \frac{dx(a+b\tan^{-1}(cx))^2}{e^2} + \frac{x^2(a+b\tan^{-1}(cx))^2}{e} - \frac{d^3}{}\right.$$

$$= \frac{d^2\int (a+b\tan^{-1}(cx))^2\,dx}{e^3} - \frac{d^3\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex}\,dx}{e^3} - \frac{d\int x(a+b\tan^{-1}(cx))^2\,}{e^2}$$

$$= \frac{d^2x(a+b\tan^{-1}(cx))^2}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))^2}{2e^2} + \frac{x^3(a+b\tan^{-1}(cx))^2}{3e} + \frac{d^3(a}{}$$

$$= \frac{id^2(a+b\tan^{-1}(cx))^2}{ce^3} + \frac{d^2x(a+b\tan^{-1}(cx))^2}{e^3} - \frac{dx^2(a+b\tan^{-1}(cx))^2}{2e^2} + \frac{x^3(}{}$$

$$= \frac{abdx}{ce^2} - \frac{bx^2(a+b\tan^{-1}(cx))}{3ce} + \frac{id^2(a+b\tan^{-1}(cx))^2}{ce^3} - \frac{d(a+b\tan^{-1}(cx))^2}{2c^2e^2} -$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2(a+b\tan^{-1}(cx))}{3ce} + \frac{id^2(a+b\tan^{-1}(cx))^2}{ce^3}$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} - \frac{b^2\tan^{-1}(cx)}{3c^3e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2(a+b\tan^{-1}(cx))}{3ce} + \frac{id^2(a}{}$$

$$= \frac{abdx}{ce^2} + \frac{b^2x}{3c^2e} - \frac{b^2\tan^{-1}(cx)}{3c^3e} + \frac{b^2dx\tan^{-1}(cx)}{ce^2} - \frac{bx^2(a+b\tan^{-1}(cx))}{3ce} + \frac{id^2(a}{}$$

Mathematica [F]
time = 156.36, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 35.82, size = 2208, normalized size = 3.69

method	result	size
derivativedivides	Expression too large to display	2208
default	Expression too large to display	2208

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/c^4*(4/3*b*a*c^3/e^3*d^2+1/3*b*a*c/e*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-2/3*b^2*c/e*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/3*b^2*c/e*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b^2*c^4*d^3/e^4*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+b^2*c^2/e^2*d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/2*b^2*c^2/e^2*d*arctan(c*x)^2+1/3*I*b^2*c/e*arctan(c*x)^2+2/3*I*b^2*c/e*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2/3*I*b^2*c/e*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2*c^5*d^4/e^4/(-I*e+c*d)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*b^2*c^4/e^4*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*arctan(c*x)^2+I*b^2*c^4*d^3/e^3/(-I*e+c*d)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I*b^2*c^5*d^4/e^4/(-I*e+c*d)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*b^2*c^4/e^4*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*b*a*c^4/e^4*d^3*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))-I*b*a*c^4/e^4*d^3*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))+2*b*a*c^4*arctan(c*x)/e^3*d^2*x-b*a*c^4*arctan(c*x)/e^2*d*x^2+1/2*I*b^2*c^4/e^4*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2*c^4/e^4*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*arctan(c*x)^2-1/3*b^2*c*arctan(c*x)/e+1/3*I*b^2*c/e-a^2*c^4*d^3/e^4*ln(c*e*x+c*d)+1/3*a^2*c^4/e*x^3+b*a*c^3/e^2*d*x+2/3*b*a*c^4*arctan(c*x)/e*x^3-2*b*a*c^4*arctan(c*x)*d^3/e^4*ln(c*e*x+c*d)+I*b*a*c^4/e^4*d^3*dilog((I*e+c*e*x)/(I*e-c*d))-I*b*a*c^4/e^4*d^3*dilog((I*e-c*e*x)/(c*d+I*e))+b^2*c^4*d^3/e^3/(-I*e+c*d)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-b^2*c^5*d^4/e^4/(-I*e+c*d)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*b^2*c^4*d^3/e^4*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2*c^4*d^3/e^3/(-I*e+c*d)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/3*b^2*c^2/e*x+b^2*c^4*arctan(c*x)^2/e^3*d^2*x-1/2*b^2*c^4*arctan(c*x)^2/e^2*d*x^2+b^2*c^3*d*arctan(c*x)/e^2*x+2*b^2*c^3/e^3*d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*c^3/e^3*d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*c^4*d^3/e^4*arctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)-b^2*c^4*arctan(c*x)^2*d^3/e^4*ln(c*e*x+c*d)-1/2*a^2*c^4/e^2*d*x^2+a^2*c^4/e^3*d^2*x-1/3*b*a*c^3/e*x^2+1/3*b^2*c^4*arctan(c*x)^2/e*x^3-1/3*b^2*c^3*arctan(c*x)/e*x^2-2*I*b^2*c^3/e^3*d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*c^3/e^3*d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b^2*c^3/e^3*d^2*arctan(c*x)^2-I*b^2*c^2*d*arctan(c*x)/e^2-b*a*c^3/e^3*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)*d^2-b*a*c^2/e^2*arctan(c*x)*d)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*(6*d^3*e^(-4)*log(x*e + d) - (2*x^3*e^2 - 3*d*x^2*e + 6*d^2*x)*e^(-3))
*a^2 + 1/96*(4*(2*b^2*x^3*e^2 - 3*b^2*d*x^2*e + 6*b^2*d^2*x)*arctan(c*x)^2
- (2*b^2*x^3*e^2 - 3*b^2*d*x^2*e + 6*b^2*d^2*x)*log(c^2*x^2 + 1)^2 + 96*e^3
*integrate(1/48*(36*(b^2*c^2*x^5*e^3 + b^2*x^3*e^3)*arctan(c*x)^2 + 3*(b^2*
c^2*x^5*e^3 + b^2*x^3*e^3)*log(c^2*x^2 + 1)^2 + 4*(24*a*b*c^2*x^5*e^3 - 2*b
^2*c*x^4*e^3 - 3*b^2*c*d^2*x^2*e - 6*b^2*c*d^3*x + (b^2*c*d*e^2 + 24*a*b*e^
3)*x^3)*arctan(c*x) + 2*(2*b^2*c^2*x^5*e^3 - b^2*c^2*d*x^4*e^2 + 3*b^2*c^2*
d^2*x^3*e + 6*b^2*c^2*d^3*x^2)*log(c^2*x^2 + 1))/(c^2*x^3*e^4 + c^2*d*x^2*e
^3 + x*e^4 + d*e^3), x))*e^(-3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(x*e + d
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{atan}\,(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (a + b \operatorname{atan}(c x))^2}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + e*x),x)
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.142 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=430

$$-\frac{abx}{ce}-\frac{b^2x\text{ArcTan}(cx)}{ce}-\frac{id(a+b\text{ArcTan}(cx))^2}{ce^2}+\frac{(a+b\text{ArcTan}(cx))^2}{2c^2e}-\frac{dx(a+b\text{ArcTan}(cx))^2}{e^2}+\frac{x^2(a+b\text{Arc}}{2e}$$

```
[Out] -a*b*x/c/e-b^2*x*arctan(c*x)/c/e-I*d*(a+b*arctan(c*x))^2/c/e^2+1/2*(a+b*arc
tan(c*x))^2/c^2/e-d*x*(a+b*arctan(c*x))^2/e^2+1/2*x^2*(a+b*arctan(c*x))^2/e
-d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^3-2*b*d*(a+b*arctan(c*x))*ln(2/(
1+I*c*x))/c/e^2+d^2*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))
/e^3+1/2*b^2*ln(c^2*x^2+1)/c^2/e+I*b*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1
-I*c*x))/e^3-I*b^2*d*polylog(2,1-2/(1+I*c*x))/c/e^2-I*b*d^2*(a+b*arctan(c*x
))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3-1/2*b^2*d^2*polylog(3,1
-2/(1-I*c*x))/e^3+1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/
e^3
```

Rubi [A]
time = 0.30, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 4968}

$$\frac{(a+b\text{ArcTan}(cx))^2}{2c^2e}+\frac{ibd^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^3}-\frac{ibd^2(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e^3}+\frac{d^2(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}-\frac{dx(a+b\text{ArcTan}(cx))^2}{e^2}-\frac{id(a+b\text{ArcTan}(cx))^2}{ce^2}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{ce^2}+\frac{x^2(a+b\text{ArcTan}(cx))^2}{2e}-\frac{abx}{ce}-\frac{b^2x\text{ArcTan}(cx)}{ce}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2e}-\frac{b^2d^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e^3}+\frac{b^2d^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{ib^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) - (I*d*(a + b*ArcTan[c*x])^2)/
(c*e^2) + (a + b*ArcTan[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTan[c*x])^2)/e^2
 + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) - (d^2*(a + b*ArcTan[c*x])^2*Log[2/(1
- I*c*x)])/e^3 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e^2) + (
d^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
^3 + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) + (I*b*d^2*(a + b*ArcTan[c*x])*PolyLo
g[2, 1 - 2/(1 - I*c*x)])/e^3 - (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e
^2) - (I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^3 - (b^2*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^3) +
(b^2*d^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{d+ex}\,dx = \int \left(-\frac{d(a+b\tan^{-1}(cx))^2}{e^2}+\frac{x(a+b\tan^{-1}(cx))^2}{e}+\frac{d^2(a+b\tan^{-1}(cx))^2}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int(a+b\tan^{-1}(cx))^2\,dx}{e^2}+\frac{d^2\int\frac{(a+b\tan^{-1}(cx))^2}{d+ex}\,dx}{e^2}+\frac{\int x(a+b\tan^{-1}(cx))^2\,d}{e}$$

$$= -\frac{dx(a+b\tan^{-1}(cx))^2}{e^2}+\frac{x^2(a+b\tan^{-1}(cx))^2}{2e}-\frac{d^2(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-i}\right.}{e^3}$$

$$= -\frac{id(a+b\tan^{-1}(cx))^2}{ce^2}-\frac{dx(a+b\tan^{-1}(cx))^2}{e^2}+\frac{x^2(a+b\tan^{-1}(cx))^2}{2e}-\frac{d^2(a}{}$$

$$= -\frac{abx}{ce}-\frac{id(a+b\tan^{-1}(cx))^2}{ce^2}+\frac{(a+b\tan^{-1}(cx))^2}{2c^2e}-\frac{dx(a+b\tan^{-1}(cx))^2}{e^2}+$$

$$= -\frac{abx}{ce}-\frac{b^2x\tan^{-1}(cx)}{ce}-\frac{id(a+b\tan^{-1}(cx))^2}{ce^2}+\frac{(a+b\tan^{-1}(cx))^2}{2c^2e}-\frac{dx(a+}{}$$

$$= -\frac{abx}{ce}-\frac{b^2x\tan^{-1}(cx)}{ce}-\frac{id(a+b\tan^{-1}(cx))^2}{ce^2}+\frac{(a+b\tan^{-1}(cx))^2}{2c^2e}-\frac{dx(a+}{}$$

Mathematica [F]
time = 83.16, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.64, size = 1855, normalized size = 4.31

method	result	size
derivativedivides	Expression too large to display	1855
default	Expression too large to display	1855

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(2*I*b^2*c^2*d/e^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2*c
^3/e^3*d^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2
+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2*c^4*d^3/e^3
/(-I*e+c*d)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+
1))-I*b^2*c^3*d^2/e^2/(-I*e+c*d)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+
I*c*x)^2/(c^2*x^2+1))+I*b*a*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I
*e))-I*b*a*c^3/e^3*d^2*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))-2*b*a*c^3*ar
ctan(c*x)/e^2*d*x+I*b^2*c^2/e^2*d*arctan(c*x)^2-b^2*c^3*d^2/e^3*arctan(c*x)
^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)+b^2
*c^3*arctan(c*x)^2*d^2/e^3*ln(c*e*x+c*d)+1/2*b^2*c^4*d^3/e^3/(-I*e+c*d)*pol
ylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-2*b^2*c^2*d/e^2*arctan(
c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*b^2*c^2*d/e^2*dilog(1+I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-a^2*c^3/e^2*d*x+b*a*c^2/e^2*d*ln(c^2*d^2-2*c*d*(c*e*
x+c*d)+e^2+(c*e*x+c*d)^2)-b*a*c^2/e*x+1/2*b^2*c^3*arctan(c*x)^2/e*x^2-b^2*c
^2*arctan(c*x)/e*x-2*b^2*c^2*d/e^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-b^2*c/e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2*b^2*c/e*arctan(c*x)^2+a^2
*c^3*d^2/e^3*ln(c*e*x+c*d)+1/2*a^2*c^3/e*x^2-b*a*c^2/e^2*d+b*a*c/e*arctan(c
*x)+I*b^2*c*arctan(c*x)/e-1/2*b^2*c^3*d^2/e^3*polylog(3,-(1+I*c*x)^2/(c^2*x
^2+1))-1/2*I*b^2*c^3/e^3*d^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*e*
(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*arctan(c*x)^2
-1/2*I*b^2*c^3/e^3*d^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x
)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2
/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2*c^3/e^3*d^2*Pi*csgn(I*(-I*e*(1+I*c
```

```
*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^
2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*
x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*arctan(c*x)^2+I*b^2*c^3*d^2/e^
3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+b^2*c^4*d^3/e^3/(-I*e+c*d
)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-b^2*c^3*d
^2/e^2/(-I*e+c*d)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^
2*x^2+1))-1/2*I*b^2*c^3*d^2/e^2/(-I*e+c*d)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1
+I*c*x)^2/(c^2*x^2+1))+b*a*c^3*arctan(c*x)/e*x^2-b^2*c^3*arctan(c*x)^2/e^2*
d*x+2*b*a*c^3*arctan(c*x)*d^2/e^3*ln(c*e*x+c*d)+I*b*a*c^3/e^3*d^2*dilog((I*
e-c*e*x)/(c*d+I*e))-I*b*a*c^3/e^3*d^2*dilog((I*e+c*e*x)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^2*e^(-3)*log(x*e + d) + (x^2*e - 2*d*x)*e^(-2))*a^2 + 1/32*(4*(b^2
*x^2*e - 2*b^2*d*x)*arctan(c*x)^2 - (b^2*x^2*e - 2*b^2*d*x)*log(c^2*x^2 + 1
)^2 + 32*e^2*integrate(1/16*(12*(b^2*c^2*x^4*e^2 + b^2*x^2*e^2)*arctan(c*x)
^2 + (b^2*c^2*x^4*e^2 + b^2*x^2*e^2)*log(c^2*x^2 + 1)^2 + 4*(8*a*b*c^2*x^4*
e^2 - b^2*c*x^3*e^2 + 2*b^2*c*d^2*x + (b^2*c*d*e + 8*a*b*e^2)*x^2)*arctan(c
*x) + 2*(b^2*c^2*x^4*e^2 - b^2*c^2*d*x^3*e - 2*b^2*c^2*d^2*x^2)*log(c^2*x^2
 + 1))/(c^2*x^3*e^3 + c^2*d*x^2*e^2 + x*e^3 + d*e^2), x))*e^(-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(x*e + d
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{atan}\,(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.143 $\int \frac{x(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=323

$$\frac{i(a+b\text{ArcTan}(cx))^2}{ce}+\frac{x(a+b\text{ArcTan}(cx))^2}{e}+\frac{d(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{2b(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-\right.}}{ce}$$

```
[Out] I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln(
2/(1-I*c*x))/e^2+2*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e-d*(a+b*arctan(c*
x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-I*b*d*(a+b*arctan(c*x))*polyl
og(2,1-2/(1-I*c*x))/e^2+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e+I*b*d*(a+b*arcta
n(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2+1/2*b^2*d*polylog(
3,1-2/(1-I*c*x))/e^2-1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))
/e^2
```

Rubi [A]
time = 0.19, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {4996, 4930, 5040, 4964, 2449, 2352, 4968}

$$-\frac{ibd\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^2}+\frac{ibd(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e^2}-\frac{d(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{x(a+b\text{ArcTan}(cx))^2}{e}+\frac{i(a+b\text{ArcTan}(cx))^2}{ce}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{ce}+\frac{b^2d\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e^2}-\frac{b^2d\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (d*(a + b*A
rcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 + (2*b*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/(c*e) - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/e^2 - (I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*
x)])/e^2 + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e) + (I*b*d*(a + b*ArcT
an[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^2 + (
b^2*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{e} - \frac{d(a+b\tan^{-1}(cx))^2}{e(d+ex)}\right) dx$$

$$= \frac{\int (a+b\tan^{-1}(cx))^2\,dx}{e} - \frac{d\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex}\,dx}{e}$$

$$= \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{d(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d(a+b\tan^{-1}(cx))^2\log}{e^2}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{d(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} -$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{d(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} +$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{d(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} +$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{d(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} +$$

Mathematica [**F**]
time = 148.84, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

```
[Out] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.03, size = 16245, normalized size = 50.29

method	result	size
derivativedivides	Expression too large to display	16245
default	Expression too large to display	16245

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -(d*e^(-2)*log(x*e + d) - x*e^(-1))*a^2 + 1/16*(4*b^2*x*arctan(c*x)^2 - b^2
*x*log(c^2*x^2 + 1)^2 + 16*e*integrate(1/16*(12*(b^2*c^2*x^3*e + b^2*x*e)*a
rctan(c*x)^2 + (b^2*c^2*x^3*e + b^2*x*e)*log(c^2*x^2 + 1)^2 + 8*(4*a*b*c^2*
x^3*e - b^2*c*x^2*e - (b^2*c*d - 4*a*b*e)*x)*arctan(c*x) + 4*(b^2*c^2*x^3*e
 + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*
e), x))*e^(-1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{atan}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + e*x),x)
[Out] int((x*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.144 $\int \frac{(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=223

$$-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{ib(a+b\text{ArcTan}(cx))\text{PolyLog}(2$$

```
[Out] -(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/e+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-I*b*
(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e-1/2*b^2*po
lylog(3,1-2/(1-I*c*x))/e+1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x
))/e
```

Rubi [A]
time = 0.03, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4968}

$$-\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e}+\frac{b^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}-\frac{b^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{ib(a$$

Mathematica [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{d + ex}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.00, size = 1324, normalized size = 5.94

method	result	size
derivativedivides	Expression too large to display	1324
default	Expression too large to display	1324

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*ln(c*e*x+c*d)/e*arctan(c*x)^2-b^2*c/e*arct
an(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c
*d)-1/2*I*b^2*c/e*arctan(c*x)^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1))*Pi-I*b^2*c*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)
*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*c*d)+I*a*b*c*ln(c*e*x+c*d)/e*ln((I*e-c*e*x)/
(c*d+I*e))+1/2*I*b^2*c/e*arctan(c*x)^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)
+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*Pi-I*a
*b*c*ln(c*e*x+c*d)/e*ln((I*e+c*e*x)/(I*e-c*d))-1/2*b^2*c/e*polylog(3,-(1+I*
c*x)^2/(c^2*x^2+1))+b^2*c^2/e*d/(-I*e+c*d)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*
d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+I*a*b*c/e*dilog((I*e-c*e*x)/(c*d+I*e))+1/2*
b^2*c^2/e*d/(-I*e+c*d)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1
))+b^2*c*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e
+I*c*d)+I*b^2*c/e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2*c
*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*c*d)+2*a*b*c*l
n(c*e*x+c*d)/e*arctan(c*x)-1/2*I*b^2*c/e*arctan(c*x)^2*csgn(I*(-I*e*(1+I*c*
x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+
I*e+c*d))*Pi-I*a*b*c/e*dilog((I*e+c*e*x)/(I*e-c*d))-I*b^2*c^2/e*d/(-I*e+c*d
)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*
b^2*c/e*arctan(c*x)^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/
(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)
+I*e+c*d))*Pi)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
[Out] a^2*e^(-1)*log(x*e + d) + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^
2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d),x)
[Out] Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atan}(c\,x))^2}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + e*x),x)
[Out] int((a + b*atan(c*x))^2/(d + e*x), x)
```

3.145 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x(d+ex)}\,dx$

Optimal. Leaf size=369

$$\frac{2(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+e}{(cd+ie)(1-}\right.}{d}$$

```
[Out] -2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/(
1-I*c*x))/d-(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d-I*b*(
a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylog(2
,1-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+I*b*(a+
b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*b^2*polyl
og(3,1-2/(1-I*c*x))/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/d+1/2*b^2*polylog(3,
-1+2/(1+I*c*x))/d-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d
```

Rubi [A]
time = 0.31, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4996, 4942, 5108, 5004, 5114, 6745, 4968}

$$\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{b^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (I*b*(a + b*Arc
Tan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + (b
^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)
])/(2*d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 -
 (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
```

```
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+ex)}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{e(a+b\tan^{-1}(cx))^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d} - \frac{e\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex}\,dx}{d}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b}{}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b}{}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b}{}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b}{}$$

Mathematica [F]
time = 148.18, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(cx))^2}{x(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.25, size = 2363, normalized size = 6.40

method	result	size
derivativedivides	Expression too large to display	2363
default	Expression too large to display	2363

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*ln(c*e*x+c*d)+a^2/d*ln(c*x)-b^2*e*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+
I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d/(e+I*c*d)-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^
2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
```

```
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x
)^2-1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2
+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arc
tan(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*arctan(c*x)^2+b^2/d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d
*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*arctan(c*x)^2*ln(c*x
)+b^2*arctan(c*x)^2/d*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*
x^2+1)+I*e+c*d)-b^2*arctan(c*x)^2/d*ln(c*e*x+c*d)-1/2*b^2*c/(-I*e+c*d)*poly
log(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+2*b*a/d*arctan(c*x)*ln(c
*x)+I*b*a/d*dilog(1+I*c*x)-I*b*a/d*dilog(1-I*c*x)-2*I*b^2/d*arctan(c*x)*pol
ylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/d*arctan(c*x)*polylog(2,(1+I*c
*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I*(
-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1+I*c
*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))+1/2*I*b^2/d*Pi*csgn
(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1
/2*I*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^3*arctan(c*x)^2-1/2*I*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b*a/d*ln(c*x)*ln(1+I*c*x)-I*b*a/d*ln
(c*x)*ln(1-I*c*x)-b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+2*b^2/d
*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+I*b^2*c/(-I*e+c*d)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))+I*b*a/d*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(I*e-c*d))-I*b
*a/d*ln(c*e*x+c*d)*ln((I*e-c*e*x)/(c*d+I*e))-1/2*I*b^2/d*Pi*arctan(c*x)^2*c
sgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3+I*b*a/d*dilog((I*e+c*e*x)/(I*e-c*d))-I*b*a/d*d
ilog((I*e-c*e*x)/(c*d+I*e))-b^2*c/(-I*e+c*d)*arctan(c*x)^2*ln(1-(I*e-c*d)/(
c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*b^2*e*polylog(3,(I*e-c*d)/(c*d+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))/d/(e+I*c*d)+1/2*I*b^2/d*Pi*arctan(c*x)^2-2*b*a*arct
an(c*x)/d*ln(c*e*x+c*d)+I*b^2*e*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))/d/(e+I*c*d)+1/2*I*b^2/d*Pi*arctan(c*x)^2*csgn(I*(-I
*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))+1/2*I*b^2/d*Pi*arct
an(c*x)^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+
I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1
)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(x*e + d)/d - log(x)/d) + integrate(1/16*(12*b^2*arctan(c*x)^2 + b
^2*log(c^2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(x^2*e + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atan}(cx))^2}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{atan}(c\,x))^2}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + e*x)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x*(d + e*x)), x)
```

3.146 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2(d+ex)}\,dx$

Optimal. Leaf size=473

$$-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}-\frac{2e(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2}-\frac{e(a+b\text{ArcTan}(}{d}$$

```
[Out] -I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*e*(a+b*arctan(c*x))^2*
arctanh(-1+2/(1+I*c*x))/d^2-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2+e*(a+
b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2+2*b*c*(a+b*arctan(
c*x))*ln(2-2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/
d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylog(2,1
-2/(1+I*c*x))/d^2-I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-I*b
*e*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2-1/2*b
^2*e*polylog(3,1-2/(1-I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x))/d^2-1/
2*b^2*e*polylog(3,-1+2/(1+I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2*c*(e*x+d)/(c*
d+I*e)/(1-I*c*x))/d^2
```

Rubi [A]

time = 0.43, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {4996, 4946, 5044, 4988, 2497, 4942, 5108, 5004, 5114, 6745, 4968}

$$\frac{ibe\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}+\frac{ibe\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{ibe\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{ibe(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2}-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}+\frac{e(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{2e\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{b^2e\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2d^2}+\frac{b^2e\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2d^2}-\frac{b^2e\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^2}+\frac{b^2e\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}-\frac{ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) - (2*e*(a +
b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - (e*(a + b*ArcTan[c*x])^2
*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c
*d + I*e)*(1 - I*c*x))])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*
c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 -
(I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1
+ 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d +
 e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x
)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/(2*d^2)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+ex)}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{dx^2} - \frac{e(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{e^2(a+b\tan^{-1}(cx))^2}{d^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}dx}{d} - \frac{e\int \frac{(a+b\tan^{-1}(cx))^2}{x}dx}{d^2} + \frac{e^2\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex}dx}{d^2}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{2e(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e(a+b\tan^{-1}(\ldots}{\ldots}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{2e(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\ldots\right.}{d^2}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{2e(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\ldots\right.}{d^2}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{2e(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\ldots\right.}{d^2}$$

Mathematica [F]
time = 83.70, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(cx))^2}{x^2(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.75, size = 40859, normalized size = 86.38

method	result	size
derivativedivides	Expression too large to display	40859
default	Expression too large to display	40859

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/16*(4*b^2*arctan(c*x)
^2 - b^2*log(c^2*x^2 + 1)^2 - 16*d*x*integrate(1/16*(12*(b^2*c^2*d*x^2 + b^
2*d)*arctan(c*x)^2 + (b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 + 8*(b^2*c*
d*x + 4*a*b*d + (4*a*b*c^2*d + b^2*c*e)*x^2)*arctan(c*x) - 4*(b^2*c^2*x^3*e
 + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*d*x^5*e + c^2*d^2*x^4 + d*x^3*e +
d^2*x^2), x))/(d*x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^3*e + d*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + e*x)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + e*x)), x)
```

3.147 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^3(d+ex)}\,dx$

Optimal. Leaf size=591

$$-\frac{bc(a+b\text{ArcTan}(cx))}{dx}-\frac{c^2(a+b\text{ArcTan}(cx))^2}{2d}+\frac{ice(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{(a+b\text{ArcTan}(cx))^2}{2dx^2}+\frac{e(a+b\text{ArcT}}{d^2x}$$

```
[Out] -b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d+I*c*e*(a+b*arctan(
c*x))^2/d^2-1/2*(a+b*arctan(c*x))^2/d/x^2+e*(a+b*arctan(c*x))^2/d^2/x-2*e^2
*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3+b^2*c^2*ln(x)/d+e^2*(a+b*a
rctan(c*x))^2*ln(2/(1-I*c*x))/d^3-e^2*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c
*d+I*e)/(1-I*c*x))/d^3-1/2*b^2*c^2*ln(c^2*x^2+1)/d-2*b*c*e*(a+b*arctan(c*x)
)*ln(2-2/(1-I*c*x))/d^2+I*b*e^2*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/d^3-I*b*e^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/
d^3+I*b^2*c*e*polylog(2,-1+2/(1-I*c*x))/d^2-I*b*e^2*(a+b*arctan(c*x))*polyl
og(2,1-2/(1-I*c*x))/d^3+I*b*e^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))
/d^3+1/2*b^2*e^2*polylog(3,1-2/(1-I*c*x))/d^3-1/2*b^2*e^2*polylog(3,1-2/(1+
I*c*x))/d^3+1/2*b^2*e^2*polylog(3,-1+2/(1+I*c*x))/d^3-1/2*b^2*e^2*polylog(3
,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

Rubi [A]
time = 0.54, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {4996, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 2497, 4942, 5108, 5114, 6745, 4968}

$$-\frac{c^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{ibe^2\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^3}-\frac{ibe^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{ibe^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{ibe^2(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3}+\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}-\frac{e^2(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{2e^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d^3}+\frac{ice(a+b\text{ArcTan}(cx))^2}{d^2}+\frac{e(a+b\text{ArcTan}(cx))^2}{d^2x}-\frac{2bce\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}-\frac{(a+b\text{ArcTan}(cx))^2}{2dx^2}-\frac{bc(a+b\text{ArcTan}(cx))}{dx}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d}+\frac{b^2c^2\log(x)}{d}+\frac{b^2e^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2d^3}-\frac{b^2e^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2d^3}+\frac{b^2e^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d^3}-\frac{b^2e^2\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}+\frac{ib^2ce\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) + (I
*c*e*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d*x^2) + (e*(a +
 b*ArcTan[c*x])^2)/(d^2*x) + (2*e^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
+ I*c*x)])/d^3 + (b^2*c^2*Log[x])/d + (e^2*(a + b*ArcTan[c*x])^2*Log[2/(1 -
 I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - (2*b*c*e*(a + b*Ar
cTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^3 + (I*b^2*c*e*PolyLog[2, -1 + 2/(1 - I*c*x)])/
d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3 + (I*
b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (I*b*e^2*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/d^3 + (b^2*e^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^3) - (b^2*e^2*PolyLog[
3, 1 - 2/(1 + I*c*x)])/(2*d^3) + (b^2*e^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(
```

```
2*d^3) - (b^2*e^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]
)/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+ex)}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{dx^3} - \frac{e(a+b\tan^{-1}(cx))^2}{d^2x^2} + \frac{e^2(a+b\tan^{-1}(cx))^2}{d^3x} - \frac{e^3(a+$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx}{d} - \frac{e\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^3} - \frac{e^3\int \frac{(a+b\tan^{-1}}{}}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{2e^2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\right.}{d^3}$$

$$= \frac{ice(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b\tan^{-1}(cx))^2}{2dx^2} + \frac{e(a+b\tan^{-1}(cx))^2}{d^2x} + \frac{2e^2(a+b\,\mathrm{t}}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} + \frac{ice(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} + \frac{ice(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} + \frac{ice(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} + \frac{ice(a+b\tan^{-1}(cx))^2}{d^2} - \frac{(a+b}{}$$

Mathematica [F]
time = 165.04, size = 0, normalized size = 0.00

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x^3(d + ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 33.84, size = 3004, normalized size = 5.08

method	result	size
derivativedivides	Expression too large to display	3004
default	Expression too large to display	3004

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-2*b^2/c/d^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))*e*arctan(c*x)-1/2*b^2/d
*arctan(c*x)^2+2*b^2/c^2*e^2/d^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*
b^2/c^2*e^2/d^3*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+a^2/c^2/d^3*e^2*ln(c
*x)-a^2/c^2/d^3*e^2*ln(c*e*x+c*d)+1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/c^2/d^
3*e^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I
*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*ar
ctan(c*x)^2-b*a/d*arctan(c*x)/c^2/x^2+b^2/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2
))+b^2/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-I*b*a/c^2/d^3*e^2*ln(c*e*x+c*d)*
ln((I*e-c*e*x)/(c*d+I*e))-I*b*a/c^2/d^3*e^2*ln(c*x)*ln(1-I*c*x)+I*b*a/c^2/d
^3*e^2*ln(c*x)*ln(1+I*c*x)+I*b*a/c^2/d^3*e^2*ln(c*e*x+c*d)*ln((I*e+c*e*x)/(
I*e-c*d))+2*b*a/c^2*arctan(c*x)/d^2*e/x+I*b^2/c^2*e^3*arctan(c*x)*polylog(2
,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^3/(e+I*c*d)-1/2*I*b^2/c^2/d
^3*e^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+
I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-I*b^2/c*e^2*arctan(c*
x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*c*d)+1/2*I*
b^2/c^2/d^3*e^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^3*arctan(c*x)^2+1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^2/c^2/d^3*e^2*P
i*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*
x)^2-I*b^2/d*arctan(c*x)-1/2*a^2/d/c^2/x^2+b^2/c^2*e^2/d^3*arctan(c*x)^2*ln
(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)+I*b^2/c/
```

```
d^2*e*arctan(c*x)^2-b^2/c^2*arctan(c*x)^2/d^3*e^2*ln(c*e*x+c*d)+b^2/c^2*arc
tan(c*x)^2/d^3*e^2*ln(c*x)-b^2/c^2*e^2/d^3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^
2*x^2+1)-1)+b*a/c/d^2*e*ln(c^2*x^2+1)-2*b*a/c/d^2*e*ln(c*x)+a^2/c^2/d^2*e/x
+b^2/c^2*e^2/d^3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/c^2*e^
2/d^3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2/c^2*e^3*polyl
og(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^3/(e+I*c*d)-2*I*b^2/c*d
ilog((1+I*c*x)/(c^2*x^2+1)^(1/2))/d^2*e+2*I*b^2/c/d^2*dilog(1+(1+I*c*x)/(c^
2*x^2+1)^(1/2))*e-b*a/d/c/x-b^2/d*arctan(c*x)/c/x-1/2*b^2/d*arctan(c*x)^2/c
^2/x^2-b^2/c*e^2*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2
*x^2+1))/d^2/(e+I*c*d)-b^2/c^2*e^3*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))/d^3/(e+I*c*d)-1/2*I*b^2/c*e^2*polylog(3,(I*e-c*d)/(
c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*c*d)-2*I*b^2/c^2*e^2/d^3*arctan(
c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2/c^2*e^2/d^3*arctan(c*x
)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2/c^2/d^3*e^2*Pi*arctan(c*
x)^2-2*b*a/c^2*arctan(c*x)/d^3*e^2*ln(c*e*x+c*d)+2*b*a/c^2*arctan(c*x)/d^3*
e^2*ln(c*x)+I*b*a/c^2/d^3*e^2*dilog(1+I*c*x)+I*b*a/c^2/d^3*e^2*dilog((I*e+c
*e*x)/(I*e-c*d))-I*b*a/c^2/d^3*e^2*dilog((I*e-c*e*x)/(c*d+I*e))-I*b*a/c^2/d
^3*e^2*dilog(1-I*c*x)+b^2/c^2*arctan(c*x)^2/d^2*e/x-a*b/d*arctan(c*x)+1/2*I
*b^2/c^2/d^3*e^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c
^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*(-I
*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c
^2*x^2+1)+I*e+c*d))*arctan(c*x)^2-1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)
)*arctan(c*x)^2-1/2*I*b^2/c^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(2*e^2*log(x*e + d)/d^3 - 2*e^2*log(x)/d^3 - (2*x*e - d)/(d^2*x^2)
) + 1/32*(32*d^2*x^2*integrate(1/16*(12*(b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(
c*x)^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*log(c^2*x^2 + 1)^2 - 4*(2*b^2*c*x^3*e^
2 - b^2*c*d^2*x - 8*a*b*d^2 - (8*a*b*c^2*d^2 - b^2*c*d*e)*x^2)*arctan(c*x)
```

```
+ 2*(2*b^2*c^2*x^4*e^2 + b^2*c^2*d*x^3*e - b^2*c^2*d^2*x^2)*log(c^2*x^2 + 1
))/(c^2*d^2*x^6*e + c^2*d^3*x^5 + d^2*x^4*e + d^3*x^3), x) + 4*(2*b^2*x*e -
 b^2*d)*arctan(c*x)^2 - (2*b^2*x*e - b^2*d)*log(c^2*x^2 + 1)^2)/(d^2*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^4*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x^3 (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x+d),x)
[Out] Integral((a + b*atan(c*x))**2/(x**3*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atan}(c\,x))^2}{x^3\,(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^3*(d + e*x)),x)
[Out] int((a + b*atan(c*x))^2/(x^3*(d + e*x)), x)
```

3.148 $\int \frac{1}{(d+ex)(a+b\mathbf{ArcTan}(cx))}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{(d+ex)(a+b\text{ArcTan}(cx))},x\right)$$

`[Out] Unintegrable(1/(e*x+d)/(a+b*arctan(c*x)),x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)(a+b\text{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]`

`[Out] Defer[Int][1/((d + e*x)*(a + b*ArcTan[c*x])), x]`

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(a+b\text{ArcTan}(cx))}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]`

`[Out] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]`

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\left(a+b\arctan\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
[Out] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arctan(c*x) + a)*(x*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*x*e + a*d + (b*x*e + b*d)*arctan(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{atan}(cx))\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
[Out] Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b\operatorname{atan}(c\,x))\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*atan(c*x))*(d + e*x)),x)
[Out] int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

3.149 $\int x^3(c+a^2cx^2)\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=69

$$\frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\text{ArcTan}(ax)}{12a^4}+\frac{1}{4}cx^4\text{ArcTan}(ax)+\frac{1}{6}a^2cx^6\text{ArcTan}(ax)$$

```
[Out] 1/12*c*x/a^3-1/36*c*x^3/a-1/30*a*c*x^5-1/12*c*arctan(a*x)/a^4+1/4*c*x^4*arc
tan(a*x)+1/6*a^2*c*x^6*arctan(a*x)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5070, 4946, 308, 209}

$$-\frac{c\text{ArcTan}(ax)}{12a^4}+\frac{cx}{12a^3}+\frac{1}{6}a^2cx^6\text{ArcTan}(ax)+\frac{1}{4}cx^4\text{ArcTan}(ax)-\frac{1}{30}acx^5-\frac{cx^3}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4) +
 (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx = c\int x^3\tan^{-1}(ax)\,dx + \left(a^2c\right)\int x^5\tan^{-1}(ax)\,dx$$
$$= \frac{1}{4}cx^4\tan^{-1}(ax) + \frac{1}{6}a^2cx^6\tan^{-1}(ax) - \frac{1}{4}(ac)\int\frac{x^4}{1+a^2x^2}\,dx - \frac{1}{6}\left(a^3c\right)\int$$
$$= \frac{1}{4}cx^4\tan^{-1}(ax) + \frac{1}{6}a^2cx^6\tan^{-1}(ax) - \frac{1}{4}(ac)\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\frac{1}{a^4\left(1+a^2x\right.}\right.$$
$$= \frac{cx}{12a^3} - \frac{cx^3}{36a} - \frac{1}{30}acx^5 + \frac{1}{4}cx^4\tan^{-1}(ax) + \frac{1}{6}a^2cx^6\tan^{-1}(ax) + \frac{c\int\frac{1}{1+a^2x^2}}{6a^3}$$
$$= \frac{cx}{12a^3} - \frac{cx^3}{36a} - \frac{1}{30}acx^5 - \frac{c\tan^{-1}(ax)}{12a^4} + \frac{1}{4}cx^4\tan^{-1}(ax) + \frac{1}{6}a^2cx^6\tan^{-1}(a$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.00

$$\frac{cx}{12a^3} - \frac{cx^3}{36a} - \frac{1}{30}acx^5 - \frac{c\text{ArcTan}(ax)}{12a^4} + \frac{1}{4}cx^4\text{ArcTan}(ax) + \frac{1}{6}a^2cx^6\text{ArcTan}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
[Out] (c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4) +
 (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

Maple [A]
time = 0.17, size = 60, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$
default	$\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$
risch	$-\frac{icx^4\left(2a^2x^2+3\right)\ln(iax+1)}{24}+\frac{ica^2x^6\ln(-iax+1)}{12}-\frac{acx^5}{30}+\frac{icx^4\ln(-iax+1)}{8}-\frac{cx^3}{36a}+\frac{cx}{12a^3}-\frac{c\arctan(ax}{12a^4}$

meijerg	$\frac{c\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/6*c*arctan(a*x)*a^6*x^6+1/4*c*arctan(a*x)*a^4*x^4-1/12*c*(2/5*a^5*
x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

Maxima [A]
time = 0.47, size = 64, normalized size = 0.93

$$-\frac{1}{180}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan\left(ax\right)}{a^5}\right)+\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/180*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)
+ 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)
```

Fricas [A]
time = 2.73, size = 57, normalized size = 0.83

$$-\frac{6\,a^5cx^5+5\,a^3cx^3-15\,acx-15\left(2\,a^6cx^6+3\,a^4cx^4-c\right)\arctan\left(ax\right)}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/180*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x - 15*(2*a^6*c*x^6 + 3*a^4*c*x^
4 - c)*arctan(a*x))/a^4
```

Sympy [A]
time = 0.32, size = 65, normalized size = 0.94

$$\begin{cases}\frac{a^2cx^6\operatorname{atan}\left(ax\right)}{6}-\frac{acx^5}{30}+\frac{cx^4\operatorname{atan}\left(ax\right)}{4}-\frac{cx^3}{36a}+\frac{cx}{12a^3}-\frac{c\operatorname{atan}\left(ax\right)}{12a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**6*atan(a*x)/6 - a*c*x**5/30 + c*x**4*atan(a*x)/4 - c*x
**3/(36*a) + c*x/(12*a**3) - c*atan(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.31, size = 57, normalized size = 0.83

$$-\frac{c\,\left(15\,\mathrm{atan}\left(a\,x\right)-15\,a\,x+5\,a^3\,x^3+6\,a^5\,x^5-45\,a^4\,x^4\,\mathrm{atan}\left(a\,x\right)-30\,a^6\,x^6\,\mathrm{atan}\left(a\,x\right)\right)}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2),x)
```

```
[Out] -(c*(15*atan(a*x) - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5 - 45*a^4*x^4*atan(a*x) -
 30*a^6*x^6*atan(a*x)))/(180*a^4)
```

3.150 $\int x^2(c+a^2cx^2)\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=66

$$-\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\text{ArcTan}(ax)+\frac{1}{5}a^2cx^5\text{ArcTan}(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

```
[Out] -1/15*c*x^2/a-1/20*a*c*x^4+1/3*c*x^3*arctan(a*x)+1/5*a^2*c*x^5*arctan(a*x)+
1/15*c*ln(a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5070, 4946, 272, 45}

$$\frac{1}{5}a^2cx^5\text{ArcTan}(ax)+\frac{c\log(a^2x^2+1)}{15a^3}+\frac{1}{3}cx^3\text{ArcTan}(ax)-\frac{1}{20}acx^4-\frac{cx^2}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -1/15*(c*x^2)/a - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan[
a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int x^2(c+a^2cx^2)\tan^{-1}(ax)\,dx &= c\int x^2\tan^{-1}(ax)\,dx+(a^2c)\int x^4\tan^{-1}(ax)\,dx\\ &= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{3}(ac)\int\frac{x^3}{1+a^2x^2}\,dx-\frac{1}{5}(a^3c)\int \\ &= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{6}(ac)\text{Subst}\left(\int\frac{x}{1+a^2x}\,dx,x,x^2\right) \\ &= \frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)-\frac{1}{6}(ac)\text{Subst}\left(\int\left(\frac{1}{a^2}-\frac{1}{a^2(1+a^2x}\right.\right. \\ &= -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\tan^{-1}(ax)+\frac{1}{5}a^2cx^5\tan^{-1}(ax)+\frac{c\log(1+a^2x^2)}{15a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 1.00

$$-\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\text{ArcTan}(ax)+\frac{1}{5}a^2cx^5\text{ArcTan}(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
[Out] -1/15*(c*x^2)/a - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan[
a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$
default	$\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$

risch	$-\frac{icx^3(3a^2x^2+5)\ln(iax+1)}{30}+\frac{ica^2x^5\ln(-iax+1)}{10}-\frac{acx^4}{20}+\frac{icx^3\ln(-iax+1)}{6}-\frac{cx^2}{15a}+\frac{c\ln(-a^2x^2-1)}{15a^3}-\frac{c}{45a}$
meijerg	$\frac{c\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{5}\right)}{4a^3}+\frac{c\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/5*c*arctan(a*x)*a^5*x^5+1/3*c*arctan(a*x)*a^3*x^3-1/15*c*(3/4*a^4*
x^4+a^2*x^2-ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 63, normalized size = 0.95

$$-\frac{1}{60}a\left(\frac{3a^2cx^4+4cx^2}{a^2}-\frac{4c\log(a^2x^2+1)}{a^4}\right)+\frac{1}{15}(3a^2cx^5+5cx^3)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*a*((3*a^2*c*x^4 + 4*c*x^2)/a^2 - 4*c*log(a^2*x^2 + 1)/a^4) + 1/15*(3*
a^2*c*x^5 + 5*c*x^3)*arctan(a*x)
```

Fricas [A]
time = 2.10, size = 62, normalized size = 0.94

$$-\frac{3a^4cx^4+4a^2cx^2-4(3a^5cx^5+5a^3cx^3)\arctan(ax)-4c\log(a^2x^2+1)}{60a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*c*x^4 + 4*a^2*c*x^2 - 4*(3*a^5*c*x^5 + 5*a^3*c*x^3)*arctan(a*x
) - 4*c*log(a^2*x^2 + 1))/a^3
```

Sympy [A]
time = 0.25, size = 61, normalized size = 0.92

$$\begin{cases}\frac{a^2cx^5\operatorname{atan}(ax)}{5}-\frac{acx^4}{20}+\frac{cx^3\operatorname{atan}(ax)}{3}-\frac{cx^2}{15a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{15a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**5*atan(a*x)/5 - a*c*x**4/20 + c*x**3*atan(a*x)/3 - c*x
**2/(15*a) + c*log(x**2 + a**(-2))/(15*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.23, size = 58, normalized size = 0.88

$$\frac{\frac{c\,\ln\left(a^2\,x^2+1\right)}{15}-\frac{a^2\,c\,x^2}{15}}{a^3}+\frac{c\,x^3\,\mathrm{atan}\left(a\,x\right)}{3}-\frac{a\,c\,x^4}{20}+\frac{a^2\,c\,x^5\,\mathrm{atan}\left(a\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2),x)
```

```
[Out] ((c*log(a^2*x^2 + 1))/15 - (a^2*c*x^2)/15)/a^3 + (c*x^3*atan(a*x))/3 - (a*c
*x^4)/20 + (a^2*c*x^5*atan(a*x))/5
```

3.151 $\int x(c+a^2cx^2)\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=42

$$-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)}{4a^2}$$

```
[Out] -1/4*c*x/a-1/12*a*c*x^3+1/4*c*(a^2*x^2+1)^2*arctan(a*x)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5050}

$$\frac{c(a^2x^2+1)^2\,\mathrm{ArcTan}(ax)}{4a^2}-\frac{1}{12}acx^3-\frac{cx}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -1/4*(c*x)/a - (a*c*x^3)/12 + (c*(1 + a^2*x^2)^2*ArcTan[a*x])/(4*a^2)
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx&=\frac{c(1+a^2x^2)^2\tan^{-1}(ax)}{4a^2}-\frac{\int \left(c+a^2cx^2\right)\,dx}{4a}\\&=-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c(1+a^2x^2)^2\tan^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.38

$$-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c\mathrm{ArcTan}(ax)}{4a^2}+\frac{1}{2}cx^2\mathrm{ArcTan}(ax)+\frac{1}{4}a^2cx^4\mathrm{ArcTan}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -1/4*(c*x)/a - (a*c*x^3)/12 + (c*ArcTan[a*x])/(4*a^2) + (c*x^2*ArcTan[a*x])
/2 + (a^2*c*x^4*ArcTan[a*x])/4
```

Maple [A]
time = 0.11, size = 54, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)}{2}+\frac{c\arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$
default	$\frac{\frac{c\arctan(ax)a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)}{2}+\frac{c\arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$
meijerg	$\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4a^2}+\frac{c\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4a^2}$
risch	$-\frac{ic\left(a^2x^2+1\right)^2\ln(iax+1)}{8a^2}+\frac{ic\,a^2x^4\ln(-iax+1)}{8}-\frac{acx^3}{12}+\frac{icx^2\ln(-iax+1)}{4}-\frac{cx}{4a}+\frac{ic\ln\left(a^2x^2+1\right)}{16a^2}+\frac{c\arcta}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4*c*arctan(a*x)*a^4*x^4+1/2*a^2*c*x^2*arctan(a*x)+1/4*c*arctan(a*x
)-1/4*c*(1/3*a^3*x^3+a*x))
```

Maxima [A]
time = 0.27, size = 50, normalized size = 1.19

$$\frac{\left(a^2cx^2+c\right)^2\arctan(ax)}{4\,a^2c}-\frac{a^2c^2x^3+3\,c^2x}{12\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*c*x^2 + c)^2*arctan(a*x)/(a^2*c) - 1/12*(a^2*c^2*x^3 + 3*c^2*x)/(a
*c)
```

Fricas [A]
time = 2.25, size = 44, normalized size = 1.05

$$-\frac{a^3cx^3+3\,acx-3\left(a^4cx^4+2\,a^2cx^2+c\right)\arctan(ax)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/12*(a^3*c*x^3 + 3*a*c*x - 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))/a
^2
```

Sympy [A]
time = 0.19, size = 54, normalized size = 1.29

$$\begin{cases} \frac{a^2cx^4\operatorname{atan}(ax)}{4} - \frac{acx^3}{12} + \frac{cx^2\operatorname{atan}(ax)}{2} - \frac{cx}{4a} + \frac{c\operatorname{atan}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**4*atan(a*x)/4 - a*c*x**3/12 + c*x**2*atan(a*x)/2 - c*x
/(4*a) + c*atan(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.48, size = 48, normalized size = 1.14

$$\frac{\frac{c\operatorname{atan}(a\,x)}{4} - \frac{a\,c\,x}{4}}{a^2} + \frac{c\,x^2\operatorname{atan}(a\,x)}{2} - \frac{a\,c\,x^3}{12} + \frac{a^2\,c\,x^4\operatorname{atan}(a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)*(c + a^2*c*x^2),x)
```

```
[Out] ((c*atan(a*x))/4 - (a*c*x)/4)/a^2 + (c*x^2*atan(a*x))/2 - (a*c*x^3)/12 + (a
^2*c*x^4*atan(a*x))/4
```

3.152 $\int (c+a^2cx^2) \text{ArcTan}(ax) \, dx$

Optimal. Leaf size=50

$$-\frac{1}{6}acx^2+cx\text{ArcTan}(ax)+\frac{1}{3}a^2cx^3\text{ArcTan}(ax)-\frac{c\log(1+a^2x^2)}{3a}$$

```
[Out] -1/6*a*c*x^2+c*x*arctan(a*x)+1/3*a^2*c*x^3*arctan(a*x)-1/3*c*ln(a^2*x^2+1)/
a
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4998, 4930, 266}

$$\frac{1}{3}cx(a^2x^2+1)\text{ArcTan}(ax)-\frac{c(a^2x^2+1)}{6a}-\frac{c\log(a^2x^2+1)}{3a}+\frac{2}{3}cx\text{ArcTan}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -1/6*(c*(1 + a^2*x^2))/a + (2*c*x*ArcTan[a*x])/3 + (c*x*(1 + a^2*x^2)*ArcTa
n[a*x])/3 - (c*Log[1 + a^2*x^2])/(3*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int (c+a^2cx^2)\tan^{-1}(ax)\,dx = -\frac{c(1+a^2x^2)}{6a} + \frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax) + \frac{1}{3}(2c)\int \tan^{-1}(ax)\,dx$$
$$= -\frac{c(1+a^2x^2)}{6a} + \frac{2}{3}cx\tan^{-1}(ax) + \frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax) - \frac{1}{3}(2ac)\int \frac{x}{1+a$$
$$= -\frac{c(1+a^2x^2)}{6a} + \frac{2}{3}cx\tan^{-1}(ax) + \frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax) - \frac{c\log(1+a^2x^2)}{3a}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{1}{6}acx^2 + cx\text{ArcTan}(ax) + \frac{1}{3}a^2cx^3\text{ArcTan}(ax) - \frac{c\log(1+a^2x^2)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] -1/6*(a*c*x^2) + c*x*ArcTan[a*x] + (a^2*c*x^3*ArcTan[a*x])/3 - (c*Log[1 + a
^2*x^2])/(3*a)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.98

method	result	size
derivativedivides	$\frac{\frac{c\arctan(ax)a^3x^3}{3}+acx\arctan(ax)-\frac{c\left(\frac{a^2x^2}{2}+\ln\left(a^2x^2+1\right)\right)}{3}}{a}$	49
default	$\frac{\frac{c\arctan(ax)a^3x^3}{3}+acx\arctan(ax)-\frac{c\left(\frac{a^2x^2}{2}+\ln\left(a^2x^2+1\right)\right)}{3}}{a}$	49
risch	$-\frac{icx(a^2x^2+3)\ln(iax+1)}{6}+\frac{ic\,a^2x^3\ln(-iax+1)}{6}-\frac{ac\,x^2}{6}+\frac{icx\ln(-iax+1)}{2}-\frac{c\ln(-a^2x^2-1)}{3a}$	79
meijerg	$\frac{c\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{3}\right)}{4a}+\frac{c\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/3*c*arctan(a*x)*a^3*x^3+a*c*x*arctan(a*x)-1/3*c*(1/2*a^2*x^2+ln(a^2*
x^2+1)))
```

Maxima [A]
time = 0.26, size = 45, normalized size = 0.90

$$-\frac{1}{6}\left(cx^2 + \frac{2c\log(a^2x^2+1)}{a^2}\right)a + \frac{1}{3}\left(a^2cx^3 + 3cx\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/6*(c*x^2 + 2*c*log(a^2*x^2 + 1)/a^2)*a + 1/3*(a^2*c*x^3 + 3*c*x)*arctan(
a*x)
```

Fricas [A]
time = 2.32, size = 47, normalized size = 0.94

$$-\frac{a^2cx^2 - 2\,(a^3cx^3 + 3\,acx)\arctan{(ax)} + 2\,c\log{(a^2x^2 + 1)}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*c*x^2 - 2*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1
))/a
```

Sympy [A]
time = 0.17, size = 48, normalized size = 0.96

$$\begin{cases} \frac{a^2cx^3\,\mathrm{atan}\,(ax)}{3} - \frac{acx^2}{6} + cx\,\mathrm{atan}\,(ax) - \frac{c\log\left(x^2 + \frac{1}{a^2}\right)}{3a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] Piecewise((a**2*c*x**3*atan(a*x)/3 - a*c*x**2/6 + c*x*atan(a*x) - c*log(x**
2 + a**(-2))/(3*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.16, size = 46, normalized size = 0.92

$$-\frac{c\,(2\,\ln{(a^2\,x^2 + 1)} + a^2\,x^2 - 2\,a^3\,x^3\,\mathrm{atan}(a\,x) - 6\,a\,x\,\mathrm{atan}(a\,x))}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2),x)
```

```
[Out] -(c*(2*log(a^2*x^2 + 1) + a^2*x^2 - 2*a^3*x^3*atan(a*x) - 6*a*x*atan(a*x)))
/(6*a)
```

3.153 $\int \frac{(c+a^2cx^2)\,\mathbf{ArcTan}(ax)}{x}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{2}acx+\frac{1}{2}c\text{ArcTan}(ax)+\frac{1}{2}a^2cx^2\text{ArcTan}(ax)+\frac{1}{2}ic\text{PolyLog}(2,-iax)-\frac{1}{2}ic\text{PolyLog}(2,iax)$$

```
[Out] -1/2*a*c*x+1/2*c*arctan(a*x)+1/2*a^2*c*x^2*arctan(a*x)+1/2*I*c*polylog(2,-I
*a*x)-1/2*I*c*polylog(2,I*a*x)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5070, 4940, 2438, 4946, 327, 209}

$$\frac{1}{2}a^2cx^2\text{ArcTan}(ax)+\frac{1}{2}c\text{ArcTan}(ax)+\frac{1}{2}ic\text{Li}_2(-iax)-\frac{1}{2}ic\text{Li}_2(iax)-\frac{acx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

```
[Out] -1/2*(a*c*x) + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*Poly
Log[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x}\,dx &= c\int \frac{\tan^{-1}(ax)}{x}\,dx+\left(a^2c\right)\int x\tan^{-1}(ax)\,dx\\ &= \frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}(ic)\int \frac{\log(1-iax)}{x}\,dx-\frac{1}{2}(ic)\int \frac{\log(1+iax)}{x}\,dx-\ldots\\ &= -\frac{1}{2}acx+\frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}ic\mathrm{Li}_2(-iax)-\frac{1}{2}ic\mathrm{Li}_2(iax)+\frac{1}{2}(ac)\int \frac{1}{1+a\ldots}\\ &= -\frac{1}{2}acx+\frac{1}{2}c\tan^{-1}(ax)+\frac{1}{2}a^2cx^2\tan^{-1}(ax)+\frac{1}{2}ic\mathrm{Li}_2(-iax)-\frac{1}{2}ic\mathrm{Li}_2(iax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.00

$$-\frac{1}{2}acx+\frac{1}{2}c\mathrm{ArcTan}(ax)+\frac{1}{2}a^2cx^2\mathrm{ArcTan}(ax)+\frac{1}{2}ic\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ic\mathrm{PolyLog}(2,iax)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

```
[Out] -1/2*(a*c*x) + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*Poly
Log[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

Maple [A]
time = 0.04, size = 90, normalized size = 1.45

method	result
risch	$\frac{ic\ln(-iax+1)x^2a^2}{4}+\frac{c\arctan(ax)}{2}-\frac{acx}{2}-\frac{ic\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic\ln(iax+1)x^2a^2}{4}+\frac{ic\,\mathrm{dilog}(iax+1)}{2}$
meijerg	$\frac{c\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4}+\frac{c\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}$
derivativedivides	$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}}{2}$
default	$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*c*x^2*arctan(a*x)+c*arctan(a*x)*ln(a*x)-1/2*c*(a*x-arctan(a*x)-I*ln
(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-I*a*x))
```

Maxima [A]
time = 0.50, size = 66, normalized size = 1.06

$$-\frac{1}{2}acx-\frac{1}{4}\pi c\log\left(a^2x^2+1\right)+c\arctan(ax)\log(ax)+\frac{1}{2}\left(a^2cx^2+c\right)\arctan(ax)-\frac{1}{2}ic\,\mathrm{Li}_2(iax+1)+\frac{1}{2}ic\,\mathrm{Li}_2(-iax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x - 1/4*pi*c*log(a^2*x^2 + 1) + c*arctan(a*x)*log(a*x) + 1/2*(a^2*
c*x^2 + c)*arctan(a*x) - 1/2*I*c*dilog(I*a*x + 1) + 1/2*I*c*dilog(-I*a*x +
1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{\operatorname{atan}(ax)}{x}\,dx+\int a^2x\operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x,x)
```

```
[Out] c*(Integral(atan(a*x)/x, x) + Integral(a**2*x*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.55, size = 57, normalized size = 0.92

$$\begin{cases} 0 & \text{if } a = 0 \\ a^2\, c\, \mathrm{atan}(a\, x)\, \left(\frac{1}{2\, a^2} + \frac{x^2}{2}\right) - \frac{a\, c\, x}{2} - \frac{c\, (\mathrm{Li}_2(1 - a\, x\, 1\mathrm{i}) - \mathrm{Li}_2(1 + a\, x\, 1\mathrm{i}))\, 1\mathrm{i}}{2} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2))/x,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (c*(dilog(- a*x*1i + 1) - dilog(a*x*1i + 1))
*1i)/2 - (a*c*x)/2 + a^2*c*atan(a*x)*(1/(2*a^2) + x^2/2))
```

3.154 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{c\text{ArcTan}(ax)}{x}+a^2cx\text{ArcTan}(ax)+ac\log(x)-ac\log\left(1+a^2x^2\right)$$

```
[Out] -c*arctan(a*x)/x+a^2*c*x*arctan(a*x)+a*c*ln(x)-a*c*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5070, 4946, 272, 36, 29, 31, 4930, 266}

$$a^2cx\text{ArcTan}(ax)-ac\log\left(a^2x^2+1\right)-\frac{c\text{ArcTan}(ax)}{x}+ac\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*x
^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)}{x^2}\,dx + (a^2c)\int \tan^{-1}(ax)\,dx$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) + (ac)\int \frac{1}{x(1+a^2x^2)}\,dx - (a^3c)\int \frac{x}{1+a^2
$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) - \frac{1}{2}ac\log(1+a^2x^2) + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{}{x(1
$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) - \frac{1}{2}ac\log(1+a^2x^2) + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{1}{x}\,d
$$
$$= -\frac{c\tan^{-1}(ax)}{x} + a^2cx\tan^{-1}(ax) + ac\log(x) - ac\log(1+a^2x^2)$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$-\frac{c\text{ArcTan}(ax)}{x} + a^2cx\text{ArcTan}(ax) + ac\log(x) - ac\log(1+a^2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*x
^2]
```

Maple [A]
time = 0.10, size = 45, normalized size = 1.12

method	result	size
derivativedivides	$a\left(acx\arctan\left(ax\right)-\frac{c\arctan(ax)}{ax}-c(\ln\left(a^2x^2+1\right)-\ln\left(ax\right))\right)$	45
default	$a\left(acx\arctan\left(ax\right)-\frac{c\arctan(ax)}{ax}-c(\ln\left(a^2x^2+1\right)-\ln\left(ax\right))\right)$	45
risch	$-\frac{ic(a^2x^2-1)\ln(iax+1)}{2x}+\frac{ic(a^2x^2\ln(-iax+1)-2ia\ln(x)x+2ia\ln(-2a^2x^2-2)x-\ln(-iax+1))}{2x}$	82
meijerg	$\frac{ac\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4}+\frac{ac\left(-\frac{4\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)+4\ln(x)+4\ln(a)\right)}{4}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(a*c*x*arctan(a*x)-c*arctan(a*x)/a/x-c*(ln(a^2*x^2+1)-ln(a*x)))
```

Maxima [A]
time = 0.25, size = 40, normalized size = 1.00

$$-\left(c\log\left(a^2x^2+1\right)-c\log\left(x\right)\right)a+\left(a^2cx-\frac{c}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -(c*log(a^2*x^2 + 1) - c*log(x))*a + (a^2*c*x - c/x)*arctan(a*x)
```

Fricas [A]
time = 2.25, size = 45, normalized size = 1.12

$$-\frac{acx\log\left(a^2x^2+1\right)-acx\log\left(x\right)-\left(a^2cx^2-c\right)\arctan\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*c*x*log(a^2*x^2 + 1) - a*c*x*log(x) - (a^2*c*x^2 - c)*arctan(a*x))/x
```

Sympy [A]
time = 0.30, size = 41, normalized size = 1.02

$$\begin{cases} a^2cx\,\text{atan}\,(ax) + ac\log\,(x) - ac\log\left(x^2 + \frac{1}{a^2}\right) - \frac{c\,\text{atan}\,(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**2*c*x*atan(a*x) + a*c*log(x) - a*c*log(x**2 + a**(-2)) - c*at
an(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.16, size = 42, normalized size = 1.05

$$a^2\,c\,x\,\text{atan}(a\,x) - \frac{c\,\text{atan}(a\,x)}{x} - c\,\left(a\,\ln\left(a^2\,x^2+1\right) - a\,\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2))/x^2,x)
```

```
[Out] a^2*c*x*atan(a*x) - (c*atan(a*x))/x - c*(a*log(a^2*x^2 + 1) - a*log(x))
```

3.155 $\int \frac{(c+a^2cx^2)\mathbf{ArcTan}(ax)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{ac}{2x}-\frac{1}{2}a^2c\text{ArcTan}(ax)-\frac{c\text{ArcTan}(ax)}{2x^2}+\frac{1}{2}ia^2c\text{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\text{PolyLog}(2,iax)$$

```
[Out] -1/2*a*c/x-1/2*a^2*c*arctan(a*x)-1/2*c*arctan(a*x)/x^2+1/2*I*a^2*c*polylog(
2,-I*a*x)-1/2*I*a^2*c*polylog(2,I*a*x)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5070, 4946, 331, 209, 4940, 2438}

$$-\frac{1}{2}a^2c\text{ArcTan}(ax)+\frac{1}{2}ia^2c\text{Li}_2(-iax)-\frac{1}{2}ia^2c\text{Li}_2(iax)-\frac{c\text{ArcTan}(ax)}{2x^2}-\frac{ac}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*c)/x - (a^2*c*ArcTan[a*x])/2 - (c*ArcTan[a*x])/(2*x^2) + (I/2)*a^2*
c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^3}\,dx &= c\int \frac{\tan^{-1}(ax)}{x^3}\,dx+\left(a^2c\right)\int \frac{\tan^{-1}(ax)}{x}\,dx\\ &= -\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}(ac)\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx+\frac{1}{2}\left(ia^2c\right)\int \frac{\log(1-iax)}{x}\,dx-\frac{1}{2}\\ &= -\frac{ac}{2x}-\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}ia^2c\text{Li}_2(-iax)-\frac{1}{2}ia^2c\text{Li}_2(iax)-\frac{1}{2}\left(a^3c\right)\int \frac{1}{1+a^2x}\\ &= -\frac{ac}{2x}-\frac{1}{2}a^2c\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)}{2x^2}+\frac{1}{2}ia^2c\text{Li}_2(-iax)-\frac{1}{2}ia^2c\text{Li}_2(iax)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.01, size = 74, normalized size = 1.06

$$-\frac{c\text{ArcTan}(ax)}{2x^2}-\frac{ac\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-a^2x^2\right)}{2x}+\frac{1}{2}ia^2c\text{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\text{PolyLog}(2,iax)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(c*ArcTan[a*x])/x^2 - (a*c*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)]
)/(2*x) + (I/2)*a^2*c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x]
```

Maple [A]
time = 0.05, size = 96, normalized size = 1.37

method	result
derivativedivides	$a^2\left(-\frac{c\arctan(ax)}{2a^2x^2}+c\arctan(ax)\ln(ax)-\frac{c\left(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{di}\right.}{2}\right.$
default	$a^2\left(-\frac{c\arctan(ax)}{2a^2x^2}+c\arctan(ax)\ln(ax)-\frac{c\left(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{di}\right.}{2}\right.$
meijerg	$\frac{a^2c\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}+\frac{a^2c\left(-\frac{2}{ax}-\frac{2\left(a^2x^2+1\right)\arctan(ax)}{a^2x^2}\right)}{4}$
risch	$-\frac{ic\,a^2\,\mathrm{dilog}(-iax+1)}{2}+\frac{ic\,a^2\ln(-iax)}{4}-\frac{ac}{2x}-\frac{a^2c\arctan(ax)}{2}-\frac{ic\ln(-iax+1)}{4x^2}+\frac{ic\,a^2\,\mathrm{dilog}(iax+1)}{2}-\frac{ic\,a^2\ln}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*c*arctan(a*x)/a^2/x^2+c*arctan(a*x)*ln(a*x)-1/2*c*(1/a/x+arctan(a
*x)-I*ln(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-
I*a*x)))
```

Maxima [A]
time = 0.53, size = 95, normalized size = 1.36

$$-\frac{\pi a^2cx^2\log\left(a^2x^2+1\right)-4\,a^2cx^2\arctan\left(ax\right)\log\left(ax\right)+2i\,a^2cx^2\mathrm{Li}_2(i\,ax+1)-2i\,a^2cx^2\mathrm{Li}_2(-i\,ax+1)+2\,acx+2\left(a^2cx^2+c\right)\arctan\left(ax\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(pi*a^2*c*x^2*log(a^2*x^2 + 1) - 4*a^2*c*x^2*arctan(a*x)*log(a*x) + 2*
I*a^2*c*x^2*dilog(I*a*x + 1) - 2*I*a^2*c*x^2*dilog(-I*a*x + 1) + 2*a*c*x +
2*(a^2*c*x^2 + c)*arctan(a*x))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{\operatorname{atan}\left(ax\right)}{x^3}\,dx+\int\frac{a^2\operatorname{atan}\left(ax\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**3,x)
```

```
[Out] c*(Integral(atan(a*x)/x**3, x) + Integral(a**2*atan(a*x)/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.56, size = 71, normalized size = 1.01

$$\begin{cases} 0 & \text{if } a=0 \\ -\frac{c\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c\,\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{a^2\,c\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^2\,c\,\mathrm{Li}_2(1+a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2))/x^3,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (c*atan(a*x))/(2*x^2) - (a^2*c*dilog(- a*x*1
i + 1)*1i)/2 + (a^2*c*dilog(a*x*1i + 1)*1i)/2 - (c*(a^3*atan(a*x) + a^2/x))
/(2*a))
```

3.156 $\int \frac{(c+a^2cx^2)\mathbf{ArcTan}(ax)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{ac}{6x^2}-\frac{c\text{ArcTan}(ax)}{3x^3}-\frac{a^2c\text{ArcTan}(ax)}{x}+\frac{2}{3}a^3c\log(x)-\frac{1}{3}a^3c\log\left(1+a^2x^2\right)$$

```
[Out] -1/6*a*c/x^2-1/3*c*arctan(a*x)/x^3-a^2*c*arctan(a*x)/x+2/3*a^3*c*ln(x)-1/3*
a^3*c*ln(a^2*x^2+1)
```

Rubi [A]

time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5070, 4946, 272, 46, 36, 29, 31}

$$\frac{2}{3}a^3c\log(x)-\frac{a^2c\text{ArcTan}(ax)}{x}-\frac{1}{3}a^3c\log\left(a^2x^2+1\right)-\frac{c\text{ArcTan}(ax)}{3x^3}-\frac{ac}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(a*c)/x^2 - (c*ArcTan[a*x])/(3*x^3) - (a^2*c*ArcTan[a*x])/x + (2*a^3*c
*Log[x])/3 - (a^3*c*Log[1 + a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+a^2cx^2)\tan^{-1}(ax)}{x^4}\,dx &= c\int \frac{\tan^{-1}(ax)}{x^4}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)}{x^2}\,dx \\
&= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{3}(ac)\int \frac{1}{x^3(1+a^2x^2)}\,dx + (a^3c)\int \frac{}{x(1-} \\
&= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{6}(ac)\text{Subst}\left(\int \frac{1}{x^2(1+a^2x)}\,dx, x, x^2\right) + \\
&= -\frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{1}{6}(ac)\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{a^2}{x} + \frac{a^4}{1+a^2x}\right)\,dx\right. \\
&= -\frac{ac}{6x^2} - \frac{c\tan^{-1}(ax)}{3x^3} - \frac{a^2c\tan^{-1}(ax)}{x} + \frac{2}{3}a^3c\log(x) - \frac{1}{3}a^3c\log(1+a^2x^2)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.92

$$\frac{c(-2(1+3a^2x^2)\,\text{ArcTan}(ax) + ax(-1+4a^2x^2\log(x) - 2a^2x^2\log(1+a^2x^2)))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

[Out] (c*(-2*(1 + 3*a^2*x^2)*ArcTan[a*x] + a*x*(-1 + 4*a^2*x^2*Log[x] - 2*a^2*x^2
*Log[1 + a^2*x^2])))/(6*x^3)

Maple [A]
time = 0.12, size = 60, normalized size = 0.95

method	result
derivativedivides	$a^{3}\left(-\frac{c\arctan\left(ax\right)}{ax}-\frac{c\arctan\left(ax\right)}{3a^{3}x^{3}}-\frac{c\left(\ln\left(a^{2}x^{2}+1\right)+\frac{1}{2a^{2}x^{2}}-2\ln\left(ax\right)\right)}{3}\right)$
default	$a^{3}\left(-\frac{c\arctan\left(ax\right)}{ax}-\frac{c\arctan\left(ax\right)}{3a^{3}x^{3}}-\frac{c\left(\ln\left(a^{2}x^{2}+1\right)+\frac{1}{2a^{2}x^{2}}-2\ln\left(ax\right)\right)}{3}\right)$
risch	$\frac{ic(3a^{2}x^{2}+1)\ln(iax+1)}{6x^{3}}+\frac{c(4\ln(x)a^{3}x^{3}-2\ln(-3a^{2}x^{2}-3)a^{3}x^{3}-3ia^{2}x^{2}\ln(-iax+1)-ax-i\ln(-iax+1))}{6x^{3}}$
meijerg	$\frac{a^{3}c\left(-\frac{4\arctan\left(\sqrt{a^{2}x^{2}}\right)}{\sqrt{a^{2}x^{2}}}-2\ln\left(a^{2}x^{2}+1\right)+4\ln\left(x\right)+4\ln\left(a\right)\right)}{4}+\frac{a^{3}c\left(\frac{-\frac{4a^{2}x^{2}}{9}+\frac{4}{3}}{a^{2}x^{2}}-\frac{4\arctan\left(\sqrt{a^{2}x^{2}}\right)}{3a^{2}x^{2}\sqrt{a^{2}x^{2}}}+\frac{2\ln\left(a^{2}x^{2}+1\right.}{3}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)

[Out] a^3*(-c*arctan(a*x)/a/x-1/3*c*arctan(a*x)/a^3/x^3-1/3*c*(ln(a^2*x^2+1)+1/2/
a^2/x^2-2*ln(a*x)))

Maxima [A]
time = 0.26, size = 56, normalized size = 0.89

$$-\frac{1}{6}\left(2\,a^{2}c\log\left(a^{2}x^{2}+1\right)-2\,a^{2}c\log\left(x^{2}\right)+\frac{c}{x^{2}}\right)a-\frac{\left(3\,a^{2}cx^{2}+c\right)\arctan\left(ax\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="maxima")

[Out] -1/6*(2*a^2*c*log(a^2*x^2 + 1) - 2*a^2*c*log(x^2) + c/x^2)*a - 1/3*(3*a^2*c
*x^2 + c)*arctan(a*x)/x^3

Fricas [A]
time = 1.49, size = 57, normalized size = 0.90

$$-\frac{2\,a^{3}cx^{3}\log\left(a^{2}x^{2}+1\right)-4\,a^{3}cx^{3}\log\left(x\right)+acx+2\left(3\,a^{2}cx^{2}+c\right)\arctan\left(ax\right)}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="fricas")

[Out] -1/6*(2*a^3*c*x^3*log(a^2*x^2 + 1) - 4*a^3*c*x^3*log(x) + a*c*x + 2*(3*a^2*
c*x^2 + c)*arctan(a*x))/x^3

Sympy [A]
time = 0.37, size = 61, normalized size = 0.97

$$\begin{cases} \frac{2a^3c\log(x)}{3} - \frac{a^3c\log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{a^2c\,\mathrm{atan}\,(ax)}{x} - \frac{ac}{6x^2} - \frac{c\,\mathrm{atan}\,(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((2*a**3*c*log(x)/3 - a**3*c*log(x**2 + a**(-2))/3 - a**2*c*atan(a
*x)/x - a*c/(6*x**2) - c*atan(a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.15, size = 57, normalized size = 0.90

$$\frac{c\,\left(4\,a^3\,\ln\left(x\right)-2\,a^3\,\ln\left(a^2\,x^2+1\right)\right)}{6}-\frac{\frac{c\,\mathrm{atan}\left(a\,x\right)}{3}+\frac{a\,c\,x}{6}+a^2\,c\,x^2\,\mathrm{atan}\left(a\,x\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2))/x^4,x)
```

```
[Out] (c*(4*a^3*log(x) - 2*a^3*log(a^2*x^2 + 1)))/6 - ((c*atan(a*x))/3 + (a*c*x)/
6 + a^2*c*x^2*atan(a*x))/x^3
```

3.157 $\int x^3(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=111

$$\frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\operatorname{ArcTan}(ax)}{24a^4}+\frac{1}{4}c^2x^4\operatorname{ArcTan}(ax)+\frac{1}{3}a^2c^2x^6\operatorname{ArcTan}(ax)+\frac{1}{8}a^4c^2x^8\operatorname{ArcTan}(ax)$$

```
[Out] 1/24*c^2*x/a^3-1/72*c^2*x^3/a-1/24*a*c^2*x^5-1/56*a^3*c^2*x^7-1/24*c^2*arct
an(a*x)/a^4+1/4*c^2*x^4*arctan(a*x)+1/3*a^2*c^2*x^6*arctan(a*x)+1/8*a^4*c^2
*x^8*arctan(a*x)
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5068, 4946, 308, 209}

$$\frac{1}{8}a^4c^2x^8\operatorname{ArcTan}(ax)-\frac{c^2\operatorname{ArcTan}(ax)}{24a^4}-\frac{1}{56}a^3c^2x^7+\frac{c^2x}{24a^3}+\frac{1}{3}a^2c^2x^6\operatorname{ArcTan}(ax)+\frac{1}{4}c^2x^4\operatorname{ArcTan}(ax)-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 - (
c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan[a
*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int x^3(c+a^2cx^2)^2\tan^{-1}(ax)\,dx &= \int \left(c^2x^3\tan^{-1}(ax)+2a^2c^2x^5\tan^{-1}(ax)+a^4c^2x^7\tan^{-1}(ax)\right)\,dx\\ &= c^2\int x^3\tan^{-1}(ax)\,dx+\left(2a^2c^2\right)\int x^5\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^7\tan^{-1}(a\\ &= \frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)-\frac{1}{4}\left(ac^2\right)\int\\ &= \frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)-\frac{1}{4}\left(ac^2\right)\int\\ &= \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7+\frac{1}{4}c^2x^4\tan^{-1}(ax)+\frac{1}{3}a^2c^2x^6\tan^{-1}(\\ &= \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\tan^{-1}(ax)}{24a^4}+\frac{1}{4}c^2x^4\tan^{-1}(ax)+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.61

$$\frac{c^2\left(-ax(-21+7a^2x^2+21a^4x^4+9a^6x^6)+21(1+a^2x^2)^3\left(-1+3a^2x^2\right)\text{ArcTan}(ax)\right)}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*(-21 + 7*a^2*x^2 + 21*a^4*x^4 + 9*a^6*x^6)) + 21*(1 + a^2*x^2)^
3*(-1 + 3*a^2*x^2)*ArcTan[a*x]))/(504*a^4)
```

Maple [A]
time = 0.18, size = 88, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$
default	$\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$

risch	$-\frac{ic^2x^4(3a^4x^4+8a^2x^2+6)\ln(iax+1)}{48}+\frac{ic^2a^4x^8\ln(-iax+1)}{16}-\frac{a^3c^2x^7}{56}+\frac{ic^2a^2x^6\ln(-iax+1)}{6}-\frac{a\,c^2x^5}{24}+\frac{ic^2x^4}{}$
meijerg	$\frac{c^2\left(\frac{xa(-45a^6x^6+63a^4x^4-105a^2x^2+315)}{630}-\frac{xa(-9a^8x^8+9)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c^2\left(-\frac{2xa(21a^4x^4-35a^2x^2+105)}{315}+\frac{2xa(}{}\right.}{2a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/8*c^2*arctan(a*x)*a^8*x^8+1/3*c^2*arctan(a*x)*a^6*x^6+1/4*a^4*c^2*
x^4*arctan(a*x)-1/24*c^2*(3/7*a^7*x^7+a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

Maxima [A]
time = 0.47, size = 98, normalized size = 0.88

$$-\frac{1}{504}\,a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/504*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2*
c^2*x^3 - 21*c^2*x)/a^4) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)
*arctan(a*x)
```

Fricas [A]
time = 2.03, size = 91, normalized size = 0.82

$$-\frac{9\,a^7c^2x^7+21\,a^5c^2x^5+7\,a^3c^2x^3-21\,ac^2x-21\left(3\,a^8c^2x^8+8\,a^6c^2x^6+6\,a^4c^2x^4-c^2\right)\arctan(ax)}{504\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/504*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x - 21*(3
*a^8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x))/a^4
```

Sympy [A]
time = 0.46, size = 104, normalized size = 0.94

$$\begin{cases}\frac{a^4c^2x^8\operatorname{atan}(ax)}{8}-\frac{a^3c^2x^7}{56}+\frac{a^2c^2x^6\operatorname{atan}(ax)}{3}-\frac{ac^2x^5}{24}+\frac{c^2x^4\operatorname{atan}(ax)}{4}-\frac{c^2x^3}{72a}+\frac{c^2x}{24a^3}-\frac{c^2\operatorname{atan}(ax)}{24a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**8*atan(a*x)/8 - a**3*c**2*x**7/56 + a**2*c**2*x**6*
atan(a*x)/3 - a*c**2*x**5/24 + c**2*x**4*atan(a*x)/4 - c**2*x**3/(72*a) + c
**2*x/(24*a**3) - c**2*atan(a*x)/(24*a**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.42, size = 89, normalized size = 0.80

$$\mathrm{atan}(a\,x)\,\left(\frac{a^4\,c^2\,x^8}{8}+\frac{a^2\,c^2\,x^6}{3}+\frac{c^2\,x^4}{4}\right)+\frac{c^2\,x}{24\,a^3}-\frac{a\,c^2\,x^5}{24}-\frac{c^2\,\mathrm{atan}(a\,x)}{24\,a^4}-\frac{c^2\,x^3}{72\,a}-\frac{a^3\,c^2\,x^7}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

```
[Out] atan(a*x)*((c^2*x^4)/4 + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/8) + (c^2*x)/(24*a
^3) - (a*c^2*x^5)/24 - (c^2*atan(a*x))/(24*a^4) - (c^2*x^3)/(72*a) - (a^3*c
^2*x^7)/56
```

3.158 $\int x^2(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=106

$$-\frac{4c^2x^2}{105a}-\frac{9}{140}ac^2x^4-\frac{1}{42}a^3c^2x^6+\frac{1}{3}c^2x^3\operatorname{ArcTan}(ax)+\frac{2}{5}a^2c^2x^5\operatorname{ArcTan}(ax)+\frac{1}{7}a^4c^2x^7\operatorname{ArcTan}(ax)+\frac{4c^2\log(1+a^2x^2)}{105a^3}$$

```
[Out] -4/105*c^2*x^2/a-9/140*a*c^2*x^4-1/42*a^3*c^2*x^6+1/3*c^2*x^3*arctan(a*x)+2
/5*a^2*c^2*x^5*arctan(a*x)+1/7*a^4*c^2*x^7*arctan(a*x)+4/105*c^2*ln(a^2*x^2
+1)/a^3
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5068, 4946, 272, 45}

$$\frac{1}{7}a^4c^2x^7\operatorname{ArcTan}(ax)-\frac{1}{42}a^3c^2x^6+\frac{2}{5}a^2c^2x^5\operatorname{ArcTan}(ax)+\frac{4c^2\log(a^2x^2+1)}{105a^3}+\frac{1}{3}c^2x^3\operatorname{ArcTan}(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*ArcT
an[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7 +
(4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^2(c+a^2cx^2)^2\tan^{-1}(ax)\,dx = \int \left(c^2x^2\tan^{-1}(ax)+2a^2c^2x^4\tan^{-1}(ax)+a^4c^2x^6\tan^{-1}(ax)\right)\,dx$$
$$= c^2\int x^2\tan^{-1}(ax)\,dx+\left(2a^2c^2\right)\int x^4\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^6\tan^{-1}(a$$
$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{3}\left(ac^2\right)\int$$
$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{6}\left(ac^2\right)\text{Sul}$$
$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)-\frac{1}{6}\left(ac^2\right)\text{Sul}$$
$$= -\frac{4c^2x^2}{105a}-\frac{9}{140}ac^2x^4-\frac{1}{42}a^3c^2x^6+\frac{1}{3}c^2x^3\tan^{-1}(ax)+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.75

$$\frac{c^2(-a^2x^2(16+27a^2x^2+10a^4x^4)+4a^3x^3(35+42a^2x^2+15a^4x^4)\,\text{ArcTan}(ax)+16\log(1+a^2x^2))}{420a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
[Out] (c^2*(-(a^2*x^2*(16 + 27*a^2*x^2 + 10*a^4*x^4)) + 4*a^3*x^3*(35 + 42*a^2*x^
2 + 15*a^4*x^4)*ArcTan[a*x] + 16*Log[1 + a^2*x^2]))/(420*a^3)
```

Maple [A]
time = 0.13, size = 93, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$
default	$\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$

risch	$-\frac{ic^2x^3(15a^4x^4+42a^2x^2+35)\ln(iax+1)}{210}+\frac{ic^2a^4x^7\ln(-iax+1)}{14}-\frac{a^3c^2x^6}{42}+\frac{ic^2a^2x^5\ln(-iax+1)}{5}-\frac{9a\,c^2x^4}{140}+$
meijerg	$\frac{c^2\left(-\frac{a^2x^2(4a^4x^4-6a^2x^2+12)}{42}+\frac{4a^8x^8\arctan\left(\sqrt{a^2x^2}\right)}{7\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{7}\right)}{4a^3}+\frac{c^2\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan(\sqrt{a^2}}{5\sqrt{a^2x^2}}\right.}{2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/7*c^2*arctan(a*x)*a^7*x^7+2/5*c^2*arctan(a*x)*a^5*x^5+1/3*a^3*c^2*
x^3*arctan(a*x)-1/105*c^2*(5/2*a^6*x^6+27/4*a^4*x^4+4*a^2*x^2-4*ln(a^2*x^2+
1)))
```

Maxima [A]
time = 0.27, size = 95, normalized size = 0.90

$$-\frac{1}{420}a\left(\frac{10\,a^4c^2x^6+27\,a^2c^2x^4+16\,c^2x^2}{a^2}-\frac{16\,c^2\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{105}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/420*a*((10*a^4*c^2*x^6 + 27*a^2*c^2*x^4 + 16*c^2*x^2)/a^2 - 16*c^2*log(a
^2*x^2 + 1)/a^4) + 1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arc
tan(a*x)
```

Fricas [A]
time = 5.53, size = 94, normalized size = 0.89

$$-\frac{10\,a^6c^2x^6+27\,a^4c^2x^4+16\,a^2c^2x^2-16\,c^2\log\left(a^2x^2+1\right)-4\left(15\,a^7c^2x^7+42\,a^5c^2x^5+35\,a^3c^2x^3\right)\arctan\left(ax\right)}{420\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/420*(10*a^6*c^2*x^6 + 27*a^4*c^2*x^4 + 16*a^2*c^2*x^2 - 16*c^2*log(a^2*x
^2 + 1) - 4*(15*a^7*c^2*x^7 + 42*a^5*c^2*x^5 + 35*a^3*c^2*x^3)*arctan(a*x))
/a^3
```

Sympy [A]
time = 0.41, size = 105, normalized size = 0.99

$$\begin{cases}\frac{a^4c^2x^7\operatorname{atan}(ax)}{7}-\frac{a^3c^2x^6}{42}+\frac{2a^2c^2x^5\operatorname{atan}(ax)}{5}-\frac{9ac^2x^4}{140}+\frac{c^2x^3\operatorname{atan}(ax)}{3}-\frac{4c^2x^2}{105a}+\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{105a^3} & \text{for}\; a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**7*atan(a*x)/7 - a**3*c**2*x**6/42 + 2*a**2*c**2*x**
5*atan(a*x)/5 - 9*a*c**2*x**4/140 + c**2*x**3*atan(a*x)/3 - 4*c**2*x**2/(10
5*a) + 4*c**2*log(x**2 + a**(-2))/(105*a**3), Ne(a, 0)), (0, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.54, size = 81, normalized size = 0.76

$$\frac{c^2\,\left(16\,\ln\left(a^2\,x^2+1\right)-16\,a^2\,x^2-27\,a^4\,x^4-10\,a^6\,x^6+140\,a^3\,x^3\,\mathrm{atan}(a\,x)+168\,a^5\,x^5\,\mathrm{atan}(a\,x)+60\,a^7\,x^7\,\mathrm{atan}(a\,x)\right)}{420\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

```
[Out] (c^2*(16*log(a^2*x^2 + 1) - 16*a^2*x^2 - 27*a^4*x^4 - 10*a^6*x^6 + 140*a^3*
x^3*atan(a*x) + 168*a^5*x^5*atan(a*x) + 60*a^7*x^7*atan(a*x)))/(420*a^3)
```

3.159 $\int x(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=61

$$-\frac{c^2x}{6a}-\frac{1}{9}ac^2x^3-\frac{1}{30}a^3c^2x^5+\frac{c^2(1+a^2x^2)^3\operatorname{ArcTan}(ax)}{6a^2}$$

```
[Out] -1/6*c^2*x/a-1/9*a*c^2*x^3-1/30*a^3*c^2*x^5+1/6*c^2*(a^2*x^2+1)^3*arctan(a*
x)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5050, 200}

$$-\frac{1}{30}a^3c^2x^5+\frac{c^2(a^2x^2+1)^3\operatorname{ArcTan}(ax)}{6a^2}-\frac{1}{9}ac^2x^3-\frac{c^2x}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] -1/6*(c^2*x)/a - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*(1 + a^2*x^2)^3*Ar
cTan[a*x])/(6*a^2)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x(c+a^2cx^2)^2\tan^{-1}(ax)\,dx &= \frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)}{6a^2}-\frac{\int (c+a^2cx^2)^2\,dx}{6a}\\ &= \frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)}{6a^2}-\frac{\int (c^2+2a^2c^2x^2+a^4c^2x^4)\,dx}{6a}\\ &= -\frac{c^2x}{6a}-\frac{1}{9}ac^2x^3-\frac{1}{30}a^3c^2x^5+\frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)}{6a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.82

$$\frac{c^2\left(-ax(15+10a^2x^2+3a^4x^4)+15(1+a^2x^2)^3\operatorname{ArcTan}(ax)\right)}{90a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)) + 15*(1 + a^2*x^2)^3*ArcTan[a*x]
))/(90*a^2)
```

Maple [A]
time = 0.10, size = 85, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)}{2}+\frac{a^2c^2x^2\arctan(ax)}{2}+\frac{c^2\arctan(ax)}{6}-\frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$
default	$\frac{\frac{c^2\arctan(ax)a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)}{2}+\frac{a^2c^2x^2\arctan(ax)}{2}+\frac{c^2\arctan(ax)}{6}-\frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$
risch	$-\frac{ic^2(a^2x^2+1)^3\ln(iax+1)}{12a^2}+\frac{ic^2a^4x^6\ln(-iax+1)}{12}-\frac{a^3c^2x^5}{30}+\frac{ic^2a^2x^4\ln(-iax+1)}{4}-\frac{a\,c^2x^3}{9}+\frac{ic^2x^2\ln(-iax}{4}$
meijerg	$\frac{c^2\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^2}+\frac{c^2\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan(}{5\sqrt{a^2x^2}}\right.}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/6*c^2*arctan(a*x)*a^6*x^6+1/2*a^4*c^2*x^4*arctan(a*x)+1/2*a^2*c^2*
x^2*arctan(a*x)+1/6*c^2*arctan(a*x)-1/6*c^2*(1/5*a^5*x^5+2/3*a^3*x^3+a*x))
```

Maxima [A]
time = 0.25, size = 62, normalized size = 1.02

$$\frac{(a^2cx^2+c)^3\arctan(ax)}{6\,a^2c}-\frac{3\,a^4c^3x^5+10\,a^2c^3x^3+15\,c^3x}{90\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^3*x^5 + 10*a^2*c^
3*x^3 + 15*c^3*x)/(a*c)
```

Fricas [A]
time = 2.44, size = 77, normalized size = 1.26

$$-\frac{3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x-15\,(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2)\arctan(ax)}{90\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/90*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x - 15*(a^6*c^2*x^6 + 3*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))/a^2
```

Sympy [A]
time = 0.75, size = 92, normalized size = 1.51

$$\begin{cases}\frac{a^4c^2x^6\operatorname{atan}(ax)}{6}-\frac{a^3c^2x^5}{30}+\frac{a^2c^2x^4\operatorname{atan}(ax)}{2}-\frac{ac^2x^3}{9}+\frac{c^2x^2\operatorname{atan}(ax)}{2}-\frac{c^2x}{6a}+\frac{c^2\operatorname{atan}(ax)}{6a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**6*atan(a*x)/6 - a**3*c**2*x**5/30 + a**2*c**2*x**4*
atan(a*x)/2 - a*c**2*x**3/9 + c**2*x**2*atan(a*x)/2 - c**2*x/(6*a) + c**2*a
tan(a*x)/(6*a**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.55, size = 71, normalized size = 1.16

$$\frac{c^2\,(15\operatorname{atan}(a\,x)-15\,a\,x-10\,a^3\,x^3-3\,a^5\,x^5+45\,a^2\,x^2\operatorname{atan}(a\,x)+45\,a^4\,x^4\operatorname{atan}(a\,x)+15\,a^6\,x^6\operatorname{atan}(a\,x))}{90\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

```
[Out] (c^2*(15*atan(a*x) - 15*a*x - 10*a^3*x^3 - 3*a^5*x^5 + 45*a^2*x^2*atan(a*x)
 + 45*a^4*x^4*atan(a*x) + 15*a^6*x^6*atan(a*x)))/(90*a^2)
```

3.160 $\int (c+a^2cx^2)^2 \text{ArcTan}(ax)\,dx$

Optimal. Leaf size=117

$$-\frac{2c^2(1+a^2x^2)}{15a}-\frac{c^2(1+a^2x^2)^2}{20a}+\frac{8}{15}c^2x\text{ArcTan}(ax)+\frac{4}{15}c^2x(1+a^2x^2)\text{ ArcTan}(ax)+\frac{1}{5}c^2x(1+a^2x^2)^2\text{ ArcT}$$

```
[Out] -2/15*c^2*(a^2*x^2+1)/a-1/20*c^2*(a^2*x^2+1)^2/a+8/15*c^2*x*arctan(a*x)+4/1
5*c^2*x*(a^2*x^2+1)*arctan(a*x)+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)-4/15*c^
2*ln(a^2*x^2+1)/a
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4998, 4930, 266}

$$\frac{1}{5}c^2x(a^2x^2+1)^2\text{ ArcTan}(ax)+\frac{4}{15}c^2x(a^2x^2+1)\text{ ArcTan}(ax)-\frac{c^2(a^2x^2+1)^2}{20a}-\frac{2c^2(a^2x^2+1)}{15a}-\frac{4c^2\log(a^2x^2+1)}{15a}+\frac{8}{15}c^2x\text{ArcTan}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (-2*c^2*(1 + a^2*x^2))/(15*a) - (c^2*(1 + a^2*x^2)^2)/(20*a) + (8*c^2*x*Arc
Tan[a*x])/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/15 + (c^2*x*(1 + a^2*x^2
)^2*ArcTan[a*x])/5 - (4*c^2*Log[1 + a^2*x^2])/(15*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)\,dx = -\frac{c^2(1+a^2x^2)^2}{20a} + \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax) + \frac{1}{5}(4c)\int \left(c+a^2cx^2\right)\tan^{-1}(c$$

$$= -\frac{2c^2(1+a^2x^2)}{15a} - \frac{c^2(1+a^2x^2)^2}{20a} + \frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax) + \frac{1}{5}c^2x(1+c$$

$$= -\frac{2c^2(1+a^2x^2)}{15a} - \frac{c^2(1+a^2x^2)^2}{20a} + \frac{8}{15}c^2x\tan^{-1}(ax) + \frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-}$$

$$= -\frac{2c^2(1+a^2x^2)}{15a} - \frac{c^2(1+a^2x^2)^2}{20a} + \frac{8}{15}c^2x\tan^{-1}(ax) + \frac{4}{15}c^2x\left(1+a^2x^2\right)\tan^{-}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.56

$$\frac{c^2(-14a^2x^2-3a^4x^4+4ax(15+10a^2x^2+3a^4x^4)\,\text{ArcTan}(ax)-16\log\left(1+a^2x^2\right))}{60a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
[Out] (c^2*(-14*a^2*x^2 - 3*a^4*x^4 + 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[
a*x] - 16*Log[1 + a^2*x^2]))/(60*a)
```

Maple [A]
time = 0.10, size = 80, normalized size = 0.68

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a\,c^2x\arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln\left(a^2x^2+1\right)\right)}{15}}{a}$
default	$\frac{\frac{c^2\arctan(ax)a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a\,c^2x\arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln\left(a^2x^2+1\right)\right)}{15}}{a}$
risch	$-\frac{ic^2x(3a^4x^4+10a^2x^2+15)\ln(iax+1)}{30}+\frac{ic^2a^4x^5\ln(-iax+1)}{10}-\frac{c^2a^3x^4}{20}+\frac{ic^2a^2x^3\ln(-iax+1)}{3}-\frac{7a\,c^2x^2}{30}+\frac{ic^2}{}$
meijerg	$\frac{c^2\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{5}\right)}{4a}+\frac{c^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2}{3}\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/5*c^2*arctan(a*x)*a^5*x^5+2/3*a^3*c^2*x^3*arctan(a*x)+a*c^2*x*arctan
(a*x)-1/15*c^2*(3/4*a^4*x^4+7/2*a^2*x^2+4*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.25, size = 77, normalized size = 0.66

$$-\frac{1}{60}\left(3\,a^2c^2x^4+14\,c^2x^2+\frac{16\,c^2\log\left(a^2x^2+1\right)}{a^2}\right)a+\frac{1}{15}\left(3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*(3*a^2*c^2*x^4 + 14*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1)/a^2)*a + 1/15*(
3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)
```

Fricas [A]
time = 2.31, size = 79, normalized size = 0.68

$$-\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+16\,c^2\log\left(a^2x^2+1\right)-4\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\arctan\left(ax\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*
c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a
```

Sympy [A]
time = 0.25, size = 88, normalized size = 0.75

$$\begin{cases} \frac{a^4c^2x^5\operatorname{atan}(ax)}{5}-\frac{a^3c^2x^4}{20}+\frac{2a^2c^2x^3\operatorname{atan}(ax)}{3}-\frac{7ac^2x^2}{30}+c^2x\operatorname{atan}(ax)-\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{15a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] Piecewise((a**4*c**2*x**5*atan(a*x)/5 - a**3*c**2*x**4/20 + 2*a**2*c**2*x**
3*atan(a*x)/3 - 7*a*c**2*x**2/30 + c**2*x*atan(a*x) - 4*c**2*log(x**2 + a**
(-2))/(15*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.20, size = 69, normalized size = 0.59

$$-\frac{c^2\left(16\,\ln\left(a^2\,x^2+1\right)+14\,a^2\,x^2+3\,a^4\,x^4-40\,a^3\,x^3\,\mathrm{atan}(a\,x)-12\,a^5\,x^5\,\mathrm{atan}(a\,x)-60\,a\,x\,\mathrm{atan}(a\,x)\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2)^2,x)
```

```
[Out] -(c^2*(16*log(a^2*x^2 + 1) + 14*a^2*x^2 + 3*a^4*x^4 - 40*a^3*x^3*atan(a*x)
- 12*a^5*x^5*atan(a*x) - 60*a*x*atan(a*x)))/(60*a)
```

3.161 $\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)}{x} \, dx$

Optimal. Leaf size=99

$$-\frac{3}{4}ac^2x-\frac{1}{12}a^3c^2x^3+\frac{3}{4}c^2\text{ArcTan}(ax)+a^2c^2x^2\text{ArcTan}(ax)+\frac{1}{4}a^4c^2x^4\text{ArcTan}(ax)+\frac{1}{2}ic^2\text{PolyLog}(2,-iax)-\frac{1}{2}i$$

```
[Out] -3/4*a*c^2*x-1/12*a^3*c^2*x^3+3/4*c^2*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+1
/4*a^4*c^2*x^4*arctan(a*x)+1/2*I*c^2*polylog(2,-I*a*x)-1/2*I*c^2*polylog(2,
I*a*x)
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5068, 4940, 2438, 4946, 327, 209, 308}

$$\frac{1}{4}a^4c^2x^4\text{ArcTan}(ax)-\frac{1}{12}a^3c^2x^3+a^2c^2x^2\text{ArcTan}(ax)+\frac{3}{4}c^2\text{ArcTan}(ax)+\frac{1}{2}ic^2\text{Li}_2(-iax)-\frac{1}{2}ic^2\text{Li}_2(iax)-\frac{3}{4}ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

```
[Out] (-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Arc
Tan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] - (
I/2)*c^2*PolyLog[2, I*a*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)}{x}\,dx &= \int \left(\frac{c^2\tan^{-1}(ax)}{x}+2a^2c^2x\tan^{-1}(ax)+a^4c^2x^3\tan^{-1}(ax)\right)dx\\ &= c^2\int \frac{\tan^{-1}(ax)}{x}\,dx+\left(2a^2c^2\right)\int x\tan^{-1}(ax)\,dx+\left(a^4c^2\right)\int x^3\tan^{-1}(ax)\,dx\\ &= a^2c^2x^2\tan^{-1}(ax)+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)+\frac{1}{2}\left(ic^2\right)\int \frac{\log(1-iax)}{x}\,dx-\frac{1}{2}\left(ic^2\right)\\ &= -ac^2x+a^2c^2x^2\tan^{-1}(ax)+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)+\frac{1}{2}ic^2\text{Li}_2(-iax)-\frac{1}{2}ic^2\text{Li}_2(\\ &= -\frac{3}{4}ac^2x-\frac{1}{12}a^3c^2x^3+c^2\tan^{-1}(ax)+a^2c^2x^2\tan^{-1}(ax)+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax\\ &= -\frac{3}{4}ac^2x-\frac{1}{12}a^3c^2x^3+\frac{3}{4}c^2\tan^{-1}(ax)+a^2c^2x^2\tan^{-1}(ax)+\frac{1}{4}a^4c^2x^4\tan^{-1}(a\end{aligned}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.75

$$\frac{1}{12}c^2\left(-9ax-a^3x^3+9\text{ArcTan}(ax)+12a^2x^2\text{ArcTan}(ax)+3a^4x^4\text{ArcTan}(ax)+6i\text{PolyLog}(2,-iax)-6i\text{PolyLog}(2,iax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

```
[Out] (c^2*(-9*a*x - a^3*x^3 + 9*ArcTan[a*x] + 12*a^2*x^2*ArcTan[a*x] + 3*a^4*x^4
*ArcTan[a*x] + (6*I)*PolyLog[2, (-I)*a*x] - (6*I)*PolyLog[2, I*a*x]))/12
```

Maple [A]
time = 0.07, size = 119, normalized size = 1.20

method	result
derivativedivides	$\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2i\,l\right.}{}$
default	$\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2i\,l\right.}{}$
risch	$\frac{ic^2\ln(-iax+1)x^4a^4}{8}+\frac{ic^2\ln(-iax+1)x^2a^2}{2}+\frac{3c^2\arctan(ax)}{4}-\frac{a^3c^2x^3}{12}-\frac{3a\,c^2x}{4}-\frac{ic^2\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic^2\,l}{}$
meijerg	$\frac{c^2\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4}+\frac{c^2\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{2}+\frac{c^2\left(-\frac{2iax\,\mathrm{polylo}}{\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^4*c^2*x^4*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+c^2*arctan(a*x)*ln(a*x)
-1/4*c^2*(1/3*a^3*x^3+3*a*x-3*arctan(a*x)-2*I*ln(a*x)*ln(1+I*a*x)+2*I*ln(a*
x)*ln(1-I*a*x)-2*I*dilog(1+I*a*x)+2*I*dilog(1-I*a*x))
```

Maxima [A]
time = 0.50, size = 104, normalized size = 1.05

$$-\frac{1}{12}a^3c^2x^3-\frac{3}{4}ac^2x-\frac{1}{4}\pi c^2\log\left(a^2x^2+1\right)+c^2\arctan(ax)\log(ax)-\frac{1}{2}i\,c^2\mathrm{Li}_2(i\,ax+1)+\frac{1}{2}i\,c^2\mathrm{Li}_2(-i\,ax+1)+\frac{1}{4}\left(a^4c^2x^4+4\,a^2c^2x^2+3\,c^2\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/12*a^3*c^2*x^3 - 3/4*a*c^2*x - 1/4*pi*c^2*log(a^2*x^2 + 1) + c^2*arctan(
a*x)*log(a*x) - 1/2*I*c^2*dilog(I*a*x + 1) + 1/2*I*c^2*dilog(-I*a*x + 1) +
1/4*(a^4*c^2*x^4 + 4*a^2*c^2*x^2 + 3*c^2)*arctan(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{\operatorname{atan}{(ax)}}{x}\,dx+\int 2a^{2}x\operatorname{atan}{(ax)}\,dx+\int a^{4}x^{3}\operatorname{atan}{(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)/x, x) + Integral(2*a**2*x*atan(a*x), x) + Integral
(a**4*x**3*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.61, size = 105, normalized size = 1.06

$$\begin{cases} 0 & \text{if } a=0\\ 2\,a^2\,c^2\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-a\,c^2\,x-\frac{c^2\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{12}+\frac{a^4\,c^2\,x^4\,\mathrm{atan}(a\,x)}{4}-\frac{c^2\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{c^2\,\mathrm{Li}_2(1+a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^2)/x,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (c^2*dilog(- a*x*1i + 1)*1i)/2 + (c^2*dilog(
a*x*1i + 1)*1i)/2 - (c^2*(3*atan(a*x) - 3*a*x + a^3*x^3))/12 - a*c^2*x + 2*
a^2*c^2*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^4*c^2*x^4*atan(a*x))/4)
```

3.162 $\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)}{x^2} \, dx$

Optimal. Leaf size=81

$$-\frac{1}{6}a^3c^2x^2-\frac{c^2\text{ArcTan}(ax)}{x}+2a^2c^2x\text{ArcTan}(ax)+\frac{1}{3}a^4c^2x^3\text{ArcTan}(ax)+ac^2\log(x)-\frac{4}{3}ac^2\log\left(1+a^2x^2\right)$$

```
[Out] -1/6*a^3*c^2*x^2-c^2*arctan(a*x)/x+2*a^2*c^2*x*arctan(a*x)+1/3*a^4*c^2*x^3*
arctan(a*x)+a*c^2*ln(x)-4/3*a*c^2*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5068, 4930, 266, 4946, 272, 36, 29, 31, 45}

$$\frac{1}{3}a^4c^2x^3\text{ArcTan}(ax) - \frac{1}{6}a^3c^2x^2 + 2a^2c^2x\text{ArcTan}(ax) - \frac{4}{3}ac^2\log\left(a^2x^2+1\right) - \frac{c^2\text{ArcTan}(ax)}{x} + ac^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

```
[Out] -1/6*(a^3*c^2*x^2) - (c^2*ArcTan[a*x])/x + 2*a^2*c^2*x*ArcTan[a*x] + (a^4*c
^2*x^3*ArcTan[a*x])/3 + a*c^2*Log[x] - (4*a*c^2*Log[1 + a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)}{x^2}\,dx = \int \left(2a^2c^2\tan^{-1}(ax) + \frac{c^2\tan^{-1}(ax)}{x^2} + a^4c^2x^2\tan^{-1}(ax)\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)}{x^2}\,dx + \left(2a^2c^2\right)\int \tan^{-1}(ax)\,dx + \left(a^4c^2\right)\int x^2\tan^{-1}(ax)\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) + \left(ac^2\right)\int \frac{1}{x\,(1+}$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) - ac^2\log\left(1+a^2x\right.$$

$$= -\frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) - ac^2\log\left(1+a^2x\right.$$

$$= -\frac{1}{6}a^3c^2x^2 - \frac{c^2\tan^{-1}(ax)}{x} + 2a^2c^2x\tan^{-1}(ax) + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax) + ac^2\,\mathrm{lo}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.77

$$\frac{c^2(2(-3+6a^2x^2+a^4x^4)\,\mathrm{ArcTan}(ax) - ax(a^2x^2 - 6\log(x) + 8\log\left(1+a^2x^2\right)))}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

```
[Out] (c^2*(2*(-3 + 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] - a*x*(a^2*x^2 - 6*Log[x] +
8*Log[1 + a^2*x^2])))/(6*x)
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.95

method	result
derivativedivides	$a\left(\frac{a^3c^2x^3\arctan(ax)}{3} + 2a\,c^2x\arctan(ax) - \frac{c^2\arctan(ax)}{ax} - \frac{c^2\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(ax)\right)}{3}\right)$
default	$a\left(\frac{a^3c^2x^3\arctan(ax)}{3} + 2a\,c^2x\arctan(ax) - \frac{c^2\arctan(ax)}{ax} - \frac{c^2\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(ax)\right)}{3}\right)$
risch	$-\frac{ic^2(a^4x^4+6a^2x^2-3)\ln(iax+1)}{6x} + \frac{ic^2(x^4\ln(-iax+1)a^4+ia^3x^3+6a^2x^2\ln(-iax+1)-6ia\ln(x)x+8ia\ln(7a^2x^2+7)}{6x}$
meijerg	$\frac{a\,c^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4} + \frac{a\,c^2\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{2} + -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*a^3*c^2*x^3*arctan(a*x)+2*a*c^2*x*arctan(a*x)-c^2*arctan(a*x)/a/x-1/
3*c^2*(1/2*a^2*x^2+4*ln(a^2*x^2+1)-3*ln(a*x)))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.88

$$-\frac{1}{6}\left(a^{2}c^{2}x^{2}+8\,c^{2}\log\left(a^{2}x^{2}+1\right)-6\,c^{2}\log\left(x\right)\right)a+\frac{1}{3}\left(a^{4}c^{2}x^{3}+6\,a^{2}c^{2}x-\frac{3\,c^{2}}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*c^2*x^2 + 8*c^2*log(a^2*x^2 + 1) - 6*c^2*log(x))*a + 1/3*(a^4*c^2
*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

Fricas [A]
time = 2.05, size = 75, normalized size = 0.93

$$-\frac{a^{3}c^{2}x^{3}+8\,ac^{2}x\log\left(a^{2}x^{2}+1\right)-6\,ac^{2}x\log\left(x\right)-2\left(a^{4}c^{2}x^{4}+6\,a^{2}c^{2}x^{2}-3\,c^{2}\right)\arctan\left(ax\right)}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*c^2*x^3 + 8*a*c^2*x*log(a^2*x^2 + 1) - 6*a*c^2*x*log(x) - 2*(a^4*
c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x))/x
```

Sympy [A]
time = 0.48, size = 82, normalized size = 1.01

$$\begin{cases}\frac{a^{4}c^{2}x^{3}\,\operatorname{atan}\left(ax\right)}{3}-\frac{a^{3}c^{2}x^{2}}{6}+2a^{2}c^{2}x\,\operatorname{atan}\left(ax\right)+ac^{2}\log\left(x\right)-\frac{4ac^{2}\log\left(x^{2}+\frac{1}{a^{2}}\right)}{3}-\frac{c^{2}\,\operatorname{atan}\left(ax\right)}{x} & \text{for}\: a\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**3*atan(a*x)/3 - a**3*c**2*x**2/6 + 2*a**2*c**2*x*at
an(a*x) + a*c**2*log(x) - 4*a*c**2*log(x**2 + a**(-2))/3 - c**2*atan(a*x)/x
, Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.21, size = 76, normalized size = 0.94

$$\frac{a^4\,c^2\,x^3\,\mathrm{atan}(a\,x)}{3}-\frac{c^2\,\mathrm{atan}(a\,x)}{x}-\frac{a^3\,c^2\,x^2}{6}-\frac{c^2\,\left(8\,a\,\ln\left(a^2\,x^2+1\right)-6\,a\,\ln\left(x\right)\right)}{6}+2\,a^2\,c^2\,x\,\mathrm{atan}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^2)/x^2,x)
```

```
[Out] (a^4*c^2*x^3*atan(a*x))/3 - (c^2*atan(a*x))/x - (a^3*c^2*x^2)/6 - (c^2*(8*a
*log(a^2*x^2 + 1) - 6*a*log(x)))/6 + 2*a^2*c^2*x*atan(a*x)
```

3.163 $\int \frac{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)}{x^3}\,dx$

Optimal. Leaf size=90

$$-\frac{ac^2}{2x}-\frac{1}{2}a^3c^2x-\frac{c^2\text{ArcTan}(ax)}{2x^2}+\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)+ia^2c^2\text{PolyLog}(2,-iax)-ia^2c^2\text{PolyLog}(2,iax)$$

```
[Out] -1/2*a*c^2/x-1/2*a^3*c^2*x-1/2*c^2*arctan(a*x)/x^2+1/2*a^4*c^2*x^2*arctan(a
*x)+I*a^2*c^2*polylog(2,-I*a*x)-I*a^2*c^2*polylog(2,I*a*x)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5068, 4946, 331, 209, 4940, 2438, 327}

$$\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)-\frac{1}{2}a^3c^2x+ia^2c^2\text{Li}_2(-iax)-ia^2c^2\text{Li}_2(iax)-\frac{c^2\text{ArcTan}(ax)}{2x^2}-\frac{ac^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*c^2)/x - (a^3*c^2*x)/2 - (c^2*ArcTan[a*x])/(2*x^2) + (a^4*c^2*x^2*A
rcTan[a*x])/2 + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a*x
]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)}{x^3}\,dx &= \int \left(\frac{c^2\tan^{-1}(ax)}{x^3}+\frac{2a^2c^2\tan^{-1}(ax)}{x}+a^4c^2x\tan^{-1}(ax)\right)dx\\ &= c^2\int \frac{\tan^{-1}(ax)}{x^3}\,dx+(2a^2c^2)\int \frac{\tan^{-1}(ax)}{x}\,dx+(a^4c^2)\int x\tan^{-1}(ax)\,dx\\ &= -\frac{c^2\tan^{-1}(ax)}{2x^2}+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)+\frac{1}{2}(ac^2)\int \frac{1}{x^2(1+a^2x^2)}\,dx+(ia^2c^2\\ &= -\frac{ac^2}{2x}-\frac{1}{2}a^3c^2x-\frac{c^2\tan^{-1}(ax)}{2x^2}+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)+ia^2c^2\mathrm{Li}_2(-iax)-i\end{aligned}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.84

$$\frac{c^2(-ax-a^3x^3-\mathrm{ArcTan}(ax)+a^4x^4\mathrm{ArcTan}(ax)+2ia^2x^2\mathrm{PolyLog}(2,-iax)-2ia^2x^2\mathrm{PolyLog}(2,iax))}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]

[Out] (c^2*(-(a*x) - a^3*x^3 - ArcTan[a*x] + a^4*x^4*ArcTan[a*x] + (2*I)*a^2*x^2*PolyLog[2, (-I)*a*x] - (2*I)*a^2*x^2*PolyLog[2, I*a*x]))/(2*x^2)

Maple [A]
time = 0.06, size = 117, normalized size = 1.30

method	result
derivativedivides	$a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\ln(a\right.}{}\right.$
default	$a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\ln(a\right.}{}\right.$
meijerg	$\frac{a^2c^2\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4}+\frac{a^2c^2\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{2}+\frac{a^2c^2\left(\right.}{}$
risch	$\frac{ic^2a^4\ln(-iax+1)x^2}{4}-\frac{a^3c^2x}{2}-ic^2a^2\,\mathrm{dilog}(-iax+1)+\frac{ic^2a^2\ln(-iax)}{4}-\frac{a\,c^2}{2x}-\frac{ic^2\ln(-iax+1)}{4x^2}-\frac{ic^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)

[Out] a^2*(1/2*a^2*c^2*x^2*arctan(a*x)-1/2*c^2*arctan(a*x)/a^2/x^2+2*c^2*arctan(a*x)*ln(a*x)-1/2*c^2*(a*x+1/a/x-2*I*ln(a*x)*ln(1+I*a*x)+2*I*ln(a*x)*ln(1-I*a*x)-2*I*dilog(1+I*a*x)+2*I*dilog(1-I*a*x)))

Maxima [A]
time = 0.50, size = 120, normalized size = 1.33

$$-\frac{a^3c^2x^3+\pi a^2c^2x^2\log\left(a^2x^2+1\right)-4\,a^2c^2x^2\arctan(ax)\log(ax)+2i\,a^2c^2x^2\mathrm{Li}_2(i\,ax+1)-2i\,a^2c^2x^2\mathrm{Li}_2(-i\,ax+1)+ac^2x-\left(a^4c^2x^4-c^2\right)\arctan(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="maxima")

[Out] -1/2*(a^3*c^2*x^3 + pi*a^2*c^2*x^2*log(a^2*x^2 + 1) - 4*a^2*c^2*x^2*arctan(a*x)*log(a*x) + 2*I*a^2*c^2*x^2*dilog(I*a*x + 1) - 2*I*a^2*c^2*x^2*dilog(-I*a*x + 1) + a*c^2*x - (a^4*c^2*x^4 - c^2)*arctan(a*x))/x^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="fricas")

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{\operatorname{atan}(ax)}{x^3}\,dx + \int \frac{2a^2\operatorname{atan}(ax)}{x}\,dx + \int a^4 x\operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**3,x)
```

```
[Out] c**2*(Integral(atan(a*x)/x**3, x) + Integral(2*a**2*atan(a*x)/x, x) + Integ
ral(a**4*x*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.50, size = 110, normalized size = 1.22

$$\begin{cases} 0 & \text{if } a=0 \\ a^4\,c^2\operatorname{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{c^2\operatorname{atan}(a\,x)}{2\,x^2}-\frac{c^2\,\left(a^3\operatorname{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{a^3\,c^2\,x}{2}-a^2\,c^2\operatorname{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}+a^2\,c^2\operatorname{Li}_2(1+a\,x\,1\mathrm{i})\,1\mathrm{i} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^2)/x^3,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (a^3*c^2*x)/2 - (c^2*atan(a*x))/(2*x^2) - a^
2*c^2*dilog(- a*x*1i + 1)*1i + a^2*c^2*dilog(a*x*1i + 1)*1i - (c^2*(a^3*ata
n(a*x) + a^2/x))/(2*a) + a^4*c^2*atan(a*x)*(1/(2*a^2) + x^2/2))
```

3.164 $\int \frac{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)}{x^4} dx$

Optimal. Leaf size=85

$$-\frac{ac^2}{6x^2}-\frac{c^2\text{ArcTan}(ax)}{3x^3}-\frac{2a^2c^2\text{ArcTan}(ax)}{x}+a^4c^2x\text{ArcTan}(ax)+\frac{5}{3}a^3c^2\log(x)-\frac{4}{3}a^3c^2\log\left(1+a^2x^2\right)$$

```
[Out] -1/6*a*c^2/x^2-1/3*c^2*arctan(a*x)/x^3-2*a^2*c^2*arctan(a*x)/x+a^4*c^2*x*ar
ctan(a*x)+5/3*a^3*c^2*ln(x)-4/3*a^3*c^2*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5068, 4930, 266, 4946, 272, 46, 36, 29, 31}

$$a^4c^2x\text{ArcTan}(ax)+\frac{5}{3}a^3c^2\log(x)-\frac{2a^2c^2\text{ArcTan}(ax)}{x}-\frac{4}{3}a^3c^2\log\left(a^2x^2+1\right)-\frac{c^2\text{ArcTan}(ax)}{3x^3}-\frac{ac^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(a*c^2)/x^2 - (c^2*ArcTan[a*x])/(3*x^3) - (2*a^2*c^2*ArcTan[a*x])/x +
a^4*c^2*x*ArcTan[a*x] + (5*a^3*c^2*Log[x])/3 - (4*a^3*c^2*Log[1 + a^2*x^2])
/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)}{x^4}\,dx = \int \left(a^4c^2\tan^{-1}(ax)+\frac{c^2\tan^{-1}(ax)}{x^4}+\frac{2a^2c^2\tan^{-1}(ax)}{x^2}\right)dx$$
$$= c^2\int \frac{\tan^{-1}(ax)}{x^4}\,dx + (2a^2c^2)\int \frac{\tan^{-1}(ax)}{x^2}\,dx + (a^4c^2)\int \tan^{-1}(ax)\,dx$$
$$= -\frac{c^2\tan^{-1}(ax)}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)}{x}+a^4c^2x\tan^{-1}(ax)+\frac{1}{3}(ac^2)\int \frac{1}{x^3(1+a^2\cdots}$$
$$= -\frac{c^2\tan^{-1}(ax)}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)}{x}+a^4c^2x\tan^{-1}(ax)-\frac{1}{2}a^3c^2\log(1+a^2x^2)\cdots$$
$$= -\frac{c^2\tan^{-1}(ax)}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)}{x}+a^4c^2x\tan^{-1}(ax)-\frac{1}{2}a^3c^2\log(1+a^2x^2)\cdots$$
$$= -\frac{ac^2}{6x^2}-\frac{c^2\tan^{-1}(ax)}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)}{x}+a^4c^2x\tan^{-1}(ax)+\frac{5}{3}a^3c^2\log(x)\cdots$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 0.80

$$\frac{c^2(2(-1-6a^2x^2+3a^4x^4)\operatorname{ArcTan}(ax)+ax(-1+10a^2x^2\log(x)-8a^2x^2\log(1+a^2x^2)))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
[Out] (c^2*(2*(-1 - 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + a*x*(-1 + 10*a^2*x^2*Log
[x] - 8*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

Maple [A]
time = 0.13, size = 78, normalized size = 0.92

method	result
derivativedivides	$a^3\left(a\,c^2x\arctan(ax)-\frac{2c^2\arctan(ax)}{ax}-\frac{c^2\arctan(ax)}{3a^3x^3}-\frac{c^2\left(4\ln(a^2x^2+1)+\frac{1}{2a^2x^2}-5\ln(ax)\right)}{3}\right)$
default	$a^3\left(a\,c^2x\arctan(ax)-\frac{2c^2\arctan(ax)}{ax}-\frac{c^2\arctan(ax)}{3a^3x^3}-\frac{c^2\left(4\ln(a^2x^2+1)+\frac{1}{2a^2x^2}-5\ln(ax)\right)}{3}\right)$
risch	$-\frac{ic^2(3a^4x^4-6a^2x^2-1)\ln(iax+1)}{6x^3}+\frac{ic^2(3x^4\ln(-iax+1)a^4-10i\ln(x)a^3x^3+8i\ln(-9a^2x^2-9)a^3x^3-6a^2x^2\ln(-iax+1\cdots}{6x^3}$
meijerg	$\frac{a^3c^2\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4}+\frac{a^3c^2\left(-\frac{4\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)+4\ln(x)+4\ln(a)\right)}{2}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(a*c^2*x*arctan(a*x)-2*c^2*arctan(a*x)/a/x-1/3*c^2*arctan(a*x)/a^3/x^3-
1/3*c^2*(4*ln(a^2*x^2+1)+1/2/a^2/x^2-5*ln(a*x)))
```

Maxima [A]
time = 0.25, size = 76, normalized size = 0.89

$$-\frac{1}{6}\left(8\,a^2c^2\log\left(a^2x^2+1\right)-10\,a^2c^2\log\left(x\right)+\frac{c^2}{x^2}\right)a+\frac{1}{3}\left(3\,a^4c^2x-\frac{6\,a^2c^2x^2+c^2}{x^3}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*a^2*c^2*log(a^2*x^2 + 1) - 10*a^2*c^2*log(x) + c^2/x^2)*a + 1/3*(3*
a^4*c^2*x - (6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x)
```

Fricas [A]
time = 2.27, size = 80, normalized size = 0.94

$$-\frac{8\,a^3c^2x^3\log\left(a^2x^2+1\right)-10\,a^3c^2x^3\log\left(x\right)+ac^2x-2\left(3\,a^4c^2x^4-6\,a^2c^2x^2-c^2\right)\arctan\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(8*a^3*c^2*x^3*log(a^2*x^2 + 1) - 10*a^3*c^2*x^3*log(x) + a*c^2*x - 2*
(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x))/x^3
```

Sympy [A]
time = 0.45, size = 87, normalized size = 1.02

$$\begin{cases} a^4c^2x\,\mathrm{atan}\left(ax\right)+\frac{5a^3c^2\log\left(x\right)}{3}-\frac{4a^3c^2\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{2a^2c^2\,\mathrm{atan}\left(ax\right)}{x}-\frac{ac^2}{6x^2}-\frac{c^2\,\mathrm{atan}\left(ax\right)}{3x^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((a**4*c**2*x*atan(a*x) + 5*a**3*c**2*log(x)/3 - 4*a**3*c**2*log(x
**2 + a**(-2))/3 - 2*a**2*c**2*atan(a*x)/x - a*c**2/(6*x**2) - c**2*atan(a*
x)/(3*x**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.47, size = 78, normalized size = 0.92

$$\frac{c^2\,\left(10\,a^3\,\ln\left(x\right)-8\,a^3\,\ln\left(a^2\,x^2+1\right)\right)}{6}-\frac{\frac{c^2\,\mathrm{atan}\left(a\,x\right)}{3}+\frac{a\,c^2\,x}{6}+2\,a^2\,c^2\,x^2\,\mathrm{atan}\left(a\,x\right)}{x^3}+a^4\,c^2\,x\,\mathrm{atan}\left(a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^2)/x^4,x)
```

```
[Out] (c^2*(10*a^3*log(x) - 8*a^3*log(a^2*x^2 + 1)))/6 - ((c^2*atan(a*x))/3 + (a*
c^2*x)/6 + 2*a^2*c^2*x^2*atan(a*x))/x^3 + a^4*c^2*x*atan(a*x)
```

3.165 $\int x^3(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=141

$$\frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7-\frac{1}{90}a^5c^3x^9-\frac{c^3\operatorname{ArcTan}(ax)}{40a^4}+\frac{1}{4}c^3x^4\operatorname{ArcTan}(ax)+\frac{1}{2}a^2c^3x^6\operatorname{ArcTan}(ax)+$$

```
[Out] 1/40*c^3*x/a^3-1/120*c^3*x^3/a-9/200*a*c^3*x^5-11/280*a^3*c^3*x^7-1/90*a^5*
c^3*x^9-1/40*c^3*arctan(a*x)/a^4+1/4*c^3*x^4*arctan(a*x)+1/2*a^2*c^3*x^6*ar
ctan(a*x)+3/8*a^4*c^3*x^8*arctan(a*x)+1/10*a^6*c^3*x^10*arctan(a*x)
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5068, 4946, 308, 209}

$$\frac{1}{10}a^6c^3x^{10}\operatorname{ArcTan}(ax)-\frac{1}{90}a^5c^3x^9+\frac{3}{8}a^4c^3x^8\operatorname{ArcTan}(ax)-\frac{c^3\operatorname{ArcTan}(ax)}{40a^4}-\frac{11}{280}a^3c^3x^7+\frac{c^3x}{40a^3}+\frac{1}{2}a^2c^3x^6\operatorname{ArcTan}(ax)+\frac{1}{4}c^3x^4\operatorname{ArcTan}(ax)-\frac{9}{200}ac^3x^5-\frac{c^3x^3}{120a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7)
/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*x]
)/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6*c^
3*x^10*ArcTan[a*x])/10
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^3(c+a^2cx^2)^3\tan^{-1}(ax)\,dx = \int \left(c^3x^3\tan^{-1}(ax)+3a^2c^3x^5\tan^{-1}(ax)+3a^4c^3x^7\tan^{-1}(ax)+a^6c^3x^9\tan^{-1}\right.$$
$$= c^3\int x^3\tan^{-1}(ax)\,dx+\left(3a^2c^3\right)\int x^5\tan^{-1}(ax)\,dx+\left(3a^4c^3\right)\int x^7\tan^{-1}(a$$
$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)+\frac{1}{10}a^6c^3x^{10}\mathrm{ta}$$
$$= \frac{1}{4}c^3x^4\tan^{-1}(ax)+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)+\frac{1}{10}a^6c^3x^{10}\mathrm{ta}$$
$$= \frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7-\frac{1}{90}a^5c^3x^9+\frac{1}{4}c^3x^4\tan^{-1}(ax)+$$
$$= \frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7-\frac{1}{90}a^5c^3x^9-\frac{c^3\tan^{-1}(ax)}{40a^4}+\frac{1}{4}c^3$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.54

$$\frac{c^3\left(-ax(-315+105a^2x^2+567a^4x^4+495a^6x^6+140a^8x^8)+315(1+a^2x^2)^4\,(-1+4a^2x^2)\,\mathrm{ArcTan}(ax)\right)}{12600a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
[Out] (c^3*(-(a*x*(-315 + 105*a^2*x^2 + 567*a^4*x^4 + 495*a^6*x^6 + 140*a^8*x^8))
 + 315*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]))/(12600*a^4)
```

Maple [A]
time = 0.13, size = 112, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{a^4c^3x^4\arctan(ax)}{4}-\frac{c^3\left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\mathrm{arc}\right.}{40}}{a^4}$

default	$\frac{\frac{c^3 \arctan(ax)a^{10}x^{10}}{10}+\frac{3c^3 \arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6 \arctan(ax)}{2}+\frac{a^4c^3x^4 \arctan(ax)}{4}-\frac{c^3\left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax+a\right.}{40}}{a^4}$
risch	$-\frac{ic^3x^4(4a^6x^6+15a^4x^4+20a^2x^2+10)\ln(iax+1)}{80}+\frac{ic^3a^6x^{10}\ln(-iax+1)}{20}-\frac{a^5c^3x^9}{90}+\frac{3ic^3a^4x^8\ln(-iax+1)}{16}-\frac{1}{}$
meijerg	$\frac{c^3\left(-\frac{2xa(385a^8x^8-495a^6x^6+693a^4x^4-1155a^2x^2+3465)}{17325}+\frac{2xa(11a^{10}x^{10}+11)\arctan\left(\sqrt{a^2x^2}\right)}{55\sqrt{a^2x^2}}\right)}{4a^4}+\frac{3c^3\left(\frac{xa(-45a^6x^6+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/10*c^3*arctan(a*x)*a^10*x^10+3/8*c^3*arctan(a*x)*a^8*x^8+1/2*a^6*c
^3*x^6*arctan(a*x)+1/4*a^4*c^3*x^4*arctan(a*x)-1/40*c^3*(4/9*a^9*x^9+11/7*a
^7*x^7+9/5*a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

Maxima [A]

time = 0.47, size = 120, normalized size = 0.85

$$-\frac{1}{12600}a\left(\frac{315\,c^3\arctan(ax)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)+\frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c^3x^8+20\,a^2c^3x^6+10\,c^3x^4\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/12600*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4) + 1/40*(4*a^6*c^3*x^10
+ 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)
```

Fricas [A]

time = 1.90, size = 113, normalized size = 0.80

$$-\frac{140\,a^9c^3x^9+495\,a^7c^3x^7+567\,a^5c^3x^5+105\,a^3c^3x^3-315\,ac^3x-315\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3x^6+10\,a^4c^3x^4-c^3\right)\arctan(ax)}{12600\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/12600*(140*a^9*c^3*x^9 + 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3
*x^3 - 315*a*c^3*x - 315*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20*a^6*c^3*x^6
 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x))/a^4
```

Sympy [A]

time = 0.67, size = 138, normalized size = 0.98

$$\begin{cases}\frac{a^6c^3x^{10}\operatorname{atan}(ax)}{10}-\frac{a^5c^3x^9}{90}+\frac{3a^4c^3x^8\operatorname{atan}(ax)}{8}-\frac{11a^3c^3x^7}{280}+\frac{a^2c^3x^6\operatorname{atan}(ax)}{2}-\frac{9ac^3x^5}{200}+\frac{c^3x^4\operatorname{atan}(ax)}{4}-\frac{c^3x^3}{120a}+\frac{c^3x}{40a^3}-\frac{c^3\operatorname{atan}(ax)}{40a^4} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**10*atan(a*x)/10 - a**5*c**3*x**9/90 + 3*a**4*c**3*x
**8*atan(a*x)/8 - 11*a**3*c**3*x**7/280 + a**2*c**3*x**6*atan(a*x)/2 - 9*a*
c**3*x**5/200 + c**3*x**4*atan(a*x)/4 - c**3*x**3/(120*a) + c**3*x/(40*a**3
) - c**3*atan(a*x)/(40*a**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.44, size = 111, normalized size = 0.79

$$\mathrm{atan}(a\,x)\,\left(\frac{a^6\,c^3\,x^{10}}{10}+\frac{3\,a^4\,c^3\,x^8}{8}+\frac{a^2\,c^3\,x^6}{2}+\frac{c^3\,x^4}{4}\right)+\frac{c^3\,x}{40\,a^3}-\frac{9\,a\,c^3\,x^5}{200}-\frac{c^3\,\mathrm{atan}(a\,x)}{40\,a^4}-\frac{c^3\,x^3}{120\,a}-\frac{11\,a^3\,c^3\,x^7}{280}-\frac{a^5\,c^3\,x^9}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)*((c^3*x^4)/4 + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8)/8 + (a^6*c^3*x^1
0)/10) + (c^3*x)/(40*a^3) - (9*a*c^3*x^5)/200 - (c^3*atan(a*x))/(40*a^4) -
(c^3*x^3)/(120*a) - (11*a^3*c^3*x^7)/280 - (a^5*c^3*x^9)/90
```

3.166 $\int x^2(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=136

$$-\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\operatorname{ArcTan}(ax)+\frac{3}{5}a^2c^3x^5\operatorname{ArcTan}(ax)+\frac{3}{7}a^4c^3x^7\operatorname{ArcTan}(ax)+$$

```
[Out] -8/315*c^3*x^2/a-89/1260*a*c^3*x^4-10/189*a^3*c^3*x^6-1/72*a^5*c^3*x^8+1/3*
c^3*x^3*arctan(a*x)+3/5*a^2*c^3*x^5*arctan(a*x)+3/7*a^4*c^3*x^7*arctan(a*x)
+1/9*a^6*c^3*x^9*arctan(a*x)+8/315*c^3*ln(a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5068, 4946, 272, 45}

$$\frac{1}{9}a^6c^3x^9\operatorname{ArcTan}(ax)-\frac{1}{72}a^5c^3x^8+\frac{3}{7}a^4c^3x^7\operatorname{ArcTan}(ax)-\frac{10}{189}a^3c^3x^6+\frac{3}{5}a^2c^3x^5\operatorname{ArcTan}(ax)+\frac{8c^3\log\left(a^2x^2+1\right)}{315a^3}+\frac{1}{3}c^3x^3\operatorname{ArcTan}(ax)-\frac{89ac^3x^4}{1260}-\frac{8c^3x^2}{315a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c^
3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*a^
4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 + a^2
*x^2])/(315*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx=\int\left(c^3x^2\tan^{-1}(ax)+3a^2c^3x^4\tan^{-1}(ax)+3a^4c^3x^6\tan^{-1}(ax)+a^6c^3x^8\tan\right.$$
$$=c^3\int x^2\tan^{-1}(ax)\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)\,dx+\left(3a^4c^3\right)\int x^6\tan^{-1}(a$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan$$
$$=\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)+\frac{1}{9}a^6c^3x^9\tan$$
$$=-\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\tan^{-1}(ax)+\frac{3}{5}a^2c^3x$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 0.70

$$\frac{c^3(-a^2x^2(192+534a^2x^2+400a^4x^4+105a^6x^6)+24a^3x^3(105+189a^2x^2+135a^4x^4+35a^6x^6)\operatorname{ArcTan}(ax)+192\log(1+a^2x^2))}{7560a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*(-(a^2*x^2*(192 + 534*a^2*x^2 + 400*a^4*x^4 + 105*a^6*x^6)) + 24*a^3*x
^3*(105 + 189*a^2*x^2 + 135*a^4*x^4 + 35*a^6*x^6)*ArcTan[a*x] + 192*Log[1 +
 a^2*x^2]))/(7560*a^3)
```

Maple [A]
time = 0.14, size = 116, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^9x^9}{9}+\frac{3c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+\frac{a^3c^3x^3\arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\ln\left(a\right.\right.}{315}}{a^3}$

default	$\frac{\frac{c^3 \arctan(ax) a^9 x^9}{9}+\frac{3c^3 \arctan(ax) a^7 x^7}{7}+\frac{3a^5 c^3 x^5 \arctan(ax)}{5}+\frac{a^3 c^3 x^3 \arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\ln\right.}{315}}{a^3}$
risch	$-\frac{ic^3x^3(35a^6x^6+135a^4x^4+189a^2x^2+105)\ln(iax+1)}{630}+\frac{ic^3a^6x^9\ln(-iax+1)}{18}-\frac{a^5c^3x^8}{72}+\frac{3ic^3a^4x^7\ln(-iax+1)}{14}-$
meijerg	$\frac{c^3\left(\frac{a^2x^2(-15a^6x^6+20a^4x^4-30a^2x^2+60)}{270}+\frac{4a^{10}x^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{9}\right)}{4a^3}+\frac{3c^3\left(-\frac{a^2x^2(4a^4x^4-6a^2x^2+\right.}{42}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/9*c^3*arctan(a*x)*a^9*x^9+3/7*c^3*arctan(a*x)*a^7*x^7+3/5*a^5*c^3*
x^5*arctan(a*x)+1/3*a^3*c^3*x^3*arctan(a*x)-1/315*c^3*(35/8*a^8*x^8+50/3*a^
6*x^6+89/4*a^4*x^4+8*a^2*x^2-8*ln(a^2*x^2+1)))
```

Maxima [A]

time = 0.29, size = 118, normalized size = 0.87

$$\frac{1}{7560}\,a\left(\frac{192\,c^3\log\left(a^2x^2+1\right)}{a^4}-\frac{105\,a^6c^3x^8+400\,a^4c^3x^6+534\,a^2c^3x^4+192\,c^3x^2}{a^2}\right)+\frac{1}{315}\left(35\,a^6c^3x^9+135\,a^4c^3x^7+189\,a^2c^3x^5+105\,c^3x^3\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*(192*c^3*log(a^2*x^2 + 1)/a^4 - (105*a^6*c^3*x^8 + 400*a^4*c^3*x^6
 + 534*a^2*c^3*x^4 + 192*c^3*x^2)/a^2) + 1/315*(35*a^6*c^3*x^9 + 135*a^4*c^
3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*arctan(a*x)
```

Fricas [A]

time = 2.59, size = 116, normalized size = 0.85

$$-\frac{105\,a^8c^3x^8+400\,a^6c^3x^6+534\,a^4c^3x^4+192\,a^2c^3x^2-192\,c^3\log\left(a^2x^2+1\right)-24\left(35\,a^9c^3x^9+135\,a^7c^3x^7+189\,a^5c^3x^5+105\,a^3c^3x^3\right)\arctan\left(ax\right)}{7560\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/7560*(105*a^8*c^3*x^8 + 400*a^6*c^3*x^6 + 534*a^4*c^3*x^4 + 192*a^2*c^3*
x^2 - 192*c^3*log(a^2*x^2 + 1) - 24*(35*a^9*c^3*x^9 + 135*a^7*c^3*x^7 + 189
*a^5*c^3*x^5 + 105*a^3*c^3*x^3)*arctan(a*x))/a^3
```

Sympy [A]

time = 0.59, size = 138, normalized size = 1.01

$$\begin{cases} \frac{a^6c^3x^9\operatorname{atan}(ax)}{9}-\frac{a^5c^3x^8}{72}+\frac{3a^4c^3x^7\operatorname{atan}(ax)}{7}-\frac{10a^3c^3x^6}{189}+\frac{3a^2c^3x^5\operatorname{atan}(ax)}{5}-\frac{89ac^3x^4}{1260}+\frac{c^3x^3\operatorname{atan}(ax)}{3}-\frac{8c^3x^2}{315a}+\frac{8c^3\log\left(x^2+\frac{1}{a^2}\right)}{315a^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**9*atan(a*x)/9 - a**5*c**3*x**8/72 + 3*a**4*c**3*x**
7*atan(a*x)/7 - 10*a**3*c**3*x**6/189 + 3*a**2*c**3*x**5*atan(a*x)/5 - 89*a
*c**3*x**4/1260 + c**3*x**3*atan(a*x)/3 - 8*c**3*x**2/(315*a) + 8*c**3*log(
x**2 + a**(-2))/(315*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.44, size = 108, normalized size = 0.79

$$\mathrm{atan}(a\,x)\,\left(\frac{a^6\,c^3\,x^9}{9}+\frac{3\,a^4\,c^3\,x^7}{7}+\frac{3\,a^2\,c^3\,x^5}{5}+\frac{c^3\,x^3}{3}\right)-\frac{89\,a\,c^3\,x^4}{1260}+\frac{8\,c^3\,\ln\left(a^2\,x^2+1\right)}{315\,a^3}-\frac{8\,c^3\,x^2}{315\,a}-\frac{10\,a^3\,c^3\,x^6}{189}-\frac{a^5\,c^3\,x^8}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)*((c^3*x^3)/3 + (3*a^2*c^3*x^5)/5 + (3*a^4*c^3*x^7)/7 + (a^6*c^3*x
^9)/9) - (89*a*c^3*x^4)/1260 + (8*c^3*log(a^2*x^2 + 1))/(315*a^3) - (8*c^3*
x^2)/(315*a) - (10*a^3*c^3*x^6)/189 - (a^5*c^3*x^8)/72
```

3.167 $\int x(c+a^2cx^2)^3 \textbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=74

$$-\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3(1+a^2x^2)^4\,\mathrm{ArcTan}(ax)}{8a^2}$$

```
[Out] -1/8*c^3*x/a-1/8*a*c^3*x^3-3/40*a^3*c^3*x^5-1/56*a^5*c^3*x^7+1/8*c^3*(a^2*x
^2+1)^4*arctan(a*x)/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5050, 200}

$$-\frac{1}{56}a^5c^3x^7-\frac{3}{40}a^3c^3x^5+\frac{c^3(a^2x^2+1)^4\,\mathrm{ArcTan}(ax)}{8a^2}-\frac{1}{8}ac^3x^3-\frac{c^3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] -1/8*(c^3*x)/a - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (c
^3*(1 + a^2*x^2)^4*ArcTan[a*x])/(8*a^2)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x(c+a^2cx^2)^3\tan^{-1}(ax)\,dx &= \frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)}{8a^2}-\frac{\int (c+a^2cx^2)^3\,dx}{8a}\\
&= \frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)}{8a^2}-\frac{\int (c^3+3a^2c^3x^2+3a^4c^3x^4+a^6c^3x^6)\,dx}{8a}\\
&= -\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)}{8a^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.78

$$\frac{c^3\left(-ax(35+35a^2x^2+21a^4x^4+5a^6x^6)+35(1+a^2x^2)^4\operatorname{ArcTan}(ax)\right)}{280a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*(-(a*x*(35 + 35*a^2*x^2 + 21*a^4*x^4 + 5*a^6*x^6)) + 35*(1 + a^2*x^2)^
4*ArcTan[a*x]))/(280*a^2)
```

Maple [A]
time = 0.10, size = 107, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+ax\right)}{8}}{a^2}$
default	$\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+ax\right)}{8}}{a^2}$
risch	$-\frac{ic^3\left(a^2x^2+1\right)^4\ln(iax+1)}{16a^2}+\frac{ic^3a^6x^8\ln(-iax+1)}{16}-\frac{a^5c^3x^7}{56}+\frac{ic^3a^4x^6\ln(-iax+1)}{4}-\frac{3a^3c^3x^5}{40}+\frac{3ic^3a^2x^4\ln(-$
meijerg	$\frac{c^3\left(\frac{xa\left(-45a^6x^6+63a^4x^4-105a^2x^2+315\right)}{630}-\frac{xa\left(-9a^8x^8+9\right)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^2}+\frac{3c^3\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa}{}\right.}{4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/8*c^3*arctan(a*x)*a^8*x^8+1/2*a^6*c^3*x^6*arctan(a*x)+3/4*a^4*c^3*
x^4*arctan(a*x)+1/2*a^2*c^3*x^2*arctan(a*x)+1/8*c^3*arctan(a*x)-1/8*c^3*(1/
7*a^7*x^7+3/5*a^5*x^5+a^3*x^3+a*x))
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.99

$$\frac{(a^2cx^2+c)^4\arctan(ax)}{8\,a^2c}-\frac{5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+35\,c^4x}{280\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^4*x^7 + 21*a^4*c
^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)/(a*c)
```

Fricas [A]
time = 3.48, size = 99, normalized size = 1.34

$$-\frac{5 a^{7} c^{3} x^{7} + 21 a^{5} c^{3} x^{5} + 35 a^{3} c^{3} x^{3} + 35 a c^{3} x - 35 \left(a^{8} c^{3} x^{8} + 4 a^{6} c^{3} x^{6} + 6 a^{4} c^{3} x^{4} + 4 a^{2} c^{3} x^{2} + c^{3}\right) \arctan\left(a x\right)}{280 a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/280*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x - 35*(
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a
*x))/a^2
```

Sympy [A]
time = 0.45, size = 124, normalized size = 1.68

$$\begin{cases} \frac{a^{6} c^{3} x^{8} \operatorname{atan}{\left(a x \right)}}{8} - \frac{a^{5} c^{3} x^{7}}{56} + \frac{a^{4} c^{3} x^{6} \operatorname{atan}{\left(a x \right)}}{2} - \frac{3 a^{3} c^{3} x^{5}}{40} + \frac{3 a^{2} c^{3} x^{4} \operatorname{atan}{\left(a x \right)}}{4} - \frac{a c^{3} x^{3}}{8} + \frac{c^{3} x^{2} \operatorname{atan}{\left(a x \right)}}{2} - \frac{c^{3} x}{8 a} + \frac{c^{3} \operatorname{atan}{\left(a x \right)}}{8 a^{2}} & \text{for}\: a \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**8*atan(a*x)/8 - a**5*c**3*x**7/56 + a**4*c**3*x**6*
atan(a*x)/2 - 3*a**3*c**3*x**5/40 + 3*a**2*c**3*x**4*atan(a*x)/4 - a*c**3*x
**3/8 + c**3*x**2*atan(a*x)/2 - c**3*x/(8*a) + c**3*atan(a*x)/(8*a**2), Ne(
a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.41, size = 100, normalized size = 1.35

$$\mathrm{atan}(a\,x)\,\left(\frac{a^6\,c^3\,x^8}{8}+\frac{a^4\,c^3\,x^6}{2}+\frac{3\,a^2\,c^3\,x^4}{4}+\frac{c^3\,x^2}{2}\right)-\frac{c^3\,x}{8\,a}-\frac{a\,c^3\,x^3}{8}+\frac{c^3\,\mathrm{atan}(a\,x)}{8\,a^2}-\frac{3\,a^3\,c^3\,x^5}{40}-\frac{a^5\,c^3\,x^7}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)*((c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)/2 + (a^6*c^3*x^8
)/8) - (c^3*x)/(8*a) - (a*c^3*x^3)/8 + (c^3*atan(a*x))/(8*a^2) - (3*a^3*c^3
*x^5)/40 - (a^5*c^3*x^7)/56
```

3.168 $\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=161

$$-\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}+\frac{16}{35}c^3x\text{ArcTan}(ax)+\frac{8}{35}c^3x(1+a^2x^2)\text{ ArcTan}(ax)+\frac{6}{35}c^3x($$

```
[Out] -4/35*c^3*(a^2*x^2+1)/a-3/70*c^3*(a^2*x^2+1)^2/a-1/42*c^3*(a^2*x^2+1)^3/a+1
6/35*c^3*x*arctan(a*x)+8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)+6/35*c^3*x*(a^2*x
^2+1)^2*arctan(a*x)+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)-8/35*c^3*ln(a^2*x^2
+1)/a
```

Rubi [A]
time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4998, 4930, 266}

$$\frac{1}{7}c^3x(a^2x^2+1)^3\text{ ArcTan}(ax)+\frac{6}{35}c^3x(a^2x^2+1)^2\text{ ArcTan}(ax)+\frac{8}{35}c^3x(a^2x^2+1)\text{ ArcTan}(ax)-\frac{c^3(a^2x^2+1)^3}{42a}-\frac{3c^3(a^2x^2+1)^2}{70a}-\frac{4c^3(a^2x^2+1)}{35a}-\frac{8c^3\log(a^2x^2+1)}{35a}+\frac{16}{35}c^3x\text{ArcTan}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (-4*c^3*(1 + a^2*x^2))/(35*a) - (3*c^3*(1 + a^2*x^2)^2)/(70*a) - (c^3*(1 +
a^2*x^2)^3)/(42*a) + (16*c^3*x*ArcTan[a*x])/35 + (8*c^3*x*(1 + a^2*x^2)*Arc
Tan[a*x])/35 + (6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/35 + (c^3*x*(1 + a^2*x
^2)^3*ArcTan[a*x])/7 - (8*c^3*Log[1 + a^2*x^2])/(35*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx = -\frac{c^3(1+a^2x^2)^3}{42a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)+\frac{1}{7}(6c)\int\left(c+a^2cx^2\right)^2\tan^{-}$$

$$= -\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)+\frac{1}{7}c^3x(1$$

$$= -\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}+\frac{8}{35}c^3x\left(1+a^2x^2\right)\tan^{-}$$

$$= -\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}+\frac{16}{35}c^3x\tan^{-1}(ax)+\frac{8}{35}$$

$$= -\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}+\frac{16}{35}c^3x\tan^{-1}(ax)+\frac{8}{35}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.52

$$\frac{c^3(-a^2x^2(57+24a^2x^2+5a^4x^4)+6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\,\mathrm{ArcTan}(ax)-48\log\left(1+a^2x^2\right))}{210a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*(-(a^2*x^2*(57 + 24*a^2*x^2 + 5*a^4*x^4)) + 6*a*x*(35 + 35*a^2*x^2 + 2
1*a^4*x^4 + 5*a^6*x^6)*ArcTan[a*x] - 48*Log[1 + a^2*x^2]))/(210*a)
```

Maple [A]
time = 0.09, size = 102, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+a\,c^3x\arctan(ax)-\frac{c^3\left(\frac{5a^6x^6}{6}+4a^4x^4+\frac{19a^2x^2}{2}+8\ln\left(a^2x^2+1\right)\right)}{35}}{a}$
default	$\frac{\frac{c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+a\,c^3x\arctan(ax)-\frac{c^3\left(\frac{5a^6x^6}{6}+4a^4x^4+\frac{19a^2x^2}{2}+8\ln\left(a^2x^2+1\right)\right)}{35}}{a}$
risch	$-\frac{ic^3x(5a^6x^6+21a^4x^4+35a^2x^2+35)\ln(iax+1)}{70}+\frac{ic^3a^6x^7\ln(-iax+1)}{14}-\frac{a^5c^3x^6}{42}+\frac{3ic^3a^4x^5\ln(-iax+1)}{10}-\frac{4a^3}{3}$
meijerg	$\frac{c^3\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\frac{4a^8x^8\arctan\left(\sqrt{a^2x^2}\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{7}\right)}{4a}+\frac{3c^3\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{}\right.}{5\sqrt{a^2x}}\right.}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/7*c^3*arctan(a*x)*a^7*x^7+3/5*a^5*c^3*x^5*arctan(a*x)+a^3*c^3*x^3*ar
ctan(a*x)+a*c^3*x*arctan(a*x)-1/35*c^3*(5/6*a^6*x^6+4*a^4*x^4+19/2*a^2*x^2+
8*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.25, size = 99, normalized size = 0.61

$$-\frac{1}{210}\left(5\,a^{4}c^{3}x^{6}+24\,a^{2}c^{3}x^{4}+57\,c^{3}x^{2}+\frac{48\,c^{3}\log\left(a^{2}x^{2}+1\right)}{a^{2}}\right)a+\frac{1}{35}\left(5\,a^{6}c^{3}x^{7}+21\,a^{4}c^{3}x^{5}+35\,a^{2}c^{3}x^{3}+35\,c^{3}x\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*(5*a^4*c^3*x^6 + 24*a^2*c^3*x^4 + 57*c^3*x^2 + 48*c^3*log(a^2*x^2 +
1)/a^2)*a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*
x)*arctan(a*x)
```

Fricas [A]
time = 3.57, size = 101, normalized size = 0.63

$$-\frac{5\,a^{6}c^{3}x^{6}+24\,a^{4}c^{3}x^{4}+57\,a^{2}c^{3}x^{2}+48\,c^{3}\log\left(a^{2}x^{2}+1\right)-6\left(5\,a^{7}c^{3}x^{7}+21\,a^{5}c^{3}x^{5}+35\,a^{3}c^{3}x^{3}+35\,ac^{3}x\right)\arctan\left(ax\right)}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^
2 + 1) - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x)*a
rctan(a*x))/a
```

Sympy [A]
time = 0.41, size = 117, normalized size = 0.73

$$\begin{cases}\frac{a^{6}c^{3}x^{7}\operatorname{atan}\left(ax\right)}{7}-\frac{a^{5}c^{3}x^{6}}{42}+\frac{3a^{4}c^{3}x^{5}\operatorname{atan}\left(ax\right)}{5}-\frac{4a^{3}c^{3}x^{4}}{35}+a^{2}c^{3}x^{3}\operatorname{atan}\left(ax\right)-\frac{19ac^{3}x^{2}}{70}+c^{3}x\operatorname{atan}\left(ax\right)-\frac{8c^{3}\log\left(x^{2}+\frac{1}{a^{2}}\right)}{35a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] Piecewise((a**6*c**3*x**7*atan(a*x)/7 - a**5*c**3*x**6/42 + 3*a**4*c**3*x**
5*atan(a*x)/5 - 4*a**3*c**3*x**4/35 + a**2*c**3*x**3*atan(a*x) - 19*a*c**3*
x**2/70 + c**3*x*atan(a*x) - 8*c**3*log(x**2 + a**(-2))/(35*a), Ne(a, 0)),
(0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.25, size = 89, normalized size = 0.55

$$-\frac{c^3\,\left(48\,\ln\left(a^2\,x^2+1\right)+57\,a^2\,x^2+24\,a^4\,x^4+5\,a^6\,x^6-210\,a^3\,x^3\,\mathrm{atan}(a\,x)-126\,a^5\,x^5\,\mathrm{atan}(a\,x)-30\,a^7\,x^7\,\mathrm{atan}(a\,x)-210\,a\,x\,\mathrm{atan}(a\,x)\right)}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2)^3,x)
```

```
[Out] -(c^3*(48*log(a^2*x^2 + 1) + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 210*a^3*
x^3*atan(a*x) - 126*a^5*x^5*atan(a*x) - 30*a^7*x^7*atan(a*x) - 210*a*x*atan
(a*x)))/(210*a)
```

3.169 $\int \frac{(c+a^2cx^2)^3\text{ArcTan}(ax)}{x}\,dx$

Optimal. Leaf size=132

$$-\frac{11}{12}ac^3x-\frac{7}{36}a^3c^3x^3-\frac{1}{30}a^5c^3x^5+\frac{11}{12}c^3\text{ArcTan}(ax)+\frac{3}{2}a^2c^3x^2\text{ArcTan}(ax)+\frac{3}{4}a^4c^3x^4\text{ArcTan}(ax)+\frac{1}{6}a^6c^3x^6\text{ArcT}$$

```
[Out] -11/12*a*c^3*x-7/36*a^3*c^3*x^3-1/30*a^5*c^3*x^5+11/12*c^3*arctan(a*x)+3/2*
a^2*c^3*x^2*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+1/6*a^6*c^3*x^6*arctan(
a*x)+1/2*I*c^3*polylog(2,-I*a*x)-1/2*I*c^3*polylog(2,I*a*x)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5068, 4940, 2438, 4946, 327, 209, 308}

$$\frac{1}{6}a^6c^3x^6\text{ArcTan}(ax)-\frac{1}{30}a^5c^3x^5+\frac{3}{4}a^4c^3x^4\text{ArcTan}(ax)-\frac{7}{36}a^3c^3x^3+\frac{3}{2}a^2c^3x^2\text{ArcTan}(ax)+\frac{11}{12}c^3\text{ArcTan}(ax)+\frac{1}{2}ic^3\text{Li}_2(-iax)-\frac{1}{2}ic^3\text{Li}_2(iax)-\frac{11}{12}ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

```
[Out] (-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[a
*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 + (
a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*Pol
yLog[2, I*a*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)}{x}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)}{x} + 3a^2c^3x\tan^{-1}(ax) + 3a^4c^3x^3\tan^{-1}(ax) + a^6c^3x^5\tan^{-1}(\right.$$

$$= c^3\int \frac{\tan^{-1}(ax)}{x}\,dx + \left(3a^2c^3\right)\int x\tan^{-1}(ax)\,dx + \left(3a^4c^3\right)\int x^3\tan^{-1}(ax)\,d$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax) + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax) + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax) + \frac{1}{2}\left(ic^3\right)\int$$

$$= -\frac{3}{2}ac^3x + \frac{3}{2}a^2c^3x^2\tan^{-1}(ax) + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax) + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax) +$$

$$= -\frac{11}{12}ac^3x - \frac{7}{36}a^3c^3x^3 - \frac{1}{30}a^5c^3x^5 + \frac{3}{2}c^3\tan^{-1}(ax) + \frac{3}{2}a^2c^3x^2\tan^{-1}(ax) +$$

$$= -\frac{11}{12}ac^3x - \frac{7}{36}a^3c^3x^3 - \frac{1}{30}a^5c^3x^5 + \frac{11}{12}c^3\tan^{-1}(ax) + \frac{3}{2}a^2c^3x^2\tan^{-1}(ax) +$$

Mathematica [A]

time = 0.04, size = 94, normalized size = 0.71

$$\frac{1}{180}c^3\left(-165ax - 35a^3x^3 - 6a^5x^5 + 165\text{ArcTan}(ax) + 270a^2x^2\text{ArcTan}(ax) + 135a^4x^4\text{ArcTan}(ax) + 30a^6x^6\text{ArcTan}(ax) + 90i\text{PolyLog}(2,-iax) - 90i\text{PolyLog}(2,iax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

```
[Out] (c^3*(-165*a*x - 35*a^3*x^3 - 6*a^5*x^5 + 165*ArcTan[a*x] + 270*a^2*x^2*Arc
Tan[a*x] + 135*a^4*x^4*ArcTan[a*x] + 30*a^6*x^6*ArcTan[a*x] + (90*I)*PolyLo
g[2, (-I)*a*x] - (90*I)*PolyLog[2, I*a*x]))/180
```

Maple [A]
time = 0.06, size = 143, normalized size = 1.08

method	result
derivativedivides	$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\frac{7a^3x}{3}\right.}{}$
default	$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\frac{7a^3x}{3}\right.}{}$
risch	$\frac{ic^3\ln(-iax+1)x^6a^6}{12}+\frac{3ic^3\ln(-iax+1)x^4a^4}{8}+\frac{3ic^3\ln(-iax+1)x^2a^2}{4}-\frac{a^5c^3x^5}{30}-\frac{7a^3c^3x^3}{36}-\frac{11a\,c^3x}{12}-\frac{ic^3\,\mathrm{dilo}}{}$
meijerg	$\frac{c^3\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4}+\frac{3c^3\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{}\right.}{5\sqrt{a^2x^2}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^6*c^3*x^6*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*arc
tan(a*x)+c^3*arctan(a*x)*ln(a*x)-1/12*c^3*(2/5*a^5*x^5+7/3*a^3*x^3+11*a*x-1
1*arctan(a*x)-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(1+I
*a*x)+6*I*dilog(1-I*a*x))
```

Maxima [A]
time = 0.50, size = 127, normalized size = 0.96

$$-\frac{1}{30}a^5c^3x^5-\frac{7}{36}a^3c^3x^3-\frac{11}{12}ac^3x-\frac{1}{4}\pi c^3\log\left(a^2x^2+1\right)+c^3\arctan\left(ax\right)\log\left(ax\right)-\frac{1}{2}i\,c^3\mathrm{Li}_2(i\,ax+1)+\frac{1}{2}i\,c^3\mathrm{Li}_2(-i\,ax+1)+\frac{1}{12}\left(2\,a^6c^3x^6+9\,a^4c^3x^4+18\,a^2c^3x^2+11\,c^3\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/30*a^5*c^3*x^5 - 7/36*a^3*c^3*x^3 - 11/12*a*c^3*x - 1/4*pi*c^3*log(a^2*x
^2 + 1) + c^3*arctan(a*x)*log(a*x) - 1/2*I*c^3*dilog(I*a*x + 1) + 1/2*I*c^3
*dilog(-I*a*x + 1) + 1/12*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2 +
 11*c^3)*arctan(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}(ax)\,dx + \int 3a^4x^3\operatorname{atan}(ax)\,dx + \int a^6x^5\operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)/x, x) + Integral(3*a**2*x*atan(a*x), x) + Integral
(3*a**4*x**3*atan(a*x), x) + Integral(a**6*x**5*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.67, size = 156, normalized size = 1.18

$$\begin{cases} 0 & \text{if } a=0 \\ 3\,a^2\,c^3\operatorname{atan}(a\,x)\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a^5\,c^3\left(\frac{x}{a^4}-\frac{\operatorname{atan}(a\,x)}{a^5}+\frac{x^5}{5}-\frac{x^3}{3\,a^2}\right)}{6}-\frac{3\,a\,c^3\,x}{2}-\frac{c^3\,\left(3\operatorname{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{4}+\frac{3\,a^4\,c^3\,x^4\operatorname{atan}(a\,x)}{4}+\frac{a^6\,c^3\,x^6\operatorname{atan}(a\,x)}{6}-\frac{c^3\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{c^3\,\mathrm{Li}_2(1+a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^3)/x,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (c^3*dilog(- a*x*1i + 1)*1i)/2 + (c^3*dilog(
a*x*1i + 1)*1i)/2 - (c^3*(3*atan(a*x) - 3*a*x + a^3*x^3))/4 - (a^5*c^3*(x/a
^4 - atan(a*x)/a^5 + x^5/5 - x^3/(3*a^2)))/6 - (3*a*c^3*x)/2 + 3*a^2*c^3*at
an(a*x)*(1/(2*a^2) + x^2/2) + (3*a^4*c^3*x^4*atan(a*x))/4 + (a^6*c^3*x^6*at
an(a*x))/6)
```

3.170 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)}{x^2} dx$

Optimal. Leaf size=108

$$-\frac{2}{5}a^3c^3x^2-\frac{1}{20}a^5c^3x^4-\frac{c^3\text{ArcTan}(ax)}{x}+3a^2c^3x\text{ArcTan}(ax)+a^4c^3x^3\text{ArcTan}(ax)+\frac{1}{5}a^6c^3x^5\text{ArcTan}(ax)+ac^3\log$$

```
[Out] -2/5*a^3*c^3*x^2-1/20*a^5*c^3*x^4-c^3*arctan(a*x)/x+3*a^2*c^3*x*arctan(a*x)
+a^4*c^3*x^3*arctan(a*x)+1/5*a^6*c^3*x^5*arctan(a*x)+a*c^3*ln(x)-8/5*a*c^3*
ln(a^2*x^2+1)
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5068, 4930, 266, 4946, 272, 36, 29, 31, 45}

$$\frac{1}{5}a^6c^3x^5\text{ArcTan}(ax) - \frac{1}{20}a^5c^3x^4 + a^4c^3x^3\text{ArcTan}(ax) - \frac{2}{5}a^3c^3x^2 + 3a^2c^3x\text{ArcTan}(ax) - \frac{8}{5}ac^3\log\left(a^2x^2+1\right) - \frac{c^3\text{ArcTan}(ax)}{x} + ac^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

```
[Out] (-2*a^3*c^3*x^2)/5 - (a^5*c^3*x^4)/20 - (c^3*ArcTan[a*x])/x + 3*a^2*c^3*x*A
rcTan[a*x] + a^4*c^3*x^3*ArcTan[a*x] + (a^6*c^3*x^5*ArcTan[a*x])/5 + a*c^3*
Log[x] - (8*a*c^3*Log[1 + a^2*x^2])/5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)}{x^2}\,dx = \int \left(3a^2c^3\tan^{-1}(ax) + \frac{c^3\tan^{-1}(ax)}{x^2} + 3a^4c^3x^2\tan^{-1}(ax) + a^6c^3x^4\tan^{-1}(ax\right.$$

$$= c^3\int \frac{\tan^{-1}(ax)}{x^2}\,dx + (3a^2c^3)\int \tan^{-1}(ax)\,dx + (3a^4c^3)\int x^2\tan^{-1}(ax)\,dx +$$

$$= -\frac{c^3\tan^{-1}(ax)}{x} + 3a^2c^3x\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax)$$

$$= -\frac{c^3\tan^{-1}(ax)}{x} + 3a^2c^3x\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax)$$

$$= -\frac{c^3\tan^{-1}(ax)}{x} + 3a^2c^3x\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax) + \frac{1}{5}a^6c^3x^5\tan^{-1}(ax)$$

$$= -\frac{2}{5}a^3c^3x^2 - \frac{1}{20}a^5c^3x^4 - \frac{c^3\tan^{-1}(ax)}{x} + 3a^2c^3x\tan^{-1}(ax) + a^4c^3x^3\tan^{-1}(ax$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.72

$$\frac{c^3(4(-5+15a^2x^2+5a^4x^4+a^6x^6)\,\text{ArcTan}(ax) - ax(8a^2x^2+a^4x^4-20\log(x)+32\log(1+a^2x^2)))}{20x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

```
[Out] (c^3*(4*(-5 + 15*a^2*x^2 + 5*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(8*a^2*x^
2 + a^4*x^4 - 20*Log[x] + 32*Log[1 + a^2*x^2])))/(20*x)
```

Maple [A]
time = 0.12, size = 99, normalized size = 0.92

method	result
derivativedivides	$a\left(\frac{a^5c^3x^5\arctan(ax)}{5} + a^3c^3x^3\arctan(ax) + 3a\,c^3x\arctan(ax) - \frac{c^3\arctan(ax)}{ax} - \frac{c^3\left(\frac{a^4x^4}{4}+2a^2x^2\right.}{}\right.$
default	$a\left(\frac{a^5c^3x^5\arctan(ax)}{5} + a^3c^3x^3\arctan(ax) + 3a\,c^3x\arctan(ax) - \frac{c^3\arctan(ax)}{ax} - \frac{c^3\left(\frac{a^4x^4}{4}+2a^2x^2\right.}{}\right.$
risch	$-\frac{ic^3(a^6x^6+5a^4x^4+15a^2x^2-5)\ln(iax+1)}{10x} + \frac{ic^3(2a^6x^6\ln(-iax+1)+ia^5x^5+10x^4\ln(-iax+1)a^4+8ia^3x^3+30a^2x^2\ln(}{20x}$
meijerg	$\frac{a\,c^3\left(\frac{a^2x^2(-3a^2x^2+6)}{15} + \frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}} - \frac{2\ln(a^2x^2+1)}{5}\right)}{4} + \frac{3a\,c^3\left(-\frac{2a^2x^2}{3} + \frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}} + \frac{2\ln(}{}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*a^5*c^3*x^5*arctan(a*x)+a^3*c^3*x^3*arctan(a*x)+3*a*c^3*x*arctan(a*x
)-c^3*arctan(a*x)/a/x-1/5*c^3*(1/4*a^4*x^4+2*a^2*x^2+8*ln(a^2*x^2+1)-5*ln(a
*x)))
```

Maxima [A]
time = 0.26, size = 93, normalized size = 0.86

$$-\frac{1}{20}\left(a^{4}c^{3}x^{4}+8\,a^{2}c^{3}x^{2}+32\,c^{3}\log\left(a^{2}x^{2}+1\right)-20\,c^{3}\log\left(x\right)\right)a+\frac{1}{5}\left(a^{6}c^{3}x^{5}+5\,a^{4}c^{3}x^{3}+15\,a^{2}c^{3}x-\frac{5\,c^{3}}{x}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/20*(a^4*c^3*x^4 + 8*a^2*c^3*x^2 + 32*c^3*log(a^2*x^2 + 1) - 20*c^3*log(x
))*a + 1/5*(a^6*c^3*x^5 + 5*a^4*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(a*
x)
```

Fricas [A]
time = 3.07, size = 97, normalized size = 0.90

$$-\frac{a^{5}c^{3}x^{5}+8\,a^{3}c^{3}x^{3}+32\,ac^{3}x\log\left(a^{2}x^{2}+1\right)-20\,ac^{3}x\log\left(x\right)-4\left(a^{6}c^{3}x^{6}+5\,a^{4}c^{3}x^{4}+15\,a^{2}c^{3}x^{2}-5\,c^{3}\right)\arctan\left(ax\right)}{20\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/20*(a^5*c^3*x^5 + 8*a^3*c^3*x^3 + 32*a*c^3*x*log(a^2*x^2 + 1) - 20*a*c^3
*x*log(x) - 4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan
(a*x))/x
```

Sympy [A]
time = 0.65, size = 110, normalized size = 1.02

$$\begin{cases}\frac{a^{6}c^{3}x^{5}\operatorname{atan}\left(ax\right)}{5}-\frac{a^{5}c^{3}x^{4}}{20}+a^{4}c^{3}x^{3}\operatorname{atan}\left(ax\right)-\frac{2a^{3}c^{3}x^{2}}{5}+3a^{2}c^{3}x\operatorname{atan}\left(ax\right)+ac^{3}\log\left(x\right)-\frac{8ac^{3}\log\left(x^{2}+\frac{1}{a^{2}}\right)}{5}-\frac{c^{3}\operatorname{atan}\left(ax\right)}{x} & \text{for}\: a\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**5*atan(a*x)/5 - a**5*c**3*x**4/20 + a**4*c**3*x**3*
atan(a*x) - 2*a**3*c**3*x**2/5 + 3*a**2*c**3*x*atan(a*x) + a*c**3*log(x) -
8*a*c**3*log(x**2 + a**(-2))/5 - c**3*atan(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.56, size = 85, normalized size = 0.79

$$-\frac{c^3\left(\mathrm{atan}(a\,x)+\frac{2\,a^3\,x^3}{5}+\frac{a^5\,x^5}{20}-a\,x\,\ln\left(x\right)-3\,a^2\,x^2\,\mathrm{atan}(a\,x)-a^4\,x^4\,\mathrm{atan}(a\,x)-\frac{a^6\,x^6\,\mathrm{atan}(a\,x)}{5}+\frac{8\,a\,x\,\ln\left(a^2\,x^2+1\right)}{5}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^3)/x^2,x)
```

```
[Out] -(c^3*(atan(a*x) + (2*a^3*x^3)/5 + (a^5*x^5)/20 - a*x*log(x) - 3*a^2*x^2*at
an(a*x) - a^4*x^4*atan(a*x) - (a^6*x^6*atan(a*x))/5 + (8*a*x*log(a^2*x^2 +
1))/5))/x
```

3.171 $\int \frac{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)}{x^3} \, dx$

Optimal. Leaf size=138

$$-\frac{ac^3}{2x}-\frac{5}{4}a^3c^3x-\frac{1}{12}a^5c^3x^3+\frac{3}{4}a^2c^3\text{ArcTan}(ax)-\frac{c^3\text{ArcTan}(ax)}{2x^2}+\frac{3}{2}a^4c^3x^2\text{ArcTan}(ax)+\frac{1}{4}a^6c^3x^4\text{ArcTan}(ax)+$$

```
[Out] -1/2*a*c^3/x-5/4*a^3*c^3*x-1/12*a^5*c^3*x^3+3/4*a^2*c^3*arctan(a*x)-1/2*c^3
*arctan(a*x)/x^2+3/2*a^4*c^3*x^2*arctan(a*x)+1/4*a^6*c^3*x^4*arctan(a*x)+3/
2*I*a^2*c^3*polylog(2,-I*a*x)-3/2*I*a^2*c^3*polylog(2,I*a*x)
```

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5068, 4946, 331, 209, 4940, 2438, 327, 308}

$$\frac{1}{4}a^6c^3x^4\text{ArcTan}(ax)-\frac{1}{12}a^5c^3x^3+\frac{3}{2}a^4c^3x^2\text{ArcTan}(ax)-\frac{5}{4}a^3c^3x+\frac{3}{4}a^2c^3\text{ArcTan}(ax)+\frac{3}{2}ia^2c^3\text{Li}_2(-iax)-\frac{3}{2}ia^2c^3\text{Li}_2(iax)-\frac{c^3\text{ArcTan}(ax)}{2x^2}-\frac{ac^3}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*c^3)/x - (5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (3*a^2*c^3*ArcTan[a*x
])/4 - (c^3*ArcTan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3
*x^4*ArcTan[a*x])/4 + ((3*I)/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^
2*c^3*PolyLog[2, I*a*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)}{x^3}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)}{x^3} + \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + a^6c^3x^3\tan^{-1}(ax$$

$$= c^3\int \frac{\tan^{-1}(ax)}{x^3}\,dx + (3a^2c^3)\int \frac{\tan^{-1}(ax)}{x}\,dx + (3a^4c^3)\int x\tan^{-1}(ax)\,dx$$

$$= -\frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax) + \frac{1}{2}(ac^3)\int \frac{}{x^2(}$$

$$= -\frac{ac^3}{2x} - \frac{3}{2}a^3c^3x - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax) + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax)$$

$$= -\frac{ac^3}{2x} - \frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + a^2c^3\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\tan$$

$$= -\frac{ac^3}{2x} - \frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + \frac{3}{4}a^2c^3\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2\,t$$

Mathematica [A]
time = 0.04, size = 109, normalized size = 0.79

$$\frac{c^3(-6ax-15a^3x^3-a^5x^5-6\text{ArcTan}(ax)+9a^2x^2\text{ArcTan}(ax)+18a^4x^4\text{ArcTan}(ax)+3a^6x^6\text{ArcTan}(ax)+18ia^2x^2\text{PolyLog}(2,-iax)-18ia^2x^2\text{PolyLog}(2,iax))}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

```
[Out] (c^3*(-6*a*x - 15*a^3*x^3 - a^5*x^5 - 6*ArcTan[a*x] + 9*a^2*x^2*ArcTan[a*x]
 + 18*a^4*x^4*ArcTan[a*x] + 3*a^6*x^6*ArcTan[a*x] + (18*I)*a^2*x^2*PolyLog[
2, (-I)*a*x] - (18*I)*a^2*x^2*PolyLog[2, I*a*x]))/(12*x^2)
```

Maple [A]
time = 0.08, size = 148, normalized size = 1.07

method	result
derivativedivides	$a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4} + \frac{3a^2c^3x^2\arctan(ax)}{2} - \frac{c^3\arctan(ax)}{2a^2x^2} + 3c^3\arctan(ax)\ln(ax) - \frac{c^3\left(\frac{a^3x^3}{3}+5a\right.}{}\right.$
default	$a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4} + \frac{3a^2c^3x^2\arctan(ax)}{2} - \frac{c^3\arctan(ax)}{2a^2x^2} + 3c^3\arctan(ax)\ln(ax) - \frac{c^3\left(\frac{a^3x^3}{3}+5a\right.}{}\right.$
meijerg	$\frac{a^2c^3\left(\frac{ax(-5a^2x^2+15)}{15} - \frac{ax(-5a^4x^4+5)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4} + \frac{3a^2c^3\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{4} + \frac{3a^2c^3\left(-\right.}{}$
risch	$-\frac{3ic^3a^2\,\text{dilog}(-iax+1)}{2} + \frac{ic^3a^2\ln(-iax)}{4} + \frac{ic^3a^6\ln(-iax+1)x^4}{8} + \frac{3a^2c^3\arctan(ax)}{4} - \frac{a^5c^3x^3}{12} - \frac{5a^3c^3x}{4} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*arctan(a*x)-1/2*c^3*arctan
(a*x)/a^2/x^2+3*c^3*arctan(a*x)*ln(a*x)-1/4*c^3*(1/3*a^3*x^3+5*a*x+2/a/x-3*
arctan(a*x)-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(1+I*a
*x)+6*I*dilog(1-I*a*x)))
```

Maxima [A]
time = 0.51, size = 155, normalized size = 1.12

$$-\frac{a^5c^3x^5+15\,a^3c^3x^3+9\,\pi a^2c^3x^2\log\left(a^2x^2+1\right)-36\,a^2c^3x^2\arctan\left(ax\right)\log\left(ax\right)+18i\,a^2c^3x^2\mathrm{Li}_2(i\,ax+1)-18i\,a^2c^3x^2\mathrm{Li}_2(-i\,ax+1)+6\,ac^3x-3\left(a^6c^3x^6+6\,a^4c^3x^4+3\,a^2c^3x^2-2\,c^3\right)\arctan\left(ax\right)}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^5*c^3*x^5 + 15*a^3*c^3*x^3 + 9*pi*a^2*c^3*x^2*log(a^2*x^2 + 1) - 3
6*a^2*c^3*x^2*arctan(a*x)*log(a*x) + 18*I*a^2*c^3*x^2*dilog(I*a*x + 1) - 18
*I*a^2*c^3*x^2*dilog(-I*a*x + 1) + 6*a*c^3*x - 3*(a^6*c^3*x^6 + 6*a^4*c^3*x
^4 + 3*a^2*c^3*x^2 - 2*c^3)*arctan(a*x))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x^
3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{\operatorname{atan}\left(ax\right)}{x^3}\,dx+\int\frac{3a^2\operatorname{atan}\left(ax\right)}{x}\,dx+\int 3a^4x\operatorname{atan}\left(ax\right)dx+\int a^6x^3\operatorname{atan}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)/x**3, x) + Integral(3*a**2*atan(a*x)/x, x) + Integ
ral(3*a**4*x*atan(a*x), x) + Integral(a**6*x**3*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.57, size = 152, normalized size = 1.10

$$\begin{cases} 0 & \text{if } a=0 \\ 3\,a^4\,c^3\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a^2\,c^3\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{12}-\frac{c^3\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c^3\,\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{3\,a^3\,c^3\,x}{2}+\frac{a^6\,c^3\,x^4\,\mathrm{atan}(a\,x)}{4}-\frac{a^2\,c^3\,\mathrm{Li_2}(1-a\,x\,1\mathrm{i})\,3\mathrm{i}}{2}+\frac{a^2\,c^3\,\mathrm{Li_2}(1+a\,x\,1\mathrm{i})\,3\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^3)/x^3,x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, - (3*a^3*c^3*x)/2 - (a^2*c^3*(3*atan(a*x) - 3*
a*x + a^3*x^3))/12 - (c^3*atan(a*x))/(2*x^2) - (a^2*c^3*dilog(- a*x*1i + 1)
*3i)/2 + (a^2*c^3*dilog(a*x*1i + 1)*3i)/2 - (c^3*(a^3*atan(a*x) + a^2/x))/(
2*a) + 3*a^4*c^3*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^6*c^3*x^4*atan(a*x))/4)
```

3.172 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)}{x^4} dx$

Optimal. Leaf size=116

$$-\frac{ac^3}{6x^2}-\frac{1}{6}a^5c^3x^2-\frac{c^3\text{ArcTan}(ax)}{3x^3}-\frac{3a^2c^3\text{ArcTan}(ax)}{x}+3a^4c^3x\text{ArcTan}(ax)+\frac{1}{3}a^6c^3x^3\text{ArcTan}(ax)+\frac{8}{3}a^3c^3\log(x)$$

```
[Out] -1/6*a*c^3/x^2-1/6*a^5*c^3*x^2-1/3*c^3*arctan(a*x)/x^3-3*a^2*c^3*arctan(a*x
)/x+3*a^4*c^3*x*arctan(a*x)+1/3*a^6*c^3*x^3*arctan(a*x)+8/3*a^3*c^3*ln(x)-8
/3*a^3*c^3*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5068, 4930, 266, 4946, 272, 46, 36, 29, 31, 45}

$$\frac{1}{3}a^6c^3x^3\text{ArcTan}(ax) - \frac{1}{6}a^5c^3x^2 + 3a^4c^3x\text{ArcTan}(ax) + \frac{8}{3}a^3c^3\log(x) - \frac{3a^2c^3\text{ArcTan}(ax)}{x} - \frac{8}{3}a^3c^3\log\left(a^2x^2+1\right) - \frac{c^3\text{ArcTan}(ax)}{3x^3} - \frac{ac^3}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(a*c^3)/x^2 - (a^5*c^3*x^2)/6 - (c^3*ArcTan[a*x])/(3*x^3) - (3*a^2*c^3
*ArcTan[a*x])/x + 3*a^4*c^3*x*ArcTan[a*x] + (a^6*c^3*x^3*ArcTan[a*x])/3 + (
8*a^3*c^3*Log[x])/3 - (8*a^3*c^3*Log[1 + a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)}{x^4}\,dx = \int \left(3a^4c^3\tan^{-1}(ax) + \frac{c^3\tan^{-1}(ax)}{x^4} + \frac{3a^2c^3\tan^{-1}(ax)}{x^2} + a^6c^3x^2\tan^{-1}(ax)\right)$$

$$= c^3\int \frac{\tan^{-1}(ax)}{x^4}\,dx + (3a^2c^3)\int \frac{\tan^{-1}(ax)}{x^2}\,dx + (3a^4c^3)\int \tan^{-1}(ax)\,dx + ($$

$$= -\frac{c^3\tan^{-1}(ax)}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + \frac{1}{3}a^6c^3x^3\tan^{-1}(ax)$$

$$= -\frac{c^3\tan^{-1}(ax)}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + \frac{1}{3}a^6c^3x^3\tan^{-1}(ax)$$

$$= -\frac{c^3\tan^{-1}(ax)}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + \frac{1}{3}a^6c^3x^3\tan^{-1}(ax)$$

$$= -\frac{ac^3}{6x^2} - \frac{1}{6}a^5c^3x^2 - \frac{c^3\tan^{-1}(ax)}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)}{x} + 3a^4c^3x\tan^{-1}(ax) + \frac{1}{3}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.72

$$\frac{c^3(2(-1-9a^2x^2+9a^4x^4+a^6x^6)\operatorname{ArcTan}(ax) - ax(1+a^4x^4-16a^2x^2\log(x)+16a^2x^2\log(1+a^2x^2)))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

```
[Out] (c^3*(2*(-1 - 9*a^2*x^2 + 9*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(1 + a^4*x
^4 - 16*a^2*x^2*Log[x] + 16*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

Maple [A]
time = 0.13, size = 102, normalized size = 0.88

method	result
derivativedivides	$a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3} + 3a\,c^3x\arctan(ax) - \frac{3c^3\arctan(ax)}{ax} - \frac{c^3\arctan(ax)}{3a^3x^3} - \frac{c^3\left(\frac{a^2x^2}{2}+8\ln(a^2x^2+1)+\right.}{3}\right.$
default	$a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3} + 3a\,c^3x\arctan(ax) - \frac{3c^3\arctan(ax)}{ax} - \frac{c^3\arctan(ax)}{3a^3x^3} - \frac{c^3\left(\frac{a^2x^2}{2}+8\ln(a^2x^2+1)+\right.}{3}\right.$
risch	$-\frac{ic^3(a^6x^6+9a^4x^4-9a^2x^2-1)\ln(iax+1)}{6x^3} + \frac{ic^3(a^6x^6\ln(-iax+1)+ia^5x^5+9x^4\ln(-iax+1)a^4-16i\ln(x)a^3x^3+16i\ln(2}{6x^3}$
meijerg	$\frac{a^3c^3\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4} + \frac{3a^3c^3\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4} + -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/3*a^3*c^3*x^3*arctan(a*x)+3*a*c^3*x*arctan(a*x)-3*c^3*arctan(a*x)/a/
x-1/3*c^3*arctan(a*x)/a^3/x^3-1/3*c^3*(1/2*a^2*x^2+8*ln(a^2*x^2+1)+1/2/a^2/
x^2-8*ln(a*x)))
```

Maxima [A]
time = 0.25, size = 96, normalized size = 0.83

$$-\frac{1}{6}\left(a^4c^3x^2+16\,a^2c^3\log\left(a^2x^2+1\right)-16\,a^2c^3\log\left(x\right)+\frac{c^3}{x^2}\right)a+\frac{1}{3}\left(a^6c^3x^3+9\,a^4c^3x-\frac{9\,a^2c^3x^2+c^3}{x^3}\right)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^4*c^3*x^2 + 16*a^2*c^3*log(a^2*x^2 + 1) - 16*a^2*c^3*log(x) + c^3/x
^2)*a + 1/3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arctan(
a*x)
```

Fricas [A]
time = 3.55, size = 100, normalized size = 0.86

$$-\frac{a^5c^3x^5+16\,a^3c^3x^3\log\left(a^2x^2+1\right)-16\,a^3c^3x^3\log\left(x\right)+ac^3x-2\left(a^6c^3x^6+9\,a^4c^3x^4-9\,a^2c^3x^2-c^3\right)\arctan\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^5*c^3*x^5 + 16*a^3*c^3*x^3*log(a^2*x^2 + 1) - 16*a^3*c^3*x^3*log(x)
 + a*c^3*x - 2*(a^6*c^3*x^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a
*x))/x^3
```

Sympy [A]
time = 0.63, size = 117, normalized size = 1.01

$$\begin{cases}\frac{a^6c^3x^3\,\mathrm{atan}\,(ax)}{3}-\frac{a^5c^3x^2}{6}+3a^4c^3x\,\mathrm{atan}\,(ax)+\frac{8a^3c^3\log\,(x)}{3}-\frac{8a^3c^3\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{3a^2c^3\,\mathrm{atan}\,(ax)}{x}-\frac{ac^3}{6x^2}-\frac{c^3\,\mathrm{atan}\,(ax)}{3x^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)/x**4,x)
```

```
[Out] Piecewise((a**6*c**3*x**3*atan(a*x)/3 - a**5*c**3*x**2/6 + 3*a**4*c**3*x*at
an(a*x) + 8*a**3*c**3*log(x)/3 - 8*a**3*c**3*log(x**2 + a**(-2))/3 - 3*a**2
*c**3*atan(a*x)/x - a*c**3/(6*x**2) - c**3*atan(a*x)/(3*x**3), Ne(a, 0)), (
0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 97, normalized size = 0.84

$$-\frac{c^3\,\left(2\,\mathrm{atan}(a\,x)+a\,x-a^3\,x^3+a^5\,x^5+18\,a^2\,x^2\,\mathrm{atan}(a\,x)-18\,a^4\,x^4\,\mathrm{atan}(a\,x)-2\,a^6\,x^6\,\mathrm{atan}(a\,x)+16\,a^3\,x^3\,\ln\left(a^2\,x^2+1\right)-16\,a^3\,x^3\,\ln\left(x\right)\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^3)/x^4,x)
```

```
[Out] -(c^3*(2*atan(a*x) + a*x - a^3*x^3 + a^5*x^5 + 18*a^2*x^2*atan(a*x) - 18*a^
4*x^4*atan(a*x) - 2*a^6*x^6*atan(a*x) + 16*a^3*x^3*log(a^2*x^2 + 1) - 16*a^
3*x^3*log(x)))/(6*x^3)
```

3.173 $\int \frac{x^4 \text{ArcTan}(ax)}{c+a^2cx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{x^2}{6a^3c}-\frac{x\text{ArcTan}(ax)}{a^4c}+\frac{x^3\text{ArcTan}(ax)}{3a^2c}+\frac{\text{ArcTan}(ax)^2}{2a^5c}+\frac{2\log\left(1+a^2x^2\right)}{3a^5c}$$

```
[Out] -1/6*x^2/a^3/c-x*arctan(a*x)/a^4/c+1/3*x^3*arctan(a*x)/a^2/c+1/2*arctan(a*x
)^2/a^5/c+2/3*ln(a^2*x^2+1)/a^5/c
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {5036, 4946, 272, 45, 4930, 266, 5004}

$$\frac{\text{ArcTan}(ax)^2}{2a^5c}-\frac{x\text{ArcTan}(ax)}{a^4c}-\frac{x^2}{6a^3c}+\frac{x^3\text{ArcTan}(ax)}{3a^2c}+\frac{2\log\left(a^2x^2+1\right)}{3a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] -1/6*x^2/(a^3*c) - (x*ArcTan[a*x])/(a^4*c) + (x^3*ArcTan[a*x])/(3*a^2*c) +
ArcTan[a*x]^2/(2*a^5*c) + (2*Log[1 + a^2*x^2])/(3*a^5*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\tan^{-1}(ax)}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x^2\tan^{-1}(ax)\,dx}{a^2c}\\
&= \frac{x^3\tan^{-1}(ax)}{3a^2c}+\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^4}-\frac{\int \tan^{-1}(ax)\,dx}{a^4c}-\frac{\int \frac{x^3}{1+a^2x^2}\,dx}{3ac}\\
&= -\frac{x\tan^{-1}(ax)}{a^4c}+\frac{x^3\tan^{-1}(ax)}{3a^2c}+\frac{\tan^{-1}(ax)^2}{2a^5c}+\frac{\int \frac{x}{1+a^2x^2}\,dx}{a^3c}-\frac{\text{Subst}\left(\int \frac{x}{1+a^2x}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{x\tan^{-1}(ax)}{a^4c}+\frac{x^3\tan^{-1}(ax)}{3a^2c}+\frac{\tan^{-1}(ax)^2}{2a^5c}+\frac{\log\left(1+a^2x^2\right)}{2a^5c}-\frac{\text{Subst}\left(\int \left(\frac{1}{a^2}-\frac{1}{a^2(1+a\ldots}\right.\right.}{6ac}\\
&= -\frac{x^2}{6a^3c}-\frac{x\tan^{-1}(ax)}{a^4c}+\frac{x^3\tan^{-1}(ax)}{3a^2c}+\frac{\tan^{-1}(ax)^2}{2a^5c}+\frac{2\log\left(1+a^2x^2\right)}{3a^5c}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.70

$$\frac{-a^2x^2+2ax(-3+a^2x^2)\,\text{ArcTan}(ax)+3\text{ArcTan}(ax)^2+4\log\left(1+a^2x^2\right)}{6a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] (-(a^2*x^2) + 2*a*x*(-3 + a^2*x^2)*ArcTan[a*x] + 3*ArcTan[a*x]^2 + 4*Log[1
+ a^2*x^2])/(6*a^5*c)
```

Maple [A]
time = 0.12, size = 76, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\arctan(ax)a^3x^3}{3c}-\frac{\arctan(ax)ax}{c}+\frac{\arctan(ax)^2}{c}-\frac{\frac{a^2x^2}{2}-2\ln\left(a^2x^2+1\right)+\frac{3\arctan(ax)^2}{2}}{3c}}{a^5}$
default	$\frac{\frac{\arctan(ax)a^3x^3}{3c}-\frac{\arctan(ax)ax}{c}+\frac{\arctan(ax)^2}{c}-\frac{\frac{a^2x^2}{2}-2\ln\left(a^2x^2+1\right)+\frac{3\arctan(ax)^2}{2}}{3c}}{a^5}$
risch	$-\frac{\ln(iax+1)^2}{8a^5c}-\frac{i\left(2a^3x^3+3i\ln(-iax+1)-6ax\right)\ln(iax+1)}{12a^5c}+\frac{ix^3\ln(-iax+1)}{6a^2c}-\frac{ix\ln(-iax+1)}{2a^4c}-\frac{x^2}{6a^3c}-\frac{\ln(-i}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/3/c*arctan(a*x)*a^3*x^3-1/c*arctan(a*x)*a*x+1/c*arctan(a*x)^2-1/3/
c*(1/2*a^2*x^2-2*ln(a^2*x^2+1)+3/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.49, size = 74, normalized size = 0.92

$$\frac{1}{3}\left(\frac{a^2x^3-3\,x}{a^4c}+\frac{3\,\arctan\left(ax\right)}{a^5c}\right)\arctan\left(ax\right)-\frac{a^2x^2+3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*((a^2*x^3 - 3*x)/(a^4*c) + 3*arctan(a*x)/(a^5*c))*arctan(a*x) - 1/6*(a^
2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/(a^5*c)
```

Fricas [A]
time = 2.02, size = 54, normalized size = 0.68

$$-\frac{a^2x^2-2\left(a^3x^3-3\,ax\right)\arctan\left(ax\right)-3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*x^2 - 2*(a^3*x^3 - 3*a*x)*arctan(a*x) - 3*arctan(a*x)^2 - 4*log(a
^2*x^2 + 1))/(a^5*c)
```

Sympy [A]
time = 0.46, size = 70, normalized size = 0.88

$$\begin{cases} \frac{x^3 \operatorname{atan}(ax)}{3a^2c} - \frac{x^2}{6a^3c} - \frac{x \operatorname{atan}(ax)}{a^4c} + \frac{2\log\left(x^2+\frac{1}{a^2}\right)}{3a^5c} + \frac{\operatorname{atan}^2(ax)}{2a^5c} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((x**3*atan(a*x)/(3*a**2*c) - x**2/(6*a**3*c) - x*atan(a*x)/(a**4*
c) + 2*log(x**2 + a**(-2))/(3*a**5*c) + atan(a*x)**2/(2*a**5*c), Ne(a, 0)),
 (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.17, size = 73, normalized size = 0.91

$$\frac{2\,\ln\left(a^2\,x^2+1\right)}{3\,a^5\,c}-a^2\,\mathrm{atan}(a\,x)\,\left(\frac{x}{a^6\,c}-\frac{x^3}{3\,a^4\,c}\right)-\frac{x^2}{6\,a^3\,c}+\frac{{\mathrm{atan}(a\,x)}^2}{2\,a^5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] (2*log(a^2*x^2 + 1))/(3*a^5*c) - a^2*atan(a*x)*(x/(a^6*c) - x^3/(3*a^4*c))
- x^2/(6*a^3*c) + atan(a*x)^2/(2*a^5*c)
```

3.174 $\int \frac{x^3 \textbf{ArcTan}(ax)}{c+a^2cx^2} dx$

Optimal. Leaf size=113

$$-\frac{x}{2a^3c}+\frac{\text{ArcTan}(ax)}{2a^4c}+\frac{x^2\text{ArcTan}(ax)}{2a^2c}+\frac{i\text{ArcTan}(ax)^2}{2a^4c}+\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}$$

```
[Out] -1/2*x/a^3/c+1/2*arctan(a*x)/a^4/c+1/2*x^2*arctan(a*x)/a^2/c+1/2*I*arctan(a
*x)^2/a^4/c+arctan(a*x)*ln(2/(1+I*a*x))/a^4/c+1/2*I*polylog(2,1-2/(1+I*a*x)
)/a^4/c
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5036, 4946, 327, 209, 5040, 4964, 2449, 2352}

$$\frac{i\text{ArcTan}(ax)^2}{2a^4c}+\frac{\text{ArcTan}(ax)}{2a^4c}+\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^4c}-\frac{x}{2a^3c}+\frac{x^2\text{ArcTan}(ax)}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] -1/2*x/(a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I/
2)*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + ((I/
2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)}{c+a^2cx^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x\tan^{-1}(ax)\,dx}{a^2c}$$
$$= \frac{x^2\tan^{-1}(ax)}{2a^2c} + \frac{i\tan^{-1}(ax)^2}{2a^4c} + \frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^3c} - \frac{\int \frac{x^2}{1+a^2x^2}\,dx}{2ac}$$
$$= -\frac{x}{2a^3c} + \frac{x^2\tan^{-1}(ax)}{2a^2c} + \frac{i\tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{\int \frac{1}{1+a^2x^2}\,dx}{2a^3c} - \frac{\int \frac{\log}{}}{}$$
$$= -\frac{x}{2a^3c} + \frac{\tan^{-1}(ax)}{2a^4c} + \frac{x^2\tan^{-1}(ax)}{2a^2c} + \frac{i\tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i\text{Subs}}{}$$
$$= -\frac{x}{2a^3c} + \frac{\tan^{-1}(ax)}{2a^4c} + \frac{x^2\tan^{-1}(ax)}{2a^2c} + \frac{i\tan^{-1}(ax)^2}{2a^4c} + \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i\text{Li}_2(}{}$$

Mathematica [A]
time = 0.03, size = 120, normalized size = 1.06

$$-\frac{x}{2a^3c} + \frac{\text{ArcTan}(ax)}{2a^4c} + \frac{x^2\text{ArcTan}(ax)}{2a^2c} + \frac{i\text{ArcTan}(ax)^2}{2a^4c} + \frac{\text{ArcTan}(ax)\log\left(\frac{2i}{i-ax}\right)}{a^4c} + \frac{i\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] -1/2*x/(a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I/
2)*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^4*c) + ((
I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/(a^4*c)
```

Maple [A]
time = 0.06, size = 185, normalized size = 1.64

method	result
derivativedivides	$\frac{\frac{\arctan(ax)a^2x^2}{2c} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{ax-\arctan(ax)+\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2} - \frac{i\ln(ax-i)^2}{4} - \frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2} - \frac{i\ln(ax-i}{}}{}}{a^4}$
default	$\frac{\frac{\arctan(ax)a^2x^2}{2c} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{ax-\arctan(ax)+\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2} - \frac{i\ln(ax-i)^2}{4} - \frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2} - \frac{i\ln(ax-i}{}}{}}{a^4}$
risch	$\frac{i\ln(-iax+1)x^2}{4c\,a^2} + \frac{\arctan(ax)}{2a^4c} - \frac{x}{2a^3c} - \frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c\,a^4} + \frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c\,a^4} - \frac{i\ln(-iax+1)^2}{8c\,a^4} - \frac{i\ln(i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2/c*arctan(a*x)*a^2*x^2-1/2/c*arctan(a*x)*ln(a^2*x^2+1)-1/2/c*(a*x
-arctan(a*x)+1/2*I*ln(a*x-I)*ln(a^2*x^2+1)-1/4*I*ln(a*x-I)^2-1/2*I*dilog(-1
```

```
/2*I*(I+a*x))-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*I*ln(I+a*x)*ln(a^2*x^2
+1)+1/4*I*ln(I+a*x)^2+1/2*I*dilog(1/2*I*(a*x-I))+1/2*I*ln(I+a*x)*ln(1/2*I*(
a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c),x)
[Out] Integral(x**3*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atan}(a\,x)}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2),x)
[Out] int((x^3*atan(a*x))/(c + a^2*c*x^2), x)
```

3.175 $\int \frac{x^2\text{ArcTan}(ax)}{c+a^2cx^2}\,dx$

Optimal. Leaf size=49

$$\frac{x\text{ArcTan}(ax)}{a^2c}-\frac{\text{ArcTan}(ax)^2}{2a^3c}-\frac{\log\left(1+a^2x^2\right)}{2a^3c}$$

```
[Out] x*arctan(a*x)/a^2/c-1/2*arctan(a*x)^2/a^3/c-1/2*ln(a^2*x^2+1)/a^3/c
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5036, 4930, 266, 5004}

$$-\frac{\text{ArcTan}(ax)^2}{2a^3c}+\frac{x\text{ArcTan}(ax)}{a^2c}-\frac{\log\left(a^2x^2+1\right)}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^3
*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
```

```
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)\,dx}{a^2c}\\ &= \frac{x\tan^{-1}(ax)}{a^2c}-\frac{\tan^{-1}(ax)^2}{2a^3c}-\frac{\int \frac{x}{1+a^2x^2}\,dx}{ac}\\ &= \frac{x\tan^{-1}(ax)}{a^2c}-\frac{\tan^{-1}(ax)^2}{2a^3c}-\frac{\log(1+a^2x^2)}{2a^3c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$\frac{x\text{ArcTan}(ax)}{a^2c}-\frac{\text{ArcTan}(ax)^2}{2a^3c}-\frac{\log(1+a^2x^2)}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] (x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^3
*c)
```

Maple [A]
time = 0.09, size = 53, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$	53
default	$\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$	53
risch	$\frac{\ln(iax+1)^2}{8a^3c}-\frac{i(-i\ln(-iax+1)+2ax)\ln(iax+1)}{4c\,a^3}+\frac{\ln(-iax+1)^2}{8c\,a^3}+\frac{ix\ln(-iax+1)}{2c\,a^2}-\frac{\ln(-a^2x^2-1)}{2c\,a^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/c*arctan(a*x)*a*x-1/c*arctan(a*x)^2-1/c*(1/2*ln(a^2*x^2+1)-1/2*arc
tan(a*x)^2))
```

Maxima [A]
time = 0.47, size = 54, normalized size = 1.10

$$\left(\frac{x}{a^2c}-\frac{\arctan(ax)}{a^3c}\right)\arctan(ax)+\frac{\arctan(ax)^2-\log(a^2x^2+1)}{2\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] (x/(a^2*c) - arctan(a*x)/(a^3*c))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^
2*x^2 + 1))/(a^3*c)
```

Fricas [A]
time = 3.03, size = 37, normalized size = 0.76

$$\frac{2\,ax\arctan\left(ax\right)-\arctan\left(ax\right)^{2}-\log\left(a^{2}x^{2}+1\right)}{2\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

Sympy [A]
time = 0.30, size = 42, normalized size = 0.86

$$\begin{cases}\frac{x\operatorname{atan}\left(ax\right)}{a^{2}c}-\frac{\log\left(x^{2}+\frac{1}{a^{2}}\right)}{2a^{3}c}-\frac{\operatorname{atan}^{2}\left(ax\right)}{2a^{3}c} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c),x)
[Out] Piecewise((x*atan(a*x)/(a**2*c) - log(x**2 + a**(-2))/(2*a**3*c) - atan(a*x
)**2/(2*a**3*c), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.16, size = 33, normalized size = 0.67

$$-\frac{\mathrm{atan}(a\,x)^{2}-2\,a\,x\,\mathrm{atan}(a\,x)+\ln\left(a^{2}\,x^{2}+1\right)}{2\,a^{3}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2),x)
[Out] -(log(a^2*x^2 + 1) + atan(a*x)^2 - 2*a*x*atan(a*x))/(2*a^3*c)
```

3.176 $\int \frac{x\mathbf{ArcTan}(ax)}{c+a^2cx^2}\,dx$

Optimal. Leaf size=72

$$-\frac{i\text{ArcTan}(ax)^2}{2a^2c}-\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}$$

```
[Out] -1/2*I*arctan(a*x)^2/a^2/c-arctan(a*x)*ln(2/(1+I*a*x))/a^2/c-1/2*I*polylog(
2,1-2/(1+I*a*x))/a^2/c
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5040, 4964, 2449, 2352}

$$-\frac{i\text{ArcTan}(ax)^2}{2a^2c}-\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^2*c)
 - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx &= -\frac{i\tan^{-1}(ax)^2}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{ac} \\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c} + \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac} \\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{a^2c} \\
&= -\frac{i\tan^{-1}(ax)^2}{2a^2c} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{i\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 1.07

$$-\frac{i\text{ArcTan}(ax)^2}{2a^2c} - \frac{\text{ArcTan}(ax)\log\left(\frac{2i}{i-ax}\right)}{a^2c} - \frac{i\text{PolyLog}\left(2, \frac{i+ax}{-i+ax}\right)}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^2*c) - ((I/2)*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(64) = 128.
time = 0.07, size = 159, normalized size = 2.21

method	result
risch	$\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c\,a^2} - \frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c\,a^2} + \frac{i\ln(-iax+1)^2}{8c\,a^2} - \frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c\,a^2} + \frac{i\,\text{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c\,a^2} - \frac{i\ln(}{}$
derivativedivides	$\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\text{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)-\frac{\ln(ax-i)^2}{2}\right)}{2}+\frac{i\left(\ln(ax+i)\ln\left(a^2x\right.\right.}{}}{2c}}{a^2}$
default	$\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\text{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)-\frac{\ln(ax-i)^2}{2}\right)}{2}+\frac{i\left(\ln(ax+i)\ln\left(a^2x\right.\right.}{}}{2c}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2/c*arctan(a*x)*ln(a^2*x^2+1)-1/2/c*(-1/2*I*(ln(a*x-I)*ln(a^2*x^2+
1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+1/2*
I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))
-1/2*ln(I+a*x)^2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2),x)
[Out] int((x*atan(a*x))/(c + a^2*c*x^2), x)
```

3.177 $\int \frac{\textbf{ArcTan}(ax)}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(ax)^2}{2ac}$$

```
[Out] 1/2*arctan(a*x)^2/a/c
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {5004}

$$\frac{\text{ArcTan}(ax)^2}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^2/(2*a*c)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^2}{2ac}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{\text{ArcTan}(ax)^2}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^2/(2*a*c)
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\arctan(ax)^2}{2ac}$	15
default	$\frac{\arctan(ax)^2}{2ac}$	15
risch	$-\frac{\ln(iax+1)^2}{8ca}+\frac{\ln(-iax+1)\ln(iax+1)}{4ca}-\frac{\ln(-iax+1)^2}{8ca}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(a*x)^2/a/c
```

Maxima [A]
time = 0.47, size = 14, normalized size = 0.88

$$\frac{\arctan{(ax)}^2}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(a*x)^2/(a*c)
```

Fricas [A]
time = 4.99, size = 14, normalized size = 0.88

$$\frac{\arctan{(ax)}^2}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(a*x)^2/(a*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.
time = 1.45, size = 36, normalized size = 2.25

$$\begin{cases} 0 & \text{for}\ a = 0 \\ \tilde{\infty}\left(\begin{cases} 0 & \text{for}\ a = 0 \\ \frac{ax\operatorname{atan}{(ax)} - \frac{\log{\left(a^2x^2+1\right)}}{2}}{a} & \text{otherwise} \end{cases}\right) & \text{for}\ c = 0 \\ \frac{\operatorname{atan}^2{(ax)}}{2ac} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((0, Eq(a, 0)), (zoo*Piecewise((0, Eq(a, 0)), ((a*x*atan(a*x) - lo
g(a**2*x**2 + 1)/2)/a, True)), Eq(c, 0)), (atan(a*x)**2/(2*a*c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.41, size = 35, normalized size = 2.19

$$-\frac{2\,\pi\arctan\left(ax\right)\left\lfloor\frac{\arctan(ax)}{\pi}+\frac{1}{2}\right\rfloor-\arctan\left(ax\right)^2}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*pi*arctan(a*x)*floor(arctan(a*x)/pi + 1/2) - arctan(a*x)^2)/(a*c)
```

Mupad [B]
time = 0.38, size = 14, normalized size = 0.88

$$\frac{\mathrm{atan}(a\,x)^2}{2\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2),x)
```

```
[Out] atan(a*x)^2/(2*a*c)
```

3.178 $\int \frac{\text{ArcTan}(ax)}{x(c+a^2cx^2)} \, dx$

Optimal. Leaf size=64

$$-\frac{i\text{ArcTan}(ax)^2}{2c}+\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}$$

```
[Out] -1/2*I*arctan(a*x)^2/c+arctan(a*x)*ln(2-2/(1-I*a*x))/c-1/2*I*polylog(2,-1+2
/(1-I*a*x))/c
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5044, 4988, 2497}

$$-\frac{i\text{ArcTan}(ax)^2}{2c}+\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((-1/2*I)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - ((I/2
)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}\\ &= -\frac{i\tan^{-1}(ax)^2}{2c}+\frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 103, normalized size = 1.61

$$\frac{i\mathrm{ArcTan}(ax)^2}{2c}+\frac{\mathrm{ArcTan}(ax)\log\left(\frac{2i}{i-ax}\right)}{c}+\frac{i\mathrm{PolyLog}(2,-iax)}{2c}-\frac{i\mathrm{PolyLog}(2,iax)}{2c}+\frac{i\mathrm{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*Pol
yLog[2, (-I)*a*x])/c - ((I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -((I
 + a*x)/(I - a*x))])/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(56) = 112.
time = 0.06, size = 221, normalized size = 3.45

method	result
risch	$-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}-\frac{i\,\mathrm{dilog}(-iax+1)}{2c}-\frac{i\ln(-iax+1)^2}{8c}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c}-$
derivativedivides	$-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}+\frac{\arctan(ax)\ln(ax)}{c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}}{}$
default	$-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}+\frac{\arctan(ax)\ln(ax)}{c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*arctan(a*x)*ln(a^2*x^2+1)+1/c*arctan(a*x)*ln(a*x)-1/2/c*(-I*ln(a*x)*
ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-I*a*x)+1/2*I*l
n(a*x-I)*ln(a^2*x^2+1)-1/4*I*ln(a*x-I)^2-1/2*I*dilog(-1/2*I*(I+a*x))-1/2*I*
ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*I*ln(I+a*x)*ln(a^2*x^2+1)+1/4*I*ln(I+a*x)^
2+1/2*I*dilog(1/2*I*(a*x-I))+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)/(a**2*x**3 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)), x)
```

3.179 $\int \frac{\text{ArcTan}(ax)}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=52

$$-\frac{\text{ArcTan}(ax)}{cx}-\frac{a\text{ArcTan}(ax)^2}{2c}+\frac{a\log(x)}{c}-\frac{a\log\left(1+a^2x^2\right)}{2c}$$

[Out] -arctan(a*x)/c/x-1/2*a*arctan(a*x)^2/c+a*ln(x)/c-1/2*a*ln(a^2*x^2+1)/c

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5038, 4946, 272, 36, 29, 31, 5004}

$$-\frac{a\log\left(a^2x^2+1\right)}{2c}-\frac{a\text{ArcTan}(ax)^2}{2c}-\frac{\text{ArcTan}(ax)}{cx}+\frac{a\log(x)}{c}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]

[Out] -(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 + a^2*x^2])/(2*c)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], 
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\int \frac{1}{x(1+a^2x^2)}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\text{Subst}\left(\int \frac{1}{x(1+a^2x)}\,dx,x,x^2\right)}{2c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2c}-\frac{a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx,x,x^2\right)}{2c}\\
&= -\frac{\tan^{-1}(ax)}{cx}-\frac{a\tan^{-1}(ax)^2}{2c}+\frac{a\log(x)}{c}-\frac{a\log(1+a^2x^2)}{2c}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$-\frac{\text{ArcTan}(ax)}{cx}-\frac{a\text{ArcTan}(ax)^2}{2c}+\frac{a\log(x)}{c}-\frac{a\log(1+a^2x^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

[Out] -(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 + a^2*x^2])/(2*c)

Maple [A]
time = 0.13, size = 62, normalized size = 1.19

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)}{cax}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$
default	$a\left(-\frac{\arctan(ax)}{cax}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$
risch	$\frac{a\ln(iax+1)^2}{8c}-\frac{(ax\ln(-iax+1)-2i)\ln(iax+1)}{4cx}-\frac{-a\ln(-iax+1)^2x-8ax\ln(x)+4ax\ln(-3a^2x^2-3)+4i\ln(-iax+1}{8cx}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)

[Out] a*(-1/c*arctan(a*x)/a/x-1/c*arctan(a*x)^2-1/c*(1/2*ln(a^2*x^2+1)-ln(a*x)-1/2*arctan(a*x)^2))

Maxima [A]
time = 0.48, size = 53, normalized size = 1.02

$$-\left(\frac{a\arctan(ax)}{c}+\frac{1}{cx}\right)\arctan(ax)+\frac{\left(\arctan(ax)^2-\log\left(a^2x^2+1\right)+2\log(x)\right)a}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")

[Out] -(a*arctan(a*x)/c + 1/(c*x))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^2*x^2 + 1) + 2*log(x))*a/c

Fricas [A]
time = 1.45, size = 43, normalized size = 0.83

$$-\frac{ax\arctan(ax)^2+ax\log\left(a^2x^2+1\right)-2\,ax\log(x)+2\arctan(ax)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")

[Out] -1/2*(a*x*arctan(a*x)^2 + a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) + 2*arctan(a*x))/(c*x)

Sympy [A]
time = 0.47, size = 42, normalized size = 0.81

$$\begin{cases} \frac{a\log(x)}{c} - \frac{a\log\left(x^2+\frac{1}{a^2}\right)}{2c} - \frac{a\operatorname{atan}^2(ax)}{2c} - \frac{\operatorname{atan}(ax)}{cx} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((a*log(x)/c - a*log(x**2 + a**(-2))/(2*c) - a*atan(a*x)**2/(2*c)
- atan(a*x)/(c*x), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.45, size = 48, normalized size = 0.92

$$\frac{a\,\ln(x)}{c} - \frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c} - \frac{a\,{\operatorname{atan}(a\,x)}^2}{2\,c} - \frac{\operatorname{atan}(a\,x)}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)),x)
```

```
[Out] (a*log(x))/c - (a*log(a^2*x^2 + 1))/(2*c) - (a*atan(a*x)^2)/(2*c) - atan(a*
x)/(c*x)
```

3.180 $\int \frac{\text{ArcTan}(ax)}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=113

$$-\frac{a}{2cx}-\frac{a^2\text{ArcTan}(ax)}{2c}-\frac{\text{ArcTan}(ax)}{2cx^2}+\frac{ia^2\text{ArcTan}(ax)^2}{2c}-\frac{a^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{ia^2\text{PolyLog}(2,-1$$

```
[Out] -1/2*a/c/x-1/2*a^2*arctan(a*x)/c-1/2*arctan(a*x)/c/x^2+1/2*I*a^2*arctan(a*x
)^2/c-a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c+1/2*I*a^2*polylog(2,-1+2/(1-I*a*x
))/c
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5038, 4946, 331, 209, 5044, 4988, 2497}

$$\frac{ia^2\text{ArcTan}(ax)^2}{2c}-\frac{a^2\text{ArcTan}(ax)}{2c}-\frac{a^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{ia^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c}-\frac{\text{ArcTan}(ax)}{2cx^2}-\frac{a}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -1/2*a/(c*x) - (a^2*ArcTan[a*x])/(2*c) - ArcTan[a*x]/(2*c*x^2) + ((I/2)*a^2
*ArcTan[a*x]^2)/c - (a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c + ((I/2)*a^2
*PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}+\frac{a\int \frac{1}{x^2(1+a^2x^2)}\,dx}{2c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}\\
&= -\frac{a}{2cx}-\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a^3\int \frac{1}{1+a^2x^2}\,dx}{2c}\\
&= -\frac{a}{2cx}-\frac{a^2\tan^{-1}(ax)}{2c}-\frac{\tan^{-1}(ax)}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c}-\frac{a^2\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.04, size = 142, normalized size = 1.26

$$-\frac{\text{ArcTan}(ax)}{2cx^2}-\frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-a^2x^2\right)}{2cx}-\frac{a^2\left(\frac{1}{2}i\text{ArcTan}(ax)^2+\frac{1}{2}i\text{PolyLog}(2,-iax)-\frac{1}{2}i\text{PolyLog}(2,iax)+\frac{1}{2}\left(2\text{ArcTan}(ax)\log\left(\frac{2i}{i-ax}\right)+i\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -1/2*ArcTan[a*x]/(c*x^2) - (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/
(2*c*x) - (a^2*((I/2)*ArcTan[a*x]^2 + (I/2)*PolyLog[2, (-I)*a*x] - (I/2)*Po
lyLog[2, I*a*x] + (2*ArcTan[a*x]*Log[(2*I)/(I - a*x)] + I*PolyLog[2, -((I +
 a*x)/(I - a*x))])/2))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(99) = 198.
time = 0.06, size = 252, normalized size = 2.23

method	result
risch	$-\frac{ia^2\ln(iax)}{4c}-\frac{ia^2\,\text{dilog}(iax+1)}{2c}+\frac{ia^2\,\text{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c}-\frac{ia^2\,\text{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}-\frac{a}{2cx}-\frac{ia^2\ln(iax+1)^2}{8c}-\frac{a^2\,\text{arc}}{}$
derivativedivides	$a^2\left(\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}-\frac{-\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2}+\frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2}+\right.$
default	$a^2\left(\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}-\frac{-\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2}+\frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2/c*arctan(a*x)*ln(a^2*x^2+1)-1/2/c*arctan(a*x)/a^2/x^2-1/c*arctan(a
*x)*ln(a*x)-1/2/c*(-1/2*I*ln(a*x-I)*ln(a^2*x^2+1)+1/2*I*dilog(-1/2*I*(I+a*x
))+1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+1/4*I*ln(a*x-I)^2+1/2*I*ln(I+a*x)*ln(
a^2*x^2+1)-1/2*I*dilog(1/2*I*(a*x-I))-1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/4
*I*ln(I+a*x)^2+1/a/x+arctan(a*x)+I*ln(a*x)*ln(1+I*a*x)-I*ln(a*x)*ln(1-I*a*x
)+I*dilog(1+I*a*x)-I*dilog(1-I*a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)/(a**2*x**5 + x**3), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)/(x^3*(c + a^2*c*x^2)), x)
```

3.181 $\int \frac{\text{ArcTan}(ax)}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=88

$$-\frac{a}{6cx^2}-\frac{\text{ArcTan}(ax)}{3cx^3}+\frac{a^2\text{ArcTan}(ax)}{cx}+\frac{a^3\text{ArcTan}(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log\left(1+a^2x^2\right)}{3c}$$

```
[Out] -1/6*a/c/x^2-1/3*arctan(a*x)/c/x^3+a^2*arctan(a*x)/c/x+1/2*a^3*arctan(a*x)^
2/c-4/3*a^3*ln(x)/c+2/3*a^3*ln(a^2*x^2+1)/c
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {5038, 4946, 272, 46, 36, 29, 31, 5004}

$$\frac{a^3\text{ArcTan}(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{a^2\text{ArcTan}(ax)}{cx}+\frac{2a^3\log\left(a^2x^2+1\right)}{3c}-\frac{\text{ArcTan}(ax)}{3cx^3}-\frac{a}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -1/6*a/(c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*Arc
Tan[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{3cx^3}+a^4\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx+\frac{a\int \frac{1}{x^3(1+a^2x^2)}\,dx}{3c}-\frac{a^2\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)}{3cx^3}+\frac{a^2\tan^{-1}(ax)}{cx}+\frac{a^3\tan^{-1}(ax)^2}{2c}+\frac{a\text{Subst}\left(\int \frac{1}{x^2(1+a^2x)}\,dx,x,x^2\right)}{6c}-\frac{a^3\int
\\
&= -\frac{\tan^{-1}(ax)}{3cx^3}+\frac{a^2\tan^{-1}(ax)}{cx}+\frac{a^3\tan^{-1}(ax)^2}{2c}+\frac{a\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{a^2}{x}+\frac{a^4}{1+a^2x}\right)\,dx,x,x
\\
&= -\frac{a}{6cx^2}-\frac{\tan^{-1}(ax)}{3cx^3}+\frac{a^2\tan^{-1}(ax)}{cx}+\frac{a^3\tan^{-1}(ax)^2}{2c}-\frac{a^3\log(x)}{3c}+\frac{a^3\log\left(1+a^2x^2\right)}{6c}
\\
&= -\frac{a}{6cx^2}-\frac{\tan^{-1}(ax)}{3cx^3}+\frac{a^2\tan^{-1}(ax)}{cx}+\frac{a^3\tan^{-1}(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log\left(1+a^2x
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.00

$$-\frac{a}{6cx^2}-\frac{\text{ArcTan}(ax)}{3cx^3}+\frac{a^2\text{ArcTan}(ax)}{cx}+\frac{a^3\text{ArcTan}(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log(1+a^2x^2)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -1/6*a/(c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*Arc
Tan[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

Maple [A]
time = 0.07, size = 85, normalized size = 0.97

method	result
derivativedivides	$a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$
default	$a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$
risch	$-\frac{a^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{2c}+\frac{a^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c}-\frac{a^3\operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}-\frac{2a^3\ln(-iax)}{3c}-\frac{a}{6c\,x^2}+2a^3\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/c*arctan(a*x)^2-1/3/c*arctan(a*x)/a^3/x^3+1/c*arctan(a*x)/a/x-1/3/c*
(-2*ln(a^2*x^2+1)+1/2/a^2/x^2+4*ln(a*x)+3/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.49, size = 90, normalized size = 1.02

$$\frac{1}{3}\left(\frac{3\,a^3\arctan(ax)}{c}+\frac{3\,a^2x^2-1}{cx^3}\right)\arctan(ax)-\frac{\left(3\,a^2x^2\arctan(ax)^2-4\,a^2x^2\log(a^2x^2+1)+8\,a^2x^2\log(x)+1\right)a}{6\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*arctan(a*x)/c + (3*a^2*x^2 - 1)/(c*x^3))*arctan(a*x) - 1/6*(3*a^
2*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a/
(c*x^2)
```

Fricas [A]
time = 1.86, size = 71, normalized size = 0.81

$$\frac{3\,a^3x^3\arctan(ax)^2+4\,a^3x^3\log(a^2x^2+1)-8\,a^3x^3\log(x)-ax+2\left(3\,a^2x^2-1\right)\arctan(ax)}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^3*x^3*arctan(a*x)^2 + 4*a^3*x^3*log(a^2*x^2 + 1) - 8*a^3*x^3*log(x
) - a*x + 2*(3*a^2*x^2 - 1)*arctan(a*x))/(c*x^3)
```

Sympy [A]
time = 0.69, size = 76, normalized size = 0.86

$$\begin{cases} -\frac{4a^3\log{(x)}}{3c} + \frac{2a^3\log{\left(x^2+\frac{1}{a^2}\right)}}{3c} + \frac{a^3\operatorname{atan}^2{(ax)}}{2c} + \frac{a^2\operatorname{atan}{(ax)}}{cx} - \frac{a}{6cx^2} - \frac{\operatorname{atan}{(ax)}}{3cx^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((-4*a**3*log(x)/(3*c) + 2*a**3*log(x**2 + a**(-2))/(3*c) + a**3*a
tan(a*x)**2/(2*c) + a**2*atan(a*x)/(c*x) - a/(6*c*x**2) - atan(a*x)/(3*c*x*
*3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.47, size = 78, normalized size = 0.89

$$\frac{2\,a^3\,\ln\left(a^2\,x^2+1\right)}{3\,c}-\frac{\mathrm{atan}(a\,x)}{3\,c\,x^3}-\frac{a}{6\,c\,x^2}-\frac{4\,a^3\,\ln\left(x\right)}{3\,c}+\frac{a^3\,{\mathrm{atan}(a\,x)}^2}{2\,c}+\frac{a^2\,\mathrm{atan}(a\,x)}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^4*(c + a^2*c*x^2)),x)
```

```
[Out] (2*a^3*log(a^2*x^2 + 1))/(3*c) - atan(a*x)/(3*c*x^3) - a/(6*c*x^2) - (4*a^3
*log(x))/(3*c) + (a^3*atan(a*x)^2)/(2*c) + (a^2*atan(a*x))/(c*x)
```

3.182 $\int \frac{x^5 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=157

$$-\frac{x}{2a^5c^2}+\frac{x}{4a^5c^2(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)}{4a^6c^2}+\frac{x^2\text{ArcTan}(ax)}{2a^4c^2}-\frac{\text{ArcTan}(ax)}{2a^6c^2(1+a^2x^2)}+\frac{i\text{ArcTan}(ax)^2}{a^6c^2}+\frac{2\text{ArcTan}(ax}{a^6}$$

```
[Out] -1/2*x/a^5/c^2+1/4*x/a^5/c^2/(a^2*x^2+1)+3/4*arctan(a*x)/a^6/c^2+1/2*x^2*ar
ctan(a*x)/a^4/c^2-1/2*arctan(a*x)/a^6/c^2/(a^2*x^2+1)+I*arctan(a*x)^2/a^6/c
^2+2*arctan(a*x)*ln(2/(1+I*a*x))/a^6/c^2+I*polylog(2,1-2/(1+I*a*x))/a^6/c^2
```

Rubi [A]
time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5084, 5036, 4946, 327, 209, 5040, 4964, 2449, 2352, 5050, 205, 211}

$$\frac{i\text{ArcTan}(ax)^2}{a^6c^2}+\frac{3\text{ArcTan}(ax)}{4a^6c^2}+\frac{2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^6c^2}+\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^6c^2}-\frac{x}{2a^5c^2}+\frac{x^2\text{ArcTan}(ax)}{2a^4c^2}-\frac{\text{ArcTan}(ax)}{2a^6c^2(a^2x^2+1)}+\frac{x}{4a^5c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/2*x/(a^5*c^2) + x/(4*a^5*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(4*a^6*c^2
) + (x^2*ArcTan[a*x])/(2*a^4*c^2) - ArcTan[a*x]/(2*a^6*c^2*(1 + a^2*x^2)) +
 (I*ArcTan[a*x]^2)/(a^6*c^2) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^6*c^2)
 + (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^6*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^2}+\frac{\int \frac{x^3\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c}$$

$$=\frac{\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^4}+\frac{\int x\tan^{-1}(ax)\,dx}{a^4c^2}-2\frac{\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^4c}$$

$$=\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)}+\frac{\int \frac{1}{(c+a^2cx^2)^2}\,dx}{2a^5}-2\Bigg(-\frac{i\tan^{-1}(ax)^2}{2a^6c^2}-\frac{\int \frac{\tan^{-1}(ax)}{i-ax}}{a^5c^2}$$

$$=-\frac{x}{2a^5c^2}+\frac{x}{4a^5c^2(1+a^2x^2)}+\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)}+\frac{\int \frac{1}{1+a^2x^2}\,dx}{2a^5c^2}-2\Bigg(-$$

$$=-\frac{x}{2a^5c^2}+\frac{x}{4a^5c^2(1+a^2x^2)}+\frac{3\tan^{-1}(ax)}{4a^6c^2}+\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)}-2\Bigg(-$$

$$=-\frac{x}{2a^5c^2}+\frac{x}{4a^5c^2(1+a^2x^2)}+\frac{3\tan^{-1}(ax)}{4a^6c^2}+\frac{x^2\tan^{-1}(ax)}{2a^4c^2}-\frac{\tan^{-1}(ax)}{2a^6c^2(1+a^2x^2)}-2\Bigg(-$$

Mathematica [A]
time = 0.15, size = 90, normalized size = 0.57

$$\frac{-4ax-8i\text{ArcTan}(ax)^2+2\text{ArcTan}(ax)\left(2+2a^2x^2-\cos(2\text{ArcTan}(ax))+8\log\left(1+e^{2i\text{ArcTan}(ax)}\right)\right)-8i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+\sin(2\text{ArcTan}(ax))}{8a^6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-4*a*x - (8*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(2 + 2*a^2*x^2 - Cos[2*ArcTan
[a*x]] + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) - (8*I)*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] + Sin[2*ArcTan[a*x]])/(8*a^6*c^2)
```

Maple [A]
time = 0.07, size = 220, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{i\ln(ax-i)^2}{2}-i}{}}{a^6}$
default	$\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{i\ln(ax-i)^2}{2}-i}{}}{a^6}$
risch	$-\frac{x}{2a^5c^2}+\frac{5\arctan(ax)}{8a^6c^2}+\frac{\ln(iax+1)x}{16c^2a^5(iax-1)}+\frac{\ln(-iax+1)x}{16c^2a^5(-iax-1)}+\frac{i\ln(-iax+1)x^2}{4c^2a^4}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{2c^2a^6}-\frac{i\ln(-iax}{4c^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/2/c^2*arctan(a*x)*a^2*x^2-1/2*arctan(a*x)/c^2/(a^2*x^2+1)-1/c^2*ar
ctan(a*x)*ln(a^2*x^2+1)-1/2/c^2*(a*x-1/2*a*x/(a^2*x^2+1)-3/2*arctan(a*x)+I*
ln(a*x-I)*ln(a^2*x^2+1)-1/2*I*ln(a*x-I)^2-I*dilog(-1/2*I*(I+a*x))-I*ln(a*x-
I)*ln(-1/2*I*(I+a*x))-I*ln(I+a*x)*ln(a^2*x^2+1)+1/2*I*ln(I+a*x)^2+I*dilog(1
/2*I*(a*x-I))+I*ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(x^5*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**5*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] int((x^5*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.183 $\int \frac{x^4 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=96

$$\frac{1}{4a^5c^2(1+a^2x^2)} + \frac{x\text{ArcTan}(ax)}{a^4c^2} + \frac{x\text{ArcTan}(ax)}{2a^4c^2(1+a^2x^2)} - \frac{3\text{ArcTan}(ax)^2}{4a^5c^2} - \frac{\log(1+a^2x^2)}{2a^5c^2}$$

```
[Out] 1/4/a^5/c^2/(a^2*x^2+1)+x*arctan(a*x)/a^4/c^2+1/2*x*arctan(a*x)/a^4/c^2/(a^
2*x^2+1)-3/4*arctan(a*x)^2/a^5/c^2-1/2*ln(a^2*x^2+1)/a^5/c^2
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5084, 5036, 4930, 266, 5004, 5054}

$$-\frac{3\text{ArcTan}(ax)^2}{4a^5c^2} + \frac{x\text{ArcTan}(ax)}{a^4c^2} + \frac{1}{4a^5c^2(a^2x^2+1)} - \frac{\log(a^2x^2+1)}{2a^5c^2} + \frac{x\text{ArcTan}(ax)}{2a^4c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^5*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(a^4*c^2) + (x*ArcTan[a*x])/(
2*a^4*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x]^2)/(4*a^5*c^2) - Log[1 + a^2*x^2]
/(2*a^5*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx &= -\frac{\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^2}+\frac{\int \frac{x^2\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c}\\ &= \frac{1}{4a^5c^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)}+\frac{\int \tan^{-1}(ax)\,dx}{a^4c^2}-\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{2a^4c}-\frac{\int \frac{\tan^{-1}(ax)}{c+a^2cx^2}\,d}{a^4c}\\ &= \frac{1}{4a^5c^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)}{a^4c^2}+\frac{x\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)^2}{4a^5c^2}-\frac{\int \frac{x}{1+a^2x^2}\,dx}{a^3c^2}\\ &= \frac{1}{4a^5c^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)}{a^4c^2}+\frac{x\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)^2}{4a^5c^2}-\frac{\log(1+a^2x^2)}{2a^5c^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.82

$$\frac{1+(6ax+4a^3x^3)\,\text{ArcTan}(ax)-3(1+a^2x^2)\,\text{ArcTan}(ax)^2-2(1+a^2x^2)\log(1+a^2x^2)}{4a^5c^2(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x] - 3*(1 + a^2*x^2)*ArcTan[a*x]^2 - 2*(1
 + a^2*x^2)*Log[1 + a^2*x^2])/(4*a^5*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.15, size = 86, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\ln\left(a^2x^2+1\right)-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$
default	$\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\ln\left(a^2x^2+1\right)-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$
risch	$\frac{3\ln(iax+1)^2}{16a^5c^2}-\frac{i(-3ia^2x^2\ln(-iax+1)+4a^3x^3-3i\ln(-iax+1)+6ax)\ln(iax+1)}{8a^5c^2(a^2x^2+1)}+\frac{i\left(-3ia^2x^2\ln(-iax+1)^2-3i\ln(-i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/c^2*arctan(a*x)*a*x+1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arctan
(a*x)^2/c^2-1/2/c^2*(-1/2/(a^2*x^2+1)+ln(a^2*x^2+1)-3/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.49, size = 114, normalized size = 1.19

$$\frac{1}{2}\left(\frac{x}{a^6c^2x^2+a^4c^2}+\frac{2\,x}{a^4c^2}-\frac{3\,\arctan\left(ax\right)}{a^5c^2}\right)\arctan\left(ax\right)+\frac{\left(3\left(a^2x^2+1\right)\arctan\left(ax\right)^2-2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)+1\right)a}{4\left(a^8c^2x^2+a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^6*c^2*x^2 + a^4*c^2) + 2*x/(a^4*c^2) - 3*arctan(a*x)/(a^5*c^2))*a
rctan(a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x
^2 + 1) + 1)*a/(a^8*c^2*x^2 + a^6*c^2)
```

Fricas [A]
time = 3.74, size = 81, normalized size = 0.84

$$-\frac{3\left(a^2x^2+1\right)\arctan\left(ax\right)^2-2\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)+2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)-1}{4\left(a^7c^2x^2+a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 2
*(a^2*x^2 + 1)*log(a^2*x^2 + 1) - 1)/(a^7*c^2*x^2 + a^5*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(90) = 180$.
time = 0.56, size = 223, normalized size = 2.32

$$\begin{cases} \frac{4a^3x^3\operatorname{atan}(ax)}{4a^7c^2x^2+4a^5c^2} - \frac{2a^2x^2\log\left(x^2+\frac{1}{a^2}\right)}{4a^7c^2x^2+4a^5c^2} - \frac{3a^2x^2\operatorname{atan}^2(ax)}{4a^7c^2x^2+4a^5c^2} + \frac{6ax\operatorname{atan}(ax)}{4a^7c^2x^2+4a^5c^2} - \frac{2\log\left(x^2+\frac{1}{a^2}\right)}{4a^7c^2x^2+4a^5c^2} - \frac{3\operatorname{atan}^2(ax)}{4a^7c^2x^2+4a^5c^2} + \frac{1}{4a^7c^2x^2+4a^5c^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((4*a**3*x**3*atan(a*x)/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 2*a**2*
x**2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 3*a**2*x**2*ata
n(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 6*a*x*atan(a*x)/(4*a**7*c**2*x
**2 + 4*a**5*c**2) - 2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c**2)
 - 3*atan(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 1/(4*a**7*c**2*x**2 +
4*a**5*c**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.46, size = 94, normalized size = 0.98

$$\frac{1}{2\,a^2\,\left(2\,a^5\,c^2\,x^2+2\,a^3\,c^2\right)}-\frac{\ln\left(a^2\,x^2+1\right)}{2\,a^5\,c^2}+\frac{\operatorname{atan}(a\,x)\,\left(\frac{3\,x}{2\,a^6\,c^2}+\frac{x^3}{a^4\,c^2}\right)}{\frac{1}{a^2}+x^2}-\frac{3\,{\operatorname{atan}(a\,x)}^2}{4\,a^5\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

```
[Out] 1/(2*a^2*(2*a^3*c^2 + 2*a^5*c^2*x^2)) - log(a^2*x^2 + 1)/(2*a^5*c^2) + (ata
n(a*x)*((3*x)/(2*a^6*c^2) + x^3/(a^4*c^2)))/(1/a^2 + x^2) - (3*atan(a*x)^2)
/(4*a^5*c^2)
```

3.184 $\int \frac{x^3\mathbf{ArcTan}(ax)}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{x}{4a^3c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)}{4a^4c^2}+\frac{\text{ArcTan}(ax)}{2a^4c^2(1+a^2x^2)}-\frac{i\text{ArcTan}(ax)^2}{2a^4c^2}-\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2}-\frac{i\text{PolyLog}(2,1}{2a^4c}$$

```
[Out] -1/4*x/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a^4/c^2+1/2*arctan(a*x)/a^4/c^2/
(a^2*x^2+1)-1/2*I*arctan(a*x)^2/a^4/c^2-arctan(a*x)*ln(2/(1+I*a*x))/a^4/c^2
-1/2*I*polylog(2,1-2/(1+I*a*x))/a^4/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5084, 5040, 4964, 2449, 2352, 5050, 205, 211}

$$-\frac{i\text{ArcTan}(ax)^2}{2a^4c^2}-\frac{\text{ArcTan}(ax)}{4a^4c^2}-\frac{\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2}-\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^4c^2}+\frac{\text{ArcTan}(ax)}{2a^4c^2(a^2x^2+1)}-\frac{x}{4a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/4*x/(a^3*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a^4*c^2) + ArcTan[a*x]/(2*a
^4*c^2*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/(a^4*c^2) - (ArcTan[a*x]*Log[
2/(1 + I*a*x)])/(a^4*c^2) - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{a^2c} \\
&= \frac{\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\int \frac{1}{(c+a^2cx^2)^2}\,dx}{2a^3} - \frac{\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^3c^2} \\
&= -\frac{x}{4a^3c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} + \cdots \\
&= -\frac{x}{4a^3c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{4a^4c^2} + \frac{\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\cdots\right)}{a^4c^2} \cdots \\
&= -\frac{x}{4a^3c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{4a^4c^2} + \frac{\tan^{-1}(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^2}{2a^4c^2} - \frac{\tan^{-1}(ax)\log\left(\cdots\right)}{a^4c^2} \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 0.58

$$\frac{4i\text{ArcTan}(ax)^2 + 2\text{ArcTan}(ax)\left(\cos(2\text{ArcTan}(ax)) - 4\log\left(1+e^{2i\text{ArcTan}(ax)}\right)\right) + 4i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) - \sin(2\text{ArcTan}(ax))}{8a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 4*Log[1 + E^((2*
I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - Sin[2*ArcTan
[a*x]])/(8*a^4*c^2)
```

Maple [A]
time = 0.06, size = 202, normalized size = 1.52

method	result
derivativedivides	$\frac{\frac{\arctan(ax)}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2} - \frac{\frac{ax}{2a^2x^2+2} + \frac{\arctan(ax)}{2} - \frac{i\ln(ax-i)\ln(a^2x^2+1)}{2} + \frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2} + \frac{i\ln(ax-i)\ln\left(-\frac{i(a\cdots}{\cdots}\right)}{2}\cdots}{\cdots}}{a^4}$
default	$\frac{\frac{\arctan(ax)}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2} - \frac{\frac{ax}{2a^2x^2+2} + \frac{\arctan(ax)}{2} - \frac{i\ln(ax-i)\ln(a^2x^2+1)}{2} + \frac{i\,\text{dilog}\left(-\frac{i(ax+i)}{2}\right)}{2} + \frac{i\ln(ax-i)\ln\left(-\frac{i(a\cdots}{\cdots}\right)}{2}\cdots}{\cdots}}{a^4}$
risch	$\frac{i\ln(-iax+1)}{8c^2a^4(-iax+1)} + \frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c^2a^4} - \frac{i}{8c^2a^4(iax+1)} - \frac{\arctan(ax)}{8a^4c^2} - \frac{i\ln(iax+1)}{8c^2a^4(iax+1)} - \frac{i\ln(-iax+1)}{16c^2a^4(-iax-1)}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2*arctan(a*x)/c^2/(a^2*x^2+1)+1/2/c^2*arctan(a*x)*ln(a^2*x^2+1)-1/
2/c^2*(1/2*a*x/(a^2*x^2+1)+1/2*arctan(a*x)-1/2*I*ln(a*x-I)*ln(a^2*x^2+1)+1/
2*I*dilog(-1/2*I*(I+a*x))+1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+1/4*I*ln(a*x-I
)^2+1/2*I*ln(I+a*x)*ln(a^2*x^2+1)-1/2*I*dilog(1/2*I*(a*x-I))-1/2*I*ln(I+a*x
)*ln(1/2*I*(a*x-I))-1/4*I*ln(I+a*x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(x^3*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**3*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] int((x^3*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.185 $\int \frac{x^2 \text{ArcTan}(ax)}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=64

$$-\frac{1}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\text{ArcTan}(ax)}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\text{ArcTan}(ax)^2}{4a^3c^2}$$

```
[Out] -1/4/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a
*x)^2/a^3/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5054, 5004}

$$\frac{\text{ArcTan}(ax)^2}{4a^3c^2}-\frac{x\text{ArcTan}(ax)}{2a^2c^2\left(a^2x^2+1\right)}-\frac{1}{4a^3c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/4*1/(a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*c^2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a^3*c^2)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx&=-\frac{1}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\int\frac{\tan^{-1}(ax)}{c+a^2cx^2}\,dx}{2a^2c}\\&=-\frac{1}{4a^3c^2\left(1+a^2x^2\right)}-\frac{x\tan^{-1}(ax)}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^2}{4a^3c^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.73

$$\frac{-1-2ax\text{ArcTan}(ax)+(1+a^2x^2)\,\text{ArcTan}(ax)^2}{4a^3c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-1 - 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*
x^2))
```

Maple [A]
time = 0.13, size = 66, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{\frac{\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}}{2c^2}}{a^3}$
default	$\frac{-\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{\frac{\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}}{2c^2}}{a^3}$
risch	$-\frac{\ln(iax+1)^2}{16c^2a^3}+\frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax)\ln(iax+1)}{8a^3c^2(a^2x^2+1)}-\frac{a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2+4iax\ln(-ia}{16(ax+i)c^2(ax-i)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2-1/2/c^2*(
1/2*arctan(a*x)^2+1/2/(a^2*x^2+1)))
```

Maxima [A]
time = 0.47, size = 83, normalized size = 1.30

$$-\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)-\frac{\left((a^2x^2+1)\arctan(ax)^2+1\right)a}{4\,(a^6c^2x^2+a^4c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x) - 1/4*
((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a/(a^6*c^2*x^2 + a^4*c^2)
```

Fricas [A]
time = 3.43, size = 49, normalized size = 0.77

$$-\frac{2\,ax\arctan(ax)-(a^2x^2+1)\arctan(ax)^2+1}{4\,(a^5c^2x^2+a^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x*arctan(a*x) - (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^5*c^2*x^2 + a
^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.40, size = 48, normalized size = 0.75

$$\frac{a^2\,x^2\,\operatorname{atan}(a\,x)^2-2\,a\,x\,\operatorname{atan}(a\,x)+\operatorname{atan}(a\,x)^2-1}{4\,a^3\,c^2\,\left(a^2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

```
[Out] (atan(a*x)^2 - 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 - 1)/(4*a^3*c^2*(a^2*x
^2 + 1))
```

3.186 $\int \frac{x\textbf{ArcTan}(ax)}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x}{4ac^2\left(1+a^2x^2\right)}+\frac{\operatorname{ArcTan}(ax)}{4a^2c^2}-\frac{\operatorname{ArcTan}(ax)}{2a^2c^2\left(1+a^2x^2\right)}$$

```
[Out] 1/4*x/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^2/c^2-1/2*arctan(a*x)/a^2/c^2/(a^
2*x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5050, 205, 211}

$$-\frac{\operatorname{ArcTan}(ax)}{2a^2c^2\left(a^2x^2+1\right)}+\frac{\operatorname{ArcTan}(ax)}{4a^2c^2}+\frac{x}{4ac^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] x/(4*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^2*c^2) - ArcTan[a*x]/(2*a^2*c^
2*(1 + a^2*x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx &= -\frac{\tan^{-1}(ax)}{2a^2c^2(1+a^2x^2)}+\frac{\int \frac{1}{(c+a^2cx^2)^2}\,dx}{2a}\\ &= \frac{x}{4ac^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)}{2a^2c^2(1+a^2x^2)}+\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4ac}\\ &= \frac{x}{4ac^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)}{4a^2c^2}-\frac{\tan^{-1}(ax)}{2a^2c^2(1+a^2x^2)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.63

$$\frac{ax+(-1+a^2x^2)\,\mathrm{ArcTan}(ax)}{4a^2c^2(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
[Out] (a*x + (-1 + a^2*x^2)*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.09, size = 53, normalized size = 0.85

method	result	size
derivativedivides	$\frac{-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}}{2c^2}}{a^2}$	53
default	$\frac{-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}}{2c^2}}{a^2}$	53
risch	$\frac{i\ln(iax+1)}{4a^2c^2(a^2x^2+1)}-\frac{i(2\ln(-iax+1)+\ln(ax-i)a^2x^2+\ln(ax-i)-\ln(-ax-i)a^2x^2-\ln(-ax-i)+2iax)}{8(ax+i)a^2c^2(ax-i)}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(-1/2*arctan(a*x)/c^2/(a^2*x^2+1)+1/2/c^2*(1/2*a*x/(a^2*x^2+1)+1/2*ar
ctan(a*x)))
```

Maxima [A]
time = 0.45, size = 59, normalized size = 0.95

$$\frac{\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}}{4\,ac}-\frac{\arctan(ax)}{2\,(a^2cx^2+c)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))/(a*c) - 1/2*arctan(a*x)/((a^2*c
*x^2 + c)*a^2*c)
```

Fricas [A]
time = 3.00, size = 40, normalized size = 0.65

$$\frac{ax + (a^2x^2 - 1)\arctan(ax)}{4(a^4c^2x^2 + a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x + (a^2*x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [A]
time = 0.44, size = 82, normalized size = 1.32

$$\begin{cases} \frac{a^2x^2\operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} + \frac{ax}{4a^4c^2x^2+4a^2c^2} - \frac{\operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((a**2*x**2*atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2) + a*x/(4*a*
*4*c**2*x**2 + 4*a**2*c**2) - atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2), N
e(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.17, size = 40, normalized size = 0.65

$$\frac{a\,x - \operatorname{atan}(a\,x) + a^2\,x^2\,\operatorname{atan}(a\,x)}{4\,a^2\,c^2\,\left(a^2\,x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

```
[Out] (a*x - atan(a*x) + a^2*x^2*atan(a*x))/(4*a^2*c^2*(a^2*x^2 + 1))
```

3.187 $\int \frac{\text{ArcTan}(ax)}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=61

$$\frac{1}{4ac^2(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^2}{4ac^2}$$

```
[Out] 1/4/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^2/a
/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5012, 267}

$$\frac{x\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}+\frac{1}{4ac^2(a^2x^2+1)}+\frac{\text{ArcTan}(ax)^2}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + ArcTan[
a*x]^2/(4*a*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx &= \frac{x\tan^{-1}(ax)}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^2}{4ac^2}-\frac{1}{2}a\int \frac{x}{(c+a^2cx^2)^2}\,dx\\ &= \frac{1}{4ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^2}{4ac^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.72

$$\frac{1+2ax\text{ArcTan}(ax)+(1+a^2x^2)\,\text{ArcTan}(ax)^2}{4c^2\,(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*c^2*(a + a^3*x^2))
```

Maple [A]
time = 0.12, size = 66, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2}}{2c^2}}{a}$
default	$\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2}}{2c^2}}{a}$
risch	$-\frac{\ln(iax+1)^2}{16a\,c^2}+\frac{\left(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax\right)\ln(iax+1)}{8c^2(a^2x^2+1)a}-\frac{a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln(-ia}{16c^2(ax+i)(ax-i)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2-1/2/c^2*(-1/
2/(a^2*x^2+1)+1/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.47, size = 78, normalized size = 1.28

$$\frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)-\frac{\left((a^2x^2+1)\arctan(ax)^2-1\right)a}{4\,(a^4c^2x^2+a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x) - 1/4*((a^2*x
^2 + 1)*arctan(a*x)^2 - 1)*a/(a^4*c^2*x^2 + a^2*c^2)
```

Fricas [A]
time = 1.71, size = 46, normalized size = 0.75

$$\frac{2\,ax\arctan(ax)+(a^2x^2+1)\arctan(ax)^2+1}{4\,(a^3c^2x^2+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*arctan(a*x) + (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^3*c^2*x^2 + a*
c^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded in com
parison
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.42, size = 48, normalized size = 0.79

$$\frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2+2\,a\,x\,\mathrm{atan}(a\,x)+\mathrm{atan}(a\,x)^2+1}{4\,a\,c^2\,\left(a^2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2)^2,x)
```

```
[Out] (atan(a*x)^2 + 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 + 1)/(4*a*c^2*(a^2*x^2
 + 1))
```

3.188 $\int \frac{\text{ArcTan}(ax)}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{ax}{4c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)}{4c^2}+\frac{\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}-\frac{i\text{ArcTan}(ax)^2}{2c^2}+\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{i\text{PolyLog}(2,-}{2c^2}$$

```
[Out] -1/4*a*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/c^2+1/2*arctan(a*x)/c^2/(a^2*x^2+1
)-1/2*I*arctan(a*x)^2/c^2+arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-1/2*I*polylog(2
,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5086, 5044, 4988, 2497, 5050, 205, 211}

$$\frac{\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}-\frac{ax}{4c^2(a^2x^2+1)}-\frac{i\text{ArcTan}(ax)^2}{2c^2}-\frac{\text{ArcTan}(ax)}{4c^2}+\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{i\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/4*(a*x)/(c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*c^2) + ArcTan[a*x]/(2*c^2*(
1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^2 + (ArcTan[a*x]*Log[2 - 2/(1 - I*a
*x)])/c^2 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx &= -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^2}{2c^2}-\frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx+\frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c^2}\\
&= -\frac{ax}{4c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^2}{2c^2}+\frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{a\int
\\
&= -\frac{ax}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{4c^2}+\frac{\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^2}{2c^2}+\frac{\tan^{-1}(ax)\log\left(2-\right.}{c^2}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.62

$$-\frac{4i\text{ArcTan}(ax)^2-2\text{ArcTan}(ax)\left(\cos(2\text{ArcTan}(ax))+4\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+4i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+\sin(2\text{ArcTan}(ax))}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/8*((4*I)*ArcTan[a*x]^2 - 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] + 4*Log[1 - E
^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + Sin[2*Ar
cTan[a*x]])/c^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(103) = 206.
time = 0.07, size = 262, normalized size = 2.24

method	result
derivativedivides	$\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2}+\frac{\arctan(ax)\ln(ax)}{c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln}{}$
default	$\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2}+\frac{\arctan(ax)\ln(ax)}{c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln}{}$
risch	$\frac{i\,\text{dilog}(iax+1)}{2c^2}-\frac{\ln(-iax+1)ax}{16c^2(-iax-1)}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c^2}+\frac{i\ln(iax+1)}{16c^2(iax-1)}-\frac{\arctan(ax)}{8c^2}-\frac{i\ln(iax+1)}{8c^2(iax+1)}+\frac{i\,\text{dilog}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(a*x)/c^2/(a^2*x^2+1)-1/2/c^2*arctan(a*x)*ln(a^2*x^2+1)+1/c^2*arc
tan(a*x)*ln(a*x)-1/2/c^2*(1/2*a*x/(a^2*x^2+1)+1/2*arctan(a*x)-I*ln(a*x)*ln(
1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-I*a*x)+1/2*I*ln(a
*x-I)*ln(a^2*x^2+1)-1/4*I*ln(a*x-I)^2-1/2*I*dilog(-1/2*I*(I+a*x))-1/2*I*ln(
a*x-I)*ln(-1/2*I*(I+a*x))-1/2*I*ln(I+a*x)*ln(a^2*x^2+1)+1/4*I*ln(I+a*x)^2+1
/2*I*dilog(1/2*I*(a*x-I))+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)}{x\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)^2), x)
```

3.189 $\int \frac{\mathbf{ArcTan}(ax)}{x^2(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{a}{4c^2(1+a^2x^2)}-\frac{\mathrm{ArcTan}(ax)}{c^2x}-\frac{a^2x\mathrm{ArcTan}(ax)}{2c^2(1+a^2x^2)}-\frac{3a\mathrm{ArcTan}(ax)^2}{4c^2}+\frac{a\log(x)}{c^2}-\frac{a\log(1+a^2x^2)}{2c^2}$$

```
[Out] -1/4*a/c^2/(a^2*x^2+1)-arctan(a*x)/c^2/x-1/2*a^2*x*arctan(a*x)/c^2/(a^2*x^2
+1)-3/4*a*arctan(a*x)^2/c^2+a*ln(x)/c^2-1/2*a*ln(a^2*x^2+1)/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5012, 267}

$$-\frac{a^2x\mathrm{ArcTan}(ax)}{2c^2(a^2x^2+1)}-\frac{a}{4c^2(a^2x^2+1)}-\frac{a\log(a^2x^2+1)}{2c^2}-\frac{3a\mathrm{ArcTan}(ax)^2}{4c^2}-\frac{\mathrm{ArcTan}(ax)}{c^2x}+\frac{a\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/4*a/(c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(c^2*x) - (a^2*x*ArcTan[a*x])/(2*c
^2*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1
 + a^2*x^2])/(2*c^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)}\,dx}{c}$$

$$= -\frac{a^2x\tan^{-1}(ax)}{2c^2(1+a^2x^2)} - \frac{a\tan^{-1}(ax)^2}{4c^2} + \frac{1}{2}a^3\int \frac{x}{(c+a^2cx^2)^2}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c^2} - \frac{a^2\int \frac{\tan}{c}}{}$$

$$= -\frac{a}{4c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{c^2x} - \frac{a^2x\tan^{-1}(ax)}{2c^2(1+a^2x^2)} - \frac{3a\tan^{-1}(ax)^2}{4c^2} + \frac{a\int \frac{1}{x(1+a^2x^2)}\,dx}{c^2}$$

$$= -\frac{a}{4c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{c^2x} - \frac{a^2x\tan^{-1}(ax)}{2c^2(1+a^2x^2)} - \frac{3a\tan^{-1}(ax)^2}{4c^2} + \frac{a\text{Subst}\left(\int \frac{1}{x(1+a^2x}\right.}{2c^2}$$

$$= -\frac{a}{4c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{c^2x} - \frac{a^2x\tan^{-1}(ax)}{2c^2(1+a^2x^2)} - \frac{3a\tan^{-1}(ax)^2}{4c^2} + \frac{a\text{Subst}\left(\int \frac{1}{x}\,dx, x\right.}{2c^2}$$

$$= -\frac{a}{4c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{c^2x} - \frac{a^2x\tan^{-1}(ax)}{2c^2(1+a^2x^2)} - \frac{3a\tan^{-1}(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(}{}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 0.97

$$-\frac{a}{4c^2(1+a^2x^2)} - \frac{(2+3a^2x^2)\,\text{ArcTan}(ax)}{2c^2x(1+a^2x^2)} - \frac{3a\text{ArcTan}(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(1+a^2x^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/4*a/(c^2*(1 + a^2*x^2)) - ((2 + 3*a^2*x^2)*ArcTan[a*x])/(2*c^2*x*(1 + a^
2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 + a^2*x^2
])/(2*c^2)
```

Maple [A]
time = 0.21, size = 95, normalized size = 0.98

method	result
derivativedivides	$a\left(-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{3\arctan(ax)^2}{2c^2} - \frac{\arctan(ax)}{c^2ax} - \frac{-\frac{3\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)-2\ln(ax)}{2c^2}\right)$
default	$a\left(-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{3\arctan(ax)^2}{2c^2} - \frac{\arctan(ax)}{c^2ax} - \frac{-\frac{3\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)-2\ln(ax)}{2c^2}\right)$
risch	$\frac{3a\ln(iax+1)^2}{16c^2} - \frac{(3a^3x^3\ln(-iax+1)+3ax\ln(-iax+1)-6ia^2x^2-4i)\ln(iax+1)}{8c^2x(a^2x^2+1)} - \frac{-3a^3x^3\ln(-iax+1)^2+8\ln(3a^2x^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arctan(a*x)^2/c^2-1/c^2*arctan(
a*x)/a/x-1/2/c^2*(-3/2*arctan(a*x)^2+1/2/(a^2*x^2+1)+ln(a^2*x^2+1)-2*ln(a*x
)))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 1.23

$$-\frac{1}{2}\left(\frac{3\,a^2x^2+2}{a^2c^2x^3+c^2x}+\frac{3\,a\arctan\left(ax\right)}{c^2}\right)\arctan\left(ax\right)+\frac{\left(3\left(a^2x^2+1\right)\arctan\left(ax\right)^2-2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)+4\left(a^2x^2+1\right)\log\left(x\right)-1\right)a}{4\left(a^2c^2x^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a^2*x^2 + 2)/(a^2*c^2*x^3 + c^2*x) + 3*a*arctan(a*x)/c^2)*arctan(a
*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x^2 + 1)
 + 4*(a^2*x^2 + 1)*log(x) - 1)*a/(a^2*c^2*x^2 + c^2)
```

Fricas [A]
time = 7.15, size = 97, normalized size = 1.00

$$-\frac{3\left(a^3x^3+ax\right)\arctan\left(ax\right)^2+ax+2\left(3\,a^2x^2+2\right)\arctan\left(ax\right)+2\left(a^3x^3+ax\right)\log\left(a^2x^2+1\right)-4\left(a^3x^3+ax\right)\log\left(x\right)}{4\left(a^2c^2x^3+c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(a^3*x^3 + a*x)*arctan(a*x)^2 + a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x)
 + 2*(a^3*x^3 + a*x)*log(a^2*x^2 + 1) - 4*(a^3*x^3 + a*x)*log(x))/(a^2*c^2*
x^3 + c^2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(88) = 176.
time = 0.84, size = 274, normalized size = 2.82

$$\begin{cases}\frac{4a^3x^3\log\left(x\right)}{4a^2c^2x^3+4c^2x}-\frac{2a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x}-\frac{3a^3x^3\operatorname{atan}^2\left(ax\right)}{4a^2c^2x^3+4c^2x}-\frac{6a^2x^2\operatorname{atan}\left(ax\right)}{4a^2c^2x^3+4c^2x}+\frac{4ax\log\left(x\right)}{4a^2c^2x^3+4c^2x}-\frac{2ax\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x}-\frac{3ax\operatorname{atan}^2\left(ax\right)}{4a^2c^2x^3+4c^2x}-\frac{ax}{4a^2c^2x^3+4c^2x}-\frac{4\operatorname{atan}\left(ax\right)}{4a^2c^2x^3+4c^2x} & \text{for}\: a \neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((4*a**3*x**3*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a**3*x**3*l
og(x**2 + a**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a**3*x**3*atan(a*x)**2
```

```
/(4*a**2*c**2*x**3 + 4*c**2*x) - 6*a**2*x**2*atan(a*x)/(4*a**2*c**2*x**3 +
4*c**2*x) + 4*a*x*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a*x*log(x**2 + a
**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a*x*atan(a*x)**2/(4*a**2*c**2*x**
3 + 4*c**2*x) - a*x/(4*a**2*c**2*x**3 + 4*c**2*x) - 4*atan(a*x)/(4*a**2*c**
2*x**3 + 4*c**2*x), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.48, size = 91, normalized size = 0.94

$$\frac{a\,\ln\left(x\right)}{c^2}-\frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c^2}-\frac{\mathrm{atan}(a\,x)\,\left(\frac{1}{a^2\,c^2}+\frac{3\,x^2}{2\,c^2}\right)}{\frac{x}{a^2}+x^3}-\frac{a}{2\,\left(2\,a^2\,c^2\,x^2+2\,c^2\right)}-\frac{3\,a\,\mathrm{atan}(a\,x)^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] (a*log(x))/c^2 - (a*log(a^2*x^2 + 1))/(2*c^2) - (atan(a*x)*(1/(a^2*c^2) + (
3*x^2)/(2*c^2)))/(x/a^2 + x^3) - a/(2*(2*c^2 + 2*a^2*c^2*x^2)) - (3*a*atan(
a*x)^2)/(4*c^2)
```

3.190 $\int \frac{\mathbf{ArcTan}(ax)}{x^3(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{a}{2c^2x}+\frac{a^3x}{4c^2(1+a^2x^2)}-\frac{a^2\text{ArcTan}(ax)}{4c^2}-\frac{\text{ArcTan}(ax)}{2c^2x^2}-\frac{a^2\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}+\frac{ia^2\text{ArcTan}(ax)^2}{c^2}-\frac{2a^2\text{ArcTan}(ax)}{c^2}$$

```
[Out] -1/2*a/c^2/x+1/4*a^3*x/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)/c^2-1/2*arctan(a
*x)/c^2/x^2-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)+I*a^2*arctan(a*x)^2/c^2-2*a
^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2+I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.29, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5086, 5038, 4946, 331, 209, 5044, 4988, 2497, 5050, 205, 211}

$$-\frac{a^2\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}+\frac{ia^2\text{ArcTan}(ax)^2}{c^2}-\frac{a^2\text{ArcTan}(ax)}{4c^2}-\frac{2a^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{ia^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}+\frac{a^3x}{4c^2(a^2x^2+1)}-\frac{\text{ArcTan}(ax)}{2c^2x^2}-\frac{a}{2c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/2*a/(c^2*x) + (a^3*x)/(4*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(4*c^2)
- ArcTan[a*x]/(2*c^2*x^2) - (a^2*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (I*a^
2*ArcTan[a*x]^2)/c^2 - (2*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 + (I*
a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3(c+a^2cx^2)}\,dx}{c}\\ &= a^4\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)}\,dx}{c}\\ &= -\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{1}{2}a^3\int \frac{1}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int \frac{1}{x^2(1+a^2x^2)}\,dx}{2c^2}-2\Bigg(-\\ &= -\frac{a}{2c^2x}+\frac{a^3x}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^3\int \frac{1}{1+a^2x^2}\,dx}{2c^2}-2\Bigg(-\frac{ia^2}{}\\ &= -\frac{a}{2c^2x}+\frac{a^3x}{4c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)}{4c^2}-\frac{\tan^{-1}(ax)}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-2\Bigg(-\frac{ia^2}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 93, normalized size = 0.60

$$\frac{a^2\left(-\frac{4}{ax}+8i\text{ArcTan}(ax)^2+\text{ArcTan}(ax)\left(-4-\frac{4}{a^2x^2}-2\cos(2\text{ArcTan}(ax))-16\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+8i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+\sin(2\text{ArcTan}(ax))\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*(-4/(a*x) + (8*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-4 - 4/(a^2*x^2) - 2*Co
s[2*ArcTan[a*x]] - 16*Log[1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*PolyLog[2, E^
((2*I)*ArcTan[a*x])] + Sin[2*ArcTan[a*x]]))/(8*c^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(142) = 284.
time = 0.08, size = 288, normalized size = 1.85

method	result
derivativedivides	$a^2\left(-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2}-\frac{\arctan(ax)}{2c^2a^2x^2}-\frac{2\arctan(ax)\ln(ax)}{c^2}-\frac{2i\ln(ax)\ln(iax+1)-2i\ln(}{}\right.$
default	$a^2\left(-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2}-\frac{\arctan(ax)}{2c^2a^2x^2}-\frac{2\arctan(ax)\ln(ax)}{c^2}-\frac{2i\ln(ax)\ln(iax+1)-2i\ln(}{}\right.$
risch	$-\frac{ia^2\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{2c^2}+\frac{ia^2\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{2c^2}+\frac{ia^2\ln(-iax+1)}{16c^2(-iax-1)}+\frac{ia^2\ln(iax+1)}{8c^2(iax+1)}-\frac{a}{2c^2x}-\frac{3a^2\,\mathrm{arct}}{8c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arctan(a*x)/c^2/(a^2*x^2+1)+1/c^2*arctan(a*x)*ln(a^2*x^2+1)-1/2/c
^2*arctan(a*x)/a^2/x^2-2/c^2*arctan(a*x)*ln(a*x)-1/2/c^2*(2*I*ln(a*x)*ln(1+
I*a*x)-2*I*ln(a*x)*ln(1-I*a*x)+2*I*dilog(1+I*a*x)-2*I*dilog(1-I*a*x)-I*ln(a
*x-I)*ln(a^2*x^2+1)+I*dilog(-1/2*I*(I+a*x))+I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+
1/2*I*ln(a*x-I)^2+I*ln(I+a*x)*ln(a^2*x^2+1)-I*dilog(1/2*I*(a*x-I))-I*ln(I+a
*x)*ln(1/2*I*(a*x-I))-1/2*I*ln(I+a*x)^2-1/2*a*x/(a^2*x^2+1)+1/2*arctan(a*x)
+1/a/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**2,x)
[Out] Integral(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2), x)
```

3.191 $\int \frac{\text{ArcTan}(ax)}{x^4(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{a}{6c^2x^2}+\frac{a^3}{4c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)}{3c^2x^3}+\frac{2a^2\text{ArcTan}(ax)}{c^2x}+\frac{a^4x\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}+\frac{5a^3\text{ArcTan}(ax)^2}{4c^2}-\frac{7a^3\log(x)}{3c^2}+\frac{7}{}$$

```
[Out] -1/6*a/c^2/x^2+1/4*a^3/c^2/(a^2*x^2+1)-1/3*arctan(a*x)/c^2/x^3+2*a^2*arctan
(a*x)/c^2/x+1/2*a^4*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/4*a^3*arctan(a*x)^2/c^2
-7/3*a^3*ln(x)/c^2+7/6*a^3*ln(a^2*x^2+1)/c^2
```

Rubi [A]
time = 0.28, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5086, 5038, 4946, 272, 46, 36, 29, 31, 5004, 5012, 267}

$$\frac{5a^3\text{ArcTan}(ax)^2}{4c^2}-\frac{7a^3\log(x)}{3c^2}+\frac{2a^2\text{ArcTan}(ax)}{c^2x}+\frac{a^4x\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}+\frac{a^3}{4c^2(a^2x^2+1)}+\frac{7a^3\log(a^2x^2+1)}{6c^2}-\frac{\text{ArcTan}(ax)}{3c^2x^3}-\frac{a}{6c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/6*a/(c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(3*c^2*x^3) + (2
*a^2*ArcTan[a*x])/(c^2*x) + (a^4*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + (5*
a^3*ArcTan[a*x]^2)/(4*c^2) - (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^
2])/(6*c^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{1}{2}a^5\int \frac{x}{\left(c+a^2cx^2\right)^2}\,dx+\frac{a\int \frac{1}{x^3(1+\cdots}}{3c\cdots}$$

$$= \frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}+\frac{a\text{Subst}\left(\int \frac{1}{x^2(1+a^2x)}\cdots\right.}{6c^2}$$

$$= \frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}+\frac{a\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{a^2}{x}\cdots\right.\right.}{6\cdots}$$

$$= -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{a^3\log(x\cdots}{3c^2}$$

$$= -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{3c^2x^3}+\frac{a^4x\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)^2}{4c^2}-\frac{a^3\log(x\cdots}{3c^2}$$

Mathematica [A]
time = 0.05, size = 124, normalized size = 0.91

$$-\frac{a}{6c^2x^2}+\frac{a^3}{4c^2\left(1+a^2x^2\right)}+\frac{\left(-2+10a^2x^2+15a^4x^4\right)\text{ArcTan}(ax)}{6c^2x^3\left(1+a^2x^2\right)}+\frac{5a^3\text{ArcTan}(ax)^2}{4c^2}-\frac{7a^3\log(x)}{3c^2}+\frac{7a^3\log\left(1+a^2x^2\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/6*a/(c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) + ((-2 + 10*a^2*x^2 + 15*a^4*x
^4)*ArcTan[a*x])/(6*c^2*x^3*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*x]^2)/(4*c^2)
- (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2])/(6*c^2)
```

Maple [A]
time = 0.08, size = 121, normalized size = 0.89

method	result
derivativedivides	$a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-\frac{3}{2(a^2x^2+1)}-7\ln(a^2x^2+1)+\frac{1}{a^2x^2}+14\ln(\right.$
default	$a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-\frac{3}{2(a^2x^2+1)}-7\ln(a^2x^2+1)+\frac{1}{a^2x^2}+14\ln(\right.$
risch	$-\frac{ia^4\ln(-iax+1)x}{16c^2(-iax-1)}-\frac{i\ln(-iax+1)}{6c^2x^3}+\frac{i\ln(iax+1)}{6c^2x^3}-\frac{a}{6c^2x^2}+\frac{a^3\ln(iax+1)}{8c^2(iax+1)}+\frac{5a^3\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{8c^2}+\frac{a^3}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^2/c^2-1/3/c^2*arct
an(a*x)/a^3/x^3+2/c^2*arctan(a*x)/a/x-1/6/c^2*(-3/2/(a^2*x^2+1)-7*ln(a^2*x^
2+1)+1/a^2/x^2+14*ln(a*x)+15/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.49, size = 160, normalized size = 1.18

$$\frac{1}{6}\left(\frac{15\,a^3\arctan\left(ax\right)}{c^2}+\frac{15\,a^4x^4+10\,a^2x^2-2}{a^2c^2x^5+c^2x^3}\right)\arctan\left(ax\right)+\frac{\left(a^2x^2-15\left(a^4x^4+a^2x^2\right)\arctan\left(ax\right)^2+14\left(a^4x^4+a^2x^2\right)\log\left(a^2x^2+1\right)-28\left(a^4x^4+a^2x^2\right)\log\left(x\right)-2\right)a}{12\left(a^2c^2x^4+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*arctan(a*x)/c^2 + (15*a^4*x^4 + 10*a^2*x^2 - 2)/(a^2*c^2*x^5 +
c^2*x^3))*arctan(a*x) + 1/12*(a^2*x^2 - 15*(a^4*x^4 + a^2*x^2)*arctan(a*x)^
2 + 14*(a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 28*(a^4*x^4 + a^2*x^2)*log(x)
 - 2)*a/(a^2*c^2*x^4 + c^2*x^2)
```

Fricas [A]
time = 1.73, size = 127, normalized size = 0.93

$$\frac{a^3x^3+15\left(a^5x^5+a^3x^3\right)\arctan\left(ax\right)^2-2\,ax+2\left(15\,a^4x^4+10\,a^2x^2-2\right)\arctan\left(ax\right)+14\left(a^5x^5+a^3x^3\right)\log\left(a^2x^2+1\right)-28\left(a^5x^5+a^3x^3\right)\log\left(x\right)}{12\left(a^2c^2x^5+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*x^3 + 15*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 - 2*a*x + 2*(15*a^4*x^
4 + 10*a^2*x^2 - 2)*arctan(a*x) + 14*(a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) -
 28*(a^5*x^5 + a^3*x^3)*log(x))/(a^2*c^2*x^5 + c^2*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(129) = 258$.
time = 1.18, size = 362, normalized size = 2.66

$$\begin{cases} -\frac{28a^5x^5\log(x)}{12a^2c^2x^5+12c^2x^3} + \frac{14a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{12a^2c^2x^5+12c^2x^3} + \frac{15a^5x^5\operatorname{atan}^2(ax)}{12a^2c^2x^5+12c^2x^3} + \frac{30a^4x^4\operatorname{atan}(ax)}{12a^2c^2x^5+12c^2x^3} - \frac{28a^3x^3\log(x)}{12a^2c^2x^5+12c^2x^3} + \frac{14a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{12a^2c^2x^5+12c^2x^3} + \frac{15a^3x^3\operatorname{atan}^2(ax)}{12a^2c^2x^5+12c^2x^3} + \frac{a^3x^3}{12a^2c^2x^5+12c^2x^3} + \frac{20a^2x^2\operatorname{atan}(ax)}{12a^2c^2x^5+12c^2x^3} - \frac{2ax}{12a^2c^2x^5+12c^2x^3} - \frac{4\operatorname{atan}(ax)}{12a^2c^2x^5+12c^2x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-28*a**5*x**5*log(x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 14*a**
5*x**5*log(x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 15*a**5*x**
5*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 30*a**4*x**4*atan(a*x)/
(12*a**2*c**2*x**5 + 12*c**2*x**3) - 28*a**3*x**3*log(x)/(12*a**2*c**2*x**5
 + 12*c**2*x**3) + 14*a**3*x**3*log(x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12
*c**2*x**3) + 15*a**3*x**3*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*x**3)
+ a**3*x**3/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 20*a**2*x**2*atan(a*x)/(12
*a**2*c**2*x**5 + 12*c**2*x**3) - 2*a*x/(12*a**2*c**2*x**5 + 12*c**2*x**3)
- 4*atan(a*x)/(12*a**2*c**2*x**5 + 12*c**2*x**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.55, size = 123, normalized size = 0.90

$$\frac{\operatorname{atan}(a\,x)\,\left(\frac{5\,x^2}{3\,c^2}-\frac{1}{3\,a^2\,c^2}+\frac{5\,a^2\,x^4}{2\,c^2}\right)}{x^5+\frac{x^3}{a^2}}-\frac{a-\frac{a^3\,x^2}{2}}{6\,a^2\,c^2\,x^4+6\,c^2\,x^2}+\frac{7\,a^3\,\ln\left(a^2\,x^2+1\right)}{6\,c^2}-\frac{7\,a^3\,\ln\left(x\right)}{3\,c^2}+\frac{5\,a^3\,\operatorname{atan}(a\,x)^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^2),x)
```

```
[Out] (atan(a*x)*((5*x^2)/(3*c^2) - 1/(3*a^2*c^2) + (5*a^2*x^4)/(2*c^2)))/(x^5 +
x^3/a^2) - (a - (a^3*x^2)/2)/(6*c^2*x^2 + 6*a^2*c^2*x^4) + (7*a^3*log(a^2*x
^2 + 1))/(6*c^2) - (7*a^3*log(x))/(3*c^2) + (5*a^3*atan(a*x)^2)/(4*c^2)
```

3.192 $\int \frac{x^3 \text{ArcTan}(ax)}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=86

$$\frac{x^3}{16ac^3(1+a^2x^2)^2}+\frac{3x}{32a^3c^3(1+a^2x^2)}-\frac{3\text{ArcTan}(ax)}{32a^4c^3}+\frac{x^4\text{ArcTan}(ax)}{4c^3(1+a^2x^2)^2}$$

```
[Out] 1/16*x^3/a/c^3/(a^2*x^2+1)^2+3/32*x/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)/a^
4/c^3+1/4*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5064, 294, 211}

$$-\frac{3\text{ArcTan}(ax)}{32a^4c^3}+\frac{x^4\text{ArcTan}(ax)}{4c^3(a^2x^2+1)^2}+\frac{x^3}{16ac^3(a^2x^2+1)^2}+\frac{3x}{32a^3c^3(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] x^3/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a^3*c^3*(1 + a^2*x^2)) - (3*ArcT
an[a*x])/(32*a^4*c^3) + (x^4*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx = \frac{x^4 \tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} - \frac{1}{4}a\int \frac{x^4}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{x^3}{16ac^3(1+a^2x^2)^2} + \frac{x^4 \tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} - \frac{3\int \frac{x^2}{(c+a^2cx^2)^2}\,dx}{16ac}$$
$$= \frac{x^3}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32a^3c^3(1+a^2x^2)} + \frac{x^4 \tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} - \frac{3\int \frac{1}{c+a^2cx^2}\,dx}{32a^3c^2}$$
$$= \frac{x^3}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32a^3c^3(1+a^2x^2)} - \frac{3\tan^{-1}(ax)}{32a^4c^3} + \frac{x^4 \tan^{-1}(ax)}{4c^3(1+a^2x^2)^2}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 0.67

$$\frac{ax(3+5a^2x^2)+(-3-6a^2x^2+5a^4x^4)\,\mathrm{ArcTan}(ax)}{32a^4c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a*x*(3 + 5*a^2*x^2) + (-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^
3*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.13, size = 84, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{(a^2x^2+1)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$
default	$\frac{-\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{(a^2x^2+1)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$
risch	$\frac{i(2a^2x^2+1)\ln(iax+1)}{8a^4c^3(a^2x^2+1)^2} - \frac{i(16a^2x^2\ln(-iax+1)+8\ln(-iax+1)+5\ln(-ax+i)a^4x^4+10\ln(-ax+i)a^2x^2+5\ln(-ax+i)-5}{64a^4(ax+i)^2(ax-i)^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/2*arctan(a*x)/c^3/(a^2*x^2+1)+1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2-1
/4/c^3*(-(5/8*a^3*x^3+3/8*a*x)/(a^2*x^2+1)^2-5/8*arctan(a*x)))
```

Maxima [A]
time = 0.46, size = 108, normalized size = 1.26

$$\frac{1}{32}\,a\left(\frac{5\,a^2x^3+3\,x}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\frac{5\,\arctan\left(ax\right)}{a^5c^3}\right)-\frac{\left(2\,a^2x^2+1\right)\arctan\left(ax\right)}{4\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3)) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^
3*x^2 + a^4*c^3)
```

Fricas [A]
time = 1.28, size = 69, normalized size = 0.80

$$\frac{5\,a^3x^3+3\,ax+\left(5\,a^4x^4-6\,a^2x^2-3\right)\arctan\left(ax\right)}{32\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*a^3*x^3 + 3*a*x + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x))/(a^8*c^3
*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(78) = 156$.
time = 0.73, size = 209, normalized size = 2.43

$$\begin{cases} \frac{5a^4x^4\operatorname{atan}\left(ax\right)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}+\frac{5a^3x^3}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}-\frac{6a^2x^2\operatorname{atan}\left(ax\right)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}+\frac{3ax}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3}-\frac{3\operatorname{atan}\left(ax\right)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((5*a**4*x**4*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 3
2*a**4*c**3) + 5*a**3*x**3/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4
*c**3) - 6*a**2*x**2*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*
a**4*c**3) + 3*a*x/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3) -
 3*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3), Ne(a,
0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.50, size = 62, normalized size = 0.72

$$\frac{3\,a\,x-3\,\mathrm{atan}(a\,x)+5\,a^3\,x^3-6\,a^2\,x^2\,\mathrm{atan}(a\,x)+5\,a^4\,x^4\,\mathrm{atan}(a\,x)}{32\,a^4\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (3*a*x - 3*atan(a*x) + 5*a^3*x^3 - 6*a^2*x^2*atan(a*x) + 5*a^4*x^4*atan(a*x
))/(32*a^4*c^3*(a^2*x^2 + 1)^2)
```

3.193 $\int \frac{x^2 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=111

$$-\frac{1}{16a^3c^3\left(1+a^2x^2\right)^2}+\frac{1}{16a^3c^3\left(1+a^2x^2\right)}-\frac{x\text{ArcTan}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{x\text{ArcTan}(ax)}{8a^2c^3\left(1+a^2x^2\right)}+\frac{\text{ArcTan}(ax)^2}{16a^3c^3}$$

```
[Out] -1/16/a^3/c^3/(a^2*x^2+1)^2+1/16/a^3/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)/a^2/
c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)+1/16*arctan(a*x)^2/
a^3/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5054, 5012, 267}

$$\frac{\text{ArcTan}(ax)^2}{16a^3c^3}+\frac{x\text{ArcTan}(ax)}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\text{ArcTan}(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{1}{16a^3c^3\left(a^2x^2+1\right)}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/16*1/(a^3*c^3*(1 + a^2*x^2)^2) + 1/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTa
n[a*x])/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(8*a^2*c^3*(1 + a^2*x
^2)) + ArcTan[a*x]^2/(16*a^3*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
```

```
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx = -\frac{1}{16a^3c^3(1+a^2x^2)^2} - \frac{x\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2} + \frac{\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{4a^2c}$$
$$= -\frac{1}{16a^3c^3(1+a^2x^2)^2} - \frac{x\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)}{8a^2c^3(1+a^2x^2)} + \frac{\tan^{-1}(ax)^2}{16a^3c^3} - \frac{\int \frac{x}{(c+a^2cx}}{8ac}$$
$$= -\frac{1}{16a^3c^3(1+a^2x^2)^2} + \frac{1}{16a^3c^3(1+a^2x^2)} - \frac{x\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)}{8a^2c^3(1+a^2x^2)} + \frac{\text{ta}}{}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.58

$$\frac{a^2x^2 + 2ax(-1+a^2x^2)\,\mathrm{ArcTan}(ax) + (1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^2}{16a^3c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(-1 + a^2*x^2)*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*x]^2
)/(16*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.16, size = 105, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{ax\arctan(ax)}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^2}{8c^3} - \frac{\frac{\arctan(ax)^2}{2} - \frac{1}{2(a^2x^2+1)} + \frac{1}{2(a^2x^2+1)^2}}{8c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{ax\arctan(ax)}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^2}{8c^3} - \frac{\frac{\arctan(ax)^2}{2} - \frac{1}{2(a^2x^2+1)} + \frac{1}{2(a^2x^2+1)^2}}{8c^3}}{a^3}$
risch	$-\frac{\ln(iax+1)^2}{64c^3a^3} + \frac{(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)}{32a^3c^3(a^2x^2+1)^2} - \frac{a^4x^4\ln(-iax+1)^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/c^3*arctan(a*x)*a^3*x^3/(a^2*x^2+1)^2-1/8*a*x*arctan(a*x)/c^3/(a
^2*x^2+1)^2+1/8*arctan(a*x)^2/c^3-1/8/c^3*(1/2*arctan(a*x)^2-1/2/(a^2*x^2+1
)+1/2/(a^2*x^2+1)^2))
```

Maxima [A]
time = 0.47, size = 129, normalized size = 1.16

$$\frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan(ax)}{a^3c^3}\right)\arctan(ax)+\frac{\left(a^2x^2-(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\right)a}{16\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x) + 1/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x
)^2)*a/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A]
time = 1.41, size = 83, normalized size = 0.75

$$\frac{a^2x^2+(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2+2\,(a^3x^3-ax)\arctan(ax)}{16\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(a^2*x^2 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(a^3*x^3 - a*x)
*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\,\mathrm{atan}\,(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.47, size = 80, normalized size = 0.72

$$\frac{a^4\,x^4\,\mathrm{atan}(a\,x)^2+2\,a^3\,x^3\,\mathrm{atan}(a\,x)+2\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+a^2\,x^2-2\,a\,x\,\mathrm{atan}(a\,x)+\mathrm{atan}(a\,x)^2}{16\,a^3\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (a^2*x^2 + atan(a*x)^2 + 2*a^3*x^3*atan(a*x) - 2*a*x*atan(a*x) + 2*a^2*x^2*
atan(a*x)^2 + a^4*x^4*atan(a*x)^2)/(16*a^3*c^3*(a^2*x^2 + 1)^2)
```

3.194 $\int \frac{x\textbf{ArcTan}(ax)}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{x}{16ac^3\left(1+a^2x^2\right)^2}+\frac{3x}{32ac^3\left(1+a^2x^2\right)}+\frac{3\text{ArcTan}(ax)}{32a^2c^3}-\frac{\text{ArcTan}(ax)}{4a^2c^3\left(1+a^2x^2\right)^2}$$

```
[Out] 1/16*x/a/c^3/(a^2*x^2+1)^2+3/32*x/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)/a^2/c^
3-1/4*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5050, 205, 211}

$$-\frac{\text{ArcTan}(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\text{ArcTan}(ax)}{32a^2c^3}+\frac{3x}{32ac^3\left(a^2x^2+1\right)}+\frac{x}{16ac^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] x/(16*a*c^3*(1 + a^2*x^2)^2) + (3*x)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a
*x])/(32*a^2*c^3) - ArcTan[a*x]/(4*a^2*c^3*(1 + a^2*x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx &= -\frac{\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2}+\frac{\int \frac{1}{(c+a^2cx^2)^3}\,dx}{4a}\\
&= \frac{x}{16ac^3(1+a^2x^2)^2}-\frac{\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2}+\frac{3\int \frac{1}{(c+a^2cx^2)^2}\,dx}{16ac}\\
&= \frac{x}{16ac^3(1+a^2x^2)^2}+\frac{3x}{32ac^3(1+a^2x^2)}-\frac{\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2}+\frac{3\int \frac{1}{c+a^2cx^2}\,dx}{32ac^2}\\
&= \frac{x}{16ac^3(1+a^2x^2)^2}+\frac{3x}{32ac^3(1+a^2x^2)}+\frac{3\tan^{-1}(ax)}{32a^2c^3}-\frac{\tan^{-1}(ax)}{4a^2c^3(1+a^2x^2)^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.65

$$\frac{ax(5+3a^2x^2)+(-5+6a^2x^2+3a^4x^4)\,\mathrm{ArcTan}(ax)}{32c^3(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (a*x*(5 + 3*a^2*x^2) + (-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x])/(32*c^3*(a
 + a^3*x^2)^2)
```

Maple [A]
time = 0.11, size = 68, normalized size = 0.81

method	result
derivativedivides	$\frac{-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax}{4(a^2x^2+1)^2}+\frac{3ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax}{4(a^2x^2+1)^2}+\frac{3ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$
risch	$\frac{i\ln(iax+1)}{8a^2c^3(a^2x^2+1)^2}-\frac{i(8\ln(-iax+1)+3\ln(-ax+i)a^4x^4+6\ln(-ax+i)a^2x^2+3\ln(-ax+i)-3\ln(ax+i)a^4x^4-6\ln(ax+i)a^2}{64a^2(ax+i)^2(ax-i)^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2+1/4/c^3*(1/4/(a^2*x^2+1)^2*a*x+3/
8*a*x/(a^2*x^2+1)+3/8*arctan(a*x)))
```

Maxima [A]
time = 0.47, size = 86, normalized size = 1.02

$$\frac{\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\,\arctan(ax)}{ac^2}}{32\,ac}-\frac{\arctan\left(ax\right)}{4\left(a^2cx^2+c\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))/(a*c) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A]
time = 0.99, size = 69, normalized size = 0.82

$$\frac{3\,a^3x^3+5\,ax+\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*a^3*x^3 + 5*a*x + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x))/(a^6*c^3
*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(75) = 150$.
time = 0.72, size = 209, normalized size = 2.49

$$\begin{cases} \frac{3a^4x^4\operatorname{atan}\left(ax\right)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{3a^3x^3}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{6a^2x^2\operatorname{atan}\left(ax\right)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}+\frac{5ax}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3}-\frac{5\operatorname{atan}\left(ax\right)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((3*a**4*x**4*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 3
2*a**2*c**3) + 3*a**3*x**3/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2
*c**3) + 6*a**2*x**2*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*
a**2*c**3) + 5*a*x/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3) -
 5*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3), Ne(a,
0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.48, size = 103, normalized size = 1.23

$$\frac{\frac{5\,x}{32\,a}+\frac{a\,x^3}{4}-\frac{\mathrm{atan}(a\,x)}{4\,a^2}-\frac{x^2\,\mathrm{atan}(a\,x)}{4}+\frac{3\,a^3\,x^5}{32}}{a^6\,c^3\,x^6+3\,a^4\,c^3\,x^4+3\,a^2\,c^3\,x^2+c^3}+\frac{3\,\mathrm{atan}\left(\frac{a^2\,x}{\sqrt{a^2}}\right)}{32\,a\,c^3\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] ((5*x)/(32*a) + (a*x^3)/4 - atan(a*x)/(4*a^2) - (x^2*atan(a*x))/4 + (3*a^3*
x^5)/32)/(c^3 + 3*a^2*c^3*x^2 + 3*a^4*c^3*x^4 + a^6*c^3*x^6) + (3*atan((a^2
*x)/(a^2)^(1/2)))/(32*a*c^3*(a^2)^(1/2))
```

3.195 $\int \frac{\text{ArcTan}(ax)}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=105

$$\frac{1}{16ac^3(1+a^2x^2)^2}+\frac{3}{16ac^3(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)}{4c^3(1+a^2x^2)^2}+\frac{3x\text{ArcTan}(ax)}{8c^3(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)^2}{16ac^3}$$

```
[Out] 1/16/a/c^3/(a^2*x^2+1)^2+3/16/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)/c^3/(a^2*
x^2+1)^2+3/8*x*arctan(a*x)/c^3/(a^2*x^2+1)+3/16*arctan(a*x)^2/a/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5016, 5012, 267}

$$\frac{3x\text{ArcTan}(ax)}{8c^3(a^2x^2+1)}+\frac{x\text{ArcTan}(ax)}{4c^3(a^2x^2+1)^2}+\frac{3}{16ac^3(a^2x^2+1)}+\frac{1}{16ac^3(a^2x^2+1)^2}+\frac{3\text{ArcTan}(ax)^2}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] 1/(16*a*c^3*(1 + a^2*x^2)^2) + 3/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x])
/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)) + (3*Arc
Tan[a*x]^2)/(16*a*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
```

```
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \frac{3\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{4c}$$
$$= \frac{1}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)}{8c^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^2}{16ac^3} - \frac{(3a)\int \frac{x}{(c+a^2cx^2)}}{8c}$$
$$= \frac{1}{16ac^3(1+a^2x^2)^2} + \frac{3}{16ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)}{8c^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax}{16ac^3}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 0.65

$$\frac{4+3a^2x^2+2ax(5+3a^2x^2)\,\mathrm{ArcTan}(ax)+3(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^2}{16ac^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcT
an[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.15, size = 101, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{ax\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{3}{2(a^2x^2+1)}-\frac{1}{2(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$
default	$\frac{\frac{ax\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{3}{2(a^2x^2+1)}-\frac{1}{2(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$
risch	$-\frac{3\ln(iax+1)^2}{64a\,c^3}+\frac{(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)}{32c^3(a^2x^2+1)^2a}-\frac{3a^4x^4\ln(-ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/4*a*x*arctan(a*x)/c^3/(a^2*x^2+1)^2+3/8*a*x*arctan(a*x)/c^3/(a^2*x^2
+1)+3/8*arctan(a*x)^2/c^3-1/8/c^3*(-3/2/(a^2*x^2+1)-1/2/(a^2*x^2+1)^2+3/2*a
rctan(a*x)^2))
```

Maxima [A]
time = 0.47, size = 129, normalized size = 1.23

$$\frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right)+\frac{\left(3\,a^2x^2-3\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4\right)a}{16\,\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x) + 1/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan
(a*x)^2 + 4)*a/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Fricas [A]
time = 1.11, size = 85, normalized size = 0.81

$$\frac{3\,a^2x^2+3\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+2\,\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)+4}{16\,\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*a^2*x^2 + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(3*a^3*x^3
+ 5*a*x)*arctan(a*x) + 4)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded while
calling a Python object
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.48, size = 85, normalized size = 0.81

$$\frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)+3\,\mathrm{atan}(a\,x)^2+4}{16\,a\,c^3\,\left(a^2\,x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (3*a^2*x^2 + 3*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^2
*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(16*a*c^3*(a^2*x^2 + 1)^2)
```

3.196 $\int \frac{\text{ArcTan}(ax)}{x(c+a^2cx^2)^3} dx$

Optimal. Leaf size=159

$$-\frac{ax}{16c^3(1+a^2x^2)^2}-\frac{11ax}{32c^3(1+a^2x^2)}-\frac{11\text{ArcTan}(ax)}{32c^3}+\frac{\text{ArcTan}(ax)}{4c^3(1+a^2x^2)^2}+\frac{\text{ArcTan}(ax)}{2c^3(1+a^2x^2)}-\frac{i\text{ArcTan}(ax)^2}{2c^3}+\frac{\text{Ar}}{}$$

```
[Out] -1/16*a*x/c^3/(a^2*x^2+1)^2-11/32*a*x/c^3/(a^2*x^2+1)-11/32*arctan(a*x)/c^3
+1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2*I*ar
ctan(a*x)^2/c^3+arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-1/2*I*polylog(2,-1+2/(1-I
*a*x))/c^3
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5086, 5044, 4988, 2497, 5050, 205, 211}

$$\frac{\text{ArcTan}(ax)}{2c^3(a^2x^2+1)}+\frac{\text{ArcTan}(ax)}{4c^3(a^2x^2+1)^2}-\frac{11ax}{32c^3(a^2x^2+1)}-\frac{ax}{16c^3(a^2x^2+1)^2}-\frac{i\text{ArcTan}(ax)^2}{2c^3}-\frac{11\text{ArcTan}(ax)}{32c^3}+\frac{\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{i\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/16*(a*x)/(c^3*(1 + a^2*x^2)^2) - (11*a*x)/(32*c^3*(1 + a^2*x^2)) - (11*A
rcTan[a*x])/(32*c^3) + ArcTan[a*x]/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(2
*c^3*(1 + a^2*x^2)) - ((I/2)*ArcTan[a*x]^2)/c^3 + (ArcTan[a*x]*Log[2 - 2/(1
 - I*a*x)])/c^3 - ((I/2)*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{4}a\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(ax)^2}{2c^3} + \frac{i\int \frac{\tan^{-1}(ax)}{x(i+ax)}}{c^3}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} - \frac{11ax}{32c^3\left(1+a^2x^2\right)} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(}{2c^3}$$

$$= -\frac{ax}{16c^3\left(1+a^2x^2\right)^2} - \frac{11ax}{32c^3\left(1+a^2x^2\right)} - \frac{11\tan^{-1}(ax)}{32c^3} + \frac{\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(}{2c^3\left(1+\right.}$$

Mathematica [A]
time = 0.14, size = 90, normalized size = 0.57

$$-\frac{64i\text{ArcTan}(ax)^2-4\text{ArcTan}(ax)\left(12\cos(2\text{ArcTan}(ax))+\cos(4\text{ArcTan}(ax))+32\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+64i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+24\sin(2\text{ArcTan}(ax))+\sin(4\text{ArcTan}(ax))}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/128*((64*I)*ArcTan[a*x]^2 - 4*ArcTan[a*x]*(12*Cos[2*ArcTan[a*x]] + Cos[4
*ArcTan[a*x]] + 32*Log[1 - E^((2*I)*ArcTan[a*x])]) + (64*I)*PolyLog[2, E^((
2*I)*ArcTan[a*x])] + 24*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]])/c^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(141) = 282.
time = 0.08, size = 292, normalized size = 1.84

method	result
derivativedivides	$\frac{\arctan(ax)}{2c^3(a^2x^2+1)} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3} + \frac{\arctan(ax)}{4c^3(a^2x^2+1)^2} + \frac{\arctan(ax)\ln(ax)}{c^3} - \frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2} + \frac{11\arctan(ax)}{8} - 2}{}$
default	$\frac{\arctan(ax)}{2c^3(a^2x^2+1)} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3} + \frac{\arctan(ax)}{4c^3(a^2x^2+1)^2} + \frac{\arctan(ax)\ln(ax)}{c^3} - \frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2} + \frac{11\arctan(ax)}{8} - 2}{}$
risch	$-\frac{11\arctan(ax)}{64c^3} + \frac{i\ln(-iax+1)x^2a^2}{128c^3(-iax-1)^2} + \frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c^3} - \frac{i\ln(iax+1)}{32c^3(iax+1)^2} - \frac{3i\ln(iax+1)}{128c^3(iax-1)^2} + \frac{5i\ln(iax}{64c^3(iax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2/c^3*arctan(a*x)*ln(a^2*x^2+1)+1/4*arcta
n(a*x)/c^3/(a^2*x^2+1)^2+1/c^3*arctan(a*x)*ln(a*x)-1/4/c^3*((11/8*a^3*x^3+1
3/8*a*x)/(a^2*x^2+1)^2+11/8*arctan(a*x)-2*I*ln(a*x)*ln(1+I*a*x)+2*I*ln(a*x)
*ln(1-I*a*x)-2*I*dilog(1+I*a*x)+2*I*dilog(1-I*a*x)+I*ln(a*x-I)*ln(a^2*x^2+1
)-1/2*I*ln(a*x-I)^2-I*dilog(-1/2*I*(I+a*x))-I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-
I*ln(I+a*x)*ln(a^2*x^2+1)+1/2*I*ln(I+a*x)^2+I*dilog(1/2*I*(a*x-I))+I*ln(I+a
*x)*ln(1/2*I*(a*x-I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x),
 x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)^3), x)
```

3.197 $\int \frac{\mathbf{ArcTan}(ax)}{x^2(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\mathrm{ArcTan}(ax)}{c^3x}-\frac{a^2x\mathrm{ArcTan}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\mathrm{ArcTan}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\mathrm{ArcTan}(ax)^2}{16c^3}+$$

```
[Out] -1/16*a/c^3/(a^2*x^2+1)^2-7/16*a/c^3/(a^2*x^2+1)-arctan(a*x)/c^3/x-1/4*a^2*
x*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)-15/16
*a*arctan(a*x)^2/c^3+a*ln(x)/c^3-1/2*a*ln(a^2*x^2+1)/c^3
```

Rubi [A]
time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$,
Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5012, 267, 5016}

$$-\frac{7a^2x\mathrm{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)}-\frac{a^2x\mathrm{ArcTan}(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{7a}{16c^3\left(a^2x^2+1\right)}-\frac{a}{16c^3\left(a^2x^2+1\right)^2}-\frac{a\log\left(a^2x^2+1\right)}{2c^3}-\frac{15a\mathrm{ArcTan}(ax)^2}{16c^3}-\frac{\mathrm{ArcTan}(ax)}{c^3x}+\frac{a\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/16*a/(c^3*(1 + a^2*x^2)^2) - (7*a)/(16*c^3*(1 + a^2*x^2)) - ArcTan[a*x]/
(c^3*x) - (a^2*x*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a*x
])/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a*x]^2)/(16*c^3) + (a*Log[x])/c^3 -
 (a*Log[1 + a^2*x^2])/(2*c^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)}\,dx}{c^2}-\frac{\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx}{4c}-\frac{a^2}{}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{7a\tan^{-1}(ax)^2}{16c^3}+\frac{\int \frac{\tan^{-1}(a}{x^2}}{c^3}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(}{8c^3\left(1+a^2x\right.}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(}{8c^3\left(1+a^2x\right.}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(}{8c^3\left(1+a^2x\right.}$$

$$= -\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{c^3x}-\frac{a^2x\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\tan^{-1}(}{8c^3\left(1+a^2x\right.}$$

Mathematica [A]
time = 0.05, size = 118, normalized size = 0.83

$$\frac{-2\left(8+25a^2x^2+15a^4x^4\right)\mathrm{ArcTan}(ax)-15ax\left(1+a^2x^2\right)^2\mathrm{ArcTan}(ax)^2+ax\left(-8-7a^2x^2+16\left(1+a^2x^2\right)^2\log(x)-8\left(1+a^2x^2\right)^2\log\left(1+a^2x^2\right)\right)}{16c^3x\left(1+a^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-2*(8 + 25*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] - 15*a*x*(1 + a^2*x^2)^2*ArcT
an[a*x]^2 + a*x*(-8 - 7*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[x] - 8*(1 + a^2*x^
2)^2*Log[1 + a^2*x^2]))/(16*c^3*x*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.96

method	result
derivativedivides	$a\left(-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3ax}-\frac{-\frac{15\arctan(ax)^2}{2}+\frac{7}{2(a^2x^2+1)}+4\ln(}{8c}\right.$
default	$a\left(-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3ax}-\frac{-\frac{15\arctan(ax)^2}{2}+\frac{7}{2(a^2x^2+1)}+4\ln(}{8c}\right.$
risch	$\frac{15a\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{32c^3}+\frac{15a\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{32c^3}+\frac{a}{64c^3(iax-1)}-\frac{a}{64c^3(iax+1)^2}+\frac{a\ln(iax)}{2c^3}+\frac{15a\ln(iax+1)^2}{64c^3}-\frac{}{32c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-7/8/c^3*arctan(a*x)*a^3*x^3/(a^2*x^2+1)^2-9/8*a*x*arctan(a*x)/c^3/(a^2*
x^2+1)^2-15/8*arctan(a*x)^2/c^3-1/c^3*arctan(a*x)/a/x-1/8/c^3*(-15/2*arctan
(a*x)^2+7/2/(a^2*x^2+1)+4*ln(a^2*x^2+1)+1/2/(a^2*x^2+1)^2-8*ln(a*x)))
```

Maxima [A]

time = 0.48, size = 181, normalized size = 1.27

$$-\frac{1}{8}\left(\frac{15\,a^4x^4+25\,a^2x^2+8}{a^4c^3x^5+2\,a^2c^3x^3+c^3x}+\frac{15\,a\arctan\left(ax\right)}{c^3}\right)\arctan\left(ax\right)-\frac{\left(7\,a^2x^2-15\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+8\,\left(a^4x^4+2\,a^2x^2+1\right)\log\left(a^2x^2+1\right)-16\,\left(a^4x^4+2\,a^2x^2+1\right)\log\left(x\right)+8\right)a}{16\,\left(a^4c^3x^4+2\,a^2c^3x^2+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^4*x^4 + 25*a^2*x^2 + 8)/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x) +
 15*a*arctan(a*x)/c^3)*arctan(a*x) - 1/16*(7*a^2*x^2 - 15*(a^4*x^4 + 2*a^2*
x^2 + 1)*arctan(a*x)^2 + 8*(a^4*x^4 + 2*a^2*x^2 + 1)*log(a^2*x^2 + 1) - 16*
(a^4*x^4 + 2*a^2*x^2 + 1)*log(x) + 8)*a/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3)
```

Fricas [A]

time = 1.12, size = 149, normalized size = 1.05

$$-\frac{7\,a^3x^3+15\,\left(a^5x^5+2\,a^3x^3+ax\right)\arctan\left(ax\right)^2+8\,ax+2\,\left(15\,a^4x^4+25\,a^2x^2+8\right)\arctan\left(ax\right)+8\,\left(a^5x^5+2\,a^3x^3+ax\right)\log\left(a^2x^2+1\right)-16\,\left(a^5x^5+2\,a^3x^3+ax\right)\log\left(x\right)}{16\,\left(a^4c^3x^5+2\,a^2c^3x^3+c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(7*a^3*x^3 + 15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 8*a*x + 2
*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arctan(a*x) + 8*(a^5*x^5 + 2*a^3*x^3 + a*x)*
log(a^2*x^2 + 1) - 16*(a^5*x^5 + 2*a^3*x^3 + a*x)*log(x))/(a^4*c^3*x^5 + 2*
a^2*c^3*x^3 + c^3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(134) = 268$.
time = 1.50, size = 604, normalized size = 4.25

$$\begin{cases} \frac{16a^5x^5\log(x)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{8a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{15a^5x^5\operatorname{atan}^2(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{30a^4x^4\operatorname{atan}(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} + \frac{32a^3x^3\log(x)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{16a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{30a^3x^3\operatorname{atan}^2(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{7a^3x^3}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{50a^2x^2\operatorname{atan}(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} + \frac{16ax\log(x)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{8ax\log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{15ax\operatorname{atan}^2(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{8ax}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{16\operatorname{atan}(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((16*a**5*x**5*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*
c**3*x) - 8*a**5*x**5*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a**2*c**3
*x**3 + 16*c**3*x) - 15*a**5*x**5*atan(a*x)**2/(16*a**4*c**3*x**5 + 32*a**2
*c**3*x**3 + 16*c**3*x) - 30*a**4*x**4*atan(a*x)/(16*a**4*c**3*x**5 + 32*a*
*2*c**3*x**3 + 16*c**3*x) + 32*a**3*x**3*log(x)/(16*a**4*c**3*x**5 + 32*a**
2*c**3*x**3 + 16*c**3*x) - 16*a**3*x**3*log(x**2 + a**(-2))/(16*a**4*c**3*x
**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 30*a**3*x**3*atan(a*x)**2/(16*a**4*c
**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 7*a**3*x**3/(16*a**4*c**3*x**5
+ 32*a**2*c**3*x**3 + 16*c**3*x) - 50*a**2*x**2*atan(a*x)/(16*a**4*c**3*x**
5 + 32*a**2*c**3*x**3 + 16*c**3*x) + 16*a*x*log(x)/(16*a**4*c**3*x**5 + 32*
a**2*c**3*x**3 + 16*c**3*x) - 8*a*x*log(x**2 + a**(-2))/(16*a**4*c**3*x**5
+ 32*a**2*c**3*x**3 + 16*c**3*x) - 15*a*x*atan(a*x)**2/(16*a**4*c**3*x**5 +
 32*a**2*c**3*x**3 + 16*c**3*x) - 8*a*x/(16*a**4*c**3*x**5 + 32*a**2*c**3*x
**3 + 16*c**3*x) - 16*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16
*c**3*x), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.57, size = 133, normalized size = 0.94

$$\frac{a\,\ln(x)}{c^3} - \frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c^3} - \frac{\frac{7\,a^3\,x^2}{2}+4\,a}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3} - \frac{\operatorname{atan}(a\,x)\,\left(\frac{1}{a^2\,c^3}+\frac{25\,x^2}{8\,c^3}+\frac{15\,a^2\,x^4}{8\,c^3}\right)}{\frac{x}{a^2}+2\,x^3+a^2\,x^5} - \frac{15\,a\,\operatorname{atan}(a\,x)^2}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] (a*log(x))/c^3 - (a*log(a^2*x^2 + 1))/(2*c^3) - (4*a + (7*a^3*x^2)/2)/(8*c^
3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)*(1/(a^2*c^3) + (25*x^2)/(8
*c^3) + (15*a^2*x^4)/(8*c^3)))/(x/a^2 + 2*x^3 + a^2*x^5) - (15*a*atan(a*x)^
2)/(16*c^3)
```

3.198 $\int \frac{\text{ArcTan}(ax)}{x^3(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{a}{2c^3x}+\frac{a^3x}{16c^3\left(1+a^2x^2\right)^2}+\frac{19a^3x}{32c^3\left(1+a^2x^2\right)}+\frac{3a^2\text{ArcTan}(ax)}{32c^3}-\frac{\text{ArcTan}(ax)}{2c^3x^2}-\frac{a^2\text{ArcTan}(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{a^2\text{ArcTan}(ax}{c^3\left(1+a^2x^2\right)}$$

```
[Out] -1/2*a/c^3/x+1/16*a^3*x/c^3/(a^2*x^2+1)^2+19/32*a^3*x/c^3/(a^2*x^2+1)+3/32*
a^2*arctan(a*x)/c^3-1/2*arctan(a*x)/c^3/x^2-1/4*a^2*arctan(a*x)/c^3/(a^2*x^
2+1)^2-a^2*arctan(a*x)/c^3/(a^2*x^2+1)+3/2*I*a^2*arctan(a*x)^2/c^3-3*a^2*ar
ctan(a*x)*ln(2-2/(1-I*a*x))/c^3+3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 0.55, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5086, 5038, 4946, 331, 209, 5044, 4988, 2497, 5050, 205, 211}

$$-\frac{a^2\text{ArcTan}(ax)}{c^3\left(a^2x^2+1\right)}-\frac{a^2\text{ArcTan}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3ia^2\text{ArcTan}(ax)^2}{2c^3}+\frac{3a^2\text{ArcTan}(ax)}{32c^3}-\frac{3a^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{3ia^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^3}+\frac{19a^3x}{32c^3\left(a^2x^2+1\right)}+\frac{a^3x}{16c^3\left(a^2x^2+1\right)^2}-\frac{\text{ArcTan}(ax)}{2c^3x^2}-\frac{a}{2c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/2*a/(c^3*x) + (a^3*x)/(16*c^3*(1 + a^2*x^2)^2) + (19*a^3*x)/(32*c^3*(1 +
 a^2*x^2)) + (3*a^2*ArcTan[a*x])/(32*c^3) - ArcTan[a*x]/(2*c^3*x^2) - (a^2*
ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x])/(c^3*(1 + a^2*x^2)
) + (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^3 - (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I
*a*x)])/c^3 + (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^3}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= a^4\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^3\left(c+a^2cx^2\right)}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= -\frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{1}{4}a^3\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{c^3}-\frac{a^2\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)}\,dx}{c^2}-2\\
&= \frac{a^3x}{16c^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)}{2c^3x^2}-\frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{ia^2\tan^{-1}(ax)^2}{2c^3}+\frac{a\int \frac{1}{x^2\left(1+a^2x^2\right)}}{2c^3}\\
&= -\frac{a}{2c^3x}+\frac{a^3x}{16c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x}{32c^3\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)}{2c^3x^2}-\frac{a^2\tan^{-1}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{ia^2}{}\\
&= -\frac{a}{2c^3x}+\frac{a^3x}{16c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x}{32c^3\left(1+a^2x^2\right)}-\frac{13a^2\tan^{-1}(ax)}{32c^3}-\frac{\tan^{-1}(ax)}{2c^3x^2}-\frac{a^2}{4c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 111, normalized size = 0.54

$$
\frac{a^2\left(-\frac{64}{ax}+192i\text{ArcTan}(ax)^2+\text{ArcTan}(ax)\left(-64-\frac{64}{a^2x^2}-80\cos(2\text{ArcTan}(ax))-4\cos(4\text{ArcTan}(ax))-384\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+192i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+40\sin(2\text{ArcTan}(ax))+\sin(4\text{ArcTan}(ax))\right)}{128c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

[Out] (a^2*(-64/(a*x) + (192*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-64 - 64/(a^2*x^2) - 80*Cos[2*ArcTan[a*x]] - 4*Cos[4*ArcTan[a*x]] - 384*Log[1 - E^((2*I)*ArcTan[a*x])]) + (192*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 40*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]]))/(128*c^3)

Maple [A]
time = 0.09, size = 320, normalized size = 1.56

method	result
derivativedivides	$a^2\left(-\frac{\arctan(ax)}{c^3(a^2x^2+1)}+\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{6i\ln(ax}{}\right.$
default	$a^2\left(-\frac{\arctan(ax)}{c^3(a^2x^2+1)}+\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{6i\ln(ax}{}\right.$
risch	$-\frac{a^3\ln(iax+1)x}{64c^3(iax-1)^2}+\frac{9a^3\ln(iax+1)x}{64c^3(iax-1)}-\frac{9ia^2\ln(-iax+1)}{32c^3(-iax+1)}+\frac{3ia^2\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c^3}-\frac{a^3\ln(-iax+1)x}{64c^3(-iax-1)^2}-\frac{a}{2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] a^2*(-arctan(a*x)/c^3/(a^2*x^2+1)+3/2/c^3*arctan(a*x)*ln(a^2*x^2+1)-1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2-1/2/c^3*arctan(a*x)/a^2/x^2-3/c^3*arctan(a*x)*ln(a*x)-1/4/c^3*(6*I*ln(a*x)*ln(1+I*a*x)-6*I*ln(a*x)*ln(1-I*a*x)+6*I*dilog(1+I*a*x)-6*I*dilog(1-I*a*x)-3*I*ln(a*x-I)*ln(a^2*x^2+1)+3*I*dilog(-1/2*I*(I+a*x))+3*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+3/2*I*ln(a*x-I)^2+3*I*ln(I+a*x)*ln(a^2*x^2+1)-3*I*dilog(1/2*I*(a*x-I))-3*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-3/2*I*ln(I+a*x)^2+(-19/8*a^3*x^3-21/8*a*x)/(a^2*x^2+1)^2-3/8*arctan(a*x)+2/a/x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{x^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3), x)
```

3.199 $\int \frac{\textbf{ArcTan}(ax)}{x^4(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=183

$$-\frac{a}{6c^3x^2}+\frac{a^3}{16c^3\left(1+a^2x^2\right)^2}+\frac{11a^3}{16c^3\left(1+a^2x^2\right)}-\frac{\text{ArcTan}(ax)}{3c^3x^3}+\frac{3a^2\text{ArcTan}(ax)}{c^3x}+\frac{a^4x\text{ArcTan}(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{11a^4x\text{ArcTan}}{8c^3\left(1+a^2\right.}$$

```
[Out] -1/6*a/c^3/x^2+1/16*a^3/c^3/(a^2*x^2+1)^2+11/16*a^3/c^3/(a^2*x^2+1)-1/3*arc
tan(a*x)/c^3/x^3+3*a^2*arctan(a*x)/c^3/x+1/4*a^4*x*arctan(a*x)/c^3/(a^2*x^2
+1)^2+11/8*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)+35/16*a^3*arctan(a*x)^2/c^3-10
/3*a^3*ln(x)/c^3+5/3*a^3*ln(a^2*x^2+1)/c^3
```

Rubi [A]
time = 0.51, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$,
Rules used = {5086, 5038, 4946, 272, 46, 36, 29, 31, 5004, 5012, 267, 5016}

$$\frac{35a^3\text{ArcTan}(ax)^2}{16c^3}-\frac{10a^3\log(x)}{3c^3}+\frac{3a^2\text{ArcTan}(ax)}{c^3x}+\frac{11a^4x\text{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{a^4x\text{ArcTan}(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{11a^3}{16c^3\left(a^2x^2+1\right)}+\frac{a^3}{16c^3\left(a^2x^2+1\right)^2}+\frac{5a^3\log\left(a^2x^2+1\right)}{3c^3}-\frac{\text{ArcTan}(ax)}{3c^3x^3}-\frac{a}{6c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/6*a/(c^3*x^2) + a^3/(16*c^3*(1 + a^2*x^2)^2) + (11*a^3)/(16*c^3*(1 + a^2
*x^2)) - ArcTan[a*x]/(3*c^3*x^3) + (3*a^2*ArcTan[a*x])/(c^3*x) + (a^4*x*Arc
Tan[a*x])/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan[a*x])/(8*c^3*(1 + a^2*
x^2)) + (35*a^3*ArcTan[a*x]^2)/(16*c^3) - (10*a^3*Log[x])/(3*c^3) + (5*a^3*
Log[1 + a^2*x^2])/(3*c^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)^2}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x^4(c+a^2cx^2)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^2}\,dx}{c}$$

$$= \frac{a^3}{16c^3(1+a^2x^2)^2} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \frac{\int \frac{\tan^{-1}(ax)}{x^4}\,dx}{c^3} - \frac{a^2\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)}\,dx}{c^2} + \frac{(3a^4)\int \ldots}{\ldots}$$

$$= \frac{a^3}{16c^3(1+a^2x^2)^2} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \frac{3a^4x\tan^{-1}(ax)}{8c^3(1+a^2x^2)} + \frac{3a^3\tan^{-1}(ax)^2}{16c^3} \ldots$$

$$= \frac{a^3}{16c^3(1+a^2x^2)^2} + \frac{3a^3}{16c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \ldots$$

$$= \frac{a^3}{16c^3(1+a^2x^2)^2} + \frac{3a^3}{16c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1+a^2x^2)^2} + \ldots$$

$$= -\frac{a}{6c^3x^2} + \frac{a^3}{16c^3(1+a^2x^2)^2} + \frac{3a^3}{16c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1 \ldots}$$

$$= -\frac{a}{6c^3x^2} + \frac{a^3}{16c^3(1+a^2x^2)^2} + \frac{3a^3}{16c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{3c^3x^3} + \frac{a^2\tan^{-1}(ax)}{c^3x} + \frac{a^4x\tan^{-1}(ax)}{4c^3(1 \ldots}$$

Mathematica [A]

time = 0.06, size = 142, normalized size = 0.78

$$\frac{2(-8+56a^2x^2+175a^4x^4+105a^6x^6)\,\text{ArcTan}(ax)+105a^3x^3(1+a^2x^2)^2\,\text{ArcTan}(ax)^2+ax\left(-8+20a^2x^2+25a^4x^4-160(ax+a^3x^3)^2\log(x)+80(ax+a^3x^3)^2\log\left(1+a^2x^2\right)\right)}{48c^3x^3\left(1+a^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (2*(-8 + 56*a^2*x^2 + 175*a^4*x^4 + 105*a^6*x^6)*ArcTan[a*x] + 105*a^3*x^3*
(1 + a^2*x^2)^2*ArcTan[a*x]^2 + a*x*(-8 + 20*a^2*x^2 + 25*a^4*x^4 - 160*(a*
x + a^3*x^3)^2*Log[x] + 80*(a*x + a^3*x^3)^2*Log[1 + a^2*x^2]))/(48*c^3*x^3
*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.25, size = 161, normalized size = 0.88

method	result
derivativedivides	$a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-\frac{33}{2(a^2x^2+1)}-4(}{}\right.$
default	$a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-\frac{33}{2(a^2x^2+1)}-4(}{}\right.$
risch	$-\frac{35a^3\ln(iax+1)^2}{64c^3}+\frac{(105a^7x^7\ln(-iax+1)-210ix^6a^6+210a^5x^5\ln(-iax+1)-350ix^4a^4+105a^3x^3\ln(-iax+1)-112i}{96x^3c^3(a^2x^2+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(11/8/c^3*arctan(a*x)*a^3*x^3/(a^2*x^2+1)^2+13/8*a*x*arctan(a*x)/c^3/(a
^2*x^2+1)^2+35/8*arctan(a*x)^2/c^3-1/3/c^3*arctan(a*x)/a^3/x^3+3/c^3*arctan
(a*x)/a/x-1/24/c^3*(-33/2/(a^2*x^2+1)-40*ln(a^2*x^2+1)-3/2/(a^2*x^2+1)^2+4/
a^2/x^2+80*ln(a*x)+105/2*arctan(a*x)^2))
```

Maxima [A]
time = 0.49, size = 223, normalized size = 1.22

$$\frac{1}{24}\left(\frac{105\,a^3\arctan(ax)}{c^3}+\frac{105\,a^6x^6+175\,a^4x^4+56\,a^2x^2-8}{a^4c^3x^7+2\,a^2c^3x^5+c^3x^3}\right)\arctan(ax)+\frac{\left(25\,a^4x^4+20\,a^2x^2-105\left(a^6x^6+2\,a^4x^4+a^2x^2\right)\arctan(ax)^2+80\left(a^6x^6+2\,a^4x^4+a^2x^2\right)\log\left(a^2x^2+1\right)-160\left(a^6x^6+2\,a^4x^4+a^2x^2\right)\log(x)-8\right)a}{48\left(a^4c^3x^6+2\,a^2c^3x^4+c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*a^3*arctan(a*x)/c^3 + (105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8
)/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3))*arctan(a*x) + 1/48*(25*a^4*x^4 +
 20*a^2*x^2 - 105*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*arctan(a*x)^2 + 80*(a^6*x
```

```
^6 + 2*a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 160*(a^6*x^6 + 2*a^4*x^4 + a^2
*x^2)*log(x) - 8)*a/(a^4*c^3*x^6 + 2*a^2*c^3*x^4 + c^3*x^2)
```

Fricas [A]
time = 1.15, size = 179, normalized size = 0.98

$$\frac{25\,a^5x^5 + 20\,a^3x^3 + 105\,(a^7x^7 + 2\,a^5x^5 + a^3x^3)\arctan\left(ax\right)^2 - 8\,ax + 2\,(105\,a^6x^6 + 175\,a^4x^4 + 56\,a^2x^2 - 8)\arctan\left(ax\right) + 80\,(a^7x^7 + 2\,a^5x^5 + a^3x^3)\log\left(a^2x^2 + 1\right) - 160\,(a^7x^7 + 2\,a^5x^5 + a^3x^3)\log\left(x\right)}{48\,(a^4c^3x^7 + 2\,a^2c^3x^5 + c^3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(25*a^5*x^5 + 20*a^3*x^3 + 105*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*arctan(
a*x)^2 - 8*a*x + 2*(105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8)*arctan(a*x)
 + 80*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) - 160*(a^7*x^7 + 2*a
^5*x^5 + a^3*x^3)*log(x))/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(177) = 354$.
time = 2.14, size = 724, normalized size = 3.96

$$\begin{cases} - \frac{160a^7x^7\log(x)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{80a^7x^7\log\left(x^2+\frac{1}{a^2}\right)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{105a^7x^7\operatorname{atan}^2(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{210a^6x^6\operatorname{atan}(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \frac{320a^5x^5\log(x)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{160a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{210a^5x^5\operatorname{atan}^2(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{25a^5x^5}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{350a^4x^4\operatorname{atan}(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \frac{160a^3x^3\log(x)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{80a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{105a^3x^3\operatorname{atan}^2(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{20a^3x^3}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{112a^2x^2\operatorname{atan}(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \frac{8ax}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \frac{16\operatorname{atan}(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((-160*a**7*x**7*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 4
8*c**3*x**3) + 80*a**7*x**7*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**
2*c**3*x**5 + 48*c**3*x**3) + 105*a**7*x**7*atan(a*x)**2/(48*a**4*c**3*x**7
 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*a**6*x**6*atan(a*x)/(48*a**4*c**
3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 320*a**5*x**5*log(x)/(48*a**4*
c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 160*a**5*x**5*log(x**2 + a*
*(-2))/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*a**5*x*
*5*atan(a*x)**2/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 25
*a**5*x**5/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 350*a**
4*x**4*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 1
60*a**3*x**3*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3)
+ 80*a**3*x**3*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 +
 48*c**3*x**3) + 105*a**3*x**3*atan(a*x)**2/(48*a**4*c**3*x**7 + 96*a**2*c*
*3*x**5 + 48*c**3*x**3) + 20*a**3*x**3/(48*a**4*c**3*x**7 + 96*a**2*c**3*x*
*5 + 48*c**3*x**3) + 112*a**2*x**2*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c
**3*x**5 + 48*c**3*x**3) - 8*a*x/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 4
8*c**3*x**3) - 16*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**
3*x**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.59, size = 163, normalized size = 0.89

$$\frac{\frac{25\,a^5\,x^4}{2}+10\,a^3\,x^2-4\,a}{24\,a^4\,c^3\,x^6+48\,a^2\,c^3\,x^4+24\,c^3\,x^2}+\frac{\mathrm{atan}(a\,x)\,\left(\frac{7\,x^2}{3\,c^3}-\frac{1}{3\,a^2\,c^3}+\frac{175\,a^2\,x^4}{24\,c^3}+\frac{35\,a^4\,x^6}{8\,c^3}\right)}{2\,x^5+\frac{x^3}{a^2}+a^2\,x^7}+\frac{5\,a^3\,\ln\left(a^2\,x^2+1\right)}{3\,c^3}-\frac{10\,a^3\,\ln\left(x\right)}{3\,c^3}+\frac{35\,a^3\,\mathrm{atan}(a\,x)^2}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^3),x)
```

```
[Out] (10*a^3*x^2 - 4*a + (25*a^5*x^4)/2)/(24*c^3*x^2 + 48*a^2*c^3*x^4 + 24*a^4*c
^3*x^6) + (atan(a*x)*((7*x^2)/(3*c^3) - 1/(3*a^2*c^3) + (175*a^2*x^4)/(24*c
^3) + (35*a^4*x^6)/(8*c^3)))/(2*x^5 + x^3/a^2 + a^2*x^7) + (5*a^3*log(a^2*x
^2 + 1))/(3*c^3) - (10*a^3*log(x))/(3*c^3) + (35*a^3*atan(a*x)^2)/(16*c^3)
```

3.200 $\int x^3 \sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=160

$$\frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a}-\frac{2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\,\text{A}$$

```
[Out] 11/120*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+1/24*x*(a^2*c*x
^2+c)^(1/2)/a^3-1/20*x^3*(a^2*c*x^2+c)^(1/2)/a-2/15*arctan(a*x)*(a^2*c*x^2+
c)^(1/2)/a^4+1/15*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+1/5*x^4*arctan(a*
x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5066, 5072, 327, 223, 212, 5050}

$$\frac{x^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{15a^2}+\frac{1}{5}x^4\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{x^3\sqrt{a^2cx^2+c}}{20a}-\frac{2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{15a^4}+\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{120a^4}+\frac{x\sqrt{a^2cx^2+c}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2])/(24*a^3) - (x^3*Sqrt[c + a^2*c*x^2])/(20*a) - (2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(15*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 + (11*Sqrt[c]*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx = \frac{1}{5}x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{5}c\int\frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx-\frac{1}{5}(ac)\int\frac{x^4}{\sqrt{c+a^2c}}$$

$$= -\frac{x^3\sqrt{c+a^2cx^2}}{20a}+\frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(a$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2}}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2}}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2}}{}$$

Mathematica [A]
time = 0.09, size = 105, normalized size = 0.66

$$\frac{ax(5-6a^2x^2)\sqrt{c+a^2cx^2}+8\sqrt{c+a^2cx^2}(-2+a^2x^2+3a^4x^4)\,\text{ArcTan}(ax)+11\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{120a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (a*x*(5 - 6*a^2*x^2)*Sqrt[c + a^2*c*x^2] + 8*Sqrt[c + a^2*c*x^2]*(-2 + a^2*
x^2 + 3*a^4*x^4)*ArcTan[a*x] + 11*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*
c*x^2]])/(120*a^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.47, size = 176, normalized size = 1.10

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(24\arctan(ax)a^4x^4-6a^3x^3+8\arctan(ax)a^2x^2+5ax-16\arctan(ax))}{120a^4}+\frac{11\sqrt{c(ax-i)(ax+i)}}{120a^4\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)*a^4*x^4-6*a^3*x^3+8*arc
tan(a*x)*a^2*x^2+5*a*x-16*arctan(a*x))+11/120/a^4*(c*(a*x-I)*(I+a*x))^(1/2)
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)-11/120/a^4*(c*(a*x-I)*
(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.33, size = 127, normalized size = 0.79

$$-\frac{1}{120}\left(a\left(\frac{3\left(\frac{2(a^2x^2+1)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{a^2x^2+1}\,x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^4}\right)-8\left(\frac{3(a^2x^2+1)^{\frac{3}{2}}x^2}{a^2}-\frac{2(a^2x^2+1)^{\frac{3}{2}}}{a^4}\right)\arctan(ax)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsi
nh(a*x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^
2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*sqrt(c)
```

Fricas [A]
time = 1.25, size = 94, normalized size = 0.59

$$-\frac{2\left(6a^3x^3-5ax-8(3a^4x^4+a^2x^2-2)\arctan(ax)\right)\sqrt{a^2cx^2+c}-11\sqrt{c}\log\left(-2a^2cx^2-2\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)}{240a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*(6*a^3*x^3 - 5*a*x - 8*(3*a^4*x^4 + a^2*x^2 - 2)*arctan(a*x))*sqr
t(a^2*c*x^2 + c) - 11*sqrt(c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sq
rt(c)*x - c))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.201 $\int x^2 \sqrt{c+a^2cx^2} \text{ArcTan}(ax)\,dx$

Optimal. Leaf size=298

$$\frac{\sqrt{c+a^2cx^2}}{8a^3}-\frac{(c+a^2cx^2)^{3/2}}{12a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{ic\sqrt{1+a^2x^2}\,\text{ArcTa}}{4a^3}$$

```
[Out] -1/12*(a^2*c*x^2+c)^(3/2)/a^3/c+1/4*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-1/8*I*c*polylog(
2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(
1/2)+1/8*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/a^3/(a^2*c*x^2+c)^(1/2)+1/8*(a^2*c*x^2+c)^(1/2)/a^3+1/8*x*arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/a^2+1/4*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5066, 5072, 267, 5010, 5006, 272, 45}

$$\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{8a^2}+\frac{1}{4}x^3\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a^3\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{(a^2cx^2+c)^{3/2}}{12a^3c}+\frac{\sqrt{a^2cx^2+c}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] Sqrt[c + a^2*c*x^2]/(8*a^3) - (c + a^2*c*x^2)^(3/2)/(12*a^3*c) + (x*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x])/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4
 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/8)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (
(-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*
c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*S
qrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx = \frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{4}c\int\frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx-\frac{1}{4}(ac)\int\frac{x^3}{\sqrt{c+a^2cx}}$$

$$= \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\int\frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}}{8a^2}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{8a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-$$

$$= \frac{\sqrt{c+a^2cx^2}}{8a^3}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx}$$

Mathematica [A]
time = 2.10, size = 278, normalized size = 0.93

$$\frac{\sqrt{c(1+a^2x^2)}\left(-6i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+6i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)-\frac{1}{4}(1+a^2x^2)^2\left(-\frac{2}{\sqrt{1+a^2x^2}}-6\cos(3\text{ArcTan}(ax))+3\text{ArcTan}(ax)\left(-\frac{14ax}{\sqrt{1+a^2x^2}}+3\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+4\cos(2\text{ArcTan}(ax))\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+\cos(4\text{ArcTan}(ax))\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)-3\log\left(1+ie^{i\text{ArcTan}(ax)}\right)+2\sin(3\text{ArcTan}(ax))\right)\right)\right)}{48a^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*((-6*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*P
olyLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6
*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1
- I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])]
 - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTa
n[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] +
 2*Sin[3*ArcTan[a*x]]))))/4))/(48*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.50, size = 199, normalized size = 0.67

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(6\arctan(ax)a^3x^3-2a^2x^2+3\arctan(ax)ax+1)}{24a^3}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)\ln\left(1+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*a^3*x^3-2*a^2*x^2+3*arcta
n(a*x)*a*x+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(
```

```
a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1
+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a
^3/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.202 $\int x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=86

$$-\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}{3a^2c}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{6a^2}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2/c-1/6*arctanh(a*x*c^(1/2)/(a^2*c*x^
2+c)^(1/2))*c^(1/2)/a^2-1/6*x*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 201, 223, 212}

$$\frac{\text{ArcTan}(ax)\,(a^2cx^2+c)^{3/2}}{3a^2c}-\frac{x\sqrt{a^2cx^2+c}}{6a}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] -1/6*(x*Sqrt[c + a^2*c*x^2])/a + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2
*c) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\tan^{-1}(ax)\,dx &= \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\int\sqrt{c+a^2cx^2}\,dx}{3a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{c\int\frac{1}{\sqrt{c+a^2cx^2}}\,dx}{6a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{c\text{Subst}\left(\int\frac{1}{1-a^2cx^2}\,dx,x,\frac{x}{\sqrt{c+a^2cx^2}}\right)}{6a}\\ &= -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3a^2c}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{6a^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 86, normalized size = 1.00

$$-\frac{ax\sqrt{c+a^2cx^2}-2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
[Out] -1/6*(a*x*Sqrt[c + a^2*c*x^2] - 2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x] + Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/a^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 156, normalized size = 1.81

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(2\arctan(ax)a^2x^2-ax+2\arctan(ax))}{6a^2}+\frac{\sqrt{c(ax-i)(ax+i)}\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)}{6a^2\sqrt{a^2x^2+1}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x+2*arctan(a*x))
+1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x
^2+1)^(1/2)-1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)+I)/(a^2*x^2+1)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(70) = 140$.
time = 0.54, size = 260, normalized size = 3.02

$$\frac{4\,(a^2x^2+1)^{\frac{3}{2}}\sqrt{c}\,\arctan(ax) - 2\,(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\left(ax\cos\left(\frac{1}{2}\arctan(4\,ax,-a^2x^2+3)\right)+2\sin\left(\frac{1}{2}\arctan(4\,ax,-a^2x^2+3)\right)\right)\sqrt{c} + \sqrt{c}\left(\arctan\left((a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(4\,ax,a^2x^2-3)\right)+2,ax+(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(4\,ax,a^2x^2-3)\right)\right)+\arctan\left((a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(4\,ax,a^2x^2-3)\right)-2,-ax+(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(4\,ax,a^2x^2-3)\right)\right)\right)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a^2*x^2 + 1)^(3/2)*sqrt(c)*arctan(a*x) - 2*(a^4*x^4 + 10*a^2*x^2 +
 9)^(1/4)*(a*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*sin(1/2*arctan2(4*
a*x, -a^2*x^2 + 3)))*sqrt(c) + sqrt(c)*(arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^
(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2
 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + arctan2((a^4*x^4 + 10*a
^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4
 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3)))))/a^2
```

Fricas [**A**]
time = 0.86, size = 77, normalized size = 0.90

$$-\frac{2\sqrt{a^2cx^2+c}\,(ax-2\,(a^2x^2+1)\arctan(ax)) - \sqrt{c}\,\log\left(-2\,a^2cx^2+2\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 + 1)*arctan(a*x)) - sqrt(c)*
log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/a^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.203 $\int \sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{ic\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}+\frac{ic\sqrt{1+a^2x}}{}$$

```
[Out] -I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a/(a^2*c*x^2+c)^(1/2)+1/2*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-1/2*I*c*polylog(2,I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-1/2*(a^2*c*x^2+c
)^(1/2)/a+1/2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4998, 5010, 5006}

$$-\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] -1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - (I*c*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sq
rt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - ((I/2)*c*Sqrt[1 + a^2*x^
2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx = -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{2}c\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{ic\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tan^{-1}}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.47, size = 141, normalized size = 0.58

$$\frac{\sqrt{c\left(1+a^2x^2\right)}\left(\sqrt{1+a^2x^2}\,(-1+ax\text{ArcTan}(ax))+\text{ArcTan}(ax)\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{2a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2]*(-1 + a*x*ArcTan[a*x]) + ArcTan[a
*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*PolyL
og[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(2*a*Sq
rt[1 + a^2*x^2])
```

Maple [A]
time = 0.22, size = 178, normalized size = 0.73

method	result
default	$\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(\arctan(ax)ax-1\right)}{2a}-\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\right.}{2a\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*a*x-1)-1/2*(c*(a*x-I)*(I+a*x))
^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dil
og(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.204 $\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}\,dx$

Optimal. Leaf size=229

$$\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{2c\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)+$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)-2*c*arctan(a*x)*arctanh((
1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c*p
olylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^
(1/2)-I*c*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2
*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5066, 5078, 5074, 223, 212}

$$\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - Sqrt[c]*ArcTanh[(
a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (I*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c*Sqrt[1 + a^2*x^2
]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
```

```
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}\,dx = \sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx - (ac)\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - (ac)\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) + \ldots$$

$$= \sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{2c\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \ldots$$

Mathematica [A]
time = 0.16, size = 164, normalized size = 0.72

$$\frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\,\text{ArcTan}(ax) + \text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right) - \text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right) + \log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right) - \sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right) - \log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right) + \sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right) + i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right) - i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x] + ArcTan[a*x]*Log[1 - E
^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[Cos[ArcTan
[a*x]/2] - Sin[ArcTan[a*x]/2]] - Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2
```

```
]] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*x])]))/S
qrt[1 + a^2*x^2]
```

Maple [A]
time = 0.28, size = 151, normalized size = 0.66

method	result
default	$\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\ \left(2i\arctan\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(\frac{iax+1}{\sqrt{a^2x^2}}\right.\right.}{\sqrt{a^2x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)+(c*(a*x-I)*(I+a*x))^(1/2)*(2*I*arctan
((1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(
1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\,(a^2x^2+1)}\ \mathrm{atan}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.205 $\int \frac{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)}{x^2}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{2iac\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right.$$

```
[Out] -a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-2*I*a*c*arctan(a*x)*arctan(
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a*
c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^
2+c)^(1/2)-I*a*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5070, 5064, 272, 65, 214, 5010, 5006}

$$-\frac{2iac\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}+\frac{iac\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{iac\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a*Sqr
t[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a*c*Sqrt[1 + a^2*x^2]*PolyLo
g[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (I*a*c*
Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c +
 a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + (ac)\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int}{\sqrt{c+a^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.35, size = 163, normalized size = 0.67

$$-\frac{a\sqrt{c(1+a^2x^2)}\left(\frac{\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)}{ax} - \mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right) + \mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right) + \log\left(\cos\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right) - \log\left(\sin\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right) - i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right) + i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x) - ArcTan[
a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])
] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.26, size = 221, normalized size = 0.91

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)}{x} + \frac{ia\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)/x+I*a*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a
rctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*
```

```
x)/(a^2*x^2+1)^(1/2))+I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2)-1)+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\ \sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2,x)
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.206 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)}{x^3}\,dx$

Optimal. Leaf size=240

$$-\frac{a\sqrt{c+a^2cx^2}}{2x}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2x^2}-\frac{a^2c\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}+\frac{ia^2c\sqrt{1+a^2}}{}$$

```
[Out] -a^2*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+1/2*I*a^2*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*c*polylog(2,(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^
2+c)^(1/2)/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5066, 5082, 270, 5078, 5074}

$$-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2x^2}-\frac{a^2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{ia^2c\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[c + a^2*c*x^2])/x - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2)
- (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^3}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2} - c\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx + (ac)\int \frac{1}{x^2\sqrt{c+a^2cx^2}} \\ &= -\frac{a\sqrt{c+a^2cx^2}}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} - \frac{1}{2}(ac)\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx + \\ &= -\frac{a\sqrt{c+a^2cx^2}}{2x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} + \frac{\left(a^2c\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax}{x\sqrt{1+a^2}}}{2\sqrt{c+a^2cx^2}} \\ &= -\frac{a\sqrt{c+a^2cx^2}}{2x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh}{\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 165, normalized size = 0.69

$$\frac{a^2\sqrt{c(1+a^2x^2)}\left(-2\cot\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-\mathrm{ArcTan}(ax)\csc^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)+4\mathrm{ArcTan}(ax)\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-4\mathrm{ArcTan}(ax)\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+4i\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-4i\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)+\mathrm{ArcTan}(ax)\sec^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-2\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)}{8\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[
a*x]/2]^2 + 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4*ArcTan[a*x]*Log[1
+ E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*PolyLog
[2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*
x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.37, size = 169, normalized size = 0.70

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+\arctan(ax))}{2x^2}+\frac{ia^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{arct}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(a*x+arctan(a*x))/x^2+1/2*I*a^2*(c*(a*x-I)*(
I+a*x))^(1/2)*(I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x
)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\, \operatorname{atan}\left(ax\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**3,x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3,x)
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.207 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}{3cx^3}-\frac{1}{6}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/c/x^3-1/6*a^3*arctanh((a^2*c*x^2+c)^(1
/2)/c^(1/2))*c^(1/2)-1/6*a*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5064, 272, 43, 65, 214}

$$-\frac{\text{ArcTan}(ax)\,(a^2cx^2+c)^{3/2}}{3cx^3}-\frac{a\sqrt{a^2cx^2+c}}{6x^2}-\frac{1}{6}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[c + a^2*c*x^2])/x^2 - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*c
*x^3) - (a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x^4}\,dx = -\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{3}a\int \frac{\sqrt{c+a^2cx^2}}{x^3}\,dx$$
$$= -\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{6}a\text{Subst}\left(\int \frac{\sqrt{c+a^2cx}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{12}(a^3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2$$
$$= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3cx^3}+\frac{1}{6}a\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx,$$
$$= -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3cx^3}-\frac{1}{6}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+a^2cx}}{\sqrt{c}}$$

Mathematica [A]
time = 0.08, size = 105, normalized size = 1.25

$$\frac{-2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+a^3\sqrt{c}\,x^3\log(x)-ax\left(\sqrt{c+a^2cx^2}+a^2\sqrt{c}\,x^2\log\left(c+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

```
[Out] (-2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + a^3*Sqrt[c]*x^3*Log[x]
- a*x*(Sqrt[c + a^2*c*x^2] + a^2*Sqrt[c]*x^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c
*x^2]]))/(6*x^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 153, normalized size = 1.82

method	result
default	$-\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(2\arctan\left(ax\right)a^{2}x^{2}+ax+2\arctan\left(ax\right)\right)}{6x^{3}}-\frac{a^{3}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\ln\left(1+\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)}{6\sqrt{a^{2}x^{2}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)*a^2*x^2+a*x+2*arctan(a*x))/x^
3-1/6*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
x^2+1)^(1/2)+1/6*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2)-1)/(a^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.35, size = 73, normalized size = 0.87

$$-\frac{1}{6}\left(\left(a^{2}\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\sqrt{a^{2}x^{2}+1}\,a^{2}+\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}\right)a+\frac{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^{3}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)*a^2 + (a^2*x^2 + 1)^(3
/2)/x^2)*a + 2*(a^2*x^2 + 1)^(3/2)*arctan(a*x)/x^3)*sqrt(c)
```

Fricas [A]
time = 6.81, size = 84, normalized size = 1.00

$$\frac{a^{3}\sqrt{c}\,x^{3}\log\left(-\frac{a^{2}cx^{2}-2\sqrt{a^{2}cx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\sqrt{a^{2}cx^{2}+c}\,\left(ax+2\left(a^{2}x^{2}+1\right)\arctan\left(ax\right)\right)}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c
)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + 2*(a^2*x^2 + 1)*arctan(a*x)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)\ \sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.208 $\int x^3(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=217

$$\frac{3cx\sqrt{c+a^2cx^2}}{112a^3}-\frac{23cx^3\sqrt{c+a^2cx^2}}{840a}-\frac{1}{42}acx^5\sqrt{c+a^2cx^2}-\frac{2c\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}{35a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\operatorname{Arc}}{35a^2}$$

```
[Out] 17/560*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+3/112*c*x*(a^2*
c*x^2+c)^(1/2)/a^3-23/840*c*x^3*(a^2*c*x^2+c)^(1/2)/a-1/42*a*c*x^5*(a^2*c*x
^2+c)^(1/2)-2/35*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/35*c*x^2*arctan(a*
x)*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/7*a
^2*c*x^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5070, 5066, 5072, 327, 223, 212, 5050}

$$\frac{cx^2\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{35a^2}+\frac{1}{7}a^2cx^6\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{8}{35}cx^4\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{1}{42}acx^5\sqrt{a^2cx^2+c}-\frac{23cx^3\sqrt{a^2cx^2+c}}{840a}-\frac{2c\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{35a^4}+\frac{17c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{560a^4}+\frac{3cx\sqrt{a^2cx^2+c}}{112a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2])/(112*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2])/(840
*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2])/42 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/7 + (17*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(560*a^4
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx = c\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx + \left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx$$

$$= \frac{1}{5}cx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{5}c^2\int \frac{x^3}{\sqrt{}}$$

$$= -\frac{cx^3\sqrt{c+a^2cx^2}}{20a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} + \frac{cx^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{15a^2} +$$

$$= \frac{cx\sqrt{c+a^2cx^2}}{24a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a^2}}{}$$

$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a}}{}$$

$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a}}{}$$

$$= \frac{3cx\sqrt{c+a^2cx^2}}{112a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}}{840a} - \frac{1}{42}acx^5\sqrt{c+a^2cx^2} - \frac{2c\sqrt{c+a}}{}$$

Mathematica [A]
time = 0.12, size = 119, normalized size = 0.55

$$\frac{acx\sqrt{c+a^2cx^2}\,(45-46a^2x^2-40a^4x^4)+48c(1+a^2x^2)^2\,(-2+5a^2x^2)\,\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+51c^{3/2}\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{1680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (a*c*x*Sqrt[c + a^2*c*x^2]*(45 - 46*a^2*x^2 - 40*a^4*x^4) + 48*c*(1 + a^2*x
^2)^2*(-2 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + 51*c^(3/2)*Log[a*c
*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(1680*a^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.36, size = 199, normalized size = 0.92

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\;(240\arctan(ax)a^6x^6-40a^5x^5+384\arctan(ax)a^4x^4-46a^3x^3+48\arctan(ax)a^2x^2+45ax-96\arctan(ax)}{1680a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(240*arctan(a*x)*a^6*x^6-40*a^5*x^5+
384*arctan(a*x)*a^4*x^4-46*a^3*x^3+48*arctan(a*x)*a^2*x^2+45*a*x-96*arctan(
a*x))+17/560*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
+I)/(a^2*x^2+1)^(1/2)-17/560*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.47, size = 214, normalized size = 0.99

$$-\frac{1}{1680}\left(\left(5\left(\frac{8\left(a^2x^2+1\right)^{\frac{3}{2}}x^3}{a^2}-\frac{6\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^4}+\frac{3\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\,\mathrm{arsinh}\left(ax\right)}{a^5}\right)c+\frac{18c\left(\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\mathrm{arsinh}\left(ax\right)}{a^3}\right)}{a^2}-\frac{48\left(\sqrt{a^2x^2+1}\,x+\frac{\mathrm{arsinh}\left(ax\right)}{a}\right)c}{a^4}\right)a-48\left(5\left(a^2x^2+1\right)^{\frac{3}{2}}cx^4+\frac{3\left(a^2x^2+1\right)^{\frac{3}{2}}cx^2}{a^2}-\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}c}{a^4}\right)\arctan\left(ax\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/1680*((5*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 +
3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c + 18*c*(2*(a^2*x^2 + 1)^(
3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 48*(sqrt(a^2
*x^2 + 1)*x + arcsinh(a*x)/a)*c/a^4)*a - 48*(5*(a^2*x^2 + 1)^(3/2)*c*x^4 +
3*(a^2*x^2 + 1)^(3/2)*c*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c/a^4)*arctan(a*x))
*sqrt(c)
```

Fricas [A]
time = 4.51, size = 118, normalized size = 0.54

$$\frac{51\,c^{\frac{3}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)-2\left(40\,a^5cx^5+46\,a^3cx^3-45\,acx-48\left(5\,a^6cx^6+8\,a^4cx^4+a^2cx^2-2\,c\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{3360\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/3360*(51*c^(3/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c
) - 2*(40*a^5*c*x^5 + 46*a^3*c*x^3 - 45*a*c*x - 48*(5*a^6*c*x^6 + 8*a^4*c*x
^4 + a^2*c*x^2 - 2*c)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.209 $\int x^2(c+a^2cx^2)^{3/2}\,\textbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=357

$$\frac{c\sqrt{c+a^2cx^2}}{16a^3}+\frac{(c+a^2cx^2)^{3/2}}{72a^3}-\frac{(c+a^2cx^2)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax$$

```
[Out] 1/72*(a^2*c*x^2+c)^(3/2)/a^3-1/30*(a^2*c*x^2+c)^(5/2)/a^3/c+1/8*I*c^2*arcta
n(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c
*x^2+c)^(1/2)-1/16*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2
*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/16*I*c^2*polylog(2,I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/16*c*(a^2*c*
x^2+c)^(1/2)/a^3+1/16*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+7/24*c*x^3*ar
ctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/6*a^2*c*x^5*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5066, 5072, 267, 5010, 5006, 272, 45}

$$\frac{cx\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{16a^2}+\frac{1}{6}a^2cx^5\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{7}{24}cx^3\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{16a^3\sqrt{a^2cx^2+c}}-\frac{(a^2cx^2+c)^{5/2}}{30a^3c}+\frac{(a^2cx^2+c)^{3/2}}{72a^3}+\frac{c\sqrt{a^2cx^2+c}}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2])/(16*a^3) + (c + a^2*c*x^2)^(3/2)/(72*a^3) - (c + a^
2*c*x^2)^(5/2)/(30*a^3*c) + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(16*a^2)
+ (7*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/24 + (a^2*c*x^5*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/16)*c^2*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c
 + a^2*c*x^2]) + ((I/16)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x
])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
```

```
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx = c\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx$$

$$=\frac{1}{4}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{4}c^2\int$$

$$=\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{6}a^2cx^5\sqrt{c}$$

$$=-\frac{c\sqrt{c+a^2cx^2}}{8a^3}+\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}$$

$$=\frac{c\sqrt{c+a^2cx^2}}{48a^3}+\frac{\left(c+a^2cx^2\right)^{3/2}}{36a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}}{16a^2}$$

$$=\frac{c\sqrt{c+a^2cx^2}}{16a^3}+\frac{\left(c+a^2cx^2\right)^{3/2}}{72a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}}{16a^2}$$

Mathematica [A]
time = 4.24, size = 576, normalized size = 1.61

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + (55*(1 + a^2*x^2)^3*Cos
[3*ArcTan[a*x]])/8 - (45*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 - (90*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + (90*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] -
(15*(1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan
[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos
[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])
]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcT
an[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])))/2 + (
15*(1 + a^2*x^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*
```

```
E^(I*ArcTan[a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45
*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a
*x])]) + 18*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*
Log[1 + I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]
))/16))/(1440*a^3*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.50, size = 221, normalized size = 0.62

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+210\arctan(ax)a^3x^3-38a^2x^2+45\arctan(ax)ax+31)}{720a^3}+\frac{\sqrt{c\,(ax-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/720*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+2
10*arctan(a*x)*a^3*x^3-38*a^2*x^2+45*arctan(a*x)*a*x+31)+1/16*(c*(a*x-I)*(I
+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.210 $\int x(c+a^2cx^2)^{3/2}\,\textbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=109

$$-\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x(c+a^2cx^2)^{3/2}}{20a}+\frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}{5a^2c}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{40a^2}$$

```
[Out] -1/20*x*(a^2*c*x^2+c)^(3/2)/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a^2/c-3/4
0*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2-3/40*c*x*(a^2*c*x^2+
c)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5050, 201, 223, 212}

$$\frac{\text{ArcTan}(ax)\,(a^2cx^2+c)^{5/2}}{5a^2c}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{40a^2}-\frac{x(a^2cx^2+c)^{3/2}}{20a}-\frac{3cx\sqrt{a^2cx^2+c}}{40a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (-3*c*x*Sqrt[c + a^2*c*x^2])/(40*a) - (x*(c + a^2*c*x^2)^(3/2))/(20*a) + ((
c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - (3*c^(3/2)*ArcTanh[(a*Sqrt[c]
*x)/Sqrt[c + a^2*c*x^2]])/(40*a^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx &= \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{\int\left(c+a^2cx^2\right)^{3/2}\,dx}{5a}\\ &= -\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{(3c)\int\sqrt{c+a^2cx^2}\,dx}{20a}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{(3c^2}{}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{(3c^2}{}\\ &= -\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x\left(c+a^2cx^2\right)^{3/2}}{20a}+\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}-\frac{3c^{3/2}}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 101, normalized size = 0.93

$$-\frac{acx(5+2a^2x^2)\sqrt{c+a^2cx^2}-8c(1+a^2x^2)^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)+3c^{3/2}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] -1/40*(a*c*x*(5 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 8*c*(1 + a^2*x^2)^2*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x] + 3*c^(3/2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*
x^2]])/a^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 179, normalized size = 1.64

method	result

default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,(8\arctan(ax)a^4x^4-2a^3x^3+16\arctan(ax)a^2x^2-5ax+8\arctan(ax))}{40a^2}-\frac{3c\sqrt{c\,(ax-i)\,(ax+i)}}{40a^2\sqrt{a^2x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(8*arctan(a*x)*a^4*x^4-2*a^3*x^3+16*ar
ctan(a*x)*a^2*x^2-5*a*x+8*arctan(a*x))-3/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)+3/40*c/a^2*(c*(a*x-I)*
(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(89) = 178$.
time = 0.59, size = 406, normalized size = 3.72

$$\frac{40\left(a^{2}cx^{2}+c\right)\sqrt{a^{2}x^{2}+1}\,\sqrt{c}\,\arctan\left(ax\right)-20\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\left(acx\cos\left(\frac{1}{2}\arctan\left(4\,ax,-a^{2}x^{2}+3\right)\right)+2\,c\sin\left(\frac{1}{2}\arctan\left(4\,ax,-a^{2}x^{2}+3\right)\right)\right)\sqrt{c}-\left(\left(a\left(\frac{3\left(\frac{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{a^{2}}-\frac{\sqrt{a^{2}x^{2}+1}\,x}{a^{2}}-\frac{\operatorname{arsinh}(ax)}{a^{3}}\right)}{a^{2}}-\frac{8\left(\sqrt{a^{2}x^{2}+1}\,x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^{4}}\right)-8\left(\frac{3\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}{a^{2}}-\frac{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{a^{4}}\right)\arctan\left(ax\right)\right)a^{4}c-10\,c\arctan\left(\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(4\,ax,a^{2}x^{2}-3\right)\right)+2,ax+\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(4\,ax,a^{2}x^{2}-3\right)\right)\right)-10\,c\arctan\left(\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(4\,ax,a^{2}x^{2}-3\right)\right)-2,-ax+\left(a^{4}x^{4}+10\,a^{2}x^{2}+9\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(4\,ax,a^{2}x^{2}-3\right)\right)\right)\right)\sqrt{c}}{120\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] 1/120*(40*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 20*(a^4*x
^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2
*c*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(3*(2*(a^2*x^2 + 1)
^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 8*(sqrt(a^
2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*
(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c - 10*c*arctan2((a^4*x^4 + 10*a^
2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) - 10*c*arctan2
((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2,
 -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3
))))*sqrt(c))/a^2
```

Fricas [A]
time = 4.97, size = 98, normalized size = 0.90

$$\frac{3\,c^{\frac{3}{2}}\log\left(-2\,a^{2}cx^{2}+2\,\sqrt{a^{2}cx^{2}+c}\,a\sqrt{c}\,x-c\right)-2\left(2\,a^{3}cx^{3}+5\,acx-8\left(a^{4}cx^{4}+2\,a^{2}cx^{2}+c\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}+c}}{80\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/80*(3*c^(3/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c) -
 2*(2*a^3*c*x^3 + 5*a*c*x - 8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*sq
rt(a^2*c*x^2 + c))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.211 $\int (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)\, dx$

Optimal. Leaf size=298

$$-\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{(c+a^2cx^2)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)+\frac{1}{4}x(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)-\frac{3ic^2\sqrt{1+a}}{}$$

```
[Out] -1/12*(a^2*c*x^2+c)^(3/2)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-3/4*I*c^2
*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a
^2*c*x^2+c)^(1/2)+3/8*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*polylog(2,I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/8*c*(a^2*c*x^2
+c)^(1/2)/a+3/8*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4998, 5010, 5006}

$$-\frac{3ic^2\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a\sqrt{a^2cx^2+c}}+\frac{3}{8}cx\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{4}x\operatorname{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}+\frac{3ic^2\sqrt{a^2x^2+1}\operatorname{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\operatorname{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{3c\sqrt{a^2cx^2+c}}{8a}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (-3*c*Sqrt[c + a^2*c*x^2])/(8*a) - (c + a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4
- (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
(((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
```

```
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx &= -\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\ \text{ta}\\ &= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\ \tan^{-1}(ax)+\frac{1}{4}x(c+\\ &= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\ \tan^{-1}(ax)+\frac{1}{4}x(c-\\ &= -\frac{3c\sqrt{c+a^2cx^2}}{8a}-\frac{\left(c+a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\ \tan^{-1}(ax)+\frac{1}{4}x(c-\end{aligned}$$

Mathematica [A]
time = 1.87, size = 351, normalized size = 1.18

$$\frac{c\sqrt{c+a^2cx^2}\left(2(1+a^2x^2)^{3/2}+96\sqrt{1+a^2x^2}(-1+ax\text{ArcTan}(ax))+6(1+a^2x^2)^2\cos(3\text{ArcTan}(ax))+96\text{ArcTan}(ax)\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+72i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-72i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)-3(1+a^2x^2)^2\text{ArcTan}(ax)\left(-\frac{14ax}{\sqrt{1+a^2x^2}}+3\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+4\cos(2\text{ArcTan}(ax))\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+\cos(4\text{ArcTan}(ax))\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)-3\log\left(1+ie^{i\text{ArcTan}(ax)}\right)+2\sin(3\text{ArcTan}(ax))\right)\right)}{192a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(2*(1 + a^2*x^2)^(3/2) + 96*Sqrt[1 + a^2*x^2]*(-1 +
a*x*ArcTan[a*x]) + 6*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]] + 96*ArcTan[a*x]*(L
og[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (72*I)*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 3*(
1 + a^2*x^2)^2*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*
ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1
+ I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])]
- Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*
ArcTan[a*x]])))/(192*a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.24, size = 201, normalized size = 0.67

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(6\arctan(ax)a^3x^3-2a^2x^2+15\arctan(ax)ax-11\right)}{24a}-\frac{3\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(\arctan(ax)\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*a^3*x^3-2*a^2*x^2+15*arct
an(a*x)*a*x-11)-3/8*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a*(arctan(a
*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.212 $\int \frac{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)}{x}\,dx$

Optimal. Leaf size=281

$$-\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)-\frac{2c^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\,\text{ta}}{\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-7/6*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*
x^2+c)^(1/2))-2*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a
*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*c^2*polylog(2,(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/6*a*c*x*(a^2
*c*x^2+c)^(1/2)+c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5066, 5078, 5074, 223, 212, 5050, 201}

$$-\frac{2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{3}\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{7}{6}c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{1}{6}acx\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

```
[Out] -1/6*(a*c*x*Sqrt[c + a^2*c*x^2]) + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + ((c
+ a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (7*c^(3/2)*Ar
cTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/6 + (I*c^2*Sqrt[1 + a^2*x^2]*Poly
Log[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*c^2*Sq
rt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
```

```
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}\,dx+\left(a^2c\right)\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)-\frac{1}{3}(ac)\int \sqrt{c+a^2c}$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax$$

$$= -\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax$$

Mathematica [A]
time = 0.18, size = 233, normalized size = 0.83

$$\frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}+8\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+2a^2x^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)-\tanh^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+6\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-6\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+6\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-6\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)+\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+6i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-6i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)\right)}{6\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) + 8*Sqrt[1 + a^2*x^2]*ArcT
an[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - ArcTanh[(a*x)/Sqrt[1 +
a^2*x^2]] + 6*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 6*ArcTan[a*x]*Log[1
+ E^(I*ArcTan[a*x])] + 6*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 6*L
og[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (6*I)*PolyLog[2, -E^(I*ArcTan
[a*x])] - (6*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(6*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.27, size = 174, normalized size = 0.62

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(2\arctan(ax)a^2x^2-ax+8\arctan(ax)\right)}{6}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3\arctan(ax)\ln\left(1+\frac{iax}{\sqrt{a^2x}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x+8*arctan(a*x))-1
/3*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(3*arctan(a*x)*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-7*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog((1+I
*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a^4*x^4 +
10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*c*si
n(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(c*arctan2((a^4*x^4 + 1
0*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x
^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + c*arctan
2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2
, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 -
3))) + 12*c*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x)/x, x))*sqrt(c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x,x)
```

[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x,x)

[Out] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x, x)

3.213 $\int \frac{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)}{x^2}\,dx$

Optimal. Leaf size=300

$$-\frac{1}{2}ac\sqrt{c+a^2cx^2}-\frac{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{3iac^2\sqrt{1+a^2x^2}\,\text{ArcTan}(a}{\sqrt{c+a}}$$

```
[Out] -a*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-3*I*a*c^2*arctan(a*x)*arcta
n((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3/
2*I*a*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)-3/2*I*a*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*c*(a^2*c*x^2+c)^(1/2)-c*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)/x+1/2*a^2*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5064, 272, 65, 214, 5010, 5006, 4998}

$$-\frac{3iac^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}a^2cx\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)+\frac{3iac^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{2}ac\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -1/2*(a*c*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a
^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
a*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((3*I)/2)*a*c^2*Sqrt[1 +
a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] - (((3*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
```

```
IntegerQ[q]))
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2}\,dx + (a^2c)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c^2\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}($$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}($$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}($$

$$= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}($$

Mathematica [A]
time = 0.68, size = 218, normalized size = 0.73

$$\frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+a^2x^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+3ax\text{ArcTan}(ax)\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-3ax\text{ArcTan}(ax)\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-2ax\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+2ax\log\left(\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+3iax\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-3iax\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{2x\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x] + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 3*a*x*ArcTan[a*x]*Log[1 -
 I*E^(I*ArcTan[a*x])] - 3*a*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2*
a*x*Log[Cos[ArcTan[a*x]/2]] + 2*a*x*Log[Sin[ArcTan[a*x]/2]] + (3*I)*a*x*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - (3*I)*a*x*PolyLog[2, I*E^(I*ArcTan[a*x])]
))/(2*x*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.28, size = 240, normalized size = 0.80

method	result

default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,(\arctan(ax)a^2x^2-ax-2\arctan(ax))}{2x}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3i\,\mathrm{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))/x+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a*c/(a^2*x^2+1)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(c(a^2x^2+1))^{\frac{3}{2}}\,\mathrm{atan}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**2,x)

[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**2, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.214 $\int \frac{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)}{x^3}\,dx$

Optimal. Leaf size=304

$$-\frac{ac\sqrt{c+a^2cx^2}}{2x}+a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2x^2}-\frac{3a^2c^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\,\text{tar}}{\sqrt{c+a^2cx^2}}$$

```
[Out] -a^2*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-3*a^2*c^2*arctan(a*x)
*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+3/2*I*a^2*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*c*(a^2*c*x^2+c)^(1/2)/x
+a^2*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2
)/x^2
```

Rubi [A]
time = 0.46, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5066, 5082, 270, 5078, 5074, 223, 212}

$$-\frac{3a^2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+a^2c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2x^2}-a^2c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{ac\sqrt{a^2cx^2+c}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*c*Sqrt[c + a^2*c*x^2])/x + a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] -
(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
 a^2*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((3*I)/2)*a^2*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c
+ a^2*c*x^2] - (((3*I)/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
```

```
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^3}\,dx + (a^2c)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}\,dx$$

$$= a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2} - c^2\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} -$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{2x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} -$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{2x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} -$$

Mathematica [A]
time = 1.20, size = 301, normalized size = 0.99

$$\frac{a^2c\sqrt{c+a^2cx^2}\left(-2-2\cot^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+4ax\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+12\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-12\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+8\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-8\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)+\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+12i\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-12i\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)\csc\left(\frac{1}{2}\text{ArcTan}(ax)\right)\sec\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{8\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-2 - 2*Cot[ArcTan[a*x]/2]^2 + 4*a*x*ArcTan[a*x]
*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2
 + 12*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 12*ArcTan
[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[ArcTan[a*x]/2]*
Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 8*Cot[ArcTan[a*x]/2]*Log[Cos
[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2
, -E^(I*ArcTan[a*x])] - (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*
x])] + ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2
])/(8*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.38, size = 180, normalized size = 0.59

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,(2\arctan(ax)a^2x^2-ax-\arctan(ax))}{2x^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3\arctan(ax)\ln\left(1+\frac{iax}{\sqrt{a^2x}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)*a^2*x^2-a*x-arctan(a*x))/x^2
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(3*arctan(a*x)*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog((1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a^2*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}{(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.215 $\int \frac{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)}{x^4}\,dx$

Optimal. Leaf size=310

$$-\frac{ac\sqrt{c+a^2cx^2}}{6x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{x}-\frac{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)}{3x^3}-\frac{2ia^3c^2\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)}{\sqrt{c+a^2c}}$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-7/6*a^3*c^(3/2)*arctanh((a^2*c*x^2
+c)^(1/2)/c^(1/2))-2*I*a^3*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a^3*c^2*polylog(2,-I*(1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^3*c^2*
polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)-1/6*a*c*(a^2*c*x^2+c)^(1/2)/x^2-a^2*c*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)/x
```

Rubi [A]
time = 0.31, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5064, 272, 43, 65, 214, 5010, 5006}

$$-\frac{a^2c\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-\frac{\mathrm{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{3x^3}-\frac{ac\sqrt{a^2cx^2+c}}{6x^2}-\frac{2ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{7}{6}a^3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)+\frac{ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(a*c*Sqrt[c + a^2*c*x^2])/x^2 - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/x - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2
*c*x^2] - (7*a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6 + (I*a^3*c
^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sq
rt[c + a^2*c*x^2] - (I*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^4}\,dx + (a^2c)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{3}(ac)\int \frac{\sqrt{c+a^2cx^2}}{x^3}\,dx + (a^2c^2)\int \frac{\tan^{-1}}{x^2\sqrt{c}}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{3x^3} + \frac{1}{6}(ac)\text{Subst}\Bigg($$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax}{3x^3}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax}{3x^3}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax}{3x^3}$$

Mathematica [A]
time = 0.41, size = 263, normalized size = 0.85

$$-\frac{c\sqrt{c+a^2cx^2}\left(ax\sqrt{1+a^2x^2}+2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+8a^2x^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+a^3x^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)-6a^3x^3\text{ArcTan}(ax)\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+6a^3x^3\text{ArcTan}(ax)\log\left(1+ie^{i\text{ArcTan}(ax)}\right)+6a^3x^3\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-6a^3x^3\log\left(\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-6ia^3x^3\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+6ia^3x^3\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{6x^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*(c*Sqrt[c + a^2*c*x^2]*(a*x*Sqrt[1 + a^2*x^2] + 2*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x] + 8*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a^3*x^3*ArcTanh[Sqrt[
1 + a^2*x^2]] - 6*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 6*a^3*
x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 6*a^3*x^3*Log[Cos[ArcTan[a*x
]/2]] - 6*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] - (6*I)*a^3*x^3*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] + (6*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(x^3*Sq
rt[1 + a^2*x^2])
```

Maple [A]
time = 0.49, size = 245, normalized size = 0.79

method	result
default	$-\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(8\arctan\left(ax\right)a^{2}x^{2}+ax+2\arctan\left(ax\right)\right)}{6x^{3}}+\frac{ia^{3}c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(6i\arctan\left(ax\right)\ln\left(1+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*(8*arctan(a*x)*a^2*x^2+a*x+2*arctan(a*x))/
x^3+1/6*I*a^3*c*(c*(a*x-I)*(I+a*x))^(1/2)*(6*I*arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+7*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.216 $\int x^3(c+a^2cx^2)^{5/2}\,\textbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=289

$$\frac{47c^2x\sqrt{c+a^2cx^2}}{2688a^3}-\frac{205c^2x^3\sqrt{c+a^2cx^2}}{12096a}-\frac{103ac^2x^5\sqrt{c+a^2cx^2}}{3024}-\frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2}-\frac{2c^2\sqrt{c+a^2cx^2}\,\text{Ar}}{63a^4}$$

```
[Out] 115/8064*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+47/2688*c^2*x
*(a^2*c*x^2+c)^(1/2)/a^3-205/12096*c^2*x^3*(a^2*c*x^2+c)^(1/2)/a-103/3024*a
*c^2*x^5*(a^2*c*x^2+c)^(1/2)-1/72*a^3*c^2*x^7*(a^2*c*x^2+c)^(1/2)-2/63*c^2*
arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/a^2+5/21*c^2*x^4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+19/63*a^2*c^2*x^6*ar
ctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)
```

Rubi [A]
time = 1.34, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 76, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5070, 5066, 5072, 327, 223, 212, 5050}

$$\frac{c^2x^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{63a^2}+\frac{19}{63}a^2c^2x^6\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{5}{21}c^2x^4\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{103ac^2x^5\sqrt{a^2cx^2+c}}{3024}-\frac{205c^2x^3\sqrt{a^2cx^2+c}}{12096a}-\frac{2c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{63a^4}+\frac{1}{9}a^4c^2x^8\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{115c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{8064a^4}+\frac{47c^2x\sqrt{a^2cx^2+c}}{2688a^3}-\frac{1}{72}a^3c^2x^7\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (47*c^2*x*Sqrt[c + a^2*c*x^2])/(2688*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2
])/(12096*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2])/3024 - (a^3*c^2*x^7*Sqrt
[c + a^2*c*x^2])/72 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^4) + (c
^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6
3 + (a^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/9 + (115*c^(5/2)*ArcTanh[
(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(8064*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx = c\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx + \left(a^2c\right)\int x^5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)$$

$$= c^2\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx + 2\left(\left(a^2c^2\right)\int x^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax\right.$$

$$= \frac{1}{5}c^2x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{9}a^4c^2x^8\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{5}c^3\int$$

$$= -\frac{c^2x^3\sqrt{c+a^2cx^2}}{20a} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2} + \frac{c^2x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax}{15a^2}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{24a^3} - \frac{c^2x^3\sqrt{c+a^2cx^2}}{20a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{24a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{7}$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{7}$$

$$= \frac{127c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^2}}{60480a} + \frac{41ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{7}$$

Mathematica [A]
time = 0.17, size = 129, normalized size = 0.45

$$\frac{c^2\left(-ax\sqrt{c+a^2cx^2}\left(-423+410a^2x^2+824a^4x^4+336a^6x^6\right)+384\left(1+a^2x^2\right)^3\left(-2+7a^2x^2\right)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+345\sqrt{c}\,\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)\right)}{24192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(-423 + 410*a^2*x^2 + 824*a^4*x^4 + 336*a^6
*x^6)) + 384*(1 + a^2*x^2)^3*(-2 + 7*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x] + 345*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(24192*a^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.45, size = 225, normalized size = 0.78

method	result
default	$\frac{c^{2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(2688\arctan\left(ax\right)a^{8}x^{8}-336a^{7}x^{7}+7296\arctan\left(ax\right)a^{6}x^{6}-824a^{5}x^{5}+5760\arctan\left(ax\right)a^{4}x^{4}-410a^{3}x^{3}+\right.}{24192a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24192*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(2688*arctan(a*x)*a^8*x^8-336*a^7
*x^7+7296*arctan(a*x)*a^6*x^6-824*a^5*x^5+5760*arctan(a*x)*a^4*x^4-410*a^3*
x^3+384*arctan(a*x)*a^2*x^2+423*a*x-768*arctan(a*x))-115/8064*c^2/a^4*(c*(a
*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)+11
5/8064*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/
(a^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.43, size = 338, normalized size = 1.17

$$-\frac{1}{24192}\left(\left(\left(7\left(\frac{48\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{5}}{a^{2}}-\frac{40\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{3}}{a^{4}}+\frac{30\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{a^{6}}-\frac{15\sqrt{a^{2}x^{2}+1}\,x}{a^{6}}-\frac{15\,\mathrm{arsinh}\left(ax\right)}{a^{7}}\right)a^{2}c^{2}+96\left(\frac{8\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{3}}{a^{2}}-\frac{6\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{a^{4}}+\frac{3\sqrt{a^{2}x^{2}+1}\,x}{a^{4}}+\frac{3\,\mathrm{arsinh}\left(ax\right)}{a^{5}}\right)c^{2}+\frac{144\,c^{2}\left(\frac{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{a^{2}}-\frac{\sqrt{a^{2}x^{2}+1}\,x}{a^{2}}-\frac{\mathrm{arsinh}\left(ax\right)}{a^{3}}\right)}{a^{2}}-\frac{384\left(\sqrt{a^{2}x^{2}+1}\,x+\frac{\mathrm{arsinh}\left(ax\right)}{a}\right)c^{2}}{a^{4}}\right)a-384\left(7\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}a^{2}c^{2}x^{6}+12\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{2}x^{4}+\frac{3\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{2}x^{2}}{a^{2}}-\frac{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{2}}{a^{4}}\right)\arctan\left(ax\right)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/24192*((7*(48*(a^2*x^2 + 1)^(3/2)*x^5/a^2 - 40*(a^2*x^2 + 1)^(3/2)*x^3/a
^4 + 30*(a^2*x^2 + 1)^(3/2)*x/a^6 - 15*sqrt(a^2*x^2 + 1)*x/a^6 - 15*arcsinh
(a*x)/a^7)*a^2*c^2 + 96*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3
/2)*x/a^4 + 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c^2 + 144*c^2*(
2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a
^2 - 384*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)*c^2/a^4)*a - 384*(7*(a^2*x^
2 + 1)^(3/2)*a^2*c^2*x^6 + 12*(a^2*x^2 + 1)^(3/2)*c^2*x^4 + 3*(a^2*x^2 + 1)
^(3/2)*c^2*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c^2/a^4)*arctan(a*x))*sqrt(c)
```

Fricas [A]
time = 2.18, size = 154, normalized size = 0.53

$$\frac{345\,c^{\frac{5}{2}}\log\left(-2\,a^{2}cx^{2}-2\,\sqrt{a^{2}cx^{2}+c}\,a\sqrt{c}\,x-c\right)-2\left(336\,a^{7}c^{2}x^{7}+824\,a^{5}c^{2}x^{5}+410\,a^{3}c^{2}x^{3}-423\,ac^{2}x-384\left(7\,a^{8}c^{2}x^{8}+19\,a^{6}c^{2}x^{6}+15\,a^{4}c^{2}x^{4}+a^{2}c^{2}x^{2}-2\,c^{2}\right)\arctan\left(ax\right)\right)\sqrt{a^{2}cx^{2}+c}}{48384\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/48384*(345*c^(5/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x -
 c) - 2*(336*a^7*c^2*x^7 + 824*a^5*c^2*x^5 + 410*a^3*c^2*x^3 - 423*a*c^2*x
```

```
- 384*(7*a^8*c^2*x^8 + 19*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + a^2*c^2*x^2 - 2*c^
2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.217 $\int x^2(c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)\,dx$

Optimal. Leaf size=418

$$\frac{5c^2\sqrt{c+a^2cx^2}}{128a^3}+\frac{5c(c+a^2cx^2)^{3/2}}{576a^3}+\frac{(c+a^2cx^2)^{5/2}}{240a^3}-\frac{(c+a^2cx^2)^{7/2}}{56a^3c}+\frac{5c^2x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{128a^2}+\frac{59}{192}c^2x$$

```
[Out] 5/576*c*(a^2*c*x^2+c)^(3/2)/a^3+1/240*(a^2*c*x^2+c)^(5/2)/a^3-1/56*(a^2*c*x
^2+c)^(7/2)/a^3/c+5/64*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-5/128*I*c^3*polylog(2,-I*(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+5/
128*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^
3/(a^2*c*x^2+c)^(1/2)+5/128*c^2*(a^2*c*x^2+c)^(1/2)/a^3+5/128*c^2*x*arctan(
a*x)*(a^2*c*x^2+c)^(1/2)/a^2+59/192*c^2*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
+17/48*a^2*c^2*x^5*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/8*a^4*c^2*x^7*arctan(a
*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 1.43, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5070, 5066, 5072, 267, 5010, 5006, 272, 45}

$$\frac{5c^2x\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{128a^2}+\frac{17}{48}a^2c^2x^5\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{59}{192}c^2x^3\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{8}a^4c^2x^7\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{64a^3\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{128a^3\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{128a^3}-\frac{(a^2cx^2+c)^{7/2}}{56a^3c}+\frac{(a^2cx^2+c)^{5/2}}{240a^3}+\frac{5c(a^2cx^2+c)^{3/2}}{576a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(128*a^3) + (5*c*(c + a^2*c*x^2)^(3/2))/(576*a^
3) + (c + a^2*c*x^2)^(5/2)/(240*a^3) - (c + a^2*c*x^2)^(7/2)/(56*a^3*c) + (
5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(128*a^2) + (59*c^2*x^3*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/48 + (a^4*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/8 + (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*
Sqrt[c + a^2*c*x^2]) - (((5*I)/128)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((5*I)/128)
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
3*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx = c\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(a$$

$$= c^2\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx+2\Bigg(\left(a^2c^2\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}($$

$$= \frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{4}c^3\int$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{48}a^2c^2x^5$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{8a^3}+\frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a^2}+\frac{43}{192}c^2x^3\sqrt{c+a^2cx^2}\,\mathrm{t}$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{4a^3}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{24a^3}+\frac{3\left(c+a^2cx^2\right)^{5/2}}{40a^3}-\frac{\left(c+a^2cx^2\right)^{7/2}}{56a^3c}$$

$$= \frac{73c^2\sqrt{c+a^2cx^2}}{384a^3}-\frac{7c\left(c+a^2cx^2\right)^{3/2}}{36a^3}+\frac{17\left(c+a^2cx^2\right)^{5/2}}{240a^3}-\frac{\left(c+a^2cx^2\right)^{7}}{56a^3c}$$

$$= \frac{21c^2\sqrt{c+a^2cx^2}}{128a^3}-\frac{107c\left(c+a^2cx^2\right)^{3/2}}{576a^3}+\frac{17\left(c+a^2cx^2\right)^{5/2}}{240a^3}-\frac{\left(c+a^2cx\right.}{56a^3c}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 907 vs. 2(418) = 836.
time = 10.82, size = 907, normalized size = 2.17

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]

[Out] (c^2*Sqrt[c + a^2*c*x^2]*((-19067*(1 + a^2*x^2)^(7/2))/32 - (3829*(1 + a^2*
x^2)^4*Cos[3*ArcTan[a*x]])/32 - (3150*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 + (3150*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 420*(1 + a^2*x^2)^2*(-2/Sqrt[
1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2
*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^
(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[
1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I
*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + 7*(1 + a^2*x^2)^3*(12/Sqrt[1 + a^
2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*Cos[5*ArcTan[a*x]] + 15*ArcTan[a*x]*((
156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*Cos[6*ArcT
an[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^
(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*ArcTan[a*x]]*(L
og[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 30*Log[1 + I*
E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - 94
*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]])) - (35*(1 + a^2*x^2)^4*(314*Cos
[5*ArcTan[a*x]] - 90*Cos[7*ArcTan[a*x]] + 3*ArcTan[a*x]*((-3530*a*x)/Sqrt[1
 + a^2*x^2] + 525*Log[1 - I*E^(I*ArcTan[a*x])] + 120*Cos[6*ArcTan[a*x]]*Log
[1 - I*E^(I*ArcTan[a*x])] + 15*Cos[8*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x
])] + 840*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I
*ArcTan[a*x])]) + 420*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Lo
g[1 + I*E^(I*ArcTan[a*x])]) - 525*Log[1 + I*E^(I*ArcTan[a*x])] - 120*Cos[6*
ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - 15*Cos[8*ArcTan[a*x]]*Log[1 + I
*E^(I*ArcTan[a*x])] + 1790*Sin[3*ArcTan[a*x]] - 794*Sin[5*ArcTan[a*x]] + 30
*Sin[7*ArcTan[a*x]])))/64))/(80640*a^3*Sqrt[1 + a^2*x^2])

Maple [A]
time = 0.55, size = 245, normalized size = 0.59

method	result
default	$\frac{c^2 \sqrt{c\,(ax-i)\,(ax+i)}\,(5040 \arctan(ax) a^7 x^7 - 720 a^6 x^6 + 14280 \arctan(ax) a^5 x^5 - 1992 a^4 x^4 + 12390 \arctan(ax) a^3 x^3 - 1474 a^2 x}{40320 a^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)

[Out] 1/40320*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(5040*arctan(a*x)*a^7*x^7-720*a^6
*x^6+14280*arctan(a*x)*a^5*x^5-1992*a^4*x^4+12390*arctan(a*x)*a^3*x^3-1474*
a^2*x^2+1575*arctan(a*x)*a*x+1373)+5/128*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2
+1)^(1/2)/a^3*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*
ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan
(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.218 $\int x(c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=134

$$-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx(c+a^2cx^2)^{3/2}}{168a}-\frac{x(c+a^2cx^2)^{5/2}}{42a}+\frac{(c+a^2cx^2)^{7/2}\operatorname{ArcTan}(ax)}{7a^2c}-\frac{5c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{112a^2}$$

```
[Out] -5/168*c*x*(a^2*c*x^2+c)^(3/2)/a-1/42*x*(a^2*c*x^2+c)^(5/2)/a+1/7*(a^2*c*x^
2+c)^(7/2)*arctan(a*x)/a^2/c-5/112*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c
)^(1/2))/a^2-5/112*c^2*x*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 201, 223, 212}

$$\frac{\operatorname{ArcTan}(ax)\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{5c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{112a^2}-\frac{5c^2x\sqrt{a^2cx^2+c}}{112a}-\frac{x\left(a^2cx^2+c\right)^{5/2}}{42a}-\frac{5cx\left(a^2cx^2+c\right)^{3/2}}{168a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (-5*c^2*x*Sqrt[c + a^2*c*x^2])/(112*a) - (5*c*x*(c + a^2*c*x^2)^(3/2))/(168
*a) - (x*(c + a^2*c*x^2)^(5/2))/(42*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
)/(7*a^2*c) - (5*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(112*a
^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx = \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c} - \frac{\int\left(c+a^2cx^2\right)^{5/2}\,dx}{7a}$$
$$= -\frac{x\left(c+a^2cx^2\right)^{5/2}}{42a} + \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c} - \frac{(5c)\int\left(c+a^2cx^2\right)^{3/2}\,dx}{42a}$$
$$= -\frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a} - \frac{x\left(c+a^2cx^2\right)^{5/2}}{42a} + \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c} - \frac{(5c}{}$$
$$= -\frac{5c^2x\sqrt{c+a^2cx^2}}{112a} - \frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a} - \frac{x\left(c+a^2cx^2\right)^{5/2}}{42a} + \frac{\left(c+a^2cx^2\right.}{}$$
$$= -\frac{5c^2x\sqrt{c+a^2cx^2}}{112a} - \frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a} - \frac{x\left(c+a^2cx^2\right)^{5/2}}{42a} + \frac{\left(c+a^2cx^2\right.}{}$$
$$= -\frac{5c^2x\sqrt{c+a^2cx^2}}{112a} - \frac{5cx\left(c+a^2cx^2\right)^{3/2}}{168a} - \frac{x\left(c+a^2cx^2\right)^{5/2}}{42a} + \frac{\left(c+a^2cx^2\right.}{}$$

Mathematica [A]
time = 0.15, size = 111, normalized size = 0.83

$$\frac{c^2\left(-ax\sqrt{c+a^2cx^2}\,(33+26a^2x^2+8a^4x^4)+48(1+a^2x^2)^3\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)-15\sqrt{c}\,\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)\right)}{336a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(33 + 26*a^2*x^2 + 8*a^4*x^4)) + 48*(1 + a^
2*x^2)^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - 15*Sqrt[c]*Log[a*c*x + Sqrt[c]*S
qrt[c + a^2*c*x^2]]))/(336*a^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 205, normalized size = 1.53

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(48\arctan(ax)a^6x^6-8a^5x^5+144\arctan(ax)a^4x^4-26a^3x^3+144\arctan(ax)a^2x^2-33ax+48\arctan(ax)}{336a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)

[Out] 1/336*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(48*arctan(a*x)*a^6*x^6-8*a^5*x^5+144*arctan(a*x)*a^4*x^4-26*a^3*x^3+144*arctan(a*x)*a^2*x^2-33*a*x+48*arctan(a*x))-5/112*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)+5/112*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(110) = 220$.
time = 0.64, size = 637, normalized size = 4.75

$$\frac{560\,(a^2c^2x^2+c^2)\sqrt{a^2x^2+1}\,\sqrt{c}\,\arctan(ax)-280\,(a^4x^4+10\,a^2x^2+9)^{\frac14}\left(ac^2x\cos\left(\tfrac12\arctan(4\,ax,-a^2x^2+3)\right)+2\,c^2\sin\left(\tfrac12\arctan(4\,ax,-a^2x^2+3)\right)\right)\sqrt{c}-\left(\left(a\left(\frac{5\left(\frac{8\,(a^2x^2+1)^{\frac32}x^3}{a^2}-\frac{6\,(a^2x^2+1)^{\frac32}x}{a^4}+\frac{3\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\,\operatorname{arsinh}(ax)}{a^5}\right)}{a^2}-\frac{24\left(\frac{2\,(a^2x^2+1)^{\frac32}x}{a^2}-\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)}{a^4}+\frac{64\left(\sqrt{a^2x^2+1}\,x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^6}\right)-16\left(\frac{15\,(a^2x^2+1)^{\frac32}x^4}{a^2}-\frac{12\,(a^2x^2+1)^{\frac32}x^2}{a^4}+\frac{8\,(a^2x^2+1)^{\frac32}}{a^6}\right)\arctan(ax)\right)a^6c^2+28\left(a\left(\frac{3\left(\frac{2\,(a^2x^2+1)^{\frac32}x}{a^2}-\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{a^2x^2+1}\,x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^4}\right)-8\left(\frac{3\,(a^2x^2+1)^{\frac32}x^2}{a^2}-\frac{2\,(a^2x^2+1)^{\frac32}}{a^4}\right)\arctan(ax)\right)a^4c^2-140\,c^2\arctan\left((a^4x^4+10\,a^2x^2+9)^{\frac14}\sin\left(\tfrac12\arctan(4\,ax,a^2x^2-3)\right)+2,\ ax+(a^4x^4+10\,a^2x^2+9)^{\frac14}\cos\left(\tfrac12\arctan(4\,ax,a^2x^2-3)\right)\right)-140\,c^2\arctan\left((a^4x^4+10\,a^2x^2+9)^{\frac14}\sin\left(\tfrac12\arctan(4\,ax,a^2x^2-3)\right)-2,\ -ax+(a^4x^4+10\,a^2x^2+9)^{\frac14}\cos\left(\tfrac12\arctan(4\,ax,a^2x^2-3)\right)\right)\right)\sqrt{c}}{1680\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")

[Out] 1/1680*(560*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 280*(a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(5*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 + 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)/a^2 - 24*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^4 + 64*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^6) - 16*(15*(a^2*x^2 + 1)^(3/2)*x^4/a^2 - 12*(a^2*x^2 + 1)^(3/2)*x^2/a^4 + 8*(a^2*x^2 + 1)^(3/2)/a^6)*arctan(a*x))*a^6*c^2 + 28*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 140*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) - 140*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))))*sqrt(c))/a^2

Fricas [A]
time = 1.27, size = 130, normalized size = 0.97

$$\frac{15\,c^{\frac52}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)-2\,(8\,a^5c^2x^5+26\,a^3c^2x^3+33\,ac^2x-48\,(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2)\arctan(ax))\sqrt{a^2cx^2+c}}{672\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")

[Out] 1/672*(15*c^(5/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c) - 2*(8*a^5*c^2*x^5 + 26*a^3*c^2*x^3 + 33*a*c^2*x - 48*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\big(c\big(a^2x^2+1\big)\big)^{\frac{5}{2}}\operatorname{atan}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)

[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)

[Out] int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)

3.219 $\int (c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=348

$$-\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c(c+a^2cx^2)^{3/2}}{72a}-\frac{(c+a^2cx^2)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{5}{24}cx(c+a^2cx^2)^{3/2}\,\text{A}$$

```
[Out] -5/72*c*(a^2*c*x^2+c)^(3/2)/a-1/30*(a^2*c*x^2+c)^(5/2)/a+5/24*c*x*(a^2*c*x^
2+c)^(3/2)*arctan(a*x)+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)-5/8*I*c^3*arct
an(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)+5/16*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/16*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/16*c^2*(a^2*c*x^2
+c)^(1/2)/a+5/16*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4998, 5010, 5006}

$$-\frac{5ic^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a\sqrt{a^2cx^2+c}}+\frac{5}{16}c^2x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{5}{24}cx\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}+\frac{1}{6}x\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{5/2}+\frac{5ic^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{16a\sqrt{a^2cx^2+c}}-\frac{5c^2\sqrt{a^2cx^2+c}}{16a}-\frac{5c(a^2cx^2+c)^{3/2}}{72a}-\frac{(a^2cx^2+c)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (-5*c^2*Sqrt[c + a^2*c*x^2])/(16*a) - (5*c*(c + a^2*c*x^2)^(3/2))/(72*a) -
(c + a^2*c*x^2)^(5/2)/(30*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/16
 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/24 + (x*(c + a^2*c*x^2)^(5/2)*
ArcTan[a*x])/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((5*I)/16)*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c
+ a^2*c*x^2]) - (((5*I)/16)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
```

```
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)+\frac{1}{6}(5c)\int \left(c+a^2cx^2\right)^{3/2}$$

$$= -\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)+\frac{1}{6}$$

$$= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2c}$$

$$= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2c}$$

$$= -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}+\frac{5}{16}c^2x\sqrt{c+a^2c}$$

Mathematica [A]
time = 4.24, size = 643, normalized size = 1.85

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + 720*Sqrt[1 + a^2*x^2]
*(-1 + a*x*ArcTan[a*x]) + (55*(1 + a^2*x^2)^3*Cos[3*ArcTan[a*x]])/8 - (45*(
1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 + 720*ArcTan[a*x]*(Log[1 - I*E^(I*ArcT
an[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (450*I)*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] - (450*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 15*(1 + a^2*x^2)^2*
(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqr
```

```
t[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log
[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*
x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1
 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + (15*(1 + a^2*x^2)^3*ArcT
an[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*
Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(Lo
g[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*ArcTa
n[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 30*
Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a
*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]))/16))/(1440*a*Sqrt[1
+ a^2*x^2])
```

Maple [A]
time = 0.29, size = 225, normalized size = 0.65

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+390\arctan(ax)a^3x^3-98a^2x^2+495\arctan(ax)ax-299)}{720a} - \frac{5\sqrt{c\,(ax}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/720*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+3
90*arctan(a*x)*a^3*x^3-98*a^2*x^2+495*arctan(a*x)*a*x-299)-5/16*(c*(a*x-I)*
(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}} \operatorname{atan}\left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.220 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}{x}\,dx$

Optimal. Leaf size=329

$$-\frac{29}{120}ac^2x\sqrt{c+a^2cx^2}-\frac{1}{20}acx(c+a^2cx^2)^{3/2}+c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{1}{3}c(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)+\frac{1}{5}$$

```
[Out] -1/20*a*c*x*(a^2*c*x^2+c)^(3/2)+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/5*(
a^2*c*x^2+c)^(5/2)*arctan(a*x)-149/120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x
^2+c)^(1/2))-2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*
x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*c^3*polylog(2,(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-29/120*a*c^2*x*
(a^2*c*x^2+c)^(1/2)+c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5066, 5078, 5074, 223, 212, 5050, 201}

$$-\frac{2c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{3}c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}+\frac{1}{5}\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{149}{120}c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{ic^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ic^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{29}{120}ac^2x\sqrt{a^2cx^2+c}-\frac{1}{20}acx\left(a^2cx^2+c\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

```
[Out] (-29*a*c^2*x*Sqrt[c + a^2*c*x^2])/120 - (a*c*x*(c + a^2*c*x^2)^(3/2))/20 +
c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/5 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (149
*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/120 + (I*c^3*Sqrt[1 +
a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2
] - (I*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/S
qrt[c + a^2*c*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
```

```
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}{x}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x}\,dx + (a^2c)\int x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)\,dx$$

$$= \frac{1}{5}(c+a^2cx^2)^{5/2}\tan^{-1}(ax) - \frac{1}{5}(ac)\int (c+a^2cx^2)^{3/2}\,dx + c^2\int \frac{\sqrt{c+a^2cx}}{}$$

$$= -\frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{3}c(c+a^2cx^2)^{3/2}\tan^{-}$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)$$

$$= -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)$$

Mathematica [A]
time = 0.23, size = 281, normalized size = 0.85

$$\frac{c^2\sqrt{c+a^2cx^2}\left(-35ax\sqrt{1+a^2x^2}-6a^3x^3\sqrt{1+a^2x^2}+184\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)+88a^2x^2\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)+24a^4x^4\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)-29\tanh^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+120\mathrm{ArcTan}(ax)\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-120\mathrm{ArcTan}(ax)\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+120\log\left(\cos\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-\sin\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)-120\log\left(\cos\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)+\sin\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)+120i\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-120i\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)\right)}{120\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-35*a*x*Sqrt[1 + a^2*x^2] - 6*a^3*x^3*Sqrt[1 + a^
2*x^2] + 184*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 88*a^2*x^2*Sqrt[1 + a^2*x^2]*A
rcTan[a*x] + 24*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 29*ArcTanh[(a*x)/Sq
rt[1 + a^2*x^2]] + 120*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 120*ArcTan[
a*x]*Log[1 + E^(I*ArcTan[a*x])] + 120*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a
*x]/2]] - 120*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (120*I)*PolyLo
g[2, -E^(I*ArcTan[a*x])] - (120*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(120*Sqr
t[1 + a^2*x^2])
```

Maple [A]
time = 0.31, size = 198, normalized size = 0.60

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(24\arctan(ax)a^4x^4-6a^3x^3+88\arctan(ax)a^2x^2-35ax+184\arctan(ax))}{120}-\frac{\sqrt{c(ax-i)(ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)*a^4*x^4-6*a^3*x^3+88*ar
ctan(a*x)*a^2*x^2-35*a*x+184*arctan(a*x))-1/60*(c*(a*x-I)*(I+a*x))^(1/2)/(a
^2*x^2+1)^(1/2)*(60*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*I*arc
tan((1+I*a*x)/(a^2*x^2+1)^(1/2))-60*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-60
*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/3*(a^4*x^
4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) +
2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - 1/120*((a*(3*(2*(a^2
*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 8
*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2
/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 20*c^2*arctan2((a^
4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x
 + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) -
 20*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2
*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a
*x, a^2*x^2 - 3))) - 120*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x)/x, x))
*sqrt(c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.221 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}{x^2}\,dx$

Optimal. Leaf size=355

$$-\frac{7}{8}ac^2\sqrt{c+a^2cx^2}-\frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2}-\frac{c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}+\frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{1}{4}a$$

```
[Out] -1/12*a*c*(a^2*c*x^2+c)^(3/2)+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-a
*c^(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-15/4*I*a*c^3*arctan(a*x)*arct
an((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1
5/8*I*a*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-15/8*I*a*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7/8*a*c^2*(a^2*c*x^2+c)^(1/2)-c^2
*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+7/8*a^2*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^
(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5070, 5064, 272, 65, 214, 5010, 5006, 4998}

$$-\frac{15iac^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4\sqrt{a^2cx^2+c}}+\frac{7}{8}a^2c^2x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}+\frac{1}{4}a^2cx\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}-ac^{5/2}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)+\frac{15iac^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{15iac^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{8\sqrt{a^2cx^2+c}}-\frac{7}{8}ac^2\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(a^2cx^2+c\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (-7*a*c^2*Sqrt[c + a^2*c*x^2])/8 - (a*c*(c + a^2*c*x^2)^(3/2))/12 - (c^2*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/8 + (a^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 - (((15*I)/4)*a*c^3
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqr
t[c + a^2*c*x^2] - a*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (((15*I
)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] - (((15*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}{x^2}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^2}\,dx + \left(a^2c\right)\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$$

$$= -\frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} + \frac{1}{4}a^2cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax) + c^2\int \frac{\sqrt{c+a^2cx}}{}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{7}{8}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{7}{8}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{7}{8}$$

$$= -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac\left(c+a^2cx^2\right)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{x} + \frac{7}{8}$$

Mathematica [A]
time = 2.76, size = 491, normalized size = 1.38

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (a*c^2*Sqrt[c + a^2*c*x^2]*((1 + a^2*x^2)^(3/2)/2 + 48*Sqrt[1 + a^2*x^2]*(-
1 + a*x*ArcTan[a*x]) + (3*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]])/2 + 48*ArcTan
[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (42*I
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 48*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/
(a*x) - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^
(I*ArcTan[a*x])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*Po
```

```
lyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]) - (42
*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (3*(1 + a^2*x^2)^2*ArcTan[a*x]*((-14*
a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*
x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*A
rcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) -
 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]]))/4))/(48*Sqrt[1 + a
^2*x^2])
```

Maple [A]
time = 0.32, size = 265, normalized size = 0.75

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\;(6\arctan(ax)a^4x^4-2a^3x^3+27\arctan(ax)a^2x^2-23ax-24\arctan(ax))}{24x}+\frac{\sqrt{c\,(ax-i)\,(ax+i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*a^4*x^4-2*a^3*x^3+27*arct
an(a*x)*a^2*x^2-23*a*x-24*arctan(a*x))/x+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(15*
arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+8*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-8*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-15*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*dilog
(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a*c^2/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\,\mathrm{atan}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.222 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}{x^3}\,dx$

Optimal. Leaf size=364

$$-\frac{ac^2\sqrt{c+a^2cx^2}}{2x}-\frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2}+2a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2x^2}+\frac{1}{3}a^2c(c$$

```
[Out] 1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-13/6*a^2*c^(5/2)*arctanh(a*x*c^(1
/2)/(a^2*c*x^2+c)^(1/2))-5*a^2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I
*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/2*I*a^2*c^3*polylog(2,
-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5/2
*I*a^2*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)-1/2*a*c^2*(a^2*c*x^2+c)^(1/2)/x-1/6*a^3*c^2*x*(a^2*c*x^2+c
)^(1/2)+2*a^2*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*c^2*arctan(a*x)*(a^2*
c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.82, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5070, 5066, 5082, 270, 5078, 5074, 223, 212, 5050, 201}

$$-\frac{5a^2c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+2a^2c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2x^2}+\frac{1}{3}a^2c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{13}{6}a^2c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{5ia^2c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^2c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{ac^2\sqrt{a^2cx^2+c}}{2x}-\frac{1}{6}a^3c^2x\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*(a*c^2*Sqrt[c + a^2*c*x^2])/x - (a^3*c^2*x*Sqrt[c + a^2*c*x^2])/6 + 2*
a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/3 - (5*a^2*c^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (13*a^2*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/6 + (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
```

```
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}{x^3}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x^3}\,dx + (a^2c)\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^3}\,dx + 2\left((a^2c^2)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}\,dx\right)$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2} + \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax) - \frac{1}{3}(a^3c^2)\int \sqrt{}$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a$$

$$= -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2x^2} + \frac{1}{3}a$$

Mathematica [A]
time = 1.43, size = 374, normalized size = 1.03

$$\frac{a^2c^2\sqrt{c+a^2cx^2}\left(-6-4\tanh^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)-6\cot^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-2a^2x^2\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+28ax\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+4a^3x^3\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-3\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+60\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-60\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+48\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-48\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)+\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+60i\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-60i\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+3\text{ArcTan}(ax)\csc\left(\frac{1}{2}\text{ArcTan}(ax)\right)\sec\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{24\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c + a^2*c*x^2]*(-6 - 4*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]]*Cot[A
rcTan[a*x]/2] - 6*Cot[ArcTan[a*x]/2]^2 - 2*a^2*x^2*Csc[ArcTan[a*x]/2]^2 + 2
8*a*x*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 4*a^3*x^3*ArcTan[a*x]*Csc[ArcTan[a
*x]/2]^2 - 3*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 60*ArcTa
n[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 60*ArcTan[a*x]*Cot[A
rcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 48*Cot[ArcTan[a*x]/2]*Log[Cos[Ar
cTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 48*Cot[ArcTan[a*x]/2]*Log[Cos[ArcTan[a
*x]/2] + Sin[ArcTan[a*x]/2]] + (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*A
rcTan[a*x])] - (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 3*
ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(24*
Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.42, size = 204, normalized size = 0.56

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(2\arctan(ax)a^4x^4-a^3x^3+14\arctan(ax)a^2x^2-3ax-3\arctan(ax)\right)}{6x^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)*a^4*x^4-a^3*x^3+14*arctan(
a*x)*a^2*x^2-3*a*x-3*arctan(a*x))/x^2-1/6*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^
2+1)^(1/2)*(15*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*arctan((1
+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*dil
og(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a^2*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^4*c^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a^
4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a^3*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 +
3)) + 2*a^2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(a^2*
c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2
 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a
^2*x^2 - 3))) + a^2*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*ar
ctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos
(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + 24*a^2*c^2*integrate(sqrt(a^2*x^2 + 1)
*arctan(a*x)/x, x) + 12*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x)/x^3, x)
)*sqrt(c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3,x)
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.223 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}{x^4}\,dx$

Optimal. Leaf size=372

$$-\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2}-\frac{ac^2\sqrt{c+a^2cx^2}}{6x^2}-\frac{2a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}+\frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{c(c$$

[Out] -1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-13/6*a^3*c^(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-5*I*a^3*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/2*I*a^3*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5/2*I*a^3*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a^3*c^2*(a^2*c*x^2+c)^(1/2)-1/6*a*c^2*(a^2*c*x^2+c)^(1/2)/x^2-2*a^2*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+1/2*a^4*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.70, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5070, 5064, 272, 43, 65, 214, 5010, 5006, 4998}

$$-\frac{2a^2c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-\frac{c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{3x^3}-\frac{ac^2\sqrt{a^2cx^2+c}}{6x^2}+\frac{1}{2}a^4c^2x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{13}{6}a^3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)+\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{1}{2}a^3c^2\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]

[Out] -1/2*(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (a*c^2*Sqrt[c + a^2*c*x^2])/(6*x^2) - (2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*x^3) - ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/6 + (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((5*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
```

```
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{x^4}\,dx = c\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^4}\,dx + \left(a^2c\right)\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^4}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^2}\,dx\right)$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan}{3x^3}$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c\left(c+a^2cx^2\right)^{3/2}\tan}{3x^3}$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) -$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) -$$

$$= -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) -$$

Mathematica [A]

time = 0.75, size = 313, normalized size = 0.84

$$\frac{c^2\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-3a^3x^3\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)-14a^2x^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+3a^4x^4\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)-a^3x^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)+15a^3x^3\text{ArcTan}(ax)\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-15a^3x^3\text{ArcTan}(ax)\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-12a^3x^3\log\left(\cos\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+12a^3x^3\log\left(\sin\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)+15ia^3x^3\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-15ia^3x^3\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{6x^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]

[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 3*a^3*x^3*Sqrt[1 + a^2 *x^2] - 2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 14*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcT an[a*x] + 3*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - a^3*x^3*ArcTanh[Sqrt[1 + a^2*x^2]] + 15*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - 15*a^3* x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 12*a^3*x^3*Log[Cos[ArcTan[a* x]/2]] + 12*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] + (15*I)*a^3*x^3*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (15*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(6* x^3*Sqrt[1 + a^2*x^2])

Maple [A]
time = 0.52, size = 270, normalized size = 0.73

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,(3\arctan(ax)a^4x^4-3a^3x^3-14\arctan(ax)a^2x^2-ax-2\arctan(ax))}{6x^3}-\frac{ia^3c^2\sqrt{c\,(ax-i)\,(ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)*a^4*x^4-3*a^3*x^3-14*arcta n(a*x)*a^2*x^2-a*x-2*arctan(a*x))/x^3-1/6*I*a^3*c^2*(c*(a*x-I)*(I+a*x))^(1/ 2)*(15*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)*l n(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+13*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-1 3*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1 /2))-15*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^4, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

```
[Out] int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.224 $\int \frac{x^3 \mathbf{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{x\sqrt{c+a^2cx^2}}{6a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3a^2c}+\frac{5\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{6a^4\sqrt{c}}$$

```
[Out] 5/6*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(1/2)-1/6*x*(a^2*c*x^2+c
)^(1/2)/a^3/c-2/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)
*(a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5072, 327, 223, 212, 5050}

$$\frac{x^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3a^4c}+\frac{5\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{6a^4\sqrt{c}}-\frac{x\sqrt{a^2cx^2+c}}{6a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/6*(x*Sqrt[c + a^2*c*x^2])/(a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) + (5*ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^4*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{\int \frac{x^2}{\sqrt{c+a^2cx^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^2c} + \frac{\int \frac{1}{\sqrt{c+a^2c}}}{6a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^2c} + \frac{\text{Subst}\Big(\int \frac{}{1-}}{}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^2c} + \frac{5\tanh^{-1}\Big(}{6a}$$

Mathematica [A]
time = 0.08, size = 91, normalized size = 0.76

$$\frac{-ax\sqrt{c+a^2cx^2} + 2(-2+a^2x^2)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax) + 5\sqrt{c}\,\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{6a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + 5*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*a^4*c)

Maple [C] Result contains complex when optimal does not.
time = 1.68, size = 165, normalized size = 1.38

method	result
default	$\frac{\left(2 \arctan \left(a x\right) a^{2} x^{2}-a x-4 \arctan \left(a x\right)\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{6 c \,a^{4}}+\frac{5 \ln \left(\frac{i a x+1}{\sqrt{a^{2} x^{2}+1}}+i\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{6 \sqrt{a^{2} x^{2}+1}\, a^{4} c}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(2*arctan(a*x)*a^2*x^2-a*x-4*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^4+5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c-5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c

Maxima [A]
time = 0.37, size = 89, normalized size = 0.74

$$-\frac{a \left(\frac{\frac{\sqrt{a^{2} x^{2}+1}\, x}{a^{2}}-\frac{\operatorname{arsinh}\left(a x\right)}{a^{3}}}{a^{2}}-\frac{4 \operatorname{arsinh}\left(a x\right)}{a^{5}}\right)-2 \left(\frac{\sqrt{a^{2} x^{2}+1}\, x^{2}}{a^{2}}-\frac{2 \sqrt{a^{2} x^{2}+1}}{a^{4}}\right) \arctan \left(a x\right)}{6 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] -1/6*(a*((sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 4*arcsinh(a*x)/a^5) - 2*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arctan(a*x))/sqrt(c)

Fricas [A]
time = 1.33, size = 80, normalized size = 0.67

$$-\frac{2 \sqrt{a^{2} c x^{2}+c} \left(a x-2 \left(a^{2} x^{2}-2\right) \arctan \left(a x\right)\right)-5 \sqrt{c} \log \left(-2 \, a^{2} c x^{2}-2 \sqrt{a^{2} c x^{2}+c}\, a \sqrt{c}\, x-c\right)}{12 \, a^{4} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] -1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 - 2)*arctan(a*x)) - 5*sqrt(c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^4*c)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}{(ax)}}{\sqrt{c(a^2x^2+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.225 $\int \frac{x^2\textbf{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=250

$$-\frac{\sqrt{c+a^2cx^2}}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2a^2c}+\frac{i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\,\text{P}\cdots}{2\cdots}$$

```
[Out] I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3
/(a^2*c*x^2+c)^(1/2)-1/2*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/2*I*polylog(2,I*(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-1/2*(a^2*c*x^2+c)
^(1/2)/a^3/c+1/2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.11, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5072, 267, 5010, 5006}

$$\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2a^2c}+\frac{i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a^3\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a
^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I
)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*Sqr
t[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c
 + a^2*c*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx &= \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{\int \frac{x}{\sqrt{c+a^2cx^2}}\,dx}{2a}\\ &= -\frac{\sqrt{c+a^2cx^2}}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2a^2c}-\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2\sqrt{c+a^2cx^2}}\\ &= -\frac{\sqrt{c+a^2cx^2}}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2a^2c}+\frac{i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right.}{a^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 158, normalized size = 0.63

$$-\frac{\sqrt{c\left(1+a^2x^2\right)}\left(\sqrt{1+a^2x^2}-ax\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)+\mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)+i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)-i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)\right)}{2a^3c\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/2*(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2] - a*x*Sqrt[1 + a^2*x^2]*ArcT
an[a*x] + ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + I*
E^(I*ArcTan[a*x])] + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*
E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.83, size = 184, normalized size = 0.74

method	result
default	$\frac{(\arctan(ax)ax-1)\sqrt{c(ax-i)(ax+i)}}{2ca^3} - \frac{i\left(i\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)\right.}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3-1/2*I*(I*arctan(a*x
)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.226 $\int \frac{x\mathbf{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{a^2c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a^2\sqrt{c}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2/c^(1/2)+arctan(a*x)*(a^2*c*x^
2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5050, 223, 212}

$$\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c +
a^2*c*x^2]]/(a^2*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2c} - \frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a} \\
&= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2c} - \frac{\text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)}{a} \\
&= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2c} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a^2\sqrt{c}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 60, normalized size = 1.02

$$
\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax) - \sqrt{c}\,\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{a^2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2
*c*x^2]])/(a^2*c)
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.39, size = 144, normalized size = 2.44

method	result
default	$\frac{\arctan(ax)\sqrt{c\,(ax-i)\,(ax+i)}}{a^2c} + \frac{\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)\sqrt{c\,(ax-i)\,(ax+i)}}{\sqrt{a^2x^2+1}\,a^2c} - \frac{\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)\sqrt{c}}{\sqrt{a^2x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c+ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-
I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c-ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

Maxima [**A**]
time = 0.54, size = 61, normalized size = 1.03

$$
\frac{2\sqrt{a^2x^2+1}\,\arctan(ax) - \log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a^2*x^2 + 1)*arctan(a*x) - log(a*x + sqrt(a^2*x^2 + 1)) + log(-
a*x + sqrt(a^2*x^2 + 1)))/(a^2*sqrt(c))
```

Fricas [**A**]
time = 1.55, size = 64, normalized size = 1.08

$$\frac{2\sqrt{a^2cx^2+c}\,\arctan(ax)+\sqrt{c}\,\log\left(-2\,a^2cx^2+2\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a^2*c*x^2 + c)*arctan(a*x) + sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(
a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^2*c)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.227 $\int \frac{\text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\,\text{Po}\ldots}{a\ldots}$$

```
[Out] -2*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a/(a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5010, 5006}

$$-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/(a*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2i\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} -$$

Mathematica [A]
time = 0.09, size = 118, normalized size = 0.61

$$\frac{\sqrt{c(1+a^2x^2)}\left(\text{ArcTan}(ax)\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{ac\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
 I*E^(I*ArcTan[a*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2
, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.22, size = 150, normalized size = 0.78

method	result
default	$\frac{i\left(i\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+\text{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\text{dilog}\left(1-\frac{i(iax+}{\sqrt{a^2x^2}}\right.\right.}{\sqrt{a^2x^2+1}\,ac}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)
/a/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(ax \right)}}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)/(c + a^2*c*x^2)^(1/2), x)
```

3.228 $\int \frac{\mathbf{ArcTan}(ax)}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=177

$$-\frac{2\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\,\mathrm{P}}{\sqrt{}}$$

```
[Out] -2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5078, 5074}

$$-\frac{2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/Sqrt[c + a^2*c*x^2] + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt
[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\mathrm{Li}_2\left(-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} -$$

Mathematica [**A**]
time = 0.12, size = 100, normalized size = 0.56

$$\frac{\sqrt{1+a^2x^2}\left(\mathrm{ArcTan}(ax)\left(\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)\right)+i\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-i\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)\right)}{\sqrt{c\left(1+a^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*
ArcTan[a*x])]) + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTa
n[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

Maple [**A**]
time = 0.25, size = 139, normalized size = 0.79

method	result
default	$-\frac{i\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\mathrm{polylog}\left(2,-\frac{}{\sqrt{a}}\right.\right.}{\sqrt{a^2x^2+1}\,c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I
*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.229 $\int \frac{\mathbf{ArcTan}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{cx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(
1/2)/c/x
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5064, 272, 65, 214}

$$-\frac{\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]])/Sqrt[c]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx}+a\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx\\ &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,dx,x,\sqrt{c+a^2cx^2}\right)}{ac}\\ &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 1.11

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{cx}+\frac{a\left(\log(x)-\log\left(c+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) + (a*(Log[x] - Log[c + Sqrt[c]*S
qrt[c + a^2*c*x^2]]))/Sqrt[c]
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 139, normalized size = 2.48

method	result
default	$-\frac{\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{cx}+\frac{a\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)\sqrt{c(ax-i)(ax+i)}}{\sqrt{a^2x^2+1}\,c}-\frac{a\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right.}{\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/x+a*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c-a*ln(1+(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [A]
time = 0.40, size = 36, normalized size = 0.64

$$-\frac{a \operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{\sqrt{a^2x^2+1}\arctan(ax)}{x}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*arcsinh(1/(a*abs(x))) + sqrt(a^2*x^2 + 1)*arctan(a*x)/x)/sqrt(c)
```

Fricas [A]
time = 1.63, size = 68, normalized size = 1.21

$$\frac{a\sqrt{c}\,x\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\sqrt{a^2cx^2+c}\,\arctan\left(ax\right)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c)/x^2
) - 2*sqrt(a^2*c*x^2 + c)*arctan(a*x))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}\left(ax\right)}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(a\,x)}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.230 $\int \frac{\mathbf{ArcTan}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{ia^2\sqrt{1+a^2x^2}}{\cdots}$$

```
[Out] a^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/2*I*a^2*polylog(2,(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^2+c)^(1
/2)/c/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5082, 270, 5078, 5074}

$$-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2cx^2}+\frac{a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c
*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]])/Sqrt[c + a^2*c*x^2] - ((I/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] + ((I/2)*a^2*Sqrt[1 +
 a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2cx^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2cx^2}-\frac{\left(a^2\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.54, size = 165, normalized size = 0.68

$$\frac{a^2\sqrt{1+a^2x^2}\left(-2\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-4\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)+4\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)-4i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+4i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-2\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{8\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[a*x]
/2]^2 - 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan[a*x]*Log[1 + E^
(I*ArcTan[a*x])] - (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2,
E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*x]/2
]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 0.46, size = 175, normalized size = 0.72

method	result
default	$-\frac{(ax+\arctan(ax))\sqrt{c\,(ax-i)\,(ax+i)}}{2c\,x^2}+\frac{ia^2\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2+1/2*I*a^2*(I*arctan(
a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^
2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}(ax)}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.231 $\int \frac{\text{ArcTan}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=118

$$-\frac{a\sqrt{c+a^2cx^2}}{6cx^2}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3cx}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] 5/6*a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*a*(a^2*c*x^2+c)^(1
/2)/c/x^2-1/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)*(a^
2*c*x^2+c)^(1/2)/c/x
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5082, 272, 44, 65, 214, 5064}

$$\frac{2a^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3cx}-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3cx^3}-\frac{a\sqrt{a^2cx^2+c}}{6cx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/6*(a*Sqrt[c + a^2*c*x^2])/(c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c*x) + (5*a^3*ArcTanh[
Sqrt[c + a^2*c*x^2]/Sqrt[c]])/(6*Sqrt[c])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^3}+\frac{1}{3}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx}+\frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+a^2cx}}\right.$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx}-\frac{1}{12}a^3\text{Sub}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx}-\frac{a\text{Subst}\left(\right.}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{6cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx}+\frac{5a^3\tanh}{}$$

Mathematica [A]
time = 0.09, size = 110, normalized size = 0.93

$$\frac{-ax\sqrt{c+a^2cx^2}+2(-1+2a^2x^2)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-5a^3\sqrt{c}\,x^3\log(x)+5a^3\sqrt{c}\,x^3\log\left(c+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{6cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-1 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x] - 5*a^3*Sqrt[c]*x^3*Log[x] + 5*a^3*Sqrt[c]*x^3*Log[c + Sqrt[c]*Sqrt[c
 + a^2*c*x^2]])/(6*c*x^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.87, size = 163, normalized size = 1.38

method	result
default	$\frac{(4\arctan(ax)a^2x^2-ax-2\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{6c\,x^3}+\frac{5a^3\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\sqrt{c(ax-i)(ax+i)}}{6\sqrt{a^2x^2+1}\,c}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(4*arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c/x
^3+5/6*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2
*x^2+1)^(1/2)/c-5/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(c*(a*x-I)*(I+a*x
))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [A]
time = 0.36, size = 81, normalized size = 0.69

$$\frac{\left(5\,a^2\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{\sqrt{a^2x^2+1}}{x^2}\right)a+2\left(\frac{2\sqrt{a^2x^2+1}\,a^2}{x}-\frac{\sqrt{a^2x^2+1}}{x^3}\right)\arctan(ax)}{6\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*((5*a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a + 2*(2*sqrt(a^
2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*arctan(a*x))/sqrt(c)
```

Fricas [A]
time = 7.05, size = 89, normalized size = 0.75

$$\frac{5\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2+2\sqrt{a^2cx^2+c}\,\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}\,(ax-2\,(2\,a^2x^2-1)\arctan(ax))}{12\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(5*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2
*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(2*a^2*x^2 - 1)*arctan(a*x)))/(c*
x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.232 $\int \frac{x^3 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{x}{a^3c\sqrt{c+a^2cx^2}}+\frac{\operatorname{ArcTan}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}{a^4c^2}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a^4c^{3/2}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(3/2)-x/a^3/c/(a^2*c*x^2+c)
^(1/2)+arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1/2
)/a^4/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5084, 5050, 223, 212, 197}

$$\frac{\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^4c^2}+\frac{\operatorname{ArcTan}(ax)}{a^4c\sqrt{a^2cx^2+c}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a^4c^{3/2}}-\frac{x}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(x/(a^3*c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(a^4*c*Sqrt[c + a^2*c*x^2])
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c^2) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[
c + a^2*c*x^2]]/(a^4*c^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{a^2}+\frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}\\ &= \frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{a^3}-\frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}\\ &= -\frac{x}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\mathrm{Subst}\left(\int \frac{1}{1-a^2cx^2}\right.}{}\\ &= -\frac{x}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a^4c^2}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}}{\sqrt{c+a^2}}\right.}{a^4c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 107, normalized size = 1.00

$$\frac{-ax\sqrt{c+a^2cx^2}+(2+a^2x^2)\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)-\sqrt{c}\,(1+a^2x^2)\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{a^4c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] (-(a*x*Sqrt[c + a^2*c*x^2]) + (2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
 - Sqrt[c]*(1 + a^2*x^2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^4*c^2
*(1 + a^2*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.63, size = 242, normalized size = 2.26

method	result
default	$\frac{(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a^4c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)a^4c^2}+\frac{\arctan(ax)\sqrt{c\,(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^4/c^2
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/a^4/c^
2+arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^2-ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^2+ln((1+I*a*x)/(a^
2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 2.80, size = 102, normalized size = 0.95

$$\frac{(a^2x^2+1)\sqrt{c}\,\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)-2\,\sqrt{a^2cx^2+c}\,(ax-(a^2x^2+2)\arctan{(ax)})}{2\,(a^6c^2x^2+a^4c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(
c)*x - c) - 2*sqrt(a^2*c*x^2 + c)*(a*x - (a^2*x^2 + 2)*arctan(a*x)))/(a^6*c
^2*x^2 + a^4*c^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.233 $\int \frac{x^2 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\text{ArcTan}(ax)}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}}{a^3c\sqrt{c+}}$$

```
[Out] -1/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*ar
ctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(
a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2
+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5054, 5010, 5006}

$$-\frac{x\text{ArcTan}(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a^3c\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(1/(a^3*c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I
)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*c*Sqrt[c
 + a^2*c*x^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
```

```
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{\int\frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$=-\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int\frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{a^2c\sqrt{c+a^2cx^2}}$$

$$=-\frac{1}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.17, size = 155, normalized size = 0.62

$$-\frac{\sqrt{1+a^2x^2}\left(\frac{1}{\sqrt{1+a^2x^2}}+\frac{ax\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}-\text{ArcTan}(ax)\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)}{a^3c\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*(1/Sqrt[1 + a^2*x^2] + (a*x*ArcTan[a*x])/Sqrt[1 + a^2*
x^2] - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(
I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

Maple [A]
time = 0.81, size = 247, normalized size = 0.98

method	result

default	$-\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2a^3}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2(a^2x^2+1)c^2a^3}+\frac{i\left(i\arctan(ax)\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2/a^3-
1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2/a^3+I
*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/
c^2/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.234 $\int \frac{x\text{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{x}{ac\sqrt{c+a^2cx^2}}-\frac{\text{ArcTan}(ax)}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] x/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {5050, 197}

$$\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx&=-\frac{\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{a}\\&=\frac{x}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 42, normalized size = 0.86

$$\frac{\sqrt{c+a^2cx^2}\,(ax-\text{ArcTan}(ax))}{a^2c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(a*x - ArcTan[a*x]))/(a^2*c^2*(1 + a^2*x^2))
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.35, size = 100, normalized size = 2.04

method	result	size
default	$-\frac{(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)a^2c^2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^2/c^
2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/a^2/c
^2
```

Maxima [**A**]
time = 0.54, size = 28, normalized size = 0.57

$$\frac{ax-\arctan(ax)}{\sqrt{a^2x^2+1}\,a^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] (a*x - arctan(a*x))/(sqrt(a^2*x^2 + 1)*a^2*c^(3/2))
```

Fricas [**A**]
time = 2.34, size = 43, normalized size = 0.88

$$\frac{\sqrt{a^2cx^2+c}\,(ax-\arctan(ax))}{a^4c^2x^2+a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x - arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.235 $\int \frac{\text{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{ac\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}$$

```
[Out] 1/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5014}

$$\frac{x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
[Out] 1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\frac{1}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.84

$$\frac{\sqrt{c+a^2cx^2}\,(1+ax\text{ArcTan}(ax))}{c^2\,(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[c + a^2*c*x^2]*(1 + a*x*ArcTan[a*x]))/(c^2*(a + a^3*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 98, normalized size = 2.18

method	result	size
default	$\frac{(\arctan(ax)+i)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a\,c^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2(a^2x^2+1)a\,c^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a/c^2+1/2
*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/a/c^2
```

Maxima [A]
time = 0.33, size = 41, normalized size = 0.91

$$\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}\,c}+\frac{1}{\sqrt{a^2cx^2+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

Fricas [A]
time = 2.69, size = 40, normalized size = 0.89

$$\frac{\sqrt{a^2cx^2+c}\,(ax\arctan(ax)+1)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x) + 1)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)/(c + a^2*c*x^2)^(3/2), x)
```

3.236 $\int \frac{\text{ArcTan}(ax)}{x(c+a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=229

$$-\frac{ax}{c\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\right.}{c\sqrt{c+a^2cx^2}}$$

```
[Out] -a*x/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)*
arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^
(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^
2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5086, 5078, 5074, 5050, 197}

$$\frac{\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{ax}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/(c*Sqrt[c + a^2*c*x^2])) + ArcTan[a*x]/(c*Sqrt[c + a^2*c*x^2]) - (2
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c
*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2,
 Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{ax}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 141, normalized size = 0.62

$$
\frac{\sqrt{1+a^2x^2}\left(-\frac{ax}{\sqrt{1+a^2x^2}}+\frac{\mathrm{ArcTan}(ax)}{\sqrt{1+a^2x^2}}+\mathrm{ArcTan}(ax)\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{ArcTan}(ax)\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+i\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-i\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)\right)}{c\sqrt{c\left(1+a^2x^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

[Out] (Sqrt[1 + a^2*x^2]*(-((a*x)/Sqrt[1 + a^2*x^2]) + ArcTan[a*x]/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*x])]))/(c*Sqrt[c*(1 + a^2*x^2)])

Maple [A]
time = 0.28, size = 232, normalized size = 1.01

method	result
default	$\frac{(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)c^2}-\frac{i\left(i\arctan(ax)\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2-I*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{(ax)}}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.237 $\int \frac{\mathbf{ArcTan}(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{c^2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] -a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)-a/c/(a^2*c*x^2+c)^(1/2)-a^2
*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {5086, 5064, 272, 65, 214, 5014}

$$-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{c^2x}-\frac{a^2x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a/(c*Sqrt[c + a^2*c*x^2])) - (a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
- (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a*ArcTanh[Sqrt[c + a^2*c*x^2
]/Sqrt[c]])/c^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x} + \frac{a\int \frac{1}{x\sqrt{c+a^2cx^2}}\,c}{c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{c+c}}\right.}{2c}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2c}}\,c\right.}{a}$$

$$= -\frac{a}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x} - \frac{a\tanh^{-1}\left(\frac{\sqrt{c+a}}{\sqrt{c}}\right.}{c^{3/2}}$$

Mathematica [A]

time = 0.13, size = 122, normalized size = 1.18

$$-\frac{a\sqrt{c(1+a^2x^2)}}{c^2(1+a^2x^2)}-\frac{\sqrt{c(1+a^2x^2)}\,(1+2a^2x^2)\,\mathrm{ArcTan}(ax)}{c^2x(1+a^2x^2)}+\frac{a\log(x)}{c^{3/2}}-\frac{a\log\left(c+\sqrt{c}\,\sqrt{c(1+a^2x^2)}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)])/(c^2*(1 + a^2*x^2))) - (Sqrt[c*(1 + a^2*x^2)]*(
1 + 2*a^2*x^2)*ArcTan[a*x])/(c^2*x*(1 + a^2*x^2)) + (a*Log[x])/c^(3/2) - (a
*Log[c + Sqrt[c]*Sqrt[c*(1 + a^2*x^2)]])/c^(3/2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 231, normalized size = 2.24

method	result
default	$-\frac{a(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)a}{2(a^2x^2+1)c^2}-\frac{\arctan(ax)\sqrt{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/
2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)*a/(a^2*x^2+1)/c^2-arcta
n(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x-a*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2+a*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

Fricas [A]
time = 2.18, size = 104, normalized size = 1.01

$$\frac{(a^3x^3+ax)\sqrt{c}\,\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\,\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}\,(ax+(2a^2x^2+1)\arctan(ax))}{2(a^2c^2x^3+c^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c
) + 2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + (2*a^2*x^2 + 1)*arctan(a*x)))/
(a^2*c^2*x^3 + c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.238 $\int \frac{\text{ArcTan}(ax)}{x^3(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{a^3x}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{2c^2x}-\frac{a^2\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2c^2x^2}+\frac{3a^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-}}{c\sqrt{c+a^2cx^2}}$$

```
[Out] a^3*x/c/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+3*a^2*arc
tan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*
c*x^2+c)^(1/2)-3/2*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x
^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^2+c)^(1/
2)/c^2/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^2
```

Rubi [A]
time = 0.44, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5086, 5082, 270, 5078, 5074, 5050, 197}

$$-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{2c^2x^2}-\frac{a^2\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2cx^2+c}}{2c^2x}-\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2c\sqrt{a^2cx^2+c}}+\frac{a^3x}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*x)/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(2*c^2*x) - (a^2*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2
*c^2*x^2) + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (
((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^3 (c+a^2cx^2)^{3/2}}\,dx = -\left(a^2 \int \frac{\tan^{-1}(ax)}{x (c+a^2cx^2)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2} + a^4 \int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx + \frac{a\int \frac{1}{x^2\sqrt{c+a^2cx^2}}\,dx}{2c} - \frac{a^2\int}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}}{2c^2x} - \frac{a^2\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2} + a^3 \int \frac{1}{(c+a^2cx^2)^{3/2}}\,c$$

$$= \frac{a^3x}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{2c^2x} - \frac{a^2\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{2c^2x^2} + \frac{3a^2\sqrt{}}{}$$

Mathematica [A]
time = 0.93, size = 258, normalized size = 0.86

$$-\frac{a^2\left(-8ax+8\text{ArcTan}(ax)+ax\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+12\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-12\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+12i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-12i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+2\sqrt{1+a^2x^2}\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{8c\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/8*(a^2*(-8*a*x + 8*ArcTan[a*x] + a*x*Csc[ArcTan[a*x]/2]^2 + Sqrt[1 + a^2
*x^2]*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*L
og[1 - E^(I*ArcTan[a*x])] - 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*A
rcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (1
2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, E^(I*ArcTan[a*x])] - Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*Sec[ArcTan[a*x]/2]^2 + 2*Sqrt[1 + a^2*x^2]*Tan[ArcTan[a*x]/2]))/(
c*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.49, size = 273, normalized size = 0.91

method	result
default	$-\frac{a^2(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2} + \frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)a^2}{2(a^2x^2+1)c^2} - \frac{(ax+\text{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^
2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)*a^2/(a^2*x^2+1)/c
```

```
^2-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^2+3/2*I*a^2*(I*arc
tan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2
*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(atan(a*x)/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{x^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.239 $\int \frac{\textbf{ArcTan}(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{a^3}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}}{6c^2x^2}+\frac{a^4x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3c^2x^3}+\frac{5a^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3c^2x}+\cdots$$

```
[Out] 11/6*a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+a^3/c/(a^2*c*x^2+c)^(
1/2)+a^4*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*c*x^2+c)^(1/2)/c^2/
x^2-1/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^3+5/3*a^2*arctan(a*x)*(a^2*c*
x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.38, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5086, 5082, 272, 44, 65, 214, 5064, 5014}

$$\frac{5a^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3c^2x}-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3c^2x^3}-\frac{a\sqrt{a^2cx^2+c}}{6c^2x^2}+\frac{a^4x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}+\frac{11a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{6c^{3/2}}+\frac{a^3}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] a^3/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2])/(6*c^2*x^2) + (a^4*x*
ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3
*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*c^2*x) + (11*a^3*Arc
Tanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/(6*c^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^4 (c+a^2cx^2)^{3/2}}\,dx = -\left(a^2 \int \frac{\tan^{-1}(ax)}{x^2 (c+a^2cx^2)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^3} + a^4 \int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx + \frac{a\int \frac{1}{x^3\sqrt{c+a^2cx^2}}\,dx}{3c} - \frac{(2a}{}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}} + \frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^3} + \frac{5a^2\sqrt{c+a^2cx^2}\,\tan^{-}}{3c^2x}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{6c^2x^2} + \frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^3} + \frac{5a^2}{}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{6c^2x^2} + \frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^3} + \frac{5a^2}{}$$

$$= \frac{a^3}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{6c^2x^2} + \frac{a^4x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^3} + \frac{5a^2}{}$$

Mathematica [A]
time = 0.25, size = 143, normalized size = 0.87

$$\frac{\frac{a(-1+5a^2x^2)\sqrt{c+a^2cx^2}}{x^2+a^2x^4} + \frac{2\sqrt{c+a^2cx^2}\,(-1+4a^2x^2+8a^4x^4)\mathrm{ArcTan}(ax)}{x^3+a^2x^5} - 11a^3\sqrt{c}\,\log(x) + 11a^3\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((a*(-1 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2])/(x^2 + a^2*x^4) + (2*Sqrt[c + a^2
*c*x^2]*(-1 + 4*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x])/(x^3 + a^2*x^5) - 11*a^3*
Sqrt[c]*Log[x] + 11*a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*c^
2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.86, size = 259, normalized size = 1.57

method	result
default	$\frac{a^3(\arctan(ax)+i)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2} + \frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+i)(\arctan(ax)-i)a^3}{2(a^2x^2+1)c^2} + \frac{(10\arctan(ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2+1
/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)*a^3/(a^2*x^2+1)/c^2+1/
6*(10*arctan(a*x)*a^2*x^2-a*x-2*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/
x^3+11/6*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)
*(I+a*x))^(1/2)/c^2-11/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)/(a^2*x^2+1)^
(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

Fricas [A]
time = 5.37, size = 129, normalized size = 0.78

$$\frac{11\,(a^5x^5+a^3x^3)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,\sqrt{a^2cx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(5\,a^3x^3-ax+2\,(8\,a^4x^4+4\,a^2x^2-1)\arctan\left(ax\right))\sqrt{a^2cx^2+c}}{12\,(a^2c^2x^5+c^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(a^5*x^5 + a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c
)*sqrt(c) + 2*c)/x^2) + 2*(5*a^3*x^3 - a*x + 2*(8*a^4*x^4 + 4*a^2*x^2 - 1)*
arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^2*c^2*x^5 + c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}\left(ax\right)}{x^4\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)}{x^4\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

3.240 $\int \frac{x^5 \text{ArcTan}(ax)}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=170

$$-\frac{x^3}{9a^3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}}+\frac{x^2\text{ArcTan}(ax)}{3a^4c\left(c+a^2cx^2\right)^{3/2}}+\frac{5\text{ArcTan}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{a^6c^3}$$

```
[Out] -1/9*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^
(3/2)-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^6/c^(5/2)-5/3*x/a^5/c^2/(a
^2*c*x^2+c)^(1/2)+5/3*arctan(a*x)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(
a^2*c*x^2+c)^(1/2)/a^6/c^3
```

Rubi [A]
time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5084, 5050, 223, 212, 197, 5058}

$$\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^6c^3}+\frac{5\text{ArcTan}(ax)}{3a^6c^2\sqrt{a^2cx^2+c}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a^6c^{5/2}}-\frac{5x}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{x^2\text{ArcTan}(ax)}{3a^4c\left(a^2cx^2+c\right)^{3/2}}-\frac{x^3}{9a^3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/9*x^3/(a^3*c*(c + a^2*c*x^2)^(3/2)) - (5*x)/(3*a^5*c^2*Sqrt[c + a^2*c*x^
2]) + (x^2*ArcTan[a*x])/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x])/(
3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^6*c^3
) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^6*c^(5/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{a^2}+\frac{\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3}{9a^3c\,(c+a^2cx^2)^{3/2}}+\frac{x^2\tan^{-1}(ax)}{3a^4c\,(c+a^2cx^2)^{3/2}}+\frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2}-\frac{2\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{3a^4c}$$

$$= -\frac{x^3}{9a^3c\,(c+a^2cx^2)^{3/2}}+\frac{x^2\tan^{-1}(ax)}{3a^4c\,(c+a^2cx^2)^{3/2}}+\frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\tan}{a^6c^3}$$

$$= -\frac{x^3}{9a^3c\,(c+a^2cx^2)^{3/2}}-\frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}}+\frac{x^2\tan^{-1}(ax)}{3a^4c\,(c+a^2cx^2)^{3/2}}+\frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x^3}{9a^3c\,(c+a^2cx^2)^{3/2}}-\frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}}+\frac{x^2\tan^{-1}(ax)}{3a^4c\,(c+a^2cx^2)^{3/2}}+\frac{5\tan^{-1}(ax)}{3a^6c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.14, size = 131, normalized size = 0.77

$$-\frac{ax(15+16a^2x^2)\sqrt{c+a^2cx^2}-3\sqrt{c+a^2cx^2}\,(8+12a^2x^2+3a^4x^4)\,\mathrm{ArcTan}(ax)+9\sqrt{c}\,(1+a^2x^2)^2\log\left(acx+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)}{9a^6c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
[Out] -1/9*(a*x*(15 + 16*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 3*Sqrt[c + a^2*c*x^2]*(8
+ 12*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[a*c*x
 + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^6*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.76, size = 386, normalized size = 2.27

method	result
default	$\frac{(3\arctan(ax)+i)(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c\,(ax-i)\,(ax+i)}}{72(a^2x^2+1)^2c^3a^6}+\frac{7(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{8a^6c^3(a^2x^2+1)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/72*(3*arctan(a*x)+I)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^
(1/2)/(a^2*x^2+1)^2/c^3/a^6+7/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x
))^(1/2)/a^6/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arcta
n(a*x)-I)/a^6/c^3/(a^2*x^2+1)-1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a
^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)+arctan(a
*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^6-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^
2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3+ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 3.84, size = 140, normalized size = 0.82

$$\frac{9\,(a^4x^4+2\,a^2x^2+1)\sqrt{c}\,\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}\,a\sqrt{c}\,x-c\right)-2\,(16\,a^3x^3+15\,ax-3\,(3\,a^4x^4+12\,a^2x^2+8)\arctan(ax))\sqrt{a^2cx^2+c}}{18\,(a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(9*(a^4*x^4 + 2*a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x
^2 + c)*a*sqrt(c)*x - c) - 2*(16*a^3*x^3 + 15*a*x - 3*(3*a^4*x^4 + 12*a^2*x
^2 + 8)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a
^6*c^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\mathrm{atan}(a\,x)}{{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.241 $\int \frac{x^4 \mathbf{ArcTan}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=308

$$\frac{1}{9a^5c\left(c+a^2cx^2\right)^{3/2}}-\frac{4}{3a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\text{ArcTan}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\text{ArcTan}(ax)}{a^4c^2\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{A}}{a^5c^2\sqrt{c+a^2}}$$

```
[Out] 1/9/a^5/c/(a^2*c*x^2+c)^(3/2)-1/3*x^3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)
-4/3/a^5/c^2/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-
2*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a
^5/c^2/(a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5084, 5054, 5010, 5006, 5064, 272, 45}

$$-\frac{x^3\text{ArcTan}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{4}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{1}{9a^5c\left(a^2cx^2+c\right)^{3/2}}-\frac{x\text{ArcTan}(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 1/(9*a^5*c*(c + a^2*c*x^2)^(3/2)) - 4/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^
3*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x])/(a^4*c^2*S
qrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) - (I*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} + \frac{\int \frac{x^3}{(c+a^2cx^2)^{5/2}}\,dx}{3a} + \frac{\int}{}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} + \frac{\text{Subst}\left(\int \frac{x}{(c+a^2cx)^{5/2}}\,dx\right.}{6a}$$

$$= -\frac{1}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(}{a^5c^2\sqrt{c}}$$

$$= \frac{1}{9a^5c\,(c+a^2cx^2)^{3/2}} - \frac{4}{3a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} -$$

Mathematica [A]
time = 0.28, size = 177, normalized size = 0.57

$$\frac{\sqrt{c(1+a^2x^2)}\left(-\frac{45}{\sqrt{1+a^2x^2}} - \frac{45ax\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}} + \cos(3\text{ArcTan}(ax)) + 36\text{ArcTan}(ax)\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right) - \log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right) + 36i\left(\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right) - \text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right) + 3\text{ArcTan}(ax)\sin(3\text{ArcTan}(ax))\right)}{36a^5c^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-45/Sqrt[1 + a^2*x^2] - (45*a*x*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*
x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (36*I)*(PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) + 3*ArcTan[a*x]*Sin[3*ArcTan[a*x]
]))/(36*a^5*c^3*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.95, size = 389, normalized size = 1.26

method	result
default	$-\frac{(3\arctan(ax)+i)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3a^5} - \frac{5(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8a^5c^3(a^2x^2+1)} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(3*arctan(a*x)+I)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)^2/c^3/a^5-5/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^
(1/2)/a^5/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x
)-I)/a^5/c^3/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*
(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^5+I*(I*arctan(a*x
)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^5/c^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**4*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \operatorname{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.242 $\int \frac{x^3 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=112

$$\frac{x^3}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\text{ArcTan}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\text{ArcTan}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/9*x^3/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/
2)+2/3*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2/3*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c)^
(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5058, 5050, 197}

$$-\frac{x^2\text{ArcTan}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2\text{ArcTan}(ax)}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{2x}{3a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) -
 (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x])/(3*a^4
*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
```

```
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{x^3}{9ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}}+\frac{2\int\frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{3a^2c}\\ &= \frac{x^3}{9ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}+\frac{2\int\frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{3a^3c}\\ &= \frac{x^3}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{2x}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.58

$$\frac{\sqrt{c+a^2cx^2}\,(ax(6+7a^2x^2)-3(2+3a^2x^2)\,\mathrm{ArcTan}(ax))}{9a^4c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
[Out] (Sqrt[c + a^2*c*x^2]*(a*x*(6 + 7*a^2*x^2) - 3*(2 + 3*a^2*x^2)*ArcTan[a*x]))
/(9*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.60, size = 244, normalized size = 2.18

method	result
default	$-\frac{(3\arctan(ax)+i)(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^4c^3}-\frac{3(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3a^4(a^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/72*(3*arctan(a*x)+I)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))
^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*
x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arct
an(a*x)-I)/c^3/a^4/(a^2*x^2+1)+1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*
a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [A]
time = 0.41, size = 65, normalized size = 0.58

$$\frac{7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)}}{9\,(a^6c^2x^2+a^4c^2)\sqrt{a^2x^2+1}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))/((a^6*c^2*x^2 + a^4
*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A]
time = 3.23, size = 74, normalized size = 0.66

$$\frac{(7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)})\sqrt{a^2cx^2+c}}{9\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*sqrt(a^2*c*x^2 + c)
/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.243 $\int \frac{x^2 \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{1}{9a^3c\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\operatorname{ArcTan}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}$$

```
[Out] -1/9/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/
3/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5064, 272, 45}

$$\frac{x^3\operatorname{ArcTan}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{1}{9a^3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a^3*c*(c + a^2*c*x^2)^(3/2)) + 1/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) +
(x^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}} - \frac{1}{3}a\int \frac{x^3}{(c+a^2cx^2)^{5/2}}\,dx$$
$$= \frac{x^3\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}} - \frac{1}{6}a\text{Subst}\left(\int \frac{x}{(c+a^2cx)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{x^3\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}} - \frac{1}{6}a\text{Subst}\left(\int \left(-\frac{1}{a^2(c+a^2cx)^{5/2}} + \frac{1}{a^2c(c+a^2cx)^{3/2}}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{9a^3c(c+a^2cx^2)^{3/2}} + \frac{1}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.74

$$\frac{\sqrt{c+a^2cx^2}\,(2+3a^2x^2+3a^3x^3\text{ArcTan}(ax))}{9a^3c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(2 + 3*a^2*x^2 + 3*a^3*x^3*ArcTan[a*x]))/(9*a^3*c^3*(1
 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.80, size = 240, normalized size = 3.12

method	result
default	$\frac{(3\arctan(ax)+i)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3a^3} + \frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8a^3c^3(a^2x^2+1)} + \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(3*arctan(a*x)+I)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1
/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(
1/2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)
-I)/a^3/c^3/(a^2*x^2+1)+1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(
a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.21

$$\frac{1}{9}a\left(\frac{3}{\sqrt{a^2cx^2+c}\,a^4c^2}-\frac{1}{(a^2cx^2+c)^{\frac{3}{2}}a^4c}\right)+\frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}\,a^2c^2}-\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(3/(sqrt(a^2*c*x^2 + c)*a^4*c^2) - 1/((a^2*c*x^2 + c)^(3/2)*a^4*c)) +
 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*ar
ctan(a*x)
```

Fricas [A]
time = 2.13, size = 67, normalized size = 0.87

$$\frac{(3\,a^3x^3\arctan(ax)+3\,a^2x^2+2)\sqrt{a^2cx^2+c}}{9\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*arctan(a*x) + 3*a^2*x^2 + 2)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^
4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.244 $\int \frac{x\mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{x}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\text{ArcTan}(ax)}{3a^2c\left(c+a^2cx^2\right)^{3/2}}$$

```
[Out] 1/9*x/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+2/9
*x/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5050, 198, 197}

$$-\frac{\text{ArcTan}(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x}{9ac^2\sqrt{a^2cx^2+c}}+\frac{x}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] x/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - Arc
Tan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}}\,dx}{3a}$$
$$= \frac{x}{9ac\,(c+a^2cx^2)^{3/2}} - \frac{\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{2\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{9ac}$$
$$= \frac{x}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{2x}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.65

$$\frac{\sqrt{c+a^2cx^2}\,(3ax+2a^3x^3-3\text{ArcTan}(ax))}{9c^3\,(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(3*a*x + 2*a^3*x^3 - 3*ArcTan[a*x]))/(9*c^3*(a + a^3*x
^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 244, normalized size = 3.09

method	result
default	$\frac{(3\arctan(ax)+i)(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c\,(ax-i)\,(ax+i)}}{72(a^2x^2+1)^2a^2c^3} - \frac{(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{8c^3a^2(a^2x^2+1)} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(3*arctan(a*x)+I)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^
(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x
))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arcta
n(a*x)-I)/c^3/a^2/(a^2*x^2+1)-1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a
^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [A]
time = 0.33, size = 66, normalized size = 0.84

$$\frac{(2\,a^3x^3+3\,ax-3\,\arctan(ax))\sqrt{a^2x^2+1}\,\sqrt{c}}{9\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*x^2 + 1)*sqrt(c)/(a^6*c^3*
x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Fricas [A]
time = 2.29, size = 64, normalized size = 0.81

$$\frac{(2 a^3x^3 + 3 ax - 3 \arctan(ax))\sqrt{a^2cx^2 + c}}{9 (a^6c^3x^4 + 2 a^4c^3x^2 + a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 +
2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.245 $\int \frac{\text{ArcTan}(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{1}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{ArcTan}(ax)}{3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/9/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+2/3/a/c
^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5016, 5014}

$$\frac{2x\text{ArcTan}(ax)}{3c^2\sqrt{a^2cx^2+c}}+\frac{x\text{ArcTan}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + 2/(3*a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcT
an[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*ArcTan[a*x])/(3*c^2*Sqrt[c + a^
2*c*x^2])
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{1}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}\\ &= \frac{1}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.62

$$\frac{\sqrt{c+a^2cx^2}\,(7+6a^2x^2+(9ax+6a^3x^3)\,\text{ArcTan}(ax))}{9ac^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(7 + 6*a^2*x^2 + (9*a*x + 6*a^3*x^3)*ArcTan[a*x]))/(9*
a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 240, normalized size = 2.38

method	result
default	$-\frac{(3\arctan(ax)+i)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c\,(ax-i)\,(ax+i)}}{72(a^2x^2+1)^2a\,c^3}+\frac{3(\arctan(ax)+i)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{8c^3a(a^2x^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(3*arctan(a*x)+I)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1
/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)
/c^3/a/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x
^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.85

$$\frac{1}{9}\,a\left(\frac{6}{\sqrt{a^2cx^2+c}\,a^2c^2}+\frac{1}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{a^2cx^2+c}\,c^2}+\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(6/(sqrt(a^2*c*x^2 + c)*a^2*c^2) + 1/((a^2*c*x^2 + c)^(3/2)*a^2*c)) +
 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a
*x)
```

Fricas [A]
time = 2.56, size = 72, normalized size = 0.71

$$\frac{\sqrt{a^2cx^2+c}\,(6\,a^2x^2+3\,(2\,a^3x^3+3\,ax)\arctan(ax)+7)}{9\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(a^2*c*x^2 + c)*(6*a^2*x^2 + 3*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 7)
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)/(c + a^2*c*x^2)^(5/2), x)
```

3.246 $\int \frac{\text{ArcTan}(ax)}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{11ax}{9c^2\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{\text{ArcTan}(ax)}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}}{c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/9*a*x/c/(a^2*c*x^2+c)^(3/2)+1/3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-11/9*a
*x/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x
)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2
+c)^(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c
^2/(a^2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^
2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5086, 5078, 5074, 5050, 197, 198}

$$\frac{\text{ArcTan}(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{11ax}{9c^2\sqrt{a^2cx^2+c}}-\frac{ax}{9c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/9*(a*x)/(c*(c + a^2*c*x^2)^(3/2)) - (11*a*x)/(9*c^2*Sqrt[c + a^2*c*x^2])
 + ArcTan[a*x]/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]/(c^2*Sqrt[c + a^2*
c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2
*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]
)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)}{x(c+a^2cx^2)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{3}a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax}{(c+a^2cx^2)^{3/}}}{c}$$

$$= -\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} - \frac{(2a)\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{9c} -$$

$$= -\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{11ax}{9c^2\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{}}{}$$

Mathematica [A]
time = 0.27, size = 168, normalized size = 0.60

$$\frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{45ax}{\sqrt{1+a^2x^2}}+\frac{45\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}+3\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax))+36\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-36\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+36i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-36i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-\sin(3\text{ArcTan}(ax))\right)}{36c\left(c\left(1+a^2x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-45*a*x)/Sqrt[1 + a^2*x^2] + (45*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + 3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*Log[1 - E^(
I*ArcTan[a*x])] - 36*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (36*I)*PolyLo
g[2, -E^(I*ArcTan[a*x])] - (36*I)*PolyLog[2, E^(I*ArcTan[a*x])] - Sin[3*Arc
Tan[a*x]]))/(36*c*(c*(1 + a^2*x^2))^(3/2))
```

Maple [A]
time = 0.33, size = 370, normalized size = 1.33

method	result
default	$-\frac{(3\arctan(ax)+i)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\,(ax-i)\,(ax+i)}}{72\left(a^2x^2+1\right)^2c^3}+\frac{5(\arctan(ax)+i)(iax+1)\sqrt{c\,(ax-i)\,(ax+i}}{8c^3\left(a^2x^2+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(3*arctan(a*x)+I)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))
^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^
(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-
```

```
I)/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*
a*x+1)*(-I+3*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)-I*(I*arctan(a*x)*ln(1-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))
+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a
^2*c^3*x^3 + c^3*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x/(a**2*c*x**2+c)**(5/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.247 $\int \frac{\mathbf{ArcTan}(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\text{ArcTan}(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a^2x\text{ArcTan}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{c^3x}-\frac{a\tan$$

```
[Out] -1/9*a/c/(a^2*c*x^2+c)^(3/2)-1/3*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-a*
arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(5/2)-5/3*a/c^2/(a^2*c*x^2+c)^(1/2)-
5/3*a^2*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(1/
2)/c^3/x
```

Rubi [A]
time = 0.25, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5086, 5064, 272, 65, 214, 5014, 5016}

$$-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{c^3x}-\frac{5a^2x\text{ArcTan}(ax)}{3c^2\sqrt{a^2cx^2+c}}-\frac{a^2x\text{ArcTan}(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{5/2}}-\frac{5a}{3c^2\sqrt{a^2cx^2+c}}-\frac{a}{9c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/9*a/(c*(c + a^2*c*x^2)^(3/2)) - (5*a)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2
*x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x])/(3*c^2*
Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^3*x) - (a*ArcTa
nh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx}{c}$$

$$= -\frac{a}{9c(c+a^2cx^2)^{3/2}}-\frac{a^2x\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}+\frac{\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c^2}-\frac{(2a^2)\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}}{3c}$$

$$= -\frac{a}{9c(c+a^2cx^2)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\sqrt{c}$$

$$= -\frac{a}{9c(c+a^2cx^2)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\sqrt{c}$$

$$= -\frac{a}{9c(c+a^2cx^2)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\sqrt{c}$$

$$= -\frac{a}{9c(c+a^2cx^2)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\sqrt{c}$$

Mathematica [A]
time = 0.17, size = 151, normalized size = 0.96

$$\frac{-3\sqrt{c+a^2cx^2}\,(3+12a^2x^2+8a^4x^4)\,\mathrm{ArcTan}(ax)+ax\left(-\left((16+15a^2x^2)\sqrt{c+a^2cx^2}\right)+9\sqrt{c}\,(1+a^2x^2)^2\log(x)-9\sqrt{c}\,(1+a^2x^2)^2\log\left(c+\sqrt{c}\,\sqrt{c+a^2cx^2}\right)\right)}{9c^3x\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*(3 + 12*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x] + a*x*(-((
16 + 15*a^2*x^2)*Sqrt[c + a^2*c*x^2]) + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[x] -
9*Sqrt[c]*(1 + a^2*x^2)^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(9*c^3*x*(
1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 369, normalized size = 2.34

method	result
default	$\frac{a(3\arctan(ax)+i)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3}-\frac{7a(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8c^3(a^2x^2+1)}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/72*a*(3*arctan(a*x)+I)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)*a/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-I+3*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/x-a*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3+a*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x^2), x)

Fricas [A]
time = 8.46, size = 142, normalized size = 0.90

$$\frac{9\,(a^5x^5+2\,a^3x^3+ax)\sqrt{c}\,\log\left(-\frac{a^2cx^2-2\,\sqrt{a^2cx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(15\,a^3x^3+16\,ax+3\,(8\,a^4x^4+12\,a^2x^2+3)\arctan(ax))\sqrt{a^2cx^2+c}}{18\,(a^4c^3x^5+2\,a^2c^3x^3+c^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] 1/18*(9*(a^5*x^5 + 2*a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(15*a^3*x^3 + 16*a*x + 3*(8*a^4*x^4 + 12*a^2*x^2 + 3)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{x^2\,(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.248 $\int x^m(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=270

$$\frac{c^3x^{1+m}\operatorname{ArcTan}(ax)}{1+m}+\frac{3a^2c^3x^{3+m}\operatorname{ArcTan}(ax)}{3+m}+\frac{3a^4c^3x^{5+m}\operatorname{ArcTan}(ax)}{5+m}+\frac{a^6c^3x^{7+m}\operatorname{ArcTan}(ax)}{7+m}-\frac{ac^3x^{2+m}\,{}_2F_1}{2}$$

```
[Out] c^3*x^(1+m)*arctan(a*x)/(1+m)+3*a^2*c^3*x^(3+m)*arctan(a*x)/(3+m)+3*a^4*c^3
*x^(5+m)*arctan(a*x)/(5+m)+a^6*c^3*x^(7+m)*arctan(a*x)/(7+m)-a*c^3*x^(2+m)*
hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-3*a^3*c^3*x^(4+m)*hy
pergeom([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)-3*a^5*c^3*x^(6+m)*hyp
ergeom([1, 3+1/2*m],[4+1/2*m],-a^2*x^2)/(5+m)/(6+m)-a^7*c^3*x^(8+m)*hyperge
om([1, 4+1/2*m],[5+1/2*m],-a^2*x^2)/(7+m)/(8+m)
```

Rubi [A]
time = 0.17, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5068, 4946, 371}

$$\frac{a^6c^3x^{m+7}\operatorname{ArcTan}(ax)}{m+7}+\frac{3a^4c^3x^{m+5}\operatorname{ArcTan}(ax)}{m+5}+\frac{3a^2c^3x^{m+3}\operatorname{ArcTan}(ax)}{m+3}-\frac{ac^3x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{a^7c^3x^{m+8}\,{}_2F_1\left(1,\frac{m+8}{2};\frac{m+10}{2};-a^2x^2\right)}{(m+7)(m+8)}-\frac{3a^5c^3x^{m+6}\,{}_2F_1\left(1,\frac{m+6}{2};\frac{m+8}{2};-a^2x^2\right)}{(m+5)(m+6)}-\frac{3a^3c^3x^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}+\frac{c^3x^{m+1}\operatorname{ArcTan}(ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] (c^3*x^(1 + m)*ArcTan[a*x])/(1 + m) + (3*a^2*c^3*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (3*a^4*c^3*x^(5 + m)*ArcTan[a*x])/(5 + m) + (a^6*c^3*x^(7 + m)*ArcTa
n[a*x])/(7 + m) - (a*c^3*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*c^3*x^(4 + m)*Hypergeometric2F1[1,
 (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5*c^3*x^(6 + m)
*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m))
- (a^7*c^3*x^(8 + m)*Hypergeometric2F1[1, (8 + m)/2, (10 + m)/2, -(a^2*x^2)
])/((7 + m)*(8 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)\,dx = \int \left(c^3x^m \tan^{-1}(ax) + 3a^2c^3x^{2+m}\tan^{-1}(ax) + 3a^4c^3x^{4+m}\tan^{-1}(ax) + a^6c^3\right.$$

$$= c^3\int x^m \tan^{-1}(ax)\,dx + \left(3a^2c^3\right)\int x^{2+m}\tan^{-1}(ax)\,dx + \left(3a^4c^3\right)\int x^{4+m}\,\mathrm{t}$$

$$= \frac{c^3x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{3a^2c^3x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{3a^4c^3x^{5+m}\tan^{-1}(ax)}{5+m} + \frac{a^6c^3}{}$$

$$= \frac{c^3x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{3a^2c^3x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{3a^4c^3x^{5+m}\tan^{-1}(ax)}{5+m} + \frac{a^6c^3}{}$$

Mathematica [A]
time = 0.33, size = 234, normalized size = 0.87

$$c^3x^{1+m}\left(\frac{\mathrm{ArcTan}(ax)}{1+m}+\frac{3a^2x^2\mathrm{ArcTan}(ax)}{3+m}+\frac{3a^4x^4\mathrm{ArcTan}(ax)}{5+m}+\frac{a^6x^6\mathrm{ArcTan}(ax)}{7+m}-\frac{ax\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}-\frac{3a^3x^3\,{}_2F_1\left(1,2+\frac{m}{2};3+\frac{m}{2};-a^2x^2\right)}{12+7m+m^2}-\frac{3a^5x^5\,{}_2F_1\left(1,3+\frac{m}{2};4+\frac{m}{2};-a^2x^2\right)}{(5+m)(6+m)}-\frac{a^7x^7\,{}_2F_1\left(1,4+\frac{m}{2};5+\frac{m}{2};-a^2x^2\right)}{(7+m)(8+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

```
[Out] c^3*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (3*a^2*x^2*ArcTan[a*x])/(3 + m) + (3*a
^4*x^4*ArcTan[a*x])/(5 + m) + (a^6*x^6*ArcTan[a*x])/(7 + m) - (a*x*Hypergeo
metric2F1[1, 1 + m/2, 2 + m/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*x^3*Hy
pergeometric2F1[1, 2 + m/2, 3 + m/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5
*x^5*Hypergeometric2F1[1, 3 + m/2, 4 + m/2, -(a^2*x^2)])/((5 + m)*(6 + m))
- (a^7*x^7*Hypergeometric2F1[1, 4 + m/2, 5 + m/2, -(a^2*x^2)])/((7 + m)*(8
+ m)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 1.58, size = 600, normalized size = 2.22

method	result

meijerg	$\frac{a^{-m-1}c^3\left(-\frac{4x^m a^m\left(a^6m^3x^6+6a^6m^2x^6+8a^6m\,x^6-a^4m^3x^4-8a^4m^2x^4-12a^4m\,x^4+a^2m^3x^2+10a^2m^2x^2+24a^2m\,x^2-m^3-12m^2-44m-48\right)}{(7+m)m(2+m)(4+m)(6+m)}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^6*m^3*x^6+6*a^6*m^2*x^6+8*a^6*m*x^6-a^4*m^3
*x^4-8*a^4*m^2*x^4-12*a^4*m*x^4+a^2*m^3*x^2+10*a^2*m^2*x^2+24*a^2*m*x^2-m^3
-12*m^2-44*m-48)/(7+m)/m/(2+m)/(4+m)/(6+m)+8*x^(8+m)*a^(8+m)/(14+2*m)/(a^2*
x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(8+m)*x^m*a^m*(-8-m)/(7+m)*LerchPhi(-a
^2*x^2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*
m^2*x^2-4*a^2*m*x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*
m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^
2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^
(4+m)*a^(4+m)/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a
^m*(-m-4)/(3+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^3*(4/(2+m)*x^m*a
^m*(-m-2)/(1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2
)^(1/2))+2*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

```
[Out] (((a^6*c^3*m^3 + 9*a^6*c^3*m^2 + 23*a^6*c^3*m + 15*a^6*c^3)*x^7 + 3*(a^4*c^
3*m^3 + 11*a^4*c^3*m^2 + 31*a^4*c^3*m + 21*a^4*c^3)*x^5 + 3*(a^2*c^3*m^3 +
13*a^2*c^3*m^2 + 47*a^2*c^3*m + 35*a^2*c^3)*x^3 + (c^3*m^3 + 15*c^3*m^2 + 7
1*c^3*m + 105*c^3)*x)*x^m*arctan(a*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)*integrate(((a^7*c^3*m^3 + 9*a^7*c^3*m^2 + 23*a^7*c^3*m + 15*a^7*c^3)*x^7
 + 3*(a^5*c^3*m^3 + 11*a^5*c^3*m^2 + 31*a^5*c^3*m + 21*a^5*c^3)*x^5 + 3*(a^
3*c^3*m^3 + 13*a^3*c^3*m^2 + 47*a^3*c^3*m + 35*a^3*c^3)*x^3 + (a*c^3*m^3 +
15*a*c^3*m^2 + 71*a*c^3*m + 105*a*c^3)*x)*x^m/(m^4 + 16*m^3 + (a^2*m^4 + 16
*a^2*m^3 + 86*a^2*m^2 + 176*a^2*m + 105*a^2)*x^2 + 86*m^2 + 176*m + 105), x
))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int x^m \operatorname{atan}(ax)\,dx + \int 3a^2x^2x^m \operatorname{atan}(ax)\,dx + \int 3a^4x^4x^m \operatorname{atan}(ax)\,dx + \int a^6x^6x^m \operatorname{atan}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x),x)
```

```
[Out] c**3*(Integral(x**m*atan(a*x), x) + Integral(3*a**2*x**2*x**m*atan(a*x), x)
 + Integral(3*a**4*x**4*x**m*atan(a*x), x) + Integral(a**6*x**6*x**m*atan(a
*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2)^3, x)
```

3.249 $\int x^m(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=201

$$\frac{c^2x^{1+m}\operatorname{ArcTan}(ax)}{1+m}+\frac{2a^2c^2x^{3+m}\operatorname{ArcTan}(ax)}{3+m}+\frac{a^4c^2x^{5+m}\operatorname{ArcTan}(ax)}{5+m}-\frac{ac^2x^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}-2$$

```
[Out] c^2*x^(1+m)*arctan(a*x)/(1+m)+2*a^2*c^2*x^(3+m)*arctan(a*x)/(3+m)+a^4*c^2*x
^(5+m)*arctan(a*x)/(5+m)-a*c^2*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^
2*x^2)/(m^2+3*m+2)-2*a^3*c^2*x^(4+m)*hypergeom([1, 2+1/2*m],[3+1/2*m],-a^2*
x^2)/(m^2+7*m+12)-a^5*c^2*x^(6+m)*hypergeom([1, 3+1/2*m],[4+1/2*m],-a^2*x^2
)/(5+m)/(6+m)
```

Rubi [A]
time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5068, 4946, 371}

$$\frac{a^4c^2x^{m+5}\operatorname{ArcTan}(ax)}{m+5}+\frac{2a^2c^2x^{m+3}\operatorname{ArcTan}(ax)}{m+3}-\frac{ac^2x^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{a^5c^2x^{m+6}\,{}_2F_1\left(1,\frac{m+6}{2};\frac{m+8}{2};-a^2x^2\right)}{(m+5)(m+6)}-\frac{2a^3c^2x^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}+\frac{c^2x^{m+1}\operatorname{ArcTan}(ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] (c^2*x^(1 + m)*ArcTan[a*x])/(1 + m) + (2*a^2*c^2*x^(3 + m)*ArcTan[a*x])/(3
+ m) + (a^4*c^2*x^(5 + m)*ArcTan[a*x])/(5 + m) - (a*c^2*x^(2 + m)*Hypergeom
etric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*c^2
*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*
m + m^2) - (a^5*c^2*x^(6 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(
a^2*x^2)])/((5 + m)*(6 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^m (c+a^2cx^2)^2 \tan^{-1}(ax)\,dx = \int \left(c^2x^m \tan^{-1}(ax) + 2a^2c^2x^{2+m}\tan^{-1}(ax) + a^4c^2x^{4+m}\tan^{-1}(ax)\right)\,dx$$
$$= c^2\int x^m \tan^{-1}(ax)\,dx + (2a^2c^2)\int x^{2+m}\tan^{-1}(ax)\,dx + (a^4c^2)\int x^{4+m}\,\text{ta}$$
$$= \frac{c^2x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{2a^2c^2x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{a^4c^2x^{5+m}\tan^{-1}(ax)}{5+m} - \frac{(ac^2)}{}$$
$$= \frac{c^2x^{1+m}\tan^{-1}(ax)}{1+m} + \frac{2a^2c^2x^{3+m}\tan^{-1}(ax)}{3+m} + \frac{a^4c^2x^{5+m}\tan^{-1}(ax)}{5+m} - \frac{ac^2x^{2}}{}$$

Mathematica [A]
time = 0.15, size = 175, normalized size = 0.87

$$c^2x^{1+m}\left(\frac{\text{ArcTan}(ax)}{1+m}+\frac{2a^2x^2\text{ArcTan}(ax)}{3+m}+\frac{a^4x^4\text{ArcTan}(ax)}{5+m}-\frac{ax\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}-\frac{2a^3x^3\,{}_2F_1\left(1,2+\frac{m}{2};3+\frac{m}{2};-a^2x^2\right)}{12+7m+m^2}-\frac{a^5x^5\,{}_2F_1\left(1,3+\frac{m}{2};4+\frac{m}{2};-a^2x^2\right)}{(5+m)(6+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

```
[Out] c^2*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (2*a^2*x^2*ArcTan[a*x])/(3 + m) + (a^4
*x^4*ArcTan[a*x])/(5 + m) - (a*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(a
^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*x^3*Hypergeometric2F1[1, 2 + m/2, 3 + m/
2, -(a^2*x^2)])/(12 + 7*m + m^2) - (a^5*x^5*Hypergeometric2F1[1, 3 + m/2, 4
 + m/2, -(a^2*x^2)])/((5 + m)*(6 + m)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.77, size = 376, normalized size = 1.87

method	result
meijerg	$\frac{a^{-m-1}c^2\left(-\frac{4x^m a^m\left(a^4m^2x^4+2a^4m\,x^4-a^2m^2x^2-4a^2m\,x^2+m^2+6m+8\right)}{(5+m)m(2+m)(4+m)}+\frac{8x^{6+m}a^{6+m}\arctan\left(\sqrt{a^2x^2}\right)}{(10+2m)\sqrt{a^2x^2}}+\frac{2x^m a^m\Phi\left(-a^2x^2,1,\frac{m}{2}\right)}{5+m}\right)}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^(-m-1)*c^2*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*m^2*x^2-4*a^2*m*x
^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*m)/(a^2*x^2)^(1/2
)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/2*a
^(-m-1)*c^2*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)/(6+
2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3+m)*L
erchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^2*(4/(2+m)*x^m*a^m*(-m-2)/(1+m)/m
+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^
m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

```
[Out] (((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a^2*c^2*
m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x) - (m^3
 + 9*m^2 + 23*m + 15)*integrate(((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c^2)*x^
5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a*c^2*m
+ 15*a*c^2)*x)*x^m/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^2 + 9
*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int x^{m}\operatorname{atan}{\left(ax\right)}\,dx+\int 2a^{2}x^{2}x^{m}\operatorname{atan}{\left(ax\right)}\,dx+\int a^{4}x^{4}x^{m}\operatorname{atan}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x),x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x), x) + Integral(2*a**2*x**2*x**m*atan(a*x), x)
 + Integral(a**4*x**4*x**m*atan(a*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2)^2, x)
```

3.250 $\int x^m(c+a^2cx^2) \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=124

$$\frac{cx^{1+m}\operatorname{ArcTan}(ax)}{1+m}+\frac{a^2cx^{3+m}\operatorname{ArcTan}(ax)}{3+m}-\frac{acx^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}-\frac{a^3cx^{4+m}\,{}_2F_1\left(1,\frac{4+m}{2};\frac{6+m}{2};-a\right.}{12+7m+m^2}$$

```
[Out] c*x^(1+m)*arctan(a*x)/(1+m)+a^2*c*x^(3+m)*arctan(a*x)/(3+m)-a*c*x^(2+m)*hyp
ergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-a^3*c*x^(4+m)*hypergeom
([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5070, 4946, 371}

$$\frac{a^2cx^{m+3}\operatorname{ArcTan}(ax)}{m+3}-\frac{acx^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}-\frac{a^3cx^{m+4}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-a^2x^2\right)}{m^2+7m+12}+\frac{cx^{m+1}\operatorname{ArcTan}(ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x^(1 + m)*ArcTan[a*x])/(1 + m) + (a^2*c*x^(3 + m)*ArcTan[a*x])/(3 + m) -
 (a*c*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2
+ 3*m + m^2) - (a^3*c*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2,
-(a^2*x^2)])/(12 + 7*m + m^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)\,dx &= c\int x^m\tan^{-1}(ax)\,dx+\left(a^2c\right)\int x^{2+m}\tan^{-1}(ax)\,dx\\ &= \frac{cx^{1+m}\tan^{-1}(ax)}{1+m}+\frac{a^2cx^{3+m}\tan^{-1}(ax)}{3+m}-\frac{(ac)\int\frac{x^{1+m}}{1+a^2x^2}\,dx}{1+m}-\frac{\left(a^3c\right)\int\frac{x^{3+m}}{1+a^2x^2}\\ &= \frac{cx^{1+m}\tan^{-1}(ax)}{1+m}+\frac{a^2cx^{3+m}\tan^{-1}(ax)}{3+m}-\frac{acx^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 144, normalized size = 1.16

$$\frac{cx^{1+m}\left(-a\left(12+7m+m^2\right)x\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-a^2x^2\right)+(2+m)\left(\left(12+7m+m^2+4a^2x^2+5a^2mx^2+a^2m^2x^2\right)\text{ArcTan}(ax)-a^3(1+m)x^3\,{}_2F_1\left(1,2+\frac{m}{2};3+\frac{m}{2};-a^2x^2\right)\right)\right)}{(1+m)(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

```
[Out] (c*x^(1 + m)*(-(a*(12 + 7*m + m^2)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
 -(a^2*x^2)]) + (2 + m)*((12 + 7*m + m^2 + 4*a^2*x^2 + 5*a^2*m*x^2 + a^2*m^
2*x^2)*ArcTan[a*x] - a^3*(1 + m)*x^3*Hypergeometric2F1[1, 2 + m/2, 3 + m/2,
 -(a^2*x^2)])))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.60, size = 222, normalized size = 1.79

method	result
meijerg	$\frac{a^{-m-1}c\left(-\frac{4x^m a^m\left(a^2m\,x^2-m-2\right)}{(3+m)m(2+m)}+\frac{8x^{4+m}a^{4+m}\arctan\left(\sqrt{a^2x^2}\right)}{(6+2m)\sqrt{a^2x^2}}+\frac{2x^m a^m(-m-4)\Phi\left(-a^2x^2,1,\frac{m}{2}\right)}{(4+m)(3+m)}\right)}{4}+\frac{a^{-m-1}c\left(\frac{4x^m a^m(-m-2)}{(2+m)(1+m)m}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^(-m-1)*c*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)/
(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3+m
)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c*(4/(2+m)*x^m*a^m*(-m-2)/(1+m)/
```

```
m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a
^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] (((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x) - (m^2 + 4*m + 3)*
integrate(((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m/((a^2*m^2 + 4*a^2
*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{m}\operatorname{atan}{\left(ax\right)}\,dx + \int a^{2}x^{2}x^{m}\operatorname{atan}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x),x)
```

```
[Out] c*(Integral(x**m*atan(a*x), x) + Integral(a**2*x**2*x**m*atan(a*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2),x)
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2), x)
```

3.251 $\int \frac{x^m \mathbf{ArcTan}(ax)}{c+a^2cx^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)}{c+a^2cx^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} dx = \int \frac{x^m \tan^{-1}(ax)}{c+a^2cx^2} dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)}{a^2 c x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x))/(c + a^2*c*x^2),x)
[Out] int((x^m*atan(a*x))/(c + a^2*c*x^2), x)
```

3.252 $\int \frac{x^m \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x^m\text{ArcTan}(ax)}{(c+a^2cx^2)^2},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\text{ArcTan}(ax)}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^m\text{ArcTan}(ax)}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m\arctan(ax)}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.253 $\int x^m(c+a^2cx^2)^{5/2}$ ArcTan$(ax)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax),x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx=\int x^m\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m*arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.254 $\int x^m(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax),x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx = \int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.255 $\int x^m \sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)\,dx$

Optimal. Leaf size=113

$$\frac{x^{1+m}\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{2+m}-\frac{ax^{2+m}\sqrt{c+a^2cx^2}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{4+m}{2};-a^2x^2\right)}{(2+m)^2}+\frac{c\text{Int}\left(\frac{x^m\text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}},x\right)}{2+m}$$

```
[Out] x^(1+m)*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/(2+m)-a*x^(2+m)*hypergeom([1, 3/2+1
/2*m],[2+1/2*m],-a^2*x^2)*(a^2*c*x^2+c)^(1/2)/(2+m)^2+c*Unintegrable(x^m*ar
ctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/(2+m)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2 + m) - (a*c*x^(2 + m)*Sqrt[1
 + a^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/((2 +
 m)^2*Sqrt[c + a^2*c*x^2]) + (c*Defer[Int][(x^m*ArcTan[a*x])/Sqrt[c + a^2*c
*x^2], x])/(2 + m)
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx &= \frac{x^{1+m}\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2+m}+\frac{c\int\frac{x^m\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{2+m}-\frac{(ac)\int\frac{x^{1+m}}{\sqrt{c+a^2cx^2}}\\ &= \frac{x^{1+m}\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2+m}+\frac{c\int\frac{x^m\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{2+m}-\frac{\left(ac\sqrt{1+a^2x^2}\right)\int}{(2+m)\sqrt{c}}\\ &= \frac{x^{1+m}\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{2+m}-\frac{acx^{2+m}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2\right.}{(2+m)^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x], x]
```

Maple [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int x^m \sqrt{a^2 c\,x^2+c}\ \arctan\left(a x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \sqrt{c \left(a^{2} x^{2} + 1\right)}\operatorname{atan}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.256 $\int \frac{x^m \text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} dx = \int \frac{x^m \tan^{-1}(ax)}{\sqrt{c+a^2cx^2}} dx$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2c\,x^2+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**m*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.257 $\int \frac{x^m \mathbf{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.258 $\int x^3(c+a^2cx^2)\,\textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=124

$$-\frac{cx^2}{180a^2}+\frac{cx^4}{60}+\frac{cx\text{ArcTan}(ax)}{6a^3}-\frac{cx^3\text{ArcTan}(ax)}{18a}-\frac{1}{15}acx^5\text{ArcTan}(ax)-\frac{c\text{ArcTan}(ax)^2}{12a^4}+\frac{1}{4}cx^4\text{ArcTan}(ax)^2+\frac{1}{6}$$

```
[Out] -1/180*c*x^2/a^2+1/60*c*x^4+1/6*c*x*arctan(a*x)/a^3-1/18*c*x^3*arctan(a*x)/
a-1/15*a*c*x^5*arctan(a*x)-1/12*c*arctan(a*x)^2/a^4+1/4*c*x^4*arctan(a*x)^2
+1/6*a^2*c*x^6*arctan(a*x)^2-7/90*c*ln(a^2*x^2+1)/a^4
```

Rubi [A]
time = 0.30, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5070, 4946, 5036, 272, 45, 4930, 266, 5004}

$$-\frac{c\text{ArcTan}(ax)^2}{12a^4}+\frac{cx\text{ArcTan}(ax)}{6a^3}+\frac{1}{6}a^2cx^6\text{ArcTan}(ax)^2-\frac{cx^2}{180a^2}-\frac{7c\log\left(a^2x^2+1\right)}{90a^4}-\frac{1}{15}acx^5\text{ArcTan}(ax)+\frac{1}{4}cx^4\text{ArcTan}(ax)^2-\frac{cx^3\text{ArcTan}(ax)}{18a}+\frac{cx^4}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] -1/180*(c*x^2)/a^2 + (c*x^4)/60 + (c*x*ArcTan[a*x])/(6*a^3) - (c*x^3*ArcTan
[a*x])/(18*a) - (a*c*x^5*ArcTan[a*x])/15 - (c*ArcTan[a*x]^2)/(12*a^4) + (c*
x^4*ArcTan[a*x]^2)/4 + (a^2*c*x^6*ArcTan[a*x]^2)/6 - (7*c*Log[1 + a^2*x^2])
/(90*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int x^3 (c + a^2cx^2) \tan^{-1}(ax)^2 \, dx = c \int x^3 \tan^{-1}(ax)^2 \, dx + (a^2c) \int x^5 \tan^{-1}(ax)^2 \, dx$$

$$= \frac{1}{4}cx^4 \tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6 \tan^{-1}(ax)^2 - \frac{1}{2}(ac) \int \frac{x^4 \tan^{-1}(ax)}{1 + a^2x^2} \, dx - \frac{1}{3}(a^3c$$

$$= \frac{1}{4}cx^4 \tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6 \tan^{-1}(ax)^2 - \frac{c \int x^2 \tan^{-1}(ax) \, dx}{2a} + \frac{c \int \frac{x^2 \tan^{-1}(}{1+a^2x^2}}{2a}$$

$$= -\frac{cx^3 \tan^{-1}(ax)}{6a} - \frac{1}{15}acx^5 \tan^{-1}(ax) + \frac{1}{4}cx^4 \tan^{-1}(ax)^2 + \frac{1}{6}a^2cx^6 \tan^{-1}(a$$

$$= \frac{cx \tan^{-1}(ax)}{2a^3} - \frac{cx^3 \tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5 \tan^{-1}(ax) - \frac{c \tan^{-1}(ax)^2}{4a^4} + \frac{1}{4}cx$$

$$= \frac{cx \tan^{-1}(ax)}{6a^3} - \frac{cx^3 \tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5 \tan^{-1}(ax) - \frac{c \tan^{-1}(ax)^2}{12a^4} + \frac{1}{4}cx$$

$$= \frac{cx^2}{20a^2} + \frac{cx^4}{60} + \frac{cx \tan^{-1}(ax)}{6a^3} - \frac{cx^3 \tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5 \tan^{-1}(ax) - \frac{c \tan}{1}$$

$$= -\frac{cx^2}{180a^2} + \frac{cx^4}{60} + \frac{cx \tan^{-1}(ax)}{6a^3} - \frac{cx^3 \tan^{-1}(ax)}{18a} - \frac{1}{15}acx^5 \tan^{-1}(ax) - \frac{c\,t}{}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.72

$$\frac{c(-a^2x^2 + 3a^4x^4 - 2ax(-15 + 5a^2x^2 + 6a^4x^4) \operatorname{ArcTan}(ax) + 15(-1 + 3a^4x^4 + 2a^6x^6) \operatorname{ArcTan}(ax)^2 - 14 \log(1 + a^2x^2))}{180a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
[Out] (c*(-(a^2*x^2) + 3*a^4*x^4 - 2*a*x*(-15 + 5*a^2*x^2 + 6*a^4*x^4)*ArcTan[a*x
] + 15*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^2 - 14*Log[1 + a^2*x^2]))/(
180*a^4)
```

Maple [A]
time = 0.21, size = 108, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{c \arctan(ax)^2 a^6 x^6}{6} + \frac{c \arctan(ax)^2 a^4 x^4}{4} - \frac{c\left(\frac{2\arctan(ax)a^5x^5}{5} + \frac{\arctan(ax)a^3x^3}{3} - \arctan(ax)ax + \frac{\arctan(ax)^2}{2} - \frac{a^4x^4}{10} + \frac{a^2x^2}{30} + \frac{7\ln(a}{}\right.}{6}}{a^4}$
default	$\frac{\frac{c \arctan(ax)^2 a^6 x^6}{6} + \frac{c \arctan(ax)^2 a^4 x^4}{4} - \frac{c\left(\frac{2\arctan(ax)a^5x^5}{5} + \frac{\arctan(ax)a^3x^3}{3} - \arctan(ax)ax + \frac{\arctan(ax)^2}{2} - \frac{a^4x^4}{10} + \frac{a^2x^2}{30} + \frac{7\ln(a}{}\right.}{6}}{a^4}$

risch	$-\frac{c(2a^6x^6+3a^4x^4-1)\ln(iax+1)^2}{48a^4}+\frac{c(30a^6x^6\ln(-iax+1)+12ia^5x^5+45x^4\ln(-iax+1)a^4+10ia^3x^3-30iax-15\ln(-}{360a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/6*c*arctan(a*x)^2*a^6*x^6+1/4*c*arctan(a*x)^2*a^4*x^4-1/6*c*(2/5*a
rctan(a*x)*a^5*x^5+1/3*arctan(a*x)*a^3*x^3-arctan(a*x)*a*x+1/2*arctan(a*x)^
2-1/10*a^4*x^4+1/30*a^2*x^2+7/15*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.52, size = 116, normalized size = 0.94

$$-\frac{1}{90}a\left(\frac{6a^4cx^5+5a^2cx^3-15cx}{a^4}+\frac{15c\arctan(ax)}{a^5}\right)\arctan(ax)+\frac{1}{12}(2a^2cx^6+3cx^4)\arctan(ax)^2+\frac{3a^4cx^4-a^2cx^2+15c\arctan(ax)^2-14c\log(a^2x^2+1)}{180a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/90*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)*a
rctan(a*x) + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 + 1/180*(3*a^4*c*x^
4 - a^2*c*x^2 + 15*c*arctan(a*x)^2 - 14*c*log(a^2*x^2 + 1))/a^4
```

Fricas [A]
time = 6.03, size = 97, normalized size = 0.78

$$\frac{3a^4cx^4-a^2cx^2+15(2a^6cx^6+3a^4cx^4-c)\arctan(ax)^2-2(6a^5cx^5+5a^3cx^3-15acx)\arctan(ax)-14c\log(a^2x^2+1)}{180a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*c*x^4 - a^2*c*x^2 + 15*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan(
a*x)^2 - 2*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x) - 14*c*log(a^
2*x^2 + 1))/a^4
```

Sympy [A]
time = 0.41, size = 121, normalized size = 0.98

$$\begin{cases}\frac{a^2cx^6\operatorname{atan}^2(ax)}{6}-\frac{acx^5\operatorname{atan}(ax)}{15}+\frac{cx^4\operatorname{atan}^2(ax)}{4}+\frac{cx^4}{60}-\frac{cx^3\operatorname{atan}(ax)}{18a}-\frac{cx^2}{180a^2}+\frac{cx\operatorname{atan}(ax)}{6a^3}-\frac{7c\log\left(x^2+\frac{1}{a^2}\right)}{90a^4}-\frac{c\operatorname{atan}^2(ax)}{12a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**2*c*x**6*atan(a*x)**2/6 - a*c*x**5*atan(a*x)/15 + c*x**4*atan
(a*x)**2/4 + c*x**4/60 - c*x**3*atan(a*x)/(18*a) - c*x**2/(180*a**2) + c*x*
```

```
atan(a*x)/(6*a**3) - 7*c*log(x**2 + a**(-2))/(90*a**4) - c*atan(a*x)**2/(12
*a**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.57, size = 102, normalized size = 0.82

$$-\frac{c\left(14\ln\left(a^2x^2+1\right)+a^2x^2-3a^4x^4+15\,\mathrm{atan}(a\,x)^2+10\,a^3x^3\,\mathrm{atan}(a\,x)+12\,a^5x^5\,\mathrm{atan}(a\,x)-30\,a\,x\,\mathrm{atan}(a\,x)-45\,a^4x^4\,\mathrm{atan}(a\,x)^2-30\,a^6x^6\,\mathrm{atan}(a\,x)^2\right)}{180\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2),x)
```

```
[Out] -(c*(14*log(a^2*x^2 + 1) + a^2*x^2 - 3*a^4*x^4 + 15*atan(a*x)^2 + 10*a^3*x^
3*atan(a*x) + 12*a^5*x^5*atan(a*x) - 30*a*x*atan(a*x) - 45*a^4*x^4*atan(a*x
)^2 - 30*a^6*x^6*atan(a*x)^2))/(180*a^4)
```

3.259 $\int x^2(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=156

$$\frac{cx}{30a^2}+\frac{cx^3}{30}-\frac{c\text{ArcTan}(ax)}{30a^3}-\frac{2cx^2\text{ArcTan}(ax)}{15a}-\frac{1}{10}acx^4\text{ArcTan}(ax)-\frac{2ic\text{ArcTan}(ax)^2}{15a^3}+\frac{1}{3}cx^3\text{ArcTan}(ax)^2+$$

```
[Out] 1/30*c*x/a^2+1/30*c*x^3-1/30*c*arctan(a*x)/a^3-2/15*c*x^2*arctan(a*x)/a-1/1
0*a*c*x^4*arctan(a*x)-2/15*I*c*arctan(a*x)^2/a^3+1/3*c*x^3*arctan(a*x)^2+1/
5*a^2*c*x^5*arctan(a*x)^2-4/15*c*arctan(a*x)*ln(2/(1+I*a*x))/a^3-2/15*I*c*p
olylog(2,1-2/(1+I*a*x))/a^3
```

Rubi [A]
time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {5070, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 308}

$$-\frac{2ic\text{ArcTan}(ax)^2}{15a^3}-\frac{c\text{ArcTan}(ax)}{30a^3}-\frac{4c\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^3}-\frac{2ic\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{15a^3}+\frac{1}{5}a^2cx^5\text{ArcTan}(ax)^2+\frac{cx}{30a^2}-\frac{1}{10}acx^4\text{ArcTan}(ax)+\frac{1}{3}cx^3\text{ArcTan}(ax)^2-\frac{2cx^2\text{ArcTan}(ax)}{15a}+\frac{cx^3}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*x)/(30*a^2) + (c*x^3)/30 - (c*ArcTan[a*x])/(30*a^3) - (2*c*x^2*ArcTan[a*
x])/(15*a) - (a*c*x^4*ArcTan[a*x])/10 - (((2*I)/15)*c*ArcTan[a*x]^2)/a^3 +
(c*x^3*ArcTan[a*x]^2)/3 + (a^2*c*x^5*ArcTan[a*x]^2)/5 - (4*c*ArcTan[a*x]*Lo
g[2/(1 + I*a*x)])/(15*a^3) - (((2*I)/15)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int x^2 (c+a^2cx^2) \tan^{-1}(ax)^2 \, dx = c\int x^2 \tan^{-1}(ax)^2 \, dx + (a^2c)\int x^4 \tan^{-1}(ax)^2 \, dx$$

$$= \frac{1}{3}cx^3 \tan^{-1}(ax)^2 + \frac{1}{5}a^2cx^5 \tan^{-1}(ax)^2 - \frac{1}{3}(2ac)\int \frac{x^3 \tan^{-1}(ax)}{1+a^2x^2} \, dx - \frac{1}{5}($$

$$= \frac{1}{3}cx^3 \tan^{-1}(ax)^2 + \frac{1}{5}a^2cx^5 \tan^{-1}(ax)^2 - \frac{(2c)\int x \tan^{-1}(ax) \, dx}{3a} + \frac{(2c)\int}{}$$

$$= -\frac{cx^2 \tan^{-1}(ax)}{3a} - \frac{1}{10}acx^4 \tan^{-1}(ax) - \frac{ic \tan^{-1}(ax)^2}{3a^3} + \frac{1}{3}cx^3 \tan^{-1}(ax)^2$$

$$= \frac{cx}{3a^2} - \frac{2cx^2 \tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4 \tan^{-1}(ax) - \frac{2ic \tan^{-1}(ax)^2}{15a^3} + \frac{1}{3}cx^3 \tan$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c \tan^{-1}(ax)}{3a^3} - \frac{2cx^2 \tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4 \tan^{-1}(ax) - \frac{2ic}{}$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c \tan^{-1}(ax)}{30a^3} - \frac{2cx^2 \tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4 \tan^{-1}(ax) - \frac{2ic}{}$$

$$= \frac{cx}{30a^2} + \frac{cx^3}{30} - \frac{c \tan^{-1}(ax)}{30a^3} - \frac{2cx^2 \tan^{-1}(ax)}{15a} - \frac{1}{10}acx^4 \tan^{-1}(ax) - \frac{2ic}{}$$

Mathematica [**A**]
time = 0.47, size = 104, normalized size = 0.67

$$\frac{c\left(ax + a^3x^3 + 2(2i + 5a^3x^3 + 3a^5x^5)\,\mathrm{ArcTan}(ax)^2 - \mathrm{ArcTan}(ax)\left(1 + 4a^2x^2 + 3a^4x^4 + 8\log\left(1 + e^{2i\mathrm{ArcTan}(ax)}\right)\right) + 4i\mathrm{PolyLog}\left(2, -e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{30a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(a*x + a^3*x^3 + 2*(2*I + 5*a^3*x^3 + 3*a^5*x^5)*ArcTan[a*x]^2 - ArcTan[
a*x]*(1 + 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)
*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a^3)
```

Maple [**A**]
time = 0.39, size = 227, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^2a^5x^5}{5}+\frac{c\arctan(ax)^2a^3x^3}{3}-\frac{2c\left(\frac{3\arctan(ax)a^4x^4}{4}+\arctan(ax)a^2x^2-\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{a^3x^3}{4}-\frac{ax}{4}+\frac{\arctan(ax)}{4}-\right.}{}}{}$
default	$\frac{\frac{c\arctan(ax)^2a^5x^5}{5}+\frac{c\arctan(ax)^2a^3x^3}{3}-\frac{2c\left(\frac{3\arctan(ax)a^4x^4}{4}+\arctan(ax)a^2x^2-\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{a^3x^3}{4}-\frac{ax}{4}+\frac{\arctan(ax)}{4}-\right.}{}}{}$
risch	$\frac{cx}{30a^2}-\frac{c\arctan(ax)}{30a^3}-\frac{c\ln(iax+1)^2x^3}{12}-\frac{c\ln(-iax+1)^2x^3}{12}+\frac{c\,a^2\ln(iax+1)\ln(-iax+1)x^5}{10}+\frac{ica\ln(iax+1)x^4}{20}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/5*c*arctan(a*x)^2*a^5*x^5+1/3*c*arctan(a*x)^2*a^3*x^3-2/15*c*(3/4*
arctan(a*x)*a^4*x^4+arctan(a*x)*a^2*x^2-arctan(a*x)*ln(a^2*x^2+1)-1/4*a^3*x
^3-1/4*a*x+1/4*arctan(a*x)-1/2*I*ln(a*x-I)*ln(a^2*x^2+1)+1/4*I*ln(a*x-I)^2+
1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+1/2*I*dilog(-1/2*I*(I+a*x))+1/2*I*ln(I+a
*x)*ln(a^2*x^2+1)-1/4*I*ln(I+a*x)^2-1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*I
*dilog(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^2 - 1/240*(3*a^2*c*x^5 + 5*c*x^3)*
log(a^2*x^2 + 1)^2 + integrate(1/240*(180*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)
*arctan(a*x)^2 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*log(a^2*x^2 + 1)^2 -
8*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x) + 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)*log
(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^2\operatorname{atan}^2(ax)\,dx+\int a^2x^4\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**2, x) + Integral(a**2*x**4*atan(a*x)**2, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2),x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.260 $\int x(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=96

$$\frac{c(1+a^2x^2)}{12a^2}-\frac{cx\text{ArcTan}(ax)}{3a}-\frac{cx(1+a^2x^2)\,\text{ArcTan}(ax)}{6a}+\frac{c(1+a^2x^2)^2\,\text{ArcTan}(ax)^2}{4a^2}+\frac{c\log\left(1+a^2x^2\right)}{6a^2}$$

```
[Out] 1/12*c*(a^2*x^2+1)/a^2-1/3*c*x*arctan(a*x)/a-1/6*c*x*(a^2*x^2+1)*arctan(a*x
)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^2/a^2+1/6*c*ln(a^2*x^2+1)/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5050, 4998, 4930, 266}

$$\frac{c(a^2x^2+1)^2\,\text{ArcTan}(ax)^2}{4a^2}-\frac{cx(a^2x^2+1)\,\text{ArcTan}(ax)}{6a}+\frac{c(a^2x^2+1)}{12a^2}+\frac{c\log\left(a^2x^2+1\right)}{6a^2}-\frac{cx\text{ArcTan}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(1 + a^2*x^2))/(12*a^2) - (c*x*ArcTan[a*x])/(3*a) - (c*x*(1 + a^2*x^2)*A
rcTan[a*x])/(6*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(4*a^2) + (c*Log[1 +
a^2*x^2])/(6*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x(c+a^2cx^2)\tan^{-1}(ax)^2\,dx = \frac{c(1+a^2x^2)^2\tan^{-1}(ax)^2}{4a^2} - \frac{\int (c+a^2cx^2)\tan^{-1}(ax)\,dx}{2a}$$
$$= \frac{c(1+a^2x^2)}{12a^2} - \frac{cx(1+a^2x^2)\tan^{-1}(ax)}{6a} + \frac{c(1+a^2x^2)^2\tan^{-1}(ax)^2}{4a^2} - \frac{c\int \text{ta}$$
$$= \frac{c(1+a^2x^2)}{12a^2} - \frac{cx\tan^{-1}(ax)}{3a} - \frac{cx(1+a^2x^2)\tan^{-1}(ax)}{6a} + \frac{c(1+a^2x^2)^2\tan}{4a^2}$$
$$= \frac{c(1+a^2x^2)}{12a^2} - \frac{cx\tan^{-1}(ax)}{3a} - \frac{cx(1+a^2x^2)\tan^{-1}(ax)}{6a} + \frac{c(1+a^2x^2)^2\tan}{4a^2}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.67

$$\frac{c\left(a^2x^2-2ax(3+a^2x^2)\,\text{ArcTan}(ax)+3(1+a^2x^2)^2\,\text{ArcTan}(ax)^2+2\log\left(1+a^2x^2\right)\right)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
[Out] (c*(a^2*x^2 - 2*a*x*(3 + a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcTan[a*
x]^2 + 2*Log[1 + a^2*x^2]))/(12*a^2)
```

Maple [A]
time = 0.10, size = 88, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^2a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^2}{2}+\frac{c\arctan(ax)^2}{4}-\frac{c\left(\frac{\arctan(ax)a^3x^3}{3}+\arctan(ax)ax-\frac{a^2x^2}{6}-\frac{\ln\left(a^2x^2+1\right)}{3}\right)}{2}}{a^2}$
default	$\frac{\frac{c\arctan(ax)^2a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^2}{2}+\frac{c\arctan(ax)^2}{4}-\frac{c\left(\frac{\arctan(ax)a^3x^3}{3}+\arctan(ax)ax-\frac{a^2x^2}{6}-\frac{\ln\left(a^2x^2+1\right)}{3}\right)}{2}}{a^2}$
risch	$-\frac{c(a^2x^2+1)^2\ln(iax+1)^2}{16a^2}+\frac{c(3x^4\ln(-iax+1)a^4+2ia^3x^3+6a^2x^2\ln(-iax+1)+6iax+3\ln(-iax+1))\ln(iax+1)}{24a^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4*c*arctan(a*x)^2*a^4*x^4+1/2*a^2*c*x^2*arctan(a*x)^2+1/4*c*arctan
(a*x)^2-1/2*c*(1/3*arctan(a*x)*a^3*x^3+arctan(a*x)*a*x-1/6*a^2*x^2-1/3*ln(a
^2*x^2+1)))
```

Maxima [A]
time = 0.26, size = 87, normalized size = 0.91

$$\frac{(a^2cx^2+c)^2\arctan(ax)^2}{4\,a^2c}+\frac{\left(c^2x^2+\frac{2\,c^2\log(a^2x^2+1)}{a^2}\right)a-2\,(a^2c^2x^3+3\,c^2x)\arctan(ax)}{12\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*c*x^2 + c)^2*arctan(a*x)^2/(a^2*c) + 1/12*((c^2*x^2 + 2*c^2*log(a^
2*x^2 + 1)/a^2)*a - 2*(a^2*c^2*x^3 + 3*c^2*x)*arctan(a*x))/(a*c)
```

Fricas [A]
time = 4.39, size = 74, normalized size = 0.77

$$\frac{a^2cx^2+3\,(a^4cx^4+2\,a^2cx^2+c)\arctan(ax)^2-2\,(a^3cx^3+3\,acx)\arctan(ax)+2\,c\log(a^2x^2+1)}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^2*c*x^2 + 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^2 - 2*(a^3*c*
x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1))/a^2
```

Sympy [A]
time = 0.24, size = 94, normalized size = 0.98

$$\begin{cases}\frac{a^2cx^4\operatorname{atan}^2(ax)}{4}-\frac{acx^3\operatorname{atan}(ax)}{6}+\frac{cx^2\operatorname{atan}^2(ax)}{2}+\frac{cx^2}{12}-\frac{cx\operatorname{atan}(ax)}{2a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{6a^2}+\frac{c\operatorname{atan}^2(ax)}{4a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**2*c*x**4*atan(a*x)**2/4 - a*c*x**3*atan(a*x)/6 + c*x**2*atan(
a*x)**2/2 + c*x**2/12 - c*x*atan(a*x)/(2*a) + c*log(x**2 + a**(-2))/(6*a**2
) + c*atan(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.51, size = 83, normalized size = 0.86

$$\frac{c\left(6\,x^2\,\mathrm{atan}(a\,x)^2+x^2\right)}{12}+\frac{\frac{c\left(3\,\mathrm{atan}(a\,x)^2+2\,\ln\left(a^2\,x^2+1\right)\right)}{12}-\frac{a\,c\,x\,\mathrm{atan}(a\,x)}{2}}{a^2}+\frac{a^2\,c\,x^4\,\mathrm{atan}(a\,x)^2}{4}-\frac{a\,c\,x^3\,\mathrm{atan}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2),x)
```

```
[Out] (c*(6*x^2*atan(a*x)^2 + x^2))/12 + ((c*(2*log(a^2*x^2 + 1) + 3*atan(a*x)^2)
)/12 - (a*c*x*atan(a*x))/2)/a^2 + (a^2*c*x^4*atan(a*x)^2)/4 - (a*c*x^3*atan
(a*x))/6
```

3.261 $\int (c+a^2cx^2) \textbf{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=128

$$\frac{cx}{3}-\frac{c(1+a^2x^2)\,\text{ArcTan}(ax)}{3a}+\frac{2ic\text{ArcTan}(ax)^2}{3a}+\frac{2}{3}cx\text{ArcTan}(ax)^2+\frac{1}{3}cx(1+a^2x^2)\,\text{ArcTan}(ax)^2+\frac{4c\text{ArcTan}}{}$$

```
[Out] 1/3*c*x-1/3*c*(a^2*x^2+1)*arctan(a*x)/a+2/3*I*c*arctan(a*x)^2/a+2/3*c*x*arc
tan(a*x)^2+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^2+4/3*c*arctan(a*x)*ln(2/(1+I*a*
x))/a+2/3*I*c*polylog(2,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5000, 4930, 5040, 4964, 2449, 2352, 8}

$$\frac{1}{3}cx(a^2x^2+1)\,\text{ArcTan}(ax)^2-\frac{c(a^2x^2+1)\,\text{ArcTan}(ax)}{3a}+\frac{2ic\text{ArcTan}(ax)^2}{3a}+\frac{2}{3}cx\text{ArcTan}(ax)^2+\frac{4c\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{3a}+\frac{2ic\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{3a}+\frac{cx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*x)/3 - (c*(1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (((2*I)/3)*c*ArcTan[a*x]^2)
/a + (2*c*x*ArcTan[a*x]^2)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (4*c*A
rcTan[a*x]*Log[2/(1 + I*a*x)])/(3*a) + (((2*I)/3)*c*PolyLog[2, 1 - 2/(1 + I
*a*x)])/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+a^2cx^2)\tan^{-1}(ax)^2\,dx = -\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax)^2+\frac{1}{3}c\int 1\,dx+\frac{1}{3}(2c)$$

$$= \frac{cx}{3}-\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax)$$

$$= \frac{cx}{3}-\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx(1+$$

$$= \frac{cx}{3}-\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx(1+$$

$$= \frac{cx}{3}-\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx(1+$$

$$= \frac{cx}{3}-\frac{c(1+a^2x^2)\tan^{-1}(ax)}{3a}+\frac{2ic\tan^{-1}(ax)^2}{3a}+\frac{2}{3}cx\tan^{-1}(ax)^2+\frac{1}{3}cx(1+$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.64

$$\frac{c\left(ax+(-2i+3ax+a^3x^3)\operatorname{ArcTan}(ax)^2-\operatorname{ArcTan}(ax)\left(1+a^2x^2-4\log\left(1+e^{2i\operatorname{ArcTan}(ax)}\right)\right)-2i\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(ax)}\right)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(a*x + (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 - ArcTan[a*x]*(1 + a^2*x^2
 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) - (2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*
x])]))/(3*a)
```

Maple [A]
time = 0.16, size = 202, normalized size = 1.58

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^2a^3x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{\arctan(ax)a^2x^2}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2}-\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{c\arctan(ax)^2a^3x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{\arctan(ax)a^2x^2}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2}-\cdots\right.}{\cdots}}{\cdots}$
risch	$\frac{ica\ln(iax+1)x^2}{6}-\frac{c\arctan(ax)}{3a}-\frac{c\ln(iax+1)^2x}{4}-\frac{c\ln(-iax+1)^2x}{4}+\frac{cx}{3}+\frac{c\ln(iax+1)\ln(-iax+1)x}{2}-\frac{ca^2\ln(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/3*c*arctan(a*x)^2*a^3*x^3+a*c*x*arctan(a*x)^2-2/3*c*(1/2*arctan(a*x)
*a^2*x^2+arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)+1/2*I*ln(a*x-I)*
ln(a^2*x^2+1)-1/2*I*dilog(-1/2*I*(I+a*x))-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x)
)-1/4*I*ln(a*x-I)^2-1/2*I*ln(I+a*x)*ln(a^2*x^2+1)+1/2*I*dilog(1/2*I*(a*x-I)
)+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+1/4*I*ln(I+a*x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 36*a^4*c*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^4*c*integ
rate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*integrate(1/48
```

```
*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 8*a^3*c*integrate(1/48*x^3*arctan
(a*x)/(a^2*x^2 + 1), x) + 72*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^
2 + 1), x) + 6*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 12*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/12*(
a^2*c*x^3 + 3*c*x)*arctan(a*x)^2 + 1/4*c*arctan(a*x)^3/a - 24*a*c*integrate
(1/48*x*arctan(a*x)/(a^2*x^2 + 1), x) - 1/48*(a^2*c*x^3 + 3*c*x)*log(a^2*x^
2 + 1)^2 + 3*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}x^{2}\operatorname{atan}^{2}{\left(ax\right)}\,dx + \int \operatorname{atan}^{2}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2,x)
[Out] c*(Integral(a**2*x**2*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{atan}\left(a\,x\right)}^2\,\left(c\,a^2\,x^2+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2),x)
[Out] int(atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.262 $\int \frac{(c+a^2cx^2)\mathbf{ArcTan}(ax)^2}{x}\,dx$

Optimal. Leaf size=169

$$-acx\text{ArcTan}(ax)+\frac{1}{2}c\text{ArcTan}(ax)^2+\frac{1}{2}a^2cx^2\text{ArcTan}(ax)^2+2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)+\frac{1}{2}c\log(1$$

```
[Out] -a*c*x*arctan(a*x)+1/2*c*arctan(a*x)^2+1/2*a^2*c*x^2*arctan(a*x)^2-2*c*arct
an(a*x)^2*arctanh(-1+2/(1+I*a*x))+1/2*c*ln(a^2*x^2+1)-I*c*arctan(a*x)*polyl
og(2,1-2/(1+I*a*x))+I*c*arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-1/2*c*polylog
(3,1-2/(1+I*a*x))+1/2*c*polylog(3,-1+2/(1+I*a*x))
```

Rubi [A]
time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 4930, 266}

$$\frac{1}{2}a^2cx^2\text{ArcTan}(ax)^2+\frac{1}{2}c\log\left(a^2x^2+1\right)-ic\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+ic\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{iax+1}-1\right)+\frac{1}{2}c\text{ArcTan}(ax)^2-acx\text{ArcTan}(ax)+2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{1}{2}c\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{1}{2}c\text{Li}_3\left(\frac{2}{iax+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] -(a*c*x*ArcTan[a*x]) + (c*ArcTan[a*x]^2)/2 + (a^2*c*x^2*ArcTan[a*x]^2)/2 +
2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (c*Log[1 + a^2*x^2])/2 - I*c
*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c*ArcTan[a*x]*PolyLog[2, -1
+ 2/(1 + I*a*x)] - (c*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c*PolyLog[3, -1 +
 2/(1 + I*a*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x}\,dx + (a^2c)\int x\tan^{-1}(ax)^2\,dx$$
$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (4ac)\int \frac{\tan^{-1}(}{}$$
$$= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (ac)\int \tan^{-1}(a$$
$$= -acx\tan^{-1}(ax) + \frac{1}{2}c\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh$$
$$= -acx\tan^{-1}(ax) + \frac{1}{2}c\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^2 + 2c\tan^{-1}(ax)^2\tanh$$

Mathematica [A]
time = 0.03, size = 177, normalized size = 1.05

$$-acx\text{ArcTan}(ax) + \frac{1}{2}c(1+a^2x^2)\text{ArcTan}(ax)^2 + 2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2i}{i-ax}\right) + \frac{1}{2}c\log(1+a^2x^2) + ic\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right) - ic\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right) + \frac{1}{2}c\text{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right) - \frac{1}{2}c\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] -(a*c*x*ArcTan[a*x]) + (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/2 + 2*c*ArcTan[a*x]^
2*ArcTanh[1 - (2*I)/(I - a*x)] + (c*Log[1 + a^2*x^2])/2 + I*c*ArcTan[a*x]*P
olyLog[2, (-I - a*x)/(-I + a*x)] - I*c*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I
 + a*x)] + (c*PolyLog[3, (-I - a*x)/(-I + a*x)])/2 - (c*PolyLog[3, (I + a*x
)/(-I + a*x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.43, size = 1055, normalized size = 6.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*c*x^2*arctan(a*x)^2+c*arctan(a*x)^2*ln(a*x)-c*(arctan(a*x)^2*ln((1+
I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I
*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/
(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan
```

```
(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arct
an(a*x)^2+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*Pi*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))*arctan(a*x)^2+arctan(a*x)*(a*x-I)+ln((1+I*a*x)^2/(a^2*x^2+1)+1)+1
/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-1/2*I*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-
1/2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*arctan(a*x)^2-1/2*I*Pi*arctan(a*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*c*x^2*arctan(a*x)^2 - 1/32*a^2*c*x^2*log(a^2*x^2 + 1)^2 + 12*a^4*c*
integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + a^4*c*integrate(1/16*x
^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^4*c*integrate(1/16*x^4*log(a^
2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^3*c*integrate(1/16*x^3*arctan(a*x)/(a^2*
x^3 + x), x) + 24*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x)
+ 1/48*c*log(a^2*x^2 + 1)^3 + 12*c*integrate(1/16*arctan(a*x)^2/(a^2*x^3 +
x), x) + c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{x}\, dx + \int a^{2} x \operatorname{atan}^{2}{\left(ax \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x, x) + Integral(a**2*x*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2))/x,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2))/x, x)
```

3.263 $\int \frac{(c+a^2cx^2)\text{ArcTan}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=113

$$-\frac{c\text{ArcTan}(ax)^2}{x}+a^2cx\text{ArcTan}(ax)^2+2ac\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2ac\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-$$

```
[Out] -c*arctan(a*x)^2/x+a^2*c*x*arctan(a*x)^2+2*a*c*arctan(a*x)*ln(2/(1+I*a*x))+
2*a*c*arctan(a*x)*ln(2-2/(1-I*a*x))-I*a*c*polylog(2,-1+2/(1-I*a*x))+I*a*c*p
olylog(2,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 4946, 5044, 4988, 2497, 4930, 5040, 4964, 2449, 2352}

$$a^2cx\text{ArcTan}(ax)^2-\frac{c\text{ArcTan}(ax)^2}{x}+2ac\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2ac\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-iac\text{Li}_2\left(\frac{2}{1-iax}-1\right)+iac\text{Li}_2\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] -((c*ArcTan[a*x]^2)/x) + a^2*c*x*ArcTan[a*x]^2 + 2*a*c*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + 2*a*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c*PolyLog[2, -1
 + 2/(1 - I*a*x)] + I*a*c*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^2}\,dx &= c\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx + (a^2c)\int \tan^{-1}(ax)^2\,dx \\
&= -\frac{c\tan^{-1}(ax)^2}{x} + a^2cx\tan^{-1}(ax)^2 + (2ac)\int \frac{\tan^{-1}(ax)}{x\,(1+a^2x^2)}\,dx - (2a^3c)\int \frac{x
\\
&= -\frac{c\tan^{-1}(ax)^2}{x} + a^2cx\tan^{-1}(ax)^2 + (2iac)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx + (2a^2c)\int \frac{\tan
\\
&= -\frac{c\tan^{-1}(ax)^2}{x} + a^2cx\tan^{-1}(ax)^2 + 2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right) + 2ac\tan
\\
&= -\frac{c\tan^{-1}(ax)^2}{x} + a^2cx\tan^{-1}(ax)^2 + 2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right) + 2ac\tan
\\
&= -\frac{c\tan^{-1}(ax)^2}{x} + a^2cx\tan^{-1}(ax)^2 + 2ac\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right) + 2ac\tan
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 123, normalized size = 1.09

$$ac\left(-i\text{ArcTan}(ax)^2 + ax\text{ArcTan}(ax)^2 + 2\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)\right) + ac\left(-\frac{\text{ArcTan}(ax)^2}{ax} + 2\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right) - i\left(\text{ArcTan}(ax)^2 + \text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] a*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[a*x]^2 + 2*ArcTan[a*x]*Log[1 + E^((2*I
)*ArcTan[a*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])]) + a*c*(-(ArcTan[a*x
]^2/(a*x)) + 2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] - I*(ArcTan[a*x]^
2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(107) = 214.
time = 0.13, size = 240, normalized size = 2.12

method	result
derivativedivides	$a\Bigg(acx\arctan(ax)^2 - \frac{c\arctan(ax)^2}{ax} - 2c\Bigg(\arctan(ax)\ln(a^2x^2+1) - \arctan(ax)\ln(ax)$

default	$a\left(acx\arctan(ax)^2-\frac{c\arctan(ax)^2}{ax}-2c\left(\arctan(ax)\ln(a^2x^2+1)-\arctan(ax)\ln(ax)+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)

[Out] a*(a*c*x*arctan(a*x)^2-c*arctan(a*x)^2/a/x-2*c*(arctan(a*x)*ln(a^2*x^2+1)-a
rctan(a*x)*ln(a*x)+1/2*I*ln(a*x-I)*ln(a^2*x^2+1)-1/2*I*dilog(-1/2*I*(I+a*x)
)-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/4*I*ln(a*x-I)^2-1/2*I*ln(I+a*x)*ln(a
^2*x^2+1)+1/2*I*dilog(1/2*I*(a*x-I))+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+1/4*
I*ln(I+a*x)^2-1/2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dil
og(1+I*a*x)+1/2*I*dilog(1-I*a*x)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="maxima")

[Out] 1/16*(4*(a^2*c*x^2 - c)*arctan(a*x)^2 - (a^2*c*x^2 - c)*log(a^2*x^2 + 1)^2
+ 8*(24*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 2*a^4*
c*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 8*a^4*c*integ
rate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + a*c*arctan(a*x)^3 - 16
*a^3*c*integrate(1/16*x^3*arctan(a*x)/(a^2*x^4 + x^2), x) + 4*a^2*c*integra
te(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 8*a^2*c*integrate(1/16
*x^2*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 16*a*c*integrate(1/16*x*arctan(
a*x)/(a^2*x^4 + x^2), x) + 24*c*integrate(1/16*arctan(a*x)^2/(a^2*x^4 + x^2
), x) + 2*c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2\operatorname{atan}^2(ax)\,dx+\int\frac{\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2))/x^2,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2))/x^2, x)
```

3.264 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=196

$$-\frac{ac\text{ArcTan}(ax)}{x}-\frac{1}{2}a^2c\text{ArcTan}(ax)^2-\frac{c\text{ArcTan}(ax)^2}{2x^2}+2a^2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)+a^2c\log(x)-$$

```
[Out] -a*c*arctan(a*x)/x-1/2*a^2*c*arctan(a*x)^2-1/2*c*arctan(a*x)^2/x^2-2*a^2*c*
arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a^2*c*ln(x)-1/2*a^2*c*ln(a^2*x^2+1)-I
*a^2*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*a^2*c*arctan(a*x)*polylog(2,-
1+2/(1+I*a*x))-1/2*a^2*c*polylog(3,1-2/(1+I*a*x))+1/2*a^2*c*polylog(3,-1+2/
(1+I*a*x))
```

Rubi [A]
time = 0.24, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5070, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745}

$$-ia^2c\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+ia^2c\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{1}{2}a^2c\text{ArcTan}(ax)^2+2a^2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{1}{2}a^2c\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{1}{2}a^2c\text{Li}_3\left(\frac{2}{iax+1}-1\right)-\frac{1}{2}a^2c\log\left(a^2x^2+1\right)+a^2c\log(x)-\frac{c\text{ArcTan}(ax)^2}{2x^2}-\frac{ac\text{ArcTan}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c*ArcTan[a*x])/x) - (a^2*c*ArcTan[a*x]^2)/2 - (c*ArcTan[a*x]^2)/(2*x^2
) + 2*a^2*c*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c*Log[x] - (a^2*
c*Log[1 + a^2*x^2])/2 - I*a^2*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] +
 I*a^2*c*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (a^2*c*PolyLog[3, 1 -
 2/(1 + I*a*x)])/2 + (a^2*c*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
```

```
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^3}\,dx &= c\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx+(a^2c)\int \frac{\tan^{-1}(ax)^2}{x}\,dx\\
&= -\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)+(ac)\int \frac{\tan^{-1}(a}{x^2\,(1+a^2}\\
&= -\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)+(ac)\int \frac{\tan^{-1}(ax}{x^2}\\
&= -\frac{ac\tan^{-1}(ax)}{x}-\frac{1}{2}a^2c\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}\\
&= -\frac{ac\tan^{-1}(ax)}{x}-\frac{1}{2}a^2c\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}\\
&= -\frac{ac\tan^{-1}(ax)}{x}-\frac{1}{2}a^2c\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}\\
&= -\frac{ac\tan^{-1}(ax)}{x}-\frac{1}{2}a^2c\tan^{-1}(ax)^2-\frac{c\tan^{-1}(ax)^2}{2x^2}+2a^2c\tan^{-1}(ax)^2\tanh^{-1}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 208, normalized size = 1.06

$$-\frac{ac\text{ArcTan}(ax)}{x}+\frac{c(-1-a^2x^2)\,\text{ArcTan}(ax)^2}{2x^2}+2a^2c\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2i}{i-ax}\right)+a^2c\log(x)-\frac{1}{2}a^2c\log\left(1+a^2x^2\right)+ia^2c\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right)-ia^2c\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)+\frac{1}{2}a^2c\text{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right)-\frac{1}{2}a^2c\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c*ArcTan[a*x])/x) + (c*(-1 - a^2*x^2)*ArcTan[a*x]^2)/(2*x^2) + 2*a^2*c
*ArcTan[a*x]^2*ArcTanh[1 - (2*I)/(I - a*x)] + a^2*c*Log[x] - (a^2*c*Log[1 +
 a^2*x^2])/2 + I*a^2*c*ArcTan[a*x]*PolyLog[2, (-I - a*x)/(-I + a*x)] - I*a^
2*c*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)] + (a^2*c*PolyLog[3, (-I -
a*x)/(-I + a*x)])/2 - (a^2*c*PolyLog[3, (I + a*x)/(-I + a*x)])/2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.40, size = 1134, normalized size = 5.79

method	result	size
derivativedivides	Expression too large to display	1134
default	Expression too large to display	1134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*c*arctan(a*x)^2/a^2/x^2+c*arctan(a*x)^2*ln(a*x)-c*(arctan(a*x)^2*
ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^3*arctan(a*x)^2-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(
1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^2*arctan(a*x)^2-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I
*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2
/(a^2*x^2+1))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/
2*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+1/2*I*Pi*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2+1/2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+1/2*ar
ctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arc
tan(a*x)^2-1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))^3*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))-1/2*I*Pi*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/
(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))^2*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/96*((1152*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^5 + x^3), x) + a^
2*c*log(a^2*x^2 + 1)^3 + 2304*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x
^5 + x^3), x) + 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5 +
x^3), x) - 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x
) + 384*a*c*integrate(1/16*x*arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*c*integ
rate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 96*c*integrate(1/16*log(a^2*x
^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 - 12*c*arctan(a*x)^2 + 3*c*log(a^2*x^2 +
 1)^2)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{x^{3}}\, dx + \int \frac{a^{2} \operatorname{atan}^{2}{\left(ax \right)}}{x}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**3,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x**3, x) + Integral(a**2*atan(a*x)**2/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^2\,\left(c\,a^2\,x^2+c\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2))/x^3,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2))/x^3, x)
```

3.265 $\int \frac{(c+a^2cx^2)\mathbf{ArcTan}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=135

$$-\frac{a^2c}{3x}-\frac{1}{3}a^3c\text{ArcTan}(ax)-\frac{ac\text{ArcTan}(ax)}{3x^2}-\frac{2}{3}ia^3c\text{ArcTan}(ax)^2-\frac{c\text{ArcTan}(ax)^2}{3x^3}-\frac{a^2c\text{ArcTan}(ax)^2}{x}+\frac{4}{3}a^3c\text{Arc}$$

```
[Out] -1/3*a^2*c/x-1/3*a^3*c*arctan(a*x)-1/3*a*c*arctan(a*x)/x^2-2/3*I*a^3*c*arct
an(a*x)^2-1/3*c*arctan(a*x)^2/x^3-a^2*c*arctan(a*x)^2/x+4/3*a^3*c*arctan(a*
x)*ln(2-2/(1-I*a*x))-2/3*I*a^3*c*polylog(2,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.23, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5070, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$-\frac{2}{3}ia^3c\text{ArcTan}(ax)^2-\frac{1}{3}a^3c\text{ArcTan}(ax)+\frac{4}{3}a^3c\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{2}{3}ia^3c\text{Li}_2\left(\frac{2}{1-iax}-1\right)-\frac{a^2c\text{ArcTan}(ax)^2}{x}-\frac{a^2c}{3x}-\frac{c\text{ArcTan}(ax)^2}{3x^3}-\frac{ac\text{ArcTan}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c)/x - (a^3*c*ArcTan[a*x])/3 - (a*c*ArcTan[a*x])/(3*x^2) - ((2*I)
/3)*a^3*c*ArcTan[a*x]^2 - (c*ArcTan[a*x]^2)/(3*x^3) - (a^2*c*ArcTan[a*x]^2)
/x + (4*a^3*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((2*I)/3)*a^3*c*PolyL
og[2, -1 + 2/(1 - I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx$$
$$= -\frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + \frac{1}{3}(2ac)\int \frac{\tan^{-1}(ax)}{x^3(1+a^2x^2)}\,dx + (2a^3c)\int$$
$$= -ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + \frac{1}{3}(2ac)\int \frac{\tan^{-1}(ax)}{x^3}$$
$$= -\frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x} + 2a^3$$
$$= -\frac{a^2c}{3x} - \frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3} - \frac{a^2c\tan^{-1}(ax)^2}{x}$$
$$= -\frac{a^2c}{3x} - \frac{1}{3}a^3c\tan^{-1}(ax) - \frac{ac\tan^{-1}(ax)}{3x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^2}{3x^3}$$

Mathematica [A]
time = 0.46, size = 103, normalized size = 0.76

$$\frac{c\left(-a^2x^2+(1-2iax)(-i+ax)^2\text{ArcTan}(ax)^2+ax\text{ArcTan}(ax)\left(-1-a^2x^2+4a^2x^2\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)-2ia^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c*(-(a^2*x^2) + (1 - (2*I)*a*x)*(-I + a*x)^2*ArcTan[a*x]^2 + a*x*ArcTan[a*
x]*(-1 - a^2*x^2 + 4*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) - (2*I)*a^3*x^
3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(117) = 234.
time = 0.19, size = 273, normalized size = 2.02

method	result
derivativedivides	$a^3\left(-\frac{c\arctan(ax)^2}{ax} - \frac{c\arctan(ax)^2}{3a^3x^3} - \frac{2c\left(\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\frac{1}{2ax}+\frac{\arctan}{2}\right.}{}\right.$
default	$a^3\left(-\frac{c\arctan(ax)^2}{ax} - \frac{c\arctan(ax)^2}{3a^3x^3} - \frac{2c\left(\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\frac{1}{2ax}+\frac{\arctan}{2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-c*arctan(a*x)^2/a/x-1/3*c*arctan(a*x)^2/a^3/x^3-2/3*c*(arctan(a*x)*ln
(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-2*arctan(a*x)*ln(a*x)+1/2/a/x+1/2*arcta
n(a*x)-I*ln(a*x)*ln(1+I*a*x)-1/4*I*ln(a*x-I)^2-I*dilog(1+I*a*x)+1/4*I*ln(I+
a*x)^2+I*ln(a*x)*ln(1-I*a*x)+I*dilog(1-I*a*x)+1/2*I*ln(a*x-I)*ln(a^2*x^2+1)
-1/2*I*ln(I+a*x)*ln(a^2*x^2+1)-1/2*I*dilog(-1/2*I*(I+a*x))+1/2*I*dilog(1/2*
I*(a*x-I))-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x
-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*(12*(a^3*c*arctan(a*x)^3 + 12*a^4*c*integrate(1/48*x^4*log(a^2*x^2 + 1
)^2/(a^2*x^6 + x^4), x) - 48*a^4*c*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2
*x^6 + x^4), x) + 96*a^3*c*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4),
x) + 288*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^
2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2*c*in
tegrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c*integrate(1/
48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 144*c*integrate(1/48*arctan(a*x)^2/(
a^2*x^6 + x^4), x) + 12*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4)
, x))*x^3 - 4*(3*a^2*c*x^2 + c)*arctan(a*x)^2 + (3*a^2*c*x^2 + c)*log(a^2*x
^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**2/x**4,x)
```

```
[Out] c*(Integral(atan(a*x)**2/x**4, x) + Integral(a**2*atan(a*x)**2/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2))/x^4,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2))/x^4, x)
```

3.266 $\int x^3(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=191

$$-\frac{5c^2x^2}{504a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\operatorname{ArcTan}(ax)}{12a^3}-\frac{c^2x^3\operatorname{ArcTan}(ax)}{36a}-\frac{1}{12}ac^2x^5\operatorname{ArcTan}(ax)-\frac{1}{28}a^3c^2x^7\operatorname{ArcTan}(ax)$$

```
[Out] -5/504*c^2*x^2/a^2+1/84*c^2*x^4+1/168*a^2*c^2*x^6+1/12*c^2*x*arctan(a*x)/a^
3-1/36*c^2*x^3*arctan(a*x)/a-1/12*a*c^2*x^5*arctan(a*x)-1/28*a^3*c^2*x^7*ar
ctan(a*x)-1/24*c^2*arctan(a*x)^2/a^4+1/4*c^2*x^4*arctan(a*x)^2+1/3*a^2*c^2*
x^6*arctan(a*x)^2+1/8*a^4*c^2*x^8*arctan(a*x)^2-2/63*c^2*ln(a^2*x^2+1)/a^4
```

Rubi [A]
time = 0.56, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5068, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{1}{8}a^4c^2x^8\operatorname{ArcTan}(ax)^2-\frac{c^2\operatorname{ArcTan}(ax)^2}{24a^4}-\frac{1}{28}a^3c^2x^7\operatorname{ArcTan}(ax)+\frac{c^2x\operatorname{ArcTan}(ax)}{12a^3}+\frac{1}{3}a^2c^2x^6\operatorname{ArcTan}(ax)^2+\frac{1}{168}a^2c^2x^6-\frac{5c^2x^2}{504a^2}-\frac{2c^2\log(a^2x^2+1)}{63a^4}-\frac{1}{12}ac^2x^5\operatorname{ArcTan}(ax)+\frac{1}{4}c^2x^4\operatorname{ArcTan}(ax)^2-\frac{c^2x^3\operatorname{ArcTan}(ax)}{36a}+\frac{c^2x^4}{84}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (-5*c^2*x^2)/(504*a^2) + (c^2*x^4)/84 + (a^2*c^2*x^6)/168 + (c^2*x*ArcTan[a
*x])/(12*a^3) - (c^2*x^3*ArcTan[a*x])/(36*a) - (a*c^2*x^5*ArcTan[a*x])/12 -
 (a^3*c^2*x^7*ArcTan[a*x])/28 - (c^2*ArcTan[a*x]^2)/(24*a^4) + (c^2*x^4*Arc
Tan[a*x]^2)/4 + (a^2*c^2*x^6*ArcTan[a*x]^2)/3 + (a^4*c^2*x^8*ArcTan[a*x]^2)
/8 - (2*c^2*Log[1 + a^2*x^2])/(63*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx = \int \left(c^2x^3\tan^{-1}(ax)^2+2a^2c^2x^5\tan^{-1}(ax)^2+a^4c^2x^7\tan^{-1}(ax)^2\right)\,dx$$

$$= c^2\int x^3\tan^{-1}(ax)^2\,dx+\left(2a^2c^2\right)\int x^5\tan^{-1}(ax)^2\,dx+\left(a^4c^2\right)\int x^7\tan^{-1}($$

$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{1}{2}\left(ac^2\right)$$

$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^2+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^2+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^2-\frac{c^2\int x^2\,\mathrm{t}}{}$$

$$= -\frac{c^2x^3\tan^{-1}(ax)}{6a}-\frac{2}{15}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-1}(ax)+\frac{1}{4}c^2x^4\tan$$

$$= \frac{c^2x\tan^{-1}(ax)}{2a^3}+\frac{c^2x^3\tan^{-1}(ax)}{18a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-1}($$

$$= -\frac{c^2x\tan^{-1}(ax)}{6a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2x^5\tan^{-1}(ax)-\frac{1}{28}a^3c^2x^7\tan^{-}$$

$$= \frac{29c^2x^2}{840a^2}+\frac{41c^2x^4}{1680}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac$$

$$= -\frac{13c^2x^2}{252a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac$$

$$= -\frac{5c^2x^2}{504a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\tan^{-1}(ax)}{12a^3}-\frac{c^2x^3\tan^{-1}(ax)}{36a}-\frac{1}{12}ac^2$$

Mathematica [A]
time = 0.04, size = 110, normalized size = 0.58

$$\frac{c^2\left(-5a^2x^2+6a^4x^4+3a^6x^6-2ax(-21+7a^2x^2+21a^4x^4+9a^6x^6)\,\mathrm{ArcTan}(ax)+21(1+a^2x^2)^3(-1+3a^2x^2)\,\mathrm{ArcTan}(ax)^2-16\log(1+a^2x^2)\right)}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(-5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 - 2*a*x*(-21 + 7*a^2*x^2 + 21*a^4*
x^4 + 9*a^6*x^6)*ArcTan[a*x] + 21*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcTan[a
*x]^2 - 16*Log[1 + a^2*x^2]))/(504*a^4)
```

Maple [A]
time = 0.28, size = 150, normalized size = 0.79

method	result

derivativedivides	$\frac{\frac{c^2 \arctan(ax)^2 a^8 x^8}{8}+\frac{c^2 \arctan(ax)^2 a^6 x^6}{3}+\frac{a^4 c^2 x^4 \arctan(ax)^2}{4}-\frac{c^2\left(\frac{3 \arctan(ax) a^7 x^7}{7}+\arctan(ax) a^5 x^5+\frac{\arctan(ax) a^3 x^3}{3}-\mathrm{arcta}\right.}{1}}{a^4}$
default	$\frac{\frac{c^2 \arctan(ax)^2 a^8 x^8}{8}+\frac{c^2 \arctan(ax)^2 a^6 x^6}{3}+\frac{a^4 c^2 x^4 \arctan(ax)^2}{4}-\frac{c^2\left(\frac{3 \arctan(ax) a^7 x^7}{7}+\arctan(ax) a^5 x^5+\frac{\arctan(ax) a^3 x^3}{3}-\mathrm{arcta}\right.}{1}}{a^4}$
risch	$-\frac{c^2(3a^8x^8+8a^6x^6+6a^4x^4-1)\ln(iax+1)^2}{96a^4}+\frac{c^2(63a^8x^8\ln(-iax+1)+18ia^7x^7+168a^6x^6\ln(-iax+1)+42ia^5x^5+126}{1008a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/8*c^2*arctan(a*x)^2*a^8*x^8+1/3*c^2*arctan(a*x)^2*a^6*x^6+1/4*a^4*
c^2*x^4*arctan(a*x)^2-1/12*c^2*(3/7*arctan(a*x)*a^7*x^7+arctan(a*x)*a^5*x^5
+1/3*arctan(a*x)*a^3*x^3-arctan(a*x)*a*x+1/2*arctan(a*x)^2-1/14*a^6*x^6-1/7
*a^4*x^4+5/42*a^2*x^2+8/21*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.49, size = 169, normalized size = 0.88

$$-\frac{1}{252}\,a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)\arctan(ax)+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)^2+\frac{3\,a^6c^2x^6+6\,a^4c^2x^4-5\,a^2c^2x^2+21\,c^2\arctan(ax)^2-16\,c^2\log(a^2x^2+1)}{504\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/252*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2*
c^2*x^3 - 21*c^2*x)/a^4)*arctan(a*x) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6
+ 6*c^2*x^4)*arctan(a*x)^2 + 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c
^2*x^2 + 21*c^2*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1))/a^4
```

Fricas [A]
time = 3.29, size = 148, normalized size = 0.77

$$\frac{3\,a^6c^2x^6+6\,a^4c^2x^4-5\,a^2c^2x^2+21\left(3\,a^8c^2x^8+8\,a^6c^2x^6+6\,a^4c^2x^4-c^2\right)\arctan(ax)^2-16\,c^2\log(a^2x^2+1)-2\left(9\,a^7c^2x^7+21\,a^5c^2x^5+7\,a^3c^2x^3-21\,ac^2x\right)\arctan(ax)}{504\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c^2*x^2 + 21*(3*a^8*c^2*x^8 +
8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1
) - 2*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*arctan(
a*x))/a^4
```

Sympy [A]
time = 0.58, size = 185, normalized size = 0.97

$$\begin{cases}\frac{a^4c^2x^8\operatorname{atan}^2(ax)}{8}-\frac{a^3c^2x^7\operatorname{atan}(ax)}{28}+\frac{a^2c^2x^6\operatorname{atan}^2(ax)}{3}+\frac{a^2c^2x^6}{168}-\frac{ac^2x^5\operatorname{atan}(ax)}{12}+\frac{c^2x^4\operatorname{atan}^2(ax)}{4}+\frac{c^2x^4}{84}-\frac{c^2x^3\operatorname{atan}(ax)}{36a}-\frac{5c^2x^2}{504a^2}+\frac{c^2x\operatorname{atan}(ax)}{12a^3}-\frac{2c^2\log\left(x^2+\frac{1}{a^2}\right)}{63a^4}-\frac{c^2\operatorname{atan}^2(ax)}{24a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**8*atan(a*x)**2/8 - a**3*c**2*x**7*atan(a*x)/28 + a*
*2*c**2*x**6*atan(a*x)**2/3 + a**2*c**2*x**6/168 - a*c**2*x**5*atan(a*x)/12
 + c**2*x**4*atan(a*x)**2/4 + c**2*x**4/84 - c**2*x**3*atan(a*x)/(36*a) - 5
*c**2*x**2/(504*a**2) + c**2*x*atan(a*x)/(12*a**3) - 2*c**2*log(x**2 + a**(
-2))/(63*a**4) - c**2*atan(a*x)**2/(24*a**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 145, normalized size = 0.76

$$\mathrm{atan}(a\,x)^2\,\left(\frac{c^2\,x^4}{4}-\frac{c^2}{24\,a^4}+\frac{a^2\,c^2\,x^6}{3}+\frac{a^4\,c^2\,x^8}{8}\right)+\frac{c^2\,x^4}{84}-a^2\,\mathrm{atan}(a\,x)\,\left(\frac{a\,c^2\,x^7}{28}-\frac{c^2\,x}{12\,a^5}+\frac{c^2\,x^5}{12\,a}+\frac{c^2\,x^3}{36\,a^3}\right)-\frac{2\,c^2\,\ln\left(a^2\,x^2+1\right)}{63\,a^4}-\frac{5\,c^2\,x^2}{504\,a^2}+\frac{a^2\,c^2\,x^6}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

```
[Out] atan(a*x)^2*((c^2*x^4)/4 - c^2/(24*a^4) + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/8
) + (c^2*x^4)/84 - a^2*atan(a*x)*((a*c^2*x^7)/28 - (c^2*x)/(12*a^5) + (c^2*
x^5)/(12*a) + (c^2*x^3)/(36*a^3)) - (2*c^2*log(a^2*x^2 + 1))/(63*a^4) - (5*
c^2*x^2)/(504*a^2) + (a^2*c^2*x^6)/168
```

3.267 $\int x^2(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=225

$$-\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\text{ArcTan}(ax)}{210a^3}-\frac{8c^2x^2\text{ArcTan}(ax)}{105a}-\frac{9}{70}ac^2x^4\text{ArcTan}(ax)-\frac{1}{21}a^3c^2x^6\text{ArcTan}($$

```
[Out] -1/210*c^2*x/a^2+17/630*c^2*x^3+1/105*a^2*c^2*x^5+1/210*c^2*arctan(a*x)/a^3
-8/105*c^2*x^2*arctan(a*x)/a-9/70*a*c^2*x^4*arctan(a*x)-1/21*a^3*c^2*x^6*ar
ctan(a*x)-8/105*I*c^2*arctan(a*x)^2/a^3+1/3*c^2*x^3*arctan(a*x)^2+2/5*a^2*c
^2*x^5*arctan(a*x)^2+1/7*a^4*c^2*x^7*arctan(a*x)^2-16/105*c^2*arctan(a*x)*l
n(2/(1+I*a*x))/a^3-8/105*I*c^2*polylog(2,1-2/(1+I*a*x))/a^3
```

Rubi [A]
time = 0.54, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5068, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 308}

$$\frac{1}{7}a^4c^2x^7\text{ArcTan}(ax)^2-\frac{1}{21}a^3c^2x^6\text{ArcTan}(ax)-\frac{8ic^2\text{ArcTan}(ax)^2}{105a^3}+\frac{c^2\text{ArcTan}(ax)}{210a^3}-\frac{16c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{105a^3}-\frac{8ic^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{105a^3}+\frac{2}{5}a^2c^2x^5\text{ArcTan}(ax)^2+\frac{1}{105}a^2c^2x^5-\frac{c^2x}{210a^2}-\frac{9}{70}ac^2x^4\text{ArcTan}(ax)+\frac{1}{3}c^2x^3\text{ArcTan}(ax)^2-\frac{8c^2x^2\text{ArcTan}(ax)}{105a}+\frac{17c^2x^3}{630}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] -1/210*(c^2*x)/a^2 + (17*c^2*x^3)/630 + (a^2*c^2*x^5)/105 + (c^2*ArcTan[a*x
])/(210*a^3) - (8*c^2*x^2*ArcTan[a*x])/(105*a) - (9*a*c^2*x^4*ArcTan[a*x])/
70 - (a^3*c^2*x^6*ArcTan[a*x])/21 - (((8*I)/105)*c^2*ArcTan[a*x]^2)/a^3 + (
c^2*x^3*ArcTan[a*x]^2)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^2)/5 + (a^4*c^2*x^7*A
rcTan[a*x]^2)/7 - (16*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(105*a^3) - (((8*
I)/105)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx = \int \left(c^2x^2\tan^{-1}(ax)^2+2a^2c^2x^4\tan^{-1}(ax)^2+a^4c^2x^6\tan^{-1}(ax)^2\right)\,dx$$

$$= c^2\int x^2\tan^{-1}(ax)^2\,dx+\left(2a^2c^2\right)\int x^4\tan^{-1}(ax)^2\,dx+\left(a^4c^2\right)\int x^6\tan^{-}$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^2+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{1}{3}(2ac^2$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^2+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^2+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^2-\frac{(2c^2)\int}{}$$

$$= -\frac{c^2x^2\tan^{-1}(ax)}{3a}-\frac{1}{5}ac^2x^4\tan^{-1}(ax)-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)-\frac{ic^2\tan^{-1}}{3a^3}$$

$$= \frac{c^2x}{3a^2}+\frac{c^2x^2\tan^{-1}(ax)}{15a}-\frac{9}{70}ac^2x^4\tan^{-1}(ax)-\frac{1}{21}a^3c^2x^6\tan^{-1}(ax)+\frac{ic^2}{}$$

$$= -\frac{23c^2x}{105a^2}+\frac{16c^2x^3}{315}+\frac{1}{105}a^2c^2x^5-\frac{c^2\tan^{-1}(ax)}{3a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}$$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{23c^2\tan^{-1}(ax)}{105a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-$$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\tan^{-1}(ax)}{210a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}$$

$$= -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\tan^{-1}(ax)}{210a^3}-\frac{8c^2x^2\tan^{-1}(ax)}{105a}-\frac{9}{70}$$

Mathematica [A]
time = 0.89, size = 133, normalized size = 0.59

$$\frac{c^2\left(ax(-3+17a^2x^2+6a^4x^4)+6(8i+35a^3x^3+42a^5x^5+15a^7x^7)\,\mathrm{ArcTan}(ax)^2-3\mathrm{ArcTan}(ax)\left(-1+16a^2x^2+27a^4x^4+10a^6x^6+32\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)\right)+48i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{630a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(a*x*(-3 + 17*a^2*x^2 + 6*a^4*x^4) + 6*(8*I + 35*a^3*x^3 + 42*a^5*x^5
+ 15*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(-1 + 16*a^2*x^2 + 27*a^4*x^4 +
```

```
10*a^6*x^6 + 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (48*I)*PolyLog[2, -E^((2
*I)*ArcTan[a*x])]))/(630*a^3)
```

Maple [A]
time = 0.36, size = 271, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{c^2 \arctan(ax)^2 a^7 x^7}{7}+\frac{2c^2 \arctan(ax)^2 a^5 x^5}{5}+\frac{a^3 c^2 x^3 \arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5 \arctan(ax) a^6 x^6}{2}+\frac{27 \arctan(ax) a^4 x^4}{4}+4 \arctan(ax) a^2 x^2-4}{}}{}$
default	$\frac{\frac{c^2 \arctan(ax)^2 a^7 x^7}{7}+\frac{2c^2 \arctan(ax)^2 a^5 x^5}{5}+\frac{a^3 c^2 x^3 \arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5 \arctan(ax) a^6 x^6}{2}+\frac{27 \arctan(ax) a^4 x^4}{4}+4 \arctan(ax) a^2 x^2-4}{}}{}$
risch	$\frac{17c^2x^3}{630}-\frac{c^2\ln(-iax+1)^2x^3}{12}-\frac{177151ic^2}{2315250a^3}-\frac{c^2\ln(iax+1)^2x^3}{12}-\frac{c^2x}{210a^2}+\frac{a^2c^2x^5}{105}+\frac{c^2\arctan(ax)}{210a^3}-\frac{ic^2a^3\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/7*c^2*arctan(a*x)^2*a^7*x^7+2/5*c^2*arctan(a*x)^2*a^5*x^5+1/3*a^3*
c^2*x^3*arctan(a*x)^2-2/105*c^2*(5/2*arctan(a*x)*a^6*x^6+27/4*arctan(a*x)*a
^4*x^4+4*arctan(a*x)*a^2*x^2-4*arctan(a*x)*ln(a^2*x^2+1)-1/2*a^5*x^5-17/12*
a^3*x^3+1/4*a*x-1/4*arctan(a*x)-2*I*ln(a*x-I)*ln(a^2*x^2+1)+2*I*ln(a*x-I)*l
n(-1/2*I*(I+a*x))+2*I*dilog(-1/2*I*(I+a*x))+2*I*ln(I+a*x)*ln(a^2*x^2+1)-2*I
*ln(I+a*x)*ln(1/2*I*(a*x-I))-I*ln(I+a*x)^2-2*I*dilog(1/2*I*(a*x-I))+I*ln(a*
x-I)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/420*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^2 - 1/1680
*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*log(a^2*x^2 + 1)^2 + integr
ate(1/1680*(1260*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*ar
ctan(a*x)^2 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*l
og(a^2*x^2 + 1)^2 - 8*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)*arct
an(a*x) + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*log(a^2*x^2
+ 1))/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^2\,\text{atan}^2\,(ax)\,dx+\int 2a^2x^4\,\text{atan}^2\,(ax)\,dx+\int a^4x^6\,\text{atan}^2\,(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**2*atan(a*x)**2, x) + Integral(2*a**2*x**4*atan(a*x)**2, x
) + Integral(a**4*x**6*atan(a*x)**2, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\text{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.268 $\int x(c+a^2cx^2)^2 \,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=153

$$\frac{2c^2(1+a^2x^2)}{45a^2}+\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{8c^2x\text{ArcTan}(ax)}{45a}-\frac{4c^2x(1+a^2x^2)\,\text{ArcTan}(ax)}{45a}-\frac{c^2x(1+a^2x^2)^2\,\text{ArcTan}(ax)}{15a}-$$

```
[Out] 2/45*c^2*(a^2*x^2+1)/a^2+1/60*c^2*(a^2*x^2+1)^2/a^2-8/45*c^2*x*arctan(a*x)/
a-4/45*c^2*x*(a^2*x^2+1)*arctan(a*x)/a-1/15*c^2*x*(a^2*x^2+1)^2*arctan(a*x)
/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^2/a^2+4/45*c^2*ln(a^2*x^2+1)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 4998, 4930, 266}

$$\frac{c^2(a^2x^2+1)^3\,\text{ArcTan}(ax)^2}{6a^2}-\frac{c^2x(a^2x^2+1)^2\,\text{ArcTan}(ax)}{15a}-\frac{4c^2x(a^2x^2+1)\,\text{ArcTan}(ax)}{45a}+\frac{c^2(a^2x^2+1)^2}{60a^2}+\frac{2c^2(a^2x^2+1)}{45a^2}+\frac{4c^2\log(a^2x^2+1)}{45a^2}-\frac{8c^2x\text{ArcTan}(ax)}{45a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (2*c^2*(1 + a^2*x^2))/(45*a^2) + (c^2*(1 + a^2*x^2)^2)/(60*a^2) - (8*c^2*x*
ArcTan[a*x])/(45*a) - (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x])/(45*a) - (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x])/(15*a) + (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(6
*a^2) + (4*c^2*Log[1 + a^2*x^2])/(45*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx=\frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)^2}{6a^2}-\frac{\int\left(c+a^2cx^2\right)^2\tan^{-1}(ax)\,dx}{3a}$$
$$=\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{c^2x(1+a^2x^2)^2\tan^{-1}(ax)}{15a}+\frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)^2}{6a^2}-$$
$$=\frac{2c^2(1+a^2x^2)}{45a^2}+\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{4c^2x(1+a^2x^2)\tan^{-1}(ax)}{45a}-\frac{c^2x(1+a^2}{}$$
$$=\frac{2c^2(1+a^2x^2)}{45a^2}+\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{8c^2x\tan^{-1}(ax)}{45a}-\frac{4c^2x(1+a^2x^2)\tan^{-1}}{45a}$$
$$=\frac{2c^2(1+a^2x^2)}{45a^2}+\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{8c^2x\tan^{-1}(ax)}{45a}-\frac{4c^2x(1+a^2x^2)\tan^{-1}}{45a}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 0.55

$$\frac{c^2\left(14a^2x^2+3a^4x^4-4ax(15+10a^2x^2+3a^4x^4)\,\text{ArcTan}(ax)+30(1+a^2x^2)^3\,\text{ArcTan}(ax)^2+16\log\left(1+a^2x^2\right)\right)}{180a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(14*a^2*x^2 + 3*a^4*x^4 - 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[a
*x] + 30*(1 + a^2*x^2)^3*ArcTan[a*x]^2 + 16*Log[1 + a^2*x^2]))/(180*a^2)
```

Maple [A]
time = 0.18, size = 133, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)^2a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^2}{2}+\frac{a^2c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)a^3x^3}{3}+\text{arcta}\right.}{3}}{a^2}$

default	$\frac{\frac{c^2 \arctan(ax)^2 a^6 x^6}{6}+\frac{a^4 c^2 x^4 \arctan(ax)^2}{2}+\frac{a^2 c^2 x^2 \arctan(ax)^2}{2}+\frac{c^2 \arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)a^3x^3}{3}+\arctan(\right.}{3}}{a^2}$
risch	$-\frac{c^2(a^2x^2+1)^3 \ln(iax+1)^2}{24a^2}+\frac{c^2(15a^6x^6 \ln(-iax+1)+6ia^5x^5+45x^4 \ln(-iax+1)a^4+20ia^3x^3+45a^2x^2 \ln(-iax+1)+30}{180a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/6*c^2*arctan(a*x)^2*a^6*x^6+1/2*a^4*c^2*x^4*arctan(a*x)^2+1/2*a^2*
c^2*x^2*arctan(a*x)^2+1/6*c^2*arctan(a*x)^2-1/3*c^2*(1/5*arctan(a*x)*a^5*x^
5+2/3*arctan(a*x)*a^3*x^3+arctan(a*x)*a*x-1/20*a^4*x^4-7/30*a^2*x^2-4/15*ln
(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 111, normalized size = 0.73

$$\frac{(a^2cx^2+c)^3 \arctan(ax)^2}{6a^2c}+\frac{\left(3a^2c^3x^4+14c^3x^2+\frac{16c^3\log(a^2x^2+1)}{a^2}\right)a-4(3a^4c^3x^5+10a^2c^3x^3+15c^3x)\arctan(ax)}{180ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) + 1/180*((3*a^2*c^3*x^4 + 14*c^
3*x^2 + 16*c^3*log(a^2*x^2 + 1)/a^2)*a - 4*(3*a^4*c^3*x^5 + 10*a^2*c^3*x^3
+ 15*c^3*x)*arctan(a*x))/(a*c)
```

Fricas [A]
time = 2.73, size = 123, normalized size = 0.80

$$\frac{3a^4c^2x^4+14a^2c^2x^2+30(a^6c^2x^6+3a^4c^2x^4+3a^2c^2x^2+c^2)\arctan(ax)^2+16c^2\log(a^2x^2+1)-4(3a^5c^2x^5+10a^3c^2x^3+15ac^2x)\arctan(ax)}{180a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 30*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3
*a^2*c^2*x^2 + c^2)*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*c^2*
x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a^2
```

Sympy [A]
time = 0.42, size = 158, normalized size = 1.03

$$\begin{cases} \frac{a^4c^2x^6\operatorname{atan}^2(ax)}{6}-\frac{a^3c^2x^5\operatorname{atan}(ax)}{15}+\frac{a^2c^2x^4\operatorname{atan}^2(ax)}{2}+\frac{a^2c^2x^4}{60}-\frac{2ac^2x^3\operatorname{atan}(ax)}{9}+\frac{c^2x^2\operatorname{atan}^2(ax)}{2}+\frac{7c^2x^2}{90}-\frac{c^2x\operatorname{atan}(ax)}{3a}+\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{45a^2}+\frac{c^2\operatorname{atan}^2(ax)}{6a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**6*atan(a*x)**2/6 - a**3*c**2*x**5*atan(a*x)/15 + a*
*2*c**2*x**4*atan(a*x)**2/2 + a**2*c**2*x**4/60 - 2*a*c**2*x**3*atan(a*x)/9
 + c**2*x**2*atan(a*x)**2/2 + 7*c**2*x**2/90 - c**2*x*atan(a*x)/(3*a) + 4*c
**2*log(x**2 + a**(-2))/(45*a**2) + c**2*atan(a*x)**2/(6*a**2), Ne(a, 0)),
(0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.31, size = 135, normalized size = 0.88

$$\frac{\frac{c^2\left(30\,\mathrm{atan}(a\,x)^2+16\,\ln(a^2\,x^2+1)\right)}{180}-\frac{a\,c^2\,x\,\mathrm{atan}(a\,x)}{3}}{a^2}+\frac{c^2\left(90\,x^2\,\mathrm{atan}(a\,x)^2+14\,x^2\right)}{180}+\frac{a^2\,c^2\left(90\,x^4\,\mathrm{atan}(a\,x)^2+3\,x^4\right)}{180}-\frac{a^3\,c^2\,x^5\,\mathrm{atan}(a\,x)}{15}+\frac{a^4\,c^2\,x^6\,\mathrm{atan}(a\,x)^2}{6}-\frac{2\,a\,c^2\,x^3\,\mathrm{atan}(a\,x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

```
[Out] ((c^2*(16*log(a^2*x^2 + 1) + 30*atan(a*x)^2))/180 - (a*c^2*x*atan(a*x))/3)/
a^2 + (c^2*(90*x^2*atan(a*x)^2 + 14*x^2))/180 + (a^2*c^2*(90*x^4*atan(a*x)^
2 + 3*x^4))/180 - (a^3*c^2*x^5*atan(a*x))/15 + (a^4*c^2*x^6*atan(a*x)^2)/6
- (2*a*c^2*x^3*atan(a*x))/9
```

3.269 $\int \left(c+a^2cx^2\right)^2 \textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=205

$$\frac{11c^2x}{30}+\frac{1}{30}a^2c^2x^3-\frac{4c^2(1+a^2x^2)\,\text{ArcTan}(ax)}{15a}-\frac{c^2(1+a^2x^2)^2\,\text{ArcTan}(ax)}{10a}+\frac{8ic^2\text{ArcTan}(ax)^2}{15a}+\frac{8}{15}c^2x\text{ArcTan}$$

```
[Out] 11/30*c^2*x+1/30*a^2*c^2*x^3-4/15*c^2*(a^2*x^2+1)*arctan(a*x)/a-1/10*c^2*(a
^2*x^2+1)^2*arctan(a*x)/a+8/15*I*c^2*arctan(a*x)^2/a+8/15*c^2*x*arctan(a*x)
^2+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)
^2+16/15*c^2*arctan(a*x)*ln(2/(1+I*a*x))/a+8/15*I*c^2*polylog(2,1-2/(1+I*a*
x))/a
```

Rubi [A]
time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5000, 4930, 5040, 4964, 2449, 2352, 8}

$$\frac{1}{5}c^2x(a^2x^2+1)^2\,\text{ArcTan}(ax)^2+\frac{4}{15}c^2x(a^2x^2+1)\,\text{ArcTan}(ax)^2-\frac{c^2(a^2x^2+1)^2\,\text{ArcTan}(ax)}{10a}-\frac{4c^2(a^2x^2+1)\,\text{ArcTan}(ax)}{15a}+\frac{1}{30}a^2c^2x^3+\frac{8}{15}c^2x\text{ArcTan}(ax)^2+\frac{8ic^2\text{ArcTan}(ax)^2}{15a}+\frac{16c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{15a}+\frac{8ic^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{15a}+\frac{11c^2x}{30}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (11*c^2*x)/30 + (a^2*c^2*x^3)/30 - (4*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15*a)
 - (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (((8*I)/15)*c^2*ArcTan[a*x]^2
)/a + (8*c^2*x*ArcTan[a*x]^2)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/15
 + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (16*c^2*ArcTan[a*x]*Log[2/(1 +
 I*a*x)])/(15*a) + (((8*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^2\,dx = -\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} + \frac{1}{5}c^2x(1+a^2x^2)^2\tan^{-1}(ax)^2 + \frac{1}{10}c\int \left(c+a^2c\right.$$

$$= \frac{c^2x}{10} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} + \frac{4}{15}$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} +$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} +$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} +$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} +$$

$$= \frac{11c^2x}{30} + \frac{1}{30}a^2c^2x^3 - \frac{4c^2(1+a^2x^2)\tan^{-1}(ax)}{15a} - \frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{10a} +$$

Mathematica [A]

time = 0.49, size = 112, normalized size = 0.55

$$\frac{c^2\left(ax(11+a^2x^2)+2(-8i+15ax+10a^3x^3+3a^5x^5)\operatorname{ArcTan}(ax)^2-\operatorname{ArcTan}(ax)\left(11+14a^2x^2+3a^4x^4-32\log\left(1+e^{2i\operatorname{ArcTan}(ax)}\right)\right)-16i\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(ax)}\right)\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(a*x*(11 + a^2*x^2) + 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*ArcTa
n[a*x]^2 - ArcTan[a*x]*(11 + 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*A
rcTan[a*x])]) - (16*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a)
```

Maple [A]

time = 0.15, size = 246, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)^2a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3\arctan(ax)a^4x^4}{4}+\frac{7\arctan(ax)a^2x^2}{2}+4\arctan(ax)\ln\left(a^2x^2+\right.\right.}{}}{}$
default	$\frac{\frac{c^2\arctan(ax)^2a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3\arctan(ax)a^4x^4}{4}+\frac{7\arctan(ax)a^2x^2}{2}+4\arctan(ax)\ln\left(a^2x^2+\right.\right.}{}}{}$
risch	$\frac{11c^2x}{30}+\frac{8ic^2\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{15a}+\frac{ic^2a^3\ln(iax+1)x^4}{20}+\frac{8ic^2\operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{15a}+\frac{c^2a^2\ln(iax+1)\ln(-iax+1)x^3}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/5*c^2*arctan(a*x)^2*a^5*x^5+2/3*a^3*c^2*x^3*arctan(a*x)^2+a*c^2*x*ar
ctan(a*x)^2-2/15*c^2*(3/4*arctan(a*x)*a^4*x^4+7/2*arctan(a*x)*a^2*x^2+4*arc
tan(a*x)*ln(a^2*x^2+1)-1/4*a^3*x^3-11/4*a*x+11/4*arctan(a*x)+2*I*ln(a*x-I)*
ln(a^2*x^2+1)-I*ln(a*x-I)^2-2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-2*I*dilog(-1/2
*I*(I+a*x))-2*I*ln(I+a*x)*ln(a^2*x^2+1)+I*ln(I+a*x)^2+2*I*ln(I+a*x)*ln(1/2*
I*(a*x-I))+2*I*dilog(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 180*a^6*c^2*integrate(1/240*x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a^6*c^
2*integrate(1/240*x^6*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^6*c^2*int
egrate(1/240*x^6*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 24*a^5*c^2*integrate(
1/240*x^5*arctan(a*x)/(a^2*x^2 + 1), x) + 540*a^4*c^2*integrate(1/240*x^4*a
rctan(a*x)^2/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1/240*x^4*log(a^2*x^2
 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*integrate(1/240*x^4*log(a^2*x^2 + 1)
/(a^2*x^2 + 1), x) - 80*a^3*c^2*integrate(1/240*x^3*arctan(a*x)/(a^2*x^2 +
1), x) + 540*a^2*c^2*integrate(1/240*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) +
45*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 60*a^
2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^2*arct
an(a*x)^3/a - 120*a*c^2*integrate(1/240*x*arctan(a*x)/(a^2*x^2 + 1), x) + 1
/60*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^2 + 15*c^2*inte
grate(1/240*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/240*(3*a^4*c^2*x^5 + 1
0*a^2*c^2*x^3 + 15*c^2*x)*log(a^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int 2a^{2}x^{2}\operatorname{atan}^{2}{\left(ax\right)}\,dx+\int a^{4}x^{4}\operatorname{atan}^{2}{\left(ax\right)}\,dx+\int\operatorname{atan}^{2}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**2, x) + Integral(a**4*x**4*atan(a*x)*
*2, x) + Integral(atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.270 $\int \frac{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^2}{x} dx$

Optimal. Leaf size=235

$$\frac{1}{12}a^2c^2x^2-\frac{3}{2}ac^2x\text{ArcTan}(ax)-\frac{1}{6}a^3c^2x^3\text{ArcTan}(ax)+\frac{3}{4}c^2\text{ArcTan}(ax)^2+a^2c^2x^2\text{ArcTan}(ax)^2+\frac{1}{4}a^4c^2x^4\text{ArcTa}$$

```
[Out] 1/12*a^2*c^2*x^2-3/2*a*c^2*x*arctan(a*x)-1/6*a^3*c^2*x^3*arctan(a*x)+3/4*c^
2*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+1/4*a^4*c^2*x^4*arctan(a*x)^2-2*c
^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+2/3*c^2*ln(a^2*x^2+1)-I*c^2*arctan
(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^2*arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-
1/2*c^2*polylog(3,1-2/(1+I*a*x))+1/2*c^2*polylog(3,-1+2/(1+I*a*x))
```

Rubi [A]
time = 0.35, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 4930, 266, 272, 45}

$$\frac{1}{4}a^4c^2x^4\text{ArcTan}(ax)^2-\frac{1}{6}a^3c^2x^3\text{ArcTan}(ax)+a^2c^2x^2\text{ArcTan}(ax)^2+\frac{1}{12}a^2c^2x^2+\frac{2}{3}c^2\log\left(a^2x^2+1\right)-ic^2\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+ic^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{3}{2}ac^2x\text{ArcTan}(ax)+\frac{3}{4}c^2\text{ArcTan}(ax)^2+2c^2\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{1}{2}c^2\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{1}{2}c^2\text{Li}_3\left(\frac{2}{iax+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

```
[Out] (a^2*c^2*x^2)/12 - (3*a*c^2*x*ArcTan[a*x])/2 - (a^3*c^2*x^3*ArcTan[a*x])/6
+ (3*c^2*ArcTan[a*x]^2)/4 + a^2*c^2*x^2*ArcTan[a*x]^2 + (a^4*c^2*x^4*ArcTan
[a*x]^2)/4 + 2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (2*c^2*Log[1
+ a^2*x^2])/3 - I*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c^2*Arc
Tan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^2*PolyLog[3, 1 - 2/(1 + I*a*x)
])/2 + (c^2*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}{x}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)^2}{x}+2a^2c^2x\tan^{-1}(ax)^2+a^4c^2x^3\tan^{-1}(ax)^2\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x}\,dx+\left(2a^2c^2\right)\int x\tan^{-1}(ax)^2\,dx+\left(a^4c^2\right)\int x^3\tan^{-1}(ax)$$

$$= a^2c^2x^2\tan^{-1}(ax)^2+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^2+2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{}{1-}\right.$$

$$= a^2c^2x^2\tan^{-1}(ax)^2+\frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^2+2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{}{1-}\right.$$

$$= -2ac^2x\tan^{-1}(ax)-\frac{1}{6}a^3c^2x^3\tan^{-1}(ax)+c^2\tan^{-1}(ax)^2+a^2c^2x^2\tan^{-1}(a$$

$$= -\frac{3}{2}ac^2x\tan^{-1}(ax)-\frac{1}{6}a^3c^2x^3\tan^{-1}(ax)+\frac{3}{4}c^2\tan^{-1}(ax)^2+a^2c^2x^2\tan^{-1}($$

$$= -\frac{3}{2}ac^2x\tan^{-1}(ax)-\frac{1}{6}a^3c^2x^3\tan^{-1}(ax)+\frac{3}{4}c^2\tan^{-1}(ax)^2+a^2c^2x^2\tan^{-1}($$

$$= \frac{1}{12}a^2c^2x^2-\frac{3}{2}ac^2x\tan^{-1}(ax)-\frac{1}{6}a^3c^2x^3\tan^{-1}(ax)+\frac{3}{4}c^2\tan^{-1}(ax)^2+a^2c$$

Mathematica [A]
time = 0.21, size = 218, normalized size = 0.93

$$\frac{1}{24}c^2\left(2-i\pi^3+2a^2x^2-36ax\text{ArcTan}(ax)-4a^3x^3\text{ArcTan}(ax)+18\text{ArcTan}(ax)^2+24a^2x^2\text{ArcTan}(ax)^2+6a^4x^4\text{ArcTan}(ax)^2+16i\text{ArcTan}(ax)^3+24\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)-24\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+16\log\left(1+a^2x^2\right)+24i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)-12\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c^2*(2 - I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcTan[a*x] - 4*a^3*x^3*ArcTan[a*x] +
 18*ArcTan[a*x]^2 + 24*a^2*x^2*ArcTan[a*x]^2 + 6*a^4*x^4*ArcTan[a*x]^2 + (1
6*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 24*
ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 16*Log[1 + a^2*x^2] + (24*I)
*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 12*P
olyLog[3, -E^((2*I)*ArcTan[a*x])]))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.82, size = 1185, normalized size = 5.04

method	result	size
derivativedivides	Expression too large to display	1185
default	Expression too large to display	1185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^4*c^2*x^4*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+c^2*arctan(a*x)^2*l
n(a*x)-1/2*c^2*(-I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))-3/2*arctan(a*x)^2+8/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1
/6*(I+a*x)^2+2*arctan(a*x)*(a*x-I)-2*I*arctan(a*x)*(I+a*x)*(a*x-I)-I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^
2+I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^2-I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^3+1/3*arctan(a*x)*(a*x-I)^3+polylog(3,-(1+I*a*x)^2/(a^2*x^2
+1))-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*arctan(a*x)^2*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1
)^(1/2))+1/3*I*(I+a*x)-I*Pi*arctan(a*x)^2-2*I*arctan(a*x)*polylog(2,-(1+I*a
*x)^2/(a^2*x^2+1))+4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4
*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*(a*x-I)^
2*(I+a*x)+arctan(a*x)*(a*x-I)*(I+a*x)^2+I*arctan(a*x)*(a*x-I)^2-I*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+I*Pi*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+I*arctan(a*x)^2*Pi*
csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 12*a^6*c^2*integrate(1/16*x^6*arctan(a*x)^2/(a^2*x^3 + x), x) + a^6*c^2*int
egrate(1/16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + a^6*c^2*integrate(1/
16*x^6*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 2*a^5*c^2*integrate(1/16*x^5*ar
ctan(a*x)/(a^2*x^3 + x), x) + 36*a^4*c^2*integrate(1/16*x^4*arctan(a*x)^2/(
a^2*x^3 + x), x) + 3*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3
 + x), x) + 4*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x)
 - 8*a^3*c^2*integrate(1/16*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 36*a^2*c^2*
integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/32*c^2*log(a^2*x^2 +
 1)^3 + 1/16*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^2 + 12*c^2*integrate
(1/16*arctan(a*x)^2/(a^2*x^3 + x), x) + c^2*integrate(1/16*log(a^2*x^2 + 1)
^2/(a^2*x^3 + x), x) - 1/64*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*log(a^2*x^2 + 1)^
2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{\operatorname{atan}^{2}\left(ax\right)}{x}\,dx + \int 2a^{2}x\operatorname{atan}^{2}\left(ax\right)dx + \int a^{4}x^{3}\operatorname{atan}^{2}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**2/x, x) + Integral(2*a**2*x*atan(a*x)**2, x) + In
tegral(a**4*x**3*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x, x)
```

3.271 $\int \frac{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^2}{x^2} \, dx$

Optimal. Leaf size=205

$$\frac{1}{3}a^2c^2x-\frac{1}{3}ac^2\text{ArcTan}(ax)-\frac{1}{3}a^3c^2x^2\text{ArcTan}(ax)+\frac{2}{3}iac^2\text{ArcTan}(ax)^2-\frac{c^2\text{ArcTan}(ax)^2}{x}+2a^2c^2x\text{ArcTan}(ax)^2$$

```
[Out] 1/3*a^2*c^2*x-1/3*a*c^2*arctan(a*x)-1/3*a^3*c^2*x^2*arctan(a*x)+2/3*I*a*c^2
*arctan(a*x)^2-c^2*arctan(a*x)^2/x+2*a^2*c^2*x*arctan(a*x)^2+1/3*a^4*c^2*x^
3*arctan(a*x)^2+10/3*a*c^2*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^2*arctan(a*x)*
ln(2-2/(1-I*a*x))-I*a*c^2*polylog(2,-1+2/(1-I*a*x))+5/3*I*a*c^2*polylog(2,1
-2/(1+I*a*x))
```

Rubi [A]
time = 0.30, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5068, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497, 5036, 327, 209}

$$\frac{1}{3}a^4c^2x^3\text{ArcTan}(ax)^2-\frac{1}{3}a^3c^2x^2\text{ArcTan}(ax)+2a^2c^2x\text{ArcTan}(ax)^2+\frac{1}{3}a^2c^2x+\frac{2}{3}iac^2\text{ArcTan}(ax)^2-\frac{1}{3}ac^2\text{ArcTan}(ax)-\frac{c^2\text{ArcTan}(ax)^2}{x}+\frac{10}{3}ac^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2ac^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-iac^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{5}{3}iac^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (a^2*c^2*x)/3 - (a*c^2*ArcTan[a*x])/3 - (a^3*c^2*x^2*ArcTan[a*x])/3 + ((2*I
)/3)*a*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/x + 2*a^2*c^2*x*ArcTan[a*x]^
2 + (a^4*c^2*x^3*ArcTan[a*x]^2)/3 + (10*a*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)
])/3 + 2*a*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^2*PolyLog[2, -1 +
 2/(1 - I*a*x)] + ((5*I)/3)*a*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^2}\, dx = \int \left(2a^2c^2\tan^{-1}(ax)^2 + \frac{c^2\tan^{-1}(ax)^2}{x^2} + a^4c^2x^2\tan^{-1}(ax)^2\right) dx$$

$$= c^2 \int \frac{\tan^{-1}(ax)^2}{x^2}\, dx + \left(2a^2c^2\right)\int \tan^{-1}(ax)^2\, dx + \left(a^4c^2\right)\int x^2\tan^{-1}(ax)^2\, dx$$

$$= -\frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2 + \left(2ac^2\right)\int \frac{\tan}{x\,(1}$$

$$= iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(ax)^2 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^2$$

$$= -\frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x\tan^{-1}(a$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan^{-1}(ax)^2}{x} + 2a^2c^2x$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\tan^{-1}(ax) - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan}{}$$

$$= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\tan^{-1}(ax) - \frac{1}{3}a^3c^2x^2\tan^{-1}(ax) + \frac{2}{3}iac^2\tan^{-1}(ax)^2 - \frac{c^2\tan}{}$$

Mathematica [A]

time = 0.25, size = 167, normalized size = 0.81

$$\frac{c^2\left(a^2x^2 - ax\text{ArcTan}(ax) - a^3x^3\text{ArcTan}(ax) - 3\text{ArcTan}(ax)^2 - 8iax\text{ArcTan}(ax)^2 + 6a^2x^2\text{ArcTan}(ax)^2 + a^4x^4\text{ArcTan}(ax)^2 + 6ax\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right) + 10ax\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - 5iax\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) - 3iax\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^2*(a^2*x^2 - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - 3*ArcTan[a*x]^2 - (
8*I)*a*x*ArcTan[a*x]^2 + 6*a^2*x^2*ArcTan[a*x]^2 + a^4*x^4*ArcTan[a*x]^2 +
6*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 10*a*x*ArcTan[a*x]*Log[1
 + E^((2*I)*ArcTan[a*x])] - (5*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])] -
(3*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x)
```

Maple [A]

time = 0.22, size = 287, normalized size = 1.40

method	result
derivativedivides	$a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3} + 2a\,c^2x\arctan(ax)^2 - \frac{c^2\arctan(ax)^2}{ax} - \frac{2c^2\left(\frac{\arctan(ax)a^2x^2}{2} + 4\arctan(ax)\ln(a^2x^2\right.}{}\right.$

default	$a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3}+2a\,c^2x\arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(\frac{\arctan(ax)a^2x^2}{2}+4\arctan(ax)\ln(a^2x\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)

[Out] a*(1/3*a^3*c^2*x^3*arctan(a*x)^2+2*a*c^2*x*arctan(a*x)^2-c^2*arctan(a*x)^2/a/x-2/3*c^2*(1/2*arctan(a*x)*a^2*x^2+4*arctan(a*x)*ln(a^2*x^2+1)-3*arctan(a*x)*ln(a*x)-1/2*a*x+1/2*arctan(a*x)+3/2*I*ln(a*x)*ln(1-I*a*x)-3/2*I*ln(a*x)*ln(1+I*a*x)-3/2*I*dilog(1+I*a*x)+3/2*I*dilog(1-I*a*x)+2*I*ln(a*x-I)*ln(a^2*x^2+1)+2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+I*ln(I+a*x)^2-2*I*dilog(-1/2*I*(I+a*x))-I*ln(a*x-I)^2-2*I*ln(I+a*x)*ln(a^2*x^2+1)+2*I*dilog(1/2*I*(a*x-I))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="maxima")

[Out] 1/48*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^2 - (a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*log(a^2*x^2 + 1)^2 + 12*(144*a^6*c^2*integrate(1/48*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 16*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 32*a^5*c^2*integrate(1/48*x^5*arctan(a*x)/(a^2*x^4 + x^2), x) + 432*a^4*c^2*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 3*a*c^2*arctan(a*x)^3 - 192*a^3*c^2*integrate(1/48*x^3*arctan(a*x)/(a^2*x^4 + x^2), x) + 36*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 48*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 96*a*c^2*integrate(1/48*x*arctan(a*x)/(a^2*x^4 + x^2), x) + 144*c^2*integrate(1/48*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*c^2*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="fricas")

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int a^4x^2\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) + I
ntegral(a**4*x**2*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2, x)
```

3.272 $\int \frac{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^2}{x^3} dx$

Optimal. Leaf size=207

$$-\frac{ac^2\text{ArcTan}(ax)}{x}-a^3c^2x\text{ArcTan}(ax)-\frac{c^2\text{ArcTan}(ax)^2}{2x^2}+\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)^2+4a^2c^2\text{ArcTan}(ax)^2\tanh^{-1}\left(\right.$$

```
[Out] -a*c^2*arctan(a*x)/x-a^3*c^2*x*arctan(a*x)-1/2*c^2*arctan(a*x)^2/x^2+1/2*a^
4*c^2*x^2*arctan(a*x)^2-4*a^2*c^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a^2
*c^2*ln(x)-2*I*a^2*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+2*I*a^2*c^2*arc
tan(a*x)*polylog(2,-1+2/(1+I*a*x))-a^2*c^2*polylog(3,1-2/(1+I*a*x))+a^2*c^2
*polylog(3,-1+2/(1+I*a*x))
```

Rubi [A]
time = 0.32, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5068, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745, 5036, 4930, 266}

$$\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)^2-a^3c^2x\text{ArcTan}(ax)-2ia^2c^2\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+2ia^2c^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{iax+1}-1\right)+4a^2c^2\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-a^2c^2\text{Li}_3\left(1-\frac{2}{iax+1}\right)+a^2c^2\text{Li}_3\left(\frac{2}{iax+1}-1\right)+a^2c^2\log(x)-\frac{c^2\text{ArcTan}(ax)^2}{2x^2}-\frac{ac^2\text{ArcTan}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c^2*ArcTan[a*x])/x) - a^3*c^2*x*ArcTan[a*x] - (c^2*ArcTan[a*x]^2)/(2*x
^2) + (a^4*c^2*x^2*ArcTan[a*x]^2)/2 + 4*a^2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] + a^2*c^2*Log[x] - (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2
/(1 + I*a*x)] + (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] -
a^2*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)] + a^2*c^2*PolyLog[3, -1 + 2/(1 + I*a*
x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^2}{x^3}\,dx &= \int \left(\frac{c^2\tan^{-1}(ax)^2}{x^3} + \frac{2a^2c^2\tan^{-1}(ax)^2}{x} + a^4c^2x\tan^{-1}(ax)^2\right)dx \\
&= c^2\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx + \left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(a^4c^2\right)\int x\tan^{-1}(ax)^2\,dx \\
&= -\frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+\ldots}\right. \\
&= -\frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 + 4a^2c^2\tan^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+\ldots}\right. \\
&= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 - \ldots \\
&= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 - \ldots \\
&= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 - \ldots \\
&= -\frac{ac^2\tan^{-1}(ax)}{x} - a^3c^2x\tan^{-1}(ax) - \frac{c^2\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^2 - \ldots
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 226, normalized size = 1.09

$$a^2c^2\left(-\frac{i\pi^3}{12}-\frac{\text{ArcTan}(ax)}{ax}-ax\text{ArcTan}(ax)-\frac{\text{ArcTan}(ax)^2}{2a^2x^2}+\frac{1}{2}a^2x^2\text{ArcTan}(ax)^2+\frac{4}{3}i\text{ArcTan}(ax)^3+2\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)-2\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+\frac{1}{2}\log\left(1+a^2x^2\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)-\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] a^2*c^2*((-1/12*I)*Pi^3 - ArcTan[a*x]/(a*x) - a*x*ArcTan[a*x] - ArcTan[a*x]
^2/(2*a^2*x^2) + (a^2*x^2*ArcTan[a*x]^2)/2 + ((4*I)/3)*ArcTan[a*x]^3 + 2*Ar
cTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 2*ArcTan[a*x]^2*Log[1 + E^((2
*I)*ArcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] + Log[1 + a^2*x^2]/2 + (2*
I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyL
og[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, E^((-2*I)*ArcTan[a*x])] - PolyLo
g[3, -E^((2*I)*ArcTan[a*x])])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.97, size = 1184, normalized size = 5.72

method	result	size
derivativedivides	Expression too large to display	1184
default	Expression too large to display	1184

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*a^2*c^2*x^2*arctan(a*x)^2-1/2*c^2*arctan(a*x)^2/a^2/x^2+2*c^2*arct
an(a*x)^2*ln(a*x)-c^2*(2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*arct
an(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*Pi*csgn(((1+I*a
*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-4*polylog(3,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arct
an(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^
2-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)^2*Pi*csgn(I/((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^2+polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+I*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2+4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+ln((1+I*a*x)^2/(a^2*x^2+1)+1)-I*Pi*arctan(a*x)^2-I*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-I*arctan(a*x)^2*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+4*I*arctan(a*x)*polylog(2,(
1+I*a*x)/(a^2*x^2+1)^(1/2))-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))^3*arctan(a*x)^2-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*
x^2+1))+1/2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+arctan(a*x)*(a*x-I)
+1/2*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*a^4*c^2*x^4 - 4*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^2 + x*log(a^
2*x^2 + 1)^2, x) - 8*a^3*c^2*x^2*integrate(-1/4*(12*(a^2*x^2 + 1)*a*x*arcta
n(a*x)^2 - 3*(a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 + 12*(a^2*x^2 + 1)*arctan
(a*x)*log(a^2*x^2 + 1) + (4*(a^2*x^2 + 1)^2*arctan(a*x)*cos(3*arctan(a*x))*
log(a^2*x^2 + 1) - 12*(a^2*x^2 + 1)^(3/2)*arctan(a*x)*cos(2*arctan(a*x))*lo
g(a^2*x^2 + 1) - 4*sqrt(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) + (4*(a^2
*x^2 + 1)^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^2)*sin(3*arcta
n(a*x)) - 3*(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2 - (a^2*x^2 + 1)^(3/2)*log(
```

```
a^2*x^2 + 1)^2)*sin(2*arctan(a*x)))*sqrt(a^2*x^2 + 1))/((a^2*x^2 + 1)^4*cos
(3*arctan(a*x))^2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2*x^2 + 1)^
(7/2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*cos(2*arcta
n(a*x))^2 + 9*(a^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 + 6*(a^2*x^2 +
 1)^2*cos(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 + 1)^(7/2)*cos
(2*arctan(a*x)) + (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6*((a^2*x^2 + 1
)^2*a*x*sin(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2*arctan(a*x)) +
 (a^2*x^2 + 1)^2*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*cos(2*arctan(a*
x)) - sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 8*a^3*c^2*x^2*integra
te(1/4*(4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) - (4*(a^2*x^2 + 1)*a*x
*arctan(a*x)^2 - (a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 + 4*(a^2*x^2 + 1)*arc
tan(a*x)*log(a^2*x^2 + 1))*cos(2*arctan(a*x)) - (4*(a^2*x^2 + 1)*a*x*arctan
(a*x)*log(a^2*x^2 + 1) - 4*(a^2*x^2 + 1)*arctan(a*x)^2 + (a^2*x^2 + 1)*log(
a^2*x^2 + 1)^2)*sin(2*arctan(a*x)))/(a^2*x^2 + 1), x) - 8*a^2*c^2*x^2*integ
rate((4*arctan(a*x)^2 + log(a^2*x^2 + 1)^2)/x, x) + 4*a^2*c^2*x^2*integrate
(-(4*arctan(a*x)^2 - log(a^2*x^2 + 1)^2)/x, x) + 4*a^2*c^2*x^2*integrate(lo
g(a^2*x^2 + 1)/x, x) - 8*a^2*c^2*x^2*log(x) + 8*a^2*c^2*x^2 - 4*c^2*x^2*int
egrate((4*arctan(a*x)^2 + log(a^2*x^2 + 1)^2)/x^3, x) - 4*(a^4*c^2*x^4 - c^
2)*arctan(a*x)^2 + (a^4*c^2*x^4 - c^2)*log(a^2*x^2 + 1)^2 + 8*(a^3*c^2*x^3
+ a*c^2*x)*arctan(a*x) - 2*(a^4*c^2*x^4 + a^2*c^2*x^2)*log(a^2*x^2 + 1))/x^
2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{\operatorname{atan}^2(ax)}{x^3}\,dx + \int \frac{2a^2\operatorname{atan}^2(ax)}{x}\,dx + \int a^4x\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**3,x)
[Out] c**2*(Integral(atan(a*x)**2/x**3, x) + Integral(2*a**2*atan(a*x)**2/x, x) +
 Integral(a**4*x*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3, x)
```

3.273 $\int \frac{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=216

$$-\frac{a^2c^2}{3x}-\frac{1}{3}a^3c^2\text{ArcTan}(ax)-\frac{ac^2\text{ArcTan}(ax)}{3x^2}-\frac{2}{3}ia^3c^2\text{ArcTan}(ax)^2-\frac{c^2\text{ArcTan}(ax)^2}{3x^3}-\frac{2a^2c^2\text{ArcTan}(ax)^2}{x}+a^4c$$

```
[Out] -1/3*a^2*c^2/x-1/3*a^3*c^2*arctan(a*x)-1/3*a*c^2*arctan(a*x)/x^2-2/3*I*a^3*
c^2*arctan(a*x)^2-1/3*c^2*arctan(a*x)^2/x^3-2*a^2*c^2*arctan(a*x)^2/x+a^4*c
^2*x*arctan(a*x)^2+2*a^3*c^2*arctan(a*x)*ln(2/(1+I*a*x))+10/3*a^3*c^2*arcta
n(a*x)*ln(2-2/(1-I*a*x))-5/3*I*a^3*c^2*polylog(2,-1+2/(1-I*a*x))+I*a^3*c^2*
polylog(2,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.32, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5068, 4930, 5040, 4964, 2449, 2352, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$a^4c^2x\text{ArcTan}(ax)^2-\frac{2}{3}ia^3c^2\text{ArcTan}(ax)^2-\frac{1}{3}a^3c^2\text{ArcTan}(ax)+2a^3c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+\frac{10}{3}a^3c^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{5}{3}ia^3c^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)+ia^3c^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)-\frac{2a^2c^2\text{ArcTan}(ax)^2}{x}-\frac{a^2c^2}{3x}-\frac{c^2\text{ArcTan}(ax)^2}{3x^3}-\frac{ac^2\text{ArcTan}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c^2)/x - (a^3*c^2*ArcTan[a*x])/3 - (a*c^2*ArcTan[a*x])/(3*x^2) -
((2*I)/3)*a^3*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/(3*x^3) - (2*a^2*c^2*
ArcTan[a*x]^2)/x + a^4*c^2*x*ArcTan[a*x]^2 + 2*a^3*c^2*ArcTan[a*x]*Log[2/(1
 + I*a*x)] + (10*a^3*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((5*I)/3)*
a^3*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] + I*a^3*c^2*PolyLog[2, 1 - 2/(1 + I*
a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^2}{x^4}\,dx &= \int \left(a^4c^2\tan^{-1}(ax)^2+\frac{c^2\tan^{-1}(ax)^2}{x^4}+\frac{2a^2c^2\tan^{-1}(ax)^2}{x^2}\right)dx \\
&= c^2\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx+\left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx+\left(a^4c^2\right)\int \tan^{-1}(ax)^2\,dx \\
&= -\frac{c^2\tan^{-1}(ax)^2}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^2}{x}+a^4c^2x\tan^{-1}(ax)^2+\frac{1}{3}\left(2ac^2\right)\int \frac{\tan}{x^3\,(} \\
&= -ia^3c^2\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(ax)^2}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^2}{x}+a^4c^2x\tan^{-1}(ax) \\
&= -\frac{ac^2\tan^{-1}(ax)}{3x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(ax)^2}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^2}{x} \\
&= -\frac{a^2c^2}{3x}-\frac{ac^2\tan^{-1}(ax)}{3x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(ax)^2}{3x^3}-\frac{2a^2c^2\tan}{x} \\
&= -\frac{a^2c^2}{3x}-\frac{1}{3}a^3c^2\tan^{-1}(ax)-\frac{ac^2\tan^{-1}(ax)}{3x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^2-\frac{c^2\tan^{-}}{3x}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 189, normalized size = 0.88

$$
\frac{c^2\left(-a^2x^2-ax\text{ArcTan}(ax)-a^3x^3\text{ArcTan}(ax)-\text{ArcTan}(ax)^2-6a^2x^2\text{ArcTan}(ax)^2-8ia^3x^3\text{ArcTan}(ax)^2+3a^4x^4\text{ArcTan}(ax)^2+10a^3x^3\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)+6a^3x^3\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-3ia^3x^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-5ia^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c^2*(-(a^2*x^2) - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - ArcTan[a*x]^2 -
6*a^2*x^2*ArcTan[a*x]^2 - (8*I)*a^3*x^3*ArcTan[a*x]^2 + 3*a^4*x^4*ArcTan[a*
x]^2 + 10*a^3*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3*Ar
cTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - (3*I)*a^3*x^3*PolyLog[2, -E^((2*
I)*ArcTan[a*x])] - (5*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3
)
```

Maple [A]
time = 0.29, size = 292, normalized size = 1.35

method	result
derivativedivides	$a^3\left(a\,c^2x\arctan(ax)^2-\frac{2c^2\arctan(ax)^2}{ax}-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-5\right.}{}\right.$
default	$a^3\left(a\,c^2x\arctan(ax)^2-\frac{2c^2\arctan(ax)^2}{ax}-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-5\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(a*c^2*x*arctan(a*x)^2-2*c^2*arctan(a*x)^2/a/x-1/3*c^2*arctan(a*x)^2/a^
3/x^3-2/3*c^2*(4*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-5*arctan
(a*x)*ln(a*x)+1/2/a/x+1/2*arctan(a*x)+2*I*ln(a*x-I)*ln(a^2*x^2+1)-2*I*ln(I+
a*x)*ln(a^2*x^2+1)+5/2*I*ln(a*x)*ln(1-I*a*x)-5/2*I*dilog(1+I*a*x)+I*ln(I+a*
x)^2+5/2*I*dilog(1-I*a*x)-2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-2*I*dilog(-1/2*I
*(I+a*x))-5/2*I*ln(a*x)*ln(1+I*a*x)-I*ln(a*x-I)^2+2*I*ln(I+a*x)*ln(1/2*I*(a
*x-I))+2*I*dilog(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*(12*(144*a^6*c^2*integrate(1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x)
+ 12*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 48
*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3*c^
2*arctan(a*x)^3 - 96*a^5*c^2*integrate(1/48*x^5*arctan(a*x)/(a^2*x^6 + x^4)
, x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x)
 - 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 192
*a^3*c^2*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4), x) + 432*a^2*c^2*i
ntegrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 36*a^2*c^2*integrate(
1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2*c^2*integrate(1/48
*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c^2*integrate(1/48*x*arcta
n(a*x)/(a^2*x^6 + x^4), x) + 144*c^2*integrate(1/48*arctan(a*x)^2/(a^2*x^6
+ x^4), x) + 12*c^2*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*
x^3 + 4*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x)^2 - (3*a^4*c^2*x^
4 - 6*a^2*c^2*x^2 - c^2)*log(a^2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int a^4\operatorname{atan}^2(ax)\,dx+\int\frac{\operatorname{atan}^2(ax)}{x^4}\,dx+\int\frac{2a^2\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**4,x)
```

```
[Out] c**2*(Integral(a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + Int
egral(2*a**2*atan(a*x)**2/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4, x)
```

3.274 $\int x^3(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=240

$$-\frac{107c^3x^2}{12600a^2}+\frac{53c^3x^4}{6300}+\frac{71a^2c^3x^6}{7560}+\frac{1}{360}a^4c^3x^8+\frac{c^3x\text{ArcTan}(ax)}{20a^3}-\frac{c^3x^3\text{ArcTan}(ax)}{60a}-\frac{9}{100}ac^3x^5\text{ArcTan}(ax)-\frac{11}{140}$$

```
[Out] -107/12600*c^3*x^2/a^2+53/6300*c^3*x^4+71/7560*a^2*c^3*x^6+1/360*a^4*c^3*x^
8+1/20*c^3*x*arctan(a*x)/a^3-1/60*c^3*x^3*arctan(a*x)/a-9/100*a*c^3*x^5*arc
tan(a*x)-11/140*a^3*c^3*x^7*arctan(a*x)-1/45*a^5*c^3*x^9*arctan(a*x)-1/40*c
^3*arctan(a*x)^2/a^4+1/4*c^3*x^4*arctan(a*x)^2+1/2*a^2*c^3*x^6*arctan(a*x)^
2+3/8*a^4*c^3*x^8*arctan(a*x)^2+1/10*a^6*c^3*x^10*arctan(a*x)^2-26/1575*c^3
*ln(a^2*x^2+1)/a^4
```

Rubi [A]

time = 0.84, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 72, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5068, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{1}{10}a^6c^3x^{10}\text{ArcTan}(ax)^2-\frac{1}{45}a^5c^3x^9\text{ArcTan}(ax)+\frac{3}{8}a^4c^3x^8\text{ArcTan}(ax)^2-\frac{c^3\text{ArcTan}(ax)^2}{40a^4}+\frac{1}{360}a^4c^3x^8-\frac{11}{140}a^3c^3x^7\text{ArcTan}(ax)+\frac{c^3x\text{ArcTan}(ax)}{20a^3}+\frac{1}{2}a^2c^3x^6\text{ArcTan}(ax)^2+\frac{71a^2c^3x^6}{7560}-\frac{107c^3x^2}{12600a^2}-\frac{26c^3\log\left(a^2x^2+1\right)}{1575a^4}-\frac{9}{100}ac^3x^5\text{ArcTan}(ax)+\frac{1}{4}c^3x^4\text{ArcTan}(ax)^2-\frac{c^3x^3\text{ArcTan}(ax)}{60a}+\frac{53c^3x^4}{6300}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (-107*c^3*x^2)/(12600*a^2) + (53*c^3*x^4)/6300 + (71*a^2*c^3*x^6)/7560 + (a
^4*c^3*x^8)/360 + (c^3*x*ArcTan[a*x])/(20*a^3) - (c^3*x^3*ArcTan[a*x])/(60*
a) - (9*a*c^3*x^5*ArcTan[a*x])/100 - (11*a^3*c^3*x^7*ArcTan[a*x])/140 - (a^
5*c^3*x^9*ArcTan[a*x])/45 - (c^3*ArcTan[a*x]^2)/(40*a^4) + (c^3*x^4*ArcTan[
a*x]^2)/4 + (a^2*c^3*x^6*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^2)/8
 + (a^6*c^3*x^10*ArcTan[a*x]^2)/10 - (26*c^3*Log[1 + a^2*x^2])/(1575*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx &= \int \left(c^3x^3\tan^{-1}(ax)^2+3a^2c^3x^5\tan^{-1}(ax)^2+3a^4c^3x^7\tan^{-1}(ax)^2+a^6c^3x^9\right.\\
&= c^3\int x^3\tan^{-1}(ax)^2\,dx+\left(3a^2c^3\right)\int x^5\tan^{-1}(ax)^2\,dx+\left(3a^4c^3\right)\int x^7\tan^{-}\\
&= \frac{1}{4}c^3x^4\tan^{-1}(ax)^2+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^2+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^2+\frac{1}{10}a^6c^3x\\
&= \frac{1}{4}c^3x^4\tan^{-1}(ax)^2+\frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^2+\frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^2+\frac{1}{10}a^6c^3x\\
&= -\frac{c^3x^3\tan^{-1}(ax)}{6a}-\frac{1}{5}ac^3x^5\tan^{-1}(ax)-\frac{3}{28}a^3c^3x^7\tan^{-1}(ax)-\frac{1}{45}a^5c^3x^9\\
&= \frac{c^3x\tan^{-1}(ax)}{2a^3}+\frac{c^3x^3\tan^{-1}(ax)}{6a}-\frac{1}{20}ac^3x^5\tan^{-1}(ax)-\frac{11}{140}a^3c^3x^7\tan^{-1}\\
&= -\frac{c^3x\tan^{-1}(ax)}{2a^3}-\frac{c^3x^3\tan^{-1}(ax)}{12a}-\frac{9}{100}ac^3x^5\tan^{-1}(ax)-\frac{11}{140}a^3c^3x^7\tan\\
&= \frac{13c^3x^2}{504a^2}+\frac{29c^3x^4}{1008}+\frac{107a^2c^3x^6}{7560}+\frac{1}{360}a^4c^3x^8+\frac{c^3x\tan^{-1}(ax)}{4a^3}-\frac{c^3x^3\tan^{-}}{60a}\\
&= -\frac{101c^3x^2}{1260a^2}-\frac{c^3x^4}{630}+\frac{71a^2c^3x^6}{7560}+\frac{1}{360}a^4c^3x^8+\frac{c^3x\tan^{-1}(ax)}{20a^3}-\frac{c^3x^3\tan^{-1}}{60a}\\
&= \frac{313c^3x^2}{12600a^2}+\frac{53c^3x^4}{6300}+\frac{71a^2c^3x^6}{7560}+\frac{1}{360}a^4c^3x^8+\frac{c^3x\tan^{-1}(ax)}{20a^3}-\frac{c^3x^3\tan^{-}}{60a}\\
&= -\frac{107c^3x^2}{12600a^2}+\frac{53c^3x^4}{6300}+\frac{71a^2c^3x^6}{7560}+\frac{1}{360}a^4c^3x^8+\frac{c^3x\tan^{-1}(ax)}{20a^3}-\frac{c^3x^3\tan}{60}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 126, normalized size = 0.52

$$\frac{c^3\left(-321a^2x^2+318a^4x^4+355a^6x^6+105a^8x^8-6ax(-315+105a^2x^2+567a^4x^4+495a^6x^6+140a^8x^8)\operatorname{ArcTan}(ax)+945(1+a^2x^2)^4(-1+4a^2x^2)\operatorname{ArcTan}(ax)^2-624\log\left(1+a^2x^2\right)\right)}{37800a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(-321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 - 6*a*x*(-315
+ 105*a^2*x^2 + 567*a^4*x^4 + 495*a^6*x^6 + 140*a^8*x^8)*ArcTan[a*x] + 945*
(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^2 - 624*Log[1 + a^2*x^2]))/(37
800*a^4)
```

Maple [A]
time = 0.22, size = 188, normalized size = 0.78

method	result
derivativedivides	$\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10} + \frac{3c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{a^4 c^3 x^4 \arctan(ax)^2}{4} - \frac{c^3 \left(\frac{4 \arctan(ax) a^9 x^9}{9} + \frac{11 \arctan(ax) a}{7}\right.}{a^4}$
default	$\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10} + \frac{3c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{a^4 c^3 x^4 \arctan(ax)^2}{4} - \frac{c^3 \left(\frac{4 \arctan(ax) a^9 x^9}{9} + \frac{11 \arctan(ax) a}{7}\right.}{a^4}$
risch	$-\frac{c^3 (4a^{10}x^{10} + 15a^8x^8 + 20a^6x^6 + 10a^4x^4 - 1) \ln(iax+1)^2}{160a^4} + \frac{c^3 (1260a^{10}x^{10} \ln(-iax+1) + 280ia^9x^9 + 4725a^8x^8 \ln(-iax$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/10*c^3*arctan(a*x)^2*a^10*x^10+3/8*c^3*arctan(a*x)^2*a^8*x^8+1/2*a
^6*c^3*x^6*arctan(a*x)^2+1/4*a^4*c^3*x^4*arctan(a*x)^2-1/20*c^3*(4/9*arctan
(a*x)*a^9*x^9+11/7*arctan(a*x)*a^7*x^7+9/5*arctan(a*x)*a^5*x^5+1/3*arctan(a
*x)*a^3*x^3-arctan(a*x)*a*x+1/2*arctan(a*x)^2-1/18*a^8*x^8-71/378*a^6*x^6-5
3/315*a^4*x^4+107/630*a^2*x^2+104/315*ln(a^2*x^2+1)))
```

Maxima [A]

time = 0.51, size = 202, normalized size = 0.84

$$-\frac{1}{6300}\,a\left(\frac{315\,c^3\arctan\left(ax\right)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)\arctan\left(ax\right)+\frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c^3x^8+20\,a^2c^3x^6+10\,c^3x^4\right)\arctan\left(ax\right)^2+\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2+945\,c^3\arctan\left(ax\right)^2-624\,c^3\log\left(a^2x^2+1\right)}{37800\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6300*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 + 5
67*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4)*arctan(a*x) + 1/40*(4*a^
6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)^2 +
1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3*
x^2 + 945*c^3*arctan(a*x)^2 - 624*c^3*log(a^2*x^2 + 1))/a^4
```

Fricas [A]

time = 2.93, size = 181, normalized size = 0.75

$$\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2-624\,c^3\log\left(a^2x^2+1\right)+945\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3x^6+10\,a^4c^3x^4-c^3\right)\arctan\left(ax\right)^2-6\left(140\,a^9c^3x^9+495\,a^7c^3x^7+567\,a^5c^3x^5+105\,a^3c^3x^3-315\,ac^3x\right)\arctan\left(ax\right)}{37800\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3*
x^2 - 624*c^3*log(a^2*x^2 + 1) + 945*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20
*a^6*c^3*x^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x)^2 - 6*(140*a^9*c^3*x^9 + 4
```

```
95*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3 - 315*a*c^3*x)*arctan(a*
x))/a^4
```

Sympy [A]
time = 0.89, size = 241, normalized size = 1.00

$$\begin{cases} \frac{a^{6}c^{3}x^{10}\operatorname{atan}^{2}{\left(ax \right)}}{10} - \frac{a^{5}c^{3}x^{9}\operatorname{atan}{\left(ax \right)}}{45} + \frac{3a^{4}c^{3}x^{8}\operatorname{atan}^{2}{\left(ax \right)}}{8} + \frac{a^{4}c^{3}x^{8}}{360} - \frac{11a^{3}c^{3}x^{7}\operatorname{atan}{\left(ax \right)}}{140} + \frac{a^{2}c^{3}x^{6}\operatorname{atan}^{2}{\left(ax \right)}}{2} + \frac{71a^{2}c^{3}x^{6}}{7560} - \frac{9ac^{3}x^{5}\operatorname{atan}{\left(ax \right)}}{100} + \frac{c^{3}x^{4}\operatorname{atan}^{2}{\left(ax \right)}}{4} + \frac{53c^{3}x^{4}}{6300} - \frac{c^{3}x^{3}\operatorname{atan}{\left(ax \right)}}{60a} - \frac{107c^{3}x^{2}}{12600a^{2}} + \frac{c^{3}x\operatorname{atan}{\left(ax \right)}}{20a^{3}} - \frac{26c^{3}\log{\left(x^{2} + \frac{1}{a^{2}} \right)}}{1575a^{4}} - \frac{c^{3}\operatorname{atan}^{2}{\left(ax \right)}}{40a^{4}} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**10*atan(a*x)**2/10 - a**5*c**3*x**9*atan(a*x)/45 +
3*a**4*c**3*x**8*atan(a*x)**2/8 + a**4*c**3*x**8/360 - 11*a**3*c**3*x**7*at
an(a*x)/140 + a**2*c**3*x**6*atan(a*x)**2/2 + 71*a**2*c**3*x**6/7560 - 9*a*
c**3*x**5*atan(a*x)/100 + c**3*x**4*atan(a*x)**2/4 + 53*c**3*x**4/6300 - c*
*3*x**3*atan(a*x)/(60*a) - 107*c**3*x**2/(12600*a**2) + c**3*x*atan(a*x)/(2
0*a**3) - 26*c**3*log(x**2 + a**(-2))/(1575*a**4) - c**3*atan(a*x)**2/(40*a
**4), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.51, size = 178, normalized size = 0.74

$$\mathrm{atan}(a\,x)^2\,\left(\frac{c^3\,x^4}{4}-\frac{c^3}{40\,a^4}+\frac{a^2\,c^3\,x^6}{2}+\frac{3\,a^4\,c^3\,x^8}{8}+\frac{a^6\,c^3\,x^{10}}{10}\right)+\frac{53\,c^3\,x^4}{6300}-\frac{26\,c^3\,\ln\left(a^2\,x^2+1\right)}{1575\,a^4}-\frac{107\,c^3\,x^2}{12600\,a^2}+\frac{71\,a^2\,c^3\,x^6}{7560}+\frac{a^4\,c^3\,x^8}{360}-a^2\,\mathrm{atan}(a\,x)\,\left(\frac{11\,a\,c^3\,x^7}{140}-\frac{c^3\,x}{20\,a^5}+\frac{9\,c^3\,x^5}{100\,a}+\frac{c^3\,x^3}{60\,a^3}+\frac{a^3\,c^3\,x^9}{45}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)^2*((c^3*x^4)/4 - c^3/(40*a^4) + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8)
/8 + (a^6*c^3*x^10)/10) + (53*c^3*x^4)/6300 - (26*c^3*log(a^2*x^2 + 1))/(15
75*a^4) - (107*c^3*x^2)/(12600*a^2) + (71*a^2*c^3*x^6)/7560 + (a^4*c^3*x^8)
/360 - a^2*atan(a*x)*((11*a*c^3*x^7)/140 - (c^3*x)/(20*a^5) + (9*c^3*x^5)/(
100*a) + (c^3*x^3)/(60*a^3) + (a^3*c^3*x^9)/45)
```

3.275 $\int x^2(c+a^2cx^2)^3 \,\textbf{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=274

$$-\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{47c^3\text{ArcTan}(ax)}{3780a^3}-\frac{16c^3x^2\text{ArcTan}(ax)}{315a}-\frac{89}{630}ac^3x^4\text{ArcTan}(ax)-$$

```
[Out] -47/3780*c^3*x/a^2+239/11340*c^3*x^3+59/3780*a^2*c^3*x^5+1/252*a^4*c^3*x^7+
47/3780*c^3*arctan(a*x)/a^3-16/315*c^3*x^2*arctan(a*x)/a-89/630*a*c^3*x^4*a
rctan(a*x)-20/189*a^3*c^3*x^6*arctan(a*x)-1/36*a^5*c^3*x^8*arctan(a*x)-16/3
15*I*c^3*arctan(a*x)^2/a^3+1/3*c^3*x^3*arctan(a*x)^2+3/5*a^2*c^3*x^5*arctan
(a*x)^2+3/7*a^4*c^3*x^7*arctan(a*x)^2+1/9*a^6*c^3*x^9*arctan(a*x)^2-32/315*
c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^3-16/315*I*c^3*polylog(2,1-2/(1+I*a*x))/a
^3
```

Rubi [A]
time = 0.81, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 68, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5068, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 308}

$$\frac{1}{9}a^6c^3x^9\text{ArcTan}(ax)^2-\frac{1}{36}a^5c^3x^8\text{ArcTan}(ax)+\frac{3}{7}a^4c^3x^7\text{ArcTan}(ax)^2+\frac{1}{252}a^4c^3x^7-\frac{20}{189}a^3c^3x^6\text{ArcTan}(ax)-\frac{16ic^3\text{ArcTan}(ax)^2}{315a^3}+\frac{47c^3\text{ArcTan}(ax)}{3780a^3}-\frac{32c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{315a^3}-\frac{16ic^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{315a^3}+\frac{3}{5}a^2c^3x^5\text{ArcTan}(ax)^2+\frac{59a^2c^3x^5}{3780}-\frac{47c^3x}{3780a^2}-\frac{89}{630}ac^3x^4\text{ArcTan}(ax)+\frac{1}{3}c^3x^3\text{ArcTan}(ax)^2-\frac{16c^3x^2\text{ArcTan}(ax)}{315a}+\frac{239c^3x^3}{11340}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (-47*c^3*x)/(3780*a^2) + (239*c^3*x^3)/11340 + (59*a^2*c^3*x^5)/3780 + (a^4
*c^3*x^7)/252 + (47*c^3*ArcTan[a*x])/(3780*a^3) - (16*c^3*x^2*ArcTan[a*x])/
(315*a) - (89*a*c^3*x^4*ArcTan[a*x])/630 - (20*a^3*c^3*x^6*ArcTan[a*x])/189
 - (a^5*c^3*x^8*ArcTan[a*x])/36 - (((16*I)/315)*c^3*ArcTan[a*x]^2)/a^3 + (c
^3*x^3*ArcTan[a*x]^2)/3 + (3*a^2*c^3*x^5*ArcTan[a*x]^2)/5 + (3*a^4*c^3*x^7*
ArcTan[a*x]^2)/7 + (a^6*c^3*x^9*ArcTan[a*x]^2)/9 - (32*c^3*ArcTan[a*x]*Log[
2/(1 + I*a*x)])/(315*a^3) - (((16*I)/315)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
)/a^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx &= \int \left(c^3x^2\tan^{-1}(ax)^2+3a^2c^3x^4\tan^{-1}(ax)^2+3a^4c^3x^6\tan^{-1}(ax)^2+a^6c^3x\right.\\
&= c^3\int x^2\tan^{-1}(ax)^2\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)^2\,dx+\left(3a^4c^3\right)\int x^6\tan\\
&= \frac{1}{3}c^3x^3\tan^{-1}(ax)^2+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^2+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^2+\frac{1}{9}a^6c^3x\\
&= \frac{1}{3}c^3x^3\tan^{-1}(ax)^2+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^2+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^2+\frac{1}{9}a^6c^3x\\
&= -\frac{c^3x^2\tan^{-1}(ax)}{3a}-\frac{3}{10}ac^3x^4\tan^{-1}(ax)-\frac{1}{7}a^3c^3x^6\tan^{-1}(ax)-\frac{1}{36}a^5c^3x\\
&= \frac{c^3x}{3a^2}+\frac{4c^3x^2\tan^{-1}(ax)}{15a}-\frac{3}{35}ac^3x^4\tan^{-1}(ax)-\frac{20}{189}a^3c^3x^6\tan^{-1}(ax)-\\
&= -\frac{569c^3x}{1260a^2}+\frac{233c^3x^3}{3780}+\frac{29a^2c^3x^5}{1260}+\frac{1}{252}a^4c^3x^7-\frac{c^3\tan^{-1}(ax)}{3a^3}-\frac{17c^3x^2}{1}\\
&= \frac{583c^3x}{3780a^2}+\frac{29c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{569c^3\tan^{-1}(ax)}{1260a^3}-\frac{16c^3x^2}{3}\\
&= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7-\frac{583c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c}{}\\
&= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{47c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c^3}{}\\
&= -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7+\frac{47c^3\tan^{-1}(ax)}{3780a^3}-\frac{16c^3}{}
\end{aligned}$$

Mathematica [A]
time = 1.66, size = 157, normalized size = 0.57

$$\frac{c^3\left(ax\left(-141+239a^2x^2+177a^4x^4+45a^6x^6\right)+36\left(16i+105a^3x^3+189a^5x^5+135a^7x^7+35a^9x^9\right)\text{ArcTan}(ax)^2-3\text{ArcTan}(ax)\left(-47+192a^2x^2+534a^4x^4+400a^6x^6+105a^8x^8+384\log\left(1+e^{2i\text{ArcTan}(ax)}\right)\right)+576i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)\right)}{11340a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(a*x*(-141 + 239*a^2*x^2 + 177*a^4*x^4 + 45*a^6*x^6) + 36*(16*I + 105*
a^3*x^3 + 189*a^5*x^5 + 135*a^7*x^7 + 35*a^9*x^9)*ArcTan[a*x]^2 - 3*ArcTan[
a*x]*(-47 + 192*a^2*x^2 + 534*a^4*x^4 + 400*a^6*x^6 + 105*a^8*x^8 + 384*Log
[1 + E^((2*I)*ArcTan[a*x])]) + (576*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))
/(11340*a^3)
```

Maple [A]

time = 0.61, size = 308, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)^2a^9x^9}{9}+\frac{3c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+\frac{a^3c^3x^3\arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35\arctan(ax)a^8x^8}{8}+\frac{50\arctan(ax)a^6}{3}\right.}{}}{}$
default	$\frac{\frac{c^3\arctan(ax)^2a^9x^9}{9}+\frac{3c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+\frac{a^3c^3x^3\arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35\arctan(ax)a^8x^8}{8}+\frac{50\arctan(ax)a^6}{3}\right.}{}}{}$
risch	$\frac{239c^3x^3}{11340}-\frac{47c^3x}{3780a^2}+\frac{59a^2c^3x^5}{3780}+\frac{a^4c^3x^7}{252}+\frac{47c^3\arctan(ax)}{3780a^3}+\frac{89ic^3a\ln(iax+1)x^4}{1260}+\frac{8ic^3\ln(iax+1)x^2}{315a}+\frac{ic^3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/9*c^3*arctan(a*x)^2*a^9*x^9+3/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5*
c^3*x^5*arctan(a*x)^2+1/3*a^3*c^3*x^3*arctan(a*x)^2-2/315*c^3*(35/8*arctan(
a*x)*a^8*x^8+50/3*arctan(a*x)*a^6*x^6+89/4*arctan(a*x)*a^4*x^4+8*arctan(a*x
)*a^2*x^2-8*arctan(a*x)*ln(a^2*x^2+1)-5/8*a^7*x^7-59/24*a^5*x^5-239/72*a^3*
x^3+47/24*a*x-47/24*arctan(a*x)-4*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+4*I*ln(a*x-
I)*ln(-1/2*I*(I+a*x))+4*I*ln(I+a*x)*ln(a^2*x^2+1)-2*I*ln(I+a*x)^2+4*I*dilog
(-1/2*I*(I+a*x))+2*I*ln(a*x-I)^2-4*I*ln(a*x-I)*ln(a^2*x^2+1)-4*I*dilog(1/2*
I*(a*x-I))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*a
rctan(a*x)^2 - 1/5040*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 +
 105*c^3*x^3)*log(a^2*x^2 + 1)^2 + integrate(1/5040*(3780*(a^8*c^3*x^10 + 4
*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x)^2 + 315
```

```
*(a^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*l
og(a^2*x^2 + 1)^2 - 8*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 +
 105*a*c^3*x^3)*arctan(a*x) + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^
4*c^3*x^6 + 105*a^2*c^3*x^4)*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x
)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int x^{2}\operatorname{atan}^{2}(ax)\,dx+\int 3a^{2}x^{4}\operatorname{atan}^{2}(ax)\,dx+\int 3a^{4}x^{6}\operatorname{atan}^{2}(ax)\,dx+\int a^{6}x^{8}\operatorname{atan}^{2}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x**2*atan(a*x)**2, x) + Integral(3*a**2*x**4*atan(a*x)**2, x
) + Integral(3*a**4*x**6*atan(a*x)**2, x) + Integral(a**6*x**8*atan(a*x)**2
, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

3.276 $\int x(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=200

$$\frac{c^3(1+a^2x^2)}{35a^2}+\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{4c^3x\operatorname{ArcTan}(ax)}{35a}-\frac{2c^3x(1+a^2x^2)\operatorname{ArcTan}(ax)}{35a}-\frac{3c^3x(1+a^2x$$

```
[Out] 1/35*c^3*(a^2*x^2+1)/a^2+3/280*c^3*(a^2*x^2+1)^2/a^2+1/168*c^3*(a^2*x^2+1)^
3/a^2-4/35*c^3*x*arctan(a*x)/a-2/35*c^3*x*(a^2*x^2+1)*arctan(a*x)/a-3/70*c^
3*x*(a^2*x^2+1)^2*arctan(a*x)/a-1/28*c^3*x*(a^2*x^2+1)^3*arctan(a*x)/a+1/8*
c^3*(a^2*x^2+1)^4*arctan(a*x)^2/a^2+2/35*c^3*ln(a^2*x^2+1)/a^2
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 4998, 4930, 266}

$$\frac{c^3(a^2x^2+1)^4\operatorname{ArcTan}(ax)^2}{8a^2}-\frac{c^3x(a^2x^2+1)^3\operatorname{ArcTan}(ax)}{28a}-\frac{3c^3x(a^2x^2+1)^2\operatorname{ArcTan}(ax)}{70a}-\frac{2c^3x(a^2x^2+1)\operatorname{ArcTan}(ax)}{35a}+\frac{c^3(a^2x^2+1)^3}{168a^2}+\frac{3c^3(a^2x^2+1)^2}{280a^2}+\frac{c^3(a^2x^2+1)}{35a^2}+\frac{2c^3\log(a^2x^2+1)}{35a^2}-\frac{4c^3x\operatorname{ArcTan}(ax)}{35a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(1 + a^2*x^2))/(35*a^2) + (3*c^3*(1 + a^2*x^2)^2)/(280*a^2) + (c^3*(1
+ a^2*x^2)^3)/(168*a^2) - (4*c^3*x*ArcTan[a*x])/(35*a) - (2*c^3*x*(1 + a^2*
x^2)*ArcTan[a*x])/(35*a) - (3*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x])/(70*a) - (
c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x])/(28*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x
]^2)/(8*a^2) + (2*c^3*Log[1 + a^2*x^2])/(35*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx=\frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)^2}{8a^2}-\frac{\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)\,dx}{4a}$$
$$=\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{c^3x(1+a^2x^2)^3\tan^{-1}(ax)}{28a}+\frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)^2}{8a^2}-$$
$$=\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{3c^3x(1+a^2x^2)^2\tan^{-1}(ax)}{70a}-\frac{c^3x(1+$$
$$=\frac{c^3(1+a^2x^2)}{35a^2}+\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{2c^3x(1+a^2x^2)\tan^{-1}(}{35a}$$
$$=\frac{c^3(1+a^2x^2)}{35a^2}+\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{4c^3x\tan^{-1}(ax)}{35a}-\frac{2c^3x}{}$$
$$=\frac{c^3(1+a^2x^2)}{35a^2}+\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2}-\frac{4c^3x\tan^{-1}(ax)}{35a}-\frac{2c^3x}{}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 0.50

$$\frac{c^3\left(57a^2x^2+24a^4x^4+5a^6x^6-6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\,\mathrm{ArcTan}(ax)+105(1+a^2x^2)^4\,\mathrm{ArcTan}(ax)^2+48\log\left(1+a^2x^2\right)\right)}{840a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 6*a*x*(35 + 35*a^2*x^2 + 21*a^4
*x^4 + 5*a^6*x^6)*ArcTan[a*x] + 105*(1 + a^2*x^2)^4*ArcTan[a*x]^2 + 48*Log[
1 + a^2*x^2]))/(840*a^2)
```

Maple [A]
time = 0.16, size = 169, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)^2a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^2}{2}+\frac{3a^4c^3x^4\arctan(ax)^2}{4}+\frac{a^2c^3x^2\arctan(ax)^2}{2}+\frac{c^3\arctan(ax)^2}{8}-\frac{c^3\left(\frac{\arctan(ax)a^7x^7}{7}+\frac{3\,a}{}\right.}{}}{a^2}$
default	$\frac{\frac{c^3\arctan(ax)^2a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^2}{2}+\frac{3a^4c^3x^4\arctan(ax)^2}{4}+\frac{a^2c^3x^2\arctan(ax)^2}{2}+\frac{c^3\arctan(ax)^2}{8}-\frac{c^3\left(\frac{\arctan(ax)a^7x^7}{7}+\frac{3\,a}{}\right.}{}}{a^2}$
risch	$-\frac{c^3\left(a^2x^2+1\right)^4\ln(iax+1)^2}{32a^2}+\frac{c^3\left(35a^8x^8\ln(-iax+1)+10ia^7x^7+140a^6x^6\ln(-iax+1)+42ia^5x^5+210x^4\ln(-iax+1)a^4\right.}{560a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/8*c^3*arctan(a*x)^2*a^8*x^8+1/2*a^6*c^3*x^6*arctan(a*x)^2+3/4*a^4*
c^3*x^4*arctan(a*x)^2+1/2*a^2*c^3*x^2*arctan(a*x)^2+1/8*c^3*arctan(a*x)^2-1
/4*c^3*(1/7*arctan(a*x)*a^7*x^7+3/5*arctan(a*x)*a^5*x^5+arctan(a*x)*a^3*x^3
+arctan(a*x)*a*x-1/42*a^6*x^6-4/35*a^4*x^4-19/70*a^2*x^2-8/35*ln(a^2*x^2+1)
))
```

Maxima [A]

time = 0.26, size = 133, normalized size = 0.66

$$\frac{\left(a^2cx^2+c\right)^4\arctan\left(ax\right)^2}{8\,a^2c}+\frac{\left(5\,a^4c^4x^6+24\,a^2c^4x^4+57\,c^4x^2+\frac{48\,c^4\log\left(a^2x^2+1\right)}{a^2}\right)a-6\left(5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+35\,c^4x\right)\arctan\left(ax\right)}{840\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) + 1/840*((5*a^4*c^4*x^6 + 24*a^
2*c^4*x^4 + 57*c^4*x^2 + 48*c^4*log(a^2*x^2 + 1)/a^2)*a - 6*(5*a^6*c^4*x^7
+ 21*a^4*c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)*arctan(a*x))/(a*c)
```

Fricas [A]

time = 2.86, size = 156, normalized size = 0.78

$$\frac{5\,a^6c^3x^6+24\,a^4c^3x^4+57\,a^2c^3x^2+48\,c^3\log\left(a^2x^2+1\right)+105\left(a^8c^3x^8+4\,a^6c^3x^6+6\,a^4c^3x^4+4\,a^2c^3x^2+c^3\right)\arctan\left(ax\right)^2-6\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\arctan\left(ax\right)}{840\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^2
 + 1) + 105*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 +
c^3)*arctan(a*x)^2 - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 3
5*a*c^3*x)*arctan(a*x))/a^2
```

Sympy [A]
time = 0.61, size = 207, normalized size = 1.04

$$\begin{cases} \frac{a^6c^3x^8\operatorname{atan}^2(ax)}{8} - \frac{a^5c^3x^7\operatorname{atan}(ax)}{28} + \frac{a^4c^3x^6\operatorname{atan}^2(ax)}{2} + \frac{a^4c^3x^6}{168} - \frac{3a^3c^3x^5\operatorname{atan}(ax)}{20} + \frac{3a^2c^3x^4\operatorname{atan}^2(ax)}{4} + \frac{a^2c^3x^4}{35} - \frac{ac^3x^3\operatorname{atan}(ax)}{4} + \frac{c^3x^2\operatorname{atan}^2(ax)}{2} + \frac{19c^3x^2}{280} - \frac{c^3x\operatorname{atan}(ax)}{4a} + \frac{2c^3\log\left(x^2+\frac{1}{a^2}\right)}{35a^2} + \frac{c^3\operatorname{atan}^2(ax)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] Piecewise((a**6*c**3*x**8*atan(a*x)**2/8 - a**5*c**3*x**7*atan(a*x)/28 + a*
*4*c**3*x**6*atan(a*x)**2/2 + a**4*c**3*x**6/168 - 3*a**3*c**3*x**5*atan(a*
x)/20 + 3*a**2*c**3*x**4*atan(a*x)**2/4 + a**2*c**3*x**4/35 - a*c**3*x**3*a
tan(a*x)/4 + c**3*x**2*atan(a*x)**2/2 + 19*c**3*x**2/280 - c**3*x*atan(a*x)
/(4*a) + 2*c**3*log(x**2 + a**(-2))/(35*a**2) + c**3*atan(a*x)**2/(8*a**2),
 Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.46, size = 156, normalized size = 0.78

$$\operatorname{atan}(a\,x)^2\left(\frac{c^3}{8\,a^2}+\frac{c^3\,x^2}{2}+\frac{3\,a^2\,c^3\,x^4}{4}+\frac{a^4\,c^3\,x^6}{2}+\frac{a^6\,c^3\,x^8}{8}\right)+\frac{19\,c^3\,x^2}{280}-a^2\operatorname{atan}(a\,x)\left(\frac{c^3\,x}{4\,a^3}+\frac{3\,a\,c^3\,x^5}{20}+\frac{c^3\,x^3}{4\,a}+\frac{a^3\,c^3\,x^7}{28}\right)+\frac{2\,c^3\,\ln\left(a^2\,x^2+1\right)}{35\,a^2}+\frac{a^2\,c^3\,x^4}{35}+\frac{a^4\,c^3\,x^6}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)^2*(c^3/(8*a^2) + (c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)/
2 + (a^6*c^3*x^8)/8) + (19*c^3*x^2)/280 - a^2*atan(a*x)*((c^3*x)/(4*a^3) +
(3*a*c^3*x^5)/20 + (c^3*x^3)/(4*a) + (a^3*c^3*x^7)/28) + (2*c^3*log(a^2*x^2
 + 1))/(35*a^2) + (a^2*c^3*x^4)/35 + (a^4*c^3*x^6)/168
```

3.277 $\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=268

$$\frac{38c^3x}{105}+\frac{19}{315}a^2c^3x^3+\frac{1}{105}a^4c^3x^5-\frac{8c^3(1+a^2x^2)\operatorname{ArcTan}(ax)}{35a}-\frac{3c^3(1+a^2x^2)^2\operatorname{ArcTan}(ax)}{35a}-\frac{c^3(1+a^2x^2)^3\operatorname{ArcT}}{21a}$$

```
[Out] 38/105*c^3*x+19/315*a^2*c^3*x^3+1/105*a^4*c^3*x^5-8/35*c^3*(a^2*x^2+1)*arct
an(a*x)/a-3/35*c^3*(a^2*x^2+1)^2*arctan(a*x)/a-1/21*c^3*(a^2*x^2+1)^3*arcta
n(a*x)/a+16/35*I*c^3*arctan(a*x)^2/a+16/35*c^3*x*arctan(a*x)^2+8/35*c^3*x*(
a^2*x^2+1)*arctan(a*x)^2+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^2+1/7*c^3*x*(
a^2*x^2+1)^3*arctan(a*x)^2+32/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a+16/35*I*
c^3*polylog(2,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5000, 4930, 5040, 4964, 2449, 2352, 8, 200}

$$\frac{1}{105}a^4c^3x^5+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\operatorname{ArcTan}(ax)^2+\frac{6}{35}c^3x\left(a^2x^2+1\right)^2\operatorname{ArcTan}(ax)^2+\frac{8}{35}c^3x\left(a^2x^2+1\right)\operatorname{ArcTan}(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\operatorname{ArcTan}(ax)}{21a}-\frac{3c^3\left(a^2x^2+1\right)^2\operatorname{ArcTan}(ax)}{35a}-\frac{8c^3(a^2x^2+1)\operatorname{ArcTan}(ax)}{35a}+\frac{19}{315}a^2c^3x^3+\frac{16}{35}c^3x\operatorname{ArcTan}(ax)^2+\frac{16ic^3\operatorname{ArcTan}(ax)^2}{35a}+\frac{32c^3\operatorname{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{35a}+\frac{16ic^3\operatorname{Li}_2\left(1-\frac{2}{iax+1}\right)}{35a}+\frac{38c^3x}{105}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (38*c^3*x)/105 + (19*a^2*c^3*x^3)/315 + (a^4*c^3*x^5)/105 - (8*c^3*(1 + a^2
*x^2)*ArcTan[a*x])/(35*a) - (3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(35*a) - (c
^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(21*a) + (((16*I)/35)*c^3*ArcTan[a*x]^2)/a
+ (16*c^3*x*ArcTan[a*x]^2)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/35 +
(6*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[
a*x]^2)/7 + (32*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a) + (((16*I)/35)*c
^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+a^2cx^2)^3 \tan^{-1}(ax)^2\,dx = -\frac{c^3(1+a^2x^2)^3\tan^{-1}(ax)}{21a} + \frac{1}{7}c^3x(1+a^2x^2)^3\tan^{-1}(ax)^2 + \frac{1}{21}c\int (c+a^2c$$

$$= -\frac{3c^3(1+a^2x^2)^2\tan^{-1}(ax)}{35a} - \frac{c^3(1+a^2x^2)^3\tan^{-1}(ax)}{21a} + \frac{6}{35}c^3x(1+a^2x^2)^2$$

$$= \frac{2c^3x}{15} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2)}{35}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2}{35}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2}{35}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2}{35}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2}{35}$$

$$= \frac{38c^3x}{105} + \frac{19}{315}a^2c^3x^3 + \frac{1}{105}a^4c^3x^5 - \frac{8c^3(1+a^2x^2)\tan^{-1}(ax)}{35a} - \frac{3c^3(1+a^2x^2}{35}$$

Mathematica [A]
time = 0.87, size = 137, normalized size = 0.51

$$\frac{c^3\left(ax(114+19a^2x^2+3a^4x^4)+9(-16i+35ax+35a^3x^3+21a^5x^5+5a^7x^7)\operatorname{ArcTan}(ax)^2-3\operatorname{ArcTan}(ax)\left(38+57a^2x^2+24a^4x^4+5a^6x^6-96\log\left(1+e^{2i\operatorname{ArcTan}(ax)}\right)\right)-144i\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(ax)}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

```
[Out] (c^3*(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4) + 9*(-16*I + 35*a*x + 35*a^3*x^3 +
 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 2
4*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E^((2*I)*ArcTan[a*x])]) - (144*I)*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])]))/(315*a)
```

Maple [A]
time = 0.21, size = 282, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+a\,c^3x\arctan(ax)^2-\frac{2c^3\left(\frac{5\arctan(ax)a^6x^6}{6}+4\arctan(ax)a^4x^4\right.}{}}{}$
default	$\frac{\frac{c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+a\,c^3x\arctan(ax)^2-\frac{2c^3\left(\frac{5\arctan(ax)a^6x^6}{6}+4\arctan(ax)a^4x^4\right.}{}}{}$

risch	$\frac{c^3\ln(iax+1)\ln(-iax+1)x}{2}+\frac{38c^3x}{105}+\frac{19ic^3a\ln(iax+1)x^2}{70}+\frac{20469ic^3}{42875a}-\frac{c^3\ln(-iax+1)^2x}{4}-\frac{c^3\ln(iax+1)^2x}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x^
3*arctan(a*x)^2+a*c^3*x*arctan(a*x)^2-2/35*c^3*(5/6*arctan(a*x)*a^6*x^6+4*a
rctan(a*x)*a^4*x^4+19/2*arctan(a*x)*a^2*x^2+8*arctan(a*x)*ln(a^2*x^2+1)-1/6
*a^5*x^5-19/18*a^3*x^3-19/3*a*x+19/3*arctan(a*x)-4*I*ln(a*x-I)*ln(-1/2*I*(I
+a*x))-2*I*ln(a*x-I)^2-4*I*dilog(-1/2*I*(I+a*x))+2*I*ln(I+a*x)^2+4*I*ln(a*x
-I)*ln(a^2*x^2+1)+4*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+4*I*dilog(1/2*I*(a*x-I))-
4*I*ln(I+a*x)*ln(a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 420*a^8*c^3*integrate(1/560*x^8*arctan(a*x)^2/(a^2*x^2 + 1), x) + 35*a^8*c^
3*integrate(1/560*x^8*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 20*a^8*c^3*int
egrate(1/560*x^8*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^7*c^3*integrate(
1/560*x^7*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^6*c^3*integrate(1/560*x^6*
arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^6*c^3*integrate(1/560*x^6*log(a^2*x
^2 + 1)^2/(a^2*x^2 + 1), x) + 84*a^6*c^3*integrate(1/560*x^6*log(a^2*x^2 +
1)/(a^2*x^2 + 1), x) - 168*a^5*c^3*integrate(1/560*x^5*arctan(a*x)/(a^2*x^2
 + 1), x) + 2520*a^4*c^3*integrate(1/560*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x
) + 210*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
140*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 280*a^
3*c^3*integrate(1/560*x^3*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^2*c^3*inte
grate(1/560*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*integrate(1/5
60*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*integrate(1/560*x
^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^3*arctan(a*x)^3/a - 280*a*c^3
*integrate(1/560*x*arctan(a*x)/(a^2*x^2 + 1), x) + 35*c^3*integrate(1/560*l
og(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/140*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5
 + 35*a^2*c^3*x^3 + 35*c^3*x)*arctan(a*x)^2 - 1/560*(5*a^6*c^3*x^7 + 21*a^4
*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*log(a^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2x^2\,\mathrm{atan}^2\,(ax)\,dx + \int 3a^4x^4\,\mathrm{atan}^2\,(ax)\,dx + \int a^6x^6\,\mathrm{atan}^2\,(ax)\,dx + \int \mathrm{atan}^2\,(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**2, x) + Integral(3*a**4*x**4*atan(a*x
)**2, x) + Integral(a**6*x**6*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

3.278 $\int \frac{(c+a^2cx^2)^3 \text{ArcTan}(ax)^2}{x} \, dx$

Optimal. Leaf size=287

$$\frac{29}{180}a^2c^3x^2+\frac{1}{60}a^4c^3x^4-\frac{11}{6}ac^3x\text{ArcTan}(ax)-\frac{7}{18}a^3c^3x^3\text{ArcTan}(ax)-\frac{1}{15}a^5c^3x^5\text{ArcTan}(ax)+\frac{11}{12}c^3\text{ArcTan}(ax$$

```
[Out] 29/180*a^2*c^3*x^2+1/60*a^4*c^3*x^4-11/6*a*c^3*x*arctan(a*x)-7/18*a^3*c^3*x
^3*arctan(a*x)-1/15*a^5*c^3*x^5*arctan(a*x)+11/12*c^3*arctan(a*x)^2+3/2*a^2
*c^3*x^2*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+1/6*a^6*c^3*x^6*arctan
(a*x)^2-2*c^3*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+34/45*c^3*ln(a^2*x^2+1)
-I*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^3*arctan(a*x)*polylog(2,-1+
2/(1+I*a*x))-1/2*c^3*polylog(3,1-2/(1+I*a*x))+1/2*c^3*polylog(3,-1+2/(1+I*a
*x))
```

Rubi [A]
time = 0.54, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 4930, 266, 272, 45}

$$\frac{1}{6}a^6c^3x^6\text{ArcTan}(ax)^2-\frac{1}{15}a^5c^3x^5\text{ArcTan}(ax)+\frac{3}{4}a^4c^3x^4\text{ArcTan}(ax)^2+\frac{1}{60}a^4c^3x^4-\frac{7}{18}a^3c^3x^3\text{ArcTan}(ax)+\frac{3}{2}a^2c^3x^2\text{ArcTan}(ax)^2+\frac{29}{180}a^2c^3x^2+\frac{34}{45}c^3\log\left(a^2x^2+1\right)-ic^3\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+ic^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{11}{6}ac^3x\text{ArcTan}(ax)+\frac{11}{12}c^3\text{ArcTan}(ax)^2+2c^3\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{1}{2}c^3\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{1}{2}c^3\text{Li}_3\left(\frac{2}{iax+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

```
[Out] (29*a^2*c^3*x^2)/180 + (a^4*c^3*x^4)/60 - (11*a*c^3*x*ArcTan[a*x])/6 - (7*a
^3*c^3*x^3*ArcTan[a*x])/18 - (a^5*c^3*x^5*ArcTan[a*x])/15 + (11*c^3*ArcTan[
a*x]^2)/12 + (3*a^2*c^3*x^2*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^2
)/4 + (a^6*c^3*x^6*ArcTan[a*x]^2)/6 + 2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1
+ I*a*x)] + (34*c^3*Log[1 + a^2*x^2])/45 - I*c^3*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + I*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^3*P
olyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c^3*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^2}{x}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^2}{x} + 3a^2c^3x\tan^{-1}(ax)^2 + 3a^4c^3x^3\tan^{-1}(ax)^2 + a^6c^3x^5\tan\right.$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(3a^2c^3\right)\int x\tan^{-1}(ax)^2\,dx + \left(3a^4c^3\right)\int x^3\tan^{-1}(ax$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^2 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^2 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^2 + 2c^3\tan$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^2 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^2 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^2 + 2c^3\tan$$

$$= -3ac^3x\tan^{-1}(ax) - \frac{1}{2}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{3}{2}c^3\tan^{-}$$

$$= -\frac{3}{2}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{3}{4}c^3\tan$$

$$= -\frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c^3x^5\tan^{-1}(ax) + \frac{11}{12}c^3t$$

$$= \frac{13}{60}a^2c^3x^2 + \frac{1}{60}a^4c^3x^4 - \frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5c$$

$$= \frac{29}{180}a^2c^3x^2 + \frac{1}{60}a^4c^3x^4 - \frac{11}{6}ac^3x\tan^{-1}(ax) - \frac{7}{18}a^3c^3x^3\tan^{-1}(ax) - \frac{1}{15}a^5$$

Mathematica [A]
time = 0.38, size = 252, normalized size = 0.88

$$\frac{1}{360}c^3\left(52-15i\pi^3+58a^2x^2+6a^4x^4-660ax\mathrm{ArcTan}(ax)-140a^3x^3\mathrm{ArcTan}(ax)-24a^5x^5\mathrm{ArcTan}(ax)+330\mathrm{ArcTan}(ax)^2+540a^2x^2\mathrm{ArcTan}(ax)^2+270a^4x^4\mathrm{ArcTan}(ax)^2+60a^6x^6\mathrm{ArcTan}(ax)^2+240i\mathrm{ArcTan}(ax)^3+360\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right)-360\mathrm{ArcTan}(ax)^2\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)+272\log\left(1+a^2x^2\right)+360i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right)+360i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)+180\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)-180\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c^3*(52 - (15*I)*Pi^3 + 58*a^2*x^2 + 6*a^4*x^4 - 660*a*x*ArcTan[a*x] - 140
*a^3*x^3*ArcTan[a*x] - 24*a^5*x^5*ArcTan[a*x] + 330*ArcTan[a*x]^2 + 540*a^2
*x^2*ArcTan[a*x]^2 + 270*a^4*x^4*ArcTan[a*x]^2 + 60*a^6*x^6*ArcTan[a*x]^2 +
 (240*I)*ArcTan[a*x]^3 + 360*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])]
- 360*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 272*Log[1 + a^2*x^2] +
 (360*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (360*I)*ArcTan[a*
x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 180*PolyLog[3, E^((-2*I)*ArcTan[a*x
])] - 180*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/360
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.04, size = 1405, normalized size = 4.90

method	result	size
derivativedivides	Expression too large to display	1405
default	Expression too large to display	1405

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^6*c^3*x^6*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^2
*arctan(a*x)^2+c^3*arctan(a*x)^2*ln(a*x)-1/6*c^3*(-3*I*arctan(a*x)^2*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-11/2*arctan(a*x)^
2+136/15*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-11/30*(I+a*x)^2+6*arctan(a*x)*(a*x-I
)-5/3*arctan(a*x)*(a*x-I)^3+3*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-12*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-6*arctan(a*x)^2*ln(1-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+5*a
rctan(a*x)*(a*x-I)^2*(I+a*x)-5*arctan(a*x)*(a*x-I)*(I+a*x)^2+12*I*arctan(a*
x)*(a*x-I)^2*(I+a*x)^2-8*I*arctan(a*x)*(a*x-I)^3*(I+a*x)+3*I*arctan(a*x)^2*
Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3*I*arct
an(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)^3-3*I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^3-6*I*arctan(a*x)*(I+a*x)*(a*x-I)-8*I*arctan(a*x)*(a*x-I)*(I+a*
x)^3+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x
)^2+3*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a
```

```
*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3*I*arctan(a*x)^2*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+3*I*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))^2*arctan(a*x)^2-1/10*(I+a*x)^4+2/5*I*(I+a*x)^3+23/15*I*(I+a*x)+2/5*ar
ctan(a*x)*(a*x-I)^5+2*I*arctan(a*x)*(a*x-I)^4+3*I*arctan(a*x)*(a*x-I)^2-2*a
rctan(a*x)*(a*x-I)^4*(I+a*x)+2*arctan(a*x)*(a*x-I)*(I+a*x)^4+4*arctan(a*x)*
(a*x-I)^3*(I+a*x)^2-4*arctan(a*x)*(a*x-I)^2*(I+a*x)^3-3*I*Pi*arctan(a*x)^2+
12*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*po
lylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)^
2/(a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 36*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*a^8*c^3*i
ntegrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^8*c^3*integrat
e(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^7*c^3*integrate(1/48*x^
7*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^6*c^3*integrate(1/48*x^6*arctan(a*x
)^2/(a^2*x^3 + x), x) + 12*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a
^2*x^3 + x), x) + 9*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^3 +
x), x) - 18*a^5*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^3 + x), x) + 216*
a^4*c^3*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*int
egrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*integrate
(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 36*a^3*c^3*integrate(1/48*x^
3*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^2*c^3*integrate(1/48*x^2*arctan(a*x
)^2/(a^2*x^3 + x), x) + 1/24*c^3*log(a^2*x^2 + 1)^3 + 36*c^3*integrate(1/48
*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*c^3*integrate(1/48*log(a^2*x^2 + 1)^2/
(a^2*x^3 + x), x) + 1/48*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*a
rctan(a*x)^2 - 1/192*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*log(a
^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{\operatorname{atan}^{2}(ax)}{x}\,dx + \int 3a^{2}x\operatorname{atan}^{2}(ax)\,dx + \int 3a^{4}x^{3}\operatorname{atan}^{2}(ax)\,dx + \int a^{6}x^{5}\operatorname{atan}^{2}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)**2/x, x) + Integral(3*a**2*x*atan(a*x)**2, x) + In
tegral(3*a**4*x**3*atan(a*x)**2, x) + Integral(a**6*x**5*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x, x)
```

3.279 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^2}{x^2} dx$

Optimal. Leaf size=251

$$\frac{7}{10}a^2c^3x+\frac{1}{30}a^4c^3x^3-\frac{7}{10}ac^3\text{ArcTan}(ax)-\frac{4}{5}a^3c^3x^2\text{ArcTan}(ax)-\frac{1}{10}a^5c^3x^4\text{ArcTan}(ax)+\frac{6}{5}iac^3\text{ArcTan}(ax)^2-$$

```
[Out] 7/10*a^2*c^3*x+1/30*a^4*c^3*x^3-7/10*a*c^3*arctan(a*x)-4/5*a^3*c^3*x^2*arct
an(a*x)-1/10*a^5*c^3*x^4*arctan(a*x)+6/5*I*a*c^3*arctan(a*x)^2-c^3*arctan(a
*x)^2/x+3*a^2*c^3*x*arctan(a*x)^2+a^4*c^3*x^3*arctan(a*x)^2+1/5*a^6*c^3*x^5
*arctan(a*x)^2+22/5*a*c^3*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^3*arctan(a*x)*l
n(2-2/(1-I*a*x))-I*a*c^3*polylog(2,-1+2/(1-I*a*x))+11/5*I*a*c^3*polylog(2,1
-2/(1+I*a*x))
```

Rubi [A]
time = 0.47, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5068, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497, 5036, 327, 209, 308}

$$\frac{1}{5}a^6c^3x^5\text{ArcTan}(ax)^2-\frac{1}{10}a^5c^3x^4\text{ArcTan}(ax)+a^4c^3x^3\text{ArcTan}(ax)^2+\frac{1}{30}a^4c^3x^3-\frac{4}{5}a^3c^3x^2\text{ArcTan}(ax)+3a^2c^3x\text{ArcTan}(ax)^2+\frac{7}{10}a^2c^3x+\frac{6}{5}iac^3\text{ArcTan}(ax)^2-\frac{7}{10}ac^3\text{ArcTan}(ax)-\frac{c^3\text{ArcTan}(ax)^2}{x}+\frac{22}{5}ac^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2ac^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-iac^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{11}{5}iac^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (7*a^2*c^3*x)/10 + (a^4*c^3*x^3)/30 - (7*a*c^3*ArcTan[a*x])/10 - (4*a^3*c^3
*x^2*ArcTan[a*x])/5 - (a^5*c^3*x^4*ArcTan[a*x])/10 + ((6*I)/5)*a*c^3*ArcTan
[a*x]^2 - (c^3*ArcTan[a*x]^2)/x + 3*a^2*c^3*x*ArcTan[a*x]^2 + a^4*c^3*x^3*A
rcTan[a*x]^2 + (a^6*c^3*x^5*ArcTan[a*x]^2)/5 + (22*a*c^3*ArcTan[a*x]*Log[2/
(1 + I*a*x)])/5 + 2*a*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^3*Poly
Log[2, -1 + 2/(1 - I*a*x)] + ((11*I)/5)*a*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}{x^2}\, dx = \int \left(3a^2c^3 \tan^{-1}(ax)^2 + \frac{c^3 \tan^{-1}(ax)^2}{x^2} + 3a^4c^3x^2 \tan^{-1}(ax)^2 + a^6c^3x^4 \tan^{-1}\right.$$

$$= c^3 \int \frac{\tan^{-1}(ax)^2}{x^2}\, dx + (3a^2c^3) \int \tan^{-1}(ax)^2\, dx + (3a^4c^3) \int x^2 \tan^{-1}(ax)^2$$

$$= -\frac{c^3 \tan^{-1}(ax)^2}{x} + 3a^2c^3x \tan^{-1}(ax)^2 + a^4c^3x^3 \tan^{-1}(ax)^2 + \frac{1}{5}a^6c^3x^5 \tan^{-1}($$

$$= 2iac^3 \tan^{-1}(ax)^2 - \frac{c^3 \tan^{-1}(ax)^2}{x} + 3a^2c^3x \tan^{-1}(ax)^2 + a^4c^3x^3 \tan^{-1}(ax)^2$$

$$= -a^3c^3x^2 \tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4 \tan^{-1}(ax) + iac^3 \tan^{-1}(ax)^2 - \frac{c^3 \tan^{-1}(ax)}{x}$$

$$= a^2c^3x - \frac{4}{5}a^3c^3x^2 \tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4 \tan^{-1}(ax) + \frac{6}{5}iac^3 \tan^{-1}(ax)^2 -$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - ac^3 \tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2 \tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4 \tan^{-1}$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - \frac{7}{10}ac^3 \tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2 \tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4 \tan$$

$$= \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - \frac{7}{10}ac^3 \tan^{-1}(ax) - \frac{4}{5}a^3c^3x^2 \tan^{-1}(ax) - \frac{1}{10}a^5c^3x^4 \tan$$

Mathematica [A]
time = 0.51, size = 202, normalized size = 0.80

$$\frac{c^3\left(21a^2x^2+a^4x^4-21ax\text{ArcTan}(ax)-24a^3x^3\text{ArcTan}(ax)-3a^5x^5\text{ArcTan}(ax)-30\text{ArcTan}(ax)^2-96iax\text{ArcTan}(ax)^2+90a^2x^2\text{ArcTan}(ax)^2+30a^4x^4\text{ArcTan}(ax)^2+6a^6x^6\text{ArcTan}(ax)^2+60ax\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)+132ax\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-66iax\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-30iax\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{30x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^3*(21*a^2*x^2 + a^4*x^4 - 21*a*x*ArcTan[a*x] - 24*a^3*x^3*ArcTan[a*x] -
3*a^5*x^5*ArcTan[a*x] - 30*ArcTan[a*x]^2 - (96*I)*a*x*ArcTan[a*x]^2 + 90*a^
2*x^2*ArcTan[a*x]^2 + 30*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 +
60*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 132*a*x*ArcTan[a*x]*Log
[1 + E^((2*I)*ArcTan[a*x])] - (66*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])]
 - (30*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(30*x)
```

Maple [A]
time = 0.28, size = 323, normalized size = 1.29

method	result

derivativedivides	$a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\arctan\right.}{}\right.$
default	$a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\arctan\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arctan
(a*x)^2-c^3*arctan(a*x)^2/a/x-2/5*c^3*(1/4*arctan(a*x)*a^4*x^4+2*arctan(a*x
)*a^2*x^2+8*arctan(a*x)*ln(a^2*x^2+1)-5*arctan(a*x)*ln(a*x)-1/12*a^3*x^3-7/
4*a*x+7/4*arctan(a*x)-2*I*ln(a*x-I)^2+5/2*I*ln(a*x)*ln(1-I*a*x)-4*I*ln(a*x-
I)*ln(-1/2*I*(I+a*x))-5/2*I*dilog(1+I*a*x)+5/2*I*dilog(1-I*a*x)+4*I*ln(a*x-
I)*ln(a^2*x^2+1)+2*I*ln(I+a*x)^2-5/2*I*ln(a*x)*ln(1+I*a*x)-4*I*ln(I+a*x)*ln
(a^2*x^2+1)-4*I*dilog(-1/2*I*(I+a*x))+4*I*dilog(1/2*I*(a*x-I))+4*I*ln(I+a*x
)*ln(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/80*(4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)^
2 - (a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*log(a^2*x^2 + 1)
^2 + 80*(60*a^8*c^3*integrate(1/80*x^8*arctan(a*x)^2/(a^2*x^4 + x^2), x) +
5*a^8*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 4*a^8
*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 8*a^7*c^3*in
tegrate(1/80*x^7*arctan(a*x)/(a^2*x^4 + x^2), x) + 240*a^6*c^3*integrate(1/
80*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80*x^6*lo
g(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80*x^6*log(a^
2*x^2 + 1)/(a^2*x^4 + x^2), x) - 40*a^5*c^3*integrate(1/80*x^5*arctan(a*x)/
(a^2*x^4 + x^2), x) + 360*a^4*c^3*integrate(1/80*x^4*arctan(a*x)^2/(a^2*x^4
 + x^2), x) + 30*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)^2/(a^2*x^4 + x
^2), x) + 60*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x
) + a*c^3*arctan(a*x)^3 - 120*a^3*c^3*integrate(1/80*x^3*arctan(a*x)/(a^2*x
^4 + x^2), x) + 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1)^2/(a^2*x^4 +
 x^2), x) - 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1)/(a^2*x^4 + x^2),
 x) + 40*a*c^3*integrate(1/80*x*arctan(a*x)/(a^2*x^4 + x^2), x) + 60*c^3*in
```

```
tegrate(1/80*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 5*c^3*integrate(1/80*log(a
^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int 3a^4x^2\operatorname{atan}^2(ax)\,dx + \int a^6x^4\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) + I
ntegral(3*a**4*x**2*atan(a*x)**2, x) + Integral(a**6*x**4*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2, x)
```

3.280 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^2}{x^3} \, dx$

Optimal. Leaf size=299

$$\frac{1}{12}a^4c^3x^2 - \frac{ac^3\mathrm{ArcTan}(ax)}{x} - \frac{5}{2}a^3c^3x\mathrm{ArcTan}(ax) - \frac{1}{6}a^5c^3x^3\mathrm{ArcTan}(ax) + \frac{3}{4}a^2c^3\mathrm{ArcTan}(ax)^2 - \frac{c^3\mathrm{ArcTan}(ax)}{2x^2}$$

```
[Out] 1/12*a^4*c^3*x^2-a*c^3*arctan(a*x)/x-5/2*a^3*c^3*x*arctan(a*x)-1/6*a^5*c^3*
x^3*arctan(a*x)+3/4*a^2*c^3*arctan(a*x)^2-1/2*c^3*arctan(a*x)^2/x^2+3/2*a^4
*c^3*x^2*arctan(a*x)^2+1/4*a^6*c^3*x^4*arctan(a*x)^2-6*a^2*c^3*arctan(a*x)^
2*arctanh(-1+2/(1+I*a*x))+a^2*c^3*ln(x)+2/3*a^2*c^3*ln(a^2*x^2+1)-3*I*a^2*c
^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3*I*a^2*c^3*arctan(a*x)*polylog(2,-
1+2/(1+I*a*x))-3/2*a^2*c^3*polylog(3,1-2/(1+I*a*x))+3/2*a^2*c^3*polylog(3,-
1+2/(1+I*a*x))
```

Rubi [A]

time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5068, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745, 5036, 4930, 266, 45}

$$\frac{1}{4}a^6c^3x^4\mathrm{ArcTan}(ax)^2 - \frac{1}{6}a^5c^3x^3\mathrm{ArcTan}(ax) + \frac{3}{2}a^4c^3x^2\mathrm{ArcTan}(ax)^2 + \frac{1}{12}a^4c^3x^2 - \frac{5}{2}a^3c^3x\mathrm{ArcTan}(ax) - 3ia^2c^3\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(1-\frac{2}{iax+1}\right) + 3ia^2c^3\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(\frac{2}{iax+1}-1\right) + \frac{3}{4}a^2c^3\mathrm{ArcTan}(ax)^2 + 6a^2c^3\mathrm{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - \frac{3}{2}a^2c^3\mathrm{Li}_3\left(1-\frac{2}{iax+1}\right) + \frac{3}{2}a^2c^3\mathrm{Li}_3\left(\frac{2}{iax+1}-1\right) + \frac{2}{3}a^2c^3\log\left(a^2x^2+1\right) + a^2c^3\log(x) - \frac{c^3\mathrm{ArcTan}(ax)^2}{2x^2} - \frac{ac^3\mathrm{ArcTan}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^4*c^3*x^2)/12 - (a*c^3*ArcTan[a*x])/x - (5*a^3*c^3*x*ArcTan[a*x])/2 - (a
^5*c^3*x^3*ArcTan[a*x])/6 + (3*a^2*c^3*ArcTan[a*x]^2)/4 - (c^3*ArcTan[a*x]^
2)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^2)/2 + (a^6*c^3*x^4*ArcTan[a*x]^2)/
4 + 6*a^2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c^3*Log[x] + (
2*a^2*c^3*Log[1 + a^2*x^2])/3 - (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)] + (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (
3*a^2*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*c^3*PolyLog[3, -1 + 2/(
1 + I*a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}{x^3}\,dx &= \int \left(\frac{c^3\tan^{-1}(ax)^2}{x^3} + \frac{3a^2c^3\tan^{-1}(ax)^2}{x} + 3a^4c^3x\tan^{-1}(ax)^2 + a^6c^3x^3\tan^{-1}(\right.\\
&= c^3\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx + \left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^2}{x}\,dx + \left(3a^4c^3\right)\int x\tan^{-1}(ax)^2\,d\\
&= -\frac{c^3\tan^{-1}(ax)^2}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^2 + 6a^2c^3\tan^{-1}(\\
&= -\frac{c^3\tan^{-1}(ax)^2}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^2 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^2 + 6a^2c^3\tan^{-1}(\\
&= -\frac{ac^3\tan^{-1}(ax)}{x} - 3a^3c^3x\tan^{-1}(ax) - \frac{1}{6}a^5c^3x^3\tan^{-1}(ax) + a^2c^3\tan^{-1}(ax)^2\\
&= -\frac{ac^3\tan^{-1}(ax)}{x} - \frac{5}{2}a^3c^3x\tan^{-1}(ax) - \frac{1}{6}a^5c^3x^3\tan^{-1}(ax) + \frac{3}{4}a^2c^3\tan^{-1}(a\\
&= -\frac{ac^3\tan^{-1}(ax)}{x} - \frac{5}{2}a^3c^3x\tan^{-1}(ax) - \frac{1}{6}a^5c^3x^3\tan^{-1}(ax) + \frac{3}{4}a^2c^3\tan^{-1}(a\\
&= \frac{1}{12}a^4c^3x^2 - \frac{ac^3\tan^{-1}(ax)}{x} - \frac{5}{2}a^3c^3x\tan^{-1}(ax) - \frac{1}{6}a^5c^3x^3\tan^{-1}(ax) + \frac{3}{4}a^2c
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 333, normalized size = 1.11

$$\frac{c^3\left(2a^2x^2 - 3ia^2\pi^3x^2 + 2a^4x^4 - 24ax\text{ArcTan}(ax) - 60a^3x^3\text{ArcTan}(ax) - 4a^5x^5\text{ArcTan}(ax) - 12\text{ArcTan}(ax)^2 + 18a^2x^2\text{ArcTan}(ax)^2 + 36a^4x^4\text{ArcTan}(ax)^2 + 6a^6x^6\text{ArcTan}(ax)^2 + 48ia^2x^2\text{ArcTan}(ax)^3 + 72a^2x^2\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) - 72a^2x^2\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right) + 24a^2x^2\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) + 28a^2x^2\log\left(1+a^2x^2\right) + 72ia^2x^2\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + 72ia^2x^2\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + 36a^2x^2\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) - 36a^2x^2\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{24x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (c^3*(2*a^2*x^2 - (3*I)*a^2*Pi^3*x^2 + 2*a^4*x^4 - 24*a*x*ArcTan[a*x] - 60*
a^3*x^3*ArcTan[a*x] - 4*a^5*x^5*ArcTan[a*x] - 12*ArcTan[a*x]^2 + 18*a^2*x^2
*ArcTan[a*x]^2 + 36*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 + (48*I
)*a^2*x^2*ArcTan[a*x]^3 + 72*a^2*x^2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan
[a*x])] - 72*a^2*x^2*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 24*a^2*
x^2*Log[(a*x)/Sqrt[1 + a^2*x^2]] + 28*a^2*x^2*Log[1 + a^2*x^2] + (72*I)*a^2
*x^2*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (72*I)*a^2*x^2*ArcTan
[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 36*a^2*x^2*PolyLog[3, E^((-2*I)*
ArcTan[a*x])] - 36*a^2*x^2*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(24*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.29, size = 1318, normalized size = 4.41

method	result	size
derivativedivides	Expression too large to display	1318
default	Expression too large to display	1318

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^2*arctan(a*x)^2-1/2*c^3*ar
ctan(a*x)^2/a^2/x^2+3*c^3*arctan(a*x)^2*ln(a*x)-1/2*c^3*(-3*I*arctan(a*x)^2
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-3/2*arctan
(a*x)^2+14/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1/6*(I+a*x)^2-2*ln((1+I*a*x)/(a^
2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*(a*x-I)
-2*I*arctan(a*x)*(I+a*x)*(a*x-I)+1/3*arctan(a*x)*(a*x-I)^3+3*polylog(3,-(1+
I*a*x)^2/(a^2*x^2+1))-12*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog
(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-
1)-6*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*ln(1+(
1+I*a*x)/(a^2*x^2+1)^(1/2))+1/3*I*(I+a*x)-arctan(a*x)*(a*x-I)^2*(I+a*x)+arc
tan(a*x)*(a*x-I)*(I+a*x)^2+I*arctan(a*x)*(a*x-I)^2+3*I*arctan(a*x)^2*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3*I*arctan(a*x
)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-3*
I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))^3+arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+arctan(a*x)*(I*a*x-(a^2
*x^2+1)^(1/2)+1)/a/x+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))^2*arctan(a*x)^2+3*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3*I*arc
tan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+3*I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*Pi*arctan(a*x)^2+12*I*arctan(a*x)
*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(192*a^8*c^3*integrate(1/16*x^8*arctan(a*x)^2/(a^2*x^5 + x^3), x) +
 16*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 16*
```

```
a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 32*a^7*c^
3*integrate(1/16*x^7*arctan(a*x)/(a^2*x^5 + x^3), x) + 768*a^6*c^3*integrat
e(1/16*x^6*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^6*c^3*integrate(1/16*x^
6*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 96*a^6*c^3*integrate(1/16*x^6*lo
g(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 192*a^5*c^3*integrate(1/16*x^5*arctan(
a*x)/(a^2*x^5 + x^3), x) + 1152*a^4*c^3*integrate(1/16*x^4*arctan(a*x)^2/(a
^2*x^5 + x^3), x) + a^2*c^3*log(a^2*x^2 + 1)^3 + 768*a^2*c^3*integrate(1/16
*x^2*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^2*c^3*integrate(1/16*x^2*log(
a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 32*a^2*c^3*integrate(1/16*x^2*log(a^2*
x^2 + 1)/(a^2*x^5 + x^3), x) + 64*a*c^3*integrate(1/16*x*arctan(a*x)/(a^2*x
^5 + x^3), x) + 192*c^3*integrate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) +
16*c^3*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 + 4*(a^6*
c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^2 - (a^6*c^3*x^6 + 6*a^4*c^3*x
^4 - 2*c^3)*log(a^2*x^2 + 1)^2)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{\operatorname{atan}^2(ax)}{x^3}\,dx+\int\frac{3a^2\operatorname{atan}^2(ax)}{x}\,dx+\int 3a^4x\operatorname{atan}^2(ax)\,dx+\int a^6x^3\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)**2/x**3, x) + Integral(3*a**2*atan(a*x)**2/x, x) +
 Integral(3*a**4*x*atan(a*x)**2, x) + Integral(a**6*x**3*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3,x)
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3, x)
```

3.281 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=250

$$-\frac{a^2c^3}{3x}+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\text{ArcTan}(ax)-\frac{ac^3\text{ArcTan}(ax)}{3x^2}-\frac{1}{3}a^5c^3x^2\text{ArcTan}(ax)-\frac{c^3\text{ArcTan}(ax)^2}{3x^3}-\frac{3a^2c^3\text{ArcTan}(}{x}$$

```
[Out] -1/3*a^2*c^3/x+1/3*a^4*c^3*x-2/3*a^3*c^3*arctan(a*x)-1/3*a*c^3*arctan(a*x)/
x^2-1/3*a^5*c^3*x^2*arctan(a*x)-1/3*c^3*arctan(a*x)^2/x^3-3*a^2*c^3*arctan(
a*x)^2/x+3*a^4*c^3*x*arctan(a*x)^2+1/3*a^6*c^3*x^3*arctan(a*x)^2+16/3*a^3*c
^3*arctan(a*x)*ln(2/(1+I*a*x))+16/3*a^3*c^3*arctan(a*x)*ln(2-2/(1-I*a*x))-8
/3*I*a^3*c^3*polylog(2,-1+2/(1-I*a*x))+8/3*I*a^3*c^3*polylog(2,1-2/(1+I*a*x
))
```

Rubi [A]
time = 0.45, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5068, 4930, 5040, 4964, 2449, 2352, 4946, 5038, 331, 209, 5044, 4988, 2497, 5036, 327}

$$\frac{1}{3}a^6c^3x^3\text{ArcTan}(ax)^2-\frac{1}{3}a^5c^3x^2\text{ArcTan}(ax)+3a^4c^3x\text{ArcTan}(ax)^2+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\text{ArcTan}(ax)+\frac{16}{3}a^3c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+\frac{16}{3}a^3c^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{8}{3}ia^3c^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{8}{3}ia^3c^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)-\frac{3a^2c^3\text{ArcTan}(ax)^2}{x}-\frac{a^2c^3}{3x}-\frac{c^3\text{ArcTan}(ax)^2}{3x^3}-\frac{ac^3\text{ArcTan}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c^3)/x + (a^4*c^3*x)/3 - (2*a^3*c^3*ArcTan[a*x])/3 - (a*c^3*ArcTa
n[a*x])/(3*x^2) - (a^5*c^3*x^2*ArcTan[a*x])/3 - (c^3*ArcTan[a*x]^2)/(3*x^3)
 - (3*a^2*c^3*ArcTan[a*x]^2)/x + 3*a^4*c^3*x*ArcTan[a*x]^2 + (a^6*c^3*x^3*A
rcTan[a*x]^2)/3 + (16*a^3*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/3 + (16*a^3*c
^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((8*I)/3)*a^3*c^3*PolyLog[2, -1
+ 2/(1 - I*a*x)] + ((8*I)/3)*a^3*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^2}{x^4}\,dx = \int \left(3a^4c^3\tan^{-1}(ax)^2 + \frac{c^3\tan^{-1}(ax)^2}{x^4} + \frac{3a^2c^3\tan^{-1}(ax)^2}{x^2} + a^6c^3x^2\tan^{-1}(\right.$$

$$= c^3\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx + \left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx + \left(3a^4c^3\right)\int \tan^{-1}(ax)^2\,d$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)^2}{x} + 3a^4c^3x\tan^{-1}(ax)^2 + \frac{1}{3}a^6c^3x^3\tan^{-1}$$

$$= -\frac{c^3\tan^{-1}(ax)^2}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)^2}{x} + 3a^4c^3x\tan^{-1}(ax)^2 + \frac{1}{3}a^6c^3x^3\tan^{-1}$$

$$= -\frac{ac^3\tan^{-1}(ax)}{3x^2} - \frac{1}{3}a^5c^3x^2\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)^2}{3x^3} - \frac{3a^2c^3\tan^{-1}(ax)^2}{x}$$

$$= -\frac{a^2c^3}{3x} + \frac{1}{3}a^4c^3x - \frac{ac^3\tan^{-1}(ax)}{3x^2} - \frac{1}{3}a^5c^3x^2\tan^{-1}(ax) - \frac{c^3\tan^{-1}(ax)^2}{3x^3} -$$

$$= -\frac{a^2c^3}{3x} + \frac{1}{3}a^4c^3x - \frac{2}{3}a^3c^3\tan^{-1}(ax) - \frac{ac^3\tan^{-1}(ax)}{3x^2} - \frac{1}{3}a^5c^3x^2\tan^{-1}(ax)$$

$$= -\frac{a^2c^3}{3x} + \frac{1}{3}a^4c^3x - \frac{2}{3}a^3c^3\tan^{-1}(ax) - \frac{ac^3\tan^{-1}(ax)}{3x^2} - \frac{1}{3}a^5c^3x^2\tan^{-1}(ax)$$

Mathematica [**A**]
time = 0.42, size = 221, normalized size = 0.88

$$\frac{c^3\left(-a^2x^2+a^4x^4-ax\text{ArcTan}(ax)-2a^3x^3\text{ArcTan}(ax)-a^5x^5\text{ArcTan}(ax)-\text{ArcTan}(ax)^2-9a^2x^2\text{ArcTan}(ax)^2-16ia^3x^3\text{ArcTan}(ax)^2+9a^4x^4\text{ArcTan}(ax)^2+a^6x^6\text{ArcTan}(ax)^2+16a^3x^3\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)+16a^3x^3\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-8ia^3x^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-8ia^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (c^3*(-(a^2*x^2) + a^4*x^4 - a*x*ArcTan[a*x] - 2*a^3*x^3*ArcTan[a*x] - a^5*
x^5*ArcTan[a*x] - ArcTan[a*x]^2 - 9*a^2*x^2*ArcTan[a*x]^2 - (16*I)*a^3*x^3*
ArcTan[a*x]^2 + 9*a^4*x^4*ArcTan[a*x]^2 + a^6*x^6*ArcTan[a*x]^2 + 16*a^3*x^
3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 16*a^3*x^3*ArcTan[a*x]*Log[1
 + E^((2*I)*ArcTan[a*x])] - (8*I)*a^3*x^3*PolyLog[2, -E^((2*I)*ArcTan[a*x])
] - (8*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

Maple [**A**]
time = 0.33, size = 324, normalized size = 1.30

method	result
derivativedivides	$a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3} + 3a\,c^3x\arctan(ax)^2 - \frac{3c^3\arctan(ax)^2}{ax} - \frac{c^3\arctan(ax)^2}{3a^3x^3} - \frac{2c^3\left(\frac{\arctan(ax)a^2x^2}{2}\right.}{}\right.$

default	$a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3}+3a\,c^3x\arctan(ax)^2-\frac{3c^3\arctan(ax)^2}{ax}-\frac{c^3\arctan(ax)^2}{3a^3x^3}-\frac{2c^3\left(\frac{\arctan(ax)a^2x^2}{2}+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/3*a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arctan(a*x)^2-3*c^3*arctan(a*x
)^2/a/x-1/3*c^3*arctan(a*x)^2/a^3/x^3-2/3*c^3*(1/2*arctan(a*x)*a^2*x^2+8*ar
ctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-8*arctan(a*x)*ln(a*x)-1/2*a
*x+1/2/a/x+arctan(a*x)-4*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-4*I*dilog(1+I*a*x)-
4*I*ln(I+a*x)*ln(a^2*x^2+1)-4*I*ln(a*x)*ln(1+I*a*x)+4*I*dilog(1-I*a*x)+4*I*
ln(a*x-I)*ln(a^2*x^2+1)+2*I*ln(I+a*x)^2-2*I*ln(a*x-I)^2-4*I*dilog(-1/2*I*(I
+a*x))+4*I*ln(a*x)*ln(1-I*a*x)+4*I*dilog(1/2*I*(a*x-I))+4*I*ln(I+a*x)*ln(1/
2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*(24*(72*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^6 + x^4), x) +
 6*a^8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 8*a^
8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 16*a^7*c^3*
integrate(1/48*x^7*arctan(a*x)/(a^2*x^6 + x^4), x) + 288*a^6*c^3*integrate(
1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^6*c^3*integrate(1/48*x^6*
log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 72*a^6*c^3*integrate(1/48*x^6*log(
a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3*c^3*arctan(a*x)^3 - 144*a^5*c^3*in
tegrate(1/48*x^5*arctan(a*x)/(a^2*x^6 + x^4), x) + 36*a^4*c^3*integrate(1/4
8*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 72*a^4*c^3*integrate(1/48*x^
4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 144*a^3*c^3*integrate(1/48*x^3*arc
tan(a*x)/(a^2*x^6 + x^4), x) + 288*a^2*c^3*integrate(1/48*x^2*arctan(a*x)^2
/(a^2*x^6 + x^4), x) + 24*a^2*c^3*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^
2*x^6 + x^4), x) - 8*a^2*c^3*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 +
 x^4), x) + 16*a*c^3*integrate(1/48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 72*
c^3*integrate(1/48*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 6*c^3*integrate(1/48
*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 4*(a^6*c^3*x^6 + 9*a^4*c^3*x
^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a*x)^2 - (a^6*c^3*x^6 + 9*a^4*c^3*x^4 - 9*
a^2*c^3*x^2 - c^3)*log(a^2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2/
x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^4\operatorname{atan}^2(ax)\,dx+\int\frac{\operatorname{atan}^2(ax)}{x^4}\,dx+\int\frac{3a^2\operatorname{atan}^2(ax)}{x^2}\,dx+\int a^6x^2\operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**4,x)
```

```
[Out] c**3*(Integral(3*a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + I
ntegral(3*a**2*atan(a*x)**2/x**2, x) + Integral(a**6*x**2*atan(a*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4, x)
```

3.282 $\int \frac{x^4\textbf{ArcTan}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=166

$$\frac{x}{3a^4c}-\frac{\text{ArcTan}(ax)}{3a^5c}-\frac{x^2\text{ArcTan}(ax)}{3a^3c}-\frac{4i\text{ArcTan}(ax)^2}{3a^5c}-\frac{x\text{ArcTan}(ax)^2}{a^4c}+\frac{x^3\text{ArcTan}(ax)^2}{3a^2c}+\frac{\text{ArcTan}(ax)^3}{3a^5c}-\frac{8\text{A}}{}$$

```
[Out] 1/3*x/a^4/c-1/3*arctan(a*x)/a^5/c-1/3*x^2*arctan(a*x)/a^3/c-4/3*I*arctan(a*
x)^2/a^5/c-x*arctan(a*x)^2/a^4/c+1/3*x^3*arctan(a*x)^2/a^2/c+1/3*arctan(a*x
)^3/a^5/c-8/3*arctan(a*x)*ln(2/(1+I*a*x))/a^5/c-4/3*I*polylog(2,1-2/(1+I*a*
x))/a^5/c
```

Rubi [A]

time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5036, 4946, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004}

$$\frac{\text{ArcTan}(ax)^3}{3a^5c}-\frac{4i\text{ArcTan}(ax)^2}{3a^5c}-\frac{\text{ArcTan}(ax)}{3a^5c}-\frac{8\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^5c}-\frac{4i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{3a^5c}-\frac{x\text{ArcTan}(ax)^2}{a^4c}+\frac{x}{3a^4c}-\frac{x^2\text{ArcTan}(ax)}{3a^3c}+\frac{x^3\text{ArcTan}(ax)^2}{3a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] x/(3*a^4*c) - ArcTan[a*x]/(3*a^5*c) - (x^2*ArcTan[a*x])/(3*a^3*c) - (((4*I)
/3)*ArcTan[a*x]^2)/(a^5*c) - (x*ArcTan[a*x]^2)/(a^4*c) + (x^3*ArcTan[a*x]^2
)/(3*a^2*c) + ArcTan[a*x]^3/(3*a^5*c) - (8*ArcTan[a*x]*Log[2/(1 + I*a*x)])/
(3*a^5*c) - (((4*I)/3)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x^2\tan^{-1}(ax)^2\,dx}{a^2c}$$
$$= \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^4} - \frac{\int \tan^{-1}(ax)^2\,dx}{a^4c} - \frac{2\int \frac{x^3\tan^{-1}(ax)}{1+a^2x^2}\,dx}{3ac}$$
$$= -\frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c} - \frac{2\int x\tan^{-1}(ax)\,dx}{3a^3c} + \frac{2\int \frac{x\tan^{-1}(ax)}{1+a^2x^2}\,d}{3a^3c}$$
$$= -\frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c} - 2\int$$
$$= \frac{x}{3a^4c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c} + \frac{\tan^{-1}(ax)^3}{3a^5c}$$
$$= \frac{x}{3a^4c} - \frac{\tan^{-1}(ax)}{3a^5c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c}$$
$$= \frac{x}{3a^4c} - \frac{\tan^{-1}(ax)}{3a^5c} - \frac{x^2\tan^{-1}(ax)}{3a^3c} - \frac{4i\tan^{-1}(ax)^2}{3a^5c} - \frac{x\tan^{-1}(ax)^2}{a^4c} + \frac{x^3\tan^{-1}(ax)^2}{3a^2c}$$

Mathematica [A]
time = 0.23, size = 90, normalized size = 0.54

$$\frac{ax + (4i - 3ax + a^3x^3)\,\mathrm{ArcTan}(ax)^2 + \mathrm{ArcTan}(ax)^3 - \mathrm{ArcTan}(ax)\left(1 + a^2x^2 + 8\log\left(1 + e^{2i\mathrm{ArcTan}(ax)}\right)\right) + 4i\,\mathrm{PolyLog}\left(2, -e^{2i\mathrm{ArcTan}(ax)}\right)}{3a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
[Out] (a*x + (4*I - 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + ArcTan[a*x]^3 - ArcTan[a*x]*
(1 + a^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*
I)*ArcTan[a*x])])/(3*a^5*c)
```

Maple [A]
time = 0.28, size = 226, normalized size = 1.36

method	result
derivativedivides	$\frac{\arctan(ax)^2 a^3x^3}{3c} - \frac{\arctan(ax)^2 ax}{c} + \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{\arctan(ax)a^2x^2}{2} - 2\arctan(ax)\ln\left(a^2x^2+1\right) - \frac{ax}{2} + \frac{\arctan(ax)}{2} - i\ln(ax-i)\ln\left(a^2\right.\right.}{}$

default	$\frac{\frac{\arctan(ax)^2 a^3 x^3}{3c}-\frac{\arctan(ax)^2 ax}{c}+\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{\arctan(ax)a^2x^2}{2}-2\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}-i\ln(ax-i)\ln\left(\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)

[Out] 1/a^5*(1/3/c*arctan(a*x)^2*a^3*x^3-1/c*arctan(a*x)^2*a*x+1/c*arctan(a*x)^3-2/3/c*(1/2*arctan(a*x)*a^2*x^2-2*arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)-I*ln(a*x-I)*ln(a^2*x^2+1)+1/2*I*ln(a*x-I)^2+I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+I*dilog(-1/2*I*(I+a*x))+I*ln(I+a*x)*ln(a^2*x^2+1)-1/2*I*ln(I+a*x)^2-I*ln(I+a*x)*ln(1/2*I*(a*x-I))-I*dilog(1/2*I*(a*x-I))+arctan(a*x)^3))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")

[Out] 1/48*(4*(432*a^4*integrate(1/48*x^4*arctan(a*x)^2/(a^6*c*x^2 + a^4*c), x) + 36*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x) + 48*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) - 96*a^3*integrate(1/48*x^3*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - 144*a^2*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) + 288*a*integrate(1/48*x*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - arctan(a*x)^3/(a^5*c) - 36*integrate(1/48*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 4*(a^3*x^3 - 3*a*x)*arctan(a*x)^2 + 8*arctan(a*x)^3 - (a^3*x^3 - 3*a*x)*log(a^2*x^2 + 1)^2)/(a^5*c)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")

[Out] integral(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4 \operatorname{atan}^2(ax)}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**4*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

```
[Out] int((x^4*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.283 $\int \frac{x^3 \mathbf{ArcTan}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=169

$$-\frac{x\text{ArcTan}(ax)}{a^3c}+\frac{\text{ArcTan}(ax)^2}{2a^4c}+\frac{x^2\text{ArcTan}(ax)^2}{2a^2c}+\frac{i\text{ArcTan}(ax)^3}{3a^4c}+\frac{\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{\log\left(1+a^2x^2\right)}{2a^4c}$$

```
[Out] -x*arctan(a*x)/a^3/c+1/2*arctan(a*x)^2/a^4/c+1/2*x^2*arctan(a*x)^2/a^2/c+1/
3*I*arctan(a*x)^3/a^4/c+arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c+1/2*ln(a^2*x^2+
1)/a^4/c+I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^4/c+1/2*polylog(3,1-2/(1+
I*a*x))/a^4/c
```

Rubi [A]
time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5036, 4946, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\frac{i\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^4c}+\frac{i\text{ArcTan}(ax)^3}{3a^4c}+\frac{\text{ArcTan}(ax)^2}{2a^4c}+\frac{\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^4c}-\frac{x\text{ArcTan}(ax)}{a^3c}+\frac{x^2\text{ArcTan}(ax)^2}{2a^2c}+\frac{\log\left(a^2x^2+1\right)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] -((x*ArcTan[a*x])/(a^3*c)) + ArcTan[a*x]^2/(2*a^4*c) + (x^2*ArcTan[a*x]^2)/
(2*a^2*c) + ((I/3)*ArcTan[a*x]^3)/(a^4*c) + (ArcTan[a*x]^2*Log[2/(1 + I*a*x
)])/(a^4*c) + Log[1 + a^2*x^2]/(2*a^4*c) + (I*ArcTan[a*x]*PolyLog[2, 1 - 2/
(1 + I*a*x)])/(a^4*c) + PolyLog[3, 1 - 2/(1 + I*a*x)]/(2*a^4*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x\tan^{-1}(ax)^2\,dx}{a^2c}$$
$$= \frac{x^2\tan^{-1}(ax)^2}{2a^2c} + \frac{i\tan^{-1}(ax)^3}{3a^4c} + \frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^3c} - \frac{\int \frac{x^2\tan^{-1}(ax)}{1+a^2x^2}\,dx}{ac}$$
$$= \frac{x^2\tan^{-1}(ax)^2}{2a^2c} + \frac{i\tan^{-1}(ax)^3}{3a^4c} + \frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c} - \frac{\int \tan^{-1}(ax)\,dx}{a^3c} + \frac{\int \frac{\tan^{-1}(}{1+a^2}}{a^3}$$
$$= -\frac{x\tan^{-1}(ax)}{a^3c} + \frac{\tan^{-1}(ax)^2}{2a^4c} + \frac{x^2\tan^{-1}(ax)^2}{2a^2c} + \frac{i\tan^{-1}(ax)^3}{3a^4c} + \frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+ic}\right.}{a^4c}$$
$$= -\frac{x\tan^{-1}(ax)}{a^3c} + \frac{\tan^{-1}(ax)^2}{2a^4c} + \frac{x^2\tan^{-1}(ax)^2}{2a^2c} + \frac{i\tan^{-1}(ax)^3}{3a^4c} + \frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+ic}\right.}{a^4c}$$

Mathematica [A]
time = 0.08, size = 123, normalized size = 0.73

$$\frac{-ax\text{ArcTan}(ax) + \frac{1}{2}(1+a^2x^2)\text{ArcTan}(ax)^2 - \frac{1}{3}i\text{ArcTan}(ax)^3 + \text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - \log\left(\frac{1}{\sqrt{1+a^2x^2}}\right) - i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (-(a*x*ArcTan[a*x]) + ((1 + a^2*x^2)*ArcTan[a*x]^2)/2 - (I/3)*ArcTan[a*x]^3
 + ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - Log[1/Sqrt[1 + a^2*x^2]]
- I*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, -E^((2*I)*A
rcTan[a*x])]/2)/(a^4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.57, size = 1492, normalized size = 8.83

method	result	size
derivativedivides	Expression too large to display	1492
default	Expression too large to display	1492

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2/c*arctan(a*x)^2*a^2*x^2-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(-
arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)*polylog(2,-(1+I
*a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/24*I*arctan(
a*x)*(3*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)*Pi*a*x+6*I*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
```

```
)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi*a
*x-6*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^2*Pi*a*x-3*I*arctan(a*x)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2
+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*Pi*a*x-3*I*arctan(a*x)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2
+1)+I)*Pi*a*x+12*I*arctan(a*x)+6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi-6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*Pi-
6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi+6*arctan(a*x)*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi+6*arctan(a*x)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))^3*Pi-12*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*Pi+6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi-3*arctan(a*x)*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi+6*arctan(a*x)
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
2*Pi-3*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi-3*arctan(a*x)
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(
1+I*a*x)^2/(a^2*x^2+1)+I)*Pi+6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I
)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi-3*ar
ctan(a*x)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3
*Pi+3*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*a*x+8*arctan
(a*x)^2-24*I*a*x+24*I*arctan(a*x)*ln(2)-24)+ln((1+I*a*x)^2/(a^2*x^2+1)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^2(ax)}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c),x)
[Out] Integral(x**3*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2),x)
[Out] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.284 $\int \frac{x^2 \mathbf{ArcTan}(ax)^2}{c+a^2cx^2} dx$

Optimal. Leaf size=98

$$\frac{i\text{ArcTan}(ax)^2}{a^3c}+\frac{x\text{ArcTan}(ax)^2}{a^2c}-\frac{\text{ArcTan}(ax)^3}{3a^3c}+\frac{2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}$$

```
[Out] I*arctan(a*x)^2/a^3/c+x*arctan(a*x)^2/a^2/c-1/3*arctan(a*x)^3/a^3/c+2*arcta
n(a*x)*ln(2/(1+I*a*x))/a^3/c+I*polylog(2,1-2/(1+I*a*x))/a^3/c
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$-\frac{\text{ArcTan}(ax)^3}{3a^3c}+\frac{i\text{ArcTan}(ax)^2}{a^3c}+\frac{2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^3c}+\frac{x\text{ArcTan}(ax)^2}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (I*ArcTan[a*x]^2)/(a^3*c) + (x*ArcTan[a*x]^2)/(a^2*c) - ArcTan[a*x]^3/(3*a^
3*c) + (2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^3*c) + (I*PolyLog[2, 1 - 2/(1
+ I*a*x)])/(a^3*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)^2\,dx}{a^2c}\\
&= \frac{x\tan^{-1}(ax)^2}{a^2c}-\frac{\tan^{-1}(ax)^3}{3a^3c}-\frac{2\int \frac{x\tan^{-1}(ax)}{1+a^2x^2}\,dx}{ac}\\
&= \frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{2\int \frac{\tan^{-1}(ax)}{i-ax}\,dx}{a^2c}\\
&= \frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}-\frac{2\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}}{a^2c}\\
&= \frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{(2i)\text{Subst}\Big(}{}\\
&= \frac{i\tan^{-1}(ax)^2}{a^3c}+\frac{x\tan^{-1}(ax)^2}{a^2c}-\frac{\tan^{-1}(ax)^3}{3a^3c}+\frac{2\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{i\text{Li}_2\Big(1-\frac{}{1+}}{a^3c}
\end{aligned}
$$

Mathematica [A]

time = 0.13, size = 69, normalized size = 0.70

$$
\frac{-\frac{1}{3}\text{ArcTan}(ax)\left((3i-3ax)\text{ArcTan}(ax)+\text{ArcTan}(ax)^2-6\log\left(1+e^{2i\text{ArcTan}(ax)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)}{a^3c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] (-1/3*(ArcTan[a*x]*((3*I - 3*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 6*Log[1 + E
^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])])/(a^3*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(92) = 184.
time = 0.22, size = 189, normalized size = 1.93

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2 ax}{c} - \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2} + \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{4} - \frac{i\,\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)}{4} - \frac{i\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)}{4}\right.}{}}{a^3}$
default	$\frac{\frac{\arctan(ax)^2 ax}{c} - \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2} + \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{4} - \frac{i\,\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)}{4} - \frac{i\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)}{4}\right.}{}}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/c*arctan(a*x)^2*a*x-1/c*arctan(a*x)^3-2/c*(1/2*arctan(a*x)*ln(a^2*
x^2+1)+1/4*I*ln(a*x-I)*ln(a^2*x^2+1)-1/4*I*dilog(-1/2*I*(I+a*x))-1/4*I*ln(a
*x-I)*ln(-1/2*I*(I+a*x))-1/8*I*ln(a*x-I)^2-1/4*I*ln(I+a*x)*ln(a^2*x^2+1)+1/
4*I*dilog(1/2*I*(a*x-I))+1/4*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+1/8*I*ln(I+a*x)^
2-1/3*arctan(a*x)^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/48*(4*(144*a^2*integrate(1/16*x^2*arctan(a*x)^2/(a^4*c*x^2 + a^2*c), x) +
 12*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x) + 48*
a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c*x^2 + a^2*c), x) - 96*a*inte
grate(1/16*x*arctan(a*x)/(a^4*c*x^2 + a^2*c), x) + arctan(a*x)^3/(a^3*c) +
12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c + 12*a*
x*arctan(a*x)^2 - 3*a*x*log(a^2*x^2 + 1)^2 - 8*arctan(a*x)^3)/(a^3*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^2}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

```
[Out] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.285 $\int \frac{x\textbf{ArcTan}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=102

$$-\frac{i\text{ArcTan}(ax)^3}{3a^2c}-\frac{\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^2c}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}$$

```
[Out] -1/3*I*arctan(a*x)^3/a^2/c-arctan(a*x)^2*ln(2/(1+I*a*x))/a^2/c-I*arctan(a*x
)*polylog(2,1-2/(1+I*a*x))/a^2/c-1/2*polylog(3,1-2/(1+I*a*x))/a^2/c
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5040, 4964, 5004, 5114, 6745}

$$-\frac{i\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^2c}-\frac{i\text{ArcTan}(ax)^3}{3a^2c}-\frac{\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/3*I)*ArcTan[a*x]^3)/(a^2*c) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^2*
c) - (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - PolyLog[3, 1 -
 2/(1 + I*a*x)]/(2*a^2*c)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx &= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\int\frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{ac}\\ &= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}+\frac{2\int\frac{\tan^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}\\ &= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a^2c}+\frac{i\int\frac{\text{Li}_2\left(1-\frac{2}{1+i}\right.}{1+a^2x^2}}{ac}\\ &= -\frac{i\tan^{-1}(ax)^3}{3a^2c}-\frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a^2c}-\frac{\text{Li}_3\left(1-\frac{2}{1+ia}\right.}{2a^2c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 110, normalized size = 1.08

$$-\frac{i\text{ArcTan}(ax)^3}{3a^2c}-\frac{\text{ArcTan}(ax)^2\log\left(\frac{2i}{i-ax}\right)}{a^2c}-\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)}{a^2c}-\frac{\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/3*I)*ArcTan[a*x]^3)/(a^2*c) - (ArcTan[a*x]^2*Log[(2*I)/(I - a*x)])/(a^
2*c) - (I*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c) - PolyLog[3
, (I + a*x)/(-I + a*x)]/(2*a^2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.19, size = 756, normalized size = 7.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(arctan(a*x)^2*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2))-1/4*arctan(a*x)^2*(I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+2*I*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
))^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^3-4*ln(2))-I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2
*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/3*I*arctan(a*x)^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{2}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

```
[Out] int((x*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.286 $\int \frac{\mathbf{ArcTan}(ax)^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(ax)^3}{3ac}$$

[Out] 1/3*arctan(a*x)^3/a/c

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5004}

$$\frac{\text{ArcTan}(ax)^3}{3ac}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2),x]

[Out] ArcTan[a*x]^3/(3*a*c)

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^3}{3ac}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{\text{ArcTan}(ax)^3}{3ac}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2),x]

[Out] ArcTan[a*x]^3/(3*a*c)

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\arctan(ax)^3}{3ac}$	15
default	$\frac{\arctan(ax)^3}{3ac}$	15
risch	$\frac{i\ln(iax+1)^3}{24ca} - \frac{i\ln(-iax+1)\ln(iax+1)^2}{8ca} + \frac{i\ln(-iax+1)^2\ln(iax+1)}{8ca} - \frac{i\ln(-iax+1)^3}{24ca}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(a*x)^3/a/c
```

Maxima [A]
time = 0.47, size = 14, normalized size = 0.88

$$\frac{\arctan(ax)^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(a*x)^3/(a*c)
```

Fricas [A]
time = 2.26, size = 14, normalized size = 0.88

$$\frac{\arctan(ax)^3}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(a*x)^3/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.16, size = 14, normalized size = 0.88

$$\frac{\mathrm{atan}(a\,x)^3}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2),x)
```

```
[Out] atan(a*x)^3/(3*a*c)
```

3.287 $\int \frac{\text{ArcTan}(ax)^2}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=91

$$-\frac{i\text{ArcTan}(ax)^3}{3c}+\frac{\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}+\frac{\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

```
[Out] -1/3*I*arctan(a*x)^3/c+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*poly
log(2,-1+2/(1-I*a*x))/c+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5044, 4988, 5004, 5112, 6745}

$$-\frac{i\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}-\frac{i\text{ArcTan}(ax)^3}{3c}+\frac{\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((-1/3*I)*ArcTan[a*x]^3)/c + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (I*
ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + PolyLog[3, -1 + 2/(1 - I*a*
x)]/(2*c)
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx = -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{i\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{(2a)\int \frac{\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{(ia)\int \ldots}{\ldots}$$

$$= -\frac{i\tan^{-1}(ax)^3}{3c}+\frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\tan^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1-iax}\right)}{c}+\frac{\text{Li}_3(-1 \ldots}{2 \ldots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 243 vs. 2(91) = 182.
time = 0.03, size = 243, normalized size = 2.67

$$\frac{i\text{ArcTan}(ax)^3}{3c}+\frac{2\text{ArcTan}(ax)^2\tanh^{-1}\left(1-\frac{2i}{i-ax}\right)}{c}+\frac{\text{ArcTan}(ax)^2\log\left(\frac{2i}{i-ax}\right)}{c}+\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right)}{c}+\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{c}-\frac{i\text{ArcTan}(ax)\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)}{c}+\frac{\text{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right)}{2c}+\frac{\text{PolyLog}\left(3,-\frac{i+ax}{i-ax}\right)}{2c}-\frac{\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/3)*ArcTan[a*x]^3)/c + (2*ArcTan[a*x]^2*ArcTanh[1 - (2*I)/(I - a*x)])/c
+ (ArcTan[a*x]^2*Log[(2*I)/(I - a*x)])/c + (I*ArcTan[a*x]*PolyLog[2, (-I -
a*x)/(-I + a*x)])/c + (I*ArcTan[a*x]*PolyLog[2, -((I + a*x)/(I - a*x))])/c
- (I*ArcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)])/c + PolyLog[3, (-I - a*x
)/(-I + a*x)]/(2*c) + PolyLog[3, -((I + a*x)/(I - a*x))]/(2*c) - PolyLog[3,
 (I + a*x)/(-I + a*x)]/(2*c)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.26, size = 1689, normalized size = 18.56

method	result	size
derivativedivides	Expression too large to display	1689
default	Expression too large to display	1689

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)+1/c*arctan(a*x)^2*ln(a*x)-1/c*(1/4*I*arc
tan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*
I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*
x)^2-arctan(a*x)^2*ln(2)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*
I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^2+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1))^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3+1/3*I*arctan(a*x)^3-1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2
+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*Pi*csgn(((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*ar
ctan(a*x)^2+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arc
tan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))-1/4*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/4*I*arct
an(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)-1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))*arctan(a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a
*x)^2+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**3 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)), x)
```

3.288 $\int \frac{\text{ArcTan}(ax)^2}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=92

$$-\frac{ia\text{ArcTan}(ax)^2}{c}-\frac{\text{ArcTan}(ax)^2}{cx}-\frac{a\text{ArcTan}(ax)^3}{3c}+\frac{2a\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}$$

```
[Out] -I*a*arctan(a*x)^2/c-arctan(a*x)^2/c/x-1/3*a*arctan(a*x)^3/c+2*a*arctan(a*x
)*ln(2-2/(1-I*a*x))/c-I*a*polylog(2,-1+2/(1-I*a*x))/c
```

Rubi [A]
time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5038, 4946, 5044, 4988, 2497, 5004}

$$-\frac{a\text{ArcTan}(ax)^3}{3c}-\frac{ia\text{ArcTan}(ax)^2}{c}-\frac{\text{ArcTan}(ax)^2}{cx}+\frac{2a\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{ia\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((-I)*a*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(c*x) - (a*ArcTan[a*x]^3)/(3*c) +
(2*a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (I*a*PolyLog[2, -1 + 2/(1 - I*
a*x)])/c
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)^2}{cx}-\frac{a\tan^{-1}(ax)^3}{3c}+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x(1+a^2x^2)}\,dx}{c}\\
&= -\frac{ia\tan^{-1}(ax)^2}{c}-\frac{\tan^{-1}(ax)^2}{cx}-\frac{a\tan^{-1}(ax)^3}{3c}+\frac{(2ia)\int \frac{\tan^{-1}(ax)}{x(i+ax)}\,dx}{c}\\
&= -\frac{ia\tan^{-1}(ax)^2}{c}-\frac{\tan^{-1}(ax)^2}{cx}-\frac{a\tan^{-1}(ax)^3}{3c}+\frac{2a\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{(2a^2
\\
&= -\frac{ia\tan^{-1}(ax)^2}{c}-\frac{\tan^{-1}(ax)^2}{cx}-\frac{a\tan^{-1}(ax)^3}{3c}+\frac{2a\tan^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{ia\mathrm{Li}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 73, normalized size = 0.79

$$\frac{a\left(-\frac{1}{3}\mathrm{ArcTan}(ax)\left(\frac{3\mathrm{ArcTan}(ax)}{ax}+\mathrm{ArcTan}(ax)(3i+\mathrm{ArcTan}(ax))-6\log\left(1-e^{2i\mathrm{ArcTan}(ax)}\right)\right)-i\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (a*(-1/3*(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a*
x]) - 6*Log[1 - E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[a*x
])]))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(86) = 172$.
time = 0.24, size = 254, normalized size = 2.76

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)^2}{cax}-\frac{\arctan(ax)^3}{c}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}\right.}{}\right.$
default	$a\left(-\frac{\arctan(ax)^2}{cax}-\frac{\arctan(ax)^3}{c}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/c*arctan(a*x)^2/a/x-1/c*arctan(a*x)^3-2/c*(-1/3*arctan(a*x)^3+1/2*arc
tan(a*x)*ln(a^2*x^2+1)-arctan(a*x)*ln(a*x)-1/2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*
ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*I*dilog(1-I*a*x)+1/4*I*ln(a*x-
I)*ln(a^2*x^2+1)-1/8*I*ln(a*x-I)^2-1/4*I*dilog(-1/2*I*(I+a*x))-1/4*I*ln(a*x
-I)*ln(-1/2*I*(I+a*x))-1/4*I*ln(I+a*x)*ln(a^2*x^2+1)+1/8*I*ln(I+a*x)^2+1/4*
I*dilog(1/2*I*(a*x-I))+1/4*I*ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/48*(8*a*x*arctan(a*x)^3 - 4*(a*arctan(a*x)^3/c + 12*a^2*integrate(1/16*x
^2*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 48*a^2*integrate(1/16*x^2*l
og(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 96*a*integrate(1/16*x*arctan(a*x)
/(a^2*c*x^4 + c*x^2), x) + 144*integrate(1/16*arctan(a*x)^2/(a^2*c*x^4 + c*
x^2), x) + 12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x))*c*
x + 12*arctan(a*x)^2 - 3*log(a^2*x^2 + 1)^2)/(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^4+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**2/(a**2*x**4 + x**2), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)), x)
```

3.289 $\int \frac{\mathbf{ArcTan}(ax)^2}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=178

$$-\frac{a\mathrm{ArcTan}(ax)}{cx}-\frac{a^2\mathrm{ArcTan}(ax)^2}{2c}-\frac{\mathrm{ArcTan}(ax)^2}{2cx^2}+\frac{ia^2\mathrm{ArcTan}(ax)^3}{3c}+\frac{a^2\log(x)}{c}-\frac{a^2\log\left(1+a^2x^2\right)}{2c}-\frac{a^2\mathrm{ArcTa}}{}$$

```
[Out] -a*arctan(a*x)/c/x-1/2*a^2*arctan(a*x)^2/c-1/2*arctan(a*x)^2/c/x^2+1/3*I*a^
2*arctan(a*x)^3/c+a^2*ln(x)/c-1/2*a^2*ln(a^2*x^2+1)/c-a^2*arctan(a*x)^2*ln(
2-2/(1-I*a*x))/c+I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-1/2*a^2*poly
log(3,-1+2/(1-I*a*x))/c
```

Rubi [A]

time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$\frac{ia^2\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}+\frac{ia^2\mathrm{ArcTan}(ax)^3}{3c}-\frac{a^2\mathrm{ArcTan}(ax)^2}{2c}-\frac{a^2\mathrm{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a^2\mathrm{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}-\frac{a^2\log\left(a^2x^2+1\right)}{2c}+\frac{a^2\log(x)}{c}-\frac{\mathrm{ArcTan}(ax)^2}{2cx^2}-\frac{a\mathrm{ArcTan}(ax)}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -((a*ArcTan[a*x])/(c*x)) - (a^2*ArcTan[a*x]^2)/(2*c) - ArcTan[a*x]^2/(2*c*x
^2) + ((I/3)*a^2*ArcTan[a*x]^3)/c + (a^2*Log[x])/c - (a^2*Log[1 + a^2*x^2])
/(2*c) - (a^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c + (I*a^2*ArcTan[a*x]*
PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2
*c)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x^3(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c} \\
&= -\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}+\frac{a\int \frac{\tan^{-1}(ax)}{x^2(1+a^2x^2)}\,dx}{c}-\frac{(ia^2)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c} \\
&= -\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{a\int \frac{\tan^{-1}(ax)}{x^2}\,dx}{c}- \\
&= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log}{c} \\
&= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log}{c} \\
&= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}-\frac{a^2\tan^{-1}(ax)^2\log}{c} \\
&= -\frac{a\tan^{-1}(ax)}{cx}-\frac{a^2\tan^{-1}(ax)^2}{2c}-\frac{\tan^{-1}(ax)^2}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^3}{3c}+\frac{a^2\log(x)}{c}-\frac{a^2\log}{}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 142, normalized size = 0.80

$$\frac{a^2\left(\frac{i\pi^3}{24}-\frac{\mathrm{ArcTan}(ax)}{ax}-\frac{(1+a^2x^2)\mathrm{ArcTan}(ax)^2}{2a^2x^2}-\frac{1}{3}i\mathrm{ArcTan}(ax)^3-\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right)+\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (a^2*((I/24)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - (I/3)*ArcTan[a*x]^3 - ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])
] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTa
n[a*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])]/2))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.48, size = 5037, normalized size = 28.30

method	result	size

derivativedivides	Expression too large to display	5037
default	Expression too large to display	5037

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{a^{2}x^{5}+x^{3}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**2/(a**2*x**5 + x**3), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)), x)
```

3.290 $\int \frac{\textbf{ArcTan}(ax)^2}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{a^2}{3cx}-\frac{a^3\text{ArcTan}(ax)}{3c}-\frac{a\text{ArcTan}(ax)}{3cx^2}+\frac{4ia^3\text{ArcTan}(ax)^2}{3c}-\frac{\text{ArcTan}(ax)^2}{3cx^3}+\frac{a^2\text{ArcTan}(ax)^2}{cx}+\frac{a^3\text{ArcTan}(ax)^3}{3c}$$

```
[Out] -1/3*a^2/c/x-1/3*a^3*arctan(a*x)/c-1/3*a*arctan(a*x)/c/x^2+4/3*I*a^3*arctan
(a*x)^2/c-1/3*arctan(a*x)^2/c/x^3+a^2*arctan(a*x)^2/c/x+1/3*a^3*arctan(a*x)
^3/c-8/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c+4/3*I*a^3*polylog(2,-1+2/(1-I*
a*x))/c
```

Rubi [A]
time = 0.32, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5038, 4946, 331, 209, 5044, 4988, 2497, 5004}

$$\frac{a^3\text{ArcTan}(ax)^3}{3c}+\frac{4ia^3\text{ArcTan}(ax)^2}{3c}-\frac{a^3\text{ArcTan}(ax)}{3c}-\frac{8a^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c}+\frac{4ia^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{3c}+\frac{a^2\text{ArcTan}(ax)^2}{cx}-\frac{a^2}{3cx}-\frac{\text{ArcTan}(ax)^2}{3cx^3}-\frac{a\text{ArcTan}(ax)}{3cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -1/3*a^2/(c*x) - (a^3*ArcTan[a*x])/(3*c) - (a*ArcTan[a*x])/(3*c*x^2) + (((4
*I)/3)*a^3*ArcTan[a*x]^2)/c - ArcTan[a*x]^2/(3*c*x^3) + (a^2*ArcTan[a*x]^2)
/(c*x) + (a^3*ArcTan[a*x]^3)/(3*c) - (8*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*
x)])/(3*c) + (((4*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c} \\
&= -\frac{\tan^{-1}(ax)^2}{3cx^3}+a^4\int \frac{\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3(1+a^2x^2)}\,dx}{3c}-\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c} \\
&= -\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+\frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3}\,dx}{3c}-\frac{(2a^3)\int \frac{\tan^{-1}(c}{x(1+a^2}}{3c} \\
&= -\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(ax)^3}{3c}+ \\
&= -\frac{a^2}{3cx}-\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax)^2}{cx}+\frac{a^3\tan^{-1}(a}{3c} \\
&= -\frac{a^2}{3cx}-\frac{a^3\tan^{-1}(ax)}{3c}-\frac{a\tan^{-1}(ax)}{3cx^2}+\frac{4ia^3\tan^{-1}(ax)^2}{3c}-\frac{\tan^{-1}(ax)^2}{3cx^3}+\frac{a^2\tan^{-1}(ax}{cx}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 120, normalized size = 0.72

$$
\frac{a^3\left(-\frac{1-4\text{ArcTan}(ax)^2+\frac{(1+a^2x^2)\text{ArcTan}(ax)^2}{a^2x^2}}{ax}+\text{ArcTan}(ax)\left(-\frac{1+a^2x^2}{a^2x^2}+\text{ArcTan}(ax)(4i+\text{ArcTan}(ax))-8\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+4i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)}{3c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (a^3*(-((1 - 4*ArcTan[a*x]^2 + ((1 + a^2*x^2)*ArcTan[a*x]^2)/(a^2*x^2))/(a*
x)) + ArcTan[a*x]*(-((1 + a^2*x^2)/(a^2*x^2)) + ArcTan[a*x]*(4*I + ArcTan[a
*x]) - 8*Log[1 - E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan
[a*x])]))/(3*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(146) = 292.
time = 0.42, size = 295, normalized size = 1.78

method	result
derivativedivides	$a^3\left(\frac{\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{3c\,a^3x^3}+\frac{\arctan(ax)^2}{cax}-\frac{2\left(-2\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}+4\arctan(ax)\ln(ax)+\right.}{}\right.$
default	$a^3\left(\frac{\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{3c\,a^3x^3}+\frac{\arctan(ax)^2}{cax}-\frac{2\left(-2\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}+4\arctan(ax)\ln(ax)+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)

[Out] a^3*(1/c*arctan(a*x)^3-1/3/c*arctan(a*x)^2/a^3/x^3+1/c*arctan(a*x)^2/a/x-2/3/c*(-2*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2+4*arctan(a*x)*ln(a*x)+1/2/a/x+1/2*arctan(a*x)-I*ln(I+a*x)*ln(1/2*I*(a*x-I))+1/2*I*ln(a*x-I)^2-I*ln(a*x-I)*ln(a^2*x^2+1)+2*I*dilog(1+I*a*x)+2*I*ln(a*x)*ln(1+I*a*x)-2*I*dilog(1-I*a*x)+I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+I*ln(I+a*x)*ln(a^2*x^2+1)-2*I*ln(a*x)*ln(1-I*a*x)+I*dilog(-1/2*I*(I+a*x))-1/2*I*ln(I+a*x)^2-I*dilog(1/2*I*(a*x-I))+arctan(a*x)^3))

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="maxima")

[Out] Timed out

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="fricas")

[Out] integral(arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}(ax)}{a^{2}x^{6}+x^{4}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c),x)

[Out] Integral(atan(a*x)**2/(a**2*x**6 + x**4), x)/c

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)), x)
```

3.291 $\int \frac{x^3 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=192

$$-\frac{1}{4a^4c^2(1+a^2x^2)}-\frac{x\mathrm{ArcTan}(ax)}{2a^3c^2(1+a^2x^2)}-\frac{\mathrm{ArcTan}(ax)^2}{4a^4c^2}+\frac{\mathrm{ArcTan}(ax)^2}{2a^4c^2(1+a^2x^2)}-\frac{i\mathrm{ArcTan}(ax)^3}{3a^4c^2}-\frac{\mathrm{ArcTan}(ax)^2\log(}{a^4c^2}$$

```
[Out] -1/4/a^4/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a
*x)^2/a^4/c^2+1/2*arctan(a*x)^2/a^4/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/a^4
/c^2-arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c^2-I*arctan(a*x)*polylog(2,1-2/(1+I
*a*x))/a^4/c^2-1/2*polylog(3,1-2/(1+I*a*x))/a^4/c^2
```

Rubi [A]
time = 0.22, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5084, 5040, 4964, 5004, 5114, 6745, 5050, 5012, 267}

$$-\frac{i\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^4c^2}-\frac{i\mathrm{ArcTan}(ax)^3}{3a^4c^2}-\frac{\mathrm{ArcTan}(ax)^2}{4a^4c^2}-\frac{\mathrm{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c^2}-\frac{\mathrm{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^4c^2}+\frac{\mathrm{ArcTan}(ax)^2}{2a^4c^2(a^2x^2+1)}-\frac{1}{4a^4c^2(a^2x^2+1)}-\frac{x\mathrm{ArcTan}(ax)}{2a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/4*1/(a^4*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^3*c^2*(1 + a^2*x^2))
- ArcTan[a*x]^2/(4*a^4*c^2) + ArcTan[a*x]^2/(2*a^4*c^2*(1 + a^2*x^2)) - ((I
/3)*ArcTan[a*x]^3)/(a^4*c^2) - (ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^4*c^2)
 - (I*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) - PolyLog[3, 1 -
 2/(1 + I*a*x)]/(2*a^4*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{a^2} + \frac{\int \frac{x\tan^{-1}(ax)^2}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^2}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^3}{3a^4c^2} - \frac{\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a^3} - \frac{\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^3c^2}$$

$$= -\frac{x\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)^2}{4a^4c^2} + \frac{\tan^{-1}(ax)^2}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(ax)^3}{3a^4c^2} - \frac{\tan^{-1}(ax)^2 \,\mathrm{lo}}{a^4c^2}$$

$$= -\frac{1}{4a^4c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)^2}{4a^4c^2} + \frac{\tan^{-1}(ax)^2}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(a}{3a^4c^2}$$

$$= -\frac{1}{4a^4c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)} - \frac{\tan^{-1}(ax)^2}{4a^4c^2} + \frac{\tan^{-1}(ax)^2}{2a^4c^2(1+a^2x^2)} - \frac{i\tan^{-1}(a}{3a^4c^2}$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 0.61

$$\frac{\frac{1}{3}i\mathrm{ArcTan}(ax)^3 + \frac{1}{8}(-1+2\mathrm{ArcTan}(ax)^2)\cos(2\mathrm{ArcTan}(ax)) - \mathrm{ArcTan}(ax)^2\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right) + i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right) - \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(ax)}\right) - \frac{1}{4}\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax))}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((I/3)*ArcTan[a*x]^3 + ((-1 + 2*ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]])/8 - ArcT
an[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, -E^((2*
I)*ArcTan[a*x])] - PolyLog[3, -E^((2*I)*ArcTan[a*x])]/2 - (ArcTan[a*x]*Sin[
2*ArcTan[a*x]])/4)/(a^4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 855, normalized size = 4.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1
)-1/c^2*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^3-
I*arctan(a*x)*(I+a*x)/(8*a*x-8*I)-1/16*(I+a*x)/(a*x-I)+I*arctan(a*x)*(a*x-I
)/(8*a*x+8*I)-1/16*(a*x-I)/(I+a*x)+1/4*arctan(a*x)^2*(-2*I*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*c
sgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi
```

```
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+I*Pi
*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1+4*ln(2))-I*arctan(a*x)*polylo
g(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3}\operatorname{atan}^{2}{\left(ax \right)}}{a^{4}x^{4}+2a^{2}x^{2}+1}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**3*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^2}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

3.292 $\int \frac{x^2\mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{x}{4a^2c^2(1+a^2x^2)}+\frac{\mathrm{ArcTan}(ax)}{4a^3c^2}-\frac{\mathrm{ArcTan}(ax)}{2a^3c^2(1+a^2x^2)}-\frac{x\mathrm{ArcTan}(ax)^2}{2a^2c^2(1+a^2x^2)}+\frac{\mathrm{ArcTan}(ax)^3}{6a^3c^2}$$

```
[Out] 1/4*x/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^3/c^2-1/2*arctan(a*x)/a^3/c^2/(
a^2*x^2+1)-1/2*x*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a^3/c^
2
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5056, 5050, 205, 211}

$$\frac{\mathrm{ArcTan}(ax)^3}{6a^3c^2}+\frac{\mathrm{ArcTan}(ax)}{4a^3c^2}-\frac{x\mathrm{ArcTan}(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{x}{4a^2c^2(a^2x^2+1)}-\frac{\mathrm{ArcTan}(ax)}{2a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] x/(4*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(4*a^3*c^2) - ArcTan[a*x]/(2*a^3*
c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a
*x]^3/(6*a^3*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx &= -\frac{x\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a}\\ &= -\frac{\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{1}{(c+a^2cx^2)^2}\,dx}{2a^2}\\ &= \frac{x}{4a^2c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}+\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4a^2c}\\ &= \frac{x}{4a^2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)}{4a^3c^2}-\frac{\tan^{-1}(ax)}{2a^3c^2(1+a^2x^2)}-\frac{x\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6a^3c^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.64

$$\frac{3ax+3(-1+a^2x^2)\operatorname{ArcTan}(ax)-6ax\operatorname{ArcTan}(ax)^2+2(1+a^2x^2)\operatorname{ArcTan}(ax)^3}{12a^3c^2(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*a*x + 3*(-1 + a^2*x^2)*ArcTan[a*x] - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^
2)*ArcTan[a*x]^3)/(12*a^3*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.24, size = 93, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4(a^2x^2+1)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$

default	$\frac{-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4(a^2x^2+1)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$
risch	$\frac{i\ln(iax+1)^3}{48c^2a^3}-\frac{i(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax)\ln(iax+1)^2}{16a^3c^2(a^2x^2+1)}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2+4iax\ln(-\right.}{16a^3c^2(ax+i)(ax-i)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/2*arctan(a*x)^3/c^2-1/c^2*(
1/3*arctan(a*x)^3+1/2*arctan(a*x)/(a^2*x^2+1)-1/4*a*x/(a^2*x^2+1)-1/4*arcta
n(a*x)))
```

Maxima [A]
time = 0.50, size = 151, normalized size = 1.42

$$-\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^2+\frac{(2(a^2x^2+1)\arctan(ax)^3+3ax+3(a^2x^2+1)\arctan(ax))a^2}{12(a^7c^2x^2+a^5c^2)}-\frac{((a^2x^2+1)\arctan(ax)^2+1)a\arctan(ax)}{2(a^6c^2x^2+a^4c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^2 + 1/
12*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(a^2*x^2 + 1)*arctan(a*x))*a^
2/(a^7*c^2*x^2 + a^5*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(
a*x)/(a^6*c^2*x^2 + a^4*c^2)
```

Fricas [A]
time = 2.81, size = 69, normalized size = 0.65

$$-\frac{6\,ax\arctan(ax)^2-2(a^2x^2+1)\arctan(ax)^3-3\,ax-3(a^2x^2-1)\arctan(ax)}{12(a^5c^2x^2+a^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*x*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 - 1)*arctan(a*x))/(a^5*c^2*x^2 + a^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{x^2\operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.42, size = 96, normalized size = 0.91

$$\frac{x}{2\,\left(2\,a^4\,c^2\,x^2+2\,a^2\,c^2\right)}+\frac{\mathrm{atan}\left(a\,x\right)}{4\,a^3\,c^2}+\frac{{\mathrm{atan}\left(a\,x\right)}^3}{6\,a^3\,c^2}-\frac{\mathrm{atan}\left(a\,x\right)}{2\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)}-\frac{x\,{\mathrm{atan}\left(a\,x\right)}^2}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

```
[Out] x/(2*(2*a^2*c^2 + 2*a^4*c^2*x^2)) + atan(a*x)/(4*a^3*c^2) + atan(a*x)^3/(6*
a^3*c^2) - atan(a*x)/(2*a^5*c^2*(1/a^2 + x^2)) - (x*atan(a*x)^2)/(2*a^4*c^2
*(1/a^2 + x^2))
```

3.293 $\int \frac{x\mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=91

$$\frac{1}{4a^2c^2\left(1+a^2x^2\right)}+\frac{x\text{ArcTan}(ax)}{2ac^2\left(1+a^2x^2\right)}+\frac{\text{ArcTan}(ax)^2}{4a^2c^2}-\frac{\text{ArcTan}(ax)^2}{2a^2c^2\left(1+a^2x^2\right)}$$

```
[Out] 1/4/a^2/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)
^2/a^2/c^2-1/2*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5050, 5012, 267}

$$-\frac{\text{ArcTan}(ax)^2}{2a^2c^2\left(a^2x^2+1\right)}+\frac{x\text{ArcTan}(ax)}{2ac^2\left(a^2x^2+1\right)}+\frac{\text{ArcTan}(ax)^2}{4a^2c^2}+\frac{1}{4a^2c^2\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^2*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*a*c^2*(1 + a^2*x^2)) + Arc
Tan[a*x]^2/(4*a^2*c^2) - ArcTan[a*x]^2/(2*a^2*c^2*(1 + a^2*x^2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx = -\frac{\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)} + \frac{\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{a}$$
$$= \frac{x\tan^{-1}(ax)}{2ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^2}{4a^2c^2} - \frac{\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)} - \frac{1}{2}\int \frac{x}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{1}{4a^2c^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)}{2ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^2}{4a^2c^2} - \frac{\tan^{-1}(ax)^2}{2a^2c^2(1+a^2x^2)}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.52

$$\frac{1+2ax\text{ArcTan}(ax)+(-1+a^2x^2)\,\text{ArcTan}(ax)^2}{4a^2c^2(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (1 + 2*a*x*ArcTan[a*x] + (-1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^2*c^2*(1 + a^2*
x^2))
```

Maple [A]
time = 0.19, size = 73, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$
risch	$-\frac{(a^2x^2-1)\ln(iax+1)^2}{16a^2c^2(a^2x^2+1)}+\frac{(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax)\ln(iax+1)}{8(ax+i)a^2c^2(ax-i)}-\frac{-4+a^2x^2\ln(-iax+1)^2-\ln(-iax}{16(ax+i)a^2c^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/c^2*(1/2*a*x/(a^2*x^2+1)*arctan
(a*x)+1/4*arctan(a*x)^2+1/4/(a^2*x^2+1)))
```

Maxima [A]
time = 0.47, size = 104, normalized size = 1.14

$$\frac{\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan(ax)}{2\,ac} - \frac{(a^2x^2+1)\arctan(ax)^2-1}{4\,(a^4cx^2+a^2c)c} - \frac{\arctan(ax)^2}{2\,(a^2cx^2+c)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)/(a*c) - 1/4*((a^2*x
^2 + 1)*arctan(a*x)^2 - 1)/((a^4*c*x^2 + a^2*c)*c) - 1/2*arctan(a*x)^2/((a^
2*c*x^2 + c)*a^2*c)
```

Fricas [A]
time = 4.40, size = 48, normalized size = 0.53

$$\frac{2\,ax\arctan\left(ax\right)+\left(a^2x^2-1\right)\arctan\left(ax\right)^2+1}{4\left(a^4c^2x^2+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*arctan(a*x) + (a^2*x^2 - 1)*arctan(a*x)^2 + 1)/(a^4*c^2*x^2 + a^
2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\,\mathrm{atan}^2\left(ax\right)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.42, size = 50, normalized size = 0.55

$$\frac{a^2\,x^2\,\mathrm{atan}\left(a\,x\right)^2+2\,a\,x\,\mathrm{atan}\left(a\,x\right)-\mathrm{atan}\left(a\,x\right)^2+1}{4\,a^2\,c^2\,\left(a^2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

```
[Out] (2*a*x*atan(a*x) - atan(a*x)^2 + a^2*x^2*atan(a*x)^2 + 1)/(4*a^2*c^2*(a^2*x
^2 + 1))
```

3.294 $\int \frac{\mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{x}{4c^2\left(1+a^2x^2\right)}-\frac{\text{ArcTan}(ax)}{4ac^2}+\frac{\text{ArcTan}(ax)}{2ac^2\left(1+a^2x^2\right)}+\frac{x\text{ArcTan}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{\text{ArcTan}(ax)^3}{6ac^2}$$

```
[Out] -1/4*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a/c^2+1/2*arctan(a*x)/a/c^2/(a^2*x^2
+1)+1/2*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5012, 5050, 205, 211}

$$\frac{x\text{ArcTan}(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\text{ArcTan}(ax)}{2ac^2\left(a^2x^2+1\right)}-\frac{x}{4c^2\left(a^2x^2+1\right)}+\frac{\text{ArcTan}(ax)^3}{6ac^2}-\frac{\text{ArcTan}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/4*x/(c^2*(1 + a^2*x^2)) - ArcTan[a*x]/(4*a*c^2) + ArcTan[a*x]/(2*a*c^2*(
1 + a^2*x^2)) + (x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*
a*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx &= \frac{x\tan^{-1}(ax)^2}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-a\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx\\ &= \frac{\tan^{-1}(ax)}{2ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^2}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{1}{2}\int \frac{1}{(c+a^2cx^2)^2}\,dx\\ &= -\frac{x}{4c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)}{2ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^2}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6ac^2}-\frac{\int \frac{1}{c+a^2cx^2}\,dx}{4c}\\ &= -\frac{x}{4c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)}{4ac^2}+\frac{\tan^{-1}(ax)}{2ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^2}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{6ac^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.65

$$\frac{-3ax+(3-3a^2x^2)\,\text{ArcTan}(ax)+6ax\text{ArcTan}(ax)^2+2(1+a^2x^2)\,\text{ArcTan}(ax)^3}{12c^2\,(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^
2)*ArcTan[a*x]^3)/(12*c^2*(a + a^3*x^2))
```

Maple [A]
time = 0.22, size = 93, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$
default	$\frac{\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$

risch	$\frac{i\ln(iax+1)^3}{48a\,c^2}-\frac{i(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^2}{16c^2(a^2x^2+1)a}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln(\right.}{16c^2(ax+i)(ax-i)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/2*arctan(a*x)^3/c^2-1/c^2*(1/3
*arctan(a*x)^3-1/2*arctan(a*x)/(a^2*x^2+1)+1/4*a*x/(a^2*x^2+1)+1/4*arctan(a
*x)))
```

Maxima [A]
time = 0.49, size = 146, normalized size = 1.46

$$\frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^2+\frac{(2(a^2x^2+1)\arctan(ax)^3-3\,ax-3(a^2x^2+1)\arctan(ax))a^2}{12(a^5c^2x^2+a^3c^2)}-\frac{((a^2x^2+1)\arctan(ax)^2-1)a\arctan(ax)}{2(a^4c^2x^2+a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^2 + 1/12*(2*(
a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^5*
c^2*x^2 + a^3*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a
^4*c^2*x^2 + a^2*c^2)
```

Fricas [A]
time = 4.01, size = 67, normalized size = 0.67

$$\frac{6\,ax\arctan(ax)^2+2(a^2x^2+1)\arctan(ax)^3-3\,ax-3(a^2x^2-1)\arctan(ax)}{12(a^3c^2x^2+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*
x^2 - 1)*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.52, size = 101, normalized size = 1.01

$$\frac{\mathrm{atan}(a\,x)}{2\,\left(a^3\,c^2\,x^2+a\,c^2\right)}-\frac{x}{2\,\left(2\,a^2\,c^2\,x^2+2\,c^2\right)}+\frac{x\,{\mathrm{atan}(a\,x)}^2}{2\,\left(a^2\,c^2\,x^2+c^2\right)}-\frac{\mathrm{atan}(a\,x)}{4\,a\,c^2}+\frac{{\mathrm{atan}(a\,x)}^3}{6\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2)^2,x)
```

```
[Out] atan(a*x)/(2*(a*c^2 + a^3*c^2*x^2)) - x/(2*(2*c^2 + 2*a^2*c^2*x^2)) + (x*at
an(a*x)^2)/(2*(c^2 + a^2*c^2*x^2)) - atan(a*x)/(4*a*c^2) + atan(a*x)^3/(6*a
*c^2)
```

3.295 $\int \frac{\textbf{ArcTan}(ax)^2}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{1}{4c^2(1+a^2x^2)}-\frac{ax\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)^2}{4c^2}+\frac{\text{ArcTan}(ax)^2}{2c^2(1+a^2x^2)}-\frac{i\text{ArcTan}(ax)^3}{3c^2}+\frac{\text{ArcTan}(ax)^2\log\left(2-\frac{}{1-}\right.}{c^2}$$

```
[Out] -1/4/c^2/(a^2*x^2+1)-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^2/
c^2+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/c^2+arctan(a*x)^2
*ln(2-2/(1-I*a*x))/c^2-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^2+1/2*poly
log(3,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5086, 5044, 4988, 5004, 5112, 6745, 5050, 5012, 267}

$$\frac{\text{ArcTan}(ax)^2}{2c^2(a^2x^2+1)}-\frac{ax\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}-\frac{1}{4c^2(a^2x^2+1)}-\frac{i\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}-\frac{i\text{ArcTan}(ax)^3}{3c^2}-\frac{\text{ArcTan}(ax)^2}{4c^2}+\frac{\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/4*1/(c^2*(1 + a^2*x^2)) - (a*x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) - ArcT
an[a*x]^2/(4*c^2) + ArcTan[a*x]^2/(2*c^2*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x
]^3)/c^2 + (ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*ArcTan[a*x]*Poly
Log[2, -1 + 2/(1 - I*a*x)])/c^2 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}-a\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{i\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c^2}$$

$$= -\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+\frac{\tan^{-1}(ax)^2\log\left(2\right.}{c^2}$$

$$= -\frac{1}{4c^2\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+$$

$$= -\frac{1}{4c^2\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{4c^2}+\frac{\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^3}{3c^2}+$$

Mathematica [A]
time = 0.10, size = 119, normalized size = 0.70

$$\frac{-i\pi^3+8i\mathrm{ArcTan}(ax)^3-3\cos(2\mathrm{ArcTan}(ax))+6\mathrm{ArcTan}(ax)^2\cos(2\mathrm{ArcTan}(ax))+24\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right)+24i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right)+12\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)-6\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax))}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] ((-I)*Pi^3 + (8*I)*ArcTan[a*x]^3 - 3*Cos[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*C
os[2*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (24*
I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)
*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/(24*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.12, size = 1795, normalized size = 10.56

method	result	size
derivativedivides	Expression too large to display	1795
default	Expression too large to display	1795

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/c^
2*arctan(a*x)^2*ln(a*x)-1/c^2*(1/4*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(
```

```
a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/4*arctan(a*x)^2-arctan(a*x)^
2*ln(2)-I*arctan(a*x)*(I+a*x)/(8*a*x-8*I)+I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)
-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/16*(I+a*x)/(a*x-I)-1/16*(a
*x-I)/(I+a*x)+1/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/4*I*arctan(a*x)^2
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/3*I*arctan(a*x)^3-1/2*I*Pi*csgn(((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I
*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(
a*x)^2-1/2*I*Pi*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2-1/4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/4*I*arctan(a*x)^2*Pi*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)^2-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*
x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)
+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arct
an(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^3*arctan(a*x)^2+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(
a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{x\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2), x)
```

3.296 $\int \frac{\text{ArcTan}(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=177

$$\frac{a^2x}{4c^2(1+a^2x^2)}+\frac{a\text{ArcTan}(ax)}{4c^2}-\frac{a\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}-\frac{ia\text{ArcTan}(ax)^2}{c^2}-\frac{\text{ArcTan}(ax)^2}{c^2x}-\frac{a^2x\text{ArcTan}(ax)^2}{2c^2(1+a^2x^2)}-\frac{a\text{ArcTan}}{2c^2}$$

```
[Out] 1/4*a^2*x/c^2/(a^2*x^2+1)+1/4*a*arctan(a*x)/c^2-1/2*a*arctan(a*x)/c^2/(a^2*
x^2+1)-I*a*arctan(a*x)^2/c^2-arctan(a*x)^2/c^2/x-1/2*a^2*x*arctan(a*x)^2/c^
2/(a^2*x^2+1)-1/2*a*arctan(a*x)^3/c^2+2*a*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2
-I*a*polylog(2,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.26, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5086, 5038, 4946, 5044, 4988, 2497, 5004, 5012, 5050, 205, 211}

$$-\frac{a^2x\text{ArcTan}(ax)^2}{2c^2(a^2x^2+1)}-\frac{a\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}+\frac{a^2x}{4c^2(a^2x^2+1)}-\frac{a\text{ArcTan}(ax)^3}{2c^2}-\frac{\text{ArcTan}(ax)^2}{c^2x}-\frac{ia\text{ArcTan}(ax)^2}{c^2}+\frac{a\text{ArcTan}(ax)}{4c^2}+\frac{2a\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{ia\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*x)/(4*c^2*(1 + a^2*x^2)) + (a*ArcTan[a*x])/(4*c^2) - (a*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(c^2*x) - (a
^2*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x]^3)/(2*c^2) + (2*
a*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (I*a*PolyLog[2, -1 + 2/(1 - I*a
*x)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx}{c}\\
&= -\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^3}{6c^2}+a^3\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{c^2}-\frac{a^2\int \frac{\tan}{c}}{}\\
&= -\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^3}{2c^2}+\frac{1}{2}a^2\int \frac{1}{\left(c+a^2cx\right.}\\
&= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)}-\\
&= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}+\frac{a\tan^{-1}(ax)}{4c^2}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x}{2c}\\
&= \frac{a^2x}{4c^2\left(1+a^2x^2\right)}+\frac{a\tan^{-1}(ax)}{4c^2}-\frac{a\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^2}{c^2}-\frac{\tan^{-1}(ax)^2}{c^2x}-\frac{a^2x}{2c}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 109, normalized size = 0.62

$$
-\frac{4ax\text{ArcTan}(ax)^3+2ax\text{ArcTan}(ax)\left(\cos(2\text{ArcTan}(ax))-8\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right)+8iax\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)-ax\sin(2\text{ArcTan}(ax))+2\text{ArcTan}(ax)^2(4+4iax+ax\sin(2\text{ArcTan}(ax)))}{8c^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

[Out] -1/8*(4*a*x*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 8*Log[1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a*x*Sin[2*ArcTan[a*x]] + 2*ArcTan[a*x]^2*(4 + (4*I)*a*x + a*x*Sin[2*ArcTan[a*x]]))/(c^2*x)

Maple [A]
time = 0.21, size = 315, normalized size = 1.78

method	result
derivativedivides	$a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2+2}-2\,\mathrm{ar}}{}\right.$
default	$a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2+2}-2\,\mathrm{ar}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] a*(-1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-3/2*arctan(a*x)^3/c^2-1/c^2*arctan(a*x)^2/a/x-1/c^2*(-arctan(a*x)^3+arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/(a^2*x^2+1)-2*arctan(a*x)*ln(a*x)-1/4*a*x/(a^2*x^2+1)-1/4*arctan(a*x)-I*ln(a*x)*ln(1+I*a*x)-1/2*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-I*dilog(1+I*a*x)-1/4*I*ln(a*x-I)^2+I*dilog(1-I*a*x)-1/2*I*ln(I+a*x)*ln(a^2*x^2+1)+1/2*I*ln(a*x-I)*ln(a^2*x^2+1)+1/4*I*ln(I+a*x)^2-1/2*I*dilog(-1/2*I*(I+a*x))+I*ln(a*x)*ln(1-I*a*x)+1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))+1/2*I*dilog(1/2*I*(a*x-I))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] -1/32*(6*a^3*x^3*arctan2(1, a*x) - 6*a^2*x^2 + 8*(a^3*x^3 + a*x)*arctan(a*x)^3 + 12*a*x*arctan(a*x) + 4*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6*a*x*arctan2(1, a*x) - (3*a^2*x^2 + 2)*log(a^2*x^2 + 1)^2 + 192*(a^6*c^2*x^3 + a^4*c^2*x)*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x) - 128*(a^2*c^2*x^3 + c^2*x)*integrate(1/64*(4*(a^2*x^2 + 1)^(7/2)*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(6*arctan(a*x)) - 24*(a^2*x^2 + 1)^3*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(5*arctan(a*x)) + 52*(a^2*x^2 + 1)^(5/2)*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(4*arctan(a*x)) - 48*(a^2*x^2 + 1)^2*a^2*arct

```
an(a*x)*log(a^2*x^2 + 1)*sin(3*arctan(a*x)) + 16*(a^2*x^2 + 1)^(3/2)*a^2*ar
ctan(a*x)*log(a^2*x^2 + 1)*sin(2*arctan(a*x)) - (4*(a^2*x^2 + 1)^(7/2)*a^2*
arctan(a*x)^2 - (a^2*x^2 + 1)^(7/2)*a^2*log(a^2*x^2 + 1)^2)*cos(6*arctan(a*
x)) + 6*(4*(a^2*x^2 + 1)^3*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^3*a^2*log(a^2*
x^2 + 1)^2)*cos(5*arctan(a*x)) - 13*(4*(a^2*x^2 + 1)^(5/2)*a^2*arctan(a*x)^
2 - (a^2*x^2 + 1)^(5/2)*a^2*log(a^2*x^2 + 1)^2)*cos(4*arctan(a*x)) + 12*(4*
(a^2*x^2 + 1)^2*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*a^2*log(a^2*x^2 + 1)^2)
*cos(3*arctan(a*x)) - 4*(4*(a^2*x^2 + 1)^(3/2)*a^2*arctan(a*x)^2 - (a^2*x^2
 + 1)^(3/2)*a^2*log(a^2*x^2 + 1)^2)*cos(2*arctan(a*x)))*sqrt(a^2*x^2 + 1)/(
(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^6*cos(6*arctan(a*x))^2 + (a^2*c^2*x^2 + c
^2)*(a^2*x^2 + 1)^6*sin(6*arctan(a*x))^2 + 36*(a^2*c^2*x^2 + c^2)*(a^2*x^2
+ 1)^5*cos(5*arctan(a*x))^2 + 36*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^5*sin(5*
arctan(a*x))^2 + 169*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*cos(4*arctan(a*x))
^2 + 169*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*sin(4*arctan(a*x))^2 + 144*(a^
2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^3*cos(3*arctan(a*x))^2 + 144*(a^2*c^2*x^2 +
c^2)*(a^2*x^2 + 1)^3*sin(3*arctan(a*x))^2 - 96*(a^2*c^2*x^2 + c^2)*(a^2*x^2
 + 1)^(5/2)*cos(3*arctan(a*x))*cos(2*arctan(a*x)) - 96*(a^2*c^2*x^2 + c^2)*
(a^2*x^2 + 1)^(5/2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 16*(a^2*c^2*x^2
 + c^2)*(a^2*x^2 + 1)^2*cos(2*arctan(a*x))^2 + 16*(a^2*c^2*x^2 + c^2)*(a^2*
x^2 + 1)^2*sin(2*arctan(a*x))^2 - 2*(6*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(1
1/2)*cos(5*arctan(a*x)) - 13*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^5*cos(4*arct
an(a*x)) + 12*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(9/2)*cos(3*arctan(a*x)) -
4*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*cos(2*arctan(a*x)))*cos(6*arctan(a*x)
) - 12*(13*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(9/2)*cos(4*arctan(a*x)) - 12*
(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*cos(3*arctan(a*x)) + 4*(a^2*c^2*x^2 + c
^2)*(a^2*x^2 + 1)^(7/2)*cos(2*arctan(a*x)))*cos(5*arctan(a*x)) - 104*(3*(a^
2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(7/2)*cos(3*arctan(a*x)) - (a^2*c^2*x^2 + c^
2)*(a^2*x^2 + 1)^3*cos(2*arctan(a*x)))*cos(4*arctan(a*x)) - 2*(6*(a^2*c^2*x
^2 + c^2)*(a^2*x^2 + 1)^(11/2)*sin(5*arctan(a*x)) - 13*(a^2*c^2*x^2 + c^2)*
(a^2*x^2 + 1)^5*sin(4*arctan(a*x)) + 12*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(
9/2)*sin(3*arctan(a*x)) - 4*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*sin(2*arcta
n(a*x)))*sin(6*arctan(a*x)) - 12*(13*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(9/2
)*sin(4*arctan(a*x)) - 12*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^4*sin(3*arctan(
a*x)) + 4*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(7/2)*sin(2*arctan(a*x)))*sin(5
*arctan(a*x)) - 104*(3*(a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^(7/2)*sin(3*arctan
(a*x)) - (a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^3*sin(2*arctan(a*x)))*sin(4*arct
an(a*x))), x) - 192*(a^6*c^2*x^3 + a^4*c^2*x)*integrate(1/64*(4*x^2*arctan(
a*x)^2 + x^2*log(a^2*x^2 + 1)^2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x) -
256*(a^2*c^2*x^3 + c^2*x)*integrate(1/64*(4*arctan(a*x)^2 + log(a^2*x^2 + 1
)^2)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 192*(a^4*c^2*x^3 + a^2*c
^2*x)*integrate(1/64*(4*arctan(a*x)^2 + log(a^2*x^2 + 1)^2)/(a^4*c^2*x^4 +
2*a^2*c^2*x^2 + c^2), x) - 256*(a^3*c^2*x^3 + a*c^2*x)*integrate(1/16*arcta
n(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x) + 128*(a^4*c^2*x^3 + a^2*c
^2*x)*integrate(1/16*log(a^2*x^2 + 1)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2),
x))/(a^2*c^2*x^3 + c^2*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{a^{4}x^{6}+2a^{2}x^{4}+x^{2}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**2,x)
[Out] Integral(atan(a*x)**2/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^2}{x^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2), x)
```

3.297 $\int \frac{\text{ArcTan}(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=250

$$\frac{a^2}{4c^2\left(1+a^2x^2\right)}-\frac{a\text{ArcTan}(ax)}{c^2x}+\frac{a^3x\text{ArcTan}(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{a^2\text{ArcTan}(ax)^2}{4c^2}-\frac{\text{ArcTan}(ax)^2}{2c^2x^2}-\frac{a^2\text{ArcTan}(ax)^2}{2c^2\left(1+a^2x^2\right)}+\frac{2ia^2\text{Arc}}{3}$$

```
[Out] 1/4*a^2/c^2/(a^2*x^2+1)-a*arctan(a*x)/c^2/x+1/2*a^3*x*arctan(a*x)/c^2/(a^2*
x^2+1)-1/4*a^2*arctan(a*x)^2/c^2-1/2*arctan(a*x)^2/c^2/x^2-1/2*a^2*arctan(a
*x)^2/c^2/(a^2*x^2+1)+2/3*I*a^2*arctan(a*x)^3/c^2+a^2*ln(x)/c^2-1/2*a^2*ln(
a^2*x^2+1)/c^2-2*a^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+2*I*a^2*arctan(a*x
)*polylog(2,-1+2/(1-I*a*x))/c^2-a^2*polylog(3,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.55, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745, 5050, 5012, 267}

$$\frac{2ia^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}-\frac{a^2\text{ArcTan}(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{2ia^2\text{ArcTan}(ax)^3}{3c^2}-\frac{a^2\text{ArcTan}(ax)^2}{4c^2}-\frac{2a^2\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{a^2\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{c^2}+\frac{a^2}{4c^2\left(a^2x^2+1\right)}-\frac{a^2\log\left(a^2x^2+1\right)}{2c^2}+\frac{a^2\log(x)}{c^2}+\frac{a^3x\text{ArcTan}(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\text{ArcTan}(ax)^2}{2c^2x^2}-\frac{a\text{ArcTan}(ax)}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] a^2/(4*c^2*(1 + a^2*x^2)) - (a*ArcTan[a*x])/(c^2*x) + (a^3*x*ArcTan[a*x])/(
2*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x]^2)/(4*c^2) - ArcTan[a*x]^2/(2*c^2*x
^2) - (a^2*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (((2*I)/3)*a^2*ArcTan[a*x
]^3)/c^2 + (a^2*Log[x])/c^2 - (a^2*Log[1 + a^2*x^2])/(2*c^2) - (2*a^2*ArcTa
n[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((2*I)*a^2*ArcTan[a*x]*PolyLog[2, -1
 + 2/(1 - I*a*x)])/c^2 - (a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3 (c+a^2cx^2)^2} \, dx = -\left(a^2 \int \frac{\tan^{-1}(ax)^2}{x (c+a^2cx^2)^2} \, dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3(c+a^2cx^2)} \, dx}{c}$$

$$= a^4 \int \frac{x \tan^{-1}(ax)^2}{(c+a^2cx^2)^2} \, dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^3} \, dx}{c^2} - 2\frac{a^2 \int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)} \, dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a^2 \tan^{-1}(ax)^2}{2c^2 (1+a^2x^2)} + a^3 \int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2} \, dx + \frac{a \int \frac{\tan^{-1}(ax)}{x^2(1+a^2x^2)} \, dx}{c^2} - 2\Bigg(-$$

$$= \frac{a^3 x \tan^{-1}(ax)}{2c^2 (1+a^2x^2)} + \frac{a^2 \tan^{-1}(ax)^2}{4c^2} - \frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a^2 \tan^{-1}(ax)^2}{2c^2 (1+a^2x^2)} - \frac{1}{2}a^4 \int \frac{x}{(c+a^2cx}$$

$$= \frac{a^2}{4c^2 (1+a^2x^2)} - \frac{a \tan^{-1}(ax)}{c^2x} + \frac{a^3 x \tan^{-1}(ax)}{2c^2 (1+a^2x^2)} - \frac{a^2 \tan^{-1}(ax)^2}{4c^2} - \frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a}{2}$$

$$= \frac{a^2}{4c^2 (1+a^2x^2)} - \frac{a \tan^{-1}(ax)}{c^2x} + \frac{a^3 x \tan^{-1}(ax)}{2c^2 (1+a^2x^2)} - \frac{a^2 \tan^{-1}(ax)^2}{4c^2} - \frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a}{2}$$

$$= \frac{a^2}{4c^2 (1+a^2x^2)} - \frac{a \tan^{-1}(ax)}{c^2x} + \frac{a^3 x \tan^{-1}(ax)}{2c^2 (1+a^2x^2)} - \frac{a^2 \tan^{-1}(ax)^2}{4c^2} - \frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a}{2}$$

$$= \frac{a^2}{4c^2 (1+a^2x^2)} - \frac{a \tan^{-1}(ax)}{c^2x} + \frac{a^3 x \tan^{-1}(ax)}{2c^2 (1+a^2x^2)} - \frac{a^2 \tan^{-1}(ax)^2}{4c^2} - \frac{\tan^{-1}(ax)^2}{2c^2x^2} - \frac{a}{2}$$

Mathematica [A]
time = 0.40, size = 183, normalized size = 0.73

$$\frac{a^2\left(\frac{i\pi^3}{12} - \frac{\text{ArcTan}(ax)}{ax} - \frac{(1+a^2x^2)\text{ArcTan}(ax)^2}{2a^2x^2} - \frac{2}{3}i\text{ArcTan}(ax)^3 + \frac{1}{8}\cos(2\text{ArcTan}(ax)) - \frac{1}{4}\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax)) - 2\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + \log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) - 2i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) - \text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) + \frac{1}{4}\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax))\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*((I/12)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - ((2*I)/3)*ArcTan[a*x]^3 + Cos[2*ArcTan[a*x]]/8 - (ArcTan[a*x]^2*Co
s[2*ArcTan[a*x]])/4 - 2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log
[(a*x)/Sqrt[1 + a^2*x^2]] - (2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a
*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])] + (ArcTan[a*x]*Sin[2*ArcTan[a*x]
])/4))/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.32, size = 4110, normalized size = 16.44

method	result	size
derivativedivides	Expression too large to display	4110
default	Expression too large to display	4110

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] a^2*(1/c^2*arctan(a*x)^2*ln(a^2*x^2+1)-1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/2/c^2*arctan(a*x)^2/a^2/x^2-2/c^2*arctan(a*x)^2*ln(a*x)-1/c^2*(2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/48/a/x/(I+a*x)*(-24*arctan(a*x)*a^2*x^2-48*I*Pi*arctan(a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x-24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*a*x-48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*a*x+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*a*x-48*I*Pi*arctan(a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a^3*x^3-24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*a^3*x^3-48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a^3*x^3+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*a^3*x^3+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*a^3*x^3+48*I*Pi*arctan(a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a*x+48*I*Pi*arctan(a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a^3*x^3-3*a^3*x^3+32*I*arctan(a*x)^3*a*x+32*I*arctan(a*x)^3*a^3*x^3-48*I*arctan(a*x)*a*x-48*I*arctan(a*x)*a^3*x^3-96*ln(2)*arctan(a*x)^2*a*x-96*ln(2)*arctan(a*x)^2*a^3*x^3+9*a*x-48*I*Pi*arctan(a*x)^2*a*x-48*I*Pi*arctan(a*x)^2*a^3*x^3-12*arctan(a*x)^2*a*x-12*arctan(a*x)^2*a^3*x^3-48*arctan(a*x)-48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*a*x+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a*x-48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*a^3*x^3+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)

```
^2/(a^2*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a^3*x^3-48*I*Pi*
arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))^2*a*x-24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*a*x-24*I*Pi*arctan(
a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a*x-24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x
)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1)+I)^2*a*x+48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I
*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)-I)*a*x+48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1
+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1))*a*x+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*a*x-48*I*Pi*arctan(a*x)^2*csgn(1/((1+I
*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^
2/(a^2*x^2+1)+1))*a^3*x^3+48*I*Pi*arctan(a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^
2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1
+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)
+1))*a*x+48*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*
x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*a*x+48*I*Pi*arctan(
a*x)^2*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2
*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*a^3*x^3+48*I*Pi*arctan(a*x)^2*csgn(I
*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)+I)*a^3*x^3+48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)-I)*a^3*x^3+48*I*Pi*arctan(a*x)^2*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*a^3*x^3+24*I*Pi*arctan(a*x)^2*csgn(I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*a^3*x^3-48*I*Pi*a
rctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{a^{4}x^{7}+2a^{2}x^{5}+x^{3}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^3\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2), x)
```

3.298 $\int \frac{\textbf{ArcTan}(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2}{3c^2x}-\frac{a^4x}{4c^2(1+a^2x^2)}-\frac{7a^3\text{ArcTan}(ax)}{12c^2}-\frac{a\text{ArcTan}(ax)}{3c^2x^2}+\frac{a^3\text{ArcTan}(ax)}{2c^2(1+a^2x^2)}+\frac{7ia^3\text{ArcTan}(ax)^2}{3c^2}-\frac{\text{ArcTan}(ax)^2}{3c^2x^3}$$

```
[Out] -1/3*a^2/c^2/x-1/4*a^4*x/c^2/(a^2*x^2+1)-7/12*a^3*arctan(a*x)/c^2-1/3*a*arc
tan(a*x)/c^2/x^2+1/2*a^3*arctan(a*x)/c^2/(a^2*x^2+1)+7/3*I*a^3*arctan(a*x)^
2/c^2-1/3*arctan(a*x)^2/c^2/x^3+2*a^2*arctan(a*x)^2/c^2/x+1/2*a^4*x*arctan(
a*x)^2/c^2/(a^2*x^2+1)+5/6*a^3*arctan(a*x)^3/c^2-14/3*a^3*arctan(a*x)*ln(2-
2/(1-I*a*x))/c^2+7/3*I*a^3*polylog(2,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.63, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5086, 5038, 4946, 331, 209, 5044, 4988, 2497, 5004, 5012, 5050, 205, 211}

$$\frac{5a^3\text{ArcTan}(ax)^3}{6c^2}+\frac{7ia^3\text{ArcTan}(ax)^2}{3c^2}-\frac{7a^3\text{ArcTan}(ax)}{12c^2}-\frac{14a^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^2}+\frac{7ia^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{3c^2}+\frac{2a^2\text{ArcTan}(ax)^2}{c^2x}-\frac{a^2}{3c^2x}+\frac{a^4x\text{ArcTan}(ax)^2}{2c^2(a^2x^2+1)}-\frac{a^4x}{4c^2(a^2x^2+1)}+\frac{a^3\text{ArcTan}(ax)}{2c^2(a^2x^2+1)}-\frac{\text{ArcTan}(ax)^2}{3c^2x^3}-\frac{a\text{ArcTan}(ax)}{3c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/3*a^2/(c^2*x) - (a^4*x)/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x])/(12*
c^2) - (a*ArcTan[a*x])/(3*c^2*x^2) + (a^3*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)
) + (((7*I)/3)*a^3*ArcTan[a*x]^2)/c^2 - ArcTan[a*x]^2/(3*c^2*x^3) + (2*a^2*
ArcTan[a*x]^2)/(c^2*x) + (a^4*x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + (5*a
^3*ArcTan[a*x]^3)/(6*c^2) - (14*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*
c^2) + (((7*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4(c+a^2cx^2)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^2}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^3}{6c^2} - a^5\int \frac{x\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^2}\,dx + \frac{(2a)\int \frac{\text{ta}}{x^3}}{3}$$

$$= \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^2}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^2}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^3}{6c^2} - \frac{1}{2}a^4\int \frac{1}{(c+a^2cx^2}$$

$$= -\frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}(ax)^2}{3c^2} - \frac{\tan^{-1}(ax)^2}{3c^2x^3} +$$

$$= -\frac{a^2}{3c^2x} - \frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{a^3\tan^{-1}(ax)}{4c^2} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}}{3c^2}$$

$$= -\frac{a^2}{3c^2x} - \frac{a^4x}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)}{12c^2} - \frac{a\tan^{-1}(ax)}{3c^2x^2} + \frac{a^3\tan^{-1}(ax)}{2c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}}{3c}$$

Mathematica [A]
time = 0.29, size = 166, normalized size = 0.69

$$\frac{20a^3x^3\text{ArcTan}(ax)^3 + 2ax\text{ArcTan}(ax)\left(-4-4a^2x^2+3a^2x^2\cos(2\text{ArcTan}(ax)) - 56a^2x^2\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right) + 56ia^3x^3\text{PolyLog}\left(2, e^{2i\text{ArcTan}(ax)}\right) - a^2x^2(8+3ax\sin(2\text{ArcTan}(ax))) + \text{ArcTan}(ax)^2\left(-8+48a^2x^2+56ia^3x^3+6a^3x^3\sin(2\text{ArcTan}(ax))\right)}{24c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (20*a^3*x^3*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(-4 - 4*a^2*x^2 + 3*a^2*x^2*C
os[2*ArcTan[a*x]] - 56*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) + (56*I)*a^3
*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a^2*x^2*(8 + 3*a*x*Sin[2*ArcTan[a*
x]]) + ArcTan[a*x]^2*(-8 + 48*a^2*x^2 + (56*I)*a^3*x^3 + 6*a^3*x^3*Sin[2*Ar
cTan[a*x]]))/(24*c^2*x^3)
```

Maple [A]
time = 0.27, size = 353, normalized size = 1.46

method	result

derivativedivides	$a^3\left(\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}-\frac{-7\arctan(ax)\ln(a^2x^2+1)-\frac{3\arctan(ax)}{2(a^2x^2+1)}}{}\right.$
default	$a^3\left(\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}-\frac{-7\arctan(ax)\ln(a^2x^2+1)-\frac{3\arctan(ax)}{2(a^2x^2+1)}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^3/c^2-1/3/c^2*ar
ctan(a*x)^2/a^3/x^3+2/c^2*arctan(a*x)^2/a/x-1/3/c^2*(-7*arctan(a*x)*ln(a^2*
x^2+1)-3/2*arctan(a*x)/(a^2*x^2+1)+arctan(a*x)/a^2/x^2+14*arctan(a*x)*ln(a*
x)-7*I*ln(a*x)*ln(1-I*a*x)+7/2*I*ln(I+a*x)*ln(a^2*x^2+1)+7*I*dilog(1+I*a*x)
-7*I*dilog(1-I*a*x)+7*I*ln(a*x)*ln(1+I*a*x)+7/4*I*ln(a*x-I)^2+7/2*I*ln(a*x-
I)*ln(-1/2*I*(I+a*x))-7/2*I*ln(a*x-I)*ln(a^2*x^2+1)+7/2*I*dilog(-1/2*I*(I+a
*x))-7/2*I*dilog(1/2*I*(a*x-I))-7/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-7/4*I*ln(
I+a*x)^2+3/4*a*x/(a^2*x^2+1)+7/4*arctan(a*x)+1/a/x+5*arctan(a*x)^3))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^8+2a^2x^6+x^4}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**2/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2}{x^4\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2), x)
```

3.299 $\int \frac{x^3 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=140

$$-\frac{x^4}{32c^3(1+a^2x^2)^2}+\frac{3}{32a^4c^3(1+a^2x^2)}+\frac{x^3\text{ArcTan}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\text{ArcTan}(ax)}{16a^3c^3(1+a^2x^2)}-\frac{3\text{ArcTan}(ax)^2}{32a^4c^3}+\frac{x^4\text{ArcTan}(a}{4c^3(1+a^2x}$$

```
[Out] -1/32*x^4/c^3/(a^2*x^2+1)^2+3/32/a^4/c^3/(a^2*x^2+1)+1/8*x^3*arctan(a*x)/a/
c^3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^2
/a^4/c^3+1/4*x^4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2
```

Rubi [A]

time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5064, 5058, 5054, 5004}

$$-\frac{3\text{ArcTan}(ax)^2}{32a^4c^3}+\frac{x^4\text{ArcTan}(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{x^3\text{ArcTan}(ax)}{8ac^3(a^2x^2+1)^2}-\frac{x^4}{32c^3(a^2x^2+1)^2}+\frac{3}{32a^4c^3(a^2x^2+1)}+\frac{3x\text{ArcTan}(ax)}{16a^3c^3(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/32*x^4/(c^3*(1 + a^2*x^2)^2) + 3/(32*a^4*c^3*(1 + a^2*x^2)) + (x^3*ArcTa
n[a*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a^3*c^3*(1 + a^2*
x^2)) - (3*ArcTan[a*x]^2)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^2)/(4*c^3*(1 + a^
2*x^2)^2)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5054

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dis
t[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]
+ Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
```

```
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx &= \frac{x^4\tan^{-1}(ax)^2}{4c^3(1+a^2x^2)^2}-\frac{1}{2}a\int \frac{x^4\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx\\ &= -\frac{x^4}{32c^3(1+a^2x^2)^2}+\frac{x^3\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{x^4\tan^{-1}(ax)^2}{4c^3(1+a^2x^2)^2}-\frac{3\int \frac{x^2\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{8ac}\\ &= -\frac{x^4}{32c^3(1+a^2x^2)^2}+\frac{3}{32a^4c^3(1+a^2x^2)}+\frac{x^3\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3(1+a^2x^2)}+\frac{x^4}{4c^3}\\ &= -\frac{x^4}{32c^3(1+a^2x^2)^2}+\frac{3}{32a^4c^3(1+a^2x^2)}+\frac{x^3\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\tan^{-1}(ax)}{16a^3c^3(1+a^2x^2)}-\frac{3\,\mathrm{ta}}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.53

$$\frac{4+5a^2x^2+2ax(3+5a^2x^2)\operatorname{ArcTan}(ax)+(-3-6a^2x^2+5a^4x^4)\operatorname{ArcTan}(ax)^2}{32a^4c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
[Out] (4 + 5*a^2*x^2 + 2*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x] + (-3 - 6*a^2*x^2 + 5*a^
4*x^4)*ArcTan[a*x]^2)/(32*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.28, size = 132, normalized size = 0.94

method	result

derivativedivides	$\frac{-\frac{\arctan(ax)^2}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{3ax\arctan(ax)}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^4}$
default	$\frac{-\frac{\arctan(ax)^2}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{3ax\arctan(ax)}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^4}$
risch	$-\frac{(5a^4x^4-6a^2x^2-3)\ln(iax+1)^2}{128a^4c^3(a^2x^2+1)^2}+\frac{(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(iax}{64a^4(ax+i)^2(ax-i)^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)
^2-1/2/c^3*(-5/8*arctan(a*x)*a^3*x^3/(a^2*x^2+1)^2-3/8*a*x/(a^2*x^2+1)^2*ar
ctan(a*x)-5/16*arctan(a*x)^2-5/16/(a^2*x^2+1)+1/16/(a^2*x^2+1)^2))
```

Maxima [A]
time = 0.48, size = 185, normalized size = 1.32

$$\frac{1}{16}a\left(\frac{5a^2x^3+3x}{a^8c^3x^4+2a^6c^3x^2+a^4c^3}+\frac{5\arctan(ax)}{a^5c^3}\right)\arctan(ax)+\frac{(5a^2x^2-5(a^4x^4+2a^2x^2+1)\arctan(ax)^2+4)a^2}{32(a^{10}c^3x^4+2a^8c^3x^2+a^6c^3)}-\frac{(2a^2x^2+1)\arctan(ax)^2}{4(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3))*arctan(a*x) + 1/32*(5*a^2*x^2 - 5*(a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2 + 4)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) - 1/4*(2
*a^2*x^2 + 1)*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A]
time = 3.96, size = 87, normalized size = 0.62

$$\frac{5a^2x^2+(5a^4x^4-6a^2x^2-3)\arctan(ax)^2+2(5a^3x^3+3ax)\arctan(ax)+4}{32(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*a^2*x^2 + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^2 + 2*(5*a^3*x^3
+ 3*a*x)*arctan(a*x) + 4)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3\operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.57, size = 85, normalized size = 0.61

$$\frac{5\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+10\,a^3\,x^3\,\mathrm{atan}(a\,x)-6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+5\,a^2\,x^2+6\,a\,x\,\mathrm{atan}(a\,x)-3\,\mathrm{atan}(a\,x)^2+4}{32\,a^4\,c^3\,\left(a^2\,x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (5*a^2*x^2 - 3*atan(a*x)^2 + 10*a^3*x^3*atan(a*x) + 6*a*x*atan(a*x) - 6*a^2
*x^2*atan(a*x)^2 + 5*a^4*x^4*atan(a*x)^2 + 4)/(32*a^4*c^3*(a^2*x^2 + 1)^2)
```

3.300 $\int \frac{x^2 \textbf{ArcTan}(ax)^2}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=181

$$\frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{x}{64a^2c^3(1+a^2x^2)} - \frac{\text{ArcTan}(ax)}{64a^3c^3} - \frac{\text{ArcTan}(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\text{ArcTan}(ax)}{8a^3c^3(1+a^2x^2)} - \frac{x\text{ArcTan}(ax)}{4a^2c^3(1+a^2x}$$

```
[Out] 1/32*x/a^2/c^3/(a^2*x^2+1)^2-1/64*x/a^2/c^3/(a^2*x^2+1)-1/64*arctan(a*x)/a^
3/c^3-1/8*arctan(a*x)/a^3/c^3/(a^2*x^2+1)^2+1/8*arctan(a*x)/a^3/c^3/(a^2*x^
2+1)-1/4*x*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)^2/a^2/c^3/
(a^2*x^2+1)+1/24*arctan(a*x)^3/a^3/c^3
```

Rubi [A]
time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5084, 5012, 5050, 205, 211, 5020}

$$\frac{\text{ArcTan}(ax)^3}{24a^3c^3} - \frac{\text{ArcTan}(ax)}{64a^3c^3} + \frac{x\text{ArcTan}(ax)^2}{8a^2c^3(a^2x^2+1)} - \frac{x\text{ArcTan}(ax)^2}{4a^2c^3(a^2x^2+1)^2} - \frac{x}{64a^2c^3(a^2x^2+1)} + \frac{x}{32a^2c^3(a^2x^2+1)^2} + \frac{\text{ArcTan}(ax)}{8a^3c^3(a^2x^2+1)} - \frac{\text{ArcTan}(ax)}{8a^3c^3(a^2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] x/(32*a^2*c^3*(1 + a^2*x^2)^2) - x/(64*a^2*c^3*(1 + a^2*x^2)) - ArcTan[a*x]
/(64*a^3*c^3) - ArcTan[a*x]/(8*a^3*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]/(8*a^
3*c^3*(1 + a^2*x^2)) - (x*ArcTan[a*x]^2)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*A
rcTan[a*x]^2)/(8*a^2*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(24*a^3*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
```

```
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{a^2c}$$

$$= -\frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)^2} - \frac{x\tan^{-1}(ax)^2}{4a^2c^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^2}{2a^2c^3(1+a^2x^2)} + \frac{\tan^{-1}(ax)^3}{6a^3c^3} + \frac{\int \frac{1}{(c+a^2c}}{8c}$$

$$= \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\tan^{-1}(ax)}{2a^3c^3(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^2}{4a^2c^3(1+a^2x^2)^2} + \frac{}{8c}$$

$$= \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{13x}{64a^2c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)} - \frac{}{4c}$$

$$= \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{x}{64a^2c^3(1+a^2x^2)} - \frac{13\tan^{-1}(ax)}{64a^3c^3} - \frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\mathrm{ta}}{8a^3c^3}$$

$$= \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{x}{64a^2c^3(1+a^2x^2)} - \frac{\tan^{-1}(ax)}{64a^3c^3} - \frac{\tan^{-1}(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\tan^{-}}{8a^3c^3(1}$$

Mathematica [A]
time = 0.07, size = 95, normalized size = 0.52

$$\frac{3ax - 3a^3x^3 - 3(1-6a^2x^2+a^4x^4)\operatorname{ArcTan}(ax) + 24ax(-1+a^2x^2)\operatorname{ArcTan}(ax)^2 + 8(1+a^2x^2)^2\operatorname{ArcTan}(ax)^3}{192a^3c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*a*x - 3*a^3*x^3 - 3*(1 - 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] + 24*a*x*(-1 +
 a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(192*a^3*c^3*(1
+ a^2*x^2)^2)
```

Maple [A]
time = 0.31, size = 149, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2 a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^3}{8c^3} - \frac{\frac{\arctan(ax)^3}{3} - \frac{\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2(a^2x^2+1)^2} + \frac{\frac{1}{8}a^3x^3 - \frac{1}{8}ax}{2(a^2x^2+1)^2} + \frac{\arctan(ax)}{16}}{4c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)^2 a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^3}{8c^3} - \frac{\frac{\arctan(ax)^3}{3} - \frac{\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2(a^2x^2+1)^2} + \frac{\frac{1}{8}a^3x^3 - \frac{1}{8}ax}{2(a^2x^2+1)^2} + \frac{\arctan(ax)}{16}}{4c^3}}{a^3}$

risch	$\frac{i\ln(iax+1)^3}{192c^3a^3} - \frac{i(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^2}{64a^3c^3(a^2x^2+1)^2} + \frac{i\left(a^4x^4\ln(-iax+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/c^3*arctan(a*x)^2*a^3*x^3/(a^2*x^2+1)^2-1/8*a*x*arctan(a*x)^2/c^
3/(a^2*x^2+1)^2+1/8*arctan(a*x)^3/c^3-1/4/c^3*(1/3*arctan(a*x)^3-1/2*arctan
(a*x)/(a^2*x^2+1)+1/2/(a^2*x^2+1)^2*arctan(a*x)+1/2*(1/8*a^3*x^3-1/8*a*x)/(
a^2*x^2+1)^2+1/16*arctan(a*x)))
```

Maxima [A]
time = 0.50, size = 232, normalized size = 1.28

$$\frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan\left(ax\right)}{a^3c^3}\right)\arctan\left(ax\right)^2-\frac{\left(3\,a^3x^3-8\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3-3\,ax+3\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)\right)a^2}{192\,\left(a^9c^3x^4+2\,a^7c^3x^2+a^5c^3\right)}+\frac{\left(a^2x^2-\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2\right)a\arctan\left(ax\right)}{8\,\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x)^2 - 1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arc
tan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^9*c^3*
x^4 + 2*a^7*c^3*x^2 + a^5*c^3) + 1/8*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^2)*a*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Fricas [A]
time = 2.96, size = 114, normalized size = 0.63

$$-\frac{3\,a^3x^3-8\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3-24\,\left(a^3x^3-ax\right)\arctan\left(ax\right)^2-3\,ax+3\,\left(a^4x^4-6\,a^2x^2+1\right)\arctan\left(ax\right)}{192\,\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 24*(a^3*x^3
 - a*x)*arctan(a*x)^2 - 3*a*x + 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x))/(a
^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\operatorname{atan}^2\left(ax\right)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.49, size = 150, normalized size = 0.83

$$\frac{\frac{x}{8\,a^2}-\frac{x^3}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3}-\frac{\mathrm{atan}(a\,x)^2\,\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{\mathrm{atan}(a\,x)}{64\,a^3\,c^3}+\frac{\mathrm{atan}(a\,x)^3}{24\,a^3\,c^3}+\frac{x^2\,\mathrm{atan}(a\,x)}{8\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (x/(8*a^2) - x^3/8)/(8*c^3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)^2
*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4) - atan(a*x)/(
64*a^3*c^3) + atan(a*x)^3/(24*a^3*c^3) + (x^2*atan(a*x))/(8*a^3*c^3*(1/a^2
+ 2*x^2 + a^2*x^4))
```

3.301 $\int \frac{x\mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=138

$$\frac{1}{32a^2c^3(1+a^2x^2)^2}+\frac{3}{32a^2c^3(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\text{ArcTan}(ax)}{16ac^3(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)^2}{32a^2c^3}-\frac{\text{ArcTan}(ax)^2}{4a^2c^3(1+a^2x^2)^2}$$

```
[Out] 1/32/a^2/c^3/(a^2*x^2+1)^2+3/32/a^2/c^3/(a^2*x^2+1)+1/8*x*arctan(a*x)/a/c^3
/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^2/a^2/
c^3-1/4*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 5016, 5012, 267}

$$-\frac{\text{ArcTan}(ax)^2}{4a^2c^3(a^2x^2+1)^2}+\frac{3x\text{ArcTan}(ax)}{16ac^3(a^2x^2+1)}+\frac{x\text{ArcTan}(ax)}{8ac^3(a^2x^2+1)^2}+\frac{3\text{ArcTan}(ax)^2}{32a^2c^3}+\frac{3}{32a^2c^3(a^2x^2+1)}+\frac{1}{32a^2c^3(a^2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] 1/(32*a^2*c^3*(1 + a^2*x^2)^2) + 3/(32*a^2*c^3*(1 + a^2*x^2)) + (x*ArcTan[a
*x])/(8*a*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x])/(16*a*c^3*(1 + a^2*x^2))
 + (3*ArcTan[a*x]^2)/(32*a^2*c^3) - ArcTan[a*x]^2/(4*a^2*c^3*(1 + a^2*x^2)^
2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
```

```
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{\tan^{-1}(ax)^2}{4a^2c^3(1+a^2x^2)^2}+\frac{\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx}{2a}$$
$$= \frac{1}{32a^2c^3(1+a^2x^2)^2}+\frac{x\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}-\frac{\tan^{-1}(ax)^2}{4a^2c^3(1+a^2x^2)^2}+\frac{3\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{8ac}$$
$$= \frac{1}{32a^2c^3(1+a^2x^2)^2}+\frac{x\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3(1+a^2x^2)}+\frac{3\tan^{-1}(ax)^2}{32a^2c^3}-\frac{\tan^{-}}{4a^2c^3(1}$$
$$= \frac{1}{32a^2c^3(1+a^2x^2)^2}+\frac{3}{32a^2c^3(1+a^2x^2)}+\frac{x\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3x\tan^{-1}(ax)}{16ac^3(1+a^2x^2)}+\frac{3\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.51

$$\frac{4+3a^2x^2+2ax(5+3a^2x^2)\,\mathrm{ArcTan}(ax)+(-5+6a^2x^2+3a^4x^4)\,\mathrm{ArcTan}(ax)^2}{32c^3(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + (-5 + 6*a^2*x^2 + 3*a^
4*x^4)*ArcTan[a*x]^2)/(32*c^3*(a + a^3*x^2)^2)
```

Maple [A]
time = 0.25, size = 106, normalized size = 0.77

method	result

derivativedivides	$\frac{-\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8(a^2x^2+1)}+\frac{3\arctan(ax)^2}{16}+\frac{3}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8(a^2x^2+1)}+\frac{3\arctan(ax)^2}{16}+\frac{3}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^2}$
risch	$-\frac{(3a^4x^4+6a^2x^2-5)\ln(iax+1)^2}{128a^2c^3(a^2x^2+1)^2}+\frac{(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(iax+}{64(ax+i)^2c^3(ax-i)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/2/c^3*(1/4*a*x/(a^2*x^2+1)^2*
arctan(a*x)+3/8*a*x/(a^2*x^2+1)*arctan(a*x)+3/16*arctan(a*x)^2+3/16/(a^2*x^
2+1)+1/16/(a^2*x^2+1)^2))
```

Maxima [A]
time = 0.48, size = 163, normalized size = 1.18

$$\frac{\left(\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\arctan(ax)}{ac^2}\right)\arctan\left(ax\right)}{16\,ac}+\frac{3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4}{32\left(a^6c^2x^4+2\,a^4c^2x^2+a^2c^2\right)c}-\frac{\arctan\left(ax\right)^2}{4\left(a^2cx^2+c\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))*arctan(a*x)/(a*c) + 1/32*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)
*arctan(a*x)^2 + 4)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*c) - 1/4*arcta
n(a*x)^2/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A]
time = 3.86, size = 87, normalized size = 0.63

$$\frac{3\,a^2x^2+\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)^2+2\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)+4}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*a^2*x^2 + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^2 + 2*(3*a^3*x^3
+ 5*a*x)*arctan(a*x) + 4)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{x\,\mathrm{atan}^2\left(ax\right)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.51, size = 85, normalized size = 0.62

$$\frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)-5\,\mathrm{atan}(a\,x)^2+4}{32\,a^2\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (3*a^2*x^2 - 5*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^2
*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(32*a^2*c^3*(a^2*x^2 + 1)^2)
```

3.302 $\int \frac{\text{ArcTan}(ax)^2}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=169

$$-\frac{x}{32c^3(1+a^2x^2)^2}-\frac{15x}{64c^3(1+a^2x^2)}-\frac{15\text{ArcTan}(ax)}{64ac^3}+\frac{\text{ArcTan}(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3\text{ArcTan}(ax)}{8ac^3(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)^2}{4c^3(1+a^2x^2)^2}+\frac{3}{-}$$

```
[Out] -1/32*x/c^3/(a^2*x^2+1)^2-15/64*x/c^3/(a^2*x^2+1)-15/64*arctan(a*x)/a/c^3+1
/8*arctan(a*x)/a/c^3/(a^2*x^2+1)^2+3/8*arctan(a*x)/a/c^3/(a^2*x^2+1)+1/4*x*
arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/8*arc
tan(a*x)^3/a/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5020, 5012, 5050, 205, 211}

$$\frac{3x\text{ArcTan}(ax)^2}{8c^3(a^2x^2+1)}+\frac{x\text{ArcTan}(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3\text{ArcTan}(ax)}{8ac^3(a^2x^2+1)}+\frac{\text{ArcTan}(ax)}{8ac^3(a^2x^2+1)^2}-\frac{15x}{64c^3(a^2x^2+1)}-\frac{x}{32c^3(a^2x^2+1)^2}+\frac{\text{ArcTan}(ax)^3}{8ac^3}-\frac{15\text{ArcTan}(ax)}{64ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/32*x/(c^3*(1 + a^2*x^2)^2) - (15*x)/(64*c^3*(1 + a^2*x^2)) - (15*ArcTan[
a*x])/(64*a*c^3) + ArcTan[a*x]/(8*a*c^3*(1 + a^2*x^2)^2) + (3*ArcTan[a*x])/
(8*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*
ArcTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + ArcTan[a*x]^3/(8*a*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
```

```
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^2}{4c^3(1+a^2x^2)^2} - \frac{1}{8}\int \frac{1}{(c+a^2cx^2)^3}\,dx + \frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= -\frac{x}{32c^3(1+a^2x^2)^2} + \frac{\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^2}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^2}{8c^3(1+a^2x^2)} + \frac{\tan^{-1}(ax}{8ac^3}$$

$$= -\frac{x}{32c^3(1+a^2x^2)^2} - \frac{3x}{64c^3(1+a^2x^2)} + \frac{\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)}{8ac^3(1+a^2x^2)} + \frac{x\tan^-}{4c^3(1+}$$

$$= -\frac{x}{32c^3(1+a^2x^2)^2} - \frac{15x}{64c^3(1+a^2x^2)} - \frac{3\tan^{-1}(ax)}{64ac^3} + \frac{\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(a}{8ac^3(1+a}$$

$$= -\frac{x}{32c^3(1+a^2x^2)^2} - \frac{15x}{64c^3(1+a^2x^2)} - \frac{15\tan^{-1}(ax)}{64ac^3} + \frac{\tan^{-1}(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(}{8ac^3(1+}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 0.58

$$\frac{-ax(17+15a^2x^2)+(17-6a^2x^2-15a^4x^4)\,\mathrm{ArcTan}(ax)+8ax(5+3a^2x^2)\,\mathrm{ArcTan}(ax)^2+8(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^3}{64ac^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-(a*x*(17 + 15*a^2*x^2)) + (17 - 6*a^2*x^2 - 15*a^4*x^4)*ArcTan[a*x] + 8*a
*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(64*a*c
^3*(1 + a^2*x^2)^2)
```

Maple [A]
time = 0.29, size = 143, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{ax\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)^2}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^3}{8c^3}-\frac{-\frac{3\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{15}{8}a^3x^3+\frac{17}{8}ax}{2(a^2x^2+1)^2}+\frac{15\arctan(ax)}{16}+\arctan(ax)^3}{4c^3}}{a}$
default	$\frac{\frac{ax\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)^2}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^3}{8c^3}-\frac{-\frac{3\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{15}{8}a^3x^3+\frac{17}{8}ax}{2(a^2x^2+1)^2}+\frac{15\arctan(ax)}{16}+\arctan(ax)^3}{4c^3}}{a}$
risch	$\frac{i\ln(iax+1)^3}{64a\,c^3}-\frac{i(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)^2}{64c^3(a^2x^2+1)^2a}+\frac{i\left(3a^4x^4\ln(-i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/4*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/8*a*x*arctan(a*x)^2/c^3/(a^2
*x^2+1)+3/8*arctan(a*x)^3/c^3-1/4/c^3*(-3/2*arctan(a*x)/(a^2*x^2+1)-1/2/(a^
2*x^2+1)^2*arctan(a*x)+1/2*(15/8*a^3*x^3+17/8*a*x)/(a^2*x^2+1)^2+15/16*arct
an(a*x)+arctan(a*x)^3))
```

Maxima [A]
time = 0.52, size = 232, normalized size = 1.37

$$\frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right)^2-\frac{\left(15\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3+17\,ax+15\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)\right)a^2}{64\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}+\frac{\left(3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4\right)a\arctan\left(ax\right)}{8\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x)^2 - 1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arc
tan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^7*c^
3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3) + 1/8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2
+ 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^
3)
```

Fricas [A]
time = 2.20, size = 113, normalized size = 0.67

$$-\frac{15\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3-8\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)^2+17\,ax+\left(15\,a^4x^4+6\,a^2x^2-17\right)\arctan\left(ax\right)}{64\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 8*(3*a^3*x^
3 + 5*a*x)*arctan(a*x)^2 + 17*a*x + (15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*
x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.53, size = 157, normalized size = 0.93

$$\frac{\operatorname{atan}(a\,x)\,\left(\frac{1}{2\,a^3\,c^3}+\frac{3\,x^2}{8\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{15\,\operatorname{atan}(a\,x)}{64\,a\,c^3}-\frac{\frac{15\,a^2\,x^3}{8}+\frac{17\,x}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3}+\frac{{\operatorname{atan}(a\,x)}^2\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{{\operatorname{atan}(a\,x)}^3}{8\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2)^3,x)
```

```
[Out] (atan(a*x)*(1/(2*a^3*c^3) + (3*x^2)/(8*a*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4) -
 (15*atan(a*x))/(64*a*c^3) - ((17*x)/8 + (15*a^2*x^3)/8)/(8*c^3 + 16*a^2*c^
3*x^2 + 8*a^4*c^3*x^4) + (atan(a*x)^2*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^3))
)/(1/a^2 + 2*x^2 + a^2*x^4) + atan(a*x)^3/(8*a*c^3)
```

3.303 $\int \frac{\text{ArcTan}(ax)^2}{x(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=236

$$-\frac{1}{32c^3\left(1+a^2x^2\right)^2}-\frac{11}{32c^3\left(1+a^2x^2\right)}-\frac{ax\text{ArcTan}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{11ax\text{ArcTan}(ax)}{16c^3\left(1+a^2x^2\right)}-\frac{11\text{ArcTan}(ax)^2}{32c^3}+\frac{\text{ArcTan}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}+$$

```
[Out] -1/32/c^3/(a^2*x^2+1)^2-11/32/c^3/(a^2*x^2+1)-1/8*a*x*arctan(a*x)/c^3/(a^2*
x^2+1)^2-11/16*a*x*arctan(a*x)/c^3/(a^2*x^2+1)-11/32*arctan(a*x)^2/c^3+1/4*
arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)-1/3*I*arc
tan(a*x)^3/c^3+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-I*arctan(a*x)*polylog(2,
-1+2/(1-I*a*x))/c^3+1/2*polylog(3,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 0.37, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5086, 5044, 4988, 5004, 5112, 6745, 5050, 5012, 267, 5016}

$$\frac{\text{ArcTan}(ax)^2}{2c^3\left(a^2x^2+1\right)}+\frac{\text{ArcTan}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{11ax\text{ArcTan}(ax)}{16c^3\left(a^2x^2+1\right)}-\frac{ax\text{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)^2}-\frac{11}{32c^3\left(a^2x^2+1\right)}-\frac{1}{32c^3\left(a^2x^2+1\right)^2}-\frac{i\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^3}-\frac{i\text{ArcTan}(ax)^3}{3c^3}-\frac{11\text{ArcTan}(ax)^2}{32c^3}+\frac{\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/32*1/(c^3*(1 + a^2*x^2)^2) - 11/(32*c^3*(1 + a^2*x^2)) - (a*x*ArcTan[a*x
])/(8*c^3*(1 + a^2*x^2)^2) - (11*a*x*ArcTan[a*x])/(16*c^3*(1 + a^2*x^2)) -
(11*ArcTan[a*x]^2)/(32*c^3) + ArcTan[a*x]^2/(4*c^3*(1 + a^2*x^2)^2) + ArcTa
n[a*x]^2/(2*c^3*(1 + a^2*x^2)) - ((I/3)*ArcTan[a*x]^3)/c^3 + (ArcTan[a*x]^2
*Log[2 - 2/(1 - I*a*x)])/c^3 - (I*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
])/c^3 + PolyLog[3, -1 + 2/(1 - I*a*x)]/(2*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx &= -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}-\frac{1}{2}a\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)}\,dx}{c^2}-\frac{a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= -\frac{1}{32c^3\left(1+a^2x^2\right)^2}-\frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}+\frac{\tan^{-1}(ax)^2}{2c^3\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax\\
&= -\frac{1}{32c^3\left(1+a^2x^2\right)^2}-\frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)}-\frac{11\tan^{-1}(ax)^2}{32c^3}+\frac{\tan^{-1}(a}{4c^3\left(1+a\\
&= -\frac{1}{32c^3\left(1+a^2x^2\right)^2}-\frac{11}{32c^3\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)}-\frac{11\tan^{-1}}{32c\\
&= -\frac{1}{32c^3\left(1+a^2x^2\right)^2}-\frac{11}{32c^3\left(1+a^2x^2\right)}-\frac{ax\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{11ax\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)}-\frac{11\tan^{-1}}{32c
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 156, normalized size = 0.66

$$\frac{-32i\pi^3+256i\,\text{ArcTan}(ax)^3-144\cos(2\text{ArcTan}(ax))+288\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax))-3\cos(4\text{ArcTan}(ax))+24\text{ArcTan}(ax)^2\cos(4\text{ArcTan}(ax))+768\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+768i\,\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+384\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)-288\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax))-12\text{ArcTan}(ax)\sin(4\text{ArcTan}(ax))}{768c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-32*I)*Pi^3 + (256*I)*ArcTan[a*x]^3 - 144*Cos[2*ArcTan[a*x]] + 288*ArcTan
[a*x]^2*Cos[2*ArcTan[a*x]] - 3*Cos[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Cos[4*
ArcTan[a*x]] + 768*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (768*I)*
ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 384*PolyLog[3, E^((-2*I)*A
rcTan[a*x])] - 288*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Sin[4*Ar
cTan[a*x]])/(768*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.49, size = 1840, normalized size = 7.80

method	result	size
derivativedivides	Expression too large to display	1840
default	Expression too large to display	1840

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^3*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/4*
arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/c^3*arctan(a*x)^2*ln(a*x)-1/2/c^3*(1/2*I*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi*arctan(a*x)^2-I
*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))+11/16*arctan(a*x)^2-2*arctan(a*x)^2*ln(2)-2*arctan(a*x)^2*ln((1+I*a*x)/
(a^2*x^2+1)^(1/2))+1/2*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))^2*Pi*arctan(a*x)^2-1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^2-1/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*arctan(a*x)^2*Pi*csgn(I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/2*I*arcta
n(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3/16*(I+a*x)/(a*x-I)-3/16*(a*x-I)/(I
+a*x)+1/32*arctan(a*x)*sin(4*arctan(a*x))+2/3*I*arctan(a*x)^3-I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+I*a
rctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2-I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^3+1/128*cos(4*arctan(a*x))-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*arctan(a*x)^2*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^3+1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^3+3*I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)-3*I*arctan(
a*x)*(I+a*x)/(8*a*x-8*I)+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*ar
ctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-I*Pi*arctan(a*x)^2+4*I*arctan(a*x)*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+I*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+I*arcta
n(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^
```

```
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arct
an(a*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}(ax)}{a^{6}x^{7}+3a^{4}x^{5}+3a^{2}x^{3}+x}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**3,x)
[Out] Integral(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3),x)
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3), x)
```

3.304 $\int \frac{\mathbf{ArcTan}(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=250

$$\frac{a^2x}{32c^3(1+a^2x^2)^2}+\frac{31a^2x}{64c^3(1+a^2x^2)}+\frac{31a\text{ArcTan}(ax)}{64c^3}-\frac{a\text{ArcTan}(ax)}{8c^3(1+a^2x^2)^2}-\frac{7a\text{ArcTan}(ax)}{8c^3(1+a^2x^2)}-\frac{ia\text{ArcTan}(ax)^2}{c^3}-\text{Arc}$$

```
[Out] 1/32*a^2*x/c^3/(a^2*x^2+1)^2+31/64*a^2*x/c^3/(a^2*x^2+1)+31/64*a*arctan(a*x
)/c^3-1/8*a*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a*arctan(a*x)/c^3/(a^2*x^2+1)
-I*a*arctan(a*x)^2/c^3-arctan(a*x)^2/c^3/x-1/4*a^2*x*arctan(a*x)^2/c^3/(a^2
*x^2+1)^2-7/8*a^2*x*arctan(a*x)^2/c^3/(a^2*x^2+1)-5/8*a*arctan(a*x)^3/c^3+2
*a*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-I*a*polylog(2,-1+2/(1-I*a*x))/c^3
```

Rubi [A]

time = 0.42, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5086, 5038, 4946, 5044, 4988, 2497, 5004, 5012, 5050, 205, 211, 5020}

$$-\frac{7a^2x\text{ArcTan}(ax)^2}{8c^3(a^2x^2+1)}-\frac{a^2x\text{ArcTan}(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{7a\text{ArcTan}(ax)}{8c^3(a^2x^2+1)}-\frac{a\text{ArcTan}(ax)}{8c^3(a^2x^2+1)^2}+\frac{31a^2x}{64c^3(a^2x^2+1)}+\frac{a^2x}{32c^3(a^2x^2+1)^2}-\frac{5a\text{ArcTan}(ax)^3}{8c^3}-\frac{\text{ArcTan}(ax)^2}{c^3x}-\frac{ia\text{ArcTan}(ax)^2}{c^3}+\frac{31a\text{ArcTan}(ax)}{64c^3}+\frac{2a\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{ia\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*x)/(32*c^3*(1 + a^2*x^2)^2) + (31*a^2*x)/(64*c^3*(1 + a^2*x^2)) + (31*
a*ArcTan[a*x])/(64*c^3) - (a*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) - (7*a*Ar
cTan[a*x])/(8*c^3*(1 + a^2*x^2)) - (I*a*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/
(c^3*x) - (a^2*x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (7*a^2*x*ArcTan[a
*x]^2)/(8*c^3*(1 + a^2*x^2)) - (5*a*ArcTan[a*x]^3)/(8*c^3) + (2*a*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^3 - (I*a*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\,(c+a^2cx^2)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)^2}\,dx}{c}$$

$$= -\frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{a^2x\tan^{-1}(ax)^2}{4c^3\,(1+a^2x^2)^2} + \frac{1}{8}a^2\int \frac{1}{(c+a^2cx^2)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2(c+a^2cx^2)}\,dx}{c^2} - $$

$$= \frac{a^2x}{32c^3\,(1+a^2x^2)^2} - \frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{a^2x\tan^{-1}(ax)^2}{4c^3\,(1+a^2x^2)^2} - \frac{7a^2x\tan^{-1}(ax)^2}{8c^3\,(1+a^2x^2)} - \frac{7a\tan^{-1}}{24}$$

$$= \frac{a^2x}{32c^3\,(1+a^2x^2)^2} + \frac{3a^2x}{64c^3\,(1+a^2x^2)} - \frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{7a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)} - \frac{\tan^{-1}(ax}{c^3x}$$

$$= \frac{a^2x}{32c^3\,(1+a^2x^2)^2} + \frac{31a^2x}{64c^3\,(1+a^2x^2)} + \frac{3a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{7a\tan^{-1}(}{8c^3\,(1+a^2}$$

$$= \frac{a^2x}{32c^3\,(1+a^2x^2)^2} + \frac{31a^2x}{64c^3\,(1+a^2x^2)} + \frac{31a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{7a\tan^{-1}}{8c^3\,(1+}$$

$$= \frac{a^2x}{32c^3\,(1+a^2x^2)^2} + \frac{31a^2x}{64c^3\,(1+a^2x^2)} + \frac{31a\tan^{-1}(ax)}{64c^3} - \frac{a\tan^{-1}(ax)}{8c^3\,(1+a^2x^2)^2} - \frac{7a\tan^{-1}}{8c^3\,(1+}$$

Mathematica [A]

time = 0.27, size = 139, normalized size = 0.56

$$-\frac{160ax\text{ArcTan}(ax)^3 + 4ax\text{ArcTan}(ax)\,(32\cos(2\text{ArcTan}(ax)) + \cos(4\text{ArcTan}(ax)) - 128\log\left(1-e^{2i\text{ArcTan}(ax)}\right)) + 256iax\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right) - ax(64\sin(2\text{ArcTan}(ax)) + \sin(4\text{ArcTan}(ax))) + 8\text{ArcTan}(ax)^2(32+32iax+16ax\sin(2\text{ArcTan}(ax)) + ax\sin(4\text{ArcTan}(ax)))}{256c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/256*(160*a*x*ArcTan[a*x]^3 + 4*a*x*ArcTan[a*x]*(32*Cos[2*ArcTan[a*x]] +
Cos[4*ArcTan[a*x]] - 128*Log[1 - E^((2*I)*ArcTan[a*x])]) + (256*I)*a*x*Poly
Log[2, E^((2*I)*ArcTan[a*x])] - a*x*(64*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a
*x]]) + 8*ArcTan[a*x]^2*(32 + (32*I)*a*x + 16*a*x*Sin[2*ArcTan[a*x]] + a*x*
Sin[4*ArcTan[a*x]]))/(c^3*x)
```

Maple [A]

time = 0.26, size = 372, normalized size = 1.49

method	result
derivativedivides	$a\left(-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2} - \frac{15\arctan(ax)^3}{8c^3} - \frac{\arctan(ax)^2}{c^3ax} - \frac{-5\arctan(ax)^3+4\arctan(ax)\,\mathrm{l}}{}\right.$

default	$a\left(-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{c^3ax}-\frac{-5\arctan(ax)^3+4\arctan(ax)\ln(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-7/8/c^3*arctan(a*x)^2*a^3*x^3/(a^2*x^2+1)^2-9/8*a*x*arctan(a*x)^2/c^3/(
a^2*x^2+1)^2-15/8*arctan(a*x)^3/c^3-1/c^3*arctan(a*x)^2/a/x-1/4/c^3*(-5*arc
tan(a*x)^3+4*arctan(a*x)*ln(a^2*x^2+1)+7/2*arctan(a*x)/(a^2*x^2+1)+1/2/(a^2
*x^2+1)^2*arctan(a*x)-8*arctan(a*x)*ln(a*x)+I*ln(I+a*x)^2-2*I*ln(a*x-I)*ln(
-1/2*I*(I+a*x))-4*I*dilog(1+I*a*x)+4*I*dilog(1-I*a*x)+4*I*ln(a*x)*ln(1-I*a*
x)+2*I*ln(a*x-I)*ln(a^2*x^2+1)-2*I*ln(I+a*x)*ln(a^2*x^2+1)-2*I*dilog(-1/2*I
*(I+a*x))+2*I*dilog(1/2*I*(a*x-I))+2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-I*ln(a*x
-I)^2-4*I*ln(a*x)*ln(1+I*a*x)-1/2*(31/8*a^3*x^3+33/8*a*x)/(a^2*x^2+1)^2-31/
16*arctan(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/256*(75*a^5*x^5*arctan2(1, a*x) - 75*a^4*x^4 + 150*a^3*x^3*arctan2(1, a*
x) - 85*a^2*x^2 + 80*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^3 + 8*(15*a^4*
x^4 + 25*a^2*x^2 + 8)*arctan(a*x)^2 + 75*a*x*arctan2(1, a*x) - 2*(15*a^4*x^
4 + 25*a^2*x^2 + 8)*log(a^2*x^2 + 1)^2 + 40*(3*a^3*x^3 + 4*a*x)*arctan(a*x)
 + 7680*(a^10*c^3*x^5 + 2*a^8*c^3*x^3 + a^6*c^3*x)*integrate(1/64*x^4*log(a
^2*x^2 + 1)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3), x) + 12800
*(a^8*c^3*x^5 + 2*a^6*c^3*x^3 + a^4*c^3*x)*integrate(1/64*x^2*log(a^2*x^2 +
 1)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3), x) + 4096*(a^4*c^3
*x^5 + 2*a^2*c^3*x^3 + c^3*x)*integrate(1/256*(4*(a^2*x^2 + 1)^(9/2)*a^2*ar
ctan(a*x)*log(a^2*x^2 + 1)*sin(8*arctan(a*x)) - 32*(a^2*x^2 + 1)^4*a^2*arct
an(a*x)*log(a^2*x^2 + 1)*sin(7*arctan(a*x)) + 100*(a^2*x^2 + 1)^(7/2)*a^2*a
rctan(a*x)*log(a^2*x^2 + 1)*sin(6*arctan(a*x)) - 152*(a^2*x^2 + 1)^3*a^2*ar
ctan(a*x)*log(a^2*x^2 + 1)*sin(5*arctan(a*x)) + 112*(a^2*x^2 + 1)^(5/2)*a^2
*arctan(a*x)*log(a^2*x^2 + 1)*sin(4*arctan(a*x)) - 32*(a^2*x^2 + 1)^2*a^2*a
rctan(a*x)*log(a^2*x^2 + 1)*sin(3*arctan(a*x)) - (4*(a^2*x^2 + 1)^(9/2)*a^2
*arctan(a*x)^2 - (a^2*x^2 + 1)^(9/2)*a^2*log(a^2*x^2 + 1)^2)*cos(8*arctan(a
*x)) + 8*(4*(a^2*x^2 + 1)^4*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^4*a^2*log(a^2
*x^2 + 1)^2)*cos(7*arctan(a*x)) - 25*(4*(a^2*x^2 + 1)^(7/2)*a^2*arctan(a*x)
^2 - (a^2*x^2 + 1)^(7/2)*a^2*log(a^2*x^2 + 1)^2)*cos(6*arctan(a*x)) + 38*(4
```

*(a^2*x^2 + 1)^3*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^3*a^2*log(a^2*x^2 + 1)^2
)*cos(5*arctan(a*x)) - 28*(4*(a^2*x^2 + 1)^(5/2)*a^2*arctan(a*x)^2 - (a^2*x
^2 + 1)^(5/2)*a^2*log(a^2*x^2 + 1)^2)*cos(4*arctan(a*x)) + 8*(4*(a^2*x^2 +
1)^2*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*a^2*log(a^2*x^2 + 1)^2)*cos(3*arct
an(a*x)))*sqrt(a^2*x^2 + 1)/((a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^8*cos(8*arct
an(a*x))^2 + (a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^8*sin(8*arctan(a*x))^2 + 64*
(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^7*cos(7*arctan(a*x))^2 + 64*(a^2*c^3*x^2
+ c^3)*(a^2*x^2 + 1)^7*sin(7*arctan(a*x))^2 + 625*(a^2*c^3*x^2 + c^3)*(a^2*
x^2 + 1)^6*cos(6*arctan(a*x))^2 + 625*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^6*s
in(6*arctan(a*x))^2 + 1444*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^5*cos(5*arctan
(a*x))^2 + 1444*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^5*sin(5*arctan(a*x))^2 +
784*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^4*cos(4*arctan(a*x))^2 + 784*(a^2*c^3
x^2 + c^3)(a^2*x^2 + 1)^4*sin(4*arctan(a*x))^2 - 448*(a^2*c^3*x^2 + c^3)*
(a^2*x^2 + 1)^(7/2)*cos(4*arctan(a*x))*cos(3*arctan(a*x)) - 448*(a^2*c^3*x^
2 + c^3)*(a^2*x^2 + 1)^(7/2)*sin(4*arctan(a*x))*sin(3*arctan(a*x)) + 64*(a^
2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^3*cos(3*arctan(a*x))^2 + 64*(a^2*c^3*x^2 + c
^3)*(a^2*x^2 + 1)^3*sin(3*arctan(a*x))^2 - 2*(8*(a^2*c^3*x^2 + c^3)*(a^2*x^
2 + 1)^(15/2)*cos(7*arctan(a*x)) - 25*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^7*c
os(6*arctan(a*x)) + 38*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(13/2)*cos(5*arcta
n(a*x)) - 28*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^6*cos(4*arctan(a*x)) + 8*(a^
2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(11/2)*cos(3*arctan(a*x)))*cos(8*arctan(a*x)
) - 16*(25*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(13/2)*cos(6*arctan(a*x)) - 38
*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^6*cos(5*arctan(a*x)) + 28*(a^2*c^3*x^2 +
c^3)*(a^2*x^2 + 1)^(11/2)*cos(4*arctan(a*x)) - 8*(a^2*c^3*x^2 + c^3)*(a^2*
x^2 + 1)^5*cos(3*arctan(a*x)))*cos(7*arctan(a*x)) - 100*(19*(a^2*c^3*x^2 +
c^3)*(a^2*x^2 + 1)^(11/2)*cos(5*arctan(a*x)) - 14*(a^2*c^3*x^2 + c^3)*(a^2*
x^2 + 1)^5*cos(4*arctan(a*x)) + 4*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(9/2)*c
os(3*arctan(a*x)))*cos(6*arctan(a*x)) - 304*(7*(a^2*c^3*x^2 + c^3)*(a^2*x^2
+ 1)^(9/2)*cos(4*arctan(a*x)) - 2*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^4*cos(
3*arctan(a*x)))*cos(5*arctan(a*x)) - 2*(8*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)
^(15/2)*sin(7*arctan(a*x)) - 25*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^7*sin(6*a
rctan(a*x)) + 38*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(13/2)*sin(5*arctan(a*x)
) - 28*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^6*sin(4*arctan(a*x)) + 8*(a^2*c^3*
x^2 + c^3)*(a^2*x^2 + 1)^(11/2)*sin(3*arctan(a*x)))*sin(8*arctan(a*x)) - 16
(25(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(13/2)*sin(6*arctan(a*x)) - 38*(a^2*
c^3*x^2 + c^3)*(a^2*x^2 + 1)^6*sin(5*arctan(a*x)) + 28*(a^2*c^3*x^2 + c^3)*
(a^2*x^2 + 1)^(11/2)*sin(4*arctan(a*x)) - 8*(a^2*c^3*x^2 + c^3)*(a^2*x^2 +
1)^5*sin(3*arctan(a*x)))*sin(7*arctan(a*x)) - 100*(19*(a^2*c^3*x^2 + c^3)*(
a^2*x^2 + 1)^(11/2)*sin(5*arctan(a*x)) - 14*(a^2*c^3*x^2 + c^3)*(a^2*x^2 +
1)^5*sin(4*arctan(a*x)) + 4*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^(9/2)*sin(3*a
rctan(a*x)))*sin(6*arctan(a*x)) - 304*(7*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^
(9/2)*sin(4*arctan(a*x)) - 2*(a^2*c^3*x^2 + c^3)*(a^2*x^2 + 1)^4*sin(3*arct
an(a*x)))*sin(5*arctan(a*x))), x) - 7680*(a^10*c^3*x^5 + 2*a^8*c^3*x^3 + a^
6*c^3*x)*integrate(1/256*(4*x^4*arctan(a*x)^2 + x^4*log(a^2*x^2 + 1)^2)/(a^
6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3), x) - 15360*(a^8*c^3*x^5 +

```
 2*a^6*c^3*x^3 + a^4*c^3*x)*integrate(1/256*(4*x^2*arctan(a*x)^2 + x^2*log(
a^2*x^2 + 1)^2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3), x) - 8
192*(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x)*integ...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{a^{6}x^{8}+3a^{4}x^{6}+3a^{2}x^{4}+x^{2}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^2}{x^2\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3), x)
```

3.305 $\int \frac{\textbf{ArcTan}(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=322

$$\frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{19a^2}{32c^3\left(1+a^2x^2\right)}-\frac{a\text{ArcTan}(ax)}{c^3x}+\frac{a^3x\text{ArcTan}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{19a^3x\text{ArcTan}(ax)}{16c^3\left(1+a^2x^2\right)}+\frac{3a^2\text{ArcTan}(ax)^2}{32c^3}$$

```
[Out] 1/32*a^2/c^3/(a^2*x^2+1)^2+19/32*a^2/c^3/(a^2*x^2+1)-a*arctan(a*x)/c^3/x+1/
8*a^3*x*arctan(a*x)/c^3/(a^2*x^2+1)^2+19/16*a^3*x*arctan(a*x)/c^3/(a^2*x^2+
1)+3/32*a^2*arctan(a*x)^2/c^3-1/2*arctan(a*x)^2/c^3/x^2-1/4*a^2*arctan(a*x)
^2/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^2/c^3/(a^2*x^2+1)+I*a^2*arctan(a*x)^3/
c^3+a^2*ln(x)/c^3-1/2*a^2*ln(a^2*x^2+1)/c^3-3*a^2*arctan(a*x)^2*ln(2-2/(1-I
*a*x))/c^3+3*I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^3-3/2*a^2*polylo
g(3,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 0.97, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745, 5050, 5012, 267, 5016}

$$\frac{3ia^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^3}-\frac{a^2\text{ArcTan}(ax)^2}{c^3\left(a^2x^2+1\right)}-\frac{a^2\text{ArcTan}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{ia^2\text{ArcTan}(ax)^3}{c^3}+\frac{3a^2\text{ArcTan}(ax)^2}{32c^3}-\frac{3a^2\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{3a^2\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^3}+\frac{19a^2}{32c^3\left(a^2x^2+1\right)}+\frac{a^2}{32c^3\left(a^2x^2+1\right)^2}-\frac{a^2\log\left(a^2x^2+1\right)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{19a^3x\text{ArcTan}(ax)}{16c^3\left(a^2x^2+1\right)}+\frac{a^3x\text{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)^2}-\frac{\text{ArcTan}(ax)^2}{2c^3x^2}-\frac{a\text{ArcTan}(ax)}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] a^2/(32*c^3*(1 + a^2*x^2)^2) + (19*a^2)/(32*c^3*(1 + a^2*x^2)) - (a*ArcTan[
a*x])/(c^3*x) + (a^3*x*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (19*a^3*x*Arc
Tan[a*x])/(16*c^3*(1 + a^2*x^2)) + (3*a^2*ArcTan[a*x]^2)/(32*c^3) - ArcTan[
a*x]^2/(2*c^3*x^2) - (a^2*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*Arc
Tan[a*x]^2)/(c^3*(1 + a^2*x^2)) + (I*a^2*ArcTan[a*x]^3)/c^3 + (a^2*Log[x])/
c^3 - (a^2*Log[1 + a^2*x^2])/(2*c^3) - (3*a^2*ArcTan[a*x]^2*Log[2 - 2/(1 -
I*a*x)])/c^3 + ((3*I)*a^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 -
 (3*a^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3(c+a^2cx^2)^2}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3(c+a^2cx^2)}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)^2}\,dx}{c}$$

$$= -\frac{a^2\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}+\frac{1}{2}a^3\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c^3}-\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)}\,dx}{c^2}-2$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)^2}{2c^3x^2}-\frac{a^2\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}+\frac{ia^2\tan^{-1}(ax)}{3c^3}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(ax)}{16c^3\left(1+a^2x^2\right)}+\frac{3a^2\tan^{-1}(ax)^2}{32c^3}-\frac{\tan^{-1}(ax)}{2c^3x^2}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^2}{32c^3\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^3x}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(a}{16c^3\left(1+a^2x\right.}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^2}{32c^3\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^3x}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(a}{16c^3\left(1+a^2x\right.}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^2}{32c^3\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^3x}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(a}{16c^3\left(1+a^2x\right.}$$

$$= \frac{a^2}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^2}{32c^3\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)}{c^3x}+\frac{a^3x\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(a}{16c^3\left(1+a^2x\right.}$$

Mathematica [A]
time = 0.50, size = 226, normalized size = 0.70

$$\frac{a^2\left(\frac{i\pi^3}{8}-\frac{\text{ArcTan}(ax)}{ax}-\frac{(1+a^2x^2)\text{ArcTan}(ax)^2}{2a^2x^2}-i\text{ArcTan}(ax)^3+\frac{5}{16}\cos(2\text{ArcTan}(ax))-\frac{5}{8}\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax))+\frac{1}{256}\cos(4\text{ArcTan}(ax))-\frac{1}{32}\text{ArcTan}(ax)^2\cos(4\text{ArcTan}(ax))-3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-3i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)+\frac{5}{8}\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax))+\frac{1}{64}\text{ArcTan}(ax)\sin(4\text{ArcTan}(ax))\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*((I/8)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^2
*x^2) - I*ArcTan[a*x]^3 + (5*Cos[2*ArcTan[a*x]])/16 - (5*ArcTan[a*x]^2*Cos[
2*ArcTan[a*x]])/8 + Cos[4*ArcTan[a*x]]/256 - (ArcTan[a*x]^2*Cos[4*ArcTan[a*
x]])/32 - 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log[(a*x)/Sqrt[
1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (3*P
olyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (5*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/8
 + (ArcTan[a*x]*Sin[4*ArcTan[a*x]])/64))/c^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.11, size = 1970, normalized size = 6.12

method	result	size
derivativedivides	Expression too large to display	1970
default	Expression too large to display	1970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(3/2/c^3*arctan(a*x)^2*ln(a^2*x^2+1)-arctan(a*x)^2/c^3/(a^2*x^2+1)-1/4*
arctan(a*x)^2/c^3/(a^2*x^2+1)^2-1/2/c^3*arctan(a*x)^2/a^2/x^2-3/c^3*arctan(
a*x)^2*ln(a*x)-1/2/c^3*(-3/16*arctan(a*x)^2+6*arctan(a*x)^2*ln(2)-2*I*arcta
n(a*x)^3+6*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+5/16*(I+a*x)/(a*x-
I)+5/16*(a*x-I)/(I+a*x)-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-1/32*arctan(a*x)*sin(4*arctan(a*x))+3/2*I*arctan(a*x)^
2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+3*I*Pi*arctan(a*x)^2-12*I*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-1/128*cos(4*arctan(a*x))+12*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+12*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*ln((1+I*
a*x)^2/(a^2*x^2+1)-1)+6*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*a
rctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*I*arctan(a*x)^2*Pi*csgn(
I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+3*I*arctan(a*x)^2*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-5*I*arctan(a*x)*
(a*x-I)/(8*a*x+8*I)+5*I*arctan(a*x)*(I+a*x)/(8*a*x-8*I)+3*I*arctan(a*x)^2*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+3*I*arc
tan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^3-3*I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^2+3/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
-3/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-3/2*I*arctan(a*x)
^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+arcta
n(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)
```

```
+1)/a/x-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3*I*arctan(a*x)^2*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+3/2
*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I
*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+3/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-3*I*arctan(a*x
)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2-3/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))-3*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x
^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{2}(ax)}{a^{6}x^{9}+3a^{4}x^{7}+3a^{2}x^{5}+x^{3}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3), x)
```

3.306 $\int \frac{\text{ArcTan}(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=317

$$-\frac{a^2}{3c^3x}-\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}-\frac{47a^4x}{64c^3\left(1+a^2x^2\right)}-\frac{205a^3\text{ArcTan}(ax)}{192c^3}-\frac{a\text{ArcTan}(ax)}{3c^3x^2}+\frac{a^3\text{ArcTan}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{11a^3\text{ArcTa}}{8c^3\left(1+a\right.}$$

```
[Out] -1/3*a^2/c^3/x-1/32*a^4*x/c^3/(a^2*x^2+1)^2-47/64*a^4*x/c^3/(a^2*x^2+1)-205
/192*a^3*arctan(a*x)/c^3-1/3*a*arctan(a*x)/c^3/x^2+1/8*a^3*arctan(a*x)/c^3/
(a^2*x^2+1)^2+11/8*a^3*arctan(a*x)/c^3/(a^2*x^2+1)+10/3*I*a^3*arctan(a*x)^2
/c^3-1/3*arctan(a*x)^2/c^3/x^3+3*a^2*arctan(a*x)^2/c^3/x+1/4*a^4*x*arctan(a
*x)^2/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+35/24*a^3*
arctan(a*x)^3/c^3-20/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3+10/3*I*a^3*pol
ylog(2,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 1.07, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5086, 5038, 4946, 331, 209, 5044, 4988, 2497, 5004, 5012, 5050, 205, 211, 5020}

$$\frac{35a^3\text{ArcTan}(ax)^3}{24c^3}+\frac{10ia^3\text{ArcTan}(ax)^2}{3c^3}-\frac{205a^3\text{ArcTan}(ax)}{192c^3}-\frac{20a^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^3}+\frac{10ia^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{3c^3}+\frac{3a^2\text{ArcTan}(ax)^2}{c^3x}-\frac{a^2}{3c^3x}+\frac{11a^4x\text{ArcTan}(ax)^2}{8c^3\left(a^2x^2+1\right)}+\frac{a^4x\text{ArcTan}(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{47a^4x}{64c^3\left(a^2x^2+1\right)}-\frac{a^4x}{32c^3\left(a^2x^2+1\right)^2}+\frac{11a^3\text{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{a^3\text{ArcTan}(ax)}{8c^3\left(a^2x^2+1\right)^2}-\frac{\text{ArcTan}(ax)^2}{3c^3x^3}-\frac{a\text{ArcTan}(ax)}{3c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] -1/3*a^2/(c^3*x) - (a^4*x)/(32*c^3*(1 + a^2*x^2)^2) - (47*a^4*x)/(64*c^3*(1
 + a^2*x^2)) - (205*a^3*ArcTan[a*x])/(192*c^3) - (a*ArcTan[a*x])/(3*c^3*x^2
) + (a^3*ArcTan[a*x])/(8*c^3*(1 + a^2*x^2)^2) + (11*a^3*ArcTan[a*x])/(8*c^3
*(1 + a^2*x^2)) + (((10*I)/3)*a^3*ArcTan[a*x]^2)/c^3 - ArcTan[a*x]^2/(3*c^3
*x^3) + (3*a^2*ArcTan[a*x]^2)/(c^3*x) + (a^4*x*ArcTan[a*x]^2)/(4*c^3*(1 + a
^2*x^2)^2) + (11*a^4*x*ArcTan[a*x]^2)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTa
n[a*x]^3)/(24*c^3) - (20*a^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/(3*c^3) +
(((10*I)/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
&& LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}\\
&= \frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}-\frac{1}{8}a^4\int \frac{1}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^2}{x^4}\,dx}{c^3}-\cdots\\
&= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}+\frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)^2}{3c^3x^3}+\frac{a^4x\tan^{-1}(ax)^2}{4c^3\left(1+a^2x^2\right)^2}+\cdots\\
&= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}-\frac{3a^4x}{64c^3\left(1+a^2x^2\right)}+\frac{a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)^2}+\frac{3a^3\tan^{-1}(ax)}{8c^3\left(1+a^2x^2\right)}-\cdots\\
&= -\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}-\frac{15a^4x}{64c^3\left(1+a^2x^2\right)}-\frac{3a^3\tan^{-1}(ax)}{64c^3}-\frac{a\tan^{-1}(ax)}{3c^3x^2}+\cdots\\
&= -\frac{a^2}{3c^3x}-\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}-\frac{15a^4x}{64c^3\left(1+a^2x^2\right)}-\frac{15a^3\tan^{-1}(ax)}{64c^3}-\frac{a\tan^{-1}(ax)}{3c^3x^2}+\cdots\\
&= -\frac{a^2}{3c^3x}-\frac{a^4x}{32c^3\left(1+a^2x^2\right)^2}-\frac{15a^4x}{64c^3\left(1+a^2x^2\right)}-\frac{109a^3\tan^{-1}(ax)}{192c^3}-\frac{a\tan^{-1}(ax)}{3c^3x^2}+\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.56, size = 189, normalized size = 0.60

$$
\frac{a^3\left(-\frac{256(1+a^2x^2)\text{ArcTan}(ax)}{a^2x^2}-\frac{256(1+a^2x^2)\text{ArcTan}(ax)^2}{a^3x^3}+1120\text{ArcTan}(ax)^3+\frac{256\left(-1+10\text{ArcTan}(ax)^2\right)}{ax}+576\text{ArcTan}(ax)\cos(2\text{ArcTan}(ax))+12\text{ArcTan}(ax)\cos(4\text{ArcTan}(ax))-5120\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)+2560i\left(\text{ArcTan}(ax)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right)+288(-1+2\text{ArcTan}(ax)^2)\sin(2\text{ArcTan}(ax))+3(-1+8\text{ArcTan}(ax)^2)\sin(4\text{ArcTan}(ax))\right)}{768c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^3*((-256*(1 + a^2*x^2)*ArcTan[a*x])/(a^2*x^2) - (256*(1 + a^2*x^2)*ArcTa
n[a*x]^2)/(a^3*x^3) + 1120*ArcTan[a*x]^3 + (256*(-1 + 10*ArcTan[a*x]^2))/(a
*x) + 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]
] - 5120*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + (2560*I)*(ArcTan[a*x]
```

```
^2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]) + 288*(-1 + 2*ArcTan[a*x]^2)*Sin[2*
ArcTan[a*x]] + 3*(-1 + 8*ArcTan[a*x]^2)*Sin[4*ArcTan[a*x]]))/(768*c^3)
```

Maple [A]
time = 0.14, size = 411, normalized size = 1.30

method	result
derivativedivides	$a^3\left(\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}-\frac{-40\arctan(a}{}\right.$
default	$a^3\left(\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}-\frac{-40\arctan(a}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(11/8/c^3*arctan(a*x)^2*a^3*x^3/(a^2*x^2+1)^2+13/8*a*x*arctan(a*x)^2/c^
3/(a^2*x^2+1)^2+35/8*arctan(a*x)^3/c^3-1/3/c^3*arctan(a*x)^2/a^3/x^3+3/c^3*
arctan(a*x)^2/a/x-1/12/c^3*(-40*arctan(a*x)*ln(a^2*x^2+1)-3/2/(a^2*x^2+1)^2
*arctan(a*x)-33/2*arctan(a*x)/(a^2*x^2+1)+4*arctan(a*x)/a^2/x^2+80*arctan(a
*x)*ln(a*x)+20*I*ln(I+a*x)*ln(a^2*x^2+1)+40*I*dilog(1+I*a*x)+40*I*ln(a*x)*l
n(1+I*a*x)-40*I*dilog(1-I*a*x)-20*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-10*I*ln(I+a
*x)^2+20*I*dilog(-1/2*I*(I+a*x))+20*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-20*I*ln(
a*x-I)*ln(a^2*x^2+1)-20*I*dilog(1/2*I*(a*x-I))-40*I*ln(a*x)*ln(1-I*a*x)+10*
I*ln(a*x-I)^2+1/2*(141/8*a^3*x^3+147/8*a*x)/(a^2*x^2+1)^2+205/16*arctan(a*x
)+4/a/x+35*arctan(a*x)^3))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^2/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*
x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**2/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/c
**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^4\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3), x)
```

3.307 $\int x^3 \sqrt{c+a^2 c x^2} \, \textbf{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=385

$$-\frac{11\sqrt{c+a^2cx^2}}{60a^4}+\frac{\left(c+a^2cx^2\right)^{3/2}}{30a^4c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{12a^3}-\frac{x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{10a}-\frac{2\sqrt{c+a^2cx^2}\,\text{Ar}}{15a^4}$$

```
[Out] 1/30*(a^2*c*x^2+c)^(3/2)/a^4/c-11/30*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+11/60*I*c*polyl
og(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c
)^(1/2)-11/60*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-11/60*(a^2*c*x^2+c)^(1/2)/a^4+1/12*x*arctan(a
*x)*(a^2*c*x^2+c)^(1/2)/a^3-1/10*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-2/15
*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4+1/15*x^2*arctan(a*x)^2*(a^2*c*x^2+c)
^(1/2)/a^2+1/5*x^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5070, 5072, 267, 5010, 5006, 5050, 272, 45}

$$\frac{x^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{15a^2}+\frac{1}{5}x^4\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{x^3\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{10a}-\frac{2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{15a^4}-\frac{11ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{30a^4\sqrt{a^2cx^2+c}}+\frac{11ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{a^2cx^2+c}}+\frac{\left(a^2cx^2+c\right)^{3/2}}{30a^4c}-\frac{11\sqrt{a^2cx^2+c}}{60a^4}+\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (-11*Sqrt[c + a^2*c*x^2])/(60*a^4) + (c + a^2*c*x^2)^(3/2)/(30*a^4*c) + (x*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^3) - (x^3*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(10*a) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^4) + (x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/5 - (((11*I)/30)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*
x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((11*I)/60)*c*Sqrt[1 + a^
2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^
2*c*x^2]) - (((11*I)/60)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])
/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
```

```
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx = c\int \frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^5\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{1}{5}(4c)\int \frac{x^3\,\text{ta}}{\sqrt{c}}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3} - \frac{x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a} - \frac{2\sqrt{c+a^2cx^2}\,\text{t}}{3a^4}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a} - \frac{2}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}}{12a^4} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a} -$$

$$= -\frac{11\sqrt{c+a^2cx^2}}{60a^4} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^4c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^3\sqrt{c+}}{}$$

Mathematica [A]
time = 0.84, size = 360, normalized size = 0.94

$$-\frac{(1+a^2x^2)^2\sqrt{c(1+a^2x^2)}\left(50-32\text{ArcTan}(ax)^2+72\cos(2\text{ArcTan}(ax))+160\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax))+22\cos(4\text{ArcTan}(ax))-\frac{110\text{ArcTan}(ax)\log\left(1-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}-55\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax))\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-11\text{ArcTan}(ax)\cos(5\text{ArcTan}(ax))\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+\frac{110\text{ArcTan}(ax)\log\left(1+ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}+55\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax))\log\left(1+ie^{i\text{ArcTan}(ax)}\right)+11\text{ArcTan}(ax)\cos(5\text{ArcTan}(ax))\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-\frac{176i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)}{(1+a^2x^2)^{5/2}}+\frac{176i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)}{(1+a^2x^2)^{5/2}}+4\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax))-22\text{ArcTan}(ax)\sin(4\text{ArcTan}(ax))\right)}{960a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] -1/960*((1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*C
os[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[
a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] -
55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[
a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log
[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[
a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[
1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1
```

```
+ a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^
(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]
]))/a^4
```

Maple [A]
time = 1.71, size = 235, normalized size = 0.61

method	result
default	$\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(12\arctan(ax)^2a^4x^4-6\arctan(ax)a^3x^3+4\arctan(ax)^2a^2x^2+2a^2x^2+5\arctan(ax)ax-8\arctan(ax)^2-\right.}{60a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/60/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(12*arctan(a*x)^2*a^4*x^4-6*arctan(a*x)*
a^3*x^3+4*arctan(a*x)^2*a^2*x^2+2*a^2*x^2+5*arctan(a*x)*a*x-8*arctan(a*x)^2
-9)-11/60*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*
x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.308 $\int x^2 \sqrt{c+a^2cx^2} \, \textbf{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=436

$$\frac{x\sqrt{c+a^2cx^2}}{12a^2}+\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{12a^3}-\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{6a}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{8a^2}+\frac{1}{4}x^3\sqrt{}$$

```
[Out] -1/6*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+1/4*I*c*arctan((1
+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c
)^(1/2)-1/4*I*c*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/4*I*c*arctan(a*x)*polylog(2,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/4*c*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(
1/2)-1/4*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(
a^2*c*x^2+c)^(1/2)+1/12*x*(a^2*c*x^2+c)^(1/2)/a^2+1/12*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)/a^3-1/6*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a+1/8*x*arctan(a*x)
^2*(a^2*c*x^2+c)^(1/2)/a^2+1/4*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

$$-\frac{x^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{6a}+\frac{x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{8a^2}+\frac{1}{4}x^3\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{x\sqrt{a^2cx^2+c}}{12a^2}-\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}-\frac{c\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{4a^3\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{12a^3}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2])/(12*a^2) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^
3) - (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(8*a^2) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/4 + ((I/4)*
c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c +
a^2*c*x^2]) - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a^3)
- ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx = c\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + (a^2c)\int \frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2} + \frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{1}{4}(3c)\int \frac{x^2\tan}{\sqrt{c-}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x\sqrt{c+a^2cx^2}\,\tan}{8a^2}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

$$= \frac{x\sqrt{c+a^2cx^2}}{12a^2} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^3} - \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} + \frac{x}{}$$

Mathematica [A]
time = 2.49, size = 599, normalized size = 1.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-4*(3*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 3*
Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTa
n[a*x]))] - 3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 3*ArcTan[a*x]^2*
Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 3*Pi*Ar
cTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*
```

```
x])] + 3*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2
)*ArcTan[a*x]))] - 3*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 4*L
og[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 3*ArcTan[a*x]^2*Log[Cos[ArcTa
n[a*x]/2] - Sin[ArcTan[a*x]/2]] + 4*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2]] - 3*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 3*Pi
*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (6*I)*ArcTan[a*x]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)] - 6*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 6*PolyLog[3, I*E^(I*ArcTan[a*x]
)]) + (1 + a^2*x^2)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTa
n[a*x]]) - 3*ArcTan[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]])
+ 2*(a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]))))/(96*a^3*Sqrt[1 + a^2*x^
2])
```

Maple [A]

time = 0.67, size = 302, normalized size = 0.69

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(6\arctan(ax)^2a^3x^3-4\arctan(ax)a^2x^2+3\arctan(ax)^2ax+2ax+2\arctan(ax)\right)}{24a^3}-\frac{i\sqrt{c\,(ax-i)\,(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)^2*a^3*x^3-4*arctan(a*x)*a
^2*x^2+3*arctan(a*x)^2*a*x+2*a*x+2*arctan(a*x))-1/24*I*(c*(a*x-I)*(I+a*x))^
(1/2)*(3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x
)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-8*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/
2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\,\operatorname{atan}(a\,x)^{2}\,\sqrt{c\,a^{2}\,x^{2}+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.309 $\int x\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=279

$$\frac{\sqrt{c+a^2cx^2}}{3a^2}-\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3a}+\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}{3a^2c}+\frac{2ic\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}}{3a^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a^2/c+2/3*I*c*arctan(a*x)*arctan((1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-1/3*
I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^
2*c*x^2+c)^(1/2)+1/3*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+1/3*(a^2*c*x^2+c)^(1/2)/a^2-1/3*x*arct
an(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]

time = 0.14, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5050, 4998, 5010, 5006}

$$\frac{\text{ArcTan}(ax)^2\,(a^2cx^2+c)^{3/2}}{3a^2c}+\frac{2ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{3a^2\sqrt{a^2cx^2+c}}-\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3a}-\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^2) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a) + (
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) + (((2*I)/3)*c*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2
*c*x^2]) - ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((I/3)*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
```

```
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx = \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3a^2c} - \frac{2\int\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx}{3a}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a} + \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3a^2c} - \frac{c\ \ldots}{\ldots}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a} + \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3a^2c} - \frac{(c\ \ldots}{\ldots}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a} + \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3a^2c} + \frac{2i\ \ldots}{\ldots}$$

Mathematica [A]
time = 0.42, size = 260, normalized size = 0.93

$$\frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\left(2+4\mathrm{ArcTan}(ax)^2+2\cos(2\mathrm{ArcTan}(ax))-\frac{3\mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}-\mathrm{ArcTan}(ax)\cos(3\mathrm{ArcTan}(ax))\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)+\frac{3\mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}+\mathrm{ArcTan}(ax)\cos(3\mathrm{ArcTan}(ax))\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)-\frac{4i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}+\frac{4i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}-2\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax))\right)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[
a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - Ar
```

```
cTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*
Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[
a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2
*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2)
```

Maple [A]
time = 0.45, size = 198, normalized size = 0.71

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)^2 a^2 x^2-\arctan(ax) ax+\arctan(ax)^2+1\right)}{3a^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^2*a^2*x^2-arctan(a*x)*a*x+ar
ctan(a*x)^2+1)+1/3*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/
a^2/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.310 $\int \sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{a\sqrt{c+a^2cx^2}}$$

```
[Out] arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a-I*c*arctan((1+I*a*x)/(a^
2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+I*c*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a
^2*c*x^2+c)^(1/2)-I*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+c*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c
*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212}

$$\frac{ic\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{c\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*
Sqrt[c + a^2*c*x^2]) + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])
/a + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
/(a*Sqrt[c + a^2*c*x^2]) - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*
PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2\,dx = -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{1}{2}c\int\frac{\tan^{-1}(ax}{\sqrt{c+a^2cx}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+c\text{Subst}\left(\int\frac{1}{1-c}\right.$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{\sqrt{c}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-\frac{ic\sqrt{1+a^2x^2}\tan}{a}$$

Mathematica [A]

time = 0.28, size = 201, normalized size = 0.59

$$\frac{\sqrt{c(1+a^2x^2)}\left(-2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)+ax\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2-2i\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2+2\tanh^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-2i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)-2\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)}{2a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a*x*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]^2 - (2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*Arc
```

```
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*a*Sqrt[1
 + a^2*x^2])
```

Maple [A]
time = 0.32, size = 268, normalized size = 0.79

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\arctan(ax)(\arctan(ax)ax-2)}{2a}+\frac{i\sqrt{c\,(ax-i)\,(ax+i)}\,\left(i\arctan(ax)^{2}\ln\left(1+\frac{i(iax+1)}{\sqrt{a^{2}x^{2}+1}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(arctan(a*x)*a*x-2)+1/2*I*(c*(a
*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I
*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+2*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x
^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\,(a^{2}x^{2}+1)}\,\operatorname{atan}^{2}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.311 $\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{x}\,dx$

Optimal. Leaf size=439

$$\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2+\frac{4ic\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{2c\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\,\text{tan}}{\sqrt{c+a^2cx^2}}$$

```
[Out] 4*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-2*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c*arctan(a*x)*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*c*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-2*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)+2*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*c*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*c*x^2+c)^
(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5070, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5010, 5006}

$$\frac{2ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{2c\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{4ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{\sqrt{a^2cx^2+c}}-\frac{2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

```
[Out] Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((4*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*
ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((
-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*I)*c*Sqrt[
1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*
c*x^2] - (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + (2*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x}\,dx &= c\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx+\left(a^2c\right)\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2-(2ac)\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx+\frac{\left(c\sqrt{1+a^2x^2}\right)\int \frac{}{x}}{\sqrt{c+a^2c}}\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{\left(c\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int x^2\csc(x)\,dx,x,\tan^{-1}(\right.}{\sqrt{c+a^2cx^2}}\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^2+\frac{4ic\sqrt{1+a^2x^2}\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 250, normalized size = 0.57

$$
\frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)^2+\mathrm{ArcTan}(ax)^2\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-2\mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)+2\mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{ArcTan}(ax)^2\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+2i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-2i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)+2i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)-2i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)-2\mathrm{PolyLog}\left(3,-e^{i\mathrm{ArcTan}(ax)}\right)+2\mathrm{PolyLog}\left(3,e^{i\mathrm{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^2*Log[1
 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 2*ArcT
an[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*
x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*Arc
Tan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] +
 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
```

Maple [A]
time = 0.44, size = 337, normalized size = 0.77

method	result
default	$\sqrt{c\left(ax-i\right)\left(ax+i\right)}\ \arctan\left(ax\right)^{2}-\frac{i\sqrt{c\left(ax-i\right)\left(ax+i\right)}\ \left(i\arctan(ax)^{2}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-i\arctan(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2-I*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arct
an(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*
arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))+2*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*dilog(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\ \operatorname{atan}^{2}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.312 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=458

$$-\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{x}-\frac{2iac\sqrt{1+a^2x^2}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}-\frac{4ac\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)}{\sqrt{c+a^2}}$$

```
[Out] -2*I*a*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-4*a*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a*c*arctan(a*x)*polylog(2,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*
c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+2*I*a*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*c*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*a*c*polylog(3,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*a*c*polylog(3,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-arctan(
a*x)^2*(a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.38, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5070, 5064, 5078, 5074, 5010, 5008, 4266, 2611, 2320, 6724}

$$\frac{2iac\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ac\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ac\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{x}-\frac{4ac\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2iac\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^
2] - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/Sqrt[c + a^2*c*x^2] - (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} + (2ac)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right.}{\sqrt{c+}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} + \frac{\left(ac\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.54, size = 265, normalized size = 0.58

$$-\frac{a\sqrt{c(1+a^2x^2)}\left(\frac{\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2}{ax} - 2\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right) - \text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right) + \text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right) + 2\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right) - 2i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right) - 2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right) + 2i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right) + 2i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right) + 2\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right) - 2\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] -((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*Arc
Tan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a
*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1 +
E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*
x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I
*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^
(I*ArcTan[a*x])] - 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.43, size = 309, normalized size = 0.67

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\;\arctan(ax)^{2}}{x}-\frac{a\sqrt{c\,(ax-i)\,(ax+i)}\;\left(\arctan(ax)^{2}\ln\left(1+\frac{i(iax+1)}{\sqrt{a^{2}x^{2}+1}}\right)-\arctan(ax)^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2/x-a*(c*(a*x-I)*(I+a*x))^(1/2)*(arc
tan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\,(a^{2}x^{2}+1)}\;\operatorname{atan}^{2}(ax)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.313 $\int \frac{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=328

$$-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x^2}-\frac{a^2c\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}-$$

```
[Out] -a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-a^2*c*arctan(a*x)^2*arcta
nh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a^2
*c*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-I*a^2*c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*c*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+a^2*c*polylog(3,(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/x-1/2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.61, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724}

$$\frac{ia^2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^2c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{a^2c\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2x^2}-\frac{a^2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]
] + (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] - (I*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*c*Sqrt[1 + a^2*x^2]*PolyLog
[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} + (ac)\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}\left(a^2c\right)\int \frac{\tan^{-}}{x\sqrt{c+}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} + \left(a^2c\right)\int \frac{1}{x\sqrt{c+a}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{2a^2c\sqrt{1+a^2x^2}\ \mathrm{t}}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\ \mathrm{ta}}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\ \mathrm{ta}}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\ \mathrm{ta}}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\ \mathrm{ta}}{}$$

Mathematica [A]
time = 1.36, size = 222, normalized size = 0.68

$$\frac{a^2\sqrt{c(1+a^2x^2)}\left(-4\mathrm{ArcTan}(ax)\cot\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-\mathrm{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)+4\mathrm{ArcTan}(ax)^2\left(\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)\right)+8\log\left(\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)+8i\mathrm{ArcTan}(ax)\left(\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)\right)+8\left(-\mathrm{PolyLog}\left(3,-e^{i\mathrm{ArcTan}(ax)}\right)+\mathrm{PolyLog}\left(3,e^{i\mathrm{ArcTan}(ax)}\right)\right)+\mathrm{ArcTan}(ax)^2\sec^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-4\mathrm{ArcTan}(ax)\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)}{8\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]
^2*Csc[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log
[1 + E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(P
olyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLo
g[3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*S
ec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2*x^
2])
```

Maple [A]
time = 0.48, size = 255, normalized size = 0.78

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\arctan(ax)(2ax+\arctan(ax))}{2x^{2}}+\frac{a^{2}\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)^{2}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*a*x+arctan(a*x))/x^2+1/2*a^2*
(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,(
1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\,(a^{2}x^{2}+1)}\,\operatorname{atan}^{2}(ax)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.314 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=275

$$-\frac{a^2\sqrt{c+a^2cx^2}}{3x}-\frac{a\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{3x^2}-\frac{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^2}{3cx^3}-\frac{2a^3c\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)\tan}{3\sqrt{c+a^2cx^2}}$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/c/x^3-2/3*a^3*c*arctan(a*x)*arctanh(
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*
I*a^3*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)-1/3*I*a^3*c*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^2+c)^(1/2)/x-1/3*a*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.30, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5064, 5066, 5082, 270, 5078, 5074}

$$-\frac{a\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3x^2}-\frac{\mathrm{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}-\frac{a^2\sqrt{a^2cx^2+c}}{3x}-\frac{2a^3c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{ia^3c\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*Sqrt[c + a^2*c*x^2])/x - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x
^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*c*x^3) - (2*a^3*c*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a
^2*c*x^2]) + ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((I/3)*a^3*c*Sqrt[1 + a^2*x^2]*PolyL
og[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^4}\,dx = -\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3cx^3} + \frac{1}{3}(2a)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{2a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3cx^3} - \frac{1}{3}(2ac)\int \frac{\tan}{x^3\sqrt{c}}$$

$$= -\frac{2a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3cx^3} +$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{3cx^3} -$$

Mathematica [A]
time = 1.20, size = 239, normalized size = 0.87

$$-\frac{c\sqrt{1+a^2x^2}\left(-4ia^3x^3\text{PolyLog}(2,-e^{i\text{ArcTan}(ax)})+4ia^3x^3\text{PolyLog}(2,e^{i\text{ArcTan}(ax)})+\sqrt{1+a^2x^2}\left(4a^2x^2+4(1+a^2x^2)\text{ArcTan}(ax)^2+\text{ArcTan}(ax)\left(ax\left(4-3\sqrt{1+a^2x^2}\log\left(1-e^{i\text{ArcTan}(ax)}\right)+3\sqrt{1+a^2x^2}\log\left(1+e^{i\text{ArcTan}(ax)}\right)\right)+(1+a^2x^2)\left(\log\left(1-e^{i\text{ArcTan}(ax)}\right)-\log\left(1+e^{i\text{ArcTan}(ax)}\right)\right)\sin(3\text{ArcTan}(ax))\right)\right)\right)}{12x^3\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/12*(c*Sqrt[1 + a^2*x^2]*((-4*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] +
 (4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*(4*a^2*x^2
 + 4*(1 + a^2*x^2)*ArcTan[a*x]^2 + ArcTan[a*x]*(a*x*(4 - 3*Sqrt[1 + a^2*x^2
]*Log[1 - E^(I*ArcTan[a*x])] + 3*Sqrt[1 + a^2*x^2]*Log[1 + E^(I*ArcTan[a*x]
)]) + (1 + a^2*x^2)*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])
])*Sin[3*ArcTan[a*x]]))))/(x^3*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.64, size = 195, normalized size = 0.71

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)^2a^2x^2+a^2x^2+\arctan(ax)ax+\arctan(ax)^2\right)}{3x^3} - \frac{ia^3\sqrt{c\,(ax-i)\,(ax+i)}\,\left(i\,\text{arcta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^2*a^2*x^2+a^2*x^2+arctan(a*x)*a
*x+arctan(a*x)^2)/x^3-1/3*I*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)*ln
```

```
(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{2}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**4,x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4,x)
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.315 $\int x^3(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=476

$$-\frac{17c\sqrt{c+a^2cx^2}}{280a^4}-\frac{17(c+a^2cx^2)^{3/2}}{1260a^4}+\frac{(c+a^2cx^2)^{5/2}}{105a^4c}+\frac{3cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{56a^3}-\frac{23cx^3\sqrt{c+a^2cx^2}\,\text{Arc}}{420a}$$

```
[Out] -17/1260*(a^2*c*x^2+c)^(3/2)/a^4+1/105*(a^2*c*x^2+c)^(5/2)/a^4/c-17/140*I*c
^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^
4/(a^2*c*x^2+c)^(1/2)+17/280*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-17/280*I*c^2*polylog(2,I*(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-17
/280*c*(a^2*c*x^2+c)^(1/2)/a^4+3/56*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3
-23/420*c*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-1/21*a*c*x^5*arctan(a*x)*(a
^2*c*x^2+c)^(1/2)-2/35*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4+1/35*c*x^2*a
rctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)^2*(a^2*c*x^2+c)
^(1/2)+1/7*a^2*c*x^6*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 2.79, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 75, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5070, 5072, 267, 5010, 5006, 5050, 272, 45}

$$\frac{cx^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{35a^2}+\frac{1}{7}a^2cx^6\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{1}{21}acx^5\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{8}{35}cx^4\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{23cx^3\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{420a}-\frac{17ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{140a^4\sqrt{a^2cx^2+c}}-\frac{2c\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{35a^4}+\frac{17ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{17ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{280a^4\sqrt{a^2cx^2+c}}+\frac{(a^2cx^2+c)^{5/2}}{105a^4c}-\frac{17(a^2cx^2+c)^{3/2}}{1260a^4}-\frac{17c\sqrt{a^2cx^2+c}}{280a^4}+\frac{3cx\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{56a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (-17*c*Sqrt[c + a^2*c*x^2])/(280*a^4) - (17*(c + a^2*c*x^2)^(3/2))/(1260*a^
4) + (c + a^2*c*x^2)^(5/2)/(105*a^4*c) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(56*a^3) - (23*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(420*a) - (a*c*
x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/21 - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(35*a^2) + (8*c
*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/7 - (((17*I)/140)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((17*I)/280)
*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
(a^4*Sqrt[c + a^2*c*x^2]) - (((17*I)/280)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = c\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx+\left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= c^2\int\frac{x^3\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx+2\left(\left(a^2c^2\right)\int\frac{x^5\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx\right)+\left(a^4c^2\right)\int$$

$$= \frac{cx^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^2}+\frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2-\frac{(2c^2)}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3}-\frac{1}{21}acx^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{2c\sqrt{c}}{}$$

$$= \frac{c\sqrt{c+a^2cx^2}}{3a^4}-\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3}+\frac{61cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(}{420a}$$

$$= \frac{c\sqrt{c+a^2cx^2}}{3a^4}-\frac{131cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{168a^3}+\frac{61cx^3\sqrt{c+a^2cx^2}\,\tan}{420a}$$

$$= \frac{139c\sqrt{c+a^2cx^2}}{168a^4}-\frac{2\left(c+a^2cx^2\right)^{3/2}}{63a^4}+\frac{\left(c+a^2cx^2\right)^{5/2}}{105a^4c}-\frac{131cx\sqrt{c+a^2}}{16}$$

$$= \frac{817c\sqrt{c+a^2cx^2}}{840a^4}-\frac{101\left(c+a^2cx^2\right)^{3/2}}{1260a^4}+\frac{\left(c+a^2cx^2\right)^{5/2}}{105a^4c}-\frac{131cx\sqrt{c+}}{}$$

Mathematica [A]
time = 3.24, size = 797, normalized size = 1.67

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]

[Out] (c*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*(-168*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) + (1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((19776*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) + ((19776*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) - 1266*ArcTan[a*x]*Sin[2*ArcTan[a*x]] + 360*ArcTan[a*x]*Sin[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Sin[6*ArcTan[a*x]])))/(161280*a^4)

Maple [A]
time = 1.65, size = 271, normalized size = 0.57

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(360\arctan(ax)^{2}a^{6}x^{6}-120\arctan(ax)a^{5}x^{5}+576\arctan(ax)^{2}a^{4}x^{4}+24a^{4}x^{4}-138\arctan(ax)a^{3}x^{3}+72\,a\right.}{2520a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2520*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(360*arctan(a*x)^2*a^6*x^6-120*arctan(a*x)*a^5*x^5+576*arctan(a*x)^2*a^4*x^4+24*a^4*x^4-138*arctan(a*x)*a^3*x^3+72*arctan(a*x)^2*a^2*x^2+14*a^2*x^2+135*arctan(a*x)*a*x-144*arctan(a*x)^2-163)-17/280*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(

```
1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(
a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.316 $\int x^2(c+a^2cx^2)^{3/2}\,\textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=531

$$\frac{cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2}+\frac{31c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{360a^3}-\frac{19cx^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{180a}-\frac{1}{15}acx^4$$

```
[Out] -41/360*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3+1/8*I*c^2*arct
an((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*
x^2+c)^(1/2)-1/8*I*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/8*I*c^2*arctan(a*x)*polylog(2
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1
/8*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2
*c*x^2+c)^(1/2)-1/8*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/36*c*x*(a^2*c*x^2+c)^(1/2)/a^2+1/60*c*x^3
*(a^2*c*x^2+c)^(1/2)+31/360*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-19/180*c*
x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-1/15*a*c*x^4*arctan(a*x)*(a^2*c*x^2+c
)^(1/2)+1/16*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+7/24*c*x^3*arctan(a*
x)^2*(a^2*c*x^2+c)^(1/2)+1/6*a^2*c*x^5*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 2.22, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 92, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

$$-\frac{19cx^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{180a}+\frac{cx\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{16a^2}+\frac{1}{6}a^2cx^5\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{1}{15}acx^4\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{7}{24}cx^3\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{cx\sqrt{a^2cx^2+c}}{36a^2}+\frac{1}{60}cx^3\sqrt{a^2cx^2+c}-\frac{ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}-\frac{c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{8a^3\sqrt{a^2cx^2+c}}+\frac{31c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{360a^3}-\frac{41c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{360a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(36*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2])/60 + (31*c
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(360*a^3) - (19*c*x^2*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x])/(180*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/15 + (c*x
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a^2) + (7*c*x^3*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/6 + ((I/
8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt
[c + a^2*c*x^2]) - (41*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/
(360*a^3) - ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*
x^2]) - (c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt
[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
```

```
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = c\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2$$

$$= c^2\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x^6}{\sqrt{}}$$

$$= \frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2} + \frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c^2\int \frac{\tan}{\sqrt{c}}}{}$$

$$= -\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3} - \frac{1}{15}acx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{cx\sqrt{c+a}}{}$$

$$= \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3} + \frac{41cx^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{180a}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}}{36a^2} + \frac{1}{60}cx^3\sqrt{c+a^2cx^2} - \frac{749c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{360a^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1101 vs. 2(531) = 1062.
time = 2.28, size = 1101, normalized size = 2.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(184*a*x*Sqrt[1 + a^2*x^2] + 128*a^3*x^3*Sqrt[1 + a^
2*x^2] - 56*a^5*x^5*Sqrt[1 + a^2*x^2] + 252*Sqrt[1 + a^2*x^2]*ArcTan[a*x] +
 264*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 12*a^4*x^4*Sqrt[1 + a^2*x^2]*A
rcTan[a*x] + 3690*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 4860*a^3*x^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]^2 + 1170*a^5*x^5*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 +
830*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 1770*a^2*x^2*ArcTan[a*x]*Cos[3*ArcTan[
a*x]] + 1050*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 110*a^6*x^6*ArcTan[a*
x]*Cos[3*ArcTan[a*x]] - 90*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^2*x^2*Arc
Tan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^4*x^4*ArcTan[a*x]*Cos[5*ArcTan[a*x]] -
90*a^6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 720*ArcTan[a*x]^2*Log[1 - I*E^(
I*ArcTan[a*x])] - 720*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x
])))/(2*E^((I/2)*ArcTan[a*x]))] + 720*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a
*x])] + 720*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/
2)*ArcTan[a*x])] - 720*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTa
n[a*x])))/E^((I/2)*ArcTan[a*x])] - 720*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)
*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] + 720*Pi*ArcTan[a*x]*Log[-Co
s[(Pi + 2*ArcTan[a*x])/4]] + 1312*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/
2]] - 720*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 1312
*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 720*ArcTan[a*x]^2*Log[Cos[A
rcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 720*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*Ar
cTan[a*x])/4]] - (1440*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] +
(1440*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 1440*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])] - 1440*PolyLog[3, I*E^(I*ArcTan[a*x])] + 132*Sin[3*ArcT
an[a*x]] + 156*a^2*x^2*Sin[3*ArcTan[a*x]] - 84*a^4*x^4*Sin[3*ArcTan[a*x]] -
 108*a^6*x^6*Sin[3*ArcTan[a*x]] - 1065*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 2
835*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 2475*a^4*x^4*ArcTan[a*x]^2*S
in[3*ArcTan[a*x]] - 705*a^6*x^6*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 52*Sin[5
*ArcTan[a*x]] - 156*a^2*x^2*Sin[5*ArcTan[a*x]] - 156*a^4*x^4*Sin[5*ArcTan[a
*x]] - 52*a^6*x^6*Sin[5*ArcTan[a*x]] + 45*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]]
+ 135*a^2*x^2*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 135*a^4*x^4*ArcTan[a*x]^2*
Sin[5*ArcTan[a*x]] + 45*a^6*x^6*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]]))/(11520*a
^3*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.70, size = 338, normalized size = 0.64

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(120\arctan\left(ax\right)^{2}a^{5}x^{5}-48\arctan\left(ax\right)a^{4}x^{4}+210\arctan\left(ax\right)^{2}a^{3}x^{3}+12a^{3}x^{3}-76\arctan\left(ax\right)a^{2}x^{2}+45\arctan\right.}{720a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/720*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)^2*a^5*x^5-48*arctan(
a*x)*a^4*x^4+210*arctan(a*x)^2*a^3*x^3+12*a^3*x^3-76*arctan(a*x)*a^2*x^2+45
*arctan(a*x)^2*a*x+20*a*x+62*arctan(a*x))-1/720*I*c*(c*(a*x-I)*(I+a*x))^(1/
2)*(45*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)
^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-90*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-164*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{2}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.317 $\int x(c+a^2cx^2)^{3/2}\,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=334

$$\frac{3c\sqrt{c+a^2cx^2}}{20a^2}+\frac{(c+a^2cx^2)^{3/2}}{30a^2}-\frac{3cx\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{20a}-\frac{x(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)}{10a}+\frac{(c+a^2cx^2)^{5/2}\,\mathrm{A}}{5a^2}$$

```
[Out] 1/30*(a^2*c*x^2+c)^(3/2)/a^2-1/10*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+1/5*(
a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a^2/c+3/10*I*c^2*arctan(a*x)*arctan((1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-3/20*I
*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a
^2*c*x^2+c)^(1/2)+3/20*I*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+3/20*c*(a^2*c*x^2+c)^(1/2)/a^2-3/2
0*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.17, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5050, 4998, 5010, 5006}

$$\frac{3ic^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{10a^2\sqrt{a^2cx^2+c}}+\frac{\mathrm{ArcTan}(ax)^2\,(a^2cx^2+c)^{5/2}}{5a^2c}-\frac{x\mathrm{ArcTan}(ax)\,(a^2cx^2+c)^{3/2}}{10a}-\frac{3cx\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{20a}-\frac{3ic^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{3ic^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{(a^2cx^2+c)^{3/2}}{30a^2}+\frac{3c\sqrt{a^2cx^2+c}}{20a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (3*c*Sqrt[c + a^2*c*x^2])/(20*a^2) + (c + a^2*c*x^2)^(3/2)/(30*a^2) - (3*c*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/(10*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) + (((3*I)/1
0)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2]) - (((3*I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2,
 ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) + (((3*
I)/20)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]
])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
```

```
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{5a^2c} - \frac{2\int\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)\,dx}{5a}$$

$$= \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a} + \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{5a^2c}$$

$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a} - \frac{x(c+}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a} - \frac{x(c+}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2} - \frac{3cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a} - \frac{x(c+}{}$$

Mathematica [A]
time = 2.86, size = 601, normalized size = 1.80

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(80*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Arc
Tan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
- ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a
*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 +
 a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - (1 + a^2*x^2)*(50 - 3
2*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*
x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcT
an[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])]
+ (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*Arc
Tan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*C
os[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[
a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[
a*x]*Sin[4*ArcTan[a*x]])))/(960*a^2)
```

Maple [A]
time = 0.25, size = 237, normalized size = 0.71

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(12\arctan\left(ax\right)^{2}a^{4}x^{4}-6\arctan\left(ax\right)a^{3}x^{3}+24\arctan\left(ax\right)^{2}a^{2}x^{2}+2a^{2}x^{2}-15\arctan\left(ax\right)ax+12\arctan\left(ax\right)\right.}{60a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(12*arctan(a*x)^2*a^4*x^4-6*arctan(a*x
)*a^3*x^3+24*arctan(a*x)^2*a^2*x^2+2*a^2*x^2-15*arctan(a*x)*a*x+12*arctan(a
*x)^2+11)+3/20*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2*(arctan(a*x)
*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.318 $\int (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=438

$$\frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{4a}-\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}{6a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2+$$

```
[Out] -1/6*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x
)^2+5/6*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a-3/4*I*c^2*arctan
((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+
c)^(1/2)+3/4*I*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4*I*c^2*arctan(a*x)*polylog(2,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4*c^2*p
olylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^
(1/2)+3/4*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)+1/12*c*x*(a^2*c*x^2+c)^(1/2)-3/4*c*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)/a+3/8*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212, 201}

$$\frac{3ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}+\frac{3c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{4a\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{4a\sqrt{a^2cx^2+c}}+\frac{3}{8}cx\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{3c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{4a}+\frac{1}{4}x\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{5c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{6a}+\frac{1}{12}cx\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/12 - (3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a)
- ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(6*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((3*I)/4)*c^2
*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2
*c*x^2]) + (5*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(6*a) + (
((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(a*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 +
 a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) + (
3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*
c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
```

```
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{6a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2+\frac{1}{6}c\int\sqrt{c+$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

$$= \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax}{6a}$$

Mathematica [A]
time = 1.42, size = 811, normalized size = 1.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(2*a*x*Sqrt[1 + a^2*x^2] + 2*a^3*x^3*Sqrt[1 + a^2*x^
2] - 94*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[
a*x] + 69*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 21*a^3*x^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]^2 - (96*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 96*ArcTa
nh[(a*x)/Sqrt[1 + a^2*x^2]] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^2*x^2
*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]]
- 12*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 12*Pi*ArcTan[a*x]*Log[((-
1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] + 12*ArcTan[
```

a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 12*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] + 12*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] + 16*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 12*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 16*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 12*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 12*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (72*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 72*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 72*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]] + 4*a^2*x^2*Sin[3*ArcTan[a*x]] + 2*a^4*x^4*Sin[3*ArcTan[a*x]] - 3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 6*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^4*x^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*a*Sqrt[1 + a^2*x^2])

Maple [A]
time = 0.35, size = 304, normalized size = 0.69

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(6\arctan\left(ax\right)^{2}a^{3}x^{3}-4\arctan\left(ax\right)a^{2}x^{2}+15\arctan\left(ax\right)^{2}ax+2ax-22\arctan\left(ax\right)\right)}{24a}+\frac{ic\sqrt{c\left(ax-i\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)^2*a^3*x^3-4*arctan(a*x)*a
^2*x^2+15*arctan(a*x)^2*a*x+2*a*x-22*arctan(a*x))+1/24*I*c*(c*(a*x-I)*(I+a*
x))^(1/2)*(9*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arctan
(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*I*polylog(3,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))-18*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(2,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-40*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2
+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
[Out] int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.319 $\int \frac{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}{x}\,dx$

Optimal. Leaf size=530

$$\frac{1}{3}c\sqrt{c+a^2cx^2}-\frac{1}{3}acx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2+$$

[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+14/3*I*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7/3*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+7/3*I*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*c*(a^2*c*x^2+c)^(1/2)-1/3*a*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)

Rubi [A]

time = 0.60, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5070, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5010, 5006, 4998}

$$\frac{2ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{2c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{14ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+c\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{1}{3}acx\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{3}\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{7ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{7ic^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{1}{3}c\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]

[Out] (c*Sqrt[c + a^2*c*x^2])/3 - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3 + (((14*I)/3)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((7*I)/3)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}\,dx + (a^2c)\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx$$

$$= \frac{1}{3}(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2 - \frac{1}{3}(2ac)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx + c^2$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

$$= \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax$$

Mathematica [A]
time = 2.12, size = 496, normalized size = 0.94

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((12*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^
2*Log[1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] +
 2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*Ar
cTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2
*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a
*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (1 + a^2*x^2)
*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 -
```

```
I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1
 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] -
 ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*Ar
cTan[a*x]])))/12
```

Maple [A]
time = 0.46, size = 365, normalized size = 0.69

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)^2a^2x^2-\arctan(ax)ax+4\arctan(ax)^2+1\right)}{3}-\frac{ic\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3i\arctan(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^2*a^2*x^2-arctan(a*x)*a*x+4*ar
ctan(a*x)^2+1)-1/3*I*c*(c*(a*x-I)*(I+a*x))^(1/2)*(3*I*arctan(a*x)^2*ln(1-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))+7*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*arctan(a*x)*ln(
1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-6*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+7*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{2}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.320 $\int \frac{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=556

$$-ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2-\frac{3iac^2\sqrt{1+a^2x^2}}{\ldots}$$

```
[Out] a*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-3*I*a*c^2*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-4
*a*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+3*I*a*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*arctan(a*x)*pol
ylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
+2*I*a*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-2*I*a*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a*c^2*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*c*arctan(a*x)*
(a^2*c*x^2+c)^(1/2)-c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x+1/2*a^2*c*x*arcta
n(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5070, 5064, 5078, 5074, 5010, 5008, 4266, 2611, 2320, 6724, 5000, 223, 212}

$$\frac{3iac^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ac^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ac^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3iac^2\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{4ac^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}a^2cx\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{c\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{x}-ac\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+ac^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{2iac^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] -(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^2)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - ((3*I)*a*c^2*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] -
(4*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a
*x]])/Sqrt[c + a^2*c*x^2] + a*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*
x^2]] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*a*c^2*Sqrt[
1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c
*x^2] - ((2*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx &= c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^2}\,dx+(a^2c)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2+c^2\int \frac{\tan}{x^2\sqrt{}} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2} \\
&= -ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)-\frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.74, size = 376, normalized size = 0.68

$$\frac{c\sqrt{c+a^2cx^2}\left(-2ax\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)-2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2+a^2x^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2-2iax\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2+2ax\tanh^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+4ax\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)+2ax\text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-2ax\text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-4ax\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)+4iax\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+6iax\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-6iax\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)-4iax\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-6ax\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)+6ax\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)}{2x\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-2*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 2*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]^2 + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (2*I)*a*x
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*a*x*ArcTanh[(a*x)/Sqrt[1 + a^2
```

```
*x^2]] + 4*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 2*a*x*ArcTan[a*x]^2
*Log[1 - I*E^(I*ArcTan[a*x])] - 2*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a
*x])] - 4*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*a*x*PolyLog[2,
 -E^(I*ArcTan[a*x])] + (6*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*
x])] - (6*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (4*I)*a*x*Po
lyLog[2, E^(I*ArcTan[a*x])] - 6*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 6*
a*x*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.43, size = 356, normalized size = 0.64

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)(\arctan(ax)a^2x^2-2ax-2\arctan(ax))}{2x} - \frac{iac\sqrt{c\,(ax-i)\,(ax+i)}\ \left(3i\arctan(ax)^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(arctan(a*x)*a^2*x^2-2*a*x-2*ar
ctan(a*x))/x-1/2*I*a*c*(c*(a*x-I)*(I+a*x))^(1/2)*(3*I*arctan(a*x)^2*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-4*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6
*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*
dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)))/
(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a^2x^2+1)\right)^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.321 $\int \frac{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=567

$$-\frac{ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}+a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2-\frac{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x^2}+\frac{4ia^2c^2\sqrt{1+a^2x^2}\ \ldots}{}$$

```
[Out] -a^2*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+4*I*a^2*c^2*arctan(a*x)*a
rctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-3*a^2*c^2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c^2*arctan(a*x
)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)-2*I*a^2*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a^2*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*c^2*polylog(3,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a^2*c^2*polylog
(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*c*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+a^2*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-1/
2*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 1.10, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5070, 5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5010, 5006}

$$\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3a^2c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3a^2c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{4ia^2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3a^2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+a^2c\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{ac\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-\frac{c\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2x^2}-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{2ia^2c^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ia^2c^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] -((a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) + a^2*c*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + ((4*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt
[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a^2*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]
/Sqrt[c]] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^2*c^2*S
qrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] + ((2*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*P
```

olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
```

```
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^3}\,dx + (a^2c)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}\,dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx + 2\left((a^2c^2)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx\right) + (a^4c^2)\int \frac{x\tan}{\sqrt{c}}$$

$$= a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} + (ac^2)\int \frac{\tan^{-}}{x^2\sqrt{c-}}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}}{2x}$$

Mathematica [A]
time = 2.13, size = 455, normalized size = 0.80

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-4*ArcTan[a*x] - 4*ArcTan[a*x]*Cot[ArcTan[a*x]/
2]^2 + 4*a*x*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]^2*Cot[ArcTan[
a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 -
```

```
E^(I*ArcTan[a*x])] - 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*ArcTa
n[a*x])] + 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x])] -
 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[Arc
Tan[a*x]/2]*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]
*PolyLog[2, -E^(I*ArcTan[a*x])] - (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] + (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*
x])] - (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])]
- 24*Cot[ArcTan[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 24*Cot[ArcTan[a*x]
/2]*PolyLog[3, E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]*Sec[Ar
cTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.56, size = 412, normalized size = 0.73

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)(2\arctan(ax)a^2x^2-2ax-\arctan(ax))}{2x^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\ \left(3\arctan(ax)^2\,\mathrm{l}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*arctan(a*x)*a^2*x^2-2*a*x-ar
ctan(a*x))/x^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(3*arctan(a*
x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*ar
ctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*ln(1+I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-
4*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln(1+(1+I*a*x)/(a^2*x^2+1)
^(1/2))-6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*a^2*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a^2x^2+1)\right)^{\frac{3}{2}} \operatorname{atan}^2{(ax)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.322 $\int \frac{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=579

$$-\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{x}-\frac{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^2}{3x^3}$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3-2*I*a^3*c^2*arctan((1+I*a*x)/(a^
2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-14/3*a^
3*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)+2*I*a^3*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a^3*c^2*arctan(a*x)*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)+7/3*I*a^3*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-7/3*I*a^3*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*a^3*c^2*polylog(3,-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*a^3*c^2*polyl
og(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1
/3*a^2*c*(a^2*c*x^2+c)^(1/2)/x-1/3*a*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2-
a^2*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]

time = 0.80, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5070, 5064, 5066, 5082, 270, 5078, 5074, 5010, 5008, 4266, 2611, 2320, 6724}

$$-\frac{a^2c\,\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{x}-\frac{ac\,\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3x^2}-\frac{\mathrm{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3x^3}-\frac{a^2c\sqrt{a^2cx^2+c}}{3x}+\frac{2ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2a^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{2a^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{14a^3c^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{7ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{7ia^3c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c*Sqrt[c + a^2*c*x^2])/x - (a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(3*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x - ((c + a^2*c*x^2)^(3
/2)*ArcTan[a*x]^2)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*A
rcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (14*a^3*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*
c*x^2]) + ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((7*I)/3)*a
^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sq
rt[c + a^2*c*x^2] - (((7*I)/3)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*a^3*c^2*Sqrt[1 + a^2*x^
2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*a^3*c^2*Sqr
t[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
```

```
2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^4}\,dx+\left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^2}\,dx$$

$$= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2ac)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x^3}\,dx+\left(a^2c\right.$$

$$= -\frac{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}-\frac{\left(c+a^2cx^2\right)^{3}}{3}$$

$$= -\frac{2a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(}{x}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(a}{x}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(a}{x}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(a}{x}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(a}{x}$$

Mathematica [A]
time = 5.20, size = 453, normalized size = 0.78

$$\frac{a^3c^2\sqrt{1+a^2x^2}\left(8i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-24\left(\frac{\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2}{ax}-2\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-\text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right)+2\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)-2i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)+2i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)-2\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)-\frac{2\left(1+a^2x^2\right)^{3/2}\left(2+4\text{ArcTan}(ax)^2-2\cos(2\text{ArcTan}(ax))+\frac{4ia^3x^3\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)}{\left(1+a^2x^2\right)^{3/2}}+\text{ArcTan}(ax)\left(2\sin(2\text{ArcTan}(ax))+\frac{\left(\log\left(1-e^{i\text{ArcTan}(ax)}\right)-\log\left(1+e^{i\text{ArcTan}(ax)}\right)\right)\left(-3ax+\sqrt{1+a^2x^2}\sin(3\text{ArcTan}(ax))\right)}{\sqrt{1+a^2x^2}}\right)\right)}{a^3x^3}\right)}{24\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] (a^3*c^2*Sqrt[1 + a^2*x^2]*((8*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - 24*((Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x
])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 + I*
E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (2*I)*PolyL
og[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a
*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2
, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3,
I*E^(I*ArcTan[a*x])]) - (2*(1 + a^2*x^2)^(3/2)*(2 + 4*ArcTan[a*x]^2 - 2*Cos
[2*ArcTan[a*x]] + ((4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^
2)^(3/2) + ArcTan[a*x]*(2*Sin[2*ArcTan[a*x]] + ((Log[1 - E^(I*ArcTan[a*x])]
 - Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x
]]))/Sqrt[1 + a^2*x^2]))))/(a^3*x^3)))/(24*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.71, size = 343, normalized size = 0.59

method	result
default	$-\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(4\arctan(ax)^{2}a^{2}x^{2}+a^{2}x^{2}+\arctan(ax)ax+\arctan(ax)^{2}\right)}{3x^{3}}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3\,\mathrm{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c*(c*(a*x-I)*(I+a*x))^(1/2)*(4*arctan(a*x)^2*a^2*x^2+a^2*x^2+arctan(a*
x)*a*x+arctan(a*x)^2)/x^3-1/3*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(
3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arct
an(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*dilog((1+I*a*x)/(a^2*x^2+1)^(
1/2))-7*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c*a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^2(ax)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.323 $\int x^3(c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=578

$$-\frac{115c^2\sqrt{c+a^2cx^2}}{4032a^4}-\frac{115c(c+a^2cx^2)^{3/2}}{18144a^4}-\frac{23(c+a^2cx^2)^{5/2}}{7560a^4}+\frac{(c+a^2cx^2)^{7/2}}{252a^4c}+\frac{47c^2x\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax}{1344a^3}$$

```
[Out] -115/18144*c*(a^2*c*x^2+c)^(3/2)/a^4-23/7560*(a^2*c*x^2+c)^(5/2)/a^4+1/252*
(a^2*c*x^2+c)^(7/2)/a^4/c+115/4032*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-115/4032*I*c^3*polylo
g(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^
(1/2)-115/2016*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-115/4032*c^2*(a^2*c*x^2+c)^(1/2)/a^
4+47/1344*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-205/6048*c^2*x^3*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)/a-103/1512*a*c^2*x^5*arctan(a*x)*(a^2*c*x^2+c)^(1
/2)-1/36*a^3*c^2*x^7*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-2/63*c^2*arctan(a*x)^2
*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2
+5/21*c^2*x^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+19/63*a^2*c^2*x^6*arctan(a*
x)^2*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 7.30, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 203, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5070, 5072, 267, 5010, 5006, 5050, 272, 45}

$$\frac{c^2x^2\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{63a^2}+\frac{19}{63}a^2c^2x^6\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{103ac^2x^5\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{1512}+\frac{5}{21}c^2x^4\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{205c^2x^3\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{6048a}-\frac{115ic^3\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2016a^4\sqrt{a^2cx^2+c}}-\frac{2c^2\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{63a^4}+\frac{1}{9}a^4c^2x^8\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{115ic^3\sqrt{a^2x^2+1}\operatorname{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115ic^3\sqrt{a^2x^2+1}\operatorname{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{4032a^4\sqrt{a^2cx^2+c}}-\frac{115c^2\sqrt{a^2cx^2+c}}{4032a^4}+\frac{(a^2cx^2+c)^{7/2}}{252a^4c}-\frac{23(a^2cx^2+c)^{5/2}}{7560a^4}-\frac{115c(a^2cx^2+c)^{3/2}}{18144a^4}+\frac{47c^2x\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{1344a^3}-\frac{1}{36}a^3c^2x^7\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (-115*c^2*Sqrt[c + a^2*c*x^2])/(4032*a^4) - (115*c*(c + a^2*c*x^2)^(3/2))/(
18144*a^4) - (23*(c + a^2*c*x^2)^(5/2))/(7560*a^4) + (c + a^2*c*x^2)^(7/2)/
(252*a^4*c) + (47*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(1344*a^3) - (205*
c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(6048*a) - (103*a*c^2*x^5*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x])/1512 - (a^3*c^2*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/36 - (2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^4) + (c^2*x^2*Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/63 + (a
^4*c^2*x^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/9 - (((115*I)/2016)*c^3*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt
[c + a^2*c*x^2]) + (((115*I)/4032)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*S
qrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) - (((115*I)/403
2)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3 \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^2 \, dx = c \int x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2 \, dx + \left(a^2c\right) \int x^5 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(a$$

$$= c^2 \int x^3 \sqrt{c+a^2cx^2} \tan^{-1}(ax)^2 \, dx + 2\left(\left(a^2c^2\right) \int x^5 \sqrt{c+a^2cx^2} \tan^{-1}(\right.$$

$$= c^3 \int \frac{x^3 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx + \left(a^2c^3\right) \int \frac{x^5 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} \, dx + \left(a^4c^3\right) \int \frac{x^7 \tan^{-}}{\sqrt{c+}}$$

$$= \frac{c^2x^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2}{3a^2} + \frac{1}{5}c^2x^4\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2 + \frac{1}{7}a^2c^2x^{6}$$

$$= -\frac{c^2x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{3a^3} - \frac{c^2x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{10a} - \frac{1}{21}ac^2x^5\sqrt{}$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{3a^4} + \frac{c^2x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{12a^3} + \frac{19c^2x^3\sqrt{c+a^2cx^2} \tan^{-}}{420a}$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{12a^4} - \frac{61c^2x\sqrt{c+a^2cx^2} \tan^{-1}(ax)}{168a^3} - \frac{3761c^2x^3\sqrt{c+a^2cx^{}}}{30240a}$$

$$= \frac{713c^2\sqrt{c+a^2cx^2}}{2520a^4} + \frac{37c(c+a^2cx^2)^{3/2}}{1260a^4} - \frac{(c+a^2cx^2)^{5/2}}{140a^4} + \frac{(c+a^2cx^2)^{7}}{252a^4c}$$

$$= -\frac{6299c^2\sqrt{c+a^2cx^2}}{60480a^4} + \frac{349c(c+a^2cx^2)^{3/2}}{11340a^4} - \frac{167(c+a^2cx^2)^{5/2}}{7560a^4} + \frac{(c+}{2}$$

$$= -\frac{5519c^2\sqrt{c+a^2cx^2}}{20160a^4} + \frac{7921c(c+a^2cx^2)^{3/2}}{90720a^4} - \frac{167(c+a^2cx^2)^{5/2}}{7560a^4} + \frac{(c-}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1320 vs. 2(578) = 1156.
time = 5.09, size = 1320, normalized size = 2.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*(-48384*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x
]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*Ar
cTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*
Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcT
an[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*Arc
Tan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I
*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2
) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTa
n[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) + 576*(1 + a
^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan
[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*C
os[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x]*Log[1 - I*E
^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*
Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 -
I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 - I*E^(I*Ar
cTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2
*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] +
2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 309*ArcT
an[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((19776*I)*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) + ((19776*I)*PolyLog[2, I
*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(7/2) - 1266*ArcTan[a*x]*Sin[2*ArcTan[a*
x]] + 360*ArcTan[a*x]*Sin[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Sin[6*ArcTan[a*x
]]) - (1 + a^2*x^2)^2*(657578 - 820224*ArcTan[a*x]^2 + 1083168*Cos[2*ArcTan
[a*x]] + 3276288*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 576936*Cos[4*ArcTan[a*x
]] - 580608*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 184160*Cos[6*ArcTan[a*x]] +
483840*ArcTan[a*x]^2*Cos[6*ArcTan[a*x]] + 32814*Cos[8*ArcTan[a*x]] - (20672
82*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 1378188*Ar
cTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 590652*ArcTan[a
*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 147663*ArcTan[a*x]*Co
s[7*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 16407*ArcTan[a*x]*Cos[9*Arc
Tan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (2067282*ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 1378188*ArcTan[a*x]*Cos[3*ArcTan[a*x]]
*Log[1 + I*E^(I*ArcTan[a*x])] + 590652*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1
 + I*E^(I*ArcTan[a*x])] + 147663*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E
^(I*ArcTan[a*x])] + 16407*ArcTan[a*x]*Cos[9*ArcTan[a*x]]*Log[1 + I*E^(I*Arc
Tan[a*x])] - ((4200192*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)
^(9/2) + ((4200192*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(9/2)
+ 78444*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 160452*ArcTan[a*x]*Sin[4*ArcTan[a*
x]] + 38172*ArcTan[a*x]*Sin[6*ArcTan[a*x]] - 32814*ArcTan[a*x]*Sin[8*ArcTan
[a*x]])))/(46448640*a^4)
```

Maple [A]
time = 1.75, size = 309, normalized size = 0.53

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(20160\arctan(ax)^2 a^8x^8-5040\arctan(ax)a^7x^7+54720\arctan(ax)^2a^6x^6+720a^6x^6-12360\arctan(ax)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/181440*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(20160*arctan(a*x)^2*a^8*x^8-504
0*arctan(a*x)*a^7*x^7+54720*arctan(a*x)^2*a^6*x^6+720*a^6*x^6-12360*arctan(
a*x)*a^5*x^5+43200*arctan(a*x)^2*a^4*x^4+1608*a^4*x^4-6150*arctan(a*x)*a^3*
x^3+2880*arctan(a*x)^2*a^2*x^2-94*a^2*x^2+6345*arctan(a*x)*a*x-5760*arctan(
a*x)^2-6157)-115/4032*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4*(arct
an(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.324 $\int x^2 (c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=638

$$\frac{43c^2x\sqrt{c+a^2cx^2}}{4032a^2}+\frac{29c^2x^3\sqrt{c+a^2cx^2}}{1680}+\frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2}+\frac{1373c^2\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}{20160a^3}-\frac{737c^2x^2\cdots}{}$$

```
[Out] -397/5040*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3-5/64*I*c^3*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/
(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a
*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*arctan(a*x)*poly
log(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1
/2)+5/64*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
3/(a^2*c*x^2+c)^(1/2)-5/64*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+43/4032*c^2*x*(a^2*c*x^2+c)^(1/2)/a^
2+29/1680*c^2*x^3*(a^2*c*x^2+c)^(1/2)+1/168*a^2*c^2*x^5*(a^2*c*x^2+c)^(1/2)
+1373/20160*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-737/10080*c^2*x^2*arcta
n(a*x)*(a^2*c*x^2+c)^(1/2)/a-83/840*a*c^2*x^4*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)-1/28*a^3*c^2*x^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+5/128*c^2*x*arctan(a*x)
^2*(a^2*c*x^2+c)^(1/2)/a^2+59/192*c^2*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
+17/48*a^2*c^2*x^5*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/8*a^4*c^2*x^7*arctan
(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 5.82, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 238, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

$$-\frac{737c^2x^2\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{10080a}+\frac{5c^2x\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{128a^2}+\frac{17}{48}a^2c^2x^5\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{83}{840}ac^2x^4\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{59}{192}c^2x^3\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{43c^2x\sqrt{a^2cx^2+c}}{4032a^2}+\frac{1}{168}a^2c^2x^5\sqrt{a^2cx^2+c}+\frac{29c^2x^3\sqrt{a^2cx^2+c}}{1680}+\frac{1}{8}a^4c^2x^7\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{5ic^3\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(-ie^{i\operatorname{ArcTan}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(ie^{i\operatorname{ArcTan}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5c^3\sqrt{a^2x^2+1}\operatorname{Li}_3\left(-ie^{i\operatorname{ArcTan}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}-\frac{5c^3\sqrt{a^2x^2+1}\operatorname{Li}_3\left(ie^{i\operatorname{ArcTan}(ax)}\right)}{64a^3\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\operatorname{ArcTan}\left(e^{i\operatorname{ArcTan}(ax)}\right)\operatorname{ArcTan}(ax)^2}{64a^3\sqrt{a^2cx^2+c}}+\frac{1373c^2\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{20160a^3}-\frac{1}{28}a^3c^2x^6\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{397c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{5040a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (43*c^2*x*Sqrt[c + a^2*c*x^2])/(4032*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]
)/1680 + (a^2*c^2*x^5*Sqrt[c + a^2*c*x^2])/168 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(20160*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
)/(10080*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/840 - (a^3*c^2
*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/28 + (5*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/192
+ (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/48 + (a^4*c^2*x^7*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan
[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) - (397*c^(5/2)
*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(5040*a^3) - (((5*I)/64)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[
```

```
c + a^2*c*x^2]) + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (5*c^3*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2]) - (5*c^3*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^
2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = c\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx + \left(a^2c\right)\int x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(a$$

$$= c^2\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx + 2\Bigg(\left(a^2c^2\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}($$

$$= c^3\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c^3\right)\int \frac{x^4\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \left(a^4c^3\right)\int \frac{x^6\tan^{-}}{\sqrt{c+}}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2} + \frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{1}{6}a^2c^2x^5$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3} - \frac{c^2x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a} - \frac{1}{15}ac^2x^4\sqrt{c}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}}{12a^2} + \frac{1}{60}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} + \frac{c^2\sqrt{}}{}$$

$$= -\frac{c^2x\sqrt{c+a^2cx^2}}{18a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} - \frac{3}{}$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

$$= \frac{491c^2x\sqrt{c+a^2cx^2}}{4032a^2} - \frac{9}{560}c^2x^3\sqrt{c+a^2cx^2} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1543 vs. 2(638) = 1276.
time = 3.58, size = 1543, normalized size = 2.42

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(35678*a*x*Sqrt[1 + a^2*x^2] + 24602*a^3*x^3*Sqrt[
1 + a^2*x^2] - 4070*a^5*x^5*Sqrt[1 + a^2*x^2] + 7006*a^7*x^7*Sqrt[1 + a^2*x
^2] + 21002*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 49890*a^2*x^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x] - 109026*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 38134*a^6*x^6
*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 1273965*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^
2 + 2168775*a^3*x^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 1080135*a^5*x^5*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^2 + 185325*a^7*x^7*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
 + 202902*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 439768*a^2*x^2*ArcTan[a*x]*Cos[3
*ArcTan[a*x]] + 263172*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 18648*a^6*x
^6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 7658*a^8*x^8*ArcTan[a*x]*Cos[3*ArcTan[a
*x]] - 51310*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 164920*a^2*x^2*ArcTan[a*x]*Co
s[5*ArcTan[a*x]] - 186900*a^4*x^4*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 84280*a^
6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 10990*a^8*x^8*ArcTan[a*x]*Cos[5*ArcT
an[a*x]] + 3150*ArcTan[a*x]*Cos[7*ArcTan[a*x]] + 12600*a^2*x^2*ArcTan[a*x]*
Cos[7*ArcTan[a*x]] + 18900*a^4*x^4*ArcTan[a*x]*Cos[7*ArcTan[a*x]] + 12600*a
^6*x^6*ArcTan[a*x]*Cos[7*ArcTan[a*x]] + 3150*a^8*x^8*ArcTan[a*x]*Cos[7*ArcT
an[a*x]] - 100800*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 100800*Pi*Ar
cTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x
]))] + 100800*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 100800*ArcTan[a*
x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 10
0800*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)
*ArcTan[a*x])] - 100800*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*
x]))/(2*E^((I/2)*ArcTan[a*x]))] + 100800*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*Ar
cTan[a*x])/4]] + 203264*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 1008
00*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 203264*Log[
Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 100800*ArcTan[a*x]^2*Log[Cos[Arc
Tan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 100800*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*A
rcTan[a*x])/4]] - (201600*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 + (201600*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 201600*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])] - 201600*PolyLog[3, I*E^(I*ArcTan[a*x])] + 17622
*Sin[3*ArcTan[a*x]] + 11352*a^2*x^2*Sin[3*ArcTan[a*x]] - 17916*a^4*x^4*Sin[
3*ArcTan[a*x]] + 600*a^6*x^6*Sin[3*ArcTan[a*x]] + 12246*a^8*x^8*Sin[3*ArcTa
n[a*x]] - 490455*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 1484700*a^2*x^2*ArcTan[
a*x]^2*Sin[3*ArcTan[a*x]] - 1592010*a^4*x^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]
] - 691740*a^6*x^6*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 93975*a^8*x^8*ArcTan[
a*x]^2*Sin[3*ArcTan[a*x]] - 15618*Sin[5*ArcTan[a*x]] - 39176*a^2*x^2*Sin[5*
```

ArcTan[a*x]] - 23820*a^4*x^4*Sin[5*ArcTan[a*x]] + 7416*a^6*x^6*Sin[5*ArcTan[a*x]] + 7678*a^8*x^8*Sin[5*ArcTan[a*x]] + 61845*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 227220*a^2*x^2*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 310590*a^4*x^4*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 186900*a^6*x^6*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 41685*a^8*x^8*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 2438*Sin[7*ArcTan[a*x]] + 9752*a^2*x^2*Sin[7*ArcTan[a*x]] + 14628*a^4*x^4*Sin[7*ArcTan[a*x]] + 9752*a^6*x^6*Sin[7*ArcTan[a*x]] + 2438*a^8*x^8*Sin[7*ArcTan[a*x]] - 1575*ArcTan[a*x]^2*Sin[7*ArcTan[a*x]] - 6300*a^2*x^2*ArcTan[a*x]^2*Sin[7*ArcTan[a*x]] - 9450*a^4*x^4*ArcTan[a*x]^2*Sin[7*ArcTan[a*x]] - 6300*a^6*x^6*ArcTan[a*x]^2*Sin[7*ArcTan[a*x]] - 1575*a^8*x^8*ArcTan[a*x]^2*Sin[7*ArcTan[a*x]]))/(2580480*a^3*Sqrt[1 + a^2*x^2])

Maple [A]
time = 0.76, size = 376, normalized size = 0.59

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(5040\arctan(ax)^2a^7x^7-1440\arctan(ax)a^6x^6+14280\arctan(ax)^2a^5x^5+240a^5x^5-3984\arctan(ax)a^4\right.}{40320a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40320*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(5040*arctan(a*x)^2*a^7*x^7-1440*arctan(a*x)*a^6*x^6+14280*arctan(a*x)^2*a^5*x^5+240*a^5*x^5-3984*arctan(a*x)*a^4*x^4+12390*arctan(a*x)^2*a^3*x^3+696*a^3*x^3-2948*arctan(a*x)*a^2*x^2+1575*arctan(a*x)^2*a*x+430*a*x+2746*arctan(a*x))-1/40320*I*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(1575*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1575*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6352*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.325 $\int x(c+a^2cx^2)^{5/2} \,\mathbf{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=387

$$\frac{5c^2\sqrt{c+a^2cx^2}}{56a^2}+\frac{5c(c+a^2cx^2)^{3/2}}{252a^2}+\frac{(c+a^2cx^2)^{5/2}}{105a^2}-\frac{5c^2x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{56a}-\frac{5cx(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}}{84a}$$

```
[Out] 5/252*c*(a^2*c*x^2+c)^(3/2)/a^2+1/105*(a^2*c*x^2+c)^(5/2)/a^2-5/84*c*x*(a^2
*c*x^2+c)^(3/2)*arctan(a*x)/a-1/21*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a+1/7*
(a^2*c*x^2+c)^(7/2)*arctan(a*x)^2/a^2/c+5/28*I*c^3*arctan(a*x)*arctan((1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-5/56*
I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(
a^2*c*x^2+c)^(1/2)+5/56*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+5/56*c^2*(a^2*c*x^2+c)^(1/2)/a^2-
5/56*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]

time = 0.21, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5050, 4998, 5010, 5006}

$$\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{28a^2\sqrt{a^2cx^2+c}}-\frac{5c^2x\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{56a}+\frac{\mathrm{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{x\mathrm{ArcTan}(ax)\left(a^2cx^2+c\right)^{5/2}}{21a}-\frac{5cx\mathrm{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{84a}-\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5ic^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{5c^2\sqrt{a^2cx^2+c}}{56a^2}+\frac{\left(a^2cx^2+c\right)^{5/2}}{105a^2}+\frac{5c(a^2cx^2+c)^{3/2}}{252a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2])/(56*a^2) + (5*c*(c + a^2*c*x^2)^(3/2))/(252*a^2
) + (c + a^2*c*x^2)^(5/2)/(105*a^2) - (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/(56*a) - (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(84*a) - (x*(c + a^
2*c*x^2)^(5/2)*ArcTan[a*x])/(21*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^2)/
(7*a^2*c) + (((5*I)/28)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2]) - (((5*I)/56)*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c
+ a^2*c*x^2]) + (((5*I)/56)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)^2}{7a^2c} - \frac{2\int\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)\,dx}{7a}$$

$$= \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{21a} + \frac{\left(c+a^2cx^2\right)^{7/2}\tan^{-1}(ax)}{7a^2c}$$

$$= \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{84a} - \frac{x(c}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx}}{56a}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx}}{56a}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2} + \frac{5c\left(c+a^2cx^2\right)^{3/2}}{252a^2} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c+a^2cx}}{56a}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1039 vs. 2(387) = 774.

time = 6.60, size = 1039, normalized size = 2.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(13440*(2 + 4*ArcTan[a*x]^2 + 2*Cos[
2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*
x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*Arc
Tan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[
3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])
/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - 336*(1 + a^2*x^2
)*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*
ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcT
an[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*
E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTa
n[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
 + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcT
an[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[
2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(
I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 2
2*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) + (1 + a^2*x^2)^2*(4116 + 10944*ArcTan[a*
x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 27
64*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*A
rcTan[a*x]] - (10815*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2
*x^2] - 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] -
2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcT
an[a*x]*Cos[7*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x
]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*
ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a
*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[
1 + I*E^(I*ArcTan[a*x])] - ((19776*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(
1 + a^2*x^2)^(7/2) + ((19776*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x
^2)^(7/2) - 1266*ArcTan[a*x]*Sin[2*ArcTan[a*x]] + 360*ArcTan[a*x]*Sin[4*Arc
Tan[a*x]] - 618*ArcTan[a*x]*Sin[6*ArcTan[a*x]])))/(161280*a^2)
```

Maple [A]
time = 0.53, size = 275, normalized size = 0.71

method	result

default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(360\arctan(ax)^2 a^6x^6-120\arctan(ax)a^5x^5+1080\arctan(ax)^2a^4x^4+24a^4x^4-390\arctan(ax)a^3x^3+\right.}{2520a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2520*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(360*arctan(a*x)^2*a^6*x^6-120*arc
tan(a*x)*a^5*x^5+1080*arctan(a*x)^2*a^4*x^4+24*a^4*x^4-390*arctan(a*x)*a^3*
x^3+1080*arctan(a*x)^2*a^2*x^2+98*a^2*x^2-495*arctan(a*x)*a*x+360*arctan(a*
x)^2+299)+5/56*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2*(arctan(a*x)
*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(a
*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.326 $\int (c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^2 \, dx$

Optimal. Leaf size=516

$$\frac{17}{180}c^2x\sqrt{c+a^2cx^2}+\frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2}-\frac{5c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{8a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}{36a}-\frac{(c\cdots}{}$$

```
[Out] 1/60*c*x*(a^2*c*x^2+c)^(3/2)-5/36*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a-1/15*
(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^
2+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2+259/360*c^(5/2)*arctanh(a*x*c^(1/
2)/(a^2*c*x^2+c)^(1/2))/a-5/8*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arc
tan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/8*I*c^3*arctan(a*x)*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(
1/2)-5/8*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/8*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/8*c^3*polylog(3,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+17/180*c^2*x*
(a^2*c*x^2+c)^(1/2)-5/8*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*ar
ctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212, 201}

$$\frac{5ic^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5c^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}+\frac{5c^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{8a\sqrt{a^2cx^2+c}}-\frac{5ic^3\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{8a\sqrt{a^2cx^2+c}}+\frac{5}{16}c^2x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{5c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{8a}+\frac{5}{24}cx\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{5c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{36a}+\frac{1}{6}x\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{259c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{360a}+\frac{17}{180}c^2x\sqrt{a^2cx^2+c}+\frac{1}{60}cx\left(a^2cx^2+c\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (17*c^2*x*Sqrt[c + a^2*c*x^2])/180 + (c*x*(c + a^2*c*x^2)^(3/2))/60 - (5*c^
2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(8*a) - (5*c*(c + a^2*c*x^2)^(3/2)*ArcTa
n[a*x])/(36*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(15*a) + (5*c^2*x*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x
]^2)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/6 - (((5*I)/8)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[c + a^2*c*x^2]
) + (259*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(360*a) + (((5
*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (5*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) + (5*c
^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x
^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
```

```
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}{15a} + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2 + \frac{1}{15}c\int \left(c+a\right.$$

$$= \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{36a} - \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(a\right.}{15a}$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

$$= \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx\left(c+a^2cx^2\right)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{8a} -$$

<u>**Mathematica**</u> [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1143 vs. 2(516) = 1032.
time = 2.45, size = 1143, normalized size = 2.22

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(424*a*x*Sqrt[1 + a^2*x^2] + 368*a^3*x^3*Sqrt[1 +
a^2*x^2] - 56*a^5*x^5*Sqrt[1 + a^2*x^2] - 11028*Sqrt[1 + a^2*x^2]*ArcTan[a*
x] + 504*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 12*a^4*x^4*Sqrt[1 + a^2*x^
2]*ArcTan[a*x] + 11970*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 7380*a^3*x^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 1170*a^5*x^5*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
```

```
^2 - (11520*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 11520*ArcTanh[(a*x
)/Sqrt[1 + a^2*x^2]] + 1550*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 3210*a^2*x^2*A
rcTan[a*x]*Cos[3*ArcTan[a*x]] + 1770*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]]
 + 110*a^6*x^6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 90*ArcTan[a*x]*Cos[5*ArcTan
[a*x]] - 270*a^2*x^2*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^4*x^4*ArcTan[a*
x]*Cos[5*ArcTan[a*x]] - 90*a^6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 2160*Ar
cTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 2160*Pi*ArcTan[a*x]*Log[((-1)^(1
/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] + 2160*ArcTan[a*x
]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2160*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I
+ E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 2160*Pi*ArcTan[a*x]*Log[-1/2
*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 2160*ArcTan
[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))
] + 2160*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] + 3232*Log[Cos[Ar
cTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 2160*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]
/2] - Sin[ArcTan[a*x]/2]] - 3232*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2
]] + 2160*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 2160
*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (7200*I)*ArcTan[a*x]*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - (7200*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*Ar
cTan[a*x])] - 7200*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 7200*PolyLog[3, I*E
^(I*ArcTan[a*x])] + 372*Sin[3*ArcTan[a*x]] + 636*a^2*x^2*Sin[3*ArcTan[a*x]]
 + 156*a^4*x^4*Sin[3*ArcTan[a*x]] - 108*a^6*x^6*Sin[3*ArcTan[a*x]] - 1425*A
rcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3555*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a
*x]] - 2835*a^4*x^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 705*a^6*x^6*ArcTan[a
*x]^2*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan[a*x]] - 156*a^2*x^2*Sin[5*ArcTan
[a*x]] - 156*a^4*x^4*Sin[5*ArcTan[a*x]] - 52*a^6*x^6*Sin[5*ArcTan[a*x]] + 4
5*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 135*a^2*x^2*ArcTan[a*x]^2*Sin[5*ArcTan
[a*x]] + 135*a^4*x^4*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 45*a^6*x^6*ArcTan[a
*x]^2*Sin[5*ArcTan[a*x]]))/(11520*a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.41, size = 342, normalized size = 0.66

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(120\arctan(ax)^2a^5x^5-48\arctan(ax)a^4x^4+390\arctan(ax)^2a^3x^3+12a^3x^3-196\arctan(ax)a^2x^2+49\right.}{720a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/720*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)^2*a^5*x^5-48*arctan(
a*x)*a^4*x^4+390*arctan(a*x)^2*a^3*x^3+12*a^3*x^3-196*arctan(a*x)*a^2*x^2+4
95*arctan(a*x)^2*a*x+80*a*x-598*arctan(a*x))+1/720*I*c^2*(c*(a*x-I)*(I+a*x)
)^(1/2)*(225*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-225*I*arct
an(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*I*polylog(3,-I*(1+I*a*x)/
```

```
(a^2*x^2+1)^(1/2))-450*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arcta
n(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*arctan(a*x)*polylog(2,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1036*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/
(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
[Out] int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.327 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^2}{x}\,dx$

Optimal. Leaf size=605

$$\frac{29}{60}c^2\sqrt{c+a^2cx^2}+\frac{1}{30}c(c+a^2cx^2)^{3/2}-\frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{1}{10}acx(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)+c$$

```
[Out] 1/30*c*(a^2*c*x^2+c)^(3/2)-1/10*a*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/3*c
*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2+14
9/30*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^3*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c^3*arctan(a*x)*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*c^
3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2
*c*x^2+c)^(1/2)-149/60*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/60*I*c^3*polylog(2,I*(1+I*a*x)^(1
/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^3*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*c^3*p
olylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
+29/60*c^2*(a^2*c*x^2+c)^(1/2)-29/60*a*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2
)+c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.87, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5070, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5010, 5006, 4998}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] (29*c^2*Sqrt[c + a^2*c*x^2])/60 + (c*(c + a^2*c*x^2)^(3/2))/30 - (29*a*c^2*
x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 - (a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan
[a*x])/10 + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2 + (c*(c + a^2*c*x^2)^(3/2
)*ArcTan[a*x]^2)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/5 + (((149*I)/30
)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/Sqrt[c + a^2*c*x^2] - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((2*I)*c^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((149*I)/60)*c^3*Sqrt[1 + a^2*x^2]*Po
```

```
lyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (2*c^3
*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2
*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{x}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x}\,dx + (a^2c)\int x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2\,d$$

$$= \frac{1}{5}(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2 - \frac{1}{5}(2ac)\int (c+a^2cx^2)^{3/2}\tan^{-1}(ax)\,dx + c$$

$$= \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{1}{10}acx(c+a^2cx^2)^{3/2}\tan^{-1}(ax) + \frac{1}{3}c(c+a^2cx^2)^{3/2}$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

$$= \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c(c+a^2cx^2)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)$$

Mathematica [A]
time = 6.32, size = 839, normalized size = 1.39

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*((960*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*
x]^2*Log[1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])
] + 2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I
*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] -
 (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTa
```

```
n[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + 160*(1 + a
^2*x^2)*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1
- I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*
Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])
/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a
*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((
4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*S
in[2*ArcTan[a*x]]) - (1 + a^2*x^2)^2*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcT
an[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (
110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan
[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[
5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^
(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(
I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2
)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4
*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]])))/960
```

Maple [A]
time = 0.50, size = 404, normalized size = 0.67

method	result
default	$\frac{c^2\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(12\arctan(ax)^2a^4x^4-6\arctan(ax)a^3x^3+44\arctan(ax)^2a^2x^2+2a^2x^2-35\arctan(ax)ax+92\arctan(a\right.}{60}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(12*arctan(a*x)^2*a^4*x^4-6*arctan(a*x)*
a^3*x^3+44*arctan(a*x)^2*a^2*x^2+2*a^2*x^2-35*arctan(a*x)*a*x+92*arctan(a*x
)^2+31)-1/60*I*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(60*I*arctan(a*x)^2*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-60*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+149*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*I*arctan(a*x)*l
n(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1
/2))-120*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*arctan(a*x)*polylog(
2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))+149*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*dilog(1-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x, x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x,x)

[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x,x)

[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x, x)

3.328 $\int \frac{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=655

$$\frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2}-\frac{7}{4}ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{1}{6}ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)-\frac{c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}}{x}$$

```
[Out] -1/6*a*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*ar
ctan(a*x)^2+11/6*a*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-15/4*I*
a*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)-4*a*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15/4*I*a*c^3*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15
/4*I*a*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*c^3*polylog(2,(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15/4*a*c^3*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
+15/4*a*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2
*c*x^2+c)^(1/2)+1/12*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)-7/4*a*c^2*arctan(a*x)*(a
^2*c*x^2+c)^(1/2)-c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x+7/8*a^2*c^2*x*arc
tan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 1.01, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5070, 5064, 5078, 5074, 5010, 5008, 4266, 2611, 2320, 6724, 5000, 223, 212, 201}

$$\frac{15iac^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{4\sqrt{a^2cx^2+c}}-\frac{15iac^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{4\sqrt{a^2cx^2+c}}-\frac{15ac^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{4\sqrt{a^2cx^2+c}}+\frac{15ac^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{4\sqrt{a^2cx^2+c}}-\frac{15iac^3\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{4\sqrt{a^2cx^2+c}}-\frac{4ac^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{7}{8}a^2c^2x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}-\frac{c^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{x}-\frac{7}{4}ac^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+\frac{1}{4}a^2cx\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{1}{6}ac\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}+\frac{11}{6}ac^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{2iac^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{12}a^2c^2x\sqrt{a^2cx^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c + a^2*c*x^2])/12 - (7*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/4 - (a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/6 - (c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^2)/x + (7*a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 + (a
^2*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 - (((15*I)/4)*a*c^3*Sqrt[1 +
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (4*
a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
])/Sqrt[c + a^2*c*x^2] + (11*a*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]])/6 + (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x
```

```
^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/
Sqrt[c + a^2*c*x^2] - ((2*I)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*
a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2]) + (15*a*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{x^2}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx + (a^2c)\int (c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2\,dx$$

$$= -\frac{1}{6}ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax) + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2 + c^2\int$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

$$= \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\text{t}$$

Mathematica [A]
time = 2.01, size = 968, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + 2*a^4*x^4*Sqrt[1 +
a^2*x^2] - 190*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^3*x^3*Sqrt[1 + a^2*x
^2]*ArcTan[a*x] - 96*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 117*a^2*x^2*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2 + 21*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (192
*I)*a*x*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 192*a*x*ArcTanh[(a*x)/Sqr
t[1 + a^2*x^2]] + 6*a*x*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^3*x^3*ArcTan[
```

```
a*x]*Cos[3*ArcTan[a*x]] + 6*a^5*x^5*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 192*a*
x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 84*a*x*ArcTan[a*x]^2*Log[1 - I*E
^(I*ArcTan[a*x])] - 12*a*Pi*x*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTa
n[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] - 84*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*
ArcTan[a*x])] + 12*a*x*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x
])))/E^((I/2)*ArcTan[a*x])] - 12*a*Pi*x*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-
I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 192*a*x*ArcTan[a*x]*Log[1
+ E^(I*ArcTan[a*x])] - 12*a*x*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*Arc
Tan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] + 12*a*Pi*x*ArcTan[a*x]*Log[-Cos[(Pi
+ 2*ArcTan[a*x])/4]] + 16*a*x*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]]
- 12*a*x*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 16*a*
x*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 12*a*x*ArcTan[a*x]^2*Log[C
os[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + 12*a*Pi*x*ArcTan[a*x]*Log[Sin[(Pi
 + 2*ArcTan[a*x])/4]] + (192*I)*a*x*PolyLog[2, -E^(I*ArcTan[a*x])] + (360*I
)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (360*I)*a*x*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (192*I)*a*x*PolyLog[2, E^(I*ArcTan[a*
x])] - 360*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 360*a*x*PolyLog[3, I*E^
(I*ArcTan[a*x])] + 2*a*x*Sin[3*ArcTan[a*x]] + 4*a^3*x^3*Sin[3*ArcTan[a*x]]
+ 2*a^5*x^5*Sin[3*ArcTan[a*x]] - 3*a*x*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 6
*a^3*x^3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^5*x^5*ArcTan[a*x]^2*Sin[3*A
rcTan[a*x]]))/(96*x*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.47, size = 399, normalized size = 0.61

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(6\arctan(ax)^2a^4x^4-4\arctan(ax)a^3x^3+27\arctan(ax)^2a^2x^2+2a^2x^2-46\arctan(ax)ax-24\arctan(a\right.}{24x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)^2*a^4*x^4-4*arctan(a*x)*a
^3*x^3+27*arctan(a*x)^2*a^2*x^2+2*a^2*x^2-46*arctan(a*x)*a*x-24*arctan(a*x)
^2)/x-1/24*I*a*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(45*I*arctan(a*x)^2*ln(1-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-48*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
90*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+88*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)
)-48*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*dilog((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2,x)
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.329 $\int \frac{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=661

$$\frac{1}{3}a^2c^2\sqrt{c+a^2cx^2}-\frac{ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+2a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}$$

```
[Out] 1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2-a^2*c^(5/2)*arctanh((a^2*c*x^2+
c)^(1/2)/c^(1/2))+26/3*I*a^2*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*
x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*arctan(a*x)^2*arc
tanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*I
*a^2*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-5*I*a^2*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13/3*I*a^2*c^3*polylog(2,
-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1
3/3*I*a^2*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^2*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*a^2*c^2*(a^2*c*x^2+c)^(1
/2)-a*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/3*a^3*c^2*x*arctan(a*x)*(a^2*
c*x^2+c)^(1/2)+2*a^2*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-1/2*c^2*arctan(a
*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 1.81, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5070, 5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5010, 5006, 4998}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c + a^2*c*x^2])/3 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x
 - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/3 + 2*a^2*c^2*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) + (a^
2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/3 + (((26*I)/3)*a^2*c^3*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]
 + ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
```

```
Log[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^2*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^
2*c*x^2] + (((13*I)/3)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*P
olyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{x^3}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^3}\,dx + (a^2c)\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^3}\,dx + 2\left((a^2c^2)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x}\,d\right.$$

$$= \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2 - \frac{1}{3}(2a^3c^2)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)\,dx +$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{c^2\sqrt{c+a^2cx^2}\,\tan^{-}}{2x^2}$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\,\tan$$

Mathematica [A]
time = 7.15, size = 761, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

```
[Out] 2*a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E^
(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(Arc
Tan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*
(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sq
rt[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] - Poly
Log[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcTa
n[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (a^2*c^2*(1
 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x
]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTa
n[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log
[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x
]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^
2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/12 + (a^2*c^2*Sqrt[c*(1 + a^2
*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*Csc[ArcTan[a*x]/2
]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x
])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcT
an[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLog[3, -E^(I*ArcTan[a*
x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2
- 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 0.62, size = 454, normalized size = 0.69

method	result
default	$\frac{c^{2}\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(2\arctan\left(ax\right)^{2}a^{4}x^{4}-2\arctan\left(ax\right)a^{3}x^{3}+14\arctan\left(ax\right)^{2}a^{2}x^{2}+2a^{2}x^{2}-6\arctan\left(ax\right)ax-3\arctan\left(ax\right)\right.}{6x^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^2*a^4*x^4-2*arctan(a*x)*a^
3*x^3+14*arctan(a*x)^2*a^2*x^2+2*a^2*x^2-6*arctan(a*x)*a*x-3*arctan(a*x)^2)
/x^2+1/6*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(15*arctan(a*x)^2*ln(1
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(
1/2))-30*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*arctan(a
*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I
*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-6*ln(1+(1+I*a*x)/(a^2*x^2+1)^(
```

```
1/2))-30*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*polylog(3,(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*a^2*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3,x)
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.330 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=675

$$-\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x}-a^3c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3x^2}-\frac{2a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(a}{x}$$

[Out] -1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3+a^3*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-5*I*a^3*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-26/3*a^3*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*I*a^3*c^3*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*I*a^3*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13/3*I*a^3*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13/3*I*a^3*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^3*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^3*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2*c^2*(a^2*c*x^2+c)^(1/2)/x-a^3*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/3*a*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2-2*a^2*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x+1/2*a^4*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 1.60, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5070, 5064, 5066, 5082, 270, 5078, 5074, 5010, 5008, 4266, 2611, 2320, 6724, 5000, 223, 212}

$$-\frac{2a^2c^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{x}-\frac{ac^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3x^2}-\frac{c\text{ArcTan}(ax)^2(a^2cx^2+c)^{3/2}}{3x^3}-\frac{a^2c^2\sqrt{a^2cx^2+c}}{3x}+\frac{1}{2}a^4c^2x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2(-ie^{i\text{ArcTan}(ax)})}{\sqrt{a^2cx^2+c}}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2(ie^{i\text{ArcTan}(ax)})}{\sqrt{a^2cx^2+c}}-\frac{5a^3c^3\sqrt{a^2x^2+1}\,\text{Li}_3(-ie^{i\text{ArcTan}(ax)})}{\sqrt{a^2cx^2+c}}+\frac{5a^3c^3\sqrt{a^2x^2+1}\,\text{Li}_3(ie^{i\text{ArcTan}(ax)})}{\sqrt{a^2cx^2+c}}-\frac{5ia^3c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(e^{i\text{ArcTan}(ax)})\text{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{26a^3c^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-a^3c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}+a^3c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)+\frac{13ia^3c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{13ia^3c^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]

[Out] -1/3*(a^2*c^2*Sqrt[c + a^2*c*x^2])/x - a^3*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*x^2) - (2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x + (a^4*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*x^3) - ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] - (26*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) + a^3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((5*I)*a^3*c^3*Sqrt[1 + a^2*

```
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((
13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x])])/Sqrt[c + a^2*c*x^2] - (((13*I)/3)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog
[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (5*a^3*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (5*
a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5066

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Dist[d/(m + 2), Int[(f*x)^m*((a + b*ArcTan[c*x])/Sqr
t[d + e*x^2]), x], x] - Dist[b*c*(d/(f*(m + 2))), Int[(f*x)^(m + 1)/Sqrt[d
+ e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && Ne
Q[m, -2]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{x^4}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^4}\,dx + (a^2c)\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^4}\,dx + 2\Bigg((a^2c^2)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^2}\,d$$

$$= -a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c(c+a^2$$

$$= -a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{2ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3x^2} + \frac{1}{2}a^4c^2x\sqrt{c}$$

$$= -\frac{2a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-}}{3x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{3x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{3x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{3x^2}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x} - a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{3x^2}$$

Mathematica [A]
time = 3.16, size = 644, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

```
[Out] -1/12*(c^3*Sqrt[1 + a^2*x^2]*(2*(1 + a^2*x^2)^(3/2) + 12*a^3*x^3*Sqrt[1 + a
^2*x^2]*ArcTan[a*x] + 24*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - 6*a^4*x^
4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 4*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2 + (
12*I)*a^3*x^3*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 12*a^3*x^3*ArcTanh[
(a*x)/Sqrt[1 + a^2*x^2]] - 2*(1 + a^2*x^2)^(3/2)*Cos[2*ArcTan[a*x]] - 3*a*x
*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 51*a^3*x^3*ArcTan[a*x]*Log[1 - E^
(I*ArcTan[a*x])] - 24*a^3*x^3*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] +
24*a^3*x^3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 3*a*x*ArcTan[a*x]*L
og[1 + E^(I*ArcTan[a*x])] + 51*a^3*x^3*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x]
)] - (52*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] - (60*I)*a^3*x^3*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (60*I)*a^3*x^3*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] + (52*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] +
60*a^3*x^3*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 60*a^3*x^3*PolyLog[3, I*E^(
I*ArcTan[a*x])] + 2*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sin[2*ArcTan[a*x]] + (1
 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]*Sin[3*ArcTan[a*x]]
 - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])]*Sin[3*ArcTan[
a*x]]))/(x^3*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.75, size = 401, normalized size = 0.59

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\,\left(3\arctan(ax)^2a^4x^4-6\arctan(ax)a^3x^3-14\arctan(ax)^2a^2x^2-2a^2x^2-2\arctan(ax)ax-2\arctan(ax)\right.}{6x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)^2*a^4*x^4-6*arctan(a*x)*a^
3*x^3-14*arctan(a*x)^2*a^2*x^2-2*a^2*x^2-2*arctan(a*x)*a*x-2*arctan(a*x)^2)
/x^3-1/6*I*a^3*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(15*I*arctan(a*x)^2*ln(1-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-26*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
30*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)
)-26*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*dilog((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

```
[Out] int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.331 $\int \frac{x^3 \text{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=315

$$\frac{\sqrt{c+a^2cx^2}}{3a^4c}-\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{3a^2c}-\frac{10i\sqrt{}}{}$$

```
[Out] -10/3*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/a^4/(a^2*c*x^2+c)^(1/2)+5/3*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-5/3*I*polylog(2,I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+1/3*(a^2*c*
x^2+c)^(1/2)/a^4/c-1/3*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3/c-2/3*arctan(a
*x)^2*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
^2/c
```

Rubi [A]
time = 0.32, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5072, 267, 5010, 5006, 5050}

$$\frac{x^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}-\frac{2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3a^4c}-\frac{10i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{3a^4\sqrt{a^2cx^2+c}}+\frac{5i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2cx^2+c}}{3a^4c}-\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Sqrt[c + a^2*c*x^2]/(3*a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^3*
c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (((10*I)/3)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*A
rcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2]) + (((5*I)
/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(
a^4*Sqrt[c + a^2*c*x^2]) - (((5*I)/3)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[
1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
```

```
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{2\int \frac{x^2\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}}{3a}$$

$$= \frac{\sqrt{c+a^2cx^2}}{3a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}}{3a}$$

Mathematica [A]
time = 0.47, size = 279, normalized size = 0.89

$$\frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\left(2-2\mathrm{ArcTan}(ax)^2+2\cos(2\mathrm{ArcTan}(ax))-6\mathrm{ArcTan}(ax)^2\cos(2\mathrm{ArcTan}(ax))+\frac{15\mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}+5\mathrm{ArcTan}(ax)\cos(3\mathrm{ArcTan}(ax))\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)-\frac{15\mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}-5\mathrm{ArcTan}(ax)\cos(3\mathrm{ArcTan}(ax))\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)+\frac{20i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}-\frac{20i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}-2\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax))\right)}{12a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 - 2*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[
a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + (15*ArcTan[a*x]*Log[1 - I*E^(I
*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1
- I*E^(I*ArcTan[a*x])] - (15*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt
[1 + a^2*x^2] - 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x]
)] + ((20*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - ((20
*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Si
n[2*ArcTan[a*x]]))/(12*a^4*c)
```

Maple [A]
time = 2.00, size = 206, normalized size = 0.65

method	result
default	$\frac{\left(\arctan(ax)^2a^2x^2-\arctan(ax)ax-2\arctan(ax)^2+1\right)\sqrt{c(ax-i)(ax+i)}}{3c\,a^4}+\frac{5i\left(i\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(arctan(a*x)^2*a^2*x^2-arctan(a*x)*a*x-2*arctan(a*x)^2+1)*(c*(a*x-I)*(I
+a*x))^(1/2)/c/a^4+5/3*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^
(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.332 $\int \frac{x^2 \mathbf{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2a^2c}+\frac{i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^3\sqrt{c+a^2cx^2}}+$$

```
[Out] arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3/c^(1/2)+I*arctan((1+I*a*x)/(a^
2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-I*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/
(a^2*c*x^2+c)^(1/2)+I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.26, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724}

$$\frac{x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^3\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^3c}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a^3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^3*c)) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^3*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]]/(a^3*Sqrt[c]) - (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2
*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (Sqrt
[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{\int \frac{x\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}}\,dx}{a^2} - \frac{\sqrt{1+\cdots}}{\cdots}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{\cdots}{\sqrt{c\cdots}}\right.}{a^2}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(\cdots}\right.}{a^3\sqrt{c+a^2cx\cdots}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(\cdots}\right.}{a^3\sqrt{c+a^2cx\cdots}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(\cdots}\right.}{a^3\sqrt{c+a^2cx\cdots}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(\cdots}\right.}{a^3\sqrt{c+a^2cx\cdots}}$$

Mathematica [A]
time = 0.88, size = 505, normalized size = 1.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(ArcTan[a*x]*(-2 + a*x*ArcTan[a*x]) + (-(ArcTan[a*x]^2
*Log[1 - I*E^(I*ArcTan[a*x])]) - Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I
*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] + ArcTan[a*x]^2*Log[1 + I*E^(I*A
rcTan[a*x])] + ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^(
(I/2)*ArcTan[a*x])] - Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan
[a*x])))/E^((I/2)*ArcTan[a*x])] - ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I
*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] + Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2
*ArcTan[a*x])/4]] - 2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - ArcTan
[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 2*Log[Cos[ArcTan[a*x
]/2] + Sin[ArcTan[a*x]/2]] + ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[Arc
Tan[a*x]/2]] + Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] - (2*I)*ArcT
an[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3, I
*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]))/(2*a^3*c)
```

Maple [A]
time = 1.14, size = 271, normalized size = 0.79

method	result
default	$\frac{(\arctan(ax)ax-2)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}+\frac{\left(\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1-\frac{i(iax\cdots}{\sqrt{a^2x^2\cdots}}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)*a*x-2)*arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3+1/2*(ar
ctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((
1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*p
olylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2
+1)^(1/2)/a^3/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.333 $\int \frac{x\mathbf{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{a^2c}+\frac{4i\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{}}{\sqrt{}}\right.}{a^2\sqrt{c+a^2cx^2}}$$

```
[Out] 4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a
^2/(a^2*c*x^2+c)^(1/2)-2*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+2*I*polylog(2,I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*
c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.11, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5050, 5010, 5006}

$$\frac{\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{a^2c}+\frac{4i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\mathrm{ArcTan}(ax)}{a^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
- ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x
]])/(a^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt
[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^2c} - \frac{2\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^2c} - \frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{a\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^2c} + \frac{4i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

Mathematica [A]
time = 0.18, size = 126, normalized size = 0.57

$$\frac{\sqrt{c(1+a^2x^2)}\left(\mathrm{ArcTan}(ax)^2 - \frac{2\left(\mathrm{ArcTan}(ax)\left(\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)-\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)\right)+i\left(\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)\right)\right)}{\sqrt{1+a^2x^2}}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 - (2*(ArcTan[a*x]*(Log[1 - I*E^(I*Arc
Tan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt[1 + a^2*x^2]))/(a^2*c)
```

Maple [A]
time = 0.50, size = 180, normalized size = 0.82

method	result
default	$\frac{\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{a^2c} - \frac{2i\left(i\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+\mathrm{dil}\right.}{\sqrt{a^2x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c-2*I*(I*arctan(a*x)*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \operatorname{atan}(a x)^2}{\sqrt{c a^2 x^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.334 $\int \frac{\mathbf{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=256

$$-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a\sqrt{c+a^2cx^2}}+\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{c+a^2cx^2}}-\frac{2i}{}$$

```
[Out] -2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-2*polylog(
3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+2
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)
^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5010, 5008, 4266, 2611, 2320, 6724}

$$\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a*Sqrt[
c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 +
a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (2*S
qrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int x\log\left(1-ie^{\cdots}\right.\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\tan^{-1}\cdots}\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\tan^{-1}\cdots}\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\mathrm{Li}_2\left(-ie^{i\tan^{-1}\cdots}\right.}{a\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 140, normalized size = 0.55

$$\frac{2\sqrt{c(1+a^2x^2)}\left(-i\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2+i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)-i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)-\mathrm{PolyLog}\left(3,-ie^{i\mathrm{ArcTan}(ax)}\right)+\mathrm{PolyLog}\left(3,ie^{i\mathrm{ArcTan}(ax)}\right)\right)}{ac\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[c*(1 + a^2*x^2)]*((-I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I*
ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[3, I*E^(
I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\operatorname{atan}(a\,x)}^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2),x)
[Out] int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2), x)
```

3.335 $\int \frac{\text{ArcTan}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}+\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1}}{}$$

```
[Out] -2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5078, 5076, 4268, 2611, 2320, 6724}

$$\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])])/Sqrt[c + a^2*c*x^2] - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

f, g, n}, x] && GtQ[m, 0]

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^2\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x\log\left(1-\right.\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-}}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-}}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\text{Li}_2\left(-e^{i\tan^{-}}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.12, size = 145, normalized size = 0.64

$$\frac{\sqrt{1+a^2x^2}\left(\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)-\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-2i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-2\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{c\left(1+a^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^
2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I
*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

Maple [A]
time = 0.33, size = 197, normalized size = 0.87

method	result
default	$\frac{\left(\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\arctan(ax)\,\text{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+2i\arctan\right.}{\sqrt{a^2x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*
x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2
+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}(ax)}{x\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**2/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.336 $\int \frac{\text{ArcTan}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=208

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{cx}-\frac{4a\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}+\frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\right.}{\sqrt{c+a^2cx^2}}$$

```
[Out] -4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^2*(a^2*c*x^2+c)^(
1/2)/c/x
```

Rubi [A]
time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5064, 5078, 5074}

$$-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{cx}-\frac{4a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) - (4*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + ((2*
I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt
[c + a^2*c*x^2] - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
```

```
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{cx}+(2a)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx\\ &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{cx}+\frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{cx}-\frac{4a\sqrt{1+a^2x^2}\,\tan^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}+\frac{2ia\sqrt{}}{}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 128, normalized size = 0.62

$$-\frac{a\sqrt{1+a^2x^2}\left(\text{ArcTan}(ax)\left(\frac{\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)}{ax}-2\log\left(1-e^{i\text{ArcTan}(ax)}\right)+2\log\left(1+e^{i\text{ArcTan}(ax)}\right)\right)-2i\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+2i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{c\left(1+a^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x)
- 2*Log[1 - E^(I*ArcTan[a*x])] + 2*Log[1 + E^(I*ArcTan[a*x])]) - (2*I)*Poly
Log[2, -E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[c*(
1 + a^2*x^2)])
```

Maple [A]
time = 0.34, size = 171, normalized size = 0.82

method	result

default	$-\frac{\arctan(ax)^2\sqrt{c\,(ax-i)\,(ax+i)}}{cx}-\frac{2ia\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/x-2*I*a*(I*arctan(a*x)*ln(1-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.337 $\int \frac{\text{ArcTan}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=328

$$-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{cx}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}-$$

```
[Out] -a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+a^2*arctan(a*x)^2*arctanh
((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^2*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)+I*a^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+a^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/c/x-1/2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.34, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724}

$$-\frac{ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{a^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}+\frac{a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/
Sqrt[c] - (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x]
)])/Sqrt[c + a^2*c*x^2] + (I*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^2*Sqrt[1 + a^2*x^2]*PolyLog[3
, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
```

```
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + a\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + a^2\int \frac{1}{x\sqrt{c+a^2cx^2}}\,dx - \frac{(a}{}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+a^2cx}}\right.$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\,\text{ta}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\,\text{ta}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\,\text{ta}}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\,\text{ta}}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.89, size = 231, normalized size = 0.70

$$\frac{a^2\sqrt{1+a^2x^2}\left(-4\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\text{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-4\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)+4\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+8\log\left(\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-8i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+8i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+8\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-8\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)^2\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-4\text{ArcTan}(ax)\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{8\sqrt{c\left(1+a^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*C
sc[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan
[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Log[Tan[ArcTan[a*x]/2]] - (8*I)*ArcT
an[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (8*I)*ArcTan[a*x]*PolyLog[2, E^(I*
ArcTan[a*x])] + 8*PolyLog[3, -E^(I*ArcTan[a*x])] - 8*PolyLog[3, E^(I*ArcTan
[a*x])] + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x
]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 0.60, size = 261, normalized size = 0.80

method	result
default	$-\frac{(2ax+\arctan(ax))\arctan(ax)\sqrt{c\,(ax-i)\,(ax+i)}}{2c\,x^2}-\frac{a^2\left(\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1+\frac{}{\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*a*x+arctan(a*x))*arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2-1/2*a^
2*(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/
2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arcta
nh((1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2
)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.338 $\int \frac{\textbf{ArcTan}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=311

$$-\frac{a^2\sqrt{c+a^2cx^2}}{3cx}-\frac{a\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{3cx^2}-\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{3cx}+\frac{1}{}$$

```
[Out] 10/3*a^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-5/3*I*a^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/3*I*a^3*polylog(2,(1+I*a*x)^(1/
2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^
2+c)^(1/2)/c/x-1/3*a*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*arctan(a*x)^
2*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x
```

Rubi [A]
time = 0.43, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5082, 270, 5078, 5074, 5064}

$$\frac{2a^2\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3cx}-\frac{a\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3cx^2}-\frac{\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}-\frac{a^2\sqrt{a^2cx^2+c}}{3cx}+\frac{10a^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}-\frac{5ia^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/3*(a^2*Sqrt[c + a^2*c*x^2])/(c*x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(3*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x^3) + (2*a^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^2)/(3*c*x) + (10*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*
ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(3*Sqrt[c + a^2*c*x^2]) - (((5*I)
/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/S
qrt[c + a^2*c*x^2] + (((5*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3cx^3}+\frac{1}{3}(2a)\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3cx}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+}}{}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}}{3cx}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+}}{}$$

Mathematica [A]

time = 1.84, size = 228, normalized size = 0.73

$$\frac{a^3\sqrt{c+a^2cx^2}\left(-20i\,\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(\mathrm{ArcTan}(ax)^2(2-6\cos(2\mathrm{ArcTan}(ax)))+2(-1+\cos(2\mathrm{ArcTan}(ax)))+\frac{20ia^3x^3\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}+\mathrm{ArcTan}(ax)\left(-2\sin(2\mathrm{ArcTan}(ax))+\frac{5\left(\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)\right)\left(-3ax+\sqrt{1+a^2x^2}\sin(3\mathrm{ArcTan}(ax))\right)}{\sqrt{1+a^2x^2}}\right)\right)}{a^3x^3}\right)}{12c\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^3*Sqrt[c + a^2*c*x^2]*((-20*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^
2*x^2)^(3/2)*(ArcTan[a*x]^2*(2 - 6*Cos[2*ArcTan[a*x]]) + 2*(-1 + Cos[2*ArcT
an[a*x]]) + ((20*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3
/2) + ArcTan[a*x]*(-2*Sin[2*ArcTan[a*x]] + (5*(Log[1 - E^(I*ArcTan[a*x])] -
 Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]
))/Sqrt[1 + a^2*x^2]))))/(a^3*x^3)))/(12*c*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 1.10, size = 206, normalized size = 0.66

method	result
default	$\frac{\left(2\arctan(ax)^2a^2x^2-a^2x^2-\arctan(ax)ax-\arctan(ax)^2\right)\sqrt{c\,(ax-i)\,(ax+i)}}{3c\,x^3}+\frac{5ia^3\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*arctan(a*x)^2*a^2*x^2-a^2*x^2-arctan(a*x)*a*x-arctan(a*x)^2)*(c*(a*x
-I)*(I+a*x))^(1/2)/c/x^3+5/3*I*a^3*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(
1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**2/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.339 $\int \frac{x^3 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=305

$$-\frac{2}{a^4c\sqrt{c+a^2cx^2}}-\frac{2x\text{ArcTan}(ax)}{a^3c\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)^2}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{a^4c^2}+\frac{4i\sqrt{1+a^2x^2}\,\text{ArcTan}(a}{a^4c\sqrt{c-}$$

```
[Out] -2/a^4/c/(a^2*c*x^2+c)^(1/2)-2*x*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)+arct
an(a*x)^2/a^4/c/(a^2*c*x^2+c)^(1/2)+4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2)-2*I*polylog(2,
-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(
1/2)+2*I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4
/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4/c^2
```

Rubi [A]
time = 0.28, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5084, 5050, 5010, 5006, 5014}

$$\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{a^4c^2}+\frac{\text{ArcTan}(ax)^2}{a^4c\sqrt{a^2cx^2+c}}+\frac{4i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a^4c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{a^2cx^2+c}}-\frac{2}{a^4c\sqrt{a^2cx^2+c}}-\frac{2x\text{ArcTan}(ax)}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -2/(a^4*c*Sqrt[c + a^2*c*x^2]) - (2*x*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x^
2]) + ArcTan[a*x]^2/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(a^4*c^2) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2
*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^4*c*Sqrt[c + a
^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
```

```
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^2}+\frac{\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^4c^2}-\frac{2\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{a^3}-\frac{2\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}$$

$$= -\frac{2}{a^4c\sqrt{c+a^2cx^2}}-\frac{2x\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^4c^2}-$$

$$= -\frac{2}{a^4c\sqrt{c+a^2cx^2}}-\frac{2x\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{a^4c\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{a^4c^2}+$$

Mathematica [A]
time = 0.58, size = 209, normalized size = 0.69

$$\frac{\sqrt{c(1+a^2x^2)}\left(-2+3\mathrm{ArcTan}(ax)^2-2\cos(2\mathrm{ArcTan}(ax))+\mathrm{ArcTan}(ax)^2\cos(2\mathrm{ArcTan}(ax))-\frac{4\mathrm{ArcTan}(ax)\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}+\frac{4\mathrm{ArcTan}(ax)\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}-\frac{4i\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}+\frac{4i\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}}-2\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax))\right)}{2a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-2 + 3*ArcTan[a*x]^2 - 2*Cos[2*ArcTan[a*x]] + ArcTa
n[a*x]^2*Cos[2*ArcTan[a*x]] - (4*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/
Sqrt[1 + a^2*x^2] + (4*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((
4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 2*ArcTan[a*x]*Sin
[2*ArcTan[a*x]]))/(2*a^4*c^2)
```

Maple [A]
time = 1.93, size = 294, normalized size = 0.96

method	result
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a^4c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-\right.}{2(a^2x^2+1)a^4c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(
a^2*x^2+1)/a^4/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2
-2*I*arctan(a*x))/(a^2*x^2+1)/a^4/c^2+arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/
2)/a^4/c^2-2*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(
a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(
I+a*x))^(1/2)/a^4/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^2}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.340 $\int \frac{x^2 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2x}{a^2c\sqrt{c+a^2cx^2}}-\frac{2\text{ArcTan}(ax)}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\text{ArcTan}(ax)^2}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^3c\sqrt{c+a^2cx^2}}+2i\sqrt{}$$

```
[Out] 2*x/a^2/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arc
tan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)
)*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x
^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+2*polylog(3,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5084, 5010, 5008, 4266, 2611, 2320, 6724, 5018, 197}

$$-\frac{x\text{ArcTan}(ax)^2}{a^2c\sqrt{a^2cx^2+c}}+\frac{2x}{a^2c\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^3c\sqrt{a^2cx^2+c}}-\frac{2\text{ArcTan}(ax)}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*x)/(a^2*c*Sqrt[c + a^2*c*x^2]) - (2*ArcTan[a*x])/(a^3*c*Sqrt[c + a^2*c*x
^2]) - (x*ArcTan[a*x]^2)/(a^2*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*
x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) +
 ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2
, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
```

```
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{a^2} + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)}{\sqrt{1+a^2x}}}{a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^2\sec(\right.}{a^3c\sqrt{c+a^2}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-}}\right.}{a^3c\sqrt{c+a^2c}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-}}\right.}{a^3c\sqrt{c+a^2c}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-}}\right.}{a^3c\sqrt{c+a^2c}}$$

$$= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-}}\right.}{a^3c\sqrt{c+a^2c}}$$

Mathematica [A]
time = 0.25, size = 228, normalized size = 0.65

$$-\frac{\sqrt{1+a^2x^2}\left(-\frac{2ax}{\sqrt{1+a^2x^2}}+\frac{2\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}+\frac{ax\text{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}}-\text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+\text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)-2\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)}{a^3c\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*((-2*a*x)/Sqrt[1 + a^2*x^2] + (2*ArcTan[a*x])/Sqrt[1 +
 a^2*x^2] + (a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - ArcTan[a*x]^2*Log[1 - I
```

```
*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (2*I)*Ar
cTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2,
 I*E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3,
 I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

Maple [F]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arctan(ax)^2}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
[Out] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^2}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.341 $\int \frac{x\mathbf{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{a^2c\sqrt{c+a^2cx^2}}+\frac{2x\mathrm{ArcTan}(ax)}{ac\sqrt{c+a^2cx^2}}-\frac{\mathrm{ArcTan}(ax)^2}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] 2/a^2/c/(a^2*c*x^2+c)^(1/2)+2*x*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)-arctan(
a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5050, 5014}

$$-\frac{\mathrm{ArcTan}(ax)^2}{a^2c\sqrt{a^2cx^2+c}}+\frac{2x\mathrm{ArcTan}(ax)}{ac\sqrt{a^2cx^2+c}}+\frac{2}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] 2/(a^2*c*Sqrt[c + a^2*c*x^2]) + (2*x*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
 - ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\frac{\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}}+\frac{2\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{a}\\ &= \frac{2}{a^2c\sqrt{c+a^2cx^2}}+\frac{2x\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^2}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.64

$$\frac{\sqrt{c+a^2cx^2}\,(2+2ax\text{ArcTan}(ax)-\text{ArcTan}(ax)^2)}{a^2c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(2 + 2*a*x*ArcTan[a*x] - ArcTan[a*x]^2))/(a^2*c^2*(1 +
 a^2*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 116, normalized size = 1.49

method	result
default	$-\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)\left(\arctan(ax)^2-\right.}{2(a^2x^2+1)a^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-
2-2*I*arctan(a*x))/(a^2*x^2+1)/a^2/c^2
```

Maxima [A]
time = 0.68, size = 73, normalized size = 0.94

$$\sqrt{c}\left(\frac{2\,x\arctan\left(ax\right)}{\sqrt{a^2x^2+1}\,ac^2}-\frac{\arctan\left(ax\right)^2}{\sqrt{a^2x^2+1}\,a^2c^2}+\frac{2}{\sqrt{a^2x^2+1}\,a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(c)*(2*x*arctan(a*x)/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^2/(sqrt(a^
2*x^2 + 1)*a^2*c^2) + 2/(sqrt(a^2*x^2 + 1)*a^2*c^2))
```

Fricas [A]
time = 2.72, size = 51, normalized size = 0.65

$$\frac{\sqrt{a^2cx^2+c}\,\left(2\,ax\arctan\left(ax\right)-\arctan\left(ax\right)^2+2\right)}{a^4c^2x^2+a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(2*a*x*arctan(a*x) - arctan(a*x)^2 + 2)/(a^4*c^2*x^2 +
a^2*c^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^2{(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\mathrm{atan}\left(a\,x\right)}^2}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.342 $\int \frac{\mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2x}{c\sqrt{c+a^2cx^2}}+\frac{2\text{ArcTan}(ax)}{ac\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}$$

```
[Out] -2*x/c/(a^2*c*x^2+c)^(1/2)+2*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a
*x)^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5018, 197}

$$\frac{x\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\text{ArcTan}(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
+ (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{2\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-2\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx\\ &= -\frac{2x}{c\sqrt{c+a^2cx^2}}+\frac{2\tan^{-1}(ax)}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.68

$$\frac{\sqrt{c+a^2cx^2}\left(-2ax+2\mathrm{ArcTan}(ax)+ax\mathrm{ArcTan}(ax)^2\right)}{c^2\left(a+a^3x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x + 2*ArcTan[a*x] + a*x*ArcTan[a*x]^2))/(c^2*(a
+ a^3*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 114, normalized size = 1.58

method	result
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{2(a^2x^2+1)a\,c^2}+\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,(ax+i)\left(\arctan(ax)^2-2-2i\,\mathrm{ar}\right.}{2(a^2x^2+1)a\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^
2*x^2+1)/a/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*a
rctan(a*x))/(a^2*x^2+1)/a/c^2
```

Maxima [A]
time = 0.52, size = 53, normalized size = 0.74

$$\frac{x\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}\,c}-\frac{2\left(ax-\arctan\left(ax\right)\right)}{\sqrt{a^2x^2+1}\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c) - 2*(a*x - arctan(a*x))/(sqrt(a^2*x
^2 + 1)*a*c^(3/2))
```

Fricas [A]
time = 2.58, size = 51, normalized size = 0.71

$$\frac{\sqrt{a^2cx^2+c}\left(ax\arctan\left(ax\right)^2-2\,ax+2\,\arctan\left(ax\right)\right)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^2 - 2*a*x + 2*arctan(a*x))/(a^3*c^2*x^
2 + a*c^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2), x)
```

3.343 $\int \frac{\textbf{ArcTan}(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{c+a^2cx^2}}+\frac{2i\sqrt{1+}}{}$$

```
[Out] -2/c/(a^2*c*x^2+c)^(1/2)-2*a*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x
)^2/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-2*I*arctan
(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2
+c)^(1/2)-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^
2*c*x^2+c)^(1/2)+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5086, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5014}

$$\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2ax\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -2/(c*Sqrt[c + a^2*c*x^2]) - (2*a*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
ArcTan[a*x]^2/(c*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*
ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (
(2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[
c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*S
qrt[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(
c*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
```

```
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx &= -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-(2a)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^2\csc(x)\,d\right.}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-}}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-}}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-}}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-}}{c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A]

time = 0.21, size = 204, normalized size = 0.66

$$
\frac{\sqrt{1+a^2x^2}\left(-\frac{2}{\sqrt{1+a^2x^2}}-\frac{2ax\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}+\frac{\text{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}}+\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)-\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+2i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-2i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-2\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)\right)}{c\sqrt{c\left(1+a^2x^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/Sqrt[1 + a^2*x^2] - (2*a*x*ArcTan[a*x])/Sqrt[1 + a^2
*x^2] + ArcTan[a*x]^2/Sqrt[1 + a^2*x^2] + ArcTan[a*x]^2*Log[1 - E^(I*ArcTan
```

```
[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*Poly
Log[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])
] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/(c
*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 0.36, size = 306, normalized size = 0.99

method	result
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-\right.}{2(a^2x^2+1)c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(
a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I
*arctan(a*x))/(a^2*x^2+1)/c^2+(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/
2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog
(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c
^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.344 $\int \frac{\text{ArcTan}(ax)^2}{x^2(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2a^2x}{c\sqrt{c+a^2cx^2}}-\frac{2a\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{c^2x}-\frac{4a\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\,\text{t}}{c\sqrt{c+a^2c}}$$

```
[Out] 2*a^2*x/c/(a^2*c*x^2+c)^(1/2)-2*a*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a^2*x*a
rctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2,-(
1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-2*I
*a*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^
2+c)^(1/2)-arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.31, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5086, 5064, 5078, 5074, 5018, 197}

$$-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{c^2x}-\frac{a^2x\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2a\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{4a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}+\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c\sqrt{a^2cx^2+c}}+\frac{2a^2x}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*x)/(c*Sqrt[c + a^2*c*x^2]) - (2*a*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (a^2*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2)/(c^2*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt[1 + a^2*x
^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c + a^2*c*x^2])
 - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/
(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
```

```
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = -\left(a^2 \int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{c^2x} + (2a^2)\int \frac{1}{(c+a^2c}$$

$$= \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{c^2x} + $$

$$= \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{c^2x} - $$

Mathematica [A]
time = 0.79, size = 226, normalized size = 0.77

$$\frac{a\left(4ax - 4\text{ArcTan}(ax) - 2ax\text{ArcTan}(ax)^2 - \frac{1}{2}ax\text{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right) + 4\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right) - 4\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right) + 4i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right) - 4i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right) - \frac{2(1+a^2x^2)\text{ArcTan}(ax)^2\sin^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{ax}\right)}{2c\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(4*a*x - 4*ArcTan[a*x] - 2*a*x*ArcTan[a*x]^2 - (a*x*ArcTan[a*x]^2*Csc[Ar
cTan[a*x]/2]^2)/2 + 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x
])] - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*Sq
rt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Sqrt[1 + a^2*x^2]*Po
lyLog[2, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^2*Sin[ArcTan[a*x
]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.36, size = 279, normalized size = 0.95

method	result
default	$-\frac{a\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2} - \frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^2-2\right.}{2(a^2x^2+1)c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*
```

```
arctan(a*x))*a/(a^2*x^2+1)/c^2-arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/
x-2*I*a*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1
+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1
/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c
^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}(ax)}{x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.345 $\int \frac{\textbf{ArcTan}(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{c^2x}-\frac{a^2\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2c^2x^2}+$$

```
[Out] -a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+2*a^2/c/(a^2*c*x^2+c)^(1/
2)+2*a^3*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)^2/c/(a^2*c*x^2
+c)^(1/2)+3*a^2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-3*I*a^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+3*I*a^2*arctan(a*x
)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^
(1/2)+3*a^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^
2*c*x^2+c)^(1/2)-3*a^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/c/(a^2*c*x^2+c)^(1/2)-a*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x-1/2*arct
an(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x^2
```

Rubi [A]
time = 0.82, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5086, 5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5014}

$$-\frac{a\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{c^2x}-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2c^2x^2}-\frac{3ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{3a^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{a^2\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2a^2}{c\sqrt{a^2cx^2+c}}+\frac{2a^3x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*a^2)/(c*Sqrt[c + a^2*c*x^2]) + (2*a^3*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x
^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c^2*x) - (a^2*ArcTan[a*x]^2)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c^2*x^2) + (
3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c
 + a^2*c*x^2]) - (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/c^(3/2) - ((3*I
)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c*Sqrt
[c + a^2*c*x^2]) + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (3*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (3*a^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
```

```
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2c^2x^2}+a^4\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx+\frac{a\int \frac{\tan^{-1}(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}-\ldots$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2c^2x^2}+\left(2a^3\right)\int\ldots$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\ldots$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\ldots$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\ldots$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\ldots$$

$$= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{c^2x}-\frac{a^2\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}}-\ldots$$

Mathematica [A]
time = 1.58, size = 371, normalized size = 0.88

$$\frac{a^2\left(16+16ax\text{ArcTan}(ax)-8\text{ArcTan}(ax)^2-2ax\text{ArcTan}(ax)\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-12\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)+12\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+8\sqrt{1+a^2x^2}\,\log\left(\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-24i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+24i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+24\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-24\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)+\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-4\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{8c\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(16 + 16*a*x*ArcTan[a*x] - 8*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]*Csc[Arc
Tan[a*x]/2]^2 - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - 12*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 12*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Sqrt[1 + a^2*x^2]*Log[Ta
n[ArcTan[a*x]/2]] - (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])] + (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[
a*x])] + 24*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] - 24*Sqrt[1 +
a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Se
c[ArcTan[a*x]/2]^2 - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(
8*c*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.64, size = 376, normalized size = 0.89

method	result
default	$-\frac{a^{2}\left(\arctan(ax)^{2}-2+2i\arctan(ax)\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^{2}x^{2}+1)c^{2}}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\;(iax-1)\left(\arctan(ax)^{2}-\right.}{2(a^{2}x^{2}+1)c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1
/2)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-
2-2*I*arctan(a*x))*a^2/(a^2*x^2+1)/c^2-1/2*(2*a*x+arctan(a*x))*arctan(a*x)*
(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^2-1/2*a^2*(3*arctan(a*x)^2*ln(1-(1+I*a*x)/(
a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arc
tan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polyl
og(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c
^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.346 $\int \frac{\text{ArcTan}(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=397

$$-\frac{2a^4x}{c\sqrt{c+a^2cx^2}}-\frac{a^2\sqrt{c+a^2cx^2}}{3c^2x}+\frac{2a^3\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{3c^2x^2}+\frac{a^4x\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2c}}{}$$

```
[Out] -2*a^4*x/c/(a^2*c*x^2+c)^(1/2)+2*a^3*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+a^4*
x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)+22/3*a^3*arctan(a*x)*arctanh((1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-11/3*I*a^3
*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2
+c)^(1/2)+11/3*I*a^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)
^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^2+c)^(1/2)/c^2/x-1/3*a*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^2-1/3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2
/x^3+5/3*a^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.88, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5086, 5082, 270, 5078, 5074, 5064, 5018, 197}

$$\frac{5a^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3c^2x}-\frac{a\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{3c^2x^2}-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{3c^2x^3}-\frac{a^2\sqrt{a^2cx^2+c}}{3c^2x}+\frac{a^4x\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{2a^4x}{c\sqrt{a^2cx^2+c}}+\frac{2a^3\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}+\frac{22a^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}-\frac{11ia^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}+\frac{11ia^3\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (-2*a^4*x)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*Sqrt[c + a^2*c*x^2])/(3*c^2*x) +
(2*a^3*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x])/(3*c^2*x^2) + (a^4*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*c^2*x^3) + (5*a^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(3*c^2*x) + (22*a^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]])/(3*c*Sqrt[c + a^2*c*x^2]) - (((11*I)/3)*a^3*Sqr
t[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c*Sqrt[c +
a^2*c*x^2]) + (((11*I)/3)*a^3*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{3c^2x^3} + a^4\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{(2a)\int \frac{\tan^{-1}(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx}{3c} - \cdots$$

$$= \frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^2} + \frac{a^4x\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(\cdots}{3c^2x^3}$$

$$= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}} - \frac{a^2\sqrt{c+a^2cx^2}}{3c^2x} + \frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^2} + \cdots$$

$$= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}} - \frac{a^2\sqrt{c+a^2cx^2}}{3c^2x} + \frac{2a^3\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{3c^2x^2} + \cdots$$

Mathematica [A]
time = 2.39, size = 270, normalized size = 0.68

$$\frac{a^3\sqrt{1+a^2x^2}\left(-88i\,\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(-22+28\cos(2\mathrm{ArcTan}(ax))-6\cos(4\mathrm{ArcTan}(ax))+\mathrm{ArcTan}(ax)^2(25-36\cos(2\mathrm{ArcTan}(ax))+3\cos(4\mathrm{ArcTan}(ax)))+\frac{88ia^3x^3\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)}{(1+a^2x^2)^{3/2}}+\mathrm{ArcTan}(ax)\left(\frac{66ax\left(-\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)+\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}+8\sin(2\mathrm{ArcTan}(ax))+22\left(\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)\right)\sin(3\mathrm{ArcTan}(ax))-6\sin(4\mathrm{ArcTan}(ax))\right)\right)}{a^3x^3}\right)}{24c\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*Sqrt[1 + a^2*x^2]*((-88*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^2*
x^2)^(3/2)*(-22 + 28*Cos[2*ArcTan[a*x]] - 6*Cos[4*ArcTan[a*x]] + ArcTan[a*x
]^2*(25 - 36*Cos[2*ArcTan[a*x]] + 3*Cos[4*ArcTan[a*x]]) + ((88*I)*a^3*x^3*P
olyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ArcTan[a*x]*((66*a*x*(-
Log[1 - E^(I*ArcTan[a*x])] + Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
 + 8*Sin[2*ArcTan[a*x]] + 22*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*Arc
Tan[a*x])])*Sin[3*ArcTan[a*x]] - 6*Sin[4*ArcTan[a*x]])))/(a^3*x^3)))/(24*c*
Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 1.13, size = 318, normalized size = 0.80

method	result
default	$\frac{a^{3}\left(\arctan(ax)^{2}-2+2i\arctan(ax)\right)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^{2}x^{2}+1)c^{2}}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+i)\left(\arctan(ax)^{2}-2-\right.}{2(a^{2}x^{2}+1)c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)
/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I
*arctan(a*x))*a^3/(a^2*x^2+1)/c^2+1/3*(5*arctan(a*x)^2*a^2*x^2-a^2*x^2-arct
an(a*x)*a*x-arctan(a*x)^2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^3+11/3*I*a^3*(I*
arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c
^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}{\left(ax \right)}}{x^{4} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^2}{x^4\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

3.347 $\int \frac{x^5 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx$

Optimal. Leaf size=400

$$\frac{2}{27a^6c\left(c+a^2cx^2\right)^{3/2}}-\frac{32}{9a^6c^2\sqrt{c+a^2cx^2}}-\frac{2x^3\text{ArcTan}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}}-\frac{10x\text{ArcTan}(ax)}{3a^5c^2\sqrt{c+a^2cx^2}}+\frac{x^2\text{ArcTan}(ax)^2}{3a^4c\left(c+a^2cx^2\right)^{3/2}}+\frac{5}{3a}$$

```
[Out] 2/27/a^6/c/(a^2*c*x^2+c)^(3/2)-2/9*x^3*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(3/2
)+1/3*x^2*arctan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(3/2)-32/9/a^6/c^2/(a^2*c*x^2+c
)^(1/2)-10/3*x*arctan(a*x)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+5/3*arctan(a*x)^2/a^
6/c^2/(a^2*c*x^2+c)^(1/2)+4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-2*I*polylog(2,-I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+2
*I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(
a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^6/c^3
```

Rubi [A]
time = 0.59, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5084, 5050, 5010, 5006, 5014, 5060, 272, 45}

$$\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{a^6c^3}+\frac{5\text{ArcTan}(ax)^2}{3a^6c^2\sqrt{a^2cx^2+c}}+\frac{4i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{2i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{32}{9a^6c^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^6c\left(a^2cx^2+c\right)^{3/2}}-\frac{10x\text{ArcTan}(ax)}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{x^2\text{ArcTan}(ax)^2}{3a^4c\left(a^2cx^2+c\right)^{3/2}}-\frac{2x^3\text{ArcTan}(ax)}{9a^3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 2/(27*a^6*c*(c + a^2*c*x^2)^(3/2)) - 32/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (10*x*ArcTan[a*x])/(3*
a^5*c^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a^4*c*(c + a^2*c*x^2)
^(3/2)) + (5*ArcTan[a*x]^2)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(a^6*c^3) + ((4*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Arc
Tan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I
)*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^
6*c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1
+ I*a*x])/Sqrt[1 - I*a*x]])/(a^6*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\int \frac{x^3 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^3 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9a^3c\,(c+a^2cx^2)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\,(c+a^2cx^2)^{3/2}} + \frac{2\int \frac{x^3}{(c+a^2cx^2)^{5/2}}\,dx}{9a^2} + \frac{\int \frac{x \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2}$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9a^3c\,(c+a^2cx^2)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\,(c+a^2cx^2)^{3/2}} + \frac{5 \tan^{-1}(ax)^2}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan}{a^6c^3}$$

$$= -\frac{10}{3a^6c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9a^3c\,(c+a^2cx^2)^{3/2}} - \frac{10x \tan^{-1}(ax)}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2 \tan^{-1}(ax)^2}{3a^4c\,(c+a^2cx^2)^{3/}}$$

$$= \frac{2}{27a^6c\,(c+a^2cx^2)^{3/2}} - \frac{32}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9a^3c\,(c+a^2cx^2)^{3/2}} - \frac{10x \tan^{-1}(ax)}{3a^5c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.92, size = 229, normalized size = 0.57

$$\frac{8(-95+\cos(2\mathrm{ArcTan}(ax)))-9(1+a^2x^2)\,\mathrm{ArcTan}(ax)^2(-45-20\cos(2\mathrm{ArcTan}(ax))+\cos(4\mathrm{ArcTan}(ax)))-432i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right)+432i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right)+6\mathrm{ArcTan}(ax)\left(-124ax-72\sqrt{1+a^2x^2}\,\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right)+72\sqrt{1+a^2x^2}\,\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right)+(1+a^2x^2)\sin(4\mathrm{ArcTan}(ax))\right)}{216a^6c^2\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (8*(-95 + Cos[2*ArcTan[a*x]]) - 9*(1 + a^2*x^2)*ArcTan[a*x]^2*(-45 - 20*Cos
[2*ArcTan[a*x]] + Cos[4*ArcTan[a*x]]) - (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] + (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*Arc
Tan[a*x])] + 6*ArcTan[a*x]*(-124*a*x - 72*Sqrt[1 + a^2*x^2]*Log[1 - I*E^(I*
ArcTan[a*x])] + 72*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] + (1 + a^
2*x^2)*Sin[4*ArcTan[a*x]]))/(216*a^6*c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 2.02, size = 454, normalized size = 1.14

method	result
default	$\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2c^3a^6}+\frac{7\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+}{8a^6c^3(a^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(
c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3/a^6+7/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3/(a^2*x^2+1)-7/8*(c*(a
*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^6/c^3/(a
^2*x^2+1)-1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(
-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)+arctan(a*
x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^6-2*I*(I*arctan(a*x)*ln(1+I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a
^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\operatorname{atan}^2(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\mathrm{atan}(a\,x)^2}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.348 $\int \frac{x^4 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=444

$$\frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}}-\frac{2x^2\text{ArcTan}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}}-\frac{22\text{ArcTan}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\text{ArcTan}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{x\text{A}}{a^4c^2}$$

```
[Out] 2/27*x^3/a^2/c/(a^2*c*x^2+c)^(3/2)-2/9*x^2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^
(3/2)-1/3*x^3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+22/9*x/a^4/c^2/(a^2*c
*x^2+c)^(1/2)-22/9*arctan(a*x)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)^2/
a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(
a*x)^2*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylo
g(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)
^(1/2)-2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*polylog(3,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5084, 5010, 5008, 4266, 2611, 2320, 6724, 5018, 197, 5064, 5058, 5050}

$$-\frac{x^3\text{ArcTan}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3}{27a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{22\text{ArcTan}(ax)}{9a^5c^2\sqrt{a^2cx^2+c}}-\frac{x\text{ArcTan}(ax)^2}{a^4c^2\sqrt{a^2cx^2+c}}+\frac{22x}{9a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^2\text{ArcTan}(ax)}{9a^3c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*x^3)/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x)/(9*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^3*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a
*x])/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^2)/(a^4*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c
+ a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - (2
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^
2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*
Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{\int \frac{x^2 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^2 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3 \tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{x^3 \tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{3a} + \frac{\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^4c}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3 \tan^{-1}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^2c\left(c+a^2cx^2\right)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} - \frac{2x^2 \tan^{-1}(ax)}{9a^3c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{9a^5c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.41, size = 239, normalized size = 0.54

$$\frac{\sqrt{c(1+a^2x^2)}\left(-\frac{270\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}} - \frac{135ax(-2+\text{ArcTan}(ax)^2)}{\sqrt{1+a^2x^2}} + 6\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax)) + 108\text{ArcTan}(ax)^2\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right) - \log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right) + 216i\text{ArcTan}(ax)\left(\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right) - \text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right) - 216\left(\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right) - \text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right) + (-2+9\text{ArcTan}(ax)^2)\sin(3\text{ArcTan}(ax))\right)}{108a^5c^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*((-270*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (135*a*x*(-2
 + ArcTan[a*x]^2))/Sqrt[1 + a^2*x^2] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 1
08*ArcTan[a*x]^2*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x]
)]) + (216*I)*ArcTan[a*x]*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2,
I*E^(I*ArcTan[a*x])]) - 216*(PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3
, I*E^(I*ArcTan[a*x])]) + (-2 + 9*ArcTan[a*x]^2)*Sin[3*ArcTan[a*x]]))/(108*
a^5*c^3*Sqrt[1 + a^2*x^2])
```

Maple [F]

time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x^4 \arctan{(ax)}^2}{(a^2 c x^2 + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{atan}^2{(ax)}}{(c (a^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \operatorname{atan}(a x)^2}{(c a^2 x^2 + c)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.349 $\int \frac{x^3 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=172

$$-\frac{2}{27a^4c\left(c+a^2cx^2\right)^{3/2}}+\frac{14}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\mathrm{ArcTan}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4x\mathrm{ArcTan}(ax)}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\mathrm{ArcTan}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2.}{3a^4}$$

```
[Out] -2/27/a^4/c/(a^2*c*x^2+c)^(3/2)+2/9*x^3*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)
-1/3*x^2*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+14/9/a^4/c^2/(a^2*c*x^2+c)
^(1/2)+4/3*x*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2/3*arctan(a*x)^2/a^4/
c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5060, 5050, 5014, 272, 45}

$$-\frac{x^2\mathrm{ArcTan}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\mathrm{ArcTan}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2\mathrm{ArcTan}(ax)^2}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{14}{9a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{27a^4c\left(a^2cx^2+c\right)^{3/2}}+\frac{4x\mathrm{ArcTan}(ax)}{3a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -2/(27*a^4*c*(c + a^2*c*x^2)^(3/2)) + 14/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(3*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^2)/(3*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
```

```
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{2x^3\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2}{9}\int\frac{x^3}{(c+a^2cx^2)^{5/2}}\,dx+\frac{2\int\frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,d}{3a^2c}\\ &= \frac{2x^3\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\tan^{-1}(ax)^2}{3a^4c^2\sqrt{c+a^2cx^2}}-\frac{1}{9}\text{Subst}\left(\int\frac{x}{(c+a^2}\right.\\ &= \frac{4}{3a^4c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\tan^{-1}(ax)^2}{3a^2c\,(c+a^2cx^2)^{3/2}}-\\ &= -\frac{2}{27a^4c\,(c+a^2cx^2)^{3/2}}+\frac{14}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{4x\tan^{-1}(ax)}{3a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 81, normalized size = 0.47

$$\frac{\sqrt{c+a^2cx^2}\,\left(40+42a^2x^2+6ax(6+7a^2x^2)\,\text{ArcTan}(ax)-9(2+3a^2x^2)\,\text{ArcTan}(ax)^2\right)}{27a^4c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]

[Out] (Sqrt[c + a^2*c*x^2]*(40 + 42*a^2*x^2 + 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x] - 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2))/(27*a^4*c^3*(1 + a^2*x^2)^2)

Maple [C] Result contains complex when optimal does not.
time = 1.92, size = 276, normalized size = 1.60

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{216\left(a^2x^2+1\right)^2a^4c^3}-\frac{3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ia}{8c^3a^4(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^4/(a^2*x^2+1)+1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)

Fricas [A]
time = 1.90, size = 92, normalized size = 0.53

$$\frac{\sqrt{a^2cx^2+c}\,\left(42\,a^2x^2-9\,(3\,a^2x^2+2)\arctan\left(ax\right)^2+6\,(7\,a^3x^3+6\,ax)\arctan\left(ax\right)+40\right)}{27\,\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] 1/27*sqrt(a^2*c*x^2 + c)*(42*a^2*x^2 - 9*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6*(7*a^3*x^3 + 6*a*x)*arctan(a*x) + 40)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^2}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.350 $\int \frac{x^2 \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2x^3}{27c(c+a^2cx^2)^{3/2}}-\frac{4x}{9a^2c^2\sqrt{c+a^2cx^2}}+\frac{2x^2\text{ArcTan}(ax)}{9ac(c+a^2cx^2)^{3/2}}+\frac{4\text{ArcTan}(ax)}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\text{ArcTan}(ax)^2}{3c(c+a^2cx^2)^{3/2}}$$

```
[Out] -2/27*x^3/c/(a^2*c*x^2+c)^(3/2)+2/9*x^2*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)
+1/3*x^3*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-4/9*x/a^2/c^2/(a^2*c*x^2+c)^(1
/2)+4/9*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5064, 5058, 5050, 197}

$$\frac{2x^2\text{ArcTan}(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{x^3\text{ArcTan}(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{4x}{9a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{27c(a^2cx^2+c)^{3/2}}+\frac{4\text{ArcTan}(ax)}{9a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x^3)/(27*c*(c + a^2*c*x^2)^(3/2)) - (4*x)/(9*a^2*c^2*Sqrt[c + a^2*c*x^2
]) + (2*x^2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(9
*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
```

```
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{x^3\tan^{-1}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}} - \frac{1}{3}(2a)\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx \\
&= -\frac{2x^3}{27c\,(c+a^2cx^2)^{3/2}} + \frac{2x^2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{x^3\tan^{-1}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}} - \frac{4\int \frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{9ac} \\
&= -\frac{2x^3}{27c\,(c+a^2cx^2)^{3/2}} + \frac{2x^2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}} - \\
&= -\frac{2x^3}{27c\,(c+a^2cx^2)^{3/2}} - \frac{4x}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x^2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{4\tan^{-1}(ax)}{9a^3c^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.58

$$\frac{\sqrt{c+a^2cx^2}\,(-2ax(6+7a^2x^2)+6(2+3a^2x^2)\,\mathrm{ArcTan}(ax)+9a^3x^3\,\mathrm{ArcTan}(ax)^2)}{27a^3c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(6 + 7*a^2*x^2) + 6*(2 + 3*a^2*x^2)*ArcTan[a*x
] + 9*a^3*x^3*ArcTan[a*x]^2))/(27*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.03, size = 272, normalized size = 1.96

method	result
default	$\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2c^3a^3}+\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)}{8a^3c^3(a^2x^2+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*
(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(a*x)^2-2+2*I*arcta
n(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I
)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a^3/c^3/(a^2*x^2
+1)+1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(I+a*x))^(1/2)*(a
^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [A]
time = 0.33, size = 117, normalized size = 0.84

$$\frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}\,a^2c^2}-\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)\arctan(ax)^2-\frac{2\left(7\,a^3x^3+6\,ax-3\left(3\,a^2x^2+2\right)\arctan(ax)\right)a}{27\left(a^6c^2x^2+a^4c^2\right)\sqrt{a^2x^2+1}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*arc
tan(a*x)^2 - 2/27*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*a/((a
^6*c^2*x^2 + a^4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A]
time = 0.93, size = 88, normalized size = 0.63

$$\frac{\left(9\,a^3x^3\arctan(ax)^2-14\,a^3x^3-12\,ax+6\left(3\,a^2x^2+2\right)\arctan(ax)\right)\sqrt{a^2cx^2+c}}{27\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arctan(a*x)^2 - 14*a^3*x^3 - 12*a*x + 6*(3*a^2*x^2 + 2)*arc
tan(a*x))*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\mathrm{atan}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^2}{{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.351 $\int \frac{x\textbf{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{27a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{4}{9a^2c^2\sqrt{c+a^2cx^2}}+\frac{2x\text{ArcTan}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4x\text{ArcTan}(ax)}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\text{ArcTan}(ax)^2}{3a^2c\left(c+a^2cx^2\right)^{3/2}}$$

```
[Out] 2/27/a^2/c/(a^2*c*x^2+c)^(3/2)+2/9*x*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)-1/
3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+4/9/a^2/c^2/(a^2*c*x^2+c)^(1/2)+4
/9*x*arctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5050, 5016, 5014}

$$\frac{4x\text{ArcTan}(ax)}{9ac^2\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x\text{ArcTan}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{4}{9a^2c^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 2/(27*a^2*c*(c + a^2*c*x^2)^(3/2)) + 4/(9*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (2
*x*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*x*ArcTan[a*x])/(9*a*c^2*
Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^2/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
```

```
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^2}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{2\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{3a}$$
$$= \frac{2}{27a^2c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}} - \frac{\tan^{-1}(ax)^2}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{4\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{9ac}$$
$$= \frac{2}{27a^2c\,(c+a^2cx^2)^{3/2}} + \frac{4}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{4x\tan^{-1}(ax)}{9ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 0.52

$$\frac{\sqrt{c+a^2cx^2}\,\left(2(7+6a^2x^2)+6ax(3+2a^2x^2)\,\mathrm{ArcTan}(ax)-9\mathrm{ArcTan}(ax)^2\right)}{27c^3\,(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
[Out] (Sqrt[c + a^2*c*x^2]*(2*(7 + 6*a^2*x^2) + 6*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]
 - 9*ArcTan[a*x]^2))/(27*c^3*(a + a^3*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.48, size = 276, normalized size = 2.01

method	result
default	$\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2a^2c^3} - \frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax\cdots}{8c^3a^2(a^2x\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(
c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a
*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^2/(a
^2*x^2+1)-1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(
-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 2.96, size = 82, normalized size = 0.60

$$\frac{\sqrt{a^2cx^2+c}\,\left(12\,a^2x^2+6\,(2\,a^3x^3+3\,ax)\arctan\left(ax\right)-9\,\arctan\left(ax\right)^2+14\right)}{27\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(a^2*c*x^2 + c)*(12*a^2*x^2 + 6*(2*a^3*x^3 + 3*a*x)*arctan(a*x) -
9*arctan(a*x)^2 + 14)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,\mathrm{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\mathrm{atan}(a\,x)^2}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.352 $\int \frac{\text{ArcTan}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{2x}{27c\left(c+a^2cx^2\right)^{3/2}}-\frac{40x}{27c^2\sqrt{c+a^2cx^2}}+\frac{2\text{ArcTan}(ax)}{9ac\left(c+a^2cx^2\right)^{3/2}}+\frac{4\text{ArcTan}(ax)}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{ArcTan}(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2/27*x/c/(a^2*c*x^2+c)^(3/2)+2/9*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x
*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-40/27*x/c^2/(a^2*c*x^2+c)^(1/2)+4/3*ar
ctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5020, 5018, 197, 198}

$$\frac{2x\text{ArcTan}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}+\frac{4\text{ArcTan}(ax)}{3ac^2\sqrt{a^2cx^2+c}}+\frac{x\text{ArcTan}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\text{ArcTan}(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}-\frac{40x}{27c^2\sqrt{a^2cx^2+c}}-\frac{2x}{27c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*c^2*Sqrt[c + a^2*c*x^2]) +
 (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a*c^2*S
qrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*
ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
```

```
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}}-\frac{2}{9}\int\frac{1}{(c+a^2cx^2)^{5/2}}\,dx+\frac{2\int\frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{3c}$$
$$=-\frac{2x}{27c\,(c+a^2cx^2)^{3/2}}+\frac{2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{4\tan^{-1}(ax)}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}}+$$
$$=-\frac{2x}{27c\,(c+a^2cx^2)^{3/2}}-\frac{40x}{27c^2\sqrt{c+a^2cx^2}}+\frac{2\tan^{-1}(ax)}{9ac\,(c+a^2cx^2)^{3/2}}+\frac{4\tan^{-1}(ax)}{3ac^2\sqrt{c+a^2cx^2}}+$$

Mathematica [A]
time = 0.05, size = 86, normalized size = 0.55

$$\frac{\sqrt{c+a^2cx^2}\,(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\,\mathrm{ArcTan}(ax)+9ax(3+2a^2x^2)\,\mathrm{ArcTan}(ax)^2)}{27ac^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[a
*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2))/(27*a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 272, normalized size = 1.73

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2a\,c^3}+\frac{3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(a}{8c^3a(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c
*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a*x)^2-2+2*I*arctan
(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(
I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a/(a^2*x^2+1)-1
/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x^
3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [A]
time = 0.34, size = 111, normalized size = 0.71

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{a^2cx^2+c}\,c^2}+\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan\left(ax\right)^2-\frac{2\left(20\,a^3x^3+21\,ax-3\left(6\,a^2x^2+7\right)\arctan\left(ax\right)\right)a}{27\left(a^4c^2x^2+a^2c^2\right)\sqrt{a^2x^2+1}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a*
x)^2 - 2/27*(20*a^3*x^3 + 21*a*x - 3*(6*a^2*x^2 + 7)*arctan(a*x))*a/((a^4*c
^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

Fricas [A]
time = 1.25, size = 93, normalized size = 0.59

$$-\frac{\left(40\,a^3x^3-9\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^2+42\,ax-6\left(6\,a^2x^2+7\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{27\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 42*a*x - 6*(6*a^2
*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a
*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^2}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2),x)
[Out] int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2), x)
```

3.353 $\int \frac{\textbf{ArcTan}(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{2}{27c\,(c+a^2cx^2)^{3/2}}-\frac{22}{9c^2\sqrt{c+a^2cx^2}}-\frac{2ax\mathrm{ArcTan}(ax)}{9c\,(c+a^2cx^2)^{3/2}}-\frac{22ax\mathrm{ArcTan}(ax)}{9c^2\sqrt{c+a^2cx^2}}+\frac{\mathrm{ArcTan}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}}+\frac{\mathrm{ArcTan}(ax}{c^2\sqrt{c+a^2cx}}$$

```
[Out] -2/27/c/(a^2*c*x^2+c)^(3/2)-2/9*a*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/3*a
rctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-22/9/c^2/(a^2*c*x^2+c)^(1/2)-22/9*a*x*ar
ctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)-2*a
rctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^
2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+2*po
lylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1
/2)
```

Rubi [A]
time = 0.57, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5086, 5078, 5076, 4268, 2611, 2320, 6724, 5050, 5014, 5016}

$$\frac{2i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-e^{i\mathrm{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(e^{i\mathrm{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-e^{i\mathrm{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(e^{i\mathrm{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{\mathrm{ArcTan}(ax)^2}{c^2\sqrt{a^2cx^2+c}}-\frac{22ax\mathrm{ArcTan}(ax)}{9c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\mathrm{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{\mathrm{ArcTan}(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}-\frac{2ax\mathrm{ArcTan}(ax)}{9c\,(a^2cx^2+c)^{3/2}}-\frac{22}{9c^2\sqrt{a^2cx^2+c}}-\frac{2}{27c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -2/(27*c*(c + a^2*c*x^2)^(3/2)) - 22/(9*c^2*Sqrt[c + a^2*c*x^2]) - (2*a*x*A
rcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*a*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*c*(c + a^2*c*x^2)^(3/2)) + ArcTan[a*x]^2
/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(
I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - ((2*I)*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a
^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2
*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{1}{3}(2a)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}}{(c+a^2cx}}{c}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^2}{c^2\sqrt{c+a^2cx^2}} - \frac{(4}{}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{}{3}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{}{3}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{}{3}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{}{3}$$

$$= -\frac{2}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{22}{9c^2\sqrt{c+a^2cx^2}} - \frac{2ax\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22ax\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{}{3}$$

Mathematica [A]
time = 0.36, size = 246, normalized size = 0.63

$$\frac{(1+a^2x^2)^{3/2}\left(-\frac{270}{\sqrt{1+a^2x^2}} - \frac{270ax\,\mathrm{ArcTan}(ax)}{\sqrt{1+a^2x^2}} + \frac{135\,\mathrm{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}} - 2\cos(3\mathrm{ArcTan}(ax)) + 9\mathrm{ArcTan}(ax)^2\cos(3\mathrm{ArcTan}(ax)) + 108\mathrm{ArcTan}(ax)^2\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right) - 108\mathrm{ArcTan}(ax)^2\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right) + 216i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right) - 216i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right) - 216\mathrm{PolyLog}\left(3,-e^{i\mathrm{ArcTan}(ax)}\right) + 216\mathrm{PolyLog}\left(3,e^{i\mathrm{ArcTan}(ax)}\right) - 6\mathrm{ArcTan}(ax)\sin(3\mathrm{ArcTan}(ax))\right)}{108c\left(c(1+a^2x^2)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*(-270/Sqrt[1 + a^2*x^2] - (270*a*x*ArcTan[a*x])/Sqrt[1
 + a^2*x^2] + (135*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 2*Cos[3*ArcTan[a*x]]
+ 9*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 108*ArcTan[a*x]^2*Log[1 - E^(I*ArcTa
n[a*x])] - 108*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (216*I)*ArcTan[a*
x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (216*I)*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] - 216*PolyLog[3, -E^(I*ArcTan[a*x])] + 216*PolyLog[3, E^(I*ArcTa
n[a*x])] - 6*ArcTan[a*x]*Sin[3*ArcTan[a*x]]))/(108*c*(c*(1 + a^2*x^2))^(3/2
))
```

Maple [A]
time = 0.41, size = 460, normalized size = 1.18

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2c^3}+\frac{5\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ia}{8c^3(a^2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)^2-2+2*I*arctan
(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(
I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/(a^2*x^2+1)+1
/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-6*I*arctan
(a*x)+9*arctan(a*x)^2-2)/c^3/(a^4*x^4+2*a^2*x^2+1)+(arctan(a*x)^2*ln(1-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*
I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-
2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+
a*x))^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3
*a^2*c^3*x^3 + c^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2}{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.354 $\int \frac{\mathbf{ArcTan}(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=381

$$\frac{2a^2x}{27c\,(c+a^2cx^2)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}-\frac{2a\text{ArcTan}(ax)}{9c\,(c+a^2cx^2)^{3/2}}-\frac{10a\text{ArcTan}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\text{ArcTan}(ax)^2}{3c\,(c+a^2cx^2)^{3/2}}-\frac{5a^2x\text{ArcTan}(}{3c^2\sqrt{c+a^2c}}$$

```
[Out] 2/27*a^2*x/c/(a^2*c*x^2+c)^(3/2)-2/9*a*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1/
3*a^2*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+94/27*a^2*x/c^2/(a^2*c*x^2+c)^(
1/2)-10/3*a*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-5/3*a^2*x*arctan(a*x)^2/c^2
/(a^2*c*x^2+c)^(1/2)-4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2,-(1+I*a*x)^(1/
2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*I*a*polylog
(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/
2)-arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^3/x
```

Rubi [A]
time = 0.50, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5086, 5064, 5078, 5074, 5018, 197, 5020, 198}

$$-\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{c^3x}-\frac{5a^2x\text{ArcTan}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}-\frac{10a\text{ArcTan}(ax)}{3c^2\sqrt{a^2cx^2+c}}-\frac{4a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{a^2x\text{ArcTan}(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}-\frac{2a\text{ArcTan}(ax)}{9c\,(a^2cx^2+c)^{3/2}}+\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2ia\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{94a^2x}{27c^2\sqrt{a^2cx^2+c}}+\frac{2a^2x}{27c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a^2*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x)/(27*c^2*Sqrt[c + a^2*c*
x^2]) - (2*a*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (10*a*ArcTan[a*x])/
(3*c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3
/2)) - (5*a^2*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^2)/(c^3*x) - (4*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTanh[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2]) + ((2*I)*a*Sqrt
[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/(c^2*Sqrt[c +
 a^2*c*x^2]) - ((2*I)*a*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{9}\left(2a^2\right)\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}}\,dx+\frac{\int \frac{\tan^{-1}(}{x^2\sqrt{c+}}}{c^2}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a}{3c}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2}{3c}$$

$$= \frac{2a^2x}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}-\frac{2a\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{10a\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2}{3c}$$

Mathematica [A]
time = 1.16, size = 296, normalized size = 0.78

$$-\frac{a\left(-378ax+378\text{ArcTan}(ax)+189ax\text{ArcTan}(ax)^2+6\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax))+27ax\text{ArcTan}(ax)^2\csc^2\left(\tfrac{1}{2}\text{ArcTan}(ax)\right)-216\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)+216\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)-216i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+216i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-2\sqrt{1+a^2x^2}\sin(3\text{ArcTan}(ax))+9\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\sin(3\text{ArcTan}(ax))+54\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tan\left(\tfrac{1}{2}\text{ArcTan}(ax)\right)\right)}{108c^2\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/108*(a*(-378*a*x + 378*ArcTan[a*x] + 189*a*x*ArcTan[a*x]^2 + 6*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^2*Csc[ArcTan[a
*x]/2]^2 - 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 2
16*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (216*I)*Sqrt[
1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] + (216*I)*Sqrt[1 + a^2*x^2]*Pol
yLog[2, E^(I*ArcTan[a*x])] - 2*Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]] + 9*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]^2*Tan[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 0.41, size = 433, normalized size = 1.14

method	result
default	$\frac{a\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2c^3}-\frac{7a\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(a}{8c^3(a^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*a*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(
c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(a*x)^2-2+2*I*arcta
n(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(I
+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))*a/c^3/(a^2*x^2+1)+1/
216*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-6*I*arctan(a*
x)+9*arctan(a*x)^2-2)*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^2*(c*(a*x-I)*
(I+a*x))^(1/2)/c^3/x-2*I*a*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*
(a*x-I)*(I+a*x))^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3
*a^2*c^3*x^4 + c^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^2}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.355 $\int x^m(c+a^2cx^2)^2 \,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx=\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx$$

Mathematica [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a
^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x)
^2 - ((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a^2*
c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*log(a^2*x^2 +
1)^2 + 16*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/16*(12*((a^6*c^2*m^3 + 9*a^
6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2*m^3 + 9*a
^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*m + 3*(a^2
*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^2)*x^m*arc
tan(a*x)^2 + ((a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6
 + c^2*m^3 + 3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^
4 + 9*c^2*m^2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m +
15*a^2*c^2)*x^2 + 15*c^2)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^5*c^2*m^2 + 4*a^5*
c^2*m + 3*a^5*c^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a
*c^2*m^2 + 8*a*c^2*m + 15*a*c^2)*x)*x^m*arctan(a*x) + 4*((a^6*c^2*m^2 + 4*a
^6*c^2*m + 3*a^6*c^2)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5*a^4*c^2)*x^4 +
 (a^2*c^2*m^2 + 8*a^2*c^2*m + 15*a^2*c^2)*x^2)*x^m*log(a^2*x^2 + 1))/(m^3 +
 (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m
^3 + 9*m^2 + 23*m + 15)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^m \operatorname{atan}^2(ax)\,dx+\int 2a^2x^2x^m \operatorname{atan}^2(ax)\,dx+\int a^4x^4x^m \operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m*atan(a*x)*
*2, x) + Integral(a**4*x**4*x**m*atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.356 $\int x^m(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(x^m\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^2,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx=\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^2\,dx$$

Mathematica [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^2 - ((a^2*c
*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*log(a^2*x^2 + 1)^2 + 16*(m^2 + 4*m + 3
)*integrate(1/16*(12*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*(a^
2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x)^2 + ((a^4
*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m + 3*a^
2*c)*x^2 + 4*c*m + 3*c)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^3*c*m + a^3*c)*x^3 +
 (a*c*m + 3*a*c)*x)*x^m*arctan(a*x) + 4*((a^4*c*m + a^4*c)*x^4 + (a^2*c*m +
 3*a^2*c)*x^2)*x^m*log(a^2*x^2 + 1))/((a^2*m^2 + 4*a^2*m + 3*a^2)*x^2 + m^2
 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^m \operatorname{atan}^2(ax)\,dx + \int a^2x^2x^m \operatorname{atan}^2(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**m*atan(a*x)**2, x) + Integral(a**2*x**2*x**m*atan(a*x)**2, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^2*(c + a^2*c*x^2),x)
```

```
[Out] int(x^m*atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.357 $\int \frac{x^m \textbf{ArcTan}(ax)^2}{c+a^2cx^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^2}{c+a^2cx^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^2}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} dx = \int \frac{x^m \tan^{-1}(ax)^2}{c+a^2cx^2} dx$$

Mathematica [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^2}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^2}{a^2 c\, x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\operatorname{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2),x)
[Out] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.358 $\int \frac{x^m \textbf{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2} dx = \int \frac{x^m \tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$$

Mathematica [A]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^2}{\left(c+a^2cx^2\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2c\,x^2+c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^2}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

3.359 $\int x^m(c+a^2cx^2)^{3/2}\,\textbf{ArcTan}(ax)^2\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx = \int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.360 $\int x^m \sqrt{c+a^2cx^2}\, \mathbf{ArcTan}(ax)^2\, dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^2, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^2\, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2\, dx = \int x^m \sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2\, dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^2\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.49, size = 0, normalized size = 0.00

$$\int x^m \sqrt{a^2c\,x^2+c}\, \arctan\left(ax\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.361 $\int \frac{x^m \mathbf{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{x^m \operatorname{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \operatorname{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} dx = \int \frac{x^m \tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2c\,x^2+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.362 $\int \frac{x^m \mathbf{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^2}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atan}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.363 $\int x^3(c+a^2cx^2)\,\textbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=219

$$\frac{cx}{15a^3}-\frac{cx^3}{60a}-\frac{c\text{ArcTan}(ax)}{15a^4}-\frac{cx^2\text{ArcTan}(ax)}{60a^2}+\frac{1}{20}cx^4\text{ArcTan}(ax)+\frac{7ic\text{ArcTan}(ax)^2}{30a^4}+\frac{cx\text{ArcTan}(ax)^2}{4a^3}-\frac{cx^3\text{A}}{}$$

```
[Out] 1/15*c*x/a^3-1/60*c*x^3/a-1/15*c*arctan(a*x)/a^4-1/60*c*x^2*arctan(a*x)/a^2
+1/20*c*x^4*arctan(a*x)+7/30*I*c*arctan(a*x)^2/a^4+1/4*c*x*arctan(a*x)^2/a^
3-1/12*c*x^3*arctan(a*x)^2/a-1/10*a*c*x^5*arctan(a*x)^2-1/12*c*arctan(a*x)^
3/a^4+1/4*c*x^4*arctan(a*x)^3+1/6*a^2*c*x^6*arctan(a*x)^3+7/15*c*arctan(a*x
)*ln(2/(1+I*a*x))/a^4+7/30*I*c*polylog(2,1-2/(1+I*a*x))/a^4
```

Rubi [A]
time = 0.82, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5070, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004, 308}

$$-\frac{c\text{ArcTan}(ax)^3}{12a^4}+\frac{7ic\text{ArcTan}(ax)^2}{30a^4}-\frac{c\text{ArcTan}(ax)}{15a^4}+\frac{7c\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^4}+\frac{7ic\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{30a^4}+\frac{cx\text{ArcTan}(ax)^2}{4a^3}+\frac{cx}{15a^3}+\frac{1}{6}a^2cx^6\text{ArcTan}(ax)^3-\frac{cx^2\text{ArcTan}(ax)}{60a^2}-\frac{1}{10}acx^5\text{ArcTan}(ax)^2+\frac{1}{4}cx^4\text{ArcTan}(ax)^3+\frac{1}{20}cx^4\text{ArcTan}(ax)-\frac{cx^3\text{ArcTan}(ax)^2}{12a}-\frac{cx^3}{60a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*x)/(15*a^3) - (c*x^3)/(60*a) - (c*ArcTan[a*x])/(15*a^4) - (c*x^2*ArcTan[
a*x])/(60*a^2) + (c*x^4*ArcTan[a*x])/20 + (((7*I)/30)*c*ArcTan[a*x]^2)/a^4
+ (c*x*ArcTan[a*x]^2)/(4*a^3) - (c*x^3*ArcTan[a*x]^2)/(12*a) - (a*c*x^5*Arc
Tan[a*x]^2)/10 - (c*ArcTan[a*x]^3)/(12*a^4) + (c*x^4*ArcTan[a*x]^3)/4 + (a^
2*c*x^6*ArcTan[a*x]^3)/6 + (7*c*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(15*a^4) +
(((7*I)/30)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx = c\int x^3\tan^{-1}(ax)^3\,dx + \left(a^2c\right)\int x^5\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)^3 + \frac{1}{6}a^2cx^6\tan^{-1}(ax)^3 - \frac{1}{4}(3ac)\int \frac{x^4\tan^{-1}(ax)^2}{1+a^2x^2}\,dx - \frac{1}{2}(\cdots$$

$$= \frac{1}{4}cx^4\tan^{-1}(ax)^3 + \frac{1}{6}a^2cx^6\tan^{-1}(ax)^3 - \frac{(3c)\int x^2\tan^{-1}(ax)^2\,dx}{4a} + \frac{(3c)\int \cdots}{\cdots}$$

$$= -\frac{cx^3\tan^{-1}(ax)^2}{4a} - \frac{1}{10}acx^5\tan^{-1}(ax)^2 + \frac{1}{4}cx^4\tan^{-1}(ax)^3 + \frac{1}{6}a^2cx^6\tan^{-1}\cdots$$

$$= \frac{3cx\tan^{-1}(ax)^2}{4a^3} - \frac{cx^3\tan^{-1}(ax)^2}{12a} - \frac{1}{10}acx^5\tan^{-1}(ax)^2 - \frac{c\tan^{-1}(ax)^3}{4a^4} + \cdots$$

$$= \frac{cx^2\tan^{-1}(ax)}{4a^2} + \frac{1}{20}cx^4\tan^{-1}(ax) + \frac{ic\tan^{-1}(ax)^2}{a^4} + \frac{cx\tan^{-1}(ax)^2}{4a^3} - \frac{cx^3\cdots}{\cdots}$$

$$= -\frac{cx}{4a^3} - \frac{cx^2\tan^{-1}(ax)}{60a^2} + \frac{1}{20}cx^4\tan^{-1}(ax) + \frac{7ic\tan^{-1}(ax)^2}{30a^4} + \frac{cx\tan^{-1}(a\cdots}{4a^3}$$

$$= \frac{cx}{15a^3} - \frac{cx^3}{60a} + \frac{c\tan^{-1}(ax)}{4a^4} - \frac{cx^2\tan^{-1}(ax)}{60a^2} + \frac{1}{20}cx^4\tan^{-1}(ax) + \frac{7ic\tan^{-1}\cdots}{30\cdots}$$

$$= \frac{cx}{15a^3} - \frac{cx^3}{60a} - \frac{c\tan^{-1}(ax)}{15a^4} - \frac{cx^2\tan^{-1}(ax)}{60a^2} + \frac{1}{20}cx^4\tan^{-1}(ax) + \frac{7ic\tan^{-1}\cdots}{30\cdots}$$

$$= \frac{cx}{15a^3} - \frac{cx^3}{60a} - \frac{c\tan^{-1}(ax)}{15a^4} - \frac{cx^2\tan^{-1}(ax)}{60a^2} + \frac{1}{20}cx^4\tan^{-1}(ax) + \frac{7ic\tan^{-1}\cdots}{30\cdots}$$

Mathematica [A]
time = 0.48, size = 135, normalized size = 0.62

$$\frac{c\left(4ax-a^3x^3-(14i-15ax+5a^3x^3+6a^5x^5)\operatorname{ArcTan}(ax)^2+5(-1+3a^4x^4+2a^6x^6)\operatorname{ArcTan}(ax)^3+\operatorname{ArcTan}(ax)\left(-4-a^2x^2+3a^4x^4+28\log\left(1+e^{2i\operatorname{ArcTan}(ax)}\right)\right)-14i\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(ax)}\right)\right)}{60a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(4*a*x - a^3*x^3 - (14*I - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5)*ArcTan[a*x]^2
 + 5*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^3 + ArcTan[a*x]*(-4 - a^2*x^2
 + 3*a^4*x^4 + 28*Log[1 + E^((2*I)*ArcTan[a*x])]) - (14*I)*PolyLog[2, -E^((
2*I)*ArcTan[a*x])]))/(60*a^4)
```

Maple [A]
time = 0.63, size = 274, normalized size = 1.25

method	result
derivativedivides	$\frac{c\arctan(ax)^3 a^6x^6}{6}+\frac{c\arctan(ax)^3 a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)^2a^5x^5}{5}+\frac{\arctan(ax)^2a^3x^3}{3}-\arctan(ax)^2ax+\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)a^4x^4}{5}\right.}{}$
default	$\frac{c\arctan(ax)^3 a^6x^6}{6}+\frac{c\arctan(ax)^3 a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)^2a^5x^5}{5}+\frac{\arctan(ax)^2a^3x^3}{3}-\arctan(ax)^2ax+\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)a^4x^4}{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/6*c*arctan(a*x)^3*a^6*x^6+1/4*c*arctan(a*x)^3*a^4*x^4-1/4*c*(2/5*a
rctan(a*x)^2*a^5*x^5+1/3*arctan(a*x)^2*a^3*x^3-arctan(a*x)^2*a*x+1/3*arctan
(a*x)^3-1/5*arctan(a*x)*a^4*x^4+1/15*arctan(a*x)*a^2*x^2+14/15*arctan(a*x)*
ln(a^2*x^2+1)+1/15*a^3*x^3-4/15*a*x+4/15*arctan(a*x)+7/15*I*ln(a*x-I)*ln(a^
2*x^2+1)+7/15*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-7/15*I*ln(I+a*x)*ln(a^2*x^2+1)-
7/15*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-7/15*I*dilog(-1/2*I*(I+a*x))-7/30*I*ln(
a*x-I)^2+7/30*I*ln(I+a*x)^2+7/15*I*dilog(1/2*I*(a*x-I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/960*(20*(23040*a^7*c*integrate(1/960*x^7*arctan(a*x)^3/(a^5*x^2 + a^3), x
) - 5760*a^6*c*integrate(1/960*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 1440
*a^6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 1152*a^
6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 46080*a^5*c*
integrate(1/960*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 2304*a^5*c*integrat
e(1/960*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 8640*a^4*c*integrate(1/960*x^
4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 2160*a^4*c*integrate(1/960*x^4*log(a^
2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 960*a^4*c*integrate(1/960*x^4*log(a^2*x^
2 + 1)/(a^5*x^2 + a^3), x) + 23040*a^3*c*integrate(1/960*x^3*arctan(a*x)^3/
(a^5*x^2 + a^3), x) + 1920*a^3*c*integrate(1/960*x^3*arctan(a*x)/(a^5*x^2 +
 a^3), x) + 2880*a^2*c*integrate(1/960*x^2*log(a^2*x^2 + 1)/(a^5*x^2 + a^3)
, x) - 5760*a*c*integrate(1/960*x*arctan(a*x)/(a^5*x^2 + a^3), x) + 720*c*i
ntegrate(1/960*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + c*arctan(a*x)^3/a^4
)*a^4 + 40*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan(a*x)^3 - 4*(6*a^5*c*x^5 +
 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x)^2 + (6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*
c*x)*log(a^2*x^2 + 1)^2)/a^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^5 + c*x^3)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{3}\operatorname{atan}^{3}(ax)\,dx + \int a^{2}x^{5}\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**3,x)
[Out] c*(Integral(x**3*atan(a*x)**3, x) + Integral(a**2*x**5*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2),x)
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.364 $\int x^2(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=211

$$-\frac{cx^2}{20a}+\frac{cx\text{ArcTan}(ax)}{10a^2}+\frac{1}{10}cx^3\text{ArcTan}(ax)-\frac{c\text{ArcTan}(ax)^2}{20a^3}-\frac{cx^2\text{ArcTan}(ax)^2}{5a}-\frac{3}{20}acx^4\text{ArcTan}(ax)^2-\frac{2ic\text{A}}{}$$

```
[Out] -1/20*c*x^2/a+1/10*c*x*arctan(a*x)/a^2+1/10*c*x^3*arctan(a*x)-1/20*c*arctan
(a*x)^2/a^3-1/5*c*x^2*arctan(a*x)^2/a-3/20*a*c*x^4*arctan(a*x)^2-2/15*I*c*a
rctan(a*x)^3/a^3+1/3*c*x^3*arctan(a*x)^3+1/5*a^2*c*x^5*arctan(a*x)^3-2/5*c*
arctan(a*x)^2*ln(2/(1+I*a*x))/a^3-2/5*I*c*arctan(a*x)*polylog(2,1-2/(1+I*a*
x))/a^3-1/5*c*polylog(3,1-2/(1+I*a*x))/a^3
```

Rubi [A]

time = 0.63, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5070, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745, 272, 45}

$$-\frac{2ic\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{5a^3}-\frac{2ic\text{ArcTan}(ax)^3}{15a^3}-\frac{c\text{ArcTan}(ax)^2}{20a^3}-\frac{2c\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^3}-\frac{c\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{5a^3}+\frac{1}{5}a^2cx^5\text{ArcTan}(ax)^3+\frac{cx\text{ArcTan}(ax)}{10a^2}-\frac{3}{20}acx^4\text{ArcTan}(ax)^2+\frac{1}{3}cx^3\text{ArcTan}(ax)^3+\frac{1}{10}cx^3\text{ArcTan}(ax)-\frac{cx^2\text{ArcTan}(ax)^2}{5a}-\frac{cx^2}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/20*(c*x^2)/a + (c*x*ArcTan[a*x])/(10*a^2) + (c*x^3*ArcTan[a*x])/10 - (c*
ArcTan[a*x]^2)/(20*a^3) - (c*x^2*ArcTan[a*x]^2)/(5*a) - (3*a*c*x^4*ArcTan[a
*x]^2)/20 - (((2*I)/15)*c*ArcTan[a*x]^3)/a^3 + (c*x^3*ArcTan[a*x]^3)/3 + (a
^2*c*x^5*ArcTan[a*x]^3)/5 - (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a^3)
- (((2*I)/5)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (c*PolyLog[
3, 1 - 2/(1 + I*a*x)])/(5*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx &= c\int x^2\tan^{-1}(ax)^3\,dx+\left(a^2c\right)\int x^4\tan^{-1}(ax)^3\,dx\\
&= \frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3-(ac)\int\frac{x^3\tan^{-1}(ax)^2}{1+a^2x^2}\,dx-\frac{1}{5}(3a^3\\
&= \frac{1}{3}cx^3\tan^{-1}(ax)^3+\frac{1}{5}a^2cx^5\tan^{-1}(ax)^3-\frac{c\int x\tan^{-1}(ax)^2\,dx}{a}+\frac{c\int\frac{x\tan^{-1}(a}{1+a^2x}}{a}\\
&= -\frac{cx^2\tan^{-1}(ax)^2}{2a}-\frac{3}{20}acx^4\tan^{-1}(ax)^2-\frac{ic\tan^{-1}(ax)^3}{3a^3}+\frac{1}{3}cx^3\tan^{-1}(ax)^3\\
&= -\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}acx^4\tan^{-1}(ax)^2-\frac{2ic\tan^{-1}(ax)^3}{15a^3}+\frac{1}{3}cx^3\tan^{-1}(ax\\
&= \frac{cx\tan^{-1}(ax)}{a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{2a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}ac\\
&= \frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}ac\\
&= \frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)^2}{5a}-\frac{3}{20}ac\\
&= -\frac{cx^2}{20a}+\frac{cx\tan^{-1}(ax)}{10a^2}+\frac{1}{10}cx^3\tan^{-1}(ax)-\frac{c\tan^{-1}(ax)^2}{20a^3}-\frac{cx^2\tan^{-1}(ax)}{5a}
\end{aligned}$$

Mathematica [A]
time = 0.42, size = 171, normalized size = 0.81

$$\frac{c\left(-3-3a^2x^2+6ax\text{ArcTan}(ax)+6a^3x^3\text{ArcTan}(ax)-3\text{ArcTan}(ax)^2-12a^2x^2\text{ArcTan}(ax)^2-9a^4x^4\text{ArcTan}(ax)^2+8i\text{ArcTan}(ax)^3+20a^3x^3\text{ArcTan}(ax)^3+12a^5x^5\text{ArcTan}(ax)^3-24\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-12\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{60a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(-3 - 3*a^2*x^2 + 6*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 3*ArcTan[a
*x]^2 - 12*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 + (8*I)*ArcTan[a
*x]^3 + 20*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*ArcTan[a*x]^3 - 24*ArcTan[a*x
]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I
)*ArcTan[a*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.18, size = 2466, normalized size = 11.69

method	result	size

derivativedivides	Expression too large to display	2466
default	Expression too large to display	2466

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/5*c*arctan(a*x)^3*a^5*x^5+1/3*c*arctan(a*x)^3*a^3*x^3-1/5*c*(3/4*a
rctan(a*x)^2*a^4*x^4+arctan(a*x)^2*a^2*x^2-arctan(a*x)^2*ln(a^2*x^2+1)+2*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I
*a*x)^2/(a^2*x^2+1))+polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/48*I*(-32*arctan
(a*x)^3-12*I-9*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a^2*x^2+18*Pi*arctan(a*x)^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*a^2*x^2+9*Pi*
arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x
^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*a^2*x^2-18*Pi*arctan(a*x)^2*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/
(a^2*x^2+1)+I)^2*a^2*x^2-9*I*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^3*a*x+3*I*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
*a^3*x^3-3*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x
)^2/(a^2*x^2+1)+I)^3*a^3*x^3+9*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x
^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*a*x+24*I*arctan(a*x)*a*x+24*I*arct
an(a*x)*a^3*x^3+24*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi*arctan(a*x)^2+24*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi*arctan(a*x)^2*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1))-12*I*arctan(a*x)^2+18*I*Pi*arctan(a*x)^2*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*a*x-
6*I*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)^2*a^3*x^3-3*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)
*a^3*x^3+6*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I
*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*a^3*x^3+9*I*Pi*arcta
n(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)
^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*a*x-18*I*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^
2+1)+I)^2*a*x-9*I*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a*x+3*I*Pi*arctan(a*x)^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a^3*x^3-12*I*
a^2*x^2-24*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*
Pi*arctan(a*x)^2-24*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+3*Pi
*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+21*Pi*arctan(a*x)^2*
csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3+48*Pi*arc
tan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-24*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/
```

```
(a^2*x^2+1)^(1/2))^2+3*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-6*Pi*arctan(a*x)^2*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+21*Pi*arctan(a
*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+
2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)-42*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2
-96*I*arctan(a*x)^2*ln(2)-9*Pi*arctan(a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3*a^2*x^2+9*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(
1+I*a*x)^2/(a^2*x^2+1)+I)^3*a^2*x^2-24*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi*arctan(a
*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^3 - 1/160*(3*a^2*c*x^5 + 5*c*x^3)
*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/160*(140*(a^4*c*x^6 + 2*a^2*c
*x^4 + c*x^2)*arctan(a*x)^3 - 4*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x)^2 + 4
*(3*a^4*c*x^6 + 5*a^2*c*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (3*a^3*c*x^5 +
5*a*c*x^3 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*arctan(a*x))*log(a^2*x^2 +
 1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{2}\operatorname{atan}^{3}(ax)\,dx+\int a^{2}x^{4}\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**3, x) + Integral(a**2*x**4*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2),x)
```

```
[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.365 $\int x(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=160

$$-\frac{cx}{4a}+\frac{c(1+a^2x^2)\,\mathrm{ArcTan}(ax)}{4a^2}-\frac{ic\mathrm{ArcTan}(ax)^2}{2a^2}-\frac{cx\mathrm{ArcTan}(ax)^2}{2a}-\frac{cx(1+a^2x^2)\,\mathrm{ArcTan}(ax)^2}{4a}+\frac{c(1+a^2x^2)^2}{4}$$

```
[Out] -1/4*c*x/a+1/4*c*(a^2*x^2+1)*arctan(a*x)/a^2-1/2*I*c*arctan(a*x)^2/a^2-1/2*
c*x*arctan(a*x)^2/a-1/4*c*x*(a^2*x^2+1)*arctan(a*x)^2/a+1/4*c*(a^2*x^2+1)^2
*arctan(a*x)^3/a^2-c*arctan(a*x)*ln(2/(1+I*a*x))/a^2-1/2*I*c*polylog(2,1-2/
(1+I*a*x))/a^2
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5050, 5000, 4930, 5040, 4964, 2449, 2352, 8}

$$\frac{c(a^2x^2+1)^2\,\mathrm{ArcTan}(ax)^3}{4a^2}-\frac{cx(a^2x^2+1)\,\mathrm{ArcTan}(ax)^2}{4a}+\frac{c(a^2x^2+1)\,\mathrm{ArcTan}(ax)}{4a^2}-\frac{ic\mathrm{ArcTan}(ax)^2}{2a^2}-\frac{c\mathrm{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}-\frac{ic\mathrm{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^2}-\frac{cx\mathrm{ArcTan}(ax)^2}{2a}-\frac{cx}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/4*(c*x)/a + (c*(1 + a^2*x^2)*ArcTan[a*x])/(4*a^2) - ((I/2)*c*ArcTan[a*x]
^2)/a^2 - (c*x*ArcTan[a*x]^2)/(2*a) - (c*x*(1 + a^2*x^2)*ArcTan[a*x]^2)/(4*
a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(4*a^2) - (c*ArcTan[a*x]*Log[2/(1 +
I*a*x)])/a^2 - ((I/2)*c*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x(c+a^2cx^2)\tan^{-1}(ax)^3\,dx = \frac{c(1+a^2x^2)^2\tan^{-1}(ax)^3}{4a^2} - \frac{3\int(c+a^2cx^2)\tan^{-1}(ax)^2\,dx}{4a}$$
$$= \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{cx(1+a^2x^2)\tan^{-1}(ax)^2}{4a} + \frac{c(1+a^2x^2)^2\tan^{-1}(ax}{4a^2}$$
$$= -\frac{cx}{4a} + \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx(1+a^2x^2)\tan^{-1}(ax)^2}{4a}$$
$$= -\frac{cx}{4a} + \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx(1+a}{}$$
$$= -\frac{cx}{4a} + \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx(1+a}{}$$
$$= -\frac{cx}{4a} + \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx(1+a}{}$$
$$= -\frac{cx}{4a} + \frac{c(1+a^2x^2)\tan^{-1}(ax)}{4a^2} - \frac{ic\tan^{-1}(ax)^2}{2a^2} - \frac{cx\tan^{-1}(ax)^2}{2a} - \frac{cx(1+a}{}$$

Mathematica [A]

time = 0.05, size = 101, normalized size = 0.63

$$\frac{c\left(-ax-(-2i+3ax+a^3x^3)\,\mathrm{ArcTan}(ax)^2+(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^3+\mathrm{ArcTan}(ax)\left(1+a^2x^2-4\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)\right)+2i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(-(a*x) - (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + (1 + a^2*x^2)^2*ArcTa
n[a*x]^3 + ArcTan[a*x]*(1 + a^2*x^2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) + (
2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(4*a^2)
```

Maple [A]

time = 0.91, size = 240, normalized size = 1.50

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{\arctan(ax)^2a^3x^3}{3}+\arctan(ax)^2ax-\frac{\arctan(ax)a^2x^2}{3}-\frac{2\arctan(ax)\ln(a}{3}\right.}{}}{}$
default	$\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{\arctan(ax)^2a^3x^3}{3}+\arctan(ax)^2ax-\frac{\arctan(ax)a^2x^2}{3}-\frac{2\arctan(ax)\ln(a}{3}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4*c*arctan(a*x)^3*a^4*x^4+1/2*a^2*c*x^2*arctan(a*x)^3+1/4*c*arctan
(a*x)^3-3/4*c*(1/3*arctan(a*x)^2*a^3*x^3+arctan(a*x)^2*a*x-1/3*arctan(a*x)*
a^2*x^2-2/3*arctan(a*x)*ln(a^2*x^2+1)+1/3*a*x-1/3*arctan(a*x)-1/3*I*ln(a*x-
I)*ln(a^2*x^2+1)+1/3*I*dilog(-1/2*I*(I+a*x))+1/3*I*ln(a*x-I)*ln(-1/2*I*(I+a
*x))+1/6*I*ln(a*x-I)^2+1/3*I*ln(I+a*x)*ln(a^2*x^2+1)-1/3*I*dilog(1/2*I*(a*x
-I))-1/3*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/6*I*ln(I+a*x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 4*(512*a^5*c*integrat
e(1/64*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) - 192*a^4*c*integrate(1/64*x^4*a
rctan(a*x)^2/(a^3*x^2 + a), x) - 48*a^4*c*integrate(1/64*x^4*log(a^2*x^2 +
1)^2/(a^3*x^2 + a), x) - 64*a^4*c*integrate(1/64*x^4*log(a^2*x^2 + 1)/(a^3*
x^2 + a), x) + 1024*a^3*c*integrate(1/64*x^3*arctan(a*x)^3/(a^3*x^2 + a), x
) + 128*a^3*c*integrate(1/64*x^3*arctan(a*x)/(a^3*x^2 + a), x) - 384*a^2*c*
integrate(1/64*x^2*arctan(a*x)^2/(a^3*x^2 + a), x) - 96*a^2*c*integrate(1/6
4*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 192*a^2*c*integrate(1/64*x^2*l
og(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 512*a*c*integrate(1/64*x*arctan(a*x)^3/
(a^3*x^2 + a), x) + 384*a*c*integrate(1/64*x*arctan(a*x)/(a^3*x^2 + a), x)
- c*arctan(a*x)^3/a^2 - 48*c*integrate(1/64*log(a^2*x^2 + 1)^2/(a^3*x^2 + a
), x))*a^2 - 4*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x)^2 + (a^3*c*x^3 + 3*a*c*x)*
log(a^2*x^2 + 1)^2)/a^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int a^{2}x^{3}\operatorname{atan}^{3}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x*atan(a*x)**3, x) + Integral(a**2*x**3*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2),x)
```

```
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.366 $\int (c+a^2cx^2)\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=172

$$cx\text{ArcTan}(ax)-\frac{c(1+a^2x^2)\,\text{ArcTan}(ax)^2}{2a}+\frac{2ic\text{ArcTan}(ax)^3}{3a}+\frac{2}{3}cx\text{ArcTan}(ax)^3+\frac{1}{3}cx\left(1+a^2x^2\right)\text{ArcTan}(ax$$

```
[Out] c*x*arctan(a*x)-1/2*c*(a^2*x^2+1)*arctan(a*x)^2/a+2/3*I*c*arctan(a*x)^3/a+2
/3*c*x*arctan(a*x)^3+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^3+2*c*arctan(a*x)^2*ln
(2/(1+I*a*x))/a-1/2*c*ln(a^2*x^2+1)/a+2*I*c*arctan(a*x)*polylog(2,1-2/(1+I*
a*x))/a+c*polylog(3,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {5000, 4930, 5040, 4964, 5004, 5114, 6745, 266}

$$\frac{1}{3}cx\left(a^2x^2+1\right)\text{ArcTan}(ax)^3-\frac{c(a^2x^2+1)\,\text{ArcTan}(ax)^2}{2a}-\frac{c\log\left(a^2x^2+1\right)}{2a}+\frac{2ic\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a}+\frac{2ic\text{ArcTan}(ax)^3}{3a}+\frac{2}{3}cx\text{ArcTan}(ax)^3+cx\text{ArcTan}(ax)+\frac{2c\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}+\frac{c\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] c*x*ArcTan[a*x] - (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a) + (((2*I)/3)*c*ArcT
an[a*x]^3)/a + (2*c*x*ArcTan[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/
3 + (2*c*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - (c*Log[1 + a^2*x^2])/(2*a) +
 ((2*I)*c*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (c*PolyLog[3, 1 -
2/(1 + I*a*x)])/a
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (c + a^2cx^2) \tan^{-1}(ax)^3 \, dx = -\frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{1}{3}cx(1 + a^2x^2) \tan^{-1}(ax)^3 + \frac{1}{3}(2c) \int \tan^{-1}(ax$$

$$= cx \tan^{-1}(ax) - \frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{2}{3}cx \tan^{-1}(ax)^3 + \frac{1}{3}cx(1 + a^2x^2)$$

$$= cx \tan^{-1}(ax) - \frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{2ic \tan^{-1}(ax)^3}{3a} + \frac{2}{3}cx \tan^{-1}(ax)^3$$

$$= cx \tan^{-1}(ax) - \frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{2ic \tan^{-1}(ax)^3}{3a} + \frac{2}{3}cx \tan^{-1}(ax)^3$$

$$= cx \tan^{-1}(ax) - \frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{2ic \tan^{-1}(ax)^3}{3a} + \frac{2}{3}cx \tan^{-1}(ax)^3$$

$$= cx \tan^{-1}(ax) - \frac{c(1 + a^2x^2) \tan^{-1}(ax)^2}{2a} + \frac{2ic \tan^{-1}(ax)^3}{3a} + \frac{2}{3}cx \tan^{-1}(ax)^3$$

Mathematica [A]
time = 0.04, size = 144, normalized size = 0.84

$$\frac{c(6ax\text{ArcTan}(ax) - 3\text{ArcTan}(ax)^2 - 3a^2x^2\text{ArcTan}(ax)^2 - 4i\text{ArcTan}(ax)^3 + 6ax\text{ArcTan}(ax)^3 + 2a^3x^3\text{ArcTan}(ax)^3 + 12\text{ArcTan}(ax)^2\log\left(1 + e^{2i\text{ArcTan}(ax)}\right) - 3\log\left(1 + a^2x^2\right) - 12i\text{ArcTan}(ax)\text{PolyLog}\left(2, -e^{2i\text{ArcTan}(ax)}\right) + 6\text{PolyLog}\left(3, -e^{2i\text{ArcTan}(ax)}\right))}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*(6*a*x*ArcTan[a*x] - 3*ArcTan[a*x]^2 - 3*a^2*x^2*ArcTan[a*x]^2 - (4*I)*A
rcTan[a*x]^3 + 6*a*x*ArcTan[a*x]^3 + 2*a^3*x^3*ArcTan[a*x]^3 + 12*ArcTan[a*
x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 3*Log[1 + a^2*x^2] - (12*I)*ArcTan[a*
x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[a*x])
]))/(6*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.41, size = 1510, normalized size = 8.78

method	result	size
derivativedivides	Expression too large to display	1510
default	Expression too large to display	1510

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/3*c*arctan(a*x)^3*a^3*x^3+c*arctan(a*x)^3*a*x-c*(1/2*arctan(a*x)^2*a
^2*x^2+arctan(a*x)^2*ln(a^2*x^2+1)-2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)
^(1/2))+1/12*I*arctan(a*x)*(-6*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
```

```
)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi*a*x-3*I*arctan(a*x)*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x
)^2/(a^2*x^2+1)+I)*Pi*a*x+3*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)^3*Pi*a*x+6*I*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I
*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi*a*x-6*I*arctan(a*
x)+3*I*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*Pi*a*x+6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*cs
gn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi-12*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*Pi+6*arctan(a*x)*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))^3*Pi-6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*Pi+6*arctan(
a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi+6*arctan(a
*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi-6*ar
ctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi-3*arctan(a*x)*csgn(I*((1+I*a*x)^2/(
a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi+6*arctan(a*x)*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi
-3*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi-3*arctan(a*x)*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*
a*x)^2/(a^2*x^2+1)+I)*Pi+6*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*cs
gn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi-3*arctan
(a*x)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*Pi+
12*I*a*x+8*arctan(a*x)^2-3*I*arctan(a*x)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2
*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*Pi*a*x+24*I*arctan(a*x)*ln(2)+12)-ln((1+I*a
*x)^2/(a^2*x^2+1)+1)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-po
lylog(3,-(1+I*a*x)^2/(a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 28*a^4*c*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 3*a^4*c*integ
rate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*in
tegrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 4*a^3*c*i
ntegrate(1/32*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + a^3*c*integrate(1/32*x^
3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/24*(a^2*c*x^3 + 3*c*x)*arctan(a*
x)^3 + 7/32*c*arctan(a*x)^4/a + 56*a^2*c*integrate(1/32*x^2*arctan(a*x)^3/(
a^2*x^2 + 1), x) + 6*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^
2/(a^2*x^2 + 1), x) + 12*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 +
 1)/(a^2*x^2 + 1), x) - 1/32*(a^2*c*x^3 + 3*c*x)*arctan(a*x)*log(a^2*x^2 +
```

```
1)^2 - 12*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a*c*inte
grate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3*c*integrate(1/32*arct
an(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}x^{2}\operatorname{atan}^{3}{\left(ax \right)}\,dx + \int \operatorname{atan}^{3}{\left(ax \right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3,x)
[Out] c*(Integral(a**2*x**2*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2),x)
[Out] int(atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.367 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)^3}{x}\,dx$

Optimal. Leaf size=276

$$-\frac{3}{2}ic\text{ArcTan}(ax)^2-\frac{3}{2}acx\text{ArcTan}(ax)^2+\frac{1}{2}c\text{ArcTan}(ax)^3+\frac{1}{2}a^2cx^2\text{ArcTan}(ax)^3+2c\text{ArcTan}(ax)^3\tanh^{-1}\Bigg(1-$$

[Out] -3/2*I*c*arctan(a*x)^2-3/2*a*c*x*arctan(a*x)^2+1/2*c*arctan(a*x)^3+1/2*a^2*c*x^2*arctan(a*x)^3-2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*c*arctan(a*x)*ln(2/(1+I*a*x))-3/2*I*c*polylog(2,1-2/(1+I*a*x))-3/2*I*c*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+3/2*I*c*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))-3/2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/2*c*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/4*I*c*polylog(4,1-2/(1+I*a*x))-3/4*I*c*polylog(4,-1+2/(1+I*a*x))

Rubi [A]
time = 0.37, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5070, 4942, 5108, 5004, 5114, 5118, 6745, 4946, 5036, 4930, 5040, 4964, 2449, 2352}

$$\frac{1}{2}a^2cx^2\text{ArcTan}(ax)^3-\frac{3}{2}ic\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}ic\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{3}{2}c\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}c\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)+\frac{1}{2}c\text{ArcTan}(ax)^3-\frac{3}{2}ic\text{ArcTan}(ax)^2-\frac{3}{2}acx\text{ArcTan}(ax)^2-3c\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2c\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ic\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{4}ic\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{3}{4}ic\text{Li}_4\left(\frac{2}{iax+1}-1\right)$$

Antiderivative was successfully verified.

[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]

[Out] ((-3*I)/2)*c*ArcTan[a*x]^2 - (3*a*c*x*ArcTan[a*x]^2)/2 + (c*ArcTan[a*x]^3)/2 + (a^2*c*x^2*ArcTan[a*x]^3)/2 + 2*c*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*c*ArcTan[a*x]*Log[2/(1 + I*a*x)] - ((3*I)/2)*c*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((3*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((3*I)/4)*c*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c*PolyLog[4, -1 + 2/(1 + I*a*x)]

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x}\,dx &= c\int \frac{\tan^{-1}(ax)^3}{x}\,dx + \left(a^2c\right)\int x\tan^{-1}(ax)^3\,dx \\
&= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - (6ac)\int \frac{\tan^{-}}{} \\
&= \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) - \frac{1}{2}(3ac)\int \tan \\
&= -\frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 + 2c\tan^{-1}(ax)^3\,t \\
&= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 \\
&= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 \\
&= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3 \\
&= -\frac{3}{2}ic\tan^{-1}(ax)^2 - \frac{3}{2}acx\tan^{-1}(ax)^2 + \frac{1}{2}c\tan^{-1}(ax)^3 + \frac{1}{2}a^2cx^2\tan^{-1}(ax)^3
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 284, normalized size = 1.03

$$\frac{1}{2}c(1+a^2x^2)\,\mathrm{ArcTan}(ax)^3 + 2c\mathrm{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2i}{i-ax}\right) - \frac{3}{2}c\left(-i\mathrm{ArcTan}(ax)^2 + ax\mathrm{ArcTan}(ax)^2 + 2\mathrm{ArcTan}(ax)\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right) - i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right) + \frac{3}{2}ic\mathrm{ArcTan}(ax)^2\mathrm{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right) - \frac{3}{2}ic\mathrm{ArcTan}(ax)^2\mathrm{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right) + \frac{3}{2}c\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right) - \frac{3}{2}c\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right) - \frac{3}{4}ic\mathrm{PolyLog}\left(4,\frac{-i-ax}{-i+ax}\right) + \frac{3}{4}ic\mathrm{PolyLog}\left(4,\frac{i+ax}{-i+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c*(1 + a^2*x^2)*ArcTan[a*x]^3)/2 + 2*c*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I
- a*x)] - (3*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[a*x]^2 + 2*ArcTan[a*x]*Log[
1 + E^((2*I)*ArcTan[a*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/2 + ((3
*I)/2)*c*ArcTan[a*x]^2*PolyLog[2, (-I - a*x)/(-I + a*x)] - ((3*I)/2)*c*ArcT
an[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] + (3*c*ArcTan[a*x]*PolyLog[3, (-
I - a*x)/(-I + a*x)])/2 - (3*c*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I + a*x)]
)/2 - ((3*I)/4)*c*PolyLog[4, (-I - a*x)/(-I + a*x)] + ((3*I)/4)*c*PolyLog[4
, (I + a*x)/(-I + a*x)]
```

Maple [A]
time = 16.67, size = 460, normalized size = 1.67

method	result

derivativedivides	$\frac{c\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2}+6ic\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3c\arctan($
default	$\frac{c\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2}+6ic\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3c\arctan($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*arctan(a*x)^2*(-3-I*arctan(a*x)+arctan(a*x)*a*x)*(I+a*x)+6*I*c*polylo
g(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1
)+1)-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-c*arctan(a*
x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*
x)^2/(a^2*x^2+1))-3/2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+6*I
*c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c*arctan(a*x)^3*ln(1-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-3/4*I*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+6*c*arctan(a*x
)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*c*arctan(a*x)^2+c*arctan(a*x)^
3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c*polylog(2,-(1+I*a*x)^2/(a^2*x^2
+1))+6*c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*arctan(a
*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*a^2*c*x^2*arctan(a*x)^3 - 3/64*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1)^
2 + integrate(1/64*(12*a^4*c*x^4*arctan(a*x)*log(a^2*x^2 + 1) - 12*a^3*c*x^
3*arctan(a*x)^2 + 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a^3*c
*x^3 + 2*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^
2*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^{3}(ax)}{x}\,dx + \int a^{2}x\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x,x)
[Out] c*(Integral(atan(a*x)**3/x, x) + Integral(a**2*x*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2))/x,x)
[Out] int((atan(a*x)^3*(c + a^2*c*x^2))/x, x)
```

3.368 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=169

$$-\frac{c\text{ArcTan}(ax)^3}{x}+a^2cx\text{ArcTan}(ax)^3+3ac\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)-$$

```
[Out] -c*arctan(a*x)^3/x+a^2*c*x*arctan(a*x)^3+3*a*c*arctan(a*x)^2*ln(2/(1+I*a*x)
)+3*a*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c*arctan(a*x)*polylog(2,-1+2/
(1-I*a*x))+3*I*a*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3/2*a*c*polylog(3,-
1+2/(1-I*a*x))+3/2*a*c*polylog(3,1-2/(1+I*a*x))
```

Rubi [A]

time = 0.28, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5070, 4946, 5044, 4988, 5004, 5112, 6745, 4930, 5040, 4964, 5114}

$$a^2cx\text{ArcTan}(ax)^3-3iac\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)+3iac\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)-\frac{c\text{ArcTan}(ax)^3}{x}+3ac\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac\text{Li}_3\left(\frac{2}{1-iax}-1\right)+\frac{3}{2}ac\text{Li}_3\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -((c*ArcTan[a*x]^3)/x) + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[2/
(1 + I*a*x)] + 3*a*c*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3*I)*a*c*ArcTa
n[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a*c*ArcTan[a*x]*PolyLog[2, 1
- 2/(1 + I*a*x)] + (3*a*c*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a*c*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/2
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]

Rule 4988

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Dist[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]

Rule 5004

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5040

Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]

Rule 5044

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]

Rule 5070

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] && IntegerQ[q]))

Rule 5112

Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d

```
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^2}\,dx &= c\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx + (a^2c)\int \tan^{-1}(ax)^3\,dx\\
&= -\frac{c\tan^{-1}(ax)^3}{x} + a^2cx\tan^{-1}(ax)^3 + (3ac)\int \frac{\tan^{-1}(ax)^2}{x(1+a^2x^2)}\,dx - (3a^3c)\int \frac{x\,\mathrm{t}}{1}\\
&= -\frac{c\tan^{-1}(ax)^3}{x} + a^2cx\tan^{-1}(ax)^3 + (3iac)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx + (3a^2c)\int \frac{\tan}{i}\\
&= -\frac{c\tan^{-1}(ax)^3}{x} + a^2cx\tan^{-1}(ax)^3 + 3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right) + 3ac\tan\\
&= -\frac{c\tan^{-1}(ax)^3}{x} + a^2cx\tan^{-1}(ax)^3 + 3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right) + 3ac\tan\\
&= -\frac{c\tan^{-1}(ax)^3}{x} + a^2cx\tan^{-1}(ax)^3 + 3ac\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right) + 3ac\tan\end{aligned}$$

Mathematica [A]
time = 0.13, size = 188, normalized size = 1.11

$$-iac\text{ArcTan}(ax)^3 + a^2cx\text{ArcTan}(ax)^3 + 3ac\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - 3iac\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + ac\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(ax)^3 - \frac{\text{ArcTan}(ax)^3}{ax} + 3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 3i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)\right) + \frac{3}{2}ac\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (-I)*a*c*ArcTan[a*x]^3 + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[1
+ E^((2*I)*ArcTan[a*x])] - (3*I)*a*c*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTa
n[a*x])] + a*c*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) + 3*A
```

```
rcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2,
 E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2) + (3*a
*c*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 37.76, size = 1765, normalized size = 10.44

method	result	size
derivativedivides	Expression too large to display	1765
default	Expression too large to display	1765

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(c*arctan(a*x)^3*a*x-c*arctan(a*x)^3/a/x-3*c*(-1/2*I*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*Pi*arctan(a*x)^2+arctan(a*x)^
2*ln(a^2*x^2+1)-2*arctan(a*x)^2*ln(2)-2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2))+1/2*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))^2*Pi*arctan(a*x)^2-1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/
2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)+I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/2*I*arctan(a*x)^2
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+2/3*I*arctan(a*x)^3-1/2*I*Pi*csgn(((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-
1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/2*I*Pi*arctan(a*x)^2+2*I*arctan(a
*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog
(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan
(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(
a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*arctan(a*x)^
2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/2*I*arctan(a*x)^2*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
```

```
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+arctan(a*x)^2*ln((1+I*
a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arcta
n(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(a*x)+I*arctan(a
*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/64*(8*(a^2*c*x^2 - c)*arctan(a*x)^3 - 6*(a^2*c*x^2 - c)*arctan(a*x)*log(a
^2*x^2 + 1)^2 + (28*a*c*arctan(a*x)^4 + 1792*a^4*c*integrate(1/32*x^4*arcta
n(a*x)^3/(a^2*x^4 + x^2), x) + 192*a^4*c*integrate(1/32*x^4*arctan(a*x)*log
(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 768*a^4*c*integrate(1/32*x^4*arctan(a
*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^3*c*integrate(1/32*x^3*arc
tan(a*x)^2/(a^2*x^4 + x^2), x) + a*c*log(a^2*x^2 + 1)^3 + 384*a^2*c*integra
te(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 768*a^2*c*
integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 768*a
*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 192*a*c*integrate(1
/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 1792*c*integrate(1/32*arctan
(a*x)^3/(a^2*x^4 + x^2), x) + 192*c*integrate(1/32*arctan(a*x)*log(a^2*x^2
+ 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}\operatorname{atan}^{3}(ax)\,dx+\int\frac{\operatorname{atan}^{3}(ax)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2))/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2))/x^2, x)
```

3.369 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=310

$$-\frac{3}{2}ia^2c\text{ArcTan}(ax)^2-\frac{3ac\text{ArcTan}(ax)^2}{2x}-\frac{1}{2}a^2c\text{ArcTan}(ax)^3-\frac{c\text{ArcTan}(ax)^3}{2x^2}+2a^2c\text{ArcTan}(ax)^3\tanh^{-1}\Bigg(1-$$

```
[Out] -3/2*I*a^2*c*arctan(a*x)^2-3/2*a*c*arctan(a*x)^2/x-1/2*a^2*c*arctan(a*x)^3-
1/2*c*arctan(a*x)^3/x^2-2*a^2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))+3*a^2
*c*arctan(a*x)*ln(2-2/(1-I*a*x))-3/2*I*a^2*c*polylog(2,-1+2/(1-I*a*x))-3/2*
I*a^2*c*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+3/2*I*a^2*c*arctan(a*x)^2*po
lylog(2,-1+2/(1+I*a*x))-3/2*a^2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/2*
a^2*c*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/4*I*a^2*c*polylog(4,1-2/(1+I*
a*x))-3/4*I*a^2*c*polylog(4,-1+2/(1+I*a*x))
```

Rubi [A]
time = 0.41, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5070, 4946, 5038, 5044, 4988, 2497, 5004, 4942, 5108, 5114, 5118, 6745}

$$-\frac{3}{2}ia^2c\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}ia^2c\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{3}{2}a^2c\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}a^2c\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)-\frac{1}{2}a^2c\text{ArcTan}(ax)^3-\frac{3}{2}ia^2c\text{ArcTan}(ax)^2+3a^2c\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)+2a^2c\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{3}{4}ia^2c\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{3}{4}ia^2c\text{Li}_4\left(\frac{2}{iax+1}-1\right)-\frac{c\text{ArcTan}(ax)^3}{2x^2}-\frac{3ac\text{ArcTan}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] ((-3*I)/2)*a^2*c*ArcTan[a*x]^2 - (3*a*c*ArcTan[a*x]^2)/(2*x) - (a^2*c*ArcTa
n[a*x]^3)/2 - (c*ArcTan[a*x]^3)/(2*x^2) + 2*a^2*c*ArcTan[a*x]^3*ArcTanh[1 -
 2/(1 + I*a*x)] + 3*a^2*c*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^
2*c*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[
2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1
+ I*a*x)] - (3*a^2*c*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*
c*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((3*I)/4)*a^2*c*PolyLog[4
, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*a^2*c*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
```

```
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)^3}{x}\,dx$$

$$= -\frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3ac)\int \frac{\tan^{-1}}{x^2\,(1}$$

$$= -\frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}(3ac)\int \frac{\tan^{-}}{x}$$

$$= -\frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} + 2a^2c\tan^{-1}(ax)^3\tan$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} +$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} +$$

$$= -\frac{3}{2}ia^2c\tan^{-1}(ax)^2 - \frac{3ac\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^2c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{2x^2} +$$

Mathematica [A]
time = 0.19, size = 337, normalized size = 1.09

$$\frac{1}{2}a^2c\text{ArcTan}(ax)^3 + \frac{c(-1-a^2x^2)\text{ArcTan}(ax)^3}{2x^2} + 2a^2c\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2i}{i-ax}\right) + \frac{3}{2}a^2c\left(-\frac{1}{3}\text{ArcTan}(ax)\left(\frac{3\text{ArcTan}(ax)}{ax} + \text{ArcTan}(ax)(3i+\text{ArcTan}(ax)) - 6\log\left(1-e^{2i\text{ArcTan}(ax)}\right)\right) - i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)\right) + \frac{3}{2}ia^2c\text{ArcTan}(ax)^2\text{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right) - \frac{3}{2}ia^2c\text{ArcTan}(ax)^2\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right) + \frac{3}{2}a^2c\text{ArcTan}(ax)\text{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right) - \frac{3}{2}a^2c\text{ArcTan}(ax)\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right) - \frac{3}{4}ia^2c\text{PolyLog}\left(4,\frac{-i-ax}{-i+ax}\right) + \frac{3}{4}ia^2c\text{PolyLog}\left(4,\frac{i+ax}{-i+ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c*ArcTan[a*x]^3)/2 + (c*(-1 - a^2*x^2)*ArcTan[a*x]^3)/(2*x^2) + 2*a^2*
c*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I - a*x)] + (3*a^2*c*(-1/3*(ArcTan[a*x]*
((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a*x]) - 6*Log[1 - E^((2*
I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/2 + ((3*I)/2)*a^
2*c*ArcTan[a*x]^2*PolyLog[2, (-I - a*x)/(-I + a*x)] - ((3*I)/2)*a^2*c*ArcTa
n[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] + (3*a^2*c*ArcTan[a*x]*PolyLog[3,
 (-I - a*x)/(-I + a*x)])/2 - (3*a^2*c*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I
+ a*x)])/2 - ((3*I)/4)*a^2*c*PolyLog[4, (-I - a*x)/(-I + a*x)] + ((3*I)/4)*
a^2*c*PolyLog[4, (I + a*x)/(-I + a*x)]
```

Maple [A]
time = 29.95, size = 525, normalized size = 1.69

method	result
derivativedivides	$a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2a^2x^2} - \frac{3ic\,\text{polylog}\left(4,-\frac{(iax+1)^2}{a^2x^2+1}\right)}{4} + 3c\arctan(a\right.$

default	$a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2a^2x^2}-\frac{3ic\,\mathrm{polylog}\left(4,-\frac{(iax+1)^2}{a^2x^2+1}\right)}{4}+3c\arctan(ax\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*c*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*a*x-3*I*a*x)*(I+a*x)/
a^2/x^2+6*I*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c*arctan(a*x)*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/4*I*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3/
2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-3*I*c*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+3*c*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I
*c*arctan(a*x)^2+c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c*po
lylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+c*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c*arctan(a*x)^2*p
olylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+6*c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-
c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3*I*c*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/64*(4*c*arctan(a*x)^3 - 3*c*arctan(a*x)*log(a^2*x^2 + 1)^2 - 64*x^2*inte
grate(-1/64*(12*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1) - 12*a*c*x*arctan(a*
x)^2 - 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a*c*x - 2*(a^4*c
*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3), x
))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx+\int \frac{a^2\operatorname{atan}^3(ax)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**3,x)
[Out] c*(Integral(atan(a*x)**3/x**3, x) + Integral(a**2*atan(a*x)**3/x, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2))/x^3,x)
[Out] int((atan(a*x)^3*(c + a^2*c*x^2))/x^3, x)
```

3.370 $\int \frac{(c+a^2cx^2)\textbf{ArcTan}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2c\text{ArcTan}(ax)}{x}-\frac{1}{2}a^3c\text{ArcTan}(ax)^2-\frac{ac\text{ArcTan}(ax)^2}{2x^2}-\frac{2}{3}ia^3c\text{ArcTan}(ax)^3-\frac{c\text{ArcTan}(ax)^3}{3x^3}-\frac{a^2c\text{ArcTan}(ax}{x}$$

```
[Out] -a^2*c*arctan(a*x)/x-1/2*a^3*c*arctan(a*x)^2-1/2*a*c*arctan(a*x)^2/x^2-2/3*
I*a^3*c*arctan(a*x)^3-1/3*c*arctan(a*x)^3/x^3-a^2*c*arctan(a*x)^3/x+a^3*c*l
n(x)-1/2*a^3*c*ln(a^2*x^2+1)+2*a^3*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-2*I*a^
3*c*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+a^3*c*polylog(3,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.42, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5070, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$-2ia^3c\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)-\frac{2}{3}ia^3c\text{ArcTan}(ax)^3-\frac{1}{2}a^3c\text{ArcTan}(ax)^2+2a^3c\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+a^3c\text{Li}_3\left(\frac{2}{1-iax}-1\right)+a^3c\log(x)-\frac{a^2c\text{ArcTan}(ax)^3}{x}-\frac{a^2c\text{ArcTan}(ax)}{x}-\frac{1}{2}a^3c\log\left(a^2x^2+1\right)-\frac{c\text{ArcTan}(ax)^3}{3x^3}-\frac{ac\text{ArcTan}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c*ArcTan[a*x])/x) - (a^3*c*ArcTan[a*x]^2)/2 - (a*c*ArcTan[a*x]^2)/(2
*x^2) - ((2*I)/3)*a^3*c*ArcTan[a*x]^3 - (c*ArcTan[a*x]^3)/(3*x^3) - (a^2*c*
ArcTan[a*x]^3)/x + a^3*c*Log[x] - (a^3*c*Log[1 + a^2*x^2])/2 + 2*a^3*c*ArcT
an[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (2*I)*a^3*c*ArcTan[a*x]*PolyLog[2, -1 +
2/(1 - I*a*x)] + a^3*c*PolyLog[3, -1 + 2/(1 - I*a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx + (a^2c)\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx$$
$$= -\frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + (ac)\int \frac{\tan^{-1}(ax)^2}{x^3(1+a^2x^2)}\,dx + (3a^3c)\int \frac{\tan}{x(1}$$
$$= -ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + (ac)\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx$$
$$= -\frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + 3a^3$$
$$= -\frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 - \frac{c\tan^{-1}(ax)^3}{3x^3} - \frac{a^2c\tan^{-1}(ax)^3}{x} + 2a^3$$
$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 -$$
$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 -$$
$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 -$$
$$= -\frac{a^2c\tan^{-1}(ax)}{x} - \frac{1}{2}a^3c\tan^{-1}(ax)^2 - \frac{ac\tan^{-1}(ax)^2}{2x^2} - \frac{2}{3}ia^3c\tan^{-1}(ax)^3 -$$

Mathematica [A]
time = 0.30, size = 177, normalized size = 0.94

$$\frac{1}{12}c\left(-ia^3\pi^3 - \frac{12a^2\text{ArcTan}(ax)}{x} - 6a^3\text{ArcTan}(ax)^2 - \frac{6a\text{ArcTan}(ax)^2}{x^2} + 8ia^3\text{ArcTan}(ax)^3 - \frac{4\text{ArcTan}(ax)^3}{x^3} - \frac{12a^2\text{ArcTan}(ax)^3}{x} + 24a^3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 12a^3\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) + 24ia^3\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + 12a^3\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c*((-I)*a^3*Pi^3 - (12*a^2*ArcTan[a*x])/x - 6*a^3*ArcTan[a*x]^2 - (6*a*Arc
Tan[a*x]^2)/x^2 + (8*I)*a^3*ArcTan[a*x]^3 - (4*ArcTan[a*x]^3)/x^3 - (12*a^2
*ArcTan[a*x]^3)/x + 24*a^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] +
12*a^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (24*I)*a^3*ArcTan[a*x]*PolyLog[2, E^(
(-2*I)*ArcTan[a*x])] + 12*a^3*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.79, size = 5426, normalized size = 28.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(7*a^3*c*arctan(a*x)^4 + 96*a^4*c*integrate(1/32*x^4*arctan(a*x)*lo
g(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 384*a^4*c*integrate(1/32*x^4*arctan(
a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 384*a^3*c*integrate(1/32*x^3*ar
ctan(a*x)^2/(a^2*x^6 + x^4), x) - 96*a^3*c*integrate(1/32*x^3*log(a^2*x^2 +
 1)^2/(a^2*x^6 + x^4), x) + 1792*a^2*c*integrate(1/32*x^2*arctan(a*x)^3/(a^
2*x^6 + x^4), x) + 192*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1
)^2/(a^2*x^6 + x^4), x) - 128*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*
x^2 + 1)/(a^2*x^6 + x^4), x) + 128*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*
x^6 + x^4), x) - 32*a*c*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4)
, x) + 896*c*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 96*c*integr
ate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 - 4*(3*a^2
*c*x^2 + c)*arctan(a*x)^3 + 3*(3*a^2*c*x^2 + c)*arctan(a*x)*log(a^2*x^2 + 1
)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**3/x**4,x)
```

```
[Out] c*(Integral(atan(a*x)**3/x**4, x) + Integral(a**2*atan(a*x)**3/x**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2))/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2))/x^4, x)
```

3.371 $\int x^3(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3 \, dx$

Optimal. Leaf size=313

$$\frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{c^2\text{ArcTan}(ax)}{21a^4}-\frac{5c^2x^2\text{ArcTan}(ax)}{168a^2}+\frac{1}{28}c^2x^4\text{ArcTan}(ax)+\frac{1}{56}a^2c^2x^6\text{ArcTan}(ax)+\frac{2}{}$$

```
[Out] 1/21*c^2*x/a^3-1/168*c^2*x^3/a-1/280*a*c^2*x^5-1/21*c^2*arctan(a*x)/a^4-5/1
68*c^2*x^2*arctan(a*x)/a^2+1/28*c^2*x^4*arctan(a*x)+1/56*a^2*c^2*x^6*arctan
(a*x)+2/21*I*c^2*arctan(a*x)^2/a^4+1/8*c^2*x*arctan(a*x)^2/a^3-1/24*c^2*x^3
*arctan(a*x)^2/a-1/8*a*c^2*x^5*arctan(a*x)^2-3/56*a^3*c^2*x^7*arctan(a*x)^2
-1/24*c^2*arctan(a*x)^3/a^4+1/4*c^2*x^4*arctan(a*x)^3+1/3*a^2*c^2*x^6*arcta
n(a*x)^3+1/8*a^4*c^2*x^8*arctan(a*x)^3+4/21*c^2*arctan(a*x)*ln(2/(1+I*a*x))
/a^4+2/21*I*c^2*polylog(2,1-2/(1+I*a*x))/a^4
```

Rubi [A]
time = 1.55, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 106, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004, 308}

$$\frac{1}{8}a^4c^2x^8\text{ArcTan}(ax)^3-\frac{c^2\text{ArcTan}(ax)^3}{24a^4}+\frac{2ic^2\text{ArcTan}(ax)^2}{21a^4}-\frac{c^2\text{ArcTan}(ax)}{21a^4}+\frac{4c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{21a^4}+\frac{2ic^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{21a^4}-\frac{3}{56}a^3c^2x^7\text{ArcTan}(ax)^2+\frac{c^2x\text{ArcTan}(ax)^2}{8a^3}+\frac{c^2x}{21a^3}+\frac{1}{3}a^2c^2x^6\text{ArcTan}(ax)^3+\frac{1}{56}a^2c^2x^6\text{ArcTan}(ax)-\frac{5c^2x^2\text{ArcTan}(ax)}{168a^2}-\frac{1}{8}ac^2x^5\text{ArcTan}(ax)^2+\frac{1}{4}c^2x^4\text{ArcTan}(ax)^3+\frac{1}{28}c^2x^4\text{ArcTan}(ax)-\frac{c^2x^3\text{ArcTan}(ax)^2}{24a}-\frac{1}{280}ac^2x^5-\frac{c^2x^3}{168a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*x)/(21*a^3) - (c^2*x^3)/(168*a) - (a*c^2*x^5)/280 - (c^2*ArcTan[a*x])/
(21*a^4) - (5*c^2*x^2*ArcTan[a*x])/(168*a^2) + (c^2*x^4*ArcTan[a*x])/28 + (
a^2*c^2*x^6*ArcTan[a*x])/56 + (((2*I)/21)*c^2*ArcTan[a*x]^2)/a^4 + (c^2*x*A
rcTan[a*x]^2)/(8*a^3) - (c^2*x^3*ArcTan[a*x]^2)/(24*a) - (a*c^2*x^5*ArcTan[
a*x]^2)/8 - (3*a^3*c^2*x^7*ArcTan[a*x]^2)/56 - (c^2*ArcTan[a*x]^3)/(24*a^4)
 + (c^2*x^4*ArcTan[a*x]^3)/4 + (a^2*c^2*x^6*ArcTan[a*x]^3)/3 + (a^4*c^2*x^8
*ArcTan[a*x]^3)/8 + (4*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(21*a^4) + (((2*
I)/21)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx = \int \left(c^2x^3\tan^{-1}(ax)^3+2a^2c^2x^5\tan^{-1}(ax)^3+a^4c^2x^7\tan^{-1}(ax)^3\right)\,dx$$

$$= c^2\int x^3\tan^{-1}(ax)^3\,dx+\left(2a^2c^2\right)\int x^5\tan^{-1}(ax)^3\,dx+\left(a^4c^2\right)\int x^7\tan^{-1}($$

$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^3+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3-\frac{1}{4}\left(3ac^2\right)$$

$$= \frac{1}{4}c^2x^4\tan^{-1}(ax)^3+\frac{1}{3}a^2c^2x^6\tan^{-1}(ax)^3+\frac{1}{8}a^4c^2x^8\tan^{-1}(ax)^3-\frac{\left(3c^2\right)\int x}{}$$

$$= -\frac{c^2x^3\tan^{-1}(ax)^2}{4a}-\frac{1}{5}ac^2x^5\tan^{-1}(ax)^2-\frac{3}{56}a^3c^2x^7\tan^{-1}(ax)^2+\frac{1}{4}c^2x^4\,\mathrm{t}$$

$$= \frac{3c^2x\tan^{-1}(ax)^2}{4a^3}+\frac{c^2x^3\tan^{-1}(ax)^2}{12a}-\frac{1}{8}ac^2x^5\tan^{-1}(ax)^2-\frac{3}{56}a^3c^2x^7\tan$$

$$= \frac{c^2x^2\tan^{-1}(ax)}{4a^2}+\frac{1}{10}c^2x^4\tan^{-1}(ax)+\frac{1}{56}a^2c^2x^6\tan^{-1}(ax)+\frac{ic^2\tan^{-1}(ax}{a^4}$$

$$= -\frac{c^2x}{4a^3}-\frac{17c^2x^2\tan^{-1}(ax)}{60a^2}+\frac{1}{28}c^2x^4\tan^{-1}(ax)+\frac{1}{56}a^2c^2x^6\tan^{-1}(ax)-\frac{8}{}$$

$$= \frac{307c^2x}{840a^3}-\frac{23c^2x^3}{840a}-\frac{1}{280}ac^2x^5+\frac{c^2\tan^{-1}(ax)}{4a^4}-\frac{5c^2x^2\tan^{-1}(ax)}{168a^2}+\frac{1}{28}c^2x$$

$$= \frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{307c^2\tan^{-1}(ax)}{840a^4}-\frac{5c^2x^2\tan^{-1}(ax)}{168a^2}+\frac{1}{28}c^2x$$

$$= \frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{c^2\tan^{-1}(ax)}{21a^4}-\frac{5c^2x^2\tan^{-1}(ax)}{168a^2}+\frac{1}{28}c^2x^4\,\mathrm{ta}$$

$$= \frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{c^2\tan^{-1}(ax)}{21a^4}-\frac{5c^2x^2\tan^{-1}(ax)}{168a^2}+\frac{1}{28}c^2x^4\,\mathrm{ta}$$

Mathematica [A]
time = 1.00, size = 165, normalized size = 0.53

$$\frac{c^2\left(40ax-5a^3x^3-3a^5x^5-5(16i-21ax+7a^3x^3+21a^5x^5+9a^7x^7)\operatorname{ArcTan}(ax)^2+35(1+a^2x^2)^3(-1+3a^2x^2)\operatorname{ArcTan}(ax)^3+5\operatorname{ArcTan}(ax)\left(-8-5a^2x^2+6a^4x^4+3a^6x^6+32\log\left(1+e^{2i\operatorname{ArcTan}(ax)}\right)\right)-80i\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(ax)}\right)\right)}{840a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(40*a*x - 5*a^3*x^3 - 3*a^5*x^5 - 5*(16*I - 21*a*x + 7*a^3*x^3 + 21*a^
5*x^5 + 9*a^7*x^7)*ArcTan[a*x]^2 + 35*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcT
an[a*x]^3 + 5*ArcTan[a*x]*(-8 - 5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 + 32*Log[
```

```
1 + E^((2*I)*ArcTan[a*x])]) - (80*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(
840*a^4)
```

Maple [A]
time = 2.01, size = 330, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+\arctan(ax)^2a^5x^5+\frac{\arctan(ax)^2a^3x^3}{3}-a\right.}{}}{}$
default	$\frac{\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+\arctan(ax)^2a^5x^5+\frac{\arctan(ax)^2a^3x^3}{3}-a\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/8*c^2*arctan(a*x)^3*a^8*x^8+1/3*c^2*arctan(a*x)^3*a^6*x^6+1/4*a^4*
c^2*x^4*arctan(a*x)^3-1/8*c^2*(3/7*arctan(a*x)^2*a^7*x^7+arctan(a*x)^2*a^5*
x^5+1/3*arctan(a*x)^2*a^3*x^3-arctan(a*x)^2*a*x+1/3*arctan(a*x)^3-1/7*arcta
n(a*x)*a^6*x^6-2/7*arctan(a*x)*a^4*x^4+5/21*arctan(a*x)*a^2*x^2+16/21*arcta
n(a*x)*ln(a^2*x^2+1)+1/35*a^5*x^5+1/21*a^3*x^3-8/21*a*x+8/21*arctan(a*x)-4/
21*I*ln(a*x-I)^2+8/21*I*ln(a*x-I)*ln(a^2*x^2+1)-8/21*I*ln(a*x-I)*ln(-1/2*I*
(I+a*x))+4/21*I*ln(I+a*x)^2-8/21*I*ln(I+a*x)*ln(a^2*x^2+1)+8/21*I*ln(I+a*x)
*ln(1/2*I*(a*x-I))-8/21*I*dilog(-1/2*I*(I+a*x))+8/21*I*dilog(1/2*I*(a*x-I))
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2688*(28*(129024*a^9*c^2*integrate(1/2688*x^9*arctan(a*x)^3/(a^5*x^2 + a^
3), x) - 24192*a^8*c^2*integrate(1/2688*x^8*arctan(a*x)^2/(a^5*x^2 + a^3),
x) - 6048*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3),
x) - 3456*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x)
 + 387072*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)^3/(a^5*x^2 + a^3), x) +
6912*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)/(a^5*x^2 + a^3), x) - 64512*a
^6*c^2*integrate(1/2688*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 16128*a^6*c
^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 8064*a^6*c
^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 387072*a^5*c
^2*integrate(1/2688*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 16128*a^5*c^2*i
```

```
ntegrate(1/2688*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 48384*a^4*c^2*integra
te(1/2688*x^4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 12096*a^4*c^2*integrate(1
/2688*x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 2688*a^4*c^2*integrate(1
/2688*x^4*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 129024*a^3*c^2*integrate(1
/2688*x^3*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 5376*a^3*c^2*integrate(1/2688
*x^3*arctan(a*x)/(a^5*x^2 + a^3), x) + 8064*a^2*c^2*integrate(1/2688*x^2*lo
g(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) - 16128*a*c^2*integrate(1/2688*x*arctan(
a*x)/(a^5*x^2 + a^3), x) + 2016*c^2*integrate(1/2688*log(a^2*x^2 + 1)^2/(a^
5*x^2 + a^3), x) + c^2*arctan(a*x)^3/a^4)*a^4 + 56*(3*a^8*c^2*x^8 + 8*a^6*c
^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^3 - 4*(9*a^7*c^2*x^7 + 21*a^5*c^2
*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*arctan(a*x)^2 + (9*a^7*c^2*x^7 + 21*a^5*
c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*log(a^2*x^2 + 1)^2)/a^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int x^{3}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int 2a^{2}x^{5}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int a^{4}x^{7}\operatorname{atan}^{3}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
[Out] c**2*(Integral(x**3*atan(a*x)**3, x) + Integral(2*a**2*x**5*atan(a*x)**3, x
) + Integral(a**4*x**7*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.372 $\int x^2(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=321

$$-\frac{11c^2x^2}{420a}-\frac{1}{140}ac^2x^4-\frac{c^2x\operatorname{ArcTan}(ax)}{70a^2}+\frac{17}{210}c^2x^3\operatorname{ArcTan}(ax)+\frac{1}{35}a^2c^2x^5\operatorname{ArcTan}(ax)+\frac{c^2\operatorname{ArcTan}(ax)^2}{140a^3}-\frac{4c^2x^2}{}$$

```
[Out] -11/420*c^2*x^2/a-1/140*a*c^2*x^4-1/70*c^2*x*arctan(a*x)/a^2+17/210*c^2*x^3
*arctan(a*x)+1/35*a^2*c^2*x^5*arctan(a*x)+1/140*c^2*arctan(a*x)^2/a^3-4/35*
c^2*x^2*arctan(a*x)^2/a-27/140*a*c^2*x^4*arctan(a*x)^2-1/14*a^3*c^2*x^6*arc
tan(a*x)^2-8/35*I*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/3*c^2*x^3*
arctan(a*x)^3+2/5*a^2*c^2*x^5*arctan(a*x)^3+1/7*a^4*c^2*x^7*arctan(a*x)^3-8
/35*c^2*arctan(a*x)^2*ln(2/(1+I*a*x))/a^3+1/30*c^2*ln(a^2*x^2+1)/a^3-8/105*
I*c^2*arctan(a*x)^3/a^3-4/35*c^2*polylog(3,1-2/(1+I*a*x))/a^3
```

Rubi [A]
time = 1.27, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 73, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745, 272, 45}

$$\frac{1}{7}a^4c^2x^7\operatorname{ArcTan}(ax)^3-\frac{8ic^2\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(1-\frac{2}{iax+1}\right)}{35a^3}-\frac{1}{14}a^3c^2x^6\operatorname{ArcTan}(ax)^2-\frac{8ic^2\operatorname{ArcTan}(ax)^3}{105a^3}+\frac{c^2\operatorname{ArcTan}(ax)^2}{140a^3}-\frac{8c^2\operatorname{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a^3}-\frac{4c^2\operatorname{Li}_3\left(1-\frac{2}{iax+1}\right)}{35a^3}+\frac{2}{5}a^2c^2x^5\operatorname{ArcTan}(ax)^3+\frac{1}{35}a^2c^2x^5\operatorname{ArcTan}(ax)-\frac{c^2x\operatorname{ArcTan}(ax)}{70a^2}+\frac{c^2\log\left(a^2x^2+1\right)}{30a^3}-\frac{27}{140}ac^2x^4\operatorname{ArcTan}(ax)^2+\frac{1}{3}c^2x^3\operatorname{ArcTan}(ax)^3+\frac{17}{210}c^2x^3\operatorname{ArcTan}(ax)-\frac{4c^2x^2\operatorname{ArcTan}(ax)^2}{35a}-\frac{1}{140}ac^2x^4-\frac{11c^2x^2}{420a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (-11*c^2*x^2)/(420*a) - (a*c^2*x^4)/140 - (c^2*x*ArcTan[a*x])/(70*a^2) + (1
7*c^2*x^3*ArcTan[a*x])/210 + (a^2*c^2*x^5*ArcTan[a*x])/35 + (c^2*ArcTan[a*x
]^2)/(140*a^3) - (4*c^2*x^2*ArcTan[a*x]^2)/(35*a) - (27*a*c^2*x^4*ArcTan[a*
x]^2)/140 - (a^3*c^2*x^6*ArcTan[a*x]^2)/14 - (((8*I)/105)*c^2*ArcTan[a*x]^3
)/a^3 + (c^2*x^3*ArcTan[a*x]^3)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^3)/5 + (a^4*
c^2*x^7*ArcTan[a*x]^3)/7 - (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(35*a^3
) + (c^2*Log[1 + a^2*x^2])/(30*a^3) - (((8*I)/35)*c^2*ArcTan[a*x]*PolyLog[2
, 1 - 2/(1 + I*a*x)])/a^3 - (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(35*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx = \int \left(c^2x^2\tan^{-1}(ax)^3+2a^2c^2x^4\tan^{-1}(ax)^3+a^4c^2x^6\tan^{-1}(ax)^3\right)\,dx$$

$$= c^2\int x^2\tan^{-1}(ax)^3\,dx+\left(2a^2c^2\right)\int x^4\tan^{-1}(ax)^3\,dx+\left(a^4c^2\right)\int x^6\tan^{-}$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^3+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^3+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^3-\left(ac^2\right)$$

$$= \frac{1}{3}c^2x^3\tan^{-1}(ax)^3+\frac{2}{5}a^2c^2x^5\tan^{-1}(ax)^3+\frac{1}{7}a^4c^2x^7\tan^{-1}(ax)^3-\frac{c^2\int x}{}$$

$$= -\frac{c^2x^2\tan^{-1}(ax)^2}{2a}-\frac{3}{10}ac^2x^4\tan^{-1}(ax)^2-\frac{1}{14}a^3c^2x^6\tan^{-1}(ax)^2-\frac{ic^2}{}$$

$$= \frac{c^2x^2\tan^{-1}(ax)^2}{10a}-\frac{27}{140}ac^2x^4\tan^{-1}(ax)^2-\frac{1}{14}a^3c^2x^6\tan^{-1}(ax)^2+\frac{ic^2}{}$$

$$= \frac{c^2x\tan^{-1}(ax)}{a^2}+\frac{1}{5}c^2x^3\tan^{-1}(ax)+\frac{1}{35}a^2c^2x^5\tan^{-1}(ax)-\frac{c^2\tan^{-1}(ax)}{2a^3}$$

$$= -\frac{4c^2x\tan^{-1}(ax)}{5a^2}+\frac{17}{210}c^2x^3\tan^{-1}(ax)+\frac{1}{35}a^2c^2x^5\tan^{-1}(ax)+\frac{2c^2\tan}{5}$$

$$= -\frac{c^2x\tan^{-1}(ax)}{70a^2}+\frac{17}{210}c^2x^3\tan^{-1}(ax)+\frac{1}{35}a^2c^2x^5\tan^{-1}(ax)+\frac{c^2\tan^{-1}}{140a}$$

$$= -\frac{3c^2x^2}{35a}-\frac{1}{140}ac^2x^4-\frac{c^2x\tan^{-1}(ax)}{70a^2}+\frac{17}{210}c^2x^3\tan^{-1}(ax)+\frac{1}{35}a^2c^2x^5$$

$$= -\frac{11c^2x^2}{420a}-\frac{1}{140}ac^2x^4-\frac{c^2x\tan^{-1}(ax)}{70a^2}+\frac{17}{210}c^2x^3\tan^{-1}(ax)+\frac{1}{35}a^2c^2x^5$$

Mathematica [A]
time = 0.82, size = 233, normalized size = 0.73

$$\frac{c^2\left(-8-11a^2x^2-3a^4x^4-6ax\text{ArcTan}(ax)+34a^3x^3\text{ArcTan}(ax)+12a^5x^5\text{ArcTan}(ax)+3\text{ArcTan}(ax)^2-48a^2x^2\text{ArcTan}(ax)^2-81a^4x^4\text{ArcTan}(ax)^2-30a^6x^6\text{ArcTan}(ax)^2+32i\text{ArcTan}(ax)^3+140a^3x^3\text{ArcTan}(ax)^3+168a^5x^5\text{ArcTan}(ax)^3+60a^7x^7\text{ArcTan}(ax)^3-96\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+14\log\left(1+a^2x^2\right)+96i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-48\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{420a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-8 - 11*a^2*x^2 - 3*a^4*x^4 - 6*a*x*ArcTan[a*x] + 34*a^3*x^3*ArcTan[a
*x] + 12*a^5*x^5*ArcTan[a*x] + 3*ArcTan[a*x]^2 - 48*a^2*x^2*ArcTan[a*x]^2 -
 81*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 + (32*I)*ArcTan[a*x]^3
 + 140*a^3*x^3*ArcTan[a*x]^3 + 168*a^5*x^5*ArcTan[a*x]^3 + 60*a^7*x^7*ArcTa
n[a*x]^3 - 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 14*Log[1 + a^2
```

```
*x^2] + (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 48*PolyLog[
3, -E^((2*I)*ArcTan[a*x])]))/(420*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 99.17, size = 1256, normalized size = 3.91

method	result	size
derivativedivides	Expression too large to display	1256
default	Expression too large to display	1256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/7*c^2*arctan(a*x)^3*a^7*x^7+2/5*c^2*arctan(a*x)^3*a^5*x^5+1/3*c^2*
arctan(a*x)^3*a^3*x^3-1/35*c^2*(-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-1/4*arctan(a*x)^2-4*arctan(a*x)^2*ln(a^
2*x^2+1)+8*arctan(a*x)^2*ln(2)+7/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+20*I*arcta
n(a*x)*(a*x-I)*(I+a*x)^3-3*I*arctan(a*x)*(a*x-I)*(I+a*x)-8*I*arctan(a*x)*po
lylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-5*I*arctan(a*x)*(a*x-I)^4+8*arctan(a*x)^2
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-7/12*(I+a*x)^2+2*I*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+3/2*I*arctan(a*x)*(a*x-I)^2+4*arctan(a*x)
*(a*x-I)+43/6*arctan(a*x)*(a*x-I)^3+4*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-2
*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arcta
n(a*x)^2-2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+20*I*arctan
(a*x)*(a*x-I)^3*(I+a*x)-30*I*arctan(a*x)*(a*x-I)^2*(I+a*x)^2-2*I*Pi*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^
2+2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+2*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*arctan(a*x)^2+27/4*arctan(a*x)^2*a^4*x^4+4*arctan(a*x)^2*a^2*x^2-43/2*
arctan(a*x)*(a*x-I)^2*(I+a*x)+43/2*arctan(a*x)*(a*x-I)*(I+a*x)^2+1/4*(I+a*x
)^4+4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1))^2*arctan(a*x)^2+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-arctan(a*x)*(
a*x-I)^5-5/6*I*(I+a*x)-8/3*I*arctan(a*x)^3+5/2*arctan(a*x)^2*a^6*x^6+5*arct
an(a*x)*(a*x-I)^4*(I+a*x)-5*arctan(a*x)*(a*x-I)*(I+a*x)^4-10*arctan(a*x)*(a
*x-I)^3*(I+a*x)^2+10*arctan(a*x)*(a*x-I)^2*(I+a*x)^3-I*(I+a*x)^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^3 - 1/1120
*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)*log(a^2*x^2 + 1
)^2 + integrate(1/1120*(980*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 +
c^2*x^2)*arctan(a*x)^3 - 4*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)
*arctan(a*x)^2 + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*arcta
n(a*x)*log(a^2*x^2 + 1) + (15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3 +
 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x))*l
og(a^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^2\operatorname{atan}^3(ax)\,dx+\int 2a^2x^4\operatorname{atan}^3(ax)\,dx+\int a^4x^6\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**2*atan(a*x)**3, x) + Integral(2*a**2*x**4*atan(a*x)**3, x
) + Integral(a**4*x**6*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.373 $\int x(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^3 \, dx$

Optimal. Leaf size=242

$$-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\,\text{ArcTan}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\,\text{ArcTan}(ax)}{20a^2}-\frac{4ic^2\text{ArcTan}(ax)^2}{15a^2}-\frac{4c^2x\text{ArcTan}}{15a}$$

```
[Out] -11/60*c^2*x/a-1/60*a*c^2*x^3+2/15*c^2*(a^2*x^2+1)*arctan(a*x)/a^2+1/20*c^2
*(a^2*x^2+1)^2*arctan(a*x)/a^2-4/15*I*c^2*arctan(a*x)^2/a^2-4/15*c^2*x*arct
an(a*x)^2/a-2/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2/a-1/10*c^2*x*(a^2*x^2+1)^2
*arctan(a*x)^2/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^3/a^2-8/15*c^2*arctan(a*
x)*ln(2/(1+I*a*x))/a^2-4/15*I*c^2*polylog(2,1-2/(1+I*a*x))/a^2
```

Rubi [A]
time = 0.14, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5050, 5000, 4930, 5040, 4964, 2449, 2352, 8}

$$-\frac{c^2x(a^2x^2+1)^2\,\text{ArcTan}(ax)^2}{10a}-\frac{2c^2x(a^2x^2+1)\,\text{ArcTan}(ax)^2}{15a}+\frac{c^2(a^2x^2+1)^3\,\text{ArcTan}(ax)^3}{6a^2}+\frac{c^2(a^2x^2+1)^2\,\text{ArcTan}(ax)}{20a^2}+\frac{2c^2(a^2x^2+1)\,\text{ArcTan}(ax)}{15a^2}-\frac{4ic^2\text{ArcTan}(ax)^2}{15a^2}-\frac{8c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^2}-\frac{4ic^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{15a^2}-\frac{4c^2x\text{ArcTan}(ax)^2}{15a}-\frac{1}{60}ac^2x^3-\frac{11c^2x}{60a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (-11*c^2*x)/(60*a) - (a*c^2*x^3)/60 + (2*c^2*(1 + a^2*x^2)*ArcTan[a*x])/(15
*a^2) + (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(20*a^2) - (((4*I)/15)*c^2*ArcTan
[a*x]^2)/a^2 - (4*c^2*x*ArcTan[a*x]^2)/(15*a) - (2*c^2*x*(1 + a^2*x^2)*ArcT
an[a*x]^2)/(15*a) - (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(10*a) + (c^2*(1
+ a^2*x^2)^3*ArcTan[a*x]^3)/(6*a^2) - (8*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)]
)/(15*a^2) - (((4*I)/15)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx=\frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)^3}{6a^2}-\frac{\int\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2\,dx}{2a}$$

$$=\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}-\frac{c^2x(1+a^2x^2)^2\tan^{-1}(ax)^2}{10a}+\frac{c^2(1+a^2x^2)^3\tan$$

$$=-\frac{c^2x}{20a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}-$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}$$

$$=-\frac{11c^2x}{60a}-\frac{1}{60}ac^2x^3+\frac{2c^2(1+a^2x^2)\tan^{-1}(ax)}{15a^2}+\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)}{20a^2}$$

Mathematica [A]

time = 0.56, size = 131, normalized size = 0.54

$$\frac{c^2\left(-ax(11+a^2x^2)-2(-8i+15ax+10a^3x^3+3a^5x^5)\,\mathrm{ArcTan}(ax)^2+10(1+a^2x^2)^3\,\mathrm{ArcTan}(ax)^3+\mathrm{ArcTan}(ax)\left(11+14a^2x^2+3a^4x^4-32\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)\right)+16i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{60a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-(a*x*(11 + a^2*x^2)) - 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*Ar
cTan[a*x]^2 + 10*(1 + a^2*x^2)^3*ArcTan[a*x]^3 + ArcTan[a*x]*(11 + 14*a^2*x
^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (16*I)*PolyLog[2, -E^
((2*I)*ArcTan[a*x])]))/(60*a^2)
```

Maple [A]

time = 3.25, size = 299, normalized size = 1.24

method	result
derivativedivides	$\frac{c^2\arctan(ax)^3a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^3}{2}+\frac{a^2c^2x^2\arctan(ax)^3}{2}+\frac{c^2\arctan(ax)^3}{6}-\frac{c^2\left(\frac{\arctan(ax)^2a^5x^5}{5}+\frac{2\arctan(ax)^2a^3x^3}{3}+\mathrm{arcta}\right.}{}$

default	$\frac{\frac{c^2 \arctan(ax)^3 a^6 x^6}{6}+\frac{a^4 c^2 x^4 \arctan(ax)^3}{2}+\frac{a^2 c^2 x^2 \arctan(ax)^3}{2}+\frac{c^2 \arctan(ax)^3}{6}-\frac{c^2\left(\frac{\arctan(ax)^2 a^5 x^5}{5}+\frac{2\arctan(ax)^2 a^3 x^3}{3}+\mathrm{arc}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/6*c^2*arctan(a*x)^3*a^6*x^6+1/2*a^4*c^2*x^4*arctan(a*x)^3+1/2*a^2*
c^2*x^2*arctan(a*x)^3+1/6*c^2*arctan(a*x)^3-1/2*c^2*(1/5*arctan(a*x)^2*a^5*
x^5+2/3*arctan(a*x)^2*a^3*x^3+arctan(a*x)^2*a*x-1/10*arctan(a*x)*a^4*x^4-7/
15*arctan(a*x)*a^2*x^2-8/15*arctan(a*x)*ln(a^2*x^2+1)+1/30*a^3*x^3+11/30*a*
x-11/30*arctan(a*x)+4/15*I*ln(I+a*x)*ln(a^2*x^2+1)-4/15*I*ln(I+a*x)*ln(1/2*
I*(a*x-I))-2/15*I*ln(I+a*x)^2+2/15*I*ln(a*x-I)^2+4/15*I*dilog(-1/2*I*(I+a*x
))-4/15*I*dilog(1/2*I*(a*x-I))+4/15*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-4/15*I*l
n(a*x-I)*ln(a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/480*(40*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^3
 + 20*(5760*a^7*c^2*integrate(1/480*x^7*arctan(a*x)^3/(a^3*x^2 + a), x) - 1
440*a^6*c^2*integrate(1/480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 360*a^6*c
^2*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 288*a^6*c^2*i
ntegrate(1/480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 17280*a^5*c^2*integ
rate(1/480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 576*a^5*c^2*integrate(1/48
0*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 4320*a^4*c^2*integrate(1/480*x^4*arct
an(a*x)^2/(a^3*x^2 + a), x) - 1080*a^4*c^2*integrate(1/480*x^4*log(a^2*x^2
+ 1)^2/(a^3*x^2 + a), x) - 960*a^4*c^2*integrate(1/480*x^4*log(a^2*x^2 + 1)
/(a^3*x^2 + a), x) + 17280*a^3*c^2*integrate(1/480*x^3*arctan(a*x)^3/(a^3*x
^2 + a), x) + 1920*a^3*c^2*integrate(1/480*x^3*arctan(a*x)/(a^3*x^2 + a), x
) - 4320*a^2*c^2*integrate(1/480*x^2*arctan(a*x)^2/(a^3*x^2 + a), x) - 1080
*a^2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 1440*a^
2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 5760*a*c^2*i
ntegrate(1/480*x*arctan(a*x)^3/(a^3*x^2 + a), x) + 2880*a*c^2*integrate(1/4
80*x*arctan(a*x)/(a^3*x^2 + a), x) - c^2*arctan(a*x)^3/a^2 - 360*c^2*integr
ate(1/480*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 - 4*(3*a^5*c^2*x^5 + 10
*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x)^2 + (3*a^5*c^2*x^5 + 10*a^3*c^2*x^3
+ 15*a*c^2*x)*log(a^2*x^2 + 1)^2)/a^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x\,\mathrm{atan}^3\left(ax\right)dx+\int 2a^2x^3\,\mathrm{atan}^3\left(ax\right)dx+\int a^4x^5\,\mathrm{atan}^3\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x*atan(a*x)**3, x) + Integral(2*a**2*x**3*atan(a*x)**3, x) +
 Integral(a**4*x**5*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.374 $\int \left(c+a^2cx^2\right)^2 \mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=289

$$-\frac{c^2(1+a^2x^2)}{20a}+c^2x\text{ArcTan}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\text{ArcTan}(ax)-\frac{2c^2(1+a^2x^2)\text{ArcTan}(ax)^2}{5a}-\frac{3c^2(1+a^2x^2$$

```
[Out] -1/20*c^2*(a^2*x^2+1)/a+c^2*x*arctan(a*x)+1/10*c^2*x*(a^2*x^2+1)*arctan(a*x
)-2/5*c^2*(a^2*x^2+1)*arctan(a*x)^2/a-3/20*c^2*(a^2*x^2+1)^2*arctan(a*x)^2/
a+8/15*I*c^2*arctan(a*x)^3/a+8/15*c^2*x*arctan(a*x)^3+4/15*c^2*x*(a^2*x^2+1
)*arctan(a*x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)^3+8/5*c^2*arctan(a*x)^2
*ln(2/(1+I*a*x))/a-1/2*c^2*ln(a^2*x^2+1)/a+8/5*I*c^2*arctan(a*x)*polylog(2,
1-2/(1+I*a*x))/a+4/5*c^2*polylog(3,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.18, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5000, 4930, 5040, 4964, 5004, 5114, 6745, 266, 4998}

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\text{ArcTan}(ax)^3+\frac{4}{15}c^2x\left(a^2x^2+1\right)\text{ArcTan}(ax)^3-\frac{3c^2(a^2x^2+1)^2\text{ArcTan}(ax)^2}{20a}-\frac{2c^2(a^2x^2+1)\text{ArcTan}(ax)^2}{5a}+\frac{1}{10}c^2x\left(a^2x^2+1\right)\text{ArcTan}(ax)-\frac{c^2(a^2x^2+1)}{20a}-\frac{c^2\log\left(a^2x^2+1\right)}{2a}+\frac{8ic^2\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{5a}+\frac{8ic^2\text{ArcTan}(ax)^3}{15a}+\frac{8}{15}c^2x\text{ArcTan}(ax)^3+c^2x\text{ArcTan}(ax)+\frac{8c^2\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a}+\frac{4c^2\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] -1/20*(c^2*(1 + a^2*x^2))/a + c^2*x*ArcTan[a*x] + (c^2*x*(1 + a^2*x^2)*ArcT
an[a*x])/10 - (2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)/(5*a) - (3*c^2*(1 + a^2*x
^2)^2*ArcTan[a*x]^2)/(20*a) + (((8*I)/15)*c^2*ArcTan[a*x]^3)/a + (8*c^2*x*A
rcTan[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/15 + (c^2*x*(1 + a
^2*x^2)^2*ArcTan[a*x]^3)/5 + (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(5*a)
 - (c^2*Log[1 + a^2*x^2])/(2*a) + (((8*I)/5)*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)])/a + (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(5*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3\,dx = -\frac{3c^2\left(1+a^2x^2\right)^2\tan^{-1}(ax)^2}{20a}+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^3+\frac{1}{10}(3c)\int (c$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}{5a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+c^2x\tan^{-1}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2(1+a}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+c^2x\tan^{-1}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2(1+a}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+c^2x\tan^{-1}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2(1+a}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+c^2x\tan^{-1}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2(1+a}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)}{20a}+c^2x\tan^{-1}(ax)+\frac{1}{10}c^2x\left(1+a^2x^2\right)\tan^{-1}(ax)-\frac{2c^2(1+a}{}$$

Mathematica [A]
time = 0.43, size = 195, normalized size = 0.67

$$\frac{c^2\left(-3-3a^2x^2+66ax\text{ArcTan}(ax)+6a^3x^3\text{ArcTan}(ax)-33\text{ArcTan}(ax)^2-42a^2x^2\text{ArcTan}(ax)^2-9a^4x^4\text{ArcTan}(ax)^2-32i\text{ArcTan}(ax)^3+60ax\text{ArcTan}(ax)^3+40a^3x^3\text{ArcTan}(ax)^3+12a^5x^5\text{ArcTan}(ax)^3+96\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-30\log\left(1+a^2x^2\right)-96i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+48\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{60a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*(-3 - 3*a^2*x^2 + 66*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 33*ArcT
an[a*x]^2 - 42*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 - (32*I)*Arc
Tan[a*x]^3 + 60*a*x*ArcTan[a*x]^3 + 40*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*A
rcTan[a*x]^3 + 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 30*Log[1 +
 a^2*x^2] - (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 48*Poly
Log[3, -E^((2*I)*ArcTan[a*x])]))/(60*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 269.12, size = 2519, normalized size = 8.72

method	result	size

derivativedivides	Expression too large to display	2519
default	Expression too large to display	2519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/5*c^2*arctan(a*x)^3*a^5*x^5+2/3*c^2*arctan(a*x)^3*a^3*x^3+c^2*arctan
(a*x)^3*a*x-1/5*c^2*(3/4*arctan(a*x)^2*a^4*x^4+7/2*arctan(a*x)^2*a^2*x^2+4*
arctan(a*x)^2*ln(a^2*x^2+1)-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))
+8*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-4*polylog(3,-(1+I*a*x)
^2/(a^2*x^2+1))+1/12*I*(24*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)*Pi*arctan(a*x)^2-24*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi*arctan(a*x)^2+42
*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)+I)*Pi*arctan(a*x)^2-21*csgn(I*(1+I*a*x)^4/(a^2*x^2+1
)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi*a
rctan(a*x)^2+6*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1))*Pi*arctan(a*x)^2-3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)^2-48*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*Pi*arctan(a*x)^2-24*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^2*Pi*arctan(a*x)^2+24*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi*arctan(a*x)^2+24*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*arctan(a*x)^2+96*I*ln(2)*arctan
(a*x)^2+24*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*Pi*arctan(a*x)^2-3*I+32*arctan
(a*x)^3-21*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^
3*Pi*arctan(a*x)^2-3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*arctan(a*x)
^2-3*I*a^2*x^2+18*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2
+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*Pi*arctan(a*x)^2*a^2*x^2-9*csgn(
I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)+I)^2*Pi*arctan(a*x)^2*a^2*x^2+66*I*arctan(a*x)*a*x+6*I*arcta
n(a*x)*a^3*x^3+60*arctan(a*x)-18*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*Pi*arctan(a*x)^2*a^2*x^2+9*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x
)^2*a^2*x^2+3*I*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1
)+I)^3*Pi*arctan(a*x)^2*a^3*x^3+9*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3
*Pi*arctan(a*x)^2*a*x-3*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*arctan
(a*x)^2*a^3*x^3-9*I*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x
^2+1)+I)^3*Pi*arctan(a*x)^2*a*x-33*I*arctan(a*x)^2-9*csgn(I*(1+I*a*x)^4/(a^
2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*Pi*arctan(a*x)^2*a^2*x^2+9*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*arctan(a*x)^2*a^2*x^2-6*I*csgn(I*(1+
I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/
```

```
(a^2*x^2+1)+I)*Pi*arctan(a*x)^2*a^3*x^3+3*I*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^
2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi*arc
tan(a*x)^2*a^3*x^3-18*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1))*Pi*arctan(a*x)^2*a*x+9*I*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)^2*a*x+6*I
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
)*Pi*arctan(a*x)^2*a^3*x^3-3*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)^2*a^3*x^3+18*I*csgn(I*(1+I*a
*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)+I)*Pi*arctan(a*x)^2*a*x-9*I*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(
1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*Pi*arctan(a*x
)^2*a*x)-5*ln((1+I*a*x)^2/(a^2*x^2+1)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 140*a^6*c^2*integrate(1/160*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 15*a^6*c^
2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12
*a^6*c^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 - 12*a^5*c^2*integrate(1/160*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^5*c
^2*integrate(1/160*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4*c^2*i
ntegrate(1/160*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1
/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*inte
grate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^3*c^2
*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 10*a^3*c^2*integrate
(1/160*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^2*arctan(a*x)^4/a
+ 420*a^2*c^2*integrate(1/160*x^2*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^2*
c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
60*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1),
x) + 1/120*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^3 - 60*a
*c^2*integrate(1/160*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a*c^2*integrate
(1/160*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/160*(3*a^4*c^2*x^5 + 10*a
^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 15*c^2*integrate(1/
160*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2\operatorname{atan}^3(ax)\,dx + \int a^4x^4\operatorname{atan}^3(ax)\,dx + \int \operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**3, x) + Integral(a**4*x**4*atan(a*x)*
*3, x) + Integral(atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.375 $\int \frac{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^3}{x} \, dx$

Optimal. Leaf size=370

$$-\frac{1}{4}ac^2x+\frac{1}{4}c^2\text{ArcTan}(ax)+\frac{1}{4}a^2c^2x^2\text{ArcTan}(ax)-2ic^2\text{ArcTan}(ax)^2-\frac{9}{4}ac^2x\text{ArcTan}(ax)^2-\frac{1}{4}a^3c^2x^3\text{ArcTan}(a$$

```
[Out] -1/4*a*c^2*x+1/4*c^2*arctan(a*x)+1/4*a^2*c^2*x^2*arctan(a*x)-2*I*c^2*arctan
(a*x)^2-9/4*a*c^2*x*arctan(a*x)^2-1/4*a^3*c^2*x^3*arctan(a*x)^2+3/4*c^2*arc
tan(a*x)^3+a^2*c^2*x^2*arctan(a*x)^3+1/4*a^4*c^2*x^4*arctan(a*x)^3-2*c^2*ar
ctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-4*c^2*arctan(a*x)*ln(2/(1+I*a*x))-3/4*I
*c^2*polylog(4,-1+2/(1+I*a*x))+3/4*I*c^2*polylog(4,1-2/(1+I*a*x))-3/2*I*c^2
*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3/2*c^2*arctan(a*x)*polylog(3,1-2/(
1+I*a*x))+3/2*c^2*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/2*I*c^2*arctan(a*
x)^2*polylog(2,-1+2/(1+I*a*x))-2*I*c^2*polylog(2,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.70, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5068, 4942, 5108, 5004, 5114, 5118, 6745, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 327, 209}

$$\frac{1}{4}a^4c^2x^4\text{ArcTan}(ax)^3-\frac{1}{4}a^3c^2x^3\text{ArcTan}(ax)^2+a^2c^2x^2\text{ArcTan}(ax)^3+\frac{1}{4}a^2c^2x^2\text{ArcTan}(ax)-\frac{3}{2}ic^2\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}ic^2\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{3}{2}c^2\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}c^2\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)-\frac{9}{4}ac^2x\text{ArcTan}(ax)^2+\frac{3}{4}c^2\text{ArcTan}(ax)^3-2ic^2\text{ArcTan}(ax)^2+\frac{1}{4}c^2\text{ArcTan}(ax)-4c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2c^2\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-2ic^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{4}ic^2\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{3}{4}ic^2\text{Li}_4\left(\frac{2}{iax+1}-1\right)-\frac{1}{4}ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

```
[Out] -1/4*(a*c^2*x) + (c^2*ArcTan[a*x])/4 + (a^2*c^2*x^2*ArcTan[a*x])/4 - (2*I)*
c^2*ArcTan[a*x]^2 - (9*a*c^2*x*ArcTan[a*x]^2)/4 - (a^3*c^2*x^3*ArcTan[a*x]^
2)/4 + (3*c^2*ArcTan[a*x]^3)/4 + a^2*c^2*x^2*ArcTan[a*x]^3 + (a^4*c^2*x^4*A
rcTan[a*x]^3)/4 + 2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 4*c^2*Ar
cTan[a*x]*Log[2/(1 + I*a*x)] - (2*I)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((
3*I)/2)*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c^2*Arc
Tan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^2*ArcTan[a*x]*PolyLog[3, 1
 - 2/(1 + I*a*x)])/2 + (3*c^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
 + ((3*I)/4)*c^2*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^2*PolyLog[4, -
1 + 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
```

```
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^3}{x}\,dx = \int \left(\frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + a^4c^2x^3\tan^{-1}(ax)^3\right)dx$$
$$= c^2\int \frac{\tan^{-1}(ax)^3}{x}\,dx + \left(2a^2c^2\right)\int x\tan^{-1}(ax)^3\,dx + \left(a^4c^2\right)\int x^3\tan^{-1}(ax)^3$$
$$= a^2c^2x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^3 + 2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+\right.$$
$$= a^2c^2x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^4c^2x^4\tan^{-1}(ax)^3 + 2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+\right.$$
$$= -3ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax)^2 + c^2\tan^{-1}(ax)^3 + a^2c^2x^2\tan^{-1}(a$$
$$= -3ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}(ax)^2 + \frac{3}{4}c^2\tan^{-1}(a$$
$$= \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3c^2x^3\tan^{-1}($$
$$= -\frac{1}{4}ac^2x + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\tan^{-1}(ax)^2 - \frac{1}{4}a^3$$
$$= -\frac{1}{4}ac^2x + \frac{1}{4}c^2\tan^{-1}(ax) + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\,t$$
$$= -\frac{1}{4}ac^2x + \frac{1}{4}c^2\tan^{-1}(ax) + \frac{1}{4}a^2c^2x^2\tan^{-1}(ax) - 2ic^2\tan^{-1}(ax)^2 - \frac{9}{4}ac^2x\,t$$

Mathematica [A]
time = 0.37, size = 302, normalized size = 0.82

$$\frac{1}{64}c^2\left(-i\pi^4 - 16ax + 16\text{ArcTan}(ax) + 16a^2x^2\text{ArcTan}(ax) + 128i\text{ArcTan}(ax)^2 - 144ax\text{ArcTan}(ax)^2 - 16a^3x^3\text{ArcTan}(ax)^2 + 48\text{ArcTan}(ax)^3 + 64a^2x^2\text{ArcTan}(ax)^3 + 16a^4x^4\text{ArcTan}(ax)^3 + 32i\text{ArcTan}(ax)^4 + 64\text{ArcTan}(ax)^3\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) - 256\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - 64\text{ArcTan}(ax)^3\log\left(1+e^{2i\text{ArcTan}(ax)}\right) + 96i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + 32i\left(4+3\text{ArcTan}(ax)^2\right)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + 96\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) - 96\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right) - 48i\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(ax)}\right) - 48i\text{PolyLog}\left(4,-e^{2i\text{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c^2*((-I)*Pi^4 - 16*a*x + 16*ArcTan[a*x] + 16*a^2*x^2*ArcTan[a*x] + (128*I
)*ArcTan[a*x]^2 - 144*a*x*ArcTan[a*x]^2 - 16*a^3*x^3*ArcTan[a*x]^2 + 48*Arc
Tan[a*x]^3 + 64*a^2*x^2*ArcTan[a*x]^3 + 16*a^4*x^4*ArcTan[a*x]^3 + (32*I)*A
rcTan[a*x]^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - 256*ArcTa
n[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log[1 + E^((2*I)*A
rcTan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3
2*I)*(4 + 3*ArcTan[a*x]^2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 96*ArcTan[a
*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, -E^((2*I
)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLo
g[4, -E^((2*I)*ArcTan[a*x])]))/64
```

Maple [A]
time = 32.36, size = 566, normalized size = 1.53

method	result
derivativedivides	$\frac{c^2\left(-8\arctan(ax)^2-3i\arctan(ax)^3+i\arctan(ax)^2ax+3\arctan(ax)^3ax-\arctan(ax)^2a^2x^2-i\arctan(ax)^3a^2x^2+\arctan(\right.}{4}$
default	$\frac{c^2\left(-8\arctan(ax)^2-3i\arctan(ax)^3+i\arctan(ax)^2ax+3\arctan(ax)^3ax-\arctan(ax)^2a^2x^2-i\arctan(ax)^3a^2x^2+\arctan(\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^2*(-8*arctan(a*x)^2-3*I*arctan(a*x)^3+I*arctan(a*x)^2*a*x+3*arctan(a*
x)^3*a*x-arctan(a*x)^2*a^2*x^2-I*arctan(a*x)^3*a^2*x^2+arctan(a*x)^3*a^3*x^
3-1-I*arctan(a*x)+arctan(a*x)*a*x)*(I+a*x)-c^2*arctan(a*x)^3*ln((1+I*a*x)^2
/(a^2*x^2+1)+1)+6*I*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*c^2*arc
tan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+c^2*arctan(a*x)^3*ln(1-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+6*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c^2*arcta
n(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c^2*arctan(a*x)^2*polylog(
2,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*I*c^2*arctan(a*x)^2+6*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*c^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3*I*c^2*arctan(a*x)^2*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/4*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^
2+1))-4*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/32*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^3 - 3/128*(a^4*c^2*x^4 + 4*a
^2*c^2*x^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/128*(112*(a^6*c^2*
x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^3 - 12*(a^5*c^2*x^5
+ 4*a^3*c^2*x^3)*arctan(a*x)^2 + 12*(a^6*c^2*x^6 + 4*a^4*c^2*x^4)*arctan(a*
x)*log(a^2*x^2 + 1) + 3*(a^5*c^2*x^5 + 4*a^3*c^2*x^3 + 4*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3
 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{\operatorname{atan}^3(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}^3(ax)\,dx + \int a^4x^3\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**3/x, x) + Integral(2*a**2*x*atan(a*x)**3, x) + In
tegral(a**4*x**3*atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x, x)
```

3.376 $\int \frac{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^3}{x^2} \, dx$

Optimal. Leaf size=284

$$a^2c^2x\text{ArcTan}(ax)-\frac{1}{2}ac^2\text{ArcTan}(ax)^2-\frac{1}{2}a^3c^2x^2\text{ArcTan}(ax)^2+\frac{2}{3}iac^2\text{ArcTan}(ax)^3-\frac{c^2\text{ArcTan}(ax)^3}{x}+2a^2c^2x$$

```
[Out] a^2*c^2*x*arctan(a*x)-1/2*a*c^2*arctan(a*x)^2-1/2*a^3*c^2*x^2*arctan(a*x)^2
+2/3*I*a*c^2*arctan(a*x)^3-c^2*arctan(a*x)^3/x+2*a^2*c^2*x*arctan(a*x)^3+1/
3*a^4*c^2*x^3*arctan(a*x)^3+5*a*c^2*arctan(a*x)^2*ln(2/(1+I*a*x))-1/2*a*c^2
*ln(a^2*x^2+1)+3*a*c^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c^2*arctan(a*x
)*polylog(2,-1+2/(1-I*a*x))+5*I*a*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+
3/2*a*c^2*polylog(3,-1+2/(1-I*a*x))+5/2*a*c^2*polylog(3,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.55, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5068, 4930, 5040, 4964, 5004, 5114, 6745, 4946, 5044, 4988, 5112, 5036, 266}

$$\frac{1}{3}a^4c^2x^3\text{ArcTan}(ax)^3-\frac{1}{2}a^3c^2x^2\text{ArcTan}(ax)^2+2a^2c^2x\text{ArcTan}(ax)^3+a^2c^2x\text{ArcTan}(ax)-\frac{1}{2}ac^2\log\left(a^2x^2+1\right)-3iac^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)+5iac^2\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{2}{3}iac^2\text{ArcTan}(ax)^3-\frac{1}{2}ac^2\text{ArcTan}(ax)^2-\frac{c^2\text{ArcTan}(ax)^3}{x}+5ac^2\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^2\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^2\text{Li}_3\left(\frac{2}{1-iax}-1\right)+\frac{5}{2}ac^2\text{Li}_3\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] a^2*c^2*x*ArcTan[a*x] - (a*c^2*ArcTan[a*x]^2)/2 - (a^3*c^2*x^2*ArcTan[a*x]^
2)/2 + ((2*I)/3)*a*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/x + 2*a^2*c^2*x*
ArcTan[a*x]^3 + (a^4*c^2*x^3*ArcTan[a*x]^3)/3 + 5*a*c^2*ArcTan[a*x]^2*Log[2
/(1 + I*a*x)] - (a*c^2*Log[1 + a^2*x^2])/2 + 3*a*c^2*ArcTan[a*x]^2*Log[2 -
2/(1 - I*a*x)] - (3*I)*a*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (
5*I)*a*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^2*PolyLog[3,
-1 + 2/(1 - I*a*x)])/2 + (5*a*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^3}{x^2}\,dx = \int \left(2a^2c^2\tan^{-1}(ax)^3 + \frac{c^2\tan^{-1}(ax)^3}{x^2} + a^4c^2x^2\tan^{-1}(ax)^3\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx + \left(2a^2c^2\right)\int \tan^{-1}(ax)^3\,dx + \left(a^4c^2\right)\int x^2\tan^{-1}(ax)^3\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^3 + \left(3ac^2\right)\int \frac{\tan}{x\,(1}$$

$$= iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(ax)^3 + \frac{1}{3}a^4c^2x^3\tan^{-1}(ax)^3$$

$$= -\frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(a$$

$$= -\frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax)^3 - \frac{c^2\tan^{-1}(ax)^3}{x} + 2a^2c^2x\tan^{-1}(a$$

$$= a^2c^2x\tan^{-1}(ax) - \frac{1}{2}ac^2\tan^{-1}(ax)^2 - \frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax$$

$$= a^2c^2x\tan^{-1}(ax) - \frac{1}{2}ac^2\tan^{-1}(ax)^2 - \frac{1}{2}a^3c^2x^2\tan^{-1}(ax)^2 + \frac{2}{3}iac^2\tan^{-1}(ax$$

Mathematica [A]
time = 0.27, size = 246, normalized size = 0.87

$$\frac{c^2\left(-3ia\pi^3x + 24a^2x^2\text{ArcTan}(ax) - 12ax\text{ArcTan}(ax)^2 - 12a^3x^3\text{ArcTan}(ax)^2 - 24\text{ArcTan}(ax)^3 - 16iax\text{ArcTan}(ax)^3 + 48a^2x^2\text{ArcTan}(ax)^3 + 8a^4x^4\text{ArcTan}(ax)^3 + 72ax\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 120ax\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - 12ax\log\left(1+a^2x^2\right) + 72iax\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) - 120iax\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + 36ax\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) + 60ax\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{24x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (c^2*((-3*I)*a*Pi^3*x + 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 - 12*
a^3*x^3*ArcTan[a*x]^2 - 24*ArcTan[a*x]^3 - (16*I)*a*x*ArcTan[a*x]^3 + 48*a^
2*x^2*ArcTan[a*x]^3 + 8*a^4*x^4*ArcTan[a*x]^3 + 72*a*x*ArcTan[a*x]^2*Log[1
- E^((-2*I)*ArcTan[a*x])] + 120*a*x*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a
*x])] - 12*a*x*Log[1 + a^2*x^2] + (72*I)*a*x*ArcTan[a*x]*PolyLog[2, E^((-2*
I)*ArcTan[a*x])] - (120*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x]
)] + 36*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, -E^((2*I
)*ArcTan[a*x])]))/(24*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 137.65, size = 5077, normalized size = 17.88

method	result	size
derivativedivides	Expression too large to display	5077

default	Expression too large to display	5077

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^3 - 3*(a^4*c^2*x^
4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 3*(896*a^6*c^2*
integrate(1/32*x^6*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 96*a^6*c^2*integrate
(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 128*a^6*c^2*
integrate(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 128*a
^5*c^2*integrate(1/32*x^5*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 32*a^5*c^2*in
tegrate(1/32*x^5*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 21*a*c^2*arctan(a
*x)^4 + 2688*a^4*c^2*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^4 + x^2), x) +
 288*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x
^2), x) + 768*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*
x^4 + x^2), x) - 768*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^4 + x^
2), x) + a*c^2*log(a^2*x^2 + 1)^3 + 288*a^2*c^2*integrate(1/32*x^2*arctan(a
*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 384*a^2*c^2*integrate(1/32*x^2
*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 384*a*c^2*integrate(1/3
2*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 96*a*c^2*integrate(1/32*x*log(a^2*x
^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*c^2*integrate(1/32*arctan(a*x)^3/(a^2*x
^4 + x^2), x) + 96*c^2*integrate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x
^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2\operatorname{atan}^3(ax)\,dx+\int\frac{\operatorname{atan}^3(ax)}{x^2}\,dx+\int a^4x^2\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) + I
ntegral(a**4*x**2*atan(a*x)**3, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2, x)
```

3.377 $\int \frac{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^3}{x^3} \, dx$

Optimal. Leaf size=399

$$-3ia^2c^2\text{ArcTan}(ax)^2-\frac{3ac^2\text{ArcTan}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\text{ArcTan}(ax)^2-\frac{c^2\text{ArcTan}(ax)^3}{2x^2}+\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)^3+4a$$

```
[Out] 3/2*I*a^2*c^2*polylog(4,1-2/(1+I*a*x))-3/2*a*c^2*arctan(a*x)^2/x-3/2*a^3*c^
2*x*arctan(a*x)^2-1/2*c^2*arctan(a*x)^3/x^2+1/2*a^4*c^2*x^2*arctan(a*x)^3-4
*a^2*c^2*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*a^2*c^2*arctan(a*x)*ln(2/(
1+I*a*x))+3*a^2*c^2*arctan(a*x)*ln(2-2/(1-I*a*x))-3*I*a^2*c^2*arctan(a*x)^2
*polylog(2,1-2/(1+I*a*x))+3*I*a^2*c^2*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x
))-3/2*I*a^2*c^2*polylog(4,-1+2/(1+I*a*x))-3*I*a^2*c^2*arctan(a*x)^2-3*a^2*
c^2*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3*a^2*c^2*arctan(a*x)*polylog(3,-1
+2/(1+I*a*x))-3/2*I*a^2*c^2*polylog(2,-1+2/(1-I*a*x))-3/2*I*a^2*c^2*polylog
(2,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.57, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {5068, 4946, 5038, 5044, 4988, 2497, 5004, 4942, 5108, 5114, 5118, 6745, 5036, 4930, 5040, 4964, 2449, 2352}

$$\frac{1}{2}a^4c^2x^2\text{ArcTan}(ax)^3-\frac{3}{2}a^3c^2x\text{ArcTan}(ax)^2-3ia^2c^2\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+3ia^2c^2\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-3a^2c^2\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+3a^2c^2\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)-3ia^2c^2\text{ArcTan}(ax)^2-3a^2c^2\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)+4a^2c^2\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)-\frac{3}{2}ia^2c^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}ia^2c^2\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{3}{2}ia^2c^2\text{Li}_4\left(\frac{2}{iax+1}-1\right)-\frac{c^2\text{ArcTan}(ax)^3}{2x^2}-\frac{3ac^2\text{ArcTan}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (-3*I)*a^2*c^2*ArcTan[a*x]^2 - (3*a*c^2*ArcTan[a*x]^2)/(2*x) - (3*a^3*c^2*x
*ArcTan[a*x]^2)/2 - (c^2*ArcTan[a*x]^3)/(2*x^2) + (a^4*c^2*x^2*ArcTan[a*x]^
3)/2 + 4*a^2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*a^2*c^2*ArcTa
n[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
((3*I)/2)*a^2*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^2*PolyLo
g[2, 1 - 2/(1 + I*a*x)] - (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 +
 I*a*x)] + (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - 3*a
^2*c^2*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Po
lyLog[3, -1 + 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c^2*PolyLog[4, 1 - 2/(1 + I*a*
x)] - ((3*I)/2)*a^2*c^2*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
```

```
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^3}{x^3}\,dx &= \int \left(\frac{c^2\tan^{-1}(ax)^3}{x^3}+\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-1}(ax)^3\right)dx \\
&= c^2\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx+\left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^3}{x}\,dx+\left(a^4c^2\right)\int x\tan^{-1}(ax)^3\,d \\
&= -\frac{c^2\tan^{-1}(ax)^3}{2x^2}+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^3+4a^2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{}{1-}\right. \\
&= -\frac{c^2\tan^{-1}(ax)^3}{2x^2}+\frac{1}{2}a^4c^2x^2\tan^{-1}(ax)^3+4a^2c^2\tan^{-1}(ax)^3\tanh^{-1}\left(1-\frac{}{1-}\right. \\
&= -\frac{3ac^2\tan^{-1}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(ax)^3}{2x^2}+\frac{1}{2}a^4c^2x^2\tan^{-1}(\\
&= -3ia^2c^2\tan^{-1}(ax)^2-\frac{3ac^2\tan^{-1}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(a}{2x^2} \\
&= -3ia^2c^2\tan^{-1}(ax)^2-\frac{3ac^2\tan^{-1}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(a}{2x^2} \\
&= -3ia^2c^2\tan^{-1}(ax)^2-\frac{3ac^2\tan^{-1}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(a}{2x^2} \\
&= -3ia^2c^2\tan^{-1}(ax)^2-\frac{3ac^2\tan^{-1}(ax)^2}{2x}-\frac{3}{2}a^3c^2x\tan^{-1}(ax)^2-\frac{c^2\tan^{-1}(a}{2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 302, normalized size = 0.76

$$\frac{1}{32}a^2c^2\left(-i\pi^4-\frac{48\text{ArcTan}(ax)^2}{ax}-48ax\text{ArcTan}(ax)^2-\frac{16\text{ArcTan}(ax)^3}{a^2x^2}+16a^2x^2\text{ArcTan}(ax)^3+32i\text{ArcTan}(ax)^4+64\text{ArcTan}(ax)^3\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+96\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)-96\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-64\text{ArcTan}(ax)^3\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+96i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+48i\left(1+2\text{ArcTan}(ax)^2\right)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+96\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)-96\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,-e^{2i\text{ArcTan}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*c^2*((-I)*Pi^4 - (48*ArcTan[a*x]^2)/(a*x) - 48*a*x*ArcTan[a*x]^2 - (16
*ArcTan[a*x]^3)/(a^2*x^2) + 16*a^2*x^2*ArcTan[a*x]^3 + (32*I)*ArcTan[a*x]^4
 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1
- E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] -
64*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2*Poly
Log[2, E^((-2*I)*ArcTan[a*x])] + (48*I)*(1 + 2*ArcTan[a*x]^2)*PolyLog[2, -E
^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 96*ArcTa
n[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, -E^((
2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*I)*Pol
yLog[4, -E^((2*I)*ArcTan[a*x])]))/32
```

Maple [A]
time = 29.61, size = 622, normalized size = 1.56

method	result
derivativedivides	$a^2\left(\frac{c^2\arctan(ax)^2(ax+i)(ax-i)(\arctan(ax)a^2x^2-\arctan(ax)-3ax)}{2a^2x^2}+12c^2\arctan(ax)\operatorname{polylog}\left(3,\frac{ia}{\sqrt{a^2}}\right.\right.$
default	$a^2\left(\frac{c^2\arctan(ax)^2(ax+i)(ax-i)(\arctan(ax)a^2x^2-\arctan(ax)-3ax)}{2a^2x^2}+12c^2\arctan(ax)\operatorname{polylog}\left(3,\frac{ia}{\sqrt{a^2}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*c^2*arctan(a*x)^2/a^2/x^2*(I+a*x)*(a*x-I)*(arctan(a*x)*a^2*x^2-arc
tan(a*x)-3*a*x)+12*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
2*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^2*arctan(a*x)*ln(1+(1+I*
a*x)/(a^2*x^2+1)^(1/2))-3*I*c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*c
^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*c^2*arctan(a*x)^
2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3*c^2*arctan(a*x)*polylog(3,-(1+I*a*x
)^2/(a^2*x^2+1))-6*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+2*c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^2*polylog(2,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1
)^(1/2))+12*I*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*c^2*arctan(a*x)
^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/2*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+
1))-3*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I*c^2*polylog(2,-(1
+I*a*x)^2/(a^2*x^2+1))+3*c^2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
6*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/64*(12*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^3 + x*arctan(a*x)*log(a^2*x^
2 + 1)^2, x) + 8*a^3*c^2*x^2*integrate(-1/8*(24*(a^2*x^2 + 1)*a*x*arctan(a*
x)^3 - 18*(a^2*x^2 + 1)*a*x*arctan(a*x)*log(a^2*x^2 + 1)^2 + 36*(a^2*x^2 +
1)*arctan(a*x)^2*log(a^2*x^2 + 1) - 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3 - sq
rt(a^2*x^2 + 1)*(12*sqrt(a^2*x^2 + 1)*arctan(a*x)^2*log(a^2*x^2 + 1) - sqrt
(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3 - (12*(a^2*x^2 + 1)^2*arctan(a*x)^2*log(a^
2*x^2 + 1) - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^3)*cos(3*arctan(a*x)) + 3*(12
*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2*log(a^2*x^2 + 1) - (a^2*x^2 + 1)^(3/2)*l
og(a^2*x^2 + 1)^3)*cos(2*arctan(a*x)) - 2*(4*(a^2*x^2 + 1)^2*arctan(a*x)^3
- 3*(a^2*x^2 + 1)^2*arctan(a*x)*log(a^2*x^2 + 1)^2)*sin(3*arctan(a*x)) + 6*
```

```
(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^3 - 3*(a^2*x^2 + 1)^(3/2)*arctan(a*x)*lo
g(a^2*x^2 + 1)^2)*sin(2*arctan(a*x))))/((a^2*x^2 + 1)^4*cos(3*arctan(a*x))^
2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2*x^2 + 1)^(7/2)*sin(3*arct
an(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*cos(2*arctan(a*x))^2 + 9*(a
^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 + 6*(a^2*x^2 + 1)^2*cos(2*arct
an(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 + 1)^(7/2)*cos(2*arctan(a*x))
+ (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6*((a^2*x^2 + 1)^2*a*x*sin(3*ar
ctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2*arctan(a*x)) + (a^2*x^2 + 1)^2
*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*cos(2*arctan(a*x)) - sqrt(a^2*x
^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 12*a^3*c^2*x^2*integrate(1/4*(8*(a^2*
x^2 + 1)*a*x*arctan(a*x)*log(a^2*x^2 + 1) - 8*(a^2*x^2 + 1)*arctan(a*x)^2 +
 2*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2 - (4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)*lo
g(a^2*x^2 + 1)*sin(2*arctan(a*x)) - 4*sqrt(a^2*x^2 + 1)*arctan(a*x)^2 + sqr
t(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2 - (4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2 -
(a^2*x^2 + 1)^(3/2)*log(a^2*x^2 + 1)^2)*cos(2*arctan(a*x)))*sqrt(a^2*x^2 +
1))/((a^2*x^2 + 1)^3*cos(2*arctan(a*x))^2 + (a^2*x^2 + 1)^3*sin(2*arctan(a*
x))^2 + a^2*x^2 + 2*(a^2*x^2 + 1)^2*cos(2*arctan(a*x)) + 4*(a^2*x^2 + 1)^2
- 4*((a^2*x^2 + 1)^(3/2)*a*x*sin(2*arctan(a*x)) + (a^2*x^2 + 1)^(3/2)*cos(2
*arctan(a*x)) + sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 8*a^3*c^2*x
^2*integrate(1/8*((8*(a^2*x^2 + 1)*a*x*arctan(a*x)^3 - 6*(a^2*x^2 + 1)*a*x*
arctan(a*x)*log(a^2*x^2 + 1)^2 + 12*(a^2*x^2 + 1)*arctan(a*x)^2*log(a^2*x^2
 + 1) - (a^2*x^2 + 1)*log(a^2*x^2 + 1)^3)*cos(2*arctan(a*x)) + (12*(a^2*x^2
 + 1)*a*x*arctan(a*x)^2*log(a^2*x^2 + 1) - (a^2*x^2 + 1)*a*x*log(a^2*x^2 +
1)^3 - 8*(a^2*x^2 + 1)*arctan(a*x)^3 + 6*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*
x^2 + 1)^2)*sin(2*arctan(a*x)) - sqrt(a^2*x^2 + 1)*(12*sqrt(a^2*x^2 + 1)*ar
ctan(a*x)^2*log(a^2*x^2 + 1) - sqrt(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3))/(a^2*
x^2 + 1), x) + 12*a^3*c^2*x^2*integrate(-1/4*((4*(a^2*x^2 + 1)*a*x*arctan(a
*x)*log(a^2*x^2 + 1) - 4*(a^2*x^2 + 1)*arctan(a*x)^2 + (a^2*x^2 + 1)*log(a^
2*x^2 + 1)^2)*cos(2*arctan(a*x)) - (4*(a^2*x^2 + 1)*a*x*arctan(a*x)^2 - (a^
2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 + 4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2
 + 1))*sin(2*arctan(a*x)) + sqrt(a^2*x^2 + 1)*(4*sqrt(a^2*x^2 + 1)*arctan(a
*x)^2 - sqrt(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2))/(a^2*x^2 + 1), x) - 6*a^2*c^
2*x^2*(I*conjugate(gamma(3, -log(-I*a*x + 1))) - I*gamma(3, -log(-I*a*x + 1
))) - 12*a^2*c^2*x^2*integrate(arctan(a*x)*log(a^2*x^2 + 1)/x, x) + 24*a^2*
c^2*x^2*integrate((4*arctan(a*x)^3 + arctan(a*x)*log(a^2*x^2 + 1)^2)/x, x)
- 4*a^2*c^2*x^2*integrate(-(4*arctan(a*x)^3 - 3*arctan(a*x)*log(a^2*x^2 + 1
)^2)/x, x) + 12*c^2*x^2*integrate((4*arctan(a*x)^3 + arctan(a*x)*log(a^2*x^
2 + 1)^2)/x^3, x) + 4*(a^4*c^2*x^4 - c^2)*arctan(a*x)^3 - 3*(a^4*c^2*x^4 -
c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx + \int \frac{2a^2\operatorname{atan}^3(ax)}{x}\,dx + \int a^4x\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**3,x)
```

```
[Out] c**2*(Integral(atan(a*x)**3/x**3, x) + Integral(2*a**2*atan(a*x)**3/x, x) +
 Integral(a**4*x*atan(a*x)**3, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3, x)
```

3.378 $\int \frac{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=311

$$-\frac{a^2c^2\text{ArcTan}(ax)}{x}-\frac{1}{2}a^3c^2\text{ArcTan}(ax)^2-\frac{ac^2\text{ArcTan}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\text{ArcTan}(ax)^3-\frac{c^2\text{ArcTan}(ax)^3}{3x^3}-\frac{2a^2c^2\text{Ar}}{}$$

```
[Out] -a^2*c^2*arctan(a*x)/x-1/2*a^3*c^2*arctan(a*x)^2-1/2*a*c^2*arctan(a*x)^2/x^
2-2/3*I*a^3*c^2*arctan(a*x)^3-1/3*c^2*arctan(a*x)^3/x^3-2*a^2*c^2*arctan(a*
x)^3/x+a^4*c^2*x*arctan(a*x)^3+a^3*c^2*ln(x)+3*a^3*c^2*arctan(a*x)^2*ln(2/(
1+I*a*x))-1/2*a^3*c^2*ln(a^2*x^2+1)+5*a^3*c^2*arctan(a*x)^2*ln(2-2/(1-I*a*x
))-5*I*a^3*c^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+3*I*a^3*c^2*arctan(a*x
)*polylog(2,1-2/(1+I*a*x))+5/2*a^3*c^2*polylog(3,-1+2/(1-I*a*x))+3/2*a^3*c^
2*polylog(3,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.58, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5068, 4930, 5040, 4964, 5004, 5114, 6745, 4946, 5038, 272, 36, 29, 31, 5044, 4988, 5112}

$$a^4c^2x\text{ArcTan}(ax)^3-5ia^3c^2\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)+3ia^3c^2\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)-\frac{2}{3}ia^3c^2\text{ArcTan}(ax)^3-\frac{1}{2}a^3c^2\text{ArcTan}(ax)^2+3a^3c^2\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+5a^3c^2\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{5}{2}a^3c^2\text{Li}_3\left(\frac{2}{1-iax}-1\right)+\frac{3}{2}a^3c^2\text{Li}_3\left(1-\frac{2}{iax+1}\right)+a^3c^2\log(x)-\frac{2a^2c^2\text{ArcTan}(ax)^3}{x}-\frac{a^2c^2\text{ArcTan}(ax)}{x}-\frac{1}{2}a^3c^2\log\left(a^2x^2+1\right)-\frac{c^2\text{ArcTan}(ax)^3}{3x^3}-\frac{ac^2\text{ArcTan}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c^2*ArcTan[a*x])/x) - (a^3*c^2*ArcTan[a*x]^2)/2 - (a*c^2*ArcTan[a*x]
^2)/(2*x^2) - ((2*I)/3)*a^3*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/(3*x^3)
 - (2*a^2*c^2*ArcTan[a*x]^3)/x + a^4*c^2*x*ArcTan[a*x]^3 + a^3*c^2*Log[x] +
 3*a^3*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)] - (a^3*c^2*Log[1 + a^2*x^2])/2
+ 5*a^3*c^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (5*I)*a^3*c^2*ArcTan[a*x
]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a^3*c^2*ArcTan[a*x]*PolyLog[2, 1 -
 2/(1 + I*a*x)] + (5*a^3*c^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*a^3*c^2
*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^3}{x^4}\,dx = \int \left(a^4c^2\tan^{-1}(ax)^3+\frac{c^2\tan^{-1}(ax)^3}{x^4}+\frac{2a^2c^2\tan^{-1}(ax)^3}{x^2}\right)dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx+\left(2a^2c^2\right)\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx+\left(a^4c^2\right)\int \tan^{-1}(ax)^3\,dx$$

$$= -\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-1}(ax)^3+\left(ac^2\right)\int \frac{\tan^{-1}(}{x^3\,(1+}$$

$$= -ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}+a^4c^2x\tan^{-1}(ax)^3$$

$$= -\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}$$

$$= -\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3-\frac{c^2\tan^{-1}(ax)^3}{3x^3}-\frac{2a^2c^2\tan^{-1}(ax)^3}{x}$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3$$

$$= -\frac{a^2c^2\tan^{-1}(ax)}{x}-\frac{1}{2}a^3c^2\tan^{-1}(ax)^2-\frac{ac^2\tan^{-1}(ax)^2}{2x^2}-\frac{2}{3}ia^3c^2\tan^{-1}(ax)^3$$

Mathematica [A]
time = 0.37, size = 289, normalized size = 0.93

$$\frac{c^2\left(-5ia^3\pi^3x^3-24a^2x^2\text{ArcTan}(ax)-12ax\text{ArcTan}(ax)^2-12a^3x^3\text{ArcTan}(ax)^2-8\text{ArcTan}(ax)^3-48a^2x^2\text{ArcTan}(ax)^3+16ia^3x^3\text{ArcTan}(ax)^3+24a^4x^4\text{ArcTan}(ax)^3+120a^3x^3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+72a^3x^3\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+24a^3x^3\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+120ia^3x^3\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)-72ia^3x^3\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+60a^3x^3\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)+36a^3x^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c^2*((-5*I)*a^3*Pi^3*x^3 - 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 -
 12*a^3*x^3*ArcTan[a*x]^2 - 8*ArcTan[a*x]^3 - 48*a^2*x^2*ArcTan[a*x]^3 + (1
6*I)*a^3*x^3*ArcTan[a*x]^3 + 24*a^4*x^4*ArcTan[a*x]^3 + 120*a^3*x^3*ArcTan[
a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 72*a^3*x^3*ArcTan[a*x]^2*Log[1 + E
^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (120*I)*a
```

```
^3*x^3*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (72*I)*a^3*x^3*ArcT
an[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a^3*x^3*PolyLog[3, E^((-2*I
)*ArcTan[a*x])] + 36*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(24*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 119.45, size = 5134, normalized size = 16.51

method	result	size
derivativedivides	Expression too large to display	5134
default	Expression too large to display	5134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/192*(3*(42*a^3*c^2*arctan(a*x)^4 + 1792*a^6*c^2*integrate(1/32*x^6*arctan
(a*x)^3/(a^2*x^6 + x^4), x) + 192*a^6*c^2*integrate(1/32*x^6*arctan(a*x)*lo
g(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 768*a^6*c^2*integrate(1/32*x^6*arcta
n(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 768*a^5*c^2*integrate(1/32*x^
5*arctan(a*x)^2/(a^2*x^6 + x^4), x) + a^3*c^2*log(a^2*x^2 + 1)^3 + 576*a^4*
c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) -
 1536*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^
4), x) + 1536*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^6 + x^4), x)
- 384*a^3*c^2*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 5
376*a^2*c^2*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 576*a^2*
c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) -
 256*a^2*c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4
), x) + 256*a*c^2*integrate(1/32*x*arctan(a*x)^2/(a^2*x^6 + x^4), x) - 64*a
*c^2*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 1792*c^2*int
egrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 192*c^2*integrate(1/32*arct
an(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 8*(3*a^4*c^2*x^4 - 6*
a^2*c^2*x^2 - c^2)*arctan(a*x)^3 - 6*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*
arctan(a*x)*log(a^2*x^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int a^4\operatorname{atan}^3(ax)\,dx+\int\frac{\operatorname{atan}^3(ax)}{x^4}\,dx+\int\frac{2a^2\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**4,x)
```

```
[Out] c**2*(Integral(a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + Int
egral(2*a**2*atan(a*x)**3/x**2, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4, x)
```

3.379 $\int x^3(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=381

$$\frac{389c^3x}{12600a^3}-\frac{17c^3x^3}{9450a}-\frac{1}{252}ac^3x^5-\frac{1}{840}a^3c^3x^7-\frac{389c^3\text{ArcTan}(ax)}{12600a^4}-\frac{107c^3x^2\text{ArcTan}(ax)}{4200a^2}+\frac{53c^3x^4\text{ArcTan}(ax)}{2100}+7$$

```
[Out] 389/12600*c^3*x/a^3-17/9450*c^3*x^3/a-1/252*a*c^3*x^5-1/840*a^3*c^3*x^7-389
/12600*c^3*arctan(a*x)/a^4-107/4200*c^3*x^2*arctan(a*x)/a^2+53/2100*c^3*x^4
*arctan(a*x)+71/2520*a^2*c^3*x^6*arctan(a*x)+1/120*a^4*c^3*x^8*arctan(a*x)+
26/525*I*c^3*arctan(a*x)^2/a^4+3/40*c^3*x*arctan(a*x)^2/a^3-1/40*c^3*x^3*ar
ctan(a*x)^2/a-27/200*a*c^3*x^5*arctan(a*x)^2-33/280*a^3*c^3*x^7*arctan(a*x)
^2-1/30*a^5*c^3*x^9*arctan(a*x)^2-1/40*c^3*arctan(a*x)^3/a^4+1/4*c^3*x^4*ar
ctan(a*x)^3+1/2*a^2*c^3*x^6*arctan(a*x)^3+3/8*a^4*c^3*x^8*arctan(a*x)^3+1/1
0*a^6*c^3*x^10*arctan(a*x)^3+52/525*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^4+26/
525*I*c^3*polylog(2,1-2/(1+I*a*x))/a^4
```

Rubi [A]

time = 2.66, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 184, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004, 308}

$$\frac{1}{10}a^6c^3x^{10}\text{ArcTan}(ax)^3-\frac{1}{30}a^5c^3x^9\text{ArcTan}(ax)^2+\frac{3}{8}a^4c^3x^8\text{ArcTan}(ax)^3+\frac{1}{120}a^4c^3x^8\text{ArcTan}(ax)-\frac{c^3\text{ArcTan}(ax)^3}{40a^4}+\frac{26ic^3\text{ArcTan}(ax)^2}{525a^4}-\frac{389c^3\text{ArcTan}(ax)}{12600a^4}+\frac{52c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{525a^4}+\frac{26ic^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{525a^4}-\frac{33}{280}a^3c^3x^7\text{ArcTan}(ax)^2+\frac{3c^3x\text{ArcTan}(ax)^2}{40a^3}-\frac{1}{840}a^3c^3x^7+\frac{389c^3x}{12600a^3}+\frac{1}{2}a^2c^3x^6\text{ArcTan}(ax)^3+\frac{71a^2c^3x^6\text{ArcTan}(ax)}{2520}-\frac{107c^3x^2\text{ArcTan}(ax)}{4200a^2}-\frac{27}{200}ac^3x^5\text{ArcTan}(ax)^2+\frac{1}{4}c^3x^4\text{ArcTan}(ax)^3+\frac{53c^3x^4\text{ArcTan}(ax)}{2100}-\frac{c^3x^3\text{ArcTan}(ax)^2}{40a}-\frac{1}{252}ac^3x^5-\frac{17c^3x^3}{9450a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (389*c^3*x)/(12600*a^3) - (17*c^3*x^3)/(9450*a) - (a*c^3*x^5)/252 - (a^3*c^
3*x^7)/840 - (389*c^3*ArcTan[a*x])/(12600*a^4) - (107*c^3*x^2*ArcTan[a*x])/
(4200*a^2) + (53*c^3*x^4*ArcTan[a*x])/2100 + (71*a^2*c^3*x^6*ArcTan[a*x])/2
520 + (a^4*c^3*x^8*ArcTan[a*x])/120 + (((26*I)/525)*c^3*ArcTan[a*x]^2)/a^4
+ (3*c^3*x*ArcTan[a*x]^2)/(40*a^3) - (c^3*x^3*ArcTan[a*x]^2)/(40*a) - (27*a
*c^3*x^5*ArcTan[a*x]^2)/200 - (33*a^3*c^3*x^7*ArcTan[a*x]^2)/280 - (a^5*c^3
*x^9*ArcTan[a*x]^2)/30 - (c^3*ArcTan[a*x]^3)/(40*a^4) + (c^3*x^4*ArcTan[a*x
]^3)/4 + (a^2*c^3*x^6*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]^3)/8 +
(a^6*c^3*x^10*ArcTan[a*x]^3)/10 + (52*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(
525*a^4) + (((26*I)/525)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int x^3 \left(c+a^2 c x^2\right)^3 \tan^{-1}(ax)^3\,dx &= \int \left(c^3 x^3 \tan^{-1}(ax)^3 + 3a^2c^3x^5 \tan^{-1}(ax)^3 + 3a^4c^3x^7 \tan^{-1}(ax)^3 + a^6c^3x^9 \cdots\right. \\
&= c^3 \int x^3 \tan^{-1}(ax)^3\,dx + \left(3a^2c^3\right) \int x^5 \tan^{-1}(ax)^3\,dx + \left(3a^4c^3\right) \int x^7 \tan^{-} \cdots \\
&= \frac{1}{4}c^3x^4\tan^{-1}(ax)^3 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^3 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{10}a^6c^3x \cdots \\
&= \frac{1}{4}c^3x^4\tan^{-1}(ax)^3 + \frac{1}{2}a^2c^3x^6\tan^{-1}(ax)^3 + \frac{3}{8}a^4c^3x^8\tan^{-1}(ax)^3 + \frac{1}{10}a^6c^3x \cdots \\
&= -\frac{c^3x^3\tan^{-1}(ax)^2}{4a} - \frac{3}{10}ac^3x^5\tan^{-1}(ax)^2 - \frac{9}{56}a^3c^3x^7\tan^{-1}(ax)^2 - \frac{1}{30}a^5c \cdots \\
&= \frac{3c^3x\tan^{-1}(ax)^2}{4a^3} + \frac{c^3x^3\tan^{-1}(ax)^2}{4a} - \frac{3}{40}ac^3x^5\tan^{-1}(ax)^2 - \frac{33}{280}a^3c^3x^7 \mathrm{ta} \cdots \\
&= \frac{c^3x^2\tan^{-1}(ax)}{4a^2} + \frac{3}{20}c^3x^4\tan^{-1}(ax) + \frac{3}{56}a^2c^3x^6\tan^{-1}(ax) + \frac{1}{120}a^4c^3x^8 \mathrm{ta} \cdots \\
&= -\frac{c^3x}{4a^3} - \frac{11c^3x^2\tan^{-1}(ax)}{20a^2} - \frac{3}{70}c^3x^4\tan^{-1}(ax) + \frac{71a^2c^3x^6\tan^{-1}(ax)}{2520} + \cdots \\
&= \frac{55c^3x}{84a^3} - \frac{11c^3x^3}{315a} - \frac{19ac^3x^5}{2100} - \frac{1}{840}a^3c^3x^7 + \frac{c^3\tan^{-1}(ax)}{4a^4} + \frac{59c^3x^2\tan^{-1}(\cdots}{280a^2} \cdots \\
&= -\frac{689c^3x}{2520a^3} + \frac{79c^3x^3}{3780a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{55c^3\tan^{-1}(ax)}{84a^4} - \frac{107c^3x \cdots}{4 \cdots} \\
&= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 + \frac{689c^3\tan^{-1}(ax)}{2520a^4} - \frac{107c^3x \cdots}{4 \cdots} \\
&= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{389c^3\tan^{-1}(ax)}{12600a^4} - \frac{107c^3x \cdots}{4 \cdots} \\
&= \frac{389c^3x}{12600a^3} - \frac{17c^3x^3}{9450a} - \frac{1}{252}ac^3x^5 - \frac{1}{840}a^3c^3x^7 - \frac{389c^3\tan^{-1}(ax)}{12600a^4} - \frac{107c^3x \cdots}{4 \cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.57, size = 191, normalized size = 0.50

$$
\frac{c^3\left(-ax(-1167+68a^2x^2+150a^4x^4+45a^6x^6)-9(208i-315ax+105a^3x^3+567a^5x^5+495a^7x^7+140a^9x^9)\,\mathrm{ArcTan}(ax)^2+945(1+a^2x^2)^4\left(-1+4a^2x^2\right)\mathrm{ArcTan}(ax)^3+3\mathrm{ArcTan}(ax)\left(-389-321a^2x^2+318a^4x^4+355a^6x^6+105a^8x^8+1248\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)\right)-1872i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{37800a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

[Out] (c^3*(-(a*x*(-1167 + 68*a^2*x^2 + 150*a^4*x^4 + 45*a^6*x^6)) - 9*(208*I - 3
15*a*x + 105*a^3*x^3 + 567*a^5*x^5 + 495*a^7*x^7 + 140*a^9*x^9)*ArcTan[a*x]
^2 + 945*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^3 + 3*ArcTan[a*x]*(-3
89 - 321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 + 1248*Log[1 + E
^((2*I)*ArcTan[a*x])]) - (1872*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(378
00*a^4)

Maple [A]
time = 2.01, size = 382, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)^3a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{a^4c^3x^4\arctan(ax)^3}{4}-\frac{3c^3\left(-\frac{389ax}{945}+\frac{4\arctan(ax)^2a^9x^9}{9}+\frac{11}{}\right.}{}}{}$
default	$\frac{\frac{c^3\arctan(ax)^3a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{a^4c^3x^4\arctan(ax)^3}{4}-\frac{3c^3\left(-\frac{389ax}{945}+\frac{4\arctan(ax)^2a^9x^9}{9}+\frac{11}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/a^4*(1/10*c^3*arctan(a*x)^3*a^10*x^10+3/8*c^3*arctan(a*x)^3*a^8*x^8+1/2*a
^6*c^3*x^6*arctan(a*x)^3+1/4*a^4*c^3*x^4*arctan(a*x)^3-3/40*c^3*(-389/945*a
*x+4/9*arctan(a*x)^2*a^9*x^9+11/7*arctan(a*x)^2*a^7*x^7+9/5*arctan(a*x)^2*a
^5*x^5+1/3*arctan(a*x)^2*a^3*x^3-arctan(a*x)^2*a*x+1/3*arctan(a*x)^3+68/283
5*a^3*x^3+389/945*arctan(a*x)+104/315*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-104/315
*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+104/315*I*ln(a*x-I)*ln(a^2*x^2+1)+52/315*I*
ln(I+a*x)^2-104/315*I*ln(I+a*x)*ln(a^2*x^2+1)-104/315*I*dilog(-1/2*I*(I+a*x
))-52/315*I*ln(a*x-I)^2+1/63*a^7*x^7+10/189*a^5*x^5+104/315*I*dilog(1/2*I*(
a*x-I))-1/9*arctan(a*x)*a^8*x^8-71/189*arctan(a*x)*a^6*x^6-106/315*arctan(a
*x)*a^4*x^4+107/315*arctan(a*x)*a^2*x^2+208/315*arctan(a*x)*ln(a^2*x^2+1)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")

[Out] 1/67200*(420*(5376000*a^11*c^3*integrate(1/67200*x^11*arctan(a*x)^3/(a^5*x^
2 + a^3), x) - 806400*a^10*c^3*integrate(1/67200*x^10*arctan(a*x)^2/(a^5*x^
2 + a^3), x) - 201600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 + 1)^2/(a
^5*x^2 + a^3), x) - 89600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 + 1)/
(a^5*x^2 + a^3), x) + 21504000*a^9*c^3*integrate(1/67200*x^9*arctan(a*x)^3/

```
(a^5*x^2 + a^3), x) + 179200*a^9*c^3*integrate(1/67200*x^9*arctan(a*x)/(a^5
*x^2 + a^3), x) - 3024000*a^8*c^3*integrate(1/67200*x^8*arctan(a*x)^2/(a^5*
x^2 + a^3), x) - 756000*a^8*c^3*integrate(1/67200*x^8*log(a^2*x^2 + 1)^2/(a
^5*x^2 + a^3), x) - 316800*a^8*c^3*integrate(1/67200*x^8*log(a^2*x^2 + 1)/(
a^5*x^2 + a^3), x) + 32256000*a^7*c^3*integrate(1/67200*x^7*arctan(a*x)^3/(
a^5*x^2 + a^3), x) + 633600*a^7*c^3*integrate(1/67200*x^7*arctan(a*x)/(a^5*
x^2 + a^3), x) - 4032000*a^6*c^3*integrate(1/67200*x^6*arctan(a*x)^2/(a^5*x
^2 + a^3), x) - 1008000*a^6*c^3*integrate(1/67200*x^6*log(a^2*x^2 + 1)^2/(a
^5*x^2 + a^3), x) - 362880*a^6*c^3*integrate(1/67200*x^6*log(a^2*x^2 + 1)/(
a^5*x^2 + a^3), x) + 21504000*a^5*c^3*integrate(1/67200*x^5*arctan(a*x)^3/(
a^5*x^2 + a^3), x) + 725760*a^5*c^3*integrate(1/67200*x^5*arctan(a*x)/(a^5*
x^2 + a^3), x) - 2016000*a^4*c^3*integrate(1/67200*x^4*arctan(a*x)^2/(a^5*x
^2 + a^3), x) - 504000*a^4*c^3*integrate(1/67200*x^4*log(a^2*x^2 + 1)^2/(a^
5*x^2 + a^3), x) - 67200*a^4*c^3*integrate(1/67200*x^4*log(a^2*x^2 + 1)/(a^
5*x^2 + a^3), x) + 5376000*a^3*c^3*integrate(1/67200*x^3*arctan(a*x)^3/(a^5
*x^2 + a^3), x) + 134400*a^3*c^3*integrate(1/67200*x^3*arctan(a*x)/(a^5*x^2
 + a^3), x) + 201600*a^2*c^3*integrate(1/67200*x^2*log(a^2*x^2 + 1)/(a^5*x^
2 + a^3), x) - 403200*a*c^3*integrate(1/67200*x*arctan(a*x)/(a^5*x^2 + a^3)
, x) + 50400*c^3*integrate(1/67200*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) +
 c^3*arctan(a*x)^3/a^4)*a^4 + 840*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20*a^
6*c^3*x^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x)^3 - 4*(140*a^9*c^3*x^9 + 495*
a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3 - 315*a*c^3*x)*arctan(a*x)^
2 + (140*a^9*c^3*x^9 + 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3
- 315*a*c^3*x)*log(a^2*x^2 + 1)^2)/a^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(a*x
)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int x^{3}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int 3a^{2}x^{5}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int 3a^{4}x^{7}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int a^{6}x^{9}\operatorname{atan}^{3}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**3*atan(a*x)**3, x) + Integral(3*a**2*x**5*atan(a*x)**3, x
) + Integral(3*a**4*x**7*atan(a*x)**3, x) + Integral(a**6*x**9*atan(a*x)**3
, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.380 $\int x^2(c+a^2cx^2)^3 \text{ArcTan}(ax)^3 \, dx$

Optimal. Leaf size=389

$$-\frac{107c^3x^2}{7560a}-\frac{11ac^3x^4}{1260}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\text{ArcTan}(ax)}{1260a^2}+\frac{239c^3x^3\text{ArcTan}(ax)}{3780}+\frac{59a^2c^3x^5\text{ArcTan}(ax)}{1260}+\frac{1}{84}a^4c^3x$$

```
[Out] -107/7560*c^3*x^2/a-11/1260*a*c^3*x^4-1/504*a^3*c^3*x^6-47/1260*c^3*x*arcta
n(a*x)/a^2+239/3780*c^3*x^3*arctan(a*x)+59/1260*a^2*c^3*x^5*arctan(a*x)+1/8
4*a^4*c^3*x^7*arctan(a*x)+47/2520*c^3*arctan(a*x)^2/a^3-8/105*c^3*x^2*arcta
n(a*x)^2/a-89/420*a*c^3*x^4*arctan(a*x)^2-10/63*a^3*c^3*x^6*arctan(a*x)^2-1
/24*a^5*c^3*x^8*arctan(a*x)^2-16/105*I*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a
*x))/a^3+1/3*c^3*x^3*arctan(a*x)^3+3/5*a^2*c^3*x^5*arctan(a*x)^3+3/7*a^4*c^
3*x^7*arctan(a*x)^3+1/9*a^6*c^3*x^9*arctan(a*x)^3-16/105*c^3*arctan(a*x)^2*
ln(2/(1+I*a*x))/a^3+31/945*c^3*ln(a^2*x^2+1)/a^3-16/315*I*c^3*arctan(a*x)^3
/a^3-8/105*c^3*polylog(3,1-2/(1+I*a*x))/a^3
```

Rubi [A]

time = 2.14, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 132, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5068, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745, 272, 45}

$$\frac{1}{9}a^6c^3x^9\text{ArcTan}(ax)^3-\frac{1}{24}a^5c^3x^8\text{ArcTan}(ax)^2+\frac{3}{7}a^4c^3x^7\text{ArcTan}(ax)^3+\frac{1}{84}a^4c^3x^7\text{ArcTan}(ax)-\frac{16ic^3\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{105a^3}-\frac{10}{63}a^3c^3x^6\text{ArcTan}(ax)^2-\frac{16ic^3\text{ArcTan}(ax)^3}{315a^3}+\frac{47c^3\text{ArcTan}(ax)^2}{2520a^3}-\frac{16c^3\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{105a^3}-\frac{8c^3\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{105a^3}-\frac{1}{504}a^3c^3x^6+\frac{3}{5}a^2c^3x^5\text{ArcTan}(ax)^3+\frac{59a^2c^3x^5\text{ArcTan}(ax)}{1260}-\frac{47c^3x\text{ArcTan}(ax)}{1260a^2}+\frac{31c^3\log\left(a^2x^2+1\right)}{945a^3}-\frac{89}{420}ac^3x^4\text{ArcTan}(ax)^2+\frac{1}{3}c^3x^3\text{ArcTan}(ax)^3+\frac{239c^3x^3\text{ArcTan}(ax)}{3780}-\frac{8c^3x^2\text{ArcTan}(ax)^2}{105a}-\frac{11ac^3x^4}{1260}-\frac{107c^3x^2}{7560a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (-107*c^3*x^2)/(7560*a) - (11*a*c^3*x^4)/1260 - (a^3*c^3*x^6)/504 - (47*c^3
*x*ArcTan[a*x])/(1260*a^2) + (239*c^3*x^3*ArcTan[a*x])/3780 + (59*a^2*c^3*x
^5*ArcTan[a*x])/1260 + (a^4*c^3*x^7*ArcTan[a*x])/84 + (47*c^3*ArcTan[a*x]^2
)/(2520*a^3) - (8*c^3*x^2*ArcTan[a*x]^2)/(105*a) - (89*a*c^3*x^4*ArcTan[a*x
]^2)/420 - (10*a^3*c^3*x^6*ArcTan[a*x]^2)/63 - (a^5*c^3*x^8*ArcTan[a*x]^2)/
24 - (((16*I)/315)*c^3*ArcTan[a*x]^3)/a^3 + (c^3*x^3*ArcTan[a*x]^3)/3 + (3*
a^2*c^3*x^5*ArcTan[a*x]^3)/5 + (3*a^4*c^3*x^7*ArcTan[a*x]^3)/7 + (a^6*c^3*x
^9*ArcTan[a*x]^3)/9 - (16*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(105*a^3) +
 (31*c^3*Log[1 + a^2*x^2])/(945*a^3) - (((16*I)/105)*c^3*ArcTan[a*x]*PolyLo
g[2, 1 - 2/(1 + I*a*x)])/a^3 - (8*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(105*a
^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx &= \int \left(c^3x^2\tan^{-1}(ax)^3+3a^2c^3x^4\tan^{-1}(ax)^3+3a^4c^3x^6\tan^{-1}(ax)^3+a^6c^3x\right.\\
&= c^3\int x^2\tan^{-1}(ax)^3\,dx+\left(3a^2c^3\right)\int x^4\tan^{-1}(ax)^3\,dx+\left(3a^4c^3\right)\int x^6\tan\\
&= \frac{1}{3}c^3x^3\tan^{-1}(ax)^3+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^3+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^3+\frac{1}{9}a^6c^3x\\
&= \frac{1}{3}c^3x^3\tan^{-1}(ax)^3+\frac{3}{5}a^2c^3x^5\tan^{-1}(ax)^3+\frac{3}{7}a^4c^3x^7\tan^{-1}(ax)^3+\frac{1}{9}a^6c^3x\\
&= -\frac{c^3x^2\tan^{-1}(ax)^2}{2a}-\frac{9}{20}ac^3x^4\tan^{-1}(ax)^2-\frac{3}{14}a^3c^3x^6\tan^{-1}(ax)^2-\frac{1}{24}a^5\\
&= \frac{2c^3x^2\tan^{-1}(ax)^2}{5a}-\frac{9}{70}ac^3x^4\tan^{-1}(ax)^2-\frac{10}{63}a^3c^3x^6\tan^{-1}(ax)^2-\frac{1}{24}a^5\\
&= \frac{c^3x\tan^{-1}(ax)}{a^2}+\frac{3}{10}c^3x^3\tan^{-1}(ax)+\frac{3}{35}a^2c^3x^5\tan^{-1}(ax)+\frac{1}{84}a^4c^3x^7\,\mathrm{ta}\\
&= -\frac{17c^3x\tan^{-1}(ax)}{10a^2}-\frac{2}{35}c^3x^3\tan^{-1}(ax)+\frac{59a^2c^3x^5\tan^{-1}(ax)}{1260}+\frac{1}{84}a^4c^3\\
&= \frac{23c^3x\tan^{-1}(ax)}{35a^2}+\frac{239c^3x^3\tan^{-1}(ax)}{3780}+\frac{59a^2c^3x^5\tan^{-1}(ax)}{1260}+\frac{1}{84}a^4c^3x\\
&= -\frac{19c^3x^2}{168a}-\frac{31ac^3x^4}{1680}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(a}{3780}\\
&= \frac{29c^3x^2}{630a}-\frac{11ac^3x^4}{1260}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(ax}{3780}\\
&= -\frac{107c^3x^2}{7560a}-\frac{11ac^3x^4}{1260}-\frac{1}{504}a^3c^3x^6-\frac{47c^3x\tan^{-1}(ax)}{1260a^2}+\frac{239c^3x^3\tan^{-1}(}{3780}
\end{aligned}
$$

Mathematica [A]
time = 1.41, size = 281, normalized size = 0.72

$$
\frac{c^3\left(-56-107a^2x^2-66a^4x^4-15a^6x^6-282ax\text{ArcTan}(ax)+478a^3x^3\text{ArcTan}(ax)+354a^5x^5\text{ArcTan}(ax)+90a^7x^7\text{ArcTan}(ax)+141\text{ArcTan}(ax)^2-576a^2x^2\text{ArcTan}(ax)^2-1602a^4x^4\text{ArcTan}(ax)^2-1200a^6x^6\text{ArcTan}(ax)^2-315a^8x^8\text{ArcTan}(ax)^2+384i\text{ArcTan}(ax)^3+2520a^3x^3\text{ArcTan}(ax)^3+4536a^5x^5\text{ArcTan}(ax)^3+3240a^7x^7\text{ArcTan}(ax)^3+840a^9x^9\text{ArcTan}(ax)^3-1152\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+248\log\left(1+a^2x^2\right)+1152i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-576\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{7560a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-56 - 107*a^2*x^2 - 66*a^4*x^4 - 15*a^6*x^6 - 282*a*x*ArcTan[a*x] + 4
78*a^3*x^3*ArcTan[a*x] + 354*a^5*x^5*ArcTan[a*x] + 90*a^7*x^7*ArcTan[a*x] +
```

```
141*ArcTan[a*x]^2 - 576*a^2*x^2*ArcTan[a*x]^2 - 1602*a^4*x^4*ArcTan[a*x]^2
 - 1200*a^6*x^6*ArcTan[a*x]^2 - 315*a^8*x^8*ArcTan[a*x]^2 + (384*I)*ArcTan[
a*x]^3 + 2520*a^3*x^3*ArcTan[a*x]^3 + 4536*a^5*x^5*ArcTan[a*x]^3 + 3240*a^7
*x^7*ArcTan[a*x]^3 + 840*a^9*x^9*ArcTan[a*x]^3 - 1152*ArcTan[a*x]^2*Log[1 +
 E^((2*I)*ArcTan[a*x])] + 248*Log[1 + a^2*x^2] + (1152*I)*ArcTan[a*x]*PolyL
og[2, -E^((2*I)*ArcTan[a*x])] - 576*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(7
560*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 75.77, size = 1576, normalized size = 4.05

method	result	size
derivativedivides	Expression too large to display	1576
default	Expression too large to display	1576

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/9*c^3*arctan(a*x)^3*a^9*x^9+3/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3*
arctan(a*x)^3*a^5*x^5+1/3*c^3*arctan(a*x)^3*a^3*x^3-1/105*c^3*(-4*I*Pi*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+4*I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+115*I*arctan(a*x)*(a*x
-I)^2*(I+a*x)^2-230/3*I*arctan(a*x)*(a*x-I)*(I+a*x)^3-4*I*Pi*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))^3*arctan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-525/4*I*arctan(a*x)*(a*x-I)^2*(
I+a*x)^4+105/2*I*arctan(a*x)*(a*x-I)*(I+a*x)^5-47/24*arctan(a*x)^2-8*arctan
(a*x)^2*ln(a^2*x^2+1)+16*arctan(a*x)^2*ln(2)+62/9*ln((1+I*a*x)^2/(a^2*x^2+1
)+1)+16*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-8/9*(I+a*x)^2+320/3*a
rctan(a*x)*(a*x-I)*(I+a*x)^4+175/4*arctan(a*x)*(a*x-I)^4*(I+a*x)^3-640/3*ar
ctan(a*x)*(a*x-I)^2*(I+a*x)^3+8*arctan(a*x)*(a*x-I)-11/9*arctan(a*x)*(a*x-I
)^3+8*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
*arctan(a*x)^2-8*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+8*I*Pi*csgn(I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2+4*I*Pi
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+5/24*(I+a*x)^6-53/24*(I+a*x)^4+35/8*arc
tan(a*x)^2*a^8*x^8+105/2*I*arctan(a*x)*(a*x-I)^5*(I+a*x)+175*I*arctan(a*x)*
(a*x-I)^3*(I+a*x)^3-525/4*I*arctan(a*x)*(a*x-I)^4*(I+a*x)^2-6*I*arctan(a*x)
*(a*x-I)*(I+a*x)-230/3*I*arctan(a*x)*(a*x-I)^3*(I+a*x)+89/4*arctan(a*x)^2*a
^4*x^4+8*arctan(a*x)^2*a^2*x^2+11/3*arctan(a*x)*(a*x-I)^2*(I+a*x)-11/3*arct
an(a*x)*(a*x-I)*(I+a*x)^2+4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-16*I*arctan(a*x)*polylog(2,
```

```
-(1+I*a*x)^2/(a^2*x^2+1))-35/4*arctan(a*x)*(a*x-I)*(I+a*x)^6-35/4*I*arctan(
a*x)*(a*x-I)^6+3*I*arctan(a*x)*(a*x-I)^2+115/6*I*arctan(a*x)*(a*x-I)^4-175/
4*arctan(a*x)*(a*x-I)^3*(I+a*x)^4+35/4*arctan(a*x)*(a*x-I)^6*(I+a*x)+640/3*
arctan(a*x)*(a*x-I)^3*(I+a*x)^2-320/3*arctan(a*x)*(a*x-I)^4*(I+a*x)+105/4*a
rctan(a*x)*(a*x-I)^2*(I+a*x)^5-105/4*arctan(a*x)*(a*x-I)^5*(I+a*x)^2+50/3*a
rctan(a*x)^2*a^6*x^6-5/9*I*(I+a*x)-5/4*I*(I+a*x)^5+1/2*I*(I+a*x)^3-16/3*I*a
rctan(a*x)^3-5/4*arctan(a*x)*(a*x-I)^7+64/3*arctan(a*x)*(a*x-I)^5))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2520*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*a
rctan(a*x)^3 - 1/3360*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 +
 105*c^3*x^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/3360*(2940*(a^8*
c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^3 - 4*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*
x^3)*arctan(a*x)^2 + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^4*c^3*x^6
 + 105*a^2*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (35*a^7*c^3*x^9 + 135*a^
5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*x^3 + 315*(a^8*c^3*x^10 + 4*a^6*c^3
*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x
)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int x^2\operatorname{atan}^3(ax)\,dx+\int 3a^2x^4\operatorname{atan}^3(ax)\,dx+\int 3a^4x^6\operatorname{atan}^3(ax)\,dx+\int a^6x^8\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**2*atan(a*x)**3, x) + Integral(3*a**2*x**4*atan(a*x)**3, x
) + Integral(3*a**4*x**6*atan(a*x)**3, x) + Integral(a**6*x**8*atan(a*x)**3
, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.381 $\int x(c+a^2cx^2)^3 \text{ArcTan}(ax)^3 dx$

Optimal. Leaf size=308

$$-\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5+\frac{3c^3(1+a^2x^2)\,\text{ArcTan}(ax)}{35a^2}+\frac{9c^3(1+a^2x^2)^2\,\text{ArcTan}(ax)}{280a^2}+\frac{c^3(1+a^2x^2)^3\,\text{A}}{56a^2}$$

```
[Out] -19/140*c^3*x/a-19/840*a*c^3*x^3-1/280*a^3*c^3*x^5+3/35*c^3*(a^2*x^2+1)*arc
tan(a*x)/a^2+9/280*c^3*(a^2*x^2+1)^2*arctan(a*x)/a^2+1/56*c^3*(a^2*x^2+1)^3
*arctan(a*x)/a^2-6/35*I*c^3*arctan(a*x)^2/a^2-6/35*c^3*x*arctan(a*x)^2/a-3/
35*c^3*x*(a^2*x^2+1)*arctan(a*x)^2/a-9/140*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^
2/a-3/56*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^2/a+1/8*c^3*(a^2*x^2+1)^4*arctan(a
*x)^3/a^2-12/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^2-6/35*I*c^3*polylog(2,1-
2/(1+I*a*x))/a^2
```

Rubi [A]
time = 0.19, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5050, 5000, 4930, 5040, 4964, 2449, 2352, 8, 200}

$$-\frac{1}{280}a^3c^3x^5-\frac{3c^3x(a^2x^2+1)^3\,\text{ArcTan}(ax)^2}{56a}-\frac{9c^3x(a^2x^2+1)^2\,\text{ArcTan}(ax)^2}{140a}-\frac{3c^3x(a^2x^2+1)\,\text{ArcTan}(ax)^2}{35a}+\frac{c^3(a^2x^2+1)^4\,\text{ArcTan}(ax)^3}{8a^2}+\frac{c^3(a^2x^2+1)^3\,\text{ArcTan}(ax)}{56a^2}+\frac{9c^3(a^2x^2+1)^2\,\text{ArcTan}(ax)}{280a^2}+\frac{3c^3(a^2x^2+1)\,\text{ArcTan}(ax)}{35a^2}-\frac{6ic^3\text{ArcTan}(ax)^2}{35a^2}-\frac{12c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{35a^2}-\frac{6ic^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{35a^2}-\frac{6c^3x\text{ArcTan}(ax)^2}{35a}-\frac{19}{840}ac^3x^3-\frac{19c^3x}{140a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (-19*c^3*x)/(140*a) - (19*a*c^3*x^3)/840 - (a^3*c^3*x^5)/280 + (3*c^3*(1 +
a^2*x^2)*ArcTan[a*x])/(35*a^2) + (9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])/(280*a
^2) + (c^3*(1 + a^2*x^2)^3*ArcTan[a*x])/(56*a^2) - (((6*I)/35)*c^3*ArcTan[a
*x]^2)/a^2 - (6*c^3*x*ArcTan[a*x]^2)/(35*a) - (3*c^3*x*(1 + a^2*x^2)*ArcTan
[a*x]^2)/(35*a) - (9*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(140*a) - (3*c^3*
x*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(56*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^
3)/(8*a^2) - (12*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(35*a^2) - (((6*I)/35)
*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx = \frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)^3}{8a^2} - \frac{3\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2\,dx}{8a}$$

$$= \frac{c^3(1+a^2x^2)^3\tan^{-1}(ax)}{56a^2} - \frac{3c^3x(1+a^2x^2)^3\tan^{-1}(ax)^2}{56a} + \frac{c^3(1+a^2x^2)^4\,\mathrm{t}}{8a^2}$$

$$= \frac{9c^3(1+a^2x^2)^2\tan^{-1}(ax)}{280a^2} + \frac{c^3(1+a^2x^2)^3\tan^{-1}(ax)}{56a^2} - \frac{9c^3x(1+a^2x^2)^2\,\mathrm{t}}{140a}$$

$$= -\frac{c^3x}{20a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+a^2}{2}$$

$$= -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+}{}$$

$$= -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+}{}$$

$$= -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+}{}$$

$$= -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+}{}$$

$$= -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 + \frac{3c^3(1+a^2x^2)\tan^{-1}(ax)}{35a^2} + \frac{9c^3(1+}{}$$

Mathematica [A]
time = 1.00, size = 157, normalized size = 0.51

$$\frac{c^3\left(-ax(114+19a^2x^2+3a^4x^4)-9(-16i+35ax+35a^3x^3+21a^5x^5+5a^7x^7)\,\mathrm{ArcTan}(ax)^2+105(1+a^2x^2)^4\,\mathrm{ArcTan}(ax)^3+3\mathrm{ArcTan}(ax)\left(38+57a^2x^2+24a^4x^4+5a^6x^6-96\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)\right)+144i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{840a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4)) - 9*(-16*I + 35*a*x + 35*a^3*x^
3 + 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 + 105*(1 + a^2*x^2)^4*ArcTan[a*x]
^3 + 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E
^((2*I)*ArcTan[a*x])]) + (144*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(840*
a^2)
```

Maple [A]
time = 2.72, size = 349, normalized size = 1.13

method	result

derivativedivides	$\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x^7}{7}+\right.}{}$
default	$\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x^7}{7}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/8*c^3*arctan(a*x)^3*a^8*x^8+1/2*a^6*c^3*x^6*arctan(a*x)^3+3/4*a^4*
c^3*x^4*arctan(a*x)^3+1/2*a^2*c^3*x^2*arctan(a*x)^3+1/8*c^3*arctan(a*x)^3-3
/8*c^3*(1/7*arctan(a*x)^2*a^7*x^7+3/5*arctan(a*x)^2*a^5*x^5+arctan(a*x)^2*a
^3*x^3+arctan(a*x)^2*a*x-1/21*arctan(a*x)*a^6*x^6-8/35*arctan(a*x)*a^4*x^4-
19/35*arctan(a*x)*a^2*x^2-16/35*arctan(a*x)*ln(a^2*x^2+1)+1/105*a^5*x^5+19/
315*a^3*x^3+38/105*a*x-38/105*arctan(a*x)+4/35*I*ln(a*x-I)^2-8/35*I*ln(a*x-
I)*ln(a^2*x^2+1)+8/35*I*dilog(-1/2*I*(I+a*x))-4/35*I*ln(I+a*x)^2-8/35*I*dil
og(1/2*I*(a*x-I))+8/35*I*ln(I+a*x)*ln(a^2*x^2+1)-8/35*I*ln(I+a*x)*ln(1/2*I*
(a*x-I))+8/35*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4480*(280*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 +
c^3)*arctan(a*x)^3 + 140*(71680*a^9*c^3*integrate(1/4480*x^9*arctan(a*x)^3/
(a^3*x^2 + a), x) - 13440*a^8*c^3*integrate(1/4480*x^8*arctan(a*x)^2/(a^3*x
^2 + a), x) - 3360*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)^2/(a^3*x^2
 + a), x) - 1920*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)/(a^3*x^2 + a
), x) + 286720*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)^3/(a^3*x^2 + a), x)
 + 3840*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)/(a^3*x^2 + a), x) - 53760*
a^6*c^3*integrate(1/4480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 13440*a^6*c^
3*integrate(1/4480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 8064*a^6*c^3*
integrate(1/4480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 430080*a^5*c^3*in
tegrate(1/4480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 16128*a^5*c^3*integrat
e(1/4480*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 80640*a^4*c^3*integrate(1/4480
*x^4*arctan(a*x)^2/(a^3*x^2 + a), x) - 20160*a^4*c^3*integrate(1/4480*x^4*l
og(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 13440*a^4*c^3*integrate(1/4480*x^4*lo
g(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 286720*a^3*c^3*integrate(1/4480*x^3*arct
```

```
an(a*x)^3/(a^3*x^2 + a), x) + 26880*a^3*c^3*integrate(1/4480*x^3*arctan(a*x
)/(a^3*x^2 + a), x) - 53760*a^2*c^3*integrate(1/4480*x^2*arctan(a*x)^2/(a^3
*x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^2*x^2 + 1)^2/(a^3*
x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^2*x^2 + 1)/(a^3*x^2
 + a), x) + 71680*a*c^3*integrate(1/4480*x*arctan(a*x)^3/(a^3*x^2 + a), x)
+ 26880*a*c^3*integrate(1/4480*x*arctan(a*x)/(a^3*x^2 + a), x) - c^3*arctan
(a*x)^3/a^2 - 3360*c^3*integrate(1/4480*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x
))*a^2 - 12*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x)*
arctan(a*x)^2 + 3*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c
^3*x)*log(a^2*x^2 + 1)^2)/a^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*x)^
3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int x\operatorname{atan}^3(ax)\,dx+\int 3a^2x^3\operatorname{atan}^3(ax)\,dx+\int 3a^4x^5\operatorname{atan}^3(ax)\,dx+\int a^6x^7\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x*atan(a*x)**3, x) + Integral(3*a**2*x**3*atan(a*x)**3, x) +
 Integral(3*a**4*x**5*atan(a*x)**3, x) + Integral(a**6*x**7*atan(a*x)**3, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.382 $\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=388

$$-\frac{13c^3(1+a^2x^2)}{210a}-\frac{c^3(1+a^2x^2)^2}{140a}+\frac{14}{15}c^3x\operatorname{ArcTan}(ax)+\frac{13}{105}c^3x(1+a^2x^2)\operatorname{ArcTan}(ax)+\frac{1}{35}c^3x(1+a^2x^2)^2\operatorname{A}$$

```
[Out] -13/210*c^3*(a^2*x^2+1)/a-1/140*c^3*(a^2*x^2+1)^2/a+14/15*c^3*x*arctan(a*x)
+13/105*c^3*x*(a^2*x^2+1)*arctan(a*x)+1/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)-
12/35*c^3*(a^2*x^2+1)*arctan(a*x)^2/a-9/70*c^3*(a^2*x^2+1)^2*arctan(a*x)^2/
a-1/14*c^3*(a^2*x^2+1)^3*arctan(a*x)^2/a+16/35*I*c^3*arctan(a*x)^3/a+16/35*
c^3*x*arctan(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)^3+6/35*c^3*x*(a^2*x^
2+1)^2*arctan(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^3+48/35*c^3*arctan
(a*x)^2*ln(2/(1+I*a*x))/a-7/15*c^3*ln(a^2*x^2+1)/a+48/35*I*c^3*arctan(a*x)*
polylog(2,1-2/(1+I*a*x))/a+24/35*c^3*polylog(3,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.26, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5000, 4930, 5040, 4964, 5004, 5114, 6745, 266, 4998}

$$\frac{1}{7}c^3x(a^2x^2+1)^3\operatorname{ArcTan}(ax)^3+\frac{6}{35}c^3x(a^2x^2+1)^2\operatorname{ArcTan}(ax)^3+\frac{8}{35}c^3x(a^2x^2+1)\operatorname{ArcTan}(ax)^3-\frac{c^3(a^2x^2+1)^3\operatorname{ArcTan}(ax)^2}{14a}-\frac{9c^3(a^2x^2+1)^2\operatorname{ArcTan}(ax)^2}{70a}-\frac{12c^3(a^2x^2+1)\operatorname{ArcTan}(ax)^2}{35a}+\frac{1}{35}c^3x(a^2x^2+1)^2\operatorname{ArcTan}(ax)+\frac{13}{105}c^3x(a^2x^2+1)\operatorname{ArcTan}(ax)-\frac{c^3(a^2x^2+1)^2}{140a}-\frac{13c^3(a^2x^2+1)}{210a}-\frac{7c^3\log(a^2x^2+1)}{15a}+\frac{48ic^3\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(1-\frac{2}{iax+1}\right)}{35a}+\frac{16}{35}c^3x\operatorname{ArcTan}(ax)^3+\frac{16ic^3\operatorname{ArcTan}(ax)^3}{35a}+\frac{14}{15}c^3x\operatorname{ArcTan}(ax)+\frac{48c^3\operatorname{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a}+\frac{24c^3\operatorname{Li}_3\left(1-\frac{2}{iax+1}\right)}{35a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (-13*c^3*(1 + a^2*x^2))/(210*a) - (c^3*(1 + a^2*x^2)^2)/(140*a) + (14*c^3*x
*ArcTan[a*x])/15 + (13*c^3*x*(1 + a^2*x^2)*ArcTan[a*x])/105 + (c^3*x*(1 + a
^2*x^2)^2*ArcTan[a*x])/35 - (12*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2)/(35*a) - (
9*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(70*a) - (c^3*(1 + a^2*x^2)^3*ArcTan[a
*x]^2)/(14*a) + (((16*I)/35)*c^3*ArcTan[a*x]^3)/a + (16*c^3*x*ArcTan[a*x]^3
)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^3)/35 + (6*c^3*x*(1 + a^2*x^2)^2*
ArcTan[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^3)/7 + (48*c^3*ArcTa
n[a*x]^2*Log[2/(1 + I*a*x)])/(35*a) - (7*c^3*Log[1 + a^2*x^2])/(15*a) + (((
48*I)/35)*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + (24*c^3*PolyLo
g[3, 1 - 2/(1 + I*a*x)])/(35*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3\,dx = -\frac{c^3\left(1+a^2x^2\right)^3\tan^{-1}(ax)^2}{14a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^3+\frac{1}{7}c\int\left(c+a^2\right.$$

$$= -\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{1}{35}c^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)-\frac{9c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax}{70a}$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{13}{105}c^3x\left(1+a^2x^2\right)\tan^{-1}(ax)+\frac{1}{35}c^3x$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)$$

$$= -\frac{13c^3\left(1+a^2x^2\right)}{210a}-\frac{c^3\left(1+a^2x^2\right)^2}{140a}+\frac{14}{15}c^3x\tan^{-1}(ax)+\frac{13}{105}c^3x\left(1+a^2x^2\right)$$

Mathematica [A]
time = 0.84, size = 243, normalized size = 0.63

$$\frac{c^3\left(-29-32a^2x^2-3a^4x^4+456ax\mathrm{ArcTan}(ax)+76a^3x^3\mathrm{ArcTan}(ax)+12a^5x^5\mathrm{ArcTan}(ax)-228\mathrm{ArcTan}(ax)^2-342a^2x^2\mathrm{ArcTan}(ax)^2-144a^4x^4\mathrm{ArcTan}(ax)^2-30a^6x^6\mathrm{ArcTan}(ax)^2-192i\mathrm{ArcTan}(ax)^3+420ax\mathrm{ArcTan}(ax)^3+420a^3x^3\mathrm{ArcTan}(ax)^3+252a^5x^5\mathrm{ArcTan}(ax)^3+60a^7x^7\mathrm{ArcTan}(ax)^3+576\mathrm{ArcTan}(ax)^2\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)-196\log\left(1+a^2x^2\right)-576i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)+288\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{420a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

```
[Out] (c^3*(-29 - 32*a^2*x^2 - 3*a^4*x^4 + 456*a*x*ArcTan[a*x] + 76*a^3*x^3*ArcTa
n[a*x] + 12*a^5*x^5*ArcTan[a*x] - 228*ArcTan[a*x]^2 - 342*a^2*x^2*ArcTan[a*
x]^2 - 144*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 - (192*I)*ArcTa
n[a*x]^3 + 420*a*x*ArcTan[a*x]^3 + 420*a^3*x^3*ArcTan[a*x]^3 + 252*a^5*x^5*
ArcTan[a*x]^3 + 60*a^7*x^7*ArcTan[a*x]^3 + 576*ArcTan[a*x]^2*Log[1 + E^((2*
```

```
I)*ArcTan[a*x])] - 196*Log[1 + a^2*x^2] - (576*I)*ArcTan[a*x]*PolyLog[2, -E
^((2*I)*ArcTan[a*x])] + 288*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(420*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 50.24, size = 1267, normalized size = 3.27

method	result	size
derivativedivides	Expression too large to display	1267
default	Expression too large to display	1267

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3*arctan(a*x)^3*a^5*x^5+c^3*arctan
(a*x)^3*a^3*x^3+c^3*arctan(a*x)^3*a*x-3/35*c^3*(4*I*arctan(a*x)^2*Pi*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+19/3*arctan(a*x)^2+8*arc
tan(a*x)^2*ln(a^2*x^2+1)-16*arctan(a*x)^2*ln(2)-98/9*ln((1+I*a*x)^2/(a^2*x^
2+1)+1)-16*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+7/18*(I+a*x)^2+4*I
*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1))-5/3*arctan(a*x)*(a*x-I)*(I+a*x)^4+10/3*arctan(a*x)*(a*x-I)^2*(
I+a*x)^3-8*arctan(a*x)*(a*x-I)+11/9*arctan(a*x)*(a*x-I)^3-8*polylog(3,-(1+I
*a*x)^2/(a^2*x^2+1))-5/3*I*arctan(a*x)*(a*x-I)^4-3*I*arctan(a*x)*(a*x-I)^2+
16*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/12*(I+a*x)^4-4*I*arc
tan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+8*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-4*I*arctan(a*x)^2*
Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-8*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-4*I*arctan(a*x)^2*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-4*I*arc
tan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+20/3*I*arctan(a*x)*(a
*x-I)^3*(I+a*x)+4*arctan(a*x)^2*a^4*x^4+19/2*arctan(a*x)^2*a^2*x^2-10*I*arc
tan(a*x)*(a*x-I)^2*(I+a*x)^2+6*I*arctan(a*x)*(a*x-I)*(I+a*x)+20/3*I*arctan(
a*x)*(a*x-I)*(I+a*x)^3+4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))
^3+4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^3-11/3*arctan(a*x)*(a*x-I)^2*(I+a*x)+11/3*arctan(a*x)*(a*x-I)*(I
+a*x)^2-10/3*arctan(a*x)*(a*x-I)^3*(I+a*x)^2+5/3*arctan(a*x)*(a*x-I)^4*(I+a
*x)+5/6*arctan(a*x)^2*a^6*x^6+16/3*I*arctan(a*x)^3-1/3*I*(I+a*x)^3-13/9*I*(
I+a*x)-1/3*arctan(a*x)*(a*x-I)^5))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 980*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)^3/(a^2*x^2 + 1), x) + 105*a^8*
c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
 60*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1)
, x) - 60*a^7*c^3*integrate(1/1120*x^7*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15
*a^7*c^3*integrate(1/1120*x^7*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3920*a
^6*c^3*integrate(1/1120*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 420*a^6*c^3*i
ntegrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 252*
a^6*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 - 252*a^5*c^3*integrate(1/1120*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 63*a^
5*c^3*integrate(1/1120*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 5880*a^4*
c^3*integrate(1/1120*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 630*a^4*c^3*inte
grate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4
*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) -
420*a^3*c^3*integrate(1/1120*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 105*a^3*
c^3*integrate(1/1120*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^3*ar
ctan(a*x)^4/a + 3920*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)^3/(a^2*x^2 +
1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a
^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^2 + 1), x) - 420*a*c^3*integrate(1/1120*x*arctan(a*x)^2/(a^2*x^
2 + 1), x) + 105*a*c^3*integrate(1/1120*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1),
 x) + 1/280*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*ar
ctan(a*x)^3 + 105*c^3*integrate(1/1120*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*
x^2 + 1), x) - 3/1120*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35
*c^3*x)*arctan(a*x)*log(a^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int 3a^{2}x^{2}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int 3a^{4}x^{4}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int a^{6}x^{6}\operatorname{atan}^{3}{\left(ax\right)}\,dx+\int \operatorname{atan}^{3}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**3, x) + Integral(3*a**4*x**4*atan(a*x
)**3, x) + Integral(a**6*x**6*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.383 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^3}{x} \, dx$

Optimal. Leaf size=447

$$-\frac{13}{30}ac^3x-\frac{1}{60}a^3c^3x^3+\frac{13}{30}c^3\text{ArcTan}(ax)+\frac{29}{60}a^2c^3x^2\text{ArcTan}(ax)+\frac{1}{20}a^4c^3x^4\text{ArcTan}(ax)-\frac{34}{15}ic^3\text{ArcTan}(ax)^2-$$

```
[Out] -13/30*a*c^3*x-1/60*a^3*c^3*x^3+13/30*c^3*arctan(a*x)+29/60*a^2*c^3*x^2*arc
tan(a*x)+1/20*a^4*c^3*x^4*arctan(a*x)-34/15*I*c^3*polylog(2,1-2/(1+I*a*x))-
11/4*a*c^3*x*arctan(a*x)^2-7/12*a^3*c^3*x^3*arctan(a*x)^2-1/10*a^5*c^3*x^5*
arctan(a*x)^2+11/12*c^3*arctan(a*x)^3+3/2*a^2*c^3*x^2*arctan(a*x)^3+3/4*a^4
*c^3*x^4*arctan(a*x)^3+1/6*a^6*c^3*x^6*arctan(a*x)^3-2*c^3*arctan(a*x)^3*ar
ctanh(-1+2/(1+I*a*x))-68/15*c^3*arctan(a*x)*ln(2/(1+I*a*x))-3/4*I*c^3*polyl
og(4,-1+2/(1+I*a*x))-34/15*I*c^3*arctan(a*x)^2+3/4*I*c^3*polylog(4,1-2/(1+I
*a*x))-3/2*c^3*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/2*c^3*arctan(a*x)*pol
ylog(3,-1+2/(1+I*a*x))-3/2*I*c^3*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+3/2
*I*c^3*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))
```

Rubi [A]
time = 1.18, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5068, 4942, 5108, 5004, 5114, 5118, 6745, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 327, 209, 308}

$$\frac{1}{6}a^6c^3x^6\text{ArcTan}(ax)^3-\frac{1}{10}a^5c^3x^5\text{ArcTan}(ax)^2+\frac{3}{4}a^4c^3x^4\text{ArcTan}(ax)^3+\frac{1}{20}a^4c^3x^4\text{ArcTan}(ax)-\frac{7}{12}a^3c^3x^3\text{ArcTan}(ax)^2-\frac{1}{60}a^3c^3x^3+\frac{3}{2}a^2c^3x^2\text{ArcTan}(ax)^3+\frac{29}{60}a^2c^3x^2\text{ArcTan}(ax)-\frac{3}{2}ic^3\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}ic^3\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{3}{2}c^3\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{3}{2}c^3\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)-\frac{11}{4}ac^3x\text{ArcTan}(ax)^2+\frac{11}{12}c^3\text{ArcTan}(ax)^3-\frac{34}{15}ic^3\text{ArcTan}(ax)^2+\frac{13}{30}c^3\text{ArcTan}(ax)-\frac{68}{15}c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+2c^3\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{34}{15}ic^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{3}{4}ic^3\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{3}{4}ic^3\text{Li}_4\left(\frac{2}{iax+1}-1\right)-\frac{13}{30}ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

```
[Out] (-13*a*c^3*x)/30 - (a^3*c^3*x^3)/60 + (13*c^3*ArcTan[a*x])/30 + (29*a^2*c^3
*x^2*ArcTan[a*x])/60 + (a^4*c^3*x^4*ArcTan[a*x])/20 - ((34*I)/15)*c^3*ArcTa
n[a*x]^2 - (11*a*c^3*x*ArcTan[a*x]^2)/4 - (7*a^3*c^3*x^3*ArcTan[a*x]^2)/12
- (a^5*c^3*x^5*ArcTan[a*x]^2)/10 + (11*c^3*ArcTan[a*x]^3)/12 + (3*a^2*c^3*x
^2*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^3)/4 + (a^6*c^3*x^6*ArcTan
[a*x]^3)/6 + 2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - (68*c^3*ArcTa
n[a*x]*Log[2/(1 + I*a*x)])/15 - ((34*I)/15)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x
)] - ((3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*
c^3*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^3*ArcTan[a*x]*PolyL
og[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a
*x)])/2 + ((3*I)/4)*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^3*PolyL
og[4, -1 + 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^3}{x} + 3a^2c^3x\tan^{-1}(ax)^3 + 3a^4c^3x^3\tan^{-1}(ax)^3 + a^6c^3x^5 \tan$$

$$= c^3\int \frac{\tan^{-1}(ax)^3}{x}\,dx + (3a^2c^3)\int x\tan^{-1}(ax)^3\,dx + (3a^4c^3)\int x^3\tan^{-1}(ax$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^3 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^3 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^3 + 2c^3\tan$$

$$= \frac{3}{2}a^2c^3x^2\tan^{-1}(ax)^3 + \frac{3}{4}a^4c^3x^4\tan^{-1}(ax)^3 + \frac{1}{6}a^6c^3x^6\tan^{-1}(ax)^3 + 2c^3\tan$$

$$= -\frac{9}{2}ac^3x\tan^{-1}(ax)^2 - \frac{3}{4}a^3c^3x^3\tan^{-1}(ax)^2 - \frac{1}{10}a^5c^3x^5\tan^{-1}(ax)^2 + \frac{3}{2}c^3\,\mathrm{ta}$$

$$= -\frac{9}{2}ic^3\tan^{-1}(ax)^2 - \frac{9}{4}ac^3x\tan^{-1}(ax)^2 - \frac{7}{12}a^3c^3x^3\tan^{-1}(ax)^2 - \frac{1}{10}a^5c^3x^5$$

$$= \frac{3}{4}a^2c^3x^2\tan^{-1}(ax) + \frac{1}{20}a^4c^3x^4\tan^{-1}(ax) - \frac{3}{2}ic^3\tan^{-1}(ax)^2 - \frac{11}{4}ac^3x\tan$$

$$= -\frac{3}{4}ac^3x + \frac{29}{60}a^2c^3x^2\tan^{-1}(ax) + \frac{1}{20}a^4c^3x^4\tan^{-1}(ax) - \frac{34}{15}ic^3\tan^{-1}(ax)^2$$

$$= -\frac{13}{30}ac^3x - \frac{1}{60}a^3c^3x^3 + \frac{3}{4}c^3\tan^{-1}(ax) + \frac{29}{60}a^2c^3x^2\tan^{-1}(ax) + \frac{1}{20}a^4c^3x^4$$

$$= -\frac{13}{30}ac^3x - \frac{1}{60}a^3c^3x^3 + \frac{13}{30}c^3\tan^{-1}(ax) + \frac{29}{60}a^2c^3x^2\tan^{-1}(ax) + \frac{1}{20}a^4c^3x^4$$

$$= -\frac{13}{30}ac^3x - \frac{1}{60}a^3c^3x^3 + \frac{13}{30}c^3\tan^{-1}(ax) + \frac{29}{60}a^2c^3x^2\tan^{-1}(ax) + \frac{1}{20}a^4c^3x^4$$

Mathematica [A]
time = 0.72, size = 350, normalized size = 0.78

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c^3*((-15*I)*Pi^4 - 416*a*x - 16*a^3*x^3 + 416*ArcTan[a*x] + 464*a^2*x^2*A
rcTan[a*x] + 48*a^4*x^4*ArcTan[a*x] + (2176*I)*ArcTan[a*x]^2 - 2640*a*x*Arc
Tan[a*x]^2 - 560*a^3*x^3*ArcTan[a*x]^2 - 96*a^5*x^5*ArcTan[a*x]^2 + 880*Arc
Tan[a*x]^3 + 1440*a^2*x^2*ArcTan[a*x]^3 + 720*a^4*x^4*ArcTan[a*x]^3 + 160*a
^6*x^6*ArcTan[a*x]^3 + (480*I)*ArcTan[a*x]^4 + 960*ArcTan[a*x]^3*Log[1 - E^
((-2*I)*ArcTan[a*x])] - 4352*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 9
60*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (1440*I)*ArcTan[a*x]^2*Po
```

```
lyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*(68 + 45*ArcTan[a*x]^2)*PolyLog[2
, -E^((2*I)*ArcTan[a*x])] + 1440*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*
x])] - 1440*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (720*I)*PolyLo
g[4, E^((-2*I)*ArcTan[a*x])] - (720*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))
/960
```

Maple [A]
time = 51.00, size = 664, normalized size = 1.49

method	result
derivativedivides	$c^3\left(-136\arctan(ax)^2-3i\arctan(ax)a^2x^2+29i\arctan(ax)^2ax+55\arctan(ax)^3ax-29\arctan(ax)^2a^2x^2+6i\arctan(ax)^2a^3\right.$
default	$c^3\left(-136\arctan(ax)^2-3i\arctan(ax)a^2x^2+29i\arctan(ax)^2ax+55\arctan(ax)^3ax-29\arctan(ax)^2a^2x^2+6i\arctan(ax)^2a^3\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*c^3*(-136*arctan(a*x)^2-3*I*arctan(a*x)*a^2*x^2+29*I*arctan(a*x)^2*a*x
+55*arctan(a*x)^3*a*x-29*arctan(a*x)^2*a^2*x^2+6*I*arctan(a*x)^2*a^3*x^3-26
*I*arctan(a*x)+35*arctan(a*x)^3*a^3*x^3-6*arctan(a*x)^2*a^4*x^4-55*I*arctan
(a*x)^3+10*arctan(a*x)^3*a^5*x^5-25-10*I*arctan(a*x)^3*a^4*x^4+I*a*x+26*arc
tan(a*x)*a*x-a^2*x^2-35*I*arctan(a*x)^3*a^2*x^2+3*arctan(a*x)*a^3*x^3)*(I+a
*x)+34/15*I*c^3*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-68/15*c^3*arctan(a*x)*l
n((1+I*a*x)^2/(a^2*x^2+1)+1)+6*I*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+68/15*I*c^3*arctan(a*x)^2
-3/2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-3*I*c^3*arctan(a*x
)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+c^3*arctan(a*x)^3*ln(1-(1+I*a*x
)/(a^2*x^2+1)^(1/2))-3/4*I*c^3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^3*ar
ctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^3*polylog(4,(1+I*a*x
)/(a^2*x^2+1)^(1/2))+c^3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*
I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^3*arctan(a*x
)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c^3*arctan(a*x)^2*polylog(2
,-(1+I*a*x)^2/(a^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/96*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)^3 - 1/128
*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)*log(a^2*x^2 +
```

```
1)^2 + integrate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 +
4*a^2*c^3*x^2 + c^3)*arctan(a*x)^3 - 4*(2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 18
*a^3*c^3*x^3)*arctan(a*x)^2 + 4*(2*a^8*c^3*x^8 + 9*a^6*c^3*x^6 + 18*a^4*c^3
*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 + 18*a^
3*c^3*x^3 + 12*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
 + c^3)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="fricas")
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{\operatorname{atan}^3(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^3(ax)\,dx + \int 3a^4x^3\operatorname{atan}^3(ax)\,dx + \int a^6x^5\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x,x)
[Out] c**3*(Integral(atan(a*x)**3/x, x) + Integral(3*a**2*x*atan(a*x)**3, x) + In
tegral(3*a**4*x**3*atan(a*x)**3, x) + Integral(a**6*x**5*atan(a*x)**3, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x,x)
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x, x)
```

3.384 $\int \frac{(c+a^2cx^2)^3 \text{ArcTan}(ax)^3}{x^2} \, dx$

Optimal. Leaf size=354

$$-\frac{1}{20}a^3c^3x^2+\frac{21}{10}a^2c^3x\text{ArcTan}(ax)+\frac{1}{10}a^4c^3x^3\text{ArcTan}(ax)-\frac{21}{20}ac^3\text{ArcTan}(ax)^2-\frac{6}{5}a^3c^3x^2\text{ArcTan}(ax)^2-\frac{3}{20}a^5c$$

```
[Out] -1/20*a^3*c^3*x^2+21/10*a^2*c^3*x*arctan(a*x)+1/10*a^4*c^3*x^3*arctan(a*x)-
21/20*a*c^3*arctan(a*x)^2-6/5*a^3*c^3*x^2*arctan(a*x)^2-3/20*a^5*c^3*x^4*ar
ctan(a*x)^2+6/5*I*a*c^3*arctan(a*x)^3-c^3*arctan(a*x)^3/x+3*a^2*c^3*x*arcta
n(a*x)^3+a^4*c^3*x^3*arctan(a*x)^3+1/5*a^6*c^3*x^5*arctan(a*x)^3+33/5*a*c^3
*arctan(a*x)^2*ln(2/(1+I*a*x))-a*c^3*ln(a^2*x^2+1)+3*a*c^3*arctan(a*x)^2*ln
(2-2/(1-I*a*x))-3*I*a*c^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+33/5*I*a*c^
3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3/2*a*c^3*polylog(3,-1+2/(1-I*a*x))+
33/10*a*c^3*polylog(3,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.91, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5068, 4930, 5040, 4964, 5004, 5114, 6745, 4946, 5044, 4988, 5112, 5036, 266, 272, 45}

$$\frac{1}{5}a^6c^3x^5\text{ArcTan}(ax)^3-\frac{3}{20}a^5c^3x^4\text{ArcTan}(ax)^2+a^4c^3x^3\text{ArcTan}(ax)^3+\frac{1}{10}a^4c^3x^3\text{ArcTan}(ax)-\frac{6}{5}a^3c^3x^2\text{ArcTan}(ax)^2-\frac{1}{20}a^3c^3x^2+3a^2c^3x\text{ArcTan}(ax)^3+\frac{21}{10}a^2c^3x\text{ArcTan}(ax)-ac^3\log\left(a^2x^2+1\right)-3iac^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{33}{5}iac^3\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{6}{5}iac^3\text{ArcTan}(ax)^3-\frac{21}{20}ac^3\text{ArcTan}(ax)^2-\frac{c^3\text{ArcTan}(ax)^3}{x}+\frac{33}{5}ac^3\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^3\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)+\frac{33}{10}ac^3\text{Li}_3\left(1-\frac{2}{iax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -1/20*(a^3*c^3*x^2) + (21*a^2*c^3*x*ArcTan[a*x])/10 + (a^4*c^3*x^3*ArcTan[a
*x])/10 - (21*a*c^3*ArcTan[a*x]^2)/20 - (6*a^3*c^3*x^2*ArcTan[a*x]^2)/5 - (
3*a^5*c^3*x^4*ArcTan[a*x]^2)/20 + ((6*I)/5)*a*c^3*ArcTan[a*x]^3 - (c^3*ArcT
an[a*x]^3)/x + 3*a^2*c^3*x*ArcTan[a*x]^3 + a^4*c^3*x^3*ArcTan[a*x]^3 + (a^6
*c^3*x^5*ArcTan[a*x]^3)/5 + (33*a*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/5 -
 a*c^3*Log[1 + a^2*x^2] + 3*a*c^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (3
*I)*a*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((33*I)/5)*a*c^3*Arc
Tan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^3*PolyLog[3, -1 + 2/(1 - I*
a*x)])/2 + (33*a*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x^2}\,dx = \int \left(3a^2c^3 \tan^{-1}(ax)^3 + \frac{c^3 \tan^{-1}(ax)^3}{x^2} + 3a^4c^3x^2 \tan^{-1}(ax)^3 + a^6c^3x^4 \tan^{-}\right.$$

$$= c^3 \int \frac{\tan^{-1}(ax)^3}{x^2}\,dx + \left(3a^2c^3\right)\int \tan^{-1}(ax)^3\,dx + \left(3a^4c^3\right)\int x^2 \tan^{-1}(ax)^{3}$$

$$= -\frac{c^3 \tan^{-1}(ax)^3}{x} + 3a^2c^3x \tan^{-1}(ax)^3 + a^4c^3x^3 \tan^{-1}(ax)^3 + \frac{1}{5}a^6c^3x^5 \tan^{-1}$$

$$= 2iac^3 \tan^{-1}(ax)^3 - \frac{c^3 \tan^{-1}(ax)^3}{x} + 3a^2c^3x \tan^{-1}(ax)^3 + a^4c^3x^3 \tan^{-1}(ax)$$

$$= -\frac{3}{2}a^3c^3x^2 \tan^{-1}(ax)^2 - \frac{3}{20}a^5c^3x^4 \tan^{-1}(ax)^2 + iac^3 \tan^{-1}(ax)^3 - \frac{c^3 \tan^{-1}}{x}$$

$$= -\frac{6}{5}a^3c^3x^2 \tan^{-1}(ax)^2 - \frac{3}{20}a^5c^3x^4 \tan^{-1}(ax)^2 + \frac{6}{5}iac^3 \tan^{-1}(ax)^3 - \frac{c^3 \tan}{}$$

$$= 3a^2c^3x \tan^{-1}(ax) + \frac{1}{10}a^4c^3x^3 \tan^{-1}(ax) - \frac{3}{2}ac^3 \tan^{-1}(ax)^2 - \frac{6}{5}a^3c^3x^2 \tan$$

$$= \frac{21}{10}a^2c^3x \tan^{-1}(ax) + \frac{1}{10}a^4c^3x^3 \tan^{-1}(ax) - \frac{21}{20}ac^3 \tan^{-1}(ax)^2 - \frac{6}{5}a^3c^3x^2\,t$$

$$= \frac{21}{10}a^2c^3x \tan^{-1}(ax) + \frac{1}{10}a^4c^3x^3 \tan^{-1}(ax) - \frac{21}{20}ac^3 \tan^{-1}(ax)^2 - \frac{6}{5}a^3c^3x^2\,t$$

$$= -\frac{1}{20}a^3c^3x^2 + \frac{21}{10}a^2c^3x \tan^{-1}(ax) + \frac{1}{10}a^4c^3x^3 \tan^{-1}(ax) - \frac{21}{20}ac^3 \tan^{-1}(ax$$

Mathematica [A]
time = 0.53, size = 298, normalized size = 0.84

$$\frac{c^3\left(-2ax - 5ia\pi^3x - 2a^3x^3 + 84a^2x^2\text{ArcTan}(ax) + 4a^4x^4\text{ArcTan}(ax) - 42ax\text{ArcTan}(ax)^2 - 48a^3x^3\text{ArcTan}(ax)^2 - 6a^5x^5\text{ArcTan}(ax)^2 - 40\text{ArcTan}(ax)^3 - 48iax\text{ArcTan}(ax)^3 + 120a^2x^2\text{ArcTan}(ax)^3 + 40a^4x^4\text{ArcTan}(ax)^3 + 8a^6x^6\text{ArcTan}(ax)^3 + 120ax\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 264ax\text{ArcTan}(ax)^2\log\left(1+e^{2i\text{ArcTan}(ax)}\right) - 40ax\log\left(1+a^2x^2\right) + 120iax\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) - 264iax\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right) + 60ax\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) + 132ax\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)\right)}{40x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (c^3*(-2*a*x - (5*I)*a*Pi^3*x - 2*a^3*x^3 + 84*a^2*x^2*ArcTan[a*x] + 4*a^4*
x^4*ArcTan[a*x] - 42*a*x*ArcTan[a*x]^2 - 48*a^3*x^3*ArcTan[a*x]^2 - 6*a^5*x
^5*ArcTan[a*x]^2 - 40*ArcTan[a*x]^3 - (48*I)*a*x*ArcTan[a*x]^3 + 120*a^2*x^
2*ArcTan[a*x]^3 + 40*a^4*x^4*ArcTan[a*x]^3 + 8*a^6*x^6*ArcTan[a*x]^3 + 120*
a*x*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 264*a*x*ArcTan[a*x]^2*L
og[1 + E^((2*I)*ArcTan[a*x])] - 40*a*x*Log[1 + a^2*x^2] + (120*I)*a*x*ArcTa
n[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (264*I)*a*x*ArcTan[a*x]*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 1
32*a*x*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(40*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 248.31, size = 9696, normalized size = 27.39

method	result	size
derivativedivides	Expression too large to display	9696
default	Expression too large to display	9696

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/320*(8*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)
^3 - 6*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)*l
og(a^2*x^2 + 1)^2 + 5*(8960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)^3/(a^2*
x^4 + x^2), x) + 960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)*log(a^2*x^2 +
1)^2/(a^2*x^4 + x^2), x) + 768*a^8*c^3*integrate(1/160*x^8*arctan(a*x)*log(
a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^7*c^3*integrate(1/160*x^7*arctan(a
*x)^2/(a^2*x^4 + x^2), x) + 192*a^7*c^3*integrate(1/160*x^7*log(a^2*x^2 + 1
)^2/(a^2*x^4 + x^2), x) + 35840*a^6*c^3*integrate(1/160*x^6*arctan(a*x)^3/(
a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*arctan(a*x)*log(a^2*x
^2 + 1)^2/(a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*arctan(a*x
)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 3840*a^5*c^3*integrate(1/160*x^5*a
rctan(a*x)^2/(a^2*x^4 + x^2), x) + 960*a^5*c^3*integrate(1/160*x^5*log(a^2*
x^2 + 1)^2/(a^2*x^4 + x^2), x) + 56*a*c^3*arctan(a*x)^4 + 53760*a^4*c^3*int
egrate(1/160*x^4*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 5760*a^4*c^3*integrate
(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 11520*a^4*c
^3*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 1
1520*a^3*c^3*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 3*a*c^
3*log(a^2*x^2 + 1)^3 + 3840*a^2*c^3*integrate(1/160*x^2*arctan(a*x)*log(a^2
*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 3840*a^2*c^3*integrate(1/160*x^2*arctan(a
*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 3840*a*c^3*integrate(1/160*x*arc
tan(a*x)^2/(a^2*x^4 + x^2), x) - 960*a*c^3*integrate(1/160*x*log(a^2*x^2 +
1)^2/(a^2*x^4 + x^2), x) + 8960*c^3*integrate(1/160*arctan(a*x)^3/(a^2*x^4
+ x^2), x) + 960*c^3*integrate(1/160*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^
4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2\operatorname{atan}^3(ax)\,dx+\int\frac{\operatorname{atan}^3(ax)}{x^2}\,dx+\int 3a^4x^2\operatorname{atan}^3(ax)\,dx+\int a^6x^4\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) + I
ntegral(3*a**4*x**2*atan(a*x)**3, x) + Integral(a**6*x**4*atan(a*x)**3, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2, x)
```

3.385 $\int \frac{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=503

$$-\frac{1}{4}a^3c^3x+\frac{1}{4}a^2c^3\text{ArcTan}(ax)+\frac{1}{4}a^4c^3x^2\text{ArcTan}(ax)-5ia^2c^3\text{ArcTan}(ax)^2-\frac{3ac^3\text{ArcTan}(ax)^2}{2x}-\frac{15}{4}a^3c^3x\text{ArcTa}$$

```
[Out] -1/4*a^3*c^3*x+1/4*a^2*c^3*arctan(a*x)+1/4*a^4*c^3*x^2*arctan(a*x)+9/2*I*a^
2*c^3*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))-3/2*a*c^3*arctan(a*x)^2/x-15/
4*a^3*c^3*x*arctan(a*x)^2-1/4*a^5*c^3*x^3*arctan(a*x)^2+3/4*a^2*c^3*arctan(
a*x)^3-1/2*c^3*arctan(a*x)^3/x^2+3/2*a^4*c^3*x^2*arctan(a*x)^3+1/4*a^6*c^3*
x^4*arctan(a*x)^3-6*a^2*c^3*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-7*a^2*c^3
*arctan(a*x)*ln(2/(1+I*a*x))+3*a^2*c^3*arctan(a*x)*ln(2-2/(1-I*a*x))-9/2*I*
a^2*c^3*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3/2*I*a^2*c^3*polylog(2,-1+2
/(1-I*a*x))-5*I*a^2*c^3*arctan(a*x)^2-7/2*I*a^2*c^3*polylog(2,1-2/(1+I*a*x)
)-9/2*a^2*c^3*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+9/2*a^2*c^3*arctan(a*x)*
polylog(3,-1+2/(1+I*a*x))+9/4*I*a^2*c^3*polylog(4,1-2/(1+I*a*x))-9/4*I*a^2*
c^3*polylog(4,-1+2/(1+I*a*x))
```

Rubi [A]
time = 0.86, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {5068, 4946, 5038, 5044, 4988, 2497, 5004, 4942, 5108, 5114, 5118, 6745, 5036, 4930, 5040, 4964, 2449, 2352, 327, 209}

$$\frac{1}{4}a^6c^3x^4\text{ArcTan}(ax)^3-\frac{1}{4}a^5c^3x^3\text{ArcTan}(ax)^2+\frac{3}{2}a^4c^3x^2\text{ArcTan}(ax)^3+\frac{1}{4}a^4c^3x^2\text{ArcTan}(ax)-\frac{15}{4}a^3c^3x\text{ArcTan}(ax)^2-\frac{1}{4}a^3c^3x-\frac{9}{2}ia^2c^3\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{9}{2}ia^2c^3\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{iax+1}-1\right)-\frac{9}{2}a^2c^3\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)+\frac{9}{2}a^2c^3\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{iax+1}-1\right)+\frac{3}{4}a^2c^3\text{ArcTan}(ax)^3-5ia^2c^3\text{ArcTan}(ax)^2+\frac{1}{4}a^2c^3\text{ArcTan}(ax)-7a^2c^3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^3\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)+6a^2c^3\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ia^2c^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)-\frac{7}{2}ia^2c^3\text{Li}_2\left(1-\frac{2}{iax+1}\right)+\frac{9}{4}ia^2c^3\text{Li}_4\left(1-\frac{2}{iax+1}\right)-\frac{9}{4}ia^2c^3\text{Li}_4\left(\frac{2}{iax+1}-1\right)-\frac{c^3\text{ArcTan}(ax)^3}{2x^2}-\frac{3ac^3\text{ArcTan}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] -1/4*(a^3*c^3*x) + (a^2*c^3*ArcTan[a*x])/4 + (a^4*c^3*x^2*ArcTan[a*x])/4 -
(5*I)*a^2*c^3*ArcTan[a*x]^2 - (3*a*c^3*ArcTan[a*x]^2)/(2*x) - (15*a^3*c^3*x
*ArcTan[a*x]^2)/4 - (a^5*c^3*x^3*ArcTan[a*x]^2)/4 + (3*a^2*c^3*ArcTan[a*x]^
3)/4 - (c^3*ArcTan[a*x]^3)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^3)/2 + (a^6
*c^3*x^4*ArcTan[a*x]^3)/4 + 6*a^2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*
x)] - 7*a^2*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^3*ArcTan[a*x]*Log[
2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^3*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((7
*I)/2)*a^2*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((9*I)/2)*a^2*c^3*ArcTan[a*x
]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((9*I)/2)*a^2*c^3*ArcTan[a*x]^2*PolyLog
[2, -1 + 2/(1 + I*a*x)] - (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*
x)])/2 + (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/2 + ((9*I)/
4)*a^2*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((9*I)/4)*a^2*c^3*PolyLog[4, -1
+ 2/(1 + I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}{x^3}\,dx = \int \left(\frac{c^3\tan^{-1}(ax)^3}{x^3} + \frac{3a^2c^3\tan^{-1}(ax)^3}{x} + 3a^4c^3x\tan^{-1}(ax)^3 + a^6c^3x^3\tan^{-1}(\right.$$

$$= c^3 \int \frac{\tan^{-1}(ax)^3}{x^3}\,dx + \left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^3}{x}\,dx + \left(3a^4c^3\right)\int x\tan^{-1}(ax)^3\,d$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^3 + 6a^2c^3\tan^{-1}($$

$$= -\frac{c^3\tan^{-1}(ax)^3}{2x^2} + \frac{3}{2}a^4c^3x^2\tan^{-1}(ax)^3 + \frac{1}{4}a^6c^3x^4\tan^{-1}(ax)^3 + 6a^2c^3\tan^{-1}($$

$$= -\frac{3ac^3\tan^{-1}(ax)^2}{2x} - \frac{9}{2}a^3c^3x\tan^{-1}(ax)^2 - \frac{1}{4}a^5c^3x^3\tan^{-1}(ax)^2 + a^2c^3\tan^{-1}$$

$$= -6ia^2c^3\tan^{-1}(ax)^2 - \frac{3ac^3\tan^{-1}(ax)^2}{2x} - \frac{15}{4}a^3c^3x\tan^{-1}(ax)^2 - \frac{1}{4}a^5c^3x^3\tan$$

$$= \frac{1}{4}a^4c^3x^2\tan^{-1}(ax) - 5ia^2c^3\tan^{-1}(ax)^2 - \frac{3ac^3\tan^{-1}(ax)^2}{2x} - \frac{15}{4}a^3c^3x\tan^{-1}$$

$$= -\frac{1}{4}a^3c^3x + \frac{1}{4}a^4c^3x^2\tan^{-1}(ax) - 5ia^2c^3\tan^{-1}(ax)^2 - \frac{3ac^3\tan^{-1}(ax)^2}{2x} - \frac{15}{4}$$

$$= -\frac{1}{4}a^3c^3x + \frac{1}{4}a^2c^3\tan^{-1}(ax) + \frac{1}{4}a^4c^3x^2\tan^{-1}(ax) - 5ia^2c^3\tan^{-1}(ax)^2 - \frac{3a}{}$$

$$= -\frac{1}{4}a^3c^3x + \frac{1}{4}a^2c^3\tan^{-1}(ax) + \frac{1}{4}a^4c^3x^2\tan^{-1}(ax) - 5ia^2c^3\tan^{-1}(ax)^2 - \frac{3a}{}$$

Mathematica [A]
time = 0.55, size = 464, normalized size = 0.92

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (c^3*((-3*I)*a^2*Pi^4*x^2 - 16*a^3*x^3 + 16*a^2*x^2*ArcTan[a*x] + 16*a^4*x^
4*ArcTan[a*x] - 96*a*x*ArcTan[a*x]^2 + (128*I)*a^2*x^2*ArcTan[a*x]^2 - 240*
a^3*x^3*ArcTan[a*x]^2 - 16*a^5*x^5*ArcTan[a*x]^2 - 32*ArcTan[a*x]^3 + 48*a^
2*x^2*ArcTan[a*x]^3 + 96*a^4*x^4*ArcTan[a*x]^3 + 16*a^6*x^6*ArcTan[a*x]^3 +
 (96*I)*a^2*x^2*ArcTan[a*x]^4 + 192*a^2*x^2*ArcTan[a*x]^3*Log[1 - E^((-2*I)
*ArcTan[a*x])] + 192*a^2*x^2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] - 4
48*a^2*x^2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 192*a^2*x^2*ArcTan[
a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (288*I)*a^2*x^2*ArcTan[a*x]^2*PolyL
og[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*a^2*x^2*(7 + 9*ArcTan[a*x]^2)*PolyLo
```

```
g[2, -E^((2*I)*ArcTan[a*x])] - (96*I)*a^2*x^2*PolyLog[2, E^((2*I)*ArcTan[a*
x])] + 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 288*a^2
*x^2*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyL
og[4, E^((-2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyLog[4, -E^((2*I)*ArcTan
[a*x])]))/(64*x^2)
```

Maple [A]
time = 73.46, size = 763, normalized size = 1.52

method	result
derivativedivides	$a^2\left(\frac{c^3\left(2i\arctan(ax)^3+6i\arctan(ax)^2ax-2\arctan(ax)^3ax-14\arctan(ax)^2a^2x^2-5i\arctan(ax)^3a^2x^2+i\arctan(ax)^2\right.}{}\right.$
default	$a^2\left(\frac{c^3\left(2i\arctan(ax)^3+6i\arctan(ax)^2ax-2\arctan(ax)^3ax-14\arctan(ax)^2a^2x^2-5i\arctan(ax)^3a^2x^2+i\arctan(ax)^2\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
[Out] a^2*(1/4*c^3*(2*I*arctan(a*x)^3+6*I*arctan(a*x)^2*a*x-2*arctan(a*x)^3*a*x-1
4*arctan(a*x)^2*a^2*x^2-5*I*arctan(a*x)^3*a^2*x^2+I*arctan(a*x)^2*a^3*x^3+5
*arctan(a*x)^3*a^3*x^3-arctan(a*x)^2*a^4*x^4-I*arctan(a*x)^3*a^4*x^4+arctan
(a*x)^3*a^5*x^5-a^2*x^2-I*arctan(a*x)*a^2*x^2+arctan(a*x)*a^3*x^3)*(I+a*x)/
a^2/x^2-9*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3
*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*c^3*arctan(a*x)^2*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))-3*I*c^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3*arct
an(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))+18*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
+9/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-9/2*c^3*arctan
(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+7/2*I*c^3*polylog(2,-(1+I*a*x)^2/
(a^2*x^2+1))+3*c^3*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-9/4*I*c^3*
polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*c^3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x
^2+1)^(1/2))+18*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*c^3*arctan(a
*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+18*I*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))-3*c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+4*I*c^3*arctan(a*
x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^3 - 3*(a^6*c^3*x
^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 128*x^2*integr
ate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
 + c^3)*arctan(a*x)^3 - 12*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x)*arctan
(a*x)^2 + 12*(a^8*c^3*x^8 + 6*a^6*c^3*x^6 - 2*a^2*c^3*x^2)*arctan(a*x)*log(
a^2*x^2 + 1) + 3*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x + 4*(a^8*c^3*x^8
+ 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x))*log(a^2
*x^2 + 1)^2)/(a^2*x^5 + x^3), x))/x^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx + \int \frac{3a^2\operatorname{atan}^3(ax)}{x}\,dx + \int 3a^4x\operatorname{atan}^3(ax)\,dx + \int a^6x^3\operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**3,x)
```

```
[Out] c**3*(Integral(atan(a*x)**3/x**3, x) + Integral(3*a**2*atan(a*x)**3/x, x) +
 Integral(3*a**4*x*atan(a*x)**3, x) + Integral(a**6*x**3*atan(a*x)**3, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3, x)
```

3.386 $\int \frac{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^3}{x^4} dx$

Optimal. Leaf size=336

$$-\frac{a^2c^3\text{ArcTan}(ax)}{x}+a^4c^3x\text{ArcTan}(ax)-a^3c^3\text{ArcTan}(ax)^2-\frac{ac^3\text{ArcTan}(ax)^2}{2x^2}-\frac{1}{2}a^5c^3x^2\text{ArcTan}(ax)^2-\frac{c^3\text{ArcTa}}{3x}$$

```
[Out] -a^2*c^3*arctan(a*x)/x+a^4*c^3*x*arctan(a*x)-a^3*c^3*arctan(a*x)^2-1/2*a*c^
3*arctan(a*x)^2/x^2-1/2*a^5*c^3*x^2*arctan(a*x)^2-1/3*c^3*arctan(a*x)^3/x^3
-3*a^2*c^3*arctan(a*x)^3/x+3*a^4*c^3*x*arctan(a*x)^3+1/3*a^6*c^3*x^3*arctan
(a*x)^3+a^3*c^3*ln(x)+8*a^3*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))-a^3*c^3*ln(a^
2*x^2+1)+8*a^3*c^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))-8*I*a^3*c^3*arctan(a*x)*
polylog(2,-1+2/(1-I*a*x))+8*I*a^3*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+
4*a^3*c^3*polylog(3,-1+2/(1-I*a*x))+4*a^3*c^3*polylog(3,1-2/(1+I*a*x))
```

Rubi [A]
time = 0.81, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {5068, 4930, 5040, 4964, 5004, 5114, 6745, 4946, 5038, 272, 36, 29, 31, 5044, 4988, 5112, 5036, 266}

$$\frac{1}{3}a^6c^3x^3\text{ArcTan}(ax)^3-\frac{1}{2}a^5c^3x^2\text{ArcTan}(ax)^2+3a^4c^3x\text{ArcTan}(ax)^3+a^4c^3x\text{ArcTan}(ax)-8ia^3c^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)+8ia^3c^3\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)-a^3c^3\text{ArcTan}(ax)^2+8a^3c^3\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)+8a^3c^3\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+4a^3c^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)+4a^3c^3\text{Li}_3\left(1-\frac{2}{iax+1}\right)+a^3c^3\log(x)-\frac{3a^2c^3\text{ArcTan}(ax)^3}{x}-\frac{a^2c^3\text{ArcTan}(ax)}{x}-a^3c^3\log\left(a^2x^2+1\right)-\frac{c^3\text{ArcTan}(ax)^3}{3x^3}-\frac{ac^3\text{ArcTan}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c^3*ArcTan[a*x])/x) + a^4*c^3*x*ArcTan[a*x] - a^3*c^3*ArcTan[a*x]^2
- (a*c^3*ArcTan[a*x]^2)/(2*x^2) - (a^5*c^3*x^2*ArcTan[a*x]^2)/2 - (c^3*ArcT
an[a*x]^3)/(3*x^3) - (3*a^2*c^3*ArcTan[a*x]^3)/x + 3*a^4*c^3*x*ArcTan[a*x]^
3 + (a^6*c^3*x^3*ArcTan[a*x]^3)/3 + a^3*c^3*Log[x] + 8*a^3*c^3*ArcTan[a*x]^
2*Log[2/(1 + I*a*x)] - a^3*c^3*Log[1 + a^2*x^2] + 8*a^3*c^3*ArcTan[a*x]^2*L
og[2 - 2/(1 - I*a*x)] - (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*
a*x)] + (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + 4*a^3*c^3
*PolyLog[3, -1 + 2/(1 - I*a*x)] + 4*a^3*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5068

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^3}{x^4}\,dx = \int \left(3a^4c^3\tan^{-1}(ax)^3+\frac{c^3\tan^{-1}(ax)^3}{x^4}+\frac{3a^2c^3\tan^{-1}(ax)^3}{x^2}+a^6c^3x^2\tan^{-1}(\right.$$

$$= c^3\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx+\left(3a^2c^3\right)\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx+\left(3a^4c^3\right)\int \tan^{-1}(ax)^3\,d$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)^3}{x}+3a^4c^3x\tan^{-1}(ax)^3+\frac{1}{3}a^6c^3x^3\tan^{-1}$$

$$= -\frac{c^3\tan^{-1}(ax)^3}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax)^3}{x}+3a^4c^3x\tan^{-1}(ax)^3+\frac{1}{3}a^6c^3x^3\tan^{-1}$$

$$= -\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-\frac{1}{2}a^5c^3x^2\tan^{-1}(ax)^2-\frac{c^3\tan^{-1}(ax)^3}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax}{x}$$

$$= -\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-\frac{1}{2}a^5c^3x^2\tan^{-1}(ax)^2-\frac{c^3\tan^{-1}(ax)^3}{3x^3}-\frac{3a^2c^3\tan^{-1}(ax}{x}$$

$$= -\frac{a^2c^3\tan^{-1}(ax)}{x}+a^4c^3x\tan^{-1}(ax)-a^3c^3\tan^{-1}(ax)^2-\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-$$

$$= -\frac{a^2c^3\tan^{-1}(ax)}{x}+a^4c^3x\tan^{-1}(ax)-a^3c^3\tan^{-1}(ax)^2-\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-$$

$$= -\frac{a^2c^3\tan^{-1}(ax)}{x}+a^4c^3x\tan^{-1}(ax)-a^3c^3\tan^{-1}(ax)^2-\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-$$

$$= -\frac{a^2c^3\tan^{-1}(ax)}{x}+a^4c^3x\tan^{-1}(ax)-a^3c^3\tan^{-1}(ax)^2-\frac{ac^3\tan^{-1}(ax)^2}{2x^2}-$$

Mathematica [A]

time = 0.52, size = 331, normalized size = 0.99

$$\frac{c^3\left(-2ia^3\pi^3x^3-6a^2x^2\mathrm{ArcTan}(ax)+6a^4x^4\mathrm{ArcTan}(ax)-3ax\mathrm{ArcTan}(ax)^2-6a^3x^3\mathrm{ArcTan}(ax)^2-3a^5x^5\mathrm{ArcTan}(ax)^2-2\mathrm{ArcTan}(ax)^3-18a^2x^2\mathrm{ArcTan}(ax)^3+18a^4x^4\mathrm{ArcTan}(ax)^3+2a^6x^6\mathrm{ArcTan}(ax)^3+48a^3x^3\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right)+48a^3x^3\mathrm{ArcTan}(ax)^2\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right)+6a^3x^3\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-3a^3x^3\log\left(1+a^2x^2\right)+48ia^3x^3\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right)-48ia^3x^3\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right)+24a^3x^3\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)+24a^3x^3\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(ax)}\right)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (c^3*((-2*I)*a^3*Pi^3*x^3 - 6*a^2*x^2*ArcTan[a*x] + 6*a^4*x^4*ArcTan[a*x] -
 3*a*x*ArcTan[a*x]^2 - 6*a^3*x^3*ArcTan[a*x]^2 - 3*a^5*x^5*ArcTan[a*x]^2 -
2*ArcTan[a*x]^3 - 18*a^2*x^2*ArcTan[a*x]^3 + 18*a^4*x^4*ArcTan[a*x]^3 + 2*a
^6*x^6*ArcTan[a*x]^3 + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*
x])] + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3*
Log[(a*x)/Sqrt[1 + a^2*x^2]] - 3*a^3*x^3*Log[1 + a^2*x^2] + (48*I)*a^3*x^3*
ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*a^3*x^3*ArcTan[a*x]
*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*PolyLog[3, E^((-2*I)*ArcTa
n[a*x])] + 24*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(6*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.90, size = 7948, normalized size = 23.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/192*(3*(1792*a^8*c^3*integrate(1/32*x^8*arctan(a*x)^3/(a^2*x^6 + x^4), x)
 + 192*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 +
 x^4), x) + 256*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)/(a^
2*x^6 + x^4), x) - 256*a^7*c^3*integrate(1/32*x^7*arctan(a*x)^2/(a^2*x^6 +
x^4), x) + 64*a^7*c^3*integrate(1/32*x^7*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4)
, x) + 84*a^3*c^3*arctan(a*x)^4 + 7168*a^6*c^3*integrate(1/32*x^6*arctan(a*
x)^3/(a^2*x^6 + x^4), x) + 768*a^6*c^3*integrate(1/32*x^6*arctan(a*x)*log(a
^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 2304*a^6*c^3*integrate(1/32*x^6*arctan(
a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 2304*a^5*c^3*integrate(1/32*x^5
*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 3*a^3*c^3*log(a^2*x^2 + 1)^3 + 1152*a^
4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x)
 - 2304*a^4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 +
x^4), x) + 2304*a^3*c^3*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^6 + x^4), x
) - 576*a^3*c^3*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) +
```

```
7168*a^2*c^3*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 768*a^
2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x)
 - 256*a^2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x
^4), x) + 256*a*c^3*integrate(1/32*x*arctan(a*x)^2/(a^2*x^6 + x^4), x) - 64
*a*c^3*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 1792*c^3*i
ntegrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 192*c^3*integrate(1/32*ar
ctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 8*(a^6*c^3*x^6 + 9*
a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a*x)^3 - 6*(a^6*c^3*x^6 + 9*a^4*c
^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a*x)*log(a^2*x^2 + 1)^2)/x^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3/
x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int 3a^{4}\operatorname{atan}^{3}{\left(ax \right)}\, dx + \int \frac{\operatorname{atan}^{3}{\left(ax \right)}}{x^{4}}\, dx + \int \frac{3a^{2}\operatorname{atan}^{3}{\left(ax \right)}}{x^{2}}\, dx + \int a^{6}x^{2}\operatorname{atan}^{3}{\left(ax \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**4,x)
```

```
[Out] c**3*(Integral(3*a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + I
ntegral(3*a**2*atan(a*x)**3/x**2, x) + Integral(a**6*x**2*atan(a*x)**3, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^3}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4, x)
```

3.387 $\int \frac{x^4 \text{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=217

$$\frac{x\text{ArcTan}(ax)}{a^4c}-\frac{\text{ArcTan}(ax)^2}{2a^5c}-\frac{x^2\text{ArcTan}(ax)^2}{2a^3c}-\frac{4i\text{ArcTan}(ax)^3}{3a^5c}-\frac{x\text{ArcTan}(ax)^3}{a^4c}+\frac{x^3\text{ArcTan}(ax)^3}{3a^2c}+\frac{\text{ArcTa}}{4a}$$

```
[Out] x*arctan(a*x)/a^4/c-1/2*arctan(a*x)^2/a^5/c-1/2*x^2*arctan(a*x)^2/a^3/c-4/3
*I*arctan(a*x)^3/a^5/c-x*arctan(a*x)^3/a^4/c+1/3*x^3*arctan(a*x)^3/a^2/c+1/
4*arctan(a*x)^4/a^5/c-4*arctan(a*x)^2*ln(2/(1+I*a*x))/a^5/c-1/2*ln(a^2*x^2+
1)/a^5/c-4*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^5/c-2*polylog(3,1-2/(1+
I*a*x))/a^5/c
```

Rubi [A]
time = 0.46, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5036, 4946, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$-\frac{4i\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^5c}+\frac{\text{ArcTan}(ax)^4}{4a^5c}-\frac{4i\text{ArcTan}(ax)^3}{3a^5c}-\frac{\text{ArcTan}(ax)^2}{2a^5c}-\frac{4\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^5c}-\frac{2\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{a^5c}-\frac{x\text{ArcTan}(ax)^3}{a^4c}+\frac{x\text{ArcTan}(ax)}{a^4c}-\frac{x^2\text{ArcTan}(ax)^2}{2a^3c}+\frac{x^3\text{ArcTan}(ax)^3}{3a^2c}-\frac{\log\left(a^2x^2+1\right)}{2a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (x*ArcTan[a*x])/(a^4*c) - ArcTan[a*x]^2/(2*a^5*c) - (x^2*ArcTan[a*x]^2)/(2*
a^3*c) - (((4*I)/3)*ArcTan[a*x]^3)/(a^5*c) - (x*ArcTan[a*x]^3)/(a^4*c) + (x
^3*ArcTan[a*x]^3)/(3*a^2*c) + ArcTan[a*x]^4/(4*a^5*c) - (4*ArcTan[a*x]^2*Lo
g[2/(1 + I*a*x)])/(a^5*c) - Log[1 + a^2*x^2]/(2*a^5*c) - ((4*I)*ArcTan[a*x]
*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^5*c) - (2*PolyLog[3, 1 - 2/(1 + I*a*x)])
/(a^5*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2} + \frac{\int x^2\tan^{-1}(ax)^3\,dx}{a^2c}$$

$$= \frac{x^3\tan^{-1}(ax)^3}{3a^2c} + \frac{\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^4} - \frac{\int \tan^{-1}(ax)^3\,dx}{a^4c} - \frac{\int \frac{x^3\tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{ac}$$

$$= -\frac{x\tan^{-1}(ax)^3}{a^4c} + \frac{x^3\tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} - \frac{\int x\tan^{-1}(ax)^2\,dx}{a^3c} + \frac{\int \frac{x\tan^{-1}(ax)^2}{1+a^2x^2}\,d}{a^3c}$$

$$= -\frac{x^2\tan^{-1}(ax)^2}{2a^3c} - \frac{4i\tan^{-1}(ax)^3}{3a^5c} - \frac{x\tan^{-1}(ax)^3}{a^4c} + \frac{x^3\tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} -$$

$$= -\frac{x^2\tan^{-1}(ax)^2}{2a^3c} - \frac{4i\tan^{-1}(ax)^3}{3a^5c} - \frac{x\tan^{-1}(ax)^3}{a^4c} + \frac{x^3\tan^{-1}(ax)^3}{3a^2c} + \frac{\tan^{-1}(ax)^4}{4a^5c} -$$

$$= \frac{x\tan^{-1}(ax)}{a^4c} - \frac{\tan^{-1}(ax)^2}{2a^5c} - \frac{x^2\tan^{-1}(ax)^2}{2a^3c} - \frac{4i\tan^{-1}(ax)^3}{3a^5c} - \frac{x\tan^{-1}(ax)^3}{a^4c} + \frac{x^3\,\mathrm{t}}{}$$

$$= \frac{x\tan^{-1}(ax)}{a^4c} - \frac{\tan^{-1}(ax)^2}{2a^5c} - \frac{x^2\tan^{-1}(ax)^2}{2a^3c} - \frac{4i\tan^{-1}(ax)^3}{3a^5c} - \frac{x\tan^{-1}(ax)^3}{a^4c} + \frac{x^3\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.19, size = 154, normalized size = 0.71

$$\frac{12ax\mathrm{ArcTan}(ax) - 6\mathrm{ArcTan}(ax)^2 - 6a^2x^2\mathrm{ArcTan}(ax)^2 + 16i\mathrm{ArcTan}(ax)^3 - 12ax\mathrm{ArcTan}(ax)^3 + 4a^3x^3\mathrm{ArcTan}(ax)^3 + 3\mathrm{ArcTan}(ax)^4 - 48\mathrm{ArcTan}(ax)^2\log\left(1+e^{2i\mathrm{ArcTan}(ax)}\right) - 6\log\left(1+a^2x^2\right) + 48i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(ax)}\right) - 24\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(ax)}\right)}{12a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 6*a^2*x^2*ArcTan[a*x]^2 + (16*I)*Ar
cTan[a*x]^3 - 12*a*x*ArcTan[a*x]^3 + 4*a^3*x^3*ArcTan[a*x]^3 + 3*ArcTan[a*x
]^4 - 48*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 6*Log[1 + a^2*x^2]
+ (48*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*PolyLog[3, -E^
((2*I)*ArcTan[a*x])])/(12*a^5*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 44.53, size = 1536, normalized size = 7.08

method	result	size
derivativedivides	Expression too large to display	1536
default	Expression too large to display	1536

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/3/c*arctan(a*x)^3*a^3*x^3-1/c*arctan(a*x)^3*a*x+1/c*arctan(a*x)^4-
1/c*(1/2*arctan(a*x)^2*a^2*x^2-2*arctan(a*x)^2*ln(a^2*x^2+1)+4*arctan(a*x)^
2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a
^2*x^2+1))+2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/6*I*arctan(a*x)*(3*I*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)*Pi*a*x+3*I*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)*Pi
*a*x-3*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)
^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*arctan(a*x)*Pi*a*x-3*I*csgn(I*(1+I*a*x)^4
/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*arctan(a*x)*Pi*a*x-6*I*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arc
tan(a*x)*Pi*a*x+3*I*arctan(a*x)+6*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(
a*x)*Pi-12*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)*Pi*csgn(I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-6*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)*Pi+6*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)*Pi+6*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1))*arctan(a*x)*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2-3*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*ar
ctan(a*x)*Pi+6*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2*arctan(a*x)*Pi-3*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*ar
ctan(a*x)*Pi+6*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^3*arctan(a*x)*Pi-6*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)*Pi-3*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/
(a^2*x^2+1)+I)*arctan(a*x)*Pi+6*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1
+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*arctan(a*x)*Pi-3*c
sgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*arctan(a*x
)*Pi+6*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^2
+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*arctan(a*x)*Pi*a*x+8*arctan(a*x)^2+24*I*a
rctan(a*x)*ln(2)-6*I*a*x-6)-ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/4*arctan(a*x)^4
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3072*(48*(7168*a^4*integrate(1/128*x^4*arctan(a*x)^3/(a^6*c*x^2 + a^4*c),
 x) + 768*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c*x^2
 + a^4*c), x) + 1024*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(
a^6*c*x^2 + a^4*c), x) - 1024*a^3*integrate(1/128*x^3*arctan(a*x)^2/(a^6*c*
x^2 + a^4*c), x) + 256*a^3*integrate(1/128*x^3*log(a^2*x^2 + 1)^2/(a^6*c*x^
```

```
2 + a^4*c), x) - 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)/
(a^6*c*x^2 + a^4*c), x) + 768*a*integrate(1/128*x*arctan(a*x)^2*log(a^2*x^2
 + 1)/(a^6*c*x^2 + a^4*c), x) + 192*a*integrate(1/128*x*log(a^2*x^2 + 1)^3/
(a^6*c*x^2 + a^4*c), x) + 3072*a*integrate(1/128*x*arctan(a*x)^2/(a^6*c*x^2
 + a^4*c), x) - 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4
*c), x) - 3*arctan(a*x)^4/(a^5*c) - 384*integrate(1/128*arctan(a*x)*log(a^2
*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 128*(a^3*x^3 - 3*a*x)*arctan(a
*x)^3 + 240*arctan(a*x)^4 - 9*log(a^2*x^2 + 1)^4 - 24*(4*(a^3*x^3 - 3*a*x)*
arctan(a*x) + 3*arctan(a*x)^2)*log(a^2*x^2 + 1)^2)/(a^5*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(x**4*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^3)/(c + a^2*c*x^2),x)
[Out] int((x^4*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.388 $\int \frac{x^3 \mathbf{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=260

$$-\frac{3i\text{ArcTan}(ax)^2}{2a^4c}-\frac{3x\text{ArcTan}(ax)^2}{2a^3c}+\frac{\text{ArcTan}(ax)^3}{2a^4c}+\frac{x^2\text{ArcTan}(ax)^3}{2a^2c}+\frac{i\text{ArcTan}(ax)^4}{4a^4c}-\frac{3\text{ArcTan}(ax)\log\left(\frac{2}{1+ia}\right.}{a^4c}$$

```
[Out] -3/2*I*arctan(a*x)^2/a^4/c-3/2*x*arctan(a*x)^2/a^3/c+1/2*arctan(a*x)^3/a^4/
c+1/2*x^2*arctan(a*x)^3/a^2/c+1/4*I*arctan(a*x)^4/a^4/c-3*arctan(a*x)*ln(2/
(1+I*a*x))/a^4/c+arctan(a*x)^3*ln(2/(1+I*a*x))/a^4/c-3/2*I*polylog(2,1-2/(1
+I*a*x))/a^4/c+3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^4/c+3/2*arcta
n(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c-3/4*I*polylog(4,1-2/(1+I*a*x))/a^4/c
```

Rubi [A]
time = 0.33, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5036, 4946, 4930, 5040, 4964, 2449, 2352, 5004, 5114, 5118, 6745}

$$\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^4c}+\frac{3\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^4c}+\frac{i\text{ArcTan}(ax)^4}{4a^4c}+\frac{\text{ArcTan}(ax)^3}{2a^4c}-\frac{3i\text{ArcTan}(ax)^2}{2a^4c}+\frac{\text{ArcTan}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c}-\frac{3\text{ArcTan}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c}-\frac{3i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^4c}-\frac{3i\text{Li}_4\left(1-\frac{2}{iax+1}\right)}{4a^4c}-\frac{3x\text{ArcTan}(ax)^2}{2a^3c}+\frac{x^2\text{ArcTan}(ax)^3}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (((-3*I)/2)*ArcTan[a*x]^2)/(a^4*c) - (3*x*ArcTan[a*x]^2)/(2*a^3*c) + ArcTan
[a*x]^3/(2*a^4*c) + (x^2*ArcTan[a*x]^3)/(2*a^2*c) + ((I/4)*ArcTan[a*x]^4)/(
a^4*c) - (3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(a^4*c) + (ArcTan[a*x]^3*Log[2/
(1 + I*a*x)])/(a^4*c) - (((3*I)/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) +
 (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c) + (3*ArcTa
n[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c) - (((3*I)/4)*PolyLog[4, 1 -
 2/(1 + I*a*x)])/(a^4*c)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\tan^{-1}(ax)^3\,dx}{a^2c}\\
&= \frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}+\frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}\,dx}{a^3c}-\frac{3\int \frac{x^2\tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{2ac}\\
&= \frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}+\frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c}-\frac{3\int \tan^{-1}(ax)^2\,dx}{2a^3c}+\cdots\\
&= -\frac{3x\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan^{-1}(ax)^3}{2a^4c}+\frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}+\cdots\\
&= -\frac{3i\tan^{-1}(ax)^2}{2a^4c}-\frac{3x\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan^{-1}(ax)^3}{2a^4c}+\frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}+\cdots\\
&= -\frac{3i\tan^{-1}(ax)^2}{2a^4c}-\frac{3x\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan^{-1}(ax)^3}{2a^4c}+\frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}-\cdots\\
&= -\frac{3i\tan^{-1}(ax)^2}{2a^4c}-\frac{3x\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan^{-1}(ax)^3}{2a^4c}+\frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}-\cdots\\
&= -\frac{3i\tan^{-1}(ax)^2}{2a^4c}-\frac{3x\tan^{-1}(ax)^2}{2a^3c}+\frac{\tan^{-1}(ax)^3}{2a^4c}+\frac{x^2\tan^{-1}(ax)^3}{2a^2c}+\frac{i\tan^{-1}(ax)^4}{4a^4c}-\cdots
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 162, normalized size = 0.62

$$\frac{6i\text{ArcTan}(ax)^2-6ax\text{ArcTan}(ax)^2+2(1+a^2x^2)\,\text{ArcTan}(ax)^3-i\text{ArcTan}(ax)^4-12\text{ArcTan}(ax)\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+4\text{ArcTan}(ax)^3\log\left(1+e^{2i\text{ArcTan}(ax)}\right)-6i(-1+\text{ArcTan}(ax)^2)\,\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+6\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)+3i\text{PolyLog}\left(4,-e^{2i\text{ArcTan}(ax)}\right)}{4a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((6*I)*ArcTan[a*x]^2 - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^2)*ArcTan[a*x]^3
- I*ArcTan[a*x]^4 - 12*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] + 4*ArcTa
n[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] - (6*I)*(-1 + ArcTan[a*x]^2)*PolyLo
g[2, -E^((2*I)*ArcTan[a*x])] + 6*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*
x])] + (3*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])])/(4*a^4*c)
```

Maple [A]
time = 73.76, size = 259, normalized size = 1.00

method	result
derivativedivides	$-\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3)(ax+i)}{2c}+\frac{\arctan(ax)^3\ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2\,\mathrm{polylog}\left(2,-\right.}{2c}$
default	$-\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3)(ax+i)}{2c}+\frac{\arctan(ax)^3\ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2\,\mathrm{polylog}\left(2,-\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/4*I*arctan(a*x)^4/c+1/2/c*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*
x)*a*x-3)*(I+a*x)+1/c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/2*I/c*a
rctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+3/2/c*arctan(a*x)*polylog(
3,-(1+I*a*x)^2/(a^2*x^2+1))+3/4*I/c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*I
/c*arctan(a*x)^2-3/c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I/c*poly
log(2,-(1+I*a*x)^2/(a^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(x**3*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2),x)
[Out] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.389 $\int \frac{x^2 \mathbf{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=130

$$\frac{i\text{ArcTan}(ax)^3}{a^3c}+\frac{x\text{ArcTan}(ax)^3}{a^2c}-\frac{\text{ArcTan}(ax)^4}{4a^3c}+\frac{3\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{3i\text{ArcTan}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}$$

[Out] I*arctan(a*x)^3/a^3/c+x*arctan(a*x)^3/a^2/c-1/4*arctan(a*x)^4/a^3/c+3*arctan(a*x)^2*ln(2/(1+I*a*x))/a^3/c+3*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^3/c+3/2*polylog(3,1-2/(1+I*a*x))/a^3/c

Rubi [A]
time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5036, 4930, 5040, 4964, 5004, 5114, 6745}

$$\frac{3i\text{ArcTan}(ax)\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^3c}-\frac{\text{ArcTan}(ax)^4}{4a^3c}+\frac{i\text{ArcTan}(ax)^3}{a^3c}+\frac{3\text{ArcTan}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{3\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^3c}+\frac{x\text{ArcTan}(ax)^3}{a^2c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]

[Out] (I*ArcTan[a*x]^3)/(a^3*c) + (x*ArcTan[a*x]^3)/(a^2*c) - ArcTan[a*x]^4/(4*a^3*c) + (3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(a^3*c) + ((3*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^3*c) + (3*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^3*c)

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)^3\,dx}{a^2c}\\
&= \frac{x\tan^{-1}(ax)^3}{a^2c}-\frac{\tan^{-1}(ax)^4}{4a^3c}-\frac{3\int \frac{x\tan^{-1}(ax)^2}{1+a^2x^2}\,dx}{ac}\\
&= \frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{3\int \frac{\tan^{-1}(ax)^2}{i-ax}\,dx}{a^2c}\\
&= \frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c}-\frac{6\int \frac{\tan^{-1}(ax)}{1+\cdots}}{a^{\cdots}}\\
&= \frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{3i\tan^{-1}(ax\cdots}{\cdots}\\
&= \frac{i\tan^{-1}(ax)^3}{a^3c}+\frac{x\tan^{-1}(ax)^3}{a^2c}-\frac{\tan^{-1}(ax)^4}{4a^3c}+\frac{3\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{3i\tan^{-1}(ax\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 93, normalized size = 0.72

$$\frac{-\frac{1}{4}\text{ArcTan}(ax)^2\left((4i-4ax)\text{ArcTan}(ax)+\text{ArcTan}(ax)^2-12\log\left(1+e^{2i\text{ArcTan}(ax)}\right)\right)-3i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] (-1/4*(ArcTan[a*x]^2*((4*I - 4*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 12*Log[1
+ E^((2*I)*ArcTan[a*x])])) - (3*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[
a*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2)/(a^3*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.48, size = 785, normalized size = 6.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/c*arctan(a*x)^3*a*x-1/c*arctan(a*x)^4-3/c*(1/2*arctan(a*x)^2*ln(a^
2*x^2+1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*arctan(a*x)^2*(I
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
3+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)-2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-4*ln(2))+I*a
rctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(1+I*a*x)^2/(
a^2*x^2+1))+1/3*I*arctan(a*x)^3-1/4*arctan(a*x)^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/1024*(16*(7168*a^2*integrate(1/128*x^2*arctan(a*x)^3/(a^4*c*x^2 + a^2*c),
 x) + 768*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c*x^2
 + a^2*c), x) + 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(
a^4*c*x^2 + a^2*c), x) - 768*a*integrate(1/128*x*arctan(a*x)^2*log(a^2*x^2
```

```
+ 1)/(a^4*c*x^2 + a^2*c), x) - 192*a*integrate(1/128*x*log(a^2*x^2 + 1)^3/(
a^4*c*x^2 + a^2*c), x) - 3072*a*integrate(1/128*x*arctan(a*x)^2/(a^4*c*x^2
+ a^2*c), x) + 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*
c), x) + 3*arctan(a*x)^4/(a^3*c) + 384*integrate(1/128*arctan(a*x)*log(a^2*
x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c + 128*a*x*arctan(a*x)^3 - 80*arct
an(a*x)^4 + 3*log(a^2*x^2 + 1)^4 - 24*(4*a*x*arctan(a*x) - arctan(a*x)^2)*l
og(a^2*x^2 + 1)^2)/(a^3*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(x**2*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^3}{c\,a^2\,x^2+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2),x)
[Out] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.390 $\int \frac{x\textbf{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=138

$$-\frac{i\text{ArcTan}(ax)^4}{4a^2c}-\frac{\text{ArcTan}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}-\frac{3\text{ArcTan}(ax)\text{PolyLog}(}{2a^2c}$$

```
[Out] -1/4*I*arctan(a*x)^4/a^2/c-arctan(a*x)^3*ln(2/(1+I*a*x))/a^2/c-3/2*I*arctan
(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^2/c-3/2*arctan(a*x)*polylog(3,1-2/(1+I*a
*x))/a^2/c+3/4*I*polylog(4,1-2/(1+I*a*x))/a^2/c
```

Rubi [A]
time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5040, 4964, 5004, 5114, 5118, 6745}

$$-\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^2c}-\frac{3\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^2c}-\frac{i\text{ArcTan}(ax)^4}{4a^2c}-\frac{\text{ArcTan}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c}+\frac{3i\text{Li}_4\left(1-\frac{2}{iax+1}\right)}{4a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/4*I)*ArcTan[a*x]^4)/(a^2*c) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^2*
c) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^2*c) - (3*A
rcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^2*c) + (((3*I)/4)*PolyLog[4,
 1 - 2/(1 + I*a*x)])/(a^2*c)
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}\,dx}{ac}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} + \frac{3\int \frac{\tan^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{ac}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} + \frac{(3i)\int \frac{\tan^{-1}(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3\tan^{-1}(ax)\ldots}{2\ldots}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4a^2c} - \frac{\tan^{-1}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\tan^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3\tan^{-1}(ax)\ldots}{2\ldots}$$

Mathematica [A]
time = 0.01, size = 149, normalized size = 1.08

$$-\frac{i\text{ArcTan}(ax)^4}{4a^2c} - \frac{\text{ArcTan}(ax)^3\log\left(\frac{2i}{i-ax}\right)}{a^2c} - \frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)}{2a^2c} - \frac{3\text{ArcTan}(ax)\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)}{2a^2c} + \frac{3i\text{PolyLog}\left(4,\frac{i+ax}{-i+ax}\right)}{4a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/4*I)*ArcTan[a*x]^4)/(a^2*c) - (ArcTan[a*x]^3*Log[(2*I)/(I - a*x)])/(a^
2*c) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c) -
 (3*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I + a*x)])/(2*a^2*c) + (((3*I)/4)*Po
lyLog[4, (I + a*x)/(-I + a*x)])/(a^2*c)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.96, size = 789, normalized size = 5.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(2/3*arctan(a*x)^3*ln((1+I*a
*x)/(a^2*x^2+1)^(1/2))+1/6*arctan(a*x)^3*(I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+4*ln(2))-I*arctan(a*x)^2*polylog(2,-(1+I*a*x
)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*poly
log(4,-(1+I*a*x)^2/(a^2*x^2+1))-1/6*I*arctan(a*x)^4))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{3}(ax)}{a^{2}x^{2}+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c),x)
[Out] Integral(x*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2),x)
[Out] int((x*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.391 $\int \frac{\text{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(ax)^4}{4ac}$$

```
[Out] 1/4*arctan(a*x)^4/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{5004\}$

$$\frac{\text{ArcTan}(ax)^4}{4ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^4/(4*a*c)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^4}{4ac}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{\text{ArcTan}(ax)^4}{4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ArcTan[a*x]^4/(4*a*c)
```

Maple [A]
time = 0.62, size = 15, normalized size = 0.94

method	result
derivativedivides	$\frac{\arctan(ax)^4}{4ac}$
default	$\frac{\arctan(ax)^4}{4ac}$
risch	$\frac{\ln(iax+1)^4}{64ca} - \frac{\ln(-iax+1)\ln(iax+1)^3}{16ca} + \frac{3\ln(-iax+1)^2\ln(iax+1)^2}{32ca} - \frac{\ln(-iax+1)^3\ln(iax+1)}{16ca} + \frac{\ln(-iax+1)^4}{64ca}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctan(a*x)^4/a/c
```

Maxima [A]
time = 0.46, size = 14, normalized size = 0.88

$$\frac{\arctan(ax)^4}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*arctan(a*x)^4/(a*c)
```

Fricas [A]
time = 13.79, size = 14, normalized size = 0.88

$$\frac{\arctan(ax)^4}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(a*x)^4/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.13, size = 14, normalized size = 0.88

$$\frac{\operatorname{atan}(a\,x)^4}{4\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2),x)
[Out] atan(a*x)^4/(4*a*c)
```

3.392 $\int \frac{\text{ArcTan}(ax)^3}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=124

$$-\frac{i\text{ArcTan}(ax)^4}{4c}+\frac{\text{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3\text{ArcTan}(ax)\text{PolyL}}{2}$$

```
[Out] -1/4*I*arctan(a*x)^4/c+arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*arctan(a*x)^
2*polylog(2,-1+2/(1-I*a*x))/c+3/2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c+3
/4*I*polylog(4,-1+2/(1-I*a*x))/c
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5044, 4988, 5004, 5112, 5116, 6745}

$$-\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c}+\frac{3\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}-\frac{i\text{ArcTan}(ax)^4}{4c}+\frac{\text{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{3i\text{Li}_4\left(\frac{2}{1-iax}-1\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((-1/4*I)*ArcTan[a*x]^4)/c + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c - (((
3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (3*ArcTan[a*x]*Po
lyLog[3, -1 + 2/(1 - I*a*x)])/(2*c) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a
*x)])/c
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{i\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{(3a)\int \frac{\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx}{c}$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{(3ia$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{3\tan$$

$$= -\frac{i\tan^{-1}(ax)^4}{4c} + \frac{\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\tan^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right)}{2c} + \frac{3\tan$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 354 vs. 2(124) = 248.
time = 0.04, size = 354, normalized size = 2.85

$$\frac{i\text{ArcTan}(ax)^4}{4c} + \frac{2\text{ArcTan}(ax)^3\tanh^{-1}\left(1-\frac{2i}{i-ax}\right)}{c} + \frac{\text{ArcTan}(ax)^3\log\left(\frac{2i}{i-ax}\right)}{c} + \frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,\frac{-i-ax}{-i+ax}\right)}{2c} + \frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c} - \frac{3i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)}{2c} + \frac{3\text{ArcTan}(ax)\text{PolyLog}\left(3,\frac{-i-ax}{-i+ax}\right)}{2c} + \frac{3\text{ArcTan}(ax)\text{PolyLog}\left(3,-\frac{i+ax}{i-ax}\right)}{2c} - \frac{3\text{ArcTan}(ax)\text{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)}{2c} - \frac{3i\text{PolyLog}\left(4,\frac{-i-ax}{-i+ax}\right)}{4c} - \frac{3i\text{PolyLog}\left(4,-\frac{i+ax}{i-ax}\right)}{4c} + \frac{3i\text{PolyLog}\left(4,\frac{i+ax}{-i+ax}\right)}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]

[Out] ((I/4)*ArcTan[a*x]^4)/c + (2*ArcTan[a*x]^3*ArcTanh[1 - (2*I)/(I - a*x)])/c + (ArcTan[a*x]^3*Log[(2*I)/(I - a*x)])/c + (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, (-I - a*x)/(-I + a*x)])/c + (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -((I + a*x)/(I - a*x))])/c - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)])/c + (3*ArcTan[a*x]*PolyLog[3, (-I - a*x)/(-I + a*x)])/(2*c) + (3*ArcTan[a*x]*PolyLog[3, -((I + a*x)/(I - a*x))])/(2*c) - (3*ArcTan[a*x]*PolyLog[3, (I + a*x)/(-I + a*x)])/(2*c) - (((3*I)/4)*PolyLog[4, (-I - a*x)/(-I + a*x)])/c - (((3*I)/4)*PolyLog[4, -((I + a*x)/(I - a*x))])/c + (((3*I)/4)*PolyLog[4, (I + a*x)/(-I + a*x)])/c

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 85.83, size = 1751, normalized size = 14.12

method	result	size
derivativedivides	Expression too large to display	1751
default	Expression too large to display	1751

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)

[Out] 1/c*arctan(a*x)^3*ln(a*x)-1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+1/6*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/3*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/6*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/3*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1/6*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/6*I*arctan(a*

```
x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/6*I*arctan(a*x)^3*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-2/3*arctan(a*x)
^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-4*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/6*I*arctan(a*x)^4+2/3*ar
ctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2/3*arctan(a*x)^3*ln(2)-4*arctan(
a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2/3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-
1/3*I*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))^3+1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-1/3*I*
arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))^3+1/3*I*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^
2/(a^2*x^2+1)+1))^2-2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*a
rctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^3*Pi+
2*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}{\left(ax \right)}}{a^{2}x^{3}+x}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**3/(a**2*x**3 + x), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)), x)
```

3.393 $\int \frac{\mathbf{ArcTan}(ax)^3}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=122

$$-\frac{ia\mathrm{ArcTan}(ax)^3}{c}-\frac{\mathrm{ArcTan}(ax)^3}{cx}-\frac{a\mathrm{ArcTan}(ax)^4}{4c}+\frac{3a\mathrm{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia\mathrm{ArcTan}(ax)\mathrm{PolyLo}}{c}$$

[Out] -I*a*arctan(a*x)^3/c-arctan(a*x)^3/c/x-1/4*a*arctan(a*x)^4/c+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-3*I*a*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c+3/2*a*polylog(3,-1+2/(1-I*a*x))/c

Rubi [A]
time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5038, 4946, 5044, 4988, 5004, 5112, 6745}

$$-\frac{3ia\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}-\frac{a\mathrm{ArcTan}(ax)^4}{4c}-\frac{ia\mathrm{ArcTan}(ax)^3}{c}-\frac{\mathrm{ArcTan}(ax)^3}{cx}+\frac{3a\mathrm{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{3a\mathrm{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]

[Out] ((-I)*a*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(c*x) - (a*ArcTan[a*x]^4)/(4*c) + (3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - ((3*I)*a*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c)

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\,(c+a^2cx^2)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{cx} - \frac{a\tan^{-1}(ax)^4}{4c} + \frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x(1+a^2x^2)}\,dx}{c}$$

$$= -\frac{ia\tan^{-1}(ax)^3}{c} - \frac{\tan^{-1}(ax)^3}{cx} - \frac{a\tan^{-1}(ax)^4}{4c} + \frac{(3ia)\int \frac{\tan^{-1}(ax)^2}{x(i+ax)}\,dx}{c}$$

$$= -\frac{ia\tan^{-1}(ax)^3}{c} - \frac{\tan^{-1}(ax)^3}{cx} - \frac{a\tan^{-1}(ax)^4}{4c} + \frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - (6$$

$$= -\frac{ia\tan^{-1}(ax)^3}{c} - \frac{\tan^{-1}(ax)^3}{cx} - \frac{a\tan^{-1}(ax)^4}{4c} + \frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - 3i$$

$$= -\frac{ia\tan^{-1}(ax)^3}{c} - \frac{\tan^{-1}(ax)^3}{cx} - \frac{a\tan^{-1}(ax)^4}{4c} + \frac{3a\tan^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - 3i$$

Mathematica [A]
time = 0.12, size = 108, normalized size = 0.89

$$\frac{a\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(ax)^3 - \frac{\text{ArcTan}(ax)^3}{ax} - \frac{1}{4}\text{ArcTan}(ax)^4 + 3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 3i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - ArcTan[a*x]^4/4
 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*Poly
Log[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2))
/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 40.05, size = 1722, normalized size = 14.11

method	result	size
derivativedivides	Expression too large to display	1722
default	Expression too large to display	1722

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/c*arctan(a*x)^3/a/x-1/c*arctan(a*x)^4-3/c*(1/4*I*arctan(a*x)^2*Pi*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(
```

```
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*arctan(a
*x)^2*ln(a^2*x^2+1)-arctan(a*x)^2*ln(2)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2))+1/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1/4*I*arctan(a*x)^2*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1))^3+1/4*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/3*I*arctan(a*x)^3-1/2*I*Pi*csgn(((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*Pi
*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x
)^2-1/2*I*Pi*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^2-1/4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/4*I*arctan(a*x)^2*Pi*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^
2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(
a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^3*arctan(a*x)^2+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x
)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2
+1)^(1/2))-arctan(a*x)^2*ln(a*x)-1/4*arctan(a*x)^4))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/1024*(80*a*x*arctan(a*x)^4 - 3*a*x*log(a^2*x^2 + 1)^4 - (48*a*arctan(a*x
)^4/c - 12288*a^3*integrate(1/128*x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^2*c
*x^4 + c*x^2), x) - 3*a*log(a^2*x^2 + 1)^4/c + 6144*a^2*integrate(1/128*x^2
*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 49152*a^2*integra
```

```
te(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 49152*a
*integrate(1/128*x*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x) - 12288*a*integrat
e(1/128*x*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) + 114688*integrate(1/1
28*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x) + 12288*integrate(1/128*arctan(a*x
)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x))*c*x + 128*arctan(a*x)^3 - 24*
(a*x*arctan(a*x)^2 + 4*arctan(a*x))*log(a^2*x^2 + 1)^2)/(c*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{2}x^{4}+x^{2}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**3/(a**2*x**4 + x**2), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)), x)
```

3.394 $\int \frac{\text{ArcTan}(ax)^3}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=262

$$-\frac{3ia^2\text{ArcTan}(ax)^2}{2c}-\frac{3a\text{ArcTan}(ax)^2}{2cx}-\frac{a^2\text{ArcTan}(ax)^3}{2c}-\frac{\text{ArcTan}(ax)^3}{2cx^2}+\frac{ia^2\text{ArcTan}(ax)^4}{4c}+\frac{3a^2\text{ArcTan}(ax)\,\text{lo}}{c}$$

```
[Out] -3/2*I*a^2*arctan(a*x)^2/c-3/2*a*arctan(a*x)^2/c/x-1/2*a^2*arctan(a*x)^3/c-
1/2*arctan(a*x)^3/c/x^2+1/4*I*a^2*arctan(a*x)^4/c+3*a^2*arctan(a*x)*ln(2-2/
(1-I*a*x))/c-a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*a^2*polylog(2,-1+2
/(1-I*a*x))/c+3/2*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c-3/2*a^2*a
rctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c-3/4*I*a^2*polylog(4,-1+2/(1-I*a*x))/
c
```

Rubi [A]
time = 0.37, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5038, 4946, 5044, 4988, 2497, 5004, 5112, 5116, 6745}

$$\frac{3ia^2\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c}-\frac{3a^2\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c}+\frac{ia^2\text{ArcTan}(ax)^4}{4c}-\frac{a^2\text{ArcTan}(ax)^3}{2c}-\frac{3ia^2\text{ArcTan}(ax)^2}{2c}-\frac{a^2\text{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{3a^2\text{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c}-\frac{3ia^2\text{Li}_4\left(\frac{2}{1-iax}-1\right)}{4c}-\frac{\text{ArcTan}(ax)^3}{2cx^2}-\frac{3a\text{ArcTan}(ax)^2}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((-3*I)/2)*a^2*ArcTan[a*x]^2)/c - (3*a*ArcTan[a*x]^2)/(2*c*x) - (a^2*ArcTa
n[a*x]^3)/(2*c) - ArcTan[a*x]^3/(2*c*x^2) + ((I/4)*a^2*ArcTan[a*x]^4)/c + (
3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/c - (a^2*ArcTan[a*x]^3*Log[2 - 2/
(1 - I*a*x)])/c - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c + (((3*I
)/2)*a^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (3*a^2*ArcTan[a*
x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c) - (((3*I)/4)*a^2*PolyLog[4, -1 + 2
/(1 - I*a*x)])/c
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2(1+a^2x^2)}\,dx}{2c}-\frac{(ia^2)\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}-\frac{a^2\tan^{-1}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2}\,dx}{2c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}-\frac{a^2\tan^{-1}(ax)^3\log}{c}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}$$

$$= -\frac{3ia^2\tan^{-1}(ax)^2}{2c}-\frac{3a\tan^{-1}(ax)^2}{2cx}-\frac{a^2\tan^{-1}(ax)^3}{2c}-\frac{\tan^{-1}(ax)^3}{2cx^2}+\frac{ia^2\tan^{-1}(ax)^4}{4c}$$

Mathematica [A]
time = 0.29, size = 189, normalized size = 0.72

$$\frac{ia^2\left(\pi^4-96\text{ArcTan}(ax)^2+\frac{96i\text{ArcTan}(ax)^2}{ax}+\frac{32i(1+a^2x^2)\text{ArcTan}(ax)^3}{a^2x^2}-16\text{ArcTan}(ax)^4+64i\text{ArcTan}(ax)^3\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)-192i\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)-96\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)-96\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)+96i\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)+48\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(ax)}\right)\right)}{64c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] ((I/64)*a^2*(Pi^4 - 96*ArcTan[a*x]^2 + ((96*I)*ArcTan[a*x]^2)/(a*x) + ((32*
I)*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - 16*ArcTan[a*x]^4 + (64*I)*ArcTa
n[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (192*I)*ArcTan[a*x]*Log[1 - E^((
2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] -
96*PolyLog[2, E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog[3, E^((-2
*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])]))/c
```

Maple [A]
time = 82.51, size = 441, normalized size = 1.68

method	result
derivativedivides	$a^2\left(\frac{3i\arctan(ax)^2\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)-3iax+\arctan(ax)ax)(ax+i)}{2c\,a^2x^2}-\right.$
default	$a^2\left(\frac{3i\arctan(ax)^2\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)-3iax+\arctan(ax)ax)(ax+i)}{2c\,a^2x^2}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(3*I/c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2/c*arct
an(a*x)^2*(-I*arctan(a*x)-3*I*a*x+arctan(a*x)*a*x)*(I+a*x)/a^2/x^2-3*I/c*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2))+3*I/c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6/
c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c*arctan(a*x)^2-1
/c*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6/c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-6*I/c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I/c*polylog(4,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+3/c*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*I/c
*arctan(a*x)^4+3/c*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**5 + x**3), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)), x)
```

3.395 $\int \frac{\text{ArcTan}(ax)^3}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=227

$$-\frac{a^2\text{ArcTan}(ax)}{cx}-\frac{a^3\text{ArcTan}(ax)^2}{2c}-\frac{a\text{ArcTan}(ax)^2}{2cx^2}+\frac{4ia^3\text{ArcTan}(ax)^3}{3c}-\frac{\text{ArcTan}(ax)^3}{3cx^3}+\frac{a^2\text{ArcTan}(ax)^3}{cx}+\frac{a^3}{}$$

```
[Out] -a^2*arctan(a*x)/c/x-1/2*a^3*arctan(a*x)^2/c-1/2*a*arctan(a*x)^2/c/x^2+4/3*
I*a^3*arctan(a*x)^3/c-1/3*arctan(a*x)^3/c/x^3+a^2*arctan(a*x)^3/c/x+1/4*a^3
*arctan(a*x)^4/c+a^3*ln(x)/c-1/2*a^3*ln(a^2*x^2+1)/c-4*a^3*arctan(a*x)^2*ln
(2-2/(1-I*a*x))/c+4*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-2*a^3*pol
ylog(3,-1+2/(1-I*a*x))/c
```

Rubi [A]
time = 0.51, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$\frac{4ia^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c}+\frac{a^3\text{ArcTan}(ax)^4}{4c}+\frac{4ia^3\text{ArcTan}(ax)^3}{3c}-\frac{a^3\text{ArcTan}(ax)^2}{2c}-\frac{4a^3\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{2a^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{c}+\frac{a^3\log(x)}{c}+\frac{a^2\text{ArcTan}(ax)^3}{cx}-\frac{a^2\text{ArcTan}(ax)}{cx}-\frac{a^3\log\left(a^2x^2+1\right)}{2c}-\frac{\text{ArcTan}(ax)^3}{3cx^3}-\frac{a\text{ArcTan}(ax)^2}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] -((a^2*ArcTan[a*x])/(c*x)) - (a^3*ArcTan[a*x]^2)/(2*c) - (a*ArcTan[a*x]^2)/
(2*c*x^2) + (((4*I)/3)*a^3*ArcTan[a*x]^3)/c - ArcTan[a*x]^3/(3*c*x^3) + (a^
2*ArcTan[a*x]^3)/(c*x) + (a^3*ArcTan[a*x]^4)/(4*c) + (a^3*Log[x])/c - (a^3*
Log[1 + a^2*x^2])/(2*c) - (4*a^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c +
((4*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c - (2*a^3*PolyLog[3
, -1 + 2/(1 - I*a*x)])/c
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3cx^3} + a^4\int \frac{\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx + \frac{a\int \frac{\tan^{-1}(ax)^2}{x^3(1+a^2x^2)}\,dx}{c} - \frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} + \frac{a\int \frac{\tan^{-1}(ax)^2}{x^3}\,dx}{c} - \frac{a^3\int \frac{\tan^{-1}(ax)^2}{x(1+a^2x^2)}\,\mathrm{d}}{c}$$

$$= -\frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} -$$

$$= -\frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} + \frac{a^2\tan^{-1}(ax)^3}{cx} + \frac{a^3\tan^{-1}(ax)^4}{4c} -$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} +$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} +$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} +$$

$$= -\frac{a^2\tan^{-1}(ax)}{cx} - \frac{a^3\tan^{-1}(ax)^2}{2c} - \frac{a\tan^{-1}(ax)^2}{2cx^2} + \frac{4ia^3\tan^{-1}(ax)^3}{3c} - \frac{\tan^{-1}(ax)^3}{3cx^3} +$$

Mathematica [A]
time = 0.37, size = 180, normalized size = 0.79

$$\frac{a^3\left(\frac{i\pi^3}{6} - \frac{\mathrm{ArcTan}(ax)}{ax} - \frac{1}{2}\mathrm{ArcTan}(ax)^2 - \frac{\mathrm{ArcTan}(ax)^2}{2a^2x^2} - \frac{4}{3}i\mathrm{ArcTan}(ax)^3 - \frac{\mathrm{ArcTan}(ax)^3}{3a^3x^3} + \frac{\mathrm{ArcTan}(ax)^3}{ax} + \frac{1}{4}\mathrm{ArcTan}(ax)^4 - 4\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right) + \log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) - 4i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right) - 2\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (a^3*((I/6)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2/(2*a
^2*x^2) - ((4*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + ArcTan[a*x]
```

```
^3/(a*x) + ArcTan[a*x]^4/4 - 4*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])
] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*A
rcTan[a*x])] - 2*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 127.36, size = 5082, normalized size = 22.39

method	result	size
derivativedivides	Expression too large to display	5082
default	Expression too large to display	5082

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}{\left(ax \right)}}{a^{2}x^{6}+x^{4}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**3/(a**2*x**6 + x**4), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{x^4\,\left(c\,a^2\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)), x)
```

3.396 $\int \frac{x^3 \text{ArcTan}(ax)^3}{(c+a^2cx^2)^2} \, dx$

Optimal. Leaf size=270

$$\frac{3x}{8a^3c^2(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)}{8a^4c^2}-\frac{3\text{ArcTan}(ax)}{4a^4c^2(1+a^2x^2)}-\frac{3x\text{ArcTan}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)^3}{4a^4c^2}+\frac{\text{ArcTan}(ax)^3}{2a^4c^2(1+a^2x^2)}-\frac{i\text{Ar}}{}$$

```
[Out] 3/8*x/a^3/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/a^4/c^2-3/4*arctan(a*x)/a^4/c^2/(
a^2*x^2+1)-3/4*x*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/a^4/c^
2+1/2*arctan(a*x)^3/a^4/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/a^4/c^2-arctan(
a*x)^3*ln(2/(1+I*a*x))/a^4/c^2-3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))
/a^4/c^2-3/2*arctan(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c^2+3/4*I*polylog(4,1
-2/(1+I*a*x))/a^4/c^2
```

Rubi [A]
time = 0.29, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5084, 5040, 4964, 5004, 5114, 5118, 6745, 5050, 5012, 205, 211}

$$-\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^4c^2}-\frac{3\text{ArcTan}(ax)\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^4c^2}-\frac{i\text{ArcTan}(ax)^4}{4a^4c^2}-\frac{\text{ArcTan}(ax)^3}{4a^4c^2}+\frac{3\text{ArcTan}(ax)}{8a^4c^2}-\frac{\text{ArcTan}(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c^2}+\frac{3i\text{Li}_4\left(1-\frac{2}{iax+1}\right)}{4a^4c^2}+\frac{\text{ArcTan}(ax)^3}{2a^4c^2(a^2x^2+1)}-\frac{3\text{ArcTan}(ax)}{4a^4c^2(a^2x^2+1)}-\frac{3x\text{ArcTan}(ax)^2}{4a^3c^2(a^2x^2+1)}+\frac{3x}{8a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*x)/(8*a^3*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*a^4*c^2) - (3*ArcTan[a
*x])/(4*a^4*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^
2)) - ArcTan[a*x]^3/(4*a^4*c^2) + ArcTan[a*x]^3/(2*a^4*c^2*(1 + a^2*x^2)) -
 ((I/4)*ArcTan[a*x]^4)/(a^4*c^2) - (ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/(a^4*
c^2) - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/(a^4*c^2) -
(3*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/(2*a^4*c^2) + (((3*I)/4)*Poly
Log[4, 1 - 2/(1 + I*a*x)])/(a^4*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx = -\frac{\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{a^2}+\frac{\int \frac{x\tan^{-1}(ax)^3}{c+a^2cx^2}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^3}{2a^4c^2(1+a^2x^2)}-\frac{i\tan^{-1}(ax)^4}{4a^4c^2}-\frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{2a^3}-\frac{\int \frac{\tan^{-1}(ax)^3}{i-ax}\,dx}{a^3c^2}$$

$$= -\frac{3x\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)^3}{4a^4c^2}+\frac{\tan^{-1}(ax)^3}{2a^4c^2(1+a^2x^2)}-\frac{i\tan^{-1}(ax)^4}{4a^4c^2}-\frac{\tan^{-1}(ax)^3\log}{a^4c^2}$$

$$= -\frac{3\tan^{-1}(ax)}{4a^4c^2(1+a^2x^2)}-\frac{3x\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)^3}{4a^4c^2}+\frac{\tan^{-1}(ax)^3}{2a^4c^2(1+a^2x^2)}-\frac{i\tan^{-1}(ax}{4a^4c^2}$$

$$= \frac{3x}{8a^3c^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)}{4a^4c^2(1+a^2x^2)}-\frac{3x\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)^3}{4a^4c^2}+\frac{\tan^{-1}(ax}{2a^4c^2(1+a}$$

$$= \frac{3x}{8a^3c^2(1+a^2x^2)}+\frac{3\tan^{-1}(ax)}{8a^4c^2}-\frac{3\tan^{-1}(ax)}{4a^4c^2(1+a^2x^2)}-\frac{3x\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\tan^{-1}(ax)^3}{4a^4c^2}$$

Mathematica [A]
time = 0.11, size = 156, normalized size = 0.58

$$\frac{4i\text{ArcTan}(ax)^4-6\text{ArcTan}(ax)\cos(2\text{ArcTan}(ax))+4\text{ArcTan}(ax)^3\cos(2\text{ArcTan}(ax))-16\text{ArcTan}(ax)^3\log\left(1+e^{2i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(ax)}\right)-24\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(ax)}\right)-12i\text{PolyLog}\left(4,-e^{2i\text{ArcTan}(ax)}\right)+3\sin(2\text{ArcTan}(ax))-6\text{ArcTan}(ax)^2\sin(2\text{ArcTan}(ax))}{16a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*I)*ArcTan[a*x]^4 - 6*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 4*ArcTan[a*x]^3*C
os[2*ArcTan[a*x]] - 16*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I
)*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*ArcTan[a*x]*PolyLog
[3, -E^((2*I)*ArcTan[a*x])] - (12*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])] + 3
*Sin[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])/(16*a^4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 35.67, size = 936, normalized size = 3.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)+1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+1
)-3/2/c^2*(2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan(a
*x)^4-I*arctan(a*x)^2*(I+a*x)/(8*a*x-8*I)-1/8*arctan(a*x)*(I+a*x)/(a*x-I)+I
*(I+a*x)/(16*a*x-16*I)+I*arctan(a*x)^2*(a*x-I)/(8*a*x+8*I)-1/8*arctan(a*x)*
(a*x-I)/(I+a*x)-I*(a*x-I)/(16*a*x+16*I)-1/6*arctan(a*x)^3*(I*Pi*csgn(I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+I*Pi*csgn(I/((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))^3-2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-1-4*ln(2))-I*arctan(a*x)^2*
polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2
*x^2+1))+1/2*I*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^3}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

3.397 $\int \frac{x^2 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=135

$$\frac{3}{8a^3c^2(1+a^2x^2)}+\frac{3x\text{ArcTan}(ax)}{4a^2c^2(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)^2}{8a^3c^2}-\frac{3\text{ArcTan}(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{x\text{ArcTan}(ax)^3}{2a^2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^4}{8a^3c^2}$$

```
[Out] 3/8/a^3/c^2/(a^2*x^2+1)+3/4*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+3/8*arctan(a*
x)^2/a^3/c^2-3/4*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)^3/a^2/
c^2/(a^2*x^2+1)+1/8*arctan(a*x)^4/a^3/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5056, 5050, 5012, 267}

$$\frac{\text{ArcTan}(ax)^4}{8a^3c^2}+\frac{3\text{ArcTan}(ax)^2}{8a^3c^2}-\frac{x\text{ArcTan}(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{3x\text{ArcTan}(ax)}{4a^2c^2(a^2x^2+1)}-\frac{3\text{ArcTan}(ax)^2}{4a^3c^2(a^2x^2+1)}+\frac{3}{8a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] 3/(8*a^3*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2)) +
 (3*ArcTan[a*x]^2)/(8*a^3*c^2) - (3*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2*x^2)
) - (x*ArcTan[a*x]^3)/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a^3*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx = -\frac{x\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^4}{8a^3c^2} + \frac{3\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{2a}$$
$$= -\frac{3\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^4}{8a^3c^2} + \frac{3\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx}{2a^2}$$
$$= \frac{3x\tan^{-1}(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^2}{8a^3c^2} - \frac{3\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^4}{8a^3c^2}$$
$$= \frac{3}{8a^3c^2(1+a^2x^2)} + \frac{3x\tan^{-1}(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^2}{8a^3c^2} - \frac{3\tan^{-1}(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(a}{2a^2c^2(1+}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.55

$$\frac{3+6ax\text{ArcTan}(ax)+3(-1+a^2x^2)\text{ArcTan}(ax)^2-4ax\text{ArcTan}(ax)^3+(1+a^2x^2)\text{ArcTan}(ax)^4}{8a^3c^2(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3 + 6*a*x*ArcTan[a*x] + 3*(-1 + a^2*x^2)*ArcTan[a*x]^2 - 4*a*x*ArcTan[a*x]
^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*a^3*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.77, size = 114, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{\arctan(ax)^3 ax}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{ax\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4(a^2x^2+1)}\right)}{2c^2}}{a^3}$

default	$\frac{-\frac{\arctan(ax)^3 ax}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{ax\arctan(ax)}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4\left(a^2x^2+1\right)}\right)}{2c^2}}{a^3}$
risch	$\frac{\ln(iax+1)^4}{128c^2a^3}-\frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax)\ln(iax+1)^3}{32a^3c^2(a^2x^2+1)}+\frac{3\left(a^2x^2\ln(-iax+1)^2+4iax\ln(-iax+1)-2a^2x\right.}{64a^3c^2(ax+i)(ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)+1/2/c^2*arctan(a*x)^4-3/2/c^2
*(1/4*arctan(a*x)^4+1/2*arctan(a*x)^2/(a^2*x^2+1)-1/2*a*x/(a^2*x^2+1)*arcta
n(a*x)-1/4*arctan(a*x)^2-1/4/(a^2*x^2+1)))
```

Maxima [A]
time = 0.52, size = 218, normalized size = 1.61

$$-\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^3-\frac{3\left((a^2x^2+1)\arctan(ax)^2+1\right)a\arctan(ax)^2}{4\left(a^6c^2x^2+a^4c^2\right)}-\frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4+3\,(a^2x^2+1)\arctan(ax)^2-3\right)a^2}{a^8c^2x^2+a^6c^2}-\frac{2\left(2\,(a^2x^2+1)\arctan(ax)^3+3\,ax+3\,(a^2x^2+1)\arctan(ax)\right)a\arctan(ax)}{a^7c^2x^2+a^5c^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^3 - 3/
4*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(a*x)^2/(a^6*c^2*x^2 + a^4*c^2)
 - 1/8*(((a^2*x^2 + 1)*arctan(a*x)^4 + 3*(a^2*x^2 + 1)*arctan(a*x)^2 - 3)*a
^2/(a^8*c^2*x^2 + a^6*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(
a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^2*x^2 + a^5*c^2))*a
```

Fricas [A]
time = 3.25, size = 76, normalized size = 0.56

$$-\frac{4\,ax\arctan(ax)^3-(a^2x^2+1)\arctan(ax)^4-6\,ax\arctan(ax)-3\,(a^2x^2-1)\arctan(ax)^2-3}{8\,(a^5c^2x^2+a^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*arctan(a*x)^3 - (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
 - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^5*c^2*x^2 + a^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**2*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.45, size = 119, normalized size = 0.88

$$\frac{3}{2\,a^2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)}+\mathrm{atan}(a\,x)^2\,\left(\frac{3}{8\,a^3\,c^2}-\frac{3}{4\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)+\frac{\mathrm{atan}(a\,x)^4}{8\,a^3\,c^2}+\frac{3\,x\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}-\frac{x\,\mathrm{atan}(a\,x)^3}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
[Out] 3/(2*a^2*(4*a*c^2 + 4*a^3*c^2*x^2)) + atan(a*x)^2*(3/(8*a^3*c^2) - 3/(4*a^5
*c^2*(1/a^2 + x^2))) + atan(a*x)^4/(8*a^3*c^2) + (3*x*atan(a*x))/(4*a^4*c^2
*(1/a^2 + x^2)) - (x*atan(a*x)^3)/(2*a^4*c^2*(1/a^2 + x^2))
```

3.398 $\int \frac{x\mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{3x}{8ac^2(1+a^2x^2)}-\frac{3\text{ArcTan}(ax)}{8a^2c^2}+\frac{3\text{ArcTan}(ax)}{4a^2c^2(1+a^2x^2)}+\frac{3x\text{ArcTan}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^3}{4a^2c^2}-\frac{\text{ArcTan}(ax)^3}{2a^2c^2(1+a^2x^2)}$$

```
[Out] -3/8*x/a/c^2/(a^2*x^2+1)-3/8*arctan(a*x)/a^2/c^2+3/4*arctan(a*x)/a^2/c^2/(a
^2*x^2+1)+3/4*x*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^3/a^2/c^2-1
/2*arctan(a*x)^3/a^2/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5050, 5012, 205, 211}

$$-\frac{\text{ArcTan}(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{3x\text{ArcTan}(ax)^2}{4ac^2(a^2x^2+1)}+\frac{3\text{ArcTan}(ax)}{4a^2c^2(a^2x^2+1)}+\frac{\text{ArcTan}(ax)^3}{4a^2c^2}-\frac{3\text{ArcTan}(ax)}{8a^2c^2}-\frac{3x}{8ac^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a*c^2*(1 + a^2*x^2)) - (3*ArcTan[a*x])/(8*a^2*c^2) + (3*ArcTan[a*
x])/(4*a^2*c^2*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2))
 + ArcTan[a*x]^3/(4*a^2*c^2) - ArcTan[a*x]^3/(2*a^2*c^2*(1 + a^2*x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx &= -\frac{\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)}+\frac{3\int\frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx}{2a}\\ &= \frac{3x\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)}-\frac{3}{2}\int\frac{x\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx\\ &= \frac{3\tan^{-1}(ax)}{4a^2c^2(1+a^2x^2)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2(1+a^2x^2)}-\frac{3\int\frac{1}{(c+a^2cx^2)^2}\,d}{4a}\\ &= -\frac{3x}{8ac^2(1+a^2x^2)}+\frac{3\tan^{-1}(ax)}{4a^2c^2(1+a^2x^2)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}-\frac{\tan^{-1}(ax)^3}{2a^2c^2(1+a^2}\\ &= -\frac{3x}{8ac^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)}{8a^2c^2}+\frac{3\tan^{-1}(ax)}{4a^2c^2(1+a^2x^2)}+\frac{3x\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^3}{4a^2c^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.51

$$\frac{-3ax+(3-3a^2x^2)\,\mathrm{ArcTan}(ax)+6ax\mathrm{ArcTan}(ax)^2+2(-1+a^2x^2)\,\mathrm{ArcTan}(ax)^3}{8a^2c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(-1 + a^2*x
^2)*ArcTan[a*x]^3)/(8*a^2*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.82, size = 101, normalized size = 0.76

method	result

derivativedivides	$\frac{-\frac{\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\frac{3\arctan(ax)^2ax}{2(2a^2x^2+2)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2(2a^2x^2+2)}-\frac{3ax}{8(a^2x^2+1)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\frac{3\arctan(ax)^2ax}{2(2a^2x^2+2)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2(2a^2x^2+2)}-\frac{3ax}{8(a^2x^2+1)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$
risch	$\frac{i(a^2x^2-1)\ln(iax+1)^3}{32a^2c^2(a^2x^2+1)}-\frac{3i(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax)\ln(iax+1)^2}{32(ax+i)a^2c^2(ax-i)}+\frac{3i\left(-4+a^2x^2\ln(-iax+1)^2-\ln(\right.}{32(ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)+3/2/c^2*(1/2*arctan(a*x)^2*a*x/(a
^2*x^2+1)+1/6*arctan(a*x)^3+1/2*arctan(a*x)/(a^2*x^2+1)-1/4*a*x/(a^2*x^2+1)
-1/4*arctan(a*x)))
```

Maxima [A]
time = 0.51, size = 174, normalized size = 1.31

$$\frac{3\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan(ax)^2}{4ac}+\frac{\frac{\left(2(a^2x^2+1)\arctan(ax)^3-3ax-3(a^2x^2+1)\arctan(ax)\right)a^2}{a^5cx^2+a^3c}-\frac{6\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)}{a^4cx^2+a^2c}}{8ac}-\frac{\arctan(ax)^3}{2(a^2cx^2+c)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 3/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)^2/(a*c) + 1/8*((2*(
a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^5*
c*x^2 + a^3*c) - 6*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a^4*c*x
^2 + a^2*c))/(a*c) - 1/2*arctan(a*x)^3/((a^2*c*x^2 + c)*a^2*c)
```

Fricas [A]
time = 5.04, size = 69, normalized size = 0.52

$$\frac{6\,ax\arctan(ax)^2+2\,(a^2x^2-1)\arctan(ax)^3-3\,ax-3\,(a^2x^2-1)\arctan(ax)}{8\,(a^4c^2x^2+a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 - 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x
^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{x\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.43, size = 114, normalized size = 0.86

$$\mathrm{atan}(a\,x)^3\,\left(\frac{1}{4\,a^2\,c^2}-\frac{1}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)-\frac{3\,x}{2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)}-\frac{3\,\mathrm{atan}(a\,x)}{8\,a^2\,c^2}+\frac{3\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}+\frac{3\,x\,\mathrm{atan}(a\,x)^2}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

```
[Out] atan(a*x)^3*(1/(4*a^2*c^2) - 1/(2*a^4*c^2*(1/a^2 + x^2))) - (3*x)/(2*(4*a*c
^2 + 4*a^3*c^2*x^2)) - (3*atan(a*x))/(8*a^2*c^2) + (3*atan(a*x))/(4*a^4*c^2
*(1/a^2 + x^2)) + (3*x*atan(a*x)^2)/(4*a^3*c^2*(1/a^2 + x^2))
```

3.399 $\int \frac{\text{ArcTan}(ax)^3}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=129

$$-\frac{3}{8ac^2(1+a^2x^2)}-\frac{3x\text{ArcTan}(ax)}{4c^2(1+a^2x^2)}-\frac{3\text{ArcTan}(ax)^2}{8ac^2}+\frac{3\text{ArcTan}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)^3}{2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^4}{8ac^2}$$

```
[Out] -3/8/a/c^2/(a^2*x^2+1)-3/4*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/8*arctan(a*x)^2/
a/c^2+3/4*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)^3/c^2/(a^2*x^2+
1)+1/8*arctan(a*x)^4/a/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5012, 5050, 267}

$$\frac{x\text{ArcTan}(ax)^3}{2c^2(a^2x^2+1)}+\frac{3\text{ArcTan}(ax)^2}{4ac^2(a^2x^2+1)}-\frac{3x\text{ArcTan}(ax)}{4c^2(a^2x^2+1)}-\frac{3}{8ac^2(a^2x^2+1)}+\frac{\text{ArcTan}(ax)^4}{8ac^2}-\frac{3\text{ArcTan}(ax)^2}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] -3/(8*a*c^2*(1 + a^2*x^2)) - (3*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) - (3*A
rcTan[a*x]^2)/(8*a*c^2) + (3*ArcTan[a*x]^2)/(4*a*c^2*(1 + a^2*x^2)) + (x*Ar
cTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
```

```
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{x\tan^{-1}(ax)^3}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{1}{2}(3a)\int \frac{x\tan^{-1}(ax)^2}{(c+a^2cx^2)^2}\,dx$$
$$=\frac{3\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^3}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^4}{8ac^2}-\frac{3}{2}\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^2}\,dx$$
$$=-\frac{3x\tan^{-1}(ax)}{4c^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)^2}{8ac^2}+\frac{3\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^3}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^4}{8ac^2}+\frac{1}{4}$$
$$=-\frac{3}{8ac^2(1+a^2x^2)}-\frac{3x\tan^{-1}(ax)}{4c^2(1+a^2x^2)}-\frac{3\tan^{-1}(ax)^2}{8ac^2}+\frac{3\tan^{-1}(ax)^2}{4ac^2(1+a^2x^2)}+\frac{x\tan^{-1}(ax)^3}{2c^2(1+a^2x^2)}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.55

$$\frac{-3-6ax\text{ArcTan}(ax)+(3-3a^2x^2)\,\text{ArcTan}(ax)^2+4ax\text{ArcTan}(ax)^3+(1+a^2x^2)\,\text{ArcTan}(ax)^4}{8c^2(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
[Out] (-3 - 6*a*x*ArcTan[a*x] + (3 - 3*a^2*x^2)*ArcTan[a*x]^2 + 4*a*x*ArcTan[a*x]
^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*c^2*(a + a^3*x^2))
```

Maple [A]
time = 0.77, size = 114, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^3ax}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$
default	$\frac{\frac{\arctan(ax)^3ax}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$
risch	$\frac{\ln(iax+1)^4}{128a\,c^2}-\frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^3}{32c^2(a^2x^2+1)a}+\frac{3\left(a^2x^2\ln(-iax+1)^2+2a^2x^2+\ln(-iax+1)^2-4ia\right.}{64c^2(ax+i)(ax-i)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)+1/2/c^2*arctan(a*x)^4-3/2/c^2*(1
/4*arctan(a*x)^4-1/2*arctan(a*x)^2/(a^2*x^2+1)+1/2*a*x/(a^2*x^2+1)*arctan(a
*x)+1/4*arctan(a*x)^2+1/4/(a^2*x^2+1)))
```

Maxima [A]
time = 0.52, size = 213, normalized size = 1.65

$$\frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^3-\frac{3\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)^2}{4\left(a^4c^2x^2+a^2c^2\right)}-\frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4-3\left(a^2x^2+1\right)\arctan(ax)^2+3\right)a^2}{a^6c^2x^2+a^4c^2}-\frac{2\left(2\left(a^2x^2+1\right)\arctan(ax)^3-3\,ax-3\left(a^2x^2+1\right)\arctan(ax)\right)a\arctan(ax)}{a^5c^2x^2+a^3c^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^3 - 3/4*((a^2
*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)^2/(a^4*c^2*x^2 + a^2*c^2) - 1/8*
(((a^2*x^2 + 1)*arctan(a*x)^4 - 3*(a^2*x^2 + 1)*arctan(a*x)^2 + 3)*a^2/(a^6
*c^2*x^2 + a^4*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2
 + 1)*arctan(a*x))*a*arctan(a*x)/(a^5*c^2*x^2 + a^3*c^2))*a
```

Fricas [A]
time = 3.59, size = 73, normalized size = 0.57

$$\frac{4\,ax\arctan(ax)^3+(a^2x^2+1)\arctan(ax)^4-6\,ax\arctan(ax)-3\left(a^2x^2-1\right)\arctan(ax)^2-3}{8\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*x*arctan(a*x)^3 + (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
- 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.43, size = 119, normalized size = 0.92

$$\frac{\operatorname{atan}(a\,x)^4}{8\,a\,c^2}-\operatorname{atan}(a\,x)^2\left(\frac{3}{8\,a\,c^2}-\frac{3}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)-\frac{3}{2\,a\,\left(4\,a^2\,c^2\,x^2+4\,c^2\right)}-\frac{3\,x\,\operatorname{atan}(a\,x)}{4\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)}+\frac{x\,\operatorname{atan}(a\,x)^3}{2\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2)^2,x)
```

```
[Out] atan(a*x)^4/(8*a*c^2) - atan(a*x)^2*(3/(8*a*c^2) - 3/(4*a^3*c^2*(1/a^2 + x^
2))) - 3/(2*a*(4*c^2 + 4*a^2*c^2*x^2)) - (3*x*atan(a*x))/(4*a^2*c^2*(1/a^2
+ x^2)) + (x*atan(a*x)^3)/(2*a^2*c^2*(1/a^2 + x^2))
```

3.400 $\int \frac{\textbf{ArcTan}(ax)^3}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=240

$$\frac{3ax}{8c^2(1+a^2x^2)}+\frac{3\text{ArcTan}(ax)}{8c^2}-\frac{3\text{ArcTan}(ax)}{4c^2(1+a^2x^2)}-\frac{3ax\text{ArcTan}(ax)^2}{4c^2(1+a^2x^2)}-\frac{\text{ArcTan}(ax)^3}{4c^2}+\frac{\text{ArcTan}(ax)^3}{2c^2(1+a^2x^2)}-\frac{i\text{ArcTan}}{4c}$$

```
[Out] 3/8*a*x/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/c^2-3/4*arctan(a*x)/c^2/(a^2*x^2+1)
-3/4*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/c^2+1/2*arctan(a*x
)^3/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^2+arctan(a*x)^3*ln(2-2/(1-I*a*x))
/c^2-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2+3/2*arctan(a*x)*poly
log(3,-1+2/(1-I*a*x))/c^2+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.31, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5086, 5044, 4988, 5004, 5112, 5116, 6745, 5050, 5012, 205, 211}

$$\frac{\text{ArcTan}(ax)^3}{2c^2(a^2x^2+1)}-\frac{3ax\text{ArcTan}(ax)^2}{4c^2(a^2x^2+1)}-\frac{3\text{ArcTan}(ax)}{4c^2(a^2x^2+1)}+\frac{3ax}{8c^2(a^2x^2+1)}-\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^2}+\frac{3\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^2}-\frac{i\text{ArcTan}(ax)^4}{4c^2}-\frac{\text{ArcTan}(ax)^3}{4c^2}+\frac{3\text{ArcTan}(ax)}{8c^2}+\frac{\text{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{3i\text{Li}_4\left(\frac{2}{1-iax}-1\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] (3*a*x)/(8*c^2*(1 + a^2*x^2)) + (3*ArcTan[a*x])/(8*c^2) - (3*ArcTan[a*x])/(
4*c^2*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) - ArcTan
[a*x]^3/(4*c^2) + ArcTan[a*x]^3/(2*c^2*(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^
4)/c^2 + (ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*ArcTan[a*x
]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/
(1 - I*a*x)])/(2*c^2) + (((3*I)/4)*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
```

```
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx &= -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)}\,dx}{c}\\
&= \frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^4}{4c^2}-\frac{1}{2}(3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{i\int \frac{\tan^{-1}(ax)^3}{x(i+ax)}\,dx}{c^2}\\
&= -\frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{4c^2}+\frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^4}{4c^2}+\frac{\tan^{-1}(ax)^3\log(}{c^2}\\
&= -\frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{4c^2}+\frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{i\tan^{-1}(ax)^4}{4c^2}+\\
&= \frac{3ax}{8c^2\left(1+a^2x^2\right)}-\frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{4c^2}+\frac{\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\\
&= \frac{3ax}{8c^2\left(1+a^2x^2\right)}+\frac{3\tan^{-1}(ax)}{8c^2}-\frac{3\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ax\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{4c^2}+\frac{}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 156, normalized size = 0.65

$$
\frac{-i\pi^4+16i\text{ArcTan}(ax)^4-24\text{ArcTan}(ax)\cos(2\text{ArcTan}(ax))+16\text{ArcTan}(ax)^3\cos(2\text{ArcTan}(ax))+64\text{ArcTan}(ax)^3\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+96i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+96\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(ax)}\right)+12\sin(2\text{ArcTan}(ax))-24\text{ArcTan}(ax)^2\sin(2\text{ArcTan}(ax))}{64c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] ((-I)*Pi^4 + (16*I)*ArcTan[a*x]^4 - 24*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 16*
ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 96*Arc
Tan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*A
rcTan[a*x])] + 12*Sin[2*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])
/(64*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 53.58, size = 1905, normalized size = 7.94

method	result	size
derivativedivides	Expression too large to display	1905
default	Expression too large to display	1905

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)-1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+1)+1/c^2
*arctan(a*x)^3*ln(a*x)-3/2/c^2*(1/6*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/6*I*arctan(a*x)^3*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+1/3*I
*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-1/6*I*arctan(a*x)^3*Pi*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))+1/6*arctan(a*x)^3-1/3*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/8*arctan(a*x)*(I+a*x)/(a
*x-I)+I*(I+a*x)/(16*a*x-16*I)-1/8*arctan(a*x)*(a*x-I)/(I+a*x)-I*(a*x-I)/(16
*a*x+16*I)+1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))^2-1/6*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/3*I*arctan(
a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^2+1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1))-I*arctan(a*x)^2*(I+a*x)/(8*a*x-8*I)+I*arctan(a
*x)^2*(a*x-I)/(8*a*x+8*I)+1/3*I*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2
+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))^3+1/6*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/
```

```
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-1/3*I*arctan(a*x)^3*Pi*csgn((
(1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-1/6*I*arctan(a*x)
^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+1/3*I*arctan(a*x)^3*Pi*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)
^2/(a^2*x^2+1)+1))^2-1/3*I*arctan(a*x)^3*Pi+2*I*arctan(a*x)^2*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*po
lylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*polylog(3,(1+I*a*x)/(a^
2*x^2+1)^(1/2))-2/3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/3*arc
tan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/3*arctan(a*x)^3*ln(2)+1/6*I*
arctan(a*x)^4-4*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,(1+
I*a*x)/(a^2*x^2+1)^(1/2))+2/3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}{\left(ax \right)}}{a^{4}x^{5}+2a^{2}x^{3}+x}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**2,x)
[Out] Integral(atan(a*x)**3/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{x\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2), x)
```

3.401 $\int \frac{\text{ArcTan}(ax)^3}{x^2(c+a^2cx^2)^2} \, dx$

Optimal. Leaf size=234

$$\frac{3a}{8c^2(1+a^2x^2)}+\frac{3a^2x\text{ArcTan}(ax)}{4c^2(1+a^2x^2)}+\frac{3a\text{ArcTan}(ax)^2}{8c^2}-\frac{3a\text{ArcTan}(ax)^2}{4c^2(1+a^2x^2)}-\frac{ia\text{ArcTan}(ax)^3}{c^2}-\frac{\text{ArcTan}(ax)^3}{c^2x}-\frac{a^2x}{2c}$$

```
[Out] 3/8*a/c^2/(a^2*x^2+1)+3/4*a^2*x*arctan(a*x)/c^2/(a^2*x^2+1)+3/8*a*arctan(a*
x)^2/c^2-3/4*a*arctan(a*x)^2/c^2/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^2-arctan(a
*x)^3/c^2/x-1/2*a^2*x*arctan(a*x)^3/c^2/(a^2*x^2+1)-3/8*a*arctan(a*x)^4/c^2
+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2-3*I*a*arctan(a*x)*polylog(2,-1+2/(
1-I*a*x))/c^2+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.33, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5086, 5038, 4946, 5044, 4988, 5004, 5112, 6745, 5012, 5050, 267}

$$-\frac{a^2x\text{ArcTan}(ax)^3}{2c^2(a^2x^2+1)}-\frac{3a\text{ArcTan}(ax)^2}{4c^2(a^2x^2+1)}+\frac{3a^2x\text{ArcTan}(ax)}{4c^2(a^2x^2+1)}+\frac{3a}{8c^2(a^2x^2+1)}-\frac{3ia\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}-\frac{3a\text{ArcTan}(ax)^4}{8c^2}-\frac{\text{ArcTan}(ax)^3}{c^2x}-\frac{ia\text{ArcTan}(ax)^3}{c^2}+\frac{3a\text{ArcTan}(ax)^2}{8c^2}+\frac{3a\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{3a\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] (3*a)/(8*c^2*(1 + a^2*x^2)) + (3*a^2*x*ArcTan[a*x])/(4*c^2*(1 + a^2*x^2)) +
 (3*a*ArcTan[a*x]^2)/(8*c^2) - (3*a*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(I*a*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(c^2*x) - (a^2*x*ArcTan[a*x]^3)/(2*
c^2*(1 + a^2*x^2)) - (3*a*ArcTan[a*x]^4)/(8*c^2) + (3*a*ArcTan[a*x]^2*Log[2
 - 2/(1 - I*a*x)])/c^2 - ((3*I)*a*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
])/c^2 + (3*a*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
```

```
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx}{c}$$

$$= -\frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{a\tan^{-1}(ax)^4}{8c^2}+\frac{1}{2}\left(3a^3\right)\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^2}\,dx}{c^2}-$$

$$= -\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{\tan^{-1}(ax)^3}{c^2x}-\frac{a^2x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^4}{8c^2}+\frac{1}{2}\left(3a^2\right)\int$$

$$= \frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}+\frac{3a\tan^{-1}(ax)^2}{8c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)^3}{c^2}-\frac{\tan^{-1}(ax)^3}{c^2x}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}+\frac{3a\tan^{-1}(ax)^2}{8c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)}{c^2}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}+\frac{3a\tan^{-1}(ax)^2}{8c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)}{c^2}$$

$$= \frac{3a}{8c^2\left(1+a^2x^2\right)}+\frac{3a^2x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}+\frac{3a\tan^{-1}(ax)^2}{8c^2}-\frac{3a\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{ia\tan^{-1}(ax)}{c^2}$$

Mathematica [A]
time = 0.23, size = 157, normalized size = 0.67

$$\frac{a\left(-2i\pi^3+16i\text{ArcTan}(ax)^3-\frac{16\text{ArcTan}(ax)^3}{ax}-6\text{ArcTan}(ax)^4+3\cos(2\text{ArcTan}(ax))-6\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax))+48\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+48i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)+24\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)+6\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax))-4\text{ArcTan}(ax)^3\sin(2\text{ArcTan}(ax))\right)}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a*((-2*I)*Pi^3 + (16*I)*ArcTan[a*x]^3 - (16*ArcTan[a*x]^3)/(a*x) - 6*ArcTa
n[a*x]^4 + 3*Cos[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 48*A
rcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[2
, E^((-2*I)*ArcTan[a*x])] + 24*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 6*ArcTa
n[a*x]*Sin[2*ArcTan[a*x]] - 4*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]]))/(16*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.32, size = 1849, normalized size = 7.90

method	result	size
derivativedivides	Expression too large to display	1849
default	Expression too large to display	1849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)-3/2/c^2*arctan(a*x)^4-1/c^2*arcta
n(a*x)^3/a/x-3/2/c^2*(1/2*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)*Pi*arctan(a*x)^2-I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-1/4*arctan(a*x)^2+arctan(a*x)^2*ln(a^2*x^2+
1)-2*arctan(a*x)^2*ln(2)+1/2*arctan(a*x)^2/(a^2*x^2+1)-2*arctan(a*x)^2*ln((
1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi*arctan(a*x)^2-1/2*I*arctan(a*x)^2*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2-1/2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*arctan(a*x)^2*
Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-1/
2*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+1/16*(I+a*x)/(a*x-I)+1/16*(
a*x-I)/(I+a*x)+2/3*I*arctan(a*x)^3-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+I*arctan(a*x)^2*Pi*csgn(((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-I*arctan(a*x)^2*Pi*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-4*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*
arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/2*I*arctan(a*x)^2*Pi*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-1/2*I*arctan
(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+2*arctan(a*x)^2*ln((1+I*
a*x)^2/(a^2*x^2+1)-1)-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*a
```

```
rctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*arctan(a*x)^2+4*I*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*arctan(a*x)^2+I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-2*arctan(a*x)^2*ln(a*x)+I*arctan(a*x)*(
I+a*x)/(8*a*x-8*I)-I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)-3/4*arctan(a*x)^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] -1/2048*(240*(a^3*x^3 + a*x)*arctan(a*x)^4 - 9*(a^3*x^3 + a*x)*log(a^2*x^2
+ 1)^4 + 128*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 24*(3*(a^3*x^3 + a*x)*arctan(a
*x)^2 + 4*(3*a^2*x^2 + 2)*arctan(a*x))*log(a^2*x^2 + 1)^2 - 4*(a^2*c^2*x^3
+ c^2*x)*(72*a^5*(a^2/(a^8*c^2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c^2*x
^2 + a^4*c^2)) - 18432*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2*x^2 +
1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 4608*a^5*integrate(1/256*x
^5*log(a^2*x^2 + 1)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) + 36864*a
^4*integrate(1/256*x^4*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2
), x) + 9216*a^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^
2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 73728*a^4*integrate(1/256*x^4*arctan
(a*x)*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) + 9*a^3*
log(a^2*x^2 + 1)^3/(a^4*c^2*x^2 + a^2*c^2) + 27*(2*a^4*(a^2/(a^10*c^2*x^2 +
 a^8*c^2) + log(a^2*x^2 + 1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*log(a^2*x^2 + 1
)^2/(a^6*c^2*x^2 + a^4*c^2))*a^3 - 18432*a^3*integrate(1/256*x^3*arctan(a*x
)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) + 73728*a^
3*integrate(1/256*x^3*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)
, x) + 36*a^3*log(a^2*x^2 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) + 36864*a^2*integr
ate(1/256*x^2*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) + 9
216*a^2*integrate(1/256*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^6 + 2
*a^2*c^2*x^4 + c^2*x^2), x) - 49152*a^2*integrate(1/256*x^2*arctan(a*x)*log
(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) + 49152*a*integra
te(1/256*x*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 1228
8*a*integrate(1/256*x*log(a^2*x^2 + 1)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2
*x^2), x) + 114688*integrate(1/256*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x
^4 + c^2*x^2), x) + 12288*integrate(1/256*arctan(a*x)*log(a^2*x^2 + 1)^2/(a
^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)))/(a^2*c^2*x^3 + c^2*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^6+2a^2x^4+x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**2,x)
[Out] Integral(atan(a*x)**3/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2), x)
```

3.402 $\int \frac{\mathbf{ArcTan}(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=374

$$-\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}-\frac{3a^2\mathrm{ArcTan}(ax)}{8c^2}+\frac{3a^2\mathrm{ArcTan}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\mathrm{ArcTan}(ax)^2}{2c^2}-\frac{3a\mathrm{ArcTan}(ax)^2}{2c^2x}+\frac{3a^3x\mathrm{ArcTan}(ax}{4c^2\left(1+a^2x^2\right)}$$

```
[Out] -3/8*a^3*x/c^2/(a^2*x^2+1)-3/8*a^2*arctan(a*x)/c^2+3/4*a^2*arctan(a*x)/c^2/
(a^2*x^2+1)-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2-3/2*a*arctan(a*x)^2/c^2
/x+3/4*a^3*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)^3/c^2-1/2*ar
ctan(a*x)^3/c^2/x^2-1/2*a^2*arctan(a*x)^3/c^2/(a^2*x^2+1)-3/2*I*a^2*arctan(
a*x)^2/c^2+3*a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-2*a^2*arctan(a*x)^3*ln(2
-2/(1-I*a*x))/c^2-3/2*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^2+1/2*I*a^2*arctan(
a*x)^4/c^2-3*a^2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c^2+3*I*a^2*arctan(a
*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.73, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5086, 5038, 4946, 5044, 4988, 2497, 5004, 5112, 5116, 6745, 5050, 5012, 205, 211}

$$\frac{3ia^2\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}-\frac{3a^2\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(\frac{2}{1-iax}-1\right)}{c^2}-\frac{a^2\mathrm{ArcTan}(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{3a^2\mathrm{ArcTan}(ax)}{4c^2\left(a^2x^2+1\right)}+\frac{ia^2\mathrm{ArcTan}(ax)^4}{2c^2}-\frac{a^2\mathrm{ArcTan}(ax)^3}{4c^2}-\frac{3ia^2\mathrm{ArcTan}(ax)^2}{2c^2}-\frac{3a^2\mathrm{ArcTan}(ax)}{8c^2}-\frac{2a^2\mathrm{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{3a^2\mathrm{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{3ia^2\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^2}-\frac{3ia^2\mathrm{Li}_4\left(\frac{2}{1-iax}-1\right)}{2c^2}+\frac{3a^3x\mathrm{ArcTan}(ax)^2}{4c^2\left(a^2x^2+1\right)}-\frac{3a^3x}{8c^2\left(a^2x^2+1\right)}-\frac{\mathrm{ArcTan}(ax)^3}{2c^2x^2}-\frac{3a\mathrm{ArcTan}(ax)^2}{2c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (-3*a^3*x)/(8*c^2*(1 + a^2*x^2)) - (3*a^2*ArcTan[a*x])/(8*c^2) + (3*a^2*Arc
Tan[a*x])/(4*c^2*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan[a*x]^2)/c^2 - (3*a*
ArcTan[a*x]^2)/(2*c^2*x) + (3*a^3*x*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) -
(a^2*ArcTan[a*x]^3)/(4*c^2) - ArcTan[a*x]^3/(2*c^2*x^2) - (a^2*ArcTan[a*x]^
3)/(2*c^2*(1 + a^2*x^2)) + ((I/2)*a^2*ArcTan[a*x]^4)/c^2 + (3*a^2*ArcTan[a*
x]*Log[2 - 2/(1 - I*a*x)])/c^2 - (2*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)
])/c^2 - (((3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 + ((3*I)*a^2*Ar
cTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^2 - (3*a^2*ArcTan[a*x]*PolyLo
g[3, -1 + 2/(1 - I*a*x)])/c^2 - (((3*I)/2)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x
)])/c^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^2}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx}{c}\\
&= a^4\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)}\,dx}{c}\\
&= -\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}+\frac{1}{2}\left(3a^3\right)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx+\frac{(3a)\int \frac{\tan^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx}{2c^2}-\\
&= \frac{3a^3x\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}+\frac{a^2\tan^{-1}(ax)^3}{4c^2}-\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\frac{a^2\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)}-\frac{1}{2}\left(3a^4\right)\int \frac{x\tan}{(c+}\\
&= \frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)}-\frac{a^2\tan^{-1}(ax)^3}{4c^2}-\frac{\tan^{-1}(ax)^3}{2c^2x^2}-\\
&= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(ax)^2}{2c^2x}+\frac{3a^3x\tan^{-1}(}{4c^2\left(1+a^2\right.}\\
&= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}-\frac{3a^2\tan^{-1}(ax)}{8c^2}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(a}{2c^2x}\\
&= -\frac{3a^3x}{8c^2\left(1+a^2x^2\right)}-\frac{3a^2\tan^{-1}(ax)}{8c^2}+\frac{3a^2\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)}-\frac{3ia^2\tan^{-1}(ax)^2}{2c^2}-\frac{3a\tan^{-1}(a}{2c^2x}
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 243, normalized size = 0.65

$$\frac{a^2\left(i\pi^4-48i\text{ArcTan}(ax)^2-\frac{48\text{ArcTan}(ax)^2}{ax}-\frac{16(1+a^2x^2)\text{ArcTan}(ax)^3}{a^2x^2}-16i\text{ArcTan}(ax)^4+12\text{ArcTan}(ax)\cos(2\text{ArcTan}(ax))-8\text{ArcTan}(ax)^3\cos(2\text{ArcTan}(ax))-64\text{ArcTan}(ax)^3\log\left(1-e^{-2i\text{ArcTan}(ax)}\right)+96\text{ArcTan}(ax)\log\left(1-e^{2i\text{ArcTan}(ax)}\right)-96i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(ax)}\right)-96\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right)+48i\text{PolyLog}\left(4,e^{-2i\text{ArcTan}(ax)}\right)-6\sin(2\text{ArcTan}(ax))+12\text{ArcTan}(ax)^2\sin(2\text{ArcTan}(ax))\right)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^2*(I*Pi^4 - (48*I)*ArcTan[a*x]^2 - (48*ArcTan[a*x]^2)/(a*x) - (16*(1 + a
^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (16*I)*ArcTan[a*x]^4 + 12*ArcTan[a*x]*Co
s[2*ArcTan[a*x]] - 8*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 64*ArcTan[a*x]^3*Lo
g[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x]
)] - (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyL
og[2, E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a
```

```
*x])] + (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - 6*Sin[2*ArcTan[a*x]] +
12*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]]))/(32*c^2)
```

Maple [A]
time = 45.96, size = 531, normalized size = 1.42

method	result
derivativedivides	$a^2\left(-\frac{3i\,\mathrm{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\,\mathrm{arct}\right.}{}\right.$
default	$a^2\left(-\frac{3i\,\mathrm{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\,\mathrm{arct}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-3*I/c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/32*(6*I*arctan(a*x)^
2+4*arctan(a*x)^3-3*I-6*arctan(a*x))*(a*x-I)/c^2/(I+a*x)+1/32*(-6*I*arctan(
a*x)^2+4*arctan(a*x)^3+3*I-6*arctan(a*x))*(I+a*x)/c^2/(a*x-I)-1/2/c^2*arcta
n(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*a*x-3*I*a*x)*(I+a*x)/a^2/x^2-12*I/c^2*
polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^2*arctan(a*x)*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3*I/c^2*arctan(a*x)^2+3/c^2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2
*x^2+1)^(1/2))+6*I/c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-3*I/c^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/c^2*arctan(a*x)^3*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I/c^2*arctan(a*x)^4-12/c^2*arctan(a*x)*pol
ylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I/c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))-2/c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c^2*arc
tan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12/c^2*arctan(a*x)*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] integral(arctan(a*x)^3/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**2,x)

[Out] Integral(atan(a*x)**3/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^3\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2),x)

[Out] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2), x)

3.403 $\int \frac{\text{ArcTan}(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{3a^3}{8c^2(1+a^2x^2)}-\frac{a^2\text{ArcTan}(ax)}{c^2x}-\frac{3a^4x\text{ArcTan}(ax)}{4c^2(1+a^2x^2)}-\frac{7a^3\text{ArcTan}(ax)^2}{8c^2}-\frac{a\text{ArcTan}(ax)^2}{2c^2x^2}+\frac{3a^3\text{ArcTan}(ax)^2}{4c^2(1+a^2x^2)}-$$

```
[Out] -3/8*a^3/c^2/(a^2*x^2+1)-a^2*arctan(a*x)/c^2/x-3/4*a^4*x*arctan(a*x)/c^2/(a
^2*x^2+1)-7/8*a^3*arctan(a*x)^2/c^2-1/2*a*arctan(a*x)^2/c^2/x^2+3/4*a^3*arc
tan(a*x)^2/c^2/(a^2*x^2+1)+7*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^
2-1/3*arctan(a*x)^3/c^2/x^3+2*a^2*arctan(a*x)^3/c^2/x+1/2*a^4*x*arctan(a*x)
^3/c^2/(a^2*x^2+1)+5/8*a^3*arctan(a*x)^4/c^2+a^3*ln(x)/c^2-1/2*a^3*ln(a^2*x
^2+1)/c^2-7*a^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+7/3*I*a^3*arctan(a*x)^3
/c^2-7/2*a^3*polylog(3,-1+2/(1-I*a*x))/c^2
```

Rubi [A]
time = 0.91, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745, 5012, 5050, 267}

$$\frac{7ia^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^2}+\frac{5a^3\text{ArcTan}(ax)^4}{8c^2}+\frac{7ia^3\text{ArcTan}(ax)^3}{3c^2}-\frac{7a^3\text{ArcTan}(ax)^2}{8c^2}-\frac{7a^3\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{7a^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^2}+\frac{a^3\log(x)}{c^2}+\frac{2a^2\text{ArcTan}(ax)^3}{c^2x}-\frac{a^2\text{ArcTan}(ax)}{c^2x}+\frac{a^4x\text{ArcTan}(ax)^3}{2c^2(a^2x^2+1)}-\frac{3a^4x\text{ArcTan}(ax)}{4c^2(a^2x^2+1)}+\frac{3a^3\text{ArcTan}(ax)^2}{4c^2(a^2x^2+1)}-\frac{3a^3}{8c^2(a^2x^2+1)}-\frac{a^3\log(a^2x^2+1)}{2c^2}-\frac{\text{ArcTan}(ax)^3}{3c^2x^3}-\frac{a\text{ArcTan}(ax)^2}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (-3*a^3)/(8*c^2*(1 + a^2*x^2)) - (a^2*ArcTan[a*x])/(c^2*x) - (3*a^4*x*ArcTa
n[a*x])/(4*c^2*(1 + a^2*x^2)) - (7*a^3*ArcTan[a*x]^2)/(8*c^2) - (a*ArcTan[a
*x]^2)/(2*c^2*x^2) + (3*a^3*ArcTan[a*x]^2)/(4*c^2*(1 + a^2*x^2)) + (((7*I)/
3)*a^3*ArcTan[a*x]^3)/c^2 - ArcTan[a*x]^3/(3*c^2*x^3) + (2*a^2*ArcTan[a*x]^
3)/(c^2*x) + (a^4*x*ArcTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*
x]^4)/(8*c^2) + (a^3*Log[x])/c^2 - (a^3*Log[1 + a^2*x^2])/(2*c^2) - (7*a^3*
ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^2 + ((7*I)*a^3*ArcTan[a*x]*PolyLog[
2, -1 + 2/(1 - I*a*x)])/c^2 - (7*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4(c+a^2cx^2)}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)}\,dx}{c}$$

$$= -\frac{\tan^{-1}(ax)^3}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^4}{8c^2} - \frac{1}{2}\left(3a^5\right)\int \frac{x\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx + \frac{a\int$$

$$= \frac{3a^3\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} - \frac{\tan^{-1}(ax)^3}{3c^2x^3} + \frac{a^4x\tan^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} + \frac{a^3\tan^{-1}(ax)^4}{8c^2} - \frac{1}{2}\left(3a^4\right)\int \frac{\tan^{-}}{(c+a}$$

$$= -\frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{3a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)^2}{2c^2x^2} + \frac{3a^3\tan^{-1}(ax)^2}{4c^2\left(1+a^2x^2\right)} + \frac{ia^3\tan^{-1}(a}{3c^2}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)} - \frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{3a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)^2}{2c^2x^2} + \frac{3a^3\tan^{-1}(a}{4c^2\left(1+a^2x\right.}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^2x} - \frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)}{2c^2x^2}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^2x} - \frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)}{2c^2x^2}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^2x} - \frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)}{2c^2x^2}$$

$$= -\frac{3a^3}{8c^2\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^2x} - \frac{3a^4x\tan^{-1}(ax)}{4c^2\left(1+a^2x^2\right)} - \frac{7a^3\tan^{-1}(ax)^2}{8c^2} - \frac{a\tan^{-1}(ax)}{2c^2x^2}$$

Mathematica [A]
time = 0.63, size = 243, normalized size = 0.73

$$\frac{a^3\left(\frac{7i\pi^3}{24} - \frac{\mathrm{ArcTan}(ax)}{ax} - \frac{1}{2}\mathrm{ArcTan}(ax)^2 - \frac{\mathrm{ArcTan}(ax)^2}{2a^2x^2} - \frac{7}{3}i\mathrm{ArcTan}(ax)^3 - \frac{\mathrm{ArcTan}(ax)^3}{3a^3x^3} + \frac{2\mathrm{ArcTan}(ax)^3}{ax} + \frac{5}{8}\mathrm{ArcTan}(ax)^4 - \frac{3}{16}\cos(2\mathrm{ArcTan}(ax)) + \frac{3}{8}\mathrm{ArcTan}(ax)^2\cos(2\mathrm{ArcTan}(ax)) - 7\mathrm{ArcTan}(ax)^2\log\left(1-e^{-2i\mathrm{ArcTan}(ax)}\right) + \log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) - 7i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(ax)}\right) - \frac{7}{2}\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(ax)}\right) - \frac{3}{8}\mathrm{ArcTan}(ax)\sin(2\mathrm{ArcTan}(ax)) + \frac{1}{4}\mathrm{ArcTan}(ax)^3\sin(2\mathrm{ArcTan}(ax))\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

```
[Out] (a^3*(((7*I)/24)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2
/(2*a^2*x^2) - ((7*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (2*Arc
```

```
Tan[a*x]^3)/(a*x) + (5*ArcTan[a*x]^4)/8 - (3*Cos[2*ArcTan[a*x]])/16 + (3*Ar
cTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - 7*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (7*I)*ArcTan[a*x]*PolyLog[2, E^((
-2*I)*ArcTan[a*x])] - (7*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (3*ArcTan[
a*x]*Sin[2*ArcTan[a*x]])/8 + (ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4))/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 308.89, size = 4175, normalized size = 12.58

method	result	size
derivativedivides	Expression too large to display	4175
default	Expression too large to display	4175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)+5/2/c^2*arctan(a*x)^4-1/3/c^2*ar
ctan(a*x)^3/a^3/x^3+2/c^2*arctan(a*x)^3/a/x-1/2/c^2*(-7*arctan(a*x)^2*ln(a^
2*x^2+1)-3/2*arctan(a*x)^2/(a^2*x^2+1)+arctan(a*x)^2/a^2/x^2+14*arctan(a*x)
^2*ln(a*x)+14*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-14*arctan(a*x)^
2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-1/48/a/x/(I+a*x)*(-168*arctan(a*x)*a^2*x^2-
336*I*Pi*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I
*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2*a^3*x^3-168*I*Pi*csgn(I*(1+I*a*x)^4
/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*arctan(a*x)^2*a^3*x^3+336*I
*Pi*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)
^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2*a^3*x^3-336*I*Pi*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan
(a*x)^2*a*x+168*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3*arctan(a*x)^2*a*x+168*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arct
an(a*x)^2*a*x+9*a^3*x^3-672*ln(2)*arctan(a*x)^2*a*x-672*ln(2)*arctan(a*x)^2
*a^3*x^3-336*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2*a^3*x^3+168*I*Pi*csgn(I/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2*a^3*x^3-336*I*Pi*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)
^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+
1)+1))*arctan(a*x)^2*a*x+168*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1))*arctan(a*x)^2*a*x-336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+
1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(1/((1+I*a*x)
^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))*ar
ctan(a*x)^2*a*x-27*a*x-84*arctan(a*x)^2*a*x-84*arctan(a*x)^2*a^3*x^3-336*I*
Pi*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^
2/(a^2*x^2+1)+1))^3*arctan(a*x)^2*a*x-168*I*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+
```

```
1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*arctan(a*x)^2*a*x+336*I*Pi*csgn(1/((1
+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2*a*x-336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a
*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2*a^3*x^3+16
8*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arct
an(a*x)^2*a^3*x^3+168*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2*
a^3*x^3-96*arctan(a*x)+336*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/((
1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1
)+1))^2*arctan(a*x)^2*a^3*x^3+336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(
1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1))*arctan(a*x)^2*a^3*x^3-336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(1/((1+I
*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1
))*arctan(a*x)^2*a^3*x^3+336*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*csgn(I/
((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2
+1)+1))^2*arctan(a*x)^2*a*x+336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+
I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*arctan(a*x)^2*a*x+336*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1
)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(1/((1+I*a*x)^
2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a
rctan(a*x)^2*a*x-168*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2*a*x-336*
I*Pi*arctan(a*x)^2*a^3*x^3-336*I*Pi*arctan(a*x)^2*a*x+336*I*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^
2/(a^2*x^2+1)+1))^2*arctan(a*x)^2*a^3*x^3-168*I*Pi*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*arctan(a*x)^2*a^3*x^3-168*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2*a^3*x^
3+168*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1))*arctan(a*x)^2*a^3*x^3-336*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2*a^3*x^3+336*I*Pi*csgn(I*(1+I*
a*x)^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1)+I)*arctan(a*x)^2*a^3*x^3-168*I*Pi*csgn(I*(1+I*a*x)^4/(a^2*x^2+1)^
2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*arctan
(a*x)^2*a^3*x^3-168*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2*a*x+168*I*Pi*cs
gn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6144*(1200*(a^5*x^5 + a^3*x^3)*arctan(a*x)^4 - 45*(a^5*x^5 + a^3*x^3)*log
(a^2*x^2 + 1)^4 + 128*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arctan(a*x)^3 - 24*(15*
(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 + 4*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arctan(
a*x))*log(a^2*x^2 + 1)^2 - 12*(a^2*c^2*x^5 + c^2*x^3)*(120*a^7*(a^2/(a^8*c^
2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c^2*x^2 + a^4*c^2)) - 30720*a^7*in
tegrate(1/256*x^7*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2*c^2*x
^6 + c^2*x^4), x) - 7680*a^7*integrate(1/256*x^7*log(a^2*x^2 + 1)^3/(a^4*c^
2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 61440*a^6*integrate(1/256*x^6*arctan
(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a^6*integrate(1
/256*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*
x^4), x) - 122880*a^6*integrate(1/256*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^4
*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15*a^5*log(a^2*x^2 + 1)^3/(a^4*c^
2*x^2 + a^2*c^2) + 45*(2*a^4*(a^2/(a^10*c^2*x^2 + a^8*c^2) + log(a^2*x^2 +
1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*log(a^2*x^2 + 1)^2/(a^6*c^2*x^2 + a^4*c^2
))*a^5 - 30720*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*
c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 122880*a^5*integrate(1/256*x^5*arc
tan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 60*a^5*log(a^2*x^2
 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) + 61440*a^4*integrate(1/256*x^4*arctan(a*x)
^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a^4*integrate(1/256*
x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4),
 x) - 81920*a^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^4*c^2*x
^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 81920*a^3*integrate(1/256*x^3*arctan(a*
x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) - 20480*a^3*integrate(1/25
6*x^3*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 1638
4*a^2*integrate(1/256*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2
*c^2*x^6 + c^2*x^4), x) - 16384*a*integrate(1/256*x*arctan(a*x)^2/(a^4*c^2*
x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 4096*a*integrate(1/256*x*log(a^2*x^2 +
 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) - 114688*integrate(1/256*
arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) - 12288*integrate
(1/256*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^
4), x)))/(a^2*c^2*x^5 + c^2*x^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^8+2a^2x^6+x^4}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**2,x)
[Out] Integral(atan(a*x)**3/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^4\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2), x)
```

3.404 $\int \frac{x^3 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=212

$$-\frac{3x^3}{128ac^3(1+a^2x^2)^2}-\frac{45x}{256a^3c^3(1+a^2x^2)}-\frac{27\text{ArcTan}(ax)}{256a^4c^3}-\frac{3x^4\text{ArcTan}(ax)}{32c^3(1+a^2x^2)^2}+\frac{9\text{ArcTan}(ax)}{32a^4c^3(1+a^2x^2)}+\frac{3x^3\text{ArcT}}{16ac^3(1}$$

```
[Out] -3/128*x^3/a/c^3/(a^2*x^2+1)^2-45/256*x/a^3/c^3/(a^2*x^2+1)-27/256*arctan(a
*x)/a^4/c^3-3/32*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^4/c^3
/(a^2*x^2+1)+3/16*x^3*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*x)^
2/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^3/a^4/c^3+1/4*x^4*arctan(a*x)^3/c^3/
(a^2*x^2+1)^2
```

Rubi [A]
time = 0.21, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5064, 5060, 5056, 5050, 205, 211, 294}

$$-\frac{3\text{ArcTan}(ax)^3}{32a^4c^3}-\frac{27\text{ArcTan}(ax)}{256a^4c^3}+\frac{x^4\text{ArcTan}(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3x^4\text{ArcTan}(ax)}{32c^3(a^2x^2+1)^2}+\frac{3x^3\text{ArcTan}(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{3x^3}{128ac^3(a^2x^2+1)^2}+\frac{9\text{ArcTan}(ax)}{32a^4c^3(a^2x^2+1)}+\frac{9x\text{ArcTan}(ax)^2}{32a^3c^3(a^2x^2+1)}-\frac{45x}{256a^3c^3(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*x^3)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a^3*c^3*(1 + a^2*x^2)) -
 (27*ArcTan[a*x])/(256*a^4*c^3) - (3*x^4*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)
^2) + (9*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)) + (3*x^3*ArcTan[a*x]^2)/(1
6*a*c^3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a^3*c^3*(1 + a^2*x^2)) -
 (3*ArcTan[a*x]^3)/(32*a^4*c^3) + (x^4*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^
2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{x^4 \tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} - \frac{1}{4}(3a)\int \frac{x^4 \tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx$$

$$= -\frac{3x^4 \tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{3x^3 \tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{x^4 \tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{1}{32}(3a)\int \frac{x^4}{(c+a^2cx^2)^3}$$

$$= -\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{3x^4 \tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{3x^3 \tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9x \tan^{-1}(ax)^2}{32a^3c^3(1+a^2x^2)} -$$

$$= -\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{9x}{256a^3c^3(1+a^2x^2)} - \frac{3x^4 \tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{9\tan^{-1}(ax)}{32a^4c^3(1+a^2x^2)} -$$

$$= -\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256a^3c^3(1+a^2x^2)} + \frac{9\tan^{-1}(ax)}{256a^4c^3} - \frac{3x^4 \tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{9}{32a}$$

$$= -\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256a^3c^3(1+a^2x^2)} - \frac{27\tan^{-1}(ax)}{256a^4c^3} - \frac{3x^4 \tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{}{32}$$

Mathematica [A]

time = 0.16, size = 105, normalized size = 0.50

$$\frac{-3ax(15+17a^2x^2) + (45+18a^2x^2-51a^4x^4)\,\text{ArcTan}(ax) + 24ax(3+5a^2x^2)\,\text{ArcTan}(ax)^2 + 8(-3-6a^2x^2+5a^4x^4)\,\text{ArcTan}(ax)^3}{256a^4c^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a*x*(15 + 17*a^2*x^2) + (45 + 18*a^2*x^2 - 51*a^4*x^4)*ArcTan[a*x] + 24
*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x]^2 + 8*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[
a*x]^3)/(256*a^4*c^3*(1 + a^2*x^2)^2)
```

Maple [A]

time = 1.00, size = 176, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2} - \frac{\arctan(ax)^3}{2c^3(a^2x^2+1)} - \frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2} - \frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2} - \frac{5\arctan(ax)^3}{24} + \frac{\arctan(ax)}{8(a^2x^2+1)^2} - \frac{5\arctan(ax)}{8(a^2x^2+1)} + \frac{\frac{17}{8}a^3x}{8(a^2}\right.}{4c^3}}{a^4}$
default	$\frac{\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2} - \frac{\arctan(ax)^3}{2c^3(a^2x^2+1)} - \frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2} - \frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2} - \frac{5\arctan(ax)^3}{24} + \frac{\arctan(ax)}{8(a^2x^2+1)^2} - \frac{5\arctan(ax)}{8(a^2x^2+1)} + \frac{\frac{17}{8}a^3x}{8(a^2}\right.}{4c^3}}{a^4}$

risch	$\frac{i(5a^4x^4-6a^2x^2-3)\ln(iax+1)^3}{256a^4c^3(a^2x^2+1)^2} - \frac{3i(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(iax+}{256a^4(ax+i)^2(ax-i)^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2-1/2*arctan(a*x)^3/c^3/(a^2*x^2+1
)-3/4/c^3*(-5/8*arctan(a*x)^2/(a^2*x^2+1)^2*a^3*x^3-3/8*arctan(a*x)^2*a*x/(
a^2*x^2+1)^2-5/24*arctan(a*x)^3+1/8/(a^2*x^2+1)^2*arctan(a*x)-5/8*arctan(a*
x)/(a^2*x^2+1)+1/8*(17/8*a^3*x^3+15/8*a*x)/(a^2*x^2+1)^2+17/64*arctan(a*x))
)
```

Maxima [A]
time = 0.51, size = 289, normalized size = 1.36

$$\frac{3}{32}a\left(\frac{5a^2x^3+3x}{a^8c^3x^4+2a^6c^3x^2+a^4c^3}+\frac{5\arctan(ax)}{a^5c^3}\right)\arctan(ax)^2-\frac{(2a^2x^2+1)\arctan(ax)^3}{4(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}-\frac{1}{256}\left(\frac{(51a^3x^3-40(a^4x^4+2a^2x^2+1)\arctan(ax)^3+45ax+51(a^4x^4+2a^2x^2+1)\arctan(ax))a^2}{a^{11}c^3x^4+2a^9c^3x^2+a^7c^3}-\frac{24(5a^2x^2-5(a^4x^4+2a^2x^2+1)\arctan(ax)^2+4)a\arctan(ax)}{a^{10}c^3x^4+2a^8c^3x^2+a^6c^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arcta
n(a*x)/(a^5*c^3))*arctan(a*x)^2 - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)^3/(a^8*c^
3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/256*((51*a^3*x^3 - 40*(a^4*x^4 + 2*a^2
*x^2 + 1)*arctan(a*x)^3 + 45*a*x + 51*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)
)*a^2/(a^11*c^3*x^4 + 2*a^9*c^3*x^2 + a^7*c^3) - 24*(5*a^2*x^2 - 5*(a^4*x^4
 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^10*c^3*x^4 + 2*a^8*c^
3*x^2 + a^6*c^3))*a
```

Fricas [A]
time = 4.82, size = 117, normalized size = 0.55

$$-\frac{51a^3x^3-8(5a^4x^4-6a^2x^2-3)\arctan(ax)^3-24(5a^3x^3+3ax)\arctan(ax)^2+45ax+3(17a^4x^4-6a^2x^2-15)\arctan(ax)}{256(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(51*a^3*x^3 - 8*(5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^3 - 24*(5*a^
3*x^3 + 3*a*x)*arctan(a*x)^2 + 45*a*x + 3*(17*a^4*x^4 - 6*a^2*x^2 - 15)*arc
tan(a*x))/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3\operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.59, size = 205, normalized size = 0.97

$$\frac{\mathrm{atan}(a\,x)^2\,\left(\frac{9\,x}{32\,a^5\,c^3}+\frac{15\,x^3}{32\,a^3\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\mathrm{atan}(a\,x)^3\,\left(\frac{\frac{1}{4\,a^6\,c^3}+\frac{x^2}{2\,a^4\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{5}{32\,a^4\,c^3}\right)-\frac{\frac{51\,a^2\,x^3}{8}+\frac{45\,x}{8}}{32\,a^7\,c^3\,x^4+64\,a^5\,c^3\,x^2+32\,a^3\,c^3}-\frac{51\,\mathrm{atan}(a\,x)}{256\,a^4\,c^3}+\frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^6\,c^3}+\frac{15\,x^2}{32\,a^4\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (atan(a*x)^2*((9*x)/(32*a^5*c^3) + (15*x^3)/(32*a^3*c^3)))/(1/a^2 + 2*x^2 +
 a^2*x^4) - atan(a*x)^3*((1/(4*a^6*c^3) + x^2/(2*a^4*c^3))/(1/a^2 + 2*x^2 +
 a^2*x^4) - 5/(32*a^4*c^3)) - ((45*x)/8 + (51*a^2*x^3)/8)/(32*a^3*c^3 + 64*
a^5*c^3*x^2 + 32*a^7*c^3*x^4) - (51*atan(a*x))/(256*a^4*c^3) + (atan(a*x)*(
3/(8*a^6*c^3) + (15*x^2)/(32*a^4*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

3.405 $\int \frac{x^2 \mathbf{ArcTan}(ax)^3}{\left(c+a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=237

$$\frac{3}{128a^3c^3\left(1+a^2x^2\right)^2}-\frac{3}{128a^3c^3\left(1+a^2x^2\right)}+\frac{3x\text{ArcTan}(ax)}{32a^2c^3\left(1+a^2x^2\right)^2}-\frac{3x\text{ArcTan}(ax)}{64a^2c^3\left(1+a^2x^2\right)}-\frac{3\text{ArcTan}(ax)^2}{128a^3c^3}-\frac{3\text{ArcTa}}{16a^3c^3\,(1}$$

```
[Out] 3/128/a^3/c^3/(a^2*x^2+1)^2-3/128/a^3/c^3/(a^2*x^2+1)+3/32*x*arctan(a*x)/a^
2/c^3/(a^2*x^2+1)^2-3/64*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)-3/128*arctan(a*x
)^2/a^3/c^3-3/16*arctan(a*x)^2/a^3/c^3/(a^2*x^2+1)^2+3/16*arctan(a*x)^2/a^3
/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)^3/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x
)^3/a^2/c^3/(a^2*x^2+1)+1/32*arctan(a*x)^4/a^3/c^3
```

Rubi [A]
time = 0.29, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5084, 5012, 5050, 267, 5020, 5016}

$$\frac{\text{ArcTan}(ax)^4}{32a^3c^3}-\frac{3\text{ArcTan}(ax)^2}{128a^3c^3}+\frac{x\text{ArcTan}(ax)^3}{8a^2c^3\left(a^2x^2+1\right)}-\frac{x\text{ArcTan}(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{3x\text{ArcTan}(ax)}{64a^2c^3\left(a^2x^2+1\right)}+\frac{3x\text{ArcTan}(ax)}{32a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\text{ArcTan}(ax)^2}{16a^3c^3\left(a^2x^2+1\right)}-\frac{3\text{ArcTan}(ax)^2}{16a^3c^3\left(a^2x^2+1\right)^2}-\frac{3}{128a^3c^3\left(a^2x^2+1\right)}+\frac{3}{128a^3c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] 3/(128*a^3*c^3*(1 + a^2*x^2)^2) - 3/(128*a^3*c^3*(1 + a^2*x^2)) + (3*x*ArcT
an[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x])/(64*a^2*c^3*(1 +
a^2*x^2)) - (3*ArcTan[a*x]^2)/(128*a^3*c^3) - (3*ArcTan[a*x]^2)/(16*a^3*c^3
*(1 + a^2*x^2)^2) + (3*ArcTan[a*x]^2)/(16*a^3*c^3*(1 + a^2*x^2)) - (x*ArcTa
n[a*x]^3)/(4*a^2*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(8*a^2*c^3*(1 + a
^2*x^2)) + ArcTan[a*x]^4/(32*a^3*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{a^2c}$$

$$= -\frac{3\tan^{-1}(ax)^2}{16a^3c^3(1+a^2x^2)^2} - \frac{x\tan^{-1}(ax)^3}{4a^2c^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^3}{2a^2c^3(1+a^2x^2)} + \frac{\tan^{-1}(ax)^4}{8a^3c^3} + \frac{3\int \frac{\tan^{-1}}{(c+a}}{8}$$

$$= \frac{3}{128a^3c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} - \frac{3\tan^{-1}(ax)^2}{16a^3c^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)^2}{4a^3c^3(1+a^2x^2)} -$$

$$= \frac{3}{128a^3c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} - \frac{39x\tan^{-1}(ax)}{64a^2c^3(1+a^2x^2)} - \frac{39\tan^{-1}(ax)^2}{128a^3c^3} - \frac{}{16a}$$

$$= \frac{3}{128a^3c^3(1+a^2x^2)^2} - \frac{39}{128a^3c^3(1+a^2x^2)} + \frac{3x\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{64a^2c^3(1+a^2x^2)} -$$

$$= \frac{3}{128a^3c^3(1+a^2x^2)^2} - \frac{3}{128a^3c^3(1+a^2x^2)} + \frac{3x\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{64a^2c^3(1+a^2x^2)} -$$

Mathematica [A]

time = 0.04, size = 111, normalized size = 0.47

$$\frac{-3a^2x^2 + (6ax - 6a^3x^3)\,\text{ArcTan}(ax) - 3(1 - 6a^2x^2 + a^4x^4)\,\text{ArcTan}(ax)^2 + 16ax(-1 + a^2x^2)\,\text{ArcTan}(ax)^3 + 4(1 + a^2x^2)^2\,\text{ArcTan}(ax)^4}{128a^3c^3(1 + a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a^2*x^2 + (6*a*x - 6*a^3*x^3)*ArcTan[a*x] - 3*(1 - 6*a^2*x^2 + a^4*x^4)
*ArcTan[a*x]^2 + 16*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^3 + 4*(1 + a^2*x^2)^2*Ar
cTan[a*x]^4)/(128*a^3*c^3*(1 + a^2*x^2)^2)
```

Maple [A]

time = 0.99, size = 197, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^3 a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{\arctan(ax)^3 ax}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^4}{8c^3} - \frac{3\left(\frac{\arctan(ax)^4}{4} + \frac{\arctan(ax)^2}{2(a^2x^2+1)^2} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2} - \frac{ax\arctan(ax)}{8(a^2x^2+1)^2} + \right.}{8c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)^3 a^3x^3}{8c^3(a^2x^2+1)^2} - \frac{\arctan(ax)^3 ax}{8c^3(a^2x^2+1)^2} + \frac{\arctan(ax)^4}{8c^3} - \frac{3\left(\frac{\arctan(ax)^4}{4} + \frac{\arctan(ax)^2}{2(a^2x^2+1)^2} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2} - \frac{ax\arctan(ax)}{8(a^2x^2+1)^2} + \right.}{8c^3}}{a^3}$

risch	$\frac{\ln(iax+1)^4}{512c^3a^3} - \frac{(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^3}{128a^3c^3(a^2x^2+1)^2} + \frac{3\left(2a^4x^4\ln(-iax\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a^3*x^3-1/8/c^3*arctan(a*x)^3/(a
^2*x^2+1)^2*a*x+1/8/c^3*arctan(a*x)^4-3/8/c^3*(1/4*arctan(a*x)^4+1/2*arctan
(a*x)^2/(a^2*x^2+1)^2-1/2*arctan(a*x)^2/(a^2*x^2+1)+1/8*arctan(a*x)*a^3*x^3
/(a^2*x^2+1)^2-1/8*a*x/(a^2*x^2+1)^2*arctan(a*x)+1/16*arctan(a*x)^2-1/16/(a
^2*x^2+1)^2+1/16/(a^2*x^2+1)))
```

Maxima [A]
time = 0.54, size = 334, normalized size = 1.41

$$\frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan\left(ax\right)}{a^3c^3}\right)\arctan\left(ax\right)^3+\frac{3\left(a^2x^2-\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2\right)a\arctan\left(ax\right)^2}{16\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}-\frac{1}{128}\left(\frac{\left(4\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^4+3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2\right)a^2}{a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3}+\frac{2\left(3\,a^3x^3-8\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^3-3\,ax+3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)\right)a\arctan\left(ax\right)}{a^9c^3x^4+2\,a^7c^3x^2+a^5c^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(a
^3*c^3))*arctan(a*x)^3 + 3/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a
*x)^2)*a*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/128*((4*
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^
2 + 1)*arctan(a*x)^2)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) + 2*(3*a
^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^9*c^3*x^4 + 2*a^7*c^3*x^2 + a^5
*c^3))*a
```

Fricas [A]
time = 4.76, size = 130, normalized size = 0.55

$$\frac{4\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^4-3\,a^2x^2+16\left(a^3x^3-ax\right)\arctan\left(ax\right)^3-3\left(a^4x^4-6\,a^2x^2+1\right)\arctan\left(ax\right)^2-6\left(a^3x^3-ax\right)\arctan\left(ax\right)}{128\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 3*a^2*x^2 + 16*(a^3*x^3
- a*x)*arctan(a*x)^3 - 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x)^2 - 6*(a^3*x
^3 - a*x)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\operatorname{atan}^3\left(ax\right)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 188, normalized size = 0.79

$$\frac{\mathrm{atan}(a\,x)\,\left(\frac{3\,x}{64\,a^4\,c^3}-\frac{3\,x^3}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{3\,x^2}{2\,\left(64\,a^5\,c^3\,x^4+128\,a^3\,c^3\,x^2+64\,a\,c^3\right)}-{\mathrm{atan}(a\,x)}^2\,\left(\frac{3}{128\,a^3\,c^3}-\frac{3\,x^2}{16\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right)-\frac{{\mathrm{atan}(a\,x)}^3\,\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{{\mathrm{atan}(a\,x)}^4}{32\,a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

```
[Out] (atan(a*x)*((3*x)/(64*a^4*c^3) - (3*x^3)/(64*a^2*c^3)))/(1/a^2 + 2*x^2 + a^
2*x^4) - (3*x^2)/(2*(64*a*c^3 + 128*a^3*c^3*x^2 + 64*a^5*c^3*x^4)) - atan(a
*x)^2*(3/(128*a^3*c^3) - (3*x^2)/(16*a^3*c^3*(1/a^2 + 2*x^2 + a^2*x^4))) -
(atan(a*x)^3*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4) +
 atan(a*x)^4/(32*a^3*c^3)
```

3.406 $\int \frac{x\mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{3x}{128ac^3(1+a^2x^2)^2}-\frac{45x}{256ac^3(1+a^2x^2)}-\frac{45\text{ArcTan}(ax)}{256a^2c^3}+\frac{3\text{ArcTan}(ax)}{32a^2c^3(1+a^2x^2)^2}+\frac{9\text{ArcTan}(ax)}{32a^2c^3(1+a^2x^2)}+\frac{3x\text{ArcT}}{16ac^3(1}$$

```
[Out] -3/128*x/a/c^3/(a^2*x^2+1)^2-45/256*x/a/c^3/(a^2*x^2+1)-45/256*arctan(a*x)/
a^2/c^3+3/32*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^2/c^3/(a^
2*x^2+1)+3/16*x*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*x)^2/a/c^
3/(a^2*x^2+1)+3/32*arctan(a*x)^3/a^2/c^3-1/4*arctan(a*x)^3/a^2/c^3/(a^2*x^2
+1)^2
```

Rubi [A]

time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5050, 5020, 5012, 205, 211}

$$-\frac{\text{ArcTan}(ax)^3}{4a^2c^3(a^2x^2+1)^2}+\frac{9x\text{ArcTan}(ax)^2}{32ac^3(a^2x^2+1)}+\frac{3x\text{ArcTan}(ax)^2}{16ac^3(a^2x^2+1)^2}+\frac{9\text{ArcTan}(ax)}{32a^2c^3(a^2x^2+1)}+\frac{3\text{ArcTan}(ax)}{32a^2c^3(a^2x^2+1)^2}+\frac{3\text{ArcTan}(ax)^3}{32a^2c^3}-\frac{45\text{ArcTan}(ax)}{256a^2c^3}-\frac{45x}{256ac^3(a^2x^2+1)}-\frac{3x}{128ac^3(a^2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*x)/(128*a*c^3*(1 + a^2*x^2)^2) - (45*x)/(256*a*c^3*(1 + a^2*x^2)) - (45
*ArcTan[a*x])/(256*a^2*c^3) + (3*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)^2)
+ (9*ArcTan[a*x])/(32*a^2*c^3*(1 + a^2*x^2)) + (3*x*ArcTan[a*x]^2)/(16*a*c^
3*(1 + a^2*x^2)^2) + (9*x*ArcTan[a*x]^2)/(32*a*c^3*(1 + a^2*x^2)) + (3*ArcT
an[a*x]^3)/(32*a^2*c^3) - ArcTan[a*x]^3/(4*a^2*c^3*(1 + a^2*x^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
```

```
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{\tan^{-1}(ax)^3}{4a^2c^3(1+a^2x^2)^2} + \frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^3}\,dx}{4a}$$

$$= \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} - \frac{\tan^{-1}(ax)^3}{4a^2c^3(1+a^2x^2)^2} - \frac{3\int \frac{1}{(c+a^2cx^2)^3}\,dx}{32a} + \frac{9\int \frac{\mathrm{t}}{(}}{}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9x\tan^{-1}(ax)^2}{32ac^3(1+a^2x^2)} + \frac{3}{}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{9x}{256ac^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)} +$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{9\tan^{-1}(ax)}{256a^2c^3} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\,\mathrm{t}}{32a^2c}$$

$$= -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{45\tan^{-1}(ax)}{256a^2c^3} + \frac{3\tan^{-1}(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9}{32a}$$

Mathematica [A]

time = 0.06, size = 103, normalized size = 0.50

$$\frac{-3ax(17+15a^2x^2)-3(-17+6a^2x^2+15a^4x^4)\operatorname{ArcTan}(ax)+24ax(5+3a^2x^2)\operatorname{ArcTan}(ax)^2+8(-5+6a^2x^2+3a^4x^4)\operatorname{ArcTan}(ax)^3}{256c^3(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*a*x*(17 + 15*a^2*x^2) - 3*(-17 + 6*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] +
24*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTa
n[a*x]^3)/(256*c^3*(a + a^3*x^2)^2)
```

Maple [A]
time = 0.99, size = 150, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9\arctan(ax)^2ax}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3\left(\frac{15}{8}a^3x^3+\frac{17}{8}ax\right)}{32(a^2x^2+1)^2}-45\,\mathrm{arc}}{c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9\arctan(ax)^2ax}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3\left(\frac{15}{8}a^3x^3+\frac{17}{8}ax\right)}{32(a^2x^2+1)^2}-45\,\mathrm{arc}}{c^3}}{a^2}$
risch	$\frac{i(3a^4x^4+6a^2x^2-5)\ln(iax+1)^3}{256a^2c^3(a^2x^2+1)^2}-\frac{3i(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(iax}{256(ax+i)^2c^3(ax-i)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3/4/c^3*(1/4*arctan(a*x)^2*a*x/
(a^2*x^2+1)^2+3/8*arctan(a*x)^2*a*x/(a^2*x^2+1)+1/8*arctan(a*x)^3+3/8*arcta
n(a*x)/(a^2*x^2+1)+1/8/(a^2*x^2+1)^2*arctan(a*x)-1/8*(15/8*a^3*x^3+17/8*a*x
)/(a^2*x^2+1)^2-15/64*arctan(a*x)))
```

Maxima [A]
time = 0.51, size = 272, normalized size = 1.31

$$\frac{3\left(\frac{3a^2x^3+5x}{a^4c^2x^4+2a^2c^2x^2+c^2}+\frac{3\arctan(ax)}{ac^2}\right)\arctan(ax)^2}{32ac}-\frac{3\left(\frac{\left(15a^3x^3-8(a^4x^4+2a^2x^2+1)\arctan(ax)^3+17ax+15(a^4x^4+2a^2x^2+1)\arctan(ax)\right)a^2}{a^7c^2x^4+2a^5c^2x^2+a^3c^2}-\frac{8\left(3a^2x^2-3(a^4x^4+2a^2x^2+1)\arctan(ax)^2+4\right)a\arctan(ax)}{a^6c^2x^4+2a^4c^2x^2+a^2c^2}\right)}{256ac}-\frac{\arctan(ax)^3}{4(a^2cx^2+c)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x)
/(a*c^2))*arctan(a*x)^2/(a*c) - 3/256*((15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2
 + 1)*arctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^
```

```
2/(a^7*c^2*x^4 + 2*a^5*c^2*x^2 + a^3*c^2) - 8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a
^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 +
 a^2*c^2))/(a*c) - 1/4*arctan(a*x)^3/((a^2*c*x^2 + c)^2*a^2*c)
```

Fricas [A]
time = 2.92, size = 117, normalized size = 0.56

$$-\frac{45\,a^3x^3-8\,(3\,a^4x^4+6\,a^2x^2-5)\arctan{(ax)}^3-24\,(3\,a^3x^3+5\,ax)\arctan{(ax)}^2+51\,ax+3\,(15\,a^4x^4+6\,a^2x^2-17)\arctan{(ax)}}{256\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] -1/256*(45*a^3*x^3 - 8*(3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^3 - 24*(3*a^
3*x^3 + 5*a*x)*arctan(a*x)^2 + 51*a*x + 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arc
tan(a*x))/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\,\mathrm{atan}^3\,(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
[Out] Integral(x*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**
3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.53, size = 189, normalized size = 0.91

$$\mathrm{atan}(a\,x)^3\,\left(\frac{3}{32\,a^2\,c^3}-\frac{1}{4\,a^4\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right)-\frac{\frac{45\,a^2\,x^3}{8}+\frac{51\,x}{8}}{32\,a^5\,c^3\,x^4+64\,a^3\,c^3\,x^2+32\,a\,c^3}+\frac{\mathrm{atan}(a\,x)^2\,\left(\frac{15\,x}{32\,a^3\,c^3}+\frac{9\,x^3}{32\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{45\,\mathrm{atan}(a\,x)}{256\,a^2\,c^3}+\frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^4\,c^3}+\frac{9\,x^2}{32\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
[Out] atan(a*x)^3*(3/(32*a^2*c^3) - 1/(4*a^4*c^3*(1/a^2 + 2*x^2 + a^2*x^4))) - ((
51*x)/8 + (45*a^2*x^3)/8)/(32*a*c^3 + 64*a^3*c^3*x^2 + 32*a^5*c^3*x^4) + (a
tan(a*x)^2*((15*x)/(32*a^3*c^3) + (9*x^3)/(32*a*c^3)))/(1/a^2 + 2*x^2 + a^2
*x^4) - (45*atan(a*x))/(256*a^2*c^3) + (atan(a*x)*(3/(8*a^4*c^3) + (9*x^2)/
(32*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

3.407 $\int \frac{\textbf{ArcTan}(ax)^3}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{3}{128ac^3(1+a^2x^2)^2}-\frac{45}{128ac^3(1+a^2x^2)}-\frac{3x\text{ArcTan}(ax)}{32c^3(1+a^2x^2)^2}-\frac{45x\text{ArcTan}(ax)}{64c^3(1+a^2x^2)}-\frac{45\text{ArcTan}(ax)^2}{128ac^3}+\frac{3\text{ArcTan}}{16ac^3(1+}$$

```
[Out] -3/128/a/c^3/(a^2*x^2+1)^2-45/128/a/c^3/(a^2*x^2+1)-3/32*x*arctan(a*x)/c^3/
(a^2*x^2+1)^2-45/64*x*arctan(a*x)/c^3/(a^2*x^2+1)-45/128*arctan(a*x)^2/a/c^
3+3/16*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/16*arctan(a*x)^2/a/c^3/(a^2*x^2+
1)+1/4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^3/c^3/(a^2*x^2+1
)+3/32*arctan(a*x)^4/a/c^3
```

Rubi [A]
time = 0.15, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5020, 5012, 5050, 267, 5016}

$$\frac{3x\text{ArcTan}(ax)^3}{8c^3(a^2x^2+1)}+\frac{x\text{ArcTan}(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{9\text{ArcTan}(ax)^2}{16ac^3(a^2x^2+1)}+\frac{3\text{ArcTan}(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{45x\text{ArcTan}(ax)}{64c^3(a^2x^2+1)}-\frac{3x\text{ArcTan}(ax)}{32c^3(a^2x^2+1)^2}-\frac{45}{128ac^3(a^2x^2+1)}-\frac{3}{128ac^3(a^2x^2+1)^2}+\frac{3\text{ArcTan}(ax)^4}{32ac^3}-\frac{45\text{ArcTan}(ax)^2}{128ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -3/(128*a*c^3*(1 + a^2*x^2)^2) - 45/(128*a*c^3*(1 + a^2*x^2)) - (3*x*ArcTan
[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (45*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2))
 - (45*ArcTan[a*x]^2)/(128*a*c^3) + (3*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^
2)^2) + (9*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^3)/(4*c
^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan
[a*x]^4)/(32*a*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{3\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} - \frac{3}{8}\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^3}\,dx + \frac{3\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} + \frac{3\tan^{-1}(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} + \cdots$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{9x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \frac{9\tan^{-1}(ax)^2}{128ac^3} + \cdots$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{9}{128ac^3(1+a^2x^2)} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{45x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \cdots$$

$$= -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{45}{128ac^3(1+a^2x^2)} - \frac{3x\tan^{-1}(ax)}{32c^3(1+a^2x^2)^2} - \frac{45x\tan^{-1}(ax)}{64c^3(1+a^2x^2)} - \cdots$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 0.51

$$-\frac{48+45a^2x^2+6ax(17+15a^2x^2)\,\mathrm{ArcTan}(ax)+3(-17+6a^2x^2+15a^4x^4)\,\mathrm{ArcTan}(ax)^2-16ax(5+3a^2x^2)\,\mathrm{ArcTan}(ax)^3-12(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^4}{128ac^3(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/128*(48 + 45*a^2*x^2 + 6*a*x*(17 + 15*a^2*x^2)*ArcTan[a*x] + 3*(-17 + 6*
a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x]^2 - 16*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^3
- 12*(1 + a^2*x^2)^2*ArcTan[a*x]^4)/(a*c^3*(1 + a^2*x^2)^2)
```

Maple [A]

time = 0.92, size = 193, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^3 ax}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^3 ax}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{3\arctan(ax)^2}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)^2}+\frac{15\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}+\frac{17ax\arctan(ax)}{8\left(a^2x^2+1\right)^2}+15a\cdots\right.}{8c^3}}{a}$
default	$\frac{\frac{\arctan(ax)^3 ax}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^3 ax}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{3\arctan(ax)^2}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)^2}+\frac{15\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}+\frac{17ax\arctan(ax)}{8\left(a^2x^2+1\right)^2}+15a\cdots\right.}{8c^3}}{a}$
risch	$\frac{3\ln(iax+1)^4}{512a\,c^3}-\frac{\left(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax\right)\ln(iax+1)^3}{128c^3\left(a^2x^2+1\right)^2a}+\frac{3\left(6a^4x^4\ln(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/4/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a*x+3/8/c^3*arctan(a*x)^3*a*x/(a^2
*x^2+1)+3/8/c^3*arctan(a*x)^4-3/8/c^3*(-3/2*arctan(a*x)^2/(a^2*x^2+1)-1/2*a
rctan(a*x)^2/(a^2*x^2+1)^2+15/8*arctan(a*x)*a^3*x^3/(a^2*x^2+1)^2+17/8*a*x/
(a^2*x^2+1)^2*arctan(a*x)+15/16*arctan(a*x)^2+15/16/(a^2*x^2+1)+1/16/(a^2*x
^2+1)^2+3/4*arctan(a*x)^4))
```

Maxima [A]

time = 0.53, size = 335, normalized size = 1.49

$$\frac{1}{8}\left(\frac{3a^2x^3+5x}{a^4c^3x^4+2a^2c^3x^2+c^3}+\frac{3\arctan(ax)}{ac^3}\right)\arctan(ax)^3+\frac{3\left(3a^2x^2-3\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^2+4\right)a\arctan(ax)^2}{16\left(a^6c^3x^4+2a^4c^3x^2+a^2c^3\right)}-\frac{3}{128}\left(\frac{\left(4\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^4+15a^2x^2-15\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^2+16\right)a^2}{a^8c^3x^4+2a^6c^3x^2+a^4c^3}+\frac{2\left(15a^3x^3-8\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)^3+17ax+15\left(a^4x^4+2a^2x^2+1\right)\arctan(ax)\right)a\arctan(ax)}{a^7c^3x^4+2a^5c^3x^2+a^3c^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)/
(a*c^3))*arctan(a*x)^3 + 3/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2 + 4)*a*arctan(a*x)^2/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) - 3/
128*((4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 15*a^2*x^2 - 15*(a^4*x^4
+ 2*a^2*x^2 + 1)*arctan(a*x)^2 + 16)*a^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4
*c^3) + 2*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 + 17*a*x
```

```
+ 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^3*x^4 + 2*
a^5*c^3*x^2 + a^3*c^3))*a
```

Fricas [A]
time = 2.40, size = 132, normalized size = 0.59

$$\frac{12\left(a^{4}x^{4}+2\,a^{2}x^{2}+1\right)\arctan\left(ax\right)^{4}-45\,a^{2}x^{2}+16\left(3\,a^{3}x^{3}+5\,ax\right)\arctan\left(ax\right)^{3}-3\left(15\,a^{4}x^{4}+6\,a^{2}x^{2}-17\right)\arctan\left(ax\right)^{2}-6\left(15\,a^{3}x^{3}+17\,ax\right)\arctan\left(ax\right)-48}{128\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(12*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 45*a^2*x^2 + 16*(3*a^3*
x^3 + 5*a*x)*arctan(a*x)^3 - 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*x)^2
- 6*(15*a^3*x^3 + 17*a*x)*arctan(a*x) - 48)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 +
a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}\left(ax\right)}{a^{6}x^{6}+3a^{4}x^{4}+3a^{2}x^{2}+1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 199, normalized size = 0.88

$$\operatorname{atan}(a\,x)^2\,\left(\frac{\frac{3}{4\,a^3\,c^3}+\frac{9\,x^2}{16\,a\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{45}{128\,a\,c^3}\right)-\frac{\frac{45\,a\,x^2}{2}+\frac{24}{a}}{64\,a^4\,c^3\,x^4+128\,a^2\,c^3\,x^2+64\,c^3}-\frac{\operatorname{atan}(a\,x)\,\left(\frac{45\,x^3}{64\,c^3}+\frac{51\,x}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\operatorname{atan}(a\,x)^3\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{3\,\operatorname{atan}(a\,x)^4}{32\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2)^3,x)
```

```
[Out] atan(a*x)^2*((3/(4*a^3*c^3) + (9*x^2)/(16*a*c^3))/(1/a^2 + 2*x^2 + a^2*x^4)
 - 45/(128*a*c^3)) - ((45*a*x^2)/2 + 24/a)/(64*c^3 + 128*a^2*c^3*x^2 + 64*a
^4*c^3*x^4) - (atan(a*x)*((45*x^3)/(64*c^3) + (51*x)/(64*a^2*c^3)))/(1/a^2
+ 2*x^2 + a^2*x^4) + (atan(a*x)^3*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^3)))/(1
/a^2 + 2*x^2 + a^2*x^4) + (3*atan(a*x)^4)/(32*a*c^3)
```

3.408 $\int \frac{\text{ArcTan}(ax)^3}{x(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=332

$$\frac{3ax}{128c^3\left(1+a^2x^2\right)^2}+\frac{141ax}{256c^3\left(1+a^2x^2\right)}+\frac{141\text{ArcTan}(ax)}{256c^3}-\frac{3\text{ArcTan}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{33\text{ArcTan}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3ax\text{ArcTan}(ax}{16c^3\left(1+a^2x^2\right)}$$

```
[Out] 3/128*a*x/c^3/(a^2*x^2+1)^2+141/256*a*x/c^3/(a^2*x^2+1)+141/256*arctan(a*x)
/c^3-3/32*arctan(a*x)/c^3/(a^2*x^2+1)^2-33/32*arctan(a*x)/c^3/(a^2*x^2+1)-3
/16*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-33/32*a*x*arctan(a*x)^2/c^3/(a^2*x^
2+1)-11/32*arctan(a*x)^3/c^3+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1/2*arctan
(a*x)^3/c^3/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^3+arctan(a*x)^3*ln(2-2/(1-I*a
*x))/c^3-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3+3/2*arctan(a*x)*
polylog(3,-1+2/(1-I*a*x))/c^3+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 0.50, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {5086, 5044, 4988, 5004, 5112, 5116, 6745, 5050, 5012, 205, 211, 5020}

$$\frac{\text{ArcTan}(ax)^3}{2c^3\left(a^2x^2+1\right)}+\frac{\text{ArcTan}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{33ax\text{ArcTan}(ax)^2}{32c^3\left(a^2x^2+1\right)}-\frac{3ax\text{ArcTan}(ax)^2}{16c^3\left(a^2x^2+1\right)^2}-\frac{33\text{ArcTan}(ax)}{32c^3\left(a^2x^2+1\right)}-\frac{3\text{ArcTan}(ax)}{32c^3\left(a^2x^2+1\right)^2}+\frac{141ax}{256c^3\left(a^2x^2+1\right)}+\frac{3ax}{128c^3\left(a^2x^2+1\right)^2}-\frac{3i\text{ArcTan}(ax)^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^3}+\frac{3\text{ArcTan}(ax)\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^3}-\frac{i\text{ArcTan}(ax)^4}{4c^3}-\frac{11\text{ArcTan}(ax)^3}{32c^3}+\frac{141\text{ArcTan}(ax)}{256c^3}+\frac{\text{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{3i\text{Li}_4\left(\frac{2}{1-iax}-1\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] (3*a*x)/(128*c^3*(1 + a^2*x^2)^2) + (141*a*x)/(256*c^3*(1 + a^2*x^2)) + (14
1*ArcTan[a*x])/(256*c^3) - (3*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) - (33*A
rcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (3*a*x*ArcTan[a*x]^2)/(16*c^3*(1 + a^2
*x^2)^2) - (33*a*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) - (11*ArcTan[a*x]^
3)/(32*c^3) + ArcTan[a*x]^3/(4*c^3*(1 + a^2*x^2)^2) + ArcTan[a*x]^3/(2*c^3*
(1 + a^2*x^2)) - ((I/4)*ArcTan[a*x]^4)/c^3 + (ArcTan[a*x]^3*Log[2 - 2/(1 -
I*a*x)])/c^3 - (((3*I)/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3
 + (3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/(2*c^3) + (((3*I)/4)*Poly
Log[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
```

```
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)^2}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{4}(3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{c}$$

$$= -\frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{3ax\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{\tan^{-1}(ax)^3}{2c^3\left(1+a^2x^2\right)} - \frac{i\tan^{-1}(a}{4c^3}$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2} - \frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{3ax\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{33ax\tan^{-1}(ax)^2}{32c^3\left(1+a^2x^2\right)} - \frac{11\tan}{3}$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2} + \frac{9ax}{256c^3\left(1+a^2x^2\right)} - \frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{33\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)} - \frac{3ax\,\mathrm{ta}}{16c^3(1}$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2} + \frac{141ax}{256c^3\left(1+a^2x^2\right)} + \frac{9\tan^{-1}(ax)}{256c^3} - \frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{33\tan^{-1}(}{32c^3(1+}$$

$$= \frac{3ax}{128c^3\left(1+a^2x^2\right)^2} + \frac{141ax}{256c^3\left(1+a^2x^2\right)} + \frac{141\tan^{-1}(ax)}{256c^3} - \frac{3\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{33\tan^{-}}{32c^3(1-}$$

Mathematica [A]
time = 0.16, size = 208, normalized size = 0.63

$$\frac{-16i\pi^4 + 256i\,\mathrm{ArcTan}(ax)^4 - 576\,\mathrm{ArcTan}(ax)\cos(2\,\mathrm{ArcTan}(ax)) + 384\,\mathrm{ArcTan}(ax)^3\cos(2\,\mathrm{ArcTan}(ax)) - 12\,\mathrm{ArcTan}(ax)\cos(4\,\mathrm{ArcTan}(ax)) + 32\,\mathrm{ArcTan}(ax)^3\cos(4\,\mathrm{ArcTan}(ax)) + 1024\,\mathrm{ArcTan}(ax)^3\log\left(1-e^{-2i\,\mathrm{ArcTan}(ax)}\right) + 1536i\,\mathrm{ArcTan}(ax)^2\,\mathrm{PolyLog}\left(2,e^{-2i\,\mathrm{ArcTan}(ax)}\right) + 1536\,\mathrm{ArcTan}(ax)\,\mathrm{PolyLog}\left(3,e^{-2i\,\mathrm{ArcTan}(ax)}\right) - 768i\,\mathrm{PolyLog}\left(4,e^{-2i\,\mathrm{ArcTan}(ax)}\right) + 288\sin(2\,\mathrm{ArcTan}(ax)) - 576\,\mathrm{ArcTan}(ax)^2\sin(2\,\mathrm{ArcTan}(ax)) + 3\sin(4\,\mathrm{ArcTan}(ax)) - 24\,\mathrm{ArcTan}(ax)^2\sin(4\,\mathrm{ArcTan}(ax))}{1024c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-16*I)*Pi^4 + (256*I)*ArcTan[a*x]^4 - 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]]
+ 384*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]]
+ 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 1024*ArcTan[a*x]^3*Log[1 - E^((-2*I
)*ArcTan[a*x])] + (1536*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])]
 + 1536*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (768*I)*PolyLog[4,
 E^((-2*I)*ArcTan[a*x])] + 288*Sin[2*ArcTan[a*x]] - 576*ArcTan[a*x]^2*Sin[2
*ArcTan[a*x]] + 3*Sin[4*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]])
/(1024*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 75.35, size = 1959, normalized size = 5.90

method	result	size

derivativedivides	Expression too large to display	1959
default	Expression too large to display	1959

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)^3/c^3/(a^2*x^2+1)-1/2/c
^3*arctan(a*x)^3*ln(a^2*x^2+1)+1/c^3*arctan(a*x)^3*ln(a*x)-3/4/c^3*(-2/3*I*
arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a
^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))+1/3*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)+1/3*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+1/3*I*arct
an(a*x)^3*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
3+11/24*arctan(a*x)^3-2/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+2/3*I*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-3*I*arctan(a*x)^2*(I+a*x)/(
8*a*x-8*I)+3*I*arctan(a*x)^2*(a*x-I)/(8*a*x+8*I)-4/3*arctan(a*x)^3*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2))+1/3*I*arctan(a*x)^4-8*I*polylog(4,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))-8*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/64*arctan(a*x)*co
s(4*arctan(a*x))-2/3*I*arctan(a*x)^3*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^3-1/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^3+1/256*(8*arctan(a*x)^2-1)*sin(4*arctan(a*x))-4/3*arctan(a*
x)^3*ln(2)-8*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*arctan(a
*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4/3*arctan(a*x)^3*ln(1+(1+I*a*x)
/(a^2*x^2+1)^(1/2))-4/3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4/3
*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-1/3*I*arctan(a*x)^3*Pi*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2+2/3*I*arctan(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+2/3*I*arcta
n(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^2+2/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^2+2/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-1/3*I*arcta
n(a*x)^3*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-2/3*I*arctan(a*x)^3*Pi*csgn(I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+1/3*I*arctan(a*x)^3
*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-1
/3*I*arctan(a*x)^3*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)-2/3*I*arctan(a*x)^3*Pi+4*I*arctan(a*x)^2*polylog(2,-
```

```
(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2
+1)^(1/2))+3*I*(I+a*x)/(16*a*x-16*I)-3*I*(a*x-I)/(16*a*x+16*I)-3/8*arctan(a
*x)*(I+a*x)/(a*x-I)-3/8*arctan(a*x)*(a*x-I)/(I+a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**3,x)
[Out] Integral(atan(a*x)**3/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3),x)
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3), x)
```

3.409 $\int \frac{\text{ArcTan}(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=332

$$\frac{3a}{128c^3(1+a^2x^2)^2}+\frac{93a}{128c^3(1+a^2x^2)}+\frac{3a^2x\text{ArcTan}(ax)}{32c^3(1+a^2x^2)^2}+\frac{93a^2x\text{ArcTan}(ax)}{64c^3(1+a^2x^2)}+\frac{93a\text{ArcTan}(ax)^2}{128c^3}-\frac{3a\text{ArcTan}(a}{16c^3(1+a^2}$$

```
[Out] 3/128*a/c^3/(a^2*x^2+1)^2+93/128*a/c^3/(a^2*x^2+1)+3/32*a^2*x*arctan(a*x)/c
^3/(a^2*x^2+1)^2+93/64*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)+93/128*a*arctan(a*
x)^2/c^3-3/16*a*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-21/16*a*arctan(a*x)^2/c^3/(
a^2*x^2+1)-I*a*arctan(a*x)^3/c^3-arctan(a*x)^3/c^3/x-1/4*a^2*x*arctan(a*x)^
3/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)^3/c^3/(a^2*x^2+1)-15/32*a*arctan(
a*x)^4/c^3+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-3*I*a*arctan(a*x)*polylo
g(2,-1+2/(1-I*a*x))/c^3+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 0.55, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5086, 5038, 4946, 5044, 4988, 5004, 5112, 6745, 5012, 5050, 267, 5020, 5016}

$$-\frac{7a^2x\text{ArcTan}(ax)^3}{8c^3(a^2x^2+1)}-\frac{a^2x\text{ArcTan}(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{21a\text{ArcTan}(ax)^2}{16c^3(a^2x^2+1)}-\frac{3a\text{ArcTan}(ax)^2}{16c^3(a^2x^2+1)^2}+\frac{93a^2x\text{ArcTan}(ax)}{64c^3(a^2x^2+1)}+\frac{3a^2x\text{ArcTan}(ax)}{32c^3(a^2x^2+1)^2}+\frac{93a}{128c^3(a^2x^2+1)}+\frac{3a}{128c^3(a^2x^2+1)^2}-\frac{3ia\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^3}-\frac{15a\text{ArcTan}(ax)^4}{32c^3}-\frac{\text{ArcTan}(ax)^3}{c^3x}-\frac{ia\text{ArcTan}(ax)^3}{c^3}+\frac{93a\text{ArcTan}(ax)^2}{128c^3}+\frac{3a\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{3a\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (3*a)/(128*c^3*(1 + a^2*x^2)^2) + (93*a)/(128*c^3*(1 + a^2*x^2)) + (3*a^2*x
*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) + (93*a^2*x*ArcTan[a*x])/(64*c^3*(1
+ a^2*x^2)) + (93*a*ArcTan[a*x]^2)/(128*c^3) - (3*a*ArcTan[a*x]^2)/(16*c^3*
(1 + a^2*x^2)^2) - (21*a*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) - (I*a*ArcTa
n[a*x]^3)/c^3 - ArcTan[a*x]^3/(c^3*x) - (a^2*x*ArcTan[a*x]^3)/(4*c^3*(1 + a
^2*x^2)^2) - (7*a^2*x*ArcTan[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (15*a*ArcTan[a
*x]^4)/(32*c^3) + (3*a*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c^3 - ((3*I)*a
*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (3*a*PolyLog[3, -1 + 2/(
1 - I*a*x)])/(2*c^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)^2}\,dx}{c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{a^2x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{1}{8}\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)}\,d}{c^2}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{3a\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{a^2x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} - \frac{7a^2x}{8c^3}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{9a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} + \frac{9a\tan^{-1}(ax)^2}{128c^3} - \frac{3a\tan}{16c^3(1}$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{9a}{128c^3\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} + 93a$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{93a}{128c^3\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} + 93a$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{93a}{128c^3\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} + 93a$$

$$= \frac{3a}{128c^3\left(1+a^2x^2\right)^2} + \frac{93a}{128c^3\left(1+a^2x^2\right)} + \frac{3a^2x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{93a^2x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} + 93a$$

Mathematica [**A**]
time = 0.36, size = 232, normalized size = 0.70

$$\frac{a\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(ax)^3 - \frac{\text{ArcTan}(ax)^3}{ax} - \frac{ax\text{ArcTan}(ax)^3}{1+a^2x^2} - \frac{15}{32}\text{ArcTan}(ax)^4 + \frac{3}{8}\cos(2\text{ArcTan}(ax)) - \frac{3}{4}\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax)) + \frac{3\cos(4\text{ArcTan}(ax))}{1024} - \frac{3}{128}\text{ArcTan}(ax)^2\cos(4\text{ArcTan}(ax)) + 3\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + 3i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) + \frac{3}{4}\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax)) + \frac{3}{256}\text{ArcTan}(ax)\sin(4\text{ArcTan}(ax)) - \frac{1}{32}\text{ArcTan}(ax)^3\sin(4\text{ArcTan}(ax))\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - (a*x*ArcTan[a*x
]^3)/(1 + a^2*x^2) - (15*ArcTan[a*x]^4)/32 + (3*Cos[2*ArcTan[a*x]])/8 - (3*
ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/4 + (3*Cos[4*ArcTan[a*x]])/1024 - (3*ArcT
an[a*x]^2*Cos[4*ArcTan[a*x]])/128 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLo
g[3, E^((-2*I)*ArcTan[a*x])])/2 + (3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/4 + (3
*ArcTan[a*x]*Sin[4*ArcTan[a*x]])/256 - (ArcTan[a*x]^3*Sin[4*ArcTan[a*x]])/3
2))/c^3
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 41.55, size = 1917, normalized size = 5.77

method	result	size
derivativedivides	Expression too large to display	1917
default	Expression too large to display	1917

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/c^3*arctan(a*x)^3/a/x-7/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a^3*x^3-9/8
/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a*x-15/8/c^3*arctan(a*x)^4-3/8/c^3*(-4*I*a
rctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)+2*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)-2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)+4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I
*a*x)^2/(a^2*x^2+1)+1))^2+4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-4
*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1))^2+4*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I*arctan(a
*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-31/16*arctan(a*x)^2+4*arctan(a*x)^2*ln(
a^2*x^2+1)-8*arctan(a*x)^2*ln(2)+7/2*arctan(a*x)^2/(a^2*x^2+1)+1/2*arctan(a
*x)^2/(a^2*x^2+1)^2-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*(I+
a*x)/(a*x-I)+1/2*(a*x-I)/(I+a*x)-1/32*arctan(a*x)*sin(4*arctan(a*x))-1/128*
cos(4*arctan(a*x))-16*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-16*polylog(3,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+2*I*
arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))+4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))+8*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1
)-8*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*arctan(a*x)^2*ln(1+(1
+I*a*x)/(a^2*x^2+1)^(1/2))+8/3*I*arctan(a*x)^3-8*arctan(a*x)^2*ln(a*x)-4*I*
arctan(a*x)^2*Pi+16*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
6*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*Pi
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*arctan(a
*x)*(I+a*x)/(a*x-I)-I*arctan(a*x)*(a*x-I)/(I+a*x)+2*I*arctan(a*x)^2*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))^3-4*I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-2*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^3+4*I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1
```

```
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-4*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-15/4*arctan(a*x)^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16384*(2400*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^4 - 90*(a^5*x^5 + 2*
a^3*x^3 + a*x)*log(a^2*x^2 + 1)^4 + 256*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arcta
n(a*x)^3 - 48*(15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 4*(15*a^4*x^4
 + 25*a^2*x^2 + 8)*arctan(a*x))*log(a^2*x^2 + 1)^2 - (a^4*c^3*x^5 + 2*a^2*c
^3*x^3 + c^3*x)*(360*((8*a^2*x^2 + 7)*a^2/(a^12*c^3*x^4 + 2*a^10*c^3*x^2 +
a^8*c^3) + 2*(4*a^2*x^2 + 3)*log(a^2*x^2 + 1)/(a^10*c^3*x^4 + 2*a^8*c^3*x^2
 + a^6*c^3))*a^7 - 2949120*a^7*integrate(1/1024*x^7*arctan(a*x)^2*log(a^2*x
^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) - 73728
0*a^7*integrate(1/1024*x^7*log(a^2*x^2 + 1)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6
+ 3*a^2*c^3*x^4 + c^3*x^2), x) + 360*(2*a^2*x^2 + 1)*a^5*log(a^2*x^2 + 1)^3
/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5898240*a^6*integrate(1/1024*x^6
*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)
+ 1474560*a^6*integrate(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c^3*
x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) - 11796480*a^6*integrate
(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a
^2*c^3*x^4 + c^3*x^2), x) + 720*(2*a^2*x^2 + 1)*a^5*log(a^2*x^2 + 1)^2/(a^8
*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 270*(((16*a^2*x^2 + 15)*a^2/(a^14*c^3
*x^4 + 2*a^12*c^3*x^2 + a^10*c^3) + 2*(8*a^2*x^2 + 7)*log(a^2*x^2 + 1)/(a^1
2*c^3*x^4 + 2*a^10*c^3*x^2 + a^8*c^3))*a^4 + 2*(4*a^2*x^2 + 3)*a^2*log(a^2*
x^2 + 1)^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3))*a^5 + 600*a^5*(a^2/(a^
10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) + 2*log(a^2*x^2 + 1)/(a^8*c^3*x^4 + 2
*a^6*c^3*x^2 + a^4*c^3)) - 5898240*a^5*integrate(1/1024*x^5*arctan(a*x)^2*l
og(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)
 + 11796480*a^5*integrate(1/1024*x^5*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3
*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 11796480*a^4*integrate(1/1024*x^4*arc
tan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 29
49120*a^4*integrate(1/1024*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c^3*x^8
+ 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) - 19660800*a^4*integrate(1/1
024*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c
^3*x^4 + c^3*x^2), x) + 180*a^3*log(a^2*x^2 + 1)^3/(a^6*c^3*x^4 + 2*a^4*c^3
*x^2 + a^2*c^3) + 135*(a^4*(a^2/(a^12*c^3*x^4 + 2*a^10*c^3*x^2 + a^8*c^3) +
 2*log(a^2*x^2 + 1)/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3)) + 2*a^2*log(a
^2*x^2 + 1)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3))*a^3 - 2949120*a^3*in
tegrate(1/1024*x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*
```

```
x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 19660800*a^3*integrate(1/1024*x^3*arct
an(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 120
0*a^3*log(a^2*x^2 + 1)^2/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + 5898240*
a^2*integrate(1/1024*x^2*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2
*c^3*x^4 + c^3*x^2), x) + 1474560*a^2*integrate(1/1024*x^2*arctan(a*x)*log(
a^2*x^2 + 1)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)
- 6291456*a^2*integrate(1/1024*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^6*c^3*x^
8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 6291456*a*integrate(1/10
24*x*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2),
 x) - 1572864*a*integrate(1/1024*x*log(a^2*x^2 + 1)^2/(a^6*c^3*x^8 + 3*a^4*
c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 14680064*integrate(1/1024*arctan(a
*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 1572864
*integrate(1/1024*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x
^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)))/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{6}x^{8}+3a^{4}x^{6}+3a^{2}x^{4}+x^{2}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^2\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3), x)
```

3.410 $\int \frac{\textbf{ArcTan}(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=478

$$-\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{237a^3x}{256c^3\left(1+a^2x^2\right)}-\frac{237a^2\mathrm{ArcTan}(ax)}{256c^3}+\frac{3a^2\mathrm{ArcTan}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{57a^2\mathrm{ArcTan}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3ia^2\mathrm{ArcTa}}{2c^3}$$

```
[Out] -3/128*a^3*x/c^3/(a^2*x^2+1)^2-237/256*a^3*x/c^3/(a^2*x^2+1)-237/256*a^2*ar
ctan(a*x)/c^3+3/32*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2+57/32*a^2*arctan(a*x)/
c^3/(a^2*x^2+1)-9/4*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^3-3/2*a*arctan(a*x)^2
/c^3/x+3/16*a^3*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+57/32*a^3*x*arctan(a*x)^2
/c^3/(a^2*x^2+1)+3/32*a^2*arctan(a*x)^3/c^3-1/2*arctan(a*x)^3/c^3/x^2-1/4*a
^2*arctan(a*x)^3/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^3/c^3/(a^2*x^2+1)+9/2*I*
a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3+3*a^2*arctan(a*x)*ln(2-2/(1
-I*a*x))/c^3-3*a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c^3+3/4*I*a^2*arctan(a*x
)^4/c^3-3/2*I*a^2*arctan(a*x)^2/c^3-9/2*a^2*arctan(a*x)*polylog(3,-1+2/(1-I
*a*x))/c^3-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 1.30, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5086, 5038, 4946, 5044, 4988, 2497, 5004, 5112, 5116, 6745, 5050, 5012, 205, 211, 5020}

$$\frac{9ia^2\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^3}-\frac{9a^2\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(\frac{2}{1-iax}-1\right)}{2c^3}-\frac{a^2\mathrm{ArcTan}(ax)^3}{c^3\left(a^2x^2+1\right)}-\frac{a^2\mathrm{ArcTan}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{57a^2\mathrm{ArcTan}(ax)}{32c^3\left(a^2x^2+1\right)}+\frac{3a^2\mathrm{ArcTan}(ax)}{32c^3\left(a^2x^2+1\right)^2}+\frac{3ia^2\mathrm{ArcTan}(ax)^4}{4c^3}+\frac{3a^2\mathrm{ArcTan}(ax)^3}{32c^3}-\frac{3ia^2\mathrm{ArcTan}(ax)^2}{2c^3}-\frac{237a^2\mathrm{ArcTan}(ax)}{256c^3}-\frac{3a^2\mathrm{ArcTan}(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{3a^2\mathrm{ArcTan}(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{3ia^2\mathrm{Li}_2\left(\frac{2}{1-iax}-1\right)}{2c^3}-\frac{9ia^2\mathrm{Li}_4\left(\frac{2}{1-iax}-1\right)}{4c^3}+\frac{57a^3x\mathrm{ArcTan}(ax)^2}{32c^3\left(a^2x^2+1\right)}+\frac{3a^3x\mathrm{ArcTan}(ax)^2}{16c^3\left(a^2x^2+1\right)^2}-\frac{237a^3x}{256c^3\left(a^2x^2+1\right)}-\frac{3a^3x}{128c^3\left(a^2x^2+1\right)^2}-\frac{\mathrm{ArcTan}(ax)^3}{2c^3x^2}-\frac{3a\mathrm{ArcTan}(ax)^2}{2c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-3*a^3*x)/(128*c^3*(1 + a^2*x^2)^2) - (237*a^3*x)/(256*c^3*(1 + a^2*x^2))
- (237*a^2*ArcTan[a*x])/(256*c^3) + (3*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^
2)^2) + (57*a^2*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)) - (((3*I)/2)*a^2*ArcTan
[a*x]^2)/c^3 - (3*a*ArcTan[a*x]^2)/(2*c^3*x) + (3*a^3*x*ArcTan[a*x]^2)/(16*
c^3*(1 + a^2*x^2)^2) + (57*a^3*x*ArcTan[a*x]^2)/(32*c^3*(1 + a^2*x^2)) + (3
*a^2*ArcTan[a*x]^3)/(32*c^3) - ArcTan[a*x]^3/(2*c^3*x^2) - (a^2*ArcTan[a*x]
^3)/(4*c^3*(1 + a^2*x^2)^2) - (a^2*ArcTan[a*x]^3)/(c^3*(1 + a^2*x^2)) + (((
3*I)/4)*a^2*ArcTan[a*x]^4)/c^3 + (3*a^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])
/c^3 - (3*a^2*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)])/c^3 - (((3*I)/2)*a^2*Po
lyLog[2, -1 + 2/(1 - I*a*x)])/c^3 + (((9*I)/2)*a^2*ArcTan[a*x]^2*PolyLog[2,
 -1 + 2/(1 - I*a*x)])/c^3 - (9*a^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x
)])/(2*c^3) - (((9*I)/4)*a^2*PolyLog[4, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5116

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3(c+a^2cx^2)^2}\,dx}{c}$$

$$= a^4\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3(c+a^2cx^2)}\,dx}{c^2}-2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)^2}\,dx}{c}$$

$$= -\frac{a^2\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{1}{4}\left(3a^3\right)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx+\frac{\int \frac{\tan^{-1}(ax)^3}{x^3}\,dx}{c^3}-\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x(c+a^2cx^2)}\,dx}{c^2}$$

$$= \frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}-\frac{\tan^{-1}(ax)^3}{2c^3x^2}-\frac{a^2\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2}+\frac{ia^2\tan^{-1}(ax}{4c^3}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}+\frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{3a^3x\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2}+\frac{9a^3x\tan^{-1}(ax)^2}{32c^3\left(1+a^2x^2\right)}+\frac{3a^2}{}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{9a^3x}{256c^3\left(1+a^2x^2\right)}+\frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3a\,t}{}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{45a^3x}{256c^3\left(1+a^2x^2\right)}-\frac{9a^2\tan^{-1}(ax)}{256c^3}+\frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9a^2\,t}{32c^3\,(}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{45a^3x}{256c^3\left(1+a^2x^2\right)}-\frac{45a^2\tan^{-1}(ax)}{256c^3}+\frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9a^2}{32c^3}$$

$$= -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{45a^3x}{256c^3\left(1+a^2x^2\right)}-\frac{45a^2\tan^{-1}(ax)}{256c^3}+\frac{3a^2\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{9a^2}{32c^3}$$

Mathematica [A]
time = 0.72, size = 295, normalized size = 0.62

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^2*((48*I)*Pi^4 - (1536*I)*ArcTan[a*x]^2 - (1536*ArcTan[a*x]^2)/(a*x) - (
512*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (768*I)*ArcTan[a*x]^4 + 960*Ar
cTan[a*x]*Cos[2*ArcTan[a*x]] - 640*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 12*Ar
cTan[a*x]*Cos[4*ArcTan[a*x]] - 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 3072*A
```

```
rcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 3072*ArcTan[a*x]*Log[1 - E^(
(2*I)*ArcTan[a*x])] - (4608*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*
x])] - (1536*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] - 4608*ArcTan[a*x]*PolyLo
g[3, E^((-2*I)*ArcTan[a*x])] + (2304*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])]
- 480*Sin[2*ArcTan[a*x]] + 960*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 3*Sin[4*A
rcTan[a*x]] + 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(1024*c^3)
```

Maple [A]
time = 51.24, size = 579, normalized size = 1.21

method	result
derivativedivides	$a^2\left(-\frac{18i\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\right.}{}\right.$
default	$a^2\left(-\frac{18i\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(9*I/c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+5/64*(6*
I*arctan(a*x)^2+4*arctan(a*x)^3-3*I-6*arctan(a*x))*(a*x-I)/c^3/(I+a*x)+5/64
*(-6*I*arctan(a*x)^2+4*arctan(a*x)^3+3*I-6*arctan(a*x))*(I+a*x)/c^3/(a*x-I)
-1/2/c^3*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*a*x-3*I*a*x)*(I+a*x)/a^2
/x^2-18*I/c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/c^3*arctan(a*x)^3*ln
(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I/c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-18/c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c^3*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+3/4*I/c^3*arctan(a*x)^4-18/c^3*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-3*I/c^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/
c^3*arctan(a*x)^2+3/c^3*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+9*I/c
^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^3*arctan(a*x)*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/256*arctan(a*x)*(8*arctan(a*x)^2-3)/c^3*
cos(4*arctan(a*x))+3/1024*(8*arctan(a*x)^2-1)/c^3*sin(4*arctan(a*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x
^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{6}x^{9}+3a^{4}x^{7}+3a^{2}x^{5}+x^{3}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3), x)
```

3.411 $\int \frac{\text{ArcTan}(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=432

$$-\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2}-\frac{141a^3}{128c^3\left(1+a^2x^2\right)}-\frac{a^2\text{ArcTan}(ax)}{c^3x}-\frac{3a^4x\text{ArcTan}(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{141a^4x\text{ArcTan}(ax)}{64c^3\left(1+a^2x^2\right)}-\frac{205a^3\text{Arc}}{128}$$

```
[Out] -3/128*a^3/c^3/(a^2*x^2+1)^2-141/128*a^3/c^3/(a^2*x^2+1)-a^2*arctan(a*x)/c^
3/x-3/32*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)^2-141/64*a^4*x*arctan(a*x)/c^3/(
a^2*x^2+1)-205/128*a^3*arctan(a*x)^2/c^3-1/2*a*arctan(a*x)^2/c^3/x^2+3/16*a
^3*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+33/16*a^3*arctan(a*x)^2/c^3/(a^2*x^2+1)+
10/3*I*a^3*arctan(a*x)^3/c^3-1/3*arctan(a*x)^3/c^3/x^3+3*a^2*arctan(a*x)^3/
c^3/x+1/4*a^4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x)^3/c^
3/(a^2*x^2+1)+35/32*a^3*arctan(a*x)^4/c^3+a^3*ln(x)/c^3-1/2*a^3*ln(a^2*x^2+
1)/c^3-10*a^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3+10*I*a^3*arctan(a*x)*poly
log(2,-1+2/(1-I*a*x))/c^3-5*a^3*polylog(3,-1+2/(1-I*a*x))/c^3
```

Rubi [A]
time = 1.54, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5086, 5038, 4946, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745, 5012, 5050, 267, 5020, 5016}

$$\frac{10ia^3\text{ArcTan}(ax)\text{Li}_2\left(\frac{2}{1-iax}-1\right)}{c^3}+\frac{35a^3\text{ArcTan}(ax)^4}{32c^3}+\frac{10ia^3\text{ArcTan}(ax)^3}{3c^3}-\frac{205a^3\text{ArcTan}(ax)^2}{128c^3}-\frac{10a^3\text{ArcTan}(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{5a^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)}{c^3}+\frac{a^3\log(x)}{c^3}+\frac{3a^2\text{ArcTan}(ax)^3}{c^3x}-\frac{a^2\text{ArcTan}(ax)}{c^3x}+\frac{11a^4x\text{ArcTan}(ax)^3}{8c^3\left(a^2x^2+1\right)}+\frac{a^4x\text{ArcTan}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{141a^4x\text{ArcTan}(ax)}{64c^3\left(a^2x^2+1\right)}-\frac{3a^4x\text{ArcTan}(ax)}{32c^3\left(a^2x^2+1\right)^2}+\frac{33a^3\text{ArcTan}(ax)^2}{16c^3\left(a^2x^2+1\right)}+\frac{3a^3\text{ArcTan}(ax)^2}{16c^3\left(a^2x^2+1\right)^2}-\frac{141a^3}{128c^3\left(a^2x^2+1\right)}-\frac{3a^3}{128c^3\left(a^2x^2+1\right)^2}-\frac{a^3\log\left(a^2x^2+1\right)}{2c^3}-\frac{\text{ArcTan}(ax)^3}{3c^3x^3}-\frac{a\text{ArcTan}(ax)^2}{2c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-3*a^3)/(128*c^3*(1 + a^2*x^2)^2) - (141*a^3)/(128*c^3*(1 + a^2*x^2)) - (a
^2*ArcTan[a*x])/(c^3*x) - (3*a^4*x*ArcTan[a*x])/(32*c^3*(1 + a^2*x^2)^2) -
(141*a^4*x*ArcTan[a*x])/(64*c^3*(1 + a^2*x^2)) - (205*a^3*ArcTan[a*x]^2)/(1
28*c^3) - (a*ArcTan[a*x]^2)/(2*c^3*x^2) + (3*a^3*ArcTan[a*x]^2)/(16*c^3*(1
+ a^2*x^2)^2) + (33*a^3*ArcTan[a*x]^2)/(16*c^3*(1 + a^2*x^2)) + (((10*I)/3)
*a^3*ArcTan[a*x]^3)/c^3 - ArcTan[a*x]^3/(3*c^3*x^3) + (3*a^2*ArcTan[a*x]^3)
/(c^3*x) + (a^4*x*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (11*a^4*x*ArcTan
[a*x]^3)/(8*c^3*(1 + a^2*x^2)) + (35*a^3*ArcTan[a*x]^4)/(32*c^3) + (a^3*Log
[x])/c^3 - (a^3*Log[1 + a^2*x^2])/(2*c^3) - (10*a^3*ArcTan[a*x]^2*Log[2 - 2
/(1 - I*a*x)])/c^3 + ((10*I)*a^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)]
)/c^3 - (5*a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/c^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
```

p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= a^4\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx}{c^2} - 2\frac{a^2\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx}{c}$$

$$= \frac{3a^3\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} + \frac{a^4x\tan^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} - \frac{1}{8}\left(3a^4\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^3}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x^4}\,dx}{c^3} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} + \frac{3a^3\tan^{-1}(ax)^2}{16c^3\left(1+a^2x^2\right)^2} - \frac{\tan^{-1}(ax)^3}{3c^3x^3} +$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{9a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} - \frac{9a^3\tan^{-1}(ax)^2}{128c^3} +$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{9a^3}{128c^3\left(1+a^2x^2\right)} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} - \frac{45a^4x\tan^{-1}(ax)}{64c^3\left(1+a^2x^2\right)} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} -$$

$$= -\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2} - \frac{45a^3}{128c^3\left(1+a^2x^2\right)} - \frac{a^2\tan^{-1}(ax)}{c^3x} - \frac{3a^4x\tan^{-1}(ax)}{32c^3\left(1+a^2x^2\right)^2} -$$

Mathematica [A]
time = 0.82, size = 301, normalized size = 0.70

$$\frac{a^3\left(\frac{5i\pi^3}{12} - \frac{\text{ArcTan}(ax)}{ax} - \frac{1}{2}\text{ArcTan}(ax)^2 - \frac{\text{ArcTan}(ax)^2}{2a^2x^2} - \frac{10}{3}i\text{ArcTan}(ax)^3 - \frac{\text{ArcTan}(ax)^3}{3a^3x^3} + \frac{3\text{ArcTan}(ax)^3}{ax} + \frac{35}{32}\text{ArcTan}(ax)^4 - \frac{9}{16}\cos(2\text{ArcTan}(ax)) + \frac{9}{8}\text{ArcTan}(ax)^2\cos(2\text{ArcTan}(ax)) - \frac{3\cos(4\text{ArcTan}(ax))}{1024} + \frac{3}{128}\text{ArcTan}(ax)^2\cos(4\text{ArcTan}(ax)) - 10\text{ArcTan}(ax)^2\log\left(1-e^{-2i\text{ArcTan}(ax)}\right) + \log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) - 10i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(ax)}\right) - 5\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(ax)}\right) - \frac{9}{8}\text{ArcTan}(ax)\sin(2\text{ArcTan}(ax)) + \frac{3}{4}\text{ArcTan}(ax)^3\sin(2\text{ArcTan}(ax)) - \frac{3}{256}\text{ArcTan}(ax)\sin(4\text{ArcTan}(ax)) + \frac{1}{32}\text{ArcTan}(ax)^3\sin(4\text{ArcTan}(ax))\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

```
[Out] (a^3*(((5*I)/12)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2
/(2*a^2*x^2) - ((10*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (3*Ar
cTan[a*x]^3)/(a*x) + (35*ArcTan[a*x]^4)/32 - (9*Cos[2*ArcTan[a*x]])/16 + (9
*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - (3*Cos[4*ArcTan[a*x]])/1024 + (3*Arc
Tan[a*x]^2*Cos[4*ArcTan[a*x]])/128 - 10*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (10*I)*ArcTan[a*x]*PolyLog[2, E
^((-2*I)*ArcTan[a*x])] - 5*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (9*ArcTan[a
*x]*Sin[2*ArcTan[a*x]])/8 + (3*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4 - (3*Arc
Tan[a*x]*Sin[4*ArcTan[a*x]])/256 + (ArcTan[a*x]^3*Sin[4*ArcTan[a*x]])/32))/
c^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 43.81, size = 2062, normalized size = 4.77

method	result	size
derivativedivides	Expression too large to display	2062
default	Expression too large to display	2062

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/c^3*arctan(a*x)^3/a^3/x^3+3/c^3*arctan(a*x)^3/a/x+11/8/c^3*arctan
(a*x)^3/(a^2*x^2+1)^2*a^3*x^3+13/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a*x+35/8
/c^3*arctan(a*x)^4-1/8/c^3*(205/16*arctan(a*x)^2-40*arctan(a*x)^2*ln(a^2*x^
2+1)+80*arctan(a*x)^2*ln(2)+4*arctan(a*x)^2/a^2/x^2-33/2*arctan(a*x)^2/(a^2
*x^2+1)-20*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)+40*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(
I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))-3/2*arctan(a*x)^2/(a^2*x^2+1)^2+80*arctan(a*x)^2*ln((1+
I*a*x)/(a^2*x^2+1)^(1/2))-9/4*(I+a*x)/(a*x-I)-9/4*(a*x-I)/(I+a*x)-8*ln((1+I
*a*x)/(a^2*x^2+1)^(1/2)-1)-8*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/32*arctan(
a*x)*sin(4*arctan(a*x))+3/128*cos(4*arctan(a*x))+160*polylog(3,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+160*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-80*arctan(a*x)
^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+80*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+80*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*(
I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+4*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/
x+20*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+20*I*arctan(a*x)^2*Pi*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)^2-40*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2+40*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
```

```
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-20*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-40*I*arctan(a*x)^2*Pi*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
+40*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^3-20*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^3+9*I*arctan(a*x)*(a*x-I)/(2*a*x+2*I)-40*I*arctan(a*
x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-20*
I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+20*I*arctan(a*x)^2*Pi*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-9*I*(I+a*x)*arctan(a*x)/(2*a*x-2*I)
-40*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+80*arctan(a*x)^2*ln(a*x)+
40*I*arctan(a*x)^2*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))^3+105/4*arctan(a*x)^4-80/3*I*arctan(a*x)^3+20*I*arctan(a*x)^2*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+
40*I*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-40
*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-160*I*arctan(a*x)*polylog(2,
(1+I*a*x)/(a^2*x^2+1)^(1/2))+40*I*arctan(a*x)^2*Pi-160*I*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^3/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*
x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{3}(ax)}{a^{6}x^{10}+3a^{4}x^{8}+3a^{2}x^{6}+x^{4}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**3/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/c
**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^4\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3), x)
```

3.412 $\int x^3\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=523

$$-\frac{x\sqrt{c+a^2cx^2}}{20a^3}-\frac{9\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{20a^4}+\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{10a^2}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{8a^3}-3x$$

[Out] 1/2*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4-11/20*I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+11/20*I*c*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-11/20*I*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-11/20*c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+11/20*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-1/20*x*(a^2*c*x^2+c)^(1/2)/a^3-9/20*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/10*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+1/8*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-3/20*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-2/15*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4+1/15*x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+1/5*x^4*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 1.70, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

$$\frac{x^2\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{15a^2}+\frac{x^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{10a^2}+\frac{1}{5}x^4\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}-\frac{3x^3\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{20a}+\frac{11ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{11c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}+\frac{11c\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{20a^4\sqrt{a^2cx^2+c}}-\frac{2\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{15a^4}-\frac{11ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{20a^4\sqrt{a^2cx^2+c}}-\frac{9\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{20a^4}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{2a^4}+\frac{x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{8a^3}-\frac{x\sqrt{a^2cx^2+c}}{20a^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]

[Out] -1/20*(x*Sqrt[c + a^2*c*x^2])/a^3 - (9*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(20*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(10*a^2) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a^3) - (3*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(20*a) - (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*a^4) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(15*a^2) + (x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/5 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a^4) + (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((11*I)/20)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^4*Sqrt[c + a^2*c*x^2]) + (11*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*a^4*Sqrt[c + a^2*c*x^2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
```

```
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx = c\int \frac{x^3\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x^5\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{1}{5}(4c)\int \frac{x^3 \text{ta}}{\sqrt{c}}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3} - \frac{3x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{20a} - \frac{2\sqrt{c+a^2cx}}{3}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-}}{8a^3}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{x\sqrt{c+a^2cx^2}}{20a^3} - \frac{9\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^4} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{10a^2}$$

Mathematica [A]
time = 4.66, size = 594, normalized size = 1.14

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((24*(11*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] +
11*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*Ar
```

```
cTan[a*x]))] - 11*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*
x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 11
*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*Arc
Tan[a*x])] + 11*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*
E^((I/2)*ArcTan[a*x]))] - 11*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4
]] - 20*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 11*ArcTan[a*x]^2*Log
[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 20*Log[Cos[ArcTan[a*x]/2] + Sin
[ArcTan[a*x]/2]] - 11*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2]] - 11*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (22*I)*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (22*I)*ArcTan[a*x]*PolyLog[2, I*E
^(I*ArcTan[a*x])] - 22*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 22*PolyLog[3, I
*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (1 + a^2*x^2)^2*((48*a*x)/(1 + a^
2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25
 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*
ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]]))))/(960*a^4)
```

Maple [A]
time = 7.31, size = 417, normalized size = 0.80

method	result
default	$\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(24\arctan\left(ax\right)^{3}a^{4}x^{4}-18\arctan\left(ax\right)^{2}a^{3}x^{3}+8\arctan\left(ax\right)^{3}a^{2}x^{2}+12\arctan\left(ax\right)a^{2}x^{2}+15\arctan\left(ax\right)^{2}ax\right.}{120a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)^3*a^4*x^4-18*arctan(a*x
)^2*a^3*x^3+8*arctan(a*x)^3*a^2*x^2+12*arctan(a*x)*a^2*x^2+15*arctan(a*x)^2
*a*x-16*arctan(a*x)^3-6*a*x-54*arctan(a*x))+11/120*(c*(a*x-I)*(I+a*x))^(1/2
)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-11/120*(c*(a*x-I)*(I+a*x))^(1
/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-I/a^4*(c*(a*x-I)*(I+a*x))^(1/
2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\mathrm{atan}\left(a\,x\right)}^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.413 $\int x^2 \sqrt{c+a^2cx^2}\, \textbf{ArcTan}(ax)^3\, dx$

Optimal. Leaf size=747

$$-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{8a^3}-\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{4a}+x\sqrt{c$$

```
[Out] 1/2*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/
(a^2*c*x^2+c)^(1/2)+3/8*I*c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c*arctan(a*x)^2*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^
(1/2)-1/2*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/a^3/(a^2*c*x^2+c)^(1/2)-3/4*I*c*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/4*I*c*polylog(4,-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/4*c*arctan(a*
x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x
^2+c)^(1/2)-3/4*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/4*I*c*arctan
((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^
2+c)^(1/2)-1/4*(a^2*c*x^2+c)^(1/2)/a^3+1/4*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2
)/a^2+1/8*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-1/4*x^2*arctan(a*x)^2*(a^2*
c*x^2+c)^(1/2)/a+1/8*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+1/4*x^3*arctan
(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 1.29, antiderivative size = 747, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 5010, 5006, 5008, 4266, 2611, 6744, 2320, 6724, 267}

$$\frac{x\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{8a^2}-\frac{x^2\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{4a}+\frac{x\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{4a^2}+\frac{1}{4}x^3\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}-\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{8a^3\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}-\frac{3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{4a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{4a^3\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{8a^3}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a^3\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] -1/4*Sqrt[c + a^2*c*x^2]/a^3 + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2)
+ (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a^3) - (x^2*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)/(4*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(8*a^2) + (x^3*S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/4 + ((I/4)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(
I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (I*c*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - (((3*I)/8)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

```
 - ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + ((I/2)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*a^3*Sqrt[c +
 a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[
a*x])])/(4*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*c*Sqrt[1 + a^2*x^2]*PolyLo
g[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*c*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

GtQ[d, 0]

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx = c\int\frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx+\left(a^2c\right)\int\frac{x^4\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$=\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3-\frac{1}{4}(3c)\int\frac{x^2\tan}{\sqrt{c}}$$

$$=-\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3}-\frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{4a}+\frac{x\sqrt{c+a^2cx^2}}{8a}$$

$$=\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-\frac{x^2\sqrt{c+a^2cx^2}\,\tan}{4a}$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

$$=-\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{8a^3}-$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. 2(747) = 1494.
time = 12.10, size = 1844, normalized size = 2.47

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] ((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt
[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*
E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I
*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-1/8
*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(L
og[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) +
 I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTa
n[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan
[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x]
)*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan
[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(
Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/
2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Log[1
 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + A
rcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + ((
3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3
*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + ArcT
an[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + (I/2)*P
olyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi +
ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan
[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (Pi/2 + (
-1/2*Pi + ArcTan[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x]
)/2))] + I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 +
(-1/2*Pi + ArcTan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcT
an[a*x])/2))]/2))/2 - (3*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[3, -E^(
(2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*
(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/2*Pi +
ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[
a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^4)
+ (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2 - ArcTan[a*x]^3))/(
16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c
*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos
[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*
x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^4) +
```

```
(Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^
2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-
2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[
ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTa
n[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[Ar
cTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[a
*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTa
n[a*x]/2] - Sin[ArcTan[a*x]/2])))/a^3
```

Maple [A]
time = 3.89, size = 460, normalized size = 0.62

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\left(2\arctan(ax)^3a^3x^3-2\arctan(ax)^2a^2x^2+\arctan(ax)^3ax+2\arctan(ax)ax+\arctan(ax)^2-2\right)}{8a^3}+\frac{\sqrt{c(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2*
a^2*x^2+arctan(a*x)^3*a*x+2*arctan(a*x)*a*x+arctan(a*x)^2-2)+1/8*(c*(a*x-I)
*(I+a*x))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a
*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*
polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I
*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1+I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c (a^2x^2 + 1)} \operatorname{atan}^3 (ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atan}(a\,x)^3 \sqrt{c\,a^2\,x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.414 $\int x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=373

$$\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{a^2}-\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2a}+\frac{ic\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^2\sqrt{c+a^2cx^2}}+\frac{(c$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/a^2/c-arctanh(a*x*c^(1/2)/(a^2*c*x^2+
c)^(1/2))*c^(1/2)/a^2+I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2
*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*c*arctan(a*x)*polylog(2,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+I*c*arc
tan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^
2*c*x^2+c)^(1/2)+c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a^2/(a^2*c*x^2+c)^(1/2)-c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2-1
/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]

time = 0.25, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5050, 5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212}

$$-\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}-\frac{c\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}+\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^2\sqrt{a^2cx^2+c}}-\frac{x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2a}+\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^2}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a^2 - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/(2*a) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/
(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c)
 - (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a^2 - (I*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2
*c*x^2]) + (I*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)])/(a^2*Sqrt[c + a^2*c*x^2]) + (c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (c*Sqrt[1 + a^2*x^2]*PolyLog[3, I
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx = \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{3a^2c} - \frac{\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx}{a}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{(c+a^2cx^2)^{3/2}\tan}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{(c+a^2cx^2)^{3/2}\tan}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{(c+a^2cx^2)^{3/2}\tan}{3a^2c}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\,\tan}{a^2}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\,\tan}{a^2}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\,\tan}{a^2}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^2} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{ic\sqrt{1+a^2x^2}\,\tan}{a^2}$$

Mathematica [A]
time = 2.09, size = 535, normalized size = 1.43

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((-6*(ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + Pi*
ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a
*x]))] - ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[((1
/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + Pi*ArcTan[a*x]
*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + Ar
cTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*
x]))] - Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 2*Log[Cos[ArcTan
[a*x]/2] - Sin[ArcTan[a*x]/2]] + ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin
```

```
[ArcTan[a*x]/2]] + 2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - ArcTan[
a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - Pi*ArcTan[a*x]*Log[Si
n[(Pi + 2*ArcTan[a*x])/4]] + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x
^2] + (1 + a^2*x^2)*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]]
 - 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(12*a^2)
```

Maple [A]
time = 2.94, size = 370, normalized size = 0.99

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)\left(2\arctan(ax)^2 a^2x^2-3\arctan(ax)ax+2\arctan(ax)^2+6\right)}{6a^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\ (}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*a^2*x^2-3*ar
ctan(a*x)*a*x+2*arctan(a*x)^2+6)-1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*
x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x
)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*pol
ylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a^2/(a^2*x^2+1)^(1/2)+2*I/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.415 $\int \sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=626

$$-\frac{3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3-\frac{ic\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a\sqrt{c+a^2cx^2}}$$

```
[Out] -I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)-6*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+3/2*I*c*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/2
*I*c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/a/(a^2*c*x^2+c)^(1/2)+3*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*c*polylog(2,I*(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*c*arctan(a*x)*p
olylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^
(1/2)+3*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*c*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+3*I*c*polylog(4,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/2*arctan(a*x)^2*(a^2
*c*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5000, 5010, 5008, 4266, 2611, 6744, 2320, 6724, 5006}

$$\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{2a\sqrt{a^2cx^2+c}}-\frac{3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}-\frac{3\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2a}-\frac{6ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a\sqrt{a^2cx^2+c}}+\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/2 - (I*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
3)/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c
 + a^2*c*x^2]) - (((3*I)/2)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*
E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*Po
lyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/(a*Sqrt[c + a^2*c*x^2]) - (3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (3*c*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)
```

```
*c*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*
x^2]) + ((3*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt
[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx = -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 + \frac{1}{2}c\int \frac{\tan^{-1}(a}{\sqrt{c+a^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 + \frac{\left(c\sqrt{1+a^2x^2}\right.}{2\sqrt{c}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{6ic\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{ic\sqrt{1+a^2x^2}}{}$$

Mathematica [A]
time = 0.54, size = 258, normalized size = 0.41

$$-\frac{i\sqrt{c(1+a^2x^2)}\left(12\text{ArcTan}(e^{i\text{ArcTan}(ax)})\text{ArcTan}(ax) - 3i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2 + iax\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3 + 2\text{ArcTan}(e^{i\text{ArcTan}(ax)})\text{ArcTan}(ax)^3 - 3(2+\text{ArcTan}(ax)^2)\text{PolyLog}(2,-ie^{i\text{ArcTan}(ax)}) + 3(2+\text{ArcTan}(ax)^2)\text{PolyLog}(2,ie^{i\text{ArcTan}(ax)}) - 6i\text{ArcTan}(ax)\text{PolyLog}(3,-ie^{i\text{ArcTan}(ax)}) + 6i\text{ArcTan}(ax)\text{PolyLog}(3,ie^{i\text{ArcTan}(ax)}) + 6\text{PolyLog}(4,-ie^{i\text{ArcTan}(ax)}) - 6\text{PolyLog}(4,ie^{i\text{ArcTan}(ax)})\right)}{2a\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] ((-1/2*I)*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] -
 (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x
]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I*A
rcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)
*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTa
n[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.32, size = 422, normalized size = 0.67

method	result
default	$\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\arctan(ax)^2(\arctan(ax)ax-3)}{2a} - \frac{\sqrt{c\,(ax-i)\,(ax+i)}\,\left(\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(arctan(a*x)*a*x-3)-1/2*(c*(a
*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arc
tan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(
a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*dilog(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^3{(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.416 $\int \frac{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}{x}\,dx$

Optimal. Leaf size=600

$$\frac{6ic\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}+\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3-\frac{2c\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3\,\text{tan}}{\sqrt{c+a^2cx^2}}$$

```
[Out] 6*I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)-2*c*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c*arctan(a*
x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)+6*I*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c*arctan(a*x)*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*c*
polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-6*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*c*polylog(4,(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)
^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5078, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5010, 5008, 4266}

$$\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6ic\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6c\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6ic\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}+\frac{6ic\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{2c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

```
[Out] ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
+ a^2*c*x^2] + Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (2*c*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*c*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*c*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c +
```

```
a^2*c*x^2] - (6*c*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + (6*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*c*Sqrt[1 + a^2*x^2]*PolyLog[4, E^
(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x}\,dx &= c\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx \\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - (3ac)\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx + \frac{\left(c\sqrt{1+a^2x^2}\right)\int -}{\sqrt{c+a^2c}} \\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 + \frac{\left(c\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^3\csc(x)\,dx, x, \tan^{-1}(\right.}{\sqrt{c+a^2cx^2}} \\
&= \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 \\
&= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 \\
&= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3 \\
&= \frac{6ic\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\tan^{-1}(ax)^3
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 366, normalized size = 0.61

$$\frac{\sqrt{c+a^2cx^2}\left(-i\pi^4+8\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3+2i\text{ArcTan}(ax)^4+8\text{ArcTan}(ax)^3\log\left(1-e^{-i\text{ArcTan}(ax)}\right)-24\text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right)+24\text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right)-8\text{ArcTan}(ax)^3\log\left(1+e^{i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-48i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+48i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)+48\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-i\text{ArcTan}(ax)}\right)-48\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)+48\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)-48\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,e^{-i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,-e^{i\text{ArcTan}(ax)}\right)\right)}{8\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*((-I)*Pi^4 + 8*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (2*I)
*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 24*ArcTan[
a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 24*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTa
n[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^
```

```
2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*
ArcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (48
*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3,
 E^((-I)*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 48*PolyLog[3, I*E^(I*ArcTan[a*x])] -
(48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*
x])]))/(8*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 3.02, size = 453, normalized size = 0.76

method	result
default	$\sqrt{c\left(ax-i\right)\left(ax+i\right)}\ \arctan\left(ax\right)^{3}+\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\ \left(\arctan(ax)^{3}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-\arctan(ax)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^3+(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a
*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x
^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*a
rctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*ln(1+I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan
(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,(a^2x^2+1)}\ \operatorname{atan}^3{(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,\sqrt{c\,a^2\,x^2+c}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.417 $\int \frac{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=622

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{x}-\frac{2iac\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}-\frac{6ac\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)}{\sqrt{c+a^2c}}$$

```
[Out] -2*I*a*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-6*a*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*c*arctan(a*x)*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a*c*ar
ctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)-3*I*a*c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c*arctan(a*x)*polylog(2,(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6
*a*c*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)+6*a*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c*polylog(3,(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c*polylog(4,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*c*po
lylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.49, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5064, 5078, 5076, 4268, 2611, 2320, 6724, 5010, 5008, 4266, 6744}

$$\frac{3iac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3iac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6iac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6iac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6ac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ac\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6ac\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6iac\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6iac\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2iac\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{x}-\frac{6ac\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x) - ((2*I)*a*c*Sqrt[1 + a^2*x^2]*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (6*a*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (
(6*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] + ((3*I)*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] - (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
```

```
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c*Sqrt[1 + a^2*x^
2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c*Sqrt[1 +
 a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*
a*c*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + (3ac)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx + \frac{\left(a^2c\sqrt{1+a^2x^2}\right.}{\sqrt{c+}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} + \frac{\left(ac\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^3\sec(x)\,dx, x, \tan\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 2.61, size = 768, normalized size = 1.23

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (a*Sqrt[c + a^2*c*x^2]*Csc[ArcTan[a*x]/2]*((-7*I)*a*Pi^4*x - (8*I)*a*Pi^3*x
*ArcTan[a*x] + (24*I)*a*Pi^2*x*ArcTan[a*x]^2 - (32*I)*a*Pi*x*ArcTan[a*x]^3
- 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (16*I)*a*x*ArcTan[a*x]^4 + 48*a*Pi^2
*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*a*Pi*x*ArcTan[a*x]^2*Log[1
```

```
 - I/E^(I*ArcTan[a*x])] - 8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])] + 64*a*x*
ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 192*a*x*ArcTan[a*x]^2*Log[1 -
E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])] - 48*a*Pi^2*x*
ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*a*Pi*x*ArcTan[a*x]^2*Log[1 +
I*E^(I*ArcTan[a*x])] - 64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] -
192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[Tan[(Pi +
 2*ArcTan[a*x])/4]] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan
[a*x])] + (48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])
] + (384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (48*I)*a*Pi^2*
x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])] - (384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 384
*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*a*Pi*x*PolyLog[3,
 I/E^(I*ArcTan[a*x])] - 384*a*x*PolyLog[3, -E^(I*ArcTan[a*x])] + 192*a*Pi*x
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 384*a*x*ArcTan[a*x]*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])] + 384*a*x*PolyLog[3, E^(I*ArcTan[a*x])] - (384*I)*a*x*Pol
yLog[4, (-I)/E^(I*ArcTan[a*x])] - (384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a
*x])])*Sec[ArcTan[a*x]/2])/(128*(1 + a^2*x^2))
```

Maple [A]
time = 3.01, size = 466, normalized size = 0.75

method	result
default	$-\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)^{3}}{x}+\frac{ia\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(i\arctan\left(ax\right)^{3}\ln\left(1+\frac{i\left(iax+1\right)}{\sqrt{a^{2}x^{2}+1}}\right)-i\arctan(a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^3/x+I*a*(c*(a*x-I)*(I+a*x))^(1/2)*(I
*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+3*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)
*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{3}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,\sqrt{c\,a^2\,x^2+c}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.418 $\int \frac{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=602

$$-\frac{3a\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}{2x}-\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^3}{2x^2}-\frac{a^2c\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

```
[Out] -a^2*c*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-6*a^2*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*c*arctan(a*x)^2*poly
log(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3
/2*I*a^2*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*polylog(2,(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*c*arctan(
a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)+3*a^2*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*polylog(4,(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*arctan(a*x)^2*
(a^2*c*x^2+c)^(1/2)/x-1/2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.77, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5070, 5082, 5064, 5078, 5074, 5076, 4268, 2611, 6744, 2320, 6724}

$$\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(-e^{i\mathrm{ArcTan}(ax)}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(e^{i\mathrm{ArcTan}(ax)}\right)}{2\sqrt{a^2cx^2+c}}-\frac{3a^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(-e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3a^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{Li}_4\left(-e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{Li}_4\left(e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{\mathrm{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{2x^2}-\frac{3a\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2x}-\frac{a^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\mathrm{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6a^2c\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2c\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) - (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(2*x^2) - (a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((3*I)/2)*a^
2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] - (((3*I)/2)*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*Sqrt[1 + a^2*x^2]*Po
lyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*
a^2*c*Sqrt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c
 + a^2*c*x^2] - (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] + (3*a^2*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
```

```
lyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c*S
qrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3ac)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx - \frac{1}{2}\left(a^2c\right)\int \frac{\text{ta}}{x\sqrt{}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} + \left(3a^2c\right)\int \frac{\text{tan}}{x\sqrt{c}}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{2a^2c\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}}{}$$

$$= -\frac{3a\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}}{}$$

Mathematica [A]
time = 3.84, size = 345, normalized size = 0.57

$$\frac{a^2\sqrt{c(1+a^2x^2)}\left(-i\pi^4+2i\text{ArcTan}(ax)^4-12\text{ArcTan}(ax)^2\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)-2\text{ArcTan}(ax)^3\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+8\text{ArcTan}(ax)^3\log\left(1-e^{-i\text{ArcTan}(ax)}\right)+48\text{ArcTan}(ax)\log\left(1-e^{i\text{ArcTan}(ax)}\right)-48\text{ArcTan}(ax)\log\left(1+e^{i\text{ArcTan}(ax)}\right)-8\text{ArcTan}(ax)^3\log\left(1+e^{i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-i\text{ArcTan}(ax)}\right)+24i\left(2+\text{ArcTan}(ax)^2\right)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+48\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-i\text{ArcTan}(ax)}\right)-48\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,e^{-i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,-e^{i\text{ArcTan}(ax)}\right)+2\text{ArcTan}(ax)^3\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-12\text{ArcTan}(ax)^2\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{16\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] (a^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x
]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*ArcTan[a*
x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x
])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E
^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] +
```

```
(24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog
[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] -
48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-I)*A
rcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Sec[
ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[1 + a^2*x
^2])
```

Maple [A]
time = 3.02, size = 404, normalized size = 0.67

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)^2(3ax+\arctan(ax))}{2x^2}-\frac{ia^2\sqrt{c\,(ax-i)\,(ax+i)}\ \left(i\arctan(ax)^3\ln\left(1-\frac{iax+}{\sqrt{a^2x^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(3*a*x+arctan(a*x))/x^2-1/2*I*
a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*pol
ylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arct
an(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*ln(1+(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,(1+I*a*x)/(a^2*x^2+
1)^(1/2))-6*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,-(1+I*a*x)/
(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{3}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,\sqrt{c\,a^2\,x^2+c}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.419 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=361

$$-\frac{a^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^3}{3cx^3}-\frac{a^3c\sqrt{1+a^2x^2}\,\text{Arc}$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/c/x^3-a^3*arctanh((a^2*c*x^2+c)^(1/2
)/c^(1/2))*c^(1/2)-a^3*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a^3*c*arctan(a*x)*polylog(2,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^3*c*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+
c)^(1/2)-a^3*c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+a^3*c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/2*a*arcta
n(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 0.69, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5064, 5070, 5082, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724}

$$-\frac{a^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{x}-\frac{a\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2x^2}-\frac{\text{ArcTan}(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+\frac{ia^3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{ia^3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^3c\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{a^3c\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^3c\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(2*x^2) - ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*c*x^3) - (a^3*c*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] - a^3*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + (I*a^3*c*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] -
 (I*a^3*c*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + (a^3*c*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^4}\,dx &= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3}+a\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^3}\,dx \\
&= -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3}+(ac)\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx+\left(a^3c\right)\int \frac{\tan^{-}}{x\sqrt{c+}} \\
&= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{3cx^3}+\left(a^2c\right)\int \frac{\tan}{x^2\sqrt{c}} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3} \\
&= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\mathrm{t}}{3cx^3}
\end{aligned}
$$

Mathematica [A]
time = 2.60, size = 341, normalized size = 0.94

$$
\frac{a^3c\sqrt{1+a^2x^2}\left(-12\mathrm{ArcTan}(ax)\cot\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-2\mathrm{ArcTan}(ax)^3\cot\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-3\mathrm{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-\frac{ax\mathrm{ArcTan}(ax)^3\csc^4\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)}{2\sqrt{1+a^2x^2}}+12\mathrm{ArcTan}(ax)^2\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-12\mathrm{ArcTan}(ax)^2\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+24\log\left(\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)+24i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-24i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)-24\mathrm{PolyLog}\left(3,-e^{i\mathrm{ArcTan}(ax)}\right)+24\mathrm{PolyLog}\left(3,e^{i\mathrm{ArcTan}(ax)}\right)+3\mathrm{ArcTan}(ax)^2\sec^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-\frac{8(1+a^2x^2)^{3/2}\mathrm{ArcTan}(ax)^3\sin^4\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)}{a^3x^3}-12\mathrm{ArcTan}(ax)\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-2\mathrm{ArcTan}(ax)^3\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)}{24\sqrt{c(1+a^2x^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (a^3*c*Sqrt[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x
]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan
[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[
1 - E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*L
og[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])]
- (24*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*Ar
cTan[a*x])] + 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan
```

```
[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^
3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x
]/2]))/(24*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 3.35, size = 462, normalized size = 1.28

method	result
default	$-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\;\arctan(ax)\left(2\arctan(ax)^2a^2x^2+6a^2x^2+3\arctan(ax)ax+2\arctan(ax)^2\right)}{6x^3}+\frac{a^3\sqrt{c\,(ax-i)\,(ax}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*a^2*x^2+6*a^2*x
^2+3*arctan(a*x)*a*x+2*arctan(a*x)^2)/x^3+1/2*a^3*(c*(a*x-I)*(I+a*x))^(1/2)
*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)-I*a^3*(c
*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*x^2+1)^(1/2)+a^3*(c*(a*x-I)*(I+a*x))^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(1/2)-1/2*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)
^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)+I*a^3*(c*(a*x-I)*(I+
a*x))^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)
^(1/2)-a^3*(c*(a*x-I)*(I+a*x))^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/(a^2*x^2+1)^(1/2)-2*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*arctanh((1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{3}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,\sqrt{c\,a^2\,x^2+c}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.420 $\int x^3(c+a^2cx^2)^{3/2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=652

$$\frac{cx\sqrt{c+a^2cx^2}}{420a^3}-\frac{cx^3\sqrt{c+a^2cx^2}}{140a}-\frac{163c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{840a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{60a^2}+\frac{1}{35}cx^4\sqrt{c+$$

```
[Out] 23/120*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4-51/280*I*c^2*ar
ctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a
^2*c*x^2+c)^(1/2)+51/280*I*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-51/280*I*c^2*arctan((1+
I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)
^(1/2)-51/280*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/a^4/(a^2*c*x^2+c)^(1/2)+51/280*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+1/420*c*x*(a^2*c*x^2+c)^(1/2)
/a^3-1/140*c*x^3*(a^2*c*x^2+c)^(1/2)/a-163/840*c*arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/a^4+1/60*c*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+1/35*c*x^4*arctan(
a*x)*(a^2*c*x^2+c)^(1/2)+9/112*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-23
/280*c*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/14*a*c*x^5*arctan(a*x)^2*(
a^2*c*x^2+c)^(1/2)-2/35*c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4+1/35*c*x^2*
arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)^3*(a^2*c*x^2+c
)^(1/2)+1/7*a^2*c*x^6*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 4.92, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 200, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

$$\frac{cx^2\mathrm{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{35a^2}+\frac{cx^2\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{60a^2}+\frac{1}{7}a^2cx^6\mathrm{ArcTan}(ax)^3\sqrt{a^2cx^2+c}-\frac{1}{14}acx^5\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+\frac{8}{35}cx^4\mathrm{ArcTan}(ax)^3\sqrt{a^2cx^2+c}+\frac{1}{35}cx^4\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}-\frac{23cx^3\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{280a}-\frac{cx^3\sqrt{a^2cx^2+c}}{140a}+\frac{51ic^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{51ic^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{51c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{280a^4\sqrt{a^2cx^2+c}}+\frac{51c^2\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{280a^4\sqrt{a^2cx^2+c}}-\frac{51ic^2\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{280a^4\sqrt{a^2cx^2+c}}-\frac{2c\mathrm{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{35a^4}-\frac{163c\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{840a^4}+\frac{23c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{120a^4}+\frac{9cx\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{112a^3}+\frac{cx\sqrt{a^2cx^2+c}}{420a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*x*Sqrt[c + a^2*c*x^2])/(420*a^3) - (c*x^3*Sqrt[c + a^2*c*x^2])/(140*a) -
 (163*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(840*a^4) + (c*x^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/(60*a^2) + (c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/35 + (
9*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a^3) - (23*c*x^3*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(280*a) - (a*c*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2
)/14 - (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(35*a^4) + (c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(35*a^2) + (8*c*x^4*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/35 + (a^2*c*x^6*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^3)/7 + (23*c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(120
*a^4) + (((51*I)/280)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
```

```
I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (((51*I)/280)*c^2*Sqrt[1 + a^2
*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]
) - (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(280*a^4*
Sqrt[c + a^2*c*x^2]) + (51*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a
*x])])/(280*a^4*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx = c\int x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx + \left(a^2c\right)\int x^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3$$

$$= c^2\int \frac{x^3\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{x^5\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x^7}{\sqrt{}}$$

$$= \frac{cx^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^2} + \frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{\left(2c^2\right)\int}{}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3} - \frac{1}{14}acx^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{2c\sqrt{c}}{}$$

$$= \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4} + \frac{1}{35}cx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{cx\sqrt{c+a^2c}}{2}$$

$$= -\frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4} - \frac{11cx^2\sqrt{c+a^2cx^2}\,\tan^{-}}{60a^2}$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

$$= \frac{43cx\sqrt{c+a^2cx^2}}{420a^3} - \frac{cx^3\sqrt{c+a^2cx^2}}{140a} + \frac{2273c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{840a^4} -$$

Mathematica [A]
time = 7.72, size = 1296, normalized size = 1.99

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*((Sqrt[c*(1 + a^2*x^2)]*(11*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] +
 11*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*A
rcTan[a*x]))] - 11*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 11*ArcTan[a
*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 1
1*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*Ar
cTan[a*x])] + 11*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2
*E^((I/2)*ArcTan[a*x]))] - 11*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/
4]] - 20*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 11*ArcTan[a*x]^2*Lo
g[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 20*Log[Cos[ArcTan[a*x]/2] + Si
n[ArcTan[a*x]/2]] - 11*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*
x]/2]] - 11*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (22*I)*ArcTan
[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (22*I)*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])] - 22*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 22*PolyLog[3,
I*E^(I*ArcTan[a*x])]))/(40*Sqrt[1 + a^2*x^2]) - ((1 + a^2*x^2)^2*Sqrt[c*(1
+ a^2*x^2)]*(150*ArcTan[a*x] - 32*ArcTan[a*x]^3 + 8*ArcTan[a*x]*(27 + 20*Ar
cTan[a*x]^2)*Cos[2*ArcTan[a*x]] + 66*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 12*Si
n[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] + 6*Sin[4*ArcTan[a*x]
] - 33*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/960))/a^4 + (c*(-1/1680*(Sqrt[c*(
1 + a^2*x^2)]*(309*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 309*Pi*ArcT
an[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x])
)] - 309*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]^2*Log
[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 309*Pi*Arc
Tan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x
])] + 309*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/
2)*ArcTan[a*x]))] - 309*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] -
518*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 309*ArcTan[a*x]^2*Log[Co
s[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 518*Log[Cos[ArcTan[a*x]/2] + Sin[A
rcTan[a*x]/2]] - 309*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]
/2]] - 309*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (618*I)*ArcTan
[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (618*I)*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] - 618*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 618*PolyLog[
3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - ((1 + a^2*x^2)^3*Sqrt[c*(1 +
a^2*x^2)]*(-4116*ArcTan[a*x] - 3648*ArcTan[a*x]^3 + 2*ArcTan[a*x]*(-3131 +
896*ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]] - 4*ArcTan[a*x]*(691 + 560*ArcTan[a*x
]^2)*Cos[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 404*Sin[2*Ar
cTan[a*x]] + 633*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 352*Sin[4*ArcTan[a*x]]
```

```
- 180*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 100*Sin[6*ArcTan[a*x]] + 309*ArcTa
n[a*x]^2*Sin[6*ArcTan[a*x]]))/53760))/a^4
```

Maple [A]
time = 7.88, size = 469, normalized size = 0.72

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(240\arctan(ax)^3 a^6 x^6-120\arctan(ax)^2 a^5 x^5+384\arctan(ax)^3 a^4 x^4+48\arctan(ax)a^4 x^4-138\arctan(a\right.}{1680a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/1680*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*arcta
n(a*x)^2*a^5*x^5+384*arctan(a*x)^3*a^4*x^4+48*arctan(a*x)*a^4*x^4-138*arcta
n(a*x)^2*a^3*x^3+48*arctan(a*x)^3*a^2*x^2-12*a^3*x^3+28*arctan(a*x)*a^2*x^2
+135*arctan(a*x)^2*a*x-96*arctan(a*x)^3+4*a*x-326*arctan(a*x))+17/560*c*(c*
(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-
6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-17/560*c
*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-23/60*
I*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.421 $\int x^2(c+a^2cx^2)^{3/2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=882

$$-\frac{c\sqrt{c+a^2cx^2}}{30a^3}-\frac{(c+a^2cx^2)^{3/2}}{60a^3}+\frac{cx\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{12a^2}+\frac{1}{20}cx^3\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)+\frac{31c\sqrt{c+a^2c\cdots}}{2\cdots}$$

```
[Out] -1/60*(a^2*c*x^2+c)^(3/2)/a^3-41/120*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/8*I*c^2*arctan((1
+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c
)^(1/2)+3/8*I*c^2*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/16*I*c^2*arctan(a*x)^2*polyl
og(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/
2)+41/120*I*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/8*c^2*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/8*c^2*arctan(a*x
)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2
+c)^(1/2)-3/16*I*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+41/60*I*c^2*arctan(a*x)*arctan(
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-
1/30*c*(a^2*c*x^2+c)^(1/2)/a^3+1/12*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2
+1/20*c*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+31/240*c*arctan(a*x)^2*(a^2*c*x
^2+c)^(1/2)/a^3-19/120*c*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/10*a*c*x
^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/16*c*x*arctan(a*x)^3*(a^2*c*x^2+c)^(
1/2)/a^2+7/24*c*x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+1/6*a^2*c*x^5*arctan(
a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 3.72, antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 108, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5070, 5072, 5050, 5010, 5006, 5008, 4266, 2611, 6744, 2320, 6724, 267, 272, 45}

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/30*(c*Sqrt[c + a^2*c*x^2])/a^3 - (c + a^2*c*x^2)^(3/2)/(60*a^3) + (c*x*S
qrt[c + a^2*c*x^2]*ArcTan[a*x])/(12*a^2) + (c*x^3*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x])/20 + (31*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(240*a^3) - (19*c*x^2
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(120*a) - (a*c*x^4*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2)/10 + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(16*a^2) + (7*c
```

```
*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/24 + (a^2*c*x^5*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^3)/6 + ((I/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*
ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/60)*c^2*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/16)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/16)*c^2*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2
]) - (((41*I)/120)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (((41*I)/120)*c^2*Sqrt[1 + a^
2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c
*x^2]) + (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) - (3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*P
olyLog[3, I*E^(I*ArcTan[a*x])])/(8*a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/8)*c^
2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c
*x^2]) - (((3*I)/8)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
```

```
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3\,dx = c\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx + \left(a^2c\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3$$

$$= c^2\int \frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{x^4\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x^6}{\sqrt{}}$$

$$= \frac{cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2} + \frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c^2\int \frac{\tan}{\sqrt{c}}}{}$$

$$= -\frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3} - \frac{1}{10}acx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{cx\sqrt{c}}{}$$

$$= \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3} + \frac{41cx^2\sqrt{c-}}{}$$

$$= -\frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{749c\sqrt{c}}{}$$

$$= \frac{5c\sqrt{c+a^2cx^2}}{12a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3\sqrt{c+a^2cx^2}\,\tan^{-1}$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3$$

$$= \frac{7c\sqrt{c+a^2cx^2}}{15a^3} - \frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3} - \frac{5cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{12a^2} + \frac{1}{20}cx^3$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4015 vs. 2(882) = 1764.
time = 18.19, size = 4015, normalized size = 4.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (S
qrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
 I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2
, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-
1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x])
*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]
) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - Ar
cTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - Arc
Tan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a
*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - Arc
Tan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(
I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(
Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Lo
g[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi
+ ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] +
 ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] +
 (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + A
rcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + (I/2
)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I)/2)
*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi
 + ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - Arc
Tan[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (Pi/2
+ (-1/2*Pi + ArcTan[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a
*x])/2))] + I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + A
rcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[3, -
E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^
(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/2*Pi
 + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcT
an[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^
4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2 - ArcTan[a*x]^3)
)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqr
t[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(
Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan
[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^4)
```

```
 + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2
*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]
*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(C
os[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[Ar
cTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos
[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTa
n[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[Ar
cTan[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a^3 + (c*((Sqrt[c*(1 + a^2*x^2)]*(50
- 19*ArcTan[a*x]^2))/(240*Sqrt[1 + a^2*x^2]) + (19*Sqrt[c*(1 + a^2*x^2)]*(A
rcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) +
I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/
(120*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*((Pi^3*Log[Cot[(Pi/2 - Arc
Tan[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcT
an[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi
/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 - (3*Pi*(
(Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*
(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/
2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[
3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))
/2 + 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x
])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 -
(Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/
2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 +
 E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a
*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1
/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((
2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/
2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-1/2*Pi + ArcTan[
a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] - (3*(
Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/
3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (Pi/2...
```

Maple [A]
time = 3.95, size = 514, normalized size = 0.58

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(40\arctan(ax)^3a^5x^5-24\arctan(ax)^2a^4x^4+70\arctan(ax)^3a^3x^3+12\arctan(ax)a^3x^3-38\arctan(ax)^2a^{\cdots}\right.}{240a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*arctan(a
*x)^2*a^4*x^4+70*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*a^3*x^3-38*arctan(a*x
```

```
)^2*a^2*x^2+15*arctan(a*x)^3*a*x-4*a^2*x^2+20*arctan(a*x)*a*x+31*arctan(a*x
)^2-12)+1/240*c*(c*(a*x-I)*(I+a*x))^(1/2)*(15*arctan(a*x)^3*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4
5*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*I*arctan(a*x
)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*arctan(a*x)*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-82*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylo
g(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-90*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-82*I*dilog(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+82*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2
*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.422 $\int x(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^3 \, dx$

Optimal. Leaf size=477

$$-\frac{cx\sqrt{c+a^2cx^2}}{20a}+\frac{9c\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}{20a^2}+\frac{(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)}{10a^2}-\frac{9cx\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)^2}{40a}-$$

```
[Out] 1/10*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2-3/20*x*(a^2*c*x^2+c)^(3/2)*arctan(
a*x)^2/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/a^2/c-1/2*c^(3/2)*arctanh(a*
x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2+9/20*I*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^
(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-9/20*I*c^2*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/
(a^2*c*x^2+c)^(1/2)+9/20*I*c^2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+9/20*c^2*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-9/20*c
^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^
2+c)^(1/2)-1/20*c*x*(a^2*c*x^2+c)^(1/2)/a+9/20*c*arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/a^2-9/40*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]

time = 0.30, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5050, 5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212, 201}

$$-\frac{9ic^2\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(-ie^{i\operatorname{ArcTan}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\operatorname{ArcTan}(ax)\operatorname{Li}_2\left(ie^{i\operatorname{ArcTan}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9c^2\sqrt{a^2x^2+1}\operatorname{Li}_3\left(-ie^{i\operatorname{ArcTan}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}-\frac{9c^2\sqrt{a^2x^2+1}\operatorname{Li}_3\left(ie^{i\operatorname{ArcTan}(ax)}\right)}{20a^2\sqrt{a^2cx^2+c}}+\frac{9ic^2\sqrt{a^2x^2+1}\operatorname{ArcTan}\left(e^{i\operatorname{ArcTan}(ax)}\right)\operatorname{ArcTan}(ax)^2}{20a^2\sqrt{a^2cx^2+c}}+\frac{\operatorname{ArcTan}(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3x\operatorname{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}{20a}-\frac{9cx\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{40a}+\frac{\operatorname{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{10a^2}+\frac{9c\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}{20a^2}-\frac{c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^2}-\frac{cx\sqrt{a^2cx^2+c}}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/20*(c*x*Sqrt[c + a^2*c*x^2])/a + (9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(
20*a^2) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(10*a^2) - (9*c*x*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x]^2)/(40*a) - (3*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/
(20*a) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/
(5*a^2*c) - (c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a^2) -
(((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a
*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((9*I)/20)*c^2*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (9*c^2*S
qrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*a^2*Sqrt[c + a^2*c
*x^2]) - (9*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*a^2*
Sqrt[c + a^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
```

```
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx = \frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}{5a^2c} - \frac{3\int\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2\,dx}{5a}$$

$$= \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2} - \frac{3x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{20a} + \frac{\left(c+a^2cx^2\right)^{5/2}}{5a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

$$= -\frac{cx\sqrt{c+a^2cx^2}}{20a} + \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{20a^2} + \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}{10a^2}$$

Mathematica [A]
time = 5.58, size = 806, normalized size = 1.69

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/960*(c*Sqrt[c + a^2*c*x^2]*(-480*(1 + a^2*x^2)^(3/2)*ArcTan[a*x] + 150*(
1 + a^2*x^2)^(5/2)*ArcTan[a*x] - 320*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3 - 32
*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3 - 480*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Co
s[2*ArcTan[a*x]] + 216*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*Cos[2*ArcTan[a*x]] +
 160*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 66*(1 + a^2*x^2
```

```
)^(5/2)*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 216*ArcTan[a*x]^2*Log[1 - I*E^(I*A
rcTan[a*x])] + 216*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x]))
)/(2*E^((I/2)*ArcTan[a*x]))] - 216*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x]
)] - 216*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*
ArcTan[a*x])] + 216*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a
*x])))/E^((I/2)*ArcTan[a*x])] + 216*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^
(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] - 216*Pi*ArcTan[a*x]*Log[-Cos[(
Pi + 2*ArcTan[a*x])/4]] - 480*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]]
+ 216*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 480*Log[
Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 216*ArcTan[a*x]^2*Log[Cos[ArcTan
[a*x]/2] + Sin[ArcTan[a*x]/2]] - 216*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[
a*x])/4]] + (432*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (432*I
)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 432*PolyLog[3, (-I)*E^(I*Ar
cTan[a*x])] + 432*PolyLog[3, I*E^(I*ArcTan[a*x])] + 12*(1 + a^2*x^2)^(5/2)*
Sin[2*ArcTan[a*x]] + 240*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x
]] + 6*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] + 6*(1 + a^2*x^
2)^(5/2)*Sin[4*ArcTan[a*x]] - 33*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2*Sin[4*Ar
cTan[a*x]]))/(a^2*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.83, size = 421, normalized size = 0.88

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(8\arctan\left(ax\right)^{3}a^{4}x^{4}-6\arctan\left(ax\right)^{2}a^{3}x^{3}+16\arctan\left(ax\right)^{3}a^{2}x^{2}+4\arctan\left(ax\right)a^{2}x^{2}-15\arctan\left(ax\right)^{2}ax\right.}{40a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(8*arctan(a*x)^3*a^4*x^4-6*arctan(a*x)
^2*a^3*x^3+16*arctan(a*x)^3*a^2*x^2+4*arctan(a*x)*a^2*x^2-15*arctan(a*x)^2*
a*x+8*arctan(a*x)^3-2*a*x+22*arctan(a*x))-3/40*c*(c*(a*x-I)*(I+a*x))^(1/2)*
(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*ar
ctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+3/40*c*(c*(a*x-I)*(I+a*x))^(1/2
)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+I*c/a^2*(c*(a*x-I)*(I+a*x))^(1/
2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.423 $\int (c+a^2cx^2)^{3/2} \textbf{ArcTan}(ax)^3 \, dx$

Optimal. Leaf size=760

$$-\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{9c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{8a}-\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}{4a}+$$

```
[Out] -1/4*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a
*x)^3-5*I*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+
1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/2*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+9/4*I*c^2*polylog(4,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/2*I*c
^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)+9/8*I*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-9/8*I*c^2*arctan(a*x)^2*polylog
(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-9
/4*c^2*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a/(a^2*c*x^2+c)^(1/2)+9/4*c^2*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-9/4*I*c^2*polylog(4,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4*I*
c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a/(
a^2*c*x^2+c)^(1/2)-1/4*c*(a^2*c*x^2+c)^(1/2)/a+1/4*c*x*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)-9/8*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arctan(a*x)^3
*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5000, 5010, 5008, 4266, 2611, 6744, 2320, 6724, 5006, 4998}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/4*(c*Sqrt[c + a^2*c*x^2])/a + (c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 -
(9*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(8*a) - ((c + a^2*c*x^2)^(3/2)*ArcT
an[a*x]^2)/(4*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (x*(c + a^
2*c*x^2)^(3/2)*ArcTan[a*x]^3)/4 - (((3*I)/4)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E
^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - ((5*I)*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c
 + a^2*c*x^2]) + (((9*I)/8)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((9*I)/8)*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x
```

```
^2]) + (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((5*I)/2)*c^2*Sqrt[1 + a^2*x^2]
*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) -
 (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(
4*a*Sqrt[c + a^2*c*x^2]) + (9*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3,
I*E^(I*ArcTan[a*x])])/(4*a*Sqrt[c + a^2*c*x^2]) - (((9*I)/4)*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((9*
I)/4)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
```

```
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}{4a} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3 + \frac{1}{2}c\int \sqrt{c+$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

$$= -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{9c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{8a}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2105 vs. $2(760) = 1520$.
time = 12.54, size = 2105, normalized size = 2.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

```
[Out] ((-1/2*I)*c*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
 - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I
```

*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2]) + (c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2 - ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(Sin[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[ArcTan[

```
a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a
```

Maple [A]
time = 2.21, size = 466, normalized size = 0.61

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\,\left(2\arctan(ax)^{3}a^{3}x^{3}-2\arctan(ax)^{2}a^{2}x^{2}+5\arctan(ax)^{3}ax+2\arctan(ax)ax-11\arctan(ax)^{2}-2\right)}{8a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2*
a^2*x^2+5*arctan(a*x)^3*a*x+2*arctan(a*x)*a*x-11*arctan(a*x)^2-2)-1/8*c*(c*
(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-3*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arctan(a*x)^2*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+9*I*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+20*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18
*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*arctan(a*x)*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+18*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*polylog(4,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
0*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.424 $\int \frac{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^3}{x}\,dx$

Optimal. Leaf size=726

$$c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{1}{2}acx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2+\frac{7ic^2\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)}{\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3-c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^
2+c)^(1/2))+3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*arctan(a*x)^3*arctanh((1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+7*I*c^2*arctan((1+
I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)+7*I*c^2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c^2*arctan(a*x)*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+7*c^2*
polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-7*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)+6*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*I*c^2*arctan(a
*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+
c)^(1/2)+c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*a*c*x*arctan(a*x)^2*(a^2*c*x
^2+c)^(1/2)+c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 726, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5070, 5078, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5010, 5008, 4266, 5000, 223, 212}

$$\frac{3ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{7ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{7ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{7c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{7c^2\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ic^2\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6ic^2\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{7ic^2\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{\sqrt{a^2cx^2+c}}-\frac{2c^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{3}\text{ArcTan}(ax)^3\left(a^2cx^2+c\right)^{3/2}+c\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}-\frac{1}{2}acx\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}+c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}-c^{3/2}\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^
2)/2 + ((7*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
)/Sqrt[c + a^2*c*x^2] + c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + ((c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^3)/3 - (2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh
[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - c^(3/2)*ArcTanh[(a*Sqrt[c]*x)/Sq
rt[c + a^2*c*x^2]] + ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*c^2*Sqrt[1 + a^2*x^2]*Arc
```

```
Tan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((7*I)*c
^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - ((3*I)*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*Arc
Tan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*c^2*Sqrt[1 + a^2*x^2]*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (7*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*c^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
 - ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((6*I)*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
```

```
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x}\,dx + (a^2c)\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{3}(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3 - (ac)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx + c^2\int \frac{}{x}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\,\text{ta}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\,\text{ta}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + c\sqrt{c+a^2cx^2}\,\text{ta}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}}{}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}}{}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}}{}$$

$$= c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{7ic^2\sqrt{1+a^2x^2}}{}$$

Mathematica [A]
time = 1.66, size = 854, normalized size = 1.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*((-3*I)*Pi^4 + (12*ArcTan[a*x])/Sqrt[1 + a^2*x^2] -
(12*a^4*x^4*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + 12*Sqrt[1 + a^2*x^2]*ArcTan[a*
x] + 12*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - (12*a*x*ArcTan[a*x]^2)/Sqrt
[1 + a^2*x^2] - (12*a^3*x^3*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + 32*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^3 + 8*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (6*I)*
ArcTan[a*x]^4 + 24*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 84*ArcTan[
a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 12*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1
- I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] + 84*ArcTan[a*x]^2*Log[1
```

```
 + I*E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcT
an[a*x])))/E^((I/2)*ArcTan[a*x])] - 12*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*
(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] - 24*ArcTan[a*x]^3*Log[1 +
 E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a
*x]))/(2*E^((I/2)*ArcTan[a*x]))] + 12*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTa
n[a*x])/4]] + 24*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 12*ArcTan[a
*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 24*Log[Cos[ArcTan[a*x]
/2] + Sin[ArcTan[a*x]/2]] + 12*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[A
rcTan[a*x]/2]] + 12*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (72*I
)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*Pol
yLog[2, -E^(I*ArcTan[a*x])] - (168*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] + (168*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 144*ArcTa
n[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*PolyLog[3, -E^(I*
ArcTan[a*x])] + 168*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 168*PolyLog[3, I*E
^(I*ArcTan[a*x])] - (144*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*Poly
Log[4, -E^(I*ArcTan[a*x])]))/(24*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 2.73, size = 511, normalized size = 0.70

method	result
default	$\frac{c\sqrt{c\,(ax-i)\,(ax+i)}\;\arctan(ax)\left(2\arctan(ax)^{2}a^{2}x^{2}-3\arctan(ax)ax+8\arctan(ax)^{2}+6\right)}{6}+\frac{c\sqrt{c\,(ax-i)\,(ax+i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*a^2*x^2-3*arct
an(a*x)*a*x+8*arctan(a*x)^2+6)+1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*
x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2))-6*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)^2*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-14*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*I*arc
tan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*arctan(a*x)*polylog(3,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+12*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*polylog(4,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+14*polylog(3
,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.425 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\mathbf{ArcTan}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=901

$$-\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2-\frac{c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^3}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^3-\frac{3iac^2\sqrt{1+}}{}$$

```
[Out] -3*I*a*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-9/2*I*a*c^2*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^2*arctan(a*x)^2
*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-6*I*a*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c^2*arctan(a*x)*polylog(
2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*
c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)+9*I*a*c^2*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I
*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^2*polylog(3,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-9*a*c^2*arcta
n(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+9*a*c^2*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-9*I*a*c^2*polylog(4,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9/2*I*a*c^2*ar
ctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)-3/2*a*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-c*arctan(a*x)^3*
(a^2*c*x^2+c)^(1/2)/x+1/2*a^2*c*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.88, antiderivative size = 901, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5070, 5064, 5078, 5076, 4268, 2611, 2320, 6724, 5010, 5008, 4266, 6744, 5000, 5006}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 - (c*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/x + (a^2*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/2 - ((3*I)*a*c^2*
Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x
```

```
^2] - ((6*I)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)/2)*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^2*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*
x^2] + ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqr
t[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^2*Sq
rt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (9*a*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[
c + a^2*c*x^2] + (9*a*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a*c^2*Sqrt[1 + a^2*x^2]*Pol
yLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9*I)*a*c^2*Sqrt[1
+ a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^2}\,dx + (a^2c)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 + c^2\int \frac{}{x}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

$$= -\frac{3}{2}ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+}$$

Mathematica [A]
time = 4.40, size = 1387, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] (a*c*Sqrt[c + a^2*c*x^2]*((-7*I)*Pi^4*Sqrt[1 + a^2*x^2] - (8*I)*Pi^3*Sqrt[1
 + a^2*x^2]*ArcTan[a*x] - (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x]
)]*ArcTan[a*x] - 96*ArcTan[a*x]^2 - 96*a^2*x^2*ArcTan[a*x]^2 + (24*I)*Pi^2*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (64*ArcTan[a*x]^3)/(a*x) - 32*a*x*ArcTan[
a*x]^3 + 32*a^3*x^3*ArcTan[a*x]^3 - (32*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
```

```
^3 - (64*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + (16
*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^4 + 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*Log[1 - I/E^(I*ArcTan[a*x])] - 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1
 - I/E^(I*ArcTan[a*x])] - 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I/E^(I*ArcTan[a*
x])] + 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 19
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 8*Pi^3*Sqrt[
1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] - 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
*Log[1 + I*E^(I*ArcTan[a*x])] - 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 +
I*E^(I*ArcTan[a*x])] - 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*Arc
Tan[a*x])] + 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[2*Sqrt[1 + a^2*x^2]*Sin[(Pi + 2*A
rcTan[a*x])/4]^2] + (192*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)
/E^(I*ArcTan[a*x])] + (48*I)*Pi*Sqrt[1 + a^2*x^2]*(Pi - 4*ArcTan[a*x])*Poly
Log[2, I/E^(I*ArcTan[a*x])] + (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog
[2, -E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] + (48*I)*Pi^2*Sqrt[1 + a^2*x^2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])] - (192*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] + (288*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])] - (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (96*
I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (384*I
)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 384*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*Pi*Sqrt[1 +
 a^2*x^2]*PolyLog[3, I/E^(I*ArcTan[a*x])] - 384*Sqrt[1 + a^2*x^2]*PolyLog[3
, -E^(I*ArcTan[a*x])] + 192*Pi*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])] - 576*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] +
384*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (384*I)*Sqrt[1 + a^2*
x^2]*PolyLog[4, (-I)/E^(I*ArcTan[a*x])] - (576*I)*Sqrt[1 + a^2*x^2]*PolyLog
[4, (-I)*E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*A
rcTan[a*x])]))/(64*(1 + a^2*x^2))
```

Maple [A]
time = 2.92, size = 602, normalized size = 0.67

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)^{2}\left(\arctan\left(ax\right)a^{2}x^{2}-3ax-2\arctan\left(ax\right)\right)}{2x}+\frac{3iac\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(-i\arctan(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(arctan(a*x)*a^2*x^2-3*a*x-2*
arctan(a*x))/x+3/2*I*a*c*(c*(a*x-I)*(I+a*x))^(1/2)*(-4*I*polylog(3,(1+I*a*x
)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
```

```
I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)^2*l
n(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*
I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*polylog(2,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+4*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)
^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(2,-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)
^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^3(ax)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**2,x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.426 $\int \frac{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=919

$$-\frac{3ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x}+\frac{6ia^2c^2\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{\sqrt{c+a^2cx^2}}+a^2c\sqrt{c+a^2cx^2}\,\text{ArcT}$$

[Out] -9*I*a^2*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2
*c*x^2+c)^(1/2)-3*a^2*c^2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*arctan(a*x)*arctanh((1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-9/2*I*a^2
*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9/2*I*a^2*c^2*arctan(a*x)^2*polylog
(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*
a^2*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)+9*I*a^2*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a^2*c^2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-9*a^2*c^2*arctan(a*
x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^
(1/2)+6*a^2*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9*a^2*c^2*arctan(a*x)*polylog(3,(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a^2*c^2*arct
an(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)+6*I*a^2*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*c*arctan(a*x)^2*(a^2*c*x^2+c)^(
1/2)/x+a^2*c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)-1/2*c*arctan(a*x)^3*(a^2*c*x
^2+c)^(1/2)/x^2

Rubi [A]
time = 1.38, antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5070, 5082, 5064, 5078, 5074, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5010, 5008, 4266}

Antiderivative was successfully verified.

[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]

[Out] (-3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x) + ((6*I)*a^2*c^2*Sqrt[1 +
a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2] + a^2

```
*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(2*x^2) - (3*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcT
anh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] + (((9*I)/2)*a^2*
c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c
+ a^2*c*x^2] - ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a^2*c^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((9*I)
/2)*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^2*Sqrt[1 + a
^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] -
(9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sq
rt[c + a^2*c*x^2] + (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E
^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (9*a^2*c^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((9*I)*a^2*c^2*
Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((9
*I)*a^2*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^3}\,dx + \left(a^2c\right)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x}\,dx$$

$$= c^2\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx + 2\left(\left(a^2c^2\right)\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx\right) + \left(a^4c^2\right)\int \frac{x\,\mathrm{t}}{\sqrt{}}$$

$$= a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2x^2} + \frac{1}{2}\left(3ac^2\right)\int \frac{\mathrm{t}}{x^2}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2c}}{}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c\sqrt{c+a^2c}}{}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 6.24, size = 691, normalized size = 0.75

Antiderivative was successfully verified.

[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]

[Out] (a^2*c*Sqrt[c + a^2*c*x^2]*(-12*ArcTan[a*x]^2 - (3*I)*Pi^4*Cot[ArcTan[a*x]/2] + (6*I)*ArcTan[a*x]^4*Cot[ArcTan[a*x]/2] - 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]^2 + 8*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 2*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 24*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] - 24*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*(2 + 3*ArcTan[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (96*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (48*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 96*Cot[ArcTan[a*x]/2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 96*Cot[ArcTan[a*x]/2]*PolyLog[3, I*E^(I*ArcTan[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(16*Sqrt[1 + a^2*x^2])

Maple [A]
time = 3.75, size = 592, normalized size = 0.64

method	result
default	$\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan(ax)^{2}\left(2\arctan(ax)a^{2}x^{2}-3ax-\arctan(ax)\right)}{2x^{2}}+\frac{3a^{2}c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(\arctan(ax)^{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)

[Out] 1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(2*arctan(a*x)*a^2*x^2-3*a*x-arctan(a*x))/x^2+3/2*a^2*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*pol

```
ylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+4*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**3,x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3,x)
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.427 $\int \frac{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=788

$$-\frac{a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{x}-\frac{(c+a^2cx^2)^{3/2}}{3x}$$

```
[Out] -1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3-a^3*c^(3/2)*arctanh((a^2*c*x^2+c
)^(1/2)/c^(1/2))+7*I*a^3*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*a^3*c^2*arctan(a*x)^2*arctanh
((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a^3
*c^2*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+
c)^(1/2)-6*I*a^3*c^2*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-7*I*a^3*c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a^3*c^2*arctan((1+I
*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-7*a^3*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)-6*a^3*c^2*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a^3*c^2*arctan(a*x)*polylog(3,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+7*a^3*c
^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-3*I*a^3*c^2*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^3*c^2*arctan(a*x)^2*polylog(2,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*c*arc
tan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/2*a*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^
2-a^2*c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 1.29, antiderivative size = 788, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5070, 5064, 5082, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724, 5010, 5008, 4266, 6744}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (a*c*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*x^2) - (a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x - ((c + a
^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) - ((2*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*A
rcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - (7*a^3*c^2*Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] - a^3*c^(3/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((7*I)*a^3*c^2*Sqrt
```

[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((7*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (7*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*a^3*c^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (7*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a^3*c^2*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
```

```
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^4}\,dx + (a^2c)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^2}\,dx$$

$$= -\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{3x^3} + (ac)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{x^3}\,dx + (a^2c$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} - \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{3x^3} + (ac^2)\int \frac{\mathrm{t}}{x^3}$$

$$= -\frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x} - \frac{(c+a^2cx^2}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

$$= -\frac{a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} - \frac{a^2c\sqrt{c+a^2}}{}$$

Mathematica [A]
time = 8.95, size = 1508, normalized size = 1.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] (a^3*c*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1 +
 a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi^2
*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*Arc
Tan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2
```

] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x*A
rcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*Arc
Tan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Log[
1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1 +
 I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 + E
^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan[a
*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^
(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*P
olyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a*x
]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*Poly
Log[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*A
rcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*
I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*
a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (19
2*a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTan[
a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt
[1 + a^2*x^2])*Sec[ArcTan[a*x]/2])/(128*Sqrt[1 + a^2*x^2]) + (a^3*c^2*Sqrt[
1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Cot[ArcT
an[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan[a*x]^3*Csc[
ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[1 - E^(I*Arc
Tan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*Log[Tan[ArcTa
n[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (24*I)*Arc
Tan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*ArcTan[a*x])]
+ 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 -
 (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^3*x^3) - 12*
ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x]/2]))/(24*S
qrt[c*(1 + a^2*x^2)])

Maple [A]

time = 4.48, size = 557, normalized size = 0.71

method	result
default	$-\frac{c\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)\left(8\arctan\left(ax\right)^{2}a^{2}x^{2}+6a^{2}x^{2}+3\arctan\left(ax\right)ax+2\arctan\left(ax\right)^{2}\right)}{6x^{3}}-\frac{ia^{3}c\sqrt{c\left(ax-i\right)\left(\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(8*arctan(a*x)^2*a^2*x^2+6*a^2
*x^2+3*arctan(a*x)*a*x+2*arctan(a*x)^2)/x^3-1/2*I*a^3*c*(c*(a*x-I)*(I+a*x))
^(1/2)*(14*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^3*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))+7*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan
(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-2*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*arctan(a*x)*polylog
(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-7*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*ln((1+I
*a*x)/(a^2*x^2+1)^(1/2)-1)-14*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I
*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(4,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+12*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+
1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^3(ax)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.428 $\int x^3(c+a^2cx^2)^{5/2}\,\textbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=798

$$\frac{85c^2x\sqrt{c+a^2cx^2}}{12096a^3}-\frac{c^2x^3\sqrt{c+a^2cx^2}}{240a}-\frac{1}{504}ac^2x^5\sqrt{c+a^2cx^2}-\frac{6157c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{60480a^4}-\frac{47c^2x^2\sqrt{c}}{}$$

```
[Out] 1433/15120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4-115/1344*I*
c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4
/(a^2*c*x^2+c)^(1/2)-115/1344*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+115/1344*I*c^3*arct
an(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^
2*c*x^2+c)^(1/2)-115/1344*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+115/1344*c^3*polylog(3,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+85/12096*c^2*x*
(a^2*c*x^2+c)^(1/2)/a^3-1/240*c^2*x^3*(a^2*c*x^2+c)^(1/2)/a-1/504*a*c^2*x^5
*(a^2*c*x^2+c)^(1/2)-6157/60480*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4-47/
30240*c^2*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+67/2520*c^2*x^4*arctan(a*
x)*(a^2*c*x^2+c)^(1/2)+1/84*a^2*c^2*x^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+47/
896*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-205/4032*c^2*x^3*arctan(a*x
)^2*(a^2*c*x^2+c)^(1/2)/a-103/1008*a*c^2*x^5*arctan(a*x)^2*(a^2*c*x^2+c)^(1
/2)-1/24*a^3*c^2*x^7*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-2/63*c^2*arctan(a*x)
^3*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a
^2+5/21*c^2*x^4*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+19/63*a^2*c^2*x^6*arctan(
a*x)^3*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2
)
```

Rubi [A]
time = 13.57, antiderivative size = 798, normalized size of antiderivative = 1.00, number of steps used = 547, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5070, 5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724, 327}

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (85*c^2*x*Sqrt[c + a^2*c*x^2])/(12096*a^3) - (c^2*x^3*Sqrt[c + a^2*c*x^2])/
(240*a) - (a*c^2*x^5*Sqrt[c + a^2*c*x^2])/504 - (6157*c^2*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x])/(60480*a^4) - (47*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(30240*a^2) + (67*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2520 + (a^2*c^2*
x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/84 + (47*c^2*x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(896*a^3) - (205*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(40
32*a) - (103*a*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/1008 - (a^3*c^2*x
```

```
^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/24 - (((115*I)/1344)*c^3*Sqrt[1 + a^2
*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) -
(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(63*a^4) + (c^2*x^2*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/(63*a^2) + (5*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]
^3)/21 + (19*a^2*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/63 + (a^4*c^2*x
^8*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/9 + (1433*c^(5/2)*ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]])/(15120*a^4) + (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
 - (((115*I)/1344)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) - (115*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2]) + (115*c^3*Sqrt[1
 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(1344*a^4*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
```

```
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx=c\int x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx+\left(a^2c\right)\int x^5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(a$$

Mathematica [A]
time = 7.48, size = 1466, normalized size = 1.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-2419200*(1 + a^2*x^2)^(5/2)*ArcTan[a*x] + 237081
6*(1 + a^2*x^2)^(7/2)*ArcTan[a*x] - 657578*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]
+ 516096*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3 + 2101248*(1 + a^2*x^2)^(7/2)*Ar
cTan[a*x]^3 + 273408*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^3 - 3483648*(1 + a^2*x
^2)^(5/2)*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 3606912*(1 + a^2*x^2)^(7/2)*ArcT
an[a*x]*Cos[2*ArcTan[a*x]] - 1083168*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]*Cos[2*
ArcTan[a*x]] - 2580480*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]]
 - 1032192*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 1092096*(
1 + a^2*x^2)^(9/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 1064448*(1 + a^2*x^2)
^(5/2)*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 1592064*(1 + a^2*x^2)^(7/2)*ArcTan[
a*x]*Cos[4*ArcTan[a*x]] - 576936*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]*Cos[4*ArcT
an[a*x]] + 1290240*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 1
93536*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 355968*(1 + a^
2*x^2)^(7/2)*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 184160*(1 + a^2*x^2)^(9/2)*Ar
cTan[a*x]*Cos[6*ArcTan[a*x]] - 161280*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^3*Cos
[6*ArcTan[a*x]] - 32814*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]*Cos[8*ArcTan[a*x]]
+ 662400*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 662400*Pi*ArcTan[a*x]
*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] - 66
2400*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 662400*ArcTan[a*x]^2*Log[
```

((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 662400*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 662400*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] - 662400*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 1467392*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 662400*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 1467392*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 662400*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 662400*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (1324800*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (1324800*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 1324800*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 1324800*PolyLog[3, I*E^(I*ArcTan[a*x])] - 193536*(1 + a^2*x^2)^(5/2)*Sin[2*ArcTan[a*x]] + 232704*(1 + a^2*x^2)^(7/2)*Sin[2*ArcTan[a*x]] - 74932*(1 + a^2*x^2)^(9/2)*Sin[2*ArcTan[a*x]] - 96768*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 364608*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 39222*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 96768*(1 + a^2*x^2)^(5/2)*Sin[4*ArcTan[a*x]] + 202752*(1 + a^2*x^2)^(7/2)*Sin[4*ArcTan[a*x]] - 77908*(1 + a^2*x^2)^(9/2)*Sin[4*ArcTan[a*x]] + 532224*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] + 103680*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] + 80226*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] + 57600*(1 + a^2*x^2)^(7/2)*Sin[6*ArcTan[a*x]] - 36612*(1 + a^2*x^2)^(9/2)*Sin[6*ArcTan[a*x]] - 177984*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^2*Sin[6*ArcTan[a*x]] - 19086*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^2*Sin[6*ArcTan[a*x]] - 7238*(1 + a^2*x^2)^(9/2)*Sin[8*ArcTan[a*x]] + 16407*(1 + a^2*x^2)^(9/2)*ArcTan[a*x]^2*Sin[8*ArcTan[a*x]]))/(15482880*a^4*Sqrt[1 + a^2*x^2])

Maple [A]
time = 7.73, size = 525, normalized size = 0.66

method	result
default	$c^2 \sqrt{c\left(ax-i\right)\left(ax+i\right)}\ \left(13440 \arctan(ax)^3 a^8 x^8 - 5040 \arctan(ax)^2 a^7 x^7 + 36480 \arctan(ax)^3 a^6 x^6 + 1440 \arctan(ax) a^6 x^6 - 1\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/120960*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(13440*arctan(a*x)^3*a^8*x^8-5040*arctan(a*x)^2*a^7*x^7+36480*arctan(a*x)^3*a^6*x^6+1440*arctan(a*x)*a^6*x^6-12360*arctan(a*x)^2*a^5*x^5+28800*arctan(a*x)^3*a^4*x^4-240*a^5*x^5+3216*arctan(a*x)*a^4*x^4-6150*arctan(a*x)^2*a^3*x^3+1920*arctan(a*x)^3*a^2*x^2-504*a^3*x^3-188*arctan(a*x)*a^2*x^2+6345*arctan(a*x)^2*a*x-3840*arctan(a*x)^3+850*a*x-12314*arctan(a*x))+115/8064*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*

```
x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-115/8064*c^2*(c*(a*x-I)*(I+a*x))^(1/2)
*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*a
rctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-1433/7560*I*c^2/a^4*(c*(a*x-I)*(
I+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arctan
(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
[Out] Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atan}(a x)^3 \left(c a^2 x^2 + c\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.429 $\int x^2(c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=1019

$$\frac{13c^2\sqrt{c+a^2cx^2}}{6720a^3}-\frac{3c(c+a^2cx^2)^{3/2}}{560a^3}-\frac{(c+a^2cx^2)^{5/2}}{280a^3}+\frac{43c^2x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{1344a^2}+\frac{29}{560}c^2x^3\sqrt{c+a^2cx^2}\,\mathrm{A}$$

```
[Out] -3/560*c*(a^2*c*x^2+c)^(3/2)/a^3-1/280*(a^2*c*x^2+c)^(5/2)/a^3+397/1680*I*c
^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*
c*x^2+c)^(1/2)-15/64*I*c^3*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-15/128*I*c^3*arctan(a*x)^2*polylog(2,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+15/6
4*I*c^3*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^
2*c*x^2+c)^(1/2)+397/840*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+15/128*I*c^3*arctan(a*x)^
2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2
+c)^(1/2)+15/64*c^3*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-15/64*c^3*arctan(a*x)*polylog(3,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-397/
1680*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a^3/(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arct
an(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+13/6720*c^2*(a^2*c*x^2+
c)^(1/2)/a^3+43/1344*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+29/560*c^2*x
^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/56*a^2*c^2*x^5*arctan(a*x)*(a^2*c*x^2+
c)^(1/2)+1373/13440*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-737/6720*c^2*
x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-83/560*a*c^2*x^4*arctan(a*x)^2*(a^2
*c*x^2+c)^(1/2)-3/56*a^3*c^2*x^6*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+5/128*c^
2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+59/192*c^2*x^3*arctan(a*x)^3*(a^2
*c*x^2+c)^(1/2)+17/48*a^2*c^2*x^5*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+1/8*a^4
*c^2*x^7*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 10.43, antiderivative size = 1019, normalized size of antiderivative = 1.00, number of steps used = 293, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5070, 5072, 5050, 5010, 5006, 5008, 4266, 2611, 6744, 2320, 6724, 267, 272, 45}

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (13*c^2*Sqrt[c + a^2*c*x^2])/(6720*a^3) - (3*c*(c + a^2*c*x^2)^(3/2))/(560*
a^3) - (c + a^2*c*x^2)^(5/2)/(280*a^3) + (43*c^2*x*Sqrt[c + a^2*c*x^2]*ArcT
```

```
an[a*x])/(1344*a^2) + (29*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/560 + (a
^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/56 + (1373*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/(13440*a^3) - (737*c^2*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x]^2)/(6720*a) - (83*a*c^2*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/560 - (3*
a^3*c^2*x^6*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/56 + (5*c^2*x*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^3)/(128*a^2) + (59*c^2*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
]^3)/192 + (17*a^2*c^2*x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/48 + (a^4*c^2
*x^7*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (((5*I)/64)*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) + (((
397*I)/840)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]
) + (((15*I)/128)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*Arc
Tan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^
2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*
x^2]) + (((397*I)/1680)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(64*a^3*Sqrt[c + a^2*c*x^2])
 - (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(
64*a^3*Sqrt[c + a^2*c*x^2]) + (((15*I)/64)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4,
 (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (((15*I)/64)*c^3*Sqrt
[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
```

```
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx = c\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx+\left(a^2c\right)\int x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(a$$

$$= c^2\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3\,dx+2\left(\left(a^2c^2\right)\int x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(\right.$$

$$= c^3\int\frac{x^2\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx+\left(a^2c^3\right)\int\frac{x^4\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx+\left(a^4c^3\right)\int\frac{x^6\tan^{-}}{\sqrt{c+}}$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2}+\frac{1}{4}c^2x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3+\frac{1}{6}a^2c^2x^5$$

$$= -\frac{3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3}-\frac{c^2x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{4a}-\frac{1}{10}ac^2x^4$$

$$= \frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{4a^2}+\frac{1}{20}c^2x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+\frac{1}{56}a^2c^2x^5$$

$$= -\frac{c^2\sqrt{c+a^2cx^2}}{4a^3}-\frac{c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{6a^2}-\frac{27}{560}c^2x^3\sqrt{c+a^2cx^2}\,\tan$$

$$= \frac{c^2\sqrt{c+a^2cx^2}}{6a^3}+\frac{491c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{1344a^2}-\frac{27}{560}c^2x^3\sqrt{c+a^2cx^2}$$

$$= -\frac{2239c^2\sqrt{c+a^2cx^2}}{6720a^3}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{210a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3}+\frac{491c^2x\sqrt{c+}}{}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3}+\frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3}+\frac{491c^2x\sqrt{}}{}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3}+\frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3}+\frac{491c^2x\sqrt{}}{}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3}+\frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3}+\frac{491c^2x\sqrt{}}{}$$

$$= -\frac{2899c^2\sqrt{c+a^2cx^2}}{6720a^3}+\frac{47c\left(c+a^2cx^2\right)^{3/2}}{1680a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{280a^3}+\frac{491c^2x\sqrt{}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6517 vs. $2(1019) = 2038$.
time = 24.30, size = 6517, normalized size = 6.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 3.98, size = 566, normalized size = 0.56

method	result
default	$c^2\sqrt{c(ax-i)(ax+i)}\,(1680\arctan(ax)^3a^7x^7-720\arctan(ax)^2a^6x^6+4760\arctan(ax)^3a^5x^5+240\arctan(ax)a^5x^5-1992\,a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13440*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(1680*arctan(a*x)^3*a^7*x^7-720*a
rctan(a*x)^2*a^6*x^6+4760*arctan(a*x)^3*a^5*x^5+240*arctan(a*x)*a^5*x^5-199
2*arctan(a*x)^2*a^4*x^4+4130*arctan(a*x)^3*a^3*x^3-48*a^4*x^4+696*arctan(a*
x)*a^3*x^3-1474*arctan(a*x)^2*a^2*x^2+525*arctan(a*x)^3*a*x-168*a^2*x^2+430
*arctan(a*x)*a*x+1373*arctan(a*x)^2-94)+1/13440*c^2*(c*(a*x-I)*(I+a*x))^(1/
2)*(525*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-525*arctan(a*x)^3
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1575*I*arctan(a*x)^2*polylog(2,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))+1575*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+3176*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*arcta
n(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*arctan(a*x)*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+3150*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*I*polylog(4
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))+3176*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")

[Out] integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^{3}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)

[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)

[Out] int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)

3.430 $\int x(c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=561

$$-\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx(c+a^2cx^2)^{3/2}}{140a}+\frac{15c^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}{56a^2}+\frac{5c(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)}{84a^2}+\frac{(c+$$

```
[Out] -1/140*c*x*(a^2*c*x^2+c)^(3/2)/a+5/84*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2
+1/35*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a^2-5/56*c*x*(a^2*c*x^2+c)^(3/2)*arct
an(a*x)^2/a-1/14*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a+1/7*(a^2*c*x^2+c)^(7
/2)*arctan(a*x)^3/a^2/c-37/120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1
/2))/a^2+15/56*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2
*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-15/56*I*c^3*arctan(a*x)*polylog(2,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+15/5
6*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a^2/(a^2*c*x^2+c)^(1/2)+15/56*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-15/56*c^3*polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-17/420*c^2
*x*(a^2*c*x^2+c)^(1/2)/a+15/56*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2-15/1
12*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.37, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5050, 5000, 5010, 5008, 4266, 2611, 2320, 6724, 223, 212, 201}

$$-\frac{15ic^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15ic^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15c^3\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}-\frac{15c^3\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{56a^2\sqrt{a^2cx^2+c}}+\frac{15ic^3\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^2}{56a^2\sqrt{a^2cx^2+c}}-\frac{15c^2x\mathrm{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{112a}+\frac{15c^2\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}{56a^2}+\frac{\mathrm{ArcTan}(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{x\mathrm{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{5/2}}{14a}+\frac{\mathrm{ArcTan}(ax)\left(a^2cx^2+c\right)^{5/2}}{35a^2}-\frac{5cx\mathrm{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}{56a}+\frac{5c\mathrm{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}{84a^2}-\frac{37c^{5/2}\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{120a^2}-\frac{17c^2x\sqrt{a^2cx^2+c}}{420a}-\frac{cx\left(a^2cx^2+c\right)^{3/2}}{140a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (-17*c^2*x*Sqrt[c + a^2*c*x^2])/(420*a) - (c*x*(c + a^2*c*x^2)^(3/2))/(140*
a) + (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(56*a^2) + (5*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x])/(84*a^2) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(35*a^
2) - (15*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(112*a) - (5*c*x*(c + a^2
*c*x^2)^(3/2)*ArcTan[a*x]^2)/(56*a) - (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^
2)/(14*a) + (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^2)/(a^2*Sqrt[c + a^2*c*x^2]) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]
^3)/(7*a^2*c) - (37*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(12
0*a^2) - (((15*I)/56)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (((15*I)/56)*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
 + (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(56*a^2*Sq
```

rt[c + a^2*c*x^2]) - (15*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x
])])/(56*a^2*Sqrt[c + a^2*c*x^2])

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3\,dx = \frac{(c+a^2cx^2)^{7/2}\tan^{-1}(ax)^3}{7a^2c} - \frac{3\int (c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2\,dx}{7a}$$

$$= \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}{35a^2} - \frac{x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}{14a} + \frac{(c+a^2cx^2)^{7/2}}{7a^2c}$$

$$= -\frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{5c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}{84a^2} + \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(}{35a^2}$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

$$= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a} - \frac{cx(c+a^2cx^2)^{3/2}}{140a} + \frac{15c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)}{56a^2} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1699 vs. 2(561) = 1122.
time = 4.27, size = 1699, normalized size = 3.03

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] -1/53760*(c^2*Sqrt[c + a^2*c*x^2]*(1880*a*x + 2952*a^3*x^3 + 264*a^5*x^5 -
808*a^7*x^7 - 14196*ArcTan[a*x] - 19824*a^2*x^2*ArcTan[a*x] - 1176*a^4*x^4*
```

ArcTan[a*x] + 336*a^6*x^6*ArcTan[a*x] - 4116*a^8*x^8*ArcTan[a*x] + 29490*a*x*ArcTan[a*x]^2 + 33366*a^3*x^3*ArcTan[a*x]^2 + 5142*a^5*x^5*ArcTan[a*x]^2 + 1266*a^7*x^7*ArcTan[a*x]^2 - 25152*ArcTan[a*x]^3 - 61184*a^2*x^2*ArcTan[a*x]^3 - 50560*a^4*x^4*ArcTan[a*x]^3 - 18176*a^6*x^6*ArcTan[a*x]^3 - 3648*a^8*x^8*ArcTan[a*x]^3 - 8950*ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 6232*a^2*x^2*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 8124*a^4*x^4*ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 856*a^6*x^6*ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 6262*a^8*x^8*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 19712*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 60928*a^2*x^2*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 64512*a^4*x^4*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 25088*a^6*x^6*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 1792*a^8*x^8*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 4628*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 11120*a^2*x^2*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 5592*a^4*x^4*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 3664*a^6*x^6*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 2764*a^8*x^8*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 2240*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 8960*a^2*x^2*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 13440*a^4*x^4*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 8960*a^6*x^6*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 2240*a^8*x^8*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 618*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 2472*a^2*x^2*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 3708*a^4*x^4*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 2472*a^6*x^6*ArcTan[a*x]*Cos[6*ArcTan[a*x]] - 618*a^8*x^8*ArcTan[a*x]*Cos[6*ArcTan[a*x]] + 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 7200*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] - 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 7200*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] - 7200*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 16576*Sqrt[1 + a^2*x^2]*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 16576*Sqrt[1 + a^2*x^2]*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 7200*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 7200*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (14400*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (14400*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 14400*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 14400*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 320*Sin[4*ArcTan[a*x]] + 608*a^2*x^2*Sin[4*ArcTan[a*x]] - 96*a^4*x^4*Sin[4*ArcTan[a*x]] - 736*a^6*x^6*Sin[4*ArcTan[a*x]] - 352*a^8*x^8*Sin[4*ArcTan[a*x]] - 3876*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 11808*a^2*x^2*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 12168*a^4*x^4*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 4416*a^6*x^6*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 180*a^8*x^8*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 100*Sin[6*ArcTan[a*x]] - 400*a^2*x^2*Sin[6*ArcTan[a*x]] - 600*a^4*x^4*Sin[6*ArcTan[a*x]] - 400*a^6*x^6*Sin[6*ArcTan[a*x]] - 100*a^8*x^8*Sin[6*ArcTan[a*x]] + 309*ArcTan[a*x]^2*Sin[6*ArcTan[a*x]] + 1236*a^2*x^2*ArcTan[a*x]^2*Sin[6*ArcTan

```
[a*x]] + 1854*a^4*x^4*ArcTan[a*x]^2*Sin[6*ArcTan[a*x]] + 1236*a^6*x^6*ArcTa
n[a*x]^2*Sin[6*ArcTan[a*x]] + 309*a^8*x^8*ArcTan[a*x]^2*Sin[6*ArcTan[a*x]])
)/(a^2*(1 + a^2*x^2))
```

Maple [A]
time = 2.96, size = 477, normalized size = 0.85

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(240\arctan(ax)^3a^6x^6-120\arctan(ax)^2a^5x^5+720\arctan(ax)^3a^4x^4+48\arctan(ax)a^4x^4-390\arctan(\right.}{1680a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*arc
tan(a*x)^2*a^5*x^5+720*arctan(a*x)^3*a^4*x^4+48*arctan(a*x)*a^4*x^4-390*arc
tan(a*x)^2*a^3*x^3+720*arctan(a*x)^3*a^2*x^2-12*a^3*x^3+196*arctan(a*x)*a^2
*x^2-495*arctan(a*x)^2*a*x+240*arctan(a*x)^3-80*a*x+598*arctan(a*x))-5/112*
c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)-5
/112*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(-I*arctan(a*x)^3+3*arctan(a*x)^2*ln(1-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/
2)+37/60*I*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(a
*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\big(c\big(a^2x^2+1\big)\big)^{\frac{5}{2}}\operatorname{atan}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.431 $\int (c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=870

$$-\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c(c+a^2cx^2)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)+\frac{1}{20}cx\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)-\frac{15c^2}{}$$

```
[Out] -1/60*c*(a^2*c*x^2+c)^(3/2)/a+1/20*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-5/24
*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a-1/10*(a^2*c*x^2+c)^(5/2)*arctan(a*x)
^2/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2)*a
rctan(a*x)^3-259/60*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+15/16*I*c^3*arctan(a*x)^2*polylo
g(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
-259/120*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/a/(a^2*c*x^2+c)^(1/2)-15/16*I*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/8*I*c^3*arctan((
1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)
^(1/2)+259/120*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2
+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-15/8*c^3*arctan(a*x)*polylog(3,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+15/8*c^3*arctan
(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x
^2+c)^(1/2)+15/8*I*c^3*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/a/(a^2*c*x^2+c)^(1/2)-15/8*I*c^3*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-17/60*c^2*(a^2*c*x^2+c)^(1/2
)/a+17/60*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-15/16*c^2*arctan(a*x)^2*(a^
2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 870, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5000, 5010, 5008, 4266, 2611, 6744, 2320, 6724, 5006, 4998}

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

```
[Out] (-17*c^2*Sqrt[c + a^2*c*x^2])/(60*a) - (c*(c + a^2*c*x^2)^(3/2))/(60*a) + (
17*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/60 + (c*x*(c + a^2*c*x^2)^(3/2)*A
rcTan[a*x])/20 - (15*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(16*a) - (5*c*(
c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(24*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[
a*x]^2)/(10*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x
]^3)/6 - (((5*I)/8)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[
```

```
a*x]^3)/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/60)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) + (
((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (((15*I)/16)*c^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((259*
I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*
a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (((259*I)/120)*c^3*Sqrt[1 + a^2*x^2]*PolyL
og[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a*Sqrt[c + a^2*c*x^2]) - (15*c
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(8*a*S
qrt[c + a^2*c*x^2]) + (15*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^
(I*ArcTan[a*x])])/(8*a*Sqrt[c + a^2*c*x^2]) - (((15*I)/8)*c^3*Sqrt[1 + a^2*
x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + (((15*I)
/8)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*
c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}{10a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3+\frac{1}{5}c\int (c+$$

$$= -\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{1}{20}cx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)-\frac{5c\left(c+a^2cx^2\right)^{3/2}\tan}{24a}$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

$$= -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}-\frac{c\left(c+a^2cx^2\right)^{3/2}}{60a}+\frac{17}{60}c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4281 vs. 2(870) = 1740.
time = 18.67, size = 4281, normalized size = 4.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

[Out] ((-1/2*I)*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*
x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^
(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (
6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*A
rcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2]) +
(2*c^2*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2])
+ (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log
[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyL
og[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)
]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a
x])(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x
]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2
- ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 -
 ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcT
an[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 -
 ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3,
 E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4
)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^
(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)
- Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2
*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)
)] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x])
)] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi
 + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] +
(I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I
)/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/
2*Pi + ArcTan[a*x])/2))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 -
 ArcTan[a*x]))])/4 - (3*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcT
an[a*x])/2))] + I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(
Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi
 + ArcTan[a*x])/2))]/2))/2 - (3*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[
3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4,
 -E^(I*(Pi/2 - ArcTan[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/
2*Pi + ArcTan[a*x])/2))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*
ArcTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/
2])^4) + (Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[a*x] - ArcTan[a*x]^2 - ArcTan[a*x
]^3))/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2) -
(Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^
2]*(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^3) - (Sqrt[c*(1 + a^2*x^2)]*Ar
cTan[a*x]^3)/(16*Sqrt[1 + a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]
)^4) + (Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(8*Sqrt[1 +
 a^2*x^2]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^3) + (Sqrt[c*(1 + a^2*x

```
^2)]*(-2*ArcTan[a*x] - ArcTan[a*x]^2 + ArcTan[a*x]^3))/(16*Sqrt[1 + a^2*x^2
]*(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])^2) + (Sqrt[c*(1 + a^2*x^2)]*(Si
n[ArcTan[a*x]/2] - ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*
(Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2])) + (Sqrt[c*(1 + a^2*x^2)]*(-Sin[A
rcTan[a*x]/2] + ArcTan[a*x]^2*Sin[ArcTan[a*x]/2]))/(4*Sqrt[1 + a^2*x^2]*(Co
s[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]))))/a + (c^2*((Sqrt[c*(1 + a^2*x^2)]*
(50 - 19*ArcTan[a*x]^2))/(240*Sqrt[1 + a^2*x^2]) + (19*Sqrt[c*(1 + a^2*x^2)
]*(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]
) + I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]
)))/(120*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*((Pi^3*Log[Cot[(Pi/2 -
 ArcTan[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 -
ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I
*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 - (3*
Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E
^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*
(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-Poly
Log[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))
])))/2 + 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan
[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/
8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 +
(-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log
[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2...
```

Maple [A]

time = 2.32, size = 518, normalized size = 0.60

method	result
default	$\frac{c^{2} \sqrt{c \left(a x -i\right) \left(a x +i\right)}\, \left(40 \arctan \left(a x \right)^{3} a^{5} x^{5}-24 \arctan \left(a x \right)^{2} a^{4} x^{4}+130 \arctan \left(a x \right)^{3} a^{3} x^{3}+12 \arctan \left(a x \right) a^{3} x^{3}-98 \arctan(a x\right.}{240 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*arctan(a
*x)^2*a^4*x^4+130*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*a^3*x^3-98*arctan(a*
x)^2*a^2*x^2+165*arctan(a*x)^3*a*x-4*a^2*x^2+80*arctan(a*x)*a*x-299*arctan(
a*x)^2-72)-1/240*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(75*arctan(a*x)^3*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-75*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-225*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+225*I*arc
tan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*arctan(a*x)*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-518*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*arctan(
a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*I*polylog(4,-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-450*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-518*I*dil
```

```
og(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^{3}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
[Out] int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.432 $\int \frac{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}{x}\,dx$

Optimal. Leaf size=845

$$-\frac{1}{20}ac^2x\sqrt{c+a^2cx^2}+\frac{29}{20}c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)+\frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)-\frac{29}{40}ac^2x\sqrt{c+a^2cx^2}\,\text{A}$$

```
[Out] 1/10*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-3/20*a*c*x*(a^2*c*x^2+c)^(3/2)*arcta
n(a*x)^2+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3+1/5*(a^2*c*x^2+c)^(5/2)*ar
ctan(a*x)^3-3/2*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))+3*I*c^3*ar
ctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)-2*c^3*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-149/20*I*c^3*arc
tan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-6*I*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)+6*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/20*c^3*polylog(
3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-149
/20*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+6*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/20*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/20*I*c^3*a
rctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)-1/20*a*c^2*x*(a^2*c*x^2+c)^(1/2)+29/20*c^2*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)-29/40*a*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+c^2*arctan(a*x)
^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 1.25, antiderivative size = 845, normalized size of antiderivative = 1.00, number of steps used = 54, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5070, 5078, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5010, 5008, 4266, 5000, 223, 212, 201}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] -1/20*(a*c^2*x*Sqrt[c + a^2*c*x^2]) + (29*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/20 + (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/10 - (29*a*c^2*x*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x]^2)/40 - (3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/
20 + (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a
```

```
*x]^2)/Sqrt[c + a^2*c*x^2] + c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 + (c*(c
+ a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/3 + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)
/5 - (2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] - (3*c^(5/2)*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/2
 + ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2] - (((149*I)/20)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((149*I)/20)*c^3*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*
c*x^2] - ((3*I)*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[
a*x])])/Sqrt[c + a^2*c*x^2] - (6*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[
3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (149*c^3*Sqrt[1 + a^2*x^2]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) - (149*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(20*Sqrt[c + a^2*c*x^2]) + (6
*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] - ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/S
qrt[c + a^2*c*x^2] + ((6*I)*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}{x}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x}\,dx + (a^2c)\int x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3\,dx$$

$$= \frac{1}{5}(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3 - \frac{1}{5}(3ac)\int (c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2\,dx + c^2$$

$$= \frac{1}{10}c(c+a^2cx^2)^{3/2}\tan^{-1}(ax) - \frac{3}{20}acx(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{3}c(c+$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

$$= -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}$$

Mathematica [A]
time = 5.74, size = 1400, normalized size = 1.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

```
[Out] -1/960*(c^2*Sqrt[c + a^2*c*x^2]*((120*I)*Pi^4 + (24*a*x)/Sqrt[1 + a^2*x^2]
+ (48*a^3*x^3)/Sqrt[1 + a^2*x^2] + (24*a^5*x^5)/Sqrt[1 + a^2*x^2] - 810*Sqr
t[1 + a^2*x^2]*ArcTan[a*x] - 660*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 15
0*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + (972*a*x*ArcTan[a*x]^2)/Sqrt[1 +
a^2*x^2] + (984*a^3*x^3*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (12*a^5*x^5*ArcT
```

```
an[a*x]^2)/Sqrt[1 + a^2*x^2] - 1632*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 - 704*a
^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 - 32*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^3 - (240*I)*ArcTan[a*x]^4 - 744*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*
ArcTan[a*x]] - 528*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]]
 + 216*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 160*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 320*a^2*x^2*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 160*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[
a*x]^3*Cos[2*ArcTan[a*x]] + 66*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[4*ArcTan[a
*x]] + 132*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 66*a^
4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 960*ArcTan[a*x]^3*
Log[1 - E^((-I)*ArcTan[a*x])] + 3576*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*
x])] + 696*Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^(
(I/2)*ArcTan[a*x]))] - 3576*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 69
6*ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[
a*x])] + 696*Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/
E^((I/2)*ArcTan[a*x])] + 960*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + 696
*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan
[a*x]))] - 696*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 1440*Log[
Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 696*ArcTan[a*x]^2*Log[Cos[ArcTan
[a*x]/2] - Sin[ArcTan[a*x]/2]] + 1440*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a
*x]/2]] - 696*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] -
696*Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] - (2880*I)*ArcTan[a*x]^
2*PolyLog[2, E^((-I)*ArcTan[a*x])] - (2880*I)*ArcTan[a*x]^2*PolyLog[2, -E^(
I*ArcTan[a*x])] + (7152*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] -
 (7152*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 5760*ArcTan[a*x]*Po
lyLog[3, E^((-I)*ArcTan[a*x])] + 5760*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a
*x])] - 7152*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 7152*PolyLog[3, I*E^(I*Ar
cTan[a*x])] + (5760*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] + (5760*I)*PolyLog[
4, -E^(I*ArcTan[a*x])] + 6*Sqrt[1 + a^2*x^2]*Sin[4*ArcTan[a*x]] + 12*a^2*x^
2*Sqrt[1 + a^2*x^2]*Sin[4*ArcTan[a*x]] + 6*a^4*x^4*Sqrt[1 + a^2*x^2]*Sin[4*
ArcTan[a*x]] - 33*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 66*a
^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]] - 33*a^4*x^4*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/Sqrt[1 + a^2*x^2]
```

Maple [A]
time = 2.94, size = 562, normalized size = 0.67

method	result
default	$\frac{c^2\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\left(24\arctan\left(ax\right)^3a^4x^4-18\arctan\left(ax\right)^2a^3x^3+88\arctan\left(ax\right)^3a^2x^2+12\arctan\left(ax\right)a^2x^2-105\arctan\left(ax\right)\cdots\right.}{120}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)^3*a^4*x^4-18*arctan(a*x
)^2*a^3*x^3+88*arctan(a*x)^3*a^2*x^2+12*arctan(a*x)*a^2*x^2-105*arctan(a*x)
^2*a*x+184*arctan(a*x)^3-6*a*x+186*arctan(a*x))+1/40*c^2*(c*(a*x-I)*(I+a*x)
)^(1/2)*(40*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-40*arctan(a*x)^
3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*arctan(a*x)^2*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+120*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+149*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*arctan(a*x)^
2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-298*I*arctan(a*x)*polylog(2,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))+298*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+240*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-240*arctan(a*
x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+240*I*polylog(4,(1+I*a*x)/(a^2*x
^2+1)^(1/2))-240*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*I*arctan((1+
I*a*x)/(a^2*x^2+1)^(1/2))+298*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-298
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.433 $\int \frac{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=1027

$$-\frac{1}{4}ac^2\sqrt{c+a^2cx^2}+\frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{21}{8}ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2-\frac{1}{4}ac\left(c+a^2cx^2\right)^{3/2}\,\text{Ar}$$

```
[Out] -1/4*a*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*
arctan(a*x)^3-15/4*I*a*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^
3*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c^3*arctan(a*x)*polylog(2,(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^3*arc
tan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^
2+c)^(1/2)+11/2*I*a*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-11/2*I*a*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*c^3*arctan(a*x)
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)-45/8*I*a*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+45/4*I*a*c^3*polylog(4,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-11*I*a*c^3*arctan(a*x)*
arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)-6*a*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-45/4*a*c^3*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+45/4*a*c^3*arctan(a*x)*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c^
3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)+45/8*I*a*c^3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-45/4*I*a*c^3*polylog(4,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/4*a*c^2*(a^2*c*x^2+
c)^(1/2)+1/4*a^2*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-21/8*a*c^2*arctan(a*
x)^2*(a^2*c*x^2+c)^(1/2)-c^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x+7/8*a^2*c^
2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 1.43, antiderivative size = 1027, normalized size of antiderivative = 1.00, number of steps used = 56, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5070, 5064, 5078, 5076, 4268, 2611, 2320, 6724, 5010, 5008, 4266, 6744, 5000, 5006, 4998}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

```
[Out] -1/4*(a*c^2*Sqrt[c + a^2*c*x^2]) + (a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/4 - (21*a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/8 - (a*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x]^2)/4 - (c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (7*
a^2*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/8 + (a^2*c*x*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x]^3)/4 - (((15*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*Arc
Tan[a*x])]*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2] - ((11*I)*a*c^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^
2] - (6*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/S
qrt[c + a^2*c*x^2] + ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2,
-E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/8)*a*c^3*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] -
 (((45*I)/8)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan
[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*P
olyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((11*I)/2)*a*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a
^2*c*x^2] - (((11*I)/2)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*
x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (6*a*c^3*Sqrt[1 + a^2*x^2]*Poly
Log[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (45*a*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(4*Sqrt[c + a^2*c*x^2])
+ (45*a*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/
(4*Sqrt[c + a^2*c*x^2]) + (6*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan
[a*x])])/Sqrt[c + a^2*c*x^2] - (((45*I)/4)*a*c^3*Sqrt[1 + a^2*x^2]*PolyLog[
4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (((45*I)/4)*a*c^3*Sqrt[1
+ a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4998

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q +
 1)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x
^2)^q*((a + b*ArcTan[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}{x^2}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx + (a^2c)\int (c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3\,dx$$

$$= -\frac{1}{4}ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2 + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3 + c^2\int$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

$$= -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\tan^{-1}(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3267 vs. 2(1027) = 2054.
time = 14.28, size = 3267, normalized size = 3.18

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]

[Out] ((-I)*a*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (a*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan[a*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2])*Sec[ArcTan[a*x]/2])/(128*Sqrt[1 + a^2*x^2]) + a*c^2*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-1/8*(Pi^3*Log[Cot

```
[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x])*(Log[1 - E^(I*(P
i/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + I*(PolyLog[2,
 -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4
 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log
[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2,
-E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*
(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[
a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi +
ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcTan[a*x
]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2) - Log[1 + E^((2*I)*(P
i/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)
^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + ((3*I)/8)*(Pi/2
- ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] + (3*Pi^2*((I/2)*(
Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*Lo
g[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + (I/2)*PolyLog[2, -E^(
(2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-1/2*Pi
 + ArcTan[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2
))] - (3*(Pi/2 - ArcTan[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))])/4 -
(3*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^3 - (Pi/2 + (-1/2*Pi + ArcT
an[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))] + I*(Pi
/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + Arc
Tan[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]/
2))/2 - (3*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 +
(-1/2*Pi + ArcTan[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcTan
[a*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2
))])))/(8*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2...
```

Maple [A]
time = 2.99, size = 655, normalized size = 0.64

method	result
default	$\frac{c^{2} \sqrt{c \left(a x -i\right) \left(a x +i\right)}\, \left(2 \arctan \left(a x \right)^{3} a^{4} x^{4}-2 \arctan \left(a x \right)^{2} a^{3} x^{3}+9 \arctan \left(a x \right)^{3} a^{2} x^{2}+2 \arctan \left(a x \right) a^{2} x^{2}-23 \arctan \left(a x \right)^{2} a x -8\right.}{8 x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^3*a^4*x^4-2*arctan(a*x)^2*
a^3*x^3+9*arctan(a*x)^3*a^2*x^2+2*arctan(a*x)*a^2*x^2-23*arctan(a*x)^2*a*x-
8*arctan(a*x)^3-2*a*x)/x-1/8*I*a*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(48*I*polylo
g(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-90*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+24*
I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*arctan(a*x)^2*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a
```

```
^2*x^2+1)^(1/2))-24*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+44*I*
arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-44*I*arctan(a*x)*ln(1+I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+90*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+48*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*arctan(a*
x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*I*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))-15*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+44*poly
log(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-44*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+90*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*polylog(4,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{3}\left(ax\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**2,x)
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.434 $\int \frac{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=1043

$$a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)-\frac{3ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x}-\frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2+\frac{13ia^2c^3\sqrt{}}{}$$

```
[Out] 1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3-a^2*c^(5/2)*arctanh(a*x*c^(1/2)
/(a^2*c*x^2+c)^(1/2))-15/2*I*a^2*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*arctan(a*x)^
3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-6*a^2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)+15/2*I*a^2*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*I*a^2*c^3*ar
ctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-3*I*a^2*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15*I*a^2*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13*I*a^2*c^3*arctan(a*x)*pol
ylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)+3*I*a^2*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-15*a^2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*a^2*c^3*polylog(3,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13*a^2*c^3*p
olylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)+15*a^2*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)-15*I*a^2*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*I*a^2*c^3*arctan((1+I*a*x)/(
a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+a^2*c
^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-3/2*a*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1
/2)/x-1/2*a^3*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+2*a^2*c^2*arctan(a*x)
^3*(a^2*c*x^2+c)^(1/2)-1/2*c^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A]
time = 2.42, antiderivative size = 1043, normalized size of antiderivative = 1.00, number of steps used = 87, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5070, 5082, 5064, 5078, 5074, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5010, 5008, 4266, 5000, 223, 212}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

```
[Out] a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - (3*a*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(2*x) - (a^3*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + ((13*I
)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/Sqrt[c
 + a^2*c*x^2] + 2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3 - (c^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^3)/(2*x^2) + (a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*
x]^3)/3 - (5*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - a^2*c^(5/2)*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (((15*I)/2)*a^2*c^3*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13
*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])
])/Sqrt[c + a^2*c*x^2] + ((13*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Poly
Log[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x
^2] + ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[
2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (15*a^2*c^3*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt
[c + a^2*c*x^2] - (13*a^2*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*
x])])/Sqrt[c + a^2*c*x^2] + (15*a^2*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyL
og[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a^2*c^3*Sqrt[1 + a^
2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((15*I)*a^2*c^
3*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}{x^3}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^3}\,dx + (a^2c)\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^3}\,dx + 2\left((a^2c^2)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x}\,d\right.$$

$$= \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3 - (a^3c^2)\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2\,dx + c$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{c^2\sqrt{c+a^2}}{}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

$$= a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax) - \frac{3ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x} - \frac{1}{2}a^3c^2x\sqrt{c+}$$

Mathematica [A]
time = 9.35, size = 1460, normalized size = 1.40

Warning: Unable to verify antiderivative.

[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]

[Out] (a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(((-I)*Pi^4)/Sqrt[1 + a^2*x^2] + 8*ArcTan[a*x]^3 + ((2*I)*ArcTan[a*x]^4)/Sqrt[1 + a^2*x^2] + (8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (24*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (24*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((24*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (48*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((48*I)*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]))/4 + a^2*c^2*(-1/2*(Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTan[a*x]))] - ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + Pi*ArcTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2)*ArcTan[a*x]))] - Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] - 2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - Pi*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + ((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/12) + (a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Sec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[1 + a^2*x^2])

Maple [A]

time = 3.85, size = 660, normalized size = 0.63

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\ \arctan(ax)\left(2\arctan(ax)^2a^4x^4-3\arctan(ax)a^3x^3+14\arctan(ax)^2a^2x^2+6a^2x^2-9\arctan(ax)ax-3\arctan\right.}{6x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*a^4*x^4-3*ar
ctan(a*x)*a^3*x^3+14*arctan(a*x)^2*a^2*x^2+6*a^2*x^2-9*arctan(a*x)*a*x-3*ar
ctan(a*x)^2)/x^2+1/2*a^2*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(5*arctan(a*x)^3*ln(
1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(
1/2))+6*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+13*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-13*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*arctan
(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(4,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a
*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*
I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan((1+I*a*x)/(a^2*x^2+1)^
(1/2))+15*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arct
an(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+26*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^3(ax)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**3,x)
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3,x)
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.435 $\int \frac{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=1061

$$-\frac{a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{x}-\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2-\frac{ac^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2x^2}-\frac{2a^2c^2\sqrt{c+}}{}$$

```
[Out] -1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3-a^3*c^(5/2)*arctanh((a^2*c*x^2
+c)^(1/2)/c^(1/2))-3*I*a^3*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15/2*I*a^3*c^3*arctan(a*x)^2*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13
*a^3*c^3*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+13*I*a^3*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13*I*a^3*c^3*arctan(a*x)
*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)+3*I*a^3*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-15*I*a^3*c^3*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a^3*c^3*arctan(a*x)*arctan((1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15*I*a
^3*c^3*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^
2+c)^(1/2)-13*a^3*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-15*a^3*c^3*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15*a^3*c^3*arctan(a*x)
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+13*a^3*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-5*I*a^3*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*
x)^3*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15/2*I*a^3*c^3*arctan(a*x)^2*pol
ylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-a^2*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-3/2*a^3*c^2*arctan(a*x)^2*(a^2*c
*x^2+c)^(1/2)-1/2*a*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2-2*a^2*c^2*arc
tan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x+1/2*a^4*c^2*x*arctan(a*x)^3*(a^2*c*x^2+c)^
(1/2)
```

Rubi [A]
time = 2.34, antiderivative size = 1061, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5070, 5064, 5082, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724, 5010, 5008, 4266, 6744, 5000, 5006}

Antiderivative was successfully verified.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] -((a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x) - (3*a^3*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^2)/2 - (a*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*x^2) -
 (2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x + (a^4*c^2*x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^3)/2 - (c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*x^3) -
 ((5*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/
Sqrt[c + a^2*c*x^2] - ((6*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (13*a^3*c^3*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - a
^3*c^(5/2)*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]] + ((13*I)*a^3*c^3*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] +
(((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])])/Sqrt[c + a^2*c*x^2] - (((15*I)/2)*a^3*c^3*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((13*I)*
a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c
 + a^2*c*x^2] + ((3*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 +
I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^3*c^3*Sqrt[1 + a^2
*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2]
- (13*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] - (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (15*a^3*c^3*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (13*a^3*c^3*Sqrt[
1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((15*I)*a
^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c
*x^2] + ((15*I)*a^3*c^3*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
```

```
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
```

```
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}{x^4}\,dx = c\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^4}\,dx + \left(a^2c\right)\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}{x^2}\,dx$$

$$= c^2\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^4}\,dx + 2\Bigg(\left(a^2c^2\right)\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{x^2}$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c(c}{}$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3 - \frac{c(c}{}$$

$$= -\frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

$$= -\frac{a^2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2 - \frac{ac^2\sqrt{c}}{}$$

Mathematica [A]

time = 9.46, size = 1771, normalized size = 1.67

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

```
[Out] ((-1/2*I)*a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTa
n[a*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x
]^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2,
I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
 + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^
(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] +
(a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1
 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi
^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*A
rcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2*x
^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x
^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x
^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (64*a*x
*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*x*A
rcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Lo
g[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan[a*x]*Lo
g[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[a*x]^2*Lo
g[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*x]^3*Log[1
 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x]^2*Log[1 +
 E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(Pi + 2*ArcTan
[a*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/
E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])
*PolyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*I)*a*x*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi^2*x*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((192*I)*a*Pi*x*ArcTa
n[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((38
4*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (38
4*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (
192*a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*Po
lyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192*a*Pi*x*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (384*a*x*ArcTan[a*x]*PolyLog[3,
(-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (384*a*x*PolyLog[3, E^(I*ArcTa
n[a*x])])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)/E^(I*ArcTan[a*x]
)])/Sqrt[1 + a^2*x^2] - ((384*I)*a*x*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2])*Sec[ArcTan[a*x]/2])/(64*Sqrt[1 + a^2*x^2]) + (a^3*c^3*Sqrt
[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Cot[Arc
```

```
Tan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcTan[a*x]^3*Csc
[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*Log[1 - E^(I*Ar
cTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 24*Log[Tan[ArcT
an[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (24*I)*Ar
cTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E^(I*ArcTan[a*x])]
 + 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2
- (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]^4)/(a^3*x^3) - 12
*ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[ArcTan[a*x]/2]))/(24*
Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]
time = 4.53, size = 699, normalized size = 0.66

method	result
default	$\frac{c^2\sqrt{c\,(ax-i)\,(ax+i)}\;\arctan(ax)\left(3\arctan(ax)^2a^4x^4-9\arctan(ax)a^3x^3-14\arctan(ax)^2a^2x^2-6a^2x^2-3\arctan(ax)ax-2\right.}{6x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(3*arctan(a*x)^2*a^4*x^4-9*ar
ctan(a*x)*a^3*x^3-14*arctan(a*x)^2*a^2*x^2-6*a^2*x^2-3*arctan(a*x)*a*x-2*ar
ctan(a*x)^2)/x^3-1/2*I*a^3*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(5*I*arctan(a*x)^3
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(
1/2))-30*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+13*I*arcta
n(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^2*polylog(2,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+15*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+2*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-13*I*arctan(a*x)^2*ln(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-6*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))+30*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-5*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(2,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+30*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*polylog(4,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x
)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{3}\left(ax\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**4,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

```
[Out] int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.436 $\int \frac{x^3 \mathbf{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=408

$$\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{a^4c}-\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2a^3c}-\frac{5i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^4\sqrt{c+a^2cx^2}}-2$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(1/2)-5*I*arctan((1+I*a*x)/
(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+
5*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/a^4/(a^2*c*x^2+c)^(1/2)-5*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-5*polylog(3,-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+5*polylog(3,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+arcta
n(a*x)*(a^2*c*x^2+c)^(1/2)/a^4/c-1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^
3/c-2/3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^3*(a^2*
c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.52, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5072, 5050, 223, 212, 5010, 5008, 4266, 2611, 2320, 6724}

$$\frac{x^2\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}+\frac{5i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{5\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}+\frac{5\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^4\sqrt{a^2cx^2+c}}-\frac{2\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{3a^4c}-\frac{5i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^4\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{a^4c}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^4\sqrt{c}}-\frac{x\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^4*c) - (x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)/(2*a^3*c) - ((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTa
n[a*x]^2)/(a^4*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)
/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*a^2*c) - ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^4*Sqrt[c]) + ((5*I)*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2]) -
((5*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*
Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])])/(a^4*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*Arc
Tan[a*x])])/(a^4*Sqrt[c + a^2*c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
```

```
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^2c} - \frac{2\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{3a^2} - \frac{\int \frac{x^2\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= -\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^2c} \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^4c} + \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3a^4c} + \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax\ldots}\right.}{a^4\sqrt{c+a^2cx^2}} \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax\ldots}\right.}{a^4\sqrt{c+a^2cx^2}} \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax\ldots}\right.}{a^4\sqrt{c+a^2cx^2}} \ldots$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} - \frac{5i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax\ldots}\right.}{a^4\sqrt{c+a^2cx^2}} \ldots$$

Mathematica [A]
time = 2.28, size = 552, normalized size = 1.35

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]

[Out] (Sqrt[c + a^2*c*x^2]*((6*(5*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 5*
Pi*ArcTan[a*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcTan[a*x])))/(2*E^((I/2)*ArcTa
n[a*x]))] - 5*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 5*ArcTan[a*x]^2*
Log[((1/2 + I/2)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*x])] + 5*Pi*Ar
cTan[a*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcTan[a*x])))/E^((I/2)*ArcTan[a*
x])] + 5*ArcTan[a*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcTan[a*x]))/(2*E^((I/2
)*ArcTan[a*x]))] - 5*Pi*ArcTan[a*x]*Log[-Cos[(Pi + 2*ArcTan[a*x])/4]] + 2*L
```

```
og[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] + 5*ArcTan[a*x]^2*Log[Cos[ArcTa
n[a*x]/2] - Sin[ArcTan[a*x]/2]] - 2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2]] - 5*ArcTan[a*x]^2*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] - 5*Pi
*ArcTan[a*x]*Log[Sin[(Pi + 2*ArcTan[a*x])/4]] + (10*I)*ArcTan[a*x]*PolyLog[
2, (-I)*E^(I*ArcTan[a*x])] - (10*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*
x])] - 10*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 10*PolyLog[3, I*E^(I*ArcTan[
a*x])]))/Sqrt[1 + a^2*x^2] - (1 + a^2*x^2)*ArcTan[a*x]*(-6 + 2*ArcTan[a*x]^
2 + 6*(-1 + ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]] + 3*ArcTan[a*x]*Sin[2*ArcTan[
a*x]])))/(12*a^4*c)
```

Maple [A]
time = 8.60, size = 380, normalized size = 0.93

method	result
default	$\frac{\left(2 \arctan(ax)^2 a^2 x^2-3 \arctan(ax) ax-4 \arctan(ax)^2+6\right) \arctan(ax) \sqrt{c\,(ax-i)\,(ax+i)}}{6c\,a^4}+\frac{5i\left(3i \arctan(ax)^2 \ln\left(1+\frac{}{\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*arctan(a*x)^2*a^2*x^2-3*arctan(a*x)*a*x-4*arctan(a*x)^2+6)*arctan(a*
x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^4+5/6*I*(3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+arctan(a*x)^3+6*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(
I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c-5/6*I*(3*I*arctan(a*x)^2*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+arctan(a*x)^3+6*arctan(a*x)*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+6*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*
(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c+2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/
2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.437 $\int \frac{x^2 \mathbf{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=625

$$-\frac{3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{2a^2c}+\frac{i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a^3\sqrt{c+a^2cx^2}}$$

```
[Out] I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/(
a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/2*I*arctan(a*x)^2*polylog(2,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/2*
I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
a^3/(a^2*c*x^2+c)^(1/2)+3*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3*I*polylog(2,I*(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3*arctan(a*x)*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(
1/2)-3*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/a^3/(a^2*c*x^2+c)^(1/2)+3*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/2*arctan(a*x)^2*(a^2*
c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.35, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5072, 5050, 5010, 5006, 5008, 4266, 2611, 6744, 2320, 6724}

$$\frac{x\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{2a^3\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a^3\sqrt{a^2cx^2+c}}-\frac{3\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2a^3c}-\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)\text{ArcTan}(ax)}{a^3\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^3)/(2*a^2*c) + (I*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^3)/(a^3*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) - (((
3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])
/(a^3*Sqrt[c + a^2*c*x^2]) + (((3*I)/2)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Pol
yLog[2, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a
^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/(a^3*Sqrt[c + a
^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/(a^3*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a
```

```
^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*Sqrt[c + a^2*c*x^
2]) + ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*Sqr
t[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])
])/(a^3*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} - \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{2a^2} - \frac{3\int \frac{x\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{3\int \frac{\tan^{-1}(ax)}{\sqrt{c+a^2cx^2}}\,dx}{a^2} - \sqrt{1$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^3 \sec\right.}{2a^3\sqrt{c+a}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^3\sqrt{c+a^2cx}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^3\sqrt{c+a^2cx}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^3\sqrt{c+a^2cx}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^3\sqrt{c+a^2cx}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^3\sqrt{c+a^2cx}}$$

Mathematica [A]
time = 4.26, size = 812, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(((7*I)/32)*Pi^4 + (I/4)*Pi^3*ArcTan[a*x] - 6*ArcTan
[a*x]^2 - ((3*I)/4)*Pi^2*ArcTan[a*x]^2 + I*Pi*ArcTan[a*x]^3 - (I/2)*ArcTan[
a*x]^4 - (3*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/2 + 3*Pi*ArcTan[
a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])] + (Pi^3*Log[1 + I/E^(I*ArcTan[a*x])])/4
 - 2*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]*Log[1 - I*
E^(I*ArcTan[a*x])] - (Pi^3*Log[1 + I*E^(I*ArcTan[a*x])])/4 - 12*ArcTan[a*x]
*Log[1 + I*E^(I*ArcTan[a*x])] + (3*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a
```

```
*x])])/2 - 3*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^
3*Log[1 + I*E^(I*ArcTan[a*x])] - (Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]])/4
- (6*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi*(Pi
 - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (12*I)*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6
*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]^2
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (12*I)*PolyLog[2, I*E^(I*ArcTan[a*x])
] - 12*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 6*Pi*PolyLog[3, I/E
^(I*ArcTan[a*x])] - 6*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 12*ArcTan[a*x
]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (-I)/E^(I*ArcTan[a
*x])] + (12*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + ArcTan[a*x]^3/(Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2])^2 - (6*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(
Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]) - ArcTan[a*x]^3/(Cos[ArcTan[a*x]/2
] + Sin[ArcTan[a*x]/2])^2 + (6*ArcTan[a*x]^2*Sin[ArcTan[a*x]/2])/(Cos[ArcTa
n[a*x]/2] + Sin[ArcTan[a*x]/2])))/(4*a^3*c*Sqrt[1 + a^2*x^2])
```

Maple [A]
time = 7.14, size = 430, normalized size = 0.69

method	result
default	$\frac{(\arctan(ax)ax-3)\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2c\,a^3} - \frac{i\left(i\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(1-\frac{}{\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)*a*x-3)*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3-1/2*I
*(I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*
arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(
4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-6*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.438 $\int \frac{x\mathbf{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=283

$$\frac{6i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{a^2c}-\frac{6i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{Pol}}{a^2\sqrt{c+a^2c}}$$

```
[Out] 6*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
/(a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*polylog(2,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+6*pol
ylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^
(1/2)-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2
*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A]
time = 0.18, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5050, 5010, 5008, 4266, 2611, 2320, 6724}

$$-\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^2\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{a^2c}+\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - ((6*I)*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c
+ a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcT
an[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyL
og[3, I*E^(I*ArcTan[a*x])])/(a^2*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{a\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} + \frac{\left(6\sqrt{1+a\ldots\right.}{\ldots}$$

$$= \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2\ldots}}{\ldots}$$

$$= \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2\ldots}}{\ldots}$$

$$= \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^2}{a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^2c} - \frac{6i\sqrt{1+a^2\ldots}}{\ldots}$$

Mathematica [A]
time = 0.19, size = 168, normalized size = 0.59

$$\frac{\sqrt{c(1+a^2x^2)}\left(\text{ArcTan}(ax)^3 - \frac{3\left(\text{ArcTan}(ax)^2\left(\log\left(1-ie^{i\text{ArcTan}(ax)}\right)-\log\left(1+ie^{i\text{ArcTan}(ax)}\right)\right)+2i\text{ArcTan}(ax)\left(\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)-\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)\right)-2\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)+2\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{1+a^2x^2}}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^3 - (3*(ArcTan[a*x]^2*(Log[1 - I*E^(I*A
rcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (2*I)*ArcTan[a*x]*(PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) - 2*PolyLog[3,
 (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2
*x^2]))/(a^2*c)
```

Maple [F]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{x\arctan(ax)^3}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^{3}(ax)}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \operatorname{atan}(a\,x)^{3}}{\sqrt{c\,a^{2}\,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.439 $\int \frac{\mathbf{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a\sqrt{c+a^2cx^2}}+\frac{3i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{c+a^2cx^2}}-3i$$

```
[Out] -2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-6*arct
an(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*
c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5010, 5008, 4266, 2611, 6744, 2320, 6724}

$$\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{a\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3)/(a*Sqrt[
c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])
])/(a*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*
ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4
, I*E^(I*ArcTan[a*x])])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^3\sec(x)\,dx, x, \tan^{-1}(ax)\right)}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\log\left(1-i\right.\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-}}\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-}}\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-}}\right.}{a\sqrt{c+a^2cx^2}} \\
&= -\frac{2i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}(ax)}\right)\tan^{-1}(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-ie^{i\tan^{-}}\right.}{a\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 190, normalized size = 0.52

$$
-\frac{i\sqrt{c(1+a^2x^2)}\left(2\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3-3\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)+3\text{ArcTan}(ax)^2\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)-6i\text{ArcTan}(ax)\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)+6i\text{ArcTan}(ax)\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)+6\text{PolyLog}\left(4,-ie^{i\text{ArcTan}(ax)}\right)-6\text{PolyLog}\left(4,ie^{i\text{ArcTan}(ax)}\right)\right)}{ac\sqrt{1+a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*PolyLog[
2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)
*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2
*x^2])
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$
\int \frac{\arctan(ax)^3}{\sqrt{a^2c\,x^2+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}(ax)}{\sqrt{c(a^{2}x^{2}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^{3}}{\sqrt{c\,a^{2}\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
[Out] int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

3.440 $\int \frac{\text{ArcTan}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=327

$$-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}+\frac{3i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}-3i$$

```
[Out] -2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+
6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2
*c*x^2+c)^(1/2)-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5078, 5076, 4268, 2611, 6744, 2320, 6724}

$$\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^
2*c*x^2] + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a
*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog
[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + (6*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((6*I)
*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((
6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int x^3\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\log\left(1\right.\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-e^{it}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-e^{it}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-e^{it}\right.}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\text{Li}_2\left(-e^{it}\right.}{\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.16, size = 208, normalized size = 0.64

$$-\frac{i\sqrt{1+a^2x^2}\left(\pi^4-2\text{ArcTan}(ax)^4+8i\text{ArcTan}(ax)^3\log\left(1-e^{-i\text{ArcTan}(ax)}\right)-8i\text{ArcTan}(ax)^3\log\left(1+e^{i\text{ArcTan}(ax)}\right)-24\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-i\text{ArcTan}(ax)}\right)-24\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+48i\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-i\text{ArcTan}(ax)}\right)-48i\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)+48\text{PolyLog}\left(4,e^{-i\text{ArcTan}(ax)}\right)+48\text{PolyLog}\left(4,-e^{i\text{ArcTan}(ax)}\right)\right)}{8\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1/8*I)*Sqrt[1 + a^2*x^2]*(Pi^4 - 2*ArcTan[a*x]^4 + (8*I)*ArcTan[a*x]^3*L
og[1 - E^((-I)*ArcTan[a*x])] - (8*I)*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x]
)] - 24*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - 24*ArcTan[a*x]^2*P
olyLog[2, -E^(I*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTa
n[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48*PolyLog[4
, E^((-I)*ArcTan[a*x])] + 48*PolyLog[4, -E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a
^2*x^2)]
```

Maple [A]
time = 2.55, size = 261, normalized size = 0.80

method	result

default	$-\frac{i\left(i \arctan(ax)^3 \ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i \arctan(ax)^3 \ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+3 \arctan(ax)^2 \operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3 \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1+
(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan
(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylo
g(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1
/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^3(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.441 $\int \frac{\text{ArcTan}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{cx}-\frac{6a\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}+\frac{6ia\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{Po}}{\sqrt{c+a^2c}}$$

```
[Out] -6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*arctan(a*x)*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*poly
log(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-ar
ctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x
```

Rubi [A]
time = 0.27, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5064, 5078, 5076, 4268, 2611, 2320, 6724}

$$\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{cx}-\frac{6a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) - (6*a*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((6*I)*a*Sqrt[1
+ a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
- ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqr
t[c + a^2*c*x^2] - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/S
qrt[c + a^2*c*x^2] + (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/
Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} + (3a)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} + \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{\tan^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} + \frac{\left(3a\sqrt{1+a^2x^2}\right)\operatorname{Subst}\left(\int x^2\csc(x)\,dx, x, \tan^{-1}(ax)\right)}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{\left(6a\sqrt{\right.}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1}}{}$$

$$= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^2\tanh^{-1}\left(e^{i\tan^{-1}(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1}}{}$$

Mathematica [A]

time = 0.24, size = 174, normalized size = 0.67

$$-\frac{a\sqrt{1+a^2x^2}\left(\frac{\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3}{ax} - 3\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right) + 3\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right) - 6i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right) + 6i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right) + 6\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right) - 6\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)\right)}{\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3)/(a*x) - 3*ArcTan[
a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x
])] - (6*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*
PolyLog[2, E^(I*ArcTan[a*x])] + 6*PolyLog[3, -E^(I*ArcTan[a*x])] - 6*PolyLo
g[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)])
```

Maple [A]

time = 2.44, size = 230, normalized size = 0.88

method	result
default	$-\frac{\arctan(ax)^3\sqrt{c(ax-i)(ax+i)}}{cx} + \frac{3a\left(\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - \arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - 2i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(a*x)^3*(c*(a*x-I)*(I+a*x))^(1/2)/c/x+3*a*(arctan(a*x)^2*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*
arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*
x))^(1/2)/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}(ax)}{x^{2}\sqrt{c\,(a^{2}x^{2}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.442 $\int \frac{\text{ArcTan}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=597

$$-\frac{3a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

```
[Out] a^2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-6*a^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2*arctan(a*x)^2*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3/2*I*a^
2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+3*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a^2*arctan(a*x)*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*a
rctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)+3*I*a^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c
/x-1/2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.49, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5082, 5064, 5078, 5074, 5076, 4268, 2611, 6744, 2320, 6724}

$$-\frac{3ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{2\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{2\sqrt{a^2cx^2+c}}+\frac{3a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}-\frac{3a\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2cx}+\frac{a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{6a^2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\tanh^{-1}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}+\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}-\frac{3ia^2\sqrt{a^2x^2+1}\,\text{Li}_2\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*c*x) - (Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/(2*c*x^2) + (a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*A
rcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (6*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Ar
cTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c*x^2] - (((3*I)/2)*a^
2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c +
a^2*c*x^2] + (((3*I)/2)*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, E^(I
*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sq
rt[1 + a^2*x^2]*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/Sqrt[c + a^2*c
*x^2] + (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])]
)/Sqrt[c + a^2*c*x^2] - (3*a^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(
```

```
I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((3*I)*a^2*Sqrt[1 + a^2*x^2]*PolyLog
[4, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - ((3*I)*a^2*Sqrt[1 + a^2*x^2]
*PolyLog[4, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5074

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{1}{2}(3a)\int \frac{\tan^{-1}(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\left(3a^2\right)\int \frac{\tan^{-1}(ax)}{x\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}-\frac{\left(a^2\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int\right.}{2\sqrt{c+}} \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3}{\sqrt{c+a^2c}} \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3}{\sqrt{c+a^2c}} \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3}{\sqrt{c+a^2c}} \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3}{\sqrt{c+a^2c}} \\
&= -\frac{3a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{2cx^2}+\frac{a^2\sqrt{1+a^2x^2}\,\tan^{-1}(ax)^3}{\sqrt{c+a^2c}}
\end{aligned}
$$

Mathematica [A]
time = 2.63, size = 345, normalized size = 0.58

$$
\frac{a^2\sqrt{1+a^2x^2}\left(i\pi^4-2i\mathrm{ArcTan}(ax)^4-12\mathrm{ArcTan}(ax)^2\cot\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-2\mathrm{ArcTan}(ax)^3\csc^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-8\mathrm{ArcTan}(ax)^3\log\left(1-e^{-i\mathrm{ArcTan}(ax)}\right)+48\mathrm{ArcTan}(ax)\log\left(1-e^{i\mathrm{ArcTan}(ax)}\right)-48\mathrm{ArcTan}(ax)\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)+8\mathrm{ArcTan}(ax)^3\log\left(1+e^{i\mathrm{ArcTan}(ax)}\right)-24i\mathrm{ArcTan}(ax)^2\mathrm{PolyLog}\left(2,e^{-i\mathrm{ArcTan}(ax)}\right)-24i\left(-2+\mathrm{ArcTan}(ax)^2\right)\mathrm{PolyLog}\left(2,-e^{i\mathrm{ArcTan}(ax)}\right)-48i\mathrm{PolyLog}\left(2,e^{i\mathrm{ArcTan}(ax)}\right)-48\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(3,e^{-i\mathrm{ArcTan}(ax)}\right)+48\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(3,-e^{i\mathrm{ArcTan}(ax)}\right)+48i\mathrm{PolyLog}\left(4,e^{-i\mathrm{ArcTan}(ax)}\right)+48i\mathrm{PolyLog}\left(4,-e^{i\mathrm{ArcTan}(ax)}\right)+2\mathrm{ArcTan}(ax)^3\sec^2\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)-12\mathrm{ArcTan}(ax)^2\tan\left(\frac{1}{2}\mathrm{ArcTan}(ax)\right)\right)}{16\sqrt{c\left(1+a^2x^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^2*Sqrt[1 + a^2*x^2]*(I*Pi^4 - (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]^2*Cot
[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 8*ArcTan[a*x]^3*Lo
g[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4
8*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 8*ArcTan[a*x]^3*Log[1 + E^(I*Arc
Tan[a*x])] - (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - (24*I)
*(-2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[2, E^
(I*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] + 48*Arc
Tan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + (48*I)*PolyLog[4, E^((-I)*ArcTan[
```

```
a*x])] + (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*Sec[ArcTan
[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[c*(1 + a^2*x^2)
])
```

Maple [A]
time = 3.28, size = 410, normalized size = 0.69

method	result
default	$-\frac{(3ax+\arctan(ax))\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2cx^2}+\frac{ia^2\left(i\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(1\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(3*a*x+arctan(a*x))*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2+1/2*
I*a^2*(I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln
(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arc
tan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(
a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}(ax)}{x^{3}\sqrt{c\,(a^{2}x^{2}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\operatorname{atan}(a\,x)}^3}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.443 $\int \frac{\text{ArcTan}(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=396

$$-\frac{a^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}{cx}-\frac{a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}{2cx^2}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\text{Arc}}{3cx}$$

```
[Out] -a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+5*a^3*arctan(a*x)^2*arcta
nh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*I*a
^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+5*I*a^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^3*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^3*polylog(3,(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)*(a
^2*c*x^2+c)^(1/2)/c/x-1/2*a*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*arc
tan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1
/2)/c/x
```

Rubi [A]
time = 0.70, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5082, 5064, 272, 65, 214, 5078, 5076, 4268, 2611, 2320, 6724}

$$\frac{2a^2\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{3cx}-\frac{a^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}-\frac{5ia^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5ia^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5a^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{5a^3\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}+\frac{5a^3\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{\sqrt{a^2cx^2+c}}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/(2*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x^3) + (2*
a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*c*x) + (5*a^3*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] - (a^3*ArcTanh
[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c] - ((5*I)*a^3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^2] + ((5*I)*a^3*Sqr
t[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])])/Sqrt[c + a^2*c*x^
2] + (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])])/Sqrt[c + a^2*
c*x^2] - (5*a^3*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/Sqrt[c + a
^2*c*x^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5082

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(m
+ 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[c^2*((m +
 2)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] &
& LtQ[m, -1] && NeQ[m, -2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3} + a\int \frac{\tan^{-1}(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx - \frac{1}{3}(2a^2)\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$$

$$= -\frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

$$= -\frac{a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}{cx} - \frac{a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}{2cx^2} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{3cx^3}$$

Mathematica [A]
time = 4.10, size = 343, normalized size = 0.87

$$\frac{a^3\sqrt{c(1+a^2x^2)}\left(-12\text{ArcTan}(ax)\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)+10\text{ArcTan}(ax)^3\cot\left(\frac{1}{2}\text{ArcTan}(ax)\right)-3\text{ArcTan}(ax)^2\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\frac{ax\text{ArcTan}(ax)^3\csc^4\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{2\sqrt{1+a^2x^2}}-60\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)+60\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+24\log\left(\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)-120i\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+120i\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)+120\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-120\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)+3\text{ArcTan}(ax)^2\sec^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)-\frac{8(1+a^2x^2)^{3/2}\text{ArcTan}(ax)^3\sin^4\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{a^3x^3}-12\text{ArcTan}(ax)\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)+10\text{ArcTan}(ax)^3\tan\left(\frac{1}{2}\text{ArcTan}(ax)\right)\right)}{24c\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (a^3*Sqrt[c*(1 + a^2*x^2)]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] + 10*ArcTan[
a*x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*Arc
Tan[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) - 60*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] + 60*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 2
4*Log[Tan[ArcTan[a*x]/2]] - (120*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])] + (120*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 120*PolyLog[3, -E
^(I*ArcTan[a*x])] - 120*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec
[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2]
```

^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] + 10*ArcTan[a*x]^3*Tan[ArcTan[a*x]/2]))/(24*c*Sqrt[1 + a^2*x^2])

Maple [A]
time = 4.83, size = 487, normalized size = 1.23

method	result
default	$\frac{\left(4 \arctan(ax)^2 a^2 x^2-6 a^2 x^2-3 \arctan(ax) a x-2 \arctan(ax)^2\right) \arctan(ax) \sqrt{c\,(ax-i)\,(ax+i)}}{6 c\, x^3}-\frac{2 a^3 \operatorname{arctanh}\left(\frac{iax+1}{\sqrt{a^2x^2}}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(4*arctan(a*x)^2*a^2*x^2-6*a^2*x^2-3*arctan(a*x)*a*x-2*arctan(a*x)^2)*arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^3-2*a^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c-5/2*a^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c+5*I*a^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c-5*a^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c+5/2*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c-5*I*a^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c+5*a^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}(ax)}{x^{4}\sqrt{c(a^{2}x^{2}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^{3}}{x^{4}\,\sqrt{c\,a^{2}\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.444 $\int \frac{x^3 \text{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=403

$$\frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\text{ArcTan}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\text{ArcTan}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\text{Ar}}{a^4c}$$

```
[Out] 6*x/a^3/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)-3*x*a
rctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/a^4/c/(a^2*c*x^2+c)^(1
/2)+6*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)
/a^4/c/(a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*polyl
og(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(
1/2)+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a
^2*c*x^2+c)^(1/2)-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4/c^2
```

Rubi [A]

time = 0.38, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5084, 5050, 5010, 5008, 4266, 2611, 2320, 6724, 5018, 197}

$$\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{a^4c^2} - \frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}} + \frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}} + \frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}} - \frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^4c\sqrt{a^2cx^2+c}} + \frac{\text{ArcTan}(ax)^3}{a^4c\sqrt{a^2cx^2+c}} + \frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^4c\sqrt{a^2cx^2+c}} - \frac{6\text{ArcTan}(ax)}{a^4c\sqrt{a^2cx^2+c}} - \frac{3x\text{ArcTan}(ax)^2}{a^3c\sqrt{a^2cx^2+c}} + \frac{6x}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (6*x)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(a^4*c*Sqrt[c + a^2*c*x
^2]) - (3*x*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^
2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^4*c*Sqrt[c + a^2*c*x^2])
 + ArcTan[a*x]^3/(a^4*c*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^3)/(a^4*c^2) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*E^
(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (6
*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt[c + a^2*
c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^4*c*Sqrt
[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{x \tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{a^2} + \frac{\int \frac{x \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^4c^2} - \frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a^3} - \frac{3\int \frac{\tan^{-1}(ax)^2}{\sqrt{c+a^2cx^2}}\,dx}{a^3c}$$

$$= -\frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{a^4c^2} +$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2c}}{}$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^4c\sqrt{c+a^2cx}}$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^4c\sqrt{c+a^2cx}}$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^4c\sqrt{c+a^2cx}}$$

$$= \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\,\tan^{-1}\left(e^{i\tan^{-1}}\right.}{a^4c\sqrt{c+a^2cx}}$$

Mathematica [A]
time = 0.58, size = 308, normalized size = 0.76

$$\frac{\sqrt{1+a^2x^2}\left(\frac{6ax}{\sqrt{1+a^2x^2}} - 3\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax) - \frac{3ax\mathrm{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}} + \frac{3}{2}\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)^3 - 3\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)\cos(2\mathrm{ArcTan}(ax)) + \frac{1}{2}\sqrt{1+a^2x^2}\,\mathrm{ArcTan}(ax)^3\cos(2\mathrm{ArcTan}(ax)) - 3\mathrm{ArcTan}(ax)^2\log\left(1-ie^{i\mathrm{ArcTan}(ax)}\right) + 3\mathrm{ArcTan}(ax)^2\log\left(1+ie^{i\mathrm{ArcTan}(ax)}\right) - 6i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,-ie^{i\mathrm{ArcTan}(ax)}\right) + 6i\mathrm{ArcTan}(ax)\mathrm{PolyLog}\left(2,ie^{i\mathrm{ArcTan}(ax)}\right) + 6\mathrm{PolyLog}\left(3,-ie^{i\mathrm{ArcTan}(ax)}\right) - 6\mathrm{PolyLog}\left(3,ie^{i\mathrm{ArcTan}(ax)}\right)\right)}{a^4c\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((6*a*x)/Sqrt[1 + a^2*x^2] - 3*Sqrt[1 + a^2*x^2]*ArcTan[
a*x] - (3*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (3*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^3)/2 - 3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + (Sqrt[1
+ a^2*x^2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]])/2 - 3*ArcTan[a*x]^2*Log[1 - I*
E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (6*I)*A
rcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*PolyLog[2
, I*E^(I*ArcTan[a*x])] + 6*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[3
, I*E^(I*ArcTan[a*x])]))/(a^4*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [F]
time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \arctan \left(a x \right)^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
[Out] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \operatorname{atan}^{3}{\left(a x \right)}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \operatorname{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.445 $\int \frac{x^2 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=495

$$\frac{6}{a^3c\sqrt{c+a^2cx^2}}+\frac{6x\text{ArcTan}(ax)}{a^2c\sqrt{c+a^2cx^2}}-\frac{3\text{ArcTan}(ax)^2}{a^3c\sqrt{c+a^2cx^2}}-\frac{x\text{ArcTan}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}$$

```
[Out] 6/a^3/c/(a^2*c*x^2+c)^(1/2)+6*x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-3*arc
tan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(1
/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)
/a^3/c/(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*p
olylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+
c)^(1/2)-6*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+6*
I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x
^2+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5084, 5010, 5008, 4266, 2611, 6744, 2320, 6724, 5018, 5014}

$$-\frac{x\text{ArcTan}(ax)^3}{a^2c\sqrt{a^2cx^2+c}}+\frac{6x\text{ArcTan}(ax)}{a^2c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(ie^{i\text{ArcTan}(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^3}{a^3c\sqrt{a^2cx^2+c}}-\frac{3\text{ArcTan}(ax)^2}{a^3c\sqrt{a^2cx^2+c}}+\frac{6}{a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] 6/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^2
]) - (3*ArcTan[a*x]^2)/(a^3*c*Sqrt[c + a^2*c*x^2]) - (x*ArcTan[a*x]^3)/(a^2
*c*Sqrt[c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])
]*ArcTan[a*x]^3)/(a^3*c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2])
- ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(
a^3*c*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-
I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c + a^2*c*x^2]) - (
(6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^3*c*Sqrt[c +
 a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])])/(a
^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
```

```
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{a^2} + \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^2c}$$

$$= -\frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} + \frac{6\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{3/2}}\,dx}{a^2} + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{\sqrt{1+a^2x^2}}}{a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x}}{}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

$$= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\tan^{-1}(ax)^2}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a}}{}$$

Mathematica [A]
time = 1.24, size = 639, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/64*(Sqrt[1 + a^2*x^2]*((7*I)*Pi^4 - 384/Sqrt[1 + a^2*x^2] + (8*I)*Pi^3*A
rcTan[a*x] - (384*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (24*I)*Pi^2*ArcTan[a
*x]^2 + (192*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (32*I)*Pi*ArcTan[a*x]^3 + (
64*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (16*I)*ArcTan[a*x]^4 - 48*Pi^2*Ar
cTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 96*Pi*ArcTan[a*x]^2*Log[1 - I/E^(I
*ArcTan[a*x])] + 8*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log
[1 + I/E^(I*ArcTan[a*x])] - 8*Pi^3*Log[1 + I*E^(I*ArcTan[a*x])] + 48*Pi^2*A
rcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 96*Pi*ArcTan[a*x]^2*Log[1 + I*E^(
I*ArcTan[a*x])] + 64*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - 8*Pi^3*Lo
g[Tan[(Pi + 2*ArcTan[a*x])/4]] - (192*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I
```

```
*ArcTan[a*x])] - (48*I)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*
x])] - (48*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)*Pi*ArcTan[a
*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*ArcTan[a*x]^2*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] - 384*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])]
 + 192*Pi*PolyLog[3, I/E^(I*ArcTan[a*x])] - 192*Pi*PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])] + 384*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (384*I)*P
olyLog[4, (-I)/E^(I*ArcTan[a*x])] + (384*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x
])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [F]
time = 5.64, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arctan(ax)^3}{(a^2 c\, x^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
[Out] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.446 $\int \frac{x\mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{6x}{ac\sqrt{c+a^2cx^2}}+\frac{6\text{ArcTan}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{3x\text{ArcTan}(ax)^2}{ac\sqrt{c+a^2cx^2}}-\frac{\text{ArcTan}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] -6*x/a/c/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)+3*x*ar
ctan(a*x)^2/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5050, 5018, 197}

$$-\frac{\text{ArcTan}(ax)^3}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\text{ArcTan}(ax)^2}{ac\sqrt{a^2cx^2+c}}+\frac{6\text{ArcTan}(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{6x}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-6*x)/(a*c*Sqrt[c + a^2*c*x^2]) + (6*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^
2]) + (3*x*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(a^2*c*
Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= -\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}+\frac{3\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx}{a}\\ &= \frac{6\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}-\frac{6\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx}{a}\\ &= -\frac{6x}{ac\sqrt{c+a^2cx^2}}+\frac{6\tan^{-1}(ax)}{a^2c\sqrt{c+a^2cx^2}}+\frac{3x\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}-\frac{\tan^{-1}(ax)^3}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 0.57

$$\frac{\sqrt{c+a^2cx^2}\,(-6ax+6\text{ArcTan}(ax)+3ax\text{ArcTan}(ax)^2-\text{ArcTan}(ax)^3)}{a^2c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-6*a*x + 6*ArcTan[a*x] + 3*a*x*ArcTan[a*x]^2 - ArcTan
[a*x]^3))/(a^2*c^2*(1 + a^2*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 2.49, size = 134, normalized size = 1.25

method	result
default	$-\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)(\ldots}{2(a\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-
I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-
1)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/a^2/c^2
```

Maxima [A]
time = 0.77, size = 98, normalized size = 0.92

$$\sqrt{c}\left(\frac{3\,x\arctan\left(ax\right)^2}{\sqrt{a^2x^2+1}\,ac^2}-\frac{\arctan\left(ax\right)^3}{\sqrt{a^2x^2+1}\,a^2c^2}-\frac{6\left(\frac{x}{\sqrt{a^2x^2+1}}-\frac{\arctan(ax)}{\sqrt{a^2x^2+1}\,a}\right)}{ac^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(c)*(3*x*arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^3/(sqrt(
a^2*x^2 + 1)*a^2*c^2) - 6*(x/sqrt(a^2*x^2 + 1) - arctan(a*x)/(sqrt(a^2*x^2
+ 1)*a))/(a*c^2))
```

Fricas [A]
time = 3.76, size = 62, normalized size = 0.58

$$\frac{\sqrt{a^2cx^2+c}\left(3\,ax\arctan\left(ax\right)^2-\arctan\left(ax\right)^3-6\,ax+6\,\arctan\left(ax\right)\right)}{a^4c^2x^2+a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(3*a*x*arctan(a*x)^2 - arctan(a*x)^3 - 6*a*x + 6*arctan
(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,\mathrm{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\mathrm{atan}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.447 $\int \frac{\text{ArcTan}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6}{ac\sqrt{c+a^2cx^2}}-\frac{6x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{3\text{ArcTan}(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)^3}{c\sqrt{c+a^2cx^2}}$$

```
[Out] -6/a/c/(a^2*c*x^2+c)^(1/2)-6*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+3*arctan(a
*x)^2/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5018, 5014}

$$\frac{x\text{ArcTan}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\text{ArcTan}(ax)^2}{ac\sqrt{a^2cx^2+c}}-\frac{6x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{6}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -6/(a*c*Sqrt[c + a^2*c*x^2]) - (6*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) +
(3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt[c +
 a^2*c*x^2])
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{3\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}-6\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\ &= -\frac{6}{ac\sqrt{c+a^2cx^2}}-\frac{6x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}}+\frac{3\tan^{-1}(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.56

$$\frac{\sqrt{c+a^2cx^2}\,(-6-6ax\text{ArcTan}(ax)+3\text{ArcTan}(ax)^2+ax\text{ArcTan}(ax)^3)}{c^2\,(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-6 - 6*a*x*ArcTan[a*x] + 3*ArcTan[a*x]^2 + a*x*ArcTan
[a*x]^3))/(c^2*(a + a^3*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.56, size = 132, normalized size = 1.32

method	result
default	$\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2a}+\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+i)\left(\text{arc}\right.}{2(a^2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*
(I+a*x))^(1/2)/(a^2*x^2+1)/c^2/a+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arc
tan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2/a
```

Maxima [A]
time = 0.69, size = 99, normalized size = 0.99

$$\frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}\,c}-\frac{3\,a\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2x^2+1}\,ac}-\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}\,a^2c}+\frac{2}{\sqrt{a^2x^2+1}\,a^2c}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*x^
2 + 1)*a*c) - arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a^2*c) + 2/(sqrt(a^2*x^2 + 1
)*a^2*c))/sqrt(c)
```

Fricas [A]
time = 2.68, size = 58, normalized size = 0.58

$$\frac{\sqrt{a^2cx^2+c}\,\left(ax\arctan(ax)^3-6\,ax\arctan(ax)+3\,\arctan(ax)^2-6\right)}{a^3c^2x^2+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^3 - 6*a*x*arctan(a*x) + 3*arctan(a*x)^
2 - 6)/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

3.448 $\int \frac{\text{ArcTan}(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=443

$$\frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTa}}\right.}{c\sqrt{c+a^2cx^2}}$$

```
[Out] 6*a*x/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*x*arcta
n(a*x)^2/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)-2*arctan
(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2
+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+6
*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^
2*c*x^2+c)^(1/2)-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/c/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5086, 5078, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5018, 197}

$$\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3ax\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}-\frac{6\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{6ax}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (6*a*x)/(c*Sqrt[c + a^2*c*x^2]) - (6*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]) -
 (3*a*x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(c*Sqrt[c +
a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])]
)/(c*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[
2, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*S
qrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^
2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])
/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan
[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I
*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - (3a)\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \frac{\sqrt{1+a^2x^2}\int \frac{\tan^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + (6a)\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx + \cdots$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\ \cdots}{\cdots}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\ \cdots}{\cdots}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\ \cdots}{\cdots}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\ \cdots}{\cdots}$$

$$= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} + \frac{\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\ \cdots}{\cdots}$$

Mathematica [A]
time = 0.31, size = 295, normalized size = 0.67

$$\frac{\sqrt{1+a^2x^2}\left(-i\pi^4+\frac{48ax}{\sqrt{1+a^2x^2}}-\frac{48\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}-\frac{24ax\text{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}}+\frac{8\text{ArcTan}(ax)^3}{\sqrt{1+a^2x^2}}+2i\text{ArcTan}(ax)^4+8\text{ArcTan}(ax)^3\log\left(1-e^{-i\text{ArcTan}(ax)}\right)-8\text{ArcTan}(ax)^3\log\left(1+e^{i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-i\text{ArcTan}(ax)}\right)+24i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+48\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-i\text{ArcTan}(ax)}\right)-48\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,e^{-i\text{ArcTan}(ax)}\right)-48i\text{PolyLog}\left(4,-e^{i\text{ArcTan}(ax)}\right)\right)}{8c\sqrt{c(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((-I)*Pi^4 + (48*a*x)/Sqrt[1 + a^2*x^2] - (48*ArcTan[a*x
])/Sqrt[1 + a^2*x^2] - (24*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (8*ArcTan
[a*x]^3)/Sqrt[1 + a^2*x^2] + (2*I)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 -
E^((-I)*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)
*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*Poly
Log[2, -E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])
] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-
I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])]))/(8*c*Sqrt[c*(1 +
 a^2*x^2)])
```

Maple [A]
time = 2.29, size = 388, normalized size = 0.88

method	result
default	$\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(iax-1)\,(}{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I
)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(a
rctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2-I*(I*arct
an(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1+(1+I*a*x)/
(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*poly
log(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,-(1+I*
a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c
^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}{\left(ax \right)}}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.449 $\int \frac{\text{ArcTan}(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=377

$$\frac{6a}{c\sqrt{c+a^2cx^2}}+\frac{6a^2x\text{ArcTan}(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3a\text{ArcTan}(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\text{ArcTan}(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}{c^2x}-\frac{6a\sqrt{1+}}{}$$

```
[Out] 6*a/c/(a^2*c*x^2+c)^(1/2)+6*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*arc
tan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-a^2*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)-
6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/
(a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*I*a*arctan(a*x)*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*a*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+6
*a*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)
^(1/2)-arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.40, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5086, 5064, 5078, 5076, 4268, 2611, 2320, 6724, 5018, 5014}

$$-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{c^2x}+\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{a^2x\text{ArcTan}(ax)^3}{c\sqrt{a^2cx^2+c}}-\frac{3a\text{ArcTan}(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{6a^2x\text{ArcTan}(ax)}{c\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c\sqrt{a^2cx^2+c}}+\frac{6a}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (6*a)/(c*Sqrt[c + a^2*c*x^2]) + (6*a^2*x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]) - (3*a*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(c
*Sqrt[c + a^2*c*x^2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^2*x) - (6*a*
Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2
*c*x^2]) + ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a
*x])])/(c*Sqrt[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Pol
yLog[2, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2
]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a
^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])])/(c*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx}{c}$$

$$= -\frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}{c^2x} + \left(6a^2\right)\int \frac{\tan^{-1}(}{(c+a^2cx}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

$$= \frac{6a}{c\sqrt{c+a^2cx^2}} + \frac{6a^2x\tan^{-1}(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3a\tan^{-1}(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\tan^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}}{c}$$

Mathematica [A]
time = 1.05, size = 301, normalized size = 0.80

$$\frac{a\left(12+12ax\text{ArcTan}(ax)-6\text{ArcTan}(ax)^2-2ax\text{ArcTan}(ax)^3-\frac{1}{2}ax\text{ArcTan}(ax)^3\csc^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)+6\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\log\left(1-e^{i\text{ArcTan}(ax)}\right)-6\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\log\left(1+e^{i\text{ArcTan}(ax)}\right)+12i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)-12i\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{PolyLog}\left(2,e^{i\text{ArcTan}(ax)}\right)-12\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)+12\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\text{ArcTan}(ax)}\right)-\frac{2(1+a^2x^2)\text{ArcTan}(ax)^3\sin^2\left(\frac{1}{2}\text{ArcTan}(ax)\right)}{ax}\right)}{2c\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(12 + 12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]^3 - (a*x*
ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2)/2 + 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*
ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTa
n[a*x])] - (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x]
)] - 12*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 12*Sqrt[1 + a^2*
x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^3*Sin[Arc
Tan[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 2.14, size = 356, normalized size = 0.94

method	result
default	$-\frac{a\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(ax-i)\sqrt{c\,(ax-i)\,(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c\,(ax-i)\,(ax+i)}\,(ax+i)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-
I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(ar
ctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/(a^2*x^2+1)/c^2-arctan(a
*x)^3*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x+3*a*(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(
a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/
2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c
^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^3(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.450 $\int \frac{x^5 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=534

$$\frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}}+\frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}}-\frac{2x^2\text{ArcTan}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}}-\frac{94\text{ArcTan}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}-\frac{x^3\text{ArcTan}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}}-\frac{5x}{a^5c}$$

```
[Out] 2/27*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)-2/9*x^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^
(3/2)-1/3*x^3*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^2*arctan(a*x)^3
/a^4/c/(a^2*c*x^2+c)^(3/2)+94/9*x/a^5/c^2/(a^2*c*x^2+c)^(1/2)-94/9*arctan(a
*x)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-5*x*arctan(a*x)^2/a^5/c^2/(a^2*c*x^2+c)^(1/
2)+5/3*arctan(a*x)^3/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan((1+I*a*x)/(a^2*
x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-6
*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*polylog(3,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-6*po
lylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2
+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^6/c^3
```

Rubi [A]
time = 0.77, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5084, 5050, 5010, 5008, 4266, 2611, 2320, 6724, 5018, 197, 5060, 5058}

$$\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{a^6c^3}-\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(ie^{i\text{ArcTan}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(-ie^{i\text{ArcTan}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{Li}_3\left(ie^{i\text{ArcTan}(ax)}\right)}{a^6c^2\sqrt{a^2cx^2+c}}+\frac{5\text{ArcTan}(ax)^3}{3a^6c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{ArcTan}\left(e^{i\text{ArcTan}(ax)}\right)\text{ArcTan}(ax)^2}{a^6c^2\sqrt{a^2cx^2+c}}-\frac{94\text{ArcTan}(ax)}{9a^6c^2\sqrt{a^2cx^2+c}}-\frac{5x\text{ArcTan}(ax)^2}{a^5c^2\sqrt{a^2cx^2+c}}+\frac{94x}{9a^5c^2\sqrt{a^2cx^2+c}}+\frac{x^2\text{ArcTan}(ax)^3}{3a^4c\,(a^2cx^2+c)^{3/2}}-\frac{2x^2\text{ArcTan}(ax)}{9a^4c\,(a^2cx^2+c)^{3/2}}-\frac{x^3\text{ArcTan}(ax)^2}{3a^3c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3}{27a^3c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*x^3)/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) + (94*x)/(9*a^5*c^2*Sqrt[c + a^2*c
*x^2]) - (2*x^2*ArcTan[a*x])/(9*a^4*c*(c + a^2*c*x^2)^(3/2)) - (94*ArcTan[a
*x])/(9*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan[a*x]^2)/(3*a^3*c*(c + a^
2*c*x^2)^(3/2)) - (5*x*ArcTan[a*x]^2)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2)/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^3)/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + (5*
ArcTan[a*x]^3)/(3*a^6*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^3)/(a^6*c^3) - ((6*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2
]*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[c + a^2*c*x^2]
) + (6*Sqrt[1 + a^2*x^2]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])])/(a^6*c^2*Sqrt[
c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^
6*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
```

```
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\int \frac{x^3\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^3\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} + \frac{x^2\tan^{-1}(ax)^3}{3a^4c\,(c+a^2cx^2)^{3/2}} + \frac{2\int \frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx}{3a^2} + \frac{\int \frac{x\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{x^3\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} + \frac{x^2\tan^{-1}(ax)^3}{3a^4c\,(c+a^2cx^2)^{3/2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{2x^2\tan^{-1}(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}} - \frac{94\tan^{-1}(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}$$

Mathematica [**A**]
time = 1.75, size = 367, normalized size = 0.69

$$-\frac{(1+a^2x^2)^2\left(1134\text{ArcTan}(ax) - 405\text{ArcTan}(ax)^3 + 1128\text{ArcTan}(ax)\cos(2\text{ArcTan}(ax)) - 180\text{ArcTan}(ax)^3\cos(2\text{ArcTan}(ax)) - 6\text{ArcTan}(ax)\cos(4\text{ArcTan}(ax)) + 9\text{ArcTan}(ax)^3\cos(4\text{ArcTan}(ax)) + \frac{648\text{ArcTan}(ax)^2\log\left(1-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} - \frac{648\text{ArcTan}(ax)^2\log\left(1+ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} + \frac{1296i\text{ArcTan}(ax)\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} - \frac{1296i\text{ArcTan}(ax)\text{PolyLog}\left(2,ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} - \frac{1296\text{PolyLog}\left(3,-ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} + \frac{1296\text{PolyLog}\left(3,ie^{i\text{ArcTan}(ax)}\right)}{\sqrt{1+a^2x^2}} - 1132\sin(2\text{ArcTan}(ax)) + 558\text{ArcTan}(ax)^2\sin(2\text{ArcTan}(ax)) + 2\sin(4\text{ArcTan}(ax)) - 9\text{ArcTan}(ax)^2\sin(4\text{ArcTan}(ax))\right)}{216a^6c\,(c\,(1+a^2x^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/216*((1 + a^2*x^2)^2*(1134*ArcTan[a*x] - 405*ArcTan[a*x]^3 + 1128*ArcTan
[a*x]*Cos[2*ArcTan[a*x]] - 180*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 6*ArcTan[
a*x]*Cos[4*ArcTan[a*x]] + 9*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + (648*ArcTan[
a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (648*ArcTan[a*x]^2
```

```
*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((1296*I)*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((1296*I)*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (1296*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (1296*PolyLog[3, I*E^(I*ArcTan[a*x
])])/Sqrt[1 + a^2*x^2] - 1132*Sin[2*ArcTan[a*x]] + 558*ArcTan[a*x]^2*Sin[2*
ArcTan[a*x]] + 2*Sin[4*ArcTan[a*x]] - 9*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/
(a^6*c*(c*(1 + a^2*x^2))^(3/2))
```

Maple [F]
time = 10.27, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \arctan \left(a x\right)^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
[Out] int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \operatorname{atan}^{3}{\left(a x \right)}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\mathrm{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.451 $\int \frac{x^4 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$

Optimal. Leaf size=622

$$-\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}}+\frac{68}{9a^5c^2\sqrt{c+a^2cx^2}}+\frac{2x^3\mathrm{ArcTan}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{22x\mathrm{ArcTan}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}-\frac{x^2\mathrm{ArcTan}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}}-$$

```
[Out] -2/27/a^5/c/(a^2*c*x^2+c)^(3/2)+2/9*x^3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/
2)-1/3*x^2*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)-1/3*x^3*arctan(a*x)^3/a^
2/c/(a^2*c*x^2+c)^(3/2)+68/9/a^5/c^2/(a^2*c*x^2+c)^(1/2)+22/3*x*arctan(a*x)
/a^4/c^2/(a^2*c*x^2+c)^(1/2)-11/3*arctan(a*x)^2/a^5/c^2/(a^2*c*x^2+c)^(1/2)
-x*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+
1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+3*I*a
rctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
5/c^2/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)*polyl
og(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c
)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5084, 5010, 5008, 4266, 2611, 6744, 2320, 6724, 5018, 5014, 5064, 5060, 5050, 272, 45}

$$-\frac{x^3\mathrm{ArcTan}(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\mathrm{ArcTan}(ax)}{9a^2c\,(a^2cx^2+c)^{3/2}}+\frac{3i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)^2\mathrm{Li}_2\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\mathrm{ArcTan}(ax)\mathrm{Li}_3\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\mathrm{Li}_4\left(-ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\mathrm{Li}_4\left(ie^{i\mathrm{ArcTan}(ax)}\right)}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}\,\mathrm{ArcTan}\left(e^{i\mathrm{ArcTan}(ax)}\right)\mathrm{ArcTan}(ax)^3}{a^5c^2\sqrt{a^2cx^2+c}}-\frac{11\mathrm{ArcTan}(ax)^2}{3a^5c^2\sqrt{a^2cx^2+c}}+\frac{68}{9a^5c^2\sqrt{a^2cx^2+c}}-\frac{2}{27a^5c\,(a^2cx^2+c)^{3/2}}-\frac{x\mathrm{ArcTan}(ax)^3}{a^4c^2\sqrt{a^2cx^2+c}}+\frac{22x\mathrm{ArcTan}(ax)}{3a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^2\mathrm{ArcTan}(ax)^2}{3a^3c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -2/(27*a^5*c*(c + a^2*c*x^2)^(3/2)) + 68/(9*a^5*c^2*Sqrt[c + a^2*c*x^2]) +
(2*x^3*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (22*x*ArcTan[a*x])/(3
*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^2)/(3*a^3*c*(c + a^2*c*x^2
)^(3/2)) - (11*ArcTan[a*x]^2)/(3*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (x^3*ArcTan
[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (x*ArcTan[a*x]^3)/(a^4*c^2*Sqrt[
c + a^2*c*x^2]) - ((2*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^3)/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*
x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) - ((
3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])])/(a^5*
c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, (-I)
*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt[1 + a^2*x^2]*A
```

```
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt[c + a^2*c*x^2]) -
 ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, (-I)*E^(I*ArcTan[a*x])])/(a^5*c^2*Sqrt
[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, I*E^(I*ArcTan[a*x])]
)/(a^5*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
```

[d, 0]

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
```

```
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\int \frac{x^2\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx}{a^2} + \frac{\int \frac{x^2\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{a^2c}$$

$$= -\frac{x^3\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\int \frac{x^3\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{a} + \frac{\int \frac{\tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx}{a^4c^2} - \frac{\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{a^4c}$$

$$= \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} - \frac{3\tan^{-1}(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}$$

$$= \frac{6}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{6x\tan^{-1}(ax)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} -$$

$$= \frac{22}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}}$$

$$= -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx}}$$

$$= -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx}}$$

$$= -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx}}$$

$$= -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\tan^{-1}(ax)}{3a^4c^2\sqrt{c+a^2cx}}$$

Mathematica [A]
time = 1.71, size = 691, normalized size = 1.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/1728*(Sqrt[c*(1 + a^2*x^2)]*((189*I)*Pi^4 - 12960/Sqrt[1 + a^2*x^2] + (2
16*I)*Pi^3*ArcTan[a*x] - (12960*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (648*I
)*Pi^2*ArcTan[a*x]^2 + (6480*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (864*I)*Pi*
ArcTan[a*x]^3 + (2160*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (432*I)*ArcTan
[a*x]^4 + 32*Cos[3*ArcTan[a*x]] - 144*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] - 12
96*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 2592*Pi*ArcTan[a*x]^2*Lo
```

```
g[1 - I/E^(I*ArcTan[a*x])] + 216*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 1728*A
rcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] - 216*Pi^3*Log[1 + I*E^(I*ArcTan[
a*x])] + 1296*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2592*Pi*ArcTa
n[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 1728*ArcTan[a*x]^3*Log[1 + I*E^(I*A
rcTan[a*x])] - 216*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (5184*I)*ArcTan[
a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - (1296*I)*Pi*(Pi - 4*ArcTan[a*x]
)*PolyLog[2, I/E^(I*ArcTan[a*x])] - (1296*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] + (5184*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (5
184*I)*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 10368*ArcTan[a*x]
*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 5184*Pi*PolyLog[3, I/E^(I*ArcTan[a*x]
)] - 5184*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 10368*ArcTan[a*x]*PolyLog
[3, (-I)*E^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)/E^(I*ArcTan[a*x])]
+ (10368*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + 96*ArcTan[a*x]*Sin[3*ArcTa
n[a*x]] - 144*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]]))/(a^5*c^3*Sqrt[1 + a^2*x^2]
)
```

Maple [**F**]
time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{x^4 \arctan(ax)^3}{(a^2 c x^2 + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
[Out] int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**4*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \operatorname{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.452 $\int \frac{x^3 \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$

Optimal. Leaf size=237

$$-\frac{2x^3}{27ac\left(c+a^2cx^2\right)^{3/2}}-\frac{40x}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{2x^2\text{ArcTan}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{40\text{ArcTan}(ax)}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{x^3\text{ArcTan}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\text{A}}{a^3c^2}$$

```
[Out] -2/27*x^3/a/c/(a^2*c*x^2+c)^(3/2)+2/9*x^2*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(
3/2)+1/3*x^3*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)^3/a^
2/c/(a^2*c*x^2+c)^(3/2)-40/9*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)+40/9*arctan(a*x)
/a^4/c^2/(a^2*c*x^2+c)^(1/2)+2*x*arctan(a*x)^2/a^3/c^2/(a^2*c*x^2+c)^(1/2)-
2/3*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5060, 5050, 5018, 197, 5058}

$$-\frac{x^2\text{ArcTan}(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^2\text{ArcTan}(ax)}{9a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\text{ArcTan}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2x^3}{27ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2\text{ArcTan}(ax)^3}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{40\text{ArcTan}(ax)}{9a^4c^2\sqrt{a^2cx^2+c}}+\frac{2x\text{ArcTan}(ax)^2}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{40x}{9a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x^3)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(9*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (2*x^2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (40*ArcTan[a*
x])/(9*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c
*x^2)^(3/2)) + (2*x*ArcTan[a*x]^2)/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*Arc
Tan[a*x]^3)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^3)/(3*a^4*c^2*
Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5058

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A
rcTan[c*x])/(c^2*d*m)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{x^3\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2}{3}\int\frac{x^3\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx+\frac{2\int\frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,d\ldots}{3a^2c}\\ &= -\frac{2x^3}{27ac\,(c+a^2cx^2)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{x^3\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}-\frac{x^2\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{\ldots}}\\ &= -\frac{2x^3}{27ac\,(c+a^2cx^2)^{3/2}}+\frac{2x^2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{x^3\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{\ldots}}\\ &= -\frac{2x^3}{27ac\,(c+a^2cx^2)^{3/2}}-\frac{40x}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{2x^2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{40\tan^{-1}(ax)}{9a^4c^2\sqrt{c+a^2c\ldots}}\end{aligned}$$

Mathematica [A]

time = 0.10, size = 104, normalized size = 0.44

$$\frac{\sqrt{c+a^2cx^2}\,(-2ax(60+61a^2x^2)+6(20+21a^2x^2)\,\mathrm{ArcTan}(ax)+9ax(6+7a^2x^2)\,\mathrm{ArcTan}(ax)^2-9(2+3a^2x^2)\,\mathrm{ArcTan}(ax)^3)}{27a^4c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]

[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(60 + 61*a^2*x^2) + 6*(20 + 21*a^2*x^2)*ArcTan[a*x] + 9*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]^2 - 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^3))/(27*a^4*c^3*(1 + a^2*x^2)^2)

Maple [C] Result contains complex when optimal does not.
time = 7.63, size = 312, normalized size = 1.32

method	result
default	$-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216\left(a^2x^2+1\right)^2a^4c^3}-\frac{3\left(\arctan(ax)^3-6\,\mathrm{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^4/(a^2*x^2+1)+1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)

Fricas [A]
time = 9.42, size = 113, normalized size = 0.48

$$-\frac{\left(122\,a^3x^3+9\,(3\,a^2x^2+2)\arctan\,(ax)^3-9\,(7\,a^3x^3+6\,ax)\arctan\,(ax)^2+120\,ax-6\,(21\,a^2x^2+20)\arctan\,(ax)\right)\sqrt{a^2cx^2+c}}{27\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] -1/27*(122*a^3*x^3 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 9*(7*a^3*x^3 + 6*a*x)*arctan(a*x)^2 + 120*a*x - 6*(21*a^2*x^2 + 20)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^3}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.453 $\int \frac{x^2 \text{ArcTan}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{2}{27a^3c\,(c+a^2cx^2)^{3/2}}-\frac{14}{9a^3c^2\sqrt{c+a^2cx^2}}-\frac{2x^3\text{ArcTan}(ax)}{9c\,(c+a^2cx^2)^{3/2}}-\frac{4x\text{ArcTan}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}}+\frac{x^2\text{ArcTan}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}+\frac{2\text{ArcT}}{3a^3c^2\sqrt{}}$$

```
[Out] 2/27/a^3/c/(a^2*c*x^2+c)^(3/2)-2/9*x^3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/
3*x^2*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)^3/c/(a^2*c*
x^2+c)^(3/2)-14/9/a^3/c^2/(a^2*c*x^2+c)^(1/2)-4/3*x*arctan(a*x)/a^2/c^2/(a^
2*c*x^2+c)^(1/2)+2/3*arctan(a*x)^2/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5064, 5060, 5050, 5014, 272, 45}

$$-\frac{4x\text{ArcTan}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}+\frac{x^2\text{ArcTan}(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}+\frac{x^3\text{ArcTan}(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-\frac{2x^3\text{ArcTan}(ax)}{9c\,(a^2cx^2+c)^{3/2}}+\frac{2\text{ArcTan}(ax)^2}{3a^3c^2\sqrt{a^2cx^2+c}}-\frac{14}{9a^3c^2\sqrt{a^2cx^2+c}}+\frac{2}{27a^3c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] 2/(27*a^3*c*(c + a^2*c*x^2)^(3/2)) - 14/(9*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (
2*x^3*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (4*x*ArcTan[a*x])/(3*a^2*c
^2*Sqrt[c + a^2*c*x^2]) + (x^2*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2))
 + (2*ArcTan[a*x]^2)/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^3)/
(3*c*(c + a^2*c*x^2)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
```

```
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3 \tan^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} - a\int \frac{x^3 \tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{x^3 \tan^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{1}{9}(2a)\int \frac{x^3}{(c+a^2cx^2)^{5/2}}$$

$$= -\frac{2x^3 \tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} + \frac{x^2 \tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{2\tan^{-1}(ax)^2}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \tan^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} + \frac{1}{9}$$

$$= -\frac{4}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} - \frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x^2 \tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} +$$

$$= \frac{2}{27a^3c\,(c+a^2cx^2)^{3/2}} - \frac{14}{9a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3 \tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} - \frac{4x\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} +$$

Mathematica [A]
time = 0.08, size = 95, normalized size = 0.48

$$\frac{\sqrt{c+a^2cx^2}\,(-40-42a^2x^2-6ax(6+7a^2x^2)\,\mathrm{ArcTan}(ax)+9(2+3a^2x^2)\,\mathrm{ArcTan}(ax)^2+9a^3x^3\mathrm{ArcTan}(ax)^3)}{27a^3c^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-40 - 42*a^2*x^2 - 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
+ 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2 + 9*a^3*x^3*ArcTan[a*x]^3))/(27*a^3*c^3*(
1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 5.59, size = 308, normalized size = 1.55

method	result
default	$\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2c^3a^3} + \frac{\left(\arctan(ax)^3-6\arctan(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a^
2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3/a^3+1/8*(arctan(
a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/
2)/a^3/c^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^3
```

```
-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/a^3/c^3/(a^2*x^2+1)+1/216*(-9*I*arcta
n(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*
x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 4.98, size = 106, normalized size = 0.53

$$\frac{\left(9\,a^3x^3\arctan\left(ax\right)^3-42\,a^2x^2+9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^2-6\left(7\,a^3x^3+6\,ax\right)\arctan\left(ax\right)-40\right)\sqrt{a^2cx^2+c}}{27\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/27*(9*a^3*x^3*arctan(a*x)^3 - 42*a^2*x^2 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^
2 - 6*(7*a^3*x^3 + 6*a*x)*arctan(a*x) - 40)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^
4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\operatorname{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.454 $\int \frac{x\textbf{ArcTan}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{2x}{27ac\left(c+a^2cx^2\right)^{3/2}}-\frac{40x}{27ac^2\sqrt{c+a^2cx^2}}+\frac{2\text{ArcTan}(ax)}{9a^2c\left(c+a^2cx^2\right)^{3/2}}+\frac{4\text{ArcTan}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2x}{3a}$$

```
[Out] -2/27*x/a/c/(a^2*c*x^2+c)^(3/2)+2/9*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+1
/3*x*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)^3/a^2/c/(a^2*c*x
^2+c)^(3/2)-40/27*x/a/c^2/(a^2*c*x^2+c)^(1/2)+4/3*arctan(a*x)/a^2/c^2/(a^2*
c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^2/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5050, 5020, 5018, 197, 198}

$$\frac{2x\text{ArcTan}(ax)^2}{3ac^2\sqrt{a^2cx^2+c}}+\frac{4\text{ArcTan}(ax)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x\text{ArcTan}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\text{ArcTan}(ax)}{9a^2c\left(a^2cx^2+c\right)^{3/2}}-\frac{40x}{27ac^2\sqrt{a^2cx^2+c}}-\frac{2x}{27ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*x)/(27*a*c*(c + a^2*c*x^2)^(3/2)) - (40*x)/(27*a*c^2*Sqrt[c + a^2*c*x^2
]) + (2*ArcTan[a*x])/(9*a^2*c*(c + a^2*c*x^2)^(3/2)) + (4*ArcTan[a*x])/(3*a
^2*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2
)) + (2*x*ArcTan[a*x]^2)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^3/(3*a
^2*c*(c + a^2*c*x^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
```

```
x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}+\frac{\int \frac{\tan^{-1}(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx}{a}\\ &= \frac{2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{x\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}-\frac{\tan^{-1}(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\int \frac{1}{(c+a^2cx^2)^{5/2}}\,dx}{9a}+\\ &= -\frac{2x}{27ac\,(c+a^2cx^2)^{3/2}}+\frac{2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}}+\frac{x\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}\\ &= -\frac{2x}{27ac\,(c+a^2cx^2)^{3/2}}-\frac{40x}{27ac^2\sqrt{c+a^2cx^2}}+\frac{2\tan^{-1}(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{4\tan^{-1}(ax)}{3a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 91, normalized size = 0.46

$$\frac{\sqrt{c+a^2cx^2}\,\left(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\operatorname{ArcTan}(ax)+9ax(3+2a^2x^2)\operatorname{ArcTan}(ax)^2-9\operatorname{ArcTan}(ax)^3\right)}{27c^3\,(a+a^3x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

[Out] (Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[a*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2 - 9*ArcTan[a*x]^3))/(27*c^3*(a + a^3*x^2)^2)

Maple [C] Result contains complex when optimal does not.
time = 2.48, size = 312, normalized size = 1.57

method	result
default	$\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216\left(a^2x^2+1\right)^2a^2c^3}-\frac{\left(\arctan(ax)^3-6\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^2/(a^2*x^2+1)-1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)

Fricas [A]
time = 6.95, size = 103, normalized size = 0.52

$$-\frac{\left(40\,a^3x^3-9\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^2+9\,\arctan\left(ax\right)^3+42\,ax-6\left(6\,a^2x^2+7\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{27\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] -1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 9*arctan(a*x)^3 + 42*a*x - 6*(6*a^2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^3 (ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\, \operatorname{atan}(a\, x)^3}{{\left(c\, a^2\, x^2+c\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.455 $\int \frac{\text{ArcTan}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{2}{27ac\left(c+a^2cx^2\right)^{3/2}}-\frac{40}{9ac^2\sqrt{c+a^2cx^2}}-\frac{2x\text{ArcTan}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{40x\text{ArcTan}(ax)}{9c^2\sqrt{c+a^2cx^2}}+\frac{\text{ArcTan}(ax)^2}{3ac\left(c+a^2cx^2\right)^{3/2}}+\frac{2\text{ArcTa}}{ac^2\sqrt{c-}}$$

```
[Out] -2/27/a/c/(a^2*c*x^2+c)^(3/2)-2/9*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/3*a
rctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3
/2)-40/9/a/c^2/(a^2*c*x^2+c)^(1/2)-40/9*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/
2)+2*arctan(a*x)^2/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^3/c^2/(a^2*c
*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5020, 5018, 5014, 5016}

$$\frac{2x\text{ArcTan}(ax)^3}{3c^2\sqrt{a^2cx^2+c}}+\frac{2\text{ArcTan}(ax)^2}{ac^2\sqrt{a^2cx^2+c}}-\frac{40x\text{ArcTan}(ax)}{9c^2\sqrt{a^2cx^2+c}}+\frac{x\text{ArcTan}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\text{ArcTan}(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2x\text{ArcTan}(ax)}{9c\left(a^2cx^2+c\right)^{3/2}}-\frac{40}{9ac^2\sqrt{a^2cx^2+c}}-\frac{2}{27ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -2/(27*a*c*(c + a^2*c*x^2)^(3/2)) - 40/(9*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*x
*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (40*x*ArcTan[a*x])/(9*c^2*Sqrt[
c + a^2*c*x^2]) + ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (2*ArcTan[a
*x]^2)/(a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)
^(3/2)) + (2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{x\tan^{-1}(ax)^3}{3c\,(c+a^2cx^2)^{3/2}} - \frac{2}{3}\int \frac{\tan^{-1}(ax)}{(c+a^2cx^2)^{5/2}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx}{3c}$$

$$= -\frac{2}{27ac\,(c+a^2cx^2)^{3/2}} - \frac{2x\tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} + \frac{\tan^{-1}(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{2\tan^{-1}(ax)^2}{ac^2\sqrt{c+a^2cx^2}} + \cdots$$

$$= -\frac{2}{27ac\,(c+a^2cx^2)^{3/2}} - \frac{40}{9ac^2\sqrt{c+a^2cx^2}} - \frac{2x\tan^{-1}(ax)}{9c\,(c+a^2cx^2)^{3/2}} - \frac{40x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} + \cdots$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 0.48

$$\frac{\sqrt{c+a^2cx^2}\,\left(-2(61+60a^2x^2) - 6ax(21+20a^2x^2)\,\mathrm{ArcTan}(ax) + 9(7+6a^2x^2)\,\mathrm{ArcTan}(ax)^2 + 9ax(3+2a^2x^2)\,\mathrm{ArcTan}(ax)^3\right)}{27ac^3\,(1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(-2*(61 + 60*a^2*x^2) - 6*a*x*(21 + 20*a^2*x^2)*ArcTan
[a*x] + 9*(7 + 6*a^2*x^2)*ArcTan[a*x]^2 + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]
^3))/(27*a*c^3*(1 + a^2*x^2)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.55, size = 308, normalized size = 1.43

method	result
default	$-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216(a^2x^2+1)^2a\,c^3}+\frac{3\left(\arctan(ax)^3-6\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a
^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a
*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2
)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^3-6*
arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a/(a^2*x^2+1)-1/216*(-9*I*arctan(a*x
)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x^3+3
*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 4.11, size = 111, normalized size = 0.52

$$-\frac{\sqrt{a^2cx^2+c}\,\left(120\,a^2x^2-9\,(2\,a^3x^3+3\,ax)\arctan\left(ax\right)^3-9\,(6\,a^2x^2+7)\arctan\left(ax\right)^2+6\,(20\,a^3x^3+21\,ax)\arctan\left(ax\right)+122\right)}{27\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*sqrt(a^2*c*x^2 + c)*(120*a^2*x^2 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^
3 - 9*(6*a^2*x^2 + 7)*arctan(a*x)^2 + 6*(20*a^3*x^3 + 21*a*x)*arctan(a*x) +
 122)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

3.456 $\int \frac{\textbf{ArcTan}(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=553

$$\frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{202ax}{27c^2\sqrt{c+a^2cx^2}}-\frac{2\text{ArcTan}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{22\text{ArcTan}(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{ax\text{ArcTan}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{11ax\text{ArcTan}}{3c^2\sqrt{c+a^2}}$$

```
[Out] 2/27*a*x/c/(a^2*c*x^2+c)^(3/2)-2/9*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1/3*a*
x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+1/3*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3/
2)+202/27*a*x/c^2/(a^2*c*x^2+c)^(1/2)-22/3*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1
/2)-11/3*a*x*arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/c^2/(a^2*c
*x^2+c)^(1/2)-2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^
2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c
)^(1/2)-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5086, 5078, 5076, 4268, 2611, 6744, 2320, 6724, 5050, 5018, 197, 5020, 198}

$$\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{6\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(-e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{6i\sqrt{a^2x^2+1}\,\text{Li}_4\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^3}{c^2\sqrt{a^2cx^2+c}}-\frac{11ax\text{ArcTan}(ax)^2}{3c^2\sqrt{a^2cx^2+c}}-\frac{22\text{ArcTan}(ax)}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^3\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{\text{ArcTan}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{ax\text{ArcTan}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2\text{ArcTan}(ax)}{9c\left(a^2cx^2+c\right)^{3/2}}+\frac{202ax}{27c^2\sqrt{a^2cx^2+c}}+\frac{2ax}{27c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a*x)/(27*c*(c + a^2*c*x^2)^(3/2)) + (202*a*x)/(27*c^2*Sqrt[c + a^2*c*x^2
]) - (2*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) - (22*ArcTan[a*x])/(3*c^2*
Sqrt[c + a^2*c*x^2]) - (a*x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (1
1*a*x*ArcTan[a*x]^2)/(3*c^2*Sqrt[c + a^2*c*x^2]) + ArcTan[a*x]^3/(3*c*(c +
a^2*c*x^2)^(3/2)) + ArcTan[a*x]^3/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a
^2*x^2]*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
 + ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])])/(
c^2*Sqrt[c + a^2*c*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*PolyLog[2
, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*
c*x^2]) - ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, -E^(I*ArcTan[a*x])])/(c^2*Sqr
```

```
t[c + a^2*c*x^2]) + ((6*I)*Sqrt[1 + a^2*x^2]*PolyLog[4, E^(I*ArcTan[a*x])])
/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} - a\int \frac{\tan^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx}{c^2} - \frac{a^2\int \frac{x\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,d}{c}$$

$$= -\frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\tan^{-1}(ax)^3}{c^2\sqrt{c+a^2cx^2}} + \frac{1}{9}(2a$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{11a}{3c^2}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax}{3c(}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax}{3c(}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax}{3c(}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax}{3c(}$$

$$= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{22\tan^{-1}(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{ax}{3c(}$$

Mathematica [A]
time = 0.54, size = 347, normalized size = 0.63

$$\frac{\left(1+a^2x^2\right)^{3/2}\left(-27i\pi^4+\frac{1620ax}{\sqrt{1+a^2x^2}}-\frac{1620\text{ArcTan}(ax)}{\sqrt{1+a^2x^2}}-\frac{810ax\text{ArcTan}(ax)^2}{\sqrt{1+a^2x^2}}+\frac{270\text{ArcTan}(ax)^3}{\sqrt{1+a^2x^2}}+54i\text{ArcTan}(ax)^4-12\text{ArcTan}(ax)\cos(3\text{ArcTan}(ax))+18\text{ArcTan}(ax)^3\cos(3\text{ArcTan}(ax))+216\text{ArcTan}(ax)^3\log\left(1-e^{-i\text{ArcTan}(ax)}\right)-216\text{ArcTan}(ax)^3\log\left(1+e^{i\text{ArcTan}(ax)}\right)+648i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,e^{-i\text{ArcTan}(ax)}\right)+648i\text{ArcTan}(ax)^2\text{PolyLog}\left(2,-e^{i\text{ArcTan}(ax)}\right)+1296\text{ArcTan}(ax)\text{PolyLog}\left(3,e^{-i\text{ArcTan}(ax)}\right)-1296\text{ArcTan}(ax)\text{PolyLog}\left(3,-e^{i\text{ArcTan}(ax)}\right)-1296i\text{PolyLog}\left(4,e^{-i\text{ArcTan}(ax)}\right)-1296i\text{PolyLog}\left(4,-e^{i\text{ArcTan}(ax)}\right)+4\sin(3\text{ArcTan}(ax))-18\text{ArcTan}(ax)^2\sin(3\text{ArcTan}(ax))\right)}{216c\left(c\left(1+a^2x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-27*I)*Pi^4 + (1620*a*x)/Sqrt[1 + a^2*x^2] - (1620*A
rcTan[a*x])/Sqrt[1 + a^2*x^2] - (810*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] +
 (270*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (54*I)*ArcTan[a*x]^4 - 12*ArcTan[a
*x]*Cos[3*ArcTan[a*x]] + 18*ArcTan[a*x]^3*Cos[3*ArcTan[a*x]] + 216*ArcTan[a
*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 216*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan
[a*x])] + (648*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (648*I)*
ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] + 1296*ArcTan[a*x]*PolyLog[3,
E^((-I)*ArcTan[a*x])] - 1296*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (
1296*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (1296*I)*PolyLog[4, -E^(I*ArcTan
[a*x])] + 4*Sin[3*ArcTan[a*x]] - 18*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(216
*c*(c*(1 + a^2*x^2))^(3/2))
```

Maple [A]
time = 2.27, size = 560, normalized size = 1.01

method	result
default	$-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{216\left(a^2x^2+1\right)^2c^3}+\frac{5\left(\arctan(ax)^3-6a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*a
^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a
*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1
/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^3-
6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*(I+a*
x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9*arctan(a*x)
^3+2*I-6*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)-I*(I*arctan(a*x)^3*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))
+3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*pol
ylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(
1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3
*a^2*c^3*x^3 + c^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^3(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.457 $\int \frac{\text{ArcTan}(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=493

$$\frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a}{9c^2\sqrt{c+a^2cx^2}}+\frac{2a^2x\text{ArcTan}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}+\frac{94a^2x\text{ArcTan}(ax)}{9c^2\sqrt{c+a^2cx^2}}-\frac{a\text{ArcTan}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a\text{ArcTan}(}{c^2\sqrt{c+a^2}}$$

```
[Out] 2/27*a/c/(a^2*c*x^2+c)^(3/2)+2/9*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1/
3*a*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-1/3*a^2*x*arctan(a*x)^3/c/(a^2*c*x^
2+c)^(3/2)+94/9*a/c^2/(a^2*c*x^2+c)^(1/2)+94/9*a^2*x*arctan(a*x)/c^2/(a^2*c
*x^2+c)^(1/2)-5*a*arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)-5/3*a^2*x*arctan(a*
x)^3/c^2/(a^2*c*x^2+c)^(1/2)-6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)
-6*I*a*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/c^2/(a^2*c*x^2+c)^(1/2)-6*a*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*a*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^3*(a^2*c*x^2+c)^(
1/2)/c^3/x
```

Rubi [A]
time = 0.65, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5086, 5064, 5078, 5076, 4268, 2611, 2320, 6724, 5018, 5014, 5020, 5016}

$$-\frac{\text{ArcTan}(ax)^3\sqrt{a^2cx^2+c}}{c^3x}+\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(-e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6ia\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)\text{Li}_2\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}+\frac{6a\sqrt{a^2x^2+1}\,\text{Li}_3\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{5a^2x\text{ArcTan}(ax)^3}{3c^2\sqrt{a^2cx^2+c}}-\frac{5a\text{ArcTan}(ax)^2}{c^2\sqrt{a^2cx^2+c}}+\frac{94a^2x\text{ArcTan}(ax)}{9c^2\sqrt{a^2cx^2+c}}-\frac{6a\sqrt{a^2x^2+1}\,\text{ArcTan}(ax)^2\tanh^{-1}\left(e^{i\text{ArcTan}(ax)}\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{a^2x\text{ArcTan}(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{a\text{ArcTan}(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2a^2x\text{ArcTan}(ax)}{9c\left(a^2cx^2+c\right)^{3/2}}+\frac{94a}{9c^2\sqrt{a^2cx^2+c}}+\frac{2a}{27c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*a)/(27*c*(c + a^2*c*x^2)^(3/2)) + (94*a)/(9*c^2*Sqrt[c + a^2*c*x^2]) + (
2*a^2*x*ArcTan[a*x])/(9*c*(c + a^2*c*x^2)^(3/2)) + (94*a^2*x*ArcTan[a*x])/(
9*c^2*Sqrt[c + a^2*c*x^2]) - (a*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2))
- (5*a*ArcTan[a*x]^2)/(c^2*Sqrt[c + a^2*c*x^2]) - (a^2*x*ArcTan[a*x]^3)/(3*
c*(c + a^2*c*x^2)^(3/2)) - (5*a^2*x*ArcTan[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c^3*x) - (6*a*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + ((6*I
)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/(c^2*Sqrt
[c + a^2*c*x^2]) - ((6*I)*a*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) - (6*a*Sqrt[1 + a^2*x^2]*PolyLog[3,
 -E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2]) + (6*a*Sqrt[1 + a^2*x^2]*Po
lyLog[3, E^(I*ArcTan[a*x])])/(c^2*Sqrt[c + a^2*c*x^2])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5014

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5016

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3)/(
2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[x*
(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5076

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTan
[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && G
tQ[d, 0]
```

Rule 5078

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[
c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5086

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*
x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p
, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q]
 && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = -\left(a^2\int \frac{\tan^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx\right) + \frac{\int \frac{\tan^{-1}(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx}{c}$$

$$= -\frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{a^2x\tan^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx + \frac{\int \frac{\tan^{-1}(\ldots}{x^2\sqrt{c+\ldots}}}{c^{\ldots}}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{a\tan^{-1}(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{5a\tan^{-1}(ax)^2}{c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\ldots}{3c(\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

$$= \frac{2a}{27c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a}{9c^2\sqrt{c+a^2cx^2}} + \frac{2a^2x\tan^{-1}(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} + \frac{94a^2x\tan^{-1}(ax)}{9c^2\sqrt{c+a^2cx^2}} - \frac{a\ldots}{3c\ldots}$$

Mathematica [A]
time = 1.66, size = 399, normalized size = 0.81

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/108*(a*(-1134 - 1134*a*x*ArcTan[a*x] + 567*ArcTan[a*x]^2 + 189*a*x*ArcTa
n[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^2*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 -
 324*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 324*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] - (648*I)*Sqrt[1 + a^
```

```
2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (648*I)*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 648*Sqrt[1 + a^2*x^2]*PolyLo
g[3, -E^(I*ArcTan[a*x])] - 648*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x
])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Sin[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x
^2]*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*T
an[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

Maple [A]
time = 2.17, size = 528, normalized size = 1.07

method	result
default	$\frac{a\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c\,(ax-i)\,(ax+i)}}{216\left(a^2x^2+1\right)^2c^3}-\frac{7a\left(\arctan(ax)^3-6\,\mathrm{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*a*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*
a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arctan(
a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/
2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^3-6*a
rctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*(I+a*x
))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-9*I*arctan(a*x)^2+9*arctan(a*x)^3+
2*I-6*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^3*(c*(a*x-I)*(I+
a*x))^(1/2)/c^3/x+3*a*(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arct
an(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,(1+I*
a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3
*a^2*c^3*x^4 + c^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{3}(ax)}{x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^3}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.458 $\int x^m(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^3,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx=\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3\,dx$$

Mathematica [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

Maple [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a
^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x)
^3 - 3*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a^
2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x)*
log(a^2*x^2 + 1)^2 + 32*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/32*(28*((a^6*
c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 + 3*(a^4
*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^2 + 23*
c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)*x^2 + 1
5*c^2)*x^m*arctan(a*x)^3 - 12*((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c^2)*x^5
+ 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a*c^2*m +
15*a*c^2)*x)*x^m*arctan(a*x)^2 + 12*((a^6*c^2*m^2 + 4*a^6*c^2*m + 3*a^6*c^2
)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5*a^4*c^2)*x^4 + (a^2*c^2*m^2 + 8*a^
2*c^2*m + 15*a^2*c^2)*x^2)*x^m*arctan(a*x)*log(a^2*x^2 + 1) + 3*(((a^6*c^2*
m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2
*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*
m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^
2)*x^m*arctan(a*x) + ((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c^2)*x^5 + 2*(a^3*
c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a*c^2*m + 15*a*c^2)
*x)*x^m)*log(a^2*x^2 + 1)^2)/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^
2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^m \operatorname{atan}^3(ax)\,dx + \int 2a^2x^2x^m \operatorname{atan}^3(ax)\,dx + \int a^4x^4x^m \operatorname{atan}^3(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**m*atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m*atan(a*x)*
*3, x) + Integral(a**4*x**4*x**m*atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.459 $\int x^m(c+a^2cx^2)\,\textbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx=\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^3\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.82, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^3 - 3*((a^2
*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)*log(a^2*x^2 + 1)^2 + 32*
(m^2 + 4*m + 3)*integrate(1/32*(28*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 +
 c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(
a*x)^3 - 12*((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m*arctan(a*x)^2 +
 12*((a^4*c*m + a^4*c)*x^4 + (a^2*c*m + 3*a^2*c)*x^2)*x^m*arctan(a*x)*log(a
^2*x^2 + 1) + 3*(((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*(a^2*c*
m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x) + ((a^3*c*m +
 a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m)*log(a^2*x^2 + 1)^2)/((a^2*m^2 + 4*a^2
*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{m}\operatorname{atan}^{3}(ax)\,dx+\int a^{2}x^{2}x^{m}\operatorname{atan}^{3}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**m*atan(a*x)**3, x) + Integral(a**2*x**2*x**m*atan(a*x)**3, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^3*(c + a^2*c*x^2),x)
```

```
[Out] int(x^m*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.460 $\int \frac{x^m \textbf{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^3}{c+a^2cx^2},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \tan^{-1}(ax)^3}{c+a^2cx^2}\,dx$$

Mathematica [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^3}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan\left(ax\right)^3}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\operatorname{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2),x)
[Out] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.461 $\int \frac{x^m \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x^m \mathrm{ArcTan}(ax)^3}{(c+a^2cx^2)^2},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \mathrm{ArcTan}(ax)^3}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \tan^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m \mathrm{ArcTan}(ax)^3}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^3}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

3.462 $\int x^m(c+a^2cx^2)^{3/2}\,\textbf{ArcTan}(ax)^3\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^3,x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3\,dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.18, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.463 $\int x^m \sqrt{c+a^2cx^2}\, \mathbf{ArcTan}(ax)^3\, dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^3, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^3\, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3\, dx = \int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3\, dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c+a^2cx^2}\, \text{ArcTan}(ax)^3\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

```
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.19, size = 0, normalized size = 0.00

$$\int x^m \arctan\left(ax\right)^3 \sqrt{a^2 c\, x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
[Out] int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.464 $\int \frac{x^m \text{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m\text{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\text{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^m\text{ArcTan}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 2.15, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan\left(ax\right)^3}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.465 $\int \frac{x^m \mathbf{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 2.17, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^3}{(a^2c\,x^2+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{atan}^{3}(ax)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.466 $\int \frac{x(c+a^2cx^2)}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx = \int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Maple [A]
time = 15.57, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x}{\operatorname{atan}\left(ax\right)}\, dx + \int \frac{a^{2}x^{3}}{\operatorname{atan}\left(ax\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(x/atan(a*x), x) + Integral(a**2*x**3/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x),x)
[Out] int((x*(c + a^2*c*x^2))/atan(a*x), x)
```

3.467 $\int \frac{c+a^2cx^2}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=20

$$\operatorname{Int}\left(\frac{c+a^2cx^2}{\operatorname{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x],x]
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x],x]
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x], x]
```

Maple [A]
time = 14.79, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x), x) + Integral(1/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2+c}{\operatorname{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)/atan(a*x), x)
```

3.468 $\int \frac{c+a^2cx^2}{x\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{c+a^2cx^2}{x\operatorname{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

Maple [A]
time = 27.67, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x),x)
[Out] int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{1}{x\operatorname{atan}(ax)}\,dx + \int \frac{a^2x}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x),x)
[Out] c*(Integral(1/(x*atan(a*x)), x) + Integral(a**2*x/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\operatorname{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)),x)
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)), x)
```

3.469 $\int \frac{x(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)},x\right)$$

[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x),x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx = \int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]

[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]

Maple [A]
time = 32.19, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{x}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{2a^{2}x^{3}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{a^{4}x^{5}}{\operatorname{atan}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(x/atan(a*x), x) + Integral(2*a**2*x**3/atan(a*x), x) + Integ
ral(a**4*x**5/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x),x)
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

3.470 $\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)} dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)} dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)} dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

Maple [A]
time = 25.16, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x), x) + Integral(a**4*x**4/atan(a*x), x)
 + Integral(1/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^2/atan(a*x), x)
```

3.471 $\int \frac{(c+a^2cx^2)^2}{x\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

Maple [A]
time = 57.40, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{1}{x\operatorname{atan}(ax)}\,dx+\int\frac{2a^2x}{\operatorname{atan}(ax)}\,dx+\int\frac{a^4x^3}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)), x) + Integral(2*a**2*x/atan(a*x), x) + Inte
gral(a**4*x**3/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)),x)
```

```
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)), x)
```

3.472 $\int \frac{x(c+a^2cx^2)^3}{\textbf{ArcTan}(ax)} \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)} \, dx = \int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

Maple [A]
time = 59.75, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{3}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{5}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{7}}{\operatorname{atan}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(x/atan(a*x), x) + Integral(3*a**2*x**3/atan(a*x), x) + Integ
ral(3*a**4*x**5/atan(a*x), x) + Integral(a**6*x**7/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^3}{\mathrm{atan}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x),x)
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

3.473 $\int \frac{(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

Maple [A]
time = 45.91, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{4}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{6}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{1}{\operatorname{atan}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x), x) + Integral(3*a**4*x**4/atan(a*x),
x) + Integral(a**6*x**6/atan(a*x), x) + Integral(1/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x), x)
```

3.474 $\int \frac{(c+a^2cx^2)^3}{x\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

Maple [A]
time = 107.76, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{1}{x\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^5}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)), x) + Integral(3*a**2*x/atan(a*x), x) + Inte
gral(3*a**4*x**3/atan(a*x), x) + Integral(a**6*x**5/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)),x)
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)), x)
```

3.475 $\int \frac{x^2}{(c+a^2cx^2)\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx = \int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A]
time = 3.81, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\mathrm{atan}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)*(c + a^2*c*x^2)),x)
[Out] int(x^2/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.476 $\int \frac{x}{(c+a^2cx^2)\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Defer[Int][x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx = \int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A]
time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^2x^2 \operatorname{atan}(ax) + \operatorname{atan}(ax)}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.477 $\int \frac{1}{(c+a^2cx^2)\,\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=12

$$\frac{\log(\text{ArcTan}(ax))}{ac}$$

```
[Out] ln(arctan(a*x))/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5002}

$$\frac{\log(\text{ArcTan}(ax))}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Log[ArcTan[a*x]]/(a*c)
```

Rule 5002

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx = \frac{\log\left(\tan^{-1}(ax)\right)}{ac}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{\log(\text{ArcTan}(ax))}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

```
[Out] Log[ArcTan[a*x]]/(a*c)
```

Maple [A]
time = 0.66, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(\arctan(ax))}{ac}$	13
default	$\frac{\ln(\arctan(ax))}{ac}$	13
risch	$\frac{\ln(-\ln(-iax+1)+\ln(iax+1))}{ca}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(arctan(a*x))/a/c
```

Maxima [A]
time = 0.26, size = 15, normalized size = 1.25

$$\frac{\log(2|\arctan(ax)|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] log(2*abs(arctan(a*x)))/(a*c)
```

Fricas [A]
time = 5.83, size = 12, normalized size = 1.00

$$\frac{\log(\arctan(ax))}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] log(arctan(a*x))/(a*c)
```

Sympy [A]
time = 0.26, size = 8, normalized size = 0.67

$$\frac{\log(\operatorname{atan}(ax))}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] log(atan(a*x))/(a*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.09, size = 12, normalized size = 1.00

$$\frac{\ln\left(\mathrm{atan}(a\,x)\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)),x)
```

```
[Out] log(atan(a*x))/(a*c)
```

3.478 $\int \frac{1}{x(c+a^2cx^2)\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A]
time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)\arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^3 \operatorname{atan}(ax)+x \operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x) + x*atan(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)),x)
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)), x)
```

3.479 $\int \frac{1}{x^2(c+a^2cx^2)\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A]
time = 4.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^4 \operatorname{atan}(ax)+x^2 \operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x),x)
[Out] Integral(1/(a**2*x**4*atan(a*x) + x**2*atan(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{atan}(a x) \left(c a^2 x^2 + c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)),x)
[Out] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)), x)
```

3.480 $\int \frac{x^4}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^2 \text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^2 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Defer[Int][x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx = \int \frac{x^4}{(c+a^2cx^2)^2 \tan^{-1}(ax)} dx$$

Mathematica [A]
time = 2.83, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c+a^2cx^2)^2 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A]
time = 4.27, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2c\,x^2+c)^2 \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**4/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.481 $\int \frac{x^3}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\,\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^2\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Defer[Int][x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 5.33, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}(ax) + 2a^2x^2 \operatorname{atan}(ax) + \operatorname{atan}(ax)}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.482 $\int \frac{x^2}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=33

$$-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2a^3c^2}+\frac{\log(\text{ArcTan}(ax))}{2a^3c^2}$$

```
[Out] -1/2*Ci(2*arctan(a*x))/a^3/c^2+1/2*ln(arctan(a*x))/a^3/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5090, 3393, 3383}

$$\frac{\log(\text{ArcTan}(ax))}{2a^3c^2}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] -1/2*CosIntegral[2*ArcTan[a*x]]/(a^3*c^2) + Log[ArcTan[a*x]]/(2*a^3*c^2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$
$$= \frac{\log\left(\tan^{-1}(ax)\right)}{2a^3c^2} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2}$$
$$= -\frac{\text{Ci}\left(2\tan^{-1}(ax)\right)}{2a^3c^2} + \frac{\log\left(\tan^{-1}(ax)\right)}{2a^3c^2}$$

Mathematica [A]
time = 0.05, size = 25, normalized size = 0.76

$$\frac{-\text{CosIntegral}(2\text{ArcTan}(ax)) + \log(\text{ArcTan}(ax))}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] (-CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a^3*c^2)
```

Maple [A]
time = 2.02, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{\ln(\arctan(ax))}{2c^2}-\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^2}}{a^3}$	28
default	$\frac{\frac{\ln(\arctan(ax))}{2c^2}-\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^2}}{a^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/2*ln(arctan(a*x))/c^2-1/2*Ci(2*arctan(a*x))/c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.97, size = 74, normalized size = 2.24

$$\frac{2 \log \left(\arctan \left(ax\right)\right) - \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(arctan(a*x)) - log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
1)) - log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}\left(ax\right)+2a^2x^2 \operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.483 $\int \frac{x}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=17

$$\frac{\text{Si}(2\text{ArcTan}(ax))}{2a^2c^2}$$

```
[Out] 1/2*Si(2*arctan(a*x))/a^2/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5090, 4491, 12, 3380}

$$\frac{\text{Si}(2\text{ArcTan}(ax))}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2}$$
$$= \frac{\text{Si}(2\tan^{-1}(ax))}{2a^2c^2}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 1.00

$$\frac{\text{Si}(2\text{ArcTan}(ax))}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

Maple [A]
time = 2.00, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\text{sinIntegral}(2\arctan(ax))}{2a^2c^2}$	16
default	$\frac{\text{sinIntegral}(2\arctan(ax))}{2a^2c^2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*Si(2*arctan(a*x))/a^2/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.27, size = 67, normalized size = 3.94

$$\frac{i\ \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - i\ \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(I*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - I*log_integra
l(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\,\text{atan}\,(ax)+2a^2x^2\,\text{atan}\,(ax)+\text{atan}\,(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{\text{atan}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.484 $\int \frac{1}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=33

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2ac^2} + \frac{\log(\text{ArcTan}(ax))}{2ac^2}$$

```
[Out] 1/2*Ci(2*arctan(a*x))/a/c^2+1/2*ln(arctan(a*x))/a/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5024, 3393, 3383}

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2ac^2} + \frac{\log(\text{ArcTan}(ax))}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^2) + Log[ArcTan[a*x]]/(2*a*c^2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cos(2x)}{2x}\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$
$$= \frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{2ac^2}$$
$$= \frac{\text{Ci}(2\tan^{-1}(ax))}{2ac^2} + \frac{\log\left(\tan^{-1}(ax)\right)}{2ac^2}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.70

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax)) + \log(\text{ArcTan}(ax))}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] (CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a*c^2)
```

Maple [A]
time = 2.02, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{\ln(\arctan(ax))}{2c^2}+\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^2}}{a}$	28
default	$\frac{\frac{\ln(\arctan(ax))}{2c^2}+\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^2}}{a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*ln(arctan(a*x))/c^2+1/2*Ci(2*arctan(a*x))/c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

[Out] integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)

Fricas [C] Result contains complex when optimal does not.
time = 1.97, size = 70, normalized size = 2.12

$$\frac{2 \log \left(\arctan \left(ax\right)\right) + \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) + \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(arctan(a*x)) + log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
1)) + log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{atan}\left(ax\right)+2a^2x^2\operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.485 $\int \frac{1}{x(c+a^2cx^2)^2 \text{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)} dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A]
time = 3.90, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{5}\operatorname{atan}(ax)+2a^{2}x^{3}\operatorname{atan}(ax)+x\operatorname{atan}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x) + 2*a**2*x**3*atan(a*x) + x*atan(a*x)), x)/
c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{atan}(a x) \left(c a^2 x^2 + c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.486 $\int \frac{1}{x^2(c+a^2cx^2)^2\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A]
time = 2.49, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)

[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")

[Out] integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")

[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{6}\operatorname{atan}{\left(ax \right)}+2a^{2}x^{4}\operatorname{atan}{\left(ax \right)}+x^{2}\operatorname{atan}{\left(ax \right)}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x),x)

[Out] Integral(1/(a**4*x**6*atan(a*x) + 2*a**2*x**4*atan(a*x) + x**2*atan(a*x)), x)/c**2

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.487 $\int \frac{x^6}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^6}{(c+a^2cx^2)^3 \text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^6}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Defer[Int][x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^6}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx = \int \frac{x^6}{(c+a^2cx^2)^3 \tan^{-1}(ax)} dx$$

Mathematica [A]
time = 5.61, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A]
time = 7.37, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a^2c\,x^2+c)^3 \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^6}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**6/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.488 $\int \frac{x^5}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx = \int \frac{x^5}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 5.38, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A]
time = 9.48, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a^2c\,x^2+c)^3 \arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**5/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3),x)
[Out] int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.489 $\int \frac{x^4}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=50

$$-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2a^5c^3}+\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8a^5c^3}+\frac{3\log(\text{ArcTan}(ax))}{8a^5c^3}$$

```
[Out] -1/2*Ci(2*arctan(a*x))/a^5/c^3+1/8*Ci(4*arctan(a*x))/a^5/c^3+3/8*ln(arctan(
a*x))/a^5/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5090, 3393, 3383}

$$-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2a^5c^3}+\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8a^5c^3}+\frac{3\log(\text{ArcTan}(ax))}{8a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] -1/2*CosIntegral[2*ArcTan[a*x]]/(a^5*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a
^5*c^3) + (3*Log[ArcTan[a*x]])/(8*a^5*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx = \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{x} \, dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} - \frac{\cos(2x)}{2x} + \frac{\cos(4x)}{8x}\right) \, dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x} \, dx, x, \tan^{-1}(ax)\right)}{8a^5c^3} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x} \, dx\right.}{2a^5c}$$

$$= -\frac{\text{Ci}(2\tan^{-1}(ax))}{2a^5c^3} + \frac{\text{Ci}(4\tan^{-1}(ax))}{8a^5c^3} + \frac{3\log\left(\tan^{-1}(ax)\right)}{8a^5c^3}$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 0.68

$$\frac{-4\text{CosIntegral}(2\text{ArcTan}(ax)) + \text{CosIntegral}(4\text{ArcTan}(ax)) + 3\log(\text{ArcTan}(ax))}{8a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] (-4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[
a*x]])/(8*a^5*c^3)
```

Maple [A]
time = 2.12, size = 40, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{3\ln(\arctan(ax))}{8c^3} - \frac{\text{cosineIntegral}(2\arctan(ax))}{2c^3} + \frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a^5}$	40
default	$\frac{\frac{3\ln(\arctan(ax))}{8c^3} - \frac{\text{cosineIntegral}(2\arctan(ax))}{2c^3} + \frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a^5}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(3/8*ln(arctan(a*x))/c^3-1/2*Ci(2*arctan(a*x))/c^3+1/8*Ci(4*arctan(a*
x))/c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.32, size = 174, normalized size = 3.48

$$\frac{6\log\left(\arctan\left(ax\right)\right)+\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-4\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-4\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{16\,a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
- 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*log_integral
(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - 4*log_integral(-(a^2*x^2 - 2*I*a
*x - 1)/(a^2*x^2 + 1)))/(a^5*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{a^6x^6\,\mathrm{atan}\left(ax\right)+3a^4x^4\,\mathrm{atan}\left(ax\right)+3a^2x^2\,\mathrm{atan}\left(ax\right)+\mathrm{atan}\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**4/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.490 $\int \frac{x^3}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} dx$

Optimal. Leaf size=35

$$\frac{\text{Si}(2\text{ArcTan}(ax))}{4a^4c^3} - \frac{\text{Si}(4\text{ArcTan}(ax))}{8a^4c^3}$$

```
[Out] 1/4*Si(2*arctan(a*x))/a^4/c^3-1/8*Si(4*arctan(a*x))/a^4/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5090, 4491, 3380}

$$\frac{\text{Si}(2\text{ArcTan}(ax))}{4a^4c^3} - \frac{\text{Si}(4\text{ArcTan}(ax))}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^4*c^3) - SinIntegral[4*ArcTan[a*x]]/(8*a^4*
c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^3 \tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x} - \frac{\sin(4x)}{8x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^3}$$
$$= \frac{\text{Si}(2\tan^{-1}(ax))}{4a^4c^3} - \frac{\text{Si}(4\tan^{-1}(ax))}{8a^4c^3}$$

Mathematica [A]
time = 0.08, size = 27, normalized size = 0.77

$$-\frac{-2\text{Si}(2\text{ArcTan}(ax)) + \text{Si}(4\text{ArcTan}(ax))}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] -1/8*(-2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(a^4*c^3)
```

Maple [A]
time = 1.64, size = 26, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{\text{sinIntegral}(4\arctan(ax))-2\,\text{sinIntegral}(2\arctan(ax))}{8a^4c^3}$	26
default	$-\frac{\text{sinIntegral}(4\arctan(ax))-2\,\text{sinIntegral}(2\arctan(ax))}{8a^4c^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] -1/8/a^4*(Si(4*arctan(a*x))-2*Si(2*arctan(a*x)))/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.94, size = 171, normalized size = 4.89

$$\frac{-i\ \text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+i\ \text{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\ \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-2i\ \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{16\,a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(-I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^
4*x^4 + 2*a^2*x^2 + 1)) + I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 + 2
*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2
*x^2 + 1)))/(a^4*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^6x^6\,\text{atan}\,(ax)+3a^4x^4\,\text{atan}\,(ax)+3a^2x^2\,\text{atan}\,(ax)+\text{atan}\,(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\text{atan}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.491 $\int \frac{x^2}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=33

$$-\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8a^3c^3}+\frac{\log(\text{ArcTan}(ax))}{8a^3c^3}$$

```
[Out] -1/8*Ci(4*arctan(a*x))/a^3/c^3+1/8*ln(arctan(a*x))/a^3/c^3
```

Rubi [A]
time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5090, 4491, 3383}

$$\frac{\log(\text{ArcTan}(ax))}{8a^3c^3}-\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] -1/8*CosIntegral[4*ArcTan[a*x]]/(a^3*c^3) + Log[ArcTan[a*x]]/(8*a^3*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{8x}-\frac{\cos(4x)}{8x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$
$$= \frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}$$
$$= -\frac{\text{Ci}(4\tan^{-1}(ax))}{8a^3c^3} + \frac{\log\left(\tan^{-1}(ax)\right)}{8a^3c^3}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.76

$$\frac{-\text{CosIntegral}(4\text{ArcTan}(ax)) + \log(\text{ArcTan}(ax))}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] (-CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]])/(8*a^3*c^3)
```

Maple [A]
time = 1.78, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{\ln(\arctan(ax))}{8c^3}-\frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a^3}$	28
default	$\frac{\frac{\ln(\arctan(ax))}{8c^3}-\frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/8*ln(arctan(a*x))/c^3-1/8*Ci(4*arctan(a*x))/c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.82, size = 120, normalized size = 3.64

$$\frac{2 \log\left(\arctan\left(ax\right)\right) - \text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) - \text{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(2*log(arctan(a*x)) - log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
- 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/(a^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}\left(ax\right)+3a^4x^4 \operatorname{atan}\left(ax\right)+3a^2x^2 \operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.492 $\int \frac{x}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=35

$$\frac{\mathrm{Si}(2\mathrm{ArcTan}(ax))}{4a^2c^3}+\frac{\mathrm{Si}(4\mathrm{ArcTan}(ax))}{8a^2c^3}$$

```
[Out] 1/4*Si(2*arctan(a*x))/a^2/c^3+1/8*Si(4*arctan(a*x))/a^2/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5090, 4491, 3380}

$$\frac{\mathrm{Si}(2\mathrm{ArcTan}(ax))}{4a^2c^3}+\frac{\mathrm{Si}(4\mathrm{ArcTan}(ax))}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] SinIntegral[2*ArcTan[a*x]]/(4*a^2*c^3) + SinIntegral[4*ArcTan[a*x]]/(8*a^2*
c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x}+\frac{\sin(4x)}{8x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^3}$$
$$= \frac{\text{Si}(2\tan^{-1}(ax))}{4a^2c^3} + \frac{\text{Si}(4\tan^{-1}(ax))}{8a^2c^3}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.77

$$\frac{2\text{Si}(2\text{ArcTan}(ax)) + \text{Si}(4\text{ArcTan}(ax))}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] (2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(8*a^2*c^3)
```

Maple [A]
time = 1.72, size = 26, normalized size = 0.74

method	result	size
derivativedivides	$\frac{\text{sinIntegral}(4\arctan(ax))+2\,\text{sinIntegral}(2\arctan(ax))}{8a^2c^3}$	26
default	$\frac{\text{sinIntegral}(4\arctan(ax))+2\,\text{sinIntegral}(2\arctan(ax))}{8a^2c^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/8/a^2*(Si(4*arctan(a*x))+2*Si(2*arctan(a*x)))/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.90, size = 171, normalized size = 4.89

$$\frac{i\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-i\,\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-2i\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{16\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4
*x^4 + 2*a^2*x^2 + 1)) - I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
+ 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 + 2*
I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*
x^2 + 1)))/(a^2*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan(
a*x) + atan(a*x)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\mathrm{atan}\,(a\,x)\,{\left(c\,a^2\,x^2+c\right)}^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.493 $\int \frac{1}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=50

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2ac^3}+\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8ac^3}+\frac{3\log(\text{ArcTan}(ax))}{8ac^3}$$

```
[Out] 1/2*Ci(2*arctan(a*x))/a/c^3+1/8*Ci(4*arctan(a*x))/a/c^3+3/8*ln(arctan(a*x))
/a/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5024, 3393, 3383}

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{2ac^3}+\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{8ac^3}+\frac{3\log(\text{ArcTan}(ax))}{8ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] CosIntegral[2*ArcTan[a*x]]/(2*a*c^3) + CosIntegral[4*ArcTan[a*x]]/(8*a*c^3)
 + (3*Log[ArcTan[a*x]])/(8*a*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cos(2x)}{2x}+\frac{\cos(4x)}{8x}\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8ac^3} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx\right.}{2ac^3}$$
$$= \frac{\text{Ci}(2\tan^{-1}(ax))}{2ac^3} + \frac{\text{Ci}(4\tan^{-1}(ax))}{8ac^3} + \frac{3\log\left(\tan^{-1}(ax)\right)}{8ac^3}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.68

$$\frac{4\text{CosIntegral}(2\text{ArcTan}(ax)) + \text{CosIntegral}(4\text{ArcTan}(ax)) + 3\log(\text{ArcTan}(ax))}{8ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] (4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[a
*x]])/(8*a*c^3)
```

Maple [A]
time = 1.98, size = 40, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{3\ln(\arctan(ax))}{8c^3}+\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^3}+\frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a}$	40
default	$\frac{\frac{3\ln(\arctan(ax))}{8c^3}+\frac{\text{cosineIntegral}(2\arctan(ax))}{2c^3}+\frac{\text{cosineIntegral}(4\arctan(ax))}{8c^3}}{a}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(3/8*ln(arctan(a*x))/c^3+1/2*Ci(2*arctan(a*x))/c^3+1/8*Ci(4*arctan(a*x)
)/c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.42, size = 174, normalized size = 3.48

$$\frac{6 \log\left(\arctan\left(ax\right)\right) + \text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \text{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + 4\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) + 4\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{16\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
- 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*log_integral
(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + 4*log_integral(-(a^2*x^2 - 2*I*a
*x - 1)/(a^2*x^2 + 1)))/(a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^6 \operatorname{atan}\left(ax\right)+3a^4x^4 \operatorname{atan}\left(ax\right)+3a^2x^2 \operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan(
a*x) + atan(a*x)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.494 $\int \frac{1}{x(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3 \text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A]
time = 5.74, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{7}\operatorname{atan}{(ax)}+3a^{4}x^{5}\operatorname{atan}{(ax)}+3a^{2}x^{3}\operatorname{atan}{(ax)}+x\operatorname{atan}{(ax)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x) + 3*a**4*x**5*atan(a*x) + 3*a**2*x**3*atan(
a*x) + x*atan(a*x)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.495 $\int \frac{1}{x^2(c+a^2cx^2)^3\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A]
time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{8}\operatorname{atan}{\left(ax \right)} + 3a^{4}x^{6}\operatorname{atan}{\left(ax \right)} + 3a^{2}x^{4}\operatorname{atan}{\left(ax \right)} + x^{2}\operatorname{atan}{\left(ax \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x) + 3*a**4*x**6*atan(a*x) + 3*a**2*x**4*atan(
a*x) + x**2*atan(a*x)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.496 $\int \frac{x\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Maple [A]
time = 9.08, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

Maple [A]
time = 13.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x),x)
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x), x)
```

3.498 $\int \frac{\sqrt{c+a^2cx^2}}{x\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

Maple [A]
time = 25.58, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{x\operatorname{atan}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)),x)
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)), x)
```

3.499 $\int \frac{x(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Maple [A]
time = 20.22, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

3.500 $\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

Maple [A]
time = 20.35, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x),x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{3/2}}{x\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\operatorname{ArcTan}(ax)},x\right)$$

`[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)`

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

`[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]`

`[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]`

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]`

`[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]`

Maple [A]
time = 36.54, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)), x)
```

3.502 $\int \frac{x(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)} dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)} dx$$

Mathematica [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Maple [A]
time = 20.45, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

3.503 $\int \frac{(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

Maple [A]
time = 23.25, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x), x)
```

3.504 $\int \frac{(c+a^2cx^2)^{5/2}}{x\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]

[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]

[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]

Maple [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}{x\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)),x)
[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)), x)
```

3.505 $\int \frac{x}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.506 $\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.507 $\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan\left(ax\right)\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.508 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 4.39, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}} \operatorname{atan}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.509 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.510 $\int \frac{x}{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5091, 5090, 3380}

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{(1+a^2x^2)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Si}(\tan^{-1}(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 0.95

$$\frac{(1+a^2x^2)^{3/2}\,\text{Si}(\text{ArcTan}(ax))}{a^2\,(c\,(1+a^2x^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*SinIntegral[ArcTan[a*x]])/(a^2*(c*(1 + a^2*x^2))^(3/2)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 82, normalized size = 2.10

method	result	size
default	$-\frac{\operatorname{csgn}(\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{2\sqrt{a^2x^2+1}\;a^2c^2}+\frac{\operatorname{sinIntegral}(\arctan(ax))\sqrt{c\,(ax-i)\,(ax+i)}}{\sqrt{a^2x^2+1}\;a^2c^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c
^2+Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.511 $\int \frac{1}{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{ac\sqrt{c+a^2cx^2}}$$

```
[Out] Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5025, 5024, 3383}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 1.00

$$\frac{\sqrt{c+a^2cx^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{ac^2\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*CosIntegral[ArcTan[a*x]])/(a*c^2*Sqrt[1 + a^2*x^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 136, normalized size = 3.49

method	result
default	$-\frac{i\,\text{csgn}(\arctan(ax))\,\text{csgn}(i\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+\frac{i\,\text{csgn}(i\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+$ co

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I
)*(I+a*x))^(1/2)/a/c^2+1/2*I*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a
*x-I)*(I+a*x))^(1/2)/a/c^2+Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+
a*x))^(1/2)/a/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^{2}\,x^{2}+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.512 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.513 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{atan}(a x) \left(c a^2 x^2 + c\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.514 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 5.45, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)

[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")

[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)), x)

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)

[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.515 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{x^4}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.516 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\mathrm{ArcTan}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 3/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/4*Si(3*
arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5091, 5090, 3393, 3380}

$$\frac{3\sqrt{a^2x^2+1}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(3\mathrm{ArcTan}(ax))}{4a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^
2]) - (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^
2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^3}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x}-\frac{\sin(3x)}{4x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$
$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \right.}{4a^4c^2\sqrt{c-}}$$
$$= \frac{3\sqrt{1+a^2x^2}\,\text{Si}(\tan^{-1}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Si}(3\tan^{-1}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 0.60

$$\frac{(1+a^2x^2)^{5/2}\,(3\text{Si}(\text{ArcTan}(ax))-\text{Si}(3\text{ArcTan}(ax)))}{4a^4\,(c\,(1+a^2x^2))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 + a^2*x^2)^(5/2)*(3*SinIntegral[ArcTan[a*x]] - SinIntegral[3*ArcTan[a*x
]]))/(4*a^4*(c*(1 + a^2*x^2))^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 125, normalized size = 1.44

method	result
default	$-\frac{\text{csgn}(\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{4\sqrt{a^2x^2+1}\,a^4c^3} - \frac{\text{sinIntegral}(3\arctan(ax))\sqrt{c\,(ax-i)\,(ax+i)}}{4\sqrt{a^2x^2+1}\,a^4c^3} + \frac{3\,\text{sinIntegral}(\text{arctan}}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c
^3-1/4*Si(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^
3+3/4*Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.517 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/4*Ci(3*
arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5091, 5090, 4491, 3383}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{4a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*c*x^2]
) - (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a^3*c^2*Sqrt[c + a^2*
c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\cos(x)}{4x}-\frac{\cos(3x)}{4x}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\right.}{4a^3c^2\sqrt{c+a^2c}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Ci}(3\tan^{-1}(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 0.61

$$\frac{\sqrt{c(1+a^2x^2)}\,(\text{CosIntegral}(\text{ArcTan}(ax)) - \text{CosIntegral}(3\text{ArcTan}(ax)))}{4a^3c^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(CosIntegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x
]]))/(4*a^3*c^3*Sqrt[1 + a^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.48, size = 84, normalized size = 0.97

method	result	size

default	$-\frac{\text{cosineIntegral}(3\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^3c^3}+\frac{\text{cosineIntegral}(\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^3c^3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Ci(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^3/c^3+
1/4*Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^3/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.518 $\int \frac{x}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\mathrm{ArcTan}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/4*Si(3*
arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5091, 5090, 4491, 3380}

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(3\mathrm{ArcTan}(ax))}{4a^2c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*
c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{x}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{x}\,d\right.}{4a^2c^2\sqrt{c+a^2c}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Si}(\tan^{-1}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Si}(3\tan^{-1}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.59

$$\frac{\sqrt{c\,(1+a^2x^2)}\,(\text{Si}(\text{ArcTan}(ax))+\text{Si}(3\text{ArcTan}(ax)))}{4a^2c^3\sqrt{1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(SinIntegral[ArcTan[a*x]] + SinIntegral[3*ArcTan[a*x
]]))/(4*a^2*c^3*Sqrt[1 + a^2*x^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 125, normalized size = 1.44

method	result

default	$-\frac{\operatorname{csgn}(\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^2c^3}+\frac{\operatorname{sinIntegral}(3\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^2c^3}+\frac{\operatorname{sinIntegral}(\operatorname{arcta}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*csgn(arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c
^3+1/4*Si(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c^
3+1/4*Si(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3)*arctan(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.519 $\int \frac{1}{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 3/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/4*Ci(3*ar
ctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5025, 5024, 3393, 3383}

$$\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{4ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

```
[Out] (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]
) + (Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] && !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)\,dx,x,\tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \text{co}\right.}{4ac^2\sqrt{c+a}}\\ &= \frac{3\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{4ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{Ci}(3\tan^{-1}(ax))}{4ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.57

$$\frac{(1+a^2x^2)^{5/2}\,(3\text{CosIntegral}(\text{ArcTan}(ax))+\text{CosIntegral}(3\text{ArcTan}(ax)))}{4a\,(c\,(1+a^2x^2))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] ((1 + a^2*x^2)^(5/2)*(3*CosIntegral[ArcTan[a*x]] + CosIntegral[3*ArcTan[a*x
]]))/(4*a*(c*(1 + a^2*x^2))^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 179, normalized size = 2.06

method	result
default	$-\frac{i\text{csgn}(\arctan(ax))\text{csgn}(i\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^3}+\frac{i\text{csgn}(i\arctan(ax))\pi\sqrt{c\,(ax-i)\,(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^3}+\text{cosi}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a*x-I
)*(I+a*x))^(1/2)/a/c^3+1/2*I*csgn(I*arctan(a*x))*Pi/(a^2*x^2+1)^(1/2)*(c*(a
*x-I)*(I+a*x))^(1/2)/a/c^3+1/4*Ci(3*arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-
I)*(I+a*x))^(1/2)/a/c^3+3/4*Ci(arctan(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I
+a*x))^(1/2)/a/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.520 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)},x\right)$$

[Out] Unintegrable(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]

[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]

[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.521 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c \left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{atan}(a x) \left(c a^2 x^2 + c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.522 $\int \frac{x^m (c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m (c+a^2cx^2)^3}{\text{ArcTan}(ax)}, x\right)$$

`[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)`

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2cx^2)^3}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

`[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]`

`[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]`

Rubi steps

$$\int \frac{x^m (c+a^2cx^2)^3}{\tan^{-1}(ax)} dx = \int \frac{x^m (c+a^2cx^2)^3}{\tan^{-1}(ax)} dx$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2cx^2)^3}{\text{ArcTan}(ax)} dx$$

Verification is not applicable to the result.

`[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]`

`[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]`

Maple [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c\, x^2+c)^3}{\arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x), x) + Integral(3*a**2*x**2*x**m/atan(a*x), x)
 + Integral(3*a**4*x**4*x**m/atan(a*x), x) + Integral(a**6*x**6*x**m/atan(a
*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{\mathrm{atan}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x),x)
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

3.523 $\int \frac{x^m (c+a^2 c x^2)^2}{\mathbf{ArcTan}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)} \, dx = \int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c x^2 + c)^2}{\arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x),x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x), x) + Integral(2*a**2*x**2*x**m/atan(a*x), x)
 + Integral(a**4*x**4*x**m/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{\mathrm{atan}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x),x)
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

3.524 $\int \frac{x^m(c+a^2cx^2)}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x^{m}}{\operatorname{atan}{\left(ax \right)}}\, dx + \int \frac{a^{2} x^{2} x^{m}}{\operatorname{atan}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x),x)
```

```
[Out] c*(Integral(x**m/atan(a*x), x) + Integral(a**2*x**2*x**m/atan(a*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x),x)
```

```
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x), x)
```

3.525 $\int \frac{x^m}{(c+a^2cx^2)\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
[Out] integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x),x)
[Out] Integral(x**m/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)),x)
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.526 $\int \frac{x^m}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
[Out] integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x),x)
[Out] Integral(x**m/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)
/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.527 $\int \frac{x^m}{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x),x)
```

```
[Out] Integral(x**m/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*at
an(a*x) + atan(a*x)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.528 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

3.529 $\int \frac{x^m (c+a^2cx^2)^{3/2}}{\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
[Out] Integral(x**m*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

3.530 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

3.531 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\arctan(ax)\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.532 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

```
[Out] Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.533 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.534 $\int \frac{x(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*c*x^5 + 2*a^2*c*x^3 + c*x - arctan(a*x)*integrate((5*a^4*c*x^4 + 6*a^
2*c*x^2 + c)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] c*(Integral(x/atan(a*x)**2, x) + Integral(a**2*x**3/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

```
[Out] int((x*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

3.535 $\int \frac{c+a^2cx^2}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=20

$$\mathrm{Int}\left(\frac{c+a^2cx^2}{\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^2,x)
[Out] int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
[Out] -(a^4*c*x^4 + 2*a^2*c*x^2 - a*arctan(a*x)*integrate(4*(a^3*c*x^3 + a*c*x)/a
rctan(a*x), x) + c)/(a*arctan(a*x))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**2,x)
[Out] c*(Integral(a**2*x**2/atan(a*x)**2, x) + Integral(atan(a*x)**(-2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x)^2,x)
[Out] int((c + a^2*c*x^2)/atan(a*x)^2, x)
```

3.536 $\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{c+a^2cx^2}{x\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx=\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*c*x^4 + 2*a^2*c*x^2 - x*arctan(a*x)*integrate((3*a^4*c*x^4 + 2*a^2*c*
x^2 - c)/(x^2*arctan(a*x)), x) + c)/(a*x*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{1}{x\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**2,x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**2), x) + Integral(a**2*x/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)^2),x)
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)^2), x)
```

3.537 $\int \frac{x(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^7 + 3*a^4*c^2*x^5 + 3*a^2*c^2*x^3 + c^2*x - arctan(a*x)*integra
te((7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x), x))/
(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{x}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{2a^{2}x^{3}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{a^{4}x^{5}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**2, x) + Integral(2*a**2*x**3/atan(a*x)**2, x) +
 Integral(a**4*x**5/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

3.538 $\int \frac{(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - a*arctan(a*x)*integrate(6*(
a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)/arctan(a*x), x) + c^2)/(a*arctan(a*x
))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**2, x) + Integral(a**4*x**4/atan(a*x)*
*2, x) + Integral(atan(a*x)**(-2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^2/atan(a*x)^2, x)
```

3.539 $\int \frac{(c+a^2cx^2)^2}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^2},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]

[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]

Maple [A]
time = 2.23, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - x*arctan(a*x)*integrate((5*
a^6*c^2*x^6 + 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)/(x^2*arctan(a*x)), x) +
c^2)/(a*x*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{1}{x\,\mathrm{atan}^2\left(ax\right)}\,dx + \int \frac{2a^2x}{\mathrm{atan}^2\left(ax\right)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**2), x) + Integral(2*a**2*x/atan(a*x)**2, x)
+ Integral(a**4*x**3/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)^2),x)
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)^2), x)
```

3.540 $\int \frac{x(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^2},x\right)$$

[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]

[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]

[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]

Maple [A]
time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^8*c^3*x^9 + 4*a^6*c^3*x^7 + 6*a^4*c^3*x^5 + 4*a^2*c^3*x^3 + c^3*x - arc
tan(a*x)*integrate((9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 12*a^
2*c^3*x^2 + c^3)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)^
2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{3}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{5}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{7}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**2, x) + Integral(3*a**2*x**3/atan(a*x)**2, x) +
 Integral(3*a**4*x**5/atan(a*x)**2, x) + Integral(a**6*x**7/atan(a*x)**2, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

```
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

3.541 $\int \frac{(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.89, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - a*arc
tan(a*x)*integrate(8*(a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3*x
)/arctan(a*x), x))/(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^2,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**2, x) + Integral(3*a**4*x**4/atan(a*x
)**2, x) + Integral(a**6*x**6/atan(a*x)**2, x) + Integral(atan(a*x)**(-2),
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x)^2, x)
```

3.542 $\int \frac{(c+a^2cx^2)^3}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - x*arc
tan(a*x)*integrate((7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 + 4*a^2
*c^3*x^2 - c^3)/(x^2*arctan(a*x)), x))/(a*x*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{1}{x \operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{3}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{5}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**2), x) + Integral(3*a**2*x/atan(a*x)**2, x)
+ Integral(3*a**4*x**3/atan(a*x)**2, x) + Integral(a**6*x**5/atan(a*x)**2,
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)^2),x)
```

```
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)^2), x)
```

3.543 $\int \frac{x^3}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=39

$$-\frac{x^3}{ac\text{ArcTan}(ax)}+\frac{3\text{Int}\left(\frac{x^2}{\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -x^3/a/c/arctan(a*x)+3*Unintegrable(x^2/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^3/(a*c*ArcTan[a*x])) + (3*Defer[Int][x^2/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{x^3}{ac\tan^{-1}(ax)}+\frac{3\int \frac{x^2}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^3 - 3*arctan(a*x)*integrate(x^2/arctan(a*x), x))/(a*c*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.544 $\int \frac{x^2}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=37

$$-\frac{x^2}{ac\text{ArcTan}(ax)}+\frac{2\text{Int}\left(\frac{x}{\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -x^2/a/c/arctan(a*x)+2*Unintegrable(x/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^2/(a*c*ArcTan[a*x])) + (2*Defer[Int][x/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{x^2}{ac\tan^{-1}(ax)}+\frac{2\int \frac{x}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^2 - 2*arctan(a*x)*integrate(x/arctan(a*x), x))/(a*c*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.545 $\int \frac{x}{(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{x}{ac\text{ArcTan}(ax)}+\frac{\text{Int}\left(\frac{1}{\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -x/a/c/arctan(a*x)+Unintegrable(1/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] -(x/(a*c*ArcTan[a*x])) + Defer[Int][ArcTan[a*x]^(-1), x]/(a*c)
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{x}{ac\tan^{-1}(ax)}+\frac{\int \frac{1}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] (arctan(a*x)*integrate(1/arctan(a*x), x) - x)/(a*c*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{2}x^{2}\operatorname{atan}^{2}{\left(ax \right)}+\operatorname{atan}^{2}{\left(ax \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.546 $\int \frac{1}{(c+a^2cx^2)\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{ac\text{ArcTan}(ax)}$$

```
[Out] -1/a/c/arctan(a*x)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5004}

$$-\frac{1}{ac\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*ArcTan[a*x]))
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx = -\frac{1}{ac\tan^{-1}(ax)}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{ac\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*ArcTan[a*x]))
```

Maple [A]
time = 0.08, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$-\frac{1}{ac\arctan(ax)}$	15
default	$-\frac{1}{ac\arctan(ax)}$	15
risch	$\frac{2i}{ac(\ln(-iax+1)-\ln(iax+1))}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/a/c/arctan(a*x)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 1.00

$$-\frac{1}{ac\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
[Out] -1/(a*c*arctan(a*x))
```

Fricas [A]
time = 0.75, size = 14, normalized size = 1.00

$$-\frac{1}{ac\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
[Out] -1/(a*c*arctan(a*x))
```

Sympy [A]
time = 0.37, size = 10, normalized size = 0.71

$$-\frac{1}{ac\operatorname{atan}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**2,x)
[Out] -1/(a*c*atan(a*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.34, size = 14, normalized size = 1.00

$$-\frac{1}{a\,c\,\mathrm{atan}\left(a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] -1/(a*c*atan(a*x))
```

3.547 $\int \frac{1}{x(c+a^2cx^2)\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{acx\text{ArcTan}(ax)}-\frac{\text{Int}\left(\frac{1}{x^2\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -1/a/c/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] -(1/(a*c*x*ArcTan[a*x])) - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x*arctan(a*x)*integrate(1/(x^2*arctan(a*x)), x) + 1)/(a*c*x*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2}x^{3}\operatorname{atan}^{2}(ax)+x\operatorname{atan}^{2}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**2 + x*atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\mathrm{atan}\,(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.548 $\int \frac{1}{x^2(c+a^2cx^2)\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{acx^2\text{ArcTan}(ax)}-\frac{2\text{Int}\left(\frac{1}{x^3\text{ArcTan}(ax)},x\right)}{ac}$$

[Out] -1/a/c/x^2/arctan(a*x)-2*Unintegrable(1/x^3/arctan(a*x),x)/a/c

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]

[Out] -(1/(a*c*x^2*ArcTan[a*x])) - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(a*c)

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx^2\tan^{-1}(ax)}-\frac{2\int \frac{1}{x^3\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]

[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(2*x^2*arctan(a*x)*integrate(1/(x^3*arctan(a*x)), x) + 1)/(a*c*x^2*arctan(
a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^4\operatorname{atan}^2(ax)+x^2\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2 \operatorname{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.549 $\int \frac{1}{x^3(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{acx^3\text{ArcTan}(ax)}-\frac{3\text{Int}\left(\frac{1}{x^4\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -1/a/c/x^3/arctan(a*x)-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{1}{x^3\,(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] -(1/(a*c*x^3*ArcTan[a*x])) - (3*Defer[Int][1/(x^4*ArcTan[a*x]), x])/(a*c)
```

Rubi steps

$$\int \frac{1}{x^3\,(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx^3\tan^{-1}(ax)}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(a^2c\,x^2+c)\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(3*x^3*arctan(a*x)*integrate(1/(x^4*arctan(a*x)), x) + 1)/(a*c*x^3*arctan(
a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^5\operatorname{atan}^2(ax)+x^3\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3 \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.550 $\int \frac{1}{x^4(c+a^2cx^2)\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{acx^4\text{ArcTan}(ax)}-\frac{4\text{Int}\left(\frac{1}{x^5\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -1/a/c/x^4/arctan(a*x)-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] -(1/(a*c*x^4*ArcTan[a*x])) - (4*Defer[Int][1/(x^5*ArcTan[a*x]), x])/(a*c)
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^2}\,dx=-\frac{1}{acx^4\tan^{-1}(ax)}-\frac{4\int \frac{1}{x^5\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(4*x^4*arctan(a*x)*integrate(1/(x^5*arctan(a*x)), x) + 1)/(a*c*x^4*arctan(
a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^6\operatorname{atan}^2(ax)+x^4\operatorname{atan}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^4 \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.551 $\int \frac{x^3}{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{x}{a^3c^2\text{ArcTan}(ax)}+\frac{x}{a^3c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{a^4c^2}+\frac{\text{Int}\left(\frac{1}{\text{ArcTan}(ax)},x\right)}{a^3c^2}$$

```
[Out] -x/a^3/c^2/arctan(a*x)+x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))/
a^4/c^2+Unintegrable(1/arctan(a*x),x)/a^3/c^2
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a^3*c^2*ArcTan[a*x])) + x/(a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/(a^4*c^2) + Defer[Int][ArcTan[a*x]^(-1), x]/(a^3*c^2)
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)}+\frac{x}{a^3c^2(1+a^2x^2)\tan^{-1}(ax)}-\frac{\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx}{a^3}+\frac{\int}{}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)}+\frac{x}{a^3c^2(1+a^2x^2)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx,x,\tan^{-}\right.}{a^4c^2}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)}+\frac{x}{a^3c^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)\,dx\right.}{a^4c^2}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)}+\frac{x}{a^3c^2(1+a^2x^2)\tan^{-1}(ax)}-2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx,x,\tan\right.}{2a^4c^2}$$

$$= -\frac{x}{a^3c^2\tan^{-1}(ax)}+\frac{x}{a^3c^2(1+a^2x^2)\tan^{-1}(ax)}-\frac{\text{Ci}(2\tan^{-1}(ax))}{a^4c^2}+\frac{\int \frac{1}{\tan^{-1}(}}{a^3c}$$

Mathematica [A]
time = 9.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
[Out] -(x^3 - (a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^4 + 3*x^2)/((a^5
*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x))/((a^3*c^2*x^2 + a*c^2)*
arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.552 $\int \frac{x^2}{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{x^2}{ac^2(1+a^2x^2)\operatorname{ArcTan}(ax)}+\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{a^3c^2}$$

```
[Out] -x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^3/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5062, 5090, 4491, 12, 3380}

$$\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\operatorname{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]]/(a^3*
c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx &= -\frac{x^2}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{2\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx}{a}\\
&= -\frac{x^2}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{2\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^2}\\
&= -\frac{x^2}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Si}(2\tan^{-1}(ax))}{a^3c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.93

$$
\frac{-\frac{a^2x^2}{(1+a^2x^2)\text{ArcTan}(ax)}+\text{Si}(2\text{ArcTan}(ax))}{a^3c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] (-((a^2*x^2)/((1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]])/(a^
3*c^2)
```

Maple [A]
time = 0.22, size = 37, normalized size = 0.86

method	result	size

derivativedivides	$\frac{2\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$	37
default	$\frac{2\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))-1)/arctan(a
*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] (4*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(1/2*x/((a^5*c^2*x^4 + 2*a^3*
c^2*x^2 + a*c^2)*arctan(a*x)), x) - x^2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x)
)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.96, size = 123, normalized size = 2.86

$$-\frac{2\,a^2x^2-(i\,a^2x^2+i)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-(-i\,a^2x^2-i)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{2\,(a^5c^2x^2+a^3c^2)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2 - (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*
a*x - 1)/(a^2*x^2 + 1)) - (-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x
^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/((a^5*c^2*x^2 + a^3*c^2)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\,\text{atan}^2(ax)+2a^2x^2\,\text{atan}^2(ax)+\text{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.553 $\int \frac{x}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^2} \, dx$

Optimal. Leaf size=41

$$-\frac{x}{ac^2\left(1+a^2x^2\right)\mathrm{ArcTan}(ax)}+\frac{\mathrm{CosIntegral}(2\mathrm{ArcTan}(ax))}{a^2c^2}$$

```
[Out] -x/a/c^2/(a^2*x^2+1)/arctan(a*x)+Ci(2*arctan(a*x))/a^2/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5088, 5090, 3393, 3383, 5024}

$$\frac{\mathrm{CosIntegral}(2\mathrm{ArcTan}(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\mathrm{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(a^2*c^
2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
```

```
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx &= -\frac{x}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)}\,dx}{a}-a\int \frac{x^2}{(c+a^2cx^2)^2\tan
\\
&= -\frac{x}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c^2}-\frac{\text{Subst}\left(\int
\\
&= -\frac{x}{ac^2(1+a^2x^2)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)\,dx,x,\tan^{-1}(ax)\right)}{a^2c^2}+
\\
&= -\frac{x}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{2a^2c^2}
\\
&= -\frac{x}{ac^2(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Ci}(2\tan^{-1}(ax))}{a^2c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.88

$$
\frac{-\frac{ax}{(1+a^2x^2)\text{ArcTan}(ax)}+\text{CosIntegral}(2\text{ArcTan}(ax))}{a^2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] (-((a*x)/((1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]])/(a^2*c^
2)
```

Maple [A]
time = 0.24, size = 38, normalized size = 0.93

method	result	size
derivativedivides	$\frac{2\,\mathrm{cosineIntegral}(2\arctan(ax))\arctan(ax)-\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$	38
default	$\frac{2\,\mathrm{cosineIntegral}(2\arctan(ax))\arctan(ax)-\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/a^2/c^2*(2*Ci(2*arctan(a*x))*arctan(a*x)-sin(2*arctan(a*x)))/arctan(a*x
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
[Out] -((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^2 - 1)/((a^5*c^2*x^4 +
 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x) + x)/((a^3*c^2*x^2 + a*c^2)*arctan
(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.81, size = 115, normalized size = 2.80

$$\frac{(a^2x^2+1)\arctan(ax)\operatorname{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan(ax)\operatorname{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-2\,ax}{2\,(a^4c^2x^2+a^2c^2)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
[Out] 1/2*((a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x
^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/
(a^2*x^2 + 1)) - 2*a*x)/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)**
2), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.554 $\int \frac{1}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^2} dx$

Optimal. Leaf size=41

$$-\frac{1}{ac^2(1+a^2x^2)\operatorname{ArcTan}(ax)}-\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{ac^2}$$

```
[Out] -1/a/c^2/(a^2*x^2+1)/arctan(a*x)-Si(2*arctan(a*x))/a/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5022, 5090, 4491, 12, 3380}

$$-\frac{1}{ac^2(a^2x^2+1)\operatorname{ArcTan}(ax)}-\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] -(1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx &= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-(2a)\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)}\,dx\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^2}\\
&= -\frac{1}{ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 0.83

$$-\frac{\frac{1}{\text{ArcTan}(ax)+a^2x^2\text{ArcTan}(ax)}+\text{Si}(2\text{ArcTan}(ax))}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] -(((ArcTan[a*x] + a^2*x^2*ArcTan[a*x])^(-1) + SinIntegral[2*ArcTan[a*x]])/(
a*c^2))
```

Maple [A]
time = 0.12, size = 37, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{2\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^2\arctan(ax)}$	37

default	$-\frac{2\,\mathrm{sinIntegral}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^{2}\arctan(ax)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))+1)/arctan(a*
x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(4*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)*integrate(1/2*x/((a^4*c^2*x^4 + 2*a
^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.06, size = 112, normalized size = 2.73

$$\frac{(-i\,a^{2}x^{2}-i)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}+2i\,ax-1}{a^{2}x^{2}+1}\right)+(i\,a^{2}x^{2}+i)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^{2}x^{2}-2i\,ax-1}{a^{2}x^{2}+1}\right)-2}{2\,(a^{3}c^{2}x^{2}+ac^{2})\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^
2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x
- 1)/(a^2*x^2 + 1)) - 2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{4}\operatorname{atan}^{2}(ax)+2a^{2}x^{2}\operatorname{atan}^{2}(ax)+\operatorname{atan}^{2}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)**
2), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.555 $\int \frac{1}{x(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{ac^2x\text{ArcTan}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{c^2}-\frac{\text{Int}\left(\frac{1}{x^2\text{ArcTan}(ax)},x\right)}{ac^2}$$

```
[Out] -1/a/c^2/x/arctan(a*x)+a*x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))/c^
2-Unintegrable(1/x^2/arctan(a*x),x)/a/c^2
```

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x*ArcTan[a*x])) + (a*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - CosInt
egral[2*ArcTan[a*x]]/c^2 - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(a*c^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx &= -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx}{c}\\
&= -\frac{1}{ac^2x\tan^{-1}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-a\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(a
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{ac^2x\tan^{-1}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx,x,\tan\right.}{c^2}\\
&= -\frac{1}{ac^2x\tan^{-1}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int\left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)\,d\right.}{c^2}\\
&= -\frac{1}{ac^2x\tan^{-1}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx,x,\tan\right.}{2c^2}\\
&= -\frac{1}{ac^2x\tan^{-1}(ax)}+\frac{ax}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Ci}(2\tan^{-1}(ax))}{c^2}-\frac{\int \frac{}{x^2\tan}}{}
\end{aligned}$$

Mathematica [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)*integrate((3*a^2*x^2 + 1)/((a^5*c^2*x
^6 + 2*a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^3 + a*c^2
*x)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^5\operatorname{atan}^2(ax)+2a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)

[Out] Integral(1/(a**4*x**5*atan(a*x)**2 + 2*a**2*x**3*atan(a*x)**2 + x*atan(a*x)**2), x)/c**2

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2),x)

[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2), x)

3.556 $\int \frac{1}{x^2(c+a^2cx^2)^2\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{ac^2x^2\text{ArcTan}(ax)}+\frac{a}{c^2(1+a^2x^2)\,\text{ArcTan}(ax)}+\frac{a\text{Si}(2\text{ArcTan}(ax))}{c^2}-\frac{2\text{Int}\left(\frac{1}{x^3\text{ArcTan}(ax)},x\right)}{ac^2}$$

```
[Out] -1/a/c^2/x^2/arctan(a*x)+a/c^2/(a^2*x^2+1)/arctan(a*x)+a*Si(2*arctan(a*x))/
c^2-2*Unintegrable(1/x^3/arctan(a*x),x)/a/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x^2*ArcTan[a*x])) + a/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*SinIn
tegral[2*ArcTan[a*x]])/c^2 - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^2(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx}{c}$$

$$= -\frac{1}{ac^2x^2\tan^{-1}(ax)}+\frac{a}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\left(2a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^2\tan$$

$$= -\frac{1}{ac^2x^2\tan^{-1}(ax)}+\frac{a}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\int \frac{1}{x^3\tan^{-1}(ax)}\,dx}{ac^2}+\frac{(2a)\text{S}}{}$$

$$= -\frac{1}{ac^2x^2\tan^{-1}(ax)}+\frac{a}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\int \frac{1}{x^3\tan^{-1}(ax)}\,dx}{ac^2}+\frac{(2a)\text{S}}{}$$

$$= -\frac{1}{ac^2x^2\tan^{-1}(ax)}+\frac{a}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{2\int \frac{1}{x^3\tan^{-1}(ax)}\,dx}{ac^2}+\frac{a\text{Sub}}{}$$

$$= -\frac{1}{ac^2x^2\tan^{-1}(ax)}+\frac{a}{c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{a\text{Si}\left(2\tan^{-1}(ax)\right)}{c^2}-\frac{2\int \frac{1}{x}}{}$$

Mathematica [A]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \text{ArcTan}(a x)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
[Out] -((a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)*integrate(2*(2*a^2*x^2 + 1)/((a^5*c
^2*x^7 + 2*a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)), x) + 1)/((a^3*c^2*x^4 + a
*c^2*x^2)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4 x^6 \operatorname{atan}^2(ax)+2a^2x^4 \operatorname{atan}^2(ax)+x^2 \operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x)**2 + 2*a**2*x**4*atan(a*x)**2 + x**2*atan(a
*x)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.557 $\int \frac{1}{x^3(c+a^2cx^2)^2\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{1}{ac^2x^3\text{ArcTan}(ax)}+\frac{a}{c^2x\text{ArcTan}(ax)}-\frac{a^3x}{c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{a^2\text{CosIntegral}(2\text{ArcTan}(ax))}{c^2}-\frac{3\text{Int}\Big(\frac{}{x^4 A}$$

```
[Out] -1/a/c^2/x^3/arctan(a*x)+a/c^2/x/arctan(a*x)-a^3*x/c^2/(a^2*x^2+1)/arctan(a
*x)+a^2*Ci(2*arctan(a*x))/c^2-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c^2+a*U
nintegrable(1/x^2/arctan(a*x),x)/c^2
```

Rubi [A]
time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*x^3*ArcTan[a*x])) + a/(c^2*x*ArcTan[a*x]) - (a^3*x)/(c^2*(1 + a^
2*x^2)*ArcTan[a*x]) + (a^2*CosIntegral[2*ArcTan[a*x]])/c^2 - (3*Defer[Int][
1/(x^4*ArcTan[a*x]), x])/(a*c^2) + (a*Defer[Int][1/(x^2*ArcTan[a*x]), x])/c
^2
```

Rubi steps

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x (c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^3(c+a^2cx^2)\tan^{-1}(ax)^2} dx}{c}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + a^4 \int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx - \frac{3\int \frac{1}{x^4 \tan^{-1}(ax)} dx}{ac^2}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + \frac{a}{c^2x \tan^{-1}(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \tan^{-1}(ax)} + a^3 \int \frac{}{(c}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + \frac{a}{c^2x \tan^{-1}(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \tan^{-1}(ax)} - \frac{3\int \frac{}{x^4 \tan}}{c}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + \frac{a}{c^2x \tan^{-1}(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \tan^{-1}(ax)} - \frac{3\int \frac{}{x^4 \tan}}{c}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + \frac{a}{c^2x \tan^{-1}(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \tan^{-1}(ax)} - \frac{3\int \frac{}{x^4 \tan}}{c}$$

$$= -\frac{1}{ac^2x^3 \tan^{-1}(ax)} + \frac{a}{c^2x \tan^{-1}(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \tan^{-1}(ax)} + \frac{a^2\text{Ci}(2\,\text{t}}{}$$

Mathematica [A]
time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \text{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^2 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)*integrate((5*a^2*x^2 + 3)/((a^5*c^2
*x^8 + 2*a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)), x) + 1)/((a^3*c^2*x^5 + a*c
^2*x^3)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^7\operatorname{atan}^2(ax)+2a^2x^5\operatorname{atan}^2(ax)+x^3\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**2 + 2*a**2*x**5*atan(a*x)**2 + x**3*atan(a
*x)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.558 $\int \frac{1}{x^4(c+a^2cx^2)^2\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{ac^2x^4\text{ArcTan}(ax)}+\frac{a}{c^2x^2\text{ArcTan}(ax)}-\frac{a^3}{c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}-\frac{a^3\text{Si}(2\text{ArcTan}(ax))}{c^2}-\frac{4\text{Int}\left(\frac{1}{x^5\text{ArcTan}(ax)}\right.}{ac^2}$$

[Out] -1/a/c^2/x^4/arctan(a*x)+a/c^2/x^2/arctan(a*x)-a^3/c^2/(a^2*x^2+1)/arctan(a*x)-a^3*Si(2*arctan(a*x))/c^2-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c^2+2*a*Unintegrable(1/x^3/arctan(a*x),x)/c^2

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]

[Out] -(1/(a*c^2*x^4*ArcTan[a*x])) + a/(c^2*x^2*ArcTan[a*x]) - a^3/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - (a^3*SinIntegral[2*ArcTan[a*x]])/c^2 - (4*Defer[Int][1/(x^5*ArcTan[a*x]), x])/(a*c^2) + (2*a*Defer[Int][1/(x^3*ArcTan[a*x]), x])/c^2

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^2} dx}{c}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + a^4 \int \frac{1}{(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx - \frac{4\int \frac{1}{x^5\tan^{-1}(ax)} dx}{ac^2}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + \frac{a}{c^2x^2\tan^{-1}(ax)} - \frac{a^3}{c^2(1+a^2x^2)\tan^{-1}(ax)} - (2a^5)$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + \frac{a}{c^2x^2\tan^{-1}(ax)} - \frac{a^3}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{4\int \frac{}{x^5 \mathrm{t}}}{}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + \frac{a}{c^2x^2\tan^{-1}(ax)} - \frac{a^3}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{4\int \frac{}{x^5 \mathrm{t}}}{}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + \frac{a}{c^2x^2\tan^{-1}(ax)} - \frac{a^3}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{4\int \frac{}{x^5 \mathrm{t}}}{}$$

$$= -\frac{1}{ac^2x^4\tan^{-1}(ax)} + \frac{a}{c^2x^2\tan^{-1}(ax)} - \frac{a^3}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{a^3\mathrm{Si}(2}{}$$

Mathematica [A]
time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^2 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)*integrate(2*(3*a^2*x^2 + 2)/((a^5*c
^2*x^9 + 2*a^3*c^2*x^7 + a*c^2*x^5)*arctan(a*x)), x) + 1)/((a^3*c^2*x^6 + a
*c^2*x^4)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^8\operatorname{atan}^2(ax)+2a^2x^6\operatorname{atan}^2(ax)+x^4\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**8*atan(a*x)**2 + 2*a**2*x**6*atan(a*x)**2 + x**4*atan(a
*x)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.559 $\int \frac{x^3}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=86

$$\frac{x}{a^3c^3\left(1+a^2x^2\right)^2\mathrm{ArcTan}(ax)}-\frac{x}{a^3c^3\left(1+a^2x^2\right)\mathrm{ArcTan}(ax)}+\frac{\mathrm{CosIntegral}(2\mathrm{ArcTan}(ax))}{2a^4c^3}-\frac{\mathrm{CosIntegral}(4\mathrm{ArcTan}(ax))}{2a^4c^3}$$

```
[Out] x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-x/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*C
i(2*arctan(a*x))/a^4/c^3-1/2*Ci(4*arctan(a*x))/a^4/c^3
```

Rubi [A]
time = 0.36, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5084, 5088, 5090, 3393, 3383, 5024, 4491}

$$\frac{\mathrm{CosIntegral}(2\mathrm{ArcTan}(ax))}{2a^4c^3}-\frac{\mathrm{CosIntegral}(4\mathrm{ArcTan}(ax))}{2a^4c^3}-\frac{x}{a^3c^3\left(a^2x^2+1\right)\mathrm{ArcTan}(ax)}+\frac{x}{a^3c^3\left(a^2x^2+1\right)^2\mathrm{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] x/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - x/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + CosIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) - CosIntegral[4*ArcTan[a*x]]/
(2*a^4*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)} - \frac{\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}}}{a^3}$$

$$= \frac{x}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\right.}{a}$$

$$= \frac{x}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int\left(\frac{1}{2x} - \right.\right.}{}$$

$$= \frac{x}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\right.}{8}$$

$$= \frac{x}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)} + \frac{\text{Ci}(2\tan^{-1}(ax))}{2a^4c^3}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 0.97

$$\frac{-2a^3x^3 + (1+a^2x^2)^2\,\text{ArcTan}(ax)\text{CosIntegral}(2\text{ArcTan}(ax)) - (1+a^2x^2)^2\,\text{ArcTan}(ax)\text{CosIntegral}(4\text{ArcTan}(ax))}{2a^4c^3(1+a^2x^2)^2\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a^3*x^3 + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] - (1 +
 a^2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*a^4*c^3*(1 + a^2*x^2
)^2*ArcTan[a*x])
```

Maple [A]
time = 0.27, size = 58, normalized size = 0.67

method	result
derivativedivides	$\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax) - 4\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax) - 2\sin(2\arctan(ax)) + \sin(4\arctan}{8a^4c^3\arctan(ax)}$
default	$\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax) - 4\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax) - 2\sin(2\arctan(ax)) + \sin(4\arctan}{8a^4c^3\arctan(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^4/c^3*(4*Ci(2*arctan(a*x))*arctan(a*x)-4*Ci(4*arctan(a*x))*arctan(a*x
)-2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(x^3 + (a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((a^2*x^
4 - 3*x^2)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*
x)), x))/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.30, size = 292, normalized size = 3.40

$$-\frac{4\,a^3x^3 + (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*x^3 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x
^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a
^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6
*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a^4*x^4 + 2*a^2*x^2 +
 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - (a^4
*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^
2*x^2 + 1)))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.560 $\int \frac{x^2}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{a^3c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}-\frac{1}{a^3c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{\text{Si}(4\text{ArcTan}(ax))}{2a^3c^3}$$

```
[Out] 1/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-1/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*S
i(4*arctan(a*x))/a^3/c^3
```

Rubi [A]
time = 0.20, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5084, 5022, 5090, 4491, 12, 3380}

$$\frac{\text{Si}(4\text{ArcTan}(ax))}{2a^3c^3}-\frac{1}{a^3c^3\left(a^2x^2+1\right)\text{ArcTan}(ax)}+\frac{1}{a^3c^3\left(a^2x^2+1\right)^2\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 1/(a^3*c^3*(1 + a^2*x^2)*ArcTan[a
*x]) + SinIntegral[4*ArcTan[a*x]]/(2*a^3*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
```

```
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)}-\frac{1}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)}+\frac{4\int \frac{x}{(c+a^2cx^2)^3\tan}}{a}$$

$$= \frac{1}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)}-\frac{1}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)}-\frac{2\text{Subst}\left(\int \frac{\cos(x)}{}\right.}{}$$

$$= \frac{1}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)}-\frac{1}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)}-\frac{2\text{Subst}\left(\int \frac{\sin(2x}{2x}\right.}{}$$

$$= \frac{1}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)}-\frac{1}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\right.}{2}$$

$$= \frac{1}{a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)}-\frac{1}{a^3c^3(1+a^2x^2)\tan^{-1}(ax)}+\frac{\text{Si}(4\tan^{-1}(ax))}{2a^3c^3}$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 0.88

$$\frac{-2a^2x^2+(1+a^2x^2)^2\,\text{ArcTan}(ax)\text{Si}(4\text{ArcTan}(ax))}{2a^3c^3(1+a^2x^2)^2\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a^2*x^2 + (1 + a^2*x^2)^2*ArcTan[a*x]*SinIntegral[4*ArcTan[a*x]])/(2*a^
3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])
```

Maple [A]
time = 0.23, size = 37, normalized size = 0.55

method	result	size
derivativedivides	$\frac{4\,\mathrm{sinIntegral}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$	37
default	$\frac{4\,\mathrm{sinIntegral}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^3/c^3*(4*Si(4*arctan(a*x))*arctan(a*x)+cos(4*arctan(a*x))-1)/arctan(a
*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(2*(a^2*x^3 -
x)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x)
+ x^2)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.26, size = 196, normalized size = 2.93

$$-\frac{4\,a^{2}x^{2}-\left(i\,a^{4}x^{4}+2i\,a^{2}x^{2}+i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(\frac{a^{4}x^{4}+4i\,a^{3}x^{3}-6\,a^{2}x^{2}-4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)-\left(-i\,a^{4}x^{4}-2i\,a^{2}x^{2}-i\right)\arctan\left(ax\right)\mathrm{log\_integral}\left(\frac{a^{4}x^{4}-4i\,a^{3}x^{3}-6\,a^{2}x^{2}+4i\,ax+1}{a^{4}x^{4}+2\,a^{2}x^{2}+1}\right)}{4\left(a^{7}c^{3}x^{4}+2\,a^{5}c^{3}x^{2}+a^{3}c^{3}\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*x^2 - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a
^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1))
- (-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^
3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/((a^7*c^3*x^4
+ 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.561 $\int \frac{x}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=61

$$-\frac{x}{ac^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)}+\frac{\operatorname{CosIntegral}(2\operatorname{ArcTan}(ax))}{2a^2c^3}+\frac{\operatorname{CosIntegral}(4\operatorname{ArcTan}(ax))}{2a^2c^3}$$

```
[Out] -x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)+1/2*Ci(2*arctan(a*x))/a^2/c^3+1/2*Ci(4*a
rctan(a*x))/a^2/c^3
```

Rubi [A]
time = 0.19, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5088, 5090, 4491, 3383, 5024, 3393}

$$\frac{\operatorname{CosIntegral}(2\operatorname{ArcTan}(ax))}{2a^2c^3}+\frac{\operatorname{CosIntegral}(4\operatorname{ArcTan}(ax))}{2a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2 \operatorname{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]]/(2*a^
2*c^3) + CosIntegral[4*ArcTan[a*x]]/(2*a^2*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
```

```
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx = -\frac{x}{ac^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)}\,dx}{a} - (3a)\int \frac{}{(c+a^2cx^2}$$

$$= -\frac{x}{ac^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3} - \frac{3\text{Subst}}{}$$

$$= -\frac{x}{ac^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{3}{8x} + \frac{\cos(2x)}{2x} + \frac{\cos(4x)}{8x}\right)dx, x, \tan^{-}\right.}{a^2c^3}$$

$$= -\frac{x}{ac^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3} + \frac{3\text{Subst}}{}$$

$$= -\frac{x}{ac^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{\text{Ci}(2\tan^{-1}(ax))}{2a^2c^3} + \frac{\text{Ci}(4\tan^{-1}(ax))}{2a^2c^3}$$

Mathematica [A]

time = 0.05, size = 75, normalized size = 1.23

$$\frac{-2ax + (1+a^2x^2)^2\,\text{ArcTan}(ax)\text{CosIntegral}(2\text{ArcTan}(ax)) + (1+a^2x^2)^2\,\text{ArcTan}(ax)\text{CosIntegral}(4\text{ArcTan}(ax))}{2c^3(a+a^3x^2)^2\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] (-2*a*x + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] + (1 + a^2
*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*c^3*(a + a^3*x^2)^2*ArcT
an[a*x])
```

Maple [A]
time = 0.24, size = 60, normalized size = 0.98

method	result
derivativedivides	$\frac{4\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)+4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))-2\sin(2\arctan(a}{8a^2c^3\arctan(ax)}$
default	$\frac{4\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)+4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))-2\sin(2\arctan(a}{8a^2c^3\arctan(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^2/c^3*(4*Ci(4*arctan(a*x))*arctan(a*x)+4*Ci(2*arctan(a*x))*arctan(a*x
)-sin(4*arctan(a*x))-2*sin(2*arctan(a*x)))/arctan(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((3*a^2*x^2 -
1)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x)
+ x)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.13, size = 286, normalized size = 4.69

$$\frac{(a^4x^4+2\,a^2x^2+1)\arctan(ax)\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan(ax)\log\_integral\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan(ax)\log\_integral\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan(ax)\log\_integral\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-4\,ax}{4\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 + 4*I*a^3*
x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*
I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*
```

```
x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*
x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) -
 4*a*x)/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
[Out] Integral(x/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**2
*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\operatorname{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.562 $\int \frac{1}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^2} \, dx$

Optimal. Leaf size=58

$$-\frac{1}{ac^3 (1+a^2x^2)^2 \operatorname{ArcTan}(ax)} - \frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{ac^3} - \frac{\operatorname{Si}(4\operatorname{ArcTan}(ax))}{2ac^3}$$

```
[Out] -1/a/c^3/(a^2*x^2+1)^2/arctan(a*x)-Si(2*arctan(a*x))/a/c^3-1/2*Si(4*arctan(
a*x))/a/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5022, 5090, 4491, 3380}

$$-\frac{1}{ac^3 (a^2x^2+1)^2 \operatorname{ArcTan}(ax)} - \frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{ac^3} - \frac{\operatorname{Si}(4\operatorname{ArcTan}(ax))}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^
3) - SinIntegral[4*ArcTan[a*x]]/(2*a*c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx &= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-(4a)\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx\\ &= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{ac^3}\\ &= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{4\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x}+\frac{\sin(4x)}{8x}\right)\,dx,x,\tan^{-1}(ax\right.}{ac^3}\\ &= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{2ac^3}-\frac{\text{Subst}\left(\right.}{}\\ &= -\frac{1}{ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)}-\frac{\text{Si}\left(2\tan^{-1}(ax)\right)}{ac^3}-\frac{\text{Si}\left(4\tan^{-1}(ax)\right)}{2ac^3}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.78

$$-\frac{\frac{1}{(1+a^2x^2)^2\text{ArcTan}(ax)}+\text{Si}(2\text{ArcTan}(ax))+\frac{1}{2}\text{Si}(4\text{ArcTan}(ax))}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -((1/((1 + a^2*x^2)^2*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]] + SinIntegr
al[4*ArcTan[a*x]]/2)/(a*c^3))
```

Maple [A]
time = 0.22, size = 59, normalized size = 1.02

method	result
derivativedivides	$-\frac{8\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)+4\,\text{sinIntegral}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))+4\cos(2\arctan(ax}{8a\,c^3\arctan(ax)}$
default	$-\frac{8\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)+4\,\text{sinIntegral}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))+4\cos(2\arctan(ax}{8a\,c^3\arctan(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)+4*Si(4*arctan(a*x))*arctan(a*x)
+cos(4*arctan(a*x))+4*cos(2*arctan(a*x))+3)/arctan(a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(8*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)*integrate(1/2*x/((a
^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)), x) + 1)/((a
^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.04, size = 287, normalized size = 4.95

$$\frac{\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan\left(ax\right)\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan\left(ax\right)\log\_integral\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-2\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan\left(ax\right)\log\_integral\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-2\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan\left(ax\right)\log\_integral\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-4}{4\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)\arctan\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 + 4*I
*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^4*x^4
 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2
*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(I*a^4*x^4 + 2*I*a^2*x^2
 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*
(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x
 - 1)/(a^2*x^2 + 1)) - 4)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{6}\operatorname{atan}^{2}{\left(ax \right)}+3a^{4}x^{4}\operatorname{atan}^{2}{\left(ax \right)}+3a^{2}x^{2}\operatorname{atan}^{2}{\left(ax \right)}+\operatorname{atan}^{2}{\left(ax \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**2
*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.563 $\int \frac{1}{x(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{ac^3x\text{ArcTan}(ax)}+\frac{ax}{c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}+\frac{ax}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)}-\frac{3\text{CosIntegral}(2\text{ArcTan}(ax))}{2c^3}-\text{C}$$

```
[Out] -1/a/c^3/x/arctan(a*x)+a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)+a*x/c^3/(a^2*x^2+1
)/arctan(a*x)-3/2*Ci(2*arctan(a*x))/c^3-1/2*Ci(4*arctan(a*x))/c^3-Unintegra
ble(1/x^2/arctan(a*x),x)/a/c^3
```

Rubi [A]
time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c^3*x*ArcTan[a*x])) + (a*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (a*x
)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*CosIntegral[2*ArcTan[a*x]])/(2*c^3)
- CosIntegral[4*ArcTan[a*x]]/(2*c^3) - Defer[Int][1/(x^2*ArcTan[a*x]), x]/(
a*c^3)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} - a\int \frac{1}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)}\,dx + \left(3a^3\right)\int \frac{\quad}{(c}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}$$

$$= -\frac{1}{ac^3x\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)} + \frac{ax}{c^3\left(1+a^2x^2\right)\tan^{-1}(ax)}$$

Mathematica [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x)*integrate((5*a^2*x^2
+ 1)/((a^7*c^3*x^8 + 3*a^5*c^3*x^6 + 3*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)
), x) + 1)/((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{7}\operatorname{atan}^{2}(ax)+3a^{4}x^{5}\operatorname{atan}^{2}(ax)+3a^{2}x^{3}\operatorname{atan}^{2}(ax)+x\operatorname{atan}^{2}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**2 + 3*a**4*x**5*atan(a*x)**2 + 3*a**2*x**3
*atan(a*x)**2 + x*atan(a*x)**2), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\,(a\,x)^2\,(c\,a^2\,x^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.564 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{1}{ac^3x^2\text{ArcTan}(ax)}+\frac{a}{c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}+\frac{a}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{2a\text{Si}(2\text{ArcTan}(ax))}{c^3}+\frac{a\text{Si}(4\text{Arc}}{2}$$

[Out] -1/a/c^3/x^2/arctan(a*x)+a/c^3/(a^2*x^2+1)^2/arctan(a*x)+a/c^3/(a^2*x^2+1)/arctan(a*x)+2*a*Si(2*arctan(a*x))/c^3+1/2*a*Si(4*arctan(a*x))/c^3-2*Uninteg rable(1/x^3/arctan(a*x),x)/a/c^3

Rubi [A]
time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]

[Out] -(1/(a*c^3*x^2*ArcTan[a*x])) + a/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + a/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*SinIntegral[2*ArcTan[a*x]])/c^3 + (a*SinIntegral[4*ArcTan[a*x]])/(2*c^3) - (2*Defer[Int][1/(x^3*ArcTan[a*x]), x])/(a*c^3)

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \tan^{-1}(ax)^2}\,dx = -\left(a^2 \int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^2(c+a^2cx^2)^2 \tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + (4a^3) \int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)}\,dx + \frac{\int \frac{}{x^2(c\cdot}}{}$$

$$= -\frac{1}{ac^3x^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2) \tan^{-1}(ax}$$

$$= -\frac{1}{ac^3x^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2) \tan^{-1}(ax}$$

$$= -\frac{1}{ac^3x^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2) \tan^{-1}(ax}$$

$$= -\frac{1}{ac^3x^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2) \tan^{-1}(ax}$$

$$= -\frac{1}{ac^3x^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a}{c^3 (1+a^2x^2) \tan^{-1}(ax}$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)*integrate(2*(3*a^2*
x^2 + 1)/((a^7*c^3*x^9 + 3*a^5*c^3*x^7 + 3*a^3*c^3*x^5 + a*c^3*x^3)*arctan(
a*x)), x) + 1)/((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{8}\operatorname{atan}^{2}(ax)+3a^{4}x^{6}\operatorname{atan}^{2}(ax)+3a^{2}x^{4}\operatorname{atan}^{2}(ax)+x^{2}\operatorname{atan}^{2}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x)**2 + 3*a**4*x**6*atan(a*x)**2 + 3*a**2*x**4
*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^2 (c a^2 x^2 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.565 $\int \frac{1}{x^3(c+a^2cx^2)^3\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{1}{ac^3x^3\text{ArcTan}(ax)}+\frac{2a}{c^3x\text{ArcTan}(ax)}-\frac{a^3x}{c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}-\frac{2a^3x}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{5a^2\text{CosInteg}}{}$$

[Out] -1/a/c^3/x^3/arctan(a*x)+2*a/c^3/x/arctan(a*x)-a^3*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-2*a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)+5/2*a^2*Ci(2*arctan(a*x))/c^3+1/2*a^2*Ci(4*arctan(a*x))/c^3-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c^3+2*a*Unintegrable(1/x^2/arctan(a*x),x)/c^3

Rubi [A]
time = 0.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]

[Out] -(1/(a*c^3*x^3*ArcTan[a*x])) + (2*a)/(c^3*x*ArcTan[a*x]) - (a^3*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - (2*a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (5*a^2*CosIntegral[2*ArcTan[a*x]])/(2*c^3) + (a^2*CosIntegral[4*ArcTan[a*x]])/(2*c^3) - (3*Defer[Int][1/(x^4*ArcTan[a*x]), x])/(a*c^3) + (2*a*Defer[Int][1/(x^2*ArcTan[a*x]), x])/c^3

Rubi steps

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x (c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^3(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx}{c}$$

$$= a^4 \int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx + \frac{\int \frac{1}{x^3(c+a^2cx^2) \tan^{-1}(ax)^2} dx}{c^2} - 2\frac{a^2 \int \frac{}{x(c}}{}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + a^3 \int \frac{1}{(c+a^2cx^2)^3 \tan^{-}}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4 \tan^{-1}(ax)} dx}{ac^3} + \frac{a^2}{}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4 \tan^{-1}(ax)} dx}{ac^3} + \frac{a^2}{}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{3\int \frac{1}{x^4 \tan^{-1}(ax)} dx}{ac^3} + \frac{a^2}{}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a^2\mathrm{Ci}(2\tan^{-1}(ax))}{2c^3} + \frac{a}{}$$

$$= -\frac{1}{ac^3x^3 \tan^{-1}(ax)} - \frac{a^3x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} + \frac{a^2\mathrm{Ci}(2\tan^{-1}(ax))}{2c^3} + \frac{a}{}$$

Mathematica [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c x^2+c)^3 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x)*integrate((7*a^2*x^
2 + 3)/((a^7*c^3*x^10 + 3*a^5*c^3*x^8 + 3*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a
*x)), x) + 1)/((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(
a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^9\operatorname{atan}^2(ax)+3a^4x^7\operatorname{atan}^2(ax)+3a^2x^5\operatorname{atan}^2(ax)+x^3\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**9*atan(a*x)**2 + 3*a**4*x**7*atan(a*x)**2 + 3*a**2*x**5
*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.566 $\int \frac{1}{x^4(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{ac^3x^4\text{ArcTan}(ax)}+\frac{2a}{c^3x^2\text{ArcTan}(ax)}-\frac{a^3}{c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}-\frac{2a^3}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)}-\frac{3a^3\text{Si}(2\text{ArcT}}{c^3}$$

[Out] -1/a/c^3/x^4/arctan(a*x)+2*a/c^3/x^2/arctan(a*x)-a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-2*a^3/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Si(2*arctan(a*x))/c^3-1/2*a^3*Si(4*arctan(a*x))/c^3-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c^3+4*a*Unintegrable(1/x^3/arctan(a*x),x)/c^3

Rubi [A]
time = 0.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]

[Out] -(1/(a*c^3*x^4*ArcTan[a*x])) + (2*a)/(c^3*x^2*ArcTan[a*x]) - a^3/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - (2*a^3)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*a^3*SinIntegral[2*ArcTan[a*x]])/c^3 - (a^3*SinIntegral[4*ArcTan[a*x]])/(2*c^3) - (4*Defer[Int][1/(x^5*ArcTan[a*x]), x])/(a*c^3) + (4*a*Defer[Int][1/(x^3*ArcTan[a*x]), x])/c^3

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^4(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx}{c}$$

$$= a^4 \int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx + \frac{\int \frac{1}{x^4(c+a^2cx^2) \tan^{-1}(ax)^2} dx}{c^2} - 2\frac{a^2 \int \frac{}{x^2(}}{}$$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - (4a^5) \int \frac{x}{(c+a^2cx^2)^3 \mathrm{t}}$$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{4 \int \frac{1}{x^5 \tan^{-1}(ax)} dx}{ac^3} - \frac{(4}{}$$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{4 \int \frac{1}{x^5 \tan^{-1}(ax)} dx}{ac^3} - 2\Big($$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{4 \int \frac{1}{x^5 \tan^{-1}(ax)} dx}{ac^3} - \frac{a^3}{}$$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{a^3\mathrm{Si}(2\tan^{-1}(ax))}{c^3} - \frac{a}{}$$

$$= -\frac{1}{ac^3x^4 \tan^{-1}(ax)} - \frac{a^3}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{a^3\mathrm{Si}(2\tan^{-1}(ax))}{c^3} - \frac{a}{}$$

Mathematica [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^3 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x)*integrate(4*(2*a^2*
x^2 + 1)/((a^7*c^3*x^11 + 3*a^5*c^3*x^9 + 3*a^3*c^3*x^7 + a*c^3*x^5)*arctan
(a*x)), x) + 1)/((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arctan
(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{10}\operatorname{atan}^{2}(ax)+3a^{4}x^{8}\operatorname{atan}^{2}(ax)+3a^{2}x^{6}\operatorname{atan}^{2}(ax)+x^{4}\operatorname{atan}^{2}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(1/(a**6*x**10*atan(a*x)**2 + 3*a**4*x**8*atan(a*x)**2 + 3*a**2*x**
6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

<u>**Mupad**</u> [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \operatorname{atan}(a x)^2 (c a^2 x^2 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.567 $\int \frac{x\sqrt{c+a^2cx^2}}{\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\sqrt{c\,a^2\,x^2+c}}{\operatorname{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

3.568 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2,x)
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2, x)
```

3.569 $\int \frac{\sqrt{c+a^2cx^2}}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 2.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{x\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2), x)
```

3.570 $\int \frac{x(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

3.571 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{3/2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\text{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx = \int \frac{\left(c+a^2cx^2\right)^{3/2}}{\tan^{-1}(ax)^2} dx$$

Mathematica [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\text{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2,x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2, x)
```

3.572 $\int \frac{(c+a^2cx^2)^{3/2}}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 3.02, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{x\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2), x)
```

3.573 $\int \frac{x(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

3.574 $\int \frac{(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}{\operatorname{atan}^{2}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2,x)
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2, x)
```

3.575 $\int \frac{(c+a^2cx^2)^{5/2}}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 3.32, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)^2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**2,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2), x)
```

3.576 $\int \frac{x}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2},x\right)$$

[Out] Unintegrable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]

[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]

[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)^2\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.577 $\int \frac{1}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^2\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.578 $\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+a^2cx^2}}{acx\text{ArcTan}(ax)}-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)},x\right)}{a}$$

```
[Out] -(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x)/(a^2*
c*x^2+c)^(1/2),x)/a
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] -(Sqrt[c + a^2*c*x^2]/(a*c*x*ArcTan[a*x])) - Defer[Int][1/(x^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx=-\frac{\sqrt{c+a^2cx^2}}{acx\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,dx}{a}$$

Mathematica [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan\left(ax\right)^{2} \sqrt{a^{2} c\,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{c \left(a^{2} x^{2} + 1\right)} \operatorname{atan}^{2}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.579 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=101

$$\frac{x}{a^3c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{a^4c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2},x\right)}{a^2c}$$

```
[Out] x/a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a
^4/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x
)/a^2/c
```

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] x/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[
ArcTan[a*x]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx &= -\frac{\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{a^2c}\\
&= \frac{x}{a^3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx}{a^3}+\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{a^2c}\\
&= \frac{x}{a^3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{a^2c}-\frac{\sqrt{1+a^2x^2}\int}{a^3c\sqrt{}}\\
&= \frac{x}{a^3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{a^2c}-\frac{\sqrt{1+a^2x^2}\,\text{Su}}{}\\
&= \frac{x}{a^3c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{a^4c\sqrt{c+a^2cx^2}}+\frac{\int \frac{x}{\sqrt{c+a^2cx^2}}}{a^2c}
\end{aligned}$$

Mathematica [A]
time = 9.30, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.580 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=98

$$\frac{1}{a^3c\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\operatorname{Si}(\operatorname{ArcTan}(ax))}{a^3c\sqrt{c+a^2cx^2}}+\frac{\operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)^2},x\right)}{a^2c}$$

```
[Out] 1/a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a
^3/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x
)/a^2/c
```

Rubi [A]

time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[
ArcTan[a*x]])/(a^3*c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^2), x]/(a^2*c)
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx}{a}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\ \mathrm{ta}}}{a^2c}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}+\frac{\sqrt{1+a^2x^2}\int}{ac}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^2c}+\frac{\sqrt{1+a^2x^2}\ \mathrm{S}}{}$$

$$= \frac{1}{a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\operatorname{Si}(\tan^{-1}(ax))}{a^3c\sqrt{c+a^2cx^2}}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}}}{a^2}$$

Mathematica [A]
time = 8.27, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.581 $\int \frac{x}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{x}{ac\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\mathrm{ArcTan}(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] -x/a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^
2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5062, 5025, 5024, 3383}

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\mathrm{ArcTan}(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral
[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx &= -\frac{x}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx}{a}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{3/2}\tan^{-1}(ax)}\,dx}{ac\sqrt{c+a^2cx^2}}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx,x,\tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}\\ &= -\frac{x}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{a^2c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.80

$$\frac{-ax+\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{CosIntegral}(\text{ArcTan}(ax))}{a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] (-(a*x) + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*CosIntegral[ArcTan[a*x]])/(a^2*c*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 210, normalized size = 3.04

method	result
default	$-\frac{\left(\arctan(ax)\,\text{expIntegral}(1,-i\arctan(ax))a^2x^2+\text{expIntegral}(1,-i\arctan(ax))\arctan(ax)+\sqrt{a^2x^2+1}\,ax-i\sqrt{a^2x^2+1}\right.}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(
a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I
)*(I+a*x))^(1/2)/arctan(a*x)/a^2/c^2-1/2*(arctan(a*x)*Ei(1,I*arctan(a*x))*a
^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^
(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^2/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.582 $\int \frac{1}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{ac\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{ac\sqrt{c+a^2cx^2}}$$

```
[Out] -1/a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/
c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5022, 5091, 5090, 3380}

$$-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\mathrm{ArcTan}(ax)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral
[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx &= -\frac{1}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - a\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}\,dx\\ &= -\frac{1}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^{3/2}\tan^{-1}(ax)}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= -\frac{1}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\ &= -\frac{1}{ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}(\tan^{-1}(ax))}{ac\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 0.77

$$-\frac{1+\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)\text{Si}(\text{ArcTan}(ax))}{ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((1 + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*SinIntegral[ArcTan[a*x]])/(a*c*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 212, normalized size = 3.07

method	result
default	$\frac{i\left(\arctan(ax)\,\text{expIntegral}(1,i\arctan(ax))a^2x^2+\text{expIntegral}(1,i\arctan(ax))\arctan(ax)+\sqrt{a^2x^2+1}\,ax+i\sqrt{a^2x^2+1}\right)\sqrt{c}}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a
*x)+(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)
*(I+a*x))^(1/2)/arctan(a*x)/a/c^2-1/2*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a
^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)
^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.583 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=130

$$\frac{ax}{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\text{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{c\sqrt{c+a^2cx^2}}-\frac{\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{Ar}}\right.}{ac}$$

```
[Out] a*x/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(
a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)/a/c^2/x/arctan(a*x)-Unintegrable(1/x
^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/a/c
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] (a*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^2*x*Ar
cTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c
*x^2]) - Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx=-\left(a^2\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c}$$

$$=\frac{ax}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-a\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(a}$$

$$=\frac{ax}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,d}{ac}$$

$$=\frac{ax}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}\,d}{ac}$$

$$=\frac{ax}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^2x\tan^{-1}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.584 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=94

$$\frac{a}{c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}+\frac{a\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{c\sqrt{c+a^2cx^2}}+\frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2},x\right)}{c}$$

```
[Out] a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+a*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(
a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/
c
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] a/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*SinIntegral[Ar
cTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2), x]/c
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,d}{c}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+a^3\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)}\,dx+\frac{\int \frac{}{x^2\sqrt{}}}{}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c}+\frac{\left(a^3\sqrt{1+a\right.}}{}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c}+\frac{\left(a\sqrt{1+a^2\right.}}{}$$

$$= \frac{a}{c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)}+\frac{a\sqrt{1+a^2x^2}\,\mathrm{Si}(\tan^{-1}(ax))}{c\sqrt{c+a^2cx^2}}+\frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

Mathematica [A]
time = 2.57, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c \left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.585 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=160

$$-\frac{a^3x}{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}+\frac{a\sqrt{c+a^2cx^2}}{c^2x\text{ArcTan}(ax)}+\frac{a^2\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}}\right.}{c}$$

```
[Out] -a^3*x/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+a^2*Ci(arctan(a*x))*(a^2*x^2+1)^(1
/2)/c/(a^2*c*x^2+c)^(1/2)+a*(a^2*c*x^2+c)^(1/2)/c^2/x/arctan(a*x)+Unintegra
ble(1/x^3/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c+a*Unintegrable(1/x^2/arcta
n(a*x)/(a^2*c*x^2+c)^(1/2),x)/c
```

Rubi [A]
time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((a^3*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (a*Sqrt[c + a^2*c*x^2])/(c
^2*x*ArcTan[a*x]) + (a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(c*Sqr
t[c + a^2*c*x^2]) + Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x
]/c + (a*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2}}{c}$$

$$= a^4 \int \frac{x}{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} dx + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} dx}{c} - $$

$$= -\frac{a^3x}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2x\tan^{-1}(ax)} + a^3 \int \frac{1}{(c+a^2cx^2)^{3/2} \tan}$$

$$= -\frac{a^3x}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2x\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2} \tan^{-1}(a}}{c}$$

$$= -\frac{a^3x}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2x\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2} \tan^{-1}(a}}{c}$$

$$= -\frac{a^3x}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2x\tan^{-1}(ax)} + \frac{a^2\sqrt{1+a^2x^2}\,\mathrm{Ci}(\tan^{-1}(}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 3.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arcta
n(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^2\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.586 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3}{c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{a^3\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2},x\right)}{c}-\frac{a^2\text{Int}\left(\cdots\right.}{\cdots}$$

```
[Out] -a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-a^3*Si(arctan(a*x))*(a^2*x^2+1)^(1/2
)/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^4/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2
),x)/c-a^2*Unintegrable(1/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c
```

Rubi [A]
time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(a^3/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (a^3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2), x]/c - (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^2), x])/c
```

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} \, dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} \, dx\right) + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2}}{c}$$

$$= a^4 \int \frac{1}{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2} \, dx + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2}{}$$

$$= -\frac{a^3}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} - a^5 \int \frac{x}{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)} \, dx + \frac{\int \frac{}{x^4\sqrt{}}}{}$$

$$= -\frac{a^3}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2 \int \frac{}{x^2\sqrt{c}}}{}$$

$$= -\frac{a^3}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} \, dx}{c} - \frac{a^2 \int \frac{}{x^2\sqrt{c}}}{}$$

$$= -\frac{a^3}{c\sqrt{c+a^2cx^2} \tan^{-1}(ax)} - \frac{a^3\sqrt{1+a^2x^2}\, \text{Si}(\tan^{-1}(ax))}{c\sqrt{c+a^2cx^2}} + \frac{\int \frac{}{x^4\sqrt{c+}}}{}$$

Mathematica [A]
time = 3.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \text{ArcTan}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")

[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^2), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(c \left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^2\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)

[Out] Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)

[Out] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)

3.587 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=177

$$\frac{x^3}{a^3c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}+\frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{7\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4a^6c^2\sqrt{c+a^2cx^2}}+\frac{3\sqrt{1}}{}$$

```
[Out] x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+x/a^5/c^2/arctan(a*x)/(a^2*c*x^2+
c)^(1/2)-7/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+
3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+Uninteg
rable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

Rubi [A]
time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] x^3/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + x/(a^5*c^2*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) - (7*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*a^6*c^
2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(
4*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{}{\sqrt{c+a^2cx}}}{a}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a^2}}}{}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a^2}}}{}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a^2}}}{}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{a^6c^2\sqrt{}}$$

$$= \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{7\sqrt{1+a^2x}}{4a^6c^2}$$

Mathematica [A]
time = 11.26, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.588 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}+\frac{2}{a^5c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}+\frac{5\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{4a^5c^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x}}{4a^5c}$$

```
[Out] -1/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+2/a^5/c^2/arctan(a*x)/(a^2*c*x^2+c
)^(1/2)+5/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-3
/4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+Unintegr
able(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

Rubi [A]
time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + 2/(a^5*c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^5*c
^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/
(4*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^4} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan}}{a^4c}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx}{a^3} + \frac{\int \frac{}{\sqrt{c+a^2c}}}{a}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\Bigg(-\frac{}{a^5c^2\sqrt{}}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2}\,dx}{a^4c^2} - 2\Bigg(-\frac{}{a^5c^2\sqrt{}}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - 2\Bigg(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+}}{}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - 2\Bigg(-\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+}}{}$$

$$= -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\tan^{-1}(ax))}{4a^5c^2\sqrt{c+a^2cx^2}} - 2\Bigg(-\frac{}{a^5c^2\sqrt{}}$$

Mathematica [A]
time = 10.31, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.589 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=118

$$-\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)}+\frac{3\sqrt{1+a^2x^2}\,\operatorname{CosIntegral}(\operatorname{ArcTan}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x^2}\,\operatorname{CosIntegral}(3\operatorname{ArcTa}$$

```
[Out] -x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+3/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1
/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4
/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5062, 5091, 5090, 4491, 3383}

$$-\frac{x^3}{ac\operatorname{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}+\frac{3\sqrt{a^2x^2+1}\,\operatorname{CosIntegral}(\operatorname{ArcTan}(ax))}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\operatorname{CosIntegral}(3\operatorname{ArcTan}(ax))}{4a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (3*Sqrt[1 + a^2*x^2]*CosIn
tegral[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]
*CosIntegral[3*ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx &= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{3\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx}{a}\\
&= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{x^2}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{ac^2\sqrt{c+a^2cx^2}}\\
&= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x}\,dx,x\right.}{a^4c^2\sqrt{c+a^2cx^2}}\\
&= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\cos(x)}{4x}-\frac{\cos(3x)}{4x}\right)\right.}{a^4c^2\sqrt{c+a^2cx^2}}\\
&= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx,x,\tan^{-}\right.}{4a^4c^2\sqrt{c+a^2cx^2}}\\
&= -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)}+\frac{3\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x}}{4a^4c^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 82, normalized size = 0.69

$$\frac{-\frac{4a^3cx^3}{(1+a^2x^2)\text{ArcTan}(ax)}+3c\sqrt{1+a^2x^2}\,(\text{CosIntegral}(\text{ArcTan}(ax))-\text{CosIntegral}(3\text{ArcTan}(ax)))}{4a^4c^3\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]

[Out] ((-4*a^3*c*x^3)/((1 + a^2*x^2)*ArcTan[a*x]) + 3*c*Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]]))/(4*a^4*c^3*Sqrt[c + a^2*c*x^2])

Maple [C] Result contains complex when optimal does not.
time = 1.06, size = 582, normalized size = 4.93

method	result
default	$\frac{\left(3\arctan(ax)\,\text{expIntegral}(1,-3i\arctan(ax))a^4x^4+6\arctan(ax)\,\text{expIntegral}(1,-3i\arctan(ax))a^2x^2-\sqrt{a^2x^2+1}\,a^3x^3+3i\sqrt{a^2\right.}{8\sqrt{a^2x^2+1}\,(a^4x^4+}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/8*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-(a^2*x^2+1)^(1/2)*a^3*x^3+3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))+3*(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)/a^4/c^3+1/8*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4-(a^2*x^2+1)^(1/2)*a^3*x^3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)/a^4/c^3-3/8*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^4/c^3-3/8*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^4/c^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")

[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.590 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=142

$$\frac{1}{a^3c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}-\frac{1}{a^3c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}+\frac{3\sqrt{1+a^2x^2}\,\text{S}}{4a^3c^2\sqrt{}}$$

```
[Out] 1/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-1/a^3/c^2/arctan(a*x)/(a^2*c*x^2+c)
^(1/2)-1/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+3/
4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5084, 5022, 5091, 5090, 3380, 4491}

$$-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\text{ArcTan}(ax))}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{Si}(3\text{ArcTan}(ax))}{4a^3c^2\sqrt{a^2cx^2+c}}-\frac{1}{a^3c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}+\frac{1}{a^3c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] 1/(a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(a^3*c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*a^3*c^2*Sq
rt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*a^
3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{a^2} + \frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{a^2c}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{3\int \frac{}{(c+a^2cx^2}}{}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{a}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{a^3c^2\sqrt{}}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{a^3c^2\sqrt{}}$$

$$= \frac{1}{a^3c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{4a^3c^2\sqrt{}}$$

Mathematica [A]
time = 0.20, size = 99, normalized size = 0.70

$$-\frac{4a^2x^2+(1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)\mathrm{Si}(\mathrm{ArcTan}(ax)) - 3(1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)\mathrm{Si}(3\mathrm{ArcTan}(ax))}{4a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -1/4*(4*a^2*x^2 + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[ArcTan[a*x]]
- 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[3*ArcTan[a*x]])/(a^3*c^2*(1
 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] Result contains complex when optimal does not.
time = 1.13, size = 586, normalized size = 4.13

method	result
default	$-\frac{i\left(3\arctan(ax)\,\mathrm{expIntegral}(1,3i\arctan(ax))a^4x^4-\sqrt{a^2x^2+1}\,a^3x^3+6\arctan(ax)\,\mathrm{expIntegral}(1,3i\arctan(ax))a^2x^2-3i\sqrt{a^2\right.}{8\sqrt{a^2x^2+1}\,(a^4x^4+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4-(a^2*x^2+1)^(1/2)*a^3*x
^3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-3*I*(a^2*x^2+1)^(1/2)*a^2*x^
2+3*(a^2*x^2+1)^(1/2)*a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)
^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/a
rctan(a*x)/a^3/c^3+1/8*I*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*ar
ctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-(a^2*x^2+1)^(1/2)*a^3*x^3+3*I*(a^2
*x^2+1)^(1/2)*a^2*x^2+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))+3*(a^2*x^2+1)^(1
/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a
^4*x^4+2*a^2*x^2+1)/arctan(a*x)/a^3/c^3+1/8*I*(arctan(a*x)*Ei(1,I*arctan(a*
x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^3/c^3
-1/8*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arcta
n(a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x
-I)*(I+a*x))^(1/2)/arctan(a*x)/a^3/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.591 $\int \frac{x}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{x}{ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}+\frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\text{ArcTan}(a$$

```
[Out] -x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+1/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2
)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c
^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5088, 5091, 5090, 4491, 3383, 5025, 5024, 3393}

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{4a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegr
al[ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (3*Sqrt[1 + a^2*x^2]*Cos
Integral[3*ArcTan[a*x]])/(4*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx}{a} - (2a)\int \frac{}{(c+a^2}$$

$$= -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{ac^2\sqrt{c+a^2cx^2}} - \frac{(2a}{}$$

$$= -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \tan^{-1}(ax\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)\,dx\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \tan^{-1}(ax\right.}{4a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{Ci}(\tan^{-1}(ax))}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}}{4a^2c^2\sqrt{}}$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.82

$$\frac{-4ax+(1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)\text{CosIntegral}(\text{ArcTan}(ax))+3(1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)\text{CosIntegral}(3\text{ArcTan}(ax))}{4a^2c^2\,(1+a^2x^2)\,\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] (-4*a*x + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[ArcTan[a*x]] + 3*(1 +
 a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*(1 + a^2
*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.45, size = 582, normalized size = 5.02

method	result
default	$-\frac{\left(\arctan(ax)\,\text{expIntegral}(1,-i\arctan(ax))a^2x^2+\text{expIntegral}(1,-i\arctan(ax))\arctan(ax)+\sqrt{a^2x^2+1}\,ax-i\sqrt{a^2x^2+1}\right)}{8(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a^2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(
a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I
)*(I+a*x))^(1/2)/arctan(a*x)/a^2/c^3-1/8*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*
x))*a^4*x^4+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-(a^2*x^2+1)^(1/2)*
a^3*x^3+3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))+
3*(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(
I+a*x))^(1/2)/arctan(a*x)/(a^4*x^4+2*a^2*x^2+1)/a^2/c^3-1/8*(3*arctan(a*x)*
Ei(1,3*I*arctan(a*x))*a^4*x^4-(a^2*x^2+1)^(1/2)*a^3*x^3+6*arctan(a*x)*Ei(1,
3*I*arctan(a*x))*a^2*x^2-3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*
a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1
/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/(a^4*x^4+2*a^2*x^2+1)/a^2/c^3-1/8
*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(
a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a
*x))^(1/2)/arctan(a*x)/a^2/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3)*arctan(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.592 $\int \frac{1}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{ac\,(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)}-\frac{3\sqrt{1+a^2x^2}\,\operatorname{Si}(\operatorname{ArcTan}(ax))}{4ac^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{1+a^2x^2}\,\operatorname{Si}(3\operatorname{ArcTan}(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-3/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2
)/a/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(
a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5022, 5091, 5090, 4491, 3380}

$$-\frac{3\sqrt{a^2x^2+1}\,\operatorname{Si}(\operatorname{ArcTan}(ax))}{4ac^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{a^2x^2+1}\,\operatorname{Si}(3\operatorname{ArcTan}(ax))}{4ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\operatorname{ArcTan}(ax)\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*SinInte
gral[ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Sin
Integral[3*ArcTan[a*x]])/(4*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - (3a)\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx$$

$$= -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^{5/2}\tan^{-1}(ax)}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x\right.}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)\right.}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \tan^{-}\right.}{4ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a^2x^2}\,\text{Si}(\tan^{-1}(ax))}{4ac^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x}}{4ac^2\sqrt{}}$$

Mathematica [A]
time = 0.14, size = 61, normalized size = 0.53

$$\frac{-\frac{4}{\text{ArcTan}(ax)} - 3(1+a^2x^2)^{3/2}\left(\text{Si}(\text{ArcTan}(ax)) + \text{Si}(3\text{ArcTan}(ax))\right)}{4ac\,(c+a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]

[Out] (-4/ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinInte gral[3*ArcTan[a*x]]))/(4*a*c*(c + a^2*c*x^2)^(3/2))

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 586, normalized size = 5.10

method	result
default	$\frac{i\left(3\arctan(ax)\,\mathrm{expIntegral}(1,3i\arctan(ax))a^4x^4-\sqrt{a^2x^2+1}\,a^3x^3+6\arctan(ax)\,\mathrm{expIntegral}(1,3i\arctan(ax))a^2x^2-3i\sqrt{a^2x^2+1}\cdots\right.}{8\sqrt{a^2x^2+1}\,(a^4x^4+2a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/8*I*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4-(a^2*x^2+1)^(1/2)*a^3*x^3+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)/a/c^3-1/8*I*(3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-(a^2*x^2+1)^(1/2)*a^3*x^3+3*I*(a^2*x^2+1)^(1/2)*a^2*x^2+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))+3*(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)/a/c^3+3/8*I*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a/c^3-3/8*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)+(a^2*x^2+1)^(1/2)*a*x-I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a/c^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")

[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.593 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=199

$$\frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)}+\frac{ax}{c^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^3x\mathrm{ArcTan}(ax)}-\frac{5\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\mathrm{Arc}}{4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a*x/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/
2)-5/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Ci(3*a
rctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)/a
/c^3/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/a/
c^2
```

Rubi [A]
time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] (a*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + (a*x)/(c^2*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) - Sqrt[c + a^2*c*x^2]/(a*c^3*x*ArcTan[a*x]) - (5*Sqrt[1 + a^
2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 +
a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - Defer[In
t][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx\right)+\frac{\int \frac{1}{x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx + \left(2a^3\right)\int$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)}$$

$$= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\tan^{-1}(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\tan^{-1}(ax)}$$

Mathematica [A]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\mathrm{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.594 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=164

$$\frac{a}{c\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)}+\frac{a}{c^2\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)}+\frac{7a\sqrt{1+a^2x^2}\operatorname{Si}(\operatorname{ArcTan}(ax))}{4c^2\sqrt{c+a^2cx^2}}+\frac{3a\sqrt{1+a^2x^2}\operatorname{Si}}{4c^2\sqrt{c+}}$$

```
[Out] a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+7
/4*a*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+3/4*a*Si(3*a
rctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^2/ar
ctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2
```

Rubi [A]
time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] a/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) + a/(c^2*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + (3*a*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt
[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
/c^2
```

Rubi steps

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2}\,dx = -\left(a^2 \int \frac{1}{\left(c+a^2 c x^2\right)^{5/2} \tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2}\,dx}{c}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \left(3a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)}\,dx + \cdots$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{\cdots}{x^2\sqrt{c+a^2c\cdots}}}{c\cdots}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{\cdots}{x^2\sqrt{c+a^2c\cdots}}}{c\cdots}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{\cdots}{x^2\sqrt{c+a^2c\cdots}}}{c\cdots}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{a\sqrt{1+a^2x^2}\,\mathrm{S}\cdots}{c^2\sqrt{c+\cdots}}$$

$$= \frac{a}{c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{7a\sqrt{1+a^2x^2}\cdots}{4c^2\sqrt{c\cdots}}$$

Mathematica [A]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c \left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\operatorname{atan}\left(a\,x\right)}^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.595 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=238

$$-\frac{a^3x}{c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}-\frac{2a^3x}{c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}+\frac{2a\sqrt{c+a^2cx^2}}{c^3x\text{ArcTan}(ax)}+\frac{9a^2\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{Ar}}{4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -a^3*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3*x/c^2/arctan(a*x)/(a^2*c*x^2
+c)^(1/2)+9/4*a^2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)
+3/4*a^2*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+2*a*(a
^2*c*x^2+c)^(1/2)/c^3/x/arctan(a*x)+Unintegrable(1/x^3/arctan(a*x)^2/(a^2*c
*x^2+c)^(1/2),x)/c^2+2*a*Unintegrable(1/x^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)
,x)/c^2
```

Rubi [A]
time = 1.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -((a^3*x)/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3*x)/(c^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x]) + (2*a*Sqrt[c + a^2*c*x^2])/(c^3*x*ArcTan[a*x]) + (
9*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2
]) + (3*a^2*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]])/(4*c^2*Sqrt[c + a
^2*c*x^2]) + Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/c^2 +
 (2*a*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \tan^{-1}(ax)^2} dx = -\left(a^2 \int \frac{1}{x (c+a^2cx^2)^{5/2} \tan^{-1}(ax)^2} dx\right) + \frac{\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} d}{c}$$

$$= a^4 \int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2} dx + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} dx}{c^2} - 2$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + a^3 \int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)} dx - (2$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} dx}{c^2} - 2\left(\frac{}{c^2\sqrt{}}\right.$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} dx}{c^2} + \frac{(a^2\sqrt{1}}{}$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} dx}{c^2} - 2\left(\frac{}{c^2\sqrt{}}\right.$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^3\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} dx}{c^2} - 2\left(\frac{}{c^2\sqrt{}}\right.$$

$$= -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{a^2\sqrt{1+a^2x^2}\,\mathrm{Ci}(\tan^{-1}(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{3a^2\sqrt{1}}{}$$

Mathematica [A]
time = 5.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5
+ c^3*x^3)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^{2}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \operatorname{atan}(a x)^2 \left(c a^2 x^2 + c\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.596 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}-\frac{2a^3}{c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{11a^3\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{4c^2\sqrt{c+a^2cx^2}}-\frac{3a^3\sqrt{1+a^2x}}{4c^2\sqrt{}}$$

```
[Out] -a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3/c^2/arctan(a*x)/(a^2*c*x^2+c)^
(1/2)-11/4*a^3*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3/
4*a^3*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+Unintegra
ble(1/x^4/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2-2*a^2*Unintegrable(1/x^2
/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2
```

Rubi [A]
time = 1.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

```
[Out] -(a^3/(c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*a^3)/(c^2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (11*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(4*
c^2*Sqrt[c + a^2*c*x^2]) - (3*a^3*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*
x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2), x]/c^2 - (2*a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \tan^{-1}(ax)^2}\,dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \tan^{-1}(ax)^2}\,dx\right) + \frac{\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,d}{c}$$

$$= a^4 \int \frac{1}{(c+a^2cx^2)^{5/2} \tan^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} - 2$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - (3a^5)\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)}\,dx +$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} - 2\Bigg(\frac{}{c^2\sqrt{}}$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} - \frac{\left(3a^3\sqrt{}\right.}{}$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} - 2\Bigg(\frac{}{c^2\sqrt{}}$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{c^2} - 2\Bigg(\frac{}{c^2\sqrt{}}$$

$$= -\frac{a^3}{c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3a^3\sqrt{1+a^2x^2}\,\mathrm{Si}(\tan^{-1}(ax))}{4c^2\sqrt{c+a^2cx^2}} - \frac{3a^3\sqrt{1}}{}$$

Mathematica [A]
time = 5.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6
 + c^3*x^4)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x**4*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \operatorname{atan}(a x)^2 \left(c a^2 x^2 + c\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.597 $\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\mathbf{ArcTan}(cx))^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\text{ArcTan}(cx))^2},x\right)$$

```
[Out] Unintegrable((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\text{ArcTan}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
[Out] Defer[Int][Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\tan^{-1}(cx))^2}\,dx$$

Mathematica [A]
time = 19.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\text{ArcTan}(cx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
[Out] Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{fx}}{(c^2d\,x^2+d)^2\,(a+b\arctan(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

```
[Out] int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="max
ima")
```

```
[Out] 1/2*(2*(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)
^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x))*sqrt(f)*integrate(1/4*(a*c^
2*x^2 + 4*b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)*sqrt(x)/(a^3*c^4*d^2*x^4
 + 2*a^3*c^2*d^2*x^2 + a^3*d^2 + (b^3*c^4*d^2*x^4 + 2*b^3*c^2*d^2*x^2 + b^3
*d^2)*arctan(c*x)^3 + 3*(a*b^2*c^4*d^2*x^4 + 2*a*b^2*c^2*d^2*x^2 + a*b^2*d^
2)*arctan(c*x)^2 + 3*(a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^2*d^2*x^2 + a^2*b*d^2)*
arctan(c*x)), x) + sqrt(f)*x^(3/2))/(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d
^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(f*x)/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^
4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^4*d^2*x^4
 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)/(c**2*d*x**2+d)**2/(a+b*atan(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="gia
c")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{f\,x}}{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2), x)
```

3.598 $\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^3}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*x^m -
 arctan(a*x)*integrate(((a^8*c^3*m + 8*a^8*c^3)*x^8 + 4*(a^6*c^3*m + 6*a^6*
c^3)*x^6 + 6*(a^4*c^3*m + 4*a^4*c^3)*x^4 + c^3*m + 4*(a^2*c^3*m + 2*a^2*c^3
)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x^{m}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{2}x^{m}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{4}x^{m}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{6}x^{m}}{\operatorname{atan}^{2}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x)**2, x) + Integral(3*a**2*x**2*x**m/atan(a*x)*
*2, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**2, x) + Integral(a**6*x**6*x*
*m/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

3.599 $\int \frac{x^m (c+a^2 c x^2)^2}{\operatorname{ArcTan}(ax)^2} \, dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{x^m (c+a^2 c x^2)^2}{\operatorname{ArcTan}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2 c x^2)^2}{\operatorname{ArcTan}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)^2} \, dx = \int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)^2} \, dx$$

Mathematica [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2 c x^2)^2}{\operatorname{ArcTan}(ax)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c x^2 + c)^2}{\arctan(ax)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*x^m - arctan(a*x)*int
egrate(((a^6*c^2*m + 6*a^6*c^2)*x^6 + 3*(a^4*c^2*m + 4*a^4*c^2)*x^4 + c^2*m
 + 3*(a^2*c^2*m + 2*a^2*c^2)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m/atan(a*x)*
*2, x) + Integral(a**4*x**4*x**m/atan(a*x)**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

3.600 $\int \frac{x^m(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^4*c*x^4 + 2*a^2*c*x^2 + c)*x^m - arctan(a*x)*integrate(((a^4*c*m + 4*a
^4*c)*x^4 + 2*(a^2*c*m + 2*a^2*c)*x^2 + c*m)*x^m/(x*arctan(a*x)), x))/(a*ar
ctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

```
[Out] c*(Integral(x**m/atan(a*x)**2, x) + Integral(a**2*x**2*x**m/atan(a*x)**2, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

3.601 $\int \frac{x^m}{(c+a^2cx^2)\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=41

$$-\frac{x^m}{ac\text{ArcTan}(ax)}+\frac{m\text{Int}\left(\frac{x^{-1+m}}{\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] -x^m/a/c/arctan(a*x)+m*Unintegrable(x^(-1+m)/arctan(a*x),x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] -(x^m/(a*c*ArcTan[a*x])) + (m*Defer[Int][x^(-1 + m)/ArcTan[a*x], x])/(a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^2}\,dx=-\frac{x^m}{ac\tan^{-1}(ax)}+\frac{m\int \frac{x^{-1+m}}{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
[Out] (m*arctan(a*x)*integrate(x^m/(x*arctan(a*x)), x) - x^m)/(a*c*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^2x^2\,\mathrm{atan}^2\,(ax)+\mathrm{atan}^2\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**2,x)
[Out] Integral(x**m/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)),x)
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.602 $\int \frac{x^m}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx = \int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^2} dx$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] ((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(((a^2*m - 2*a^2)*x^2 + m)*x^m/
((a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)), x) - x^m)/((a^3*c^2*
x^2 + a*c^2)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x
)**2), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.603 $\int \frac{x^m}{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx = \int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx$$

Mathematica [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] ((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(((a^2*m - 4*a^
2)*x^2 + m)*x^m/((a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3*x)*ar
ctan(a*x)), x) - x^m)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x
**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.604 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

3.605 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\mathrm{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

3.606 $\int \frac{x^m \sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx = \int \frac{x^m \sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sqrt{c\,a^2\,x^2+c}}{\operatorname{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

3.607 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\arctan(ax)^2\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.608 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

```
[Out] Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^2\,{(c\,a^2\,x^2+c)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.609 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.610 $\int \frac{x(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*a^2*arctan(a*x)^2*integrate((15*a^4*c*x^5
 + 22*a^2*c*x^3 + 7*c*x)/arctan(a*x), x) + a*c*x + (5*a^6*c*x^6 + 11*a^4*c*
x^4 + 7*a^2*c*x^2 + c)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x}{\operatorname{atan}^{3}(ax)}\, dx + \int \frac{a^{2}x^{3}}{\operatorname{atan}^{3}(ax)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(x/atan(a*x)**3, x) + Integral(a**2*x**3/atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

```
[Out] int((x*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

3.611 $\int \frac{c+a^2cx^2}{\mathbf{ArcTan}(ax)^3} \, dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{c+a^2cx^2}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{a^2 c\, x^2+c}{\arctan\left(a x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^4*c*x^4 + 2*a^2*c*x^2 - 2*a*arctan(a*x)^2*integrate(2*(5*a^4*c*x^4
+ 6*a^2*c*x^2 + c)/arctan(a*x), x) + 4*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*ar
ctan(a*x) + c)/(a*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}^3\left(ax\right)}\,dx + \int \frac{1}{\operatorname{atan}^3\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**3, x) + Integral(atan(a*x)**(-3), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2 + c}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x)^3,x)
```

```
[Out] int((c + a^2*c*x^2)/atan(a*x)^3, x)
```

3.612 $\int \frac{c+a^2cx^2}{x\mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{c+a^2cx^2}{x\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3} dx = \int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{a^2 c\, x^2+c}{x \arctan\left(a x\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*x^2*arctan(a*x)^2*integrate((6*a^6*c*x^6
+ 5*a^4*c*x^4 + c)/(x^3*arctan(a*x)), x) + a*c*x + (3*a^6*c*x^6 + 5*a^4*c*x
^4 + a^2*c*x^2 - c)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{1}{x\operatorname{atan}^{3}\left(ax\right)}\,dx + \int \frac{a^{2}x}{\operatorname{atan}^{3}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**3,x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**3), x) + Integral(a**2*x/atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)^3),x)
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)^3), x)
```

3.613 $\int \frac{x(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*a^2*arctan(a*x)^2*int
egrate(2*(14*a^6*c^2*x^7 + 33*a^4*c^2*x^5 + 24*a^2*c^2*x^3 + 5*c^2*x)/arcta
n(a*x), x) + a*c^2*x + (7*a^8*c^2*x^8 + 22*a^6*c^2*x^6 + 24*a^4*c^2*x^4 + 1
0*a^2*c^2*x^2 + c^2)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{x}{\operatorname{atan}^{3}\left(ax\right)}\, dx + \int \frac{2a^{2}x^{3}}{\operatorname{atan}^{3}\left(ax\right)}\, dx + \int \frac{a^{4}x^{5}}{\operatorname{atan}^{3}\left(ax\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**3, x) + Integral(2*a**2*x**3/atan(a*x)**3, x) +
 Integral(a**4*x**5/atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

```
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

3.614 $\int \frac{(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - 2*a*arctan(a*x)^2*integ
rate(3*(7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x),
x) + c^2 + 6*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*arctan
(a*x))/(a*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**3, x) + Integral(a**4*x**4/atan(a*x)*
*3, x) + Integral(atan(a*x)**(-3), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x)^3,x)
[Out] int((c + a^2*c*x^2)^2/atan(a*x)^3, x)
```

3.615 $\int \frac{(c+a^2cx^2)^2}{x\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^3},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]

[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]

Maple [A]
time = 4.32, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*x^2*arctan(a*x)^2*int
egrate((15*a^8*c^2*x^8 + 28*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + c^2)/(x^3*arctan
(a*x)), x) + a*c^2*x + (5*a^8*c^2*x^8 + 14*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + 2
*a^2*c^2*x^2 - c^2)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{1}{x\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^3}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**3), x) + Integral(2*a**2*x/atan(a*x)**3, x)
+ Integral(a**4*x**3/atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)^3),x)
```

```
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)^3), x)
```

3.616 $\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx = \int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 3.78, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x
 - 2*a^2*arctan(a*x)^2*integrate((45*a^8*c^3*x^9 + 148*a^6*c^3*x^7 + 174*a^
4*c^3*x^5 + 84*a^2*c^3*x^3 + 13*c^3*x)/arctan(a*x), x) + (9*a^10*c^3*x^10 +
 37*a^8*c^3*x^8 + 58*a^6*c^3*x^6 + 42*a^4*c^3*x^4 + 13*a^2*c^3*x^2 + c^3)*a
rctan(a*x))/(a^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)^
3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{3}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{5}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{7}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**3, x) + Integral(3*a**2*x**3/atan(a*x)**3, x) +
 Integral(3*a**4*x**5/atan(a*x)**3, x) + Integral(a**6*x**7/atan(a*x)**3, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

```
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

3.617 $\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

Maple [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - 2*a*arc
tan(a*x)^2*integrate(4*(9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 1
2*a^2*c^3*x^2 + c^3)/arctan(a*x), x) + c^3 + 8*(a^9*c^3*x^9 + 4*a^7*c^3*x^7
 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))/(a*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^3,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{atan}^{3}\left(ax\right)}\, dx + \int \frac{3a^{4}x^{4}}{\operatorname{atan}^{3}\left(ax\right)}\, dx + \int \frac{a^{6}x^{6}}{\operatorname{atan}^{3}\left(ax\right)}\, dx + \int \frac{1}{\operatorname{atan}^{3}\left(ax\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**3, x) + Integral(3*a**4*x**4/atan(a*x
)**3, x) + Integral(a**6*x**6/atan(a*x)**3, x) + Integral(atan(a*x)**(-3),
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x)^3,x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x)^3, x)
```

3.618 $\int \frac{(c+a^2cx^2)^3}{x\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

Maple [A]
time = 6.39, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x
 - 2*x^2*arctan(a*x)^2*integrate((28*a^10*c^3*x^10 + 81*a^8*c^3*x^8 + 76*a^
6*c^3*x^6 + 22*a^4*c^3*x^4 + c^3)/(x^3*arctan(a*x)), x) + (7*a^10*c^3*x^10
+ 27*a^8*c^3*x^8 + 38*a^6*c^3*x^6 + 22*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*a
rctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x)
^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{1}{x\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^5}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**3), x) + Integral(3*a**2*x/atan(a*x)**3, x)
+ Integral(3*a**4*x**3/atan(a*x)**3, x) + Integral(a**6*x**5/atan(a*x)**3,
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)^3),x)
```

```
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)^3), x)
```

3.619 $\int \frac{x^3}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=43

$$-\frac{x^3}{2ac\text{ArcTan}(ax)^2}+\frac{3\text{Int}\left(\frac{x^2}{\text{ArcTan}(ax)^2},x\right)}{2ac}$$

```
[Out] -1/2*x^3/a/c/arctan(a*x)^2+3/2*Unintegrable(x^2/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*x^3/(a*c*ArcTan[a*x]^2) + (3*Defer[Int][x^2/ArcTan[a*x]^2, x])/(2*a*c)
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx=-\frac{x^3}{2ac\tan^{-1}(ax)^2}+\frac{3\int \frac{x^2}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^3 - 2*arctan(a*x)^2*integrate(3*(2*a^2*x^3 + x)/arctan(a*x), x) +
 3*(a^2*x^4 + x^2)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.620 $\int \frac{x^2}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=38

$$-\frac{x^2}{2ac\text{ArcTan}(ax)^2}+\frac{\text{Int}\left(\frac{x}{\text{ArcTan}(ax)^2},x\right)}{ac}$$

[Out] -1/2*x^2/a/c/arctan(a*x)^2+Unintegrable(x/arctan(a*x)^2,x)/a/c

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]

[Out] -1/2*x^2/(a*c*ArcTan[a*x]^2) + Defer[Int][x/ArcTan[a*x]^2, x]/(a*c)

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx=-\frac{x^2}{2ac\tan^{-1}(ax)^2}+\frac{\int \frac{x}{\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]

[Out] Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 - 2*arctan(a*x)^2*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + 2
*(a^2*x^3 + x)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^2x^2\,\mathrm{atan}^3\,(ax)+\mathrm{atan}^3\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.621 $\int \frac{x}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=37

$$-\frac{x}{2ac\text{ArcTan}(ax)^2}+\frac{\text{Int}\left(\frac{1}{\text{ArcTan}(ax)^2},x\right)}{2ac}$$

```
[Out] -1/2*x/a/c/arctan(a*x)^2+1/2*Unintegrable(1/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*x/(a*c*ArcTan[a*x]^2) + Defer[Int][ArcTan[a*x]^(-2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac\tan^{-1}(ax)^2}+\frac{\int \frac{1}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*arctan(a*x)^2*integrate(x/arctan(a*x), x) - a*x - (a^2*x^2 + 1)*
arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{2}x^{2}\operatorname{atan}^{3}(ax)+\operatorname{atan}^{3}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.622 $\int \frac{1}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{2ac\text{ArcTan}(ax)^2}$$

[Out] -1/2/a/c/arctan(a*x)^2

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{5004\}$

$$-\frac{1}{2ac\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]

[Out] -1/2*1/(a*c*ArcTan[a*x]^2)

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^3} dx = -\frac{1}{2ac\tan^{-1}(ax)^2}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2ac\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]

[Out] -1/2*1/(a*c*ArcTan[a*x]^2)

Maple [A]
time = 0.09, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{1}{2ac\arctan(ax)^2}$	15
default	$-\frac{1}{2ac\arctan(ax)^2}$	15
risch	$\frac{2}{ac(\ln(-iax+1)-\ln(iax+1))^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/c/arctan(a*x)^2
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$-\frac{1}{2\,ac\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a*c*arctan(a*x)^2)
```

Fricas [A]
time = 3.35, size = 14, normalized size = 0.88

$$-\frac{1}{2\,ac\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(a*c*arctan(a*x)^2)
```

Sympy [A]
time = 0.39, size = 14, normalized size = 0.88

$$-\frac{1}{2ac\,\mathrm{atan}^2\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] -1/(2*a*c*atan(a*x)**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.35, size = 14, normalized size = 0.88

$$-\frac{1}{2\,a\,c\,\mathrm{atan}\left(a\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

```
[Out] -1/(2*a*c*atan(a*x)^2)
```

3.623 $\int \frac{1}{x(c+a^2cx^2)\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2acx\text{ArcTan}(ax)^2}-\frac{\text{Int}\left(\frac{1}{x^2\text{ArcTan}(ax)^2},x\right)}{2ac}$$

```
[Out] -1/2/a/c/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*1/(a*c*x*ArcTan[a*x]^2) - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right) \arctan \left(a x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*arctan(a*x)^2*integrate(1/(x^3*arctan(a*x)), x) - a*x + (a^2*x^2
 + 1)*arctan(a*x))/(a^2*c*x^2*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**3 + x*atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x \operatorname{atan}(a x)^{3} \left(c a^{2} x^{2}+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.624 $\int \frac{1}{x^2(c+a^2cx^2)\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2acx^2\text{ArcTan}(ax)^2}-\frac{\text{Int}\left(\frac{1}{x^3\text{ArcTan}(ax)^2},x\right)}{ac}$$

```
[Out] -1/2/a/c/x^2/arctan(a*x)^2-Unintegrable(1/x^3/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*1/(a*c*x^2*ArcTan[a*x]^2) - Defer[Int][1/(x^3*ArcTan[a*x]^2), x]/(a*c)
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^2\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^3\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^3*arctan(a*x)^2*integrate((a^2*x^2 + 3)/(x^4*arctan(a*x)), x) - a*
x + 2*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^3*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.625 $\int \frac{1}{x^3(c+a^2cx^2)\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2acx^3\mathrm{ArcTan}(ax)^2}-\frac{3\mathrm{Int}\left(\frac{1}{x^4\mathrm{ArcTan}(ax)^2},x\right)}{2ac}$$

```
[Out] -1/2/a/c/x^3/arctan(a*x)^2-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*1/(a*c*x^3*ArcTan[a*x]^2) - (3*Defer[Int][1/(x^4*ArcTan[a*x]^2), x])/(
2*a*c)
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^3\tan^{-1}(ax)^2}-\frac{3\int \frac{1}{x^4\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^4*arctan(a*x)^2*integrate(3*(a^2*x^2 + 2)/(x^5*arctan(a*x)), x) -
a*x + 3*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^4*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2}x^{5}\operatorname{atan}^{3}{(ax)}+x^{3}\operatorname{atan}^{3}{(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.626 $\int \frac{1}{x^4(c+a^2cx^2)\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=41

$$-\frac{1}{2acx^4\text{ArcTan}(ax)^2}-\frac{2\text{Int}\left(\frac{1}{x^5\text{ArcTan}(ax)^2},x\right)}{ac}$$

```
[Out] -1/2/a/c/x^4/arctan(a*x)^2-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] -1/2*1/(a*c*x^4*ArcTan[a*x]^2) - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2), x])/(
a*c)
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx=-\frac{1}{2acx^4\tan^{-1}(ax)^2}-\frac{2\int \frac{1}{x^5\tan^{-1}(ax)^2}\,dx}{ac}$$

Mathematica [A]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^5*arctan(a*x)^2*integrate(2*(3*a^2*x^2 + 5)/(x^6*arctan(a*x)), x)
- a*x + 4*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^5*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^6\operatorname{atan}^3(ax)+x^4\operatorname{atan}^3(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**2*x**6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4 \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.627 $\int \frac{x^3}{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{x}{2a^3c^2\text{ArcTan}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{\text{Si}(2\text{ArcTan}(ax))}{a^4c^2}+\frac{\text{Int}(}{}$$

```
[Out] -1/2*x/a^3/c^2/arctan(a*x)^2+1/2*x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(-
a^2*x^2+1)/a^4/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^4/c^2+1/2*Un
integrable(1/arctan(a*x)^2,x)/a^3/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,
Rules used = {}

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x/(a^3*c^2*ArcTan[a*x]^2) + x/(2*a^3*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
+ (1 - a^2*x^2)/(2*a^4*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTa
n[a*x]]/(a^4*c^2) + Defer[Int][ArcTan[a*x]^(-2), x]/(2*a^3*c^2)
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx=-\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx}{a^2}+\frac{\int \frac{x}{\left(c+a^2cx^2\right)\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$=-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}}$$

$$=-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}}$$

$$=-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}}$$

$$=-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}}$$

$$=-\frac{x}{2a^3c^2\tan^{-1}(ax)^2}+\frac{x}{2a^3c^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2a^4c^2\left(1+a^2x^2\right)\tan^{-1}}$$

Mathematica [A]
time = 9.97, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2 \arctan(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
[Out] -1/2*(a*x^3 - 2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^4*x^5 +
2*a^2*x^3 + 3*x)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x)
+ (a^2*x^4 + 3*x^2)*arctan(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
[Out] integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.628 $\int \frac{x^2}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=71

$$-\frac{x^2}{2ac^2(1+a^2x^2)\,\text{ArcTan}(ax)^2}-\frac{x}{a^2c^2(1+a^2x^2)\,\text{ArcTan}(ax)}+\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{a^3c^2}$$

```
[Out] -1/2*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2-x/a^2/c^2/(a^2*x^2+1)/arctan(a*x)+
Ci(2*arctan(a*x))/a^3/c^2
```

Rubi [A]
time = 0.20, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5062, 5088, 5090, 3393, 3383, 5024}

$$\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{a^3c^2}-\frac{x^2}{2ac^2(a^2x^2+1)\,\text{ArcTan}(ax)^2}-\frac{x}{a^2c^2(a^2x^2+1)\,\text{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - x/(a^2*c^2*(1 + a^2*x^2)*Arc
Tan[a*x]) + CosIntegral[2*ArcTan[a*x]]/(a^3*c^2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
```

```
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^3}\,dx = -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}+\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-}}}{a^2}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\right.}{}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}-\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\right.\right.}{}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cos(2}{x}\right.}{}$$

$$= -\frac{x^2}{2ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^2}-\frac{x}{a^2c^2\left(1+a^2x^2\right)\tan^{-1}(ax)}+\frac{\text{Ci}(2\tan^{-1}(ax)}{a^3c^2}$$

Mathematica [A]

time = 0.07, size = 51, normalized size = 0.72

$$\frac{-\frac{ax(ax+2\text{ArcTan}(ax))}{(1+a^2x^2)\text{ArcTan}(ax)^2}+2\text{CosIntegral}(2\text{ArcTan}(ax))}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] (-((a*x*(a*x + 2*ArcTan[a*x]))/((1 + a^2*x^2)*ArcTan[a*x]^2)) + 2*CosIntegr
al[2*ArcTan[a*x]])/(2*a^3*c^2)
```

Maple [A]
time = 0.39, size = 52, normalized size = 0.73

method	result	size
derivativedivides	$\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$	52
default	$\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/4/a^3/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(
a*x)+cos(2*arctan(a*x))-1)/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
[Out] -1/2*(2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^6
*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x) + a*x^2 + 2*x*arctan(a
*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.51, size = 132, normalized size = 1.86

$$-\frac{a^2x^2-(a^2x^2+1)\arctan(ax)^2\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-(a^2x^2+1)\arctan(ax)^2\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)+2\,ax\arctan(ax)}{2\,(a^5c^2x^2+a^3c^2)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*
x - 1)/(a^2*x^2 + 1)) - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2
- 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x*arctan(a*x))/((a^5*c^2*x^2 + a^3*c^2)
*arctan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.629 $\int \frac{x}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{x}{2ac^2(1+a^2x^2)\operatorname{ArcTan}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\operatorname{ArcTan}(ax)}-\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{a^2c^2}$$

```
[Out] -1/2*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(a^2*x^2-1)/a^2/c^2/(a^2*x^2+1)/
arctan(a*x)-Si(2*arctan(a*x))/a^2/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5052, 5090, 4491, 12, 3380}

$$-\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{a^2c^2}-\frac{x}{2ac^2(a^2x^2+1)\operatorname{ArcTan}(ax)^2}-\frac{1-a^2x^2}{2a^2c^2(a^2x^2+1)\operatorname{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*c^2*(1 +
a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/(a^2*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
```

```
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\tan^{-1}(ax)} - 2\int \frac{}{(c+a^2cx}$$

$$= -\frac{x}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{2\text{Subst}\left(\int \frac{\cos(}{}\right.}{}$$

$$= -\frac{x}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{2\text{Subst}\left(\int \frac{\sin(}{2}\right.}{}$$

$$= -\frac{x}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(2}{x}\right.}{}$$

$$= -\frac{x}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Si}(2\tan^{-1}(ax}{a^2c^2}$$

Mathematica [A]
time = 0.04, size = 70, normalized size = 0.86

$$\frac{-ax+(-1+a^2x^2)\,\text{ArcTan}(ax)-2(1+a^2x^2)\,\text{ArcTan}(ax)^2\text{Si}(2\text{ArcTan}(ax))}{2a^2c^2(1+a^2x^2)\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] (-(a*x) + (-1 + a^2*x^2)*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*SinInt
egral[2*ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
```

Maple [A]
time = 0.35, size = 51, normalized size = 0.63

method	result	size
derivativedivides	$-\frac{4\,\mathrm{sinIntegral}(2\arctan(ax))\arctan(ax)^2+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^2c^2\arctan(ax)^2}$	51
default	$-\frac{4\,\mathrm{sinIntegral}(2\arctan(ax))\arctan(ax)^2+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^2c^2\arctan(ax)^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/c^2*(4*Si(2*arctan(a*x))*arctan(a*x)^2+2*cos(2*arctan(a*x))*arctan
(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(8*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate(1/2*x/((a^4*c^2*x^4
 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + a*x - (a^2*x^2 - 1)*arctan(a*x))
/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 5.87, size = 135, normalized size = 1.67

$$\frac{(-i\,a^2x^2-i)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(i\,a^2x^2+i)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-ax+(a^2x^2-1)\arctan(ax)}{2\,(a^4c^2x^2+a^2c^2)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(
a^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*
a*x - 1)/(a^2*x^2 + 1)) - a*x + (a^2*x^2 - 1)*arctan(a*x))/((a^4*c^2*x^2 +
a^2*c^2)*arctan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\,\mathrm{atan}^3(ax)+2a^2x^2\,\mathrm{atan}^3(ax)+\mathrm{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)**
3), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.630 $\int \frac{1}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=65

$$-\frac{1}{2ac^2(1+a^2x^2)\,\text{ArcTan}(ax)^2}+\frac{x}{c^2(1+a^2x^2)\,\text{ArcTan}(ax)}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{ac^2}$$

```
[Out] -1/2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2+x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arc
tan(a*x))/a/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5022, 5088, 5090, 3393, 3383, 5024}

$$\frac{x}{c^2(a^2x^2+1)\,\text{ArcTan}(ax)}-\frac{1}{2ac^2(a^2x^2+1)\,\text{ArcTan}(ax)^2}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) + x/(c^2*(1 + a^2*x^2)*ArcTan[a*
x]) - CosIntegral[2*ArcTan[a*x]]/(a*c^2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} - a\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^2}\,dx\\ &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{x}{c^2(1+a^2x^2)\tan^{-1}(ax)} + a^2\int \frac{x}{(c+a^2cx^2)}\\ &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{x}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,d\right.}{ac}\\ &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{x}{c^2(1+a^2x^2)\tan^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \right.\right.}{}\\ &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{x}{c^2(1+a^2x^2)\tan^{-1}(ax)} - 2\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\right.}{2}\\ &= -\frac{1}{2ac^2(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{x}{c^2(1+a^2x^2)\tan^{-1}(ax)} - \frac{\text{Ci}(2\tan^{-1}(ax))}{ac^2}\end{aligned}$$

Mathematica [A]

time = 0.05, size = 58, normalized size = 0.89

$$\frac{-1+2ax\text{ArcTan}(ax)-2(1+a^2x^2)\,\text{ArcTan}(ax)^2\text{CosIntegral}(2\text{ArcTan}(ax))}{2c^2\,(a+a^3x^2)\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] (-1 + 2*a*x*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*CosIntegral[2*ArcTa
n[a*x]])/(2*c^2*(a + a^3*x^2)*ArcTan[a*x]^2)
```

Maple [A]
time = 0.18, size = 52, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$	52
default	$-\frac{4\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(a
*x)+cos(2*arctan(a*x))+1)/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^4*c^
2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 2*a*x*arctan(a*x) - 1)/((a^
3*c^2*x^2 + a*c^2)*arctan(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 4.20, size = 122, normalized size = 1.88

$$-\frac{(a^2x^2+1)\arctan(ax)^2\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan(ax)^2\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-2\,ax\arctan(ax)+1}{2\,(a^3c^2x^2+ac^2)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^
2*x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x
- 1)/(a^2*x^2 + 1)) - 2*a*x*arctan(a*x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(
a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)**
3), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.631 $\int \frac{1}{x(c+a^2cx^2)^2\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{2ac^2x\text{ArcTan}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\text{ArcTan}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\text{ArcTan}(ax)}+\frac{\text{Si}(2\text{ArcTan}(ax))}{c^2}-\frac{\text{Int}\left(\frac{\ }{x^2}\right.}{}$$

```
[Out] -1/2/a/c^2/x/arctan(a*x)^2+1/2*a*x/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(-a^2*
x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/c^2-1/2*Unintegrable(1
/x^2/arctan(a*x)^2,x)/a/c^2
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(c+a^2cx^2)^2\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^2*x*ArcTan[a*x]^2) + (a*x)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
+ (1 - a^2*x^2)/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*
x]]/c^2 - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx &= -\left(a^2\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx}{c}\\
&= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\tan^{-1}(}\\
&= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\tan^{-1}(}\\
&= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\tan^{-1}(}\\
&= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\tan^{-1}(}\\
&= -\frac{1}{2ac^2x\tan^{-1}(ax)^2}+\frac{ax}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2}+\frac{1-a^2x^2}{2c^2(1+a^2x^2)\tan^{-1}(}
\end{aligned}$$

Mathematica [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)^2*integrate((3*a^4*x^4 + 2*a
^2*x^2 + 1)/((a^6*c^2*x^7 + 2*a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)), x) -
 a*x + (3*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x
)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^5\operatorname{atan}^3(ax)+2a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**3 + 2*a**2*x**3*atan(a*x)**3 + x*atan(a*x)
**3), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\operatorname{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.632 $\int \frac{1}{x^2(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{2ac^2x^2\text{ArcTan}(ax)^2}+\frac{a}{2c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)^2}-\frac{a^2x}{c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)}+\frac{a\text{CosIntegral}(2\text{ArcTan}(ax)}{c^2}$$

```
[Out] -1/2/a/c^2/x^2/arctan(a*x)^2+1/2*a/c^2/(a^2*x^2+1)/arctan(a*x)^2-a^2*x/c^2/
(a^2*x^2+1)/arctan(a*x)+a*Ci(2*arctan(a*x))/c^2-Unintegrable(1/x^3/arctan(a
*x)^2,x)/a/c^2
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^2*x^2*ArcTan[a*x]^2) + a/(2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) -
(a^2*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) + (a*CosIntegral[2*ArcTan[a*x]])/c^
2 - Defer[Int][1/(x^3*ArcTan[a*x]^2), x]/(a*c^2)
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx = -\left(a^2 \int \frac{1}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx\right) + \frac{\int \frac{1}{x^2(c+a^2cx^2)\tan^{-1}(ax)^3} dx}{c}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} + a^3 \int \frac{x}{(c+a^2cx^2)^2 \mathrm{t}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \tan^{-1}}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \tan^{-1}}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \tan^{-1}}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \tan^{-1}}$$
$$= -\frac{1}{2ac^2x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \tan^{-1}(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \tan^{-1}}$$

Mathematica [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c)^2 \arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)^2*integrate((6*a^4*x^4 + 7*a
^2*x^2 + 3)/((a^6*c^2*x^8 + 2*a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)), x) -
 a*x + 2*(2*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a
*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^6\operatorname{atan}^3(ax)+2a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**6*atan(a*x)**3 + 2*a**2*x**4*atan(a*x)**3 + x**2*atan(a
*x)**3), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.633 $\int \frac{1}{x^3(c+a^2cx^2)^2\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{1}{2ac^2x^3\text{ArcTan}(ax)^2}+\frac{a}{2c^2x\text{ArcTan}(ax)^2}-\frac{a^3x}{2c^2(1+a^2x^2)\text{ArcTan}(ax)^2}-\frac{a^2(1-a^2x^2)}{2c^2(1+a^2x^2)\text{ArcTan}(ax)}-\frac{a^2\text{Si}(2$$

```
[Out] -1/2/a/c^2/x^3/arctan(a*x)^2+1/2*a/c^2/x/arctan(a*x)^2-1/2*a^3*x/c^2/(a^2*x
^2+1)/arctan(a*x)^2-1/2*a^2*(-a^2*x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)-a^2*Si
(2*arctan(a*x))/c^2-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c^2+1/2*a*Uni
ntegrable(1/x^2/arctan(a*x)^2,x)/c^2
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3(c+a^2cx^2)^2\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^2*x^3*ArcTan[a*x]^2) + a/(2*c^2*x*ArcTan[a*x]^2) - (a^3*x)/(2*c
^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a^2*(1 - a^2*x^2))/(2*c^2*(1 + a^2*x^2)*
ArcTan[a*x]) - (a^2*SinIntegral[2*ArcTan[a*x]])/c^2 - (3*Defer[Int][1/(x^4*
ArcTan[a*x]^2), x])/(2*a*c^2) + (a*Defer[Int][1/(x^2*ArcTan[a*x]^2), x])/(2
*c^2)
```

Rubi steps

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx = -\left(a^2 \int \frac{1}{x (c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx\right) + \frac{\int \frac{1}{x^3(c+a^2cx^2)\tan^{-1}(ax)^3} dx}{c}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + a^4 \int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx - \frac{3\int \frac{1}{x^4\tan^{-1}(ax}}{2ac^2}$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + \frac{a}{2c^2x\tan^{-1}(ax)^2} - \frac{a^3x}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + \frac{a}{2c^2x\tan^{-1}(ax)^2} - \frac{a^3x}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + \frac{a}{2c^2x\tan^{-1}(ax)^2} - \frac{a^3x}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + \frac{a}{2c^2x\tan^{-1}(ax)^2} - \frac{a^3x}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

$$= -\frac{1}{2ac^2x^3\tan^{-1}(ax)^2} + \frac{a}{2c^2x\tan^{-1}(ax)^2} - \frac{a^3x}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

Mathematica [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.39, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^2 \arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)^2*integrate(2*(5*a^4*x^4 + 7
*a^2*x^2 + 3)/((a^6*c^2*x^9 + 2*a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)), x)
 - a*x + (5*a^2*x^2 + 3)*arctan(a*x))/((a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a
*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^7\operatorname{atan}^3(ax)+2a^2x^5\operatorname{atan}^3(ax)+x^3\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**3 + 2*a**2*x**5*atan(a*x)**3 + x**3*atan(a
*x)**3), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.634 $\int \frac{1}{x^4(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{2ac^2x^4\mathrm{ArcTan}(ax)^2}+\frac{a}{2c^2x^2\mathrm{ArcTan}(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\mathrm{ArcTan}(ax)^2}+\frac{a^4x}{c^2\left(1+a^2x^2\right)\mathrm{ArcTan}(ax)}-\frac{a^3\mathrm{Cos}}{}$$

```
[Out] -1/2/a/c^2/x^4/arctan(a*x)^2+1/2*a/c^2/x^2/arctan(a*x)^2-1/2*a^3/c^2/(a^2*x
^2+1)/arctan(a*x)^2+a^4*x/c^2/(a^2*x^2+1)/arctan(a*x)-a^3*Ci(2*arctan(a*x))
/c^2-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c^2+a*Unintegrable(1/x^3/arcta
n(a*x)^2,x)/c^2
```

Rubi [A]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^2*x^4*ArcTan[a*x]^2) + a/(2*c^2*x^2*ArcTan[a*x]^2) - a^3/(2*c^2
*(1 + a^2*x^2)*ArcTan[a*x]^2) + (a^4*x)/(c^2*(1 + a^2*x^2)*ArcTan[a*x]) - (
a^3*CosIntegral[2*ArcTan[a*x]])/c^2 - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2),
x])/(a*c^2) + (a*Defer[Int][1/(x^3*ArcTan[a*x]^2), x])/c^2
```

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx\right) + \frac{\int \frac{1}{x^4(c+a^2cx^2)\tan^{-1}(ax)^3} dx}{c}$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + a^4 \int \frac{1}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx - \frac{2\int \frac{1}{x^5 \tan^{-1}(ax}$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} -$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} +$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} +$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} +$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} +$$

$$= -\frac{1}{2ac^2x^4 \tan^{-1}(ax)^2} + \frac{a}{2c^2x^2 \tan^{-1}(ax)^2} - \frac{a^3}{2c^2(1+a^2x^2)\tan^{-1}(ax)^2} +$$

Mathematica [A]
time = 4.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c x^2+c)^2 \arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")

[Out] 1/2*(2*(a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)^2*integrate((15*a^4*x^4 + 23*a^2*x^2 + 10)/((a^6*c^2*x^10 + 2*a^4*c^2*x^8 + a^2*c^2*x^6)*arctan(a*x)), x) - a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x))/((a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)^2)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^3), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^8\operatorname{atan}^3(ax)+2a^2x^6\operatorname{atan}^3(ax)+x^4\operatorname{atan}^3(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**3,x)

[Out] Integral(1/(a**4*x**8*atan(a*x)**3 + 2*a**2*x**6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c**2

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.635 $\int \frac{x^3}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=177

$$\frac{x}{2a^3c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^2}-\frac{x}{2a^3c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)^2}+\frac{2}{a^4c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)}-\frac{}{2a^4c^3\left(1+a^2\right.}$$

```
[Out] 1/2*x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2*x/a^3/c^3/(a^2*x^2+1)/arctan(
a*x)^2+2/a^4/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/a^4/c^3/(a^2*x^2+1)/arctan(a
*x)+1/2*(a^2*x^2-1)/a^4/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x))/a
^4/c^3+Si(4*arctan(a*x))/a^4/c^3
```

Rubi [A]
time = 0.44, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5084, 5052, 5090, 4491, 12, 3380, 5088, 5022}

$$-\frac{\text{Si}(2\text{ArcTan}(ax))}{2a^4c^3}+\frac{\text{Si}(4\text{ArcTan}(ax))}{a^4c^3}-\frac{1-a^2x^2}{2a^4c^3\left(a^2x^2+1\right)\text{ArcTan}(ax)}-\frac{3}{2a^4c^3\left(a^2x^2+1\right)\text{ArcTan}(ax)}+\frac{2}{a^4c^3\left(a^2x^2+1\right)^2\text{ArcTan}(ax)}-\frac{x}{2a^3c^3\left(a^2x^2+1\right)\text{ArcTan}(ax)^2}+\frac{x}{2a^3c^3\left(a^2x^2+1\right)^2\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] x/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - x/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) + 2/(a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3/(2*a^4*c^3*(1 +
a^2*x^2)*ArcTan[a*x]) - (1 - a^2*x^2)/(2*a^4*c^3*(1 + a^2*x^2)*ArcTan[a*x])
 - SinIntegral[2*ArcTan[a*x]]/(2*a^4*c^3) + SinIntegral[4*ArcTan[a*x]]/(a^4
*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{1-}{2a^4c^3(1+a^2}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{2a^4c^3(1+a^2}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{a^4c^3(1+a^2x}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{a^4c^3(1+a^2x}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{a^4c^3(1+a^2x}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{a^4c^3(1+a^2x}$$

$$= \frac{x}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{}{a^4c^3(1+a^2x}$$

Mathematica [A]
time = 0.17, size = 72, normalized size = 0.41

$$\frac{\frac{a^2x^2\left(-ax+(-3+a^2x^2)\text{ArcTan}(ax)\right)}{(1+a^2x^2)^2\text{ArcTan}(ax)^2} - \text{Si}(2\text{ArcTan}(ax)) + 2\text{Si}(4\text{ArcTan}(ax))}{2a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] ((a^2*x^2*(-(a*x) + (-3 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*
x]^2) - SinIntegral[2*ArcTan[a*x]] + 2*SinIntegral[4*ArcTan[a*x]])/(2*a^4*c
^3)
```

Maple [A]
time = 0.42, size = 90, normalized size = 0.51

method	result

derivativedivides	$-\frac{8\,\mathrm{sinIntegral}(2\arctan(ax))\arctan(ax)^2-16\,\mathrm{sinIntegral}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)-4}{16a^4c^3\arctan(ax)^2}$
default	$-\frac{8\,\mathrm{sinIntegral}(2\arctan(ax))\arctan(ax)^2-16\,\mathrm{sinIntegral}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)-4}{16a^4c^3\arctan(ax)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a^4/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)^2-16*Si(4*arctan(a*x))*arcta
n(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)-4*cos(4*arctan(a*x))*arctan(a*x)+
2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^3 + 2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integ
rate((5*a^2*x^3 - 3*x)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2*
c^3)*arctan(a*x)), x) - (a^2*x^4 - 3*x^2)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^
4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.54, size = 328, normalized size = 1.85

$$-\frac{2\,a^3x^3+2\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)-2\left(a^4x^4-3\,a^2x^2\right)\arctan(ax)}{4\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*x^3 + 2*(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integr
al((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 +
 1)) + 2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x^4
- 4*I*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (-I*a
^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x -
1)/(a^2*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integra
l(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*(a^4*x^4 - 3*a^2*x^2)*arctan(
a*x))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x
**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.636 $\int \frac{x^2}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^3} \, dx$

Optimal. Leaf size=120

$$\frac{1}{2a^3c^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\operatorname{ArcTan}(ax)^2} - \frac{2x}{a^2c^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)} + \frac{}{a^2c^3(1+a^2}$$

```
[Out] 1/2/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2/a^3/c^3/(a^2*x^2+1)/arctan(a*x)
^2-2*x/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)+x/a^2/c^3/(a^2*x^2+1)/arctan(a*x)+
Ci(4*arctan(a*x))/a^3/c^3
```

Rubi [A]
time = 0.41, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5084, 5022, 5088, 5090, 3393, 3383, 5024, 4491}

$$\frac{\operatorname{CosIntegral}(4\operatorname{ArcTan}(ax))}{a^3c^3} + \frac{x}{a^2c^3(a^2x^2+1)\operatorname{ArcTan}(ax)} - \frac{2x}{a^2c^3(a^2x^2+1)^2\operatorname{ArcTan}(ax)} - \frac{1}{2a^3c^3(a^2x^2+1)\operatorname{ArcTan}(ax)^2} + \frac{1}{2a^3c^3(a^2x^2+1)^2\operatorname{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 1/(2*a^3*c^3*(1 + a^2*x^2)*Ar
cTan[a*x]^2) - (2*x)/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + x/(a^2*c^3*(1
+ a^2*x^2)*ArcTan[a*x]) + CosIntegral[4*ArcTan[a*x]]/(a^3*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} + \frac{2\int \frac{}{(c+a^2cx^2}}{}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{}{a^2c^3(1+a}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{}{a^2c^3(1+a}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{}{a^2c^3(1+a}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{}{a^2c^3(1+a}$$

$$= \frac{1}{2a^3c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\tan^{-1}(ax)^2} - \frac{}{a^2c^3(1+a}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.50

$$\frac{\frac{ax\left(-ax+2(-1+a^2x^2)\text{ArcTan}(ax)\right)}{(1+a^2x^2)^2\text{ArcTan}(ax)^2} + 2\text{CosIntegral}(4\text{ArcTan}(ax))}{2a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] ((a*x*(-(a*x) + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*x]
^2) + 2*CosIntegral[4*ArcTan[a*x]])/(2*a^3*c^3)
```

Maple [A]
time = 0.42, size = 52, normalized size = 0.43

method	result	size
derivativedivides	$\frac{16\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)^2-4\sin(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{16a^3c^3\arctan(ax)^2}$	52
default	$\frac{16\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)^2-4\sin(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{16a^3c^3\arctan(ax)^2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a^3/c^3*(16*Ci(4*arctan(a*x))*arctan(a*x)^2-4*sin(4*arctan(a*x))*arcta
n(a*x)+cos(4*arctan(a*x))-1)/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((a^4
*x^4 - 6*a^2*x^2 + 1)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2*c
^3)*arctan(a*x)), x) - a*x^2 + 2*(a^2*x^3 - x)*arctan(a*x))/((a^6*c^3*x^4 +
 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.25, size = 215, normalized size = 1.79

$$-\frac{a^2x^2 - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^2 \operatorname{log\_integral}\left(\frac{a^4x^4 + 4i\,a^3x^3 - 6\,a^2x^2 - 4i\,ax + 1}{a^4x^4 + 2\,a^2x^2 + 1}\right) - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^2 \operatorname{log\_integral}\left(\frac{a^4x^4 - 4i\,a^3x^3 - 6\,a^2x^2 + 4i\,ax + 1}{a^4x^4 + 2\,a^2x^2 + 1}\right) - 2\,(a^3x^3 - ax)\arctan(ax)}{2\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)\arctan(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x
^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a
^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 -
 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(a^3*x^3 - a*x)*ar
ctan(a*x))/((a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^3(ax) + 3a^4x^4 \operatorname{atan}^3(ax) + 3a^2x^2 \operatorname{atan}^3(ax) + \operatorname{atan}^3(ax)}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x
**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.637 $\int \frac{x}{(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^3} dx$

Optimal. Leaf size=113

$$-\frac{x}{2ac^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)^2}-\frac{2}{a^2c^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)}+\frac{3}{2a^2c^3(1+a^2x^2) \operatorname{ArcTan}(ax)}-\frac{\operatorname{Si}(2\operatorname{ArcTan}(}{2a^2c^3}$$

```
[Out] -1/2*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-2/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x
)+3/2/a^2/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x))/a^2/c^3-Si(4*ar
ctan(a*x))/a^2/c^3
```

Rubi [A]

time = 0.32, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5088, 5084, 5022, 5090, 4491, 12, 3380}

$$-\frac{\operatorname{Si}(2\operatorname{ArcTan}(ax))}{2a^2c^3}-\frac{\operatorname{Si}(4\operatorname{ArcTan}(ax))}{a^2c^3}-\frac{x}{2ac^3(a^2x^2+1)^2 \operatorname{ArcTan}(ax)^2}+\frac{3}{2a^2c^3(a^2x^2+1) \operatorname{ArcTan}(ax)}-\frac{2}{a^2c^3(a^2x^2+1)^2 \operatorname{ArcTan}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 2/(a^2*c^3*(1 + a^2*x^2)^2*A
rcTan[a*x]) + 3/(2*a^2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTa
n[a*x]]/(2*a^2*c^3) - SinIntegral[4*ArcTan[a*x]]/(a^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
```

```
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx}{2a} - \frac{1}{2}(3a)\int \frac{}{(c+a^2c}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} - 2\int \frac{}{(c+a^2}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{}{2a^2c^3(1+a^2}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{}{2a^2c^3(1+a^2}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{}{2a^2c^3(1+a^2}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{}{2a^2c^3(1+a^2}$$

$$= -\frac{x}{2ac^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\tan^{-1}(ax)} + \frac{}{2a^2c^3(1+a^2}$$

Mathematica [A]
time = 0.14, size = 98, normalized size = 0.87

$$-\frac{ax+\text{ArcTan}(ax)-3a^2x^2\text{ArcTan}(ax)+(1+a^2x^2)^2\,\text{ArcTan}(ax)^2\text{Si}(2\text{ArcTan}(ax))+2(1+a^2x^2)^2\,\text{ArcTan}(ax)^2\text{Si}(4\text{ArcTan}(ax))}{2a^2c^3\,(1+a^2x^2)^2\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*(a*x + ArcTan[a*x] - 3*a^2*x^2*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*
x]^2*SinIntegral[2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*SinIntegr
al[4*ArcTan[a*x]])/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

Maple [A]
time = 0.36, size = 88, normalized size = 0.78

method	result
derivativedivides	$-\frac{16\,\text{sinIntegral}(4\arctan(ax))\arctan(ax)^2+8\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)^2+4\cos(4\arctan(ax))\arctan(ax)+4\,\text{c}}{16a^2c^3\arctan(ax)^2}$
default	$-\frac{16\,\text{sinIntegral}(4\arctan(ax))\arctan(ax)^2+8\,\text{sinIntegral}(2\arctan(ax))\arctan(ax)^2+4\cos(4\arctan(ax))\arctan(ax)+4\,\text{c}}{16a^2c^3\arctan(ax)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)

[Out] -1/16/a^2/c^3*(16*Si(4*arctan(a*x))*arctan(a*x)^2+8*Si(2*arctan(a*x))*arcta
n(a*x)^2+4*cos(4*arctan(a*x))*arctan(a*x)+4*cos(2*arctan(a*x))*arctan(a*x)+
sin(4*arctan(a*x))+2*sin(2*arctan(a*x)))/arctan(a*x)^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")

[Out] 1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((3*a
^2*x^3 - 5*x)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x) - a*x + (3*a^2*x^2 - 1)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^4*c^3*x^2
 + a^2*c^3)*arctan(a*x)^2)

Fricas [C] Result contains complex when optimal does not.
time = 5.84, size = 318, normalized size = 2.81

$$-\frac{2\left(i a^4x^4+2i a^2x^2+i\right)\arctan\left(ax\right)^2\log\_integral\left(\frac{a^4x^4+4i a^3x^3-6a^2x^2-4i ax+1}{a^4x^4+2a^2x^2+1}\right)+2\left(-i a^4x^4-2i a^2x^2-i\right)\arctan\left(ax\right)^2\log\_integral\left(\frac{a^4x^4-4i a^3x^3-6a^2x^2+4i ax+1}{a^4x^4+2a^2x^2+1}\right)-\left(-i a^4x^4-2i a^2x^2-i\right)\arctan\left(ax\right)^2\log\_integral\left(-\frac{a^2x^2+2i ax-1}{a^2x^2+1}\right)-\left(i a^4x^4+2i a^2x^2+i\right)\arctan\left(ax\right)^2\log\_integral\left(-\frac{a^2x^2-2i ax-1}{a^2x^2+1}\right)+2ax-2\left(3a^2x^2-1\right)\arctan\left(ax\right)}{4\left(a^6c^3x^4+2a^4c^3x^2+a^2c^3\right)\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")

[Out] -1/4*(2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x^4 +
 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*(-I*
a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^
3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (-I*a^4*x^4 - 2*I
*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2
+ 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x^2
- 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x - 2*(3*a^2*x^2 - 1)*arctan(a*x))/((a^
6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)

[Out] Integral(x/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**2
*atan(a*x)**3 + atan(a*x)**3), x)/c**3

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.638 $\int \frac{1}{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=81

$$-\frac{1}{2ac^3(1+a^2x^2)^2 \text{ArcTan}(ax)^2}+\frac{2x}{c^3(1+a^2x^2)^2 \text{ArcTan}(ax)}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{ac^3}-\frac{\text{CosIntegral}(4\text{A}}{ac^3}$$

```
[Out] -1/2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-Ci
(2*arctan(a*x))/a/c^3-Ci(4*arctan(a*x))/a/c^3
```

Rubi [A]
time = 0.20, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5022, 5088, 5090, 4491, 3383, 5024, 3393}

$$\frac{2x}{c^3(a^2x^2+1)^2 \text{ArcTan}(ax)}-\frac{1}{2ac^3(a^2x^2+1)^2 \text{ArcTan}(ax)^2}-\frac{\text{CosIntegral}(2\text{ArcTan}(ax))}{ac^3}-\frac{\text{CosIntegral}(4\text{ArcTan}(ax))}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) + (2*x)/(c^3*(1 + a^2*x^2)^2*A
rcTan[a*x]) - CosIntegral[2*ArcTan[a*x]]/(a*c^3) - CosIntegral[4*ArcTan[a*x
]]/(a*c^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx = -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} - (2a) \int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2} dx$$

$$= -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - 2 \int \frac{}{(c+a^2cx}$$

$$= -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{2\text{Subst}\left(\int \frac{\cos^4}{x}\right.}{}$$

$$= -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{2\text{Subst}\left(\int \left(\frac{3}{8x}\right.\right.}{}$$

$$= -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(4x}{x}\right.}{}$$

$$= -\frac{1}{2ac^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2x}{c^3 (1+a^2x^2)^2 \tan^{-1}(ax)} - \frac{\text{Ci}(2\tan^{-1}(ax}{ac^3}$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 1.10

$$-\frac{1-4ax\text{ArcTan}(ax)+2(1+a^2x^2)^2\text{ArcTan}(ax)^2\text{CosIntegral}(2\text{ArcTan}(ax))+2(1+a^2x^2)^2\text{ArcTan}(ax)^2\text{CosIntegral}(4\text{ArcTan}(ax))}{2ac^3(1+a^2x^2)^2\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*(1 - 4*a*x*ArcTan[a*x] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[2
*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[4*ArcTan[a*x]])
/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

Maple [A]
time = 0.37, size = 89, normalized size = 1.10

method	result
derivativedivides	$-\frac{16\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2+16\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)^2-8\sin(2\arctan(ax))\,\text{arctan}}{16a\,c^3\arctan(ax)^2}$
default	$-\frac{16\,\text{cosineIntegral}(2\arctan(ax))\arctan(ax)^2+16\,\text{cosineIntegral}(4\arctan(ax))\arctan(ax)^2-8\sin(2\arctan(ax))\,\text{arctan}}{16a\,c^3\arctan(ax)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a/c^3*(16*Ci(2*arctan(a*x))*arctan(a*x)^2+16*Ci(4*arctan(a*x))*arctan
(a*x)^2-8*sin(2*arctan(a*x))*arctan(a*x)-4*sin(4*arctan(a*x))*arctan(a*x)+4
*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)^2*integrate(2*(3*a
^2*x^2 - 1)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x
)), x) + 4*a*x*arctan(a*x) - 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arct
an(a*x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 5.75, size = 297, normalized size = 3.67

$$-\frac{\left(a^{4} x^{4}+2 a^{2} x^{2}+1\right) \arctan \left(a x\right)^{2} \operatorname{log\_integral}\left(\frac{a^{4} x^{4}+4 i a^{3} x^{3}-6 a^{2} x^{2}-4 i a x+1}{a^{4} x^{4}+2 a^{2} x^{2}+1}\right)+\left(a^{4} x^{4}+2 a^{2} x^{2}+1\right) \arctan \left(a x\right)^{2} \operatorname{log\_integral}\left(\frac{a^{4} x^{4}-4 i a^{3} x^{3}-6 a^{2} x^{2}+4 i a x+1}{a^{4} x^{4}+2 a^{2} x^{2}+1}\right)+\left(a^{4} x^{4}+2 a^{2} x^{2}+1\right) \arctan \left(a x\right)^{2} \operatorname{log\_integral}\left(-\frac{a^{2} x^{2}+2 i a x-1}{a^{2} x^{2}+1}\right)+\left(a^{4} x^{4}+2 a^{2} x^{2}+1\right) \arctan \left(a x\right)^{2} \operatorname{log\_integral}\left(-\frac{a^{2} x^{2}-2 i a x-1}{a^{2} x^{2}+1}\right)-4 a x \arctan \left(a x\right)+1}{2 \left(a^{5} c^{3} x^{4}+2 a^{3} c^{3} x^{2}+a c^{3}\right) \arctan \left(a x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 + 4*I*a
^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2
 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^
2 + 1)) - 4*a*x*arctan(a*x) + 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arc
tan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{6} \operatorname{atan}^{3}{\left(a x \right)} + 3 a^{4} x^{4} \operatorname{atan}^{3}{\left(a x \right)} + 3 a^{2} x^{2} \operatorname{atan}^{3}{\left(a x \right)} + \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**2
*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.639 $\int \frac{1}{x(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{1}{2ac^3x\mathrm{ArcTan}(ax)^2}+\frac{ax}{2c^3\left(1+a^2x^2\right)^2\mathrm{ArcTan}(ax)^2}+\frac{ax}{2c^3\left(1+a^2x^2\right)\mathrm{ArcTan}(ax)^2}+\frac{2}{c^3\left(1+a^2x^2\right)^2\mathrm{ArcTan}(ax)}$$

[Out] -1/2/a/c^3/x/arctan(a*x)^2+1/2*a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a*x/c^3/(a^2*x^2+1)/arctan(a*x)^2+2/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*(-a^2*x^2+1)/c^3/(a^2*x^2+1)/arctan(a*x)+3/2*Si(2*arctan(a*x))/c^3+Si(4*arctan(a*x))/c^3-1/2*Unintegrable(1/x^2/arctan(a*x)^2,x)/a/c^3

Rubi [A]
time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]

[Out] -1/2*1/(a*c^3*x*ArcTan[a*x]^2) + (a*x)/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) + (a*x)/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + 2/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (1 - a^2*x^2)/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (3*SinIntegral[2*ArcTan[a*x]])/(2*c^3) + SinIntegral[4*ArcTan[a*x]]/c^3 - Defer[Int][1/(x^2*ArcTan[a*x]^2), x]/(2*a*c^3)

Rubi steps

$$\int \frac{1}{x(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x(c+a^2cx^2)^2\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^2}\,dx + \frac{1}{2}(3a^3$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

$$= -\frac{1}{2ac^3x\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\tan^{-1}}$$

Mathematica [A]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+a^2cx^2)^3\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a^2c\,x^2+c)^3\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2*integrate(
(10*a^4*x^4 + 3*a^2*x^2 + 1)/((a^8*c^3*x^9 + 3*a^6*c^3*x^7 + 3*a^4*c^3*x^5
+ a^2*c^3*x^3)*arctan(a*x)), x) - a*x + (5*a^2*x^2 + 1)*arctan(a*x))/((a^6*
c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*
x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^7\operatorname{atan}^3(ax)+3a^4x^5\operatorname{atan}^3(ax)+3a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**3 + 3*a**4*x**5*atan(a*x)**3 + 3*a**2*x**3
*atan(a*x)**3 + x*atan(a*x)**3), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.640 $\int \frac{1}{x^2(c+a^2cx^2)^3\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=168

$$-\frac{1}{2ac^3x^2\text{ArcTan}(ax)^2}+\frac{a}{2c^3(1+a^2x^2)^2\,\text{ArcTan}(ax)^2}+\frac{a}{2c^3(1+a^2x^2)\,\text{ArcTan}(ax)^2}-\frac{2a^2x}{c^3(1+a^2x^2)^2\,\text{ArcTan}(a$$

```
[Out] -1/2/a/c^3/x^2/arctan(a*x)^2+1/2*a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a/c^
3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-a^2*x/c^3
/(a^2*x^2+1)/arctan(a*x)+2*a*Ci(2*arctan(a*x))/c^3+a*Ci(4*arctan(a*x))/c^3-
Unintegrable(1/x^3/arctan(a*x)^2,x)/a/c^3
```

Rubi [A]
time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(c+a^2cx^2)^3\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^3*x^2*ArcTan[a*x]^2) + a/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
+ a/(2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) - (2*a^2*x)/(c^3*(1 + a^2*x^2)^2*Ar
cTan[a*x]) - (a^2*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) + (2*a*CosIntegral[2*A
rcTan[a*x]])/c^3 + (a*CosIntegral[4*ArcTan[a*x]])/c^3 - Defer[Int][1/(x^3*A
rcTan[a*x]^2), x]/(a*c^3)
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx = -\left(a^2 \int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^2(c+a^2cx^2)^2 \tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + (2a^3) \int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)^2}\,dx + \frac{\int}{}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

$$= -\frac{1}{2ac^3x^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{a}{2c^3 (1+a^2x^2) \tan}$$

Mathematica [A]
time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c)^3 \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2*integrate(
(15*a^4*x^4 + 10*a^2*x^2 + 3)/((a^8*c^3*x^10 + 3*a^6*c^3*x^8 + 3*a^4*c^3*x^
6 + a^2*c^3*x^4)*arctan(a*x)), x) - a*x + 2*(3*a^2*x^2 + 1)*arctan(a*x))/((
a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(
a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^8\operatorname{atan}^3(ax)+3a^4x^6\operatorname{atan}^3(ax)+3a^2x^4\operatorname{atan}^3(ax)+x^2\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x)**3 + 3*a**4*x**6*atan(a*x)**3 + 3*a**2*x**4
*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.641 $\int \frac{1}{x^3(c+a^2cx^2)^3\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=248

$$-\frac{1}{2ac^3x^3\text{ArcTan}(ax)^2}+\frac{a}{c^3x\text{ArcTan}(ax)^2}-\frac{a^3x}{2c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^2}-\frac{a^3x}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)^2}-\frac{}{c^3(1+}$$

```
[Out] -1/2/a/c^3/x^3/arctan(a*x)^2+a/c^3/x/arctan(a*x)^2-1/2*a^3*x/c^3/(a^2*x^2+1
)^2/arctan(a*x)^2-a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2/c^3/(a^2*x^2+1)
^2/arctan(a*x)+3/2*a^2/c^3/(a^2*x^2+1)/arctan(a*x)-a^2*(-a^2*x^2+1)/c^3/(a^
2*x^2+1)/arctan(a*x)-5/2*a^2*Si(2*arctan(a*x))/c^3-a^2*Si(4*arctan(a*x))/c^
3-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c^3+a*Unintegrable(1/x^2/arctan
(a*x)^2,x)/c^3
```

Rubi [A]
time = 0.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c^3*x^3*ArcTan[a*x]^2) + a/(c^3*x*ArcTan[a*x]^2) - (a^3*x)/(2*c^3
*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - (a^3*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2)
 - (2*a^2)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (3*a^2)/(2*c^3*(1 + a^2*x^2)
*ArcTan[a*x]) - (a^2*(1 - a^2*x^2))/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (5*a^
2*SinIntegral[2*ArcTan[a*x]])/(2*c^3) - (a^2*SinIntegral[4*ArcTan[a*x]])/c^
3 - (3*Defer[Int][1/(x^4*ArcTan[a*x]^2), x])/(2*a*c^3) + (a*Defer[Int][1/(x
^2*ArcTan[a*x]^2), x])/c^3
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3 (c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx &= -\left(a^2\int \frac{1}{x (c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x^3 (c+a^2cx^2)^2 \tan^{-1}(ax)^3}\,dx}{c} \\
&= a^4\int \frac{x}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3}\,dx+\frac{\int \frac{1}{x^3 (c+a^2cx^2) \tan^{-1}(ax)^3}\,dx}{c^2}-2\frac{a^2\int}{} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}+\frac{1}{2}a^3\int \frac{}{(c+a^2cx^2)} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{a^2}{2c^3(1+a^2x^2)^2} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2} \\
&= -\frac{1}{2ac^3x^3\tan^{-1}(ax)^2}-\frac{a^3x}{2c^3(1+a^2x^2)^2\tan^{-1}(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2}
\end{aligned}
$$

Mathematica [A]
time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A]
time = 3.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^3 \arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)

[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")

[Out] 1/2*(2*(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2*integrate((21*a^4*x^4 + 19*a^2*x^2 + 6)/((a^8*c^3*x^11 + 3*a^6*c^3*x^9 + 3*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)), x) - a*x + (7*a^2*x^2 + 3)*arctan(a*x))/((a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(a*x)^3), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^9\operatorname{atan}^3(ax)+3a^4x^7\operatorname{atan}^3(ax)+3a^2x^5\operatorname{atan}^3(ax)+x^3\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)

[Out] Integral(1/(a**6*x**9*atan(a*x)**3 + 3*a**4*x**7*atan(a*x)**3 + 3*a**2*x**5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c**3

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.642 $\int \frac{1}{x^4(c+a^2cx^2)^3\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{1}{2ac^3x^4\text{ArcTan}(ax)^2}+\frac{a}{c^3x^2\text{ArcTan}(ax)^2}-\frac{a^3}{2c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^2}-\frac{a^3}{c^3\left(1+a^2x^2\right)\text{ArcTan}(ax)^2}+\frac{}{c^3(1-}$$

[Out] -1/2/a/c^3/x^4/arctan(a*x)^2+a/c^3/x^2/arctan(a*x)^2-1/2*a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-a^3/c^3/(a^2*x^2+1)/arctan(a*x)^2+2*a^4*x/c^3/(a^2*x^2+1)^2/arctan(a*x)+2*a^4*x/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Ci(2*arctan(a*x))/c^3-a^3*Ci(4*arctan(a*x))/c^3-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c^3+2*a*Unintegrable(1/x^3/arctan(a*x)^2,x)/c^3

Rubi [A]
time = 0.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]

[Out] -1/2*1/(a*c^3*x^4*ArcTan[a*x]^2) + a/(c^3*x^2*ArcTan[a*x]^2) - a^3/(2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - a^3/(c^3*(1 + a^2*x^2)*ArcTan[a*x]^2) + (2*a^4*x)/(c^3*(1 + a^2*x^2)^2*ArcTan[a*x]) + (2*a^4*x)/(c^3*(1 + a^2*x^2)*ArcTan[a*x]) - (3*a^3*CosIntegral[2*ArcTan[a*x]])/c^3 - (a^3*CosIntegral[4*ArcTan[a*x]])/c^3 - (2*Defer[Int][1/(x^5*ArcTan[a*x]^2), x])/(a*c^3) + (2*a*Defer[Int][1/(x^3*ArcTan[a*x]^2), x])/c^3

Rubi steps

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx = -\left(a^2 \int \frac{1}{x^2 (c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx\right) + \frac{\int \frac{1}{x^4(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx}{c}$$

$$= a^4 \int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^3} dx + \frac{\int \frac{1}{x^4(c+a^2cx^2) \tan^{-1}(ax)^3} dx}{c^2} - 2\frac{a^2 \int \frac{}{x^2(}}{}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} - (2a^5) \int \frac{}{(c+a^2cx}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

$$= -\frac{1}{2ac^3x^4 \tan^{-1}(ax)^2} - \frac{a^3}{2c^3 (1+a^2x^2)^2 \tan^{-1}(ax)^2} + \frac{2a^4x}{c^3 (1+a^2x^2)^2 \tan^{-}}$$

Mathematica [A]
time = 6.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^3 \arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2*integrate(
2*(14*a^4*x^4 + 15*a^2*x^2 + 5)/((a^8*c^3*x^12 + 3*a^6*c^3*x^10 + 3*a^4*c^3
*x^8 + a^2*c^3*x^6)*arctan(a*x)), x) - a*x + 4*(2*a^2*x^2 + 1)*arctan(a*x))
/((a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arctan
(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^3(ax)+3a^4x^8\operatorname{atan}^3(ax)+3a^2x^6\operatorname{atan}^3(ax)+x^4\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(1/(a**6*x**10*atan(a*x)**3 + 3*a**4*x**8*atan(a*x)**3 + 3*a**2*x**
6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.643 $\int \left(\frac{x^3}{(1+a^2x^2)\textbf{ArcTan}(ax)^3}-\frac{3x^2}{2a\textbf{ArcTan}(ax)^2}\right)\,dx$

Optimal. Leaf size=16

$$-\frac{x^3}{2a\text{ArcTan}(ax)^2}$$

[Out] -1/2*x^3/a/arctan(a*x)^2

Rubi [A]
time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {5046}

$$-\frac{x^3}{2a\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x]

[Out] -1/2*x^3/(a*ArcTan[a*x]^2)

Rule 5046

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Dist[f*(m/(b*c*d*(p + 1))), Int[(f*x)^(m - 1)*(a + b*ArcTan[
c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &
& LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x^3}{(1+a^2x^2)\tan^{-1}(ax)^3}-\frac{3x^2}{2a\tan^{-1}(ax)^2}\right)\,dx &= -\frac{3\int \frac{x^2}{\tan^{-1}(ax)^2}\,dx}{2a}+\int \frac{x^3}{(1+a^2x^2)\tan^{-1}(ax)^3}\,dx\\ &= -\frac{x^3}{2a\tan^{-1}(ax)^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 16, normalized size = 1.00

$$-\frac{x^3}{2a\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x
]
```

```
[Out] -1/2*x^3/(a*ArcTan[a*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 3.48, size = 30, normalized size = 1.88

method	result	size
risch	$\frac{2x^3}{a(\ln(-iax+1)-\ln(iax+1))^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/a*x^3/(ln(1-I*a*x)-ln(1+I*a*x))^2
```

Maxima [A]
time = 0.33, size = 14, normalized size = 0.88

$$-\frac{x^3}{2\,a\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="maxima")
```

```
[Out] -1/2*x^3/(a*arctan(a*x)^2)
```

Fricas [A]
time = 2.72, size = 14, normalized size = 0.88

$$-\frac{x^3}{2\,a\arctan\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="fricas")
```

```
[Out] -1/2*x^3/(a*arctan(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{2ax^3}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\right)dx+\int\frac{3x^2\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx+\int\frac{3a^2x^4\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*x**2+1)/atan(a*x)**3-3/2*x**2/a/atan(a*x)**2,x)
```

```
[Out] -(Integral(-2*a*x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral
(3*x**2*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral(3*
a**2*x**4*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x))/(2*a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.43, size = 14, normalized size = 0.88

$$-\frac{x^3}{2\,a\,\mathrm{atan}\left(a\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(a^2*x^2 + 1)) - (3*x^2)/(2*a*atan(a*x)^2),x)
```

```
[Out] -x^3/(2*a*atan(a*x)^2)
```

3.644 $\int \frac{x\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

3.645 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^3} \, dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3} \, dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\operatorname{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3,x)
```

```
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3, x)
```

3.646 $\int \frac{\sqrt{c+a^2cx^2}}{x\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{x\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3),x)
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3), x)
```

3.647 $\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 4.39, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\operatorname{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

3.648 $\int \frac{(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^{3}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,a^2\,x^2+c)^{3/2}}{\operatorname{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3,x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3, x)
```

3.649 $\int \frac{(c+a^2cx^2)^{3/2}}{x\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

Maple [A]
time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{x\,\mathrm{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3), x)
```

3.650 $\int \frac{x(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\operatorname{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 3.21, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(a
*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

```
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

3.651 $\int \frac{(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*x
)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3,x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3, x)
```

3.652 $\int \frac{(c+a^2cx^2)^{5/2}}{x\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

Maple [A]
time = 4.66, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan(
a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\,(a\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3),x)
[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3), x)
```

3.653 $\int \frac{x}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.654 $\int \frac{1}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.655 $\int \frac{1}{x\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{c+a^2cx^2}}{2acx\mathrm{ArcTan}(ax)^2}-\frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2},x\right)}{2a}$$

```
[Out] -1/2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(
a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*Sqrt[c + a^2*c*x^2]/(a*c*x*ArcTan[a*x]^2) - Defer[Int][1/(x^2*Sqrt[c +
 a^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx=-\frac{\sqrt{c+a^2cx^2}}{2acx\tan^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A]
time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan \left(a x\right)^{3} \sqrt{a^{2} c \,x^{2}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{c \left(a^{2} x^{2} + 1\right)} \operatorname{atan}^{3}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.656 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\arctan\left(ax\right)^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^3 \sqrt{c a^2 x^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.657 $\int \frac{1}{x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx = \int \frac{1}{x^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\arctan(ax)^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^3 \operatorname{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.658 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=136

$$\frac{x}{2a^3c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}+\frac{1}{2a^4c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{2a^4c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\Big(\frac{}{\sqrt{c+a^2c}}}{}$$

```
[Out] 1/2*x/a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2/a^4/c/arctan(a*x)/(a^2*c*
x^2+c)^(1/2)+1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2
)+Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^2/c
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] x/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + 1/(2*a^4*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^4*c*Sq
rt[c + a^2*c*x^2]) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(
a^2*c)
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{x}{\sqrt{c+a^2cx^2}}}{a^2c}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{x}{(c+a^2cx^2)^{3/}}}{2}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a^2c}}}{}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a^2c}}}{}$$

$$= \frac{x}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x^2}}{2a^4c\sqrt{c}}$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^3), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.659 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=135

$$\frac{1}{2a^3c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}-\frac{x}{2a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{2a^3c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\Big(}{}$$

```
[Out] 1/2/a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*x/a^2/c/arctan(a*x)/(a^2*c*
x^2+c)^(1/2)+1/2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2
)+Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^2/c
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - x/(2*a^2*c*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a^3*c*Sq
rt[c + a^2*c*x^2]) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(
a^2*c)
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} + \frac{\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a} + \frac{\int \frac{1}{\sqrt{c+a^2cx}}}{a}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{(c+a^2cx^2)}}{}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a}}}{}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{c+a}}}{}$$

$$= \frac{1}{2a^3c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\tan^{-1}(ax)} + \frac{\sqrt{1+a^2x}}{2a^3c}$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")

[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)

[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)

[Out] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)

3.660 $\int \frac{x}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{x}{2ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}-\frac{1}{2a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(ax))}{2a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*x/a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2/a^2/c/arctan(a*x)/(a^2*c*x
^2+c)^(1/2)-1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5062, 5022, 5091, 5090, 3380}

$$-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\text{ArcTan}(ax))}{2a^2c\sqrt{a^2cx^2+c}}-\frac{x}{2ac\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}-\frac{1}{2a^2c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - 1/(2*a^2*c*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*a^2*c*S
qrt[c + a^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx}{2a}$$
$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{1}{2}\int \frac{}{(c+a}$$
$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{2}$$
$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{}$$
$$= -\frac{x}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{2a^2c\sqrt{}}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.61

$$-\frac{ax+\text{ArcTan}(ax)+\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\text{Si}(\text{ArcTan}(ax))}{2a^2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*(a*x + ArcTan[a*x] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*SinIntegral[ArcTa
n[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.59, size = 294, normalized size = 2.83

method	result
default	$-\frac{i\left(\arctan(ax)^2 \operatorname{expIntegral}(1,-i\arctan(ax))a^2x^2+\arctan(ax)\sqrt{a^2x^2+1}\,ax+\operatorname{expIntegral}(1,-i\arctan(ax))\arctan(ax)^2-i\sqrt{}\right.}{4(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)^2a^2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^
(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*a*x-I*arct
an(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(
I+a*x))^(1/2)/arctan(a*x)^2/a^2/c^2+1/4*I*(arctan(a*x)^2*Ei(1,I*arctan(a*x)
)*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*
arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2
))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a^2/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(
a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.661 $\int \frac{1}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}+\frac{x}{2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2/a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*x/c/arctan(a*x)/(a^2*c*x^2+c
)^(1/2)-1/2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5022, 5062, 5025, 5024, 3383}

$$-\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{2ac\sqrt{a^2cx^2+c}}+\frac{x}{2c\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}-\frac{1}{2ac\text{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + x/(2*c*Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*a*c*Sqrt[c
+ a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{1}{2}\int \frac{}{(c+a^2c}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}}{2c}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{S}}{}$$

$$= -\frac{1}{2ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{C}}{2ac\sqrt{c}}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.64

$$\frac{-1+ax\text{ArcTan}(ax)-\sqrt{1+a^2x^2}\,\text{ArcTan}(ax)^2\text{CosIntegral}(\text{ArcTan}(ax))}{2ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-1 + a*x*ArcTan[a*x] - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*CosIntegral[ArcTan[
a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 292, normalized size = 2.89

method	result
default	$\frac{\left(\arctan(ax)^2 \operatorname{expIntegral}(1,i\arctan(ax))a^2x^2+\arctan(ax)\sqrt{a^2x^2+1}\,ax+i\sqrt{a^2x^2+1}\,ax+\operatorname{expIntegral}(1,i\arctan(ax))\operatorname{arc}\right.}{4(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)^2 a\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2
)*a*x+I*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*
x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x
))^(1/2)/arctan(a*x)^2/a/c^2+1/4*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^
2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a
^2*x^2+1)^(1/2)*a*x-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2
*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*
x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.662 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=166

$$\frac{ax}{2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\mathrm{ArcTan}(ax)^2}+\frac{1}{2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\mathrm{ArcTan}(ax))}{2c\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*a*x/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2/c/arctan(a*x)/(a^2*c*x^2+c)
^(1/2)+1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*(a^2
*c*x^2+c)^(1/2)/a/c^2/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(a*x)^2/
(a^2*c*x^2+c)^(1/2),x)/a/c
```

Rubi [A]

time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] (a*x)/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^
2*x*ArcTan[a*x]^2) + 1/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (Sqrt[1 + a^
2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) - Defer[Int][1/(
x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/(2*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx\right)+\frac{\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2}-\frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\,\mathrm{ta}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2}+\frac{1}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2}+\frac{1}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2}+\frac{1}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(}$$

$$= \frac{ax}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^2x\tan^{-1}(ax)^2}+\frac{1}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(}$$

Mathematica [A]
time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(
a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.663 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=131

$$\frac{a}{2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)}+\frac{a\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\mathrm{ArcTan}(ax))}{2c\sqrt{c+a^2cx^2}}+\frac{\mathrm{Int}\Big(\frac{\quad}{x^2\sqrt{\quad}}$$

```
[Out] 1/2*a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2*x/c/arctan(a*x)/(a^2*c*x^
2+c)^(1/2)+1/2*a*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+Un
integrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] a/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a^2*x)/(2*c*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]) + (a*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c*Sqrt[c
 + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^3}\, dx = -\left(a^2 \int \frac{1}{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^3}\, dx\right) + \frac{\int \frac{1}{x^2\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^3}\, d}{c}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2} + \frac{1}{2}a^3 \int \frac{x}{(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^2}\, dx + -$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2} - \frac{a^2x}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)} + \frac{1}{2}a^2 \int \frac{}{(c+}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2} - \frac{a^2x}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)} + \frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2} - \frac{a^2x}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)} + \frac{\int \frac{}{x^2\sqrt{c+a^2}}}{}$$

$$= \frac{a}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)^2} - \frac{a^2x}{2c\sqrt{c+a^2cx^2}\, \tan^{-1}(ax)} + \frac{a\sqrt{1+a^2x^2}}{2c\sqrt{c}}$$

Mathematica [A]
time = 1.89, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^3}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arcta
n(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.664 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=201

$$-\frac{a^3x}{2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}+\frac{a\sqrt{c+a^2cx^2}}{2c^2x\text{ArcTan}(ax)^2}-\frac{a^2}{2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{a^2\sqrt{1+a^2x^2}\,\text{Si}(\text{ArcTan}(a}{2c\sqrt{c+a^2cx^2}}$$

[Out] -1/2*a^3*x/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-1/2*a^2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+1/2*a*(a^2*c*x^2+c)^(1/2)/c^2/x/arctan(a*x)^2+Unintegrable(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c+1/2*a*Unintegrable(1/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c

Rubi [A]
time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]

[Out] -1/2*(a^3*x)/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a*Sqrt[c + a^2*c*x^2])/(2*c^2*x*ArcTan[a*x]^2) - a^2/(2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (a^2*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(2*c*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c + (a*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x])/(2*c)

Rubi steps

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3} \, dx = -\left(a^2 \int \frac{1}{x \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3} \, dx\right) + \frac{\int \frac{1}{x^3 \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3} \, d}{c}$$

$$= a^4 \int \frac{x}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3} \, dx + \frac{\int \frac{1}{x^3 \sqrt{c+a^2cx^2} \tan^{-1}(ax)^3} \, dx}{c} - \frac{a^2}{}$$

$$= -\frac{a^3 x}{2c\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2 x \tan^{-1}(ax)^2} + \frac{1}{2}a^3 \int \frac{1}{\left(c+a^2cx^2\right)^{3/}}$$

$$= -\frac{a^3 x}{2c\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2 x \tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2} \tan^{-1}}$$

$$= -\frac{a^3 x}{2c\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2 x \tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2} \tan^{-1}}$$

$$= -\frac{a^3 x}{2c\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2 x \tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2} \tan^{-1}}$$

$$= -\frac{a^3 x}{2c\sqrt{c+a^2cx^2} \tan^{-1}(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2 x \tan^{-1}(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2} \tan^{-1}}$$

Mathematica [A]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^{3/2} \text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arcta
n(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{3}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.665 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=170

$$-\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}+\frac{a^4x}{2c\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{a^3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{2c\sqrt{c+a^2cx^2}}+\frac{\text{Int}\Big(}{}$$

```
[Out] -1/2*a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*a^4*x/c/arctan(a*x)/(a^2*c
*x^2+c)^(1/2)-1/2*a^3*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/
2)+Unintegrable(1/x^4/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c-a^2*Unintegrab
le(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c
```

Rubi [A]
time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*a^3/(c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (a^4*x)/(2*c*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]) - (a^3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(2*c
*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
), x]/c - (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x])/c
```

Rubi steps

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx = -\left(a^2 \int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}}{c}$$

$$= a^4 \int \frac{1}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^3}\,dx + \frac{\int \frac{1}{x^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx}{c} - \frac{a^2}{}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{1}{2}a^5 \int \frac{x}{\left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^2}\,dx +$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{1}{2}a^4 \int$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{}{x^4\sqrt{c+a}}}{}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} + \frac{\int \frac{}{x^4\sqrt{c+a}}}{}$$

$$= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)} - \frac{a^3\sqrt{1+a^2}}{2c\sqrt{}}$$

Mathematica [A]
time = 5.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \operatorname{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.66, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arcta
n(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{3}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
[Out] Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.666 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=241

$$\frac{x^3}{2a^3c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}+\frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}-\frac{3}{2a^6c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}+\frac{}{a^6c^2\sqrt{c}}$$

```
[Out] 1/2*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2/a^6/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)+1/2*x/a^5/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+2/a^6/c^2/ar
ctan(a*x)/(a^2*c*x^2+c)^(1/2)+7/8*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2
/(a^2*c*x^2+c)^(1/2)-9/8*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c
*x^2+c)^(1/2)+Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

Rubi [A]
time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] x^3/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + x/(2*a^5*c^2*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2) - 3/(2*a^6*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) +
2/(a^6*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*Sqrt[1 + a^2*x^2]*SinInteg
ral[ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*Si
nIntegral[3*ArcTan[a*x]])/(8*a^6*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][x/(S
qrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{}{\sqrt{c+a}}}{}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{3\int \frac{}{(c+}}{}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

$$= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^6c\,(}$$

Mathematica [**A**]
time = 6.99, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [**A**]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.667 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=235

$$-\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^2}+\frac{1}{a^5c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}+\frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)}-\frac{}{a^4c^2\sqrt{}}$$

```
[Out] -1/2/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2*x/a^4/c/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)+1/a^5/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-x/a^4/c^2/arctan(
a*x)/(a^2*c*x^2+c)^(1/2)+5/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2
*c*x^2+c)^(1/2)-9/8*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+
c)^(1/2)+Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

Rubi [A]
time = 0.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a^5*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 1/(a^5*c^2*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x]^2) + (3*x)/(2*a^4*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
 - x/(a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (5*Sqrt[1 + a^2*x^2]*CosIn
tegral[ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]
*CosIntegral[3*ArcTan[a*x]])/(8*a^5*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1
/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/(a^4*c^2)
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^4} + \frac{\int \frac{1}{\sqrt{c+a^2cx^2}\tan^{-1}(ax)^3}\,dx}{a^4c^2} - 2\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\ \mathrm{ta}}}{a^4c}$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{\int \frac{}{\sqrt{c+}}}{}$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\int}{}$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - 2\Big(-$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - 2\Big(-$$

$$= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1}}{}$$

Mathematica [A]
time = 7.78, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.668 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{x^3}{2ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}+\frac{3}{2a^4c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}-\frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{3\sqrt{1+}}{8a}$$

```
[Out] -1/2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2/a^4/c/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)-3/2/a^4/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-3/8*Si(arctan(a*x
))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+9/8*Si(3*arctan(a*x))*(a^2
*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5062, 5084, 5022, 5091, 5090, 3380, 4491}

$$-\frac{x^3}{2ac\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}-\frac{3\sqrt{a^2x^2+1}\,\text{Si}(\text{ArcTan}(ax))}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\,\text{Si}(3\text{ArcTan}(ax))}{8a^4c^2\sqrt{a^2cx^2+c}}-\frac{3}{2a^4c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}+\frac{3}{2a^4c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + 3/(2*a^4*c*(c + a^2*c*
x^2)^(3/2)*ArcTan[a*x]) - 3/(2*a^4*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (
3*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a^2*c*x^2
]) + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^4*c^2*Sqrt[c + a
^2*c*x^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
```

```
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a^3} + \frac{3\int \frac{}{(c+a^2cx}}{}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

$$= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{}{2a^4c^2\sqrt{}}$$

Mathematica [A]
time = 0.22, size = 114, normalized size = 0.63

$$\frac{-4a^2x^2(ax+3\mathrm{ArcTan}(ax)) - 3(1+a^2x^2)^{3/2}\mathrm{ArcTan}(ax)^2\mathrm{Si}(\mathrm{ArcTan}(ax)) + 9(1+a^2x^2)^{3/2}\mathrm{ArcTan}(ax)^2\mathrm{Si}(3\mathrm{ArcTan}(ax))}{8a^4c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4*a^2*x^2*(a*x + 3*ArcTan[a*x]) - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Sin
Integral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*SinIntegral[3*A
rcTan[a*x]])/(8*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.77, size = 848, normalized size = 4.71

method	result

default	$\frac{i\left(9\arctan(ax)^2\,\mathrm{expIntegral}(1,-3i\arctan(ax))a^4x^4-3\arctan(ax)\sqrt{a^2x^2+1}\,a^3x^3+18\arctan(ax)^2\,\mathrm{expIntegral}(1,-3i\arctan(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*(a^2*x
^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2+I*(a^2*
x^2+1)^(1/2)*a^3*x^3+9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1
)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x-3*I*(a^2*x^2+1)^(1/2)*a
*x+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)
-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*
a^2*x^2+1)/arctan(a*x)^2/a^4/c^3-1/16*I*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*
x))*a^4*x^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,3
*I*arctan(a*x))*a^2*x^2-I*(a^2*x^2+1)^(1/2)*a^3*x^3-9*I*arctan(a*x)*(a^2*x^
2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1
/2)*a*x+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*a*x+3*I
*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x
-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/a^4/c^3+3/16*I*(arct
an(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+I*(
a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x
^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/
arctan(a*x)^2/a^4/c^3-3/16*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+ar
ctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x
^2+1)^(1/2)*a*x-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2
+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a^4/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\mathrm{atan}\left(a\,x\right)}^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.669 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=209

$$\frac{1}{2a^3c\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^2}-\frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)^2}-\frac{3x}{2a^2c\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)}+\frac{}{2a^2c^2}$$

```
[Out] 1/2/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2*x/a^2/c/(a^2*c*x^2+c)^(3/2)
/arctan(a*x)-1/2/a^3/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*x/a^2/c^2/ar
ctan(a*x)/(a^2*c*x^2+c)^(1/2)-1/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2
/(a^2*c*x^2+c)^(1/2)+9/8*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c
*x^2+c)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5084, 5022, 5062, 5025, 5024, 3383, 5088, 5091, 5090, 4491, 3393}

$$\frac{x}{2a^2c^2\operatorname{ArcTan}(ax)\sqrt{a^2cx^2+c}}-\frac{3x}{2a^2c\operatorname{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\,\operatorname{CosIntegral}(\operatorname{ArcTan}(ax))}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{9\sqrt{a^2x^2+1}\,\operatorname{CosIntegral}(3\operatorname{ArcTan}(ax))}{8a^3c^2\sqrt{a^2cx^2+c}}-\frac{1}{2a^3c^2\operatorname{ArcTan}(ax)^2\sqrt{a^2cx^2+c}}+\frac{1}{2a^3c\operatorname{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] 1/(2*a^3*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 1/(2*a^3*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - (3*x)/(2*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])
+ x/(2*a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt[1 + a^2*x^2]*CosInt
egral[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*
CosIntegral[3*ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
```

```
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx}{a^2} + \frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx}{a^2c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} + \frac{3\int \frac{}{(c+a}}{}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

$$= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2a^2c\,(c}$$

Mathematica [A]
time = 0.17, size = 119, normalized size = 0.57

$$\frac{4ax(-ax+(-2+a^2x^2)\,\text{ArcTan}(ax)) - (1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)^2\text{CosIntegral}(\text{ArcTan}(ax)) + 9(1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)^2\text{CosIntegral}(3\text{ArcTan}(ax))}{8a^3c^2\,(1+a^2x^2)\,\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (4*a*x*(-(a*x) + (-2 + a^2*x^2)*ArcTan[a*x]) - (1 + a^2*x^2)^(3/2)*ArcTan[a
*x]^2*CosIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosInt
egral[3*ArcTan[a*x]])/(8*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.55, size = 844, normalized size = 4.04

method	result

default	$-\frac{\left(9\arctan(ax)^2\,\mathrm{expIntegral}(1,3i\arctan(ax))a^4x^4-3\arctan(ax)\sqrt{a^2x^2+1}\,a^3x^3+18\arctan(ax)^2\,\mathrm{expIntegral}(1,3i\arctan(ax))\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*(a^2*x^2
+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^2*x^2-I*(a^2*x^2
+1)^(1/2)*a^3*x^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(
1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,3*I*arctan(a*x))*ar
ctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*a*x+3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^
2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*
x^2+1)/arctan(a*x)^2/a^3/c^3-1/16*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a
^4*x^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,-3*I*a
rctan(a*x))*a^2*x^2+I*(a^2*x^2+1)^(1/2)*a^3*x^3+9*I*arctan(a*x)*(a^2*x^2+1)
^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*
a*x-3*I*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*ar
ctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)
*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/a^3/c^3+1/16*(arctan(a*
x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x
^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)
^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arcta
n(a*x)^2/a^3/c^3+1/16*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+arctan(a*
x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(
1/2)*a*x-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/
2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a^3/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.670 $\int \frac{x}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{x}{2ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}-\frac{3}{2a^2c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}+\frac{1}{a^2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)}-\frac{\sqrt{1+a}}{8a}$$

```
[Out] -1/2*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2/a^2/c/(a^2*c*x^2+c)^(3/2)/
arctan(a*x)+1/a^2/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-1/8*Si(arctan(a*x))*(
a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-9/8*Si(3*arctan(a*x))*(a^2*x^2
+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5088, 5084, 5022, 5091, 5090, 3380, 4491}

$$-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\text{ArcTan}(ax))}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\text{Si}(3\text{ArcTan}(ax))}{8a^2c^2\sqrt{a^2cx^2+c}}+\frac{1}{a^2c^2\text{ArcTan}(ax)\sqrt{a^2cx^2+c}}-\frac{x}{2ac\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}-\frac{3}{2a^2c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - 3/(2*a^2*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x]) + 1/(a^2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) - (Sqrt
[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (
9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*a^2*c^2*Sqrt[c + a^2*c*x
^2])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
```

```
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5084

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} + \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^2}\,dx}{2a} - a\int \frac{}{(c+a}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} - \frac{3}{2}\int$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

$$= -\frac{x}{2ac(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^2} - \frac{3}{2a^2c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)} + \frac{}{a^2c^2\sqrt{}}$$

Mathematica [**A**]
time = 0.20, size = 118, normalized size = 0.67

$$\frac{-4ax - 4\mathrm{ArcTan}(ax) + 8a^2x^2\mathrm{ArcTan}(ax) - (1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)^2\mathrm{Si}(\mathrm{ArcTan}(ax)) - 9(1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)^2\mathrm{Si}(3\mathrm{ArcTan}(ax))}{8a^2c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4*a*x - 4*ArcTan[a*x] + 8*a^2*x^2*ArcTan[a*x] - (1 + a^2*x^2)^(3/2)*ArcTa
n[a*x]^2*SinIntegral[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Sin
Integral[3*ArcTan[a*x]])/(8*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.74, size = 867, normalized size = 4.95

method	result

default	$-\frac{i\left(\arctan(ax)^2\,\mathrm{expIntegral}(1,-i\arctan(ax))a^2x^2+\arctan(ax)\sqrt{a^2x^2+1}\,ax+\mathrm{expIntegral}(1,-i\arctan(ax))\arctan(ax)^2-i\sqrt{a\right.}{16\arctan(ax)^2(a^4x^4+2a^2x^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)
^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*a*x-I*arc
tan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*
(I+a*x))^(1/2)/arctan(a*x)^2/(a^4*x^4+2*a^2*x^2+1)/a^2/c^3-1/16*I*(9*arctan
(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*
x^3+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2+I*(a^2*x^2+1)^(1/2)*a^3
*x^3+9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+
9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x-3*I*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,-3*I*a
rctan(a*x))*arctan(a*x)^2-3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/
2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^2/(a^4*x^4+2*a^2*x^2+
1)/arctan(a*x)^2+1/16*I*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-3*ar
ctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*
a^2*x^2-I*(a^2*x^2+1)^(1/2)*a^3*x^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x
^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,3
*I*arctan(a*x))*arctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*a*x+3*I*arctan(a*x)*(a^
2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/
2)/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2+1/16*I*(arctan(a*x)^2*Ei(1,I
*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2)
*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2
*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a^
2/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3)*arctan(a*x)^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.671 $\int \frac{1}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=145

$$-\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}+\frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)}-\frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\text{ArcTan}(ax))}{8ac^2\sqrt{c+a^2cx^2}}-9$$

```
[Out] -1/2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2*x/c/(a^2*c*x^2+c)^(3/2)/arct
an(a*x)-3/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-9/8
*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5022, 5088, 5091, 5090, 4491, 3383, 5025, 5024, 3393}

$$-\frac{3\sqrt{a^2x^2+1}\,\text{CosIntegral}(\text{ArcTan}(ax))}{8ac^2\sqrt{a^2cx^2+c}}-\frac{9\sqrt{a^2x^2+1}\,\text{CosIntegral}(3\text{ArcTan}(ax))}{8ac^2\sqrt{a^2cx^2+c}}+\frac{3x}{2c\text{ArcTan}(ax)\left(a^2cx^2+c\right)^{3/2}}-\frac{1}{2ac\text{ArcTan}(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (3*x)/(2*c*(c + a^2*c*x^
2)^(3/2)*ArcTan[a*x]) - (3*Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(8*a
*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[1 + a^2*x^2]*CosIntegral[3*ArcTan[a*x]]
)/(8*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
```

```
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2}(3a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3}{2}\int \frac{}{(c+}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+}\right.}{}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+}\right.}{}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+}\right.}{}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{\left(3\sqrt{1+}\right.}{}$$

$$= -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)} - \frac{3\sqrt{1+a}}{8ac}$$

Mathematica [A]
time = 0.15, size = 102, normalized size = 0.70

$$\frac{-4+12ax\text{ArcTan}(ax)-3\left(1+a^2x^2\right)^{3/2}\text{ArcTan}(ax)^2\text{CosIntegral}(\text{ArcTan}(ax))-9\left(1+a^2x^2\right)^{3/2}\text{ArcTan}(ax)^2\text{CosIntegral}(3\text{ArcTan}(ax))}{8c^2\left(a+a^3x^2\right)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (-4 + 12*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[
ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[3*ArcTan[a*x
]])/(8*c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.60, size = 844, normalized size = 5.82

method	result

default	$\left(9\arctan(ax)^2\,\mathrm{expIntegral}(1,3i\arctan(ax))a^4x^4-3\arctan(ax)\sqrt{a^2x^2+1}\,a^3x^3+18\arctan(ax)^2\,\mathrm{expIntegral}(1,3i\arctan(ax))a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/16*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^2*x^2-I*(a^2*x^2+1)^(1/2)*a^3*x^3-9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2+3*I*(a^2*x^2+1)^(1/2)*a*x+3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/a/c^3+1/16*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2+I*(a^2*x^2+1)^(1/2)*a^3*x^3+9*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*(a^2*x^2+1)^(1/2)*a^2*x^2+9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x-3*I*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-3*I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^4*x^4+2*a^2*x^2+1)/arctan(a*x)^2/a/c^3+3/16*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+I*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a/c^3+3/16*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-I*(a^2*x^2+1)^(1/2)*a*x-I*arctan(a*x)*(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a/c^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")

[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
+ c^3)*arctan(a*x)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.672 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=263

$$\frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^2}+\frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^2}-\frac{\sqrt{c+a^2cx^2}}{2ac^3x\text{ArcTan}(ax)^2}+\frac{3}{2c\left(c+a^2cx^2\right)^{3/2}\text{ArcTa}}$$

```
[Out] 1/2*a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2/c/(a^2*c*x^2+c)^(3/2)/arcta
n(a*x)+1/2*a*x/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2/c^2/arctan(a*x)/(a
^2*c*x^2+c)^(1/2)+5/8*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(
1/2)+9/8*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-1/2*(a
^2*c*x^2+c)^(1/2)/a/c^3/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(a*x)^
2/(a^2*c*x^2+c)^(1/2),x)/a/c^2
```

Rubi [A]
time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] (a*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (a*x)/(2*c^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2) - Sqrt[c + a^2*c*x^2]/(2*a*c^3*x*ArcTan[a*x]^2) + 3/
(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - 1/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]) + (5*Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a
^2*c*x^2]) + (9*Sqrt[1 + a^2*x^2]*SinIntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c
 + a^2*c*x^2]) - Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]/(
2*a*c^2)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} - \frac{1}{2}a\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx + a^3$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

$$= \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{\sqrt{c+a^2}}{2ac^3x\tan^-}$$

Mathematica [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\,\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
+ c^3*x)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)

[Out] int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)

3.673 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=232

$$\frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^2}+\frac{a}{2c^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^2}-\frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)}-\frac{}{2c^2\sqrt{c+a^2c}}$$

```
[Out] 1/2*a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2*a^2*x/c/(a^2*c*x^2+c)^(3/2)/a
rctan(a*x)+1/2*a/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2*x/c^2/arctan
(a*x)/(a^2*c*x^2+c)^(1/2)+7/8*a*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*
c*x^2+c)^(1/2)+9/8*a*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^
(1/2)+Unintegrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c^2
```

Rubi [A]
time = 0.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

```
[Out] a/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + a/(2*c^2*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2) - (3*a^2*x)/(2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]) - (a^2*x
)/(2*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]) + (7*a*Sqrt[1 + a^2*x^2]*CosInteg
ral[ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + (9*a*Sqrt[1 + a^2*x^2]*CosI
ntegral[3*ArcTan[a*x]])/(8*c^2*Sqrt[c + a^2*c*x^2]) + Defer[Int][1/(x^2*Sqr
t[c + a^2*c*x^2]*ArcTan[a*x]^3), x]/c^2
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx = -\left(a^2\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^3}\,dx\right) + \frac{\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^3}\,dx}{c}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{1}{2}\left(3a^3\right)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^2}\,dx$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

$$= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^2} + \frac{a}{2c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^2} - \frac{}{2c\left(c+a^2\right.}$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
+ c^3*x^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^3 \left(c a^2 x^2 + c\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.674 $\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^3}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*arctan(a*x)^2*integrate(((a^10*c^3*m^2 + 17*a^10*c^3*m + 72*a^10*c^3
)*x^10 + (5*a^8*c^3*m^2 + 67*a^8*c^3*m + 224*a^8*c^3)*x^8 + 2*(5*a^6*c^3*m^
2 + 49*a^6*c^3*m + 120*a^6*c^3)*x^6 + c^3*m^2 + 2*(5*a^4*c^3*m^2 + 31*a^4*c
^3*m + 48*a^4*c^3)*x^4 - c^3*m + (5*a^2*c^3*m^2 + 13*a^2*c^3*m + 8*a^2*c^3)
*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^10*c^3*m + 8*a^10*c^3)*x^10 + (5*a^8*
c^3*m + 32*a^8*c^3)*x^8 + 2*(5*a^6*c^3*m + 24*a^6*c^3)*x^6 + 2*(5*a^4*c^3*m
 + 16*a^4*c^3)*x^4 + c^3*m + (5*a^2*c^3*m + 8*a^2*c^3)*x^2)*x^m*arctan(a*x)
 - (a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x)*
x^m)/(a^2*x*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x^{m}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{2}x^{m}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{4}x^{m}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{6}x^{m}}{\operatorname{atan}^{3}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] c**3*(Integral(x**m/atan(a*x)**3, x) + Integral(3*a**2*x**2*x**m/atan(a*x)*
*3, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**3, x) + Integral(a**6*x**6*x*
*m/atan(a*x)**3, x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

3.675 $\int \frac{x^m(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

Maple [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^2}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*arctan(a*x)^2*integrate(((a^8*c^2*m^2 + 13*a^8*c^2*m + 42*a^8*c^2)*x
^8 + 2*(2*a^6*c^2*m^2 + 19*a^6*c^2*m + 45*a^6*c^2)*x^6 + 6*(a^4*c^2*m^2 + 6
*a^4*c^2*m + 9*a^4*c^2)*x^4 + c^2*m^2 - c^2*m + 2*(2*a^2*c^2*m^2 + 5*a^2*c^
2*m + 3*a^2*c^2)*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^8*c^2*m + 6*a^8*c^2)*
x^8 + 2*(2*a^6*c^2*m + 9*a^6*c^2)*x^6 + 6*(a^4*c^2*m + 3*a^4*c^2)*x^4 + c^2
*m + 2*(2*a^2*c^2*m + 3*a^2*c^2)*x^2)*x^m*arctan(a*x) - (a^7*c^2*x^7 + 3*a^
5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{x^{m}}{\operatorname{atan}^{3}\left(ax\right)}\,dx+\int\frac{2a^{2}x^{2}x^{m}}{\operatorname{atan}^{3}\left(ax\right)}\,dx+\int\frac{a^{4}x^{4}x^{m}}{\operatorname{atan}^{3}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m/atan(a*x)*
*3, x) + Integral(a**4*x**4*x**m/atan(a*x)**3, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

3.676 $\int \frac{x^m(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*arctan(a*x)^2*integrate(((a^6*c*m^2 + 9*a^6*c*m + 20*a^6*c)*x^6 + (3
*a^4*c*m^2 + 17*a^4*c*m + 24*a^4*c)*x^4 + c*m^2 + (3*a^2*c*m^2 + 7*a^2*c*m
+ 4*a^2*c)*x^2 - c*m)*x^m/(x^2*arctan(a*x)), x) - ((a^6*c*m + 4*a^6*c)*x^6
+ (3*a^4*c*m + 8*a^4*c)*x^4 + (3*a^2*c*m + 4*a^2*c)*x^2 + c*m)*x^m*arctan(a
*x) - (a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] c*(Integral(x**m/atan(a*x)**3, x) + Integral(a**2*x**2*x**m/atan(a*x)**3, x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

3.677 $\int \frac{x^m}{(c+a^2cx^2)\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=46

$$-\frac{x^m}{2ac\text{ArcTan}(ax)^2}+\frac{m\text{Int}\left(\frac{x^{-1+m}}{\text{ArcTan}(ax)^2},x\right)}{2ac}$$

```
[Out] -1/2*x^m/a/c/arctan(a*x)^2+1/2*m*Unintegrable(x^(-1+m)/arctan(a*x)^2,x)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] -1/2*x^m/(a*c*ArcTan[a*x]^2) + (m*Defer[Int][x^(-1 + m)/ArcTan[a*x]^2, x])/
(2*a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^3}\,dx=-\frac{x^m}{2ac\tan^{-1}(ax)^2}+\frac{m\int \frac{x^{-1+m}}{\tan^{-1}(ax)^2}\,dx}{2ac}$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*arctan(a*x)^2*integrate(((a^2*m^2 + a^2*m)*x^2 + m^2 - m)*x^m/(x^2*a
rctan(a*x)), x) - a*x*x^m - (a^2*m*x^2 + m)*x^m*arctan(a*x))/(a^2*c*x*arcta
n(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^2x^2\,\mathrm{atan}^3\,(ax)+\mathrm{atan}^3\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)),x)
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.678 $\int \frac{x^m}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^3} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx = \int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2*integrate(1/2*((a^4*m^2 - 3*
a^4*m + 2*a^4)*x^4 + 2*(a^2*m^2 - 2*a^2*m - a^2)*x^2 + m^2 - m)*x^m/((a^6*c
^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)), x) - a*x*x^m - ((a^2*m
- 2*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^4x^4\,\mathrm{atan}^3\,(ax)+2a^2x^2\,\mathrm{atan}^3\,(ax)+\mathrm{atan}^3\,(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x
)**3), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.679 $\int \frac{x^m}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx=\int \frac{x^m}{(c+a^2cx^2)^3\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2*integrate(1/
2*((a^4*m^2 - 7*a^4*m + 12*a^4)*x^4 + 2*(a^2*m^2 - 4*a^2*m - 2*a^2)*x^2 + m
^2 - m)*x^m/((a^8*c^3*x^8 + 3*a^6*c^3*x^6 + 3*a^4*c^3*x^4 + a^2*c^3*x^2)*ar
ctan(a*x)), x) - a*x*x^m - ((a^2*m - 4*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^6
*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x
**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.680 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

3.681 $\int \frac{x^m (c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^3} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m (c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m (c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3} \, dx = \int \frac{x^m (c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^3} \, dx$$

Mathematica [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

Maple [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

3.682 $\int \frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Rubi steps

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3} dx = \int \frac{x^m \sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^3} dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

Maple [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{a^2 c\, x^2+c}}{\arctan(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sqrt{c\,a^2\,x^2+c}}{\operatorname{atan}\left(a\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

3.683 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\arctan(ax)^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\ \operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.684 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

```
[Out] Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.685 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^3},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^3}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.686 $\int x^m(c+a^2cx^2) \sqrt{\mathbf{ArcTan}(ax)}\, dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx = \int x^m\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{m}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int a^{2}x^{2}x^{m}\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m*sqrt(atan(a*
x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \sqrt{\operatorname{atan}(a x)} \left(c a^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

```
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.687 $\int x(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=57

$$\frac{c(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}{4a^2}-\frac{\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\text{ArcTan}(ax)}},x\right)}{8a}$$

```
[Out] 1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2-1/8*Unintegrable((a^2*c*x^2+c)/ar
ctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
[Out] (c*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])/(4*a^2) - Defer[Int][(c + a^2*c*x^2)/
Sqrt[ArcTan[a*x]], x]/(8*a)
```

Rubi steps

$$\int x(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}\,dx=\frac{c(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{4a^2}-\frac{\int\frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}}\,dx}{8a}$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int x(a^2c\,x^2+c)\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x\sqrt{\operatorname{atan}(ax)}\,dx+\int a^2x^3\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x*sqrt(atan(a*x)), x) + Integral(a**2*x**3*sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.688 $\int (c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left((c+a^2cx^2)\sqrt{\text{ArcTan}(ax)},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]

Rubi steps

$$\int (c+a^2cx^2)\sqrt{\tan^{-1}(ax)}\,dx = \int (c+a^2cx^2)\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 2.35, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]

[Out] Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]

Maple [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int (a^2c\,x^2+c)\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(a**2*x**2*sqrt(atan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\operatorname{atan}(a x)} \left(c a^2 x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.689 $\int \frac{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
[Out] Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)\sqrt{\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{\sqrt{\operatorname{atan}{(ax)}}}{x}\,dx+\int a^{2}x\sqrt{\operatorname{atan}{(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(1/2)/x,x)
```

```
[Out] c*(Integral(sqrt(atan(a*x))/x, x) + Integral(a**2*x*sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x,x)
```

```
[Out] int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x, x)
```

3.690 $\int x^m(c+a^2cx^2)^2 \sqrt{\mathbf{ArcTan}(ax)}\, dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^2\sqrt{\operatorname{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^2\sqrt{\operatorname{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\, dx = \int x^m\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^2\sqrt{\operatorname{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.95, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^m\sqrt{\operatorname{atan}(ax)}\,dx+\int 2a^2x^2x^m\sqrt{\operatorname{atan}(ax)}\,dx+\int a^4x^4x^m\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m*sqrt(at
an(a*x)), x) + Integral(a**4*x**4*x**m*sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \sqrt{\operatorname{atan}(a x)} \left(c a^2 x^2 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.691 $\int x(c+a^2cx^2)^2 \sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=61

$$\frac{c^2(1+a^2x^2)^3\sqrt{\text{ArcTan}(ax)}}{6a^2}-\frac{\text{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}},x\right)}{12a}$$

```
[Out] 1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(1/2)/a^2-1/12*Unintegrable((a^2*c*x^2+c)
^2/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] (c^2*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(6*a^2) - Defer[Int][(c + a^2*c*x^2
)^2/Sqrt[ArcTan[a*x]], x]/(12*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}\,dx=\frac{c^2(1+a^2x^2)^3\sqrt{\tan^{-1}(ax)}}{6a^2}-\frac{\int\frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx}{12a}$$

Mathematica [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int x\left(a^{2} c\,x^{2}+c\right)^{2} \sqrt{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int x\sqrt{\operatorname{atan}\left(a x\right)}\, dx+\int 2a^{2}x^{3}\sqrt{\operatorname{atan}\left(a x\right)}\, dx+\int a^{4}x^{5}\sqrt{\operatorname{atan}\left(a x\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
[Out] c**2*(Integral(x*sqrt(atan(a*x)), x) + Integral(2*a**2*x**3*sqrt(atan(a*x))
, x) + Integral(a**4*x**5*sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.692 $\int (c+a^2cx^2)^2 \sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left((c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int (c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}\,dx = \int (c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int (a^2c\,x^2+c)^2\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx+\int a^4x^4\sqrt{\operatorname{atan}(ax)}\,dx+\int\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*sqrt(atan(a*x)), x) + Integral(a**4*x**4*sqrt(at
an(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.693 $\int \frac{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
[Out] Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2\sqrt{\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx+\int 2a^2x\sqrt{\operatorname{atan}(ax)}\,dx+\int a^4x^3\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2)/x,x)
```

```
[Out] c**2*(Integral(sqrt(atan(a*x))/x, x) + Integral(2*a**2*x*sqrt(atan(a*x)), x
) + Integral(a**4*x**3*sqrt(atan(a*x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x,x)
[Out] int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.694 $\int x^m(c+a^2cx^2)^3 \sqrt{\mathbf{ArcTan}(ax)}\, dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\, dx = \int x^m\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 2.41, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*sqrt(arcta
n(a*x)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \sqrt{\operatorname{atan}(a x)} \left(c a^2 x^2+c\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.695 $\int x(c+a^2cx^2)^3 \sqrt{\mathbf{ArcTan}(ax)}\, dx$

Optimal. Leaf size=61

$$\frac{c^3(1+a^2x^2)^4\sqrt{\operatorname{ArcTan}(ax)}}{8a^2}-\frac{\operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\operatorname{ArcTan}(ax)}},x\right)}{16a}$$

```
[Out] 1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(1/2)/a^2-1/16*Unintegrable((a^2*c*x^2+c)
^3/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^3\sqrt{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] (c^3*(1 + a^2*x^2)^4*Sqrt[ArcTan[a*x]])/(8*a^2) - Defer[Int][(c + a^2*c*x^2
)^3/Sqrt[ArcTan[a*x]], x]/(16*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}\,dx=\frac{c^3(1+a^2x^2)^4\sqrt{\tan^{-1}(ax)}}{8a^2}-\frac{\int\frac{(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}}\,dx}{16a}$$

Mathematica [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^3\sqrt{\operatorname{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x\left(a^{2} c\,x^{2}+c\right)^{3} \sqrt{\arctan\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3} \left(\int x \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int 3 a^{2} x^{3} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int 3 a^{4} x^{5} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int a^{6} x^{7} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(x*sqrt(atan(a*x)), x) + Integral(3*a**2*x**3*sqrt(atan(a*x))
, x) + Integral(3*a**4*x**5*sqrt(atan(a*x)), x) + Integral(a**6*x**7*sqrt(a
tan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.696 $\int (c+a^2cx^2)^3 \sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx=\int \left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^3\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int 3a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int 3a^{4}x^{4}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int a^{6}x^{6}\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int \sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*sqrt(atan(a*x)), x) + Integral(3*a**4*x**4*sqrt(
atan(a*x)), x) + Integral(a**6*x**6*sqrt(atan(a*x)), x) + Integral(sqrt(ata
n(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.697 $\int \frac{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}{x}\,dx$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

Maple [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3\sqrt{\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{x}\, dx + \int 3a^{2}x\sqrt{\operatorname{atan}{\left(ax \right)}}\, dx + \int 3a^{4}x^{3}\sqrt{\operatorname{atan}{\left(ax \right)}}\, dx + \int a^{6}x^{5}\sqrt{\operatorname{atan}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2)/x,x)
```

```
[Out] c**3*(Integral(sqrt(atan(a*x))/x, x) + Integral(3*a**2*x*sqrt(atan(a*x)), x
) + Integral(3*a**4*x**3*sqrt(atan(a*x)), x) + Integral(a**6*x**5*sqrt(atan
(a*x)), x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x,x)
```

```
[Out] int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.698 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{c + a^2cx^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{c + a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{c + a^2cx^2} \, dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{c + a^2cx^2} \, dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{c + a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2c\,x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sqrt{\operatorname{atan}(a x)}}{c a^2 x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.699 $\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=61

$$-\frac{2x\text{ArcTan}(ax)^{3/2}}{3a^3c}+\frac{\text{Int}\left(x\sqrt{\text{ArcTan}(ax)},x\right)}{a^2c}+\frac{2\text{Int}\left(\text{ArcTan}(ax)^{3/2},x\right)}{3a^3c}$$

```
[Out] -2/3*x*arctan(a*x)^(3/2)/a^3/c+2/3*Unintegrable(arctan(a*x)^(3/2),x)/a^3/c+
Unintegrable(x*arctan(a*x)^(1/2),x)/a^2/c
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(3/2))/(3*a^3*c) + Defer[Int][x*Sqrt[ArcTan[a*x]], x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(3/2), x])/(3*a^3*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{c+a^2cx^2} \, dx &= -\frac{\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2} \, dx}{a^2}+\frac{\int x\sqrt{\tan^{-1}(ax)} \, dx}{a^2c}\\
&= -\frac{2x\tan^{-1}(ax)^{3/2}}{3a^3c}+\frac{2\int \tan^{-1}(ax)^{3/2} \, dx}{3a^3c}+\frac{\int x\sqrt{\tan^{-1}(ax)} \, dx}{a^2c}
\end{aligned}$$

Mathematica [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{\arctan \left(a x \right)}}{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3} \sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
[Out] Integral(x**3*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.700 $\int \frac{x^2 \sqrt{\mathbf{ArcTan}(ax)}}{c+a^2cx^2} dx$

Optimal. Leaf size=37

$$-\frac{2\text{ArcTan}(ax)^{3/2}}{3a^3c}+\frac{\text{Int}\left(\sqrt{\text{ArcTan}(ax)},x\right)}{a^2c}$$

```
[Out] -2/3*arctan(a*x)^(3/2)/a^3/c+Unintegrable(arctan(a*x)^(1/2),x)/a^2/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2 \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] (-2*ArcTan[a*x]^(3/2))/(3*a^3*c) + Defer[Int][Sqrt[ArcTan[a*x]], x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \sqrt{\tan^{-1}(ax)}}{c+a^2cx^2} dx &= -\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2} dx}{a^2}+\frac{\int \sqrt{\tan^{-1}(ax)} dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{3/2}}{3a^3c}+\frac{\int \sqrt{\tan^{-1}(ax)} dx}{a^2c}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{\arctan \left(a x \right)}}{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
[Out] Integral(x**2*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.701 $\int \frac{x\sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} dx$

Optimal. Leaf size=41

$$\frac{2x\text{ArcTan}(ax)^{3/2}}{3ac} - \frac{2\text{Int}\left(\text{ArcTan}(ax)^{3/2}, x\right)}{3ac}$$

```
[Out] 2/3*x*arctan(a*x)^(3/2)/a/c-2/3*Unintegrable(arctan(a*x)^(3/2),x)/a/c
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
[Out] (2*x*ArcTan[a*x]^(3/2))/(3*a*c) - (2*Defer[Int][ArcTan[a*x]^(3/2), x])/(3*a
*c)
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2} dx = \frac{2x\tan^{-1}(ax)^{3/2}}{3ac} - \frac{2\int \tan^{-1}(ax)^{3/2} dx}{3ac}$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\text{ArcTan}(ax)}}{c+a^2cx^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
[Out] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\arctan(ax)}}{a^2c\,x^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.702 $\int \frac{\sqrt{\mathbf{ArcTan}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{2\mathrm{ArcTan}(ax)^{3/2}}{3ac}$$

```
[Out] 2/3*arctan(a*x)^(3/2)/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5004\}$

$$\frac{2\mathrm{ArcTan}(ax)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(3/2))/(3*a*c)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{2\tan^{-1}(ax)^{3/2}}{3ac}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{2\mathrm{ArcTan}(ax)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(3/2))/(3*a*c)
```

Maple [A]
time = 0.17, size = 15, normalized size = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{3}{2}}}{3ac}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arctan(a*x)^(3/2)/a/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.08, size = 14, normalized size = 0.78

$$\frac{2\arctan\left(ax\right)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/3*arctan(a*x)^(3/2)/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.44, size = 14, normalized size = 0.78

$$\frac{2\arctan\left(ax\right)^{\frac{3}{2}}}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
[Out] 2/3*arctan(a*x)^(3/2)/(a*c)
```

Mupad [B]
time = 0.38, size = 14, normalized size = 0.78

$$\frac{2\,\mathrm{atan}(a\,x)^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2),x)
[Out] (2*atan(a*x)^(3/2))/(3*a*c)
```

3.703 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=49

$$-\frac{2i\text{ArcTan}(ax)^{3/2}}{3c}+\frac{i\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x(i+ax)},x\right)}{c}$$

```
[Out] -2/3*I*arctan(a*x)^(3/2)/c+I*Unintegrable(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
[Out] (((-2*I)/3)*ArcTan[a*x]^(3/2))/c + (I*Defer[Int][Sqrt[ArcTan[a*x]]/(x*(I +
a*x)), x])/c
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\,(c+a^2cx^2)}\,dx=-\frac{2i\tan^{-1}(ax)^{3/2}}{3c}+\frac{i\int\frac{\sqrt{\tan^{-1}(ax)}}{x(i+ax)}\,dx}{c}$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c),x)
[Out] Integral(sqrt(atan(a*x))/(a**2*x**3 + x), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)), x)
```

3.704 $\int \frac{\sqrt{\mathbf{ArcTan}(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{2a\text{ArcTan}(ax)^{3/2}}{3c}+\frac{\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x^2},x\right)}{c}$$

```
[Out] -2/3*a*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^2,x)/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (-2*a*ArcTan[a*x]^(3/2))/(3*c) + Defer[Int][Sqrt[ArcTan[a*x]]/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\\ &= -\frac{2a\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan \left(a x \right)}}{x^{2} \left(a^{2} c \,x^{2}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{2} x^{4} + x^{2}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**2*x**4 + x**2), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.705 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{2ia^2\text{ArcTan}(ax)^{3/2}}{3c}+\frac{\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x^3},x\right)}{c}-\frac{ia^2\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x(i+ax)},x\right)}{c}$$

```
[Out] 2/3*I*a^2*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^3,x)/c-I*a^2
*Unintegrable(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/3)*a^2*ArcTan[a*x]^(3/2))/c + Defer[Int][Sqrt[ArcTan[a*x]]/x^3, x]/
c - (I*a^2*Defer[Int][Sqrt[ArcTan[a*x]]/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3}\,dx}{c}\\ &= \frac{2ia^2\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{x(i+ax)}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)), x]
```

Maple [**A**]
time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x^{3} \left(a^{2} c \,x^{2}+c \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
[Out] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{2} x^{5} + x^{3}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c),x)
[Out] Integral(sqrt(atan(a*x))/(a**2*x**5 + x**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.706 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2a^3\text{ArcTan}(ax)^{3/2}}{3c}+\frac{\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x^4},x\right)}{c}-\frac{a^2\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x^2},x\right)}{c}$$

```
[Out] 2/3*a^3*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^4,x)/c-a^2*Uni
ntegrable(arctan(a*x)^(1/2)/x^2,x)/c
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (2*a^3*ArcTan[a*x]^(3/2))/(3*c) + Defer[Int][Sqrt[ArcTan[a*x]]/x^4, x]/c -
(a^2*Defer[Int][Sqrt[ArcTan[a*x]]/x^2, x])/c
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\,(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\,(c+a^2cx^2)}\,dx\right)+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}\\
&= a^4\int \frac{\sqrt{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}\\
&= \frac{2a^3\tan^{-1}(ax)^{3/2}}{3c}+\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2}\,dx}{c}
\end{aligned}$$

Mathematica [A]
time = 4.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan \left(a x\right)}}{x^{4} \left(a^{2} c \,x^{2}+c \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{a^{2} x^{6} + x^{4}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c),x)
[Out] Integral(sqrt(atan(a*x))/(a**2*x**6 + x**4), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.707 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2} \, dx$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sqrt{\operatorname{atan}(a x)}}{(c a^2 x^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.708 $\int \frac{x^3\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^3\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2},x\right)$$

[Out] Unintegrable(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 3.24, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]

[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]

Maple [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3 \sqrt{\operatorname{atan}(a x)}}{(c a^2 x^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.709 $\int \frac{x^2 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=80

$$-\frac{x\sqrt{\text{ArcTan}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{3/2}}{3a^3c^2}+\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}$$

```
[Out] 1/3*arctan(a*x)^(3/2)/a^3/c^2+1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi
^(1/2)/a^3/c^2-1/2*x*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5056, 5090, 4491, 12, 3386, 3432}

$$\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}+\frac{\text{ArcTan}(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\text{ArcTan}(ax)}}{2a^2c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] -1/2*(x*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a
^3*c^2) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^3*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\int \frac{x}{\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{4a}\\
&= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^3c^2}\\
&= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^3c^2}\\
&= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^3c^2}\\
&= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2}\\
&= -\frac{x\sqrt{\tan^{-1}(ax)}}{2a^2c^2\left(1+a^2x^2\right)}+\frac{\tan^{-1}(ax)^{3/2}}{3a^3c^2}+\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.82

$$\frac{4\sqrt{\text{ArcTan}(ax)}\left(-\frac{3ax}{1+a^2x^2}+2\text{ArcTan}(ax)\right)+3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{24a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[ArcTan[a*x]]*((-3*a*x)/(1 + a^2*x^2) + 2*ArcTan[a*x]) + 3*Sqrt[Pi]*
FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(24*a^3*c^2)
```

Maple [A]
time = 0.26, size = 57, normalized size = 0.71

method	result	size
default	$\frac{8\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}-6\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\sin(2\arctan(ax))+3\pi\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24c^2a^3\sqrt{\pi}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/c^2/a^3/Pi^(1/2)*(8*arctan(a*x)^(3/2)*Pi^(1/2)-6*arctan(a*x)^(1/2)*Pi^
(1/2)*sin(2*arctan(a*x))+3*Pi*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.710 $\int \frac{x\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{\text{ArcTan}(ax)}}{4a^2c^2}-\frac{\sqrt{\text{ArcTan}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}$$

```
[Out] 1/8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2+1/4*arctan(a*x)
^(1/2)/a^2/c^2-1/2*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5050, 5024, 3393, 3385, 3433}

$$\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}-\frac{\sqrt{\text{ArcTan}(ax)}}{2a^2c^2(a^2x^2+1)}+\frac{\sqrt{\text{ArcTan}(ax)}}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^2*c^2) - Sqrt[ArcTan[a*x]]/(2*a^2*c^2*(1 + a^2*x^2))
 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^2*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx &= -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\int \frac{1}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{4a}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^2c^2}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}\right)dx,x,\tan^{-1}(ax)\right)}{4a^2c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2}-\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^2c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2}-\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\text{Subst}\left(\int \cos(2x^2)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^2}-\frac{\sqrt{\tan^{-1}(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 136, normalized size = 1.72

$$
\frac{4\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+\frac{\frac{16(-1+a^2x^2)\text{ArcTan}(ax)}{1+a^2x^2}-i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{\sqrt{\text{ArcTan}(ax)}}}{64a^2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((16*(-1 + a^2*x^2)*
ArcTan[a*x])/(1 + a^2*x^2) - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a
*x]])/Sqrt[ArcTan[a*x]])/(64*a^2*c^2)
```

Maple [A]
time = 0.20, size = 46, normalized size = 0.58

method	result	size
default	$-\frac{2\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}\,\cos\left(2\arctan\left(ax\right)\right)-\pi\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)}{8c^{2}a^{2}\sqrt{\pi}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^2/a^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))-Pi*FresnelC(2
*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
[Out] Integral(x*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\operatorname{atan}(a\,x)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.711 $\int \frac{\sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{x\sqrt{\text{ArcTan}(ax)}}{2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{3/2}}{3ac^2}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}$$

```
[Out] 1/3*arctan(a*x)^(3/2)/a/c^2-1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(
1/2)/a/c^2+1/2*x*arctan(a*x)^(1/2)/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5012, 5090, 4491, 12, 3386, 3432}

$$\frac{x\sqrt{\text{ArcTan}(ax)}}{2c^2(a^2x^2+1)}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}+\frac{\text{ArcTan}(ax)^{3/2}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (x*Sqrt[ArcTan[a*x]])/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a*c^2) -
 (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx &= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{1}{4}a\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4ac^2}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4ac^2}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8ac^2}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4ac^2}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{2c^2(1+a^2x^2)}+\frac{\tan^{-1}(ax)^{3/2}}{3ac^2}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 0.88

$$\frac{4\sqrt{\text{ArcTan}(ax)}\left(\frac{3x}{1+a^2x^2}+\frac{2\text{ArcTan}(ax)}{a}\right)-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a}}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[ArcTan[a*x]]*((3*x)/(1 + a^2*x^2) + (2*ArcTan[a*x])/a) - (3*Sqrt[Pi
]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a)/(24*c^2)
```

Maple [A]
time = 0.25, size = 60, normalized size = 0.78

method	result	size
default	$\frac{8\arctan(ax)^2+6\sin(2\arctan(ax))\arctan(ax)-3\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24c^2a\sqrt{\arctan(ax)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/c^2/a/arctan(a*x)^(1/2)*(8*arctan(a*x)^2+6*sin(2*arctan(a*x))*arctan(a
*x)-3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2,x)
```

```
[Out] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2, x)
```

3.712 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^2},x\right)$$

[Out] Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]

[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx$$

Mathematica [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]

[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]

Maple [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{a^{4}x^{5}+2a^{2}x^{3}+x}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.713 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.714 $\int \frac{x^5 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^5 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
[Out] Defer[Int][(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^5 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3} \, dx$$

Mathematica [A]
time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**5*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.715 $\int \frac{x^4 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=139

$$\frac{\text{ArcTan}(ax)^{3/2}}{4a^5c^3} - \frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^5c^3} + \frac{\sqrt{\pi}\, S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3} - \frac{\sqrt{\text{ArcTan}(ax)}\, \sin(2\text{ArcT}}{4a^5c^3}$$

```
[Out] 1/4*arctan(a*x)^(3/2)/a^5/c^3-1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3+1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))
*Pi^(1/2)/a^5/c^3-1/4*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3+1/32*sin
(4*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5090, 3393, 3377, 3386, 3432}

$$-\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^5c^3} + \frac{\sqrt{\pi}\, S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3} + \frac{\text{ArcTan}(ax)^{3/2}}{4a^5c^3} - \frac{\sqrt{\text{ArcTan}(ax)}\, \sin(2\text{ArcTan}(ax))}{4a^5c^3} + \frac{\sqrt{\text{ArcTan}(ax)}\, \sin(4\text{ArcTan}(ax))}{32a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(3/2)/(4*a^5*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(64*a^5*c^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]
])/(8*a^5*c^3) - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt
[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^5*c^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\,\sin^4(x)\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}-\frac{1}{2}\sqrt{x}\,\cos(2x)+\frac{1}{8}\sqrt{x}\,\cos(4x)\right)\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}+\frac{\text{Subst}\left(\int \sqrt{x}\,\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^5c^3}-\frac{\text{Subst}\left(\int \sqrt{x}\,\cos(2x)\,dx,\right.}{2a^5c^3}\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32a^5c^3}-\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{4a^5c^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32a^5c^3}-\\
&= \frac{\tan^{-1}(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^5c^3}+\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3}-\frac{\sqrt{\text{ta}}}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 181, normalized size = 1.30

$$
\frac{-\frac{96ax\text{ArcTan}(ax)}{(1+a^2x^2)^2}-\frac{160a^3x^3\text{ArcTan}(ax)}{(1+a^2x^2)^2}+64\text{ArcTan}(ax)^2-8\sqrt{2}\,\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)-8\sqrt{2}\,\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{256a^5c^3\sqrt{\text{ArcTan}(ax)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-96*a*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (160*a^3*x^3*ArcTan[a*x])/(1 + a^2
*x^2)^2 + 64*ArcTan[a*x]^2 - 8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] - 8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a
*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTa
n[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(256*a^5*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.29, size = 102, normalized size = 0.73

method	result
default	$\frac{-\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+32\arctan(ax)^2+4\sin(4\arctan(ax))\arctan(ax)-32\sin(2\arctan}{128c^3a^5\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/c^3/a^5/arctan(a*x)^(1/2)*(-FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+32*arctan(a*x)^2+4*sin(4*arctan(a*
x))*arctan(a*x)-32*sin(2*arctan(a*x))*arctan(a*x)+16*arctan(a*x)^(1/2)*Pi^(
1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**4*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.716 $\int \frac{x^3\sqrt{\textbf{ArcTan}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{3\sqrt{\text{ArcTan}(ax)}}{32a^4c^3}+\frac{x^4\sqrt{\text{ArcTan}(ax)}}{4c^3(1+a^2x^2)^2}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^4c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}$$

```
[Out] -1/128*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/
c^3+1/16*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3-3/32*arcta
n(a*x)^(1/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)^2
```

Rubi [A]
time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5064, 5090, 3393, 3385, 3433}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^4c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}-\frac{3\sqrt{\text{ArcTan}(ax)}}{32a^4c^3}+\frac{x^4\sqrt{\text{ArcTan}(ax)}}{4c^3(a^2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(32*a^4*c^3) + (x^4*Sqrt[ArcTan[a*x]])/(4*c^3*(1 + a
^2*x^2)^2) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^4*
c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^4*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{1}{8}a\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$

$$= \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{64a^4c^3} + \frac{\text{Subst}\big(\cdots}{\cdots}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\text{Subst}\left(\int \cos\left(4x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{32a^4c^3} + \cdots$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\tan^{-1}(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^4c^3} + \frac{\sqrt{\pi}\,C\big(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.46, size = 230, normalized size = 1.95

$$\frac{-10\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+80\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+\frac{\frac{64(-3-6a^2x^2+5a^4x^4)\text{ArcTan}(ax)}{(1+a^2x^2)^2}-12i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+12i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+3i\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-3i\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{\sqrt{\text{ArcTan}(ax)}}}{2048a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-10*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 80*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*Ar
cTan[a*x])/(1 + a^2*x^2)^2 - (12*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/
2, (-2*I)*ArcTan[a*x]] + (12*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I
)*ArcTan[a*x]] + (3*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]
] - (3*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*
x]])/(2048*a^4*c^3)
```

Maple [A]
time = 0.27, size = 93, normalized size = 0.79

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+16\cos(2\arctan(ax))\arctan(ax)-4\cos(4\arctan(ax))\,a}{128c^3a^4\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/c^3/a^4/arctan(a*x)^(1/2)*(2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresne
lC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+16*cos(2*arctan(a*x))*arctan(a*x)-
4*cos(4*arctan(a*x))*arctan(a*x)-8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*ar
ctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.717 $\int \frac{x^2 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=83

$$\frac{\text{ArcTan}(ax)^{3/2}}{12a^3c^3} + \frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^3c^3} - \frac{\sqrt{\text{ArcTan}(ax)}\sin(4\text{ArcTan}(ax))}{32a^3c^3}$$

```
[Out] 1/12*arctan(a*x)^(3/2)/a^3/c^3+1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3-1/32*sin(4*arctan(a*x))*arctan(a*x)^(1/2)
/a^3/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5090, 4491, 3377, 3386, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^3c^3} + \frac{\text{ArcTan}(ax)^{3/2}}{12a^3c^3} - \frac{\sqrt{\text{ArcTan}(ax)}\sin(4\text{ArcTan}(ax))}{32a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(3/2)/(12*a^3*c^3) + (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(64*a^3*c^3) - (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(32*a^3*c
^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\,\cos^2(x)\sin^2(x)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sqrt{x}}{8}-\frac{1}{8}\sqrt{x}\,\cos(4x)\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3} \\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\text{Subst}\left(\int \sqrt{x}\,\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3} \\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{64a^3c^3} \\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}+\frac{\text{Subst}\left(\int \sin\left(4x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{32a^3c^3} \\
&= \frac{\tan^{-1}(ax)^{3/2}}{12a^3c^3}+\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^3c^3}-\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 141, normalized size = 1.70

$$
\frac{32\text{ArcTan}(ax)\left(3ax(-1+a^2x^2)+2(1+a^2x^2)^2\,\text{ArcTan}(ax)\right)-3(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-3(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{768a^3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (32*ArcTan[a*x]*(3*a*x*(-1 + a^2*x^2) + 2*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3*
(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - 3*(
1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(768*a^3*
c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.24, size = 66, normalized size = 0.80

method	result	size
default	$\frac{3\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}+32\arctan\left(a x\right)^{2}-12\sin\left(4\arctan\left(a x\right)\right)\arctan\left(a x\right)}{384 c^{3} a^{3}\sqrt{\arctan\left(a x\right)}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/c^3/a^3*(3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arc
tan(a*x)^(1/2)*Pi^(1/2)+32*arctan(a*x)^2-12*sin(4*arctan(a*x))*arctan(a*x))
/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}\sqrt{\operatorname{atan}\left(a x\right)}}{a^{6} x^{6}+3 a^{4} x^{4}+3 a^{2} x^{2}+1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.718 $\int \frac{x\sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=118

$$\frac{3\sqrt{\text{ArcTan}(ax)}}{32a^2c^3}-\frac{\sqrt{\text{ArcTan}(ax)}}{4a^2c^3\left(1+a^2x^2\right)^2}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^2c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}$$

```
[Out] 1/128*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c
^3+1/16*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3+3/32*arctan
(a*x)^(1/2)/a^2/c^3-1/4*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)^2
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5050, 5024, 3393, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64a^2c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}-\frac{\sqrt{\text{ArcTan}(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\text{ArcTan}(ax)}}{32a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(32*a^2*c^3) - Sqrt[ArcTan[a*x]]/(4*a^2*c^3*(1 + a^2*
x^2)^2) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(64*a^2*c^3
) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a^2*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\int \frac{1}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{8a}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3} - \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{64a^2c^3} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3} - \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\text{Subst}\left(\int \cos(4x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{32a^2c^3} + \frac{\text{Su}}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32a^2c^3} - \frac{\sqrt{\tan^{-1}(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64a^2c^3} + \frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{}}{}\right.}{16}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 230, normalized size = 1.95

$$\frac{-6\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) + 48\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right) + \frac{\frac{64(-5+6a^2x^2+3a^4x^4)\text{ArcTan}(ax)}{(1+a^2x^2)^2} - 20i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) + 20i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right) - 11i\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right) + 11i\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{\sqrt{\text{ArcTan}(ax)}}}{2048a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-6*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 48*Sqrt[Pi]*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*Arc
Tan[a*x])/(1 + a^2*x^2)^2 - (20*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-2*I)*ArcTan[a*x]] + (20*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)
*ArcTan[a*x]] - (11*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]
] + (11*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a
*x]])/(2048*a^2*c^3)
```

Maple [A]
time = 0.21, size = 94, normalized size = 0.80

method	result
default	$-\frac{-\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+16\cos(2\arctan(ax))\arctan(ax)-8\sqrt{\arctan(a}}{128c^3a^2\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/c^3/a^2/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+16*cos(2*arctan(a*x))*arctan(a*x)
-8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+4*cos(
4*arctan(a*x))*arctan(a*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.719 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=139

$$\frac{\text{ArcTan}(ax)^{3/2}}{4ac^3}-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\frac{\sqrt{\text{ArcTan}(ax)}\,\sin(2\text{ArcT}}{4ac^3}$$

```
[Out] 1/4*arctan(a*x)^(3/2)/a/c^3-1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*2^(1/2)*Pi^(1/2)/a/c^3-1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^
(1/2)/a/c^3+1/4*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a/c^3+1/32*sin(4*arcta
n(a*x))*arctan(a*x)^(1/2)/a/c^3
```

Rubi [A]

time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5024, 3393, 3377, 3386, 3432}

$$-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\frac{\text{ArcTan}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\text{ArcTan}(ax)}\,\sin(2\text{ArcTan}(ax))}{4ac^3}+\frac{\sqrt{\text{ArcTan}(ax)}\,\sin(4\text{ArcTan}(ax))}{32ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

```
[Out] ArcTan[a*x]^(3/2)/(4*a*c^3) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]])/(64*a*c^3) - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(
8*a*c^3) + (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(4*a*c^3) + (Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(32*a*c^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\,\cos^4(x)\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}+\frac{1}{2}\sqrt{x}\,\cos(2x)+\frac{1}{8}\sqrt{x}\,\cos(4x)\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$
$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\text{Subst}\left(\int \sqrt{x}\,\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8ac^3}+\frac{\text{Subst}\left(\int \sqrt{x}\,\cos(2x)\,dx, x,\right.}{2ac^3}$$
$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32ac^3}-$$
$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{4ac^3}+\frac{\sqrt{\tan^{-1}(ax)}\,\sin\left(4\tan^{-1}(ax)\right)}{32ac^3}-$$
$$= \frac{\tan^{-1}(ax)^{3/2}}{4ac^3}-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{64ac^3}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{8ac^3}+\sqrt{\tan^{-}}$$

Mathematica [A]
time = 0.21, size = 103, normalized size = 0.74

$$-\frac{-16\sqrt{\text{ArcTan}(ax)}\left(\frac{ax(5+3a^2x^2)}{(1+a^2x^2)^2}+2\text{ArcTan}(ax)\right)+\sqrt{2\pi}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+16\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/128*(-16*Sqrt[ArcTan[a*x]]*((a*x*(5 + 3*a^2*x^2))/(1 + a^2*x^2)^2 + 2*Ar
cTan[a*x]) + Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqrt[
Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a*c^3)
```

Maple [A]
time = 0.28, size = 102, normalized size = 0.73

method	result
default	$\frac{-S\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+32\arctan(ax)^2+32\sin(2\arctan(ax))\arctan(ax)+4\sin(4\arctan}{128c^3a\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/c^3/a/arctan(a*x)^(1/2)*(-FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+32*arctan(a*x)^2+32*sin(2*arctan(a*x
))*arctan(a*x)+4*sin(4*arctan(a*x))*arctan(a*x)-16*arctan(a*x)^(1/2)*Pi^(1/
2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3, x)
```

3.720 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^3},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^3} dx$$

Mathematica [A]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\operatorname{atan}{(ax)}}}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(sqrt(atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c*
*3
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.721 $\int x^m \sqrt{c+a^2cx^2}\,\sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m \sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx = \int x^m \sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int x^m \sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c\left(a^2x^2+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \sqrt{\operatorname{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.722 $\int x^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcTan}(ax)} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcTan}(ax)}, x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2 \sqrt{c+a^2 c x^2} \sqrt{\tan^{-1}(ax)} \, dx = \int x^2 \sqrt{c+a^2 c x^2} \sqrt{\tan^{-1}(ax)} \, dx$$

Mathematica [A]
time = 2.97, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcTan}(ax)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.77, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a^2 c \, x^2 + c} \, \sqrt{\arctan(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{c\left(a^{2}x^{2}+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{\operatorname{atan}(a x)} \sqrt{c a^2 x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.723 $\int x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}{3a^2c}-\frac{\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}},x\right)}{6a}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2/c-1/6*Unintegrable((a^2*c*x^2
+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(3*a^2*c) - Defer[Int][Sqrt[c + a
^2*c*x^2]/Sqrt[ArcTan[a*x]], x]/(6*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx=\frac{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{\int\frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{6a}$$

Mathematica [A]
time = 2.11, size = 0, normalized size = 0.00

$$\int x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int x\sqrt{a^2c\,x^2+c}\;\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\left(a^2x^2+1\right)}\;\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.724 $\int \sqrt{c+a^2cx^2}\,\sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx = \int \sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^{2}x^{2}+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\operatorname{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.725 $\int x^m(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*sqrt(arctan(a*x)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\,\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.726 $\int x^2(c+a^2cx^2)^{3/2}\sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 3.44, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.83, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{\operatorname{atan}(a x)} \left(c a^2 x^2 + c\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.727 $\int x(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}{5a^2c}-\frac{\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)}{10a}$$

```
[Out] 1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a^2/c-1/10*Unintegrable((a^2*c*x^
2+c)^(3/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])/(5*a^2*c) - Defer[Int][(c + a^2*c
*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]/(10*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\frac{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}{5a^2c}-\frac{\int\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{10a}$$

Mathematica [A]
time = 5.68, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.15, size = 0, normalized size = 0.00

$$\int x\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.728 $\int (c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx=\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.729 $\int x^m(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx=\int x^m\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.62, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*sqrt(a
rctan(a*x)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\,\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.730 $\int x^2(c+a^2cx^2)^{5/2}\sqrt{\textbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.98, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.731 $\int x(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{7/2}\sqrt{\text{ArcTan}(ax)}}{7a^2c}-\frac{\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)}{14a}$$

```
[Out] 1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(1/2)/a^2/c-1/14*Unintegrable((a^2*c*x^
2+c)^(5/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] ((c + a^2*c*x^2)^(7/2)*Sqrt[ArcTan[a*x]])/(7*a^2*c) - Defer[Int][(c + a^2*c
*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]/(14*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx=\frac{(c+a^2cx^2)^{7/2}\sqrt{\tan^{-1}(ax)}}{7a^2c}-\frac{\int\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx}{14a}$$

Mathematica [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \sqrt{\arctan \left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.732 $\int (c+a^2cx^2)^{5/2} \sqrt{\mathbf{ArcTan}(ax)}\, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\, dx=\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\, dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.733 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}} dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}} dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2c\,x^2+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \sqrt{\operatorname{atan}(a x)}}{\sqrt{c a^2 x^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.734 $\int \frac{x^3 \sqrt{\mathbf{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{c+a^2cx^2}\sqrt{\mathrm{ArcTan}(ax)}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\mathrm{ArcTan}(ax)}}{3a^2c}+\frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{3a^3}-$$

```
[Out] -2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c+1/3*x^2*(a^2*c*x^2+c)^(1/2
)*arctan(a*x)^(1/2)/a^2/c+1/3*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x
)^(1/2),x)/a^3-1/6*Unintegrable(x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x
)/a
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]])/(3*a^2*c) + Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[
ArcTan[a*x]]), x]/(3*a^3) - Defer[Int][x^2/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan
[a*x]]), x]/(6*a)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^2c}-\frac{2\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(a}}}{6a}\\ &= -\frac{2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3a^2c}+\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{t}}}{3a^3}\end{aligned}$$

Mathematica [A]
time = 3.22, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 5.14, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{\arctan \left(a x\right)}}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{\operatorname{atan}{\left(a x \right)}}}{\sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.735 $\int \frac{x^2 \sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=101

$$\frac{x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{2a^2c}-\frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)}{4a}-\frac{\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

```
[Out] 1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/4*Unintegrable(x/(a^2*c
*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-1/2*Unintegrable(arctan(a*x)^(1/2)/(a^
2*c*x^2+c)^(1/2),x)/a^2
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(2*a^2*c) - Defer[Int][x/(Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(4*a) - Defer[Int][Sqrt[ArcTan[a*x]]/Sq
rt[c + a^2*c*x^2], x]/(2*a^2)
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{x\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{2a^2c}-\frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{\int \frac{x}{\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}\,dx}{4a}$$

Mathematica [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 4.84, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{\arctan \left(a x \right)}}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}}}{\sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.736 $\int \frac{x\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{a^2c}-\frac{\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)}{2a}$$

```
[Out] (a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/2*Unintegrable(1/(a^2*c*x^2+c
)^(1/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(a^2*c) - Defer[Int][1/(Sqrt[c + a^
2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(2*a)
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{a^2c}-\frac{\int \frac{1}{\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}\,dx}{2a}$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{\sqrt{a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\operatorname{atan}\left(ax\right)}}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.737 $\int \frac{\sqrt{\mathbf{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{\sqrt{c\left(a^{2}x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
[Out] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.738 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}{(ax)}}}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\sqrt{c\,a^2\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.739 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{cx}+\frac{1}{2}a\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] -(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x+1/2*a*Unintegrable(1/x/(a^2*c*x^
2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x)) + (a*Defer[Int][1/(x*Sqrt[
c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/2
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{cx}+\frac{1}{2}a\int \frac{1}{x\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan (ax)}}{x^2\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.740 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{2cx^2}+\frac{1}{4}a\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)-\frac{1}{2}a^2\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] -1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2+1/4*a*Unintegrable(1/x^2/(
a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-1/2*a^2*Unintegrable(arctan(a*x)^(1
/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/2*(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x^2) + (a*Defer[Int][1/(x^2
*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/4 - (a^2*Defer[Int][Sqrt[ArcTa
n[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{2cx^2}+\frac{1}{4}a\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}\,dx-\frac{1}{2}a^2\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x^{3} \sqrt{a^{2} c\,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}{\left(a x \right)}}}{x^{3} \sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.741 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}{3cx}+\frac{1}{6}a\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] -1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^3+2/3*a^2*(a^2*c*x^2+c)^(1/2
)*arctan(a*x)^(1/2)/c/x+1/6*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan
(a*x)^(1/2),x)-1/3*a^3*Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/
2),x)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/3*(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x^3) + (2*a^2*Sqrt[c + a^2*
c*x^2]*Sqrt[ArcTan[a*x]])/(3*c*x) + (a*Defer[Int][1/(x^3*Sqrt[c + a^2*c*x^2
]*Sqrt[ArcTan[a*x]]), x])/6 - (a^3*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tan^{-1}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx^3}+\frac{1}{6}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx-\frac{1}{3}(2a^2)\int \frac{\sqrt{}}{x^2} \\ &= -\frac{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{3cx}+\frac{1}{6}a\int \frac{1}{x^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 14.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 4.86, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan (ax)}}{x^{4}\sqrt{a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{x^{4}\sqrt{c\left(a^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.742 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}} dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}} dx$$

Mathematica [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{\operatorname{atan}\left(ax\right)}}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \sqrt{\operatorname{atan}(a\,x)}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.743 $\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^3 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

Mathematica [A]
time = 6.03, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 4.84, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{\operatorname{atan}\left(ax\right)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.744 $\int \frac{x^2 \sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}\, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^2\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\, dx = \int \frac{x^2\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\, dx$$

Mathematica [A]
time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2\,\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.745 $\int \frac{x\sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{\text{ArcTan}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+
1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1
/2)
```

Rubi [A]
time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5050, 5025, 5024, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\text{ArcTan}(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(Sqrt[ArcTan[a*x]]/(a^2*c*Sqrt[c + a^2*c*x^2])) + (Sqrt[Pi/2]*Sqrt[1 + a^2
*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
```

```
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx &= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{2a}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{2ac\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{2a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{\tan^{-1}(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 121, normalized size = 1.30

$$
\frac{-4\text{ArcTan}(ax)-i\sqrt{1+a^2x^2}\,\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+i\sqrt{1+a^2x^2}\,\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)}{4a^2c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-4*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I
)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTa
n[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\arctan \left(a x\right)}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\operatorname{atan}{\left(a x \right)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.746 $\int \frac{\sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x\sqrt{\text{ArcTan}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5025, 5024, 3377, 3386, 3432}

$$\frac{x\sqrt{\text{ArcTan}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcTan[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/2]*Sqrt[1 + a^2*x^
2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\,\cos(x)\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}\\
&= \frac{x\sqrt{\tan^{-1}(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 78, normalized size = 0.86

$$\frac{2ax\sqrt{\text{ArcTan}(ax)} - \sqrt{2\pi}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[ArcTan[a*x]] - Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]
*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2 c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.747 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{x \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.748 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 3.63, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x^2\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.749 $\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
[Out] Defer[Int][(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx = \int \frac{x^m \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Mathematica [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.750 $\int \frac{x^4 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^4 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
[Out] Defer[Int][(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^4 \sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$$

Mathematica [A]
time = 3.65, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 6.12, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.751 $\int \frac{x^3\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{3\sqrt{\text{ArcTan}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\sqrt{\text{ArcTan}(ax)}\cos(3\text{ArcTan}(ax))}{12a^4c^2\sqrt{c+a^2cx^2}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{A}\right.}{4a^4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/72*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+3/8*FresnelC(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
-3/4*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/12*cos(3*arctan(a*x))*
(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5091, 5090, 3393, 3377, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{12a^4c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\text{ArcTan}(ax)}}{4a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\text{ArcTan}(ax)}\cos(3\text{ArcTan}(ax))}{12a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(4*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]
*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (
3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a
^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]])/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}} \\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\,\sin^3(x)\,dx,x,\tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}} \\ &= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\,\sin(x)-\frac{1}{4}\sqrt{x}\,\sin(3x)\right)\,dx,x,\tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}} \\ &= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sqrt{x}\,\sin(3x)\,dx,x,\tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}}+\frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\right.}{4a^4c^2\sqrt{c+}} \\ &= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\text{Su}}{2} \\ &= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{1+a^2x^2}\,\text{Su}}{} \\ &= -\frac{3\sqrt{\tan^{-1}(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax)\right)}{12a^4c^2\sqrt{c+a^2cx^2}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a}}{4a}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 324, normalized size = 1.51

$$\frac{-96\text{ArcTan}(ax)-144a^2x^2\text{ArcTan}(ax)-27i(1+a^2x^2)^{3/2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+27i(1+a^2x^2)^{3/2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+i\sqrt{3+3a^2x^2}\,\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+ia^2x^2\sqrt{3+3a^2x^2}\,\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-i\sqrt{3+3a^2x^2}\,\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)-ia^2x^2\sqrt{3+3a^2x^2}\,\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)}{144a^4c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-96*ArcTan[a*x] - 144*a^2*x^2*ArcTan[a*x] - (27*I)*(1 + a^2*x^2)^(3/2)*Sqr
t[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + (27*I)*(1 + a^2*x^2)^(3/
2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*
a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqrt[3 +
 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2*x^2*S
qrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(144*
a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.752 $\int \frac{x^2\sqrt{\mathbf{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{x^3\sqrt{\text{ArcTan}(ax)}}{3c\,(c+a^2cx^2)^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4a^3c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/72*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/8*FresnelS(2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+
1/3*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5064, 5091, 5090, 3393, 3386, 3432}

$$\frac{x^3\sqrt{\text{ArcTan}(ax)}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{12a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^3*Sqrt[ArcTan[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{1}{6}a\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x^3}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{6a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{6a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{24a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Su}}{}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin(3x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a}}{}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S}{12a^3c^2}$$

Mathematica [A]
time = 0.19, size = 133, normalized size = 0.82

$$\frac{24a^3x^3\sqrt{\text{ArcTan}(ax)} - 9\sqrt{2\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) + \sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{72a^3c^2\,(1+a^2x^2)\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*a^3*x^3*Sqrt[ArcTan[a*x]] - 9*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[Sqrt
[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [F]
time = 5.34, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.753 $\int \frac{x\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\text{ArcTan}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{4a^2c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\right.}{12a^2c^2\sqrt{c+a^2cx}}$$

```
[Out] 1/72*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/8*FresnelC(2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-
1/3*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5050, 5025, 5024, 3393, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{4a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{12a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\text{ArcTan}(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/3*Sqrt[ArcTan[a*x]]/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[Pi/2]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a^2*c^2*Sqrt[c + a^2*c
*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]
]])/(12*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6a}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{6ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{6a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}} + \frac{\cos(3x)}{4\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{6a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}}{}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \cos\left(3x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{12a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+}}{}$$

$$= -\frac{\sqrt{\tan^{-1}(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}}{12a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 167, normalized size = 1.02

$$\frac{-48\mathrm{ArcTan}(ax) - i(1+a^2x^2)^{3/2}\left(9\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -i\mathrm{ArcTan}(ax)\right) - 9\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, i\mathrm{ArcTan}(ax)\right) + \sqrt{3}\left(\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -3i\mathrm{ArcTan}(ax)\right) - \sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, 3i\mathrm{ArcTan}(ax)\right)\right)\right)}{144a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
[Out] (-48*ArcTan[a*x] - I*(1 + a^2*x^2)^(3/2)*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/
2, (-I)*ArcTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sq
rt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*ArcTa
n[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(144*a^2*c*(c + a^2*c*x^2)^(3/2)*S
qrt[ArcTan[a*x]])
```

Maple [F]
time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.754 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{3x\sqrt{\text{ArcTan}(ax)}}{4c^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4ac^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{12ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/72*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-3/8*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+3/4
*x*arctan(a*x)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+1/12*sin(3*arctan(a*x))*(a^2*x
^2+1)^(1/2)*arctan(a*x)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5025, 5024, 3393, 3377, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\text{ArcTan}(ax)}}{4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\text{ArcTan}(ax)}\sin(3\text{ArcTan}(ax))}{12ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(4*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c^2*Sqrt[c + a^2*c*
x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(12*a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\cos^3(x)\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\,\cos(x)+\frac{1}{4}\sqrt{x}\,\cos(3x)\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\cos(3x)\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\mathrm{Subst}\left(\int \sqrt{x}\,\mathrm{c}\right.}{4ac^2\sqrt{c+a}}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\sin\left(3\tan^{-1}(ax)\right)}{12ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\right.}{24ac}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\sin\left(3\tan^{-1}(ax)\right)}{12ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\right.}{12}$$
$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}(ax)}\right)}{4ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\,\sqrt{1+a^2x^2}\,S}{12ac^2\sqrt{}}$$

Mathematica [A]
time = 0.13, size = 137, normalized size = 0.64

$$\frac{24ax(3+2a^2x^2)\sqrt{\mathrm{ArcTan}(ax)} - 27\sqrt{2\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right) - \sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{72c^2\,(a+a^3x^2)\,\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*a*x*(3 + 2*a^2*x^2)*Sqrt[ArcTan[a*x]] - 27*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/
2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*c^2*(a + a^3*x^2)*Sqrt[c + a^2*
c*x^2])
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\operatorname{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
[Out] int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.755 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.756 $\int x^m(c+a^2cx^2) \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.60, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.757 $\int x^2(c+a^2cx^2)\,\textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 2.94, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.48, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^{2}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(a**2*x**4*atan(a*x)**(3/2)
, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.758 $\int x(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=57

$$\frac{c(1+a^2x^2)^2\,\mathrm{ArcTan}(ax)^{3/2}}{4a^2}-\frac{3\mathrm{Int}\left((c+a^2cx^2)\sqrt{\mathrm{ArcTan}(ax)},x\right)}{8a}$$

```
[Out] 1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(3/2)/a^2-3/8*Unintegrable((a^2*c*x^2+c)*ar
ctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x(c+a^2cx^2)\,\mathrm{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(4*a^2) - (3*Defer[Int][(c + a^2*c*x^
2)*Sqrt[ArcTan[a*x]], x])/(8*a)
```

Rubi steps

$$\int x(c+a^2cx^2)\tan^{-1}(ax)^{3/2}\,dx=\frac{c(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{4a^2}-\frac{3\int(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}\,dx}{8a}$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)\,\mathrm{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x*atan(a*x)**(3/2), x) + Integral(a**2*x**3*atan(a*x)**(3/2), x
))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{atan}(a x)^{3/2} \left(c a^2 x^2 + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

```
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.759 $\int (c+a^2cx^2) \mathbf{ArcTan}(ax)^{3/2} dx$

Optimal. Leaf size=81

$$-\frac{c(1+a^2x^2)\sqrt{\mathrm{ArcTan}(ax)}}{4a}+\frac{1}{3}cx(1+a^2x^2)\mathrm{ArcTan}(ax)^{3/2}+\frac{1}{8}c\mathrm{Int}\left(\frac{1}{\sqrt{\mathrm{ArcTan}(ax)}},x\right)+\frac{2}{3}c\mathrm{Int}(\mathrm{ArcTan}($$

```
[Out] 1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(3/2)-1/4*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/a
+2/3*c*Unintegrable(arctan(a*x)^(3/2),x)+1/8*c*Unintegrable(1/arctan(a*x)^(
1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2) \mathrm{ArcTan}(ax)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] -1/4*(c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/a + (c*x*(1 + a^2*x^2)*ArcTan[a*x]
^(3/2))/3 + (c*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/8 + (2*c*Defer[Int][ArcT
an[a*x]^(3/2), x])/3
```

Rubi steps

$$\int (c+a^2cx^2)\tan^{-1}(ax)^{3/2} dx = -\frac{c(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{4a}+\frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax)^{3/2}+\frac{1}{8}c\int\frac{1}{\sqrt{\tan^{-}}}$$

Mathematica [A]
time = 2.99, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2) \mathrm{ArcTan}(ax)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c \left(\int a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2),x)
[Out] c*(Integral(a**2*x**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.760 $\int \frac{(c+a^2cx^2)\text{ArcTan}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{3/2}}{x},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]

[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]

[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]

Maple [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x,x)
```

```
[Out] c*(Integral(atan(a*x)**(3/2)/x, x) + Integral(a**2*x*atan(a*x)**(3/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x,x)
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x, x)
```

3.761 $\int \frac{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Mathematica [A]
time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)\arctan(ax)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int\frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2,x)
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2, x)
```

3.762 $\int x^m(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.98, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.763 $\int x^2(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx = \int x^2\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 2.21, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
[Out] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^2 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 2a^2x^4 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^4x^6 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
[Out] c**2*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(2*a**2*x**4*atan(a*x)**
(3/2), x) + Integral(a**4*x**6*atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.764 $\int x(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{c^2(1+a^2x^2)^3 \operatorname{ArcTan}(ax)^{3/2}}{6a^2}-\frac{\operatorname{Int}\left((c+a^2cx^2)^2\sqrt{\operatorname{ArcTan}(ax)},x\right)}{4a}$$

```
[Out] 1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a^2-1/4*Unintegrable((a^2*c*x^2+c)^
2*arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] (c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(6*a^2) - Defer[Int][(c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]], x]/(4*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}\,dx=\frac{c^2(1+a^2x^2)^3 \tan^{-1}(ax)^{3/2}}{6a^2}-\frac{\int (c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}\,dx}{4a}$$

Mathematica [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 2a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(x*atan(a*x)**(3/2), x) + Integral(2*a**2*x**3*atan(a*x)**(3/
2), x) + Integral(a**4*x**5*atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.765 $\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=172

$$-\frac{c^2(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{5a}-\frac{3c^2(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}{40a}+\frac{4}{15}c^2x(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}+\frac{1}{5}c^2x(1+$$

[Out] 4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)^(3/2)-1/5*c^2*(a^2*x^2+1)*arctan(a*x)^(1/2)/a-3/40*c^2*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a+8/15*c^2*Unintegrable(arctan(a*x)^(3/2),x)+1/10*c^2*Unintegrable(1/arctan(a*x)^(1/2),x)+3/80*c*Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,
Rules used = {}

$$\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]

[Out] -1/5*(c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/a - (3*c^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])/(40*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/15 + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/5 + (c^2*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/10 + (3*c*Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/80 + (8*c^2*Defer[Int][ArcTan[a*x]^(3/2), x])/15

Rubi steps

$$\int (c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}\,dx = -\frac{3c^2(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{40a}+\frac{1}{5}c^2x(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}+\frac{1}{80}(3c)$$

$$= -\frac{c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{5a}-\frac{3c^2(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{40a}+\frac{4}{15}c^2x(1+$$

Mathematica [A]
time = 1.57, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(a^{2} c\,x^{2}+c\right)^{2} \arctan \left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2} \left(\int 2 a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int a^{4} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**(3/2), x) + Integral(a**4*x**4*atan(a
*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.766 $\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{3}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^4x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**(3/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(3/2),
 x) + Integral(a**4*x**3*atan(a*x)**(3/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.767 $\int \frac{(c+a^2cx^2)^2\textbf{ArcTan}(ax)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2\arctan(ax)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int 2a^{2}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}\,dx+\int\frac{\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}}{x^{2}}\,dx+\int a^{4}x^{2}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2
, x) + Integral(a**4*x**2*atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

3.768 $\int x^m(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.769 $\int x^2(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^3 \operatorname{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 2.06, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 2.34, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^{2}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^{4}x^{6}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^{6}x^{8}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(3*a**2*x**4*atan(a*x)**
(3/2), x) + Integral(3*a**4*x**6*atan(a*x)**(3/2), x) + Integral(a**6*x**8*
atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.770 $\int x(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{c^3(1+a^2x^2)^4 \operatorname{ArcTan}(ax)^{3/2}}{8a^2} - \frac{3\operatorname{Int}\left((c+a^2cx^2)^3 \sqrt{\operatorname{ArcTan}(ax)},x\right)}{16a}$$

[Out] 1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(3/2)/a^2-3/16*Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)/a

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]

[Out] (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(3/2))/(8*a^2) - (3*Defer[Int][(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x])/(16*a)

Rubi steps

$$\int x(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}\,dx = \frac{c^3(1+a^2x^2)^4 \tan^{-1}(ax)^{3/2}}{8a^2} - \frac{3\int (c+a^2cx^2)^3 \sqrt{\tan^{-1}(ax)}\,dx}{16a}$$

Mathematica [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]

[Out] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^7\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(x*atan(a*x)**(3/2), x) + Integral(3*a**2*x**3*atan(a*x)**(3/
2), x) + Integral(3*a**4*x**5*atan(a*x)**(3/2), x) + Integral(a**6*x**7*ata
n(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.771 $\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=259

$$-\frac{6c^3(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{35a}-\frac{9c^3(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}{140a}-\frac{c^3(1+a^2x^2)^3\sqrt{\operatorname{ArcTan}(ax)}}{28a}+\frac{8}{35}c^3x(1+$$

```
[Out] 8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)^(3/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*
x)^(3/2)+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^(3/2)-6/35*c^3*(a^2*x^2+1)*arc
tan(a*x)^(1/2)/a-9/140*c^3*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a-1/28*c^3*(a^2*
x^2+1)^3*arctan(a*x)^(1/2)/a+16/35*c^3*Unintegrable(arctan(a*x)^(3/2),x)+3/
35*c^3*Unintegrable(1/arctan(a*x)^(1/2),x)+9/280*c^2*Unintegrable((a^2*c*x^
2+c)/arctan(a*x)^(1/2),x)+1/56*c*Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(
1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-6*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(35*a) - (9*c^3*(1 + a^2*x^2)^2*Sq
rt[ArcTan[a*x]])/(140*a) - (c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(28*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(3/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/7 + (3*c^3*
Defer[Int][1/Sqrt[ArcTan[a*x]], x])/35 + (9*c^2*Defer[Int][(c + a^2*c*x^2)/
Sqrt[ArcTan[a*x]], x])/280 + (c*Defer[Int][(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*
x]], x])/56 + (16*c^3*Defer[Int][ArcTan[a*x]^(3/2), x])/35
```

Rubi steps

$$\int (c + a^2cx^2)^3 \tan^{-1}(ax)^{3/2}\,dx = -\frac{c^3(1+a^2x^2)^3\sqrt{\tan^{-1}(ax)}}{28a} + \frac{1}{7}c^3x(1+a^2x^2)^3\tan^{-1}(ax)^{3/2} + \frac{1}{56}c\int$$

$$= -\frac{9c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{140a} - \frac{c^3(1+a^2x^2)^3\sqrt{\tan^{-1}(ax)}}{28a} + \frac{6}{35}c^3x(1$$

$$= -\frac{6c^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{35a} - \frac{9c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{140a} - \frac{c^3(1+a}{}$$

Mathematica [A]
time = 1.65, size = 0, normalized size = 0.00

$$\int (c + a^2cx^2)^3 \operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
[Out] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**(3/2), x) + Integral(3*a**4*x**4*atan
(a*x)**(3/2), x) + Integral(a**6*x**6*atan(a*x)**(3/2), x) + Integral(atan(
a*x)**(3/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.772 $\int \frac{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}}{x} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{3/2}}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx = \int \frac{(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}{x} dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{3/2}}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3 \arctan(ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^4x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^6x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)**(3/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(3/2),
 x) + Integral(3*a**4*x**3*atan(a*x)**(3/2), x) + Integral(a**6*x**5*atan(a
*x)**(3/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.773 $\int \frac{(c+a^2cx^2)^3\text{ArcTan}(ax)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}{x^2}\,dx$$

Mathematica [A]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

Maple [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3\arctan(ax)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int\frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2}\,dx+\int 3a^4x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2
, x) + Integral(3*a**4*x**2*atan(a*x)**(3/2), x) + Integral(a**6*x**4*atan(
a*x)**(3/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

3.774 $\int \frac{x^m \mathbf{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{c+a^2cx^2},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**m*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.775 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{3/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=61

$$-\frac{2x\text{ArcTan}(ax)^{5/2}}{5a^3c}+\frac{\text{Int}\left(x\text{ArcTan}(ax)^{3/2},x\right)}{a^2c}+\frac{2\text{Int}\left(\text{ArcTan}(ax)^{5/2},x\right)}{5a^3c}$$

```
[Out] -2/5*x*arctan(a*x)^(5/2)/a^3/c+Unintegrable(x*arctan(a*x)^(3/2),x)/a^2/c+2/
5*Unintegrable(arctan(a*x)^(5/2),x)/a^3/c
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(5/2))/(5*a^3*c) + Defer[Int][x*ArcTan[a*x]^(3/2), x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(5/2), x])/(5*a^3*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\tan^{-1}(ax)^{3/2}\,dx}{a^2c}\\
&= -\frac{2x\tan^{-1}(ax)^{5/2}}{5a^3c}+\frac{2\int \tan^{-1}(ax)^{5/2}\,dx}{5a^3c}+\frac{\int x\tan^{-1}(ax)^{3/2}\,dx}{a^2c}
\end{aligned}$$

Mathematica [A]
time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{x^3\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{a^2 c\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.776 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{3/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=37

$$-\frac{2\text{ArcTan}(ax)^{5/2}}{5a^3c}+\frac{\text{Int}\left(\text{ArcTan}(ax)^{3/2},x\right)}{a^2c}$$

```
[Out] -2/5*arctan(a*x)^(5/2)/a^3/c+Unintegrable(arctan(a*x)^(3/2),x)/a^2/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] (-2*ArcTan[a*x]^(5/2))/(5*a^3*c) + Defer[Int][ArcTan[a*x]^(3/2), x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)^{3/2}\,dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{5/2}}{5a^3c}+\frac{\int \tan^{-1}(ax)^{3/2}\,dx}{a^2c}\end{aligned}$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{x^2\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^2\arctan\left(ax\right)^{\frac{3}{2}}}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{atan}(a x)^{3/2}}{c a^2 x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.777 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=41

$$\frac{2x\text{ArcTan}(ax)^{5/2}}{5ac}-\frac{2\text{Int}\left(\text{ArcTan}(ax)^{5/2},x\right)}{5ac}$$

```
[Out] 2/5*x*arctan(a*x)^(5/2)/a/c-2/5*Unintegrable(arctan(a*x)^(5/2),x)/a/c
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] (2*x*ArcTan[a*x]^(5/2))/(5*a*c) - (2*Defer[Int][ArcTan[a*x]^(5/2), x])/(5*a
*c)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2x\tan^{-1}(ax)^{5/2}}{5ac}-\frac{2\int \tan^{-1}(ax)^{5/2}\,dx}{5ac}$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x\text{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
[Out] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x\arctan\left(ax\right)^{\frac{3}{2}}}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.778 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{2\text{ArcTan}(ax)^{5/2}}{5ac}$$

```
[Out] 2/5*arctan(a*x)^(5/2)/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5004\}$

$$\frac{2\text{ArcTan}(ax)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(5/2))/(5*a*c)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{2\tan^{-1}(ax)^{5/2}}{5ac}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{2\text{ArcTan}(ax)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(5/2))/(5*a*c)
```

Maple [A]
time = 0.20, size = 15, normalized size = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{5}{2}}}{5ac}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*arctan(a*x)^(5/2)/a/c
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 3.13, size = 14, normalized size = 0.78

$$\frac{2\arctan\left(ax\right)^{\frac{5}{2}}}{5\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(a*x)^(5/2)/(a*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.43, size = 14, normalized size = 0.78

$$\frac{2\arctan\left(ax\right)^{\frac{5}{2}}}{5\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/5*arctan(a*x)^(5/2)/(a*c)
```

Mupad [B]
time = 0.39, size = 14, normalized size = 0.78

$$\frac{2\,\mathrm{atan}(a\,x)^{5/2}}{5\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2),x)
```

```
[Out] (2*atan(a*x)^(5/2))/(5*a*c)
```

3.779 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)} \, dx$

Optimal. Leaf size=49

$$-\frac{2i\text{ArcTan}(ax)^{5/2}}{5c}+\frac{i\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

```
[Out] -2/5*I*arctan(a*x)^(5/2)/c+I*Unintegrable(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (((-2*I)/5)*ArcTan[a*x]^(5/2))/c + (I*Defer[Int][ArcTan[a*x]^(3/2)/(x*(I + a*x)), x])/c
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx=-\frac{2i\tan^{-1}(ax)^{5/2}}{5c}+\frac{i\int\frac{\tan^{-1}(ax)^{3/2}}{x(i+ax)}\,dx}{c}$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^3+x}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**3 + x), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)), x)
```

3.780 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2(c+a^2cx^2)} dx$

Optimal. Leaf size=36

$$-\frac{2a\text{ArcTan}(ax)^{5/2}}{5c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^2},x\right)}{c}$$

```
[Out] -2/5*a*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^2,x)/c
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
[Out] (-2*a*ArcTan[a*x]^(5/2))/(5*c) + Defer[Int][ArcTan[a*x]^(3/2)/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}\\ &= -\frac{2a\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^2 (a^2 c x^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}} (ax)}{a^2 x^4 + x^2} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**4 + x**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.781 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{2ia^2\text{ArcTan}(ax)^{5/2}}{5c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^3},x\right)}{c}-\frac{ia^2\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

```
[Out] 2/5*I*a^2*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^3,x)/c-I*a^2
*Unintegrable(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/5)*a^2*ArcTan[a*x]^(5/2))/c + Defer[Int][ArcTan[a*x]^(3/2)/x^3, x]/
c - (I*a^2*Defer[Int][ArcTan[a*x]^(3/2)/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^3}\,dx}{c}\\ &= \frac{2ia^2\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^3}\,dx}{c}-\frac{\left(ia^2\right)\int \frac{\tan^{-1}(ax)^{3/2}}{x(i+ax)}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x^{3} \left(a^{2} c \,x^{2}+c\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
[Out] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{a^{2} x^{5} + x^{3}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**5 + x**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.782 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2a^3\text{ArcTan}(ax)^{5/2}}{5c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^4},x\right)}{c}-\frac{a^2\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^2},x\right)}{c}$$

```
[Out] 2/5*a^3*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^4,x)/c-a^2*Uni
ntegrable(arctan(a*x)^(3/2)/x^2,x)/c
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (2*a^3*ArcTan[a*x]^(5/2))/(5*c) + Defer[Int][ArcTan[a*x]^(3/2)/x^4, x]/c -
(a^2*Defer[Int][ArcTan[a*x]^(3/2)/x^2, x])/c
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4}\,dx}{c}\\
&= a^4\int \frac{\tan^{-1}(ax)^{3/2}}{c+a^2cx^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}\\
&= \frac{2a^3\tan^{-1}(ax)^{5/2}}{5c}+\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x^2}\,dx}{c}
\end{aligned}$$

Mathematica [A]
time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x^{4} \left(a^{2} c \,x^{2}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{a^{2} x^{6} + x^{4}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**2*x**6 + x**4), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.783 $\int \frac{x^m \textbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}, x\right)$$

[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]

[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]

[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**m*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.784 $\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^3\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2},x\right)$$

```
[Out] Unintegrable(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
[Out] Defer[Int][(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx=\int \frac{x^3\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{x^3\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{x^3\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.785 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=127

$$\frac{3\sqrt{\text{ArcTan}(ax)}}{16a^3c^2}-\frac{3\sqrt{\text{ArcTan}(ax)}}{8a^3c^2(1+a^2x^2)}-\frac{x\text{ArcTan}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{5/2}}{5a^3c^2}+\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2}$$

```
[Out] -1/2*x*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a^3/c^2+
3/32*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+3/16*arctan(a*
x)^(1/2)/a^3/c^2-3/8*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5056, 5050, 5024, 3393, 3385, 3433}

$$\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2}+\frac{\text{ArcTan}(ax)^{5/2}}{5a^3c^2}+\frac{3\sqrt{\text{ArcTan}(ax)}}{16a^3c^2}-\frac{x\text{ArcTan}(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}-\frac{3\sqrt{\text{ArcTan}(ax)}}{8a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(16*a^3*c^2) - (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*(1 +
a^2*x^2)) - (x*ArcTan[a*x]^(3/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(
5/2)/(5*a^3*c^2) + (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3
2*a^3*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = -\frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx}{4a}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\int \frac{1}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}}{16a^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan\right.}{16a^3c^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right.\right.}{16a^3c^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{16a^3c^2} - \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\pi}\,C\left(\right.}{3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 187, normalized size = 1.47

$$\frac{\frac{16\sqrt{\text{ArcTan}(ax)}\left(15(-1+a^2x^2)-40ax\text{ArcTan}(ax)+16(1+a^2x^2)\text{ArcTan}(ax)^2\right)}{1+a^2x^2}+60\left(-2\sqrt{\text{ArcTan}(ax)}+\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)\right)+\frac{15\left(8\text{ArcTan}(ax)-i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)\right)}{\sqrt{\text{ArcTan}(ax)}}}{1280a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((16*Sqrt[ArcTan[a*x]]*(15*(-1 + a^2*x^2) - 40*a*x*ArcTan[a*x] + 16*(1 + a^
2*x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) + 60*(-2*Sqrt[ArcTan[a*x]] + Sqrt[Pi]*
FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + (15*(8*ArcTan[a*x] - I*Sqrt[2]*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(1280*a^3*c^2
)
```

Maple [A]
time = 0.29, size = 75, normalized size = 0.59

method	result
default	$\frac{32\arctan(ax)^3-40\arctan(ax)^2\sin(2\arctan(ax))+15\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-30\cos(2\arctan$
	$160c^2a^3\sqrt{\arctan(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/c^2/a^3*(32*arctan(a*x)^3-40*arctan(a*x)^2*sin(2*arctan(a*x))+15*arct
an(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-30*cos(2*arct
an(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.786 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=109

$$\frac{3x\sqrt{\text{ArcTan}(ax)}}{8ac^2\,(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{3/2}}{4a^2c^2}-\frac{\text{ArcTan}(ax)^{3/2}}{2a^2c^2\,(1+a^2x^2)}-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

[Out] 1/4*arctan(a*x)^(3/2)/a^2/c^2-1/2*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)-3/32*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2+3/8*x*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)

Rubi [A]
time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5050, 5012, 5090, 4491, 12, 3386, 3432}

$$-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}-\frac{\text{ArcTan}(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}+\frac{3x\sqrt{\text{ArcTan}(ax)}}{8ac^2\,(a^2x^2+1)}+\frac{\text{ArcTan}(ax)^{3/2}}{4a^2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]

[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(4*a^2*c^2) - ArcTan[a*x]^(3/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx}{4a}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3}{16}\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\mathrm{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \mathrm{t}\right.}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\mathrm{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-}\right.}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\mathrm{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-}\right.}{32a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\mathrm{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{}\right.}{16a^2c^2}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.69

$$\frac{\frac{4\sqrt{\mathrm{ArcTan}(ax)}\left(3ax+2(-1+a^2x^2)\mathrm{ArcTan}(ax)\right)}{1+a^2x^2} - 3\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((4*Sqrt[ArcTan[a*x]]*(3*a*x + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/(1 + a^2*x^2)
 - 3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

Maple [A]
time = 0.23, size = 67, normalized size = 0.61

method	result	size
default	$-\frac{8 \arctan(ax)^2 \cos(2 \arctan(ax))+3 \sqrt{\arctan(ax)}\, \sqrt{\pi}\, \mathrm{S}\left(\frac{2 \sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-6 \sin(2 \arctan(ax)) \arctan(ax)}{32 c^2 a^2 \sqrt{\arctan(ax)}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/c^2/a^2*(8*arctan(a*x)^2*cos(2*arctan(a*x))+3*arctan(a*x)^(1/2)*Pi^(1
/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))-6*sin(2*arctan(a*x))*arctan(a*x)
)/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.787 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{3\sqrt{\text{ArcTan}(ax)}}{16ac^2}+\frac{3\sqrt{\text{ArcTan}(ax)}}{8ac^2\left(1+a^2x^2\right)}+\frac{x\text{ArcTan}(ax)^{3/2}}{2c^2\left(1+a^2x^2\right)}+\frac{\text{ArcTan}(ax)^{5/2}}{5ac^2}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32ac^2}$$

```
[Out] 1/2*x*arctan(a*x)^(3/2)/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a/c^2-3/32*Fr
esnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-3/16*arctan(a*x)^(1/2)/
a/c^2+3/8*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5012, 5050, 5024, 3393, 3385, 3433}

$$\frac{x\text{ArcTan}(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{3\sqrt{\text{ArcTan}(ax)}}{8ac^2\left(a^2x^2+1\right)}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32ac^2}+\frac{\text{ArcTan}(ax)^{5/2}}{5ac^2}-\frac{3\sqrt{\text{ArcTan}(ax)}}{16ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-3*Sqrt[ArcTan[a*x]])/(16*a*c^2) + (3*Sqrt[ArcTan[a*x]])/(8*a*c^2*(1 + a^2
*x^2)) + (x*ArcTan[a*x]^(3/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5
*a*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{1}{4}(3a)\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3}{16}\int \frac{1}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,d$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{16ac^2}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right)\,dx, x\right.}{16ac^2}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\text{Subst}\left(\int \frac{\cos(}{\sqrt{}}\right.}{}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\text{Subst}\left(\int \cos\right.}{}$$

$$= -\frac{3\sqrt{\tan^{-1}(ax)}}{16ac^2} + \frac{3\sqrt{\tan^{-1}(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{5ac^2} - \frac{3\sqrt{\pi}\,C\left(\frac{2\sqrt{t}}{}\right.}{32a}$$

Mathematica [A]
time = 0.07, size = 90, normalized size = 0.73

$$\frac{\frac{2\sqrt{\text{ArcTan}(ax)}\left(15-15a^2x^2+40ax\text{ArcTan}(ax)+16(1+a^2x^2)\text{ArcTan}(ax)^2\right)}{1+a^2x^2} - 15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{160ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((2*Sqrt[ArcTan[a*x]]*(15 - 15*a^2*x^2 + 40*a*x*ArcTan[a*x] + 16*(1 + a^2*x
^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) - 15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]])/(160*a*c^2)
```

Maple [A]
time = 0.30, size = 75, normalized size = 0.60

method	result
default	$\frac{32 \arctan(ax)^{3}+40 \arctan(ax)^{2} \sin(2 \arctan(ax))+30 \cos(2 \arctan(ax)) \arctan(ax)-15 \sqrt{\arctan(ax)}\, \sqrt{\pi}\, \mathrm{FresnelC}\left(\frac{2 \sqrt{\mathrm{a}}}{}\right.}{160 c^{2} a \sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/c^2/a/arctan(a*x)^(1/2)*(32*arctan(a*x)^3+40*arctan(a*x)^2*sin(2*arct
an(a*x))+30*cos(2*arctan(a*x))*arctan(a*x)-15*arctan(a*x)^(1/2)*Pi^(1/2)*Fr
esnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^{4}x^{4}+2a^{2}x^{2}+1}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2,x)
```

```
[Out] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2, x)
```

3.788 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^2},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^2} dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^2} dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

Maple [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x(a^2c\,x^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2),x)
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.789 $\int \frac{x^m \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.790 $\int \frac{x^5 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^5 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
[Out] Defer[Int][(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^5 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 5.06, size = 0, normalized size = 0.00

$$\int \frac{x^5 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{x^5 \arctan(ax)^{\frac{3}{2}}}{(a^2 c x^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**5*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5 \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
[Out] int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.791 $\int \frac{x^4 \textbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=230

$$\frac{27\sqrt{\text{ArcTan}(ax)}}{256a^5c^3}+\frac{3x^4\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}-\frac{9\sqrt{\text{ArcTan}(ax)}}{32a^5c^3\left(1+a^2x^2\right)}-\frac{x^3\text{ArcTan}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2}-\frac{3x\text{ArcTan}(ax)^{3/2}}{8a^4c^3\left(1+a^2x^2\right)}+\frac{3\text{ArcTan}}{20a}$$

```
[Out] -1/4*x^3*arctan(a*x)^(3/2)/a^2/c^3/(a^2*x^2+1)^2-3/8*x*arctan(a*x)^(3/2)/a^
4/c^3/(a^2*x^2+1)+3/20*arctan(a*x)^(5/2)/a^5/c^3-3/1024*FresnelC(2*2^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3+3/32*FresnelC(2*arctan
(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^5/c^3+27/256*arctan(a*x)^(1/2)/a^5/c^3+3/3
2*x^4*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)^2-9/32*arctan(a*x)^(1/2)/a^5/c^3/
(a^2*x^2+1)
```

Rubi [A]
time = 0.30, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5060, 5056, 5050, 5024, 3393, 3385, 3433, 5090}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{512a^5c^3}+\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32a^5c^3}+\frac{3\text{ArcTan}(ax)^{5/2}}{20a^5c^3}+\frac{27\sqrt{\text{ArcTan}(ax)}}{256a^5c^3}+\frac{3x^4\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\text{ArcTan}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{9\sqrt{\text{ArcTan}(ax)}}{32a^5c^3\left(a^2x^2+1\right)}-\frac{3x\text{ArcTan}(ax)^{3/2}}{8a^4c^3\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (27*Sqrt[ArcTan[a*x]])/(256*a^5*c^3) + (3*x^4*Sqrt[ArcTan[a*x]])/(32*a*c^3*
(1 + a^2*x^2)^2) - (9*Sqrt[ArcTan[a*x]])/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*
ArcTan[a*x]^(3/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(3/2))/(8
*a^4*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a^5*c^3) - (3*Sqrt[Pi/2
]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^5*c^3) + (3*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^5*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3}{64}\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx + \frac{3\int}{}$$

$$= \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)^{3/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^{5/2}}{20a^5c^3} - \frac{3\mathrm{Sub}}{}$$

$$= \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\tan^{-1}(ax)^{3/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\,\mathrm{t}}{}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3}{8a}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3}{8a}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x}{8a}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x}{8a}$$

$$= \frac{27\sqrt{\tan^{-1}(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\tan^{-1}(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x}{8a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 355, normalized size = 1.54

$$\frac{\frac{64\sqrt{\mathrm{ArcTan}(ax)}\left(15(-15-6a^2x^2+17a^4x^4)-160ax(3+5a^2x^2)\mathrm{ArcTan}(ax)+192(1+a^2x^2)^2\mathrm{ArcTan}(ax)^2\right)}{(1+a^2x^2)^2} - 510\left(12\sqrt{\mathrm{ArcTan}(ax)}+\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)-8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)\right) + 90\sqrt{\mathrm{ArcTan}(ax)}\left(8+\frac{\mathrm{Gamma}\left(\frac{1}{2},-4i\mathrm{ArcTan}(ax)\right)}{\sqrt{-i\mathrm{ArcTan}(ax)}}+\frac{\mathrm{Gamma}\left(\frac{1}{2},4i\mathrm{ArcTan}(ax)\right)}{\sqrt{i\mathrm{ArcTan}(ax)}}\right) + \frac{225\left(24\mathrm{ArcTan}(ax)-4i\sqrt{2}\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-2i\mathrm{ArcTan}(ax)\right)+4i\sqrt{2}\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},2i\mathrm{ArcTan}(ax)\right)-i\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-4i\mathrm{ArcTan}(ax)\right)+i\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},4i\mathrm{ArcTan}(ax)\right)\right)}{\sqrt{\mathrm{ArcTan}(ax)}}}{81920a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

[Out] ((64*Sqrt[ArcTan[a*x]]*(15*(-15 - 6*a^2*x^2 + 17*a^4*x^4) - 160*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2 - 510*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + 90*Sqrt[ArcTan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamma[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (225*(24*ArcTan[a*x] - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^5*c^3)

Maple [A]
time = 0.36, size = 132, normalized size = 0.57

method	result
default	$\frac{-15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+768\arctan(ax)^3-1280\arctan(ax)^2\sin(2\arctan(ax)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/5120/c^3/a^5*(-15*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+768*arctan(a*x)^3-1280*arctan(a*x)^2*sin(2*arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))+480*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-960*cos(2*arctan(a*x))*arctan(a*x)+60*cos(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**4*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.792 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=168

$$-\frac{3\text{ArcTan}(ax)^{3/2}}{32a^4c^3}+\frac{x^4\text{ArcTan}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{512a^4c^3}-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3}+\frac{3}{}$$

```
[Out] -3/32*arctan(a*x)^(3/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)^2
+3/1024*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4
/c^3-3/64*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3+3/32*sin(
2*arctan(a*x))*arctan(a*x)^(1/2)/a^4/c^3-3/256*sin(4*arctan(a*x))*arctan(a*
x)^(1/2)/a^4/c^3
```

Rubi [A]
time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5064, 5090, 3393, 3377, 3386, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{512a^4c^3}-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3}-\frac{3\text{ArcTan}(ax)^{3/2}}{32a^4c^3}+\frac{3\sqrt{\text{ArcTan}(ax)}\,\sin(2\text{ArcTan}(ax))}{32a^4c^3}-\frac{3\sqrt{\text{ArcTan}(ax)}\,\sin(4\text{ArcTan}(ax))}{256a^4c^3}+\frac{x^4\text{ArcTan}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-3*ArcTan[a*x]^(3/2))/(32*a^4*c^3) + (x^4*ArcTan[a*x]^(3/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a
^4*c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^4*c^3
) + (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^4*c^3) - (3*Sqrt[ArcTan[
a*x]]*Sin[4*ArcTan[a*x]])/(256*a^4*c^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$$

$$= \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} - \frac{3\text{Subst}\left(\int \sqrt{x}\,\sin^4(x)\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3}$$

$$= \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} - \frac{3\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8} - \frac{1}{2}\sqrt{x}\,\cos(2x) + \frac{1}{8}\sqrt{x}\,\cos(4x)\right)dx, x, \tan\right.}{8a^4c^3}$$

$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} - \frac{3\text{Subst}\left(\int \sqrt{x}\,\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^4c^3} + $$

$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} - \frac{3\sqrt{\tan^{-}}}{}$$

$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\,\sin\left(2\tan^{-1}(ax)\right)}{32a^4c^3} - \frac{3\sqrt{\tan^{-}}}{}$$

$$= -\frac{3\tan^{-1}(ax)^{3/2}}{32a^4c^3} + \frac{x^4\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^4c^3} - \frac{3\sqrt{\pi}\,S\Big(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 350, normalized size = 2.08

$$\sqrt{\text{ArcTan}(ax)}\left(\frac{3x(3+5a^2x^2)}{64a^3c^3(1+a^2x^2)^2} + \frac{(-3-6a^2x^2+5a^4x^4)\,\text{ArcTan}(ax)}{32a^4c^3(1+a^2x^2)^2}\right) - \frac{9\left(-2\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) - 2\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right) - \sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right) - \sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{4096a^4c^3\sqrt{\text{ArcTan}(ax)}} - \frac{15\left(-2\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) - 2\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right) + \sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right) + \sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{4096a^4c^3\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Sqrt[ArcTan[a*x]]*((3*x*(3 + 5*a^2*x^2))/(64*a^3*c^3*(1 + a^2*x^2)^2) + ((-
3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)^2)) - (9*
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]]))/(4096*a^4*c^3*Sqrt[ArcTan[a*x]]) - (15*(-2*Sqrt[2]*Sqrt[(-I)*A
rcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*
Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Ar
cTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(4096*a^4*
c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.32, size = 124, normalized size = 0.74

method	result
default	$-\frac{-3\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+128\arctan(ax)^{2}\cos(2\arctan(ax))-32\arctan(ax)^{2}\cos(4\,\mathrm{arct}}{1024c^{3}a^{4}\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/1024/c^3/a^4*(-3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+128*arctan(a*x)^2*cos(2*arctan(a*x))-32*arctan(a*x)^2*cos(4*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))-96*sin(2*arctan(a*x))*arctan(a*x)+12*sin(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3}\operatorname{atan}^{\frac{3}{2}}(ax)}{a^{6}x^{6}+3a^{4}x^{4}+3a^{2}x^{2}+1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.793 $\int \frac{x^2 \textbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=108

$$\frac{\text{ArcTan}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\text{ArcTan}(ax)}\cos(4\text{ArcTan}(ax))}{256a^3c^3} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{512a^3c^3} - \text{ArcTan}(a$$

[Out] 1/20*arctan(a*x)^(5/2)/a^3/c^3-1/32*arctan(a*x)^(3/2)*sin(4*arctan(a*x))/a^3/c^3+3/1024*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3-3/256*cos(4*arctan(a*x))*arctan(a*x)^(1/2)/a^3/c^3

Rubi [A]
time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5090, 4491, 3377, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{512a^3c^3} + \frac{\text{ArcTan}(ax)^{5/2}}{20a^3c^3} - \frac{\text{ArcTan}(ax)^{3/2}\sin(4\text{ArcTan}(ax))}{32a^3c^3} - \frac{3\sqrt{\text{ArcTan}(ax)}\cos(4\text{ArcTan}(ax))}{256a^3c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]

[Out] ArcTan[a*x]^(5/2)/(20*a^3*c^3) - (3*Sqrt[ArcTan[a*x]]*Cos[4*ArcTan[a*x]])/(256*a^3*c^3) + (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^3*c^3) - (ArcTan[a*x]^(3/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\text{Subst}\left(\int x^{3/2}\cos^2(x)\sin^2(x)\,dx,x,\tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{x^{3/2}}{8}-\frac{1}{8}x^{3/2}\cos(4x)\right)\,dx,x,\tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{\text{Subst}\left(\int x^{3/2}\cos(4x)\,dx,x,\tan^{-1}(ax)\right)}{8a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{3\text{Subst}\left(\int\sqrt{x}\,\sin(4x)\,dx,x,\tan^{-1}\right.}{64a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\,\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\,\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{3/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\tan^{-1}(ax)}\,\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{3\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}(ax)}\right.}{512a^3c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 353, normalized size = 3.27

$$\frac{\frac{64\sqrt{\text{ArcTan}(ax)}\left(-15(1-6a^2x^2+a^4x^4)+160ax(-1+a^2x^2)\text{ArcTan}(ax)+64(1+a^2x^2)^2\text{ArcTan}(ax)^2\right)}{(1+a^2x^2)^2}+30\left(12\sqrt{\text{ArcTan}(ax)}+\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)-8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)\right)-90\sqrt{\text{ArcTan}(ax)}\left(8+\frac{\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)}{\sqrt{-i\text{ArcTan}(ax)}}+\frac{\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{\sqrt{i\text{ArcTan}(ax)}}\right)+\frac{15\left(24\text{ArcTan}(ax)-4i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+4i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)-i\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+i\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{\sqrt{\text{ArcTan}(ax)}}}{81920a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

[Out] ((64*Sqrt[ArcTan[a*x]]*(-15*(1 - 6*a^2*x^2 + a^4*x^4) + 160*a*x*(-1 + a^2*x^2)*ArcTan[a*x] + 64*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2 + 30*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) - 90*Sqrt[ArcTan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamma[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (15*(24*ArcTan[a*x] - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^3*c^3)

Maple [A]
time = 0.28, size = 81, normalized size = 0.75

method	result
default	$\frac{15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+256\arctan(ax)^3-160\arctan(ax)^2\sin(4\arctan(ax))-60}{5120c^3a^3\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/5120/c^3/a^3*(15*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+256*arctan(a*x)^3-160*arctan(a*x)^2*sin(4*arctan(a*x))-60*cos(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.794 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{3\text{ArcTan}(ax)^{3/2}}{32a^2c^3}-\frac{\text{ArcTan}(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2}-\frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{512a^2c^3}-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3}+\frac{3\sqrt{A}}{}$$

```
[Out] 3/32*arctan(a*x)^(3/2)/a^2/c^3-1/4*arctan(a*x)^(3/2)/a^2/c^3/(a^2*x^2+1)^2-
3/1024*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/
c^3-3/64*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3+3/32*sin(2
*arctan(a*x))*arctan(a*x)^(1/2)/a^2/c^3+3/256*sin(4*arctan(a*x))*arctan(a*x
)^(1/2)/a^2/c^3
```

Rubi [A]

time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5050, 5024, 3393, 3377, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{512a^2c^3}-\frac{3\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3}-\frac{\text{ArcTan}(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\text{ArcTan}(ax)^{3/2}}{32a^2c^3}+\frac{3\sqrt{\text{ArcTan}(ax)}\,\sin(2\text{ArcTan}(ax))}{32a^2c^3}+\frac{3\sqrt{\text{ArcTan}(ax)}\,\sin(4\text{ArcTan}(ax))}{256a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*ArcTan[a*x]^(3/2))/(32*a^2*c^3) - ArcTan[a*x]^(3/2)/(4*a^2*c^3*(1 + a^2*
x^2)^2) - (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(512*a^2*
c^3) - (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(64*a^2*c^3) +
 (3*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(32*a^2*c^3) + (3*Sqrt[ArcTan[a*x
]]*Sin[4*ArcTan[a*x]])/(256*a^2*c^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx}{8a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\text{Subst}\left(\int \sqrt{x}\cos^4(x)\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8} + \frac{1}{2}\sqrt{x}\cos(2x) + \frac{1}{8}\sqrt{x}\cos(4x)\right)dx, x, \tan\right.}{8a^2c^3}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\text{Subst}\left(\int \sqrt{x}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{64a^2c^3} + \frac{3\text{Su}}{}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^2c^3} + \frac{3\sqrt{\tan^{-1}(a}}{}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{3\sqrt{\tan^{-1}(ax)}\sin\left(2\tan^{-1}(ax)\right)}{32a^2c^3} + \frac{3\sqrt{\tan^{-1}(a}}{}$$

$$= \frac{3\tan^{-1}(ax)^{3/2}}{32a^2c^3} - \frac{\tan^{-1}(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{512a^2c^3} - \frac{3\sqrt{\pi}\,S\left(\frac{2}{}\right.}{6}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 347, normalized size = 2.07

$$\frac{480ax\text{ArcTan}(ax) + 288a^3x^3\text{ArcTan}(ax) - 320\text{ArcTan}(ax)^2 + 384a^2x^2\text{ArcTan}(ax)^2 + 192a^4x^4\text{ArcTan}(ax)^2 + 24\sqrt{2}(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2i\text{ArcTan}(ax)\right) + 24\sqrt{2}(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 2i\text{ArcTan}(ax)\right) + 3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4i\text{ArcTan}(ax)\right) + 6a^2x^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4i\text{ArcTan}(ax)\right) + 3a^4x^4\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4i\text{ArcTan}(ax)\right) + 3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 4i\text{ArcTan}(ax)\right) + 6a^2x^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 4i\text{ArcTan}(ax)\right) + 3a^4x^4\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 4i\text{ArcTan}(ax)\right)}{2048c^3(a+a^3x^2)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (480*a*x*ArcTan[a*x] + 288*a^3*x^3*ArcTan[a*x] - 320*ArcTan[a*x]^2 + 384*a^
2*x^2*ArcTan[a*x]^2 + 192*a^4*x^4*ArcTan[a*x]^2 + 24*Sqrt[2]*(1 + a^2*x^2)^
2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 24*Sqrt[2]*(1 + a
^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + 3*Sqrt[(-I)*A
rcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 6*a^2*x^2*Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*a^4*x^4*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-4*I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*
x]] + 6*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]] + 3*a^4*x
^4*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(2048*c^3*(a + a^3*x^
2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.28, size = 124, normalized size = 0.74

method	result
default	$-\frac{3\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}+128\arctan(ax)^{2}\cos(2\arctan(ax))+32\arctan(ax)^{2}\cos(4\arctan}{1024c^{3}a^{2}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1024/c^3/a^2*(3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*a
rctan(a*x)^(1/2)*Pi^(1/2)+128*arctan(a*x)^2*cos(2*arctan(a*x))+32*arctan(a*
x)^2*cos(4*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x
)^(1/2)/Pi^(1/2))-96*sin(2*arctan(a*x))*arctan(a*x)-12*sin(4*arctan(a*x))*a
rctan(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)}{a^{6}x^{6}+3a^{4}x^{4}+3a^{2}x^{2}+1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.795 $\int \frac{\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{45\sqrt{\text{ArcTan}(ax)}}{256ac^3}+\frac{3\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{9\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(1+a^2x^2\right)}+\frac{x\text{ArcTan}(ax)^{3/2}}{4c^3\left(1+a^2x^2\right)^2}+\frac{3x\text{ArcTan}(ax)^{3/2}}{8c^3\left(1+a^2x^2\right)}+\frac{3\text{ArcTan}}{20a}$$

```
[Out] 1/4*x*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^(3/2)/c^3/(a^2*
x^2+1)+3/20*arctan(a*x)^(5/2)/a/c^3-3/1024*FresnelC(2*2^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3-3/32*FresnelC(2*arctan(a*x)^(1/2)/Pi^
(1/2))*Pi^(1/2)/a/c^3-45/256*arctan(a*x)^(1/2)/a/c^3+3/32*arctan(a*x)^(1/2)
/a/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)
```

Rubi [A]
time = 0.22, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5020, 5012, 5050, 5024, 3393, 3385, 3433}

$$\frac{3x\text{ArcTan}(ax)^{3/2}}{8c^3\left(a^2x^2+1\right)}+\frac{x\text{ArcTan}(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(a^2x^2+1\right)}+\frac{3\sqrt{\text{ArcTan}(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{512ac^3}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{32ac^3}+\frac{3\text{ArcTan}(ax)^{5/2}}{20ac^3}-\frac{45\sqrt{\text{ArcTan}(ax)}}{256ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-45*Sqrt[ArcTan[a*x]])/(256*a*c^3) + (3*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 +
a^2*x^2)^2) + (9*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*
x]^(3/2))/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcTan[a*x]^(3/2))/(8*c^3*(1 + a^2
*x^2)) + (3*ArcTan[a*x]^(5/2))/(20*a*c^3) - (3*Sqrt[Pi/2]*FresnelC[2*Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(512*a*c^3) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[
a*x]])/Sqrt[Pi]])/(32*a*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} - \frac{3}{64}\int \frac{1}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx + \frac{3\int \frac{\tan^{-}}{(c+}}{\cdots}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^{3/2}}{8c^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^{5/2}}{20ac^3} - \frac{3\text{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\tan^{-1}(ax)^{3/2}}{8c^3(1+a^2x^2)} + \frac{3\tan^{-1}\cdots}{20\cdots}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256ac^3} + \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\tan\cdots}{8c^3(1\cdots}$$

$$= -\frac{9\sqrt{\tan^{-1}(ax)}}{256ac^3} + \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\tan\cdots}{8c^3(1\cdots}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3} + \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\,\text{ta}\cdots}{8c^3(\cdots}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3} + \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\,\text{ta}\cdots}{8c^3(\cdots}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{256ac^3} + \frac{3\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\tan^{-1}(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\,\text{ta}\cdots}{8c^3(\cdots}$$

<u>Mathematica</u> [A]
time = 0.15, size = 142, normalized size = 0.65

$$\frac{\frac{4\sqrt{\text{ArcTan}(ax)}\left(-15(-17+6a^2x^2+15a^4x^4)+160ax(5+3a^2x^2)\text{ArcTan}(ax)+192(1+a^2x^2)^2\text{ArcTan}(ax)^2\right)}{(1+a^2x^2)^2} - 15\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) - 480\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{5120ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((4*Sqrt[ArcTan[a*x]]*(-15*(-17 + 6*a^2*x^2 + 15*a^4*x^4) + 160*a*x*(5 + 3*
a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2
```

```
- 15*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - 480*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(5120*a*c^3)
```

Maple [A]
time = 0.32, size = 132, normalized size = 0.60

method	result
default	$\frac{768\arctan(ax)^3+1280\arctan(ax)^2\sin(2\arctan(ax))+160\arctan(ax)^2\sin(4\arctan(ax))-15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5120/c^3/a/arctan(a*x)^(1/2)*(768*arctan(a*x)^3+1280*arctan(a*x)^2*sin(2*
arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))-15*2^(1/2)*arctan(a*x)^(1
/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+960*cos(2*arcta
n(a*x))*arctan(a*x)-480*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(
1/2)/Pi^(1/2))+60*cos(4*arctan(a*x))*arctan(a*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3, x)
```

3.796 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^3} dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x(a^2c\,x^2+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c
**3
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.797 $\int x^m \sqrt{c+a^2 c x^2} \text{ArcTan}(ax)^{3/2} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m \sqrt{c+a^2 c x^2} \text{ArcTan}(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{c+a^2 c x^2} \text{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx = \int x^m \sqrt{c+a^2 c x^2} \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c+a^2 c x^2} \text{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int x^m \arctan(ax)^{\frac{3}{2}} \sqrt{a^2 c \, x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a x)^{3/2} \sqrt{c a^2 x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.798 $\int x^2\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 2.23, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int x^2\arctan\left(ax\right)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.799 $\int x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}{3a^2c}-\frac{\text{Int}\left(\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)},x\right)}{2a}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a^2/c-1/2*Unintegrable((a^2*c*x^2
+c)^(1/2)*arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(3*a^2*c) - Defer[Int][Sqrt[c + a
^2*c*x^2]*Sqrt[ArcTan[a*x]], x]/(2*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{\int\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}\,dx}{2a}$$

Mathematica [A]
time = 4.39, size = 0, normalized size = 0.00

$$\int x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int x\arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^{\frac{3}{2}}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.800 $\int \sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=119

$$-\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}+\frac{3}{8}c\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)+\frac{1}{2}$$

```
[Out] 1/2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)-3/4*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^(1/2)/a+1/2*c*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)+3/
8*c*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(3/2))/2 + (3*c*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]]), x])/8 + (c*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/2
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}\,dx=-\frac{3\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}+\frac{1}{8}(3c)\int -$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \arctan(ax)^{\frac{3}{2}} \sqrt{a^2 c x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c (a^2 x^2 + 1)}\, \operatorname{atan}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.801 $\int \frac{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
[Out] Defer[Int][(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A]
time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
[Out] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.802 $\int x^m(c+a^2cx^2)^{3/2} \mathbf{ArcTan}(ax)^{3/2} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.803 $\int x^2(c+a^2cx^2)^{3/2} \text{ArcTan}(ax)^{3/2} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.80, size = 0, normalized size = 0.00

$$\int x^{2}\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.804 $\int x(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{3/2} \, dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^{3/2}}{5a^2c} - \frac{3\operatorname{Int}\left((c+a^2cx^2)^{3/2} \sqrt{\operatorname{ArcTan}(ax)}, x\right)}{10a}$$

```
[Out] 1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/a^2/c-3/10*Unintegrable((a^2*c*x^
2+c)^(3/2)*arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(5*a^2*c) - (3*Defer[Int][(c + a^
2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/(10*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2} \, dx = \frac{(c+a^2cx^2)^{5/2} \tan^{-1}(ax)^{3/2}}{5a^2c} - \frac{3\int (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}(ax)} \, dx}{10a}$$

Mathematica [A]
time = 1.73, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int x(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.805 $\int \left(c+a^2 c x^2\right)^{3/2} \textbf{ArcTan}(ax)^{3/2} \, dx$

Optimal. Leaf size=212

$$-\frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{16a}-\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}{8a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}+\frac{1}{4}x\left(c+\right.$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)+3/8*c*x*arctan(a*x)^(3/2)*(a^2*
c*x^2+c)^(1/2)-1/8*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a-9/16*c*(a^2*c*x^
2+c)^(1/2)*arctan(a*x)^(1/2)/a+3/8*c^2*Unintegrable(arctan(a*x)^(3/2)/(a^2*
c*x^2+c)^(1/2),x)+9/32*c^2*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(
1/2),x)+1/16*c*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^{3/2} \text{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(16*a) - ((c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]])/(8*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/
8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/4 + (9*c^2*Defer[Int][1/(Sq
rt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/32 + (c*Defer[Int][Sqrt[c + a^2*c
*x^2]/Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[
c + a^2*c*x^2], x])/8
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2} \tan^{-1}(ax)^{3/2} \, dx = -\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{8a}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}+\frac{1}{16}c$$

$$=-\frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{16a}-\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{8a}+\frac{3}{8}cx\sqrt{}$$

Mathematica [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^{3/2} \text{ArcTan}(ax)^{3/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

[Out] Timed out

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.806 $\int \frac{(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2)/x,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x,x)
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.807 $\int x^m(c+a^2cx^2)^{5/2}\,\mathbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.73, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x)^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.808 $\int x^2(c+a^2cx^2)^{5/2}\,\textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 2.16, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.98, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.809 $\int x(c+a^2cx^2)^{5/2}\,\textbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{(c+a^2cx^2)^{7/2}\,\text{ArcTan}(ax)^{3/2}}{7a^2c}-\frac{3\text{Int}\left((c+a^2cx^2)^{5/2}\,\sqrt{\text{ArcTan}(ax)},x\right)}{14a}$$

```
[Out] 1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(3/2)/a^2/c-3/14*Unintegrable((a^2*c*x^
2+c)^(5/2)*arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^(3/2))/(7*a^2*c) - (3*Defer[Int][(c + a^
2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x])/(14*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{7/2}\tan^{-1}(ax)^{3/2}}{7a^2c}-\frac{3\int\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}\,dx}{14a}$$

Mathematica [A]
time = 4.87, size = 0, normalized size = 0.00

$$\int x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.37, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.810 $\int (c+a^2cx^2)^{5/2} \text{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=307

$$-\frac{15c^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{32a}-\frac{5c(c+a^2cx^2)^{3/2}\,\sqrt{\text{ArcTan}(ax)}}{48a}-\frac{(c+a^2cx^2)^{5/2}\,\sqrt{\text{ArcTan}(ax)}}{20a}+\frac{5}{16}c^2x$$

```
[Out] 5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)+1/6*x*(a^2*c*x^2+c)^(5/2)*ar
ctan(a*x)^(3/2)+5/16*c^2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)-5/48*c*(a^
2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a-1/20*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(
1/2)/a-15/32*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a+5/16*c^3*Unintegra
ble(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)+1/40*c*Unintegrable((a^2*c*x^2
+c)^(3/2)/arctan(a*x)^(1/2),x)+15/64*c^3*Unintegrable(1/(a^2*c*x^2+c)^(1/2)
/arctan(a*x)^(1/2),x)+5/96*c^2*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)
^(1/2),x)
```

Rubi [A]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

```
[Out] (-15*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a) - (5*c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]])/(48*a) - ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x
]])/(20*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/16 + (5*c*x*(c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcTan
[a*x]^(3/2))/6 + (15*c^3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]
]), x])/64 + (5*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/9
6 + (c*Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x])/40 + (5*c^3*
Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}\,dx = -\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{20a} + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2} + \frac{1}{40}c$$

$$= -\frac{5c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{48a} - \frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}{20a} + \frac{5}{24}c$$

$$= -\frac{15c^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{32a} - \frac{5c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}{48a} - $$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.811 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Defer[Int][((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx=\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

```
[Out] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

Maple [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x,x)
```

```
[Out] int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.812 $\int \frac{x^m \mathbf{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.813 $\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=168

$$-\frac{x\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{4a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{3a^2c}+\frac{\text{Int}\Big(\frac{}{\sqrt{c+}}$$

```
[Out] -2/3*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^(3/2)*
(a^2*c*x^2+c)^(1/2)/a^2/c-1/4*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^3/c
+1/8*Unintegrable(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^2+5/4*Uninte
grable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)/a^3
```

Rubi [A]
time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/4*(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(a^3*c) - (2*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x]^(3/2))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3
/2))/(3*a^2*c) + Defer[Int][x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]/(
8*a^2) + (5*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/(4*a^3)
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{3a^2c}-\frac{2\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}-\frac{\int \frac{x^2\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

$$=-\frac{x\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{4a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-}}{3a^2c}$$

Mathematica [A]
time = 2.51, size = 0, normalized size = 0.00

$$\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 5.32, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan\left(a x\right)^{\frac{3}{2}}}{\sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{\sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.814 $\int \frac{x^2\textbf{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{3\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{2a^2c}+\frac{3\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)}{8a^2}-\text{Int}($$

[Out] 1/2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-3/4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^3/c-1/2*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a^2+3/8*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^2

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\text{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]

[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(2*a^2*c) + (3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(8*a^2) - Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]/(2*a^2)

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{3\int \frac{x\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{4a}\\ &= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{2a^2c}+\frac{3\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{ta}}}}{8a^2}\end{aligned}$$

Mathematica [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{x^2\text{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2 c\, x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.815 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}}{a^2c}-\frac{3\mathrm{Int}\left(\frac{\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a}$$

```
[Out] arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-3/2*Unintegrable(arctan(a*x)^(1
/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(a^2*c) - (3*Defer[Int][Sqrt[ArcTan
[a*x]]/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{a^2c}-\frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{\sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^{\frac{3}{2}} (ax)}{\sqrt{c \left(a^2 x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)

[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)

3.816 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.817 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \, dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \, dx$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\sqrt{a^2c\,x^2+c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.818 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{cx}+\frac{3}{2}a\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] -arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x+3/2*a*Unintegrable(arctan(a*x)^(
1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(c*x)) + (3*a*Defer[Int][Sqrt[Arc
Tan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{cx}+\frac{1}{2}(3a)\int \frac{\sqrt{\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^2 \sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}} (ax)}{x^2 \sqrt{c\, (a^2 x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**(3/2)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.819 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{3a\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}{4cx}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{2cx^2}+\frac{3}{8}a^2\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] -1/2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^2-3/4*a*(a^2*c*x^2+c)^(1/2)*
arctan(a*x)^(1/2)/c/x-1/2*a^2*Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c
)^(1/2),x)+3/8*a^2*Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x
)
```

Rubi [A]
time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-3*a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(3/2))/(2*c*x^2) + (3*a^2*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*
Sqrt[ArcTan[a*x]]), x])/8 - (a^2*Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a
^2*c*x^2]), x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{2cx^2}+\frac{1}{4}(3a)\int \frac{\sqrt{\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}} \\ &= -\frac{3a\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{4cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{2cx^2}+\frac{1}{8}(3a^2)\int \frac{}{x\sqrt{c+a^2}}\end{aligned}$$

Mathematica [A]
time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 2.26, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(a x\right)^{\frac{3}{2}}}{x^{3} \sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{x^{3} \sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**(3/2)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.820 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{a\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{4cx^2}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{3cx}+\frac{1}{8}a^2\text{Int}\left(\frac{}{x^2\sqrt{}}\right.$$

```
[Out] -1/3*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^(3/2)*
(a^2*c*x^2+c)^(1/2)/c/x-1/4*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2+1
/8*a^2*Unintegrable(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-5/4*a^3*
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/4*(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(c*x^2) - (Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(3/2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3
/2))/(3*c*x) + (a^2*Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]
), x])/8 - (5*a^3*Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x])
/4
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{3cx^3}+\frac{1}{2}a\int \frac{\sqrt{\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,d\\ &= -\frac{a\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{4cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}(}{3cx}\end{aligned}$$

Mathematica [A]
time = 14.22, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 5.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x^{4} \sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{x^{4} \sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(atan(a*x)**(3/2)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.821 $\int \frac{x^m \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \operatorname{atan}(a x)^{3/2}}{(c a^2 x^2 + c)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.822 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^3 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{x^3 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 5.35, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.823 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^2\mathrm{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\mathrm{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{x^2\mathrm{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 5.34, size = 0, normalized size = 0.00

$$\int \frac{x^2\arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.824 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{3x\sqrt{\operatorname{ArcTan}(ax)}}{2ac\sqrt{c+a^2cx^2}}-\frac{\operatorname{ArcTan}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\operatorname{ArcTan}(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] -arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-3/4*FresnelS(2^(1/2)/Pi^(1/2)*
arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(
1/2)+3/2*x*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5050, 5025, 5024, 3377, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\operatorname{ArcTan}(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\operatorname{ArcTan}(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\operatorname{ArcTan}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(a^
2*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} + \frac{3\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{2a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{2ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\cos(x)\,dx, x, \tan^{-1}(ax)\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax\right.}{4a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\tan\right.}{2a^2c\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 128, normalized size = 0.99

$$\frac{4(3ax-2\text{ArcTan}(ax))\text{ArcTan}(ax)+3\sqrt{1+a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{1+a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)}{8a^2c\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (4*(3*a*x - 2*ArcTan[a*x])*ArcTan[a*x] + 3*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcT
an[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, I*ArcTan[a*x]])/(8*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]])
```

Maple [F]
time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.825 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{3\sqrt{\mathrm{ArcTan}(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\mathrm{ArcTan}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] x*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*arc
tan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+
3/2*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5018, 5025, 5024, 3385, 3433}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\mathrm{ArcTan}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\mathrm{ArcTan}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(
c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
```

```
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3}{4}\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx \\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\int \frac{1}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{4c\sqrt{c+a^2cx^2}} \\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4ac\sqrt{c+a^2cx^2}} \\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos(x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}} \\
&= \frac{3\sqrt{\tan^{-1}(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.69

$$\frac{2\sqrt{\text{ArcTan}(ax)}\,(3+2ax\text{ArcTan}(ax)) - 3\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[ArcTan[a*x]]*(3 + 2*a*x*ArcTan[a*x]) - 3*Sqrt[2*Pi]*Sqrt[1 + a^2*x^
2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(a x\right)^{\frac{3}{2}}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}\left(a x\right)}{\left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.826 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\tan^{-1}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.827 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2(c+a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}} dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}} dx$$

Mathematica [A]
time = 4.22, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2 \left(c \left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.828 $\int \frac{x^m \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x^m \operatorname{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \operatorname{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx = \int \frac{x^m \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.829 $\int \frac{x^5 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^5\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Defer[Int][(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^5\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Mathematica [A]
time = 7.22, size = 0, normalized size = 0.00

$$\int \frac{x^5\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 11.09, size = 0, normalized size = 0.00

$$\int \frac{x^5\arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^5\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.830 $\int \frac{x^4 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^4\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

[Out] Unintegrable(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]

[Out] Defer[Int][(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^4\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Mathematica [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{x^4\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]

[Out] Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]

Maple [A]
time = 8.12, size = 0, normalized size = 0.00

$$\int \frac{x^4\arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4 \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.831 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{x^3\sqrt{\text{ArcTan}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}}+\frac{x\sqrt{\text{ArcTan}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\,\text{ArcTan}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\text{ArcTan}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}}-\frac{9\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\right.}{8a^4c^2\sqrt{c+a^2c}}$$

[Out] -1/3*x^2*arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(3/2)-2/3*arctan(a*x)^(3/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/144*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-9/16*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/6*x^3*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(3/2)+x*arctan(a*x)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)

Rubi [A]

time = 0.45, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5060, 5050, 5025, 5024, 3377, 3386, 3432, 5091, 5090, 3393}

$$-\frac{x^2\text{ArcTan}(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\text{ArcTan}(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}-\frac{9\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{8a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{24a^4c^2\sqrt{a^2cx^2+c}}-\frac{2\text{ArcTan}(ax)^{3/2}}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{x\sqrt{\text{ArcTan}(ax)}}{a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]

[Out] (x^3*Sqrt[ArcTan[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*Sqrt[ArcTan[a*x]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(3/2))/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*ArcTan[a*x]^(3/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^4*c^2*Sqrt[c + a^2*c*x^2])

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{1}{12}\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx +$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{a^3c}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\Big(}{12a^4c}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\Big(}{}$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} +$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} +$$

$$= \frac{x^3\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{x\sqrt{\tan^{-1}(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 272, normalized size = 1.03

$$\frac{24\text{ArcTan}(ax)\left(ax(6+7a^2x^2)-2(2+3a^2x^2)\text{ArcTan}(ax)\right)-7\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\left(3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)-S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)\right)+3(1+a^2x^2)^{3/2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{144a^4c\,(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*ArcTan[a*x]*(a*x*(6 + 7*a^2*x^2) - 2*(2 + 3*a^2*x^2)*ArcTan[a*x]) - 7*S
qrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a
^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sq
rt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*Ar
cTan[a*x]])))/(144*a^4*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 5.31, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.832 $\int \frac{x^2 \text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{3\sqrt{\text{ArcTan}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\text{ArcTan}(ax)^{3/2}}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{\sqrt{1+a^2x^2}\,\sqrt{\text{ArcTan}(ax)}\,\cos(3\text{ArcTan}(ax))}{24a^3c^2\sqrt{c+a^2cx^2}}-\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,\text{F}}{8a}$$

```
[Out] 1/3*x^3*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(3/2)+1/144*FresnelC(6^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x
^2+c)^(1/2)-3/16*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+3/8*arctan(a*x)^(1/2)/a^3
/c^2/(a^2*c*x^2+c)^(1/2)-1/24*cos(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*arctan(a
*x)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5064, 5091, 5090, 3393, 3377, 3385, 3433}

$$\frac{x^3\text{ArcTan}(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{8a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{24a^3c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\text{ArcTan}(ax)}}{8a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,\sqrt{\text{ArcTan}(ax)}\,\cos(3\text{ArcTan}(ax))}{24a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(8*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (x^3*ArcTan[a*x]^(3
/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos
[3*ArcTan[a*x]])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 +
a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^3*c^2*Sqrt[c + a^2*c*
x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/(24*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{1}{2}a\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

$$= \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{5/2}}\,dx}{2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\sin^3(x)\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\,\sin(x) - \frac{1}{4}\sqrt{x}\,\sin(3x)\right)\,dx, x, \tan^{-}\right.}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\sin(3x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^2\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+\right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax\right.}{24a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax\right.}{24a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\cos\left(3\tan^{-1}(ax\right.}{24a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 338, normalized size = 1.37

$$\frac{96\mathrm{ArcTan}(ax) + 144a^2x^2\mathrm{ArcTan}(ax) + 96a^3x^3\mathrm{ArcTan}(ax)^2 + 27i(1+a^2x^2)^{3/2}\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -i\mathrm{ArcTan}(ax)\right) - 27i(1+a^2x^2)^{3/2}\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, i\mathrm{ArcTan}(ax)\right) - i\sqrt{3+3a^2x^2}\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -3i\mathrm{ArcTan}(ax)\right) - ia^2x^2\sqrt{3+3a^2x^2}\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -3i\mathrm{ArcTan}(ax)\right) + i\sqrt{3+3a^2x^2}\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, 3i\mathrm{ArcTan}(ax)\right) + ia^2x^2\sqrt{3+3a^2x^2}\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, 3i\mathrm{ArcTan}(ax)\right)}{288a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (96*ArcTan[a*x] + 144*a^2*x^2*ArcTan[a*x] + 96*a^3*x^3*ArcTan[a*x]^2 + (27*
I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]]
- (27*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]
- I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x
]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)
*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*
ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
```

```
(3*I)*ArcTan[a*x]])/(288*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[Ar
cTan[a*x]])
```

Maple [F]
time = 6.11, size = 0, normalized size = 0.00

$$\int \frac{x^2 \arctan (ax)^{\frac{3}{2}}}{(a^2 c\, x^2 + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}} (ax)}{(c\,(a^2x^2 + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.833 $\int \frac{x\mathbf{ArcTan}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$\frac{3x\sqrt{\mathrm{ArcTan}(ax)}}{8ac^2\sqrt{c+a^2cx^2}}-\frac{\mathrm{ArcTan}(ax)^{3/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^2c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\Big(}{24a^2c^2\sqrt{}}$$

```
[Out] -1/3*arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(3/2)-1/144*FresnelS(6^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*
x^2+c)^(1/2)-3/16*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(
1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+3/8*x*arctan(a*x)^(1/2)/
a/c^2/(a^2*c*x^2+c)^(1/2)+1/24*sin(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*arctan(
a*x)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5050, 5025, 5024, 3393, 3377, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{24a^2c^2\sqrt{a^2cx^2+c}}+\frac{3x\sqrt{\mathrm{ArcTan}(ax)}}{8ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\sqrt{\mathrm{ArcTan}(ax)}\sin(3\mathrm{ArcTan}(ax))}{24a^2c^2\sqrt{a^2cx^2+c}}-\frac{\mathrm{ArcTan}(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*x*Sqrt[ArcTan[a*x]])/(8*a*c^2*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(3/2)/(
3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]
*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(24*a^2*c^2*Sqrt
[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]])
/(24*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx}{2a}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{5/2}}\,dx}{2ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\cos^3(x)\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3}{4}\sqrt{x}\,\cos(x)+\frac{1}{4}\sqrt{x}\,\cos(3x)\right)\,dx, x, \mathrm{ta}\right.}{2a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Subst}\left(\int \sqrt{x}\,\cos(3x)\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1}\right.}{}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\sin\left(3\tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{1+a^2x^2}\,\sqrt{\tan^{-1}(ax)}\,\sin\left(3\tan^{-1}(ax)\right)}{24a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{3x\sqrt{\tan^{-1}(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 261, normalized size = 1.05

$$\frac{48(3ax+2a^3x^3-2\mathrm{ArcTan}(ax))\mathrm{ArcTan}(ax)-4\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}\left(3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)-S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)\right)+3(1+a^2x^2)^{3/2}\left(3\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-i\mathrm{ArcTan}(ax)\right)+3\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},i\mathrm{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-3i\mathrm{ArcTan}(ax)\right)+\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},3i\mathrm{ArcTan}(ax)\right)\right)\right)}{288a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (48*(3*a*x + 2*a^3*x^3 - 2*ArcTan[a*x])*ArcTan[a*x] - 4*Sqrt[6*Pi]*(1 + a^2
*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a^2*x^2)^(3/2)*(3*S
qrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*
Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*
I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(288
*a^2*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
[Out] int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.834 $\int \frac{\text{ArcTan}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{\text{ArcTan}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}}+\frac{\sqrt{\text{ArcTan}(ax)}}{ac^2\sqrt{c+a^2cx^2}}+\frac{x\text{ArcTan}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\text{ArcTan}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}}-\frac{9\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\Big(\sqrt{\cdots}}{8ac^2\sqrt{c+a^2c\cdots}}$$

[Out] 1/3*x*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^(3/2)/c^2/(a^2*c*x^2+c)^(1/2)-1/144*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-9/16*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/6*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(3/2)+arctan(a*x)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.24, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5020, 5018, 5025, 5024, 3385, 3433, 3393}

$$-\frac{9\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{8ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{24ac^2\sqrt{a^2cx^2+c}}+\frac{2x\text{ArcTan}(ax)^{3/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\text{ArcTan}(ax)}}{ac^2\sqrt{a^2cx^2+c}}+\frac{x\text{ArcTan}(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\text{ArcTan}(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]

[Out] Sqrt[ArcTan[a*x]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + Sqrt[ArcTan[a*x]]/(a*c^2*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*ArcTan[a*x]^(3/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) - (9*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(24*a*c^2*Sqrt[c + a^2*c*x^2])

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{1}{12}\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx + \frac{2\int}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\int \frac{}{(c+}}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1-}}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1-}}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1-}}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}}{}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\tan^{-1}(ax)}}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.15, size = 153, normalized size = 0.61

$$\frac{24\sqrt{\text{ArcTan}(ax)}\,(7+6a^2x^2+(6ax+4a^3x^3)\,\text{ArcTan}(ax)) - 81\sqrt{2\pi}\,(1+a^2x^2)^{3/2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) - \sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{144c^2\,(a+a^3x^2)\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*Sqrt[ArcTan[a*x]]*(7 + 6*a^2*x^2 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x]) - 8
1*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - S
qrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(144*
c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [F]

time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.835 $\int \frac{\textbf{ArcTan}(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}},x\right)$$

`[Out] Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)`

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

`[In] Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]`

`[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]`

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx$$

Mathematica [A]
time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]`

`[Out] Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]`

Maple [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}{(ax)}}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.836 $\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}} dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}} dx$$

Mathematica [A]
time = 6.50, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
[Out] Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.837 $\int x^m(c+a^2cx^2)\,\textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 1.33, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.838 $\int x^2(c+a^2cx^2) \operatorname{ArcTan}(ax)^{5/2} \, dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx = \int x^2\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 2.55, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int x^{2}\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^{2}x^{4}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(a**2*x**4*atan(a*x)**(5/2)
, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.839 $\int x(c+a^2cx^2)\,\textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=117

$$\frac{5c(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}{32a^2}-\frac{5cx(1+a^2x^2)\,\text{ArcTan}(ax)^{3/2}}{24a}+\frac{c(1+a^2x^2)^2\,\text{ArcTan}(ax)^{5/2}}{4a^2}-\frac{5c\text{Int}\left(\frac{1}{\sqrt{\text{ArcTa}}}\right.}{64a}$$

```
[Out] -5/24*c*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(
5/2)/a^2+5/32*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2-5/12*c*Unintegrable(arcta
n(a*x)^(3/2),x)/a-5/64*c*Unintegrable(1/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (5*c*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(32*a^2) - (5*c*x*(1 + a^2*x^2)*ArcTa
n[a*x]^(3/2))/(24*a) + (c*(1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/(4*a^2) - (5*c
*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(64*a) - (5*c*Defer[Int][ArcTan[a*x]^(
3/2), x])/(12*a)
```

Rubi steps

$$\int x(c+a^2cx^2)\tan^{-1}(ax)^{5/2}\,dx=\frac{c(1+a^2x^2)^2\tan^{-1}(ax)^{5/2}}{4a^2}-\frac{5\int(c+a^2cx^2)\tan^{-1}(ax)^{3/2}\,dx}{8a}$$

$$=\frac{5c(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{32a^2}-\frac{5cx(1+a^2x^2)\tan^{-1}(ax)^{3/2}}{24a}+\frac{c(1+a^2x^2)^2\,\text{t}}{4a}$$

Mathematica [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int x\left(a^{2} c\,x^{2}+c\right) \arctan \left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c \left(\int x \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int a^{2} x^{3} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(x*atan(a*x)**(5/2), x) + Integral(a**2*x**3*atan(a*x)**(5/2), x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.840 $\int (c+a^2cx^2) \text{ArcTan}(ax)^{5/2} dx$

Optimal. Leaf size=81

$$-\frac{5c(1+a^2x^2)\,\text{ArcTan}(ax)^{3/2}}{12a}+\frac{1}{3}cx(1+a^2x^2)\,\text{ArcTan}(ax)^{5/2}+\frac{5}{8}c\text{Int}\left(\sqrt{\text{ArcTan}(ax)},x\right)+\frac{2}{3}c\text{Int}\left(\text{ArcTan}(\right.$$

```
[Out] -5/12*c*(a^2*x^2+1)*arctan(a*x)^(3/2)/a+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(5/
2)+2/3*c*Unintegrable(arctan(a*x)^(5/2),x)+5/8*c*Unintegrable(arctan(a*x)^(
1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-5*c*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(12*a) + (c*x*(1 + a^2*x^2)*ArcTan[a
*x]^(5/2))/3 + (5*c*Defer[Int][Sqrt[ArcTan[a*x]], x])/8 + (2*c*Defer[Int][A
rcTan[a*x]^(5/2), x])/3
```

Rubi steps

$$\int (c+a^2cx^2)\tan^{-1}(ax)^{5/2}\,dx=-\frac{5c(1+a^2x^2)\tan^{-1}(ax)^{3/2}}{12a}+\frac{1}{3}cx(1+a^2x^2)\tan^{-1}(ax)^{5/2}+\frac{1}{8}(5c)\int\sqrt{}$$

Mathematica [A]
time = 2.70, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int (a^2c\,x^2+c)\arctan(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx+\int\operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(a**2*x**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

```
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.841 $\int \frac{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx=\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)\,\mathrm{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)\arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x}\,dx + \int a^2 x \operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x,x)
```

```
[Out] c*(Integral(atan(a*x)**(5/2)/x, x) + Integral(a**2*x*atan(a*x)**(5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x,x)
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x, x)
```

3.842 $\int \frac{(c+a^2cx^2)\text{ArcTan}(ax)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
[Out] Defer[Int][((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Mathematica [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
[Out] Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

Maple [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)\arctan(ax)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int\frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] c*(Integral(a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**2, x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2,x)
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2, x)
```

3.843 $\int x^m(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.09, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m\,\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.844 $\int x^2(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 1.82, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
[Out] int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int 2a^2x^4\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^4x^6\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
[Out] c**2*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(2*a**2*x**4*atan(a*x)**
(5/2), x) + Integral(a**4*x**6*atan(a*x)**(5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.845 $\int x(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2} dx$

Optimal. Leaf size=216

$$\frac{c^2(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{12a^2}+\frac{c^2(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}{32a^2}-\frac{c^2x(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}}{9a}-\frac{c^2x(1+a^2x^2)^2}{12}$$

```
[Out] -1/9*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/12*c^2*x*(a^2*x^2+1)^2*arctan(
a*x)^(3/2)/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(5/2)/a^2+1/12*c^2*(a^2*x^2+
1)*arctan(a*x)^(1/2)/a^2+1/32*c^2*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2-2/9*c
^2*Unintegrable(arctan(a*x)^(3/2),x)/a-1/24*c^2*Unintegrable(1/arctan(a*x)^
(1/2),x)/a-1/64*c*Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2} dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] (c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(12*a^2) + (c^2*(1 + a^2*x^2)^2*Sqrt[
ArcTan[a*x]])/(32*a^2) - (c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(9*a) - (c
^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))/(12*a) + (c^2*(1 + a^2*x^2)^3*ArcTa
n[a*x]^(5/2))/(6*a^2) - (c^2*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(24*a) - (
c*Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x])/(64*a) - (2*c^2*Defer[I
nt][ArcTan[a*x]^(3/2), x])/(9*a)
```

Rubi steps

$$\int x(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2} dx = \frac{c^2(1+a^2x^2)^3\tan^{-1}(ax)^{5/2}}{6a^2}-\frac{5\int (c+a^2cx^2)^2\tan^{-1}(ax)^{3/2} dx}{12a}$$

$$= \frac{c^2(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{32a^2}-\frac{c^2x(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{12a}+\frac{c^2(1+a^2x}{}$$

$$= \frac{c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{12a^2}+\frac{c^2(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{32a^2}-\frac{c^2x(1+a^2x}{}$$

Mathematica [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int x\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int x\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx+\int 2a^2x^3\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx+\int a^4x^5\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(x*atan(a*x)**(5/2), x) + Integral(2*a**2*x**3*atan(a*x)**(5/
2), x) + Integral(a**4*x**5*atan(a*x)**(5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.846 $\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=172

$$-\frac{c^2(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}}{3a}-\frac{c^2(1+a^2x^2)^2\operatorname{ArcTan}(ax)^{3/2}}{8a}+\frac{4}{15}c^2x(1+a^2x^2)\operatorname{ArcTan}(ax)^{5/2}+\frac{1}{5}c^2x(1+a$$

```
[Out] -1/3*c^2*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/8*c^2*(a^2*x^2+1)^2*arctan(a*x)^
(3/2)/a+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(5/2)+1/5*c^2*x*(a^2*x^2+1)^2*ar
ctan(a*x)^(5/2)+8/15*c^2*Unintegrable(arctan(a*x)^(5/2),x)+1/2*c^2*Unintegr
able(arctan(a*x)^(1/2),x)+3/16*c*Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(1/
2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

```
[Out] -1/3*(c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/a - (c^2*(1 + a^2*x^2)^2*ArcTan[
a*x]^(3/2))/(8*a) + (4*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/15 + (c^2*x*(
1 + a^2*x^2)^2*ArcTan[a*x]^(5/2))/5 + (c^2*Defer[Int][Sqrt[ArcTan[a*x]], x]
)/2 + (3*c*Defer[Int][(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x])/16 + (8*c^2*De
fer[Int][ArcTan[a*x]^(5/2), x])/15
```

Rubi steps

$$\begin{aligned}\int (c+a^2cx^2)^2\tan^{-1}(ax)^{5/2}\,dx &= -\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{8a}+\frac{1}{5}c^2x(1+a^2x^2)^2\tan^{-1}(ax)^{5/2}+\frac{1}{16}(3c)\\ &= -\frac{c^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}}{3a}-\frac{c^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{8a}+\frac{4}{15}c^2x(1+a\end{aligned}$$

Mathematica [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
[Out] Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2} \left(\int 2 a^{2} x^{2} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int a^{4} x^{4} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
[Out] c**2*(Integral(2*a**2*x**2*atan(a*x)**(5/2), x) + Integral(a**4*x**4*atan(a
*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.847 $\int \frac{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2 \,\text{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2 \,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2 \,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}{x}\, dx + \int 2a^{2}x\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}\, dx + \int a^{4}x^{3}\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x,x)
```

```
[Out] c**2*(Integral(atan(a*x)**(5/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(5/2),
 x) + Integral(a**4*x**3*atan(a*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.848 $\int \frac{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}{x^2}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
[Out] Defer[Int][((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}{x^2} \, dx = \int \frac{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}{x^2} \, dx$$

Mathematica [A]
time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
[Out] Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

Maple [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{5}{2}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int 2a^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^{2}}\,dx+\int a^{4}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] c**2*(Integral(2*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**2
, x) + Integral(a**4*x**2*atan(a*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

3.849 $\int x^m(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^m\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.96, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*x
)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.850 $\int x^2(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.83, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.851 $\int x(c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=309

$$\frac{3c^3(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{56a^2}+\frac{9c^3(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}{448a^2}+\frac{5c^3(1+a^2x^2)^3\sqrt{\operatorname{ArcTan}(ax)}}{448a^2}-\frac{c^3x(1+a^2x}{}$$

```
[Out] -1/14*c^3*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-3/56*c^3*x*(a^2*x^2+1)^2*arctan
(a*x)^(3/2)/a-5/112*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+1/8*c^3*(a^2*x^
2+1)^4*arctan(a*x)^(5/2)/a^2+3/56*c^3*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2+9/4
48*c^3*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2+5/448*c^3*(a^2*x^2+1)^3*arctan(a
*x)^(1/2)/a^2-1/7*c^3*Unintegrable(arctan(a*x)^(3/2),x)/a-3/112*c^3*Uninteg
rable(1/arctan(a*x)^(1/2),x)/a-9/896*c^2*Unintegrable((a^2*c*x^2+c)/arctan(
a*x)^(1/2),x)/a-5/896*c*Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(c+a^2cx^2\right)^3 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] (3*c^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])/(56*a^2) + (9*c^3*(1 + a^2*x^2)^2*S
qrt[ArcTan[a*x]])/(448*a^2) + (5*c^3*(1 + a^2*x^2)^3*Sqrt[ArcTan[a*x]])/(44
8*a^2) - (c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(14*a) - (3*c^3*x*(1 + a^2
*x^2)^2*ArcTan[a*x]^(3/2))/(56*a) - (5*c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(3
/2))/(112*a) + (c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^(5/2))/(8*a^2) - (3*c^3*Def
er[Int][1/Sqrt[ArcTan[a*x]], x])/(112*a) - (9*c^2*Defer[Int][(c + a^2*c*x^2
)/Sqrt[ArcTan[a*x]], x])/(896*a) - (5*c*Defer[Int][(c + a^2*c*x^2)^2/Sqrt[A
rcTan[a*x]], x])/(896*a) - (c^3*Defer[Int][ArcTan[a*x]^(3/2), x])/(7*a)
```

Rubi steps

$$\int x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}\,dx=\frac{c^3(1+a^2x^2)^4\tan^{-1}(ax)^{5/2}}{8a^2}-\frac{5\int\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}\,dx}{16a}$$

$$=\frac{5c^3(1+a^2x^2)^3\sqrt{\tan^{-1}(ax)}}{448a^2}-\frac{5c^3x(1+a^2x^2)^3\tan^{-1}(ax)^{3/2}}{112a}+\frac{c^3(1+$$

$$=\frac{9c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{448a^2}+\frac{5c^3(1+a^2x^2)^3\sqrt{\tan^{-1}(ax)}}{448a^2}-\frac{3c^3x(1+$$

$$=\frac{3c^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}{56a^2}+\frac{9c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}{448a^2}+\frac{5c^3(1+$$

Mathematica [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int x\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int 3a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int 3a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^6x^7\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(x*atan(a*x)**(5/2), x) + Integral(3*a**2*x**3*atan(a*x)**(5/
2), x) + Integral(3*a**4*x**5*atan(a*x)**(5/2), x) + Integral(a**6*x**7*ata
n(a*x)**(5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.852 $\int (c+a^2cx^2)^3 \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=259

$$-\frac{2c^3(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}}{7a}-\frac{3c^3(1+a^2x^2)^2\operatorname{ArcTan}(ax)^{3/2}}{28a}-\frac{5c^3(1+a^2x^2)^3\operatorname{ArcTan}(ax)^{3/2}}{84a}+\frac{8}{35}c^3x(1+$$

```
[Out] -2/7*c^3*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-3/28*c^3*(a^2*x^2+1)^2*arctan(a*x)
^(3/2)/a-5/84*c^3*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+8/35*c^3*x*(a^2*x^2+1)*
arctan(a*x)^(5/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^(5/2)+1/7*c^3*x*(a^2
*x^2+1)^3*arctan(a*x)^(5/2)+16/35*c^3*Unintegrable(arctan(a*x)^(5/2),x)+3/7
*c^3*Unintegrable(arctan(a*x)^(1/2),x)+9/56*c^2*Unintegrable((a^2*c*x^2+c)*
arctan(a*x)^(1/2),x)+5/56*c*Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),
x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^3 \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-2*c^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2))/(7*a) - (3*c^3*(1 + a^2*x^2)^2*Arc
Tan[a*x]^(3/2))/(28*a) - (5*c^3*(1 + a^2*x^2)^3*ArcTan[a*x]^(3/2))/(84*a) +
 (8*c^3*x*(1 + a^2*x^2)*ArcTan[a*x]^(5/2))/35 + (6*c^3*x*(1 + a^2*x^2)^2*Ar
cTan[a*x]^(5/2))/35 + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^(5/2))/7 + (3*c^3*
Defer[Int][Sqrt[ArcTan[a*x]], x])/7 + (9*c^2*Defer[Int][(c + a^2*c*x^2)*Sqr
t[ArcTan[a*x]], x])/56 + (5*c*Defer[Int][(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]
], x])/56 + (16*c^3*Defer[Int][ArcTan[a*x]^(5/2), x])/35
```

Rubi steps

$$\begin{aligned}\int \left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}\,dx &= -\frac{5c^3(1+a^2x^2)^3\tan^{-1}(ax)^{3/2}}{84a}+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\tan^{-1}(ax)^{5/2}+\frac{1}{56}(5c)\\ &= -\frac{3c^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{28a}-\frac{5c^3(1+a^2x^2)^3\tan^{-1}(ax)^{3/2}}{84a}+\frac{6}{35}c^3x(1\\ &= -\frac{2c^3(1+a^2x^2)\tan^{-1}(ax)^{3/2}}{7a}-\frac{3c^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}{28a}-\frac{5c^3(1+a\end{aligned}$$

Mathematica [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \left(c+a^2cx^2\right)^3 \text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int 3a^2x^2\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx+\int 3a^4x^4\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx+\int a^6x^6\operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx+\int \operatorname{atan}^{\frac{5}{2}}\left(ax\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2*atan(a*x)**(5/2), x) + Integral(3*a**4*x**4*atan
(a*x)**(5/2), x) + Integral(a**6*x**6*atan(a*x)**(5/2), x) + Integral(atan(
a*x)**(5/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.853 $\int \frac{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3\arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}{x}\, dx + \int 3a^{2}x\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}\, dx + \int 3a^{4}x^{3}\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}\, dx + \int a^{6}x^{5}\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x,x)
```

```
[Out] c**3*(Integral(atan(a*x)**(5/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(5/2),
 x) + Integral(3*a**4*x**3*atan(a*x)**(5/2), x) + Integral(a**6*x**5*atan(a
*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.854 $\int \frac{(c+a^2cx^2)^3\textbf{ArcTan}(ax)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x^2},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]

[Out] Defer[Int][((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}{x^2}\,dx$$

Mathematica [A]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{5/2}}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]

[Out] Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]

Maple [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3\arctan(ax)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

```
[Out] int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int 3a^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int\frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^{2}}\,dx+\int 3a^{4}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^{6}x^{4}\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x**2,x)
```

```
[Out] c**3*(Integral(3*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**2
, x) + Integral(3*a**4*x**2*atan(a*x)**(5/2), x) + Integral(a**6*x**4*atan(
a*x)**(5/2), x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

3.855 $\int \frac{x^m \mathbf{ArcTan}(ax)^{5/2}}{c+a^2cx^2} \, dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m \mathrm{ArcTan}(ax)^{5/2}}{c+a^2cx^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{5/2}}{c+a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{c+a^2cx^2} \, dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m \mathrm{ArcTan}(ax)^{5/2}}{c+a^2cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{a^2 c\, x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**A**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\mathrm{atan}\left(a\,x\right)}^{5/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.856 $\int \frac{x^3 \text{ArcTan}(ax)^{5/2}}{c+a^2cx^2} dx$

Optimal. Leaf size=61

$$-\frac{2x\text{ArcTan}(ax)^{7/2}}{7a^3c}+\frac{\text{Int}\left(x\text{ArcTan}(ax)^{5/2},x\right)}{a^2c}+\frac{2\text{Int}\left(\text{ArcTan}(ax)^{7/2},x\right)}{7a^3c}$$

```
[Out] -2/7*x*arctan(a*x)^(7/2)/a^3/c+Unintegrable(x*arctan(a*x)^(5/2),x)/a^2/c+2/
7*Unintegrable(arctan(a*x)^(7/2),x)/a^3/c
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] (-2*x*ArcTan[a*x]^(7/2))/(7*a^3*c) + Defer[Int][x*ArcTan[a*x]^(5/2), x]/(a^
2*c) + (2*Defer[Int][ArcTan[a*x]^(7/2), x])/(7*a^3*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int x\tan^{-1}(ax)^{5/2}\,dx}{a^2c}\\ &= -\frac{2x\tan^{-1}(ax)^{7/2}}{7a^3c}+\frac{2\int \tan^{-1}(ax)^{7/2}\,dx}{7a^3c}+\frac{\int x\tan^{-1}(ax)^{5/2}\,dx}{a^2c}\end{aligned}$$

Mathematica [A]
time = 3.16, size = 0, normalized size = 0.00

$$\int \frac{x^3\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \arctan \left(a x \right)^{\frac{5}{2}}}{a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
[Out] Integral(x**3*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.857 $\int \frac{x^2\mathbf{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=37

$$-\frac{2\text{ArcTan}(ax)^{7/2}}{7a^3c}+\frac{\text{Int}\left(\text{ArcTan}(ax)^{5/2},x\right)}{a^2c}$$

```
[Out] -2/7*arctan(a*x)^(7/2)/a^3/c+Unintegrable(arctan(a*x)^(5/2),x)/a^2/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
[Out] (-2*ArcTan[a*x]^(7/2))/(7*a^3*c) + Defer[Int][ArcTan[a*x]^(5/2), x]/(a^2*c)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx &= -\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx}{a^2}+\frac{\int \tan^{-1}(ax)^{5/2}\,dx}{a^2c}\\ &= -\frac{2\tan^{-1}(ax)^{7/2}}{7a^3c}+\frac{\int \tan^{-1}(ax)^{5/2}\,dx}{a^2c}\end{aligned}$$

Mathematica [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^2\arctan\left(ax\right)^{\frac{5}{2}}}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**2*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.858 $\int \frac{x\mathbf{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=41

$$\frac{2x\text{ArcTan}(ax)^{7/2}}{7ac}-\frac{2\text{Int}\left(\text{ArcTan}(ax)^{7/2},x\right)}{7ac}$$

```
[Out] 2/7*x*arctan(a*x)^(7/2)/a/c-2/7*Unintegrable(arctan(a*x)^(7/2),x)/a/c
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] (2*x*ArcTan[a*x]^(7/2))/(7*a*c) - (2*Defer[Int][ArcTan[a*x]^(7/2), x])/(7*a
*c)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx=\frac{2x\tan^{-1}(ax)^{7/2}}{7ac}-\frac{2\int \tan^{-1}(ax)^{7/2}\,dx}{7ac}$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{x\text{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x\arctan\left(ax\right)^{\frac{5}{2}}}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}{(ax)}}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.859 $\int \frac{\mathbf{ArcTan}(ax)^{5/2}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{2\mathrm{ArcTan}(ax)^{7/2}}{7ac}$$

```
[Out] 2/7*arctan(a*x)^(7/2)/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5004\}$

$$\frac{2\mathrm{ArcTan}(ax)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(7/2))/(7*a*c)
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\tan^{-1}(ax)^{7/2}}{7ac}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{2\mathrm{ArcTan}(ax)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcTan[a*x]^(7/2))/(7*a*c)
```

Maple [A]
time = 0.24, size = 15, normalized size = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{7}{2}}}{7ac}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 2/7*arctan(a*x)^(7/2)/a/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.45, size = 14, normalized size = 0.78

$$\frac{2\,\arctan\left(ax\right)^{\frac{7}{2}}}{7\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] 2/7*arctan(a*x)^(7/2)/(a*c)
```

Sympy [A]
time = 9.35, size = 15, normalized size = 0.83

$$\begin{cases} \frac{2\,\operatorname{atan}^{\frac{7}{2}}\left(ax\right)}{7ac} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
[Out] Piecewise((2*atan(a*x)**(7/2)/(7*a*c), Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.39, size = 14, normalized size = 0.78

$$\frac{2\,\arctan\left(ax\right)^{\frac{7}{2}}}{7\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/7*arctan(a*x)^(7/2)/(a*c)
```

Mupad [B]
time = 0.40, size = 14, normalized size = 0.78

$$\frac{2\,\mathrm{atan}(a\,x)^{7/2}}{7\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2),x)
```

```
[Out] (2*atan(a*x)^(7/2))/(7*a*c)
```

3.860 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=49

$$-\frac{2i\text{ArcTan}(ax)^{7/2}}{7c}+\frac{i\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

[Out] -2/7*I*arctan(a*x)^(7/2)/c+I*Unintegrable(arctan(a*x)^(5/2)/x/(I+a*x),x)/c

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]

[Out] (((-2*I)/7)*ArcTan[a*x]^(7/2))/c + (I*Defer[Int][ArcTan[a*x]^(5/2)/(x*(I + a*x)), x])/c

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx=-\frac{2i\tan^{-1}(ax)^{7/2}}{7c}+\frac{i\int \frac{\tan^{-1}(ax)^{5/2}}{x(i+ax)}\,dx}{c}$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]

[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)), x]

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**3 + x), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)),x)
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)), x)
```

3.861 $\int \frac{\textbf{ArcTan}(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{2a\text{ArcTan}(ax)^{7/2}}{7c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x^2},x\right)}{c}$$

```
[Out] -2/7*a*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^2,x)/c
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^2\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] (-2*a*ArcTan[a*x]^(7/2))/(7*c) + Defer[Int][ArcTan[a*x]^(5/2)/x^2, x]/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\,(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\\ &= -\frac{2a\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^2\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{5}{2}}}{x^{2} \left(a^{2} c \,x^{2}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}{a^{2} x^{4} + x^{2}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**4 + x**2), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.862 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=74

$$\frac{2ia^2\text{ArcTan}(ax)^{7/2}}{7c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x^3},x\right)}{c}-\frac{ia^2\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

```
[Out] 2/7*I*a^2*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^3,x)/c-I*a^2
*Unintegrable(arctan(a*x)^(5/2)/x/(I+a*x),x)/c
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^3\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] (((2*I)/7)*a^2*ArcTan[a*x]^(7/2))/c + Defer[Int][ArcTan[a*x]^(5/2)/x^3, x]/
c - (I*a^2*Defer[Int][ArcTan[a*x]^(5/2)/(x*(I + a*x)), x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\,(c+a^2cx^2)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^3}\,dx}{c}\\ &= \frac{2ia^2\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^3}\,dx}{c}-\frac{(ia^2)\int \frac{\tan^{-1}(ax)^{5/2}}{x(i+ax)}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^3\,(c+a^2cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 2.76, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(a x\right)^{\frac{5}{2}}}{x^{3} \left(a^{2} c \,x^{2}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
[Out] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}{a^{2} x^{5} + x^{3}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**5 + x**3), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.863 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2a^3\text{ArcTan}(ax)^{7/2}}{7c}+\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x^4},x\right)}{c}-\frac{a^2\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x^2},x\right)}{c}$$

```
[Out] 2/7*a^3*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^4,x)/c-a^2*Uni
ntegrable(arctan(a*x)^(5/2)/x^2,x)/c
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

```
[Out] (2*a^3*ArcTan[a*x]^(7/2))/(7*c) + Defer[Int][ArcTan[a*x]^(5/2)/x^4, x]/c -
(a^2*Defer[Int][ArcTan[a*x]^(5/2)/x^2, x])/c
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx &= -\left(a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx\right)+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}\\ &= a^4\int \frac{\tan^{-1}(ax)^{5/2}}{c+a^2cx^2}\,dx+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\\ &= \frac{2a^3\tan^{-1}(ax)^{7/2}}{7c}+\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{x^4}\,dx}{c}-\frac{a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x^2}\,dx}{c}\end{aligned}$$

Mathematica [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)), x]
```

Maple [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{5}{2}}}{x^{4} \left(a^{2} c \,x^{2}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
[Out] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}{a^{2} x^{6} + x^{4}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c),x)
[Out] Integral(atan(a*x)**(5/2)/(a**2*x**6 + x**4), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)),x)
```

```
[Out] int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.864 $\int \frac{x^m \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.865 $\int \frac{x^3 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^3\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2},x\right)$$

```
[Out] Unintegrable(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Defer[Int][(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Rubi steps

$$\int \frac{x^3\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 3.66, size = 0, normalized size = 0.00

$$\int \frac{x^3\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

Maple [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{x^3\arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
[Out] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.866 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2} dx$

Optimal. Leaf size=157

$$\frac{15x\sqrt{\text{ArcTan}(ax)}}{32a^2c^2(1+a^2x^2)}+\frac{5\text{ArcTan}(ax)^{3/2}}{16a^3c^2}-\frac{5\text{ArcTan}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)}-\frac{x\text{ArcTan}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{7/2}}{7a^3c^2}-\frac{15\sqrt{\pi}\,S\left(\frac{2\sqrt{\ldots}}{\ldots}\right.}{128\ldots}$$

```
[Out] 5/16*arctan(a*x)^(3/2)/a^3/c^2-5/8*arctan(a*x)^(3/2)/a^3/c^2/(a^2*x^2+1)-1/
2*x*arctan(a*x)^(5/2)/a^2/c^2/(a^2*x^2+1)+1/7*arctan(a*x)^(7/2)/a^3/c^2-15/
128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+15/32*x*arctan(
a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

Rubi [A]

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5056, 5050, 5012, 5090, 4491, 12, 3386, 3432}

$$-\frac{15\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128a^3c^2}+\frac{\text{ArcTan}(ax)^{7/2}}{7a^3c^2}+\frac{5\text{ArcTan}(ax)^{3/2}}{16a^3c^2}-\frac{x\text{ArcTan}(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}+\frac{15x\sqrt{\text{ArcTan}(ax)}}{32a^2c^2(a^2x^2+1)}-\frac{5\text{ArcTan}(ax)^{3/2}}{8a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (15*x*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(1 + a^2*x^2)) + (5*ArcTan[a*x]^(3/2))
/(16*a^3*c^2) - (5*ArcTan[a*x]^(3/2))/(8*a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan
[a*x]^(5/2))/(2*a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a^3*c^2) - (1
5*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^3*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = -\frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} + \frac{5\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx}{4a}$$

$$= -\frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{7/2}}{7a^3c^2} + \frac{15\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx}{16a^2}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

$$= \frac{15x\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\tan^{-1}(ax)^{3/2}}{16a^3c^2} - \frac{5\tan^{-1}(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\tan^{-}}{7}$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 0.71

$$\frac{4\sqrt{\mathrm{ArcTan}(ax)}\left(105ax+70(-1+a^2x^2)\,\mathrm{ArcTan}(ax)-112ax\mathrm{ArcTan}(ax)^2+32(1+a^2x^2)\,\mathrm{ArcTan}(ax)^3\right)-105\sqrt{\pi}\,(1+a^2x^2)\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{896a^3c^2\,(1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
[Out] (4*Sqrt[ArcTan[a*x]]*(105*a*x + 70*(-1 + a^2*x^2)*ArcTan[a*x] - 112*a*x*Arc
Tan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) - 105*Sqrt[Pi]*(1 + a^2*x^2)*F
resnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*a^3*c^2*(1 + a^2*x^2))
```

Maple [A]
time = 0.33, size = 93, normalized size = 0.59

method	result
default	$-\frac{-128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}+224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))+280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))-210\sqrt{\arctan(a}}{896c^2a^3\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/896/c^2/a^3/Pi^(1/2)*(-128*arctan(a*x)^(7/2)*Pi^(1/2)+224*arctan(a*x)^(5
/2)*Pi^(1/2)*sin(2*arctan(a*x))+280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan
(a*x))-210*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+105*Pi*FresnelS(2*
arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**2*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.867 $\int \frac{x\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{15\sqrt{\text{ArcTan}(ax)}}{64a^2c^2}+\frac{15\sqrt{\text{ArcTan}(ax)}}{32a^2c^2(1+a^2x^2)}+\frac{5x\text{ArcTan}(ax)^{3/2}}{8ac^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{5/2}}{4a^2c^2}-\frac{\text{ArcTan}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)}-\frac{15\sqrt{\pi}\text{ Fres}}{}$$

[Out] 5/8*x*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^(5/2)/a^2/c^2-1/2*arctan(a*x)^(5/2)/a^2/c^2/(a^2*x^2+1)-15/128*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2-15/64*arctan(a*x)^(1/2)/a^2/c^2+15/32*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)

Rubi [A]
time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5050, 5012, 5024, 3393, 3385, 3433}

$$-\frac{15\sqrt{\pi}\text{ FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128a^2c^2}-\frac{\text{ArcTan}(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}+\frac{5x\text{ArcTan}(ax)^{3/2}}{8ac^2(a^2x^2+1)}+\frac{15\sqrt{\text{ArcTan}(ax)}}{32a^2c^2(a^2x^2+1)}+\frac{\text{ArcTan}(ax)^{5/2}}{4a^2c^2}-\frac{15\sqrt{\text{ArcTan}(ax)}}{64a^2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]

[Out] (-15*Sqrt[ArcTan[a*x]])/(64*a^2*c^2) + (15*Sqrt[ArcTan[a*x]])/(32*a^2*c^2*(1 + a^2*x^2)) + (5*x*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(4*a^2*c^2) - ArcTan[a*x]^(5/2)/(2*a^2*c^2*(1 + a^2*x^2)) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a^2*c^2)

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx}{4a}$$

$$= \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15}{16}\int \frac{x\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15\int \cdots}{(c+a^2 \cdots}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15\text{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15\text{Subst}\Big(\cdots}{\cdots}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}\cdots}{2a^2c^2(1\cdots}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}\cdots}{2a^2c^2(1\cdots}$$

$$= -\frac{15\sqrt{\tan^{-1}(ax)}}{64a^2c^2} + \frac{15\sqrt{\tan^{-1}(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^2} - \frac{\tan^{-1}\cdots}{2a^2c^2(1\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 234, normalized size = 1.50

$$\frac{240\text{ArcTan}(ax) - 240a^2x^2\text{ArcTan}(ax) + 640ax\text{ArcTan}(ax)^2 - 256\text{ArcTan}(ax)^3 + 256a^2x^2\text{ArcTan}(ax)^3 - 60\sqrt{\pi}(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right) + 15i\sqrt{2}(1+a^2x^2)\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) - 15i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right) - 15i\sqrt{2}a^2x^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{1024a^2c^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

```
[Out] (240*ArcTan[a*x] - 240*a^2*x^2*ArcTan[a*x] + 640*a*x*ArcTan[a*x]^2 - 256*Ar
cTan[a*x]^3 + 256*a^2*x^2*ArcTan[a*x]^3 - 60*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[Ar
cTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (15*I)*Sqrt[2]*(1 + a
^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - (15*I)*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - (15*I)*Sqrt[2]*a^2*
```

```
x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(1024*a^2*c^2*(1 + a
^2*x^2)*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.25, size = 82, normalized size = 0.53

method	result
default	$-\frac{32\arctan(ax)^{\frac{5}{2}}\cos(2\arctan(ax))\sqrt{\pi}-40\arctan(ax)^{\frac{3}{2}}\sin(2\arctan(ax))\sqrt{\pi}-30\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\cos(2\arctan(ax))}{128c^{2}a^{2}\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/128/c^2/a^2/Pi^(1/2)*(32*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/2)-4
0*arctan(a*x)^(3/2)*sin(2*arctan(a*x))*Pi^(1/2)-30*arctan(a*x)^(1/2)*Pi^(1/
2)*cos(2*arctan(a*x))+15*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

```
[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.868 $\int \frac{\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{15x\sqrt{\text{ArcTan}(ax)}}{32c^2(1+a^2x^2)}-\frac{5\text{ArcTan}(ax)^{3/2}}{16ac^2}+\frac{5\text{ArcTan}(ax)^{3/2}}{8ac^2(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)^{5/2}}{2c^2(1+a^2x^2)}+\frac{\text{ArcTan}(ax)^{7/2}}{7ac^2}+\frac{15\sqrt{\pi}\,S\Big(2\ldots}{\ldots}$$

```
[Out] -5/16*arctan(a*x)^(3/2)/a/c^2+5/8*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/2*x
*arctan(a*x)^(5/2)/c^2/(a^2*x^2+1)+1/7*arctan(a*x)^(7/2)/a/c^2+15/128*Fresn
elS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-15/32*x*arctan(a*x)^(1/2)/
c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5012, 5050, 5090, 4491, 12, 3386, 3432}

$$\frac{x\text{ArcTan}(ax)^{5/2}}{2c^2(a^2x^2+1)}+\frac{5\text{ArcTan}(ax)^{3/2}}{8ac^2(a^2x^2+1)}-\frac{15x\sqrt{\text{ArcTan}(ax)}}{32c^2(a^2x^2+1)}+\frac{15\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128ac^2}+\frac{\text{ArcTan}(ax)^{7/2}}{7ac^2}-\frac{5\text{ArcTan}(ax)^{3/2}}{16ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(32*c^2*(1 + a^2*x^2)) - (5*ArcTan[a*x]^(3/2))/(1
6*a*c^2) + (5*ArcTan[a*x]^(3/2))/(8*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(
5/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a*c^2) + (15*Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(128*a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} - \frac{1}{4}(5a)\int \frac{x\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(ax)^{7/2}}{7ac^2} - \frac{15}{16}\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$
$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{32c^2(1+a^2x^2)} - \frac{5\tan^{-1}(ax)^{3/2}}{16ac^2} + \frac{5\tan^{-1}(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\tan^{-1}(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\tan^{-1}(a}{7ac^2}$$

Mathematica [A]
time = 0.06, size = 108, normalized size = 0.72

$$\frac{4\sqrt{\text{ArcTan}(ax)}\left(-105ax-70(-1+a^2x^2)\,\text{ArcTan}(ax)+112ax\text{ArcTan}(ax)^2+32(1+a^2x^2)\,\text{ArcTan}(ax)^3\right)+105\sqrt{\pi}\,(1+a^2x^2)\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{896c^2\,(a+a^3x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (4*Sqrt[ArcTan[a*x]]*(-105*a*x - 70*(-1 + a^2*x^2)*ArcTan[a*x] + 112*a*x*Ar
cTan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) + 105*Sqrt[Pi]*(1 + a^2*x^2)*
FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*c^2*(a + a^3*x^2))
```

Maple [A]
time = 0.32, size = 93, normalized size = 0.62

method	result
default	$\frac{128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}+224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))+280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))-210\sqrt{\arctan(ax)}}{896c^2a\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/c^2/a/Pi^(1/2)*(128*arctan(a*x)^(7/2)*Pi^(1/2)+224*arctan(a*x)^(5/2)*
Pi^(1/2)*sin(2*arctan(a*x))+280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x
))-210*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+105*Pi*FresnelS(2*arct
an(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2,x)
[Out] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2, x)
```

3.869 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x\,(c+a^2cx^2)^2},x\right)$$

`[Out] Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)`

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

`[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]`

`[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]`

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx$$

Mathematica [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]`

`[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]`

Maple [A]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.870 $\int \frac{x^m \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$$

Mathematica [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2 c\, x^2+c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.871 $\int \frac{x^5 \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^5 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
[Out] Defer[Int][(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^5 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 6.44, size = 0, normalized size = 0.00

$$\int \frac{x^5 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

Maple [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{x^5 \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**5*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5 \operatorname{atan}(a x)^{5/2}}{\left(c a^2 x^2 + c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
[Out] int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.872 $\int \frac{x^4 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=310

$$\frac{45x\sqrt{\text{ArcTan}(ax)}}{128a^4c^3(1+a^2x^2)} + \frac{45\text{ArcTan}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\text{ArcTan}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} - \frac{15\text{ArcTan}(ax)^{3/2}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\text{ArcTan}(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\text{Arc}}{8a^4c^3}$$

```
[Out] 45/256*arctan(a*x)^(3/2)/a^5/c^3+5/32*x^4*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+
1)^2-15/32*arctan(a*x)^(3/2)/a^5/c^3/(a^2*x^2+1)-1/4*x^3*arctan(a*x)^(5/2)/
a^2/c^3/(a^2*x^2+1)^2-3/8*x*arctan(a*x)^(5/2)/a^4/c^3/(a^2*x^2+1)+3/28*arct
an(a*x)^(7/2)/a^5/c^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)
)*2^(1/2)*Pi^(1/2)/a^5/c^3-15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi
^(1/2)/a^5/c^3+45/128*x*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)+15/256*sin(2*
arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3-15/2048*sin(4*arctan(a*x))*arctan(a*
x)^(1/2)/a^5/c^3
```

Rubi [A]
time = 0.35, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5060, 5056, 5050, 5012, 5090, 4491, 12, 3386, 3432, 3393, 3377}

$$\frac{15\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096a^5c^3} - \frac{15\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128a^5c^3} + \frac{3\text{ArcTan}(ax)^{7/2}}{28a^5c^3} + \frac{45\text{ArcTan}(ax)^{3/2}}{256a^5c^3} + \frac{15\sqrt{\text{ArcTan}(ax)}\sin(2\text{ArcTan}(ax))}{256a^5c^3} - \frac{15\sqrt{\text{ArcTan}(ax)}\sin(4\text{ArcTan}(ax))}{2048a^5c^3} + \frac{5x^4\text{ArcTan}(ax)^{3/2}}{32ac^3(a^2x^2+1)^2} - \frac{x^3\text{ArcTan}(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2} - \frac{15\text{ArcTan}(ax)^{3/2}}{32a^5c^3(a^2x^2+1)} - \frac{3x\text{ArcTan}(ax)^{5/2}}{8a^4c^3(a^2x^2+1)} + \frac{45x\sqrt{\text{ArcTan}(ax)}}{128a^4c^3(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (45*x*Sqrt[ArcTan[a*x]])/(128*a^4*c^3*(1 + a^2*x^2)) + (45*ArcTan[a*x]^(3/2
))/(256*a^5*c^3) + (5*x^4*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) - (
15*ArcTan[a*x]^(3/2))/(32*a^5*c^3*(1 + a^2*x^2)) - (x^3*ArcTan[a*x]^(5/2))/
(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*x*ArcTan[a*x]^(5/2))/(8*a^4*c^3*(1 + a^2*x
^2)) + (3*ArcTan[a*x]^(7/2))/(28*a^5*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^5*c^3) - (15*Sqrt[Pi]*FresnelS[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/(128*a^5*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a
*x]])/(256*a^5*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^5*c
^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{x^3 \tan^{-1}(ax)^{5/2}}{4a^2c^3\,(1+a^2x^2)^2} - \frac{15}{64}\int \frac{x^4\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx + \frac{3\int \frac{x^2 \tan^{-1}(ax)^5}{(c+a^2cx^2)^2}}{4a^2c}$$

$$= \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{x^3 \tan^{-1}(ax)^{5/2}}{4a^2c^3\,(1+a^2x^2)^2} - \frac{3x \tan^{-1}(ax)^{5/2}}{8a^4c^3\,(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^{7/2}}{28a^5c^3} - \frac{15\mathrm{Su}}{}$$

$$= \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x^3 \tan^{-1}(ax)^{5/2}}{4a^2c^3\,(1+a^2x^2)^2} - \frac{3x \tan^{-1}(ax)^{5/2}}{8a^4c^3\,(1+a^2x^2)} + \frac{3\,\mathrm{t}}{}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

$$= \frac{45x\sqrt{\tan^{-1}(ax)}}{128a^4c^3\,(1+a^2x^2)} + \frac{45\tan^{-1}(ax)^{3/2}}{256a^5c^3} + \frac{5x^4 \tan^{-1}(ax)^{3/2}}{32ac^3\,(1+a^2x^2)^2} - \frac{15\tan^{-1}(ax)^{3/2}}{32a^5c^3\,(1+a^2x^2)} - \frac{x}{4a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 287, normalized size = 0.93

$$\frac{50400ax\mathrm{ArcTan}(ax) + 57120a^3x^3\mathrm{ArcTan}(ax) - 33600\mathrm{ArcTan}(ax)^2 - 13440a^2x^2\mathrm{ArcTan}(ax)^2 + 38080a^4x^4\mathrm{ArcTan}(ax)^2 - 43008ax\mathrm{ArcTan}(ax)^3 - 71680a^3x^3\mathrm{ArcTan}(ax)^3 + 12288(1+a^2x^2)^2\mathrm{ArcTan}(ax)^4 + 3360\sqrt{2}(1+a^2x^2)^2\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-2i\mathrm{ArcTan}(ax)\right) + 3360\sqrt{2}(1+a^2x^2)^2\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},2i\mathrm{ArcTan}(ax)\right) - 105(1+a^2x^2)^2\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-4i\mathrm{ArcTan}(ax)\right) - 105(1+a^2x^2)^2\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},4i\mathrm{ArcTan}(ax)\right)}{114688a^5c^3(1+a^2x^2)^2\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

[Out] (50400*a*x*ArcTan[a*x] + 57120*a^3*x^3*ArcTan[a*x] - 33600*ArcTan[a*x]^2 - 13440*a^2*x^2*ArcTan[a*x]^2 + 38080*a^4*x^4*ArcTan[a*x]^2 - 43008*a*x*ArcTan[a*x]^3 - 71680*a^3*x^3*ArcTan[a*x]^3 + 12288*(1 + a^2*x^2)^2*ArcTan[a*x]^4 + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(114688*a^5*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])

Maple [A]
time = 0.40, size = 168, normalized size = 0.54

method	result
default	$\frac{6144 \arctan(ax)^{\frac{7}{2}} \sqrt{\pi} - 14336 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi}\ \sin(2\arctan(ax)) + 1792 \arctan(ax)^{\frac{5}{2}} \sin(4\arctan(ax)) \sqrt{\pi} - 17920 \arctan(ax)^{\frac{3}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/57344/c^3/a^5*(6144*arctan(a*x)^(7/2)*Pi^(1/2)-14336*arctan(a*x)^(5/2)*Pi^(1/2)*sin(2*arctan(a*x))+1792*arctan(a*x)^(5/2)*sin(4*arctan(a*x))*Pi^(1/2)-17920*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+1120*arctan(a*x)^(3/2)*cos(4*arctan(a*x))*Pi^(1/2)+105*Pi*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)+13440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))-420*sin(4*arctan(a*x))*arctan(a*x)^(1/2)*Pi^(1/2)-6720*Pi*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**4*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\mathrm{atan}\left(a\,x\right)}^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.873 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

Optimal. Leaf size=256

$$-\frac{135\sqrt{\text{ArcTan}(ax)}}{2048a^4c^3}-\frac{15x^4\sqrt{\text{ArcTan}(ax)}}{256c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\text{ArcTan}(ax)}}{256a^4c^3\left(1+a^2x^2\right)}+\frac{5x^3\text{ArcTan}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\text{ArcTan}(ax)^{3/2}}{64a^3c^3\left(1+a^2x^2\right)}-$$

```
[Out] 5/32*x^3*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a^
3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^(5/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(5/2)/
c^3/(a^2*x^2+1)^2+15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)/a^4/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/
2)/a^4/c^3-135/2048*arctan(a*x)^(1/2)/a^4/c^3-15/256*x^4*arctan(a*x)^(1/2)/
c^3/(a^2*x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)
```

Rubi [A]
time = 0.36, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5064, 5060, 5056, 5050, 5024, 3393, 3385, 3433, 5090}

$$\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096a^4c^3}-\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{256a^4c^3}-\frac{3\text{ArcTan}(ax)^{5/2}}{32a^4c^3}-\frac{135\sqrt{\text{ArcTan}(ax)}}{2048a^4c^3}+\frac{x^4\text{ArcTan}(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{15x^4\sqrt{\text{ArcTan}(ax)}}{256c^3\left(a^2x^2+1\right)^2}+\frac{5x^3\text{ArcTan}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}+\frac{45\sqrt{\text{ArcTan}(ax)}}{256a^4c^3\left(a^2x^2+1\right)}+\frac{15x\text{ArcTan}(ax)^{3/2}}{64a^3c^3\left(a^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-135*Sqrt[ArcTan[a*x]])/(2048*a^4*c^3) - (15*x^4*Sqrt[ArcTan[a*x]])/(256*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^4*c^3*(1 + a^2*x^2)) +
(5*x^3*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3
/2))/(64*a^3*c^3*(1 + a^2*x^2)) - (3*ArcTan[a*x]^(5/2))/(32*a^4*c^3) + (x^4
*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a^2*x^2)^2) + (15*Sqrt[Pi/2]*FresnelC[2*Sqr
t[2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^4*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(256*a^4*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5056

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (Dist[b*(p/(2*c)), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x]
, x] - Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
```

```
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{x^4 \tan^{-1}(ax)^{5/2}}{4c^3(1+a^2x^2)^2} - \frac{1}{8}(5a)\int \frac{x^4 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{x^4\tan^{-1}(ax)^{5/2}}{4c^3(1+a^2x^2)^2} + \frac{1}{512}(15a)\int \frac{}{(c+a^2c}$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64a^3c^3(1+a^2x^2)} - \frac{3\tan^{-1}(ax)^{5/2}}{32a^4c^3} +$$

$$= -\frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64a^3c^3(1+a^2x^2)}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} +$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} +$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}$$

$$= -\frac{135\sqrt{\tan^{-1}(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\tan^{-1}(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 359, normalized size = 1.40

$$\frac{510\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) + \frac{14400\text{ArcTan}(ax)+5760a^2x^2\text{ArcTan}(ax)-16320a^4x^4\text{ArcTan}(ax)+30720ax\text{ArcTan}(ax)^2+51200a^3x^3\text{ArcTan}(ax)^2-12288\text{ArcTan}(ax)^3-24576a^2x^2\text{ArcTan}(ax)^3+20480a^4x^4\text{ArcTan}(ax)^3-4080\sqrt{\pi}(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+900i\sqrt{2}(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)-900i\sqrt{2}(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+135i(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-135i(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}}{131072a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

[Out] (510*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (14400*ArcTan[a*x] + 5760*a^2*x^2*ArcTan[a*x] - 16320*a^4*x^4*ArcTan[a*x] + 30720*a*x*ArcTan[a*x]^2 + 51200*a^3*x^3*ArcTan[a*x]^2 - 12288*ArcTan[a*x]^3 - 24576*a^2*x^2*ArcTan[a*x]^3 + 20480*a^4*x^4*ArcTan[a*x]^3 - 4080*Sqrt[Pi]*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (900*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - (900*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + (135*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - (135*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^4*c^3)

Maple [A]
time = 0.35, size = 157, normalized size = 0.61

method	result
default	$\frac{-1024 \arctan(ax)^{\frac{5}{2}} \cos(2 \arctan(ax)) \sqrt{\pi} + 256 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \cos(4 \arctan(ax)) + 1280 \arctan(ax)^{\frac{3}{2}} \sin(2 \arctan(ax)) \sqrt{\pi} - }{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/8192/c^3/a^4*(-1024*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/2)+256*arctan(a*x)^(5/2)*Pi^(1/2)*cos(4*arctan(a*x))+1280*arctan(a*x)^(3/2)*sin(2*arctan(a*x))*Pi^(1/2)-160*arctan(a*x)^(3/2)*Pi^(1/2)*sin(4*arctan(a*x))+15*Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+960*arctan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))-60*arctan(a*x)^(1/2)*Pi^(1/2)*cos(4*arctan(a*x))-480*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**3*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.874 $\int \frac{x^2 \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

Optimal. Leaf size=133

$$\frac{\text{ArcTan}(ax)^{7/2}}{28a^3c^3} - \frac{5\text{ArcTan}(ax)^{3/2}\cos(4\text{ArcTan}(ax))}{256a^3c^3} - \frac{15\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096a^3c^3} + \frac{15\sqrt{\text{ArcTan}(ax}}{20}$$

[Out] 1/28*arctan(a*x)^(7/2)/a^3/c^3-5/256*arctan(a*x)^(3/2)*cos(4*arctan(a*x))/a^3/c^3-1/32*arctan(a*x)^(5/2)*sin(4*arctan(a*x))/a^3/c^3-15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3+15/2048*sin(4*arctan(a*x))*arctan(a*x)^(1/2)/a^3/c^3

Rubi [A]

time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5090, 4491, 3377, 3386, 3432}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096a^3c^3} + \frac{\text{ArcTan}(ax)^{7/2}}{28a^3c^3} - \frac{\text{ArcTan}(ax)^{5/2}\sin(4\text{ArcTan}(ax))}{32a^3c^3} + \frac{15\sqrt{\text{ArcTan}(ax)}\sin(4\text{ArcTan}(ax))}{2048a^3c^3} - \frac{5\text{ArcTan}(ax)^{3/2}\cos(4\text{ArcTan}(ax))}{256a^3c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]

[Out] ArcTan[a*x]^(7/2)/(28*a^3*c^3) - (5*ArcTan[a*x]^(3/2)*Cos[4*ArcTan[a*x]])/(256*a^3*c^3) - (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4096*a^3*c^3) + (15*Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a^3*c^3) - (ArcTan[a*x]^(5/2)*Sin[4*ArcTan[a*x]])/(32*a^3*c^3)

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\text{Subst}\left(\int x^{5/2}\cos^2(x)\sin^2(x)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x^{5/2}}{8} - \frac{1}{8}x^{5/2}\cos(4x)\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{\text{Subst}\left(\int x^{5/2}\cos(4x)\,dx, x, \tan^{-1}(ax)\right)}{8a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} + \frac{5\text{Subst}\left(\int x^{3/2}\sin(4x)\,dx, x, \tan^{-1}\right.}{64a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{\tan^{-1}(ax)^{5/2}\sin\left(4\tan^{-1}(ax)\right)}{32a^3c^3} +$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right.}{2048a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} + \frac{15\sqrt{\tan^{-1}(ax)}\sin\left(4\tan^{-1}(ax)\right.}{2048a^3c^3}$$

$$= \frac{\tan^{-1}(ax)^{7/2}}{28a^3c^3} - \frac{5\tan^{-1}(ax)^{3/2}\cos\left(4\tan^{-1}(ax)\right)}{256a^3c^3} - \frac{15\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{4096a^3c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 185, normalized size = 1.39

$$\frac{32\text{ArcTan}(ax)\left(-105ax(-1+a^2x^2)-70(1-6a^2x^2+a^4x^4)\text{ArcTan}(ax)+448ax(-1+a^2x^2)\text{ArcTan}(ax)^2+128(1+a^2x^2)^2\text{ArcTan}(ax)^3\right)+105(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+105(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{114688a^3c^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (32*ArcTan[a*x]*(-105*a*x*(-1 + a^2*x^2) - 70*(1 - 6*a^2*x^2 + a^4*x^4)*Arc
Tan[a*x] + 448*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^2 + 128*(1 + a^2*x^2)^2*ArcTa
n[a*x]^3) + 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Ar
cTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTa
n[a*x]])/(114688*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.34, size = 96, normalized size = 0.72

method	result
default	$\frac{2048\arctan(ax)^4-1792\arctan(ax)^3\sin(4\arctan(ax))-105\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}-1120}{57344c^3a^3\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/57344/c^3/a^3*(2048*arctan(a*x)^4-1792*arctan(a*x)^3*sin(4*arctan(a*x))-1
05*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arctan(a*x)^(1/2)
*Pi^(1/2)-1120*arctan(a*x)^2*cos(4*arctan(a*x))+420*sin(4*arctan(a*x))*arct
an(a*x))/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**2*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.875 $\int \frac{x\mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{225\sqrt{\text{ArcTan}(ax)}}{2048a^2c^3}+\frac{15\sqrt{\text{ArcTan}(ax)}}{256a^2c^3\left(1+a^2x^2\right)^2}+\frac{45\sqrt{\text{ArcTan}(ax)}}{256a^2c^3\left(1+a^2x^2\right)}+\frac{5x\text{ArcTan}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2}+\frac{15x\text{ArcTan}(ax)^{3/2}}{64ac^3\left(1+a^2x^2\right)}+3$$

```
[Out] 5/32*x*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a/c^
3/(a^2*x^2+1)+3/32*arctan(a*x)^(5/2)/a^2/c^3-1/4*arctan(a*x)^(5/2)/a^2/c^3/
(a^2*x^2+1)^2-15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2
)*Pi^(1/2)/a^2/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a
^2/c^3-225/2048*arctan(a*x)^(1/2)/a^2/c^3+15/256*arctan(a*x)^(1/2)/a^2/c^3/
(a^2*x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)
```

Rubi [A]
time = 0.25, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5050, 5020, 5012, 5024, 3393, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096a^2c^3}-\frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{256a^2c^3}-\frac{\text{ArcTan}(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{15x\text{ArcTan}(ax)^{3/2}}{64ac^3\left(a^2x^2+1\right)}+\frac{5x\text{ArcTan}(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}+\frac{45\sqrt{\text{ArcTan}(ax)}}{256a^2c^3\left(a^2x^2+1\right)}+\frac{15\sqrt{\text{ArcTan}(ax)}}{256a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\text{ArcTan}(ax)^{5/2}}{32a^2c^3}-\frac{225\sqrt{\text{ArcTan}(ax)}}{2048a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-225*Sqrt[ArcTan[a*x]])/(2048*a^2*c^3) + (15*Sqrt[ArcTan[a*x]])/(256*a^2*c
^3*(1 + a^2*x^2)^2) + (45*Sqrt[ArcTan[a*x]])/(256*a^2*c^3*(1 + a^2*x^2)) +
(5*x*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*x*ArcTan[a*x]^(3/2
))/(64*a*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]^(5/2))/(32*a^2*c^3) - ArcTan[a
*x]^(5/2)/(4*a^2*c^3*(1 + a^2*x^2)^2) - (15*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(4096*a^2*c^3) - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(256*a^2*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx}{8a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} - \frac{\tan^{-1}(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{15\int \frac{1}{(c+a^2cx^2)^3\sqrt{\tan\ldots}}}{512a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3(1+a^2x^2)} + \frac{3\tan^{-1}(ax)^{5/2}}{32a^2c^3} - \ldots$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\tan^{-1}(ax)^{3/2}}{64ac^3(1+a^2x^2)}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{2048a^2c^3} + \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}$$

$$= -\frac{45\sqrt{\tan^{-1}(ax)}}{2048a^2c^3} + \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3} + \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3} + \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)}$$

$$= -\frac{225\sqrt{\tan^{-1}(ax)}}{2048a^2c^3} + \frac{15\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\tan^{-1}(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\tan^{-1}(ax)^{3/2}}{32ac^3(1+a^2x^2)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 359, normalized size = 1.41

$$\frac{450\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) + \frac{16320\text{ArcTan}(ax) - 5760a^2x^2\text{ArcTan}(ax) - 14400a^4x^4\text{ArcTan}(ax) + 51200ax\text{ArcTan}(ax)^2 + 30720a^3x^3\text{ArcTan}(ax)^2 - 20480\text{ArcTan}(ax)^3 + 24576a^2x^2\text{ArcTan}(ax)^3 + 12288a^4x^4\text{ArcTan}(ax)^3 - 3600\sqrt{\pi}(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right) + 1020i\sqrt{2}(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) - 1020i\sqrt{2}(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right) + 345i(1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right) - 345i(1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}}{131072a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

```
[Out] (450*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (16320*ArcTan[a*
x] - 5760*a^2*x^2*ArcTan[a*x] - 14400*a^4*x^4*ArcTan[a*x] + 51200*a*x*ArcTa
n[a*x]^2 + 30720*a^3*x^3*ArcTan[a*x]^2 - 20480*ArcTan[a*x]^3 + 24576*a^2*x^
2*ArcTan[a*x]^3 + 12288*a^4*x^4*ArcTan[a*x]^3 - 3600*Sqrt[Pi]*(1 + a^2*x^2)
^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (1020*I)*Sq
rt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]]
 - (1020*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*Ar
cTan[a*x]] + (345*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*
I)*ArcTan[a*x]] - (345*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4
*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^2*c^3)
```

Maple [A]
time = 0.30, size = 157, normalized size = 0.62

method	result
default	$-\frac{1024 \arctan(ax)^{\frac{5}{2}} \cos(2 \arctan(ax)) \sqrt{\pi} + 256 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi}\, \cos(4 \arctan(ax)) - 1280 \arctan(ax)^{\frac{3}{2}} \sin(2 \arctan(ax)) \sqrt{\pi} - 16}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8192/c^3/a^2/Pi^(1/2)*(1024*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/2
)+256*arctan(a*x)^(5/2)*Pi^(1/2)*cos(4*arctan(a*x))-1280*arctan(a*x)^(3/2)*
sin(2*arctan(a*x))*Pi^(1/2)-160*arctan(a*x)^(3/2)*Pi^(1/2)*sin(4*arctan(a*x
))+15*Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-960*arctan(
a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))+480*Pi*FresnelC(2*arctan(a*x)^(1/2)/
Pi^(1/2))-60*arctan(a*x)^(1/2)*Pi^(1/2)*cos(4*arctan(a*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\mathrm{atan}\left(a\,x\right)}^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.876 $\int \frac{\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=296

$$-\frac{45x\sqrt{\text{ArcTan}(ax)}}{128c^3(1+a^2x^2)}-\frac{75\text{ArcTan}(ax)^{3/2}}{256ac^3}+\frac{5\text{ArcTan}(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}+\frac{15\text{ArcTan}(ax)^{3/2}}{32ac^3(1+a^2x^2)}+\frac{x\text{ArcTan}(ax)^{5/2}}{4c^3(1+a^2x^2)^2}+\frac{3x\text{ArcTa}}{8c^3(1}$$

```
[Out] -75/256*arctan(a*x)^(3/2)/a/c^3+5/32*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+
15/32*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)^(5/2)/c^3/(a^2*
x^2+1)^2+3/8*x*arctan(a*x)^(5/2)/c^3/(a^2*x^2+1)+3/28*arctan(a*x)^(7/2)/a/c
^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/
a/c^3+15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3-45/128*x
*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)-15/256*sin(2*arctan(a*x))*arctan(a*x)^(1
/2)/a/c^3-15/2048*sin(4*arctan(a*x))*arctan(a*x)^(1/2)/a/c^3
```

Rubi [A]
time = 0.26, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5020, 5012, 5050, 5090, 4491, 12, 3386, 3432, 5024, 3393, 3377}

$$\frac{3x\text{ArcTan}(ax)^{5/2}}{8c^3(a^2x^2+1)}+\frac{x\text{ArcTan}(ax)^{5/2}}{4c^3(a^2x^2+1)^2}+\frac{15\text{ArcTan}(ax)^{3/2}}{32ac^3(a^2x^2+1)}+\frac{5\text{ArcTan}(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{45x\sqrt{\text{ArcTan}(ax)}}{128c^3(a^2x^2+1)}+\frac{15\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4096ac^3}+\frac{15\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{128ac^3}+\frac{3\text{ArcTan}(ax)^{7/2}}{28ac^3}-\frac{75\text{ArcTan}(ax)^{3/2}}{256ac^3}-\frac{15\sqrt{\text{ArcTan}(ax)}\sin(2\text{ArcTan}(ax))}{256ac^3}-\frac{15\sqrt{\text{ArcTan}(ax)}\sin(4\text{ArcTan}(ax))}{2048ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(128*c^3*(1 + a^2*x^2)) - (75*ArcTan[a*x]^(3/2))/
(256*a*c^3) + (5*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (15*ArcTan
[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)) + (x*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a
^2*x^2)^2) + (3*x*ArcTan[a*x]^(5/2))/(8*c^3*(1 + a^2*x^2)) + (3*ArcTan[a*x]
^(7/2))/(28*a*c^3) + (15*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/(4096*a*c^3) + (15*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(1
28*a*c^3) - (15*Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/(256*a*c^3) - (15*Sqr
t[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/(2048*a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5012

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c*(
p/2), Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(a +
 b*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2} - \frac{15}{64}\int \frac{\sqrt{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx + \frac{3\int \frac{\tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}(ax)^{7/2}}{28ac^3} - \frac{15\mathrm{Subst}(}{}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\tan^{-1}(ax)^{5/2}}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\tan^{-1}(ax)^{5/2}}{8c^3\left(1+a^2x^2\right)} + \frac{3\tan^{-1}}{28}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{128c^3\left(1+a^2x^2\right)} - \frac{75\tan^{-1}(ax)^{3/2}}{256ac^3} + \frac{5\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)^2} + \frac{15\tan^{-1}(ax)^{3/2}}{32ac^3\left(1+a^2x^2\right)} + \frac{x\,\mathrm{t}}{4c^3}$$

Mathematica [A]
time = 0.34, size = 149, normalized size = 0.50

$$\frac{105\sqrt{2\pi}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right) + 6720\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right) + 4\sqrt{\mathrm{ArcTan}(ax)}\left(1536\mathrm{ArcTan}(ax)^3 + 280\mathrm{ArcTan}(ax)(16\cos(2\mathrm{ArcTan}(ax)) + \cos(4\mathrm{ArcTan}(ax))) + 448\mathrm{ArcTan}(ax)^2(8\sin(2\mathrm{ArcTan}(ax)) + \sin(4\mathrm{ArcTan}(ax))) - 105(32\sin(2\mathrm{ArcTan}(ax)) + \sin(4\mathrm{ArcTan}(ax)))\right)}{57344ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

[Out] (105*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 6720*Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + 4*Sqrt[ArcTan[a*x]]*(1536*ArcTan[a
*x]^3 + 280*ArcTan[a*x]*(16*Cos[2*ArcTan[a*x]] + Cos[4*ArcTan[a*x]]) + 448*
ArcTan[a*x]^2*(8*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]]) - 105*(32*Sin[2*A
rcTan[a*x]] + Sin[4*ArcTan[a*x]])))/(57344*a*c^3)

Maple [A]
time = 0.35, size = 168, normalized size = 0.57

method	result
default	$6144 \arctan(ax)^{\frac{7}{2}} \sqrt{\pi} + 1792 \arctan(ax)^{\frac{5}{2}} \sin(4 \arctan(ax)) \sqrt{\pi} + 14336 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \sin(2 \arctan(ax)) + 1120 \arctan(ax)^{\frac{3}{2}}$ co

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/57344/c^3/a/Pi^(1/2)*(6144*arctan(a*x)^(7/2)*Pi^(1/2)+1792*arctan(a*x)^(5
/2)*sin(4*arctan(a*x))*Pi^(1/2)+14336*arctan(a*x)^(5/2)*Pi^(1/2)*sin(2*arct
an(a*x))+1120*arctan(a*x)^(3/2)*cos(4*arctan(a*x))*Pi^(1/2)+17920*arctan(a*
x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+105*Pi*FresnelS(2*2^(1/2)/Pi^(1/2)*arc
tan(a*x)^(1/2))*2^(1/2)-420*sin(4*arctan(a*x))*arctan(a*x)^(1/2)*Pi^(1/2)-1
3440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+6720*Pi*FresnelS(2*arcta
n(a*x)^(1/2)/Pi^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3,x)
```

```
[Out] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3, x)
```

3.877 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^3}, x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x(c+a^2cx^2)^3} dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$$

Mathematica [A]
time = 2.25, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

Maple [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{x(a^2c\,x^2+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c
**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3),x)
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.878 $\int x^m \sqrt{c+a^2cx^2} \text{ ArcTan}(ax)^{5/2} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m \sqrt{c+a^2cx^2} \text{ ArcTan}(ax)^{5/2}, x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{c+a^2cx^2} \text{ ArcTan}(ax)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^{5/2} \, dx = \int x^m \sqrt{c+a^2cx^2} \tan^{-1}(ax)^{5/2} \, dx$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int x^m \sqrt{c+a^2cx^2} \text{ ArcTan}(ax)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int x^m \arctan(ax)^{\frac{5}{2}} \sqrt{a^2 c\, x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a x)^{5/2} \sqrt{c a^2 x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.879 $\int x^2 \sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 2.55, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.19, size = 0, normalized size = 0.00

$$\int x^2\arctan(ax)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.880 $\int x\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{5\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{8a^2}-\frac{5x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{12a}+\frac{\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}{3a^2c}-\frac{5c\text{Int}\Big(\frac{}{\sqrt{c+}}$$

```
[Out] 1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/a^2/c-5/12*x*arctan(a*x)^(3/2)*(a
^2*c*x^2+c)^(1/2)/a+5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-5/12*c*Un
integrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a-5/16*c*Unintegrable(1
/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^2) - (5*x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^(3/2))/(12*a) + ((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/(3*
a^2*c) - (5*c*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(16
*a) - (5*c*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a)
```

Rubi steps

$$\int x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}\,dx=\frac{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}{3a^2c}-\frac{5\int\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}\,dx}{6a}$$

$$=\frac{5\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{8a^2}-\frac{5x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{12a}+\frac{(c+a^2cx}{}$$

Mathematica [A]
time = 3.49, size = 0, normalized size = 0.00

$$\int x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int x \arctan(ax)^{\frac{5}{2}} \sqrt{a^2 c\,x^2 + c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.881 $\int \sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=119

$$-\frac{5\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}+\frac{15}{8}c\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}},x\right)+\frac{1}{2}c\text{Int}\left(\frac{\text{ArcT}}{\sqrt{c}}\right.$$

```
[Out] -5/4*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^(5/2)*(a^2*c
*x^2+c)^(1/2)+1/2*c*Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)+1
5/8*c*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a) + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x]^(5/2))/2 + (15*c*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2
], x])/8 + (c*Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/2
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}\,dx=-\frac{5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}+\frac{1}{2}c\int\frac{\tan^{-}}{\sqrt{c}}$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \arctan(ax)^{\frac{5}{2}} \sqrt{a^2 c\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.882 $\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.883 $\int x^m(c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int x^m\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.884 $\int x^2(c+a^2cx^2)^{3/2}\,\mathbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 3.42, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.99, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a x)^{5/2} \left(c a^2 x^2 + c\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.885 $\int x(c+a^2cx^2)^{3/2}\,\mathbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=259

$$\frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{32a^2}+\frac{(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}{16a^2}-\frac{3cx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{16a}-\frac{x(c+a^2cx}{}$$

[Out] -1/8*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/a^2/c-3/16*c*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+1/16*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2+9/32*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-3/16*c^2*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a-9/64*c^2*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-1/32*c*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

[In] Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]

[Out] (9*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(32*a^2) + ((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(16*a^2) - (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(8*a) + ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/(5*a^2*c) - (9*c^2*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(64*a) - (c*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/(32*a) - (3*c^2*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(16*a)

Rubi steps

$$\int x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}\,dx=\frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}{5a^2c}-\frac{\int(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}\,dx}{2a}$$

$$=\frac{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}{16a^2}-\frac{x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{8a}+\frac{(c+a^2c}{}$$

$$=\frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{32a^2}+\frac{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}{16a^2}-\frac{3cx\sqrt{c-}}{}$$

Mathematica [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int x\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.886 $\int (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=212

$$-\frac{15c\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)^{3/2}}{16a}-\frac{5(c+a^2cx^2)^{3/2}\operatorname{ArcTan}(ax)^{3/2}}{24a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\operatorname{ArcTan}(ax)^{5/2}+\frac{1}{4}x(c+$$

```
[Out] -5/24*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arc
tan(a*x)^(5/2)-15/16*c*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arct
an(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)+3/8*c^2*Unintegrable(arctan(a*x)^(5/2)/(a
^2*c*x^2+c)^(1/2),x)+45/32*c^2*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)
^(1/2),x)+5/16*c*Unintegrable((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-15*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(16*a) - (5*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x]^(3/2))/(24*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/4 + (45*c^2*Defer[Int][
Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x])/32 + (5*c*Defer[Int][Sqrt[c + a^
2*c*x^2]*Sqrt[ArcTan[a*x]], x])/16 + (3*c^2*Defer[Int][ArcTan[a*x]^(5/2)/Sq
rt[c + a^2*c*x^2], x])/8
```

Rubi steps

$$\int (c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}\,dx = -\frac{5(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{24a}+\frac{1}{4}x(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}+\frac{1}{16}($$

$$= -\frac{15c\sqrt{c+a^2cx^2}\tan^{-1}(ax)^{3/2}}{16a}-\frac{5(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{24a}+\frac{3}{8}cx$$

Mathematica [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.887 $\int \frac{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x,x)
```

```
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.888 $\int x^m(c+a^2cx^2)^{5/2} \textbf{ArcTan}(ax)^{5/2} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2} \, dx = \int x^m \left(c+a^2cx^2\right)^{5/2} \tan^{-1}(ax)^{5/2} \, dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.88, size = 0, normalized size = 0.00

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arctan
(a*x)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(a x)^{5/2} \left(c a^2 x^2 + c\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.889 $\int x^2(c+a^2cx^2)^{5/2}\,\textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2},x\right)$$

```
[Out] Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx$$

Mathematica [A]
time = 2.47, size = 0, normalized size = 0.00

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.19, size = 0, normalized size = 0.00

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
[Out] int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.890 $\int x(c+a^2cx^2)^{5/2} \mathbf{ArcTan}(ax)^{5/2} dx$

Optimal. Leaf size=360

$$\frac{75c^2\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}{448a^2}+\frac{25c(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}{672a^2}+\frac{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}{56a^2}-\frac{25c^2}{}$$

[Out] -25/336*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a-5/84*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/a+1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(5/2)/a^2/c-25/224*c^2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+25/672*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2+1/56*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a^2+75/448*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-25/224*c^3*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a-1/112*c*Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-75/896*c^3*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-25/1344*c^2*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(c+a^2cx^2)^{5/2} \text{ArcTan}(ax)^{5/2} dx$$

Verification is not applicable to the result.

[In] Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]

[Out] (75*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(448*a^2) + (25*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])/(672*a^2) + ((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])/(56*a^2) - (25*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(224*a) - (25*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/(336*a) - (5*x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/(84*a) + ((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^(5/2))/(7*a^2*c) - (75*c^3*Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(896*a) - (25*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x])/(1344*a) - (c*Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x])/(112*a) - (25*c^3*Defer[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(224*a)

Rubi steps

$$\int x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}\,dx = \frac{(c+a^2cx^2)^{7/2}\tan^{-1}(ax)^{5/2}}{7a^2c} - \frac{5\int (c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}\,dx}{14a}$$

$$= \frac{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}{56a^2} - \frac{5x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}{84a} + \frac{(c+a^2}{}$$

$$= \frac{25c(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}{672a^2} + \frac{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}{56a^2} - \frac{25cx}{}$$

$$= \frac{75c^2\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}{448a^2} + \frac{25c(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}{672a^2} + \frac{(c}{}$$

Mathematica [A]
time = 4.17, size = 0, normalized size = 0.00

$$\int x(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
[Out] Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
[Out] int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.891 $\int (c+a^2cx^2)^{5/2} \textbf{ArcTan}(ax)^{5/2}\,dx$

Optimal. Leaf size=307

$$-\frac{25c^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{32a}-\frac{25c(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}{144a}-\frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}{12a}+\frac{5}{16}c^2x$$

```
[Out] -25/144*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a-1/12*(a^2*c*x^2+c)^(5/2)*
arctan(a*x)^(3/2)/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)+1/6*x*(a
^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)-25/32*c^2*arctan(a*x)^(3/2)*(a^2*c*x^2+
c)^(1/2)/a+5/16*c^2*x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)+5/16*c^3*Uninte
grable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)+1/8*c*Unintegrable((a^2*c*x
^2+c)^(3/2)*arctan(a*x)^(1/2),x)+75/64*c^3*Unintegrable(arctan(a*x)^(1/2)/(
a^2*c*x^2+c)^(1/2),x)+25/96*c^2*Unintegrable((a^2*c*x^2+c)^(1/2)*arctan(a*x
)^(1/2),x)
```

Rubi [A]

time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] (-25*c^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(32*a) - (25*c*(c + a^2*c*x
^2)^(3/2)*ArcTan[a*x]^(3/2))/(144*a) - ((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(
3/2))/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/16 + (5*c*x*
(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/24 + (x*(c + a^2*c*x^2)^(5/2)*ArcT
an[a*x]^(5/2))/6 + (75*c^3*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2]
, x])/64 + (25*c^2*Defer[Int][Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x])/96
 + (c*Defer[Int][(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x])/8 + (5*c^3*De
fer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x])/16
```

Rubi steps

$$\begin{aligned}\int \left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}\,dx &= -\frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}{12a}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}+\frac{1}{8}c\int (\\ &= -\frac{25c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{144a}-\frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}{12a}+\frac{5}{24}cx(\\ &= -\frac{25c^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{32a}-\frac{25c(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}{144a}-(c\end{aligned}$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int (c+a^2cx^2)^{5/2} \operatorname{ArcTan}(ax)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.892 $\int \frac{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^{5/2}}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Defer[Int][((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx=\int \frac{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}{x}\,dx$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{5/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
[Out] Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

Maple [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x,x)
[Out] int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.893 $\int \frac{x^m \mathbf{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x^m \operatorname{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \operatorname{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.894 $\int \frac{x^3 \text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=199

$$\frac{5\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{8a^4c}-\frac{5x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{12a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}$$

```
[Out] -5/12*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^3/c-2/3*arctan(a*x)^(5/2)*(
a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a^2/
c+5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c+25/12*Unintegrable(arctan
(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a^3-5/16*Unintegrable(1/(a^2*c*x^2+c)^(1
/2)/arctan(a*x)^(1/2),x)/a^3
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3 \text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (5*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*a^4*c) - (5*x*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^(3/2))/(12*a^3*c) - (2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2
))/(3*a^4*c) + (x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*a^2*c) - (5*D
efer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/(16*a^3) + (25*Def
er[Int][ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x])/(12*a^3)
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{3a^2c}-\frac{2\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx}{3a^2}-\frac{5\int \frac{x^2\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{6a}$$

$$=-\frac{5x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{12a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\,\tan}{3a^2c}$$

$$=\frac{5\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{8a^4c}-\frac{5x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{12a^3c}-\frac{2\sqrt{c+a^2cx^2}\,\tan^{-1}(}{3a^4c}$$

Mathematica [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{x^3\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 5.89, size = 0, normalized size = 0.00

$$\int \frac{x^3\arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.895 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{5\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{2a^2c}+\frac{15\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{8a^2}-\frac{\text{Int}\left(\frac{\text{ArcTan}(ax)}{\sqrt{c+a^2cx}}\right.}{2a^2}$$

```
[Out] -5/4*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^(5/2)*(a
^2*c*x^2+c)^(1/2)/a^2/c-1/2*Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1
/2),x)/a^2+15/8*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)/a^2
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (-5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*a^3*c) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^(5/2))/(2*a^2*c) + (15*Defer[Int][Sqrt[ArcTan[a*x]]/Sqrt[c +
 a^2*c*x^2], x])/(8*a^2) - Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2]
, x]/(2*a^2)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}-\frac{5\int \frac{x\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{4a}\\ &= -\frac{5\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{4a^3c}+\frac{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{2a^2c}-\frac{\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a^2}+\end{aligned}$$

Mathematica [A]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 5.89, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \arctan \left(a x\right)^{\frac{5}{2}}}{\sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.896 $\int \frac{x\mathbf{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{5/2}}{a^2c}-\frac{5\mathrm{Int}\left(\frac{\mathrm{ArcTan}(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a}$$

```
[Out] arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-5/2*Unintegrable(arctan(a*x)^(3
/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\mathrm{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(a^2*c) - (5*Defer[Int][ArcTan[a*x]
^(3/2)/Sqrt[c + a^2*c*x^2], x])/(2*a)
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{a^2c}-\frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx}{2a}$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x\mathrm{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.897 $\int \frac{\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
[Out] Defer[Int][ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.898 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\sqrt{a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.899 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{cx}+\frac{5}{2}a\text{Int}\left(\frac{\text{ArcTan}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

```
[Out] -arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/c/x+5/2*a*Unintegrable(arctan(a*x)^(
3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(c*x)) + (5*a*Defer[Int][ArcTan[a
*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/2
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx=-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{cx}+\frac{1}{2}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{x^2\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.900 $\int \frac{\mathbf{ArcTan}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{5a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{4cx}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{2cx^2}+\frac{15}{8}a^2\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x\sqrt{c+a^2cx^2}},x\right)-\frac{1}{2}a^2\text{Int}\left(\frac{\text{Arc}}{x\sqrt{}}\right.$$

```
[Out] -5/4*a*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x-1/2*arctan(a*x)^(5/2)*(a^2
*c*x^2+c)^(1/2)/c/x^2-1/2*a^2*Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c
)^(1/2),x)+15/8*a^2*Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-5*a*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/(4*c*x) - (Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^(5/2))/(2*c*x^2) + (15*a^2*Defer[Int][Sqrt[ArcTan[a*x]]/(x*Sqr
t[c + a^2*c*x^2]), x])/8 - (a^2*Defer[Int][ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^
2*c*x^2]), x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{2cx^2}+\frac{1}{4}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}} \\ &= -\frac{5a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{4cx}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{2cx^2}-\frac{1}{2}a^2\int \frac{\tan^{-1}(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 3.55, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 2.26, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(a x\right)^{\frac{5}{2}}}{x^{3} \sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.901 $\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=208

$$-\frac{5a^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}{8cx}-\frac{5a\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}{12cx^2}-\frac{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2}}{}$$

```
[Out] -5/12*a*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*arctan(a*x)^(5/2)*(
a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/c/x-
5/8*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x-25/12*a^3*Unintegrable(ar
ctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)+5/16*a^3*Unintegrable(1/x/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]

time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-5*a^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])/(8*c*x) - (5*a*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x]^(3/2))/(12*c*x^2) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/
2))/(3*c*x^3) + (2*a^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/(3*c*x) + (5*
a^3*Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/16 - (25*a^
3*Defer[Int][ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x])/12
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{3cx^3}+\frac{1}{6}(5a)\int \frac{\tan^{-1}(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx-\frac{1}{3}(2a^2)\int \frac{\tan^{-1}(ax)^{5/}}{x^2\sqrt{c+a^2cx}}\\ &= -\frac{5a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{12cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\,\tan^{-1}}{3cx}\\ &= -\frac{5a^2\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}{8cx}-\frac{5a\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}{12cx^2}-\frac{\sqrt{c+a^2cx^2}\,\tan^{-1}}{3cx^3}\end{aligned}$$

Mathematica [A]
time = 11.88, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
[Out] Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

Maple [A]
time = 5.42, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{x^4\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
[Out] int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.902 $\int \frac{x^m \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.903 $\int \frac{x^2 \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^2\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Mathematica [A]
time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A]
time = 5.81, size = 0, normalized size = 0.00

$$\int \frac{x^2\arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.904 $\int \frac{x\mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{15\sqrt{\text{ArcTan}(ax)}}{4a^2c\sqrt{c+a^2cx^2}}+\frac{5x\text{ArcTan}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}}-\frac{\text{ArcTan}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax}\right.}{4a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] 5/2*x*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(5/2)/a^2/c/(a^
2*c*x^2+c)^(1/2)-15/8*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*
Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)+15/4*arctan(a*x)^(1/2)
/a^2/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5050, 5018, 5025, 5024, 3385, 3433}

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4a^2c\sqrt{a^2cx^2+c}}-\frac{\text{ArcTan}(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}+\frac{5x\text{ArcTan}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}+\frac{15\sqrt{\text{ArcTan}(ax)}}{4a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (15*Sqrt[ArcTan[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/
2))/(2*a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]^(5/2)/(a^2*c*Sqrt[c + a^2*c*x
^2]) - (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/(4*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
```

```
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx}{2a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(}}}{8a}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\int \frac{}{(1+a^2}}{8ac\sqrt{c+}}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\Big(}{8a^2c\sqrt{c}}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\Big(}{4a^2c}$$

$$= \frac{15\sqrt{\tan^{-1}(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\tan^{-1}(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\Big(\sqrt{}}{4a^2c\sqrt{c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 139, normalized size = 0.86

$$\frac{4\text{ArcTan}(ax)\left(15+10ax\text{ArcTan}(ax)-4\text{ArcTan}(ax)^2\right)+15i\sqrt{1+a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-15i\sqrt{1+a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)}{16a^2c\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (4*ArcTan[a*x]*(15 + 10*a*x*ArcTan[a*x] - 4*ArcTan[a*x]^2) + (15*I)*Sqrt[1
+ a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (15*I)*Sqr
t[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(16*a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x\arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.905 $\int \frac{\mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{15x\sqrt{\mathrm{ArcTan}(ax)}}{4c\sqrt{c+a^2cx^2}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\mathrm{ArcTan}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}+\frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

```
[Out] 5/2*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(5/2)/c/(a^2*c*
x^2+c)^(1/2)+15/8*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(
1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-15/4*x*arctan(a*x)^(1/2)/c/(
a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5018, 5025, 5024, 3377, 3386, 3432}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\mathrm{ArcTan}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}-\frac{15x\sqrt{\mathrm{ArcTan}(ax)}}{4c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-15*x*Sqrt[ArcTan[a*x]])/(4*c*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/2))
/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]
) + (15*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
)/(4*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{15}{4}\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\int \frac{\sqrt{\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{4c\sqrt{c+a^2cx^2}}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} - \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sqrt{x}\,\cos(x)\,dx, x, \tan^{-}\right.}{4ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\right.}{8ac\sqrt{c+}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{\left(15\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\right.}{4ac\sqrt{}}$$

$$= -\frac{15x\sqrt{\tan^{-1}(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{}\right.}{4ac\sqrt{c+a^2}}$$

Mathematica [A]
time = 0.07, size = 97, normalized size = 0.63

$$\frac{2\sqrt{\text{ArcTan}(ax)}\left(-15ax+10\text{ArcTan}(ax)+4ax\text{ArcTan}(ax)^2\right)+15\sqrt{2\pi}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{8ac\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[ArcTan[a*x]]*(-15*a*x + 10*ArcTan[a*x] + 4*a*x*ArcTan[a*x]^2) + 15*
Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a*c
*Sqrt[c + a^2*c*x^2])
```

Maple [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.906 $\int \frac{\operatorname{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{\operatorname{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

Maple [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.907 $\int \frac{x^m \textbf{ArcTan}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \text{ArcTan}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Defer[Int][(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx = \int \frac{x^m \tan^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{x^m \text{ArcTan}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.908 $\int \frac{x^4 \mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^4\,\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\,\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Mathematica [A]
time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A]
time = 8.68, size = 0, normalized size = 0.00

$$\int \frac{x^4\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4 \operatorname{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.909 $\int \frac{x^3 \mathbf{ArcTan}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=350

$$\frac{45\sqrt{\text{ArcTan}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}}+\frac{5x^3\text{ArcTan}(ax)^{3/2}}{18ac\left(c+a^2cx^2\right)^{3/2}}+\frac{5x\text{ArcTan}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\text{ArcTan}(ax)^{5/2}}{3a^2c\left(c+a^2cx^2\right)^{3/2}}-\frac{2\text{ArcTan}(ax)^{5/2}}{3a^4c^2\sqrt{c+a^2cx^2}}-5\sqrt{1}$$

```
[Out] 5/18*x^3*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)^(5/2
)/a^2/c/(a^2*c*x^2+c)^(3/2)+5/3*x*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)^(
1/2)-2/3*arctan(a*x)^(5/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+5/864*FresnelC(6^(1/
2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(
a^2*c*x^2+c)^(1/2)-45/32*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/
2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+45/16*arctan(a*x)
^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-5/144*cos(3*arctan(a*x))*(a^2*x^2+1)^(1/
2)*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5060, 5050, 5018, 5025, 5024, 3385, 3433, 5091, 5090, 3393, 3377}

$$-\frac{x^2\text{ArcTan}(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\text{ArcTan}(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}-\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{16a^4c^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{144a^4c^2\sqrt{a^2cx^2+c}}-\frac{2\text{ArcTan}(ax)^{5/2}}{3a^4c^2\sqrt{a^2cx^2+c}}+\frac{45\sqrt{\text{ArcTan}(ax)}}{16a^4c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{a^2x^2+1}\sqrt{\text{ArcTan}(ax)}\cos(3\text{ArcTan}(ax))}{144a^4c^2\sqrt{a^2cx^2+c}}+\frac{5x\text{ArcTan}(ax)^{3/2}}{3a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (45*Sqrt[ArcTan[a*x]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^3*ArcTan[a*x
]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(3*a^3*c^
2*Sqrt[c + a^2*c*x^2]) - (x^2*ArcTan[a*x]^(5/2))/(3*a^2*c*(c + a^2*c*x^2)^(
3/2)) - (2*ArcTan[a*x]^(5/2))/(3*a^4*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 +
 a^2*x^2]*Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/(144*a^4*c^2*Sqrt[c + a^2*c
*x^2]) - (45*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/(16*a^4*c^2*Sqrt[c + a^2*c*x^2]) + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*F
resnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(144*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{5}{12}\int \frac{x^3\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx + \frac{2\int \frac{x\tan^{-1}}{(c+a^2c}}{3a}$$

$$= \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\tan^{-1}(ax)^{5/2}}{3a^4c^2\sqrt{c+a^2cx^2}} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx}{3a^3c}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/}}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/}}{3a^2c\,(c+a^2cx^2)}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/}}{3a^2c\,(c+a^2cx^2)}$$

$$= \frac{45\sqrt{\tan^{-1}(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\tan^{-1}(ax)^{5/}}{3a^2c\,(c+a^2cx^2)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 370, normalized size = 1.06

$$\frac{4800\text{ArcTan}(ax)+5040a^2x^2\text{ArcTan}(ax)+2880ax\text{ArcTan}(ax)^2+3360a^3x^3\text{ArcTan}(ax)^2-1152\text{ArcTan}(ax)^3-1728a^2x^2\text{ArcTan}(ax)^3+1215i(1+a^2x^2)^{3/2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-1215i(1+a^2x^2)^{3/2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)-5i\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-5ia^2x^2\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+5i\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)+5ia^2x^2\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)}{1728a^4c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (4800*ArcTan[a*x] + 5040*a^2*x^2*ArcTan[a*x] + 2880*a*x*ArcTan[a*x]^2 + 336
0*a^3*x^3*ArcTan[a*x]^2 - 1152*ArcTan[a*x]^3 - 1728*a^2*x^2*ArcTan[a*x]^3 +
 (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] - (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcT
```

```
an[a*x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*
I)*ArcTan[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(1728*a^4*c^2*(1 + a^2*x^2)*S
qrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 6.10, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{(a^2 c x^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
[Out] int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.910 $\int \frac{x^2\text{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{5x^3\sqrt{\text{ArcTan}(ax)}}{36c\,(c+a^2cx^2)^{3/2}}-\frac{5x\sqrt{\text{ArcTan}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x^2\text{ArcTan}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}}+\frac{5\text{ArcTan}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\text{ArcTan}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{15\sqrt{}}{}$$

```
[Out] 5/18*x^2*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)^(5/2
)/c/(a^2*c*x^2+c)^(3/2)+5/9*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-5
/864*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+15/32*FresnelS(2^(1/2)/Pi^(1/2)*arcta
n(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2
)-5/36*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)-5/6*x*arctan(a*x)^(1/2)/
a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5064, 5060, 5050, 5025, 5024, 3377, 3386, 3432, 5091, 5090, 3393}

$$-\frac{5x\sqrt{\text{ArcTan}(ax)}}{6a^2c^2\sqrt{a^2cx^2+c}}+\frac{5x^2\text{ArcTan}(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}+\frac{x^3\text{ArcTan}(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{5x^3\sqrt{\text{ArcTan}(ax)}}{36c\,(a^2cx^2+c)^{3/2}}+\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{16a^3c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{144a^3c^2\sqrt{a^2cx^2+c}}+\frac{5\text{ArcTan}(ax)^{3/2}}{9a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-5*x^3*Sqrt[ArcTan[a*x]])/(36*c*(c + a^2*c*x^2)^(3/2)) - (5*x*Sqrt[ArcTan[
a*x]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x^2*ArcTan[a*x]^(3/2))/(18*a*c*
(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(9*a^3*c^2*Sqrt[c + a^2*c*x^
2]) + (x^3*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (15*Sqrt[Pi/2]*
Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^3*c^2*Sqrt[
c + a^2*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[
ArcTan[a*x]]])/(144*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5060

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x
)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((
p - 1)/m^2), Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]
- Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)
), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q +
 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2 \tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3 \tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{1}{6}(5a)\int \frac{x^3 \tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{1}{72}(5a)\int \frac{}{(c+a^2}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx}}$$

$$= -\frac{5x^3\sqrt{\tan^{-1}(ax)}}{36c\,(c+a^2cx^2)^{3/2}} - \frac{5x\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x^2\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.77, size = 287, normalized size = 0.97

$$\frac{-24\text{ArcTan}(ax)\left(5ax(6+7a^2x^2)-10(2+3a^2x^2)\text{ArcTan}(ax)-12a^3x^3\text{ArcTan}(ax)^2\right)+35\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\sqrt{\text{ArcTan}(ax)}\left(3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)-S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)\right)-15(1+a^2x^2)^{3/2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{864a^3c\,(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-24*ArcTan[a*x]*(5*a*x*(6 + 7*a^2*x^2) - 10*(2 + 3*a^2*x^2)*ArcTan[a*x] -
12*a^3*x^3*ArcTan[a*x]^2) + 35*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a
*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi
]*Sqrt[ArcTan[a*x]]]) - 15*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Ga
mma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]
```

```
] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I
*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(864*a^3*c*(c + a^2*c*x^2)^(
3/2)*Sqrt[ArcTan[a*x]])
```

Maple [**F**]
time = 5.91, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \arctan \left(a x \right)^{\frac{5}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.911 $\int \frac{x\mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{5\sqrt{\mathrm{ArcTan}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}}+\frac{5\sqrt{\mathrm{ArcTan}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}+\frac{5x\mathrm{ArcTan}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}}+\frac{5x\mathrm{ArcTan}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}-\frac{\mathrm{ArcTan}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{15\sqrt{}}{}$$

```
[Out] 5/18*x*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)^(5/2)/a^2/
c/(a^2*c*x^2+c)^(3/2)+5/9*x*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-5/8
64*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-15/32*FresnelC(2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+
5/36*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)+5/6*arctan(a*x)^(1/2)/a^2/
c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5050, 5020, 5018, 5025, 5024, 3385, 3433, 3393}

$$-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{16a^2c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{144a^2c^2\sqrt{a^2cx^2+c}}+\frac{5x\mathrm{ArcTan}(ax)^{3/2}}{9ac^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\mathrm{ArcTan}(ax)}}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\mathrm{ArcTan}(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x\mathrm{ArcTan}(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}+\frac{5\sqrt{\mathrm{ArcTan}(ax)}}{36a^2c\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (5*Sqrt[ArcTan[a*x]])/(36*a^2*c*(c + a^2*c*x^2)^(3/2)) + (5*Sqrt[ArcTan[a*x
]])/(6*a^2*c^2*Sqrt[c + a^2*c*x^2]) + (5*x*ArcTan[a*x]^(3/2))/(18*a*c*(c +
a^2*c*x^2)^(3/2)) + (5*x*ArcTan[a*x]^(3/2))/(9*a*c^2*Sqrt[c + a^2*c*x^2]) -
 ArcTan[a*x]^(5/2)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (15*Sqrt[Pi/2]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a^2*c^2*Sqrt[c + a^2
*c*x^2]) - (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[
a*x]]])/(144*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\int \frac{\tan^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx}{6a}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} - \frac{\tan^{-1}(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{5\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{t}}}}{72a}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{5\sqrt{\tan^{-1}(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\tan^{-1}(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\tan^{-1}(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 356, normalized size = 1.22

$$\frac{1680\text{ArcTan}(ax) + 1440a^2x^2\text{ArcTan}(ax) + 1440ax\text{ArcTan}(ax)^2 + 960a^3x^3\text{ArcTan}(ax)^2 - 576\text{ArcTan}(ax)^3 + 405i(1+a^2x^2)^{3/2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -i\text{ArcTan}(ax)\right) - 405i(1+a^2x^2)^{3/2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, i\text{ArcTan}(ax)\right) + 5i\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -3i\text{ArcTan}(ax)\right) + 5ia^2x^2\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -3i\text{ArcTan}(ax)\right) - 5i\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 3i\text{ArcTan}(ax)\right) - 5ia^2x^2\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 3i\text{ArcTan}(ax)\right)}{1728a^2c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (1680*ArcTan[a*x] + 1440*a^2*x^2*ArcTan[a*x] + 1440*a*x*ArcTan[a*x]^2 + 960
*a^3*x^3*ArcTan[a*x]^2 - 576*ArcTan[a*x]^3 + (405*I)*(1 + a^2*x^2)^(3/2)*Sq
```

```
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (405*I)*(1 + a^2*x^2)^(
3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x
^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*S
qrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] -
(5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]
 - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*A
rcTan[a*x]])/(1728*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]])
```

Maple [F]
time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
[Out] int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] sage0*x

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)

[Out] int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)

3.912 $\int \frac{\mathbf{ArcTan}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=337

$$-\frac{45x\sqrt{\mathrm{ArcTan}(ax)}}{16c^2\sqrt{c+a^2cx^2}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\mathrm{ArcTan}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\mathrm{ArcTan}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}}+\frac{45\sqrt{}}{}$$

```
[Out] 5/18*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)^(5/2)/c/(a
^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arc
tan(a*x)^(5/2)/c^2/(a^2*c*x^2+c)^(1/2)+5/864*FresnelS(6^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
+45/32*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-45/16*x*arctan(a*x)^(1/2)/c^2/(a^2*c*
x^2+c)^(1/2)-5/144*sin(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)/a
/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5020, 5018, 5025, 5024, 3377, 3386, 3432, 3393}

$$\frac{45\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{16ac^2\sqrt{a^2cx^2+c}}+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{144ac^2\sqrt{a^2cx^2+c}}+\frac{2x\mathrm{ArcTan}(ax)^{5/2}}{3c^2\sqrt{a^2cx^2+c}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{3ac^2\sqrt{a^2cx^2+c}}-\frac{45x\sqrt{\mathrm{ArcTan}(ax)}}{16c^2\sqrt{a^2cx^2+c}}-\frac{5\sqrt{a^2x^2+1}\sqrt{\mathrm{ArcTan}(ax)}\sin(3\mathrm{ArcTan}(ax))}{144ac^2\sqrt{a^2cx^2+c}}+\frac{x\mathrm{ArcTan}(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\mathrm{ArcTan}(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (-45*x*Sqrt[ArcTan[a*x]])/(16*c^2*Sqrt[c + a^2*c*x^2]) + (5*ArcTan[a*x]^(3/
2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (5*ArcTan[a*x]^(3/2))/(3*a*c^2*Sqrt[c
 + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Ar
cTan[a*x]^(5/2))/(3*c^2*Sqrt[c + a^2*c*x^2]) + (45*Sqrt[Pi/2]*Sqrt[1 + a^2*
x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(16*a*c^2*Sqrt[c + a^2*c*x^2])
 + (5*Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/
(144*a*c^2*Sqrt[c + a^2*c*x^2]) - (5*Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]*Si
n[3*ArcTan[a*x]])/(144*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5018

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (-Dist[b^2*p*(p - 1), Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2),
 x], x] + Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

Rule 5020

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a
 + b*ArcTan[c*x])^p, x], x] - Dist[b^2*p*((p - 1)/(4*(q + 1)^2)), Int[(d +
e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x] - Simp[x*(d + e*x^2)^(q + 1)*(
(a + b*ArcTan[c*x])^p/(2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{5}{12}\int \frac{\sqrt{\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx + \frac{2\int \frac{\tan^{-1}(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,d}{3c}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{5}{}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{(5}{}$$

$$= \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\tan^{-1}(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} - \frac{(5}{}$$

$$= -\frac{5x\sqrt{\tan^{-1}(ax)}}{2c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} +$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} +$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} +$$

$$= -\frac{45x\sqrt{\tan^{-1}(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\tan^{-1}(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\tan^{-1}(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\tan^{-1}(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} +$$

Mathematica [A]
time = 0.19, size = 176, normalized size = 0.52

$$\frac{24\sqrt{\text{ArcTan}(ax)}\left(-5ax(21+20a^2x^2)+10(7+6a^2x^2)\,\text{ArcTan}(ax)+12ax(3+2a^2x^2)\,\text{ArcTan}(ax)^2\right)+1215\sqrt{2\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+5\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{864c^2\,(a+a^3x^2)\sqrt{c+a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (24*Sqrt[ArcTan[a*x]]*(-5*a*x*(21 + 20*a^2*x^2) + 10*(7 + 6*a^2*x^2)*ArcTan
[a*x] + 12*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2) + 1215*Sqrt[2*Pi]*(1 + a^2*x^
```

```
2)^(3/2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 5*Sqrt[6*Pi]*(1 + a^2*x^2
)^(3/2)*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(864*c^2*(a + a^3*x^2)*Sqrt
[c + a^2*c*x^2])
```

Maple [**F**]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.913 $\int \frac{\mathbf{ArcTan}(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\tan^{-1}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

Maple [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atan}(a\,x)^{5/2}}{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.914 $\int \frac{x^m(c+a^2cx^2)}{\sqrt{\text{ArcTan}(ax)}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)}{\sqrt{\text{ArcTan}(ax)}}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\text{ArcTan}(ax)}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}} dx = \int \frac{x^m(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}} dx$$

Mathematica [A]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\text{ArcTan}(ax)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\sqrt{\arctan(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m/sqrt(atan(a*
x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

3.915 $\int \frac{x(c+a^2cx^2)}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{x(c+a^2cx^2)}{\sqrt{\operatorname{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x(c+a^2cx^2)}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{a^{2} x^{3}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(x/sqrt(atan(a*x)), x) + Integral(a**2*x**3/sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(c\,a^2\,x^2+c)}{\sqrt{\operatorname{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
[Out] int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

3.916 $\int \frac{c+a^2cx^2}{\sqrt{\text{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{c+a^2cx^2}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{\sqrt{\arctan\left(ax\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(a**2*x**2/sqrt(atan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2+c}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)/atan(a*x)^(1/2), x)
```

3.917 $\int \frac{c+a^2cx^2}{x\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{c+a^2cx^2}{x\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{c+a^2cx^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{x\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(1/2),x)
```

```
[Out] c*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(a**2*x/sqrt(atan(a*x)), x)
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)), x)
```

3.918 $\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^2}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{x^{m}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx+\int\frac{2a^{2}x^{2}x^{m}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx+\int\frac{a^{4}x^{4}x^{m}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m/sqrt(at
an(a*x)), x) + Integral(a**4*x**4*x**m/sqrt(atan(a*x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

3.919 $\int \frac{x(c+a^2cx^2)^2}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{x}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx+\int\frac{2a^{2}x^{3}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx+\int\frac{a^{4}x^{5}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(x/sqrt(atan(a*x)), x) + Integral(2*a**2*x**3/sqrt(atan(a*x))
, x) + Integral(a**4*x**5/sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

3.920 $\int \frac{(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{2a^{2}x^{2}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{a^{4}x^{4}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{1}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/sqrt(atan(a*x)), x) + Integral(a**4*x**4/sqrt(at
an(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x)^(1/2),x)
```

```
[Out] int((c + a^2*c*x^2)^2/atan(a*x)^(1/2), x)
```

3.921 $\int \frac{(c+a^2cx^2)^2}{x\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{(c+a^2cx^2)^2}{x\sqrt{\operatorname{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{2a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(1/2),x)
```

```
[Out] c**2*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(2*a**2*x/sqrt(atan(a*x)
), x) + Integral(a**4*x**3/sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)), x)
```

3.922 $\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

[In] Int[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]

Rubi steps

$$\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]

[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]

Maple [A]
time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(a^2 c\,x^2+c\right)^3}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/sqrt(arcta
n(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{3a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{3a^4x^4x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{a^6x^6x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(3*a**2*x**2*x**m/sqrt(at
an(a*x)), x) + Integral(3*a**4*x**4*x**m/sqrt(atan(a*x)), x) + Integral(a**
6*x**6*x**m/sqrt(atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

3.923 $\int \frac{x(c+a^2cx^2)^3}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{3a^{2}x^{3}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{3a^{4}x^{5}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx + \int \frac{a^{6}x^{7}}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(x/sqrt(atan(a*x)), x) + Integral(3*a**2*x**3/sqrt(atan(a*x))
, x) + Integral(3*a**4*x**5/sqrt(atan(a*x)), x) + Integral(a**6*x**7/sqrt(a
tan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

3.924 $\int \frac{(c+a^2cx^2)^3}{\sqrt{\text{ArcTan}(ax)}} dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\text{ArcTan}(ax)}} dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]

[Out] Defer[Int][(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}} dx = \int \frac{(c+a^2cx^2)^3}{\sqrt{\tan^{-1}(ax)}} dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\text{ArcTan}(ax)}} dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]

[Out] Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]

Maple [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\sqrt{\arctan(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{3a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{3a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{a^6x^6}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int\frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/sqrt(atan(a*x)), x) + Integral(3*a**4*x**4/sqrt(
atan(a*x)), x) + Integral(a**6*x**6/sqrt(atan(a*x)), x) + Integral(1/sqrt(a
tan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x)^(1/2),x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x)^(1/2), x)
```

3.925 $\int \frac{(c+a^2cx^2)^3}{x\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{x\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{3a^{2}x}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{3a^{4}x^{3}}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{a^{6}x^{5}}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(1/2),x)
```

```
[Out] c**3*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(3*a**2*x/sqrt(atan(a*x)
), x) + Integral(3*a**4*x**3/sqrt(atan(a*x)), x) + Integral(a**6*x**5/sqrt(
atan(a*x)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)), x)
```

3.926 $\int \frac{x^m}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.927 $\int \frac{x}{(c+a^2cx^2)\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=37

$$\frac{2x\sqrt{\text{ArcTan}(ax)}}{ac}-\frac{2\text{Int}\left(\sqrt{\text{ArcTan}(ax)},x\right)}{ac}$$

```
[Out] 2*x*arctan(a*x)^(1/2)/a/c-2*Unintegrable(arctan(a*x)^(1/2),x)/a/c
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] (2*x*Sqrt[ArcTan[a*x]])/(a*c) - (2*Defer[Int][Sqrt[ArcTan[a*x]], x])/(a*c)
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx=\frac{2x\sqrt{\tan^{-1}(ax)}}{ac}-\frac{2\int\sqrt{\tan^{-1}(ax)}\,dx}{ac}$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.928 $\int \frac{1}{(c+a^2cx^2)\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{\text{ArcTan}(ax)}}{ac}$$

[Out] 2*arctan(a*x)^(1/2)/a/c

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5004\}$

$$\frac{2\sqrt{\text{ArcTan}(ax)}}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]

[Out] (2*Sqrt[ArcTan[a*x]])/(a*c)

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\tan^{-1}(ax)}}\,dx = \frac{2\sqrt{\tan^{-1}(ax)}}{ac}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{2\sqrt{\text{ArcTan}(ax)}}{ac}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]

```
[Out] (2*Sqrt[ArcTan[a*x]])/(a*c)
```

Maple [A]
time = 0.13, size = 15, normalized size = 0.94

method	result	size
default	$\frac{2\sqrt{\arctan\left(a x\right)}}{a c}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(a*x)^(1/2)/a/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.96, size = 14, normalized size = 0.88

$$\frac{2\sqrt{\arctan\left(ax\right)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(arctan(a*x))/(a*c)
```

Sympy [A]
time = 0.76, size = 12, normalized size = 0.75

$$\frac{2\sqrt{\operatorname{atan}\left(ax\right)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] 2*sqrt(atan(a*x))/(a*c)
```

Giac [A]
time = 0.44, size = 14, normalized size = 0.88

$$\frac{2\sqrt{\arctan(ax)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(arctan(a*x))/(a*c)
```

Mupad [B]
time = 0.34, size = 14, normalized size = 0.88

$$\frac{2\sqrt{\operatorname{atan}(a\,x)}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
[Out] (2*atan(a*x)^(1/2))/(a*c)
```

3.929 $\int \frac{1}{x(c+a^2cx^2)\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2}x^{3}\sqrt{\operatorname{atan}\left(ax\right)}+x\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.930 $\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(arctan(a*x))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sq
rt(atan(a*x))), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.931 $\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sq
rt(atan(a*x))), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.932 $\int \frac{x^2}{(c+a^2cx^2)^2 \sqrt{\mathbf{ArcTan}(ax)}} dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\text{ArcTan}(ax)}}{a^3c^2} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

```
[Out] -1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+arctan(a*x)^(1
/2)/a^3/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5090, 3393, 3385, 3433}

$$\frac{\sqrt{\text{ArcTan}(ax)}}{a^3c^2} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(a^3*c^2) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt
[Pi]])/(2*a^3*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}-\frac{\cos(2x)}{2\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^2}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{a^3c^2}-\frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 122, normalized size = 2.60

$$\frac{16\text{ArcTan}(ax)-4\sqrt{\pi}\sqrt{\text{ArcTan}(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)-i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{16a^3c^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (16*ArcTan[a*x] - 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]
])/Sqrt[Pi]] + I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*
x]] - I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a^3*
c^2*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.20, size = 38, normalized size = 0.81

method	result	size
default	$-\frac{\pi \,\mathrm{FresnelC}\left(\frac{2 \sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)-2 \sqrt{\arctan \left(a x \right)}\, \sqrt{\pi}}{2 c^{2} a^{3} \sqrt{\pi}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2/a^3/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-2*arctan(a
*x)^(1/2)*Pi^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}}{a^{4} x^{4} \sqrt{\operatorname{atan}{\left(a x \right)}} + 2 a^{2} x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}} + \sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sq
rt(atan(a*x))), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.933 $\int \frac{x}{(c+a^2cx^2)^2\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

```
[Out] 1/2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5090, 4491, 12, 3386, 3432}

$$\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^2c^2}$$
$$= \frac{\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^2c^2}$$
$$= \frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

Maple [A]
time = 0.14, size = 24, normalized size = 0.77

method	result	size
default	$\frac{\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}}{2a^2c^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt(
atan(a*x))), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.934 $\int \frac{1}{(c+a^2cx^2)^2 \sqrt{\mathbf{ArcTan}(ax)}} dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\text{ArcTan}(ax)}}{ac^2} + \frac{\sqrt{\pi}\ \text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}$$

```
[Out] 1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2+arctan(a*x)^(1/2)
/a/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5024, 3393, 3385, 3433}

$$\frac{\sqrt{\pi}\ \text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2ac^2} + \frac{\sqrt{\text{ArcTan}(ax)}}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(a*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/(2*a*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
```

```
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2ac^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2} + \frac{\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^2}$$

$$= \frac{\sqrt{\tan^{-1}(ax)}}{ac^2} + \frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.91

$$\frac{2\sqrt{\text{ArcTan}(ax)}+\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] (2*Sqrt[ArcTan[a*x]] + Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(
2*a*c^2)
```

Maple [A]
time = 0.20, size = 38, normalized size = 0.81

method	result	size

default	$\frac{\pi \,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+2\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}}{2c^{2}a\sqrt{\pi}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2/a/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*arctan(a*x)
^(1/2)*Pi^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{4}\sqrt{\operatorname{atan}\left(ax\right)}+2a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}+\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt(
atan(a*x))), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.935 $\int \frac{1}{x(c+a^2cx^2)^2 \sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^5\sqrt{\operatorname{atan}(ax)}+2a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**4*x**5*sqrt(atan(a*x)) + 2*a**2*x**3*sqrt(atan(a*x)) + x*sqr
t(atan(a*x))), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.936 $\int \frac{x^m}{(c+a^2cx^2)^3 \sqrt{\text{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \sqrt{\text{ArcTan}(ax)}}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3 \sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3 \sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*sqrt(arct
an(a*x))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.937 $\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a^2c\,x^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**5/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.938 $\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=89

$$\frac{3\sqrt{\text{ArcTan}(ax)}}{4a^5c^3}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}$$

```
[Out] 1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^
3-1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^5/c^3+3/4*arctan(a*
x)^(1/2)/a^5/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5090, 3393, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}+\frac{3\sqrt{\text{ArcTan}(ax)}}{4a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a^5*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[
ArcTan[a*x]]])/(8*a^5*c^3) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[
Pi]])/(2*a^5*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}-\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^5c^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3}+\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^5c^3}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\right.}{2a}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3}+\frac{\text{Subst}\left(\int \cos\left(4x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^5c^3}-\frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4a^5c^3}+\frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(}}{\sqrt{\pi}}\right.}{2a^5c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 230, normalized size = 2.58

$$\frac{10\sqrt{2\pi}\sqrt{\text{ArcTan}(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)-80\sqrt{\pi}\sqrt{\text{ArcTan}(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+3\sqrt{\text{ArcTan}(ax)}\left(64\sqrt{\text{ArcTan}(ax)^2}+4\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+4\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)-\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{256a^5c^3\sqrt{\text{ArcTan}(ax)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (10*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
 - 80*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
 + 3*Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (2*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x
]] - Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^5*c^3*Sq
rt[ArcTan[a*x]^2])
```

Maple [A]
time = 0.27, size = 59, normalized size = 0.66

method	result	size
default	$\frac{\pi\sqrt{2}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+12\sqrt{\arctan(ax)}\,\sqrt{\pi}-8\pi\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16c^{3}a^{5}\sqrt{\pi}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/a^5*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+12*
arctan(a*x)^(1/2)*Pi^(1/2)-8*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^
(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{4}}{a^{6}x^{6}\sqrt{\operatorname{atan}(ax)}+3a^{4}x^{4}\sqrt{\operatorname{atan}(ax)}+3a^{2}x^{2}\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.939 $\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^4c^3}+\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}$$

```
[Out] -1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c
^3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5090, 4491, 3386, 3432}

$$\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}-\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^3) + (Sqr
t[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^4*c^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} - \frac{\sin(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^4c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^4c^3} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right.}{4a^4c^3}$$

$$= -\frac{\text{Subst}\left(\int \sin(4x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^4c^3} + \frac{\text{Subst}\left(\int \sin(2x^2)\,dx, x, \right.}{2a^4c^3}$$

$$= -\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^4c^3} + \frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 131, normalized size = 1.85

$$\frac{-2\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)-2\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{32a^4c^3\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]])/(32*a^4*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.20, size = 47, normalized size = 0.66

method	result	size
default	$\frac{\sqrt{\pi}\,\left(-\sqrt{2}\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+4\,\mathrm{S}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\right)}{16c^{3}a^{4}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/a^4*Pi^(1/2)*(-2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3}}{a^{6}x^{6}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{4}x^{4}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}+\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.940 $\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\mathrm{ArcTan}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^3c^3}$$

```
[Out] -1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c
^3+1/4*arctan(a*x)^(1/2)/a^3/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5090, 4491, 3385, 3433}

$$\frac{\sqrt{\mathrm{ArcTan}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Sqrt[ArcTan[a*x]]/(4*a^3*c^3) - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(8*a^3*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin^2(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{8\sqrt{x}}-\frac{\cos(4x)}{8\sqrt{x}}\right)\,dx,x,\tan^{-1}(ax)\right)}{a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{8a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\text{Subst}\left(\int \cos\left(4x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{4a^3c^3}\\
&= \frac{\sqrt{\tan^{-1}(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^3c^3}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 229, normalized size = 3.95

$$\frac{-2\sqrt{2\pi}\sqrt{\text{ArcTan}(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+16\sqrt{\pi}\sqrt{\text{ArcTan}(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+\sqrt{\text{ArcTan}(ax)}\left(64\sqrt{\text{ArcTan}(ax)^2}+4\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+4\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+7\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+7\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{256a^3c^3\sqrt{\text{ArcTan}(ax)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
 + 16*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
 + Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (2*I)*ArcTan[a*x]] + 7*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x
]] + 7*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^3*c^3*
Sqrt[ArcTan[a*x]^2])
```

Maple [A]
time = 0.21, size = 48, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{2}\,\left(2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}-\pi\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\right)}{16c^{3}a^{3}\sqrt{\pi}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/a^3*2^(1/2)*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)-Pi*FresnelC(2*2^
(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)))/Pi^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}}{a^{6}x^{6}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{4}x^{4}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}+\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*
a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.941 $\int \frac{x}{(c+a^2cx^2)^3 \sqrt{\mathbf{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^2c^3}+\frac{\sqrt{\pi}\, S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

```
[Out] 1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^
3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5090, 4491, 3386, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{8a^2c^3}+\frac{\sqrt{\pi}\, S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a^2*c^3) + (Sqrt[P
i]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(4*a^2*c^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} + \frac{\sin(4x)}{8\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8a^2c^3} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^3}$$

$$= \frac{\text{Subst}\left(\int \sin\left(4x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4a^2c^3} + \frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\mathrm{t}\right.}{2a^2c^3}$$

$$= \frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8a^2c^3} + \frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 133, normalized size = 1.87

$$\frac{-2\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)-2\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)-\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{32a^2c^3\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a*x
]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]])/(32*a^2*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.19, size = 46, normalized size = 0.65

method	result	size
default	$\frac{\sqrt{\pi}\,\left(\sqrt{2}\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+4\,\mathrm{S}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\right)}{16c^{3}a^{2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/a^2*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2
))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{6}x^{6}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{4}x^{4}\sqrt{\operatorname{atan}\left(ax\right)}+3a^{2}x^{2}\sqrt{\operatorname{atan}\left(ax\right)}+\sqrt{\operatorname{atan}\left(ax\right)}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a**
2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.942 $\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\mathbf{ArcTan}(ax)}} dx$

Optimal. Leaf size=89

$$\frac{3\sqrt{\text{ArcTan}(ax)}}{4ac^3}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{8ac^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2ac^3}$$

```
[Out] 1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3+
1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3+3/4*arctan(a*x)^(
1/2)/a/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5024, 3393, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{8ac^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{2ac^3}+\frac{3\sqrt{\text{ArcTan}(ax)}}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[ArcTan[a*x]])/(4*a*c^3) + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[Ar
cTan[a*x]]])/(8*a*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]
)/(2*a*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{8ac^3} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,d\right.}{2a}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3} + \frac{\text{Subst}\left(\int \cos\left(4x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{4ac^3} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\sqrt{\tan^{-1}(ax)}}{4ac^3} + \frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{8ac^3} + \frac{\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(a}}{\sqrt{\pi}}\right.}{2ac^3}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.81

$$\frac{12\sqrt{\text{ArcTan}(ax)}+\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] (12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]
 + 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a*c^3)
```

Maple [A]
time = 0.26, size = 59, normalized size = 0.66

method	result	size
default	$\frac{\pi\sqrt{2}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+8\pi\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+12\sqrt{\arctan(ax)}\,\sqrt{\pi}}{16c^3a\sqrt{\pi}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^3/a/Pi^(1/2)*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))+8*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+12*arctan(a*x)^(1/2)*Pi^(1/
2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a**
2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.943 $\int \frac{1}{x(c+a^2cx^2)^3\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^7\sqrt{\operatorname{atan}(ax)}+3a^4x^5\sqrt{\operatorname{atan}(ax)}+3a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(a**6*x**7*sqrt(atan(a*x)) + 3*a**4*x**5*sqrt(atan(a*x)) + 3*a**
2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.944 $\int \frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\text{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\text{ArcTan}(ax)}}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{a^2 c\, x^2 + c}}{\sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c\left(a^2x^2+1\right)}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

3.945 $\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

3.946 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2), x)
```

3.947 $\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{x\sqrt{\operatorname{atan}\left(ax\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*sqrt(atan(a*x))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)),x)
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)), x)
```

3.948 $\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\textbf{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c\, x^2 + c)^{\frac{3}{2}}}{\sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/sqrt(arctan(a*x)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(c\,a^2\,x^2+c\right)^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

3.949 $\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

3.950 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2), x)
```

3.951 $\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 2.05, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*sqrt(atan(a*x))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)), x)
```

3.952 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/sqrt(a
rctan(a*x)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

3.953 $\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

3.954 $\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2), x)
```

3.955 $\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(5/2)/(x*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)),x)

[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)), x)

3.956 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c\,(a^2x^2+1)}\,\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{\operatorname{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.957 $\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

`[Out] Unintegrable(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)`

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

`[In] Int[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]`

`[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]`

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]`

`[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]`

Maple [A]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.958 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.959 $\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\,\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.960 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(
arctan(a*x))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.961 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 6.67, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.962 $\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

[Out] FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5091, 5090, 3386, 3432}

$$\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]

[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}}$$
$$= \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 97, normalized size = 1.62

$$-\frac{\sqrt{1+a^2x^2}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)\right)}{2a^2c\sqrt{c\,(1+a^2x^2)}\,\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -1/2*(Sqrt[1 + a^2*x^2]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]))/(a^2*c*Sqrt[c*(1 + a^2*
x^2)]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.963 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(
1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5025, 5024, 3385, 3433}

$$\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*
Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 1.02

$$\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac\sqrt{c\,(1+a^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
[Out] (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*
Sqrt[c*(1 + a^2*x^2)])
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.964 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.965 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*sqrt(arctan(a*x))), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.966 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\mathrm{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 8.43, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.967 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/12*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+3/4*FresnelS(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5091, 5090, 3393, 3386, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[
6/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^3}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}}-\frac{\sin(3x)}{4\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Sub}}{4a^4c}$$

$$= -\frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(3x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right.}{}$$

$$= \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\right.}{2a^4c^2\sqrt{c+a}}$$

Mathematica [A]

time = 0.15, size = 95, normalized size = 0.73

$$\frac{\sqrt{\frac{\pi}{6}}\,(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)-S\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)\right)}{2a^4c\,(c\,(1+a^2x^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^4*c*(c*(1 + a^2*x^2)
)^(3/2))
```

Maple [**F**]
time = 6.24, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}}\,\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.968 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/12*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+1/4*FresnelC(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5091, 5090, 4491, 3385, 3433}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
3*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}}-\frac{\cos(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\mathrm{c}}{}\right.}{4a^3c^2\sqrt{c}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{Sub}}{}$$

$$= \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{6}{\pi}}\right.}{2a^3c^2\sqrt{c+a^2}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.14, size = 159, normalized size = 1.21

$$-\frac{i\sqrt{1+a^2x^2}\left(3\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-i\mathrm{ArcTan}(ax)\right)-3\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},i\mathrm{ArcTan}(ax)\right)+\sqrt{3}\left(-\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-3i\mathrm{ArcTan}(ax)\right)+\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},3i\mathrm{ArcTan}(ax)\right)\right)\right)}{24a^3c^2\sqrt{c\left(1+a^2x^2\right)}\,\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] ((-1/24*I)*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcT
an[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(-(Sqr
t[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]) + Sqrt[I*ArcTan[a*x]]*G
amma[1/2, (3*I)*ArcTan[a*x]])))/(a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Sqrt[ArcTan[
a*x]])
```

Maple [F]
time = 7.19, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{5}{2}}\,\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\operatorname{atan}\,(a\,x)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.969 $\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/12*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/4*FresnelS(2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5091, 5090, 4491, 3386, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2a^2c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a^
2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{x}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}}+\frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \tan^{-1}(ax)\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\text{s}}{}\right.}{4a^2c^2\sqrt{c}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin\left(x^2\right)dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{Subs}}{}$$

$$= \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\right.}{2a^2c^2\sqrt{c+a^2}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.16, size = 156, normalized size = 1.19

$$-\frac{(1+a^2x^2)^{3/2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{24a^2c\,(c\,(1+a^2x^2))^{3/2}\,\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] -1/24*((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[
1/2, (3*I)*ArcTan[a*x]])))/(a^2*c*(c*(1 + a^2*x^2))^(3/2)*Sqrt[ArcTan[a*x]]
)
```

Maple [F]
time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{5}{2}}\,\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
[Out] Integral(x/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\operatorname{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.970 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=131

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/12*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+3/4*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5025, 5024, 3393, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{2ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*
a*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}}+\frac{\cos(3x)}{4\sqrt{x}}\right)\,dx, x, \tan^{-1}(ax)\right)}{ac^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subs}}{4ac^2}$$

$$= \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\left(3\sqrt{1+a^2x^2}\right.}{}$$

$$= \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\right.}{2ac^2\sqrt{c+a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 159, normalized size = 1.21

$$-\frac{i\sqrt{c\left(1+a^2x^2\right)}\left(9\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-9\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{24ac^3\sqrt{1+a^2x^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] ((-1/24*I)*Sqrt[c*(1 + a^2*x^2)]*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*
ArcTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(S
qrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*
Gamma[1/2, (3*I)*ArcTan[a*x]])))/(a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[ArcTan[a*x]]
)
```

Maple [F]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.971 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.972 $\int \frac{x^m(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(a**2*x**2*x**m/atan(a*x)**
(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

```
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

3.973 $\int \frac{x(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(x/atan(a*x)**(3/2), x) + Integral(a**2*x**3/atan(a*x)**(3/2), x
))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

3.974 $\int \frac{c+a^2cx^2}{\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{c+a^2cx^2}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{a^{2}x^{2}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx+\int\frac{1}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(a**2*x**2/atan(a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x)
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)/atan(a*x)^(3/2), x)
```

3.975 $\int \frac{c+a^2cx^2}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{1}{x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{a^2x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(3/2),x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(a**2*x/atan(a*x)**(3/2),
x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)),x)
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)), x)
```

3.976 $\int \frac{x^m (c+a^2 c x^2)^2}{\mathbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{x^m (c+a^2 c x^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2 c x^2)^2}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 2.26, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c\, x^2 + c)^2}{\arctan(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{2a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int \frac{a^4x^4x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(2*a**2*x**2*x**m/atan(a
*x)**(3/2), x) + Integral(a**4*x**4*x**m/atan(a*x)**(3/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

3.977 $\int \frac{x(c+a^2cx^2)^2}{\textbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{2a^{2}x^{3}}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{a^{4}x^{5}}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**(3/2), x) + Integral(2*a**2*x**3/atan(a*x)**(3/
2), x) + Integral(a**4*x**5/atan(a*x)**(3/2), x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

3.978 $\int \frac{(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{3/2}} dx$$

Mathematica [A]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\text{ArcTan}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]

[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]

Maple [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{2a^{2}x^{2}}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx + \int \frac{a^{4}x^{4}}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**(3/2), x) + Integral(a**4*x**4/atan(a
*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^2/atan(a*x)^(3/2), x)
```

3.979 $\int \frac{(c+a^2cx^2)^2}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{1}{x\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{2a^{2}x}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{a^{4}x^{3}}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(3/2),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(2*a**2*x/atan(a*x)**(3
/2), x) + Integral(a**4*x**3/atan(a*x)**(3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)),x)
```

```
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)), x)
```

3.980 $\int \frac{x^m\left(c+a^2cx^2\right)^3}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m\left(c+a^2cx^2\right)^3}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m\left(c+a^2cx^2\right)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(c+a^2cx^2\right)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 3.22, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a^2c\,x^2+c\right)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

3.981 $\int \frac{x\left(c+a^2cx^2\right)^3}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^{2}x^{3}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^{4}x^{5}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^{6}x^{7}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**(3/2), x) + Integral(3*a**2*x**3/atan(a*x)**(3/
2), x) + Integral(3*a**4*x**5/atan(a*x)**(3/2), x) + Integral(a**6*x**7/ata
n(a*x)**(3/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

3.982 $\int \frac{(c+a^2cx^2)^3}{\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}},x\right)$$

[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]

[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]

[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]

Maple [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{3a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{3a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{a^6x^6}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx+\int\frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**(3/2), x) + Integral(3*a**4*x**4/atan
(a*x)**(3/2), x) + Integral(a**6*x**6/atan(a*x)**(3/2), x) + Integral(atan(
a*x)**(-3/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x)^(3/2), x)
```

3.983 $\int \frac{(c+a^2cx^2)^3}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{1}{x\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{3}}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{5}}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(3/2),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(3*a**2*x/atan(a*x)**(3
/2), x) + Integral(3*a**4*x**3/atan(a*x)**(3/2), x) + Integral(a**6*x**5/at
an(a*x)**(3/2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)),x)
```

```
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)), x)
```

3.984 $\int \frac{x^m}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=46

$$-\frac{2x^m}{ac\sqrt{\text{ArcTan}(ax)}}+\frac{2m\text{Int}\left(\frac{x^{-1+m}}{\sqrt{\text{ArcTan}(ax)}},x\right)}{ac}$$

```
[Out] -2*x^m/a/c/arctan(a*x)^(1/2)+2*m*Unintegrable(x^(-1+m)/arctan(a*x)^(1/2),x)
/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^m)/(a*c*Sqrt[ArcTan[a*x]]) + (2*m*Defer[Int][x^(-1 + m)/Sqrt[ArcTan[a
*x]], x])/(a*c)
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}} \, dx = -\frac{2x^m}{ac\sqrt{\tan^{-1}(ax)}}+\frac{(2m)\int \frac{x^{-1+m}}{\sqrt{\tan^{-1}(ax)}} \, dx}{ac}$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{m}}{a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
[Out] Integral(x**m/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

```
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.985 $\int \frac{x}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2x}{ac\sqrt{\operatorname{ArcTan}(ax)}}+\frac{2\operatorname{Int}\left(\frac{1}{\sqrt{\operatorname{ArcTan}(ax)}},x\right)}{ac}$$

```
[Out] -2*x/a/c/arctan(a*x)^(1/2)+2*Unintegrable(1/arctan(a*x)^(1/2),x)/a/c
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c*Sqrt[ArcTan[a*x]]) + (2*Defer[Int][1/Sqrt[ArcTan[a*x]], x])/(a*
c)
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x}{ac\sqrt{\tan^{-1}(ax)}}+\frac{2\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx}{ac}$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

```
[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.986 $\int \frac{1}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{ac\sqrt{\text{ArcTan}(ax)}}$$

[Out] -2/a/c/arctan(a*x)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5004}

$$-\frac{2}{ac\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]

[Out] -2/(a*c*Sqrt[ArcTan[a*x]])

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{ac\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]

[Out] -2/(a*c*Sqrt[ArcTan[a*x]])

Maple [A]
time = 0.23, size = 15, normalized size = 0.94

method	result	size
default	$-\frac{2}{ac\sqrt{\arctan\left(ax\right)}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/c/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.44, size = 14, normalized size = 0.88

$$-\frac{2}{ac\sqrt{\arctan\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/(a*c*sqrt(arctan(a*x)))
```

Sympy [A]
time = 1.57, size = 14, normalized size = 0.88

$$-\frac{2}{ac\sqrt{\operatorname{atan}\left(ax\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

```
[Out] -2/(a*c*sqrt(atan(a*x)))
```

Giac [**A**]
time = 0.40, size = 14, normalized size = 0.88

$$-\frac{2}{ac\sqrt{\arctan(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(a*c*sqrt(arctan(a*x)))
```

Mupad [**B**]
time = 0.33, size = 14, normalized size = 0.88

$$-\frac{2}{a\,c\,\sqrt{\mathrm{atan}\left(a\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

```
[Out] -2/(a*c*atan(a*x)^(1/2))
```

3.987 $\int \frac{1}{x(c+a^2cx^2)\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2}{acx\sqrt{\mathrm{ArcTan}(ax)}}-\frac{2\mathrm{Int}\left(\frac{1}{x^2\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{ac}$$

```
[Out] -2/a/c/x/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/arctan(a*x)^(1/2),x)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Defer[Int][1/(x^2*Sqrt[ArcTan[a*x]]), x])
/(a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx\sqrt{\tan^{-1}(ax)}}-\frac{2\int \frac{1}{x^2\sqrt{\tan^{-1}(ax)}}\,dx}{ac}$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)+x\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
[Out] Integral(1/(a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.988 $\int \frac{x^m}{(c+a^2cx^2)^2 \textbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(3/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.989 $\int \frac{x^4}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2x^4}{ac^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+\frac{8\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)}{a}+4a\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] -2*x^4/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+8*Unintegrable(x^3/(a^2*c*x^2+c)
^2/arctan(a*x)^(1/2),x)/a+4*a*Unintegrable(x^5/(a^2*c*x^2+c)^2/arctan(a*x)^
(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4}{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^4)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (8*Defer[Int][x^3/((c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Defer[Int][x^5/((c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^4}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x^4}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}+\frac{8\int\frac{x^3}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}+(4a)\int \frac{x^5}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 3.71, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{4}}{a^{4} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.990 $\int \frac{x^3}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=106

$$-\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+\frac{6\sqrt{\text{ArcTan}(ax)}}{a^4c^2}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^4c^2}+2a\text{Int}\left(\frac{}{(c+a^2cx}\right.$$

[Out] -3*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^2-2*x^3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+6*arctan(a*x)^(1/2)/a^4/c^2+2*a*Unintegrable(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]

[Out] (-2*x^3)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[ArcTan[a*x]])/(a^4*c^2) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) + 2*a*Defer[Int][x^4/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{6\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} + (2a)\int -$$

$$= -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^4}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx + -$$

$$= -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^4}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx + -$$

$$= -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} + (2a)\int \frac{x^4}{(c+a^2cx^2)^2\sqrt{}}$$

$$= -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} + (2a)\int \frac{x^4}{(c+a^2cx^2)^2\sqrt{}}$$

$$= -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{6\sqrt{\tan^{-1}(ax)}}{a^4c^2} - \frac{3\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^{-1}(ax}}{\sqrt{\pi}}\right.}{a^4c^2}$$

Mathematica [A]
time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.991 $\int \frac{x^2}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=60

$$-\frac{2x^2}{ac^2\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}+\frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

```
[Out] 2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2-2*x^2/a/c^2/(a^2*
x^2+1)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5062, 5090, 4491, 12, 3386, 3432}

$$\frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*
Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{2\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^2}$$

$$= -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 1.00

$$-\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}} + \frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*
Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

Maple [A]
time = 0.30, size = 46, normalized size = 0.77

method	result	size
default	$\frac{2\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(2\arctan(ax))-1}{c^{2}a^{3}\sqrt{\arctan(ax)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2/a^3*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))+cos(2*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}}{a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)+2a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
atan(a*x)**(3/2)), x)/c**2
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.992 $\int \frac{x}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=138

$$-\frac{2x}{ac^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+\frac{4\sqrt{\text{ArcTan}(ax)}}{a^2c^2}-\frac{8\sqrt{\text{ArcTan}(ax)}}{a^2c^2(1+a^2x^2)}+\frac{4(1-a^2x^2)\sqrt{\text{ArcTan}(ax)}}{a^2c^2(1+a^2x^2)}+\frac{2\sqrt{\pi}\,\text{F}}{}$$

[Out] 2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2-2*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4*arctan(a*x)^(1/2)/a^2/c^2-8*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)+4*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5052, 5050, 5024, 3393, 3385, 3433}

$$\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^2c^2}-\frac{2x}{ac^2(a^2x^2+1)\sqrt{\text{ArcTan}(ax)}}+\frac{4(1-a^2x^2)\sqrt{\text{ArcTan}(ax)}}{a^2c^2(a^2x^2+1)}-\frac{8\sqrt{\text{ArcTan}(ax)}}{a^2c^2(a^2x^2+1)}+\frac{4\sqrt{\text{ArcTan}(ax)}}{a^2c^2}$$

Antiderivative was successfully verified.

[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]

[Out] (-2*x)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt[ArcTan[a*x]])/(a^2*c^2) - (8*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (4*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^2*c^2)

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4(1-a^2x^2)\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + 16\int \frac{x\sqrt{\mathrm{ta}}}{(c+}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1-a^2x^2)\sqrt{\tan^{-1}(a}}{a^2c^2(1+a^2x^2)}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1-a^2x^2)\sqrt{\tan^{-1}(a}}{a^2c^2(1+a^2x^2)}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1-a^2x^2)\sqrt{\tan^{-1}(a}}{a^2c^2(1+a^2x^2)}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1}{}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1}{}$$

$$= -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} + \frac{4\sqrt{\tan^{-1}(ax)}}{a^2c^2} - \frac{8\sqrt{\tan^{-1}(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 158, normalized size = 1.14

$$\frac{-8ax+4\sqrt{\pi}\,(1+a^2x^2)\sqrt{\mathrm{ArcTan}(ax)}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)-i\sqrt{2}\,(1+a^2x^2)\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-2i\mathrm{ArcTan}(ax)\right)+i\sqrt{2}\,(1+a^2x^2)\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},2i\mathrm{ArcTan}(ax)\right)}{4a^2c^2(1+a^2x^2)\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8*a*x + 4*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTa
n[a*x]])/Sqrt[Pi]] - I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[a*x]]*Gamma
[1/2, (2*I)*ArcTan[a*x]])/(4*a^2*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.27, size = 46, normalized size = 0.33

method	result	size
default	$-\frac{-2\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sin(2\arctan(ax))}{c^{2}a^{2}\sqrt{\arctan(ax)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2/a^2/arctan(a*x)^(1/2)*(-2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arct
an(a*x)^(1/2)/Pi^(1/2))+sin(2*arctan(a*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)+2a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

[Out] Integral(x/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**2

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")

[Out] sage0*x

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)

[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)

3.993 $\int \frac{1}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=57

$$-\frac{2}{ac^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}-\frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

```
[Out] -2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-2/a/c^2/(a^2*x^2+1
)/arctan(a*x)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5022, 5090, 4491, 12, 3386, 3432}

$$-\frac{2}{ac^2(a^2x^2+1)\sqrt{\text{ArcTan}(ax)}}-\frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[A
rcTan[a*x]])/Sqrt[Pi]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - (4a)\int \frac{x}{(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{4\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{4\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{2\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^2}$$

$$= -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{4\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{ac^2}$$

$$= -\frac{2}{ac^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{2\sqrt{\pi}\,S\left(\frac{2\sqrt{\tan^{-1}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.91

$$\frac{-\frac{2}{(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}} - 2\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
[Out] (-2/((1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - 2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[
a*x]])/Sqrt[Pi]])/(a*c^2)
```

Maple [A]
time = 0.27, size = 47, normalized size = 0.82

method	result	size

default	$-\frac{2\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)+\cos\left(2\arctan\left(ax\right)\right)+1}{c^{2}a\sqrt{\arctan\left(ax\right)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2/a*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))+cos(2*arctan(a*x))+1)/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{4}\operatorname{atan}^{\frac{3}{2}}(ax)+2a^{2}x^{2}\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + ata
n(a*x)**(3/2)), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.994 $\int \frac{1}{x(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=102

$$-\frac{2}{ac^2x(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}-\frac{6\sqrt{\text{ArcTan}(ax)}}{c^2}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{c^2}-\frac{2\text{Int}\left(\frac{}{x^2(c+a^2cx^2)^2}\right.}{}$$

```
[Out] -3*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/c^2-2/a/c^2/x/(a^2*x^2+1
)/arctan(a*x)^(1/2)-6*arctan(a*x)^(1/2)/c^2-2*Unintegrable(1/x^2/(a^2*c*x^2
+c)^2/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[ArcTan[a*x]])/c^2 -
(3*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 - (2*Defer[Int][1
/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (6$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{6S}{}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{6S}{}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\text{ta}}}}{a}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\text{ta}}}}{a}$$

$$= -\frac{2}{ac^2x\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{\tan^{-1}(ax)}}{c^2} - \frac{3\sqrt{\pi}\,C\left(\frac{2\sqrt{\tan^-}}{\sqrt{\pi}}\right.}{c^2}$$

Mathematica [A]
time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4}x^{5}\operatorname{atan}^{\frac{3}{2}}(ax)+2a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**(3/2) + 2*a**2*x**3*atan(a*x)**(3/2) + x*a
tan(a*x)**(3/2)), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \operatorname{atan}(a x)^{3/2} (c a^2 x^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.995 $\int \frac{1}{x^2(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^2x^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}-\frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-8a\text{Int}\left(\frac{1}{x\,(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\right.$$

```
[Out] -2/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*c*x^2+
c)^2/arctan(a*x)^(1/2),x)/a-8*a*Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x
)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Defer[Int][1/(x*(c + a^2*c*x^2
)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^2x^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}-\frac{4\int\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(8$$

Mathematica [A]
time = 3.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]

Maple [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)

[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)

[Out] Integral(1/(a**4*x**6*atan(a*x)**(3/2) + 2*a**2*x**4*atan(a*x)**(3/2) + x**2*atan(a*x)**(3/2)), x)/c**2

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.996 $\int \frac{1}{x^3(c+a^2cx^2)^2 \text{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^2x^3\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}-\frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-10a\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^2\sqrt{\text{ArcT}}}\right.$$

```
[Out] -2/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*c*x^2+
c)^2/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x^2/(a^2*c*x^2+c)^2/arctan(
a*x)^(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x^2*(c + a^2*c*
x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}} \, dx = -\frac{2}{ac^2x^3\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{6\int \frac{1}{x^4(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}} \, dx}{a} -$$

Mathematica [A]
time = 5.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2 \text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]

Maple [A]
time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)

[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{7} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{5} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)

[Out] Integral(1/(a**4*x**7*atan(a*x)**(3/2) + 2*a**2*x**5*atan(a*x)**(3/2) + x**3*atan(a*x)**(3/2)), x)/c**2

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.997 $\int \frac{1}{x^4(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^2x^4(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}-\frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-12a\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\text{ArcTan}(ax)}}\right.$$

```
[Out] -2/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*c*x^2+
c)^2/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^2/arctan(
a*x)^(1/2),x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4(c+a^2cx^2)^2\text{ArcTan}(ax)^{3/2}}dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^2*x^4*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(c +
a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x^3*(c + a^2*c*
x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}dx = -\frac{2}{ac^2x^4(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5(c+a^2cx^2)^2\sqrt{\tan^{-1}(ax)}}dx}{a}-(1$$

Mathematica [A]
time = 4.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(c+a^2cx^2)^2\text{ArcTan}(ax)^{3/2}}dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{8} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
[Out] Integral(1/(a**4*x**8*atan(a*x)**(3/2) + 2*a**2*x**6*atan(a*x)**(3/2) + x**
4*atan(a*x)**(3/2)), x)/c**2
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.998 $\int \frac{x^m}{(c+a^2cx^2)^3\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^(3/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.999 $\int \frac{x^3}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=96

$$-\frac{2x^3}{ac^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^4c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}$$

```
[Out] -1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c^
3+FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3-2*x^3/a/c^3/(a^2*
x^2+1)^2/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5088, 5090, 3393, 3385, 3433, 4491}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^4c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}-\frac{2x^3}{ac^3(a^2x^2+1)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^3)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelC[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/(a^4*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}+\frac{6\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-(2a)\int$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{a^4c^3}+\frac{6\text{S}}{}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{2\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}-\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)\,d}{a^4c^3}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx,x,\tan^{-1}(ax)\right)}{4a^4c^3}-\frac{3\text{S}}{}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{\text{Subst}\left(\int \cos\left(4x^2\right)\,dx,x,\sqrt{\tan^{-1}(ax)}\right)}{2a^4c^3}$$

$$= -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^4c^3}+\frac{\sqrt{\pi}\,C}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 148, normalized size = 1.54

$$\frac{-2\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+\frac{-\frac{32a^3x^3}{(1+a^2x^2)^2}+3i\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)-3i\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{\sqrt{\text{ArcTan}(ax)}}}{16a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqrt[Pi]*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((-32*a^3*x^3)/(1 + a^2*x^2)^2 + (3*I
)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - (3*I)*Sqrt[I*ArcT
an[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*x]])/(16*a^4*c^3)
```

Maple [A]
time = 0.34, size = 86, normalized size = 0.90

method	result
default	$-\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan}}{\sqrt{\pi}}\right.}{4c^3a^4\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^3/a^4*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^
(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1000 $\int \frac{x^2}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=67

$$-\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^3}$$

```
[Out] 1/2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3
-2*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5088, 5090, 4491, 3386, 3432}

$$\frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^3}-\frac{2x^2}{ac^3(a^2x^2+1)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelS[2
*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (4a)\int$$

$$= -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{4\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^3c^3}$$

$$= -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} - \frac{\sin(4x)}{8\sqrt{x}}\right)dx, x, \tan\right.}{a^3c^3}$$

$$= -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + 2\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{2a^3c^3}$$

$$= -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + 2\frac{\text{Subst}\left(\int \sin(4x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}$$

$$= -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^3c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 112, normalized size = 1.67

$$\frac{-8a^2x^2 - (1+a^2x^2)^2\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4i\text{ArcTan}(ax)\right) - (1+a^2x^2)^2\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 4i\text{ArcTan}(ax)\right)}{4a^3c^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8*a^2*x^2 - (1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcT
an[a*x]] - (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]
])/(4*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.34, size = 53, normalized size = 0.79

method	result	size

default	$\frac{2\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan\left(ax\right)}\,\sqrt{\pi}+\cos\left(4\arctan\left(ax\right)\right)-1}{4c^{3}a^{3}\sqrt{\arctan\left(ax\right)}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c^3/a^3*(2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arcta
n(a*x)^(1/2)*Pi^(1/2)+cos(4*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}}{a^{6}x^{6}\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)+3a^{4}x^{4}\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)+3a^{2}x^{2}\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)+\mathrm{atan}^{\frac{3}{2}}\left(ax\right)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1001 $\int \frac{x}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

```
[Out] 1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^3
+FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3-2*x/a/c^3/(a^2*x^2
+1)^2/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5088, 5090, 4491, 3385, 3433, 5024, 3393}

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c^3}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (Sqrt[Pi/2]*FresnelC[2*S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^3) + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(a^2*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\int \frac{1}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (6a)\int$$

$$= -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{a^2c^3} - \frac{6}{}$$

$$= -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{2\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)d\right.}{a^2c^3}$$

$$= -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{4a^2c^3} + \frac{3\text{S}}{}$$

$$= -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{\text{Subst}\left(\int \cos(4x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right)}{2a^2c^3}$$

$$= -\frac{2x}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{a^2c^3} + \frac{\sqrt{\pi}\,C}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 156, normalized size = 1.68

$$\frac{-\frac{8ax}{(1+a^2x^2)^2} - i\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -2i\text{ArcTan}(ax)\right) + i\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 2i\text{ArcTan}(ax)\right) - i\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, -4i\text{ArcTan}(ax)\right) + i\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2}, 4i\text{ArcTan}(ax)\right)}{4a^2c^3\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-8*a*x)/(1 + a^2*x^2)^2 - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2
*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*
x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a^2*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.29, size = 84, normalized size = 0.90

method	result
default	$-\frac{-2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan}}{\sqrt{}}\right.}{4c^3a^2\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^3/a^2/arctan(a*x)^(1/2)*(-2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*Fres
nelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*a
**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1002 $\int \frac{1}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=94

$$-\frac{2}{ac^3 (1+a^2x^2)^2 \sqrt{\text{ArcTan}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac^3}-\frac{2\sqrt{\pi}\, S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

```
[Out] -1/2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3-
2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3-2/a/c^3/(a^2*x^2+1)
^2/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5022, 5090, 4491, 3386, 3432}

$$-\frac{2}{ac^3 (a^2x^2+1)^2 \sqrt{\text{ArcTan}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\, S\left(2\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac^3}-\frac{2\sqrt{\pi}\, S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^3) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]])/(a*c^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - (8a)\int \frac{x}{(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{8\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3}$$

$$= -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{8\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} + \frac{\sin(4x)}{8\sqrt{x}}\right)\,dx, x, \tan^{-}\right.}{ac^3}$$

$$= -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \tan^{-1}(ax)\right)}{ac^3} - \frac{2\text{S}}{}$$

$$= -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\text{Subst}\left(\int \sin(4x^2)\,dx, x, \sqrt{\tan^{-1}(ax)}\right.}{ac^3}$$

$$= -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax)}\right)}{ac^3} - \frac{2\sqrt{\pi}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 144, normalized size = 1.53

$$\frac{-\frac{8}{(1+a^2x^2)^2}+2\sqrt{2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+2\sqrt{2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{4ac^3\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8/(1 + a^2*x^2)^2 + 2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*Ar
cTan[a*x]] + 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] +
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]
*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a*c^3*Sqrt[ArcTan[a*x]])
```

Maple [A]
time = 0.32, size = 85, normalized size = 0.90

method	result
default	$-\frac{2\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+8\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\,\mathrm{c}}{4c^3a\sqrt{\arctan(ax)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^3/a*(2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arctan
(a*x)^(1/2)*Pi^(1/2)+8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1
/2)/Pi^(1/2))+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*a
**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1003 $\int \frac{1}{x(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{3/2}} dx$

Optimal. Leaf size=140

$$-\frac{2}{ac^3x\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}-\frac{15\sqrt{\text{ArcTan}(ax)}}{2c^3}-\frac{5\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{4c^3}-\frac{5\sqrt{\pi}\,\text{F}}{}$$

```
[Out] -5/8*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/c^3-5*
FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/c^3-2/a/c^3/x/(a^2*x^2+1)^2
/arctan(a*x)^(1/2)-15/2*arctan(a*x)^(1/2)/c^3-2*Unintegrable(1/x^2/(a^2*c*x
^2+c)^3/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3 \text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (15*Sqrt[ArcTan[a*x]])/(2*
c^3) - (5*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*c^3) - (5
*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^3 - (2*Defer[Int][1/(
x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x(c+a^2cx^2)^3 \tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (10$$

$$= -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{10}{}$$

$$= -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a} - \frac{10}{}$$

$$= -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^3\sqrt{\mathrm{t}}}}{a}$$

$$= -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^3\sqrt{\mathrm{t}}}}{a}$$

$$= -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{15\sqrt{\tan^{-1}(ax)}}{2c^3} - \frac{5\sqrt{\frac{\pi}{2}}\,C\left(2\sqrt{\frac{2}{\pi}}\right.}{4c^3}$$

Mathematica [A]
time = 4.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+a^2cx^2)^3\,\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a^2c\,x^2+c)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{7}\operatorname{atan}^{\frac{3}{2}}(ax)+3a^{4}x^{5}\operatorname{atan}^{\frac{3}{2}}(ax)+3a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**(3/2) + 3*a**4*x**5*atan(a*x)**(3/2) + 3*a
**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \operatorname{atan}(a x)^{3/2} (c a^2 x^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1004 $\int \frac{1}{x^2(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^3x^2\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}-\frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-12a\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTa}}}\right.$$

```
[Out] -2/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*c*x^
2+c)^3/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(
a*x)^(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x*(c + a^2*c*
x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^3x^2\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{4\int\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-$$

Mathematica [A]
time = 4.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]

Maple [A]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)

[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{8} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)

[Out] Integral(1/(a**6*x**8*atan(a*x)**(3/2) + 3*a**4*x**6*atan(a*x)**(3/2) + 3*a**2*x**4*atan(a*x)**(3/2) + x**2*atan(a*x)**(3/2)), x)/c**3

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1005 $\int \frac{1}{x^3(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^3x^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}-\frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^3\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-14a\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\text{ArcTa}}}\right.$$

```
[Out] -2/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*c*x^
2+c)^3/arctan(a*x)^(1/2),x)/a-14*a*Unintegrable(1/x^2/(a^2*c*x^2+c)^3/arcta
n(a*x)^(1/2),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Defer[Int][1/(x^2*(c + a^2*
c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^3x^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-($$

Mathematica [A]
time = 5.69, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]

Maple [A]
time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)

[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{9} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{7} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{5} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)

[Out] Integral(1/(a**6*x**9*atan(a*x)**(3/2) + 3*a**4*x**7*atan(a*x)**(3/2) + 3*a **2*x**5*atan(a*x)**(3/2) + x**3*atan(a*x)**(3/2)), x)/c**3

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1006 $\int \frac{1}{x^4(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2}{ac^3x^4\left(1+a^2x^2\right)^2\sqrt{\mathrm{ArcTan}(ax)}}-\frac{8\mathrm{Int}\left(\frac{1}{x^5(c+a^2cx^2)^3\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{a}-16a\mathrm{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^3\sqrt{\mathrm{ArcT}}}\right.$$

```
[Out] -2/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*c*x^
2+c)^3/arctan(a*x)^(1/2),x)/a-16*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^3/arcta
n(a*x)^(1/2),x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c^3*x^4*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(c
+ a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a - 16*a*Defer[Int][1/(x^3*(c + a^2*
c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{ac^3x^4\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}-\frac{8\int\frac{1}{x^5(c+a^2cx^2)^3\sqrt{\tan^{-1}(ax)}}\,dx}{a}-$$

Mathematica [A]
time = 5.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a^2 c\,x^2+c\right)^3 \arctan\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{10} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{8} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
[Out] Integral(1/(a**6*x**10*atan(a*x)**(3/2) + 3*a**4*x**8*atan(a*x)**(3/2) + 3*
a**2*x**6*atan(a*x)**(3/2) + x**4*atan(a*x)**(3/2)), x)/c**3
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1007 $\int \frac{x^m \sqrt{c+a^2 c x^2}}{\mathbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m \sqrt{c+a^2 c x^2}}{\text{ArcTan}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{x^m \sqrt{c+a^2 c x^2}}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c+a^2 c x^2}}{\text{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{a^2 c \, x^2 + c}}{\arctan\left(a x\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

3.1008 $\int \frac{x\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

3.1009 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\operatorname{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\operatorname{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2), x)
```

3.1010 $\int \frac{\sqrt{c+a^2cx^2}}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 3.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)),x)
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)), x)
```

3.1011 $\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\textbf{ArcTan}(ax)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x^m (c+a^2 c x^2)^{3/2}}{\operatorname{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\operatorname{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\tan^{-1}(ax)^{3/2}} \, dx = \int \frac{x^m (c+a^2 c x^2)^{3/2}}{\tan^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\operatorname{ArcTan}(ax)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{x^m (a^2 c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

3.1012 $\int \frac{x(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

3.1013 $\int \frac{(c+a^2cx^2)^{3/2}}{\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2), x)
```

3.1014 $\int \frac{(c+a^2cx^2)^{3/2}}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{x\,\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)), x)
```

3.1015 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\mathrm{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

3.1016 $\int \frac{x(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

3.1017 $\int \frac{(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2), x)
```

3.1018 $\int \frac{(c+a^2cx^2)^{5/2}}{x\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 3.93, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)),x)
[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)), x)
```

3.1019 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\arctan\left(ax\right)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1020 $\int \frac{x}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\operatorname{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan\left(ax\right)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1021 $\int \frac{1}{\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,(a^2x^2+1)}\ \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1022 $\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{c+a^2cx^2}}{acx\sqrt{\text{ArcTan}(ax)}}-\frac{2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}},x\right)}{a}$$

```
[Out] -2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/(a^2*c*
x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(a*c*x*Sqrt[ArcTan[a*x]]) - (2*Defer[Int][1/(x^2*S
qrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{acx\sqrt{\tan^{-1}(ax)}}-\frac{2\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan \left(a x\right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{c \left(a^{2} x^{2} + 1\right)} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1023 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 7.89, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{c(a^2x^2+1)}\, \operatorname{atan}^{\frac{3}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1024 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^(3/2)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1025 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2x^3}{ac\sqrt{c+a^2cx^2}\sqrt{\mathrm{ArcTan}(ax)}}+\frac{6\mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{a}+4a\mathrm{Int}\left(\frac{x^4}{(c+a^2cx^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}\right.$$

```
[Out] -2*x^3/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+6*Unintegrable(x^2/(a^2*c*
x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a+4*a*Unintegrable(x^4/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^3)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (6*Defer[Int][x^2/((
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a + 4*a*Defer[Int][x^4/((c + a
^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2x^3}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{6\int\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}+(4a)$$

Mathematica [A]
time = 5.48, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 6.05, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1026 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2x^2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{4\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}+2a\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\,\sqrt{\text{A}}}\right.$$

```
[Out] 4*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)
^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-2*x^2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^
(1/2)+2*a*Unintegrable(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1
 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c*Sqrt[c + a^2*c*x
^2]) + 2*a*Defer[Int][x^3/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} + (2a)$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + (2a)\int \frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}} + \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax}\right.}{a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 5.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 6.47, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,\text{atan}^{\frac{3}{2}}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1027 $\int \frac{x}{(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}+\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] 2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)
^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-2*x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1
/2)
```

Rubi [A]

time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = $\{5062, 5025, 5024, 3385, 3433\}$

$$\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\text{ArcTan}(ax)}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2
])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
```

```
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{2\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{1}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(a}}}{ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \text{ta}\right.}{a^2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x\right.}{a^2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}+\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(ax}\right.}{a^2c\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.11, size = 116, normalized size = 1.25

$$\frac{-2ax - i\sqrt{1+a^2x^2}\,\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, -i\mathrm{ArcTan}(ax)\right) + i\sqrt{1+a^2x^2}\,\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2}, i\mathrm{ArcTan}(ax)\right)}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*a*x - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])
/(a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1028 $\int \frac{1}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}-\frac{2\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5022, 5091, 5090, 3386, 3432}

$$-\frac{2\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\mathrm{ArcTan}(ax)}\,\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^2
*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - (2a)\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx$$

$$= -\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^{3/2}\sqrt{\tan^{-1}(a}}}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \text{ta}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{2\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}(ax}\right.}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 107, normalized size = 1.16

$$\frac{-2+\sqrt{1+a^2x^2}\,\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+\sqrt{1+a^2x^2}\,\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)}{ac\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2 + Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]]
 + Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(a*c*Sq
rt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan \left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1029 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}-\frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{c\sqrt{c+a^2cx^2}}-\frac{2\text{Int}\left(\frac{}{x^2(c+a^2cx^2)^{3/}}\right.}{}$$

[Out] -4*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-2/a/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]

[Out] -2/(a*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) - (2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{acx\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} -$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} -$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} -$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} -$$

$$= -\frac{2}{acx\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\tan^{-1}\right.}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 4.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1030 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^2\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}-\frac{4\mathrm{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{a}-6a\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcT}}}\right.$$

```
[Out] -2/a/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*
c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-6*a*Unintegrable(1/x/(a^2*c*x^2+c)^(3
/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 6*a*Defer[Int][1/(x*(c + a
^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^2\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}-\frac{4\int\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

Mathematica [A]
time = 10.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1031 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^3\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}-\frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-8a\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcT}}}\right.$$

```
[Out] -2/a/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*
c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-8*a*Unintegrable(1/x^2/(a^2*c*x^2+c)^
(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 8*a*Defer[Int][1/(x^2*(c +
 a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^3\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{6\int\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}-$$

Mathematica [A]
time = 12.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 2.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1032 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^4\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}-\frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-10a\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{Ar}}}\right.$$

```
[Out] -2/a/c/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*
c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x^3/(a^2*c*x^2+c)
^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^4*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x^3*(c
+ a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^4\sqrt{c+a^2cx^2}\sqrt{\tan^{-1}(ax)}}-\frac{8\int \frac{1}{x^5(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

Mathematica [A]
time = 14.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 6.66, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a^2 c x^2 + c)^{\frac{3}{2}} \arctan(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1033 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^(3/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1034 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{2x^3}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\ldots}{}$$

[Out] 3/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)

Rubi [A]
time = 0.32, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5062, 5091, 5090, 4491, 3385, 3433}

$$-\frac{2x^3}{ac\sqrt{\text{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]

[Out] (-2*x^3)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2])

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{6\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\int \frac{x^2}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}}}}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)\sin^2(x}{\sqrt{x}}\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int\left(\frac{\cos(x)}{4\sqrt{x}} -\right.\right.}{a^4c^2\sqrt{c+a^2}}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx,\right.}{2a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \cos\left(x^2\right)\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^3}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}}\right.}{a^4c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 182, normalized size = 1.14

$$\frac{-\frac{8a^3cx^3}{1+a^2x^2} - ic\sqrt{1+a^2x^2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right) - 3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right) + \sqrt{3}\left(-\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right) + \sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{4a^4c^3\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-8*a^3*c*x^3)/(1 + a^2*x^2) - I*c*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcT
an[a*x]] + Sqrt[3]*(-(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]
```

```
) + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(4*a^4*c^3*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 6.23, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1035 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{2x^2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\right.}{a^3c^2\sqrt{c+a^2}}$$

```
[Out] 1/2*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+
1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2
*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5088, 5091, 5090, 3393, 3386, 3432, 4491}

$$-\frac{2x^2}{ac\sqrt{\text{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x^2)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt
[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2
*c*x^2]) + (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*Fresnel
S[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*P
i)/3]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sq
rt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{4\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (2a$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(4\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-1}}}}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}}\,dx, x\right.}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} -\right.\right.}{a^3c^2\sqrt{c+a^2c}}$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \tan\right.}{2a^3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \sin(3x^2)\,dx, x,\right.}{a^3c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(\right.}{a^3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 241, normalized size = 0.86

$$\frac{-\frac{12a^2x^2}{\sqrt{\text{ArcTan}(ax)}} + \sqrt{6\pi}\,(1+a^2x^2)^{3/2}\left(-3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right) + S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)\right) - \frac{(1+a^2x^2)^{3/2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{\sqrt{\text{ArcTan}(ax)}}}{6a^3c\,(c+a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-12*a^2*x^2)/Sqrt[ArcTan[a*x]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*(-3*Sqrt[
3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan
[a*x]]]) - ((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*A
rcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sq
```

```
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*G
amma[1/2, (3*I)*ArcTan[a*x]])))/Sqrt[ArcTan[a*x]])/(6*a^3*c*(c + a^2*c*x^2)
^(3/2))
```

Maple [F]
time = 7.62, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1036 $\int \frac{x}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\,\sqrt{\mathrm{ArcTan}(ax)}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}}{}$$

```
[Out] 1/2*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+
1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-2
*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5088, 5091, 5090, 4491, 3385, 3433, 5025, 5024, 3393}

$$-\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\frac{\pi}{2}}\,\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{2\pi}{3}}\,\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{\frac{\pi}{6}}\,\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\mathrm{ArcTan}(ax)}\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c
*x^2]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[Pi/6]*Sqrt[1 + a^2*x^2]*FresnelC[
Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[(2*Pi)
/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt
[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
```

```
Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{2\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a} - (4$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\int \frac{1}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-}}}}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx,\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\left(2\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}}\right.\right.}{a^2c^2\sqrt{c+a}}$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \text{ta}\right.}{2a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{\sqrt{1+a^2x^2}\,\text{Subst}\left(\int \cos\left(3x^2\right)\,dx,\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}}\right.}{a^2c^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 299, normalized size = 1.07

$$-\frac{i\left(-8iax+(1+a^2x^2)^{3/2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-(1+a^2x^2)^{3/2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+a^2x^2\sqrt{3+3a^2x^2}\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)-a^2x^2\sqrt{3+3a^2x^2}\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)}{4a^2c^2\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] ((-1/4*I)*((-8*I)*a*x + (1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/
2, (-I)*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I
*ArcTan[a*x]] + Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I
)*ArcTan[a*x]] + a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-3*I)*ArcTan[a*x]] - Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2
, (3*I)*ArcTan[a*x]] - a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (3*I)*ArcTan[a*x]]))/(a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]])
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1037 $\int \frac{1}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{2}{ac\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] -3/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2/a/
c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5022, 5091, 5090, 4491, 3386, 3432}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\mathrm{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (3*Sqrt[Pi/2]*Sqrt[1 + a
^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2])
 - (Sqrt[(3*Pi)/2]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/(a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - (6a)\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,d$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)^{5/2}\sqrt{\tan^{-}}}}{c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\right.}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}}+\right.\right.}{ac^2\sqrt{c+a^2cx}}$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x,\right.}{2ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \sin\left(x^2\right)\,dx\right.}{ac^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan^{-1}(\right.}{ac^2\sqrt{c+a^2cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 158, normalized size = 1.01

$$\frac{-8+(1+a^2x^2)^{3/2}\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{4ac\,(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] (-8 + (1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[
a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I
)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (3*I)*ArcTan[a*x]])))/(4*a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

Maple [F]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1038 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{2}{acx\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}}-\frac{6\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\operatorname{ArcTan}(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{\frac{2\pi}{3}}\,\sqrt{1+a}}{}$$

```
[Out] -2/3*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-6*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2/a/c/x/(a
^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/(a^2*c*x^2+c)^(5/2
)/arctan(a*x)^(1/2),x)/a
```

Rubi [A]
time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[2*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2])
 - (2*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]
]])/(c^2*Sqrt[c + a^2*c*x^2]) - (2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*
Sqrt[ArcTan[a*x]]), x])/a
```

Rubi steps

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx = -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

$$= -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - \frac{6\sqrt{2\pi}\sqrt{1+a^2x^2}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\tan\right.}{c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A]
time = 4.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\,\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1039 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}-\frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-10a\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{A}}\right.$$

[Out] -2/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]

[Out] -2/(a*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 10*a*Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]

Rubi steps

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{4\int\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}}{a}$$

Mathematica [A]
time = 6.87, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]

[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1040 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^3(c+a^2cx^2)^{3/2}\sqrt{\text{ArcTan}(ax)}}-\frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\text{ArcTan}(ax)}},x\right)}{a}-12a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\text{A}}}\right.$$

```
[Out] -2/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*
c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x^2/(a^2*c*x^2+c)
^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] -2/(a*c*x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Defer[Int][1/(x^4
*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 12*a*Defer[Int][1/(x^2*(
c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^3(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{6\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}\,dx}{a}$$

Mathematica [A]
time = 19.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\,\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 2.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1041 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{acx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\mathrm{ArcTan}(ax)}}-\frac{8\mathrm{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\mathrm{ArcTan}(ax)}},x\right)}{a}-14a\mathrm{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\sqrt{}}\right.$$

[Out] -2/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-14*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]

[Out] -2/(a*c*x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (8*Defer[Int][1/(x^5*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a - 14*a*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx=-\frac{2}{acx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}-\frac{8\int\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\tan^{-1}(ax)}}}{a}$$

Mathematica [A]
time = 16.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 5.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1042 $\int \frac{x^m(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
[Out] int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

3.1043 $\int \frac{x(c+a^2cx^2)}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{x(c+a^2cx^2)}{\operatorname{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int\frac{x}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx+\int\frac{a^2x^3}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(x/atan(a*x)**(5/2), x) + Integral(a**2*x**3/atan(a*x)**(5/2), x
))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(c\,a^2\,x^2+c)}{\operatorname{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

3.1044 $\int \frac{c+a^2cx^2}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{c+a^2cx^2}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{c+a^2cx^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
[Out] int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/atan(a*x)**(5/2),x)
[Out] c*(Integral(a**2*x**2/atan(a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x)
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/atan(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)/atan(a*x)^(5/2), x)
```

3.1045 $\int \frac{c+a^2cx^2}{x\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{c+a^2cx^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{c+a^2cx^2}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^2x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/x/atan(a*x)**(5/2),x)
```

```
[Out] c*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(a**2*x/atan(a*x)**(5/2),
x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)),x)
[Out] int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)), x)
```

3.1046 $\int \frac{x^m(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\operatorname{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^2}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.51, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

```
[Out] int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

3.1047 $\int \frac{x(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{x}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx+\int\frac{2a^{2}x^{3}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx+\int\frac{a^{4}x^{5}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(x/atan(a*x)**(5/2), x) + Integral(2*a**2*x**3/atan(a*x)**(5/
2), x) + Integral(a**4*x**5/atan(a*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

3.1048 $\int \frac{(c+a^2cx^2)^2}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\frac{2a^{2}x^{2}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int\frac{a^{4}x^{4}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int\frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/atan(a*x)**(5/2), x) + Integral(a**4*x**4/atan(a
*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/atan(a*x)^(5/2),x)
```

```
[Out] int((c + a^2*c*x^2)^2/atan(a*x)^(5/2), x)
```

3.1049 $\int \frac{(c+a^2cx^2)^2}{x\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^2}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \frac{1}{x \operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\, dx + \int \frac{2a^{2}x}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\, dx + \int \frac{a^{4}x^{3}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(5/2),x)
```

```
[Out] c**2*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(2*a**2*x/atan(a*x)**(5
/2), x) + Integral(a**4*x**3/atan(a*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^2}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)),x)
[Out] int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)), x)
```

3.1050 $\int \frac{x^m(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*x
)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

3.1051 $\int \frac{x(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^3}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}\, dx + \int \frac{3a^{2}x^{3}}{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}\, dx + \int \frac{3a^{4}x^{5}}{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}\, dx + \int \frac{a^{6}x^{7}}{\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(x/atan(a*x)**(5/2), x) + Integral(3*a**2*x**3/atan(a*x)**(5/
2), x) + Integral(3*a**4*x**5/atan(a*x)**(5/2), x) + Integral(a**6*x**7/ata
n(a*x)**(5/2), x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
```

```
[Out] int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

3.1052 $\int \frac{(c+a^2cx^2)^3}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int\frac{3a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int\frac{3a^4x^4}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int\frac{a^6x^6}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int\frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/atan(a*x)**(5/2), x) + Integral(3*a**4*x**4/atan
(a*x)**(5/2), x) + Integral(a**6*x**6/atan(a*x)**(5/2), x) + Integral(atan(
a*x)**(-5/2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/atan(a*x)^(5/2),x)
```

```
[Out] int((c + a^2*c*x^2)^3/atan(a*x)^(5/2), x)
```

3.1053 $\int \frac{(c+a^2cx^2)^3}{x\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 2.79, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^3}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{3a^{2}x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{3a^{4}x^{3}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{a^{6}x^{5}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(5/2),x)
```

```
[Out] c**3*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(3*a**2*x/atan(a*x)**(5
/2), x) + Integral(3*a**4*x**3/atan(a*x)**(5/2), x) + Integral(a**6*x**5/at
an(a*x)**(5/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)),x)
[Out] int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)), x)
```

3.1054 $\int \frac{x^m}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{2x^m}{3ac\text{ArcTan}(ax)^{3/2}}+\frac{2m\text{Int}\left(\frac{x^{-1+m}}{\text{ArcTan}(ax)^{3/2}},x\right)}{3ac}$$

[Out] -2/3*x^m/a/c/arctan(a*x)^(3/2)+2/3*m*Unintegrable(x^(-1+m)/arctan(a*x)^(3/2),x)/a/c

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]

[Out] (-2*x^m)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*m*Defer[Int][x^(-1 + m)/ArcTan[a*x]^(3/2), x])/(3*a*c)

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^m}{3ac\tan^{-1}(ax)^{3/2}}+\frac{(2m)\int \frac{x^{-1+m}}{\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]

[Out] Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1055 $\int \frac{x}{(c+a^2cx^2)\mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{2x}{3ac\text{ArcTan}(ax)^{3/2}}+\frac{2\text{Int}\left(\frac{1}{\text{ArcTan}(ax)^{3/2}},x\right)}{3ac}$$

[Out] -2/3*x/a/c/arctan(a*x)^(3/2)+2/3*Unintegrable(1/arctan(a*x)^(3/2),x)/a/c

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]

[Out] (-2*x)/(3*a*c*ArcTan[a*x]^(3/2)) + (2*Defer[Int][ArcTan[a*x]^(-3/2), x])/(3*a*c)

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac\tan^{-1}(ax)^{3/2}}+\frac{2\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+a^2cx^2)\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]

[Out] Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x/(a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1056 $\int \frac{1}{(c+a^2cx^2)\,\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3ac\text{ArcTan}(ax)^{3/2}}$$

```
[Out] -2/3/a/c/arctan(a*x)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5004}

$$-\frac{2}{3ac\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*ArcTan[a*x]^(3/2))
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac\tan^{-1}(ax)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2}{3ac\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*ArcTan[a*x]^(3/2))
```

Maple [A]
time = 0.25, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{2}{3ac\arctan(ax)^{\frac{3}{2}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/c/arctan(a*x)^(3/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 3.09, size = 14, normalized size = 0.78

$$-\frac{2}{3\,ac\arctan\left(ax\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/(a*c*arctan(a*x)^(3/2))
```

Sympy [**A**]
time = 4.97, size = 15, normalized size = 0.83

$$-\frac{2}{3ac\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] -2/(3*a*c*atan(a*x)**(3/2))
```

Giac [**A**]
time = 0.52, size = 14, normalized size = 0.78

$$-\frac{2}{3\,ac\arctan\left(ax\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/(a*c*arctan(a*x)^(3/2))
```

Mupad [B]
time = 0.33, size = 14, normalized size = 0.78

$$-\frac{2}{3\,a\,c\,\mathrm{atan}\left(a\,x\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

```
[Out] -2/(3*a*c*atan(a*x)^(3/2))
```

3.1057 $\int \frac{1}{x(c+a^2cx^2)\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2}{3acx\text{ArcTan}(ax)^{3/2}}-\frac{2\text{Int}\left(\frac{1}{x^2\text{ArcTan}(ax)^{3/2}},x\right)}{3ac}$$

```
[Out] -2/3/a/c/x/arctan(a*x)^(3/2)-2/3*Unintegrable(1/x^2/arctan(a*x)^(3/2),x)/a/
c
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Defer[Int][1/(x^2*ArcTan[a*x]^(3/2)), x
])/(3*a*c)
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3acx\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^2\tan^{-1}(ax)^{3/2}}\,dx}{3ac}$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1058 $\int \frac{x^m}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}} dx = \int \frac{x^m}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}} dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(5/2)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1059 $\int \frac{x^3}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{5/2}}\, dx$

Optimal. Leaf size=186

$$-\frac{2x^3}{3ac^2(1+a^2x^2)\,\text{ArcTan}(ax)^{3/2}}-\frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}-\frac{4x^4}{3c^2(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+\frac{4\sqrt{\pi}\,S}{}$$

```
[Out] -2/3*x^3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+4*FresnelS(2*arctan(a*x)^(1/2)
/Pi^(1/2))*Pi^(1/2)/a^4/c^2-4*x^2/a^2/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-4/3
*x^4/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+16/3*Unintegrable(x^3/(a^2*c*x^2+c)^
2/arctan(a*x)^(1/2),x)+8/3*a^2*Unintegrable(x^5/(a^2*c*x^2+c)^2/arctan(a*x)
^(1/2),x)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{5/2}}\, dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) - (4*x^4)/(3*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]
]) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^4*c^2) + (16*
Defer[Int][x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (8*a^2*Defer[
Int][x^5/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(2a)\int \frac{}{(c}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

$$= -\frac{2x^3}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{}{3c^2(1+}$$

Mathematica [A]
time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

[Out] int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)

[Out] Integral(x**3/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**2

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1060 $\int \frac{x^2}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=180

$$-\frac{2x^2}{3ac^2(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c^2(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}+\frac{16\sqrt{\operatorname{ArcTan}(ax)}}{3a^3c^2}-\frac{32\sqrt{\operatorname{ArcTan}(ax)}}{3a^3c^2(1+a^2x^2)}$$

```
[Out] -2/3*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8/3*FresnelC(2*arctan(a*x)^(1/
2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2-8/3*x/a^2/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+1
6/3*arctan(a*x)^(1/2)/a^3/c^2-32/3*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)+16
/3*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)
```

Rubi [A]
time = 0.17, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5062, 5052, 5050, 5024, 3393, 3385, 3433}

$$\frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\operatorname{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3a^3c^2}+\frac{16\sqrt{\operatorname{ArcTan}(ax)}}{3a^3c^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\operatorname{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c^2(a^2x^2+1)\sqrt{\operatorname{ArcTan}(ax)}}+\frac{16(1-a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{3a^3c^2(a^2x^2+1)}-\frac{32\sqrt{\operatorname{ArcTan}(ax)}}{3a^3c^2(a^2x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^2)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^2*(1 +
a^2*x^2)*Sqrt[ArcTan[a*x]]) + (16*Sqrt[ArcTan[a*x]])/(3*a^3*c^2) - (32*Sqrt
[ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*
x]])/(3*a^3*c^2*(1 + a^2*x^2)) + (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/(3*a^3*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^2 \tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16(1 \cdots}{\cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\cdots}}{3a^3c^2 \cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\cdots}}{3a^3c^2 \cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{32\sqrt{\cdots}}{3a^3c^2 \cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{16\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 162, normalized size = 0.90

$$\frac{-2ax(ax+4\text{ArcTan}(ax)) + 4\sqrt{\pi}\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right) + \sqrt{2}\left(1+a^2x^2\right)(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right) + \sqrt{2}\left(1+a^2x^2\right)(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{3a^3c^2\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*a*x*(a*x + 4*ArcTan[a*x]) + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*
FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*((-I)*ArcT
an[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]] + Sqrt[2]*(1 + a^2*x^2)*(I*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2)*Ar
cTan[a*x]^(3/2))
```

Maple [A]
time = 0.35, size = 62, normalized size = 0.34

method	result	size
default	$-\frac{-8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\sin(2\arctan(ax))\arctan(ax)-\cos(2\arctan(ax))+1}{3c^2a^3\arctan(ax)^{\frac{3}{2}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/a^3*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x
)^(3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))+1)/arctan(a*x)^
(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\,\mathrm{atan}^{\frac{5}{2}}(ax)+2a^2x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)+\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) +
atan(a*x)**(5/2)), x)/c**2
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1061 $\int \frac{x}{(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2x}{3ac^2\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}}-\frac{4(1-a^2x^2)}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}-\frac{8\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}$$

```
[Out] -2/3*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-8/3*FresnelS(2*arctan(a*x)^(1/2)
/Pi^(1/2))*Pi^(1/2)/a^2/c^2-4/3*(-a^2*x^2+1)/a^2/c^2/(a^2*x^2+1)/arctan(a*x
)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5052, 5090, 4491, 12, 3386, 3432}

$$-\frac{8\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\text{ArcTan}(ax)^{3/2}}-\frac{4(1-a^2x^2)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*(1 - a^2*x^2))/(3*a^2
*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(3*a^2*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{16}{3}\int$$

$$= -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{16\mathrm{Subs}}{}$$

$$= -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{16\mathrm{Subs}}{}$$

$$= -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{8\mathrm{Subst}}{}$$

$$= -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{16\mathrm{Subs}}{}$$

$$= -\frac{2x}{3ac^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\tan^{-1}(ax)}} - \frac{8\sqrt{\pi}\,S}{}$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.87

$$-\frac{2\left(ax+(2-2a^2x^2)\,\mathrm{ArcTan}(ax)+4\sqrt{\pi}\,(1+a^2x^2)\,\mathrm{ArcTan}(ax)^{3/2}S\left(\frac{2\sqrt{\mathrm{ArcTan}(ax)}}{\sqrt{\pi}}\right)\right)}{3a^2c^2(1+a^2x^2)\,\mathrm{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*(a*x + (2 - 2*a^2*x^2)*ArcTan[a*x] + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*
x]^(3/2)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]))/(3*a^2*c^2*(1 + a^2*x^2
)*ArcTan[a*x]^(3/2))
```

Maple [A]
time = 0.32, size = 59, normalized size = 0.58

method	result	size
default	$-\frac{8\sqrt{\pi}\, \mathrm{S}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right) \arctan(ax)^{\frac{3}{2}}+4\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{3c^{2}a^{2}\arctan(ax)^{\frac{3}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/a^2*(8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)
^(3/2)+4*cos(2*arctan(a*x))*arctan(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^(3/
2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{4}x^{4}\operatorname{atan}^{\frac{5}{2}}(ax)+2a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + ata
n(a*x)**(5/2)), x)/c**2
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1062 $\int \frac{1}{(c+a^2cx^2)^2 \mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=174

$$-\frac{2}{3ac^2(1+a^2x^2)\operatorname{ArcTan}(ax)^{3/2}}+\frac{8x}{3c^2(1+a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}-\frac{16\sqrt{\operatorname{ArcTan}(ax)}}{3ac^2}+\frac{32\sqrt{\operatorname{ArcTan}(ax)}}{3ac^2(1+a^2x^2)}-$$

```
[Out] -2/3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-8/3*FresnelC(2*arctan(a*x)^(1/2)/P
i^(1/2))*Pi^(1/2)/a/c^2+8/3*x/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-16/3*arctan
(a*x)^(1/2)/a/c^2+32/3*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)-16/3*(-a^2*x^2+1
)*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)
```

Rubi [A]

time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5022, 5052, 5050, 5024, 3393, 3385, 3433}

$$\frac{8x}{3c^2(a^2x^2+1)\sqrt{\operatorname{ArcTan}(ax)}}-\frac{16(1-a^2x^2)\sqrt{\operatorname{ArcTan}(ax)}}{3ac^2(a^2x^2+1)}+\frac{32\sqrt{\operatorname{ArcTan}(ax)}}{3ac^2(a^2x^2+1)}-\frac{2}{3ac^2(a^2x^2+1)\operatorname{ArcTan}(ax)^{3/2}}-\frac{8\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\operatorname{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3ac^2}-\frac{16\sqrt{\operatorname{ArcTan}(ax)}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + (8*x)/(3*c^2*(1 + a^2*x^2)*S
qrt[ArcTan[a*x]]) - (16*Sqrt[ArcTan[a*x]])/(3*a*c^2) + (32*Sqrt[ArcTan[a*x]
])/(3*a*c^2*(1 + a^2*x^2)) - (16*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(3*a*c^2*
(1 + a^2*x^2)) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a
*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5052

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Dist[4/(b^2*(p + 1)*(p + 2)), Int[x*((a + b*ArcTan[c*x])^(p + 2
)/(d + e*x^2)^2), x], x] - Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/
(b^2*e*(p + 1)*(p + 2)*(d + e*x^2))), x]) /; FreeQ[{a, b, c, d, e}, x] && E
qQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(4a)\int \frac{x}{\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16(1-}{3a}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan}}{3ac^2\,(1}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan}}{3ac^2\,(1}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{32\sqrt{\tan}}{3ac^2\,(1}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan}}{3a}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan}}{3a}$$

$$= -\frac{2}{3ac^2\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} - \frac{16\sqrt{\tan}}{3a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 170, normalized size = 0.98

$$\frac{-2+8ax\text{ArcTan}(ax)-4\sqrt{\pi}\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)+\sqrt{2}\left(1+a^2x^2\right)\sqrt{i\text{ArcTan}(ax)}\sqrt{\text{ArcTan}(ax)^2}\,\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+\frac{\sqrt{2}\left(1+a^2x^2\right)\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{\sqrt{i\text{ArcTan}(ax)}}}{3c^2\left(a+a^3x^2\right)\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2 + 8*a*x*ArcTan[a*x] - 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*Fresne
lC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[a*
```

```
x]]*Sqrt[ArcTan[a*x]^2]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (Sqrt[2]*(1 + a^2*
x^2)*ArcTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(3*c
^2*(a + a^3*x^2)*ArcTan[a*x]^(3/2))
```

Maple [A]
time = 0.34, size = 62, normalized size = 0.36

method	result	size
default	$\frac{-8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\sin(2\arctan(ax))\arctan(ax)-\cos(2\arctan(ax))-1}{3c^2a\arctan(ax)^{\frac{3}{2}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c^2/a*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^(
3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))-1)/arctan(a*x)^(3/
2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + ata
n(a*x)**(5/2)), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1063 $\int \frac{1}{x(c+a^2cx^2)^2\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{2}{3ac^2x\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}}+\frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}+\frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}+\frac{4\sqrt{\pi}}{}$$

```
[Out] -2/3/a/c^2/x/(a^2*x^2+1)/arctan(a*x)^(3/2)+4*FresnelS(2*arctan(a*x)^(1/2)/P
i^(1/2))*Pi^(1/2)/c^2+4/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4/3/a^2/c^2/x^2/(
a^2*x^2+1)/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2+c)^2/arctan(
a*x)^(1/2),x)/a^2+16/3*Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x
)
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(c^2*(1 + a^2*x^2)*Sqrt[
ArcTan[a*x]]) + 4/(3*a^2*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*Sqrt
[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/c^2 + (8*Defer[Int][1/(x^3*(
c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (16*Defer[Int][1/(x*(c +
 a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (2a)$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

$$= -\frac{2}{3ac^2x\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2c^2}$$

Mathematica [A]
time = 3.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**4*x**5*atan(a*x)**(5/2) + 2*a**2*x**3*atan(a*x)**(5/2) + x*a
tan(a*x)**(5/2)), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \operatorname{atan}(a x)^{5/2} (c a^2 x^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1064 $\int \frac{1}{x^2(c+a^2cx^2)^2\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{2}{3ac^2x^2(1+a^2x^2)\,\text{ArcTan}(ax)^{3/2}}+\frac{8}{3a^2c^2x^3(1+a^2x^2)\,\sqrt{\text{ArcTan}(ax)}}+\frac{16}{3c^2x(1+a^2x^2)\,\sqrt{\text{ArcTan}(ax)}}+\frac{1\cdots}{\cdots}$$

```
[Out] -2/3/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8*a*FresnelC(2*arctan(a*x)^(1/
2)/Pi^(1/2))*Pi^(1/2)/c^2+8/3/a^2/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)+16/
3/c^2/x/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*a*arctan(a*x)^(1/2)/c^2+8*Unintegr
able(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+56/3*Unintegrable(1/x^2
/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\,(c+a^2cx^2)^2\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^2*x^3*(1 + a^
2*x^2)*Sqrt[ArcTan[a*x]]) + 16/(3*c^2*x*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*a*Sqrt[ArcTan[a*x]])/c^2 + (8*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/c^2 + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])
, x])/a^2 + (56*Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])
/3
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(8$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^2x^2(1+a^2x^2)\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 4.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

[Out] int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^6\,\mathrm{atan}^{\frac{5}{2}}(ax)+2a^2x^4\,\mathrm{atan}^{\frac{5}{2}}(ax)+x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)

[Out] Integral(1/(a**4*x**6*atan(a*x)**(5/2) + 2*a**2*x**4*atan(a*x)**(5/2) + x**2*atan(a*x)**(5/2)), x)/c**2

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1065 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\text{ArcTan}(ax)^{3/2}}+\frac{4}{a^2c^2x^4\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}+\frac{20}{3c^2x^2\left(1+a^2x^2\right)\sqrt{\text{ArcTan}(ax)}}+\frac{16}{}$$

```
[Out] -2/3/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(3/2)+4/a^2/c^2/x^4/(a^2*x^2+1)/arct
an(a*x)^(1/2)+20/3/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*Unintegrable(1/
x^5/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+112/3*Unintegrable(1/x^3/(a^2*
c*x^2+c)^2/arctan(a*x)^(1/2),x)+80/3*a^2*Unintegrable(1/x/(a^2*c*x^2+c)^2/a
rctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^3*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c^2*x^4*(1 + a^2*
x^2)*Sqrt[ArcTan[a*x]]) + 20/(3*c^2*x^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/a^2 + (112*
Defer[Int][1/(x^3*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + (80*a^2*Def
er[Int][1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\tan^{-1}(ax)^{3/2}}\,dx}{a}-\frac{1}{3}(10$$

$$=-\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\tan^{-1}(ax)^{3/2}}+\frac{4}{a^2c^2x^4\left(1+a^2x^2\right)\sqrt{\tan^{-1}(ax)}}+\frac{}{3}$$

Mathematica [A]
time = 3.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{7} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 2 a^{2} x^{5} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**4*x**7*atan(a*x)**(5/2) + 2*a**2*x**5*atan(a*x)**(5/2) + x**
3*atan(a*x)**(5/2)), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1066 $\int \frac{1}{x^4(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{2}{3ac^2x^4(1+a^2x^2)\text{ArcTan}(ax)^{3/2}}+\frac{16}{3a^2c^2x^5(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+\frac{8}{c^2x^3(1+a^2x^2)\sqrt{\text{ArcTan}(ax)}}+$$

```
[Out] -2/3/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(3/2)+16/3/a^2/c^2/x^5/(a^2*x^2+1)/a
rctan(a*x)^(1/2)+8/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)+80/3*Unintegrable(
1/x^6/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+184/3*Unintegrable(1/x^4/(a^
2*c*x^2+c)^2/arctan(a*x)^(1/2),x)+40*a^2*Unintegrable(1/x^2/(a^2*c*x^2+c)^2
/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,
Rules used = {}

$$\int \frac{1}{x^4(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^2*x^4*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^2*x^5*(1 + a
^2*x^2)*Sqrt[ArcTan[a*x]]) + 8/(c^2*x^3*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) +
(80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (
184*Defer[Int][1/(x^4*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x])/3 + 40*a^2*
Defer[Int][1/(x^2*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4(c+a^2cx^2)^2\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^4(1+a^2x^2)\tan^{-1}(ax)^{3/2}}-\frac{8\int \frac{1}{x^5(c+a^2cx^2)^2\tan^{-1}(ax)^{3/2}}\,dx}{3a}-(4a$$

$$= -\frac{2}{3ac^2x^4(1+a^2x^2)\tan^{-1}(ax)^{3/2}}+\frac{16}{3a^2c^2x^5(1+a^2x^2)\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 8.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(c+a^2cx^2)^2 \text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]

[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]

Maple [A]
time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

[Out] int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{8} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 2 a^{2} x^{6} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + x^{4} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)

```
[Out] Integral(1/(a**4*x**8*atan(a*x)**(5/2) + 2*a**2*x**6*atan(a*x)**(5/2) + x**
4*atan(a*x)**(5/2)), x)/c**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1067 $\int \frac{x^m}{(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}} dx = \int \frac{x^m}{(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}} dx$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(a^2 c\, x^2 + c)^3 \arctan(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*
x)^(5/2)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1068 $\int \frac{x^3}{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{2x^3}{3ac^3(1+a^2x^2)^2 \operatorname{ArcTan}(ax)^{3/2}}-\frac{4x^2}{a^2c^3(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}+\frac{4x^4}{3c^3(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}+\frac{4\sqrt{2}\cdots}{\cdots}$$

[Out] -2/3*x^3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4/3*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3+4/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c^3-4*x^2/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4/3*x^4/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)

Rubi [A]
time = 0.41, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5088, 5062, 5090, 4491, 3386, 3432}

$$\frac{4\sqrt{2\pi}\, S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\operatorname{ArcTan}(ax)}\right)}{3a^4c^3}-\frac{4\sqrt{\pi}\, S\left(\frac{2\sqrt{\operatorname{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3a^4c^3}-\frac{4x^2}{a^2c^3(a^2x^2+1)^2\sqrt{\operatorname{ArcTan}(ax)}}+\frac{4x^4}{3c^3(a^2x^2+1)^2\sqrt{\operatorname{ArcTan}(ax)}}-\frac{2x^3}{3ac^3(a^2x^2+1)^2\operatorname{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]

[Out] (-2*x^3)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^4)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^4*c^3) - (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^4*c^3)

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx}{a} - \frac{1}{3}(2a)\int$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

$$= -\frac{2x^3}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 227, normalized size = 1.42

$$\frac{i\sqrt{2}\left(1+a^2x^2\right)^2(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+\sqrt{2}\left(1+a^2x^2\right)^2\sqrt{i\text{ArcTan}(ax)}\,\text{ArcTan}(ax)\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)-2\left(a^2x^2(ax+\left(6-2a^2x^2\right)\text{ArcTan}(ax))+i\left(1+a^2x^2\right)^2(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+\left(1+a^2x^2\right)^2\sqrt{i\text{ArcTan}(ax)}\,\text{ArcTan}(ax)\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{3a^4c^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTa
n[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2
, (2*I)*ArcTan[a*x]] - 2*(a^2*x^2*(a*x + (6 - 2*a^2*x^2)*ArcTan[a*x]) + I*(
1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]] + (1
 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[a*x]]
))/(3*a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))
```

Maple [A]
time = 0.44, size = 112, normalized size = 0.70

method	result
default	$-\frac{-16\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\cos(2\arctan(a}{12c^3a^4\arctan(ax)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/c^3/a^4*(-16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)
)*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)-8*cos(4*arctan(a*x))*a
rctan(a*x)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^6x^6\,\mathrm{atan}^{\frac{5}{2}}(ax)+3a^4x^4\,\mathrm{atan}^{\frac{5}{2}}(ax)+3a^2x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)+\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1069 $\int \frac{x^2}{(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{8x^3}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{4\sqrt{2}}{}$$

[Out] -2/3*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+4/3*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3-8/3*x/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+8/3*x^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)

Rubi [A]
time = 0.48, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5088, 5090, 3393, 3385, 3433, 4491, 5024}

$$\frac{4\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{3a^3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\text{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c^3\left(a^2x^2+1\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{8x^3}{3c^3\left(a^2x^2+1\right)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]

[Out] (-2*x^2)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (8*x^3)/(3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^3*c^3)

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(4a)\int \frac{}{(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

$$= -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3c^3\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 259, normalized size = 2.01

$$\frac{\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3} - \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{a^3} + \frac{-\frac{8x^2}{a(1+a^2x^2)^2} + \frac{32x^3\text{ArcTan}(ax)}{(1+a^2x^2)^2} - \frac{32x\,\text{ArcTan}(ax)}{(a+a^3x^2)^2} + \frac{4\sqrt{2}(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)}{a^3} + \frac{4\sqrt{2}(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{a^3} + \frac{7(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)}{a^3} + \frac{7(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{a^3}}{\text{ArcTan}(ax)^{3/2}}}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] ((2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/a^3 - (16*Sqrt[Pi]
*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a^3 + ((-8*x^2)/(a*(1 + a^2*x^2)
^2) + (32*x^3*ArcTan[a*x])/(1 + a^2*x^2)^2 - (32*x*ArcTan[a*x])/(a + a^3*x^
2)^2 + (4*Sqrt[2]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/
```

```
a^3 + (4*Sqrt[2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/a^3 +
 (7*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a^3 + (7*(I*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcTan[a*x]])/a^3)/ArcTan[a*x]^(3/2))/(12
*c^3)
```

Maple [A]
time = 0.38, size = 68, normalized size = 0.53

method	result
default	$-\frac{-16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\sin(4\arctan(ax))\arctan(ax)-\cos(4\arctan(ax))+1}{12c^{3}a^{3}\arctan(ax)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12/c^3/a^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+8*sin(4*arctan(a*x))*arctan(a*x)-cos(4*arctan(a*x
))+1)/arctan(a*x)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{2}}{a^{6}x^{6}\operatorname{atan}^{\frac{5}{2}}(ax)+3a^{4}x^{4}\operatorname{atan}^{\frac{5}{2}}(ax)+3a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1070 $\int \frac{x}{(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2x}{3ac^3(1+a^2x^2)^2\text{ArcTan}(ax)^{3/2}}-\frac{4}{3a^2c^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}+\frac{4x^2}{c^3(1+a^2x^2)^2\sqrt{\text{ArcTan}(ax)}}-\frac{4\sqrt{2}\cdots}{\cdots}$$

[Out] -2/3*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4/3*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3-4/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^3-4/3/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4*x^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)

Rubi [A]
time = 0.36, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5088, 5090, 4491, 3386, 3432, 5022}

$$-\frac{4\sqrt{2\pi}\,S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{3a^2c^3}-\frac{4\sqrt{\pi}\,S\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3a^2c^3}+\frac{4x^2}{c^3(a^2x^2+1)^2\sqrt{\text{ArcTan}(ax)}}-\frac{2x}{3ac^3(a^2x^2+1)^2\text{ArcTan}(ax)^{3/2}}-\frac{4}{3a^2c^3(a^2x^2+1)^2\sqrt{\text{ArcTan}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]

[Out] (-2*x)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (4*x^2)/(c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c^3) - (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*a^2*c^3)

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\left(c+a^2cx^2\right)^3 \tan^{-1}(ax)^{3/2}}\,dx}{3a} - (2a)\int \frac{}{(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

$$= -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{c^3\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 220, normalized size = 1.42

$$\frac{i\sqrt{2}\left(1+a^2x^2\right)^2(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)+\sqrt{2}\left(1+a^2x^2\right)^2\sqrt{i\text{ArcTan}(ax)}\,\text{ArcTan}(ax)\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)+2\left(-ax-2\text{ArcTan}(ax)+6a^2x^2\text{ArcTan}(ax)+i\left(1+a^2x^2\right)^2(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)+\left(1+a^2x^2\right)^2\sqrt{i\text{ArcTan}(ax)}\,\text{ArcTan}(ax)\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)\right)}{3c^3\left(a+a^3x^2\right)^2\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTa
n[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2
, (2*I)*ArcTan[a*x]] + 2*(-(a*x) - 2*ArcTan[a*x] + 6*a^2*x^2*ArcTan[a*x] +
I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]] +
 (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[a*
x]]))/(3*c^3*(a + a^3*x^2)^2*ArcTan[a*x]^(3/2))
```

Maple [A]
time = 0.36, size = 110, normalized size = 0.71

method	result
default	$-\frac{16\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\mathrm{S}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\cos(2\arctan(ax}{12c^3a^2\arctan(ax)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/c^3/a^2*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))
*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)+8*cos(4*arctan(a*x))*ar
ctan(a*x)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*a
**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1071 $\int \frac{1}{(c+a^2cx^2)^3 \textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2}{3ac^3\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^{3/2}}+\frac{16x}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}-\frac{4\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{3ac^3}$$

```
[Out] -2/3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-8/3*FresnelC(2*arctan(a*x)^(1/2)
/Pi^(1/2))*Pi^(1/2)/a/c^3-4/3*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)
)*2^(1/2)*Pi^(1/2)/a/c^3+16/3*x/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5022, 5088, 5090, 4491, 3385, 3433, 5024, 3393}

$$\frac{16x}{3c^3\left(a^2x^2+1\right)^2\sqrt{\text{ArcTan}(ax)}}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\text{ArcTan}(ax)^{3/2}}-\frac{4\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{3ac^3}-\frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\text{ArcTan}(ax)}}{\sqrt{\pi}}\right)}{3ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + (16*x)/(3*c^3*(1 + a^2*x^2
)^2*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*
x]]])/(3*a*c^3) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*
a*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(8a)\int \frac{x}{(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{16}{3}\int$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{16\text{Subs}}{}$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{16\text{Subs}}{}$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{2\text{Subst}}{}$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\text{Subst}}{}$$

$$= -\frac{2}{3ac^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 186, normalized size = 1.49

$$\frac{2\left(-\frac{1}{a(1+a^2x^2)^2}+\frac{8x\text{ArcTan}(ax)}{(1+a^2x^2)^2}-\frac{\sqrt{2}\,(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-2i\text{ArcTan}(ax)\right)}{a}+\frac{\sqrt{2}\,\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},2i\text{ArcTan}(ax)\right)}{a\sqrt{i\text{ArcTan}(ax)}}-\frac{(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-4i\text{ArcTan}(ax)\right)}{a}+\frac{\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},4i\text{ArcTan}(ax)\right)}{a\sqrt{i\text{ArcTan}(ax)}}\right)}{3c^3\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (2*(-(1/(a*(1 + a^2*x^2)^2)) + (8*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (Sqrt[2]
*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/a + (Sqrt[2]*ArcT
an[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]]) - (((-I)*A
rcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a + (ArcTan[a*x]^2*Gamma[
1/2, (4*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]])))/(3*c^3*ArcTan[a*x]^(3/2)
)
```

Maple [A]
time = 0.40, size = 113, normalized size = 0.90

method	result
default	$\frac{-16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\arctan\left(ax\right)^{\frac{3}{2}}-32\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan\left(ax\right)}}{\sqrt{\pi}}\right)\arctan\left(ax\right)^{\frac{3}{2}}+}{12c^{3}a\arctan\left(ax\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/c^3/a*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))*arctan(a*x)^(3/2)-32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*a
rctan(a*x)^(3/2)+16*sin(2*arctan(a*x))*arctan(a*x)+8*sin(4*arctan(a*x))*arc
tan(a*x)-4*cos(2*arctan(a*x))-cos(4*arctan(a*x))-3)/arctan(a*x)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6}x^{6}\operatorname{atan}^{\frac{5}{2}}(ax)+3a^{4}x^{4}\operatorname{atan}^{\frac{5}{2}}(ax)+3a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)

[Out] Integral(1/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)

[Out] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)

3.1072 $\int \frac{1}{x(c+a^2cx^2)^3 \mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^{3/2}}+\frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{4}{3a^2c^3x^2\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+$$

```
[Out] -2/3/a/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+20/3*FresnelS(2*arctan(a*x)^(1
/2)/Pi^(1/2))*Pi^(1/2)/c^3+5/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2
))*2^(1/2)*Pi^(1/2)/c^3+20/3/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4/3/a^2/c^
3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2+c)^
3/arctan(a*x)^(1/2),x)/a^2+8*Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(
1/2),x)
```

Rubi [A]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 20/(3*c^3*(1 + a^2*x^2)^
2*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
+ (5*Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^3) + (20*Sqr
t[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(3*c^3) + (8*Defer[Int][1/(
x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 8*Defer[Int][1/(x*(
c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(10$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3ac^3x\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}} + \frac{20}{3c^3\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

Mathematica [A]
time = 4.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^7 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^5 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^3 \operatorname{atan}^{\frac{5}{2}}(ax)+x \operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**6*x**7*atan(a*x)**(5/2) + 3*a**4*x**5*atan(a*x)**(5/2) + 3*a
**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c**3
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

[Out] Timed out

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1073 $\int \frac{1}{x^2(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=231

$$-\frac{2}{3ac^3x^2(1+a^2x^2)^2 \text{ArcTan}(ax)^{3/2}}+\frac{8}{3a^2c^3x^3(1+a^2x^2)^2 \sqrt{\text{ArcTan}(ax)}}+\frac{8}{c^3x(1+a^2x^2)^2 \sqrt{\text{ArcTan}(ax)}}\cdot$$

```
[Out] -2/3/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+5/2*a*FresnelC(2*2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/c^3+20*a*FresnelC(2*arctan(a*x)^(
1/2)/Pi^(1/2))*Pi^(1/2)/c^3+8/3/a^2/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
+8/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+30*a*arctan(a*x)^(1/2)/c^3+8*Unint
egrable(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80/3*Unintegrable(1/
x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x^2*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c^3*x^3*(1 +
a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 8/(c^3*x*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
 + (30*a*Sqrt[ArcTan[a*x]])/c^3 + (5*a*Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqr
t[ArcTan[a*x]]])/c^3 + (20*a*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/c^3 + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a
^2 + (80*Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a} - (4a$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 5.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^6x^8\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^4\operatorname{atan}^{\frac{5}{2}}(ax)+x^2\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**6*x**8*atan(a*x)**(5/2) + 3*a**4*x**6*atan(a*x)**(5/2) + 3*a
**2*x**4*atan(a*x)**(5/2) + x**2*atan(a*x)**(5/2)), x)/c**3
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
[Out] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1074 $\int \frac{1}{x^3(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{2}{3ac^3x^3(1+a^2x^2)^2\operatorname{ArcTan}(ax)^{3/2}}+\frac{4}{a^2c^3x^4(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}+\frac{28}{3c^3x^2(1+a^2x^2)^2\sqrt{\operatorname{ArcTan}(ax)}}$$

```
[Out] -2/3/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+4/a^2/c^3/x^4/(a^2*x^2+1)^2/
arctan(a*x)^(1/2)+28/3/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+16*Unintegra
ble(1/x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+152/3*Unintegrable(1/x^3
/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+56*a^2*Unintegrable(1/x/(a^2*c*x^2+c)
^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3(c+a^2cx^2)^3\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 4/(a^2*c^3*x^4*(1 + a^
2*x^2)^2*Sqrt[ArcTan[a*x]]) + 28/(3*c^3*x^2*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x
]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/a^2 +
 (152*Defer[Int][1/(x^3*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3 + 56*a^
2*Defer[Int][1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3(c+a^2cx^2)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^4(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx}{a}-\frac{1}{3}$$

$$= -\frac{2}{3ac^3x^3(1+a^2x^2)^2\tan^{-1}(ax)^{3/2}}+\frac{4}{a^2c^3x^4(1+a^2x^2)^2\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 4.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(c+a^2cx^2)^3\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 3.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{9} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{4} x^{7} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{2} x^{5} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**6*x**9*atan(a*x)**(5/2) + 3*a**4*x**7*atan(a*x)**(5/2) + 3*a
**2*x**5*atan(a*x)**(5/2) + x**3*atan(a*x)**(5/2)), x)/c**3
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1075 $\int \frac{1}{x^4(c+a^2cx^2)^3\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\text{ArcTan}(ax)^{3/2}}+\frac{16}{3a^2c^3x^5\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}+\frac{32}{3c^3x^3\left(1+a^2x^2\right)^2\sqrt{\text{ArcTan}(ax)}}$$

```
[Out] -2/3/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+16/3/a^2/c^3/x^5/(a^2*x^2+1)
^2/arctan(a*x)^(1/2)+32/3/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+80/3*Unin
tegrable(1/x^6/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80*Unintegrable(1/x
^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+224/3*a^2*Unintegrable(1/x^2/(a^2*c
*x^2+c)^3/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c^3*x^4*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c^3*x^5*(1 +
 a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + 32/(3*c^3*x^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[
a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/(3
*a^2) + 80*Defer[Int][1/(x^4*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x] + (22
4*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}-\frac{8\int \frac{1}{x^5(c+a^2cx^2)^3\tan^{-1}(ax)^{3/2}}\,dx}{3a}-\frac{1}{3}(1$$

$$=-\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\tan^{-1}(ax)^{3/2}}+\frac{16}{3a^2c^3x^5\left(1+a^2x^2\right)^2\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 9.95, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{6} x^{10} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{4} x^{8} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{2} x^{6} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + x^{4} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

```
[Out] Integral(1/(a**6*x**10*atan(a*x)**(5/2) + 3*a**4*x**8*atan(a*x)**(5/2) + 3*
a**2*x**6*atan(a*x)**(5/2) + x**4*atan(a*x)**(5/2)), x)/c**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

```
[Out] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1076 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

3.1077 $\int \frac{x\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
[Out] int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

3.1078 $\int \frac{\sqrt{c+a^2cx^2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2), x)
```

3.1079 $\int \frac{\sqrt{c+a^2cx^2}}{x\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 9.60, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)),x)
```

```
[Out] int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)), x)
```

3.1080 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(5/2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

3.1081 $\int \frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^{3/2}}{\operatorname{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
[Out] int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

3.1082 $\int \frac{(c+a^2cx^2)^{3/2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2), x)
```

3.1083 $\int \frac{(c+a^2cx^2)^{3/2}}{x\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 7.79, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)),x)
[Out] int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)), x)
```

3.1084 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arctan
(a*x)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
[Out] int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

3.1085 $\int \frac{x\left(c+a^2cx^2\right)^{5/2}}{\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Defer[Int][(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 2.22, size = 0, normalized size = 0.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

```
[Out] Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
[Out] int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

3.1086 $\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}} dx = \int \frac{(c+a^2cx^2)^{5/2}}{\tan^{-1}(ax)^{5/2}} dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

Maple [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2), x)
```

3.1087 $\int \frac{(c+a^2cx^2)^{5/2}}{x\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 4.51, size = 0, normalized size = 0.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)),x)
[Out] int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)), x)
```

3.1088 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\arctan(ax)^{\frac{5}{2}}\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1089 $\int \frac{x}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.98, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan(ax)^{\frac{5}{2}}\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1090 $\int \frac{1}{\sqrt{c+a^2cx^2}\,\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^{\frac{5}{2}}\,\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^{2}x^{2}+1\right)}\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1091 $\int \frac{1}{x\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\sqrt{c+a^2cx^2}}{3acx\text{ArcTan}(ax)^{3/2}}-\frac{2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}},x\right)}{3a}$$

```
[Out] -2/3*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(3/2)-2/3*Unintegrable(1/x^2/arc
tan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2])/(3*a*c*x*ArcTan[a*x]^(3/2)) - (2*Defer[Int][1/(x^2
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x])/(3*a)
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{3acx\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

Mathematica [A]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arctan(ax)^{\frac{5}{2}} \sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
[Out] int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{c\,(a^2x^2+1)}\, \operatorname{atan}^{\frac{5}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1092 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 10.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\arctan\left(ax\right)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1093 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcta
n(a*x)^(5/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1094 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}-\frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}-\frac{8x^4}{3c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{8\sqrt{2\pi}}{}$$

```
[Out] -2/3*x^3/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8*FresnelS(2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+
c)^(1/2)-4*x^2/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-8/3*x^4/c/(a^2*c
*x^2+c)^(1/2)/arctan(a*x)^(1/2)+44/3*Unintegrable(x^3/(a^2*c*x^2+c)^(3/2)/a
rctan(a*x)^(1/2),x)+8*a^2*Unintegrable(x^5/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^
(1/2),x)
```

Rubi [A]
time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (8*x^4)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(a^4*c*Sqrt[c + a^2*c*x^2]) + (44*Defer[Int][x^3/((c + a^2*c*
x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3 + 8*a^2*Defer[Int][x^5/((c + a^2*c*x^2
)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(4a)$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{c}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{c}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{c}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{c}}$$

$$= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{c}}$$

Mathematica [A]
time = 1.89, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 6.42, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1095 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}-\frac{4x^3}{3c\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}+\frac{8\sqrt{2\pi}}{\quad}$$

```
[Out] -2/3*x^2/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/3*FresnelC(2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^
2+c)^(1/2)-8/3*x/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4/3*x^3/c/(a^2
*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4*Unintegrable(x^2/(a^2*c*x^2+c)^(3/2)/ar
ctan(a*x)^(1/2),x)+8/3*a^2*Unintegrable(x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)
^(1/2),x)
```

Rubi [A]

time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^2)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*x^3)/(3*c*Sqrt[c + a^2*c*x^2]*Sqrt
[ArcTan[a*x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[A
rcTan[a*x]]])/(3*a^3*c*Sqrt[c + a^2*c*x^2]) + 4*Defer[Int][x^2/((c + a^2*c*
x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x] + (8*a^2*Defer[Int][x^4/((c + a^2*c*x^2)^
(3/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(2a)$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{}}$$

$$= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{}{3c\sqrt{}}$$

Mathematica [A]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 6.79, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1096 $\int \frac{x}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}-\frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}-\frac{4\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}}\right.}{3a^2c\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*x/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)-4/3*FresnelS(2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+
c)^(1/2)-4/3/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5062, 5022, 5091, 5090, 3386, 3432}

$$-\frac{4\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{3a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{3ac\text{ArcTan}(ax)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4}{3a^2c\sqrt{\text{ArcTan}(ax)}\,\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*Sqrt[c +
a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sq
rt[2/Pi]*Sqrt[ArcTan[a*x]]])/(3*a^2*c*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{4}{3}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{(4}{}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{(4}{}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{(8}{}$$

$$= -\frac{2x}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 124, normalized size = 0.96

$$-\frac{2\left(ax+2\mathrm{ArcTan}(ax)-i\sqrt{1+a^2x^2}\,(-i\mathrm{ArcTan}(ax))^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-i\mathrm{ArcTan}(ax)\right)+i\sqrt{1+a^2x^2}\,(i\mathrm{ArcTan}(ax))^{3/2}\mathrm{Gamma}\left(\frac{1}{2},i\mathrm{ArcTan}(ax)\right)\right)}{3a^2c\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] (-2*(a*x + 2*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gam
ma[1/2, (-I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma
[1/2, I*ArcTan[a*x]]))/(3*a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1097 $\int \frac{1}{(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{2}{3ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}+\frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}-\frac{4\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcT}}\right.}{3ac\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)-4/3*FresnelC(2^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(
1/2)+4/3*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5022, 5062, 5025, 5024, 3385, 3433}

$$-\frac{4\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{3ac\sqrt{a^2cx^2+c}}+\frac{4x}{3c\sqrt{\text{ArcTan}(ax)}\,\sqrt{a^2cx^2+c}}-\frac{2}{3ac\text{ArcTan}(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + (4*x)/(3*c*Sqrt[c + a^2*
c*x^2]*Sqrt[ArcTan[a*x]]) - (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(3*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
```

```
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{1}{3}(2a)\int \frac{x}{(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}}\,d$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{4}{3}\int \frac{}{(}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1}\right.}{}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(4\sqrt{1}\right.}{}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{\left(8\sqrt{1}\right.}{}$$

$$= -\frac{2}{3ac\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} - \frac{4\sqrt{2\pi}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 120, normalized size = 0.95

$$\frac{-2+4ax\text{ArcTan}(ax)-2\sqrt{1+a^2x^2}\,(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-2\sqrt{1+a^2x^2}\,(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)}{3ac\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2 + 4*a*x*ArcTan[a*x] - 2*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gamm
a[1/2, (-I)*ArcTan[a*x]] - 2*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma[
1/2, I*ArcTan[a*x]])/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1098 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2}{3acx\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}+\frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{8}{}$$

```
[Out] -2/3/a/c/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/3*FresnelS(2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(
1/2)+8/3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^2+c
)^(1/2)/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan
(a*x)^(1/2),x)/a^2+4*Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
,x)
```

Rubi [A]
time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*c*Sqrt[c + a^2*c*
x^2]*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*
x]]) + (8*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/(3*c*Sqrt[c + a^2*c*x^2]) + (8*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(3/2)*
Sqrt[ArcTan[a*x]]), x])/(3*a^2) + 4*Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*S
qrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}($$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

$$= -\frac{2}{3acx\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}} + \frac{}{3a^2}$$

Mathematica [A]
time = 7.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1099 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2}{3acx^2\sqrt{c+a^2cx^2}\,\mathrm{ArcTan}(ax)^{3/2}}+\frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}+\frac{4}{cx\sqrt{c+a^2cx^2}\,\sqrt{\mathrm{ArcTan}(ax)}}+$$

[Out] -2/3/a/c/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8*a*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+8/3/a^2/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+8*Unintegrable(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+44/3*Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)

Rubi [A]
time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]

[Out] -2/(3*a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 4/(c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (8*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(c*Sqrt[c + a^2*c*x^2]) + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/a^2 + (44*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x])/3

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{5/2}} dx = -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} dx}{3a} - ($$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx^2\sqrt{c+a^2cx^2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2} \sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 10.48, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \operatorname{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
[Out] int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \operatorname{atan}(a x)^{5/2} \left(c a^2 x^2 + c\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1100 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2}{3acx^3\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}+\frac{4}{a^2cx^4\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{16}{3cx^2\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+$$

```
[Out] -2/3/a/c/x^3/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+4/a^2/c/x^4/(a^2*c*x^2+c
)^(1/2)/arctan(a*x)^(1/2)+16/3/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+
16*Unintegrable(1/x^5/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+92/3*Uni
ntegrable(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+16*a^2*Unintegrabl
e(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 16/(3*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcT
an[a*x]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),
 x])/a^2 + (92*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),
x])/3 + 16*a^2*Defer[Int][1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{a}-$$

$$= -\frac{2}{3acx^3\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}+\frac{4}{a^2cx^4\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 11.96, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 2.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1101 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2}{3acx^4\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}+\frac{16}{3a^2cx^5\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+\frac{20}{3cx^3\sqrt{c+a^2cx^2}\,\sqrt{\text{ArcTan}(ax)}}+$$

```
[Out] -2/3/a/c/x^4/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+16/3/a^2/c/x^5/(a^2*c*x^
2+c)^(1/2)/arctan(a*x)^(1/2)+20/3/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/
2)+80/3*Unintegrable(1/x^6/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+52*
Unintegrable(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+80/3*a^2*Uninte
grable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*Sqrt
[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + 20/(3*c*x^3*Sqrt[c + a^2*c*x^2]*Sqrt[A
rcTan[a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x])/(3*a^2) + 52*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*
x]]), x] + (80*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
]), x])/3
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3acx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}-\frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a}-$$

$$=-\frac{2}{3acx^4\sqrt{c+a^2cx^2}\,\tan^{-1}(ax)^{3/2}}+\frac{16}{3a^2cx^5\sqrt{c+a^2cx^2}\,\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 20.71, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{3/2} \operatorname{ArcTan}(a x)^{5/2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 7.65, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan \left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1102 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
[Out] Defer[Int][x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\mathrm{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
[Out] Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*
x^2 + c^3)*arctan(a*x)^(5/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\operatorname{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1103 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{3/2}}-\frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}}-\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\operatorname{ArcTan}(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-FresnelS(2^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+
c)^(1/2)+FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2
*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-4*x^2/a^2/c/(a^2*c*x^2+c)^(3/2)/a
rctan(a*x)^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5062, 5088, 5091, 5090, 3393, 3386, 3432, 4491}

$$-\frac{4x^2}{a^2c\sqrt{\operatorname{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2x^3}{3ac\operatorname{ArcTan}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\operatorname{ArcTan}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\operatorname{ArcTan}(ax)}\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^3)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (4*x^2)/(a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*Fre
snelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[
6*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^4*c^2*Sq
rt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{a}$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} - 4$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 0.60, size = 255, normalized size = 1.34

$$\frac{-2a^2x^2(ax+6\text{ArcTan}(ax))+\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}\left(-3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)+S\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)\right)-(1+a^2x^2)^{3/2}\,\text{ArcTan}(ax)\left(3\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+3\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)+\sqrt{i\text{ArcTan}(ax)}\,\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)\right)\right)}{3a^4c\,(c+a^2cx^2)^{3/2}\,\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*a^2*x^2*(a*x + 6*ArcTan[a*x]) + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*ArcTan[a
*x]^(3/2)*(-3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqr
```

```
t[6/Pi]*Sqrt[ArcTan[a*x]]]) - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*Sqrt[(-I)*
ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2
, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan
[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(3*a^4*c*(c +
 a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 7.09, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
[Out] int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1104 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}-\frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}+\frac{4x^3}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

```
[Out] -2/3*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-1/3*FresnelC(2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*
x^2+c)^(1/2)+FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*
(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-8/3*x/a^2/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)^(1/2)+4/3*x^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5088, 5062, 5091, 5090, 4491, 3385, 3433, 5025, 5024, 3393}

$$-\frac{2x^2}{3ac\text{ArcTan}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}-\frac{8x}{3a^2c\sqrt{\text{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}+\frac{4x^3}{3c\sqrt{\text{ArcTan}(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{3a^3c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{6\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{ArcTan}(ax)}\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (8*x)/(3*a^2*c*(
c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (4*x^3)/(3*c*(c + a^2*c*x^2)^(3/2
)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(3*a^3*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[6*Pi]*Sqrt[1 + a^
2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^3*c^2*Sqrt[c + a^2*c*x^2]
)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5062

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Dist[f*(m/(b*c*(p + 1))), Int[(f*x)
^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
```

```
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(2a$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.59, size = 311, normalized size = 1.39

$$\frac{-(1+a^2x^2)^{3/2}(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)+\frac{-4a^2x^2\sqrt{i\text{ArcTan}(ax)}+16iax(i\text{ArcTan}(ax))^{3/2}-8ia^3x^3(i\text{ArcTan}(ax))^{3/2}+(1+a^2x^2)^{3/2}\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)-3i\sqrt{3}(1+a^2x^2)^{3/2}\text{ArcTan}(ax)\sqrt{\text{ArcTan}(ax)^2}\,\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-3\sqrt{3+3a^2x^2}\,\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)-3a^2x^2\sqrt{3+3a^2x^2}\,\text{ArcTan}(ax)^2\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)}{\sqrt{i\text{ArcTan}(ax)}}}{6a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-((1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-I)*ArcTan[a*x]
]) + (-4*a^2*x^2*Sqrt[I*ArcTan[a*x]] + (16*I)*a*x*(I*ArcTan[a*x])^(3/2) - (
```

```
8*I)*a^3*x^3*(I*ArcTan[a*x])^(3/2) + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Gamm
a[1/2, I*ArcTan[a*x]] - (3*I)*Sqrt[3]*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sqrt[
ArcTan[a*x]^2]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*ArcTa
n[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*ArcT
an[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(6*a^3*c^2*(1
 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1105 $\int \frac{x}{(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{2x}{3ac\,(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}}-\frac{4}{3a^2c\,(c+a^2cx^2)^{3/2}\,\sqrt{\mathrm{ArcTan}(ax)}}+\frac{8x^2}{3c\,(c+a^2cx^2)^{3/2}\,\sqrt{\mathrm{ArcTan}(ax)}}$$

```
[Out] -2/3*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-1/3*FresnelS(2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^
2+c)^(1/2)-FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a
^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-4/3/a^2/c/(a^2*c*x^2+c)^(3/2)/a
rctan(a*x)^(1/2)+8/3*x^2/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5088, 5091, 5090, 3393, 3386, 3432, 4491, 5022}

$$-\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{3a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\,\sqrt{a^2x^2+1}\,S\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\mathrm{ArcTan}(ax)}\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{8x^2}{3c\sqrt{\mathrm{ArcTan}(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2x}{3ac\mathrm{ArcTan}(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}-\frac{4}{3a^2c\sqrt{\mathrm{ArcTan}(ax)}\,(a^2cx^2+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-2*x)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - 4/(3*a^2*c*(c + a^
2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (8*x^2)/(3*c*(c + a^2*c*x^2)^(3/2)*Sqrt
[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]])/(3*a^2*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 + a^2*x^2]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} - \frac{1}{3}(4$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

$$= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 261, normalized size = 1.18

$$\frac{-12(ax+(2-4a^2x^2)\,\mathrm{ArcTan}(ax))+4\sqrt{6\pi}\,(1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}\left(3\sqrt{3}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)-S\left(\sqrt{\frac{6}{\pi}}\sqrt{\mathrm{ArcTan}(ax)}\right)\right)+7(1+a^2x^2)^{3/2}\,\mathrm{ArcTan}(ax)\left(3\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-i\mathrm{ArcTan}(ax)\right)+3\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},i\mathrm{ArcTan}(ax)\right)+\sqrt{3}\left(\sqrt{-i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},-3i\mathrm{ArcTan}(ax)\right)+\sqrt{i\mathrm{ArcTan}(ax)}\,\mathrm{Gamma}\left(\frac{1}{2},3i\mathrm{ArcTan}(ax)\right)\right)\right)}{18a^2c\,(c+a^2cx^2)^{3/2}\,\mathrm{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-12*(a*x + (2 - 4*a^2*x^2)*ArcTan[a*x]) + 4*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)
*ArcTan[a*x]^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Fres
```

```
nelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 7*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]
*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3
*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(18
*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
[Out] int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1106 $\int \frac{1}{(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}+\frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}-\frac{\sqrt{2\pi}\,\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{A}\right.}{ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-FresnelC(2^(1/2)/Pi^(1/2)*ar
ctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/
2)-FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+4*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/
2)
```

Rubi [A]
time = 0.43, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5022, 5088, 5091, 5090, 4491, 3385, 3433, 5025, 5024, 3393}

$$-\frac{\sqrt{2\pi}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{6\pi}\,\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\,\sqrt{\text{ArcTan}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{4x}{c\sqrt{\text{ArcTan}(ax)}\,\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3ac\text{ArcTan}(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + (4*x)/(c*(c + a^2*c*x^
2)^(3/2)*Sqrt[ArcTan[a*x]]) - (Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2]) - (Sqrt[6*Pi]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(a*c^2*Sqrt[c + a^2*c*x^2
])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5022

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Dist[2*c*((q + 1)/(b*(p + 1))), Int[x*(d + e*x^2)^q*(a + b*ArcT
an[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && L
tQ[q, -1] && LtQ[p, -1]
```

Rule 5024

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Arc
Tan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(q
 + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5025

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]), Int[(1 + c
^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

Rule 5088

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Dist[c*((m + 2*q + 2)/(b*(p + 1))), Int[x^(m
+ 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p + 1
)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; Fre
eQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &&
LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5090

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[(a + b*x)^p*(Sin[x]^m/C
```

```
os[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}
, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q]
 || GtQ[d, 0])
```

Rule 5091

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]),
 Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d
, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(In
tegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - (2a)\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} - 4$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

$$= -\frac{2}{3ac\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{4x}{c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 300, normalized size = 1.64

$$\frac{-4+24ax\text{ArcTan}(ax)-3(1+a^2x^2)^{3/2}(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-i\text{ArcTan}(ax)\right)-3(1+a^2x^2)^{3/2}(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},i\text{ArcTan}(ax)\right)-3\sqrt{3+3a^2x^2}(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-3a^2x^2\sqrt{3+3a^2x^2}(-i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},-3i\text{ArcTan}(ax)\right)-3\sqrt{3+3a^2x^2}(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)-3a^2x^2\sqrt{3+3a^2x^2}(i\text{ArcTan}(ax))^{3/2}\text{Gamma}\left(\frac{1}{2},3i\text{ArcTan}(ax)\right)}{6c^2(a+a^3x^2)\sqrt{c+a^2cx^2}\text{ArcTan}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] (-4 + 24*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*G
amma[1/2, (-I)*ArcTan[a*x]] - 3*(1 + a^2*x^2)^(3/2)*(I*ArcTan[a*x])^(3/2)*G
amma[1/2, I*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*G
```

```
amma[1/2, (-3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[
a*x])^(3/2)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*(I*ArcTa
n[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]
*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]])/(6*c^2*(a + a^3*x^2)*
Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1107 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\textbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}+\frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}+\frac{4}{3a^2cx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

```
[Out] -2/3/a/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4/3*FresnelS(6^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)
^(1/2)+4*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2
*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+16/3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x
)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+8/3*Unintegrabl
e(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+20/3*Unintegrable(1/x/
(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*c*(c + a^2*c*x
^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 4/(3*a^2*c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[Ar
cTan[a*x]]) + (4*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[(2*Pi)/3]*Sqrt[1 + a^2*x^2]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (8*Defe
r[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/(3*a^2) + (20*D
efer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx = -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} - \frac{2\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a} -$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

$$= -\frac{2}{3acx\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}} + \frac{16}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Mathematica [**A**]
time = 9.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^{2} c\, x^{2}+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1108 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{2}{3acx^2\left(c+a^2cx^2\right)^{3/2}\operatorname{ArcTan}(ax)^{3/2}}+\frac{8}{3a^2cx^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{ArcTan}(ax)}}+\frac{20}{3cx\left(c+a^2cx^2\right)^{3/2}\sqrt{\operatorname{ArcTan}(}$$

```
[Out] -2/3/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+20/9*a*FresnelC(6^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x
^2+c)^(1/2)+20*a*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+8/3/a^2/c/x^3/(a^2*c*x^2+c)^(
3/2)/arctan(a*x)^(1/2)+20/3/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+8*Uni
ntegrable(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+68/3*Unintegra
ble(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\operatorname{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 8/(3*a^2*c*x^3*(c
+ a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 20/(3*c*x*(c + a^2*c*x^2)^(3/2)*Sqr
t[ArcTan[a*x]]) + (20*a*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sq
rt[ArcTan[a*x]]])/(c^2*Sqrt[c + a^2*c*x^2]) + (20*a*Sqrt[(2*Pi)/3]*Sqrt[1 +
 a^2*x^2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(3*c^2*Sqrt[c + a^2*c*x^2
]) + (8*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/a^2
 + (68*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x])/3
```

Rubi steps

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \tan^{-1}(ax)^{5/2}} dx = -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} - \frac{4\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\tan^{-1}(ax)^{3/2}} dx}{3a}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

$$= -\frac{2}{3acx^2 (c+a^2cx^2)^{3/2} \tan^{-1}(ax)^{3/2}} + \frac{8}{3a^2cx^3 (c+a^2cx^2)^{3/2} \sqrt{\tan^{-1}}}$$

Mathematica [A]
time = 11.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \text{ArcTan}(ax)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1109 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\mathbf{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}+\frac{4}{a^2cx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}+\frac{8}{cx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}$$

```
[Out] -2/3/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4/a^2/c/x^4/(a^2*c*x^2+c
)^(3/2)/arctan(a*x)^(1/2)+8/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+16*
Unintegrable(1/x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+44*Unintegr
able(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+40*a^2*Unintegrable(1/x
/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 4/(a^2*c*x^4*(c +
a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 8/(c*x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[A
rcTan[a*x]]) + (16*Defer[Int][1/(x^5*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]
]), x])/a^2 + 44*Defer[Int][1/(x^3*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]])
, x] + 40*a^2*Defer[Int][1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}-\frac{2\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{a}$$

$$=-\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}+\frac{4}{a^2cx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}(ax)}}$$

Mathematica [A]
time = 13.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1110 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\text{ArcTan}(ax)^{5/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\text{ArcTan}(ax)^{3/2}}+\frac{16}{3a^2cx^5\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTan}(ax)}}+\frac{28}{3cx^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\text{ArcTa}}}$$

```
[Out] -2/3/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+16/3/a^2/c/x^5/(a^2*c*x^
2+c)^(3/2)/arctan(a*x)^(1/2)+28/3/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/
2)+80/3*Unintegrable(1/x^6/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+212
/3*Unintegrable(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+56*a^2*Unint
egrable(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] -2/(3*a*c*x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + 16/(3*a^2*c*x^5*(c
 + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + 28/(3*c*x^3*(c + a^2*c*x^2)^(3/2)*
Sqrt[ArcTan[a*x]]) + (80*Defer[Int][1/(x^6*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTa
n[a*x]]), x])/(3*a^2) + (212*Defer[Int][1/(x^4*(c + a^2*c*x^2)^(5/2)*Sqrt[A
rcTan[a*x]]), x])/3 + 56*a^2*Defer[Int][1/(x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[A
rcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{5/2}}\,dx=-\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}-\frac{8\int \frac{1}{x^5\left(c+a^2cx^2\right)^{5/2}\tan^{-1}(ax)^{3/2}}\,dx}{3a}$$

$$=-\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\tan^{-1}(ax)^{3/2}}+\frac{16}{3a^2cx^5\left(c+a^2cx^2\right)^{3/2}\sqrt{\tan^{-1}}}$$

Mathematica [A]
time = 23.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{5/2} \text{ArcTan}(ax)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

Maple [A]
time = 7.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

```
[Out] int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1111 $\int \frac{x\mathbf{ArcTan}(ax)^n}{c+a^2cx^2}\,dx$

Optimal. Leaf size=46

$$\frac{x\text{ArcTan}(ax)^{1+n}}{ac(1+n)}-\frac{\text{Int}(\text{ArcTan}(ax)^{1+n},x)}{ac(1+n)}$$

```
[Out] x*arctan(a*x)^(1+n)/a/c/(1+n)-Unintegrable(arctan(a*x)^(1+n),x)/a/c/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x\text{ArcTan}(ax)^n}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
[Out] (x*ArcTan[a*x]^(1 + n))/(a*c*(1 + n)) - Defer[Int][ArcTan[a*x]^(1 + n), x]/
(a*c*(1 + n))
```

Rubi steps

$$\int \frac{x\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx=\frac{x\tan^{-1}(ax)^{1+n}}{ac(1+n)}-\frac{\int \tan^{-1}(ax)^{1+n}\,dx}{ac(1+n)}$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x\text{ArcTan}(ax)^n}{c+a^2cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
[Out] Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x\arctan(ax)^n}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

```
[Out] int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x \operatorname{atan}^{n}{\left(a x \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**n/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x*atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(a\,x)^n}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(a*x)^n)/(c + a^2*c*x^2),x)
```

```
[Out] int((x*atan(a*x)^n)/(c + a^2*c*x^2), x)
```

3.1112 $\int \frac{\text{ArcTan}(ax)^n}{c+a^2cx^2}\,dx$

Optimal. Leaf size=20

$$\frac{\text{ArcTan}(ax)^{1+n}}{ac(1+n)}$$

[Out] arctan(a*x)^(1+n)/a/c/(1+n)

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5004}

$$\frac{\text{ArcTan}(ax)^{n+1}}{ac(n+1)}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[a*x]^n/(c + a^2*c*x^2),x]

[Out] ArcTan[a*x]^(1 + n)/(a*c*(1 + n))

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^n}{c+a^2cx^2}\,dx = \frac{\tan^{-1}(ax)^{1+n}}{ac(1+n)}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\text{ArcTan}(ax)^{1+n}}{ac(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[a*x]^n/(c + a^2*c*x^2),x]

[Out] ArcTan[a*x]^(1 + n)/(a*c*(1 + n))

Maple [A]
time = 0.18, size = 21, normalized size = 1.05

method	result	size
default	$\frac{\arctan(ax)^{1+n}}{ac(1+n)}$	21
risch	$\frac{i(\ln(-iax+1)-\ln(iax+1))\left(\frac{i(\ln(-iax+1)-\ln(iax+1))}{2}\right)^{n}}{2ca(1+n)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^n/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(a*x)^(1+n)/a/c/(1+n)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.09, size = 21, normalized size = 1.05

$$\frac{\arctan\left(ax\right)^{n}\arctan\left(ax\right)}{acn+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] arctan(a*x)^n*arctan(a*x)/(a*c*n + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}^{n}\left(ax\right)}{a^{2}x^{2}+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**n/(a**2*c*x**2+c),x)
```

```
[Out] Integral(atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

Giac [A]
time = 0.40, size = 20, normalized size = 1.00

$$\frac{\arctan(ax)^{n+1}}{ac(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] arctan(a*x)^(n + 1)/(a*c*(n + 1))
```

Mupad [B]
time = 0.36, size = 20, normalized size = 1.00

$$\frac{\mathrm{atan}(a\,x)^{n+1}}{a\,c\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^n/(c + a^2*c*x^2),x)
```

```
[Out] atan(a*x)^(n + 1)/(a*c*(n + 1))
```

3.1113 $\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\textbf{ArcTan}(cx))^p\,dx$

Optimal. Leaf size=31

$$\text{Int}\left((fx)^m \left(d+c^2dx^2\right)^q (a+b\text{ArcTan}(cx))^p, x\right)$$

```
[Out] Unintegrable((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx = \int (fx)^m \left(d+c^2dx^2\right)^q \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

Maple [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int (fx)^m \left(c^2d\,x^2+d\right)^q (a+b\arctan(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
[Out] int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="maxima"
)
[Out] integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="fricas"
)
[Out] integral((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c**2*d*x**2+d)**q*(a+b*atan(c*x))**p,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a+b\,\mathrm{atan}(c\,x))^p\,\left(d\,c^2\,x^2+d\right)^q\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m,x)
[Out] int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m, x)
```

3.1114 $\int x^3(d+ex^2)\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=107

$$\frac{b(3c^2d-2e)\,x}{12c^5}-\frac{b(3c^2d-2e)\,x^3}{36c^3}-\frac{bex^5}{30c}-\frac{b(3c^2d-2e)\,\text{ArcTan}(cx)}{12c^6}+\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))+\frac{1}{6}ex^6(a+b\text{ArcT}$$

```
[Out] 1/12*b*(3*c^2*d-2*e)*x/c^5-1/36*b*(3*c^2*d-2*e)*x^3/c^3-1/30*b*e*x^5/c-1/12
*b*(3*c^2*d-2*e)*arctan(c*x)/c^6+1/4*d*x^4*(a+b*arctan(c*x))+1/6*e*x^6*(a+b
*arctan(c*x))
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 5096, 470, 308, 209}

$$\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))+\frac{1}{6}ex^6(a+b\text{ArcTan}(cx))-\frac{b\text{ArcTan}(cx)\,(3c^2d-2e)}{12c^6}+\frac{bx(3c^2d-2e)}{12c^5}-\frac{bx^3(3c^2d-2e)}{36c^3}-\frac{bex^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(3*c^2*d - 2*e)*x)/(12*c^5) - (b*(3*c^2*d - 2*e)*x^3)/(36*c^3) - (b*e*x^
5)/(30*c) - (b*(3*c^2*d - 2*e)*ArcTan[c*x])/(12*c^6) + (d*x^4*(a + b*ArcTan
[c*x]))/4 + (e*x^6*(a + b*ArcTan[c*x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{x^4(3d+2e}{12+12c^2}$$
$$=-\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)+\left(bc\left(-3\right.\right.$$
$$=-\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)+\left(bc\left(-3\right.\right.$$
$$=\frac{b\left(3d-\frac{2e}{c^2}\right)x}{12c^3}-\frac{b\left(3d-\frac{2e}{c^2}\right)x^3}{36c}-\frac{bex^5}{30c}+\frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{6}e$$
$$=\frac{b\left(3d-\frac{2e}{c^2}\right)x}{12c^3}-\frac{b\left(3d-\frac{2e}{c^2}\right)x^3}{36c}-\frac{bex^5}{30c}-\frac{b\left(3d-\frac{2e}{c^2}\right)\tan^{-1}(cx)}{12c^4}+\frac{1}{4}dx$$

Mathematica [A]
time = 0.01, size = 127, normalized size = 1.19

$$\frac{bdx}{4c^3}-\frac{bex}{6c^5}-\frac{bdx^3}{12c}+\frac{bex^3}{18c^3}+\frac{1}{4}adx^4-\frac{bex^5}{30c}+\frac{1}{6}aex^6-\frac{bd\text{ArcTan}(cx)}{4c^4}+\frac{be\text{ArcTan}(cx)}{6c^6}+\frac{1}{4}bdx^4\text{ArcTan}(cx)+\frac{1}{6}bex^6\text{ArcTan}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*x^3)/(12*c) + (b*e*x^3)/(18*c^3) +
 (a*d*x^4)/4 - (b*e*x^5)/(30*c) + (a*e*x^6)/6 - (b*d*ArcTan[c*x])/(4*c^4) +
 (b*e*ArcTan[c*x])/(6*c^6) + (b*d*x^4*ArcTan[c*x])/4 + (b*e*x^6*ArcTan[c*x]
)/6
```

Maple [A]
time = 0.42, size = 119, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b\arctan(cx)d\,c^4x^4}{4}+\frac{b\,c^4\arctan(cx)e\,x^6}{6}-\frac{bd\,c^3x^3}{12}-\frac{b\,c^3e\,x^5}{30}+\frac{bcdx}{4}+\frac{bce\,x^3}{18}-\frac{bex}{6c}-\frac{\arctan(cx)bd}{4}+}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b\arctan(cx)d\,c^4x^4}{4}+\frac{b\,c^4\arctan(cx)e\,x^6}{6}-\frac{bd\,c^3x^3}{12}-\frac{b\,c^3e\,x^5}{30}+\frac{bcdx}{4}+\frac{bce\,x^3}{18}-\frac{bex}{6c}-\frac{\arctan(cx)bd}{4}+}{c^4}$
risch	$-\frac{ib\left(2e\,x^6+3d\,x^4\right)\ln(icx+1)}{24}+\frac{ibe\,x^6\ln(-icx+1)}{12}+\frac{x^6ea}{6}+\frac{ibd\,x^4\ln(-icx+1)}{8}+\frac{x^4da}{4}-\frac{be\,x^5}{30c}-\frac{bd\,x^3}{12c}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^2*(1/4*d*c^6*x^4+1/6*e*c^6*x^6)+1/4*b*arctan(c*x)*d*c^4*x^4+1/6*
b*c^4*arctan(c*x)*e*x^6-1/12*b*d*c^3*x^3-1/30*b*c^3*e*x^5+1/4*b*c*d*x+1/18*
b*c*e*x^3-1/6*b*e*x/c-1/4*arctan(c*x)*b*d+1/6*b*e*arctan(c*x)/c^2)
```

Maxima [A]

time = 0.46, size = 110, normalized size = 1.03

$$\frac{1}{6}ax^6e+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd+\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e + 1/4*a*d*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^
4 + 3*arctan(c*x)/c^5))*b*d + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*
c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e
```

Fricas [A]

time = 3.80, size = 109, normalized size = 1.02

$$\frac{45\,ac^6dx^4-15\,bc^5dx^3+45\,bc^3dx+15\left(3\,bc^6dx^4-3\,bc^2d+2\left(bc^6x^6+b\right)e\right)\arctan(cx)+2\left(15\,ac^6x^6-3\,bc^5x^5+5\,bc^3x^3-15\,bcx\right)e}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(45*a*c^6*d*x^4 - 15*b*c^5*d*x^3 + 45*b*c^3*d*x + 15*(3*b*c^6*d*x^4 -
 3*b*c^2*d + 2*(b*c^6*x^6 + b)*e)*arctan(c*x) + 2*(15*a*c^6*x^6 - 3*b*c^5*x
^5 + 5*b*c^3*x^3 - 15*b*c*x)*e)/c^6
```

Sympy [A]

time = 0.39, size = 138, normalized size = 1.29

$$\begin{cases}\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{atan}(cx)}{4}+\frac{bex^6\operatorname{atan}(cx)}{6}-\frac{bdx^3}{12c}-\frac{bex^5}{30c}+\frac{bdx}{4c^3}+\frac{bex^3}{18c^3}-\frac{bd\operatorname{atan}(cx)}{4c^4}-\frac{bex}{6c^5}+\frac{be\operatorname{atan}(cx)}{6c^6} & \text{for } c\neq 0\\ a\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*atan(c*x)/4 + b*e*x**6*atan(c
*x)/6 - b*d*x**3/(12*c) - b*e*x**5/(30*c) + b*d*x/(4*c**3) + b*e*x**3/(18*c
**3) - b*d*atan(c*x)/(4*c**4) - b*e*x/(6*c**5) + b*e*atan(c*x)/(6*c**6), Ne
(c, 0)), (a*(d*x**4/4 + e*x**6/6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.62, size = 105, normalized size = 0.98

$$\frac{a\,d\,x^4}{4}+\frac{a\,e\,x^6}{6}+\frac{b\,d\,x}{4\,c^3}-\frac{b\,e\,x}{6\,c^5}-\frac{b\,d\,\mathrm{atan}(c\,x)}{4\,c^4}+\frac{b\,e\,\mathrm{atan}(c\,x)}{6\,c^6}+\frac{b\,d\,x^4\,\mathrm{atan}(c\,x)}{4}+\frac{b\,e\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2),x)
```

```
[Out] (a*d*x^4)/4 + (a*e*x^6)/6 + (b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*atan(c
*x))/(4*c^4) + (b*e*atan(c*x))/(6*c^6) + (b*d*x^4*atan(c*x))/4 + (b*e*x^6*a
tan(c*x))/6 - (b*d*x^3)/(12*c) - (b*e*x^5)/(30*c) + (b*e*x^3)/(18*c^3)
```

3.1115 $\int x^2(d+ex^2)(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=94

$$-\frac{b(5c^2d-3e)\,x^2}{30c^3}-\frac{bex^4}{20c}+\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))+\frac{1}{5}ex^5(a+b\text{ArcTan}(cx))+\frac{b(5c^2d-3e)\log\left(1+c^2x^2\right)}{30c^5}$$

```
[Out] -1/30*b*(5*c^2*d-3*e)*x^2/c^3-1/20*b*e*x^4/c+1/3*d*x^3*(a+b*arctan(c*x))+1/
5*e*x^5*(a+b*arctan(c*x))+1/30*b*(5*c^2*d-3*e)*ln(c^2*x^2+1)/c^5
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {14, 5096, 457, 78}

$$\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))+\frac{1}{5}ex^5(a+b\text{ArcTan}(cx))+\frac{b(5c^2d-3e)\log\left(c^2x^2+1\right)}{30c^5}-\frac{bx^2(5c^2d-3e)}{30c^3}-\frac{bex^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/30*(b*(5*c^2*d - 3*e)*x^2)/c^3 - (b*e*x^4)/(20*c) + (d*x^3*(a + b*ArcTan
[c*x]))/3 + (e*x^5*(a + b*ArcTan[c*x]))/5 + (b*(5*c^2*d - 3*e)*Log[1 + c^2*
x^2])/(30*c^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^2(d+ex^2)\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-(bc)\int\frac{x^3(5d+3e}{15+15c^2}$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\text{Subst}\left(\int\frac{x(}{1}\right.$$

$$=\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\text{Subst}\left(\int\left(\right.\right.$$

$$=-\frac{b(5c^2d-3e)\,x^2}{30c^3}-\frac{bex^4}{20c}+\frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\tan\right.$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 1.27

$$-\frac{bdx^2}{6c}+\frac{bex^2}{10c^3}+\frac{1}{3}adx^3-\frac{bex^4}{20c}+\frac{1}{5}aex^5+\frac{1}{3}bdx^3\text{ArcTan}(cx)+\frac{1}{5}bex^5\text{ArcTan}(cx)+\frac{bd\log(1+c^2x^2)}{6c^3}-\frac{be\log(1+c^2x^2)}{10c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*d*x^2)/c + (b*e*x^2)/(10*c^3) + (a*d*x^3)/3 - (b*e*x^4)/(20*c) + (a
*e*x^5)/5 + (b*d*x^3*ArcTan[c*x])/3 + (b*e*x^5*ArcTan[c*x])/5 + (b*d*Log[1
+ c^2*x^2])/(6*c^3) - (b*e*Log[1 + c^2*x^2])/(10*c^5)
```

Maple [A]
time = 0.26, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}dc^5x^3+\frac{1}{5}ec^5x^5\right)}{c^2}+\frac{b\arctan(cx)dc^3x^3}{3}+\frac{bc^3\arctan(cx)ex^5}{5}-\frac{bdc^2x^2}{6}-\frac{bc^2ex^4}{20}+\frac{bex^2}{10}+\frac{b\ln\left(c^2x^2+1\right)d}{6}-\frac{b\ln\left(c^2x^2+1\right)e}{10c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}dc^5x^3+\frac{1}{5}ec^5x^5\right)}{c^2}+\frac{b\arctan(cx)dc^3x^3}{3}+\frac{bc^3\arctan(cx)ex^5}{5}-\frac{bdc^2x^2}{6}-\frac{bc^2ex^4}{20}+\frac{bex^2}{10}+\frac{b\ln\left(c^2x^2+1\right)d}{6}-\frac{b\ln\left(c^2x^2+1\right)e}{10c^2}}{c^3}$

risch	$-\frac{ib(3e\,x^5+5d\,x^3)\ln(icx+1)}{30}+\frac{ibe\,x^5\ln(-icx+1)}{10}+\frac{ibd\,x^3\ln(-icx+1)}{6}+\frac{ae\,x^5}{5}+\frac{x^3da}{3}-\frac{be\,x^4}{20c}-\frac{bd\,x^2}{6c}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^2*(1/3*d*c^5*x^3+1/5*e*c^5*x^5)+1/3*b*arctan(c*x)*d*c^3*x^3+1/5*
b*c^3*arctan(c*x)*e*x^5-1/6*b*d*c^2*x^2-1/20*b*c^2*e*x^4+1/10*b*e*x^2+1/6*b
*ln(c^2*x^2+1)*d-1/10*b/c^2*ln(c^2*x^2+1)*e)
```

Maxima [A]
time = 0.26, size = 107, normalized size = 1.14

$$\frac{1}{5}ax^5e+\frac{1}{3}adx^3+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bd+\frac{1}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e + 1/3*a*d*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x
^2 + 1)/c^4))*b*d + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*
log(c^2*x^2 + 1)/c^6))*b*e
```

Fricas [A]
time = 2.08, size = 111, normalized size = 1.18

$$\frac{20\,ac^5dx^3-10\,bc^4dx^2+4\,(3\,bc^5x^5e+5\,bc^5dx^3)\arctan(cx)+3\,(4\,ac^5x^5-bc^4x^4+2\,bc^2x^2)e+2\,(5\,bc^2d-3\,be)\log(c^2x^2+1)}{60\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(20*a*c^5*d*x^3 - 10*b*c^4*d*x^2 + 4*(3*b*c^5*x^5*e + 5*b*c^5*d*x^3)*a
rctan(c*x) + 3*(4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2)*e + 2*(5*b*c^2*d - 3
*b*e)*log(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.34, size = 128, normalized size = 1.36

$$\begin{cases}\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{atan}(cx)}{3}+\frac{bex^5\operatorname{atan}(cx)}{5}-\frac{bdx^2}{6c}-\frac{bex^4}{20c}+\frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{6c^3}+\frac{bex^2}{10c^3}-\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for}\ c\neq 0\\ a\left(\frac{dx^3}{3}+\frac{ex^5}{5}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*atan(c*x)/3 + b*e*x**5*atan(c
*x)/5 - b*d*x**2/(6*c) - b*e*x**4/(20*c) + b*d*log(x**2 + c**(-2))/(6*c**3)
```

```
 + b*e*x**2/(10*c**3) - b*e*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(d
*x**3/3 + e*x**5/5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.57, size = 101, normalized size = 1.07

$$\frac{a\,d\,x^3}{3}+\frac{a\,e\,x^5}{5}+\frac{b\,d\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,e\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,d\,x^2}{6\,c}-\frac{b\,e\,x^4}{20\,c}+\frac{b\,e\,x^2}{10\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2),x)
```

```
[Out] (a*d*x^3)/3 + (a*e*x^5)/5 + (b*d*x^3*atan(c*x))/3 + (b*e*x^5*atan(c*x))/5 +
 (b*d*log(c^2*x^2 + 1))/(6*c^3) - (b*e*log(c^2*x^2 + 1))/(10*c^5) - (b*d*x^
2)/(6*c) - (b*e*x^4)/(20*c) + (b*e*x^2)/(10*c^3)
```

3.1116 $\int x(d+ex^2)\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=82

$$-\frac{b(2c^2d-e)\,x}{4c^3}-\frac{bex^3}{12c}-\frac{b(c^2d-e)^2\,\mathrm{ArcTan}(cx)}{4c^4e}+\frac{(d+ex^2)^2\,(a+b\mathrm{ArcTan}(cx))}{4e}$$

```
[Out] -1/4*b*(2*c^2*d-e)*x/c^3-1/12*b*e*x^3/c-1/4*b*(c^2*d-e)^2*arctan(c*x)/c^4/e
+1/4*(e*x^2+d)^2*(a+b*arctan(c*x))/e
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {5094, 398, 209}

$$\frac{(d+ex^2)^2\,(a+b\mathrm{ArcTan}(cx))}{4e}-\frac{b\mathrm{ArcTan}(cx)\,(c^2d-e)^2}{4c^4e}-\frac{bx(2c^2d-e)}{4c^3}-\frac{bex^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/4*(b*(2*c^2*d - e)*x)/c^3 - (b*e*x^3)/(12*c) - (b*(c^2*d - e)^2*ArcTan[c
*x])/(4*c^4*e) + ((d + e*x^2)^2*(a + b*ArcTan[c*x]))/(4*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int x(d+ex^2)\left(a+b\tan^{-1}(cx)\right)\,dx = \frac{(d+ex^2)^2\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{(bc)\int\frac{(d+ex^2)^2}{1+c^2x^2}dx}{4e}$$
$$= \frac{(d+ex^2)^2\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{(bc)\int\left(\frac{(2c^2d-e)e}{c^4}+\frac{e^2x^2}{c^2}+\frac{c^4d^2-2c^2de+e^2}{c^4(1+c^2x^2)}\right)}{4e}$$
$$= -\frac{b(2c^2d-e)x}{4c^3} - \frac{bex^3}{12c} + \frac{(d+ex^2)^2\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{\left(b(c^2d-e)^2\right.}{4c}$$
$$= -\frac{b(2c^2d-e)x}{4c^3} - \frac{bex^3}{12c} - \frac{b(c^2d-e)^2\tan^{-1}(cx)}{4c^4e} + \frac{(d+ex^2)^2\left(a+b\,\mathrm{ta}\right.}{4e}$$

Mathematica [A]
time = 0.01, size = 103, normalized size = 1.26

$$-\frac{bdx}{2c}+\frac{bex}{4c^3}+\frac{1}{2}adx^2-\frac{bex^3}{12c}+\frac{1}{4}aex^4+\frac{bd\mathrm{ArcTan}(cx)}{2c^2}-\frac{be\mathrm{ArcTan}(cx)}{4c^4}+\frac{1}{2}bdx^2\mathrm{ArcTan}(cx)+\frac{1}{4}bex^4\mathrm{ArcTan}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*d*x)/c + (b*e*x)/(4*c^3) + (a*d*x^2)/2 - (b*e*x^3)/(12*c) + (a*e*x^
4)/4 + (b*d*ArcTan[c*x])/(2*c^2) - (b*e*ArcTan[c*x])/(4*c^4) + (b*d*x^2*Arc
Tan[c*x])/2 + (b*e*x^4*ArcTan[c*x])/4
```

Maple [A]
time = 0.34, size = 100, normalized size = 1.22

method	result
derivativedivides	$\frac{\frac{(c^2ex^2+c^2d)^2a}{4c^2e}+\frac{\arctan(cx)bc^2dx^2}{2}+\frac{\arctan(cx)bc^2ex^4}{4}-\frac{bcdx}{2}-\frac{bcex^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$
default	$\frac{\frac{(c^2ex^2+c^2d)^2a}{4c^2e}+\frac{\arctan(cx)bc^2dx^2}{2}+\frac{\arctan(cx)bc^2ex^4}{4}-\frac{bcdx}{2}-\frac{bcex^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$
risch	$-\frac{i(ex^2+d)^2b\ln(icx+1)}{8e}+\frac{iebx^4\ln(-icx+1)}{8}+\frac{ibd^2\ln(c^2x^2+1)}{16e}+\frac{aex^4}{4}+\frac{ibdx^2\ln(-icx+1)}{4}-\frac{bd^2\arctan(cx)}{8e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*(c^2*e*x^2+c^2*d)^2*a/c^2/e+1/2*arctan(c*x)*b*c^2*d*x^2+1/4*arct
an(c*x)*b*c^2*e*x^4-1/2*b*c*d*x-1/12*b*c*e*x^3+1/4*b*e*x/c+1/2*arctan(c*x)*
b*d-1/4*b*e*arctan(c*x)/c^2)
```

Maxima [A]
time = 0.46, size = 90, normalized size = 1.10

$$\frac{1}{4}ax^4e+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c
^3))*b*d + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)
/c^5))*b*e
```

Fricas [A]
time = 3.79, size = 90, normalized size = 1.10

$$\frac{6\,ac^4dx^2-6\,bc^3dx+3\,(2\,bc^4dx^2+2\,bc^2d+(bc^4x^4-b)e)\arctan\left(cx\right)+(3\,ac^4x^4-bc^3x^3+3\,bcx)e}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*c^4*d*x^2 - 6*b*c^3*d*x + 3*(2*b*c^4*d*x^2 + 2*b*c^2*d + (b*c^4*x
^4 - b)*e)*arctan(c*x) + (3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x)*e)/c^4
```

Sympy [A]
time = 0.27, size = 114, normalized size = 1.39

$$\begin{cases}\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{atan}\left(cx\right)}{2}+\frac{bex^4\operatorname{atan}\left(cx\right)}{4}-\frac{bdx}{2c}-\frac{bex^3}{12c}+\frac{bd\operatorname{atan}\left(cx\right)}{2c^2}+\frac{bex}{4c^3}-\frac{be\operatorname{atan}\left(cx\right)}{4c^4} & \text{for}\: c\neq 0\\a\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*atan(c*x)/2 + b*e*x**4*atan(c
*x)/4 - b*d*x/(2*c) - b*e*x**3/(12*c) + b*d*atan(c*x)/(2*c**2) + b*e*x/(4*c
**3) - b*e*atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d*x**2/2 + e*x**4/4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.29, size = 85, normalized size = 1.04

$$\frac{a\,d\,x^2}{2}+\frac{a\,e\,x^4}{4}-\frac{b\,d\,x}{2\,c}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,d\,\mathrm{atan}(c\,x)}{2\,c^2}-\frac{b\,e\,\mathrm{atan}(c\,x)}{4\,c^4}+\frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,e\,x^3}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2),x)
```

```
[Out] (a*d*x^2)/2 + (a*e*x^4)/4 - (b*d*x)/(2*c) + (b*e*x)/(4*c^3) + (b*d*atan(c*x
))/(2*c^2) - (b*e*atan(c*x))/(4*c^4) + (b*d*x^2*atan(c*x))/2 + (b*e*x^4*ata
n(c*x))/4 - (b*e*x^3)/(12*c)
```

3.1117 $\int (d+ex^2)(a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=68

$$-\frac{bex^2}{6c}+dx(a+b\text{ArcTan}(cx))+\frac{1}{3}ex^3(a+b\text{ArcTan}(cx))-\frac{b(3c^2d-e)\log(1+c^2x^2)}{6c^3}$$

```
[Out] -1/6*b*e*x^2/c+d*x*(a+b*arctan(c*x))+1/3*e*x^3*(a+b*arctan(c*x))-1/6*b*(3*c
^2*d-e)*ln(c^2*x^2+1)/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5032, 1607, 455, 45}

$$dx(a+b\text{ArcTan}(cx))+\frac{1}{3}ex^3(a+b\text{ArcTan}(cx))-\frac{b(3c^2d-e)\log(c^2x^2+1)}{6c^3}-\frac{bex^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*e*x^2)/c + d*x*(a + b*ArcTan[c*x]) + (e*x^3*(a + b*ArcTan[c*x]))/3
- (b*(3*c^2*d - e)*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
```

```
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)\,dx = dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int \frac{dx+\frac{ex^3}{3}}{1+c^2x^2}\,dx$$
$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-(bc)\int \frac{x\left(d+\frac{ex^2}{3}\right)}{1+c^2x^2}\,dx$$
$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\text{Subst}\left(\int \frac{d+\frac{ex}{3}}{1+c^2x}\right.$$
$$= dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\text{Subst}\left(\int\left(\frac{e}{3c^2}+\right.\right.$$
$$= -\frac{bex^2}{6c}+dx\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)-\frac{b(3c^2d-e)\log}{6c}$$

Mathematica [A]
time = 0.01, size = 85, normalized size = 1.25

$$adx-\frac{bex^2}{6c}+\frac{1}{3}aex^3+bdx\text{ArcTan}(cx)+\frac{1}{3}bex^3\text{ArcTan}(cx)-\frac{bd\log\left(1+c^2x^2\right)}{2c}+\frac{be\log\left(1+c^2x^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
[Out] a*d*x - (b*e*x^2)/(6*c) + (a*e*x^3)/3 + b*d*x*ArcTan[c*x] + (b*e*x^3*ArcTan
[c*x])/3 - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1 + c^2*x^2])/(6*c^3)
```

Maple [A]
time = 0.05, size = 86, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+b\arctan(cx)dcx+\frac{bc\arctan(cx)e\,x^3}{3}-\frac{be\,x^2}{6}-\frac{b\ln\left(c^2x^2+1\right)d}{2}+\frac{b\ln\left(c^2x^2+1\right)e}{6c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+b\arctan(cx)dcx+\frac{bc\arctan(cx)e\,x^3}{3}-\frac{be\,x^2}{6}-\frac{b\ln\left(c^2x^2+1\right)d}{2}+\frac{b\ln\left(c^2x^2+1\right)e}{6c^2}}{c}$
risch	$-\frac{ib(e\,x^3+3dx)\ln(icx+1)}{6}+\frac{ibe\,x^3\ln(-icx+1)}{6}+\frac{ibdx\ln(-icx+1)}{2}+\frac{ae\,x^3}{3}+adx-\frac{be\,x^2}{6c}-\frac{\ln(-c^2x^2-1)bd}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^2*(d*c^3*x+1/3*e*c^3*x^3)+b*arctan(c*x)*d*c*x+1/3*b*c*arctan(c*x)*
e*x^3-1/6*b*e*x^2-1/2*b*ln(c^2*x^2+1)*d+1/6*b/c^2*ln(c^2*x^2+1)*e)
```

Maxima [A]
time = 0.26, size = 82, normalized size = 1.21

$$\frac{1}{3}ax^3e + adx + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)be + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bd}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e + a*d*x + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1
)/c^4))*b*e + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [A]
time = 2.90, size = 86, normalized size = 1.26

$$\frac{6ac^3dx + 2(bc^3x^3e + 3bc^3dx)\arctan(cx) + (2ac^3x^3 - bc^2x^2)e - (3bc^2d - be)\log(c^2x^2+1)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^3*d*x + 2*(b*c^3*x^3*e + 3*b*c^3*d*x)*arctan(c*x) + (2*a*c^3*x^3
 - b*c^2*x^2)*e - (3*b*c^2*d - b*e)*log(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.20, size = 94, normalized size = 1.38

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\operatorname{atan}(cx) + \frac{bex^3\operatorname{atan}(cx)}{3} - \frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bex^2}{6c} + \frac{be\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*atan(c*x) + b*e*x**3*atan(c*x)/3 - b*
d*log(x**2 + c**(-2))/(2*c) - b*e*x**2/(6*c) + b*e*log(x**2 + c**(-2))/(6*c
**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.52, size = 75, normalized size = 1.10

$$a\,d\,x+\frac{a\,e\,x^3}{3}+b\,d\,x\,\mathrm{atan}(c\,x)+\frac{b\,e\,x^3\,\mathrm{atan}(c\,x)}{3}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c}+\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,e\,x^2}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2),x)
```

```
[Out] a*d*x + (a*e*x^3)/3 + b*d*x*atan(c*x) + (b*e*x^3*atan(c*x))/3 - (b*d*log(c^
2*x^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/(6*c^3) - (b*e*x^2)/(6*c)
```

3.1118 $\int \frac{(d+ex^2)(a+b\mathbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{bex}{2c}+\frac{be\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))+ad\log(x)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)$$

```
[Out] -1/2*b*e*x/c+1/2*b*e*arctan(c*x)/c^2+1/2*e*x^2*(a+b*arctan(c*x))+a*d*ln(x)+
1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5100, 4940, 2438, 4946, 327, 209}

$$\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))+ad\log(x)+\frac{be\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}ibd\text{Li}_2(-icx)-\frac{1}{2}ibd\text{Li}_2(icx)-\frac{bex}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -1/2*(b*e*x)/c + (b*e*ArcTan[c*x])/(2*c^2) + (e*x^2*(a + b*ArcTan[c*x]))/2
+ a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x}\,dx &= \int \left(\frac{d(a+b\tan^{-1}(cx))}{x}+ex(a+b\tan^{-1}(cx))\right)dx\\ &= d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+e\int x(a+b\tan^{-1}(cx))\,dx\\ &= \frac{1}{2}ex^2(a+b\tan^{-1}(cx))+ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\\ &= -\frac{bex}{2c}+\frac{1}{2}ex^2(a+b\tan^{-1}(cx))+ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-icx)-\frac{1}{2}ibd\mathrm{Li}_2(\\ &= -\frac{bex}{2c}+\frac{be\tan^{-1}(cx)}{2c^2}+\frac{1}{2}ex^2(a+b\tan^{-1}(cx))+ad\log(x)+\frac{1}{2}ibd\mathrm{Li}_2(-\end{aligned}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.08

$$-\frac{bex}{2c}+\frac{1}{2}aex^2+\frac{be\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}bex^2\text{ArcTan}(cx)+ad\log(x)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -1/2*(b*e*x)/c + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + (b*e*x^2*ArcTan[
c*x])/2 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2
, I*c*x]
```

Maple [A]
time = 0.12, size = 117, normalized size = 1.52

method	result
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{\arctan(cx)be\,x^2}{2}+b\arctan(cx)\,d\ln(cx)+\frac{be\arctan(cx)}{2c^2}-\frac{bex}{2c}+\frac{ibd\ln(cx)\ln(}{2}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{\arctan(cx)be\,x^2}{2}+b\arctan(cx)\,d\ln(cx)+\frac{be\arctan(cx)}{2c^2}-\frac{bex}{2c}+\frac{ibd\ln(cx)\ln(}{2}$
risch	$-\frac{ib\ln(-icx+1)(-icx+1)^2e}{4c^2}-\frac{bex}{2c}+\frac{ib\ln(-icx+1)(-icx+1)e}{2c^2}-\frac{ibd\,\mathrm{dilog}(-icx+1)}{2}+\frac{ae\,x^2}{2}+\frac{ae}{2c^2}+ad\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*e*x^2+a*d*ln(c*x)+1/2*arctan(c*x)*b*e*x^2+b*arctan(c*x)*d*ln(c*x)+1/2
*b*e*arctan(c*x)/c^2-1/2*b*e*x/c+1/2*I*b*d*ln(c*x)*ln(1+I*c*x)-1/2*I*b*d*ln
(c*x)*ln(1-I*c*x)+1/2*I*b*d*dilog(1+I*c*x)-1/2*I*b*d*dilog(1-I*c*x)
```

Maxima [A]
time = 0.59, size = 108, normalized size = 1.40

$$\frac{1}{2}\,ax^2e+ad\log(x)-\frac{\pi bc^2d\log(c^2x^2+1)-4\,bc^2d\arctan(cx)\log(cx)+2i\,bc^2d\mathrm{Li}_2(i\,cx+1)-2i\,bc^2d\mathrm{Li}_2(-i\,cx+1)+2\,bcxe-2\,(bc^2x^2e+be)\arctan(cx)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*log(x) - 1/4*(pi*b*c^2*d*log(c^2*x^2 + 1) - 4*b*c^2*d*arc
tan(c*x)*log(c*x) + 2*I*b*c^2*d*dilog(I*c*x + 1) - 2*I*b*c^2*d*dilog(-I*c*x
 + 1) + 2*b*c*x*e - 2*(b*c^2*x^2*e + b*e)*arctan(c*x))/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.68, size = 88, normalized size = 1.14

$$\begin{cases} \frac{a\,\left(e\,x^2+2\,d\,\ln(x)\right)}{2} & \text{if}\ \ c=0 \\ \frac{a\,\left(e\,x^2+2\,d\,\ln(x)\right)}{2}-b\,e\,\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)-\frac{b\,d\,\left(\mathrm{Li_2}(1-c\,x\,1\mathrm{i})-\mathrm{Li_2}(1+c\,x\,1\mathrm{i})\right)\,1\mathrm{i}}{2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x,x)
```

```
[Out] piecewise(c == 0, (a*(e*x^2 + 2*d*log(x)))/2, c ~= 0, (a*(e*x^2 + 2*d*log(x
)))/2 - b*e*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (b*d*(dilog(- c*x*1
i + 1) - dilog(c*x*1i + 1))*1i)/2)
```

3.1119 $\int \frac{(d+ex^2)(a+b\mathbf{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{d(a+b\text{ArcTan}(cx))}{x}+ex(a+b\text{ArcTan}(cx))+bcd\log(x)-\frac{b(c^2d+e)\log\left(1+c^2x^2\right)}{2c}$$

```
[Out] -d*(a+b*arctan(c*x))/x+e*x*(a+b*arctan(c*x))+b*c*d*ln(x)-1/2*b*(c^2*d+e)*ln
(c^2*x^2+1)/c
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {14, 5096, 457, 78}

$$-\frac{d(a+b\text{ArcTan}(cx))}{x}+ex(a+b\text{ArcTan}(cx))-\frac{b(c^2d+e)\log\left(c^2x^2+1\right)}{2c}+bcd\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTan[c*x]))/x) + e*x*(a + b*ArcTan[c*x]) + b*c*d*Log[x] - (b*
(c^2*d + e)*Log[1 + c^2*x^2])/(2*c)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\tan^{-1}(cx))}{x} + ex(a+b\tan^{-1}(cx)) - (bc)\int \frac{-d+ex^2}{x(1+c^2x^2)}\,dx$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{x} + ex(a+b\tan^{-1}(cx)) - \frac{1}{2}(bc)\text{Subst}\left(\int \frac{-d+ex}{x(1+c^2x}\right.$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{x} + ex(a+b\tan^{-1}(cx)) - \frac{1}{2}(bc)\text{Subst}\left(\int \left(-\frac{d}{x}+\frac{c}{1}\right.\right.$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{x} + ex(a+b\tan^{-1}(cx)) + bcd\log(x) - \frac{b(c^2d+e)\log}{2c}$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.28

$$-\frac{ad}{x} + aex - \frac{bd\text{ArcTan}(cx)}{x} + bex\text{ArcTan}(cx) + bcd\log(x) - \frac{1}{2}bcd\log(1+c^2x^2) - \frac{be\log(1+c^2x^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
[Out] -((a*d)/x) + a*e*x - (b*d*ArcTan[c*x])/x + b*e*x*ArcTan[c*x] + b*c*d*Log[x]
 - (b*c*d*Log[1 + c^2*x^2])/2 - (b*e*Log[1 + c^2*x^2])/(2*c)
```

Maple [A]
time = 0.10, size = 84, normalized size = 1.47

method	result
derivativedivides	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\arctan(cx)ex}{c}-\frac{b\arctan(cx)d}{cx}-\frac{b\ln(c^2x^2+1)d}{2}-\frac{b\ln(c^2x^2+1)e}{2c^2}+bd\ln(cx)\right)$
default	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\arctan(cx)ex}{c}-\frac{b\arctan(cx)d}{cx}-\frac{b\ln(c^2x^2+1)d}{2}-\frac{b\ln(c^2x^2+1)e}{2c^2}+bd\ln(cx)\right)$

risch	$\frac{ib(-e\,x^2+d)\ln(icx+1)}{2x}+\frac{ibce\,x^2\ln(-icx+1)+2b\,c^2d\ln(x)x-\ln(c^2x^2+1)b\,c^2dx-ibcd\ln(-icx+1)+2ae\,x^2c-\ln(c^2x^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^2*(e*c*x-d*c/x)+b/c*arctan(c*x)*e*x-b*arctan(c*x)*d/c/x-1/2*b*ln(c^2
*x^2+1)*d-1/2*b/c^2*ln(c^2*x^2+1)*e+b*d*ln(c*x))
```

Maxima [A]
time = 0.25, size = 75, normalized size = 1.32

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bd+axe+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be}{2\,c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + a*x*e + 1/2*
(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e/c - a*d/x
```

Fricas [A]
time = 2.05, size = 78, normalized size = 1.37

$$\frac{2\,bc^2dx\log\left(x\right)+2\,acx^2e-2\,acd+2\left(bcx^2e-bcd\right)\arctan\left(cx\right)-\left(bc^2dx+bxe\right)\log\left(c^2x^2+1\right)}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^2*d*x*log(x) + 2*a*c*x^2*e - 2*a*c*d + 2*(b*c*x^2*e - b*c*d)*arc
tan(c*x) - (b*c^2*d*x + b*x*e)*log(c^2*x^2 + 1))/(c*x)
```

Sympy [A]
time = 0.36, size = 80, normalized size = 1.40

$$\begin{cases} -\frac{ad}{x}+aex+bcd\log\left(x\right)-\frac{bcd\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd\operatorname{atan}\left(cx\right)}{x}+bex\operatorname{atan}\left(cx\right)-\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for}\ c\neq 0\\ a\left(-\frac{d}{x}+ex\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d/x + a*e*x + b*c*d*log(x) - b*c*d*log(x**2 + c**(-2))/2 - b*
d*atan(c*x)/x + b*e*x*atan(c*x) - b*e*log(x**2 + c**(-2))/(2*c), Ne(c, 0)),
 (a*(-d/x + e*x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.23, size = 69, normalized size = 1.21

$$a\,e\,x-\frac{a\,d}{x}+b\,e\,x\,\mathrm{atan}(c\,x)-\frac{b\,c\,d\,\ln\left(c^2\,x^2+1\right)}{2}+b\,c\,d\,\ln\left(x\right)-\frac{b\,d\,\mathrm{atan}(c\,x)}{x}-\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^2,x)
```

```
[Out] a*e*x - (a*d)/x + b*e*x*atan(c*x) - (b*c*d*log(c^2*x^2 + 1))/2 + b*c*d*log(
x) - (b*d*atan(c*x))/x - (b*e*log(c^2*x^2 + 1))/(2*c)
```

3.1120 $\int \frac{(d+ex^2)(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{bcd}{2x}-\frac{1}{2}bc^2d\text{ArcTan}(cx)-\frac{d(a+b\text{ArcTan}(cx))}{2x^2}+ae\log(x)+\frac{1}{2}ibe\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\text{PolyLog}(2,icx)$$

```
[Out] -1/2*b*c*d/x-1/2*b*c^2*d*arctan(c*x)-1/2*d*(a+b*arctan(c*x))/x^2+a*e*ln(x)+
1/2*I*b*e*polylog(2,-I*c*x)-1/2*I*b*e*polylog(2,I*c*x)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5100, 4946, 331, 209, 4940, 2438}

$$-\frac{d(a+b\text{ArcTan}(cx))}{2x^2}+ae\log(x)-\frac{1}{2}bc^2d\text{ArcTan}(cx)-\frac{bcd}{2x}+\frac{1}{2}ibe\text{Li}_2(-icx)-\frac{1}{2}ibe\text{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d)/x - (b*c^2*d*ArcTan[c*x])/2 - (d*(a + b*ArcTan[c*x]))/(2*x^2)
+ a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)\left(a+b\tan^{-1}(cx)\right)}{x^3}\,dx &= \int \left(\frac{d(a+b\tan^{-1}(cx))}{x^3}+\frac{e(a+b\tan^{-1}(cx))}{x}\right)dx\\
&= d\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx\\
&= -\frac{d(a+b\tan^{-1}(cx))}{2x^2}+ae\log(x)+\frac{1}{2}(bcd)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx+\frac{1}{2}(ibe)\\
&= -\frac{bcd}{2x}-\frac{d(a+b\tan^{-1}(cx))}{2x^2}+ae\log(x)+\frac{1}{2}ibe\text{Li}_2(-icx)-\frac{1}{2}ibe\text{Li}_2(icx)\\
&= -\frac{bcd}{2x}-\frac{1}{2}bc^2d\tan^{-1}(cx)-\frac{d(a+b\tan^{-1}(cx))}{2x^2}+ae\log(x)+\frac{1}{2}ibe\text{Li}_2(-i
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.01, size = 86, normalized size = 1.12

$$-\frac{ad}{2x^2}-\frac{bd\text{ArcTan}(cx)}{2x^2}-\frac{bcd\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^2\right)}{2x}+ae\log(x)+\frac{1}{2}ibe\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\text{PolyLog}(2,icx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

[Out] -1/2*(a*d)/x^2 - (b*d*ArcTan[c*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x) + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*x]

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.08, size = 144, normalized size = 1.87

method	result
risch	$\frac{ic^2bd\ln(-icx)}{4} - \frac{bcd}{2x} - \frac{b\,c^2d\arctan(cx)}{2} - \frac{ibd\ln(-icx+1)}{4x^2} - \frac{ib\,\mathrm{dilog}(-icx+1)e}{2} - \frac{ad}{2x^2} + ae\ln(-icx) -$
derivativedivides	$c^2\Big(-\frac{ad}{2c^2x^2} + \frac{ae\ln(cx)}{c^2} - \frac{b\arctan(cx)d}{2c^2x^2} + \frac{b\arctan(cx)e\ln(cx)}{c^2} + \frac{ibe\ln(cx)\ln(icx+1)}{2c^2} - \frac{ibe\ln(cx)\ln(-icx+1}{2c^2}$
default	$c^2\Big(-\frac{ad}{2c^2x^2} + \frac{ae\ln(cx)}{c^2} - \frac{b\arctan(cx)d}{2c^2x^2} + \frac{b\arctan(cx)e\ln(cx)}{c^2} + \frac{ibe\ln(cx)\ln(icx+1)}{2c^2} - \frac{ibe\ln(cx)\ln(-icx+1}{2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)

[Out] c^2*(-1/2*a*d/c^2/x^2+a/c^2*e*ln(c*x)-1/2*b*arctan(c*x)*d/c^2/x^2+b/c^2*arctan(c*x)*e*ln(c*x)+1/2*I*b/c^2*e*ln(c*x)*ln(1+I*c*x)-1/2*I*b/c^2*e*ln(c*x)*ln(1-I*c*x)-1/2*I*b/c^2*e*dilog(1-I*c*x)+1/2*I*b/c^2*e*dilog(1+I*c*x)-1/2*arctan(c*x)*b*d-1/2*b*d/c/x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")

[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d + b*e*integrate(arctan(c*x)/x, x) + a*e*log(x) - 1/2*a*d/x^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")

[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))/x^3, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.72, size = 91, normalized size = 1.18

$$\begin{cases} a\,e\,\ln\left(x\right)-\frac{a\,d}{2\,x^2} & \text{ if } c=0\\ a\,e\,\ln\left(x\right)-\frac{a\,d}{2\,x^2}-\frac{b\,d\,\mathrm{atan}\left(c\,x\right)}{2\,x^2}-\frac{b\,d\,\left(c^3\,\mathrm{atan}\left(c\,x\right)+\frac{c^2}{x}\right)}{2\,c}-\frac{b\,e\,\left(\mathrm{Li_2}\left(1-c\,x\,1{}\mathrm{i}\right)-\mathrm{Li_2}\left(1+c\,x\,1{}\mathrm{i}\right)\right)\,1{}\mathrm{i}}{2} & \text{ if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^3,x)
```

```
[Out] piecewise(c == 0, a*e*log(x) - (a*d)/(2*x^2), c ~= 0, a*e*log(x) - (b*e*(di
log(- c*x*1i + 1) - dilog(c*x*1i + 1))*1i)/2 - (a*d)/(2*x^2) - (b*d*atan(c*
x))/(2*x^2) - (b*d*(c^3*atan(c*x) + c^2/x))/(2*c))
```

3.1121 $\int \frac{(d+ex^2)(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{bcd}{6x^2}-\frac{d(a+b\text{ArcTan}(cx))}{3x^3}-\frac{e(a+b\text{ArcTan}(cx))}{x}-\frac{1}{3}bc(c^2d-3e)\log(x)+\frac{1}{6}bc(c^2d-3e)\log\left(1+c^2x^2\right)$$

```
[Out] -1/6*b*c*d/x^2-1/3*d*(a+b*arctan(c*x))/x^3-e*(a+b*arctan(c*x))/x-1/3*b*c*(c
^2*d-3*e)*ln(x)+1/6*b*c*(c^2*d-3*e)*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5096, 12, 457, 78}

$$-\frac{d(a+b\text{ArcTan}(cx))}{3x^3}-\frac{e(a+b\text{ArcTan}(cx))}{x}+\frac{1}{6}bc(c^2d-3e)\log\left(c^2x^2+1\right)-\frac{1}{3}bc\log(x)\left(c^2d-3e\right)-\frac{bcd}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d)/x^2 - (d*(a + b*ArcTan[c*x]))/(3*x^3) - (e*(a + b*ArcTan[c*x])
)/x - (b*c*(c^2*d - 3*e)*Log[x])/3 + (b*c*(c^2*d - 3*e)*Log[1 + c^2*x^2])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)(a+b\tan^{-1}(cx))}{x^4}\,dx &= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{e(a+b\tan^{-1}(cx))}{x}-(bc)\int\frac{-d-3ex^2}{3x^3(1+c^2x^2)}\,dx\\
&= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{e(a+b\tan^{-1}(cx))}{x}-\frac{1}{3}(bc)\int\frac{-d-3ex^2}{x^3(1+c^2x^2)}\,dx\\
&= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{e(a+b\tan^{-1}(cx))}{x}-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{-d-3ex}{x^2(1+c^2x}\right.\\
&= -\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{e(a+b\tan^{-1}(cx))}{x}-\frac{1}{6}(bc)\text{Subst}\left(\int\left(-\frac{d}{x^2}+\frac{c}{}\right.\right.\\
&= -\frac{bcd}{6x^2}-\frac{d(a+b\tan^{-1}(cx))}{3x^3}-\frac{e(a+b\tan^{-1}(cx))}{x}-\frac{1}{3}bc(c^2d-3e)\log(x
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 1.18

$$-\frac{ad}{3x^3}-\frac{ae}{x}-\frac{bd\text{ArcTan}(cx)}{3x^3}-\frac{be\text{ArcTan}(cx)}{x}+bce\log(x)-\frac{1}{2}bce\log(1+c^2x^2)+\frac{1}{6}bcd\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log(1+c^2x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d)/x^3 - (a*e)/x - (b*d*ArcTan[c*x])/(3*x^3) - (b*e*ArcTan[c*x])/x
+ b*c*e*Log[x] - (b*c*e*Log[1 + c^2*x^2])/2 + (b*c*d*(-x^(-2) - 2*c^2*Log[x
] + c^2*Log[1 + c^2*x^2]))/6
```

Maple [A]
time = 0.08, size = 117, normalized size = 1.41

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}-\frac{b\arctan(cx)d}{3c^3x^3}-\frac{b\arctan(cx)e}{c^3x}+\frac{b\ln(c^2x^2+1)d}{6}-\frac{b\ln(c^2x^2+1)e}{2c^2}-\frac{bd\ln(cx)}{3}+\frac{b\ln(c}{c^2}\right.$
default	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}-\frac{b\arctan(cx)d}{3c^3x^3}-\frac{b\arctan(cx)e}{c^3x}+\frac{b\ln(c^2x^2+1)d}{6}-\frac{b\ln(c^2x^2+1)e}{2c^2}-\frac{bd\ln(cx)}{3}+\frac{b\ln(c}{c^2}\right.$
risch	$\frac{ib(3e\,x^2+d)\ln(icx+1)}{6x^3}-\frac{2\ln(x)b\,c^3d\,x^3-\ln(c^2x^2+1)b\,c^3d\,x^3-6\ln(x)bce\,x^3+3\ln(c^2x^2+1)bce\,x^3+3ibe\,x^2\ln(-icx+1)}{6x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^2*(-1/3*d/c/x^3-e/c/x)-1/3*b*arctan(c*x)*d/c^3/x^3-b/c^3*arctan(c*
x)*e/x+1/6*b*ln(c^2*x^2+1)*d-1/2*b/c^2*ln(c^2*x^2+1)*e-1/3*b*d*ln(c*x)+b/c^
2*ln(c*x)*e-1/6*b*d/c^2/x^2)
```

Maxima [A]
time = 0.27, size = 95, normalized size = 1.14

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e - a*e/x -
1/3*a*d/x^3
```

Fricas [A]
time = 5.13, size = 95, normalized size = 1.14

$$-\frac{bcdx+6\,ax^2e+2\,ad+2\,(3\,bx^2e+bd)\arctan\left(cx\right)-\left(bc^3dx^3-3\,bcx^3e\right)\log\left(c^2x^2+1\right)+2\,\left(bc^3dx^3-3\,bcx^3e\right)\log\left(x\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*d*x + 6*a*x^2*e + 2*a*d + 2*(3*b*x^2*e + b*d)*arctan(c*x) - (b*c^
3*d*x^3 - 3*b*c*x^3*e)*log(c^2*x^2 + 1) + 2*(b*c^3*d*x^3 - 3*b*c*x^3*e)*log
(x))/x^3
```

Sympy [A]
time = 0.43, size = 116, normalized size = 1.40

$$\begin{cases} -\frac{ad}{3x^3}-\frac{ae}{x}-\frac{bc^3d\log\left(x\right)}{3}+\frac{bc^3d\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bcd}{6x^2}+bce\log\left(x\right)-\frac{bce\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd\,\mathrm{atan}\left(cx\right)}{3x^3}-\frac{be\,\mathrm{atan}\left(cx\right)}{x} & \text{for}\ c\neq 0 \\ a\left(-\frac{d}{3x^3}-\frac{e}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d/(3*x**3) - a*e/x - b*c**3*d*log(x)/3 + b*c**3*d*log(x**2 +
c**(-2))/6 - b*c*d/(6*x**2) + b*c*e*log(x) - b*c*e*log(x**2 + c**(-2))/2 -
b*d*atan(c*x)/(3*x**3) - b*e*atan(c*x)/x, Ne(c, 0)), (a*(-d/(3*x**3) - e/x)
, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.55, size = 92, normalized size = 1.11

$$b\,c\,e\,\ln\left(x\right)-\frac{a\,e}{x}-\frac{b\,c\,e\,\ln\left(c^2\,x^2+1\right)}{2}-\frac{b\,c\,d}{6\,x^2}-\frac{a\,d}{3\,x^3}-\frac{b\,d\,\mathrm{atan}(c\,x)}{3\,x^3}-\frac{b\,e\,\mathrm{atan}(c\,x)}{x}+\frac{b\,c^3\,d\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{b\,c^3\,d\,\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^4,x)
```

```
[Out] b*c*e*log(x) - (a*e)/x - (b*c*e*log(c^2*x^2 + 1))/2 - (b*c*d)/(6*x^2) - (a*
d)/(3*x^3) - (b*d*atan(c*x))/(3*x^3) - (b*e*atan(c*x))/x + (b*c^3*d*log(c^2
*x^2 + 1))/6 - (b*c^3*d*log(x))/3
```

3.1122 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=82

$$-\frac{bcd}{12x^3}+\frac{bc(c^2d-2e)}{4x}+\frac{1}{4}bc^2(c^2d-2e)\,\text{ArcTan}(cx)-\frac{d(a+b\text{ArcTan}(cx))}{4x^4}-\frac{e(a+b\text{ArcTan}(cx))}{2x^2}$$

```
[Out] -1/12*b*c*d/x^3+1/4*b*c*(c^2*d-2*e)/x+1/4*b*c^2*(c^2*d-2*e)*arctan(c*x)-1/4
*d*(a+b*arctan(c*x))/x^4-1/2*e*(a+b*arctan(c*x))/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 5096, 12, 464, 331, 209}

$$-\frac{d(a+b\text{ArcTan}(cx))}{4x^4}-\frac{e(a+b\text{ArcTan}(cx))}{2x^2}+\frac{1}{4}bc^2\text{ArcTan}(cx)\,(c^2d-2e)+\frac{bc(c^2d-2e)}{4x}-\frac{bcd}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d)/x^3 + (b*c*(c^2*d - 2*e))/(4*x) + (b*c^2*(c^2*d - 2*e)*ArcTan
[c*x])/4 - (d*(a + b*ArcTan[c*x]))/(4*x^4) - (e*(a + b*ArcTan[c*x]))/(2*x^2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\tan^{-1}(cx))}{x^5}\,dx &= -\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{e(a+b\tan^{-1}(cx))}{2x^2}-(bc)\int \frac{-d-2ex^2}{4x^4\,(1+c^2x^2)}\,dx \\
&= -\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{e(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{4}(bc)\int \frac{-d-2ex^2}{x^4\,(1+c^2x^2)}\,dx \\
&= -\frac{bcd}{12x^3}-\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{e(a+b\tan^{-1}(cx))}{2x^2}-\frac{1}{4}\left(bc(c^2d-2e)\right)\int \\
&= -\frac{bcd}{12x^3}+\frac{bc(c^2d-2e)}{4x}-\frac{d(a+b\tan^{-1}(cx))}{4x^4}-\frac{e(a+b\tan^{-1}(cx))}{2x^2}+\frac{1}{4}(\\
&= -\frac{bcd}{12x^3}+\frac{bc(c^2d-2e)}{4x}+\frac{1}{4}bc^2\left(c^2d-2e\right)\tan^{-1}(cx)-\frac{d(a+b\tan^{-1}(cx))}{4x^4}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 97, normalized size = 1.18

$$-\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd\text{ArcTan}(cx)}{4x^4}-\frac{be\text{ArcTan}(cx)}{2x^2}-\frac{bcd\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-c^2x^2\right)}{12x^3}-\frac{bce\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/4*(a*d)/x^4 - (a*e)/(2*x^2) - (b*d*ArcTan[c*x])/(4*x^4) - (b*e*ArcTan[c*
x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
 - (b*c*e*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x)
```

Maple [A]
time = 0.12, size = 107, normalized size = 1.30

method	result
derivativedivides	$c^4\left(\frac{a\left(-\frac{d}{4c^2x^4}-\frac{e}{2c^2x^2}\right)}{c^2}-\frac{b\arctan(cx)d}{4c^4x^4}-\frac{b\arctan(cx)e}{2c^4x^2}+\frac{\arctan(cx)bd}{4}-\frac{be\arctan(cx)}{2c^2}+\frac{bd}{4cx}-\frac{be}{2c^3x}-\right.$
default	$c^4\left(\frac{a\left(-\frac{d}{4c^2x^4}-\frac{e}{2c^2x^2}\right)}{c^2}-\frac{b\arctan(cx)d}{4c^4x^4}-\frac{b\arctan(cx)e}{2c^4x^2}+\frac{\arctan(cx)bd}{4}-\frac{be\arctan(cx)}{2c^2}+\frac{bd}{4cx}-\frac{be}{2c^3x}-\right.$
risch	$\frac{ib(2e\,x^2+d)\ln(icx+1)}{8x^4}-\frac{3i\ln(-cx+i)b\,c^4d\,x^4-3i\ln(-cx-i)b\,c^4d\,x^4-6i\ln(-cx+i)b\,c^2e\,x^4+6i\ln(-cx-i)b\,c^2e\,x^4-6b}{24x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(a/c^2*(-1/4*d/c^2/x^4-1/2*e/c^2/x^2)-1/4*b*arctan(c*x)*d/c^4/x^4-1/2*b
/c^4*arctan(c*x)*e/x^2+1/4*arctan(c*x)*b*d-1/2*b*e*arctan(c*x)/c^2+1/4*b*d/
c/x-1/2*b/c^3*e/x-1/12*b*d/c^3/x^3)
```

Maxima [A]
time = 0.46, size = 82, normalized size = 1.00

$$\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)be-\frac{ae}{2\,x^2}-\frac{ad}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
- 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e - 1/2*a*e/x^2 - 1/4*a
*d/x^4
```

Fricas [A]
time = 6.35, size = 80, normalized size = 0.98

$$\frac{3\,bc^3dx^3-bcdx-3\,ad+3\left(bc^4dx^4-bd-2\left(bc^2x^4+bx^2\right)e\right)\arctan\left(cx\right)-6\left(bcx^3+ax^2\right)e}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*d*x^3 - b*c*d*x - 3*a*d + 3*(b*c^4*d*x^4 - b*d - 2*(b*c^2*x^4
 + b*x^2)*e)*arctan(c*x) - 6*(b*c*x^3 + a*x^2)*e)/x^4
```

Sympy [A]
time = 0.33, size = 99, normalized size = 1.21

$$-\frac{ad}{4x^4} - \frac{ae}{2x^2} + \frac{bc^4 d\,\mathrm{atan}\,(cx)}{4} + \frac{bc^3 d}{4x} - \frac{bc^2 e\,\mathrm{atan}\,(cx)}{2} - \frac{bcd}{12x^3} - \frac{bce}{2x} - \frac{bd\,\mathrm{atan}\,(cx)}{4x^4} - \frac{be\,\mathrm{atan}\,(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] -a*d/(4*x**4) - a*e/(2*x**2) + b*c**4*d*atan(c*x)/4 + b*c**3*d/(4*x) - b*c*
*2*e*atan(c*x)/2 - b*c*d/(12*x**3) - b*c*e/(2*x) - b*d*atan(c*x)/(4*x**4) -
 b*e*atan(c*x)/(2*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.59, size = 162, normalized size = 1.98

$$-\frac{\frac{a\,d}{4}+\frac{a\,x^2\,\left(d\,c^2+2\,e\right)}{4}+\frac{b\,d\,\mathrm{atan}\left(c\,x\right)}{4}+\frac{b\,c\,d\,x}{12}+\frac{b\,c^3\,x^5\,\left(2\,e-c^2\,d\right)}{4}+\frac{b\,c\,x^3\,\left(3\,e-c^2\,d\right)}{6}-\frac{a\,c^4\,e\,x^6}{2}+\frac{b\,x^2\,\mathrm{atan}\left(c\,x\right)\,\left(d\,c^2+2\,e\right)}{4}+\frac{b\,c^2\,e\,x^4\,\mathrm{atan}\left(c\,x\right)}{2}}{c^2\,x^6+x^4}-\frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\left(2\,b\,e-b\,c^2\,d\right)\,{\left(c^2\right)}^{3/2}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^5,x)
```

```
[Out] - ((a*d)/4 + (a*x^2*(2*e + c^2*d))/4 + (b*d*atan(c*x))/4 + (b*c*d*x)/12 + (
b*c^3*x^5*(2*e - c^2*d))/4 + (b*c*x^3*(3*e - c^2*d))/6 - (a*c^4*e*x^6)/2 +
(b*x^2*atan(c*x)*(2*e + c^2*d))/4 + (b*c^2*e*x^4*atan(c*x))/2)/(x^4 + c^2*x
^6) - (atan((c^2*x)/(c^2)^(1/2))*(2*b*e - b*c^2*d)*(c^2)^(3/2))/(4*c)
```

3.1123 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=110

$$-\frac{bcd}{20x^4}+\frac{bc(3c^2d-5e)}{30x^2}-\frac{d(a+b\text{ArcTan}(cx))}{5x^5}-\frac{e(a+b\text{ArcTan}(cx))}{3x^3}+\frac{1}{15}bc^3(3c^2d-5e)\log(x)-\frac{1}{30}bc^3(3c^2d$$

```
[Out] -1/20*b*c*d/x^4+1/30*b*c*(3*c^2*d-5*e)/x^2-1/5*d*(a+b*arctan(c*x))/x^5-1/3*
e*(a+b*arctan(c*x))/x^3+1/15*b*c^3*(3*c^2*d-5*e)*ln(x)-1/30*b*c^3*(3*c^2*d-
5*e)*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 5096, 12, 457, 78}

$$-\frac{d(a+b\text{ArcTan}(cx))}{5x^5}-\frac{e(a+b\text{ArcTan}(cx))}{3x^3}+\frac{bc(3c^2d-5e)}{30x^2}-\frac{1}{30}bc^3(3c^2d-5e)\log(c^2x^2+1)+\frac{1}{15}bc^3\log(x)(3c^2d-5e)-\frac{bcd}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d)/x^4 + (b*c*(3*c^2*d - 5*e))/(30*x^2) - (d*(a + b*ArcTan[c*x])
)/(5*x^5) - (e*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c^3*(3*c^2*d - 5*e)*Log[x]
)/15 - (b*c^3*(3*c^2*d - 5*e)*Log[1 + c^2*x^2])/30
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx = -\frac{d(a+b\tan^{-1}(cx))}{5x^5} - \frac{e(a+b\tan^{-1}(cx))}{3x^3} - (bc)\int \frac{-3d-5ex^2}{15x^5\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{5x^5} - \frac{e(a+b\tan^{-1}(cx))}{3x^3} - \frac{1}{15}(bc)\int \frac{-3d-5ex^2}{x^5\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{5x^5} - \frac{e(a+b\tan^{-1}(cx))}{3x^3} - \frac{1}{30}(bc)\text{Subst}\left(\int \frac{-3d-5e}{x^3\left(1+c^2\right.}\right.$$
$$= -\frac{d(a+b\tan^{-1}(cx))}{5x^5} - \frac{e(a+b\tan^{-1}(cx))}{3x^3} - \frac{1}{30}(bc)\text{Subst}\left(\int\left(-\frac{3d}{x^3}+\right.\right.$$
$$= -\frac{bcd}{20x^4} + \frac{bc(3c^2d-5e)}{30x^2} - \frac{d(a+b\tan^{-1}(cx))}{5x^5} - \frac{e(a+b\tan^{-1}(cx))}{3x^3} + \frac{1}{15}$$

Mathematica [A]
time = 0.03, size = 123, normalized size = 1.12

$$-\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\text{ArcTan}(cx)}{5x^5}-\frac{be\text{ArcTan}(cx)}{3x^3}+\frac{1}{6}bce\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)+\frac{1}{10}bcd\left(-\frac{1}{2x^4}+\frac{c^2}{x^2}+2c^4\log(x)-c^4\log\left(1+c^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/5*(a*d)/x^5 - (a*e)/(3*x^3) - (b*d*ArcTan[c*x])/(5*x^5) - (b*e*ArcTan[c*
x])/(3*x^3) + (b*c*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log[1 + c^2*x^2]))/6 + (
b*c*d*(-1/2*1/x^4 + c^2/x^2 + 2*c^4*Log[x] - c^4*Log[1 + c^2*x^2]))/10
```

Maple [A]
time = 0.10, size = 138, normalized size = 1.25

method	result
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}-\frac{b\arctan(cx)d}{5c^5x^5}-\frac{b\arctan(cx)e}{3c^5x^3}-\frac{b\ln(c^2x^2+1)d}{10}+\frac{b\ln(c^2x^2+1)e}{6c^2}+\frac{bd\ln(cx)}{5}-b\right.$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}-\frac{b\arctan(cx)d}{5c^5x^5}-\frac{b\arctan(cx)e}{3c^5x^3}-\frac{b\ln(c^2x^2+1)d}{10}+\frac{b\ln(c^2x^2+1)e}{6c^2}+\frac{bd\ln(cx)}{5}-b\right.$
risch	$\frac{ib(5e\,x^2+3d)\ln(icx+1)}{30x^5}-\frac{-12\ln(x)b\,c^5d\,x^5+6\ln(-c^2x^2-1)b\,c^5d\,x^5+20\ln(x)b\,c^3e\,x^5-10\ln(-c^2x^2-1)b\,c^3e\,x^5-6bd}{60x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^2*(-1/5*d/c^3/x^5-1/3*e/c^3/x^3)-1/5*b*arctan(c*x)*d/c^5/x^5-1/3*b
/c^5*arctan(c*x)*e/x^3-1/10*b*ln(c^2*x^2+1)*d+1/6*b/c^2*ln(c^2*x^2+1)*e+1/5
*b*d*ln(c*x)-1/3*b/c^2*ln(c*x)*e+1/10*b*d/c^2/x^2-1/6*b/c^4*e/x^2-1/20*b*d/
c^4/x^4)
```

Maxima [A]

time = 0.26, size = 118, normalized size = 1.07

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)be-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)
*c - 2*arctan(c*x)/x^3)*b*e - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Fricas [A]

time = 1.87, size = 122, normalized size = 1.11

$$\frac{6\,bc^3dx^3-3\,bcdx-12\,ad-4\left(5\,bx^2e+3\,bd\right)\arctan\left(cx\right)-10\left(bcx^3+2\,ax^2\right)e-2\left(3\,bc^5dx^5-5\,bc^3x^5e\right)\log\left(c^2x^2+1\right)+4\left(3\,bc^5dx^5-5\,bc^3x^5e\right)\log\left(x\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(6*b*c^3*d*x^3 - 3*b*c*d*x - 12*a*d - 4*(5*b*x^2*e + 3*b*d)*arctan(c*x
) - 10*(b*c*x^3 + 2*a*x^2)*e - 2*(3*b*c^5*d*x^5 - 5*b*c^3*x^5*e)*log(c^2*x^
2 + 1) + 4*(3*b*c^5*d*x^5 - 5*b*c^3*x^5*e)*log(x))/x^5
```

Sympy [A]

time = 0.61, size = 153, normalized size = 1.39

$$\begin{cases} -\frac{ad}{5x^5}-\frac{ae}{3x^3}+\frac{bc^5d\log\left(x\right)}{5}-\frac{bc^5d\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3d}{10x^2}-\frac{bc^3e\log\left(x\right)}{3}+\frac{bc^3e\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bcd}{20x^4}-\frac{bce}{6x^2}-\frac{bd\,\mathrm{atan}\left(cx\right)}{5x^5}-\frac{be\,\mathrm{atan}\left(cx\right)}{3x^3} & \text{for}\: c \neq 0 \\ a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d/(5*x**5) - a*e/(3*x**3) + b*c**5*d*log(x)/5 - b*c**5*d*log(
x**2 + c**(-2))/10 + b*c**3*d/(10*x**2) - b*c**3*e*log(x)/3 + b*c**3*e*log(
x**2 + c**(-2))/6 - b*c*d/(20*x**4) - b*c*e/(6*x**2) - b*d*atan(c*x)/(5*x**
5) - b*e*atan(c*x)/(3*x**3), Ne(c, 0)), (a*(-d/(5*x**5) - e/(3*x**3)), True
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.24, size = 111, normalized size = 1.01

$$\frac{b\,c^3\,e\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{b\,c^5\,d\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{x^3\,\left(\frac{b\,c\,e}{6}-\frac{b\,c^3\,d}{10}\right)+\frac{a\,d}{5}+x^2\,\left(\frac{a\,e}{3}+\frac{b\,e\,\mathrm{atan}\left(c\,x\right)}{3}\right)+\frac{b\,d\,\mathrm{atan}\left(c\,x\right)}{5}+\frac{b\,c\,d\,x}{20}}{x^5}+\frac{b\,c^5\,d\,\ln\left(x\right)}{5}-\frac{b\,c^3\,e\,\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^6,x)
```

```
[Out] (b*c^3*e*log(c^2*x^2 + 1))/6 - (b*c^5*d*log(c^2*x^2 + 1))/10 - (x^3*((b*c*e
)/6 - (b*c^3*d)/10) + (a*d)/5 + x^2*((a*e)/3 + (b*e*atan(c*x))/3) + (b*d*at
an(c*x))/5 + (b*c*d*x)/20)/x^5 + (b*c^5*d*log(x))/5 - (b*c^3*e*log(x))/3
```

3.1124 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x}-\frac{1}{12}bc^4(2c^2d-3e)\,\text{ArcTan}(cx)-\frac{d(a+b\text{ArcTan}(cx))}{6x^6}-\frac{e(a+b\text{ArcTan}(cx))}{4x^4}$$

```
[Out] -1/30*b*c*d/x^5+1/36*b*c*(2*c^2*d-3*e)/x^3-1/12*b*c^3*(2*c^2*d-3*e)/x-1/12*
b*c^4*(2*c^2*d-3*e)*arctan(c*x)-1/6*d*(a+b*arctan(c*x))/x^6-1/4*e*(a+b*arct
an(c*x))/x^4
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 5096, 12, 464, 331, 209}

$$-\frac{d(a+b\text{ArcTan}(cx))}{6x^6}-\frac{e(a+b\text{ArcTan}(cx))}{4x^4}-\frac{1}{12}bc^4\text{ArcTan}(cx)\,(2c^2d-3e)+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x}-\frac{bcd}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d)/x^5 + (b*c*(2*c^2*d - 3*e))/(36*x^3) - (b*c^3*(2*c^2*d - 3*e)
)/(12*x) - (b*c^4*(2*c^2*d - 3*e)*ArcTan[c*x])/12 - (d*(a + b*ArcTan[c*x]))
/(6*x^6) - (e*(a + b*ArcTan[c*x]))/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\tan^{-1}(cx))}{x^7}\,dx &= -\frac{d(a+b\tan^{-1}(cx))}{6x^6}-\frac{e(a+b\tan^{-1}(cx))}{4x^4}-(bc)\int \frac{-2d-3ex^2}{12x^6\,(1+c^2x^2)}\,dx \\
&= -\frac{d(a+b\tan^{-1}(cx))}{6x^6}-\frac{e(a+b\tan^{-1}(cx))}{4x^4}-\frac{1}{12}(bc)\int \frac{-2d-3ex^2}{x^6\,(1+c^2x^2)}\,dx \\
&= -\frac{bcd}{30x^5}-\frac{d(a+b\tan^{-1}(cx))}{6x^6}-\frac{e(a+b\tan^{-1}(cx))}{4x^4}-\frac{1}{12}\left(bc\left(2c^2d-3e\right)\right) \\
&= -\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{d(a+b\tan^{-1}(cx))}{6x^6}-\frac{e(a+b\tan^{-1}(cx))}{4x^4}+\frac{1}{12} \\
&= -\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x}-\frac{d(a+b\tan^{-1}(cx))}{6x^6}-\frac{e(a+ \\
&= -\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x}-\frac{1}{12}bc^4\left(2c^2d-3e\right)\tan^{-1}(cx)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 97, normalized size = 0.92

$$
-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd\text{ArcTan}(cx)}{6x^6}-\frac{be\text{ArcTan}(cx)}{4x^4}-\frac{bcd\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-c^2x^2\right)}{30x^5}-\frac{bce\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-c^2x^2\right)}{12x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/6*(a*d)/x^6 - (a*e)/(4*x^4) - (b*d*ArcTan[c*x])/(6*x^6) - (b*e*ArcTan[c*
x])/(4*x^4) - (b*c*d*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/(30*x^5)
 - (b*c*e*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
```

Maple [A]
time = 0.12, size = 127, normalized size = 1.21

method	result
derivativedivides	$c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}-\frac{b\arctan(cx)e}{4c^6x^4}-\frac{b\arctan(cx)d}{6c^6x^6}-\frac{\arctan(cx)bd}{6}+\frac{be\arctan(cx)}{4c^2}-\frac{bd}{6cx}+\frac{be}{4c^3x}+\right.$
default	$c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}-\frac{b\arctan(cx)e}{4c^6x^4}-\frac{b\arctan(cx)d}{6c^6x^6}-\frac{\arctan(cx)bd}{6}+\frac{be\arctan(cx)}{4c^2}-\frac{bd}{6cx}+\frac{be}{4c^3x}+\right.$
risch	$\frac{ib(3e\,x^2+2d)\ln(icx+1)}{24x^6}-\frac{30i\ln(-cx-i)b\,c^6d\,x^6-30i\ln(-cx+i)b\,c^6d\,x^6-45i\ln(-cx-i)b\,c^4e\,x^6+45i\ln(-cx+i)b\,c^4e\,x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(a/c^2*(-1/4*e/c^4/x^4-1/6*d/c^4/x^6)-1/4*b/c^6*arctan(c*x)*e/x^4-1/6*b
*arctan(c*x)*d/c^6/x^6-1/6*arctan(c*x)*b*d+1/4*b*e*arctan(c*x)/c^2-1/6*b*d/
c/x+1/4*b/c^3*e/x+1/18*b*d/c^3/x^3-1/12*b/c^5*e/x^3-1/30*b*d/c^5/x^5)
```

Maxima [A]
time = 0.47, size = 105, normalized size = 1.00

$$-\frac{1}{90}\left(\left(15\,c^5\arctan(cx)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd+\frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)be-\frac{ae}{4\,x^4}-\frac{ad}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arc
tan(c*x)/x^4)*b*e - 1/4*a*e/x^4 - 1/6*a*d/x^6
```

Fricas [A]
time = 2.60, size = 103, normalized size = 0.98

$$-\frac{30\,bc^5dx^5-10\,bc^3dx^3+6\,bcdx+30\,ad+15\,(2\,bc^6dx^6+2\,bd-3\,(bc^4x^6-bx^2)e)\arctan(cx)-15\,(3\,bc^3x^5-bcx^3-3\,ax^2)e}{180\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/180*(30*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 6*b*c*d*x + 30*a*d + 15*(2*b*c^6*
d*x^6 + 2*b*d - 3*(b*c^4*x^6 - b*x^2)*e)*arctan(c*x) - 15*(3*b*c^3*x^5 - b*
c*x^3 - 3*a*x^2)*e)/x^6
```

Sympy [A]
time = 0.45, size = 122, normalized size = 1.16

$$-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bc^6d\operatorname{atan}(cx)}{6}-\frac{bc^5d}{6x}+\frac{bc^4e\operatorname{atan}(cx)}{4}+\frac{bc^3d}{18x^3}+\frac{bc^3e}{4x}-\frac{bcd}{30x^5}-\frac{bce}{12x^3}-\frac{bd\operatorname{atan}(cx)}{6x^6}-\frac{be\operatorname{atan}(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))/x**7,x)
```

```
[Out] -a*d/(6*x**6) - a*e/(4*x**4) - b*c**6*d*atan(c*x)/6 - b*c**5*d/(6*x) + b*c*
*4*e*atan(c*x)/4 + b*c**3*d/(18*x**3) + b*c**3*e/(4*x) - b*c*d/(30*x**5) -
b*c*e/(12*x**3) - b*d*atan(c*x)/(6*x**6) - b*e*atan(c*x)/(4*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.59, size = 130, normalized size = 1.24

$$\frac{b\,c^4\operatorname{atan}\left(\frac{b\,c^2\,x\,(3\,e-2\,c^2\,d)}{3\,b\,c\,e-2\,b\,c^3\,d}\right)\,(3\,e-2\,c^2\,d)}{12}-\frac{\operatorname{atan}(c\,x)\,\left(\frac{b\,e\,x^2}{4}+\frac{b\,d}{6}\right)}{x^6}-\frac{x^3\,\left(b\,c\,e-\frac{2\,b\,c^3\,d}{3}\right)+2\,a\,d-c^2\,x^5\,(3\,b\,c\,e-2\,b\,c^3\,d)+3\,a\,e\,x^2+\frac{2\,b\,c\,d\,x}{5}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2))/x^7,x)
```

```
[Out] (b*c^4*atan((b*c^2*x*(3*e - 2*c^2*d))/(3*b*c*e - 2*b*c^3*d))*(3*e - 2*c^2*d
))/12 - (atan(c*x)*((b*d)/6 + (b*e*x^2)/4))/x^6 - (x^3*(b*c*e - (2*b*c^3*d)
/3) + 2*a*d - c^2*x^5*(3*b*c*e - 2*b*c^3*d) + 3*a*e*x^2 + (2*b*c*d*x)/5)/(1
2*x^6)
```

3.1125 $\int x^3(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=185

$$\frac{b(6c^4d^2-8c^2de+3e^2)\,x}{24c^7}-\frac{b(6c^4d^2-8c^2de+3e^2)\,x^3}{72c^5}-\frac{b(8c^2d-3e)\,ex^5}{120c^3}-\frac{be^2x^7}{56c}-\frac{b(6c^4d^2-8c^2de+3e^2)\,\mathrm{Arc}}{24c^8}$$

```
[Out] 1/24*b*(6*c^4*d^2-8*c^2*d*e+3*e^2)*x/c^7-1/72*b*(6*c^4*d^2-8*c^2*d*e+3*e^2)
*x^3/c^5-1/120*b*(8*c^2*d-3*e)*e*x^5/c^3-1/56*b*e^2*x^7/c-1/24*b*(6*c^4*d^2
-8*c^2*d*e+3*e^2)*arctan(c*x)/c^8+1/4*d^2*x^4*(a+b*arctan(c*x))+1/3*d*e*x^6
*(a+b*arctan(c*x))+1/8*e^2*x^8*(a+b*arctan(c*x))
```

Rubi [A]
time = 0.12, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {272, 45, 5096, 1275, 209}

$$\frac{1}{4}d^2x^4(a+b\mathrm{ArcTan}(cx))+\frac{1}{3}dex^6(a+b\mathrm{ArcTan}(cx))+\frac{1}{8}e^2x^8(a+b\mathrm{ArcTan}(cx))-\frac{b\mathrm{ArcTan}(cx)\,(6c^4d^2-8c^2de+3e^2)}{24c^8}-\frac{bex^5(8c^2d-3e)}{120c^3}+\frac{bx(6c^4d^2-8c^2de+3e^2)}{24c^7}-\frac{bx^3(6c^4d^2-8c^2de+3e^2)}{72c^5}-\frac{be^2x^7}{56c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x)/(24*c^7) - (b*(6*c^4*d^2 - 8*c^2*d*e
+ 3*e^2)*x^3)/(72*c^5) - (b*(8*c^2*d - 3*e)*e*x^5)/(120*c^3) - (b*e^2*x^7)/
(56*c) - (b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan[c*x])/(24*c^8) + (d^2*x^
4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x^8*(a +
b*ArcTan[c*x]))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan\right. \\
&= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\tan\right. \\
&= \frac{b(6c^4d^2-8c^2de+3e^2)\,x}{24c^7}-\frac{b(6c^4d^2-8c^2de+3e^2)\,x^3}{72c^5}-\frac{b(8c^2d-3e}{120c^3} \\
&= \frac{b(6c^4d^2-8c^2de+3e^2)\,x}{24c^7}-\frac{b(6c^4d^2-8c^2de+3e^2)\,x^3}{72c^5}-\frac{b(8c^2d-3e}{120c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 174, normalized size = 0.94

$$\frac{105ac^8x^4(6d^2+8dex^2+3e^2x^4)+bcx(315e^2-105c^2e(8d+ex^2)+7c^4(90d^2+40dex^2+9e^2x^4)-3c^6(70d^2x^2+56dex^4+15e^2x^6))+105b(-6c^4d^2+8c^2de-3e^2+c^8(6d^2x^4+8dex^6+3e^2x^8))\,\mathrm{ArcTan}(cx)}{2520c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (105*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b*c*x*(315*e^2 - 105*c^2*e
*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4) - 3*c^6*(70*d^2*x^
2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*b*(-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8
*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8))*ArcTan[c*x])/(2520*c^8)
```

Maple [A]
time = 0.37, size = 219, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b\arctan(cx)d^2c^4x^4}{4}+\frac{b\,c^4\arctan(cx)de\,x^6}{3}+\frac{b\,c^4\arctan(cx)e^2x^8}{8}-\frac{b\,d^2c^3x^3}{12}-\frac{b\,c^3de\,x^5}{15}-\frac{b\,c^3}{5}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b\arctan(cx)d^2c^4x^4}{4}+\frac{b\,c^4\arctan(cx)de\,x^6}{3}+\frac{b\,c^4\arctan(cx)e^2x^8}{8}-\frac{b\,d^2c^3x^3}{12}-\frac{b\,c^3de\,x^5}{15}-\frac{b\,c^3}{5}}{c^4}$
risch	$\frac{ib\,e^2x^8\ln(-icx+1)}{16}+\frac{ib\,d^2x^4\ln(-icx+1)}{8}+\frac{ibde\,x^6\ln(-icx+1)}{6}+\frac{x^8e^2a}{8}-\frac{ib\left(3e^2x^8+8de\,x^6+6d^2x^4\right)\ln(icx+1)}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^4*(1/4*d^2*c^8*x^4+1/3*d*c^8*e*x^6+1/8*e^2*c^8*x^8)+1/4*b*arctan
(c*x)*d^2*c^4*x^4+1/3*b*c^4*arctan(c*x)*d*e*x^6+1/8*b*c^4*arctan(c*x)*e^2*x
^8-1/12*b*d^2*c^3*x^3-1/15*b*c^3*d*e*x^5-1/56*b*c^3*e^2*x^7+1/4*b*c*d^2*x+1
/9*b*c*d*e*x^3+1/40*b*c*e^2*x^5-1/3*b*d*e*x/c-1/24*b*e^2*x^3/c+1/8*b*e^2*x/
c^3-1/4*b*d^2*arctan(c*x)+1/3*b*d*e*arctan(c*x)/c^2-1/8*b*e^2*arctan(c*x)/c
^4)
```

Maxima [A]

time = 0.46, size = 184, normalized size = 0.99

$$\frac{1}{8}\,ax^8e^2+\frac{1}{3}\,adx^6e+\frac{1}{4}\,ad^2x^4+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^2+\frac{1}{45}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bde+\frac{1}{840}\left(105\,x^8\arctan\left(cx\right)-c\left(\frac{15\,c^6x^7-21\,c^4x^5+35\,c^2x^3-105\,x}{c^8}+\frac{105\,\arctan\left(cx\right)}{c^9}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^2 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arctan(c*x) - c
*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2 + 1/45*(15*x^6*arctan(c*x
) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*e + 1/
840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x
)/c^8 + 105*arctan(c*x)/c^9))*b*e^2
```

Fricas [A]

time = 2.49, size = 193, normalized size = 1.04

$$\frac{630\,ac^8d^2x^4-210\,bc^7d^2x^3+630\,bc^5d^2x+105\left(6\,bc^8d^2x^4-6\,bc^4d^2+3\left(bc^8x^8-b\right)e^2+8\left(bc^8dx^6+bc^2d\right)e\right)\arctan\left(cx\right)+3\left(105\,ac^8x^8-15\,bc^7x^7+21\,bc^5x^5-35\,bc^3x^3+105\,bcx\right)e^2+56\left(15\,ac^8dx^6-3\,bc^7dx^5+5\,bc^5dx^3-15\,bc^3dx\right)e}{2520\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(630*a*c^8*d^2*x^4 - 210*b*c^7*d^2*x^3 + 630*b*c^5*d^2*x + 105*(6*b*
c^8*d^2*x^4 - 6*b*c^4*d^2 + 3*(b*c^8*x^8 - b)*e^2 + 8*(b*c^8*d*x^6 + b*c^2*
d)*e)*arctan(c*x) + 3*(105*a*c^8*x^8 - 15*b*c^7*x^7 + 21*b*c^5*x^5 - 35*b*c
^3*x^3 + 105*b*c*x)*e^2 + 56*(15*a*c^8*d*x^6 - 3*b*c^7*d*x^5 + 5*b*c^5*d*x^
3 - 15*b*c^3*d*x)*e)/c^8
```

Sympy [A]
time = 0.58, size = 260, normalized size = 1.41

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{atan}(cx)}{4} + \frac{bdex^6\operatorname{atan}(cx)}{3} + \frac{be^2x^8\operatorname{atan}(cx)}{8} - \frac{bd^2x^3}{12c} - \frac{bdex^5}{15c} - \frac{be^2x^7}{56c} + \frac{bd^2x}{4c^3} + \frac{bdex^3}{9c^3} + \frac{be^2x^5}{40c^3} - \frac{bd^2\operatorname{atan}(cx)}{4c^4} - \frac{bdex}{3c^5} - \frac{be^2x^3}{24c^5} + \frac{bde\operatorname{atan}(cx)}{3c^6} + \frac{be^2x}{8c^7} - \frac{be^2\operatorname{atan}(cx)}{8c^8} & \text{for}\ c \neq 0 \\ a\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*atan(
c*x)/4 + b*d*e*x**6*atan(c*x)/3 + b*e**2*x**8*atan(c*x)/8 - b*d**2*x**3/(12
*c) - b*d*e*x**5/(15*c) - b*e**2*x**7/(56*c) + b*d**2*x/(4*c**3) + b*d*e*x*
*3/(9*c**3) + b*e**2*x**5/(40*c**3) - b*d**2*atan(c*x)/(4*c**4) - b*d*e*x/(
3*c**5) - b*e**2*x**3/(24*c**5) + b*d*e*atan(c*x)/(3*c**6) + b*e**2*x/(8*c*
*7) - b*e**2*atan(c*x)/(8*c**8), Ne(c, 0)), (a*(d**2*x**4/4 + d*e*x**6/3 +
e**2*x**8/8), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.54, size = 374, normalized size = 2.02

$$x^4\left(\frac{\frac{ae^2}{c^2}-\frac{ae\,(2\,d\,c^2+e)}{c^2}}{4\,c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{4\,c^2}\right)-x^6\left(\frac{a\,e^2}{6\,c^2}-\frac{a\,e\,(2\,d\,c^2+e)}{6\,c^2}\right)+x^5\left(\frac{b\,e^2}{40\,c^3}-\frac{b\,d\,e}{15\,c}\right)+\operatorname{atan}(c\,x)\left(\frac{b\,d^2\,x^4}{4}+\frac{b\,d\,e\,x^6}{3}+\frac{b\,e^2\,x^8}{8}\right)-x^2\left(\frac{\frac{\frac{ae^2}{c^2}-\frac{ae\,(2\,d\,c^2+e)}{c^2}}{c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{c^2}}{2\,c^2}-\frac{a\,d^2}{2\,c^2}\right)-x^3\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{3\,c^2}+\frac{b\,d^2}{12\,c}\right)+\frac{x\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{c^2}+\frac{b\,d^2}{4\,c}\right)}{c^2}+\frac{a\,e^2\,x^8}{8}-\frac{b\operatorname{atan}\left(\frac{b\,c\,x\,(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{6\,b\,c^4\,d^2-8\,b\,c^2\,d\,e+3\,b\,e^2}\right)(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{24\,c^8}-\frac{b\,e^2\,x^7}{56\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

```
[Out] x^4*(((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/(4*c^2) + (a*d*(2*e + c^2*d))/
(4*c^2)) - x^6*((a*e^2)/(6*c^2) - (a*e*(e + 2*c^2*d))/(6*c^2)) + x^5*((b*e^
2)/(40*c^3) - (b*d*e)/(15*c)) + atan(c*x)*((b*d^2*x^4)/4 + (b*e^2*x^8)/8 +
(b*d*e*x^6)/3) - x^2*((((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/c^2 + (a*d*(
2*e + c^2*d))/c^2)/(2*c^2) - (a*d^2)/(2*c^2)) - x^3*(((b*e^2)/(8*c^3) - (b*
d*e)/(3*c))/(3*c^2) + (b*d^2)/(12*c)) + (x*(((b*e^2)/(8*c^3) - (b*d*e)/(3*c
))/c^2 + (b*d^2)/(4*c)))/c^2 + (a*e^2*x^8)/8 - (b*atan((b*c*x*(3*e^2 + 6*c^
4*d^2 - 8*c^2*d*e))/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e))*(3*e^2 + 6*c^4*d
^2 - 8*c^2*d*e))/(24*c^8) - (b*e^2*x^7)/(56*c)
```

3.1126 $\int x^2(d+ex^2)^2 (a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=161

$$-\frac{b(35c^4d^2-42c^2de+15e^2)\,x^2}{210c^5}-\frac{b(14c^2d-5e)\,ex^4}{140c^3}-\frac{be^2x^6}{42c}+\frac{1}{3}d^2x^3(a+b\text{ArcTan}(cx))+\frac{2}{5}dex^5(a+b\text{ArcTan}(c$$

```
[Out] -1/210*b*(35*c^4*d^2-42*c^2*d*e+15*e^2)*x^2/c^5-1/140*b*(14*c^2*d-5*e)*e*x^
4/c^3-1/42*b*e^2*x^6/c+1/3*d^2*x^3*(a+b*arctan(c*x))+2/5*d*e*x^5*(a+b*arcta
n(c*x))+1/7*e^2*x^7*(a+b*arctan(c*x))+1/210*b*(35*c^4*d^2-42*c^2*d*e+15*e^2
)*ln(c^2*x^2+1)/c^7
```

Rubi [A]
time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1265, 785}

$$\frac{1}{3}d^2x^3(a+b\text{ArcTan}(cx))+\frac{2}{5}dex^5(a+b\text{ArcTan}(cx))+\frac{1}{7}e^2x^7(a+b\text{ArcTan}(cx))-\frac{bex^4(14c^2d-5e)}{140c^3}+\frac{b(35c^4d^2-42c^2de+15e^2)\log(c^2x^2+1)}{210c^7}-\frac{bx^2(35c^4d^2-42c^2de+15e^2)}{210c^5}-\frac{be^2x^6}{42c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/210*(b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*x^2)/c^5 - (b*(14*c^2*d - 5*e)
*e*x^4)/(140*c^3) - (b*e^2*x^6)/(42*c) + (d^2*x^3*(a + b*ArcTan[c*x]))/3 +
(2*d*e*x^5*(a + b*ArcTan[c*x]))/5 + (e^2*x^7*(a + b*ArcTan[c*x]))/7 + (b*(3
5*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(210*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan\right.\\ &= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan\right.\\ &= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan\right.\\ &= \frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\tan\right.\\ &= -\frac{b(35c^4d^2-42c^2de+15e^2)\,x^2}{210c^5}-\frac{b(14c^2d-5e)\,ex^4}{140c^3}-\frac{be^2x^6}{42c}+\frac{1}{3}d^2x\end{aligned}$$

Mathematica [A]

time = 0.07, size = 162, normalized size = 1.01

$$\frac{c^2x^2(-30be^2+3bc^2e(28d+5ex^2)-2bc^4(35d^2+21dex^2+5e^2x^4)+4ac^5x(35d^2+42dex^2+15e^2x^4))+4bc^7x^3(35d^2+42dex^2+15e^2x^4)\,\mathrm{ArcTan}(cx)+2b(35c^4d^2-42c^2de+15e^2)\log\left(1+c^2x^2\right)}{420c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x^2*(-30*b*e^2 + 3*b*c^2*e*(28*d + 5*e*x^2) - 2*b*c^4*(35*d^2 + 21*d*e
*x^2 + 5*e^2*x^4) + 4*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)) + 4*b*c^7
*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcTan[c*x] + 2*b*(35*c^4*d^2 - 42*
c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(420*c^7)
```

Maple [A]

time = 0.28, size = 208, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\arctan(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\arctan(cx)de\,x^5}{5}+\frac{b\,c^3\arctan(cx)e^2x^7}{7}-\frac{b\,d^2c^2x^2}{6}-\frac{b\,c^2de\,x^4}{10}+bd}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\arctan(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\arctan(cx)de\,x^5}{5}+\frac{b\,c^3\arctan(cx)e^2x^7}{7}-\frac{b\,d^2c^2x^2}{6}-\frac{b\,c^2de\,x^4}{10}+bd}{c^3}$
risch	$\frac{ib\,e^2x^7\ln(-icx+1)}{14}+\frac{ibde\,x^5\ln(-icx+1)}{5}+\frac{ib\,d^2x^3\ln(-icx+1)}{6}+\frac{x^7e^2a}{7}-\frac{ib\left(15e^2x^7+42de\,x^5+35d^2x^3\right)\ln(icx+}{210}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^4*(1/3*d^2*c^7*x^3+2/5*d*c^7*e*x^5+1/7*e^2*c^7*x^7)+1/3*b*arctan
(c*x)*d^2*c^3*x^3+2/5*b*c^3*arctan(c*x)*d*e*x^5+1/7*b*c^3*arctan(c*x)*e^2*x
^7-1/6*b*d^2*c^2*x^2-1/10*b*c^2*d*e*x^4+1/5*b*d*e*x^2-1/42*b*c^2*e^2*x^6+1/
28*b*e^2*x^4-1/14*b/c^2*e^2*x^2+1/6*b*ln(c^2*x^2+1)*d^2-1/5*b/c^2*ln(c^2*x^
2+1)*d*e+1/14*b/c^4*ln(c^2*x^2+1)*e^2)
```

Maxima [A]

time = 0.26, size = 181, normalized size = 1.12

$$\frac{1}{7}ax^7e^2+\frac{2}{5}adx^5e+\frac{1}{3}ad^2x^3+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2+\frac{1}{10}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)bde+\frac{1}{84}\left(12x^7\arctan(cx)-c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6}-\frac{6\log(c^2x^2+1)}{c^8}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^2 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arctan(c*x) - c*
(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2 + 1/10*(4*x^5*arctan(c*x) - c*((c^2
*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e + 1/84*(12*x^7*arctan(c*
x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*e^
2
```

Fricas [A]

time = 3.00, size = 184, normalized size = 1.14

$$\frac{140\,ac^7d^2x^3-70\,bc^6d^2x^2+4\,(15\,bc^7x^7e^2+42\,bc^7dx^5e+35\,bc^7d^2x^3)\arctan(cx)+5\,(12\,ac^7x^7-2\,bc^6x^6+3\,bc^4x^4-6\,bc^2x^2)e^2+42\,(4\,ac^7dx^5-bc^6dx^4+2\,bc^4dx^2)e+2\,(35\,bc^4d^2-42\,bc^2de+15\,be^2)\log(c^2x^2+1)}{420\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/420*(140*a*c^7*d^2*x^3 - 70*b*c^6*d^2*x^2 + 4*(15*b*c^7*x^7*e^2 + 42*b*c^
7*d*x^5*e + 35*b*c^7*d^2*x^3)*arctan(c*x) + 5*(12*a*c^7*x^7 - 2*b*c^6*x^6 +
 3*b*c^4*x^4 - 6*b*c^2*x^2)*e^2 + 42*(4*a*c^7*d*x^5 - b*c^6*d*x^4 + 2*b*c^4
*d*x^2)*e + 2*(35*b*c^4*d^2 - 42*b*c^2*d*e + 15*b*e^2)*log(c^2*x^2 + 1))/c^
7
```

Sympy [A]
time = 0.50, size = 245, normalized size = 1.52

$$\begin{cases} \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3\operatorname{atan}(cx)}{3} + \frac{2bdex^5\operatorname{atan}(cx)}{5} + \frac{be^2x^7\operatorname{atan}(cx)}{7} - \frac{bd^2x^2}{6c} - \frac{bdex^4}{10c} - \frac{be^2x^6}{42c} + \frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} + \frac{bdex^2}{5c^3} + \frac{be^2x^4}{28c^3} - \frac{bde\log\left(x^2+\frac{1}{c^2}\right)}{5c^5} - \frac{be^2x^2}{14c^5} + \frac{be^2\log\left(x^2+\frac{1}{c^2}\right)}{14c^7} & \text{for } c \neq 0 \\ a\left(\frac{d^2x^3}{3} + \frac{2dex^5}{5} + \frac{e^2x^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*ata
n(c*x)/3 + 2*b*d*e*x**5*atan(c*x)/5 + b*e**2*x**7*atan(c*x)/7 - b*d**2*x**2
/(6*c) - b*d*e*x**4/(10*c) - b*e**2*x**6/(42*c) + b*d**2*log(x**2 + c**(-2)
)/(6*c**3) + b*d*e*x**2/(5*c**3) + b*e**2*x**4/(28*c**3) - b*d*e*log(x**2 +
 c**(-2))/(5*c**5) - b*e**2*x**2/(14*c**5) + b*e**2*log(x**2 + c**(-2))/(14
*c**7), Ne(c, 0)), (a*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.79, size = 191, normalized size = 1.19

$$\frac{a\,d^2\,x^3}{3}+\frac{a\,e^2\,x^7}{7}+\frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}+\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{b\,d^2\,x^2}{6\,c}-\frac{b\,e^2\,x^6}{42\,c}+\frac{b\,e^2\,x^4}{28\,c^3}-\frac{b\,e^2\,x^2}{14\,c^5}+\frac{2\,a\,d\,e\,x^5}{5}+\frac{b\,d^2\,x^3\,\mathrm{atan}\left(c\,x\right)}{3}+\frac{b\,e^2\,x^7\,\mathrm{atan}\left(c\,x\right)}{7}-\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{5\,c^5}-\frac{b\,d\,e\,x^4}{10\,c}+\frac{b\,d\,e\,x^2}{5\,c^3}+\frac{2\,b\,d\,e\,x^5\,\mathrm{atan}\left(c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

```
[Out] (a*d^2*x^3)/3 + (a*e^2*x^7)/7 + (b*d^2*log(c^2*x^2 + 1))/(6*c^3) + (b*e^2*l
og(c^2*x^2 + 1))/(14*c^7) - (b*d^2*x^2)/(6*c) - (b*e^2*x^6)/(42*c) + (b*e^2
*x^4)/(28*c^3) - (b*e^2*x^2)/(14*c^5) + (2*a*d*e*x^5)/5 + (b*d^2*x^3*atan(c
*x))/3 + (b*e^2*x^7*atan(c*x))/7 - (b*d*e*log(c^2*x^2 + 1))/(5*c^5) - (b*d*
e*x^4)/(10*c) + (b*d*e*x^2)/(5*c^3) + (2*b*d*e*x^5*atan(c*x))/5
```

3.1127 $\int x(d+ex^2)^2 (a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=115

$$-\frac{b(3c^4d^2-3c^2de+e^2)\,x}{6c^5}-\frac{b(3c^2d-e)\,ex^3}{18c^3}-\frac{be^2x^5}{30c}-\frac{b(c^2d-e)^3\,\text{ArcTan}(cx)}{6c^6e}+\frac{(d+ex^2)^3\,(a+b\text{ArcTan}(cx))}{6e}$$

```
[Out] -1/6*b*(3*c^4*d^2-3*c^2*d*e+e^2)*x/c^5-1/18*b*(3*c^2*d-e)*e*x^3/c^3-1/30*b*
e^2*x^5/c-1/6*b*(c^2*d-e)^3*arctan(c*x)/c^6/e+1/6*(e*x^2+d)^3*(a+b*arctan(c
*x))/e
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5094, 398, 209}

$$\frac{(d+ex^2)^3\,(a+b\text{ArcTan}(cx))}{6e}-\frac{b\text{ArcTan}(cx)\,(c^2d-e)^3}{6c^6e}-\frac{bex^3(3c^2d-e)}{18c^3}-\frac{bx(3c^4d^2-3c^2de+e^2)}{6c^5}-\frac{be^2x^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x)/c^5 - (b*(3*c^2*d - e)*e*x^3)/(18*
c^3) - (b*e^2*x^5)/(30*c) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*c^6*e) + ((d +
 e*x^2)^3*(a + b*ArcTan[c*x]))/(6*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^3}{1+c^2x^2}\,dx}{6e}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{6e}-\frac{(bc)\int\left(\frac{e(3c^4d^2-3c^2de+e^2)}{c^6}+\frac{(3c^2d-e)e^2x^2}{c^4}\right.}{6e}$$

$$=-\frac{b(3c^4d^2-3c^2de+e^2)\,x}{6c^5}-\frac{b(3c^2d-e)\,ex^3}{18c^3}-\frac{be^2x^5}{30c}+\frac{\left(d+ex^2\right)^3(a}{6}$$

$$=-\frac{b(3c^4d^2-3c^2de+e^2)\,x}{6c^5}-\frac{b(3c^2d-e)\,ex^3}{18c^3}-\frac{be^2x^5}{30c}-\frac{b\left(c^2d-e\right)^3\text{ta}}{6c^6e}$$

Mathematica [A]
time = 0.20, size = 140, normalized size = 1.22

$$\frac{cx(-15be^2+5bc^2e(9d+ex^2)+15ac^5x(3d^2+3dex^2+e^2x^4)-3bc^4(15d^2+5dex^2+e^2x^4))+15b(3c^4d^2-3c^2de+e^2+c^6(3d^2x^2+3dex^4+e^2x^6))\,\text{ArcTan}(cx)}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(-15*b*e^2 + 5*b*c^2*e*(9*d + e*x^2) + 15*a*c^5*x*(3*d^2 + 3*d*e*x^2 +
 e^2*x^4) - 3*b*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)) + 15*b*(3*c^4*d^2 - 3*c
^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x])/(90*c^6)
```

Maple [A]
time = 0.33, size = 171, normalized size = 1.49

method	result
derivativedivides	$\frac{\frac{\left(c^2ex^2+c^2d\right)^3a}{6c^4e}+\frac{b\arctan(cx)d^2c^2x^2}{2}+\frac{bc^2e\arctan(cx)dx^4}{2}+\frac{bc^2e^2\arctan(cx)x^6}{6}-\frac{bcd^2x}{2}-\frac{bcdex^3}{6}-\frac{bce^2x^5}{30}+\frac{bdex}{2c}+\frac{be^2x^3}{18c}-\frac{be}{6}}{c^2}$
default	$\frac{\frac{\left(c^2ex^2+c^2d\right)^3a}{6c^4e}+\frac{b\arctan(cx)d^2c^2x^2}{2}+\frac{bc^2e\arctan(cx)dx^4}{2}+\frac{bc^2e^2\arctan(cx)x^6}{6}-\frac{bcd^2x}{2}-\frac{bcdex^3}{6}-\frac{bce^2x^5}{30}+\frac{bdex}{2c}+\frac{be^2x^3}{18c}-\frac{be}{6}}{c^2}$
risch	$\frac{iebdx^4\ln(-icx+1)}{4}+\frac{bd^3\arctan\left(\frac{\left(-c^7d^3+6c^5d^2e-6c^3de^2+2ce^3\right)x}{c^6d^3-6c^4d^2e+6c^2de^2-2e^3}\right)}{12e}+\frac{ebd\arctan\left(\frac{\left(-c^7d^3+6c^5d^2e-6c^3de^2+2ce^3\right)x}{c^6d^3-6c^4d^2e+6c^2de^2-2e^3}\right)}{4c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*(c^2*e*x^2+c^2*d)^3*a/c^4/e+1/2*b*arctan(c*x)*d^2*c^2*x^2+1/2*b*
c^2*e*arctan(c*x)*d*x^4+1/6*b*c^2*e^2*arctan(c*x)*x^6-1/2*b*c*d^2*x-1/6*b*c
*d*e*x^3-1/30*b*c*e^2*x^5+1/2*b*d*e*x/c+1/18*b*e^2*x^3/c-1/6*b*e^2*x/c^3+1/
2*b*d^2*arctan(c*x)-1/2*b*d*e*arctan(c*x)/c^2+1/6*b*e^2*arctan(c*x)/c^4)
```

Maxima [A]
time = 0.47, size = 156, normalized size = 1.36

$$\frac{1}{6}ax^6e^2 + \frac{1}{2}adx^4e + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^2 + \frac{1}{6}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bde + \frac{1}{90}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5 - 5c^2x^3 + 15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^2 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2 + 1/2*(x^2*arctan(c*x) - c*(x
/c^2 - arctan(c*x)/c^3))*b*d^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x
)/c^4 + 3*arctan(c*x)/c^5))*b*d*e + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^
5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e^2
```

Fricas [A]
time = 2.63, size = 159, normalized size = 1.38

$$\frac{45\,ac^6d^2x^2 - 45\,bc^5d^2x + 15\,(3\,bc^6d^2x^2 + 3\,bc^4d^2 + (bc^6x^6 + b)e^2 + 3\,(bc^6dx^4 - bc^2d)e)\arctan(cx) + (15\,ac^6x^6 - 3\,bc^5x^5 + 5\,bc^3x^3 - 15\,bcx)e^2 + 15\,(3\,ac^6dx^4 - bc^5dx^3 + 3\,bc^3dx)e}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(45*a*c^6*d^2*x^2 - 45*b*c^5*d^2*x + 15*(3*b*c^6*d^2*x^2 + 3*b*c^4*d^2
 + (b*c^6*x^6 + b)*e^2 + 3*(b*c^6*d*x^4 - b*c^2*d)*e)*arctan(c*x) + (15*a*c
^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x)*e^2 + 15*(3*a*c^6*d*x^4 - b*
c^5*d*x^3 + 3*b*c^3*d*x)*e)/c^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(102) = 204$.
time = 0.41, size = 219, normalized size = 1.90

$$\begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2\operatorname{atan}(cx)}{2} + \frac{bdex^4\operatorname{atan}(cx)}{2} + \frac{be^2x^6\operatorname{atan}(cx)}{6} - \frac{bd^2x}{2c} - \frac{bdex^3}{6c} - \frac{be^2x^5}{30c} + \frac{bd^2\operatorname{atan}(cx)}{2c^2} + \frac{bdex}{2c^3} + \frac{be^2x^3}{18c^3} - \frac{bde\operatorname{atan}(cx)}{2c^4} - \frac{be^2x}{6c^5} + \frac{be^2\operatorname{atan}(cx)}{6c^6} & \text{for}\ c \neq 0 \\ a\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*atan(
c*x)/2 + b*d*e*x**4*atan(c*x)/2 + b*e**2*x**6*atan(c*x)/6 - b*d**2*x/(2*c)
- b*d*e*x**3/(6*c) - b*e**2*x**5/(30*c) + b*d**2*atan(c*x)/(2*c**2) + b*d*e
*x/(2*c**3) + b*e**2*x**3/(18*c**3) - b*d*e*atan(c*x)/(2*c**4) - b*e**2*x/(
6*c**5) + b*e**2*atan(c*x)/(6*c**6), Ne(c, 0)), (a*(d**2*x**2/2 + d*e*x**4/
2 + e**2*x**6/6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.46, size = 167, normalized size = 1.45

$$\frac{a\,d^2\,x^2}{2}+\frac{a\,e^2\,x^6}{6}-\frac{b\,e^2\,x^5}{30\,c}+\frac{b\,e^2\,x^3}{18\,c^3}+\frac{a\,d\,e\,x^4}{2}-\frac{b\,d^2\,x}{2\,c}-\frac{b\,e^2\,x}{6\,c^5}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{2\,c^2}+\frac{b\,e^2\,\mathrm{atan}(c\,x)}{6\,c^6}+\frac{b\,d^2\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e^2\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,e\,x^3}{6\,c}+\frac{b\,d\,e\,x}{2\,c^3}-\frac{b\,d\,e\,\mathrm{atan}(c\,x)}{2\,c^4}+\frac{b\,d\,e\,x^4\,\mathrm{atan}(c\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

```
[Out] (a*d^2*x^2)/2 + (a*e^2*x^6)/6 - (b*e^2*x^5)/(30*c) + (b*e^2*x^3)/(18*c^3) +
 (a*d*e*x^4)/2 - (b*d^2*x)/(2*c) - (b*e^2*x)/(6*c^5) + (b*d^2*atan(c*x))/(2
*c^2) + (b*e^2*atan(c*x))/(6*c^6) + (b*d^2*x^2*atan(c*x))/2 + (b*e^2*x^6*at
an(c*x))/6 - (b*d*e*x^3)/(6*c) + (b*d*e*x)/(2*c^3) - (b*d*e*atan(c*x))/(2*c
^4) + (b*d*e*x^4*atan(c*x))/2
```

3.1128 $\int (d+ex^2)^2 (a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=124

$$-\frac{b(10c^2d-3e)\,ex^2}{30c^3}-\frac{be^2x^4}{20c}+d^2x(a+b\text{ArcTan}(cx))+\frac{2}{3}dex^3(a+b\text{ArcTan}(cx))+\frac{1}{5}e^2x^5(a+b\text{ArcTan}(cx))-\frac{b(1$$

```
[Out] -1/30*b*(10*c^2*d-3*e)*e*x^2/c^3-1/20*b*e^2*x^4/c+d^2*x*(a+b*arctan(c*x))+2
/3*d*e*x^3*(a+b*arctan(c*x))+1/5*e^2*x^5*(a+b*arctan(c*x))-1/30*b*(15*c^4*d
^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)/c^5
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {200, 5032, 1608, 1261, 712}

$$d^2x(a+b\text{ArcTan}(cx))+\frac{2}{3}dex^3(a+b\text{ArcTan}(cx))+\frac{1}{5}e^2x^5(a+b\text{ArcTan}(cx))-\frac{bex^2(10c^2d-3e)}{30c^3}-\frac{b(15c^4d^2-10c^2de+3e^2)\log(c^2x^2+1)}{30c^5}-\frac{be^2x^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/30*(b*(10*c^2*d - 3*e)*e*x^2)/c^3 - (b*e^2*x^4)/(20*c) + d^2*x*(a + b*Ar
cTan[c*x]) + (2*d*e*x^3*(a + b*ArcTan[c*x]))/3 + (e^2*x^5*(a + b*ArcTan[c*x
]))/5 - (b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(30*c^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)\,dx = d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(c\right.$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(c\right.$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(c\right.$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\tan^{-1}(c\right.$$

$$= -\frac{b(10c^2d-3e)\,ex^2}{30c^3}-\frac{be^2x^4}{20c}+d^2x\left(a+b\tan^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\tan\right.$$

Mathematica [A]
time = 0.04, size = 130, normalized size = 1.05

$$\frac{c^2x(4ac^3(15d^2+10dex^2+3e^2x^4)+bex(6e-c^2(20d+3ex^2)))+4bc^5x(15d^2+10dex^2+3e^2x^4)\,\mathrm{ArcTan}(cx)-2b(15c^4d^2-10c^2de+3e^2)\log\left(1+c^2x^2\right)}{60c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x*(4*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*x*(6*e - c^2*(20*d
+ 3*e*x^2))) + 4*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcTan[c*x] - 2*
b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(60*c^5)
```

Maple [A]
time = 0.15, size = 161, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\arctan(cx)d^2cx+\frac{2bc\arctan(cx)dex^3}{3}+\frac{bc\arctan(cx)e^2x^5}{5}-\frac{bdex^2}{3}-\frac{be^2x^4}{20}+\frac{be^2x^2}{10c^2}-\frac{b\ln(c^2x^2}{2}}{c}$

default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\arctan(cx)d^2cx+\frac{2bc\arctan(cx)dex^3}{3}+\frac{bc\arctan(cx)e^2x^5}{5}-\frac{bdex^2}{3}-\frac{be^2x^4}{20}+\frac{be^2x^2}{10c^2}-\frac{b\ln(c^2}{}}{c}$
risch	$-\frac{ib(3e^2x^5+10dex^3+15d^2x)\ln(icx+1)}{30}+\frac{ibe^2x^5\ln(-icx+1)}{10}+\frac{ibdex^3\ln(-icx+1)}{3}+\frac{ae^2x^5}{5}+\frac{ibd^2x\ln(-icx+}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b*arctan(c*x)*d^2*c*
x+2/3*b*c*arctan(c*x)*d*e*x^3+1/5*b*c*arctan(c*x)*e^2*x^5-1/3*b*d*e*x^2-1/2
0*b*e^2*x^4+1/10*b/c^2*e^2*x^2-1/2*b*ln(c^2*x^2+1)*d^2+1/3*b/c^2*ln(c^2*x^2
+1)*d*e-1/10*b/c^4*ln(c^2*x^2+1)*e^2)
```

Maxima [A]
time = 0.27, size = 147, normalized size = 1.19

$$\frac{1}{5}ax^5e^2+\frac{2}{3}adx^3e+ad^2x+\frac{1}{3}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bde+\frac{(2cx\arctan(cx)-\log(c^2x^2+1))bd^2}{2c}+\frac{1}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^2 + 2/3*a*d*x^3*e + a*d^2*x + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c
^2 - log(c^2*x^2 + 1)/c^4))*b*d*e + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 +
1))*b*d^2/c + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^
2*x^2 + 1)/c^6))*b*e^2
```

Fricas [A]
time = 3.93, size = 149, normalized size = 1.20

$$\frac{60ac^5d^2x+4(3bc^5x^5e^2+10bc^5dx^3e+15bc^5d^2x)\arctan(cx)+3(4ac^5x^5-bc^4x^4+2bc^2x^2)e^2+20(2ac^5dx^3-bc^4dx^2)e-2(15bc^4d^2-10bc^2de+3be^2)\log(c^2x^2+1)}{60c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*c^5*d^2*x + 4*(3*b*c^5*x^5*e^2 + 10*b*c^5*d*x^3*e + 15*b*c^5*d^2
*x)*arctan(c*x) + 3*(4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2)*e^2 + 20*(2*a*c
^5*d*x^3 - b*c^4*d*x^2)*e - 2*(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*log(c
^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.36, size = 194, normalized size = 1.56

$$\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\operatorname{atan}(cx)+\frac{2bdex^3\operatorname{atan}(cx)}{3}+\frac{be^2x^5\operatorname{atan}(cx)}{5}-\frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bdex^2}{3c}-\frac{be^2x^4}{20c}+\frac{bde\log\left(x^2+\frac{1}{c^2}\right)}{3c^3}+\frac{be^2x^2}{10c^3}-\frac{be^2\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c\neq 0 \\ a\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*atan(c*x) +
 2*b*d*e*x**3*atan(c*x)/3 + b*e**2*x**5*atan(c*x)/5 - b*d**2*log(x**2 + c**
(-2))/(2*c) - b*d*e*x**2/(3*c) - b*e**2*x**4/(20*c) + b*d*e*log(x**2 + c**(
-2))/(3*c**3) + b*e**2*x**2/(10*c**3) - b*e**2*log(x**2 + c**(-2))/(10*c**5
), Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.66, size = 150, normalized size = 1.21

$$\frac{a\,e^2\,x^5}{5}+a\,d^2\,x-\frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{2\,c}-\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,e^2\,x^4}{20\,c}+\frac{b\,e^2\,x^2}{10\,c^3}+\frac{2\,a\,d\,e\,x^3}{3}+b\,d^2\,x\,\mathrm{atan}(c\,x)+\frac{b\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^3}-\frac{b\,d\,e\,x^2}{3\,c}+\frac{2\,b\,d\,e\,x^3\,\mathrm{atan}(c\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2)^2,x)
```

```
[Out] (a*e^2*x^5)/5 + a*d^2*x - (b*d^2*log(c^2*x^2 + 1))/(2*c) - (b*e^2*log(c^2*x
^2 + 1))/(10*c^5) - (b*e^2*x^4)/(20*c) + (b*e^2*x^2)/(10*c^3) + (2*a*d*e*x^
3)/3 + b*d^2*x*atan(c*x) + (b*e^2*x^5*atan(c*x))/5 + (b*d*e*log(c^2*x^2 + 1
))/(3*c^3) - (b*d*e*x^2)/(3*c) + (2*b*d*e*x^3*atan(c*x))/3
```

3.1129 $\int \frac{(d+ex^2)^2(a+b\text{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=137

$$-\frac{bdex}{c}+\frac{be^2x}{4c^3}-\frac{be^2x^3}{12c}+\frac{bde\text{ArcTan}(cx)}{c^2}-\frac{be^2\text{ArcTan}(cx)}{4c^4}+dex^2(a+b\text{ArcTan}(cx))+\frac{1}{4}e^2x^4(a+b\text{ArcTan}(cx))+$$

```
[Out] -b*d*e*x/c+1/4*b*e^2*x/c^3-1/12*b*e^2*x^3/c+b*d*e*arctan(c*x)/c^2-1/4*b*e^2
*arctan(c*x)/c^4+d*e*x^2*(a+b*arctan(c*x))+1/4*e^2*x^4*(a+b*arctan(c*x))+a*
d^2*ln(x)+1/2*I*b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5100, 4940, 2438, 4946, 327, 209, 308}

$$dex^2(a+b\text{ArcTan}(cx))+\frac{1}{4}e^2x^4(a+b\text{ArcTan}(cx))+ad^2\log(x)-\frac{be^2\text{ArcTan}(cx)}{4c^4}+\frac{bde\text{ArcTan}(cx)}{c^2}+\frac{be^2x}{4c^3}+\frac{1}{2}ibd^2\text{Li}_2(-icx)-\frac{1}{2}ibd^2\text{Li}_2(icx)-\frac{bdex}{c}-\frac{be^2x^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] -((b*d*e*x)/c) + (b*e^2*x)/(4*c^3) - (b*e^2*x^3)/(12*c) + (b*d*e*ArcTan[c*x
])/c^2 - (b*e^2*ArcTan[c*x])/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x
^4*(a + b*ArcTan[c*x]))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x]
 - (I/2)*b*d^2*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x}\,dx = \int \left(\frac{d^2(a+b\tan^{-1}(cx))}{x}+2dex\left(a+b\tan^{-1}(cx)\right)+e^2x^3\left(a+b\tan^{-1}(c\right.\right.$$

$$= d^2\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+(2de)\int x\left(a+b\tan^{-1}(cx)\right)\,dx+e^2\int x^3(a$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)+\frac{1}{2}(ibd^2$$

$$= -\frac{bdex}{c}+dex^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)+ad^2\log(x)$$

$$= -\frac{bdex}{c}+\frac{be^2x}{4c^3}-\frac{be^2x^3}{12c}+\frac{bde\tan^{-1}(cx)}{c^2}+dex^2\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}e$$

$$= -\frac{bdex}{c}+\frac{be^2x}{4c^3}-\frac{be^2x^3}{12c}+\frac{bde\tan^{-1}(cx)}{c^2}-\frac{be^2\tan^{-1}(cx)}{4c^4}+dex^2(a+b$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 0.88

$$adex^2+\frac{1}{4}ae^2x^4+\frac{bde(-cx+(1+c^2x^2)\,\text{ArcTan}(cx))}{c^2}+\frac{be^2(3cx-c^3x^3+3(-1+c^4x^4)\,\text{ArcTan}(cx))}{12c^4}+ad^2\log(x)+\frac{1}{2}ibd^2(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^
2 + (b*e^2*(3*c*x - c^3*x^3 + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(12*c^4) + a*d
^2*Log[x] + (I/2)*b*d^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x])
```

Maple [A]
time = 0.20, size = 187, normalized size = 1.36

method	result
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+b\arctan(cx)\,de\,x^2+\frac{b\arctan(cx)e^2x^4}{4}+b\arctan(cx)\,d^2\ln($
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+b\arctan(cx)\,de\,x^2+\frac{b\arctan(cx)e^2x^4}{4}+b\arctan(cx)\,d^2\ln($
risch	$\frac{ade}{c^2}+\frac{ib\ln(-icx+1)(-icx+1)de}{c^2}+\frac{ib\ln(-icx+1)(-icx+1)^4e^2}{8c^4}-\frac{ibed\ln(icx+1)x^2}{2}-\frac{ibde\ln(c^2x^2+1)}{4c^2}+\frac{3ib\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*d*e*x^2+1/4*a*e^2*x^4+a*d^2*ln(c*x)+b*arctan(c*x)*d*e*x^2+1/4*b*arctan(c*
x)*e^2*x^4+b*arctan(c*x)*d^2*ln(c*x)-b*d*e*x/c-1/12*b*e^2*x^3/c+1/4*b*e^2*x
/c^3+b*d*e*arctan(c*x)/c^2-1/4*b*e^2*arctan(c*x)/c^4+1/2*I*b*d^2*ln(c*x)*ln
(1+I*c*x)-1/2*I*b*d^2*ln(c*x)*ln(1-I*c*x)-1/2*I*b*d^2*dilog(1-I*c*x)+1/2*I*
b*d^2*dilog(1+I*c*x)
```

Maxima [A]
time = 0.61, size = 171, normalized size = 1.25

$$\frac{1}{4}ax^4e^2+adx^2e+ad^2\log(x)-\frac{3\,\pi bc^4d^2\log(c^2x^2+1)-12\,bc^4d^2\arctan(cx)\log(cx)+6i\,bc^4d^2\text{Li}_2(i\,cx+1)-6i\,bc^4d^2\text{Li}_2(-i\,cx+1)+bc^3x^3e^2+3\,(4\,bc^3de-bce^2)x-3\,(bc^4x^4e^2+4\,bc^4dx^2e+4\,bc^2de-be^2)\arctan(cx)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^2 + a*d*x^2*e + a*d^2*log(x) - 1/12*(3*pi*b*c^4*d^2*log(c^2*x^2
 + 1) - 12*b*c^4*d^2*arctan(c*x)*log(c*x) + 6*I*b*c^4*d^2*dilog(I*c*x + 1)
- 6*I*b*c^4*d^2*dilog(-I*c*x + 1) + b*c^3*x^3*e^2 + 3*(4*b*c^3*d*e - b*c*e^
2)*x - 3*(b*c^4*x^4*e^2 + 4*b*c^4*d*x^2*e + 4*b*c^2*d*e - b*e^2)*arctan(c*x
))/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.71, size = 157, normalized size = 1.15

$$\begin{cases} \frac{a\,\left(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2\right)}{4} & \text{if}\ \ c=0\\ \frac{a\,\left(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2\right)}{4}-2\,b\,d\,e\,\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)-\frac{b\,e^2\,\left(3\,\mathrm{atan}(c\,x)-3\,c\,x+c^3\,x^3\right)}{12\,c^4}+\frac{b\,e^2\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,d^2\,\mathrm{Li_2}(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,d^2\,\mathrm{Li_2}(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x,x)
```

```
[Out] piecewise(c == 0, (a*(4*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4, c ~= 0, (a*(4
*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2 +
(b*d^2*dilog(c*x*1i + 1)*1i)/2 - 2*b*d*e*(x/(2*c) - atan(c*x)*(1/(2*c^2) +
x^2/2)) - (b*e^2*(3*atan(c*x) - 3*c*x + c^3*x^3))/(12*c^4) + (b*e^2*x^4*ata
n(c*x))/4)
```

3.1130 $\int \frac{(d+ex^2)^2(a+b\text{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{be^2x^2}{6c}-\frac{d^2(a+b\text{ArcTan}(cx))}{x}+2dex(a+b\text{ArcTan}(cx))+\frac{1}{3}e^2x^3(a+b\text{ArcTan}(cx))+bcd^2\log(x)-\frac{b(3c^4d^2+6$$

```
[Out] -1/6*b*e^2*x^2/c-d^2*(a+b*arctan(c*x))/x+2*d*e*x*(a+b*arctan(c*x))+1/3*e^2*
x^3*(a+b*arctan(c*x))+b*c*d^2*ln(x)-1/6*b*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^2*
x^2+1)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {276, 5096, 1265, 907}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{x}+2dex(a+b\text{ArcTan}(cx))+\frac{1}{3}e^2x^3(a+b\text{ArcTan}(cx))-\frac{b(3c^4d^2+6c^2de-e^2)\log(c^2x^2+1)}{6c^3}+bcd^2\log(x)-\frac{be^2x^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -1/6*(b*e^2*x^2)/c - (d^2*(a + b*ArcTan[c*x]))/x + 2*d*e*x*(a + b*ArcTan[c*
x]) + (e^2*x^3*(a + b*ArcTan[c*x]))/3 + b*c*d^2*Log[x] - (b*(3*c^4*d^2 + 6*
c^2*d*e - e^2)*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^2}\,dx = -\frac{d^2(a+b\tan^{-1}(cx))}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{d^2(a+b\tan^{-1}(cx))}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{d^2(a+b\tan^{-1}(cx))}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{be^2x^2}{6c} - \frac{d^2(a+b\tan^{-1}(cx))}{x} + 2dex\left(a+b\tan^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\right.$$

Mathematica [A]
time = 0.06, size = 114, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{6ad^2}{x} + 12adex - \frac{be^2x^2}{c} + 2ae^2x^3 + \frac{2b(-3d^2+6dex^2+e^2x^4)\operatorname{ArcTan}(cx)}{x} + 6bcd^2\log(x) + \frac{b(-3c^4d^2-6c^2de+e^2)\log(1+c^2x^2)}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] ((-6*a*d^2)/x + 12*a*d*e*x - (b*e^2*x^2)/c + 2*a*e^2*x^3 + (2*b*(-3*d^2 + 6
*d*e*x^2 + e^2*x^4)*ArcTan[c*x])/x + 6*b*c*d^2*Log[x] + (b*(-3*c^4*d^2 - 6*
c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^3)/6
```

Maple [A]
time = 0.19, size = 159, normalized size = 1.46

method	result
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4} + \frac{2b\arctan(cx)dex}{c} + \frac{b\arctan(cx)e^2x^3}{3c} - \frac{b\arctan(cx)d^2}{cx} - \frac{be^2x^2}{6c^2} - \frac{b\ln(c^2x^2+1)}{2}\right.$

default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\arctan(cx)dex}{c}+\frac{b\arctan(cx)e^2x^3}{3c}-\frac{b\arctan(cx)d^2}{cx}-\frac{b\,e^2x^2}{6c^2}-\frac{b\ln(c^2x^2+}{2}\right.$
risch	$\frac{ib(-e^2x^4-6de\,x^2+3d^2)\ln(icx+1)}{6x}+\frac{ib\,c^3e^2x^4\ln(-icx+1)+6ib\,c^3de\,x^2\ln(-icx+1)+2a\,c^3e^2x^4+6b\,c^4d^2\ln(x)x-3\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^4*(2*c^3*d*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+2*b/c*arctan(c*x)*d*e*x+1/
3*b/c*arctan(c*x)*e^2*x^3-b*arctan(c*x)*d^2/c/x-1/6*b/c^2*e^2*x^2-1/2*b*ln(
c^2*x^2+1)*d^2-b/c^2*ln(c^2*x^2+1)*d*e+1/6*b/c^4*ln(c^2*x^2+1)*e^2+b*d^2*ln
(c*x))
```

Maxima [A]
time = 0.27, size = 130, normalized size = 1.19

$$\frac{1}{3}ax^3e^2-\frac{1}{2}\left(c(\log(c^2x^2+1)-\log(x^2))+\frac{2\arctan(cx)}{x}\right)bd^2+2adxe+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)be^2+\frac{(2cx\arctan(cx)-\log(c^2x^2+1))bde}{c}-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d
^2 + 2*a*d*x*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4
))*b*e^2 + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e/c - a*d^2/x
```

Fricas [A]
time = 3.86, size = 141, normalized size = 1.29

$$\frac{6\,bc^4d^2x\log(x)+12\,ac^3dx^2e-6\,ac^3d^2+2\,(bc^3x^4e^2+6\,bc^3dx^2e-3\,bc^3d^2)\arctan(cx)+(2\,ac^3x^4-bc^2x^3)e^2-(3\,bc^4d^2x+6\,bc^2dxe-bxe^2)\log(c^2x^2+1)}{6\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*c^4*d^2*x*log(x) + 12*a*c^3*d*x^2*e - 6*a*c^3*d^2 + 2*(b*c^3*x^4*e
^2 + 6*b*c^3*d*x^2*e - 3*b*c^3*d^2)*arctan(c*x) + (2*a*c^3*x^4 - b*c^2*x^3)
*e^2 - (3*b*c^4*d^2*x + 6*b*c^2*d*x*e - b*x*e^2)*log(c^2*x^2 + 1))/(c^3*x)
```

Sympy [A]
time = 0.53, size = 165, normalized size = 1.51

$$\begin{cases} -\frac{ad^2}{x}+2adex+\frac{ae^2x^3}{3}+bcd^2\log(x)-\frac{bcd^2\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd^2\operatorname{atan}(cx)}{x}+2bdex\operatorname{atan}(cx)+\frac{be^2x^3\operatorname{atan}(cx)}{3}-\frac{bde\log\left(x^2+\frac{1}{c^2}\right)}{c}-\frac{be^2x^2}{6c}+\frac{be^2\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c\neq 0 \\ a\left(-\frac{d^2}{x}+2dex+\frac{e^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*log(x) - b*c*d*
*2*log(x**2 + c**(-2))/2 - b*d**2*atan(c*x)/x + 2*b*d*e*x*atan(c*x) + b*e**
2*x**3*atan(c*x)/3 - b*d*e*log(x**2 + c**(-2))/c - b*e**2*x**2/(6*c) + b*e*
*2*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**2/x + 2*d*e*x + e**2*x*
*3/3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.70, size = 135, normalized size = 1.24

$$\frac{a\,e^2\,x^3}{3}-\frac{a\,d^2}{x}+2\,a\,d\,e\,x+\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,e^2\,x^2}{6\,c}-\frac{b\,c\,d^2\,\ln\left(c^2\,x^2+1\right)}{2}+b\,c\,d^2\,\ln\left(x\right)-\frac{b\,d^2\,\mathrm{atan}\left(c\,x\right)}{x}+\frac{b\,e^2\,x^3\,\mathrm{atan}\left(c\,x\right)}{3}-\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{c}+2\,b\,d\,e\,x\,\mathrm{atan}\left(c\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^2,x)
```

```
[Out] (a*e^2*x^3)/3 - (a*d^2)/x + 2*a*d*e*x + (b*e^2*log(c^2*x^2 + 1))/(6*c^3) -
(b*e^2*x^2)/(6*c) - (b*c*d^2*log(c^2*x^2 + 1))/2 + b*c*d^2*log(x) - (b*d^2*
atan(c*x))/x + (b*e^2*x^3*atan(c*x))/3 - (b*d*e*log(c^2*x^2 + 1))/c + 2*b*d
*e*x*atan(c*x)
```

3.1131 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{bcd^2}{2x}-\frac{be^2x}{2c}-\frac{1}{2}bc^2d^2\text{ArcTan}(cx)+\frac{be^2\text{ArcTan}(cx)}{2c^2}-\frac{d^2(a+b\text{ArcTan}(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\text{ArcTan}(cx))+2ade$$

```
[Out] -1/2*b*c*d^2/x-1/2*b*e^2*x/c-1/2*b*c^2*d^2*arctan(c*x)+1/2*b*e^2*arctan(c*x
)/c^2-1/2*d^2*(a+b*arctan(c*x))/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))+2*a*d*e*l
n(x)+I*b*d*e*polylog(2,-I*c*x)-I*b*d*e*polylog(2,I*c*x)
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5100, 4946, 331, 209, 4940, 2438, 327}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\text{ArcTan}(cx))+2ade\log(x)-\frac{1}{2}bc^2d^2\text{ArcTan}(cx)+\frac{be^2\text{ArcTan}(cx)}{2c^2}-\frac{bcd^2}{2x}+ibde\text{Li}_2(-icx)-ibde\text{Li}_2(icx)-\frac{be^2x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^2)/x - (b*e^2*x)/(2*c) - (b*c^2*d^2*ArcTan[c*x])/2 + (b*e^2*Arc
Tan[c*x])/(2*c^2) - (d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (e^2*x^2*(a + b*Arc
Tan[c*x]))/2 + 2*a*d*e*Log[x] + I*b*d*e*PolyLog[2, (-I)*c*x] - I*b*d*e*Poly
Log[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2\,(a+b\tan^{-1}(cx))}{x^3}\,dx &= \int \left(\frac{d^2(a+b\tan^{-1}(cx))}{x^3}+\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x\big(a+b\tan^{-1}(cx)\right.\\ &= d^2\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+(2de)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+e^2\int x\big(a+b\,\mathrm{t}\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{2x^2}+\frac{1}{2}e^2x^2\big(a+b\tan^{-1}(cx)\big)+2ade\log(x)+\frac{1}{2}\big(bcd\\ &= -\frac{bcd^2}{2x}-\frac{be^2x}{2c}-\frac{d^2(a+b\tan^{-1}(cx))}{2x^2}+\frac{1}{2}e^2x^2\big(a+b\tan^{-1}(cx)\big)+2ade\\ &= -\frac{bcd^2}{2x}-\frac{be^2x}{2c}-\frac{1}{2}bc^2d^2\tan^{-1}(cx)+\frac{be^2\tan^{-1}(cx)}{2c^2}-\frac{d^2(a+b\tan^{-1}(cx}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 114, normalized size = 0.89

$$\frac{1}{2}\left(-\frac{ad^2}{x^2}+ae^2x^2-\frac{bd^2\text{ArcTan}(cx)}{x^2}-\frac{bcd^2(1+cx\text{ArcTan}(cx))}{x}+\frac{be^2(-cx+(1+c^2x^2)\,\text{ArcTan}(cx))}{c^2}+4ade\log(x)+2ibde(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (-((a*d^2)/x^2) + a*e^2*x^2 - (b*d^2*ArcTan[c*x])/x^2 - (b*c*d^2*(1 + c*x*A
rcTan[c*x]))/x + (b*e^2*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^2 + 4*a*d*e
*Log[x] + (2*I)*b*d*e*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]))/2
```

Maple [A]
time = 0.17, size = 211, normalized size = 1.65

method	result
derivativedivides	$c^2\left(\frac{a\,e^2x^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{b\arctan(cx)e^2x^2}{2c^2}-\frac{b\arctan(cx)d^2}{2c^2x^2}+\frac{2b\arctan(cx)de\ln(cx)}{c^2}-\frac{b\,e^2x}{2c^3}-\right.$
default	$c^2\left(\frac{a\,e^2x^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{b\arctan(cx)e^2x^2}{2c^2}-\frac{b\arctan(cx)d^2}{2c^2x^2}+\frac{2b\arctan(cx)de\ln(cx)}{c^2}-\frac{b\,e^2x}{2c^3}-\right.$
risch	$\frac{ib\ln(-icx+1)e^2x^2}{4}+\frac{b\,e^2\arctan(cx)}{2c^2}+\frac{ib\,d^2\ln(icx+1)}{4x^2}+ibde\,\text{dilog}\,(icx+1)-\frac{b\,c^2d^2\arctan(cx)}{2}-\frac{b\,e^2x}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/c^2*e^2*x^2-1/2*a*d^2/c^2/x^2+2*a/c^2*d*e*ln(c*x)+1/2*b/c^2*arct
an(c*x)*e^2*x^2-1/2*b*arctan(c*x)*d^2/c^2/x^2+2*b/c^2*arctan(c*x)*d*e*ln(c*
x)-1/2*b*e^2*x/c^3-1/2*b*d^2*arctan(c*x)+1/2*b*e^2*arctan(c*x)/c^4-1/2*b*d^
2/c/x+I*b/c^2*d*e*ln(c*x)*ln(1+I*c*x)-I*b/c^2*d*e*ln(c*x)*ln(1-I*c*x)+I*b/c
^2*d*e*dilog(1+I*c*x)-I*b/c^2*d*e*dilog(1-I*c*x))
```

Maxima [A]
time = 0.60, size = 154, normalized size = 1.20

$$-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd^2+\frac{1}{2}ax^2e^2+2ade\log(x)-\frac{ad^2}{2x^2}-\frac{\pi bc^2de\log(c^2x^2+1)-4bc^2d\arctan(cx)e\log(cx)+2ibc^2d\text{Li}_2(icx+1)e-2ibc^2d\text{Li}_2(-icx+1)e+bcxe^2-(bc^2x^2e^2+be^2)\arctan(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2 + 1/2*a*x^2*e^2 + 2*
a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/2*(pi*b*c^2*d*e*log(c^2*x^2 + 1) - 4*b*c^2
*d*arctan(c*x)*e*log(c*x) + 2*I*b*c^2*d*dilog(I*c*x + 1)*e - 2*I*b*c^2*d*di
log(-I*c*x + 1)*e + b*c*x*e^2 - (b*c^2*x^2*e^2 + b*e^2)*arctan(c*x))/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))\,(d + ex^2)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.68, size = 157, normalized size = 1.23

$$\begin{cases} \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} & \text{if } c=0 \\ \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} - b\,e^2\left(\frac{x}{2\,c} - \operatorname{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,d^2\left(c^3\operatorname{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,d^2\operatorname{atan}(c\,x)}{2\,x^2} - b\,d\,e\,(\operatorname{Li}_2(1-c\,x\,1\mathrm{i}) - \operatorname{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^3,x)
```

```
[Out] piecewise(c == 0, (a*(- d^2 + e^2*x^4 + 4*d*e*x^2*log(x)))/(2*x^2), c ~= 0,
 - b*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) + (a*(- d^2 + e^2*x^4 +
4*d*e*x^2*log(x)))/(2*x^2) - b*d*e*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)
)*1i - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (b*d^2*atan(c*x))/(2*x^2))
```

3.1132 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{bcd^2}{6x^2}-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{2de(a+b\mathrm{ArcTan}(cx))}{x}+e^2x(a+b\mathrm{ArcTan}(cx))-\frac{1}{3}bcd(c^2d-6e)\log(x)+\frac{b(c^4}{}$$

```
[Out] -1/6*b*c*d^2/x^2-1/3*d^2*(a+b*arctan(c*x))/x^3-2*d*e*(a+b*arctan(c*x))/x+e^
2*x*(a+b*arctan(c*x))-1/3*b*c*d*(c^2*d-6*e)*ln(x)+1/6*b*(c^4*d^2-6*c^2*d*e-
3*e^2)*ln(c^2*x^2+1)/c
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1265, 907}

$$-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{3x^3}-\frac{2de(a+b\mathrm{ArcTan}(cx))}{x}+e^2x(a+b\mathrm{ArcTan}(cx))-\frac{1}{3}bcd\log(x)\left(c^2d-6e\right)+\frac{b(c^4d^2-6c^2de-3e^2)\log\left(c^2x^2+1\right)}{6c}-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^2)/x^2 - (d^2*(a + b*ArcTan[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcT
an[c*x]))/x + e^2*x*(a + b*ArcTan[c*x]) - (b*c*d*(c^2*d - 6*e)*Log[x])/3 +
(b*(c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/(6*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\tan^{-1}(cx))}{x^4}\,dx &= -\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}-\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x(a+b\tan^{-1}(cx)) -\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}-\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x(a+b\tan^{-1}(cx)) -\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}-\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x(a+b\tan^{-1}(cx)) -\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}-\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x(a+b\tan^{-1}(cx)) -\\ &= -\frac{bcd^2}{6x^2}-\frac{d^2(a+b\tan^{-1}(cx))}{3x^3}-\frac{2de(a+b\tan^{-1}(cx))}{x}+e^2x(a+b\tan^{-}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 119, normalized size = 1.03

$$\frac{1}{6}\left(-\frac{2ad^2}{x^3}-\frac{bcd^2}{x^2}-\frac{12ade}{x}+6ae^2x-\frac{2b(d^2+6dex^2-3e^2x^4)\operatorname{ArcTan}(cx)}{x^3}-2bcd(c^2d-6e)\log(x)+\frac{b(c^4d^2-6c^2de-3e^2)\log(1+c^2x^2)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^2)/x^3 - (b*c*d^2)/x^2 - (12*a*d*e)/x + 6*a*e^2*x - (2*b*(d^2 + 6*
d*e*x^2 - 3*e^2*x^4)*ArcTan[c*x])/x^3 - 2*b*c*d*(c^2*d - 6*e)*Log[x] + (b*(
c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c)/6
```

Maple [A]
time = 0.19, size = 166, normalized size = 1.44

method	result
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\arctan(cx)e^2x}{c^3}-\frac{b\arctan(cx)d^2}{3c^3x^3}-\frac{2b\arctan(cx)de}{c^3x}+\frac{b\ln(c^2x^2+1)d^2}{6}-\frac{b\ln(c^2x}{c}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\arctan(cx)e^2x}{c^3}-\frac{b\arctan(cx)d^2}{3c^3x^3}-\frac{2b\arctan(cx)de}{c^3x}+\frac{b\ln(c^2x^2+1)d^2}{6}-\frac{b\ln(c^2x}{c}\right.$
risch	$\frac{ib(-3e^2x^4+6de\,x^2+d^2)\ln(icx+1)}{6x^3}-\frac{2\ln(x)b\,c^4d^2x^3-\ln(-c^2x^2-1)b\,c^4d^2x^3-3ibc\,e^2x^4\ln(-icx+1)-12\ln(x)b\,c^2de\,x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^4*(e^2*c*x-1/3*c*d^2/x^3-2*c*d*e/x)+b/c^3*arctan(c*x)*e^2*x-1/3*b*
arctan(c*x)*d^2/c^3/x^3-2*b/c^3*arctan(c*x)*d*e/x+1/6*b*ln(c^2*x^2+1)*d^2-b
/c^2*ln(c^2*x^2+1)*d*e-1/2*b/c^4*ln(c^2*x^2+1)*e^2-1/3*b*d^2*ln(c*x)+2*b/c^
2*ln(c*x)*d*e-1/6*b*d^2/c^2/x^2)
```

Maxima [A]

time = 0.26, size = 135, normalized size = 1.17

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^2-\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bde+axe^2+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be^2}{2\,c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d^2 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d*e + a*x*e^2
+ 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^
2/x^3
```

Fricas [A]

time = 2.26, size = 149, normalized size = 1.30

$$\frac{6\,acx^4e^2-bc^2d^2x-12\,acdx^2e-2\,acd^2+2\left(3\,bcx^4e^2-6\,bcdx^2e-bcd^2\right)\arctan\left(cx\right)+\left(bc^4d^2x^3-6\,bc^2dx^3e-3\,bx^3e^2\right)\log\left(c^2x^2+1\right)-2\left(bc^4d^2x^3-6\,bc^2dx^3e\right)\log\left(x\right)}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c*x^4*e^2 - b*c^2*d^2*x - 12*a*c*d*x^2*e - 2*a*c*d^2 + 2*(3*b*c*x^
4*e^2 - 6*b*c*d*x^2*e - b*c*d^2)*arctan(c*x) + (b*c^4*d^2*x^3 - 6*b*c^2*d*x
^3*e - 3*b*x^3*e^2)*log(c^2*x^2 + 1) - 2*(b*c^4*d^2*x^3 - 6*b*c^2*d*x^3*e)*
log(x))/(c*x^3)
```

Sympy [A]

time = 0.56, size = 180, normalized size = 1.57

$$\begin{cases}-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-\frac{bc^3d^2\log\left(x\right)}{3}+\frac{bc^3d^2\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bcd^2}{6x^2}+2bcde\log\left(x\right)-bcde\log\left(x^2+\frac{1}{c^2}\right)-\frac{bd^2\operatorname{atan}\left(cx\right)}{3x^3}-\frac{2bde\operatorname{atan}\left(cx\right)}{x}+be^2x\operatorname{atan}\left(cx\right)-\frac{be^2\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for}\: c\neq 0\\a\left(-\frac{d^2}{3x^3}-\frac{2de}{x}+e^2x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*c**3*d**2*log(x)/3 +
 b*c**3*d**2*log(x**2 + c**(-2))/6 - b*c*d**2/(6*x**2) + 2*b*c*d*e*log(x) -
 b*c*d*e*log(x**2 + c**(-2)) - b*d**2*atan(c*x)/(3*x**3) - 2*b*d*e*atan(c*x
)/x + b*e**2*x*atan(c*x) - b*e**2*log(x**2 + c**(-2))/(2*c), Ne(c, 0)), (a*
(-d**2/(3*x**3) - 2*d*e/x + e**2*x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.68, size = 142, normalized size = 1.23

$$a\,e^2\,x-\frac{a\,d^2}{3\,x^3}+\frac{b\,c^3\,d^2\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{2\,c}-\frac{b\,c^3\,d^2\,\ln\left(x\right)}{3}-\frac{2\,a\,d\,e}{x}+b\,e^2\,x\,\mathrm{atan}(c\,x)-\frac{b\,c\,d^2}{6\,x^2}-\frac{b\,d^2\,\mathrm{atan}(c\,x)}{3\,x^3}-b\,c\,d\,e\,\ln\left(c^2\,x^2+1\right)+2\,b\,c\,d\,e\,\ln\left(x\right)-\frac{2\,b\,d\,e\,\mathrm{atan}(c\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^4,x)
```

```
[Out] a*e^2*x - (a*d^2)/(3*x^3) + (b*c^3*d^2*log(c^2*x^2 + 1))/6 - (b*e^2*log(c^2
*x^2 + 1))/(2*c) - (b*c^3*d^2*log(x))/3 - (2*a*d*e)/x + b*e^2*x*atan(c*x) -
 (b*c*d^2)/(6*x^2) - (b*d^2*atan(c*x))/(3*x^3) - b*c*d*e*log(c^2*x^2 + 1) +
 2*b*c*d*e*log(x) - (2*b*d*e*atan(c*x))/x
```

3.1133 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=139

$$-\frac{bcd^2}{12x^3}+\frac{bc^3d^2}{4x}-\frac{bcde}{x}+\frac{1}{4}bc^4d^2\mathrm{ArcTan}(cx)-bc^2de\mathrm{ArcTan}(cx)-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{4x^4}-\frac{de(a+b\mathrm{ArcTan}(cx))}{x^2}$$

```
[Out] -1/12*b*c*d^2/x^3+1/4*b*c^3*d^2/x-b*c*d*e/x+1/4*b*c^4*d^2*arctan(c*x)-b*c^2
*d*e*arctan(c*x)-1/4*d^2*(a+b*arctan(c*x))/x^4-d*e*(a+b*arctan(c*x))/x^2+a*
e^2*ln(x)+1/2*I*b*e^2*polylog(2,-I*c*x)-1/2*I*b*e^2*polylog(2,I*c*x)
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {5100, 4946, 331, 209, 4940, 2438}

$$-\frac{d^2(a+b\mathrm{ArcTan}(cx))}{4x^4}-\frac{de(a+b\mathrm{ArcTan}(cx))}{x^2}+ae^2\log(x)+\frac{1}{4}bc^4d^2\mathrm{ArcTan}(cx)-bc^2de\mathrm{ArcTan}(cx)+\frac{bc^3d^2}{4x}-\frac{bcd^2}{12x^3}-\frac{bcde}{x}+\frac{1}{2}ibe^2\mathrm{Li}_2(-icx)-\frac{1}{2}ibe^2\mathrm{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^2)/x^3 + (b*c^3*d^2)/(4*x) - (b*c*d*e)/x + (b*c^4*d^2*ArcTan[c
*x])/4 - b*c^2*d*e*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e*(
a + b*ArcTan[c*x]))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*PolyLog[2, (-I)*c*x] -
 (I/2)*b*e^2*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)}{x^5}\,dx &= \int \left(\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{x^5}+\frac{2de\left(a+b\tan^{-1}(cx)\right)}{x^3}+\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{x}\right.\\
&= d^2\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx+(2de)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+e^2\int \frac{a+b\tan\\
&= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{de\left(a+b\tan^{-1}(cx)\right)}{x^2}+ae^2\log(x)+\frac{1}{4}\left(bcd^2\right)\int\\
&= -\frac{bcd^2}{12x^3}-\frac{bcde}{x}-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{de\left(a+b\tan^{-1}(cx)\right)}{x^2}+ae^2\log(\\
&= -\frac{bcd^2}{12x^3}+\frac{bc^3d^2}{4x}-\frac{bcde}{x}-bc^2de\tan^{-1}(cx)-\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{4x^4}-\frac{de(\\
&= -\frac{bcd^2}{12x^3}+\frac{bc^3d^2}{4x}-\frac{bcde}{x}+\frac{1}{4}bc^4d^2\tan^{-1}(cx)-bc^2de\tan^{-1}(cx)-\frac{d^2(a+
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 121, normalized size = 0.87

$$-\frac{ad^2}{4x^4}-\frac{ade}{x^2}+\frac{bd^2\left(cx\left(-1+3c^2x^2\right)+3\left(-1+c^4x^4\right)\mathrm{ArcTan}(cx)\right)}{12x^4}-\frac{bde(\mathrm{ArcTan}(cx)+cx(1+cx\mathrm{ArcTan}(cx)))}{x^2}+ae^2\log(x)+\frac{1}{2}ibe^2(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/4*(a*d^2)/x^4 - (a*d*e)/x^2 + (b*d^2*(c*x*(-1 + 3*c^2*x^2) + 3*(-1 + c^4
*x^4)*ArcTan[c*x]))/(12*x^4) - (b*d*e*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*
x])))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I
*c*x])
```

Maple [A]
time = 0.18, size = 225, normalized size = 1.62

method	result
derivativedivides	$c^4\left(-\frac{a\,d^2}{4c^4x^4}-\frac{ade}{c^4x^2}+\frac{a\,e^2\ln(cx)}{c^4}-\frac{b\arctan(cx)d^2}{4c^4x^4}-\frac{b\arctan(cx)de}{c^4x^2}+\frac{b\arctan(cx)e^2\ln(cx)}{c^4}+\frac{ib\,e^2\ln(cx)\ln}{2c^4}\right.$
default	$c^4\left(-\frac{a\,d^2}{4c^4x^4}-\frac{ade}{c^4x^2}+\frac{a\,e^2\ln(cx)}{c^4}-\frac{b\arctan(cx)d^2}{4c^4x^4}-\frac{b\arctan(cx)de}{c^4x^2}+\frac{b\arctan(cx)e^2\ln(cx)}{c^4}+\frac{ib\,e^2\ln(cx)\ln}{2c^4}\right.$
risch	$a\,e^2\ln(-icx)-\frac{a\,d^2}{4x^4}-\frac{bc\,d^2}{12x^3}+\frac{b\,c^3d^2}{4x}-\frac{ade}{x^2}-\frac{bcde}{x}+\frac{ib\,c^4d^2\ln(icx)}{8}-b\,c^2de\arctan(cx)+\frac{ibde}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a*d^2/c^4/x^4-a/c^4*d*e/x^2+a/c^4*e^2*ln(c*x)-1/4*b*arctan(c*x)*d
^2/c^4/x^4-b/c^4*arctan(c*x)*d*e/x^2+b/c^4*arctan(c*x)*e^2*ln(c*x)+1/2*I*b/
c^4*e^2*ln(c*x)*ln(1+I*c*x)-1/2*I*b/c^4*e^2*ln(c*x)*ln(1-I*c*x)+1/2*I*b/c^4
*e^2*dilog(1+I*c*x)-1/2*I*b/c^4*e^2*dilog(1-I*c*x)+1/4*b*d^2*arctan(c*x)-b*
d*e*arctan(c*x)/c^2+1/4*b*d^2/c/x-b/c^3*d*e/x-1/12*b*d^2/c^3/x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^
2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d*e + b*e^2*integrate(arc
tan(c*x)/x, x) + a*e^2*log(x) - a*d*e/x^2 - 1/4*a*d^2/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.76, size = 177, normalized size = 1.27

$$\begin{cases} a\,e^2\,\ln(x)-\frac{\frac{a\,d^2}{4}+a\,e\,d\,x^2}{x^4} & \text{if}\ \ c=0\\ a\,e^2\,\ln(x)-\frac{\frac{a\,d^2}{4}+a\,e\,d\,x^2}{x^4}-\frac{b\,d^2\,\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c}-2\,b\,d\,e\,\left(\frac{c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}}{2\,c}+\frac{\mathrm{atan}(c\,x)}{2\,x^2}\right)-\frac{b\,d^2\,\mathrm{atan}(c\,x)}{4\,x^4}-\frac{b\,e^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,e^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^5,x)
```

```
[Out] piecewise(c == 0, - ((a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x), c ~= 0, - (
(a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x) - (b*e^2*dilog(- c*x*1i + 1)*1i)/
2 + (b*e^2*dilog(c*x*1i + 1)*1i)/2 - (b*d^2*((c^2/3 - c^4*x^2)/x^3 - c^5*at
an(c*x)))/(4*c) - 2*b*d*e*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x)/(2*x^2
)) - (b*d^2*atan(c*x))/(4*x^4))
```

3.1134 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{bcd^2}{20x^4}+\frac{bcd(3c^2d-10e)}{30x^2}-\frac{d^2(a+b\text{ArcTan}(cx))}{5x^5}-\frac{2de(a+b\text{ArcTan}(cx))}{3x^3}-\frac{e^2(a+b\text{ArcTan}(cx))}{x}+\frac{1}{15}bc(3c^4$$

```
[Out] -1/20*b*c*d^2/x^4+1/30*b*c*d*(3*c^2*d-10*e)/x^2-1/5*d^2*(a+b*arctan(c*x))/x
^5-2/3*d*e*(a+b*arctan(c*x))/x^3-e^2*(a+b*arctan(c*x))/x+1/15*b*c*(3*c^4*d^
2-10*c^2*d*e+15*e^2)*ln(x)-1/30*b*c*(3*c^4*d^2-10*c^2*d*e+15*e^2)*ln(c^2*x^
2+1)
```

Rubi [A]

time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1265, 907}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{5x^5}-\frac{2de(a+b\text{ArcTan}(cx))}{3x^3}-\frac{e^2(a+b\text{ArcTan}(cx))}{x}+\frac{bcd(3c^2d-10e)}{30x^2}-\frac{1}{30}bc(3c^4d^2-10c^2de+15e^2)\log(c^2x^2+1)+\frac{1}{15}bc\log(x)(3c^4d^2-10c^2de+15e^2)-\frac{bcd^2}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^2)/x^4 + (b*c*d*(3*c^2*d - 10*e))/(30*x^2) - (d^2*(a + b*ArcTa
n[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcTan[c*x]))/(3*x^3) - (e^2*(a + b*ArcTan
[c*x]))/x + (b*c*(3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*Log[x])/15 - (b*c*(3*c^4
*d^2 - 10*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/30
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^6}\,dx &= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5}-\frac{2de(a+b\tan^{-1}(cx))}{3x^3}-\frac{e^2(a+b\tan^{-1}(cx))}{x}-\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5}-\frac{2de(a+b\tan^{-1}(cx))}{3x^3}-\frac{e^2(a+b\tan^{-1}(cx))}{x}-\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5}-\frac{2de(a+b\tan^{-1}(cx))}{3x^3}-\frac{e^2(a+b\tan^{-1}(cx))}{x}-\\ &= -\frac{d^2(a+b\tan^{-1}(cx))}{5x^5}-\frac{2de(a+b\tan^{-1}(cx))}{3x^3}-\frac{e^2(a+b\tan^{-1}(cx))}{x}-\\ &= -\frac{bcd^2}{20x^4}+\frac{bcd(3c^2d-10e)}{30x^2}-\frac{d^2(a+b\tan^{-1}(cx))}{5x^5}-\frac{2de(a+b\tan^{-1}(cx}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 153, normalized size = 1.02

$$-\frac{12ad^2+3bcd^2x+40adex^2-2bcd(3c^2d-10e)\,x^3+60ae^2x^4+4b(3d^2+10dex^2+15e^2x^4)\,\mathrm{ArcTan}(cx)-4bc(3c^4d^2-10c^2de+15e^2)\,x^5\log(x)+2bc(3c^4d^2-10c^2de+15e^2)\,x^5\log\left(1+c^2x^2\right)}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/60*(12*a*d^2 + 3*b*c*d^2*x + 40*a*d*e*x^2 - 2*b*c*d*(3*c^2*d - 10*e)*x^3
 + 60*a*e^2*x^4 + 4*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcTan[c*x] - 4*b*c
*(3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*x^5*Log[x] + 2*b*c*(3*c^4*d^2 - 10*c^2*d
*e + 15*e^2)*x^5*Log[1 + c^2*x^2])/x^5
```

Maple [A]
time = 0.20, size = 213, normalized size = 1.42

method	result
derivativedivides	$c^{5}\left(\frac{a\left(-\frac{2de}{3c\,x^{3}}-\frac{d^{2}}{5c\,x^{5}}-\frac{e^{2}}{cx}\right)}{c^{4}}-\frac{2b\arctan(cx)de}{3c^{5}x^{3}}-\frac{b\arctan(cx)d^{2}}{5c^{5}x^{5}}-\frac{b\arctan(cx)e^{2}}{c^{5}x}-\frac{b\ln(c^{2}x^{2}+1)d^{2}}{10}+\frac{b\ln(c^{2}x}{3}\right.$
default	$c^{5}\left(\frac{a\left(-\frac{2de}{3c\,x^{3}}-\frac{d^{2}}{5c\,x^{5}}-\frac{e^{2}}{cx}\right)}{c^{4}}-\frac{2b\arctan(cx)de}{3c^{5}x^{3}}-\frac{b\arctan(cx)d^{2}}{5c^{5}x^{5}}-\frac{b\arctan(cx)e^{2}}{c^{5}x}-\frac{b\ln(c^{2}x^{2}+1)d^{2}}{10}+\frac{b\ln(c^{2}x}{3}\right.$
risch	$\frac{ib(15e^{2}x^{4}+10de\,x^{2}+3d^{2})\ln(icx+1)}{30x^{5}}-\frac{-12\ln(x)b\,c^{5}d^{2}x^{5}+6\ln(-c^{2}x^{2}-1)b\,c^{5}d^{2}x^{5}+40\ln(x)b\,c^{3}de\,x^{5}-20\ln(-c^{2}x^{2}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^4*(-2/3/c*d*e/x^3-1/5/c*d^2/x^5-e^2/c/x)-2/3*b/c^5*arctan(c*x)*d*e
/x^3-1/5*b*arctan(c*x)*d^2/c^5/x^5-b/c^5*arctan(c*x)*e^2/x-1/10*b*ln(c^2*x^
2+1)*d^2+1/3*b/c^2*ln(c^2*x^2+1)*d*e-1/2*b/c^4*ln(c^2*x^2+1)*e^2+1/5*b*d^2*
ln(c*x)-2/3*b/c^2*ln(c*x)*d*e+b/c^4*ln(c*x)*e^2+1/10*b*d^2/c^2/x^2-1/3*b/c^
4*d*e/x^2-1/20*b*d^2/c^4/x^4)
```

Maxima [A]
time = 0.25, size = 166, normalized size = 1.11

$$-\frac{1}{20}\left(\left(2c^{4}\log\left(c^{2}x^{2}+1\right)-2c^{4}\log\left(x^{2}\right)-\frac{2c^{2}x^{2}-1}{x^{4}}\right)c+\frac{4\arctan\left(cx\right)}{x^{5}}\right)bd^{2}+\frac{1}{3}\left(\left(c^{2}\log\left(c^{2}x^{2}+1\right)-c^{2}\log\left(x^{2}\right)-\frac{1}{x^{2}}\right)c-\frac{2\arctan\left(cx\right)}{x^{3}}\right)bde-\frac{1}{2}\left(c\left(\log\left(c^{2}x^{2}+1\right)-\log\left(x^{2}\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)be^{2}-\frac{ae^{2}}{x}-\frac{2ade}{3x^{3}}-\frac{ad^{2}}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d^2 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*d*e - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*
arctan(c*x)/x)*b*e^2 - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

Fricas [A]
time = 2.55, size = 173, normalized size = 1.15

$$\frac{6\,bc^{3}d^{2}x^{3}-60\,ax^{4}e^{2}-3\,bcd^{2}x-12\,ad^{2}-4\left(15\,bx^{4}e^{2}+10\,bdx^{2}e+3\,bd^{2}\right)\arctan\left(cx\right)-20\left(bcdx^{3}+2\,adx^{2}\right)e-2\left(3\,bc^{5}d^{2}x^{5}-10\,bc^{3}dx^{5}e+15\,bcx^{5}e^{2}\right)\log\left(c^{2}x^{2}+1\right)+4\left(3\,bc^{5}d^{2}x^{5}-10\,bc^{3}dx^{5}e+15\,bcx^{5}e^{2}\right)\log\left(x\right)}{60\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(6*b*c^3*d^2*x^3 - 60*a*x^4*e^2 - 3*b*c*d^2*x - 12*a*d^2 - 4*(15*b*x^4
*e^2 + 10*b*d*x^2*e + 3*b*d^2)*arctan(c*x) - 20*(b*c*d*x^3 + 2*a*d*x^2)*e -
 2*(3*b*c^5*d^2*x^5 - 10*b*c^3*d*x^5*e + 15*b*c*x^5*e^2)*log(c^2*x^2 + 1) +
 4*(3*b*c^5*d^2*x^5 - 10*b*c^3*d*x^5*e + 15*b*c*x^5*e^2)*log(x))/x^5
```

Sympy [A]
time = 0.64, size = 235, normalized size = 1.57

$$\begin{cases} -\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} + \frac{bc^5d^2\log(x)}{5} - \frac{bc^5d^2\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^2}{10x^2} - \frac{2bc^3de\log(x)}{3} + \frac{bc^3de\log\left(x^2+\frac{1}{c^2}\right)}{3} - \frac{bcd^2}{20x^4} - \frac{bcde}{3x^2} + bce^2\log(x) - \frac{bce^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^2\operatorname{atan}(cx)}{5x^5} - \frac{2bde\operatorname{atan}(cx)}{3x^3} - \frac{be^2\operatorname{atan}(cx)}{x} & \text{for } c \neq 0 \\ a\left(-\frac{d^2}{5x^5} - \frac{2de}{3x^3} - \frac{e^2}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c**5*d**2*log
(x)/5 - b*c**5*d**2*log(x**2 + c**(-2))/10 + b*c**3*d**2/(10*x**2) - 2*b*c*
*3*d*e*log(x)/3 + b*c**3*d*e*log(x**2 + c**(-2))/3 - b*c*d**2/(20*x**4) - b
*c*d*e/(3*x**2) + b*c*e**2*log(x) - b*c*e**2*log(x**2 + c**(-2))/2 - b*d**2
*atan(c*x)/(5*x**5) - 2*b*d*e*atan(c*x)/(3*x**3) - b*e**2*atan(c*x)/x, Ne(c
, 0)), (a*(-d**2/(5*x**5) - 2*d*e/(3*x**3) - e**2/x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.49, size = 179, normalized size = 1.19

$$\frac{b\,c^3\,d^2}{10\,x^2} - \frac{a\,e^2}{x} - \frac{b\,c^5\,d^2\,\ln\left(c^2\,x^2+1\right)}{10} - \frac{a\,d^2}{5\,x^5} + \frac{b\,c^5\,d^2\,\ln\left(x\right)}{5} - \frac{2\,a\,d\,e}{3\,x^3} - \frac{b\,c\,e^2\,\ln\left(c^2\,x^2+1\right)}{2} - \frac{b\,c\,d^2}{20\,x^4} + b\,c\,e^2\,\ln\left(x\right) - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{5\,x^5} - \frac{b\,e^2\,\mathrm{atan}(c\,x)}{x} + \frac{b\,c^3\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3} - \frac{2\,b\,c^3\,d\,e\,\ln\left(x\right)}{3} - \frac{b\,c\,d\,e}{3\,x^2} - \frac{2\,b\,d\,e\,\mathrm{atan}(c\,x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^6,x)
```

```
[Out] (b*c^3*d^2)/(10*x^2) - (a*e^2)/x - (b*c^5*d^2*log(c^2*x^2 + 1))/10 - (a*d^2
)/(5*x^5) + (b*c^5*d^2*log(x))/5 - (2*a*d*e)/(3*x^3) - (b*c*e^2*log(c^2*x^2
 + 1))/2 - (b*c*d^2)/(20*x^4) + b*c*e^2*log(x) - (b*d^2*atan(c*x))/(5*x^5)
- (b*e^2*atan(c*x))/x + (b*c^3*d*e*log(c^2*x^2 + 1))/3 - (2*b*c^3*d*e*log(x
))/3 - (b*c*d*e)/(3*x^2) - (2*b*d*e*atan(c*x))/(3*x^3)
```

3.1135 $\int \frac{(d+ex^2)^2(a+b\text{ArcTan}(cx))}{x^7} dx$

Optimal. Leaf size=111

$$-\frac{bcd^2}{30x^5}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x}-\frac{b(c^2d-e)^3\,\text{ArcTan}(cx)}{6d}-\frac{(d+ex^2)^3\,(a+b\text{ArcTan}(cx))}{6dx^6}$$

```
[Out] -1/30*b*c*d^2/x^5+1/18*b*c*d*(c^2*d-3*e)/x^3-1/6*b*c*(c^4*d^2-3*c^2*d*e+3*e
^2)/x-1/6*b*(c^2*d-e)^3*arctan(c*x)/d-1/6*(e*x^2+d)^3*(a+b*arctan(c*x))/d/x
^6
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {270, 5096, 12, 472, 209}

$$-\frac{(d+ex^2)^3\,(a+b\text{ArcTan}(cx))}{6dx^6}-\frac{b\text{ArcTan}(cx)\,(c^2d-e)^3}{6d}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x}-\frac{bcd^2}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^2)/x^5 + (b*c*d*(c^2*d - 3*e))/(18*x^3) - (b*c*(c^4*d^2 - 3*c^
2*d*e + 3*e^2))/(6*x) - (b*(c^2*d - e)^3*ArcTan[c*x])/(6*d) - ((d + e*x^2)^
3*(a + b*ArcTan[c*x]))/(6*d*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2\,(a+b\tan^{-1}(cx))}{x^7}\,dx &= -\frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{6dx^6}-(bc)\int \frac{(d+ex^2)^3}{6x^6\,(-d-c^2dx^2)}\,dx\\ &= -\frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{6dx^6}-\frac{1}{6}(bc)\int \frac{(d+ex^2)^3}{x^6\,(-d-c^2dx^2)}\,dx\\ &= -\frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{6dx^6}-\frac{1}{6}(bc)\int \left(-\frac{d^2}{x^6}+\frac{d(c^2d-3e)}{x^4}+\frac{-c^4d\right.\\ &= -\frac{bcd^2}{30x^5}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x}-\frac{(d+ex^2)^3\,(a+b\,\mathrm{t}}{6dx^6}\\ &= -\frac{bcd^2}{30x^5}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x}-\frac{b(c^2d-e)^3\tan^{-1}(}{6d}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 136, normalized size = 1.23

$$-\frac{15a(d^2+3dex^2+3e^2x^4)+bcx(45e^2x^4+15dex^2(1-3c^2x^2)+d^2(3-5c^2x^2+15c^4x^4))+15b(3e^2x^4(1+c^2x^2)-3dex^2(-1+c^4x^4)+d^2(1+c^6x^6))\,\mathrm{ArcTan}(cx)}{90x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/90*(15*a*(d^2 + 3*d*e*x^2 + 3*e^2*x^4) + b*c*x*(45*e^2*x^4 + 15*d*e*x^2*
(1 - 3*c^2*x^2) + d^2*(3 - 5*c^2*x^2 + 15*c^4*x^4)) + 15*b*(3*e^2*x^4*(1 +
c^2*x^2) - 3*d*e*x^2*(-1 + c^4*x^4) + d^2*(1 + c^6*x^6))*ArcTan[c*x])/x^6
```

Maple [A]
time = 0.25, size = 196, normalized size = 1.77

method	result
derivativedivides	$c^6\left(\frac{a\left(-\frac{de}{2c^2x^4}-\frac{d^2}{6c^2x^6}-\frac{e^2}{2c^2x^2}\right)}{c^4}-\frac{b\arctan(cx)de}{2c^6x^4}-\frac{b\arctan(cx)d^2}{6c^6x^6}-\frac{b\arctan(cx)e^2}{2c^6x^2}-\frac{b\,d^2\arctan(cx)}{6}+\frac{bde\,a}{}\right.$
default	$c^6\left(\frac{a\left(-\frac{de}{2c^2x^4}-\frac{d^2}{6c^2x^6}-\frac{e^2}{2c^2x^2}\right)}{c^4}-\frac{b\arctan(cx)de}{2c^6x^4}-\frac{b\arctan(cx)d^2}{6c^6x^6}-\frac{b\arctan(cx)e^2}{2c^6x^2}-\frac{b\,d^2\arctan(cx)}{6}+\frac{bde\,a}{}\right.$
risch	$\frac{ib(3e^2x^4+3de\,x^2+d^2)\ln(icx+1)}{12x^6}-\frac{15ib\,d^2\ln(-icx+1)+45ib\,e^2x^4\ln(-icx+1)-15i\ln(-cx+i)b\,c^6d^2x^6-45i\ln(-cx-i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(a/c^4*(-1/2/c^2*d*e/x^4-1/6/c^2*d^2/x^6-1/2*e^2/c^2/x^2)-1/2*b/c^6*arc
tan(c*x)*d*e/x^4-1/6*b*arctan(c*x)*d^2/c^6/x^6-1/2*b/c^6*arctan(c*x)*e^2/x^
2-1/6*b*d^2*arctan(c*x)+1/2*b*d*e*arctan(c*x)/c^2-1/2*b*e^2*arctan(c*x)/c^4
-1/6*b*d^2/c/x+1/2*b/c^3*d*e/x-1/2*b/c^5*e^2/x+1/18*b*d^2/c^3/x^3-1/6*b/c^5
*d*e/x^3-1/30*b*d^2/c^5/x^5)
```

Maxima [A]
time = 0.48, size = 145, normalized size = 1.31

$$-\frac{1}{90}\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\,\arctan\left(cx\right)}{x^6}\right)bd^2+\frac{1}{6}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)bde-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)be^2-\frac{ae^2}{2\,x^2}-\frac{ade}{2\,x^4}-\frac{ad^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d^2 + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*ar
ctan(c*x)/x^4)*b*d*e - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^
2 - 1/2*a*e^2/x^2 - 1/2*a*d*e/x^4 - 1/6*a*d^2/x^6
```

Fricas [A]
time = 2.74, size = 152, normalized size = 1.37

$$-\frac{15\,bc^5d^2x^5-5\,bc^3d^2x^3+3\,bcd^2x+15\,ad^2+15\left(bc^6d^2x^6+bd^2+3\left(bc^2x^6+bx^4\right)e^2-3\left(bc^4dx^6-bdx^2\right)e\right)\arctan\left(cx\right)+45\left(bcx^5+ax^4\right)e^2-15\left(3\,bc^3dx^5-bcdx^3-3\,adx^2\right)e}{90\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(15*b*c^5*d^2*x^5 - 5*b*c^3*d^2*x^3 + 3*b*c*d^2*x + 15*a*d^2 + 15*(b*
c^6*d^2*x^6 + b*d^2 + 3*(b*c^2*x^6 + b*x^4)*e^2 - 3*(b*c^4*d*x^6 - b*d*x^2)
*e)*arctan(c*x) + 45*(b*c*x^5 + a*x^4)*e^2 - 15*(3*b*c^3*d*x^5 - b*c*d*x^3
- 3*a*d*x^2)*e)/x^6
```

Sympy [A]
time = 0.48, size = 192, normalized size = 1.73

$$-\frac{ad^2}{6x^6}-\frac{ade}{2x^4}-\frac{ae^2}{2x^2}-\frac{bc^6d^2\operatorname{atan}(cx)}{6}-\frac{bc^5d^2}{6x}+\frac{bc^4de\operatorname{atan}(cx)}{2}+\frac{bc^3d^2}{18x^3}+\frac{bc^3de}{2x}-\frac{bc^2e^2\operatorname{atan}(cx)}{2}-\frac{bcd^2}{30x^5}-\frac{bcde}{6x^3}-\frac{bce^2}{2x}-\frac{bd^2\operatorname{atan}(cx)}{6x^6}-\frac{bde\operatorname{atan}(cx)}{2x^4}-\frac{be^2\operatorname{atan}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**7,x)
```

```
[Out] -a*d**2/(6*x**6) - a*d*e/(2*x**4) - a*e**2/(2*x**2) - b*c**6*d**2*atan(c*x)
/6 - b*c**5*d**2/(6*x) + b*c**4*d*e*atan(c*x)/2 + b*c**3*d**2/(18*x**3) + b
*c**3*d*e/(2*x) - b*c**2*e**2*atan(c*x)/2 - b*c*d**2/(30*x**5) - b*c*d*e/(6
*x**3) - b*c*e**2/(2*x) - b*d**2*atan(c*x)/(6*x**6) - b*d*e*atan(c*x)/(2*x*
*4) - b*e**2*atan(c*x)/(2*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.81, size = 256, normalized size = 2.31

$$-\frac{\frac{a\,d^2}{6}+\frac{b\,d^2\,\operatorname{atan}(c\,x)}{6}-\frac{a\,c^4\,e^2\,x^8}{2}+\frac{a\,e\,x^4\,\left(d\,c^2+e\right)}{2}+\frac{b\,c\,x^5\,\left(2\,c^4\,d^2-6\,c^2\,d\,e+9\,e^2\right)}{18}+\frac{b\,c\,d^2\,x}{30}+\frac{a\,d\,x^2\,\left(d\,c^2+3\,e\right)}{6}+\frac{b\,c^3\,x^7\,\left(c^4\,d^2-3\,c^2\,d\,e+3\,e^2\right)}{6}+\frac{b\,c\,d\,x^3\,\left(15\,e-2\,c^2\,d\right)}{90}+\frac{b\,d\,x^2\,\operatorname{atan}(c\,x)\,\left(d\,c^2+3\,e\right)}{6}+\frac{b\,c^2\,e^2\,x^6\,\operatorname{atan}(c\,x)}{2}+\frac{b\,e\,x^4\,\operatorname{atan}(c\,x)\,\left(d\,c^2+e\right)}{2}}{c^2\,x^8+x^6}-\frac{\operatorname{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,{\left(c^2\right)}^{5/2}\,\left(b\,c^4\,d^2-3\,b\,c^2\,d\,e+3\,b\,e^2\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^7,x)
```

```
[Out] - ((a*d^2)/6 + (b*d^2*atan(c*x))/6 - (a*c^4*e^2*x^8)/2 + (a*e*x^4*(e + c^2*
d))/2 + (b*c*x^5*(9*e^2 + 2*c^4*d^2 - 6*c^2*d*e))/18 + (b*c*d^2*x)/30 + (a*
d*x^2*(3*e + c^2*d))/6 + (b*c^3*x^7*(3*e^2 + c^4*d^2 - 3*c^2*d*e))/6 + (b*c
*d*x^3*(15*e - 2*c^2*d))/90 + (b*d*x^2*atan(c*x)*(3*e + c^2*d))/6 + (b*c^2*
e^2*x^6*atan(c*x))/2 + (b*e*x^4*atan(c*x)*(e + c^2*d))/2)/(x^6 + c^2*x^8) -
 (atan((c^2*x)/(c^2)^(1/2))*(c^2)^(5/2)*(3*b*e^2 + b*c^4*d^2 - 3*b*c^2*d*e)
)/(6*c^3)
```

3.1136 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))}{x^8}\,dx$

Optimal. Leaf size=186

$$-\frac{bcd^2}{42x^6}+\frac{bcd(5c^2d-14e)}{140x^4}-\frac{bc(15c^4d^2-42c^2de+35e^2)}{210x^2}-\frac{d^2(a+b\text{ArcTan}(cx))}{7x^7}-\frac{2de(a+b\text{ArcTan}(cx))}{5x^5}-\frac{e^2}{}$$

```
[Out] -1/42*b*c*d^2/x^6+1/140*b*c*d*(5*c^2*d-14*e)/x^4-1/210*b*c*(15*c^4*d^2-42*c
^2*d*e+35*e^2)/x^2-1/7*d^2*(a+b*arctan(c*x))/x^7-2/5*d*e*(a+b*arctan(c*x))/
x^5-1/3*e^2*(a+b*arctan(c*x))/x^3-1/105*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*e^2
)*ln(x)+1/210*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*e^2)*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1265, 907}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{7x^7}-\frac{2de(a+b\text{ArcTan}(cx))}{5x^5}-\frac{e^2(a+b\text{ArcTan}(cx))}{3x^3}+\frac{bcd(5c^2d-14e)}{140x^4}-\frac{bc(15c^4d^2-42c^2de+35e^2)}{210x^2}+\frac{1}{210}bc^3(15c^4d^2-42c^2de+35e^2)\log(c^2x^2+1)-\frac{1}{105}bc^3\log(x)(15c^4d^2-42c^2de+35e^2)-\frac{bcd^2}{42x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d^2)/x^6 + (b*c*d*(5*c^2*d - 14*e))/(140*x^4) - (b*c*(15*c^4*d^2
 - 42*c^2*d*e + 35*e^2))/(210*x^2) - (d^2*(a + b*ArcTan[c*x]))/(7*x^7) - (2
*d*e*(a + b*ArcTan[c*x]))/(5*x^5) - (e^2*(a + b*ArcTan[c*x]))/(3*x^3) - (b*
c^3*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*Log[x])/105 + (b*c^3*(15*c^4*d^2 - 4
2*c^2*d*e + 35*e^2)*Log[1 + c^2*x^2])/210
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

```
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\tan^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{3x^3}-$$

$$= -\frac{bcd^2}{42x^6}+\frac{bcd\left(5c^2d-14e\right)}{140x^4}-\frac{bc\left(15c^4d^2-42c^2de+35e^2\right)}{210x^2}-\frac{d^2\left(a+b\tan\right.}{7x^7}$$

Mathematica [A]

time = 0.07, size = 185, normalized size = 0.99

$$-\frac{60ad^2+10bcd^2x+168adex^2-3bcd\left(5c^2d-14e\right)x^3+140ae^2x^4+2bc\left(15c^4d^2-42c^2de+35e^2\right)x^5+4b\left(15d^2+42dex^2+35e^2x^4\right)\mathrm{ArcTan}(cx)+4bc^3\left(15c^4d^2-42c^2de+35e^2\right)x^7\log(x)-2bc^3\left(15c^4d^2-42c^2de+35e^2\right)x^7\log\left(1+c^2x^2\right)}{420x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/420*(60*a*d^2 + 10*b*c*d^2*x + 168*a*d*e*x^2 - 3*b*c*d*(5*c^2*d - 14*e)*
x^3 + 140*a*e^2*x^4 + 2*b*c*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*x^5 + 4*b*(1
```

```
5*d^2 + 42*d*e*x^2 + 35*e^2*x^4)*ArcTan[c*x] + 4*b*c^3*(15*c^4*d^2 - 42*c^2
*d*e + 35*e^2)*x^7*Log[x] - 2*b*c^3*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*x^7*
Log[1 + c^2*x^2])/x^7
```

Maple [A]
time = 0.21, size = 249, normalized size = 1.34

method	result
derivativedivides	$c^7\left(\frac{a\left(-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}\right)}{c^4}-\frac{b\arctan(cx)d^2}{7c^7x^7}-\frac{b\arctan(cx)e^2}{3c^7x^3}-\frac{2b\arctan(cx)de}{5c^7x^5}+\frac{b\ln(c^2x^2+1)d^2}{14}-\frac{b\ln}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}\right)}{c^4}-\frac{b\arctan(cx)d^2}{7c^7x^7}-\frac{b\arctan(cx)e^2}{3c^7x^3}-\frac{2b\arctan(cx)de}{5c^7x^5}+\frac{b\ln(c^2x^2+1)d^2}{14}-\frac{b\ln}{}\right.$
risch	$\frac{ib(35e^2x^4+42de\,x^2+15d^2)\ln(icx+1)}{210x^7}-\frac{60\ln(x)b\,c^7d^2x^7-30\ln(c^2x^2+1)b\,c^7d^2x^7-168\ln(x)b\,c^5de\,x^7+84\ln(c^2x^2+1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^4*(-1/7/c^3*d^2/x^7-1/3*e^2/c^3/x^3-2/5/c^3*d*e/x^5)-1/7*b*arctan(
c*x)*d^2/c^7/x^7-1/3*b/c^7*arctan(c*x)*e^2/x^3-2/5*b/c^7*arctan(c*x)*d*e/x^
5+1/14*b*ln(c^2*x^2+1)*d^2-1/5*b/c^2*ln(c^2*x^2+1)*d*e+1/6*b/c^4*ln(c^2*x^2
+1)*e^2-1/14*b*d^2/c^2/x^2+1/5*b/c^4*d*e/x^2-1/6*b/c^6*e^2/x^2-1/7*b*d^2*ln
(c*x)+2/5*b/c^2*ln(c*x)*d*e-1/3*b/c^4*ln(c*x)*e^2-1/42*b*d^2/c^6/x^6+1/28*b
*d^2/c^4/x^4-1/10*b/c^6*d*e/x^4)
```

Maxima [A]
time = 0.26, size = 197, normalized size = 1.06

$$\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^2-\frac{1}{10}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bde+\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)be^2-\frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2
)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^2 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^
4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d*e + 1/6*((c^2*
log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*e^2 - 1/3
*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Fricas [A]
time = 2.57, size = 209, normalized size = 1.12

$$-\frac{30\,bc^5d^2x^5-15\,bc^3d^2x^3+10\,bcd^2x+60\,ad^2+4\left(35\,bx^4e^2+42\,bdx^2e+15\,bd^2\right)\arctan\left(cx\right)+70\left(bcx^5+2\,ax^4\right)e^2-42\left(2\,bc^3dx^5-bcdx^3-4\,adx^2\right)e-2\left(15\,bc^7d^2x^7-42\,bc^5dx^7e+35\,bc^3x^7e^2\right)\log\left(c^2x^2+1\right)+4\left(15\,bc^7d^2x^7-42\,bc^5dx^7e+35\,bc^3x^7e^2\right)\log\left(x\right)}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/420*(30*b*c^5*d^2*x^5 - 15*b*c^3*d^2*x^3 + 10*b*c*d^2*x + 60*a*d^2 + 4*(
35*b*x^4*e^2 + 42*b*d*x^2*e + 15*b*d^2)*arctan(c*x) + 70*(b*c*x^5 + 2*a*x^4
)*e^2 - 42*(2*b*c^3*d*x^5 - b*c*d*x^3 - 4*a*d*x^2)*e - 2*(15*b*c^7*d^2*x^7
- 42*b*c^5*d*x^7*e + 35*b*c^3*x^7*e^2)*log(c^2*x^2 + 1) + 4*(15*b*c^7*d^2*x
^7 - 42*b*c^5*d*x^7*e + 35*b*c^3*x^7*e^2)*log(x))/x^7
```

Sympy [A]
time = 0.87, size = 289, normalized size = 1.55

$$\begin{cases} - \frac{a d^{2}}{7 x^{7}} - \frac{2 a d e}{5 x^{5}} - \frac{a e^{2}}{3 x^{3}} - \frac{b c^{7} d^{2} \log{\left(x \right)}}{7} + \frac{b c^{7} d^{2} \log{\left(x^{2} + \frac{1}{c^{2}} \right)}}{14} - \frac{b c^{5} d^{2}}{14 x^{2}} + \frac{2 b c^{5} d e \log{\left(x \right)}}{5} - \frac{b c^{5} d e \log{\left(x^{2} + \frac{1}{c^{2}} \right)}}{5} + \frac{b c^{3} d^{2}}{28 x^{4}} + \frac{b c^{3} d e}{5 x^{2}} - \frac{b c^{3} e^{2} \log{\left(x \right)}}{3} + \frac{b c^{3} e^{2} \log{\left(x^{2} + \frac{1}{c^{2}} \right)}}{6} - \frac{b c d^{2}}{42 x^{6}} - \frac{b c d e}{10 x^{4}} - \frac{b c e^{2}}{6 x^{2}} - \frac{b d^{2} \operatorname{atan}{\left(c x \right)}}{7 x^{7}} - \frac{2 b d e \operatorname{atan}{\left(c x \right)}}{5 x^{5}} - \frac{b e^{2} \operatorname{atan}{\left(c x \right)}}{3 x^{3}} & \text{for}\: c \neq 0 \\a \left(- \frac{d^{2}}{7 x^{7}} - \frac{2 d e}{5 x^{5}} - \frac{e^{2}}{3 x^{3}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*c**7*d
**2*log(x)/7 + b*c**7*d**2*log(x**2 + c**(-2))/14 - b*c**5*d**2/(14*x**2) +
 2*b*c**5*d*e*log(x)/5 - b*c**5*d*e*log(x**2 + c**(-2))/5 + b*c**3*d**2/(28
*x**4) + b*c**3*d*e/(5*x**2) - b*c**3*e**2*log(x)/3 + b*c**3*e**2*log(x**2
+ c**(-2))/6 - b*c*d**2/(42*x**6) - b*c*d*e/(10*x**4) - b*c*e**2/(6*x**2) -
 b*d**2*atan(c*x)/(7*x**7) - 2*b*d*e*atan(c*x)/(5*x**5) - b*e**2*atan(c*x)/
(3*x**3), Ne(c, 0)), (a*(-d**2/(7*x**7) - 2*d*e/(5*x**5) - e**2/(3*x**3)),
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.63, size = 232, normalized size = 1.25

$$-\frac{60\,a\,d^2+60\,b\,d^2\,\mathrm{atan}(c\,x)+140\,a\,e^2\,x^4-15\,b\,c^3\,d^2\,x^3+30\,b\,c^5\,d^2\,x^5+10\,b\,c\,d^2\,x+168\,a\,d\,e\,x^2+70\,b\,c\,e^2\,x^5+140\,b\,e^2\,x^4\,\mathrm{atan}(c\,x)+60\,b\,c^7\,d^2\,x^7\,\ln\left(x\right)+140\,b\,c^3\,e^2\,x^7\,\ln\left(x\right)-84\,b\,c^3\,d\,e\,x^5+42\,b\,c\,d\,e\,x^3-30\,b\,c^7\,d^2\,x^7\,\ln\left(c^2\,x^2+1\right)-70\,b\,c^3\,e^2\,x^7\,\ln\left(c^2\,x^2+1\right)+168\,b\,d\,e\,x^2\,\mathrm{atan}(c\,x)-168\,b\,c^5\,d\,e\,x^7\,\ln\left(x\right)+84\,b\,c^5\,d\,e\,x^7\,\ln\left(c^2\,x^2+1\right)}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^8,x)
```

```
[Out] -(60*a*d^2 + 60*b*d^2*atan(c*x) + 140*a*e^2*x^4 - 15*b*c^3*d^2*x^3 + 30*b*c
^5*d^2*x^5 + 10*b*c*d^2*x + 168*a*d*e*x^2 + 70*b*c*e^2*x^5 + 140*b*e^2*x^4*
atan(c*x) + 60*b*c^7*d^2*x^7*log(x) + 140*b*c^3*e^2*x^7*log(x) - 84*b*c^3*d
*e*x^5 + 42*b*c*d*e*x^3 - 30*b*c^7*d^2*x^7*log(c^2*x^2 + 1) - 70*b*c^3*e^2*
x^7*log(c^2*x^2 + 1) + 168*b*d*e*x^2*atan(c*x) - 168*b*c^5*d*e*x^7*log(x) +
 84*b*c^5*d*e*x^7*log(c^2*x^2 + 1))/(420*x^7)
```

3.1137 $\int x^3(d+ex^2)^3 (a+b\mathbf{ArcTan}(cx))\, dx$

Optimal. Leaf size=240

$$\frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x}{40c^9}-\frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x^3}{120c^7}-\frac{be(20c^4d^2-15c^2de+4e^2}{200c^5}$$

```
[Out] 1/40*b*(10*c^6*d^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x/c^9-1/120*b*(10*c^6*d
^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x^3/c^7-1/200*b*e*(20*c^4*d^2-15*c^2*d*
e+4*e^2)*x^5/c^5-1/280*b*(15*c^2*d-4*e)*e^2*x^7/c^3-1/90*b*e^3*x^9/c+1/40*b
*(c^2*d-e)^4*(c^2*d+4*e)*arctan(c*x)/c^10/e^2-1/8*d*(e*x^2+d)^4*(a+b*arctan
(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arctan(c*x))/e^2
```

Rubi [A]
time = 0.32, antiderivative size = 285, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 45, 5096, 12, 542, 396, 209}

$$\frac{(d+ex^2)^5(a+b\text{ArcTan}(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\text{ArcTan}(cx))}{8e^2}+\frac{b\text{ArcTan}(cx)(c^2d-e)^4(c^2d+4e)}{40c^{10}e^2}-\frac{bx(23c^2d-36e)(d+ex^2)^3}{2520c^3e}-\frac{bx(25c^4d^2-135c^2de+84e^2)(d+ex^2)^2}{4200c^5e}+\frac{bx(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3)(d+ex^2)}{12600c^7e}+\frac{bx(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4)}{12600c^9e}-\frac{bx(d+ex^2)^4}{90ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(325*c^8*d^4 + 1815*c^6*d^3*e - 4977*c^4*d^2*e^2 + 4305*c^2*d*e^3 - 1260
*e^4)*x)/(12600*c^9*e) + (b*(5*c^6*d^3 + 750*c^4*d^2*e - 1071*c^2*d*e^2 + 4
20*e^3)*x*(d + e*x^2))/(12600*c^7*e) - (b*(25*c^4*d^2 - 135*c^2*d*e + 84*e^
2)*x*(d + e*x^2)^2)/(4200*c^5*e) - (b*(23*c^2*d - 36*e)*x*(d + e*x^2)^3)/(2
520*c^3*e) - (b*x*(d + e*x^2)^4)/(90*c*e) + (b*(c^2*d - e)^4*(c^2*d + 4*e)*
ArcTan[c*x])/(40*c^10*e^2) - (d*(d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e^2)
+ ((d + e*x^2)^5*(a + b*ArcTan[c*x]))/(10*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^3 (d+ex^2)^3 (a+b\tan^{-1}(cx))\,dx = -\frac{d(d+ex^2)^4 (a+b\tan^{-1}(cx))}{8e^2} + \frac{(d+ex^2)^5 (a+b\tan^{-1}(cx))}{10e^2} - (\cdots$$

$$= -\frac{d(d+ex^2)^4 (a+b\tan^{-1}(cx))}{8e^2} + \frac{(d+ex^2)^5 (a+b\tan^{-1}(cx))}{10e^2} - \frac{(\cdots}{\cdots}$$

$$= -\frac{bx(d+ex^2)^4}{90ce} - \frac{d(d+ex^2)^4 (a+b\tan^{-1}(cx))}{8e^2} + \frac{(d+ex^2)^5 (a+b\cdots}{10e^2\cdots}$$

$$= -\frac{b(23c^2d-36e)\,x(d+ex^2)^3}{2520c^3e} - \frac{bx(d+ex^2)^4}{90ce} - \frac{d(d+ex^2)^4 (a+b\cdots}{8e^2}$$

$$= -\frac{b(25c^4d^2-135c^2de+84e^2)\,x(d+ex^2)^2}{4200c^5e} - \frac{b(23c^2d-36e)\,x(d+\cdots}{2520c^3e}$$

$$= \frac{b(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3)\,x(d+ex^2)}{12600c^7e} - \frac{b(25c^4d^2-\cdots}{\cdots}$$

$$= \frac{b(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4)\,x}{12600c^9e} + \frac{b(\cdots}{\cdots}$$

$$= \frac{b(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4)\,x}{12600c^9e} + \frac{b(\cdots}{\cdots}$$

Mathematica [A]
time = 3.37, size = 262, normalized size = 1.09

$$\frac{cx(315ac^9x^3(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6)-b(1260e^3-105c^2e^2(45d+4ex^2)+63c^4e(100d^2+25dex^2+4e^2x^4)-15c^6(210d^3+140d^2ex^2+63de^2x^4+12e^3x^6)+5c^8(210d^3x^2+252d^2ex^4+135de^2x^6+28e^3x^8)))+315b(-10c^6d^3+20c^4d^2e-15c^2de^2+4e^3+c^{10}x^4(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6))\,\mathrm{ArcTan}(cx)}{12600c^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(315*a*c^9*x^3*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - b*
(1260*e^3 - 105*c^2*e^2*(45*d + 4*e*x^2) + 63*c^4*e*(100*d^2 + 25*d*e*x^2 +
 4*e^2*x^4) - 15*c^6*(210*d^3 + 140*d^2*e*x^2 + 63*d*e^2*x^4 + 12*e^3*x^6)
+ 5*c^8*(210*d^3*x^2 + 252*d^2*e*x^4 + 135*d*e^2*x^6 + 28*e^3*x^8))) + 315*
b*(-10*c^6*d^3 + 20*c^4*d^2*e - 15*c^2*d*e^2 + 4*e^3 + c^10*x^4*(10*d^3 + 2
0*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6))*ArcTan[c*x])/(12600*c^10)
```

Maple [A]
time = 0.31, size = 334, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}d^3c^{10}x^4+\frac{1}{2}d^2c^{10}e\,x^6+\frac{3}{8}d\,c^{10}e^2x^8+\frac{1}{10}e^3c^{10}x^{10}\right)}{c^6}+\frac{b\arctan(cx)d^3c^4x^4}{4}+\frac{b\,c^4\arctan(cx)d^2e\,x^6}{2}+\frac{3b\,c^4\arctan(cx)d\,e^2x^8}{8}+\frac{b\,c^{\cdots}}{\cdots}}{\cdots}$

default	$\frac{a\left(\frac{1}{4}d^{3}c^{10}x^{4}+\frac{1}{2}d^{2}c^{10}e\,x^{6}+\frac{3}{8}d\,c^{10}e^{2}x^{8}+\frac{1}{10}e^{3}c^{10}x^{10}\right)}{c^{6}}+\frac{b\arctan(cx)d^{3}c^{4}x^{4}}{4}+\frac{b\,c^{4}\arctan(cx)d^{2}e\,x^{6}}{2}+\frac{3b\,c^{4}\arctan(cx)d\,e^{2}x^{8}}{8}+\frac{b\,c^{4}\,\mathrm{a}}{}$
risch	$\frac{b\,d^{2}e\,x^{3}}{6c^{3}}-\frac{bd\,e^{2}x^{3}}{8c^{5}}-\frac{b\,d^{2}ex}{2c^{5}}+\frac{3bd\,e^{2}x}{8c^{7}}+\frac{b\,d^{2}e\arctan(cx)}{2c^{6}}-\frac{3bd\,e^{2}\arctan(cx)}{8c^{8}}+\frac{x^{10}e^{3}a}{10}+\frac{x^{4}d^{3}a}{4}+\frac{3ibd\,e^{2}x^{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^6*(1/4*d^3*c^10*x^4+1/2*d^2*c^10*e*x^6+3/8*d*c^10*e^2*x^8+1/10*e
^3*c^10*x^10)+1/4*b*arctan(c*x)*d^3*c^4*x^4+1/2*b*c^4*arctan(c*x)*d^2*e*x^6
+3/8*b*c^4*arctan(c*x)*d*e^2*x^8+1/10*b*c^4*arctan(c*x)*e^3*x^10-1/12*b*d^3
*c^3*x^3-1/10*b*c^3*d^2*e*x^5-3/56*b*c^3*d*e^2*x^7-1/90*b*c^3*e^3*x^9+1/4*b
*c*d^3*x+1/6*b*c*d^2*e*x^3+3/40*b*c*d*e^2*x^5+1/70*b*c*e^3*x^7-1/2*b*d^2*e*
x/c-1/8*b*d*e^2*x^3/c-1/50*b*e^3*x^5/c+3/8*b*d*e^2*x/c^3+1/30*b*e^3*x^3/c^3
-1/10*b*e^3*x/c^5-1/4*b*d^3*arctan(c*x)+1/2*b*d^2*e*arctan(c*x)/c^2-3/8*b*d
*e^2*arctan(c*x)/c^4+1/10*b*e^3*arctan(c*x)/c^6)
```

Maxima [A]
time = 0.47, size = 266, normalized size = 1.11

$$\frac{1}{10}ax^{10}e^{3}+\frac{3}{8}adx^{8}e^{2}+\frac{1}{2}ad^{2}x^{6}e+\frac{1}{4}ad^{3}x^{4}+\frac{1}{12}\left(3x^{4}\arctan(cx)-c\left(\frac{c^{2}x^{3}-3x}{c^{4}}+\frac{3\arctan(cx)}{c^{5}}\right)\right)bd^{3}+\frac{1}{30}\left(15x^{6}\arctan(cx)-c\left(\frac{3c^{4}x^{5}-5c^{2}x^{3}+15x}{c^{6}}-\frac{15\arctan(cx)}{c^{7}}\right)\right)bd^{2}e+\frac{1}{280}\left(105x^{8}\arctan(cx)-c\left(\frac{15c^{6}x^{7}-21c^{4}x^{5}+35c^{2}x^{3}-105x}{c^{8}}+\frac{105\arctan(cx)}{c^{9}}\right)\right)bde^{2}+\frac{1}{3150}\left(315x^{10}\arctan(cx)-c\left(\frac{35c^{8}x^{9}-45c^{6}x^{7}+63c^{4}x^{5}-105c^{2}x^{3}+315x}{c^{10}}-\frac{315\arctan(cx)}{c^{11}}\right)\right)be^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*x^10*e^3 + 3/8*a*d*x^8*e^2 + 1/2*a*d^2*x^6*e + 1/4*a*d^3*x^4 + 1/12*
(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^3 + 1
/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan
(c*x)/c^7))*b*d^2*e + 1/280*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*
x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*d*e^2 + 1/3150*(315
*x^10*arctan(c*x) - c*((35*c^8*x^9 - 45*c^6*x^7 + 63*c^4*x^5 - 105*c^2*x^3
+ 315*x)/c^10 - 315*arctan(c*x)/c^11))*b*e^3
```

Fricas [A]
time = 2.20, size = 286, normalized size = 1.19

$$\frac{3150\,ac^{10}d^{3}x^{4}-1050\,bc^{9}d^{3}x^{3}+3150\,bc^{7}d^{3}x+315\,(10\,bc^{10}d^{3}x^{4}-10\,bc^{6}d^{3}+4\,(bc^{10}x^{10}+b)e^{3}+15\,(bc^{10}dx^{8}-bc^{2}d)e^{2}+20\,(bc^{10}d^{2}x^{6}+bc^{4}d^{2})e)\arctan(cx)+4\,(315\,ac^{10}x^{10}-35\,bc^{9}x^{9}+45\,bc^{7}x^{7}-63\,bc^{5}x^{5}+105\,bc^{3}x^{3}-315\,bcx)e^{3}+45\,(105\,ac^{10}dx^{8}-15\,bc^{9}dx^{7}+21\,bc^{7}dx^{5}-35\,bc^{5}dx^{3}+105\,bc^{3}dx)e^{2}+420\,(15\,ac^{10}d^{2}x^{6}-3\,bc^{9}d^{2}x^{5}+5\,bc^{7}d^{2}x^{3}-15\,bc^{5}d^{2}x)e}{12600\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12600*(3150*a*c^10*d^3*x^4 - 1050*b*c^9*d^3*x^3 + 3150*b*c^7*d^3*x + 315*
(10*b*c^10*d^3*x^4 - 10*b*c^6*d^3 + 4*(b*c^10*x^10 + b)*e^3 + 15*(b*c^10*d*
x^8 - b*c^2*d)*e^2 + 20*(b*c^10*d^2*x^6 + b*c^4*d^2)*e)*arctan(c*x) + 4*(31
5*a*c^10*x^10 - 35*b*c^9*x^9 + 45*b*c^7*x^7 - 63*b*c^5*x^5 + 105*b*c^3*x^3
```

```
- 315*b*c*x)*e^3 + 45*(105*a*c^10*d*x^8 - 15*b*c^9*d*x^7 + 21*b*c^7*d*x^5 -
 35*b*c^5*d*x^3 + 105*b*c^3*d*x)*e^2 + 420*(15*a*c^10*d^2*x^6 - 3*b*c^9*d^2
*x^5 + 5*b*c^7*d^2*x^3 - 15*b*c^5*d^2*x)*e)/c^10
```

Sympy [A]
time = 0.86, size = 411, normalized size = 1.71

$$\begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\operatorname{atan}(cx)}{4} + \frac{bd^2ex^6\operatorname{atan}(cx)}{2} + \frac{3bde^2x^8\operatorname{atan}(cx)}{8} + \frac{be^3x^{10}\operatorname{atan}(cx)}{10} - \frac{bd^3x^3}{12c} - \frac{bd^2ex^5}{10c} - \frac{3bde^2x^7}{56c} - \frac{be^3x^9}{90c} + \frac{bd^3x}{4c^3} + \frac{bd^2ex^3}{6c^3} + \frac{3bde^2x^5}{40c^3} + \frac{be^3x^7}{70c^3} - \frac{bd^3\operatorname{atan}(cx)}{4c^4} - \frac{bd^2ex}{2c^5} - \frac{bde^2x^3}{8c^5} - \frac{be^3x^5}{50c^5} + \frac{bd^2e\operatorname{atan}(cx)}{2c^6} + \frac{3bde^2x}{8c^7} + \frac{be^3x^3}{30c^7} - \frac{3bde^2\operatorname{atan}(cx)}{8c^8} - \frac{be^3x}{10c^9} + \frac{be^3\operatorname{atan}(cx)}{10c^{10}} & \text{for}\: c \neq 0 \\ a\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**
10/10 + b*d**3*x**4*atan(c*x)/4 + b*d**2*e*x**6*atan(c*x)/2 + 3*b*d*e**2*x*
*8*atan(c*x)/8 + b*e**3*x**10*atan(c*x)/10 - b*d**3*x**3/(12*c) - b*d**2*e*
x**5/(10*c) - 3*b*d*e**2*x**7/(56*c) - b*e**3*x**9/(90*c) + b*d**3*x/(4*c**
3) + b*d**2*e*x**3/(6*c**3) + 3*b*d*e**2*x**5/(40*c**3) + b*e**3*x**7/(70*c
**3) - b*d**3*atan(c*x)/(4*c**4) - b*d**2*e*x/(2*c**5) - b*d*e**2*x**3/(8*c
**5) - b*e**3*x**5/(50*c**5) + b*d**2*e*atan(c*x)/(2*c**6) + 3*b*d*e**2*x/(
8*c**7) + b*e**3*x**3/(30*c**7) - 3*b*d*e**2*atan(c*x)/(8*c**8) - b*e**3*x/
(10*c**9) + b*e**3*atan(c*x)/(10*c**10), Ne(c, 0)), (a*(d**3*x**4/4 + d**2*
e*x**6/2 + 3*d*e**2*x**8/8 + e**3*x**10/10), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.62, size = 599, normalized size = 2.50

$$x^3\left(\frac{\frac{\frac{be^3}{10c^3}-\frac{3bde^2}{8c}}{c^2}+\frac{bd^2e}{2c}}{3c^2}-\frac{bd^3}{12c}\right)-x^8\left(\frac{ae^3}{8c^2}-\frac{ae^2\left(3dc^2+e\right)}{8c^2}\right)+x^6\left(\frac{\frac{ae^3}{c^2}-\frac{ae^2\left(3dc^2+e\right)}{c^2}}{6c^2}+\frac{ade\left(dc^2+e\right)}{2c^2}\right)+x^7\left(\frac{be^3}{70c^3}-\frac{3bde^2}{56c}\right)+\operatorname{atan}(cx)\left(\frac{bd^3x^4}{4}+\frac{bd^2ex^6}{2}+\frac{3bde^2x^8}{8}+\frac{be^3x^{10}}{10}\right)-x^5\left(\frac{\frac{be^3}{10c^3}-\frac{3bde^2}{8c}}{5c^2}+\frac{bd^2e}{10c}\right)+x^2\left(\frac{\frac{\frac{\frac{ae^3}{c^2}-\frac{ae^2\left(3dc^2+e\right)}{c^2}}{c^2}+\frac{3ade\left(dc^2+e\right)}{c^2}}{c^2}-\frac{ad^2\left(dc^2+3e\right)}{c^2}}{2c^2}+\frac{ad^3}{2c^2}\right)-x^4\left(\frac{\frac{\frac{ae^3}{c^2}-\frac{ae^2\left(3dc^2+e\right)}{c^2}}{c^2}+\frac{3ade\left(dc^2+e\right)}{c^2}}{4c^2}-\frac{ad^2\left(dc^2+3e\right)}{4c^2}\right)+\frac{ae^3x^{10}}{10}-\frac{x\left(\frac{\frac{\frac{be^3}{10c^3}-\frac{3bde^2}{8c}}{c^2}+\frac{bd^2e}{2c}}{c^2}-\frac{bd^3}{4c}\right)}{c^2}-\frac{be^3x^9}{90c}+\frac{b\operatorname{atan}\left(\frac{bcx\left(-10c^6d^3+20c^4d^2e-15c^2de^2+4e^3\right)}{-10bc^6d^3+20bc^4d^2e-15bc^2de^2+4be^3}\right)\left(-10c^6d^3+20c^4d^2e-15c^2de^2+4e^3\right)}{40c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

```
[Out] x^3*((((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/(3*c^2)
 - (b*d^3)/(12*c)) - x^8*((a*e^3)/(8*c^2) - (a*e^2*(e + 3*c^2*d))/(8*c^2))
+ x^6*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(6*c^2) + (a*d*e*(e + c^2*
d))/(2*c^2)) + x^7*((b*e^3)/(70*c^3) - (3*b*d*e^2)/(56*c)) + atan(c*x)*((b*
d^3*x^4)/4 + (b*e^3*x^10)/10 + (b*d^2*e*x^6)/2 + (3*b*d*e^2*x^8)/8) - x^5*(
```

```
((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/(5*c^2) + (b*d^2*e)/(10*c)) + x^2*((
(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^2)
/c^2 - (a*d^2*(3*e + c^2*d))/c^2)/(2*c^2) + (a*d^3)/(2*c^2)) - x^4*((((a*e^
3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^2)/(4*c^2
) - (a*d^2*(3*e + c^2*d))/(4*c^2)) + (a*e^3*x^10)/10 - (x*((((b*e^3)/(10*c^
3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/c^2 - (b*d^3)/(4*c)))/c^2 -
(b*e^3*x^9)/(90*c) + (b*atan((b*c*x*(4*e^3 - 10*c^6*d^3 - 15*c^2*d*e^2 + 20
*c^4*d^2*e))/(4*b*e^3 - 10*b*c^6*d^3 - 15*b*c^2*d*e^2 + 20*b*c^4*d^2*e))*(4
*e^3 - 10*c^6*d^3 - 15*c^2*d*e^2 + 20*c^4*d^2*e))/(40*c^10)
```

3.1138 $\int x^2(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=239

$$-\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\,x^2}{630c^7}-\frac{be(189c^4d^2-135c^2de+35e^2)\,x^4}{1260c^5}-\frac{b(27c^2d-7e)\,e^2x^6}{378c^3}-\frac{be^3x^8}{72c}$$

```
[Out] -1/630*b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*x^2/c^7-1/1260*b*
e*(189*c^4*d^2-135*c^2*d*e+35*e^2)*x^4/c^5-1/378*b*(27*c^2*d-7*e)*e^2*x^6/c
^3-1/72*b*e^3*x^8/c+1/3*d^3*x^3*(a+b*arctan(c*x))+3/5*d^2*e*x^5*(a+b*arctan
(c*x))+3/7*d*e^2*x^7*(a+b*arctan(c*x))+1/9*e^3*x^9*(a+b*arctan(c*x))+1/630*
b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)/c^9
```

Rubi [A]
time = 0.26, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1813, 1634}

$$\frac{1}{3}d^3x^3(a+b\text{ArcTan}(cx))+\frac{3}{5}d^2ex^5(a+b\text{ArcTan}(cx))+\frac{3}{7}de^2x^7(a+b\text{ArcTan}(cx))+\frac{1}{9}e^3x^9(a+b\text{ArcTan}(cx))-\frac{be^2x^6(27c^2d-7e)}{378c^3}-\frac{bex^4(189c^4d^2-135c^2de+35e^2)}{1260c^5}+\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\log(c^2x^2+1)}{630c^9}-\frac{bx^2(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)}{630c^7}-\frac{be^3x^8}{72c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/630*(b*(105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*e^3)*x^2)/c^7 -
 (b*e*(189*c^4*d^2 - 135*c^2*d*e + 35*e^2)*x^4)/(1260*c^5) - (b*(27*c^2*d -
 7*e)*e^2*x^6)/(378*c^3) - (b*e^3*x^8)/(72*c) + (d^3*x^3*(a + b*ArcTan[c*x]
))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcTan[c*x
]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*(105*c^6*d^3 - 189*c^4*d^2*e +
 135*c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^2])/(630*c^9)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\right.\\ &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\right.\\ &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\right.\\ &= \frac{1}{3}d^3x^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\tan^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\right.\\ &= -\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\,x^2}{630c^7}-\frac{be(189c^4d^2-135c^2}{1260c^5}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 236, normalized size = 0.99

$$\frac{c^2x^2(420be^3-30bc^2e^2(54d+7ex^2)+2bc^4e(1134d^2+405dex^2+70e^2x^4)+24ac^7x(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)-3bc^6(420d^3+378d^2ex^2+180de^2x^4+35e^3x^6))+24bc^9x^3(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)\,\mathrm{ArcTan}(cx)+12b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\log\left(1+c^2x^2\right)}{7560c^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*x^2*(420*b*e^3 - 30*b*c^2*e^2*(54*d + 7*e*x^2) + 2*b*c^4*e*(1134*d^2 +
 405*d*e*x^2 + 70*e^2*x^4) + 24*a*c^7*x*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^
```

```
2*x^4 + 35*e^3*x^6) - 3*b*c^6*(420*d^3 + 378*d^2*e*x^2 + 180*d*e^2*x^4 + 35
*e^3*x^6)) + 24*b*c^9*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3
*x^6)*ArcTan[c*x] + 12*b*(105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*
e^3)*Log[1 + c^2*x^2])/(7560*c^9)
```

Maple [A]
time = 0.28, size = 315, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\arctan(cx)d^3c^3x^3}{3}+\frac{3b\,c^3\arctan(cx)d^2e\,x^5}{5}+\frac{3b\,c^3\arctan(cx)d\,e^2x^7}{7}+\frac{b\,c^3\,\mathrm{arcta}}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\arctan(cx)d^3c^3x^3}{3}+\frac{3b\,c^3\arctan(cx)d^2e\,x^5}{5}+\frac{3b\,c^3\arctan(cx)d\,e^2x^7}{7}+\frac{b\,c^3\,\mathrm{arcta}}{}}{}$
risch	$\frac{3ibd\,e^2x^7\ln(-icx+1)}{14}-\frac{ib(35e^3x^9+135e^2d\,x^7+189d^2e\,x^5+105d^3x^3)\ln(icx+1)}{630}+\frac{3ib\,d^2e\,x^5\ln(-icx+1)}{10}+\frac{x^9e^3a}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^6*(1/3*d^3*c^9*x^3+3/5*d^2*c^9*e*x^5+3/7*d*c^9*e^2*x^7+1/9*e^3*c
^9*x^9)+1/3*b*arctan(c*x)*d^3*c^3*x^3+3/5*b*c^3*arctan(c*x)*d^2*e*x^5+3/7*b
*c^3*arctan(c*x)*d*e^2*x^7+1/9*b*c^3*arctan(c*x)*e^3*x^9-1/6*b*d^3*c^2*x^2-
3/20*b*c^2*d^2*e*x^4+3/10*b*d^2*e*x^2-1/14*b*c^2*d*e^2*x^6+3/28*b*d*e^2*x^4
-1/72*b*c^2*e^3*x^8-3/14*b/c^2*d*e^2*x^2+1/54*b*e^3*x^6-1/36*b/c^2*e^3*x^4+
1/18*b/c^4*e^3*x^2+1/6*b*ln(c^2*x^2+1)*d^3-3/10*b/c^2*ln(c^2*x^2+1)*d^2*e+3
/14*b/c^4*ln(c^2*x^2+1)*d*e^2-1/18*b/c^6*ln(c^2*x^2+1)*e^3)
```

Maxima [A]
time = 0.27, size = 263, normalized size = 1.10

$$\frac{1}{9}ax^9e^3+\frac{3}{7}adx^7e^2+\frac{3}{5}ad^2x^5e+\frac{1}{3}ad^3x^3+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^3+\frac{3}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)bd^2e+\frac{1}{28}\left(12x^7\arctan(cx)-c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6}-\frac{6\log(c^2x^2+1)}{c^8}\right)\right)bde^2+\frac{1}{216}\left(24x^9\arctan(cx)-c\left(\frac{3c^6x^8-4c^4x^6+6c^2x^4-12x^2}{c^8}+\frac{12\log(c^2x^2+1)}{c^{10}}\right)\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9*e^3 + 3/7*a*d*x^7*e^2 + 3/5*a*d^2*x^5*e + 1/3*a*d^3*x^3 + 1/6*(2*
x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3 + 3/20*(4*x^5*a
rctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d^2*e +
1/28*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c
^2*x^2 + 1)/c^8))*b*d*e^2 + 1/216*(24*x^9*arctan(c*x) - c*((3*c^6*x^8 - 4*c
^4*x^6 + 6*c^2*x^4 - 12*x^2)/c^8 + 12*log(c^2*x^2 + 1)/c^10))*b*e^3
```

Fricas [A]
time = 3.14, size = 269, normalized size = 1.13

$$\frac{2520\,ac^9d^3x^3-1260\,bc^8d^3x^2+24\,(35\,bc^9x^9e^3+135\,bc^9dx^7e^2+189\,bc^9d^2x^5e+105\,bc^9d^3x^3)\arctan(cx)+35\,(24\,ac^9x^9-3\,bc^8x^8+4\,bc^6x^6-6\,bc^4x^4+12\,bc^2x^2)e^3+270\,(12\,ac^9dx^7-2\,bc^8dx^6+3\,bc^6dx^4-6\,bc^4dx^2)e^2+1134\,(4\,ac^9d^2x^5-bc^8d^2x^4+2\,bc^6d^2x^2)e+12\,(105\,bc^6d^3-189\,bc^4d^2e+135\,bc^2de^2-35\,be^3)\log(c^2x^2+1)}{7560\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/7560*(2520*a*c^9*d^3*x^3 - 1260*b*c^8*d^3*x^2 + 24*(35*b*c^9*x^9*e^3 + 13
5*b*c^9*d*x^7*e^2 + 189*b*c^9*d^2*x^5*e + 105*b*c^9*d^3*x^3)*arctan(c*x) +
35*(24*a*c^9*x^9 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 6*b*c^4*x^4 + 12*b*c^2*x^2)*
e^3 + 270*(12*a*c^9*d*x^7 - 2*b*c^8*d*x^6 + 3*b*c^6*d*x^4 - 6*b*c^4*d*x^2)*
e^2 + 1134*(4*a*c^9*d^2*x^5 - b*c^8*d^2*x^4 + 2*b*c^6*d^2*x^2)*e + 12*(105*
b*c^6*d^3 - 189*b*c^4*d^2*e + 135*b*c^2*d*e^2 - 35*b*e^3)*log(c^2*x^2 + 1))
/c^9
```

Sympy [A]
time = 0.73, size = 389, normalized size = 1.63

$$\begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3\operatorname{atan}(cx)}{3} + \frac{3bd^2ex^5\operatorname{atan}(cx)}{5} + \frac{3bde^2x^7\operatorname{atan}(cx)}{7} + \frac{be^3x^9\operatorname{atan}(cx)}{9} - \frac{bd^3x^2}{6c} - \frac{3bd^2ex^4}{20c} - \frac{bde^2x^6}{14c} - \frac{be^3x^8}{72c} + \frac{bd^3\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} + \frac{3bd^2ex^2}{10c^3} + \frac{3bde^2x^4}{28c^3} + \frac{be^3x^6}{54c^3} - \frac{3bd^2e\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} - \frac{3bde^2x^2}{14c^5} - \frac{be^3x^4}{36c^5} + \frac{3bde^2\log\left(x^2+\frac{1}{c^2}\right)}{14c^7} + \frac{be^3x^2}{18c^7} - \frac{be^3\log\left(x^2+\frac{1}{c^2}\right)}{18c^9} & \text{for}\: c \neq 0 \\ a\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x
**9/9 + b*d**3*x**3*atan(c*x)/3 + 3*b*d**2*e*x**5*atan(c*x)/5 + 3*b*d*e**2*
x**7*atan(c*x)/7 + b*e**3*x**9*atan(c*x)/9 - b*d**3*x**2/(6*c) - 3*b*d**2*e
*x**4/(20*c) - b*d*e**2*x**6/(14*c) - b*e**3*x**8/(72*c) + b*d**3*log(x**2
+ c**(-2))/(6*c**3) + 3*b*d**2*e*x**2/(10*c**3) + 3*b*d*e**2*x**4/(28*c**3)
 + b*e**3*x**6/(54*c**3) - 3*b*d**2*e*log(x**2 + c**(-2))/(10*c**5) - 3*b*d
*e**2*x**2/(14*c**5) - b*e**3*x**4/(36*c**5) + 3*b*d*e**2*log(x**2 + c**(-2
))/(14*c**7) + b*e**3*x**2/(18*c**7) - b*e**3*log(x**2 + c**(-2))/(18*c**9)
, Ne(c, 0)), (a*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e**2*x**7/7 + e**3*x**
9/9), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.97, size = 296, normalized size = 1.24

$$\frac{a\,d^3\,x^3}{3}+\frac{a\,e^3\,x^9}{9}+\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,e^3\,\ln\left(c^2\,x^2+1\right)}{18\,c^9}-\frac{b\,d^3\,x^2}{6\,c}-\frac{b\,e^3\,x^8}{72\,c}+\frac{b\,e^3\,x^6}{54\,c^3}-\frac{b\,e^3\,x^4}{36\,c^5}+\frac{b\,e^3\,x^2}{18\,c^7}+\frac{3\,a\,d^2\,e\,x^5}{5}+\frac{3\,a\,d\,e^2\,x^7}{7}+\frac{b\,d^3\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,e^3\,x^9\,\mathrm{atan}(c\,x)}{9}+\frac{3\,b\,d^2\,e\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{3\,b\,d\,e^2\,x^7\,\mathrm{atan}(c\,x)}{7}-\frac{3\,b\,d^2\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}+\frac{3\,b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{3\,b\,d^2\,e\,x^4}{20\,c}+\frac{3\,b\,d^2\,e\,x^2}{10\,c^3}-\frac{b\,d\,e^2\,x^6}{14\,c}+\frac{3\,b\,d\,e^2\,x^4}{28\,c^3}-\frac{3\,b\,d\,e^2\,x^2}{14\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

```
[Out] (a*d^3*x^3)/3 + (a*e^3*x^9)/9 + (b*d^3*log(c^2*x^2 + 1))/(6*c^3) - (b*e^3*l
og(c^2*x^2 + 1))/(18*c^9) - (b*d^3*x^2)/(6*c) - (b*e^3*x^8)/(72*c) + (b*e^3
*x^6)/(54*c^3) - (b*e^3*x^4)/(36*c^5) + (b*e^3*x^2)/(18*c^7) + (3*a*d^2*e*x
^5)/5 + (3*a*d*e^2*x^7)/7 + (b*d^3*x^3*atan(c*x))/3 + (b*e^3*x^9*atan(c*x))
/9 + (3*b*d^2*e*x^5*atan(c*x))/5 + (3*b*d*e^2*x^7*atan(c*x))/7 - (3*b*d^2*e
*log(c^2*x^2 + 1))/(10*c^5) + (3*b*d*e^2*log(c^2*x^2 + 1))/(14*c^7) - (3*b*
d^2*e*x^4)/(20*c) + (3*b*d^2*e*x^2)/(10*c^3) - (b*d*e^2*x^6)/(14*c) + (3*b*
d*e^2*x^4)/(28*c^3) - (3*b*d*e^2*x^2)/(14*c^5)
```

3.1139 $\int x(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=158

$$-\frac{b(2c^2d-e)(2c^4d^2-2c^2de+e^2)x}{8c^7}-\frac{be(6c^4d^2-4c^2de+e^2)x^3}{24c^5}-\frac{b(4c^2d-e)e^2x^5}{40c^3}-\frac{be^3x^7}{56c}-\frac{b(c^2d-e)^4\,\mathrm{ArcT}}{8c^8e}$$

[Out] -1/8*b*(2*c^2*d-e)*(2*c^4*d^2-2*c^2*d*e+e^2)*x/c^7-1/24*b*e*(6*c^4*d^2-4*c^2*d*e+e^2)*x^3/c^5-1/40*b*(4*c^2*d-e)*e^2*x^5/c^3-1/56*b*e^3*x^7/c-1/8*b*(c^2*d-e)^4*arctan(c*x)/c^8/e+1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/e

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {5094, 398, 209}

$$\frac{(d+ex^2)^4(a+b\mathrm{ArcTan}(cx))}{8e}-\frac{b\mathrm{ArcTan}(cx)(c^2d-e)^4}{8c^8e}-\frac{be^2x^5(4c^2d-e)}{40c^3}-\frac{bx(2c^2d-e)(2c^4d^2-2c^2de+e^2)}{8c^7}-\frac{bex^3(6c^4d^2-4c^2de+e^2)}{24c^5}-\frac{be^3x^7}{56c}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]

[Out] -1/8*(b*(2*c^2*d - e)*(2*c^4*d^2 - 2*c^2*d*e + e^2)*x)/c^7 - (b*e*(6*c^4*d^2 - 4*c^2*d*e + e^2)*x^3)/(24*c^5) - (b*(4*c^2*d - e)*e^2*x^5)/(40*c^3) - (b*e^3*x^7)/(56*c) - (b*(c^2*d - e)^4*ArcTan[c*x])/(8*c^8*e) + ((d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^4}{1+c^2x^2}\,dx}{8e}\\
&= \frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8e}-\frac{(bc)\int\left(\frac{(2c^2d-e)e(2c^4d^2-2c^2de+e^2)}{c^8}+\frac{e^2(6}{}\right.}{}\\
&= -\frac{b(2c^2d-e)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7}-\frac{be(6c^4d^2-4c^2de+e^2)\,x^3}{24c^5}-\\
&= -\frac{b(2c^2d-e)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7}-\frac{be(6c^4d^2-4c^2de+e^2)\,x^3}{24c^5}-
\end{aligned}
$$

Mathematica [A]
time = 1.84, size = 217, normalized size = 1.37

$$
\frac{cx(105be^3-35bc^2e^2(12d+ex^2)+7bc^4e(90d^2+20dex^2+3e^2x^4)+105ac^7x(4d^3+6d^2ex^2+4de^2x^4+e^3x^6)-3bc^6(140d^3+70d^2ex^2+28de^2x^4+5e^3x^6))+105b(4c^6d^3-6c^4d^2e+4c^2de^2-e^3+c^8(4d^3x^2+6d^2ex^4+4de^2x^6+e^3x^8))\,\mathrm{ArcTan}(cx)}{840c^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c*x*(105*b*e^3 - 35*b*c^2*e^2*(12*d + e*x^2) + 7*b*c^4*e*(90*d^2 + 20*d*e*
x^2 + 3*e^2*x^4) + 105*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6
) - 3*b*c^6*(140*d^3 + 70*d^2*e*x^2 + 28*d*e^2*x^4 + 5*e^3*x^6)) + 105*b*(4
*c^6*d^3 - 6*c^4*d^2*e + 4*c^2*d*e^2 - e^3 + c^8*(4*d^3*x^2 + 6*d^2*e*x^4 +
 4*d*e^2*x^6 + e^3*x^8))*ArcTan[c*x])/(840*c^8)
```

Maple [A]
time = 0.57, size = 257, normalized size = 1.63

method	result
derivativedivides	$\frac{\frac{\left(c^2 e x^2+c^2 d\right)^4 a}{8c^6 e}+\frac{b\arctan(cx)d^3c^2x^2}{2}+\frac{3b\,c^2 e\arctan(cx)d^2x^4}{4}+\frac{b\,c^2e^2\arctan(cx)d\,x^6}{2}+\frac{b\,c^2e^3\arctan(cx)x^8}{8}-\frac{bc\,d^3x}{2}-\frac{bc\,d^2e\,x^3}{4}-}{}$
default	$\frac{\frac{\left(c^2 e x^2+c^2 d\right)^4 a}{8c^6 e}+\frac{b\arctan(cx)d^3c^2x^2}{2}+\frac{3b\,c^2 e\arctan(cx)d^2x^4}{4}+\frac{b\,c^2e^2\arctan(cx)d\,x^6}{2}+\frac{b\,c^2e^3\arctan(cx)x^8}{8}-\frac{bc\,d^3x}{2}-\frac{bc\,d^2e\,x^3}{4}-}{}$
risch	$\frac{b\,d^3\arctan\left(\frac{\left(c^9d^4-8c^7d^3e+12c^5d^2e^2-8c^3d\,e^3+2c\,e^4\right)x}{c^8d^4-8c^6d^3e+12c^4d^2e^2-8c^2d\,e^3+2e^4}\right)}{4c^2}+\frac{b\,d^4\arctan\left(\frac{\left(-c^9d^4+8c^7d^3e-12c^5d^2e^2+8c^3d\,e^3-2c\,e^4\right)x}{c^8d^4-8c^6d^3e+12c^4d^2e^2-8c^2d\,e^3+2e^4}\right)}{16e}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/8*(c^2*e*x^2+c^2*d)^4*a/c^6/e+1/2*b*arctan(c*x)*d^3*c^2*x^2+3/4*b*
c^2*e*arctan(c*x)*d^2*x^4+1/2*b*c^2*e^2*arctan(c*x)*d*x^6+1/8*b*c^2*e^3*arc
tan(c*x)*x^8-1/2*b*c*d^3*x-1/4*b*c*d^2*e*x^3-1/10*b*c*d*e^2*x^5-1/56*b*c*e^
3*x^7+3/4*b*d^2*e*x/c+1/6*b*d*e^2*x^3/c+1/40*b*e^3*x^5/c-1/2*b*d*e^2*x/c^3-
1/24*b*e^3*x^3/c^3+1/8*b*e^3*x/c^5+1/2*b*d^3*arctan(c*x)-3/4*b*d^2*e*arctan
(c*x)/c^2+1/2*b*d*e^2*arctan(c*x)/c^4-1/8*b*e^3*arctan(c*x)/c^6)
```

Maxima [A]
time = 0.47, size = 230, normalized size = 1.46

$$\frac{1}{8}\,ax^8e^3+\frac{1}{2}\,adx^6e^2+\frac{3}{4}\,ad^2x^4e+\frac{1}{2}\,ad^3x^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^3+\frac{1}{4}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^2e+\frac{1}{30}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bde^2+\frac{1}{840}\left(105\,x^8\arctan\left(cx\right)-c\left(\frac{15\,c^6x^7-21\,c^4x^5+35\,c^2x^3-105\,x}{c^8}+\frac{105\,\arctan\left(cx\right)}{c^9}\right)\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^3 + 1/2*a*d*x^6*e^2 + 3/4*a*d^2*x^4*e + 1/2*a*d^3*x^2 + 1/2*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3 + 1/4*(3*x^4*arctan(c*x)
 - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2*e + 1/30*(15*x^6*arct
an(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*
e^2 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3
 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^3
```

Fricas [A]
time = 3.14, size = 243, normalized size = 1.54

$$\frac{420\,ac^8d^3x^2-420\,bc^7d^3x+105\,(4\,bc^8d^3x^2+4\,bc^6d^3+(bc^8x^8-b)e^3+4\,(bc^8dx^6+bc^2d)e^2+6\,(bc^8d^2x^4-bc^4d^2)e)\arctan\left(cx\right)+(105\,ac^8x^8-15\,bc^7x^7+21\,bc^5x^5-35\,bc^3x^3+105\,bcx)e^3+28\,(15\,ac^8dx^6-3\,bc^7dx^5+5\,bc^5dx^3-15\,bc^3dx)e^2+210\,(3\,ac^8d^2x^4-bc^7d^2x^3+3\,bc^5d^2x)e}{840\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(420*a*c^8*d^3*x^2 - 420*b*c^7*d^3*x + 105*(4*b*c^8*d^3*x^2 + 4*b*c^6
*d^3 + (b*c^8*x^8 - b)*e^3 + 4*(b*c^8*d*x^6 + b*c^2*d)*e^2 + 6*(b*c^8*d^2*x
^4 - b*c^4*d^2)*e)*arctan(c*x) + (105*a*c^8*x^8 - 15*b*c^7*x^7 + 21*b*c^5*x
^5 - 35*b*c^3*x^3 + 105*b*c*x)*e^3 + 28*(15*a*c^8*d*x^6 - 3*b*c^7*d*x^5 + 5
*b*c^5*d*x^3 - 15*b*c^3*d*x)*e^2 + 210*(3*a*c^8*d^2*x^4 - b*c^7*d^2*x^3 + 3
*b*c^5*d^2*x)*e)/c^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(144) = 288$.
time = 0.60, size = 350, normalized size = 2.22

$$\begin{cases} \frac{ad^3x^2}{2}+\frac{3ad^2ex^4}{4}+\frac{ade^2x^6}{2}+\frac{ae^3x^8}{8}+\frac{bd^3x^2\operatorname{atan}\left(cx\right)}{2}+\frac{3bd^2ex^4\operatorname{atan}\left(cx\right)}{4}+\frac{bde^2x^6\operatorname{atan}\left(cx\right)}{2}+\frac{be^3x^8\operatorname{atan}\left(cx\right)}{8}-\frac{bd^3x}{2c}-\frac{bd^2ex^3}{4c}-\frac{bde^2x^5}{10c}-\frac{be^3x^7}{56c}+\frac{bd^3\operatorname{atan}\left(cx\right)}{2c^2}+\frac{3bd^2ex}{4c^3}+\frac{bde^2x^3}{6c^3}+\frac{be^3x^5}{40c^3}-\frac{3bd^2e\operatorname{atan}\left(cx\right)}{4c^4}-\frac{bde^2x}{2c^5}-\frac{be^3x^3}{24c^5}+\frac{bde^2\operatorname{atan}\left(cx\right)}{2c^6}+\frac{be^3x}{8c^7}-\frac{be^3\operatorname{atan}\left(cx\right)}{8c^8} & \text{for}\: c \neq 0 \\ a\left(\frac{d^3x^2}{2}+\frac{3d^2ex^4}{4}+\frac{de^2x^6}{2}+\frac{e^3x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**
8/8 + b*d**3*x**2*atan(c*x)/2 + 3*b*d**2*e*x**4*atan(c*x)/4 + b*d*e**2*x**6
*atan(c*x)/2 + b*e**3*x**8*atan(c*x)/8 - b*d**3*x/(2*c) - b*d**2*e*x**3/(4*
c) - b*d*e**2*x**5/(10*c) - b*e**3*x**7/(56*c) + b*d**3*atan(c*x)/(2*c**2)
+ 3*b*d**2*e*x/(4*c**3) + b*d*e**2*x**3/(6*c**3) + b*e**3*x**5/(40*c**3) -
3*b*d**2*e*atan(c*x)/(4*c**4) - b*d*e**2*x/(2*c**5) - b*e**3*x**3/(24*c**5)
 + b*d*e**2*atan(c*x)/(2*c**6) + b*e**3*x/(8*c**7) - b*e**3*atan(c*x)/(8*c*
*8), Ne(c, 0)), (a*(d**3*x**2/2 + 3*d**2*e*x**4/4 + d*e**2*x**6/2 + e**3*x*
*8/8), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.56, size = 442, normalized size = 2.80

$$x\left(\frac{\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{c^2}+\frac{3\,b\,d^2\,e}{4\,c}}{c^2}-\frac{b\,d^3}{2\,c}\right)-x^6\left(\frac{a\,e^3}{6\,c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{6\,c^2}\right)+x^4\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{c^2}}{4\,c^2}+\frac{3\,a\,d\,e\,\left(d\,c^2+e\right)}{4\,c^2}\right)+x^5\left(\frac{b\,e^3}{40\,c^3}-\frac{b\,d\,e^2}{10\,c}\right)+\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3\,x^2}{2}+\frac{3\,b\,d^2\,e\,x^4}{4}+\frac{b\,d\,e^2\,x^6}{2}+\frac{b\,e^3\,x^8}{8}\right)-x^3\left(\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{3\,c^2}+\frac{b\,d^2\,e}{4\,c}\right)-x^2\left(\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,\left(d\,c^2+e\right)}{c^2}}{2\,c^2}-\frac{a\,d^2\,\left(d\,c^2+3\,e\right)}{2\,c^2}\right)+\frac{a\,e^3\,x^8}{8}-\frac{b\,e^3\,x^7}{56\,c}-\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,\left(e-2\,c^2\,d\right)\,\left(2\,c^4\,d^2-2\,c^2\,d\,e+e^2\right)}{-4\,b\,c^6\,d^3+6\,b\,c^4\,d^2\,e-4\,b\,c^2\,d\,e^2+b\,e^3}\right)\,\left(e-2\,c^2\,d\right)\,\left(2\,c^4\,d^2-2\,c^2\,d\,e+e^2\right)}{8\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

```
[Out] x*((((b*e^3)/(8*c^3) - (b*d*e^2)/(2*c))/c^2 + (3*b*d^2*e)/(4*c))/c^2 - (b*d
^3)/(2*c)) - x^6*((a*e^3)/(6*c^2) - (a*e^2*(e + 3*c^2*d))/(6*c^2)) + x^4*((
(a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(4*c^2) + (3*a*d*e*(e + c^2*d))/(4
*c^2)) + x^5*((b*e^3)/(40*c^3) - (b*d*e^2)/(10*c)) + atan(c*x)*((b*d^3*x^2)
/2 + (b*e^3*x^8)/8 + (3*b*d^2*e*x^4)/4 + (b*d*e^2*x^6)/2) - x^3*(((b*e^3)/(
8*c^3) - (b*d*e^2)/(2*c))/(3*c^2) + (b*d^2*e)/(4*c)) - x^2*((((a*e^3)/c^2 -
 (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^2)/(2*c^2) - (a*d
^2*(3*e + c^2*d))/(2*c^2)) + (a*e^3*x^8)/8 - (b*e^3*x^7)/(56*c) - (b*atan((
b*c*x*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2*c^2*d*e))/(b*e^3 - 4*b*c^6*d^3 - 4
*b*c^2*d*e^2 + 6*b*c^4*d^2*e))*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2*c^2*d*e))
/(8*c^8)
```

3.1140 $\int (d+ex^2)^3 (a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=188

$$-\frac{be(35c^4d^2-21c^2de+5e^2)\,x^2}{70c^5}-\frac{b(21c^2d-5e)\,e^2x^4}{140c^3}-\frac{be^3x^6}{42c}+d^3x(a+b\text{ArcTan}(cx))+d^2ex^3(a+b\text{ArcTan}(cx))-$$

```
[Out] -1/70*b*e*(35*c^4*d^2-21*c^2*d*e+5*e^2)*x^2/c^5-1/140*b*(21*c^2*d-5*e)*e^2*
x^4/c^3-1/42*b*e^3*x^6/c+d^3*x*(a+b*arctan(c*x))+d^2*e*x^3*(a+b*arctan(c*x)
)+3/5*d*e^2*x^5*(a+b*arctan(c*x))+1/7*e^3*x^7*(a+b*arctan(c*x))-1/70*b*(35*
c^6*d^3-35*c^4*d^2*e+21*c^2*d*e^2-5*e^3)*ln(c^2*x^2+1)/c^7
```

Rubi [A]
time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {200, 5032, 1824, 266}

$$d^3x(a+b\text{ArcTan}(cx))+d^2ex^3(a+b\text{ArcTan}(cx))+\frac{3}{5}de^2x^5(a+b\text{ArcTan}(cx))+\frac{1}{7}e^3x^7(a+b\text{ArcTan}(cx))-\frac{be^2x^4(21c^2d-5e)}{140c^3}-\frac{bex^2(35c^4d^2-21c^2de+5e^2)}{70c^5}-\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log(c^2x^2+1)}{70c^7}-\frac{be^3x^6}{42c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/70*(b*e*(35*c^4*d^2 - 21*c^2*d*e + 5*e^2)*x^2)/c^5 - (b*(21*c^2*d - 5*e)
*e^2*x^4)/(140*c^3) - (b*e^3*x^6)/(42*c) + d^3*x*(a + b*ArcTan[c*x]) + d^2*
e*x^3*(a + b*ArcTan[c*x]) + (3*d*e^2*x^5*(a + b*ArcTan[c*x]))/5 + (e^3*x^7*
(a + b*ArcTan[c*x]))/7 - (b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e
^3)*Log[1 + c^2*x^2])/(70*c^7)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
```

```
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+b\tan^{-1}(cx)\right)\,dx &= d^3x\left(a+b\tan^{-1}(cx)\right)+d^2ex^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}\right.\\ &= d^3x\left(a+b\tan^{-1}(cx)\right)+d^2ex^3\left(a+b\tan^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\tan^{-1}\right.\\ &= -\frac{be\left(35c^4d^2-21c^2de+5e^2\right)x^2}{70c^5}-\frac{b\left(21c^2d-5e\right)e^2x^4}{140c^3}-\frac{be^3x^6}{42c}+d^3x\left(\right.\\ &= -\frac{be\left(35c^4d^2-21c^2de+5e^2\right)x^2}{70c^5}-\frac{b\left(21c^2d-5e\right)e^2x^4}{140c^3}-\frac{be^3x^6}{42c}+d^3x\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 192, normalized size = 1.02

$$\frac{c^2x(12ac^5(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-bex(30e^2-3c^2e(42d+5ex^2)+c^4(210d^2+63dex^2+10e^2x^4)))+12bc^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)\,\mathrm{ArcTan}(cx)-6b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log\left(1+c^2x^2\right)}{420c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
[Out] (c^2*x*(12*a*c^5*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - b*e*x
*(30*e^2 - 3*c^2*e*(42*d + 5*e*x^2) + c^4*(210*d^2 + 63*d*e*x^2 + 10*e^2*x^
4))) + 12*b*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6)*ArcTan
[c*x] - 6*b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Log[1 + c^2*
x^2])/(420*c^7)
```

Maple [A]
time = 0.17, size = 249, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+b\arctan(cx)d^3cx+bc\arctan(cx)d^2e\,x^3+\frac{3bc\arctan(cx)d\,e^2x^5}{5}+\frac{bc\arctan(cx)e^3x^7}{7}}{}$
default	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+b\arctan(cx)d^3cx+bc\arctan(cx)d^2e\,x^3+\frac{3bc\arctan(cx)d\,e^2x^5}{5}+\frac{bc\arctan(cx)e^3x^7}{7}}{}$
risch	$\frac{ib\,d^2e\,x^3\ln(-icx+1)}{2}+\frac{ib\,d^3x\ln(-icx+1)}{2}+\frac{3ibd\,e^2x^5\ln(-icx+1)}{10}+\frac{a\,e^3x^7}{7}-\frac{ib\left(5e^3x^7+21e^2d\,x^5+35d^2e\,x^3+35\right.}{70}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b*ar
ctan(c*x)*d^3*c*x+b*c*arctan(c*x)*d^2*e*x^3+3/5*b*c*arctan(c*x)*d*e^2*x^5+1
/7*b*c*arctan(c*x)*e^3*x^7-1/2*b*d^2*e*x^2-3/20*b*d*e^2*x^4+3/10*b/c^2*d*e^
2*x^2-1/42*b*e^3*x^6+1/28*b/c^2*e^3*x^4-1/14*b/c^4*e^3*x^2-1/2*b*ln(c^2*x^2
+1)*d^3+1/2*b/c^2*ln(c^2*x^2+1)*d^2*e-3/10*b/c^4*ln(c^2*x^2+1)*d*e^2+1/14*b
/c^6*ln(c^2*x^2+1)*e^3)
```

Maxima [A]
time = 0.26, size = 220, normalized size = 1.17

$$\frac{1}{7}ax^7e^3+\frac{3}{5}adx^5e^2+ad^2x^3e+ad^3x+\frac{1}{2}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2e+\frac{(2cx\arctan(cx)-\log(c^2x^2+1))bd^3}{2c}+\frac{3}{20}\left(4x^5\arctan(cx)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)bde^2+\frac{1}{84}\left(12x^7\arctan(cx)-c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6}-\frac{6\log(c^2x^2+1)}{c^8}\right)\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x + 1/2*(2*x^3*arctan
(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/2*(2*c*x*arctan(c*x
) - log(c^2*x^2 + 1))*b*d^3/c + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x
^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e^2 + 1/84*(12*x^7*arctan(c*x) - c*(
(2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*e^3
```

Fricas [A]
time = 2.37, size = 222, normalized size = 1.18

$$\frac{420ac^7d^3x+12(5bc^7x^7e^3+21bc^7dx^5e^2+35bc^7d^2x^3e+35bc^7d^3x)\arctan(cx)+5(12ac^7x^7-2bc^6x^6+3bc^4x^4-6bc^2x^2)e^3+63(4ac^7dx^5-bc^6dx^4+2bc^4dx^2)e^2+210(2ac^7d^2x^3-bc^6d^2x^2)e-6(35bc^6d^3-35bc^4d^2e+21bc^2de^2-5be^3)\log(c^2x^2+1)}{420c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*c^7*d^3*x + 12*(5*b*c^7*x^7*e^3 + 21*b*c^7*d*x^5*e^2 + 35*b*c^
7*d^2*x^3*e + 35*b*c^7*d^3*x)*arctan(c*x) + 5*(12*a*c^7*x^7 - 2*b*c^6*x^6 +
 3*b*c^4*x^4 - 6*b*c^2*x^2)*e^3 + 63*(4*a*c^7*d*x^5 - b*c^6*d*x^4 + 2*b*c^4
*d*x^2)*e^2 + 210*(2*a*c^7*d^2*x^3 - b*c^6*d^2*x^2)*e - 6*(35*b*c^6*d^3 - 3
5*b*c^4*d^2*e + 21*b*c^2*d*e^2 - 5*b*e^3)*log(c^2*x^2 + 1))/c^7
```

Sympy [A]
time = 0.53, size = 306, normalized size = 1.63

$$\begin{cases} ad^3x+ad^2ex^3+\frac{3ade^2x^5}{5}+\frac{ae^3x^7}{7}+bd^3x\,\mathrm{atan}(cx)+bd^2ex^3\,\mathrm{atan}(cx)+\frac{3bde^2x^5\,\mathrm{atan}(cx)}{5}+\frac{be^3x^7\,\mathrm{atan}(cx)}{7}-\frac{bd^3\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bd^2ex^2}{2c}-\frac{3bde^2x^4}{20c}-\frac{be^3x^6}{42c}+\frac{bd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2c^3}+\frac{3bde^2x^2}{10c^3}+\frac{be^3x^4}{28c^3}-\frac{3bde^2\log\left(x^2+\frac{1}{c^2}\right)}{10c^5}-\frac{be^3x^2}{14c^5}+\frac{be^3\log\left(x^2+\frac{1}{c^2}\right)}{14c^7} & \text{for } c\neq 0 \\ a\left(d^3x+d^2ex^3+\frac{3de^2x^5}{5}+\frac{e^3x^7}{7}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*atan(c*x) + b*d**2*e*x**3*atan(c*x) + 3*b*d*e**2*x**5*atan(c*x)/5 +
```

```
 b*e**3*x**7*atan(c*x)/7 - b*d**3*log(x**2 + c**(-2))/(2*c) - b*d**2*e*x**2
/(2*c) - 3*b*d*e**2*x**4/(20*c) - b*e**3*x**6/(42*c) + b*d**2*e*log(x**2 +
c**(-2))/(2*c**3) + 3*b*d*e**2*x**2/(10*c**3) + b*e**3*x**4/(28*c**3) - 3*b
*d*e**2*log(x**2 + c**(-2))/(10*c**5) - b*e**3*x**2/(14*c**5) + b*e**3*log(
x**2 + c**(-2))/(14*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x
**5/5 + e**3*x**7/7), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.43, size = 238, normalized size = 1.27

$$\frac{a\,e^3\,x^7}{7}+a\,d^3\,x-\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{2\,c}+\frac{b\,e^3\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{b\,e^3\,x^6}{42\,c}+\frac{b\,e^3\,x^4}{28\,c^3}-\frac{b\,e^3\,x^2}{14\,c^5}+b\,d^3\,x\,\mathrm{atan}(c\,x)+a\,d^2\,e\,x^3+\frac{3\,a\,d\,e^2\,x^5}{5}+\frac{b\,e^3\,x^7\,\mathrm{atan}(c\,x)}{7}+b\,d^2\,e\,x^3\,\mathrm{atan}(c\,x)+\frac{3\,b\,d\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d^2\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c^3}-\frac{3\,b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,d^2\,e\,x^2}{2\,c}-\frac{3\,b\,d\,e^2\,x^4}{20\,c}+\frac{3\,b\,d\,e^2\,x^2}{10\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2)^3,x)
```

```
[Out] (a*e^3*x^7)/7 + a*d^3*x - (b*d^3*log(c^2*x^2 + 1))/(2*c) + (b*e^3*log(c^2*x
^2 + 1))/(14*c^7) - (b*e^3*x^6)/(42*c) + (b*e^3*x^4)/(28*c^3) - (b*e^3*x^2)
/(14*c^5) + b*d^3*x*atan(c*x) + a*d^2*e*x^3 + (3*a*d*e^2*x^5)/5 + (b*e^3*x^
7*atan(c*x))/7 + b*d^2*e*x^3*atan(c*x) + (3*b*d*e^2*x^5*atan(c*x))/5 + (b*d
^2*e*log(c^2*x^2 + 1))/(2*c^3) - (3*b*d*e^2*log(c^2*x^2 + 1))/(10*c^5) - (b
*d^2*e*x^2)/(2*c) - (3*b*d*e^2*x^4)/(20*c) + (3*b*d*e^2*x^2)/(10*c^3)
```

3.1141 $\int \frac{(d+ex^2)^3(a+b\textbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=228

$$-\frac{3bd^2ex}{2c}+\frac{3bde^2x}{4c^3}-\frac{be^3x}{6c^5}-\frac{bde^2x^3}{4c}+\frac{be^3x^3}{18c^3}-\frac{be^3x^5}{30c}+\frac{3bd^2e\mathrm{ArcTan}(cx)}{2c^2}-\frac{3bde^2\mathrm{ArcTan}(cx)}{4c^4}+\frac{be^3\mathrm{ArcTan}(cx)}{6c^6}+$$

```
[Out] -3/2*b*d^2*e*x/c+3/4*b*d*e^2*x/c^3-1/6*b*e^3*x/c^5-1/4*b*d*e^2*x^3/c+1/18*b
*e^3*x^3/c^3-1/30*b*e^3*x^5/c+3/2*b*d^2*e*arctan(c*x)/c^2-3/4*b*d*e^2*arcta
n(c*x)/c^4+1/6*b*e^3*arctan(c*x)/c^6+3/2*d^2*e*x^2*(a+b*arctan(c*x))+3/4*d*
e^2*x^4*(a+b*arctan(c*x))+1/6*e^3*x^6*(a+b*arctan(c*x))+a*d^3*ln(x)+1/2*I*b
*d^3*polylog(2,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.15, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5100, 4940, 2438, 4946, 327, 209, 308}

$$\frac{3}{2}d^2ex^2(a+b\mathrm{ArcTan}(cx))+\frac{3}{4}de^2x^4(a+b\mathrm{ArcTan}(cx))+\frac{1}{6}e^3x^6(a+b\mathrm{ArcTan}(cx))+ad^3\log(x)+\frac{be^3\mathrm{ArcTan}(cx)}{6c^6}-\frac{3bde^2\mathrm{ArcTan}(cx)}{4c^4}+\frac{3bd^2e\mathrm{ArcTan}(cx)}{2c^2}-\frac{be^3x}{6c^5}+\frac{3bde^2x}{4c^3}+\frac{be^3x^3}{18c^3}+\frac{1}{2}ibd^3\mathrm{Li}_2(-icx)-\frac{1}{2}ibd^3\mathrm{Li}_2(icx)-\frac{3bd^2ex}{2c}-\frac{bde^2x^3}{4c}-\frac{be^3x^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (-3*b*d^2*e*x)/(2*c) + (3*b*d*e^2*x)/(4*c^3) - (b*e^3*x)/(6*c^5) - (b*d*e^2
*x^3)/(4*c) + (b*e^3*x^3)/(18*c^3) - (b*e^3*x^5)/(30*c) + (3*b*d^2*e*ArcTan
[c*x])/(2*c^2) - (3*b*d*e^2*ArcTan[c*x])/(4*c^4) + (b*e^3*ArcTan[c*x])/(6*c
^6) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcTan[c*x]
))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*d^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x}\,dx &= \int \left(\frac{d^3(a+b\tan^{-1}(cx))}{x} + 3d^2ex(a+b\tan^{-1}(cx)) + 3de^2x^3(a+b\tan^{-1}\right. \\
&= d^3\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + (3d^2e)\int x(a+b\tan^{-1}(cx))\,dx + (3de^2)\int \\
&= \frac{3}{2}d^2ex^2(a+b\tan^{-1}(cx)) + \frac{3}{4}de^2x^4(a+b\tan^{-1}(cx)) + \frac{1}{6}e^3x^6(a+b\tan \\
&= -\frac{3bd^2ex}{2c} + \frac{3}{2}d^2ex^2(a+b\tan^{-1}(cx)) + \frac{3}{4}de^2x^4(a+b\tan^{-1}(cx)) + \frac{1}{6}e^3 \\
&= -\frac{3bd^2ex}{2c} + \frac{3bde^2x}{4c^3} - \frac{be^3x}{6c^5} - \frac{bde^2x^3}{4c} + \frac{be^3x^3}{18c^3} - \frac{be^3x^5}{30c} + \frac{3bd^2e\tan^{-1}(c}{2c^2} \\
&= -\frac{3bd^2ex}{2c} + \frac{3bde^2x}{4c^3} - \frac{be^3x}{6c^5} - \frac{bde^2x^3}{4c} + \frac{be^3x^3}{18c^3} - \frac{be^3x^5}{30c} + \frac{3bd^2e\tan^{-1}(c}{2c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 190, normalized size = 0.83

$$\frac{3}{2}ad^2ex^2+\frac{3}{4}ade^2x^4+\frac{1}{6}ae^3x^6+\frac{3bd^2e(-cx+(1+c^2x^2)\operatorname{ArcTan}(cx))}{2c^2}+\frac{bde^2(3cx-c^3x^3+3(-1+c^4x^4)\operatorname{ArcTan}(cx))}{4c^4}+\frac{be^3(-15cx+5c^3x^3-3c^5x^5+15(1+c^6x^6)\operatorname{ArcTan}(cx))}{90c^6}+ad^3\log(x)+\frac{1}{2}ibd^3(\operatorname{PolyLog}(2,-icx)-\operatorname{PolyLog}(2,icx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 + (a*e^3*x^6)/6 + (3*b*d^2*e*(-(c*x)
+ (1 + c^2*x^2)*ArcTan[c*x]))/(2*c^2) + (b*d*e^2*(3*c*x - c^3*x^3 + 3*(-1 +
 c^4*x^4)*ArcTan[c*x]))/(4*c^4) + (b*e^3*(-15*c*x + 5*c^3*x^3 - 3*c^5*x^5 +
 15*(1 + c^6*x^6)*ArcTan[c*x]))/(90*c^6) + a*d^3*Log[x] + (I/2)*b*d^3*(Poly
Log[2, (-I)*c*x] - PolyLog[2, I*c*x])
```

Maple [A]
time = 0.17, size = 272, normalized size = 1.19

method	result
derivativedivides	$\frac{3a\,d^2e\,x^2}{2}+\frac{3ad\,e^2x^4}{4}+\frac{a\,e^3x^6}{6}+a\,d^3\ln(cx)+\frac{3b\arctan(cx)d^2e\,x^2}{2}+\frac{3b\arctan(cx)d\,e^2x^4}{4}+\frac{b\arctan(cx)e^3x}{6}$
default	$\frac{3a\,d^2e\,x^2}{2}+\frac{3ad\,e^2x^4}{4}+\frac{a\,e^3x^6}{6}+a\,d^3\ln(cx)+\frac{3b\arctan(cx)d^2e\,x^2}{2}+\frac{3b\arctan(cx)d\,e^2x^4}{4}+\frac{b\arctan(cx)e^3x}{6}$
risch	$-\frac{ib\,e^3\ln(c^2x^2+1)}{24c^6}-\frac{ib\,e^3\ln(icx+1)x^6}{12}+a\,d^3\ln(-icx)-\frac{3ib\,d^2e(-icx+1)^2\ln(-icx+1)}{4c^2}-\frac{3ibd\,e^2(-icx+1)^3}{2c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*a*d^2*e*x^2+3/4*a*d*e^2*x^4+1/6*a*e^3*x^6+a*d^3*ln(c*x)+3/2*b*arctan(c*
x)*d^2*e*x^2+3/4*b*arctan(c*x)*d*e^2*x^4+1/6*b*arctan(c*x)*e^3*x^6+b*arctan
```

```
(c*x)*d^3*ln(c*x)-3/2*b*d^2*e*x/c-1/4*b*d*e^2*x^3/c-1/30*b*e^3*x^5/c+3/4*b*
d*e^2*x/c^3+1/18*b*e^3*x^3/c^3-1/6*b*e^3*x/c^5+3/2*b*d^2*e*arctan(c*x)/c^2-
3/4*b*d*e^2*arctan(c*x)/c^4+1/6*b*e^3*arctan(c*x)/c^6-1/2*I*b*d^3*ln(c*x)*l
n(1-I*c*x)+1/2*I*b*d^3*ln(c*x)*ln(1+I*c*x)-1/2*I*b*d^3*dilog(1-I*c*x)+1/2*I
*b*d^3*dilog(1+I*c*x)
```

Maxima [A]
time = 0.61, size = 244, normalized size = 1.07

$$\frac{1}{6}ax^6e^3+\frac{3}{4}adx^4e^2+\frac{3}{2}ad^2x^2e+ad^3\log(x)-\frac{6\,bc^5x^5e^3+45\,\pi bc^6d^3\log(c^2x^2+1)-180\,bc^6d^3\arctan(cx)\log(cx)+90i\,bc^6d^3\mathrm{Li}_2(i\,cx+1)-90i\,bc^6d^3\mathrm{Li}_2(-i\,cx+1)+5\,(9\,bc^5de^2-2\,bc^3e^3)x^3+15\,(18\,bc^5d^2e-9\,bc^3de^2+2\,bce^3)x-15\,(2\,bc^6x^6e^3+9\,bc^6dx^4e^2+18\,bc^6d^2x^2e+18\,bc^4d^2e-9\,bc^2de^2+2\,be^3)\arctan(cx)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^3 + 3/4*a*d*x^4*e^2 + 3/2*a*d^2*x^2*e + a*d^3*log(x) - 1/180*(6
*b*c^5*x^5*e^3 + 45*pi*b*c^6*d^3*log(c^2*x^2 + 1) - 180*b*c^6*d^3*arctan(c*
x)*log(c*x) + 90*I*b*c^6*d^3*dilog(I*c*x + 1) - 90*I*b*c^6*d^3*dilog(-I*c*x
 + 1) + 5*(9*b*c^5*d*e^2 - 2*b*c^3*e^3)*x^3 + 15*(18*b*c^5*d^2*e - 9*b*c^3*
d*e^2 + 2*b*c*e^3)*x - 15*(2*b*c^6*x^6*e^3 + 9*b*c^6*d*x^4*e^2 + 18*b*c^6*d
^2*x^2*e + 18*b*c^4*d^2*e - 9*b*c^2*d*e^2 + 2*b*e^3)*arctan(c*x))/c^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arctan(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atan}(cx))(d+ex^2)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.78, size = 232, normalized size = 1.02

$$\begin{cases} \frac{a\,e^3\,x^6}{6}+a\,d^3\,\ln\left(x\right)+\frac{3\,a\,d^2\,e\,x^2}{2}+\frac{3\,a\,d\,e^2\,x^4}{4} & \text{if}\ \ c=0\\ \frac{a\,e^3\,x^6}{6}+a\,d^3\,\ln\left(x\right)-\frac{b\,e^3\,\left(\frac{x}{c^4}-\frac{\mathrm{atan}\left(c\,x\right)}{c^5}+\frac{x^5}{5}-\frac{x^3}{3\,c^2}\right)}{6\,c}-3\,b\,d^2\,e\,\left(\frac{x}{2\,c}-\mathrm{atan}\left(c\,x\right)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)+\frac{3\,a\,d^2\,e\,x^2}{2}+\frac{3\,a\,d\,e^2\,x^4}{4}-3\,b\,d\,e^2\,\left(\frac{3\,\mathrm{atan}\left(c\,x\right)-3\,c\,x+c^3\,x^3}{12\,c^4}-\frac{x^4\,\mathrm{atan}\left(c\,x\right)}{4}\right)+\frac{b\,e^3\,x^6\,\mathrm{atan}\left(c\,x\right)}{6}-\frac{b\,d^3\,\mathrm{Li}_2\left(1-c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\frac{b\,d^3\,\mathrm{Li}_2\left(1+c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x,x)
```

```
[Out] piecewise(c == 0, (a*e^3*x^6)/6 + a*d^3*log(x) + (3*a*d^2*e*x^2)/2 + (3*a*d
*e^2*x^4)/4, c ~= 0, (a*e^3*x^6)/6 + a*d^3*log(x) - (b*d^3*dilog(- c*x*1i +
 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)*1i)/2 - (b*e^3*(x/c^4 - atan(c*x)/c^5
+ x^5/5 - x^3/(3*c^2)))/(6*c) - 3*b*d^2*e*(x/(2*c) - atan(c*x)*(1/(2*c^2) +
 x^2/2)) + (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 - 3*b*d*e^2*((3*atan(c*x)
- 3*c*x + c^3*x^3)/(12*c^4) - (x^4*atan(c*x))/4) + (b*e^3*x^6*atan(c*x))/6)
```

3.1142 $\int \frac{(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=160

$$-\frac{b(5c^2d-e)\,e^2x^2}{10c^3}-\frac{be^3x^4}{20c}-\frac{d^3(a+b\text{ArcTan}(cx))}{x}+3d^2ex(a+b\text{ArcTan}(cx))+de^2x^3(a+b\text{ArcTan}(cx))+\frac{1}{5}e^3x^5$$

```
[Out] -1/10*b*(5*c^2*d-e)*e^2*x^2/c^3-1/20*b*e^3*x^4/c-d^3*(a+b*arctan(c*x))/x+3*
d^2*e*x*(a+b*arctan(c*x))+d*e^2*x^3*(a+b*arctan(c*x))+1/5*e^3*x^5*(a+b*arct
an(c*x))+b*c*d^3*ln(x)-1/10*b*(5*c^6*d^3+15*c^4*d^2*e-5*c^2*d*e^2+e^3)*ln(c
^2*x^2+1)/c^5
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {276, 5096, 1813, 1634}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{x}+3d^2ex(a+b\text{ArcTan}(cx))+de^2x^3(a+b\text{ArcTan}(cx))+\frac{1}{5}e^3x^5(a+b\text{ArcTan}(cx))-\frac{be^2x^2(5c^2d-e)}{10c^3}-\frac{b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log(c^2x^2+1)}{10c^5}+bcd^3\log(x)-\frac{be^3x^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] -1/10*(b*(5*c^2*d - e)*e^2*x^2)/c^3 - (b*e^3*x^4)/(20*c) - (d^3*(a + b*ArcT
an[c*x]))/x + 3*d^2*e*x*(a + b*ArcTan[c*x]) + d*e^2*x^3*(a + b*ArcTan[c*x])
 + (e^3*x^5*(a + b*ArcTan[c*x]))/5 + b*c*d^3*Log[x] - (b*(5*c^6*d^3 + 15*c^
4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/(10*c^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{x^2}\,dx = -\frac{d^3(a+b\tan^{-1}(cx))}{x} + 3d^2ex\big(a+b\tan^{-1}(cx)\big) + de^2x^3\big(a+b\tan^{-1}(c$$

$$= -\frac{d^3(a+b\tan^{-1}(cx))}{x} + 3d^2ex\big(a+b\tan^{-1}(cx)\big) + de^2x^3\big(a+b\tan^{-1}(c$$

$$= -\frac{d^3(a+b\tan^{-1}(cx))}{x} + 3d^2ex\big(a+b\tan^{-1}(cx)\big) + de^2x^3\big(a+b\tan^{-1}(c$$

$$= -\frac{b(5c^2d-e)\,e^2x^2}{10c^3} - \frac{be^3x^4}{20c} - \frac{d^3(a+b\tan^{-1}(cx))}{x} + 3d^2ex\big(a+b\tan^{-1}($$

Mathematica [A]
time = 0.08, size = 169, normalized size = 1.06

$$\frac{1}{20}\left(-\frac{20ad^3}{x}+60ad^2ex+\frac{2be^2(-5c^2d+e)\,x^2}{c^3}+20ade^2x^3-\frac{be^3x^4}{c}+4ae^3x^5+\frac{4b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\,\mathrm{ArcTan}(cx)}{x}+20bcd^3\log(x)-\frac{2b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log(1+c^2x^2)}{c^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] ((-20*a*d^3)/x + 60*a*d^2*e*x + (2*b*e^2*(-5*c^2*d + e)*x^2)/c^3 + 20*a*d*e
^2*x^3 - (b*e^3*x^4)/c + 4*a*e^3*x^5 + (4*b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^
2*x^4 + e^3*x^6)*ArcTan[c*x])/x + 20*b*c*d^3*Log[x] - (2*b*(5*c^6*d^3 + 15*
c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^5)/20
```

Maple [A]
time = 0.21, size = 237, normalized size = 1.48

method	result
derivativedivides	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{3b\arctan(cx)d^2ex}{c}+\frac{b\arctan(cx)d\,e^2x^3}{c}+\frac{b\arctan(cx)e^3x^5}{5c}-\frac{b\arcta}{c}\right.$

default	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{3b\arctan(cx)d^2ex}{c}+\frac{b\arctan(cx)d\,e^2x^3}{c}+\frac{b\arctan(cx)e^3x^5}{5c}-\frac{b\,\text{arc}}{}\right.$
risch	$\frac{ib(-e^3x^6-5e^2d\,x^4-15d^2e\,x^2+5d^3)\ln(icx+1)}{10x}+\frac{-10ib\,c^5d^3\ln(-icx+1)+30ib\,c^5d^2e\,x^2\ln(-icx+1)+4a\,c^5e^3x^6+10ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^6*(3*c^5*d^2*e*x+c^5*d*e^2*x^3+1/5*e^3*c^5*x^5-c^5*d^3/x)+3*b/c*arct
an(c*x)*d^2*e*x+b/c*arctan(c*x)*d*e^2*x^3+1/5*b/c*arctan(c*x)*e^3*x^5-b*arc
tan(c*x)*d^3/c/x-1/2*b/c^2*d*e^2*x^2-1/20*b/c^2*e^3*x^4+1/10*b/c^4*e^3*x^2-
1/2*b*ln(c^2*x^2+1)*d^3-3/2*b/c^2*ln(c^2*x^2+1)*d^2*e+1/2*b/c^4*ln(c^2*x^2+
1)*d*e^2-1/10*b/c^6*ln(c^2*x^2+1)*e^3+b*d^3*ln(c*x))
```

Maxima [A]

time = 0.27, size = 195, normalized size = 1.22

$$\frac{1}{5}ax^5e^3+adx^3e^2-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bd^3+3\,ad^2xe+\frac{1}{2}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bde^2+\frac{3\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd^2e}{2\,c}-\frac{ad^3}{x}+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^3 + a*d*x^3*e^2 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arct
an(c*x)/x)*b*d^3 + 3*a*d^2*x*e + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(
c^2*x^2 + 1)/c^4))*b*d*e^2 + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d
^2*e/c - a*d^3/x + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*l
og(c^2*x^2 + 1)/c^6))*b*e^3
```

Fricas [A]

time = 2.23, size = 202, normalized size = 1.26

$$\frac{20\,bc^6d^3x\log\left(x\right)+60\,ac^5d^2x^2e-20\,ac^5d^3+4\left(bc^5x^6e^3+5\,bc^5dx^4e^2+15\,bc^5d^2x^2e-5\,bc^5d^3\right)\arctan\left(cx\right)+\left(4\,ac^5x^6-bc^4x^5+2\,bc^2x^3\right)e^3+10\left(2\,ac^5dx^4-bc^4dx^3\right)e^2-2\left(5\,bc^6d^3x+15\,bc^4d^2xe-5\,bc^2dxe^2+bxe^3\right)\log\left(c^2x^2+1\right)}{20\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/20*(20*b*c^6*d^3*x*log(x) + 60*a*c^5*d^2*x^2*e - 20*a*c^5*d^3 + 4*(b*c^5*
x^6*e^3 + 5*b*c^5*d*x^4*e^2 + 15*b*c^5*d^2*x^2*e - 5*b*c^5*d^3)*arctan(c*x)
 + (4*a*c^5*x^6 - b*c^4*x^5 + 2*b*c^2*x^3)*e^3 + 10*(2*a*c^5*d*x^4 - b*c^4*
d*x^3)*e^2 - 2*(5*b*c^6*d^3*x + 15*b*c^4*d^2*x*e - 5*b*c^2*d*x*e^2 + b*x*e^
3)*log(c^2*x^2 + 1))/(c^5*x)
```

Sympy [A]

time = 0.73, size = 258, normalized size = 1.61

$$\begin{cases}-\frac{ad^3}{x}+3ad^2ex+ade^2x^3+\frac{ae^3x^5}{5}+bcd^3\log\left(x\right)-\frac{bcd^3\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd^3\operatorname{atan}\left(cx\right)}{x}+3bd^2ex\operatorname{atan}\left(cx\right)+bde^2x^3\operatorname{atan}\left(cx\right)+\frac{be^3x^5\operatorname{atan}\left(cx\right)}{5}-\frac{3bd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bde^2x^2}{2c}-\frac{be^3x^4}{20c}+\frac{bde^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^3}+\frac{be^3x^2}{10c^3}-\frac{be^3\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for}\: c\neq 0\\a\left(-\frac{d^3}{x}+3d^2ex+de^2x^3+\frac{e^3x^5}{5}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 + b*c*d
**3*log(x) - b*c*d**3*log(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/x + 3*b*d**2
*e*x*atan(c*x) + b*d*e**2*x**3*atan(c*x) + b*e**3*x**5*atan(c*x)/5 - 3*b*d*
*2*e*log(x**2 + c**(-2))/(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**4/(20*c) +
 b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x**2/(10*c**3) - b*e**3*log
(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(-d**3/x + 3*d**2*e*x + d*e**2*x*
*3 + e**3*x**5/5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.64, size = 236, normalized size = 1.48

$$x\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,\left(d\,c^2+e\right)}{c^2}\right)-x^3\,\left(\frac{a\,e^3}{3\,c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{3\,c^2}\right)+x^2\,\left(\frac{b\,e^3}{10\,c^3}-\frac{b\,d\,e^2}{2\,c}\right)-\frac{a\,d^3}{x}+\frac{a\,e^3\,x^5}{5}-\frac{\ln\left(c^2\,x^2+1\right)\,\left(5\,b\,c^6\,d^3+15\,b\,c^4\,d^2\,e-5\,b\,c^2\,d\,e^2+b\,e^3\right)}{10\,c^5}+\frac{\mathrm{atan}(c\,x)\,\left(-b\,d^3+3\,b\,d^2\,e\,x^2+b\,d\,e^2\,x^4+\frac{b\,e^3\,x^6}{5}\right)}{x}-\frac{b\,e^3\,x^4}{20\,c}+b\,c\,d^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^2,x)
```

```
[Out] x*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^
2) - x^3*((a*e^3)/(3*c^2) - (a*e^2*(e + 3*c^2*d))/(3*c^2)) + x^2*((b*e^3)/(
10*c^3) - (b*d*e^2)/(2*c)) - (a*d^3)/x + (a*e^3*x^5)/5 - (log(c^2*x^2 + 1)*
(b*e^3 + 5*b*c^6*d^3 - 5*b*c^2*d*e^2 + 15*b*c^4*d^2*e))/(10*c^5) + (atan(c*
x)*((b*e^3*x^6)/5 - b*d^3 + 3*b*d^2*e*x^2 + b*d*e^2*x^4))/x - (b*e^3*x^4)/(
20*c) + b*c*d^3*log(x)
```

3.1143 $\int \frac{(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=200

$$-\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\text{ArcTan}(cx)+\frac{3bde^2\text{ArcTan}(cx)}{2c^2}-\frac{be^3\text{ArcTan}(cx)}{4c^4}-\frac{d^3(a+b\text{ArcTan}(c$$

```
[Out] -1/2*b*c*d^3/x-3/2*b*d*e^2*x/c+1/4*b*e^3*x/c^3-1/12*b*e^3*x^3/c-1/2*b*c^2*d
^3*arctan(c*x)+3/2*b*d*e^2*arctan(c*x)/c^2-1/4*b*e^3*arctan(c*x)/c^4-1/2*d^
3*(a+b*arctan(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arctan(c*x))+1/4*e^3*x^4*(a+b*ar
ctan(c*x))+3*a*d^2*e*ln(x)+3/2*I*b*d^2*e*polylog(2,-I*c*x)-3/2*I*b*d^2*e*po
lylog(2,I*c*x)
```

Rubi [A]
time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5100, 4946, 331, 209, 4940, 2438, 327, 308}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\text{ArcTan}(cx))+\frac{1}{4}e^3x^4(a+b\text{ArcTan}(cx))+3ad^2e\log(x)-\frac{be^3\text{ArcTan}(cx)}{4c^4}-\frac{1}{2}bc^2d^3\text{ArcTan}(cx)+\frac{3bde^2\text{ArcTan}(cx)}{2c^2}+\frac{be^3x}{4c^3}-\frac{bcd^3}{2x}+\frac{3}{2}ibd^2e\text{Li}_2(-icx)-\frac{3}{2}ibd^2e\text{Li}_2(icx)-\frac{3bde^2x}{2c}-\frac{be^3x^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(b*c*d^3)/x - (3*b*d*e^2*x)/(2*c) + (b*e^3*x)/(4*c^3) - (b*e^3*x^3)/(1
2*c) - (b*c^2*d^3*ArcTan[c*x])/2 + (3*b*d*e^2*ArcTan[c*x])/(2*c^2) - (b*e^3
*ArcTan[c*x])/(4*c^4) - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a
 + b*ArcTan[c*x]))/2 + (e^3*x^4*(a + b*ArcTan[c*x]))/4 + 3*a*d^2*e*Log[x] +
 ((3*I)/2)*b*d^2*e*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d^2*e*PolyLog[2, I*c*
x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{x^3}\,dx &= \int \left(\frac{d^3(a+b\tan^{-1}(cx))}{x^3}+\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de^2x\big(a+b\tan^{-1}\right.\\
&= d^3\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx+\left(3d^2e\right)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+\left(3de^2\right)\int x\\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{2x^2}+\frac{3}{2}de^2x^2\big(a+b\tan^{-1}(cx)\big)+\frac{1}{4}e^3x^4\big(a+b\tan^{-}\\
&= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}-\frac{d^3(a+b\tan^{-1}(cx))}{2x^2}+\frac{3}{2}de^2x^2\big(a+b\tan^{-1}(cx)\big)+\\
&= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\tan^{-1}(cx)+\frac{3bde^2\tan^{-1}(c}{2c^2}\\
&= -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\tan^{-1}(cx)+\frac{3bde^2\tan^{-1}(c}{2c^2}
\end{aligned}
$$

Mathematica [A]

time = 0.09, size = 198, normalized size = 0.99

$$\frac{1}{4}\left(-\frac{2ad^3}{x^2}-\frac{2bcd^3}{x}-\frac{6bde^2x}{c}+\frac{be^3x}{c^3}+6ade^2x^2-\frac{be^3x^3}{3c}+ae^3x^4-2bc^2d^3\mathrm{ArcTan}(cx)+\frac{6bde^2\mathrm{ArcTan}(cx)}{c^2}-\frac{be^3\mathrm{ArcTan}(cx)}{c^4}-\frac{2bd^3\mathrm{ArcTan}(cx)}{x^2}+6bde^2x^2\mathrm{ArcTan}(cx)+be^3x^4\mathrm{ArcTan}(cx)+12ad^2e\log(x)+6ibd^2e\mathrm{PolyLog}(2,-icx)-6ibd^2e\mathrm{PolyLog}(2,icx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^3)/x^2 - (2*b*c*d^3)/x - (6*b*d*e^2*x)/c + (b*e^3*x)/c^3 + 6*a*d*e
^2*x^2 - (b*e^3*x^3)/(3*c) + a*e^3*x^4 - 2*b*c^2*d^3*ArcTan[c*x] + (6*b*d*e
^2*ArcTan[c*x])/c^2 - (b*e^3*ArcTan[c*x])/c^4 - (2*b*d^3*ArcTan[c*x])/x^2 +
 6*b*d*e^2*x^2*ArcTan[c*x] + b*e^3*x^4*ArcTan[c*x] + 12*a*d^2*e*Log[x] + (6
*I)*b*d^2*e*PolyLog[2, (-I)*c*x] - (6*I)*b*d^2*e*PolyLog[2, I*c*x])/4
```

Maple [A]

time = 0.21, size = 290, normalized size = 1.45

method	result
derivativedivides	$c^2\left(\frac{3ad\,e^2x^2}{2c^2}+\frac{a\,e^3x^4}{4c^2}-\frac{a\,d^3}{2c^2x^2}+\frac{3a\,d^2e\ln(cx)}{c^2}+\frac{3b\arctan(cx)d\,e^2x^2}{2c^2}+\frac{b\arctan(cx)e^3x^4}{4c^2}-\frac{b\arctan(cx)d^3}{2c^2x^2}\right.$
default	$c^2\left(\frac{3ad\,e^2x^2}{2c^2}+\frac{a\,e^3x^4}{4c^2}-\frac{a\,d^3}{2c^2x^2}+\frac{3a\,d^2e\ln(cx)}{c^2}+\frac{3b\arctan(cx)d\,e^2x^2}{2c^2}+\frac{b\arctan(cx)e^3x^4}{4c^2}-\frac{b\arctan(cx)d^3}{2c^2x^2}\right.$
risch	$-\frac{a\,e^3}{4c^4}-\frac{a\,d^3}{2x^2}+\frac{a\,x^4e^3}{4}-\frac{3bd\,e^2x}{2c}+\frac{3bd\,e^2\arctan(cx)}{2c^2}+\frac{ib\,d^3\ln(icx+1)}{4x^2}-\frac{ib\,c^2d^3\ln(icx)}{4}+\frac{b\,e^3x}{4c^3}-\frac{b\,e^3x^3}{12c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3/2*a/c^2*d*e^2*x^2+1/4*a/c^2*e^3*x^4-1/2*a*d^3/c^2/x^2+3*a/c^2*d^2*e*
ln(c*x)+3/2*b/c^2*arctan(c*x)*d*e^2*x^2+1/4*b/c^2*arctan(c*x)*e^3*x^4-1/2*b
*arctan(c*x)*d^3/c^2/x^2+3*b/c^2*arctan(c*x)*d^2*e*ln(c*x)-3/2*b*d*e^2*x/c^
3-1/12*b*e^3*x^3/c^3+1/4*b*e^3*x/c^5-1/2*b*d^3*arctan(c*x)+3/2*b*d*e^2*arct
an(c*x)/c^4-1/4*b*e^3*arctan(c*x)/c^6-1/2*b*d^3/c/x+3/2*I*b/c^2*d^2*e*dilog
(1+I*c*x)-3/2*I*b/c^2*d^2*e*dilog(1-I*c*x)-3/2*I*b/c^2*d^2*e*ln(c*x)*ln(1-I
*c*x)+3/2*I*b/c^2*d^2*e*ln(c*x)*ln(1+I*c*x))
```

Maxima [A]
time = 0.61, size = 219, normalized size = 1.10

$$\frac{1}{4}\,ax^4e^3-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bd^3+\frac{3}{2}\,adx^2e^2+3\,ad^2e\log\left(x\right)-\frac{ad^3}{2\,x^2}-\frac{9\,\pi bc^4d^2e\log\left(c^2x^2+1\right)-36\,bc^4d^2\arctan\left(cx\right)e\log\left(cx\right)+18i\,bc^4d^2\mathrm{Li}_2(i\,cx+1)\,e-18i\,bc^4d^2\mathrm{Li}_2(-i\,cx+1)\,e+bc^3x^3e^3+3\left(6\,bc^3de^2-bce^3\right)x-3\left(bc^4x^4e^3+6\,bc^4dx^2e^2+6\,bc^2de^2-be^3\right)\arctan\left(cx\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^3 + 3/2
*a*d*x^2*e^2 + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 - 1/12*(9*pi*b*c^4*d^2*e*lo
g(c^2*x^2 + 1) - 36*b*c^4*d^2*arctan(c*x)*e*log(c*x) + 18*I*b*c^4*d^2*dilog
(I*c*x + 1)*e - 18*I*b*c^4*d^2*dilog(-I*c*x + 1)*e + b*c^3*x^3*e^3 + 3*(6*b
*c^3*d*e^2 - b*c*e^3)*x - 3*(b*c^4*x^4*e^3 + 6*b*c^4*d*x^2*e^2 + 6*b*c^2*d*
e^2 - b*e^3)*arctan(c*x))/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arctan(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)\left(d+ex^2\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.73, size = 224, normalized size = 1.12

$$\begin{cases} \frac{a\,e^3\,x^4}{4}-\frac{a\,d^3}{2\,x^2}+\frac{3\,a\,d\,e^2\,x^2}{2}+3\,a\,d^2\,e\,\ln\left(x\right) & \text{if}\ \ c=0\\ \frac{a\,e^3\,x^4}{4}-\frac{a\,d^3}{2\,x^2}-\frac{b\,d^3\,\left(c^3\,\mathrm{atan}\left(c\,x\right)+\frac{c^2}{x}\right)}{2\,c}-3\,b\,d\,e^2\,\left(\frac{x}{2\,c}-\mathrm{atan}\left(c\,x\right)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)+\frac{3\,a\,d\,e^2\,x^2}{2}+3\,a\,d^2\,e\,\ln\left(x\right)-\frac{b\,e^3\,\left(3\,\mathrm{atan}\left(c\,x\right)-3\,c\,x+c^3\,x^3\right)}{12\,c^4}-\frac{b\,d^3\,\mathrm{atan}\left(c\,x\right)}{2\,x^2}+\frac{b\,e^3\,x^4\,\mathrm{atan}\left(c\,x\right)}{4}-\frac{b\,d^2\,e\,\mathrm{Li_2}\left(1-c\,x\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2}+\frac{b\,d^2\,e\,\mathrm{Li_2}\left(1+c\,x\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2} & \text{if}\ \ c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^3,x)
```

```
[Out] piecewise(c == 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 + (3*a*d*e^2*x^2)/2 + 3
*a*d^2*e*log(x), c ~= 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 - (b*d^3*(c^3*at
an(c*x) + c^2/x))/(2*c) - 3*b*d*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2
)) + (3*a*d*e^2*x^2)/2 + 3*a*d^2*e*log(x) - (b*e^3*(3*atan(c*x) - 3*c*x + c
^3*x^3))/(12*c^4) - (b*d^2*e*dilog(- c*x*1i + 1)*3i)/2 + (b*d^2*e*dilog(c*x
*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(2*x^2) + (b*e^3*x^4*atan(c*x))/4)
```

3.1144 $\int \frac{(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=158

$$-\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}-\frac{d^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{x}+3de^2x(a+b\text{ArcTan}(cx))+\frac{1}{3}e^3x^3(a+b\text{ArcTan}$$

```
[Out] -1/6*b*c*d^3/x^2-1/6*b*e^3*x^2/c-1/3*d^3*(a+b*arctan(c*x))/x^3-3*d^2*e*(a+b
*arctan(c*x))/x+3*d*e^2*x*(a+b*arctan(c*x))+1/3*e^3*x^3*(a+b*arctan(c*x))-1
/3*b*c*d^2*(c^2*d-9*e)*ln(x)+1/6*b*(c^2*d+e)*(c^4*d^2-10*c^2*d*e+e^2)*ln(c^
2*x^2+1)/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1813, 1634}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{3x^3}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{x}+3de^2x(a+b\text{ArcTan}(cx))+\frac{1}{3}e^3x^3(a+b\text{ArcTan}(cx))-\frac{1}{3}bcd^2\log(x)\left(c^2d-9e\right)+\frac{b(c^2d+e)\left(c^4d^2-10c^2de+e^2\right)\log\left(c^2x^2+1\right)}{6c^3}-\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*d^3)/x^2 - (b*e^3*x^2)/(6*c) - (d^3*(a + b*ArcTan[c*x]))/(3*x^3)
- (3*d^2*e*(a + b*ArcTan[c*x]))/x + 3*d*e^2*x*(a + b*ArcTan[c*x]) + (e^3*x^
3*(a + b*ArcTan[c*x]))/3 - (b*c*d^2*(c^2*d - 9*e)*Log[x])/3 + (b*(c^2*d + e
)*(c^4*d^2 - 10*c^2*d*e + e^2)*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= -\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de^2x\big(a+b\tan^{-1}(c\\ &= -\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de^2x\big(a+b\tan^{-1}(c\\ &= -\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de^2x\big(a+b\tan^{-1}(c\\ &= -\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de^2x\big(a+b\tan^{-1}(c\\ &= -\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}-\frac{d^3(a+b\tan^{-1}(cx))}{3x^3}-\frac{3d^2e(a+b\tan^{-1}(cx))}{x}+3de\end{aligned}$$

Mathematica [A]
time = 0.09, size = 166, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{2ad^3}{x^3}-\frac{bcd^3}{x^2}-\frac{18ad^2e}{x}+18ade^2x-\frac{be^3x^2}{c}+2ae^3x^3+\frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\,\mathrm{ArcTan}(cx)}{x^3}-2bcd^2\left(c^2d-9e\right)\log(x)+\frac{b(c^6d^3-9c^4d^2e-9c^2de^2+e^3)\log\left(1+c^2x^2\right)}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^3)/x^3 - (b*c*d^3)/x^2 - (18*a*d^2*e)/x + 18*a*d*e^2*x - (b*e^3*x^
2)/c + 2*a*e^3*x^3 + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcT
an[c*x])/x^3 - 2*b*c*d^2*(c^2*d - 9*e)*Log[x] + (b*(c^6*d^3 - 9*c^4*d^2*e -
 9*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^3)/6
```

Maple [A]
time = 0.21, size = 243, normalized size = 1.54

method	result
derivativedivides	$c^3\left(\frac{a\left(3c^3d\,e^2x+\frac{e^3c^3x^3}{3}-\frac{c^3d^3}{3x^3}-\frac{3c^3d^2e}{x}\right)}{c^6}+\frac{3b\arctan(cx)d\,e^2x}{c^3}+\frac{b\arctan(cx)e^3x^3}{3c^3}-\frac{b\arctan(cx)d^3}{3c^3x^3}-\frac{3b\arctan(\ldots}{c^3x\ldots}\right.$
default	$c^3\left(\frac{a\left(3c^3d\,e^2x+\frac{e^3c^3x^3}{3}-\frac{c^3d^3}{3x^3}-\frac{3c^3d^2e}{x}\right)}{c^6}+\frac{3b\arctan(cx)d\,e^2x}{c^3}+\frac{b\arctan(cx)e^3x^3}{3c^3}-\frac{b\arctan(cx)d^3}{3c^3x^3}-\frac{3b\arctan(\ldots}{c^3x\ldots}\right.$
risch	$\frac{ib(-e^3x^6-9e^2d\,x^4+9d^2e\,x^2+d^3)\ln(icx+1)}{6x^3}-\frac{9ib\,c^3d^2e\,x^2\ln(-icx+1)+2\ln(x)b\,c^6d^3x^3-\ln(-c^2x^2-1)b\,c^6d^3x^3-9ib\,c^3\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^6*(3*c^3*d*e^2*x+1/3*e^3*c^3*x^3-1/3*c^3*d^3/x^3-3*c^3*d^2*e/x)+3*
b/c^3*arctan(c*x)*d*e^2*x+1/3*b/c^3*arctan(c*x)*e^3*x^3-1/3*b*arctan(c*x)*d
^3/c^3/x^3-3*b/c^3*arctan(c*x)*d^2*e/x-1/6*b/c^4*e^3*x^2+1/6*b*ln(c^2*x^2+1
)*d^3-3/2*b/c^2*ln(c^2*x^2+1)*d^2*e-3/2*b/c^4*ln(c^2*x^2+1)*d*e^2+1/6*b/c^6
*ln(c^2*x^2+1)*e^3-1/3*b*d^3*ln(c*x)+3*b/c^2*ln(c*x)*d^2*e-1/6*b*d^3/c^2/x^
2)
```

Maxima [A]

time = 0.26, size = 191, normalized size = 1.21

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^3+\frac{1}{3}\,ax^3e^3-\frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bd^2e+3\,adxe^2+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^3+\frac{3\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bde^2}{2\,c}-\frac{3\,ad^2e}{x}-\frac{ad^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d^3 + 1/3*a*x^3*e^3 - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*b*d^2*e + 3*a*d*x*e^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x
^2 + 1)/c^4))*b*e^3 + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e^2/c
- 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

Fricas [A]

time = 1.94, size = 214, normalized size = 1.35

$$\frac{18\,ac^3dx^4e^2-bc^4d^3x-18\,ac^3d^2x^2e-2\,ac^3d^3+2\left(bc^3x^6e^3+9\,bc^3dx^4e^2-9\,bc^3d^2x^2e-bc^3d^3\right)\arctan\left(cx\right)+\left(2\,ac^3x^6-bc^2x^5\right)e^3+\left(bc^6d^3x^3-9\,bc^4d^2x^3e-9\,bc^2dx^3e^2+bx^3e^3\right)\log\left(c^2x^2+1\right)-2\left(bc^6d^3x^3-9\,bc^4d^2x^3e\right)\log\left(x\right)}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*c^3*d*x^4*e^2 - b*c^4*d^3*x - 18*a*c^3*d^2*x^2*e - 2*a*c^3*d^3 +
2*(b*c^3*x^6*e^3 + 9*b*c^3*d*x^4*e^2 - 9*b*c^3*d^2*x^2*e - b*c^3*d^3)*arcta
n(c*x) + (2*a*c^3*x^6 - b*c^2*x^5)*e^3 + (b*c^6*d^3*x^3 - 9*b*c^4*d^2*x^3*e
```

```
 - 9*b*c^2*d*x^3*e^2 + b*x^3*e^3)*log(c^2*x^2 + 1) - 2*(b*c^6*d^3*x^3 - 9*b
*c^4*d^2*x^3*e)*log(x))/(c^3*x^3)
```

Sympy [A]
time = 0.76, size = 272, normalized size = 1.72

$$\begin{cases} -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} - \frac{bc^3d^3\log(x)}{3} + \frac{bc^3d^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^3}{6x^2} + 3bcd^2e\log(x) - \frac{3bcd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{3x^3} - \frac{3bd^2e\operatorname{atan}(cx)}{x} + 3bde^2x\operatorname{atan}(cx) + \frac{be^3x^3\operatorname{atan}(cx)}{3} - \frac{3bde^2\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{be^3x^2}{6c} + \frac{be^3\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c \neq 0 \\ a\left(-\frac{d^3}{3x^3} - \frac{3d^2e}{x} + 3de^2x + \frac{e^3x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3 -
 b*c**3*d**3*log(x)/3 + b*c**3*d**3*log(x**2 + c**(-2))/6 - b*c*d**3/(6*x**
2) + 3*b*c*d**2*e*log(x) - 3*b*c*d**2*e*log(x**2 + c**(-2))/2 - b*d**3*atan
(c*x)/(3*x**3) - 3*b*d**2*e*atan(c*x)/x + 3*b*d*e**2*x*atan(c*x) + b*e**3*x
**3*atan(c*x)/3 - 3*b*d*e**2*log(x**2 + c**(-2))/(2*c) - b*e**3*x**2/(6*c)
+ b*e**3*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**3/(3*x**3) - 3*d*
*2*e/x + 3*d*e**2*x + e**3*x**3/3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.64, size = 203, normalized size = 1.28

$$\frac{a\,e^3\,x^3}{3}-\ln\left(x\right)\,\left(\frac{b\,c^3\,d^3}{3}-3\,b\,c\,d^2\,e\right)-\frac{\frac{b\,c^2\,d^3\,x}{2}+a\,c\,d^3+9\,a\,e\,c\,d^2\,x^2}{3\,c\,x^3}-x\,\left(\frac{a\,e^3}{c^2}-\frac{a\,e^2\,\left(3\,d\,c^2+e\right)}{c^2}\right)+\frac{\ln\left(c^2\,x^2+1\right)\,\left(b\,c^6\,d^3-9\,b\,c^4\,d^2\,e-9\,b\,c^2\,d\,e^2+b\,e^3\right)}{6\,c^3}-\frac{\mathrm{atan}\left(c\,x\right)\,\left(\frac{b\,d^3}{3}+3\,b\,d^2\,e\,x^2-3\,b\,d\,e^2\,x^4-\frac{b\,e^3\,x^6}{3}\right)}{x^3}-\frac{b\,e^3\,x^2}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^4,x)
```

```
[Out] (a*e^3*x^3)/3 - log(x)*((b*c^3*d^3)/3 - 3*b*c*d^2*e) - (a*c*d^3 + (b*c^2*d^
3*x)/2 + 9*a*c*d^2*e*x^2)/(3*c*x^3) - x*((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d)
)/c^2) + (log(c^2*x^2 + 1)*(b*e^3 + b*c^6*d^3 - 9*b*c^2*d*e^2 - 9*b*c^4*d^2
*e))/(6*c^3) - (atan(c*x)*((b*d^3)/3 - (b*e^3*x^6)/3 + 3*b*d^2*e*x^2 - 3*b*
d*e^2*x^4))/x^3 - (b*e^3*x^2)/(6*c)
```

3.1145 $\int \frac{(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=200

$$-\frac{bcd^3}{12x^3}+\frac{bc^3d^3}{4x}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}+\frac{1}{4}bc^4d^3\text{ArcTan}(cx)-\frac{3}{2}bc^2d^2e\text{ArcTan}(cx)+\frac{be^3\text{ArcTan}(cx)}{2c^2}-\frac{d^3(a+b\text{ArcTan}(</math>$$

```
[Out] -1/12*b*c*d^3/x^3+1/4*b*c^3*d^3/x-3/2*b*c*d^2*e/x-1/2*b*e^3*x/c+1/4*b*c^4*d
^3*arctan(c*x)-3/2*b*c^2*d^2*e*arctan(c*x)+1/2*b*e^3*arctan(c*x)/c^2-1/4*d^
3*(a+b*arctan(c*x))/x^4-3/2*d^2*e*(a+b*arctan(c*x))/x^2+1/2*e^3*x^2*(a+b*ar
ctan(c*x))+3*a*d*e^2*ln(x)+3/2*I*b*d*e^2*polylog(2,-I*c*x)-3/2*I*b*d*e^2*po
lylog(2,I*c*x)
```

Rubi [A]

time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5100, 4946, 331, 209, 4940, 2438, 327}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{4x^4}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{2x^2}+\frac{1}{2}e^3x^2(a+b\text{ArcTan}(cx))+3ade^2\log(x)+\frac{1}{4}bc^4d^3\text{ArcTan}(cx)-\frac{3}{2}bc^2d^2e\text{ArcTan}(cx)+\frac{be^3\text{ArcTan}(cx)}{2c^2}+\frac{bc^3d^3}{4x}-\frac{bcd^3}{12x^3}-\frac{3bcd^2e}{2x}+\frac{3}{2}ibde^2\text{Li}_2(-icx)-\frac{3}{2}ibde^2\text{Li}_2(icx)-\frac{be^3x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/12*(b*c*d^3)/x^3 + (b*c^3*d^3)/(4*x) - (3*b*c*d^2*e)/(2*x) - (b*e^3*x)/(
2*c) + (b*c^4*d^3*ArcTan[c*x])/4 - (3*b*c^2*d^2*e*ArcTan[c*x])/2 + (b*e^3*A
rcTan[c*x])/(2*c^2) - (d^3*(a + b*ArcTan[c*x]))/(4*x^4) - (3*d^2*e*(a + b*A
rcTan[c*x]))/(2*x^2) + (e^3*x^2*(a + b*ArcTan[c*x]))/2 + 3*a*d*e^2*Log[x] +
 ((3*I)/2)*b*d*e^2*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d*e^2*PolyLog[2, I*c*
x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex^2)^3 (a+b\tan^{-1}(cx))}{x^5}\,dx = \int \left(\frac{d^3(a+b\tan^{-1}(cx))}{x^5} + \frac{3d^2e(a+b\tan^{-1}(cx))}{x^3} + \frac{3de^2(a+b\tan^{-1}(c\right.$$

$$= d^3\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx + (3d^2e)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx + (3de^2)\int \frac{a-}{}$$

$$= -\frac{d^3(a+b\tan^{-1}(cx))}{4x^4} - \frac{3d^2e(a+b\tan^{-1}(cx))}{2x^2} + \frac{1}{2}e^3x^2(a+b\tan^{-1}(cx$$

$$= -\frac{bcd^3}{12x^3} - \frac{3bcd^2e}{2x} - \frac{be^3x}{2c} - \frac{d^3(a+b\tan^{-1}(cx))}{4x^4} - \frac{3d^2e(a+b\tan^{-1}(cx)}{2x^2}$$

$$= -\frac{bcd^3}{12x^3} + \frac{bc^3d^3}{4x} - \frac{3bcd^2e}{2x} - \frac{be^3x}{2c} - \frac{3}{2}bc^2d^2e\tan^{-1}(cx) + \frac{be^3\tan^{-1}(cx)}{2c^2}$$

$$= -\frac{bcd^3}{12x^3} + \frac{bc^3d^3}{4x} - \frac{3bcd^2e}{2x} - \frac{be^3x}{2c} + \frac{1}{4}bc^4d^3\tan^{-1}(cx) - \frac{3}{2}bc^2d^2e\tan^{-1}($$

Mathematica [A]

time = 0.11, size = 162, normalized size = 0.81

$$\frac{1}{12}\left(-\frac{3ad^3}{x^4} - \frac{18ad^2e}{x^2} + 6ae^3x^2 + \frac{6be^3(-cx+(1+c^2x^2)\operatorname{ArcTan}(cx))}{c^2} + \frac{bd^3(cx(-1+3c^2x^2)+3(-1+c^4x^4)\operatorname{ArcTan}(cx))}{x^4} - \frac{18bd^2e(\operatorname{ArcTan}(cx)+cx(1+cx\operatorname{ArcTan}(cx)))}{x^2} + 36ade^2\log(x) + 18ibde^2(\operatorname{PolyLog}(2,-icx) - \operatorname{PolyLog}(2,icx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] ((-3*a*d^3)/x^4 - (18*a*d^2*e)/x^2 + 6*a*e^3*x^2 + (6*b*e^3*(-(c*x) + (1 +
c^2*x^2)*ArcTan[c*x]))/c^2 + (b*d^3*(c*x*(-1 + 3*c^2*x^2) + 3*(-1 + c^4*x^4
)*ArcTan[c*x]))/x^4 - (18*b*d^2*e*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x]))
)/x^2 + 36*a*d*e^2*Log[x] + (18*I)*b*d*e^2*(PolyLog[2, (-I)*c*x] - PolyLog[
2, I*c*x]))/12
```

Maple [A]

time = 0.16, size = 292, normalized size = 1.46

method	result
derivativedivides	$c^4\left(\frac{a\,e^3x^2}{2c^4} - \frac{a\,d^3}{4c^4x^4} - \frac{3a\,d^2e}{2c^4x^2} + \frac{3ad\,e^2\ln(cx)}{c^4} + \frac{b\arctan(cx)e^3x^2}{2c^4} - \frac{b\arctan(cx)d^3}{4c^4x^4} - \frac{3b\arctan(cx)d^2e}{2c^4x^2} + \frac{3b\,\mathrm{a}}{}\right.$
default	$c^4\left(\frac{a\,e^3x^2}{2c^4} - \frac{a\,d^3}{4c^4x^4} - \frac{3a\,d^2e}{2c^4x^2} + \frac{3ad\,e^2\ln(cx)}{c^4} + \frac{b\arctan(cx)e^3x^2}{2c^4} - \frac{b\arctan(cx)d^3}{4c^4x^4} - \frac{3b\arctan(cx)d^2e}{2c^4x^2} + \frac{3b\,\mathrm{a}}{}\right.$
risch	$\frac{a\,e^3}{2c^2} - \frac{a\,d^3}{4x^4} + \frac{a\,x^2e^3}{2} - \frac{3bc\,d^2e}{2x} - \frac{3b\,c^2d^2e\arctan(cx)}{2} - \frac{3ibd\,e^2\,\mathrm{dilog}(-icx+1)}{2} + \frac{ib\,d^3\ln(icx+1)}{8x^4} + \frac{3ibd\,e^2\,\mathrm{dilo}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(1/2*a/c^4*e^3*x^2-1/4*a*d^3/c^4/x^4-3/2*a/c^4*d^2*e/x^2+3*a/c^4*d*e^2*
ln(c*x)+1/2*b/c^4*arctan(c*x)*e^3*x^2-1/4*b*arctan(c*x)*d^3/c^4/x^4-3/2*b/c
^4*arctan(c*x)*d^2*e/x^2+3*b/c^4*arctan(c*x)*d*e^2*ln(c*x)-1/2*b*e^3*x/c^5+
1/4*b*d^3*arctan(c*x)-3/2*b*d^2*e*arctan(c*x)/c^2+1/2*b*e^3*arctan(c*x)/c^6
-1/12*b*d^3/c^3/x^3+1/4*b*d^3/c/x-3/2*b/c^3*d^2*e/x+3/2*I*b/c^4*d*e^2*dilog
(1+I*c*x)-3/2*I*b/c^4*d*e^2*ln(c*x)*ln(1-I*c*x)-3/2*I*b/c^4*d*e^2*dilog(1-I
*c*x)+3/2*I*b/c^4*d*e^2*ln(c*x)*ln(1+I*c*x))
```

Maxima [A]
time = 0.59, size = 211, normalized size = 1.06

$$\frac{1}{12}\left(\left(3\,c^{3}\arctan\left(cx\right)+\frac{3\,c^{2}x^{2}-1}{x^{3}}\right)c-\frac{3\,\arctan\left(cx\right)}{x^{4}}\right)bd^{3}-\frac{3}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^{2}}\right)bd^{2}e+\frac{1}{2}\,ax^{2}e^{3}+3\,ade^{2}\log\left(x\right)-\frac{3\,ad^{2}e}{2\,x^{2}}-\frac{ad^{3}}{4\,x^{4}}-\frac{3\,\pi bc^{2}de^{2}\log\left(c^{2}x^{2}+1\right)-12\,bc^{2}d\arctan\left(cx\right)e^{2}\log\left(cx\right)+6i\,bc^{2}d\mathrm{Li}_{2}(i\,cx+1)\,e^{2}-6i\,bc^{2}d\mathrm{Li}_{2}(-i\,cx+1)\,e^{2}+2\,bcxe^{3}-2\,\left(bc^{2}x^{2}e^{3}+be^{3}\right)\arctan\left(cx\right)}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^
3 - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2*e + 1/2*a*x^2*e^3
 + 3*a*d*e^2*log(x) - 3/2*a*d^2*e/x^2 - 1/4*a*d^3/x^4 - 1/4*(3*pi*b*c^2*d*e
^2*log(c^2*x^2 + 1) - 12*b*c^2*d*arctan(c*x)*e^2*log(c*x) + 6*I*b*c^2*d*dil
og(I*c*x + 1)*e^2 - 6*I*b*c^2*d*dilog(-I*c*x + 1)*e^2 + 2*b*c*x*e^3 - 2*(b*
c^2*x^2*e^3 + b*e^3)*arctan(c*x))/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arctan(c*x))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{3}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.71, size = 234, normalized size = 1.17

$$\begin{cases} -\frac{a\left(d^3-2e^3x^6+6d^2ex^2-12de^2x^4\ln(x)\right)}{4x^4} & \text{if } c=0 \\ -be^3\left(\frac{x}{2c}-\operatorname{atan}(cx)\left(\frac{1}{2c^2}+\frac{x^2}{2}\right)\right)-\frac{a\left(d^3-2e^3x^6+6d^2ex^2-12de^2x^4\ln(x)\right)}{4x^4}-\frac{bd^3\left(\frac{\frac{c^2}{3}-c^4x^2}{x^3}-c^5\operatorname{atan}(cx)\right)}{4c}-3bd^2e\left(\frac{c^3\operatorname{atan}(cx)+\frac{c^2}{x}}{2c}+\frac{\operatorname{atan}(cx)}{2x^2}\right)-\frac{bd^3\operatorname{atan}(cx)}{4x^4}-\frac{bde^2\operatorname{Li}_2(1-cx\,1\mathrm{i})\,3\mathrm{i}}{2}+\frac{bde^2\operatorname{Li}_2(1+cx\,1\mathrm{i})\,3\mathrm{i}}{2} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^5,x)
```

```
[Out] piecewise(c == 0, -(a*(d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x))
)/(4*x^4), c ~= 0, - b*e^3*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (a*(
d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x)))/(4*x^4) - (b*d^3*((c^
2/3 - c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - 3*b*d^2*e*((c^3*atan(c*x) + c^
2/x)/(2*c) + atan(c*x)/(2*x^2)) - (b*d*e^2*dilog(- c*x*1i + 1)*3i)/2 + (b*d
*e^2*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(4*x^4))
```

3.1146 $\int \frac{\left(d+ex^2\right)^3 (a+b\mathbf{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=177

$$-\frac{bcd^3}{20x^4}+\frac{bcd^2(c^2d-5e)}{10x^2}-\frac{d^3(a+b\text{ArcTan}(cx))}{5x^5}-\frac{d^2e(a+b\text{ArcTan}(cx))}{x^3}-\frac{3de^2(a+b\text{ArcTan}(cx))}{x}+e^3x(a+b$$

```
[Out] -1/20*b*c*d^3/x^4+1/10*b*c*d^2*(c^2*d-5*e)/x^2-1/5*d^3*(a+b*arctan(c*x))/x^
5-d^2*e*(a+b*arctan(c*x))/x^3-3*d*e^2*(a+b*arctan(c*x))/x+e^3*x*(a+b*arctan
(c*x))+1/5*b*c*d*(c^4*d^2-5*c^2*d*e+15*e^2)*ln(x)-1/10*b*(c^6*d^3-5*c^4*d^2
*e+15*c^2*d*e^2+5*e^3)*ln(c^2*x^2+1)/c
```

Rubi [A]
time = 0.19, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1813, 1634}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{5x^5}-\frac{d^2e(a+b\text{ArcTan}(cx))}{x^3}-\frac{3de^2(a+b\text{ArcTan}(cx))}{x}+e^3x(a+b\text{ArcTan}(cx))+\frac{bcd^2(c^2d-5e)}{10x^2}+\frac{1}{5}bcd\log(x)\left(c^4d^2-5c^2de+15e^2\right)-\frac{b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log\left(c^2x^2+1\right)}{10c}-\frac{bcd^3}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] -1/20*(b*c*d^3)/x^4 + (b*c*d^2*(c^2*d - 5*e))/(10*x^2) - (d^3*(a + b*ArcTan
[c*x]))/(5*x^5) - (d^2*e*(a + b*ArcTan[c*x]))/x^3 - (3*d*e^2*(a + b*ArcTan[
c*x]))/x + e^3*x*(a + b*ArcTan[c*x]) + (b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^2
)*Log[x])/5 - (b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2
*x^2])/(10*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{x^6}\,dx &= -\frac{d^3(a+b\tan^{-1}(cx))}{5x^5}-\frac{d^2e(a+b\tan^{-1}(cx))}{x^3}-\frac{3de^2(a+b\tan^{-1}(cx))}{x} \\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{5x^5}-\frac{d^2e(a+b\tan^{-1}(cx))}{x^3}-\frac{3de^2(a+b\tan^{-1}(cx))}{x} \\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{5x^5}-\frac{d^2e(a+b\tan^{-1}(cx))}{x^3}-\frac{3de^2(a+b\tan^{-1}(cx))}{x} \\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{5x^5}-\frac{d^2e(a+b\tan^{-1}(cx))}{x^3}-\frac{3de^2(a+b\tan^{-1}(cx))}{x} \\
&= -\frac{bcd^3}{20x^4}+\frac{bcd^2(c^2d-5e)}{10x^2}-\frac{d^3(a+b\tan^{-1}(cx))}{5x^5}-\frac{d^2e(a+b\tan^{-1}(cx))}{x^3}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 184, normalized size = 1.04

$$\frac{1}{20}\left(-\frac{4ad^3}{x^5}-\frac{bcd^3}{x^4}-\frac{20ad^2e}{x^3}+\frac{2bcd^2(c^2d-5e)}{x^2}-\frac{60ade^2}{x}+20ae^3x-\frac{4b(d^3+5d^2ex^2+15de^2x^4-5e^3x^6)\,\mathrm{ArcTan}(cx)}{x^5}+4bcd(c^4d^2-5c^2de+15e^2)\log(x)-\frac{2b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(1+c^2x^2)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] ((-4*a*d^3)/x^5 - (b*c*d^3)/x^4 - (20*a*d^2*e)/x^3 + (2*b*c*d^2*(c^2*d - 5*
e))/x^2 - (60*a*d*e^2)/x + 20*a*e^3*x - (4*b*(d^3 + 5*d^2*e*x^2 + 15*d*e^2*
x^4 - 5*e^3*x^6)*ArcTan[c*x])/x^5 + 4*b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^2)*
Log[x] - (2*b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2*x^
2])/c)/20
```

Maple [A]
time = 0.21, size = 260, normalized size = 1.47

method	result
derivativedivides	$c^5\left(\frac{a\left(e^3cx-\frac{c\,d^2e}{x^3}-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}\right)}{c^6}+\frac{b\arctan(cx)e^3x}{c^5}-\frac{b\arctan(cx)d^2e}{c^5x^3}-\frac{b\arctan(cx)d^3}{5c^5x^5}-\frac{3b\arctan(cx)d\,e^2}{c^5x}-\right.$
default	$c^5\left(\frac{a\left(e^3cx-\frac{c\,d^2e}{x^3}-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}\right)}{c^6}+\frac{b\arctan(cx)e^3x}{c^5}-\frac{b\arctan(cx)d^2e}{c^5x^3}-\frac{b\arctan(cx)d^3}{5c^5x^5}-\frac{3b\arctan(cx)d\,e^2}{c^5x}-\right.$
risch	$\frac{ib\left(-5e^3x^6+15e^2d\,x^4+5d^2e\,x^2+d^3\right)\ln(icx+1)}{10x^5}+\frac{4\ln(x)b\,c^6d^3x^5-2\ln\left(c^2x^2+1\right)b\,c^6d^3x^5-20\ln(x)b\,c^4d^2e\,x^5+10\ln(c^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^6*(e^3*c*x-c*d^2*e/x^3-1/5*c*d^3/x^5-3*c*d*e^2/x)+b/c^5*arctan(c*x
)*e^3*x-b/c^5*arctan(c*x)*d^2*e/x^3-1/5*b*arctan(c*x)*d^3/c^5/x^5-3*b/c^5*a
rctan(c*x)*d*e^2/x-1/10*b*ln(c^2*x^2+1)*d^3+1/2*b/c^2*ln(c^2*x^2+1)*d^2*e-3
/2*b/c^4*ln(c^2*x^2+1)*d*e^2-1/2*b/c^6*ln(c^2*x^2+1)*e^3+1/10*b*d^3/c^2/x^2
-1/2*b/c^4*d^2*e/x^2-1/20*b*d^3/c^4/x^4+1/5*b*d^3*ln(c*x)-b/c^2*ln(c*x)*d^2
*e+3*b/c^4*ln(c*x)*d*e^2)
```

Maxima [A]
time = 0.26, size = 206, normalized size = 1.16

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd^3+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)bd^2e-\frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)bde^2+axe^3+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be^3}{2\,c}-\frac{3\,ade^2}{x}-\frac{ad^2e}{x^3}-\frac{ad^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d^3 + 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*d^2*e - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) +
2*arctan(c*x)/x)*b*d*e^2 + a*x*e^3 + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 +
 1))*b*e^3/c - 3*a*d*e^2/x - a*d^2*e/x^3 - 1/5*a*d^3/x^5
```

Fricas [A]
time = 3.36, size = 228, normalized size = 1.29

$$\frac{2\,bc^4d^3x^3+20\,acx^6e^3-60\,acdx^4e^2-bc^2d^3x-4\,acd^3+4\left(5\,bcx^6e^3-15\,bcdx^4e^2-5\,bcd^2x^2e-bcd^3\right)\arctan\left(cx\right)-10\left(bc^2d^2x^3+2\,acd^2x^2\right)e-2\left(bc^6d^3x^5-5\,bc^4d^2x^5e+15\,bc^2dx^5e^2+5\,bx^5e^3\right)\log\left(c^2x^2+1\right)+4\left(bc^6d^3x^5-5\,bc^4d^2x^5e+15\,bc^2dx^5e^2\right)\log\left(x\right)}{20\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^4*d^3*x^3 + 20*a*c*x^6*e^3 - 60*a*c*d*x^4*e^2 - b*c^2*d^3*x - 4
*a*c*d^3 + 4*(5*b*c*x^6*e^3 - 15*b*c*d*x^4*e^2 - 5*b*c*d^2*x^2*e - b*c*d^3)
```

```
*arctan(c*x) - 10*(b*c^2*d^2*x^3 + 2*a*c*d^2*x^2)*e - 2*(b*c^6*d^3*x^5 - 5*
b*c^4*d^2*x^5*e + 15*b*c^2*d*x^5*e^2 + 5*b*x^5*e^3)*log(c^2*x^2 + 1) + 4*(b
*c^6*d^3*x^5 - 5*b*c^4*d^2*x^5*e + 15*b*c^2*d*x^5*e^2)*log(x))/(c*x^5)
```

Sympy [**A**]
time = 0.77, size = 289, normalized size = 1.63

$$\begin{cases} -\frac{ad^3}{5x^5} - \frac{ad^2e}{x^3} - \frac{3ade^2}{x} + ae^3x + \frac{bc^5d^3\log(x)}{5} - \frac{bc^5d^3\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{10x^2} - bc^3d^2e\log(x) + \frac{bc^3d^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bcd^3}{20x^4} - \frac{bcd^2e}{2x^2} + 3bcde^2\log(x) - \frac{3bcde^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{5x^5} - \frac{bd^2e\operatorname{atan}(cx)}{x^3} - \frac{3bde^2\operatorname{atan}(cx)}{x} + be^3x\operatorname{atan}(cx) - \frac{be^3\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for}\ c \neq 0 \\ a\left(-\frac{d^3}{5x^5} - \frac{d^2e}{x^3} - \frac{3de^2}{x} + e^3x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*d**3/(5*x**5) - a*d**2*e/x**3 - 3*a*d*e**2/x + a*e**3*x + b*c
**5*d**3*log(x)/5 - b*c**5*d**3*log(x**2 + c**(-2))/10 + b*c**3*d**3/(10*x*
*2) - b*c**3*d**2*e*log(x) + b*c**3*d**2*e*log(x**2 + c**(-2))/2 - b*c*d**3
/(20*x**4) - b*c*d**2*e/(2*x**2) + 3*b*c*d*e**2*log(x) - 3*b*c*d*e**2*log(x
**2 + c**(-2))/2 - b*d**3*atan(c*x)/(5*x**5) - b*d**2*e*atan(c*x)/x**3 - 3*
b*d*e**2*atan(c*x)/x + b*e**3*x*atan(c*x) - b*e**3*log(x**2 + c**(-2))/(2*c
), Ne(c, 0)), (a*(-d**3/(5*x**5) - d**2*e/x**3 - 3*d*e**2/x + e**3*x), True
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.64, size = 194, normalized size = 1.10

$$\ln(x)\left(\frac{b\,c^5\,d^3}{5} - b\,c^3\,d^2\,e + 3\,b\,c\,d\,e^2\right) - \frac{a\,d^3 - x^3\left(\frac{b\,c^3\,d^3}{2} - \frac{5\,b\,c\,d^2\,e}{2}\right) + \frac{b\,c\,d^3\,x}{4} + 5\,a\,d^2\,e\,x^2 + 15\,a\,d\,e^2\,x^4}{5\,x^5} - \frac{\ln\left(c^2\,x^2+1\right)\,\left(b\,c^6\,d^3 - 5\,b\,c^4\,d^2\,e + 15\,b\,c^2\,d\,e^2 + 5\,b\,e^3\right)}{10\,c} - \frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{5} + b\,d^2\,e\,x^2 + 3\,b\,d\,e^2\,x^4 - b\,e^3\,x^6\right)}{x^5} + a\,e^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^6,x)
```

```
[Out] log(x)*((b*c^5*d^3)/5 + 3*b*c*d*e^2 - b*c^3*d^2*e) - (a*d^3 - x^3*((b*c^3*d
^3)/2 - (5*b*c*d^2*e)/2) + (b*c*d^3*x)/4 + 5*a*d^2*e*x^2 + 15*a*d*e^2*x^4)/
(5*x^5) - (log(c^2*x^2 + 1)*(5*b*e^3 + b*c^6*d^3 + 15*b*c^2*d*e^2 - 5*b*c^4
*d^2*e))/(10*c) - (atan(c*x)*((b*d^3)/5 - b*e^3*x^6 + b*d^2*e*x^2 + 3*b*d*e
^2*x^4))/x^5 + a*e^3*x
```

3.1147 $\int \frac{(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))}{x^7}\,dx$

Optimal. Leaf size=228

$$-\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}-\frac{bc^5d^3}{6x}+\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}-\frac{1}{6}bc^6d^3\text{ArcTan}(cx)+\frac{3}{4}bc^4d^2e\text{ArcTan}(cx)-\frac{3}{2}bc^2de^2\text{ArcTan}$$

```
[Out] -1/30*b*c*d^3/x^5+1/18*b*c^3*d^3/x^3-1/4*b*c*d^2*e/x^3-1/6*b*c^5*d^3/x+3/4*
b*c^3*d^2*e/x-3/2*b*c*d*e^2/x-1/6*b*c^6*d^3*arctan(c*x)+3/4*b*c^4*d^2*e*arc
tan(c*x)-3/2*b*c^2*d*e^2*arctan(c*x)-1/6*d^3*(a+b*arctan(c*x))/x^6-3/4*d^2*
e*(a+b*arctan(c*x))/x^4-3/2*d*e^2*(a+b*arctan(c*x))/x^2+a*e^3*ln(x)+1/2*I*b
*e^3*polylog(2,-I*c*x)-1/2*I*b*e^3*polylog(2,I*c*x)
```

Rubi [A]
time = 0.15, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5100, 4946, 331, 209, 4940, 2438}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{6x^6}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{4x^4}-\frac{3de^2(a+b\text{ArcTan}(cx))}{2x^2}+ae^3\log(x)-\frac{1}{6}bc^6d^3\text{ArcTan}(cx)+\frac{3}{4}bc^4d^2e\text{ArcTan}(cx)-\frac{3}{2}bc^2de^2\text{ArcTan}(cx)-\frac{bc^5d^3}{6x}+\frac{bc^3d^3}{18x^3}+\frac{3bc^3d^2e}{4x}-\frac{bcd^3}{30x^5}-\frac{bcd^2e}{4x^3}-\frac{3bcde^2}{2x}+\frac{1}{2}ibe^3\text{Li}_2(-icx)-\frac{1}{2}ibe^3\text{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/30*(b*c*d^3)/x^5 + (b*c^3*d^3)/(18*x^3) - (b*c*d^2*e)/(4*x^3) - (b*c^5*d
^3)/(6*x) + (3*b*c^3*d^2*e)/(4*x) - (3*b*c*d*e^2)/(2*x) - (b*c^6*d^3*ArcTan
[c*x])/6 + (3*b*c^4*d^2*e*ArcTan[c*x])/4 - (3*b*c^2*d*e^2*ArcTan[c*x])/2 -
(d^3*(a + b*ArcTan[c*x]))/(6*x^6) - (3*d^2*e*(a + b*ArcTan[c*x]))/(4*x^4) -
 (3*d*e^2*(a + b*ArcTan[c*x]))/(2*x^2) + a*e^3*Log[x] + (I/2)*b*e^3*PolyLog
[2, (-I)*c*x] - (I/2)*b*e^3*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^3\,(a+b\tan^{-1}(cx))}{x^7}\,dx &= \int \left(\frac{d^3(a+b\tan^{-1}(cx))}{x^7}+\frac{3d^2e(a+b\tan^{-1}(cx))}{x^5}+\frac{3de^2(a+b\tan^{-1}(c\right.\\
&= d^3\int \frac{a+b\tan^{-1}(cx)}{x^7}\,dx+\left(3d^2e\right)\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx+\left(3de^2\right)\int \frac{a-}{}\\
&= -\frac{d^3(a+b\tan^{-1}(cx))}{6x^6}-\frac{3d^2e(a+b\tan^{-1}(cx))}{4x^4}-\frac{3de^2(a+b\tan^{-1}(cx)}{2x^2}\\
&= -\frac{bcd^3}{30x^5}-\frac{bcd^2e}{4x^3}-\frac{3bcde^2}{2x}-\frac{d^3(a+b\tan^{-1}(cx))}{6x^6}-\frac{3d^2e(a+b\tan^{-1}(cx}{4x^4}\\
&= -\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}+\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}-\frac{3}{2}bc^2de^2\tan^{-1}(cx)-\frac{d^3}{}\\
&= -\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}-\frac{bc^5d^3}{6x}+\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}+\frac{3}{4}bc^4d^2e\tan^{-1}(\\
&= -\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}-\frac{bc^5d^3}{6x}+\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}-\frac{1}{6}bc^6d^3\tan^{-1}(c
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 190, normalized size = 0.83

$$-\frac{ad^3}{6x^6}-\frac{3ad^2e}{4x^4}-\frac{3ade^2}{2x^2}+\frac{bd^2e(cx(-1+3c^2x^2)+3(-1+c^4x^4)\operatorname{ArcTan}(cx))}{4x^4}-\frac{bd^3(cx(3-5c^2x^2+15c^4x^4)+15(1+c^6x^6)\operatorname{ArcTan}(cx))}{90x^6}-\frac{3bde^2(\operatorname{ArcTan}(cx)+cx(1+cx\operatorname{ArcTan}(cx)))}{2x^2}+ae^3\log(x)+\frac{1}{2}ibe^3(\operatorname{PolyLog}(2,-icx)-\operatorname{PolyLog}(2,icx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

```
[Out] -1/6*(a*d^3)/x^6 - (3*a*d^2*e)/(4*x^4) - (3*a*d*e^2)/(2*x^2) + (b*d^2*e*(c*
x*(-1 + 3*c^2*x^2) + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(4*x^4) - (b*d^3*(c*x*(
3 - 5*c^2*x^2 + 15*c^4*x^4) + 15*(1 + c^6*x^6)*ArcTan[c*x]))/(90*x^6) - (3*
b*d*e^2*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/(2*x^2) + a*e^3*Log[x] +
 (I/2)*b*e^3*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x])
```

Maple [A]
time = 0.18, size = 315, normalized size = 1.38

method	result
derivativedivides	$c^6\left(-\frac{3a\,d^2e}{4c^6x^4}-\frac{a\,d^3}{6c^6x^6}-\frac{3ad\,e^2}{2c^6x^2}+\frac{a\,e^3\ln(cx)}{c^6}-\frac{3b\arctan(cx)d^2e}{4c^6x^4}-\frac{b\arctan(cx)d^3}{6c^6x^6}-\frac{3b\arctan(cx)d\,e^2}{2c^6x^2}+\frac{b}{}\right.$
default	$c^6\left(-\frac{3a\,d^2e}{4c^6x^4}-\frac{a\,d^3}{6c^6x^6}-\frac{3ad\,e^2}{2c^6x^2}+\frac{a\,e^3\ln(cx)}{c^6}-\frac{3b\arctan(cx)d^2e}{4c^6x^4}-\frac{b\arctan(cx)d^3}{6c^6x^6}-\frac{3b\arctan(cx)d\,e^2}{2c^6x^2}+\frac{b}{}\right.$
risch	$a\,e^3\ln(-icx)+\frac{3ib\,e^2d\ln(icx+1)}{4x^2}-\frac{3ib\,c^2e^2d\ln(icx)}{4}-\frac{a\,d^3}{6x^6}-\frac{ib\,d^3\ln(-icx+1)}{12x^6}+\frac{ic^6b\,d^3\ln(-icx)}{12}-ib\,c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(-3/4*a/c^6*d^2*e/x^4-1/6*a*d^3/c^6/x^6-3/2*a/c^6*d*e^2/x^2+a/c^6*e^3*l
n(c*x)-3/4*b/c^6*arctan(c*x)*d^2*e/x^4-1/6*b*arctan(c*x)*d^3/c^6/x^6-3/2*b/
c^6*arctan(c*x)*d*e^2/x^2+b/c^6*arctan(c*x)*e^3*ln(c*x)+1/2*I*b/c^6*e^3*ln(
c*x)*ln(1+I*c*x)-1/2*I*b/c^6*e^3*dilog(1-I*c*x)+1/2*I*b/c^6*e^3*dilog(1+I*c
*x)-1/2*I*b/c^6*e^3*ln(c*x)*ln(1-I*c*x)-1/6*b*d^3*arctan(c*x)+3/4*b*d^2*e*a
rctan(c*x)/c^2-3/2*b*d*e^2*arctan(c*x)/c^4-1/6*b*d^3/c/x+3/4*b/c^3*d^2*e/x-
3/2*b/c^5*d*e^2/x-1/30*b*d^3/c^5/x^5+1/18*b*d^3/c^3/x^3-1/4*b/c^5*d^2*e/x^3
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arcta
n(c*x)/x^6)*b*d^3 + 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*ar
ctan(c*x)/x^4)*b*d^2*e - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*
d*e^2 + b*e^3*integrate(arctan(c*x)/x, x) + a*e^3*log(x) - 3/2*a*d*e^2/x^2
- 3/4*a*d^2*e/x^4 - 1/6*a*d^3/x^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arctan(c*x))/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))\,(d + ex^2)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**7,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.84, size = 261, normalized size = 1.14

$$\begin{cases} a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6} + \frac{3\,a\,d^2\,e\,x^2}{4} + \frac{3\,a\,d\,e^2\,x^4}{2}}{x^6} & \text{if } c = 0\\ a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6} + \frac{3\,a\,d^2\,e\,x^2}{4} + \frac{3\,a\,d\,e^2\,x^4}{2}}{x^6} - 3\,b\,d^2\,e\left(\frac{\operatorname{atan}(c\,x)}{4\,x^4} + \frac{\frac{\frac{c^2}{3} - c^4\,x^2}{x^3} - c^5\operatorname{atan}(c\,x)}{4\,c}\right) - \frac{b\,d^3\left(\frac{c^6\,x^4 - \frac{c^4\,x^2}{3} + \frac{c^2}{5}}{x^5} + c^7\operatorname{atan}(c\,x)\right)}{6\,c} - 3\,b\,d\,e^2\left(\frac{c^3\operatorname{atan}(c\,x) + \frac{c^2}{x}}{2\,c} + \frac{\operatorname{atan}(c\,x)}{2\,x^2}\right) - \frac{b\,d^3\operatorname{atan}(c\,x)}{6\,x^6} - \frac{b\,e^3\,\mathrm{Li}_2(1 - c\,x\,1i)\,1i}{2} + \frac{b\,e^3\,\mathrm{Li}_2(1 + c\,x\,1i)\,1i}{2} & \text{if } c \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^7,x)
```

```
[Out] piecewise(c == 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^4)/2)/x^6
 + a*e^3*log(x), c ~= 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^4)
/2)/x^6 + a*e^3*log(x) - (b*e^3*dilog(- c*x*1i + 1)*1i)/2 + (b*e^3*dilog(c*
x*1i + 1)*1i)/2 - 3*b*d^2*e*(atan(c*x)/(4*x^4) + ((c^2/3 - c^4*x^2)/x^3 - c
^5*atan(c*x))/(4*c)) - (b*d^3*((c^2/5 - (c^4*x^2)/3 + c^6*x^4)/x^5 + c^7*at
an(c*x)))/(6*c) - 3*b*d*e^2*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x)/(2*x
^2)) - (b*d^3*atan(c*x))/(6*x^6))
```

3.1148 $\int \frac{(d+ex^2)^3(a+b\textbf{ArcTan}(cx))}{x^8}\,dx$

Optimal. Leaf size=224

$$-\frac{bcd^3}{42x^6}+\frac{bcd^2(5c^2d-21e)}{140x^4}-\frac{bcd(5c^4d^2-21c^2de+35e^2)}{70x^2}-\frac{d^3(a+b\text{ArcTan}(cx))}{7x^7}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{5x^5}-\frac{de}{}$$

```
[Out] -1/42*b*c*d^3/x^6+1/140*b*c*d^2*(5*c^2*d-21*e)/x^4-1/70*b*c*d*(5*c^4*d^2-21
*c^2*d*e+35*e^2)/x^2-1/7*d^3*(a+b*arctan(c*x))/x^7-3/5*d^2*e*(a+b*arctan(c*
x))/x^5-d*e^2*(a+b*arctan(c*x))/x^3-e^3*(a+b*arctan(c*x))/x-1/35*b*c*(5*c^6
*d^3-21*c^4*d^2*e+35*c^2*d*e^2-35*e^3)*ln(x)+1/70*b*c*(5*c^6*d^3-21*c^4*d^2
*e+35*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.21, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {276, 5096, 12, 1813, 1634}

$$-\frac{d^3(a+b\text{ArcTan}(cx))}{7x^7}-\frac{3d^2e(a+b\text{ArcTan}(cx))}{5x^5}-\frac{de^2(a+b\text{ArcTan}(cx))}{x^3}-\frac{e^3(a+b\text{ArcTan}(cx))}{x}+\frac{bcd^2(5c^2d-21e)}{140x^4}-\frac{bcd(5c^4d^2-21c^2de+35e^2)}{70x^2}+\frac{1}{70}bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\log\left(c^2x^2+1\right)-\frac{1}{35}bc\log(x)\left(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3\right)-\frac{bcd^3}{42x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/42*(b*c*d^3)/x^6 + (b*c*d^2*(5*c^2*d - 21*e))/(140*x^4) - (b*c*d*(5*c^4*
d^2 - 21*c^2*d*e + 35*e^2))/(70*x^2) - (d^3*(a + b*ArcTan[c*x]))/(7*x^7) -
(3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5) - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (
e^3*(a + b*ArcTan[c*x]))/x - (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2
- 35*e^3)*Log[x])/35 + (b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e
^3)*Log[1 + c^2*x^2])/70
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^3(a+b\tan^{-1}(cx))}{7x^7} - \frac{3d^2e(a+b\tan^{-1}(cx))}{5x^5} - \frac{de^2(a+b\tan^{-1}(cx)}{x^3}$$
$$= -\frac{d^3(a+b\tan^{-1}(cx))}{7x^7} - \frac{3d^2e(a+b\tan^{-1}(cx))}{5x^5} - \frac{de^2(a+b\tan^{-1}(cx)}{x^3}$$
$$= -\frac{d^3(a+b\tan^{-1}(cx))}{7x^7} - \frac{3d^2e(a+b\tan^{-1}(cx))}{5x^5} - \frac{de^2(a+b\tan^{-1}(cx)}{x^3}$$
$$= -\frac{d^3(a+b\tan^{-1}(cx))}{7x^7} - \frac{3d^2e(a+b\tan^{-1}(cx))}{5x^5} - \frac{de^2(a+b\tan^{-1}(cx)}{x^3}$$
$$= -\frac{bcd^3}{42x^6} + \frac{bcd^2(5c^2d-21e)}{140x^4} - \frac{bcd(5c^4d^2-21c^2de+35e^2)}{70x^2} - \frac{d^3(a+b}{7}$$

Mathematica [A]
time = 0.09, size = 229, normalized size = 1.02

$$-\frac{60ad^3+10bcd^3x+252ad^2ex^2-3bcd^2(5c^2d-21e)\,x^3+420ade^2x^4+6bcd(5c^4d^2-21c^2de+35e^2)\,x^5+420ae^3x^6+12b(5d^3+21d^2ex^2+35de^2x^4+35e^3x^6)\,\mathrm{ArcTan}(cx)+12bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\,x^7\log(x)-6bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\,x^7\log\left(1+c^2x^2\right)}{420x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

```
[Out] -1/420*(60*a*d^3 + 10*b*c*d^3*x + 252*a*d^2*e*x^2 - 3*b*c*d^2*(5*c^2*d - 21
*e)*x^3 + 420*a*d*e^2*x^4 + 6*b*c*d*(5*c^4*d^2 - 21*c^2*d*e + 35*e^2)*x^5 +
```

```
420*a*e^3*x^6 + 12*b*(5*d^3 + 21*d^2*e*x^2 + 35*d*e^2*x^4 + 35*e^3*x^6)*Ar
cTan[c*x] + 12*b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*x^7*L
og[x] - 6*b*c*(5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*x^7*Log[1
+ c^2*x^2])/x^7
```

Maple [A]
time = 0.22, size = 324, normalized size = 1.45

method	result
derivativedivides	$c^7\left(\frac{a\left(-\frac{d\,e^2}{c\,x^3}-\frac{3d^2e}{5c\,x^5}-\frac{e^3}{cx}-\frac{d^3}{7c\,x^7}\right)}{c^6}-\frac{b\arctan(cx)d\,e^2}{c^7x^3}-\frac{3b\arctan(cx)d^2e}{5c^7x^5}-\frac{b\arctan(cx)e^3}{c^7x}-\frac{b\arctan(cx)d^3}{7c^7x^7}+b\right.$
default	$c^7\left(\frac{a\left(-\frac{d\,e^2}{c\,x^3}-\frac{3d^2e}{5c\,x^5}-\frac{e^3}{cx}-\frac{d^3}{7c\,x^7}\right)}{c^6}-\frac{b\arctan(cx)d\,e^2}{c^7x^3}-\frac{3b\arctan(cx)d^2e}{5c^7x^5}-\frac{b\arctan(cx)e^3}{c^7x}-\frac{b\arctan(cx)d^3}{7c^7x^7}+b\right.$
risch	$\frac{ib(35e^3x^6+35e^2d\,x^4+21d^2e\,x^2+5d^3)\ln(icx+1)}{70x^7}-\frac{60\ln(x)b\,c^7d^3x^7-30\ln(c^2x^2+1)b\,c^7d^3x^7-252\ln(x)b\,c^5d^2e\,x^7+126\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^6*(-1/c*d*e^2/x^3-3/5/c*d^2*e/x^5-e^3/c/x-1/7/c*d^3/x^7)-b/c^7*arc
tan(c*x)*d*e^2/x^3-3/5*b/c^7*arctan(c*x)*d^2*e/x^5-b/c^7*arctan(c*x)*e^3/x-
1/7*b*arctan(c*x)*d^3/c^7/x^7+1/14*b*ln(c^2*x^2+1)*d^3-3/10*b/c^2*ln(c^2*x^
2+1)*d^2*e+1/2*b/c^4*ln(c^2*x^2+1)*d*e^2-1/2*b/c^6*ln(c^2*x^2+1)*e^3-1/7*b*
d^3*ln(c*x)+3/5*b/c^2*ln(c*x)*d^2*e-b/c^4*ln(c*x)*d*e^2+b/c^6*ln(c*x)*e^3+1
/28*b*d^3/c^4/x^4-3/20*b/c^6*d^2*e/x^4-1/42*b*d^3/c^6/x^6-1/14*b*d^3/c^2/x^
2+3/10*b/c^4*d^2*e/x^2-1/2*b/c^6*d*e^2/x^2)
```

Maxima [A]
time = 0.26, size = 245, normalized size = 1.09

$$\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\,\arctan\left(cx\right)}{x^7}\right)bd^3-\frac{3}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd^2e+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)bde^2-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)be^3-\frac{ae^3}{x}-\frac{ade^2}{x^3}-\frac{3\,ad^2e}{5\,x^5}-\frac{ad^3}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 + 2
)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^3 - 3/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^
4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2*e + 1/2*((c^
2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d*e^2 -
 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e^3 - a*e^3/x -
a*d*e^2/x^3 - 3/5*a*d^2*e/x^5 - 1/7*a*d^3/x^7
```

Fricas [A]
time = 2.37, size = 260, normalized size = 1.16

$$-\frac{30\,bc^5d^3x^5-15\,bc^3d^3x^3+420\,ax^6e^3+10\,bcd^3x+60\,ad^3+12\left(35\,bx^6e^3+35\,bdx^4e^2+21\,bd^2x^2e+5\,bd^3\right)\arctan\left(cx\right)+210\left(bcdx^5+2\,adx^4\right)e^2-63\left(2\,bc^3d^2x^5-bcd^2x^3-4\,ad^2x^2\right)e-6\left(5\,bc^7d^3x^7-21\,bc^5d^2x^7e+35\,bc^3dx^7e^2-35\,bcx^7e^3\right)\log\left(c^2x^2+1\right)+12\left(5\,bc^7d^3x^7-21\,bc^5d^2x^7e+35\,bc^3dx^7e^2-35\,bcx^7e^3\right)\log\left(x\right)}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/420*(30*b*c^5*d^3*x^5 - 15*b*c^3*d^3*x^3 + 420*a*x^6*e^3 + 10*b*c*d^3*x
+ 60*a*d^3 + 12*(35*b*x^6*e^3 + 35*b*d*x^4*e^2 + 21*b*d^2*x^2*e + 5*b*d^3)*
arctan(c*x) + 210*(b*c*d*x^5 + 2*a*d*x^4)*e^2 - 63*(2*b*c^3*d^2*x^5 - b*c*d
^2*x^3 - 4*a*d^2*x^2)*e - 6*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^2*x^7*e + 35*b*c^
3*d*x^7*e^2 - 35*b*c*x^7*e^3)*log(c^2*x^2 + 1) + 12*(5*b*c^7*d^3*x^7 - 21*b
*c^5*d^2*x^7*e + 35*b*c^3*d*x^7*e^2 - 35*b*c*x^7*e^3)*log(x))/x^7
```

Sympy [A]
time = 0.92, size = 362, normalized size = 1.62

$$\begin{cases} -\frac{ad^3}{7x^7} - \frac{3ad^2e}{5x^5} - \frac{ade^2}{x^3} - \frac{ae^3}{x} - \frac{bc^7d^3\log(x)}{7} + \frac{bc^7d^3\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^3}{14x^2} + \frac{3bc^5d^2e\log(x)}{5} - \frac{3bc^5d^2e\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{28x^4} + \frac{3bc^3d^2e}{10x^2} - bc^3de^2\log(x) + \frac{bc^3de^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bcd^3}{42x^6} - \frac{3bcd^2e}{20x^4} - \frac{bcde^2}{2x^2} + bce^3\log(x) - \frac{bce^3\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{7x^7} - \frac{3bd^2e\operatorname{atan}(cx)}{5x^5} - \frac{bde^2\operatorname{atan}(cx)}{x^3} - \frac{be^3\operatorname{atan}(cx)}{x} & \text{for}\ c \neq 0 \\ a\left(-\frac{d^3}{7x^7} - \frac{3d^2e}{5x^5} - \frac{de^2}{x^3} - \frac{e^3}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*d**3/(7*x**7) - 3*a*d**2*e/(5*x**5) - a*d*e**2/x**3 - a*e**3/
x - b*c**7*d**3*log(x)/7 + b*c**7*d**3*log(x**2 + c**(-2))/14 - b*c**5*d**3
/(14*x**2) + 3*b*c**5*d**2*e*log(x)/5 - 3*b*c**5*d**2*e*log(x**2 + c**(-2))
/10 + b*c**3*d**3/(28*x**4) + 3*b*c**3*d**2*e/(10*x**2) - b*c**3*d*e**2*log
(x) + b*c**3*d*e**2*log(x**2 + c**(-2))/2 - b*c*d**3/(42*x**6) - 3*b*c*d**2
*e/(20*x**4) - b*c*d*e**2/(2*x**2) + b*c*e**3*log(x) - b*c*e**3*log(x**2 +
c**(-2))/2 - b*d**3*atan(c*x)/(7*x**7) - 3*b*d**2*e*atan(c*x)/(5*x**5) - b*
d*e**2*atan(c*x)/x**3 - b*e**3*atan(c*x)/x, Ne(c, 0)), (a*(-d**3/(7*x**7) -
 3*d**2*e/(5*x**5) - d*e**2/x**3 - e**3/x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.69, size = 236, normalized size = 1.05

$$\ln\left(c^2\,x^2+1\right)\,\left(\frac{b\,c^7\,d^3}{14}-\frac{3\,b\,c^5\,d^2\,e}{10}+\frac{b\,c^3\,d\,e^2}{2}-\frac{b\,c\,e^3}{2}\right)-\ln\left(x\right)\,\left(\frac{b\,c^7\,d^3}{7}-\frac{3\,b\,c^5\,d^2\,e}{5}+b\,c^3\,d\,e^2-b\,c\,e^3\right)-\frac{5\,a\,d^3-x^3\,\left(\frac{5\,b\,c^3\,d^3}{4}-\frac{21\,b\,c\,d^2\,e}{4}\right)+x^5\,\left(\frac{5\,b\,c^5\,d^3}{2}-\frac{21\,b\,c^3\,d^2\,e}{2}+\frac{35\,b\,c\,d\,e^2}{2}\right)+35\,a\,e^3\,x^6+\frac{5\,b\,c\,d^3\,x}{6}+21\,a\,d^2\,e\,x^2+35\,a\,d\,e^2\,x^4}{35\,x^7}-\frac{\mathrm{atan}\left(c\,x\right)\,\left(\frac{b\,d^3}{7}+\frac{3\,b\,d^2\,e\,x^2}{5}+b\,d\,e^2\,x^4+b\,e^3\,x^6\right)}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^8,x)
```

```
[Out] log(c^2*x^2 + 1)*((b*c^7*d^3)/14 - (b*c*e^3)/2 + (b*c^3*d*e^2)/2 - (3*b*c^5
*d^2*e)/10) - log(x)*((b*c^7*d^3)/7 - b*c*e^3 + b*c^3*d*e^2 - (3*b*c^5*d^2*
e)/5) - (5*a*d^3 - x^3*((5*b*c^3*d^3)/4 - (21*b*c*d^2*e)/4) + x^5*((5*b*c^5
*d^3)/2 + (35*b*c*d*e^2)/2 - (21*b*c^3*d^2*e)/2) + 35*a*e^3*x^6 + (5*b*c*d^
3*x)/6 + 21*a*d^2*e*x^2 + 35*a*d*e^2*x^4)/(35*x^7) - (atan(c*x)*((b*d^3)/7
+ b*e^3*x^6 + (3*b*d^2*e*x^2)/5 + b*d*e^2*x^4))/x^7
```

3.1149 $\int \frac{(d+ex^2)^3(a+b\text{ArcTan}(cx))}{x^9}\,dx$

Optimal. Leaf size=152

$$-\frac{bcd^3}{56x^7}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)\,(c^4d^2-2c^2de+2e^2)}{8x}+\frac{b(c^2d-e)^4\,\text{ArcTan}(}{8d}$$

```
[Out] -1/56*b*c*d^3/x^7+1/40*b*c*d^2*(c^2*d-4*e)/x^5-1/24*b*c*d*(c^4*d^2-4*c^2*d*
e+6*e^2)/x^3+1/8*b*c*(c^2*d-2*e)*(c^4*d^2-2*c^2*d*e+2*e^2)/x+1/8*b*(c^2*d-e
)^4*arctan(c*x)/d-1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/d/x^8
```

Rubi [A]
time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {270, 5096, 12, 472, 209}

$$-\frac{(d+ex^2)^4\,(a+b\text{ArcTan}(cx))}{8dx^8}+\frac{b\text{ArcTan}(cx)\,(c^2d-e)^4}{8d}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)\,(c^4d^2-2c^2de+2e^2)}{8x}-\frac{bcd^3}{56x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

```
[Out] -1/56*(b*c*d^3)/x^7 + (b*c*d^2*(c^2*d - 4*e))/(40*x^5) - (b*c*d*(c^4*d^2 -
4*c^2*d*e + 6*e^2))/(24*x^3) + (b*c*(c^2*d - 2*e)*(c^4*d^2 - 2*c^2*d*e + 2*
e^2))/(8*x) + (b*(c^2*d - e)^4*ArcTan[c*x])/(8*d) - ((d + e*x^2)^4*(a + b*A
rcTan[c*x]))/(8*d*x^8)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)}{x^9}\,dx &= -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-(bc)\int \frac{\left(d+ex^2\right)^4}{8x^8\left(-d-c^2dx^2\right)}\,dx \\
&= -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-\frac{1}{8}(bc)\int \frac{\left(d+ex^2\right)^4}{x^8\left(-d-c^2dx^2\right)}\,dx \\
&= -\frac{\left(d+ex^2\right)^4\left(a+b\tan^{-1}(cx)\right)}{8dx^8}-\frac{1}{8}(bc)\int \left(-\frac{d^3}{x^8}+\frac{d^2(c^2d-4e)}{x^6}-\frac{d(c^4 \\
&= -\frac{bcd^3}{56x^7}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)\,(c^4 \\
&= -\frac{bcd^3}{56x^7}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3}+\frac{bc(c^2d-2e)\,(c^4
\end{aligned}$$

Mathematica [A]
time = 3.01, size = 204, normalized size = 1.34

$$\frac{-105a(d^3+4d^2ex^2+6de^2x^4+4e^3x^6)+bcx(-420e^3x^6+210de^2x^4(-1+3c^2x^2)-28d^2ex^2(3-5c^2x^2+15c^4x^4)+d^3(-15+21c^2x^2-35c^4x^4+105c^6x^6))+105b(-4e^3x^6(1+c^2x^2)+6de^2x^4(-1+c^4x^4)-4d^2ex^2(1+c^6x^6)+d^3(-1+c^8x^8))\,\mathrm{ArcTan}(cx)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

```
[Out] (-105*a*(d^3 + 4*d^2*e*x^2 + 6*d*e^2*x^4 + 4*e^3*x^6) + b*c*x*(-420*e^3*x^6
 + 210*d*e^2*x^4*(-1 + 3*c^2*x^2) - 28*d^2*e*x^2*(3 - 5*c^2*x^2 + 15*c^4*x^
4) + d^3*(-15 + 21*c^2*x^2 - 35*c^4*x^4 + 105*c^6*x^6)) + 105*b*(-4*e^3*x^6
*(1 + c^2*x^2) + 6*d*e^2*x^4*(-1 + c^4*x^4) - 4*d^2*e*x^2*(1 + c^6*x^6) + d
^3*(-1 + c^8*x^8))*ArcTan[c*x])/(840*x^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(140) = 280$.
time = 0.43, size = 300, normalized size = 1.97

method	result
derivativedivides	$c^8\left(\frac{a\left(-\frac{3d\,e^2}{4c^2x^4}-\frac{d^3}{8c^2x^8}-\frac{d^2e}{2c^2x^6}-\frac{e^3}{2c^2x^2}\right)}{c^6}-\frac{3b\arctan(cx)d\,e^2}{4c^8x^4}-\frac{b\arctan(cx)d^3}{8c^8x^8}-\frac{b\arctan(cx)d^2e}{2c^8x^6}-\frac{b\arctan(cx)}{2c^8x^2}\right.$
default	$c^8\left(\frac{a\left(-\frac{3d\,e^2}{4c^2x^4}-\frac{d^3}{8c^2x^8}-\frac{d^2e}{2c^2x^6}-\frac{e^3}{2c^2x^2}\right)}{c^6}-\frac{3b\arctan(cx)d\,e^2}{4c^8x^4}-\frac{b\arctan(cx)d^3}{8c^8x^8}-\frac{b\arctan(cx)d^2e}{2c^8x^6}-\frac{b\arctan(cx)}{2c^8x^2}\right.$
risch	$\frac{ib\left(4e^3x^6+6e^2d\,x^4+4d^2e\,x^2+d^3\right)\ln(icx+1)}{16x^8}-\frac{840bc\,e^3x^7+168bc\,d^2e\,x^3+420bcd\,e^2x^5-280b\,c^3d^2e\,x^5-1260b\,c^3d\,e^2x^7+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] c^8*(a/c^6*(-3/4/c^2*d*e^2/x^4-1/8/c^2*d^3/x^8-1/2/c^2*d^2*e/x^6-1/2*e^3/c^
2/x^2)-3/4*b/c^8*arctan(c*x)*d*e^2/x^4-1/8*b*arctan(c*x)*d^3/c^8/x^8-1/2*b/
c^8*arctan(c*x)*d^2*e/x^6-1/2*b/c^8*arctan(c*x)*e^3/x^2+1/8*b*d^3*arctan(c*
x)-1/2*b*d^2*e*arctan(c*x)/c^2+3/4*b*d*e^2*arctan(c*x)/c^4-1/2*b*e^3*arctan
(c*x)/c^6+1/8*b*d^3/c/x-1/2*b/c^3*d^2*e/x+3/4*b/c^5*d*e^2/x-1/2*b/c^7*e^3/x
+1/40*b*d^3/c^5/x^5-1/10*b/c^7*d^2*e/x^5-1/56*b*d^3/c^7/x^7-1/24*b*d^3/c^3/
x^3+1/6*b/c^5*d^2*e/x^3-1/4*b/c^7*d*e^2/x^3)
```

Maxima [A]
time = 0.48, size = 216, normalized size = 1.42

$$\frac{1}{840}\left(\left(105\,c^7\arctan(cx)+\frac{105\,c^6x^6-35\,c^4x^4+21\,c^2x^2-15}{x^7}\right)c-\frac{105\arctan(cx)}{x^8}\right)bd^3-\frac{1}{30}\left(\left(15\,c^5\arctan(cx)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd^2e+\frac{1}{4}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bde^2-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)be^3-\frac{ae^3}{2\,x^2}-\frac{3\,ade^2}{4\,x^4}-\frac{ad^2e}{2\,x^6}-\frac{ad^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="maxima")
```

```
[Out] 1/840*((105*c^7*arctan(c*x) + (105*c^6*x^6 - 35*c^4*x^4 + 21*c^2*x^2 - 15)/
x^7)*c - 105*arctan(c*x)/x^8)*b*d^3 - 1/30*((15*c^5*arctan(c*x) + (15*c^4*x
^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^2*e + 1/4*((3*c^3*arct
an(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d*e^2 - 1/2*((c*arc
tan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^3 - 1/2*a*e^3/x^2 - 3/4*a*d*e^2/x^
4 - 1/2*a*d^2*e/x^6 - 1/8*a*d^3/x^8
```

Fricas [A]
time = 3.38, size = 237, normalized size = 1.56

$$\frac{105\,bc^7d^3x^7-35\,bc^5d^3x^5+21\,bc^3d^3x^3-15\,bcd^3x-105\,ad^3+105\,(bc^8d^3x^8-bd^3-4\,(bc^2x^8+bx^6)e^3+6\,(bc^4dx^8-bdx^4)e^2-4\,(bc^6d^2x^8+bd^2x^2)e)\arctan(cx)-420\,(bcx^7+ax^6)e^3+210\,(3\,bc^3dx^7-bcdx^5-3\,adx^4)e^2-28\,(15\,bc^5d^2x^7-5\,bc^3d^2x^5+3\,bcd^2x^3+15\,ad^2x^2)e}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b*c^7*d^3*x^7 - 35*b*c^5*d^3*x^5 + 21*b*c^3*d^3*x^3 - 15*b*c*d^3
*x - 105*a*d^3 + 105*(b*c^8*d^3*x^8 - b*d^3 - 4*(b*c^2*x^8 + b*x^6)*e^3 + 6
*(b*c^4*d*x^8 - b*d*x^4)*e^2 - 4*(b*c^6*d^2*x^8 + b*d^2*x^2)*e)*arctan(c*x)
 - 420*(b*c*x^7 + a*x^6)*e^3 + 210*(3*b*c^3*d*x^7 - b*c*d*x^5 - 3*a*d*x^4)*
e^2 - 28*(15*b*c^5*d^2*x^7 - 5*b*c^3*d^2*x^5 + 3*b*c*d^2*x^3 + 15*a*d^2*x^2
)*e)/x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(139) = 278$.
time = 0.66, size = 309, normalized size = 2.03

$$-\frac{ad^3}{8x^8} - \frac{ad^2e}{2x^6} - \frac{3ade^2}{4x^4} - \frac{ae^3}{2x^2} + \frac{bc^8d^3\operatorname{atan}(cx)}{8} + \frac{bc^7d^3}{8x} - \frac{bc^6d^2e\operatorname{atan}(cx)}{2} - \frac{bc^5d^3}{24x^3} - \frac{bc^5d^2e}{2x} + \frac{3bc^4de^2\operatorname{atan}(cx)}{4} + \frac{bc^3d^3}{40x^5} + \frac{bc^3d^2e}{6x^3} + \frac{3bc^3de^2}{4x} - \frac{bc^2e^3\operatorname{atan}(cx)}{2} - \frac{bcd^3}{56x^7} - \frac{bcd^2e}{10x^5} - \frac{bcde^2}{4x^3} - \frac{bce^3}{2x} - \frac{bd^3\operatorname{atan}(cx)}{8x^8} - \frac{bd^2e\operatorname{atan}(cx)}{2x^6} - \frac{3bde^2\operatorname{atan}(cx)}{4x^4} - \frac{be^3\operatorname{atan}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**9,x)
```

```
[Out] -a*d**3/(8*x**8) - a*d**2*e/(2*x**6) - 3*a*d*e**2/(4*x**4) - a*e**3/(2*x**2
) + b*c**8*d**3*atan(c*x)/8 + b*c**7*d**3/(8*x) - b*c**6*d**2*e*atan(c*x)/2
 - b*c**5*d**3/(24*x**3) - b*c**5*d**2*e/(2*x) + 3*b*c**4*d*e**2*atan(c*x)/
4 + b*c**3*d**3/(40*x**5) + b*c**3*d**2*e/(6*x**3) + 3*b*c**3*d*e**2/(4*x)
- b*c**2*e**3*atan(c*x)/2 - b*c*d**3/(56*x**7) - b*c*d**2*e/(10*x**5) - b*c
*d*e**2/(4*x**3) - b*c*e**3/(2*x) - b*d**3*atan(c*x)/(8*x**8) - b*d**2*e*at
an(c*x)/(2*x**6) - 3*b*d*e**2*atan(c*x)/(4*x**4) - b*e**3*atan(c*x)/(2*x**2
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.63, size = 301, normalized size = 1.98

$$\frac{b\,c^2\,\mathrm{atan}\left(\frac{b\,c^2\,x\,\left(2\,e-c^2\,d\right)\,\left(c^4\,d^2-2\,c^2\,d\,e+2\,e^2\right)}{b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3}\right)\,\left(2\,e-c^2\,d\right)\,\left(c^4\,d^2-2\,c^2\,d\,e+2\,e^2\right)}{8}-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{8}+\frac{b\,d^2\,e\,x^2}{2}+\frac{3\,b\,d\,e^2\,x^4}{4}+\frac{b\,e^3\,x^6}{2}\right)}{x^8}-\frac{a\,d^3-x^3\,\left(\frac{b\,c^3\,d^3}{5}-\frac{4\,b\,c\,d^2\,e}{5}\right)-x^7\,\left(b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3\right)+x^5\,\left(\frac{b\,c^5\,d^3}{3}-\frac{4\,b\,c^3\,d^2\,e}{3}+2\,b\,c\,d\,e^2\right)+4\,a\,e^3\,x^6+\frac{b\,c\,d^3\,x}{7}+4\,a\,d^2\,e\,x^2+6\,a\,d\,e^2\,x^4}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^9,x)
```

```
[Out] (b*c^2*atan((b*c^2*x*(2*e - c^2*d)*(2*e^2 + c^4*d^2 - 2*c^2*d*e))/(b*c^7*d^
3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e))*(2*e - c^2*d)*(2*e^2 + c^4*
```

```
d^2 - 2*c^2*d*e))/8 - (atan(c*x)*((b*d^3)/8 + (b*e^3*x^6)/2 + (b*d^2*e*x^2)
/2 + (3*b*d*e^2*x^4)/4))/x^8 - (a*d^3 - x^3*((b*c^3*d^3)/5 - (4*b*c*d^2*e)/
5) - x^7*(b*c^7*d^3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e) + x^5*((b*
c^5*d^3)/3 + 2*b*c*d*e^2 - (4*b*c^3*d^2*e)/3) + 4*a*e^3*x^6 + (b*c*d^3*x)/7
 + 4*a*d^2*e*x^2 + 6*a*d*e^2*x^4)/(8*x^8)
```

3.1150 $\int (c+dx^2)^4 \operatorname{ArcTan}(ax)\,dx$

Optimal. Leaf size=244

$$-\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}-\frac{d^2(378a^4c^2-180a^2cd+35d^2)\,x^4}{1260a^5}-\frac{(36a^2c-7d)\,d^3x^6}{378a^3}-\frac{d^4x^8}{72a}$$

```
[Out] -1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7-1/1260*d^
2*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5-1/378*(36*a^2*c-7*d)*d^3*x^6/a^3
-1/72*d^4*x^8/a+c^4*x*arctan(a*x)+4/3*c^3*d*x^3*arctan(a*x)+6/5*c^2*d^2*x^5
*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+1/9*d^4*x^9*arctan(a*x)-1/630*(315*a
^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1)/a^
9
```

Rubi [A]
time = 0.13, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 5032, 1824, 266}

$$-\frac{d^3x^6(36a^2c-7d)}{378a^3}-\frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{1260a^5}-\frac{dx^2(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)}{630a^7}-\frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log(a^2x^2+1)}{630a^9}+c^4x\operatorname{ArcTan}(ax)+\frac{4}{3}c^3dx^3\operatorname{ArcTan}(ax)+\frac{6}{5}c^2d^2x^5\operatorname{ArcTan}(ax)+\frac{4}{7}cd^3x^7\operatorname{ArcTan}(ax)+\frac{1}{9}d^4x^9\operatorname{ArcTan}(ax)-\frac{d^4x^8}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcTan[a*x],x]
```

```
[Out] -1/630*(d*(420*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/a^7 -
 (d^2*(378*a^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) - ((36*a^2*c - 7
*d)*d^3*x^6)/(378*a^3) - (d^4*x^8)/(72*a) + c^4*x*ArcTan[a*x] + (4*c^3*d*x^
3*ArcTan[a*x])/3 + (6*c^2*d^2*x^5*ArcTan[a*x])/5 + (4*c*d^3*x^7*ArcTan[a*x]
)/7 + (d^4*x^9*ArcTan[a*x])/9 - ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2
*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(630*a^9)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \tan^{-1}(ax)\,dx = c^4 x\tan^{-1}(ax)+\frac{4}{3}c^3dx^3\tan^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tan^{-1}(ax)+\frac{4}{7}cd^3x^7\tan^{-1}(ax)$$

$$= c^4 x\tan^{-1}(ax)+\frac{4}{3}c^3dx^3\tan^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tan^{-1}(ax)+\frac{4}{7}cd^3x^7\tan^{-1}(ax)$$

$$= -\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}-\frac{d^2(378a^4c^2-180a^2cd+35}{1260a^5}$$

$$= -\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}-\frac{d^2(378a^4c^2-180a^2cd+35}{1260a^5}$$

Mathematica [A]
time = 0.07, size = 212, normalized size = 0.87

$$-\frac{a^2dx^2(-420d^3+30a^2d^2(72c+7dx^2)-4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240cd^2x^4+35d^3x^6))-24a^9x(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8)\,\mathrm{ArcTan}(ax)+12(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log(1+a^2x^2)}{7560a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcTan[a*x],x]
```

```
[Out] -1/7560*(a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134*
c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d
^2*x^4 + 35*d^3*x^6)) - 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4
 + 180*c*d^3*x^6 + 35*d^4*x^8)*ArcTan[a*x] + 12*(315*a^8*c^4 - 420*a^6*c^3*
d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^9
```

Maple [A]
time = 0.28, size = 254, normalized size = 1.04

method	result
derivativedivides	$\frac{\arctan(ax)c^4ax+\frac{4a\arctan(ax)c^3d\,x^3}{3}+\frac{6a\arctan(ax)c^2d^2x^5}{5}+\frac{4a\arctan(ax)c\,d^3x^7}{7}+\frac{a\arctan(ax)d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2a^8}{2}}{}}{}$
default	$\frac{\arctan(ax)c^4ax+\frac{4a\arctan(ax)c^3d\,x^3}{3}+\frac{6a\arctan(ax)c^2d^2x^5}{5}+\frac{4a\arctan(ax)c\,d^3x^7}{7}+\frac{a\arctan(ax)d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2a^8}{2}}{}}{}$

meijerg	$\frac{d^4\left(\frac{x^2a^2\left(-15a^6x^6+20x^4a^4-30a^2x^2+60\right)}{270}+\frac{4x^{10}a^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{9}\right)}{4a^9}+\frac{d^3c\left(-\frac{a^2x^2\left(4x^4a^4-6a^2x^2+12\right.}{42}\right.}{}$
risch	$\frac{i\ln(-iax+1)c^4x}{2}+\frac{2i\ln(-iax+1)d\,c^3x^3}{3}+\frac{3i\ln(-iax+1)c^2d^2x^5}{5}-\frac{d^4x^8}{72a}+\frac{2i\ln(-iax+1)d^3c\,x^7}{7}-\frac{2c\,d^3x^6}{21a}+\frac{i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arctan(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arctan(a*x)*c^4*a*x+4/3*a*arctan(a*x)*c^3*d*x^3+6/5*a*arctan(a*x)*c^2*
d^2*x^5+4/7*a*arctan(a*x)*c*d^3*x^7+1/9*a*arctan(a*x)*d^4*x^9-1/315/a^8*(21
0*c^3*a^8*d*x^2+189/2*c^2*a^8*d^2*x^4-189*c^2*a^6*d^2*x^2+30*c*a^8*d^3*x^6-
45*a^6*c*d^3*x^4+35/8*d^4*a^8*x^8+90*a^4*c*d^3*x^2-35/6*d^4*a^6*x^6+35/4*d^
4*a^4*x^4-35/2*d^4*a^2*x^2+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-1
80*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.26, size = 226, normalized size = 0.93

$$-\frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3-7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2-180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d-378\,a^4c^2d^2+180\,a^2cd^3-35\,d^4)x^2}{a^8}+\frac{12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(a^2x^2+1\right)}{a^{10}}\right)+\frac{1}{315}\,(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x)\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="maxima")
```

```
[Out] -1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a^
6*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4*c
^2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
 + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1/315
*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)
*arctan(a*x)
```

Fricas [A]
time = 4.29, size = 237, normalized size = 0.97

$$-\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3-7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2-180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d-378\,a^6c^2d^2+180\,a^4cd^3-35\,a^2d^4)x^2-24\,(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x)\arctan\left(ax\right)+12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(a^2x^2+1\right)}{7560\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="fricas")
```

```
[Out] -1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*c
^2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^2*
d^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 - 24*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*
x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arctan(a*x)
+ 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^
4)*log(a^2*x^2 + 1))/a^9
```

Sympy [A]
time = 0.68, size = 314, normalized size = 1.29

$$\begin{cases} c^4x\operatorname{atan}(ax)+\frac{4c^3dx^3\operatorname{atan}(ax)}{3}+\frac{6c^2d^2x^5\operatorname{atan}(ax)}{5}+\frac{4cd^3x^7\operatorname{atan}(ax)}{7}+\frac{d^4x^9\operatorname{atan}(ax)}{9}-\frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a}-\frac{2c^3dx^2}{3a}-\frac{3c^2d^2x^4}{10a}-\frac{2cd^3x^6}{21a}-\frac{d^4x^8}{72a}+\frac{2c^3d\log\left(x^2+\frac{1}{a^2}\right)}{3a^3}+\frac{3c^2d^2x^2}{5a^3}+\frac{cd^3x^4}{7a^3}+\frac{d^4x^6}{54a^3}-\frac{3c^2d^2\log\left(x^2+\frac{1}{a^2}\right)}{5a^5}-\frac{2cd^3x^2}{7a^5}-\frac{d^4x^4}{36a^5}+\frac{2cd^3\log\left(x^2+\frac{1}{a^2}\right)}{7a^7}+\frac{d^4x^2}{18a^7}-\frac{d^4\log\left(x^2+\frac{1}{a^2}\right)}{18a^9} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*atan(a*x),x)
```

```
[Out] Piecewise((c**4*x*atan(a*x) + 4*c**3*d*x**3*atan(a*x)/3 + 6*c**2*d**2*x**5*
atan(a*x)/5 + 4*c*d**3*x**7*atan(a*x)/7 + d**4*x**9*atan(a*x)/9 - c**4*log(
x**2 + a**(-2))/(2*a) - 2*c**3*d*x**2/(3*a) - 3*c**2*d**2*x**4/(10*a) - 2*c
*d**3*x**6/(21*a) - d**4*x**8/(72*a) + 2*c**3*d*log(x**2 + a**(-2))/(3*a**3
) + 3*c**2*d**2*x**2/(5*a**3) + c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a**3)
- 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) - 2*c*d**3*x**2/(7*a**5) - d**4*
x**4/(36*a**5) + 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) + d**4*x**2/(18*a**7
) - d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.20, size = 233, normalized size = 0.95

$$\operatorname{atan}(a\,x)\,\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)+x^2\,\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)+x^6\,\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)-x^4\,\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)-\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}-\frac{d^4\,x^8}{72\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)*(c + d*x^2)^4,x)
```

```
[Out] atan(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^2
*d^2*x^5)/5) + x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a
))/(2*a^2) - (2*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) - x^4
*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) - (log(a^2*
x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a^4*c^
2*d^2))/(630*a^9) - (d^4*x^8)/(72*a)
```

3.1151 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=361

$$-\frac{bx}{2ce}+\frac{b\text{ArcTan}(cx)}{2c^2e}+\frac{x^2(a+b\text{ArcTan}(cx))}{2e}+\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2}{(c\ldots}\right.}{2e^2}$$

```
[Out] -1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e+1/2*x^2*(a+b*arctan(c*x))/e+d*(a+b*arc
tan(c*x))*ln(2/(1-I*c*x))/e^2-1/2*d*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/2*d*(a+b*arctan(c*x))*ln
(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-1/2*I*b
*d*polylog(2,1-2/(1-I*c*x))/e^2+1/4*I*b*d*polylog(2,1-2*c*((-d)^(1/2)-x*e^(
1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*I*b*d*polylog(2,1-2*c*((-
d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2
```

Rubi [A]

time = 0.29, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5036, 4946, 327, 209, 5100, 4966, 2449, 2352, 2497}

$$\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^2}-\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^2}-\frac{d(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^2}+\frac{x^2(a+b\text{ArcTan}(cx))}{2e}+\frac{b\text{ArcTan}(cx)}{2c^2e}-\frac{ibd\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e^2}+\frac{ibd\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4e^2}-\frac{bx}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/2*(b*x)/(c*e) + (b*ArcTan[c*x])/(2*c^2*e) + (x^2*(a + b*ArcTan[c*x]))/(2
*e) + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x
])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]
)/(2*e^2) - (d*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqr
t[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1
- I*c*x)])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 + ((I/4)*b*d*PolyLog[2, 1 - (2*c*
(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{d+ex^2}\,dx = \frac{\int x(a+b\tan^{-1}(cx))\,dx}{e} - \frac{d\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2(a+b\tan^{-1}(cx))}{2e} - \frac{(bc)\int \frac{x^2}{1+c^2x^2}\,dx}{2e} - \frac{d\int \left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{a+b}{2\sqrt{e}\,(\sqrt{}}\right.}{e}$$

$$= -\frac{bx}{2ce} + \frac{x^2(a+b\tan^{-1}(cx))}{2e} + \frac{d\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2e^{3/2}} + \frac{b\int}{}$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2(a+b\tan^{-1}(cx))}{2e} + \frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} -$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2(a+b\tan^{-1}(cx))}{2e} + \frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} -$$

$$= -\frac{bx}{2ce} + \frac{b\tan^{-1}(cx)}{2c^2e} + \frac{x^2(a+b\tan^{-1}(cx))}{2e} + \frac{d(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} -$$

Mathematica [A]
time = 0.19, size = 503, normalized size = 1.39

$$-\frac{bx}{2ce}+\frac{ax^2}{2e}+\frac{b\text{ArcTan}(cx)}{2c^2e}+\frac{bx^2\text{ArcTan}(cx)}{2e}+\frac{ibd\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^2}-\frac{ibd\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^2}-\frac{ibd\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^2}+\frac{ibd\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^2}-\frac{ad\log\left(d+ex^2\right)}{2e^2}-\frac{ibd\text{PolyLog}\left(2,-\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e^2}-\frac{ibd\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^2}+\frac{ibd\text{PolyLog}\left(2,-\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e^2}+\frac{ibd\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/2*(b*x)/(c*e) + (a*x^2)/(2*e) + (b*ArcTan[c*x])/(2*c^2*e) + (b*x^2*ArcTa
n[c*x])/(2*e) + ((I/4)*b*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c
```

```
*Sqrt[-d] - I*Sqrt[e])])/e^2 - ((I/4)*b*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] -
 Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^2 - ((I/4)*b*d*Log[1 - I*c*x]*Log
[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/e^2 + ((I/4)*b*d*Log
[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^2 -
 (a*d*Log[d + e*x^2])/(2*e^2) - ((I/4)*b*d*PolyLog[2, -((Sqrt[e]*(1 - I*c*x
))/(I*c*Sqrt[-d] - Sqrt[e]))])/e^2 - ((I/4)*b*d*PolyLog[2, (Sqrt[e]*(1 - I*
c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e^2 + ((I/4)*b*d*PolyLog[2, -((Sqrt[e]*(1
+ I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e^2 + ((I/4)*b*d*PolyLog[2, (Sqrt[e]*
(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e^2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 749, normalized size = 2.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a*c^4/e*x^2-1/2*a*c^4*d/e^2*ln(c^2*e*x^2+c^2*d)+1/2*b*c^4*arctan
(c*x)/e*x^2-1/2*b*c^4*arctan(c*x)*d/e^2*ln(c^2*e*x^2+c^2*d)-1/2*b*c^3/e*x+1
/2*b*c^2/e*arctan(c*x)+1/4*I*b*c^4*d/e^2*ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-1/4*
I*b*c^4*d/e^2*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/
RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b*c^4*d/e^2*ln(c*x-I)*ln((Ro
otOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e
,index=2))-1/4*I*b*c^4*d/e^2*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b*c^4*d/e^2*d
ilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z
+c^2*d-e,index=2))-1/4*I*b*c^4*d/e^2*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,
index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b*c^4*d/e^2*
dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=2))+1/4*I*b*c^4*d/e^2*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+
c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))-1/4*I*b*c^
4*d/e^2*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)+1/4*I*b*c^4*d/e^2*dilog((RootOf(e*_Z^
2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))
)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e^(-1) - d*e^(-2)*log(x^2*e + d))*a + 2*b*integrate(1/2*x^3*arctan
(c*x)/(x^2*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{atan}\,(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1152 $\int \frac{x(a+b\text{ArcTan}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=311

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{(a+b\text{ArcTan}(cx))\log}{}$$

[Out] -(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e

Rubi [A]
time = 0.19, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5100, 4966, 2449, 2352, 2497}

$$\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e}-\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e}-\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4e}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]

[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx = \int \left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{e}}+\frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{e}}$$

$$= -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}$$

$$= -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}$$

$$= -\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}$$

Mathematica [A]
time = 0.09, size = 441, normalized size = 1.42

$$-\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e}+\frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e}+\frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e}-\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e}+\frac{a\log\left(d+ex^2\right)}{2e}+\frac{ib\text{PolyLog}\left(2,-\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e}+\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,-\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e}-\frac{ib\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] ((-1/4*I)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*S
qrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqr
t[-d] + I*Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]
*x))/(c*Sqrt[-d] - I*Sqrt[e])])/e - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d
] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e + (a*Log[d + e*x^2])/(2*e) + (
(I/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e +
((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e - ((
I/4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))])/e - (
(I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 686, normalized size = 2.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a*c^2/e*ln(c^2*e*x^2+c^2*d)+1/2*b*c^2/e*ln(c^2*e*x^2+c^2*d)*arct
an(c*x)-1/4*I*b*c^2/e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)
-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))-1/4*I*b*c^2/e*ln(c*x-I)*ln
((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2
*d-e,index=2))+1/4*I*b*c^2/e*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/4*I*b*c^2/e*di
log((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+
c^2*d-e,index=1))-1/4*I*b*c^2/e*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index
=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b*c^2/e*ln(I+c*x)
*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+
c^2*d-e,index=1))+1/4*I*b*c^2/e*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-
e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b*c^2/e*ln
(I+c*x)*ln(c^2*e*x^2+c^2*d)+1/4*I*b*c^2/e*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2
*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b*c^2/e
*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*
_Z+c^2*d-e,index=2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(-1)*log(x^2*e + d) + 2*b*integrate(1/2*x*arctan(c*x)/(x^2*e + d),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctan(c*x) + a*x)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{atan}\,(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a + b\,\mathrm{atan}(c\,x))}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1153 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=353

$$\frac{a\log(x)}{d}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d}-\frac{(a+b\text{ArcTan}($$

```
[Out] a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-1/2*(a+b*arctan(c*x))*ln(2*c*
((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d-1/2*(a+b*arcta
n(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d
+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,I*c*x)/d-1/2*I*b*polylog(2,1
-2/(1-I*c*x))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c
*(-d)^(1/2)-I*e^(1/2)))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1
-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d
```

Rubi [A]
time = 0.28, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5048, 4940, 2438, 5100, 4966, 2449, 2352, 2497}

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4d}+\frac{ib\text{Li}_2(-icx)}{2d}-\frac{ib\text{Li}_2(icx)}{2d}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[
c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d -
 ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
+ ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqr
t[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ex(a+b\tan^{-1}(cx))}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d}$$

$$= \frac{a\log(x)}{d} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d} - \frac{(ib)\int \frac{\log(1+icx)}{x}\,dx}{2d} - \frac{e\int\left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \cdots\right.}{d}$$

$$= \frac{a\log(x)}{d} + \frac{ib\text{Li}_2(-icx)}{2d} - \frac{ib\text{Li}_2(icx)}{2d} + \frac{\sqrt{e}\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2d} - \frac{\sqrt{e}\int \frac{a+b\tan^{-1}(c\cdots}{\sqrt{-d}+\sqrt{e}\cdots}}{2d}$$

$$= \frac{a\log(x)}{d} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\cdots\right.}{\left(c\sqrt{-d}-i\sqrt{e}\right)(\cdots}\right.}{2d}$$

$$= \frac{a\log(x)}{d} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\cdots\right.}{\left(c\sqrt{-d}-i\sqrt{e}\right)(\cdots}\right.}{2d}$$

$$= \frac{a\log(x)}{d} + \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\cdots\right.}{\left(c\sqrt{-d}-i\sqrt{e}\right)(\cdots}\right.}{2d}$$

Mathematica [A]
time = 1.18, size = 384, normalized size = 1.09

$$-\frac{-2a\log(x)+a\log\left(d+ex^2\right)+b\left(-i\text{ArcTan}(cx)^2+2i\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)\text{ArcTan}\left(\frac{cex}{\sqrt{c^2de}}\right)-2\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)+\left(-\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)+\text{ArcTan}(cx)\right)\log\left(1+\frac{\left(c^2d+e+2\sqrt{c^2de}\right)e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)+\left(\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)+\text{ArcTan}(cx)\right)\log\left(\frac{-2\sqrt{c^2de}\,e^{2i\text{ArcTan}(cx)}+e\left(-1+e^{2i\text{ArcTan}(cx)}\right)+c^2d\left(1+e^{2i\text{ArcTan}(cx)}\right)}{c^2d-e}\right)+i\left(\text{ArcTan}(cx)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{\left(-c^2d-e+2\sqrt{c^2de}\right)e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)+\text{PolyLog}\left(2,-\frac{\left(c^2d+e+2\sqrt{c^2de}\right)e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(-2*a*Log[x] + a*Log[d + e*x^2] + b*((-I)*ArcTan[c*x]^2 + (2*I)*ArcSin
[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] - 2*ArcTan[c*x]*L
og[1 - E^((2*I)*ArcTan[c*x])] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTa
n[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*
d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^
2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 +
E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*
I)*ArcTan[c*x])]) - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^(
(2*I)*ArcTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*
e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 736, normalized size = 2.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*ln(c*x)-1/2*b*arctan(c*x)/d*ln(c^2*e*x^2+c
^2*d)+b*arctan(c*x)/d*ln(c*x)+1/4*I*b/d*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b/
d*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e
*_Z+c^2*d-e,index=2))+1/4*I*b/d*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-
e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/2*I*b/d*ln(c*x
)*ln(1+I*c*x)+1/4*I*b/d*ln(c^2*e*x^2+c^2*d)*ln(I+c*x)-1/4*I*b/d*ln(I+c*x)*l
n((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^
2*d-e,index=1))-1/2*I*b/d*dilog(1-I*c*x)-1/2*I*b/d*ln(c*x)*ln(1-I*c*x)-1/4*
I*b/d*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2
*I*e*_Z+c^2*d-e,index=1))-1/4*I*b/d*ln(c^2*e*x^2+c^2*d)*ln(c*x-I)-1/4*I*b/d
*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2
-2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b/d*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e
,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b/d*dilog((
RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d
-e,index=1))+1/2*I*b/d*dilog(1+I*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(x^2*e + d)/d - 2*log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(x^3
*e + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^3*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{x \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{x\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)), x)
```

3.1154 $\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=409

$$-\frac{bc}{2dx}-\frac{bc^2\text{ArcTan}(cx)}{2d}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}-\frac{ae\log(x)}{d^2}-\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}+\frac{e(a+b\text{ArcTan}(cx)}{}$$

```
[Out] -1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2-a*e*ln(x)
/d^2-e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2+1/2*e*(a+b*arctan(c*x))*ln(2*c
*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2+1/2*e*(a+b*
arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2
)))/d^2-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog(2,I*c*x)/d^2+1/2*
I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/4*I*b*e*polylog(2,1-2*c*((-d)^(1/2)-x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/4*I*b*e*polylog(2,1-2*c*
((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2
```

Rubi [A]
time = 0.35, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5038, 4946, 331, 209, 5048, 4940, 2438, 5100, 4966, 2449, 2352, 2497}

$$-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^2}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^2}-\frac{a+b\text{ArcTan}(cx)}{2dx^2}-\frac{ae\log(x)}{d^2}-\frac{bc^2\text{ArcTan}(cx)}{2d}-\frac{ibe\text{Li}_2(-icx)}{2d^2}+\frac{ibe\text{Li}_2(icx)}{2d^2}+\frac{ibe\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2}-\frac{ibe\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}-\frac{bc}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*(b*c)/(d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^2
) - (a*e*Log[x])/d^2 - (e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*
(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[
e])*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) - ((I/2)*b*e*Pol
yLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2 + ((I/2)*b*e*Pol
yLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d]
- Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/4)*b*e*Pol
yLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c
*x))])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3(d+ex^2)}\,dx = \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x(d+ex^2)}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2dx^2} + \frac{(bc)\int \frac{1}{x^2(1+c^2x^2)}\,dx}{2d} - \frac{e\int \left(\frac{a+b\tan^{-1}(cx)}{dx} - \frac{ex(a+b\tan^{-1}(cx))}{d(d+ex^2)}\right)dx}{d}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{(bc^3)\int \frac{1}{1+c^2x^2}\,dx}{2d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{x(a+b\tan^{-1}}{d+ex^2}}{d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{(ibe)\int \frac{\log(1-icx)}{x}\,dx}{2d^2} + \frac{(ibe}{}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{ibe\text{Li}_2(-icx)}{2d^2} + \frac{ibe\text{Li}_2(ic}{2d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{}{1-}\right.}{d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{}{1-}\right.}{d^2}$$

$$= -\frac{bc}{2dx} - \frac{bc^2\tan^{-1}(cx)}{2d} - \frac{a+b\tan^{-1}(cx)}{2dx^2} - \frac{ae\log(x)}{d^2} - \frac{e(a+b\tan^{-1}(cx))\log\left(\frac{}{1-}\right.}{d^2}$$

Mathematica [A]
time = 1.86, size = 446, normalized size = 1.09

$$-\frac{\frac{ad}{x^2}+\frac{bcd}{x}+\frac{bd(1+c^2x^2)\text{ArcTan}(cx)}{x^2}+2ae\log(x)-ae\log(d+ex^2)-ibe\left(\text{ArcTan}(cx)\left(\text{ArcTan}(cx)+2i\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)-\frac{2b(c^2d-e)e\left(-i\text{ArcTan}(cx)^2+2i\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)\text{ArcTan}\left(\frac{cex}{\sqrt{c^2de}}\right)+\left(-\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)+\text{ArcTan}(cx)\right)\log\left(1+\frac{(c^2d+e+2\sqrt{c^2de})e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)+\left(\text{ArcSin}\left(\sqrt{\frac{c^2d}{c^2d-e}}\right)+\text{ArcTan}(cx)\right)\log\left(\frac{-2\sqrt{c^2de}\,e^{2i\text{ArcTan}(cx)}+e\left(-1+e^{2i\text{ArcTan}(cx)}\right)+c^2d\left(1+e^{2i\text{ArcTan}(cx)}\right)}{c^2d-e}\right)-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{(-c^2d-e+2\sqrt{c^2de})e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)+\text{PolyLog}\left(2,-\frac{(c^2d+e+2\sqrt{c^2de})e^{2i\text{ArcTan}(cx)}}{c^2d-e}\right)\right)\right)}{2c^2d-2e}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*((a*d)/x^2 + (b*c*d)/x + (b*d*(1 + c^2*x^2)*ArcTan[c*x])/x^2 + 2*a*e*L
og[x] - a*e*Log[d + e*x^2] - I*b*e*(ArcTan[c*x]*(ArcTan[c*x] + (2*I)*Log[1
- E^((2*I)*ArcTan[c*x])]) + PolyLog[2, E^((2*I)*ArcTan[c*x])]) - (2*b*(c^2*
d - e)*e*((-I)*ArcTan[c*x]^2 + (2*I)*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcT
an[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*
x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d -
```

```
e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*
e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((
2*I)*ArcTan[c*x])))/(c^2*d - e)] - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqr
t[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e
+ 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])))/(2*c^2*d - 2*e))
/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 864, normalized size = 2.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/c^2*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*a/d/c^2/x^2-a/c^2/d^2*e*ln(c*x
)+1/2*b/c^2*arctan(c*x)*e/d^2*ln(c^2*e*x^2+c^2*d)-1/2*b*arctan(c*x)/d/c^2/x
^2-b/c^2*arctan(c*x)/d^2*e*ln(c*x)+1/2*I*b/c^2/d^2*e*ln(c*x)*ln(1-I*c*x)+1/
4*I*b/c^2/d^2*e*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))-1/2*I*b/c^2/d^2*e*ln(c*x)*ln(1+I*c*x)+1
/4*I*b/c^2/d^2*e*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-
I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/2*I*b/c^2/d^2*e*dilog(1-I*c*x
)+1/4*I*b/c^2/d^2*e*ln(c^2*e*x^2+c^2*d)*ln(c*x-I)-1/4*I*b/c^2/d^2*e*ln(c*x-
I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=1))-1/4*I*b/c^2/d^2*e*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e
,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b/c^2/d^2*e
*ln(c^2*e*x^2+c^2*d)*ln(I+c*x)-1/4*I*b/c^2/d^2*e*dilog((RootOf(e*_Z^2+2*I*e
*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))-1/2*b/
d*arctan(c*x)-1/2*b/d/c/x-1/2*I*b/c^2/d^2*e*dilog(1+I*c*x)+1/4*I*b/c^2/d^2*
e*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e
*_Z+c^2*d-e,index=2))+1/4*I*b/c^2/d^2*e*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_
Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b/
c^2/d^2*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/Root
Of(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*log(x^2*e + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + 2*b*integrate(1
/2*arctan(c*x)/(x^5*e + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^5*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atan}(cx)}{x^3 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{atan}(c\,x)}{x^3\,\left(e\,x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)), x)
```

3.1155 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=555

$$\frac{ax}{e}+\frac{bx\text{ArcTan}(cx)}{e}-\frac{a\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}}-\frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\,\log(1-$$

```
[Out] a*x/e+b*x*arctan(c*x)/e-1/2*b*ln(c^2*x^2+1)/c/e-1/4*I*b*ln(1+I*c*x)*ln(c*((
-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*l
n(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)
/e^(3/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^
(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/
(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,(I-c*x)*e^(1
/2)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,(c*x+I)*
e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/4*I*b*polylog(2,(1-I
*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/4*I*b*polylog(
2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*(-d)^(1/2)/e^(3/2)-a*arctan(x
*e^(1/2)/d^(1/2))*d^(1/2)/e^(3/2)
```

Rubi [A]
time = 0.49, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5036, 4930, 266, 5030, 211, 5028, 2456, 2441, 2440, 2438}

$$-\frac{a\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}}+\frac{ax}{e}+\frac{bx\text{ArcTan}(cx)}{e}-\frac{b\log\left(c^2x^2+1\right)}{2ce}+\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,(i-cx)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,(1-icx)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,(icx+1)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{\sqrt{e}\,(cx+i)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}-\frac{ib\sqrt{-d}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}}+\frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e + (b*x*ArcTan[c*x])/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(
3/2) - ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 - I*c*x]*Log[(c*(S
qrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - ((I/4)*b*Sqrt[-d
]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/
e^(3/2) + ((I/4)*b*Sqrt[-d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(
c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2) - (b*Log[1 + c^2*x^2])/(2*c*e) + ((I/4)*b
*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
 - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])])/e^(3/2) - ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqr
t[-d] + Sqrt[e])])/e^(3/2) + ((I/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*(I + c*x
))/(c*Sqrt[-d] + I*Sqrt[e])])/e^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{d+ex^2}\,dx = \frac{\int (a+b\tan^{-1}(cx))\,dx}{e} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{b\int \tan^{-1}(cx)\,dx}{e} - \frac{(ad)\int \frac{1}{d+ex^2}\,dx}{e} - \frac{(bd)\int \frac{\tan^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{(bc)\int \frac{x}{1+c^2x^2}\,dx}{e} - \frac{(ibd)\int \frac{\log(1-icx}{d+ex^2}}{2e}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{b\log(1+c^2x^2)}{2ce} - \frac{(ibd)\int\left(\frac{\sqrt{-}}{2d(\sqrt{}}\right.}{}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{b\log(1+c^2x^2)}{2ce} - \frac{\left(ib\sqrt{-d}\right)\int -}{4}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}\right.}{c\sqrt{-d}}\right.}{4e^{3/2}}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}\right.}{c\sqrt{-d}}\right.}{4e^{3/2}}$$

$$= \frac{ax}{e} + \frac{bx\tan^{-1}(cx)}{e} - \frac{a\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}} - \frac{ib\sqrt{-d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}\right.}{c\sqrt{-d}}\right.}{4e^{3/2}}$$

Mathematica [A]
time = 2.35, size = 766, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (b*(4*c*x*ArcTa
n[c*x] - 2*Log[1 + c^2*x^2] + (c^2*d*(-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(
c^2*d*e)]*x)] - 2*ArcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqrt[-
(c^2*d*e)]] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*x)/S
qrt[-(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d -
 e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] +
 (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((-I)*e + Sqrt[-(c^2*d
*e)])*(I + c*x))/((c^2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] + (ArcCos[(c^
2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + (2*I
)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-
(c^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*Arc
Tan[c*x]]])] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*d)/(S
qrt[-(c^2*d*e)]*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]
*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e
+ (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c
^2*d*e)]*x))] + PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c*d + Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c^2*d*e)]*x))])))/Sqrt[-(c^2*d
*e)]))/(4*c*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 2437, normalized size = 4.39

method	result
risch	$\frac{ia}{ce} - \frac{b\ln(c^2x^2+1)}{2ce} - \frac{iad\,\mathrm{arctanh}\left(\frac{2(-icx+1)e-2e}{2c\sqrt{de}}\right)}{e\sqrt{de}} + \frac{ib\ln(-icx+1)x}{2e} + \frac{b}{ce} - \frac{bd\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-i}{c\sqrt{de}}\right.}{4e\sqrt{de}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-5/8*b*c^4*d/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*
(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(
c^2*x^2+1)-e)+3/8*b*c^2*e/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*
```

```
c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*
x)^2/(c^2*x^2+1)-e)+3/4*b*c^3*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I
*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))+a*c^3/e*x+2*b*c^2/(c^2*d-e)
*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+b*c^2/e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/4*
b*c^2/e*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-
e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+1/8*b*c^2/
(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1
)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1/4*b*c^
3*(d*e)^(1/2)*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)
/c/(d*e)^(1/2))/(c^2*d-e)^2+1/4*b*c^4/e^2*d*sum((_R1^2*c^2*d-_R1^2*e+c^2*d+
3*e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^
2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((
c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-1/4*b*c^4/e^2*d*ln(c^2*d*(1+I*c*
x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1
)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+I*b*c^2*arctan(c*x)/e-1/4*b*c^4/e^
2*d*sum((_R1^2*c^2*d-_R1^2*e+c^2*d-e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arct
an(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2
*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))
+b*c^3*arctan(c*x)/e*x-a*c^3*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/8*b*
c^6/e*d^2/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^
2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)
-e)-2*b*c^4/e*d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*b*c^6/e^2*d^2
/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+
1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1/8*b*c
^5*(d*e)^(1/2)/e^3*d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2
*d+2*e)/c/(d*e)^(1/2))+1/2*b*c^3*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+
I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+3/8*b*c*(d*e)^(1
/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*
e)^(1/2))-1/4*b*c^4/e*d/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*
d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2
/(c^2*x^2+1)-e)+1/8*b*c^8/e^2*d^3/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2
+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e
*(1+I*c*x)^2/(c^2*x^2+1)-e)+3/4*b*c*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*
(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-5/4*b*c^5*(d*
e)^(1/2)/e^2*d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e
)/c/(d*e)^(1/2))/(c^2*d-e)-1/2*b*c^5*(d*e)^(1/2)*d/e*arctanh(1/4*(2*(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+1/8*b*c^
9*(d*e)^(1/2)/e^3*d^3*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^
2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+3/8*b*c*(d*e)^(1/2)/d*e*arctanh(1/4*(2*
(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+1
/4*b*c^7*(d*e)^(1/2)/e^2*d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+
1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] -(sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1))*a + 2*b*integrate(
1/2*x^2*arctan(c*x)/(x^2*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(x^2*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{atan}\,(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d),x)
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2),x)
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1156 $\int \frac{a+b\text{ArcTan}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=517

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}-\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}+\frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{ib\log(1$$

[Out] -1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^(1/2)+a*arctan(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)

Rubi [A]

time = 0.33, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5030, 211, 5028, 2456, 2441, 2440, 2438}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}+\frac{ib\text{Li}_2\left(\frac{\sqrt{e}\,(i-cx)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{ib\text{Li}_2\left(\frac{\sqrt{e}\,(1-icx)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{ib\text{Li}_2\left(\frac{\sqrt{e}\,(icx+1)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}+\frac{ib\text{Li}_2\left(\frac{\sqrt{e}\,(cx+i)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}+\frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{ib\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}+\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2),x]

[Out] (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*Sqrt[e])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx = a\int \frac{1}{d+ex^2}\,dx + b\int \frac{\tan^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} + \frac{1}{2}(ib)\int \frac{\log(1-icx)}{d+ex^2}\,dx - \frac{1}{2}(ib)\int \frac{\log(1+icx)}{d+ex^2}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} + \frac{1}{2}(ib)\int \left(\frac{\sqrt{-d}\,\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\,\log(1-icx)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right) dx - \frac{1}{2}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} - \frac{(ib)\int \frac{\log(1-icx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{4\sqrt{-d}} - \frac{(ib)\int \frac{\log(1-icx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{4\sqrt{-d}} + \frac{(ib)\int \frac{\log(1+}{\sqrt{-d}-}}{4\sqrt{-d}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c(}{c}\right.}{4\sqrt{-d}\,\sqrt{}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c(}{c}\right.}{4\sqrt{-d}\,\sqrt{}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}} + \frac{ib\log(1-icx)\log\left(\frac{c(}{c}\right.}{4\sqrt{-d}\,\sqrt{}}$$

Mathematica [A]
time = 0.17, size = 461, normalized size = 0.89

$$\frac{4a\sqrt{-d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) - ib\sqrt{d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right) + ib\sqrt{d}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right) - ib\sqrt{d}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right) + ib\sqrt{d}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right) + ib\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right) - ib\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right) - ib\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right) + ib\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}\,(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d^2}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

```
[Out] (4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - I*b*Sqrt[d]*Log[1 + I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*Sqrt[d]*Log[1 -
I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*Sqrt[
d]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])]
+ I*b*Sqrt[d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I
```

```
*Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqr
t[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e]
)] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/
(4*Sqrt[-d^2]*Sqrt[e])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(381) = 762$.
time = 0.56, size = 879, normalized size = 1.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*I*b*(c^2*d*e)^(1/2)/d/e*ar
ctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-
e))-1/2*b*(c^2*d*e)^(1/2)/d/e*arctan(c*x)^2-1/4*b*(c^2*d*e)^(1/2)/d/e*polyl
og(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))+1/2*I*
b*c^4*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*
arctan(c*x)/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*d-I*b*c^2*ln(1-(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*
d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/2*b*c^4/e/(c^4*d^2-2*c^2*d*e+e^2)*arct
an(c*x)^2*(c^2*d*e)^(1/2)*d-b*c^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^
2*d*e)^(1/2)+1/2*I*b*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*
d*e)^(1/2)-e))*arctan(c*x)/d/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*e+1/4*
b*c^4/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)*d-1/2*b*c^2/(c^4*d^2-2*c^2*d
*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/
2)-e))*(c^2*d*e)^(1/2)+1/2*b/d/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*d
*e)^(1/2)*e+1/4*b/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)*e)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + 2*b*integrate(1/2*arctan(c*x
)/(x^2*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{(cx)}}{d + ex^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{atan}(c\,x)}{e\,x^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2),x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x^2), x)
```

3.1157 $\int \frac{a+b\textbf{ArcTan}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=561

$$-\frac{a+b\text{ArcTan}(cx)}{dx}-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}}+\frac{bc\log(x)}{d}-\frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}}{}$$

```
[Out] (-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(c^2*x^2+1)/d-a*arctan(x*e^(1/
2)/d^(1/2))*e^(1/2)/d^(3/2)-1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))
/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d
)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b*ln(
1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)
^(3/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1
/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*
e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2
)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(
-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/
(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(3/2)
```

Rubi [A]

time = 0.41, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5038, 4946, 272, 36, 29, 31, 5030, 211, 5028, 2456, 2441, 2440, 2438}

$$-\frac{a+b\text{ArcTan}(cx)}{dx}-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}}-\frac{bc\log\left(c^2x^2+1\right)}{2d}+\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,(i-cx)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,(1-icx)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,(icx+1)}{i\sqrt{-d}\,c+\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{\sqrt{e}\,(cx+i)}{\sqrt{-d}\,c+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3
/2) + (b*c*Log[x])/d - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - S
qrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 -
 I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2
) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[
-d] - I*Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2) - (b*c*Log[1 + c^
2*x^2])/(2*d) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d]
 + I*Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*
x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sq
rt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(3/2) + ((I/4)*b*Sqrt[e]
*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(3/2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} + \frac{(bc)\int \frac{1}{x(1+c^2x^2)}\,dx}{d} - \frac{(ae)\int \frac{1}{d+ex^2}\,dx}{d} - \frac{(be)\int \frac{\tan^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{(bc)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx,x,x^2\right)}{2d} - \frac{(ibe)}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2d} - \frac{(bc^3)\,\text{Subst}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{bc\log\left(1+c^2x^2\right)}{2d} - \frac{(ibe)\int}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c(}{c}\right.}{4(-d)^{3/2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c(}{c}\right.}{4(-d)^{3/2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{bc\log(x)}{d} - \frac{ib\sqrt{e}\,\log(1+icx)\log\left(\frac{c(}{c}\right.}{4(-d)^{3/2}}$$

Mathematica [A]
time = 0.53, size = 468, normalized size = 0.83

$$\frac{-\frac{a+b\text{ArcTan}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)-\frac{\sqrt{e}\left(4a\sqrt{-d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+ib\sqrt{d}\left(\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{e}\,(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)\right)-ib\sqrt{d}\left(\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)\right)-ib\sqrt{d}\left(\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{e}\,(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)\right)+ib\sqrt{d}\left(\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{e}\,(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)\right)\right)}{4\sqrt{-d^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (-((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2 - (Sqrt[e
]*(4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*Sqrt[d]*(Log[1 + I*c*x]*L
og[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[
```

```
e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*
(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 + I*c*x]*Log[(
c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*(
1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) + I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[(c
*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[e]*(I
 + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])))/(4*Sqrt[-d^2]))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 2446, normalized size = 4.36

method	result
risch	$-\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4d\sqrt{de}}+\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4d\sqrt{de}}-\frac{be\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(}{c\sqrt{d}}\right.}{4d\sqrt{de}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] c*(b/d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*b/d*ln(c^2*d*(1+I*c*x)^4/(c^2*
x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+
2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+b/d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-a/d/c/
x+3/8*b*c^4*d/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c
*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^
2+1)-e)-5/8*b*c^2*e/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*
(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(
c^2*x^2+1)-e)+I*b*arctan(c*x)/d-b*arctan(c*x)/c/x/d+2*b/d*e/(c^2*d-e)*ln((1
+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*b/d*e/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^
2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*
e*(1+I*c*x)^2/(c^2*x^2+1)-e)+1/8*b/d*e^2/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(
c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^
2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1/4*b/c^2/d^2*e*sum((_R1^2*c^2*d-_R1^2*e
-c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c
^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=Roo
tOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))+1/4*b/c^2/d^2*e*sum((_R1^2*
c^2*d-_R1^2*e+3*c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_
R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2)
)/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-2*b*c^2/(c^2
*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*b*c^2/(c^2*d-e)*ln(c^2*d*(1+I*c*x
)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)
^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1/4*b*c^3*(d*e)^(1/2)*arctanh(1/4*(
2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2
```

```
-3/4*b*c^3*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2
*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+3/8*b*c*(d*e)^(1/2)/d/e*arctanh(1/4*(2
*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))-1/2*b*c*(d*e
)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(
d*e)^(1/2))/(c^2*d-e)+3/4*b/c*(d*e)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I
*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))-1/4*b/c^2/d^2*e*ln(c^2*d*(1
+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*
x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+1/4*b/c*(d*e)^(1/2)/d^2*e^2*a
rctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))
/(c^2*d-e)^2+5/4*b/c*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+1/8*b/c^3*(d*e)^(1/2)/d^
3*e^3*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)
^(1/2))/(c^2*d-e)^2-a/c*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/8*b/c^2/d
^2*e^2/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^
2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1
/8*b/c^3*(d*e)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
+2*c^2*d+2*e)/c/(d*e)^(1/2))+1/8*b/c^2/d^2*e^3/(c^2*d-e)^2*ln(c^2*d*(1+I*c*
x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1
)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+3/8*b*c^5*(d*e)^(1/2)*d/e*arctanh(
1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d
-e)^2-1/2*b*c*(d*e)^(1/2)/d*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2
+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x)) + 2*b*integrate(1/
2*arctan(c*x)/(x^4*e + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^4*e + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{x^{2} \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)), x)
```

3.1158 $\int \frac{x^3(a+b\textbf{ArcTan}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=403

$$-\frac{bc^2d\text{ArcTan}(cx)}{2(c^2d-e)e^2}+\frac{d(a+b\text{ArcTan}(cx))}{2e^2(d+ex^2)}+\frac{bc\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{(a+b\text{Ar}}{}$$

```
[Out] -1/2*b*c^2*d*arctan(c*x)/(c^2*d-e)/e^2+1/2*d*(a+b*arctan(c*x))/e^2/(e*x^2+d
)-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^2+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^
(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2*(a+b*arctan(c*
x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2+1
/2*I*b*polylog(2,1-2/(1-I*c*x))/e^2-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e
^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/4*I*b*polylog(2,1-2*c*((-
d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2+1/2*b*c*arctan(
x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d-e)/e^(3/2)
```

Rubi [A]
time = 0.33, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5100, 5094, 400, 209, 211, 4966, 2449, 2352, 2497}

$$\frac{d(a+b\text{ArcTan}(cx))}{2e^2(d+ex^2)}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}\,x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e^2}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}\,x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e^2}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^2}+\frac{bc\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{3/2}(c^2d-e)}-\frac{bc^2d\text{ArcTan}(cx)}{2e^2(c^2d-e)}-\frac{ib\text{Li}_2\left(1-\frac{2c(\sqrt{-d}-\sqrt{e}\,x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}-\frac{ib\text{Li}_2\left(1-\frac{2c(\sqrt{e}\,x+\sqrt{-d})}{(\sqrt{-d}\,c+i\sqrt{e})(1-icx)}\right)}{4e^2}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^2) + (d*(a + b*ArcTan[c*x]))/(2*e
^2*(d + e*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(c^2*d - e)*
e^(3/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 + ((a + b*ArcTan[c*
x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))
]])/(2*e^2) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt
[-d] + I*Sqrt[e])*(1 - I*c*x))]])/(2*e^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I
*c*x)])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt
[-d] - I*Sqrt[e])*(1 - I*c*x))]])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-
d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
```

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \left(-\frac{dx(a+b\tan^{-1}(cx))}{e(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{e(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx}{e}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{2e^2(d+ex^2)} - \frac{(bcd)\int \frac{1}{(1+c^2x^2)(d+ex^2)}\,dx}{2e^2} + \frac{\int \left(-\frac{a+b\tan^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \ldots\right.}{e}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{2e^2(d+ex^2)} - \frac{(bc^3d)\int \frac{1}{1+c^2x^2}\,dx}{2(c^2d-e)e^2} - \frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}} + \frac{\int \frac{a+b\tan^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,\ldots}{2e^{3/2}}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2(c^2d-e)e^2} + \frac{d(a+b\tan^{-1}(cx))}{2e^2(d+ex^2)} + \frac{bc\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}} - \frac{(a+b\tan^{-1}(\ldots}{\ldots}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2(c^2d-e)e^2} + \frac{d(a+b\tan^{-1}(cx))}{2e^2(d+ex^2)} + \frac{bc\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}} - \frac{(a+b\tan^{-1}(\ldots}{\ldots}$$

$$= -\frac{bc^2d\tan^{-1}(cx)}{2(c^2d-e)e^2} + \frac{d(a+b\tan^{-1}(cx))}{2e^2(d+ex^2)} + \frac{bc\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}} - \frac{(a+b\tan^{-1}(\ldots}{\ldots}$$

Mathematica [A]
time = 5.78, size = 522, normalized size = 1.30

$$\frac{2a\left(\frac{d}{d+ex^2}+\log(d+ex^2)\right)+b\left(-\frac{2c^2d\text{ArcTan}(cx)}{c^2d-e}+\frac{2d\text{ArcTan}(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{c^2d-e}+2\text{ArcTan}(cx)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+2\text{ArcTan}(cx)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+i\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(-1-icx)}{c\sqrt{d}-\sqrt{e}}\right)-i\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(1-icx)}{c\sqrt{d}+\sqrt{e}}\right)-i\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(-1+icx)}{c\sqrt{d}-\sqrt{e}}\right)+i\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(1+icx)}{c\sqrt{d}+\sqrt{e}}\right)-i\text{PolyLog}\left(2,\frac{c(\sqrt{d}-i\sqrt{e}\,x)}{c\sqrt{d}-\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c(\sqrt{d}-i\sqrt{e}\,x)}{c\sqrt{d}+\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c(\sqrt{d}+i\sqrt{e}\,x)}{c\sqrt{d}-\sqrt{e}}\right)-i\text{PolyLog}\left(2,\frac{c(\sqrt{d}+i\sqrt{e}\,x)}{c\sqrt{d}+\sqrt{e}}\right)\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (2*a*(d/(d + e*x^2) + Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x])/(c^2*d -
e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x
```

```
)/Sqrt[d]])/(c^2*d - e) + 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2
*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*S
qrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*
Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e]
)] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*Po
lyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2,
(c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[
d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 802, normalized size = 1.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a*c^6*d/e^2/(c^2*e*x^2+c^2*d)+1/2*a*c^4/e^2*ln(c^2*e*x^2+c^2*d)+
1/2*b*c^6*arctan(c*x)*d/e^2/(c^2*e*x^2+c^2*d)+1/2*b*c^4*arctan(c*x)/e^2*ln(
c^2*e*x^2+c^2*d)+1/4*I*b*c^4/e^2*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d
-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b*c^4/e^2
*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2
-2*I*e*_Z+c^2*d-e,index=1))-1/4*I*b*c^4/e^2*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I
*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))-1/4*
I*b*c^4/e^2*ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-1/4*I*b*c^4/e^2*dilog((RootOf(e*_
Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2
))+1/4*I*b*c^4/e^2*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-1/4*I*b*c^4/e^2*ln(c*x-I)*
ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c
^2*d-e,index=2))+1/4*I*b*c^4/e^2*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,inde
x=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))-1/4*I*b*c^4/e^2*dilog(
(RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*
d-e,index=1))+1/4*I*b*c^4/e^2*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2
)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))+1/2*b*c^5*d/e/(c^2*d-e)/(
d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c^6*d/e^2/(c^2*d-e)*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2)*log(x^2*e + d) + d/(x^2*e^3 + d*e^2))*a + 2*b*integrate(1/2*x^3
*arctan(c*x)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arctan(c*x) + a*x^3)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1159 $\int \frac{x(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=91

$$\frac{bc^2\text{ArcTan}(cx)}{2\,(c^2d-e)\,e}-\frac{a+b\text{ArcTan}(cx)}{2e\,(d+ex^2)}-\frac{bc\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,\left(c^2d-e\right)\sqrt{e}}$$

```
[Out] 1/2*b*c^2*arctan(c*x)/(c^2*d-e)/e+1/2*(-a-b*arctan(c*x))/e/(e*x^2+d)-1/2*b*
c*arctan(x*e^(1/2)/d^(1/2))/(c^2*d-e)/d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5094, 400, 209, 211}

$$-\frac{a+b\text{ArcTan}(cx)}{2e\,(d+ex^2)}+\frac{bc^2\text{ArcTan}(cx)}{2e\,(c^2d-e)}-\frac{bc\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{e}\,\left(c^2d-e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*c^2*ArcTan[c*x])/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])/(2*e*(d + e*x^2
)) - (b*c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*(c^2*d - e)*Sqrt[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx &= -\frac{a+b\tan^{-1}(cx)}{2e\,(d+ex^2)}+\frac{(bc)\int\frac{1}{(1+c^2x^2)(d+ex^2)}\,dx}{2e}\\ &= -\frac{a+b\tan^{-1}(cx)}{2e\,(d+ex^2)}-\frac{(bc)\int\frac{1}{d+ex^2}\,dx}{2\,(c^2d-e)}+\frac{(bc^3)\int\frac{1}{1+c^2x^2}\,dx}{2\,(c^2d-e)\,e}\\ &= \frac{bc^2\tan^{-1}(cx)}{2\,(c^2d-e)\,e}-\frac{a+b\tan^{-1}(cx)}{2e\,(d+ex^2)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,(c^2d-e)\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 98, normalized size = 1.08

$$\frac{a\sqrt{d}\,(c^2d-e)-b\sqrt{d}\,e(1+c^2x^2)\,\mathrm{ArcTan}(cx)+bc\sqrt{e}\,(d+ex^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e\,(-c^2d+e)\,(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*Sqrt[d]*(c^2*d - e) - b*Sqrt[d]*e*(1 + c^2*x^2)*ArcTan[c*x] + b*c*Sqrt[e
]*(d + e*x^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e*(-(c^2*d) + e)*(d +
 e*x^2))
```

Maple [A]
time = 0.31, size = 115, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{b\,c^4\arctan(cx)}{2\left(e\,c^2x^2+c^2d\right)e}-\frac{b\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\left(c^2d-e\right)\sqrt{de}}+\frac{b\,c^4\arctan(cx)}{2e\left(c^2d-e\right)}}{c^2}$
default	$\frac{-\frac{a\,c^4}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{b\,c^4\arctan(cx)}{2\left(e\,c^2x^2+c^2d\right)e}-\frac{b\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\left(c^2d-e\right)\sqrt{de}}+\frac{b\,c^4\arctan(cx)}{2e\left(c^2d-e\right)}}{c^2}$

risch	$\frac{ib\ln(icx+1)}{4e(e\,x^2+d)} - \frac{-i\ln((d\,c^3-ce)x+id\,c^2-ie)b\,c^2de\,x^2+i\ln((d\,c^3-ce)x-id\,c^2+ie)b\,c^2de\,x^2-\ln\left(\sqrt{-de}\;x+d\right)\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a*c^4/e/(c^2*e*x^2+c^2*d)-1/2*b*c^4/(c^2*e*x^2+c^2*d)*arctan(c*
x)/e-1/2*b*c^3/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c^4/e/(c
^2*d-e)*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 90, normalized size = 0.99

$$\frac{1}{2}\left(c\left(\frac{c\arctan(cx)}{c^2de-e^2}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{(c^2d-e)\sqrt{d}}\right)-\frac{\arctan(cx)}{x^2e^2+de}\right)b-\frac{a}{2(x^2e^2+de)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(c*arctan(c*x)/(c^2*d*e - e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/
((c^2*d - e)*sqrt(d))) - arctan(c*x)/(x^2*e^2 + d*e))*b - 1/2*a/(x^2*e^2 +
d*e)
```

Fricas [A]
time = 3.29, size = 237, normalized size = 2.60

$$\left[-\frac{2\,ac^2d^2-2\,ade-2\,(bc^2dx^2+bd)\arctan(cx)\,e-(bcx^2e+bcd)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)}{4\,(c^2d^3e-dx^2e^3+(c^2d^2x^2-d^2)e^2)},\;-\frac{ac^2d^2+(bcx^2e+bcd)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-ade-(bc^2dx^2+bd)\arctan(cx)\,e}{2\,(c^2d^3e-dx^2e^3+(c^2d^2x^2-d^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*e - 2*(b*c^2*d*x^2 + b*d)*arctan(c*x)*e - (b*c*x
^2*e + b*c*d)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)))/(c^
2*d^3*e - d*x^2*e^3 + (c^2*d^2*x^2 - d^2)*e^2), -1/2*(a*c^2*d^2 + (b*c*x^2*
e + b*c*d)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - a*d*e - (b*c^2*d*x^2
 + b*d)*arctan(c*x)*e)/(c^2*d^3*e - d*x^2*e^3 + (c^2*d^2*x^2 - d^2)*e^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. 2(76) = 152.
time = 118.77, size = 1054, normalized size = 11.58

$$\begin{cases} \frac{\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}(cx)}{2}-\frac{bx}{2c}+\frac{b\operatorname{atan}(cx)}{2c^2}}{d^2} & \text{for}\: e=0 \\ -\frac{2ad}{4d^2e+4de^2x^2}-\frac{bdx\sqrt{\frac{e}{d}}}{4d^2e+4de^2x^2}+\frac{bd\operatorname{atan}\left(x\sqrt{\frac{e}{d}}\right)}{4d^2e+4de^2x^2}-\frac{bex^2\operatorname{atan}\left(x\sqrt{\frac{e}{d}}\right)}{4d^2e+4de^2x^2} & \text{for}\: c=-\sqrt{\frac{e}{d}} \\ -\frac{2ad}{4d^2e+4de^2x^2}+\frac{bdx\sqrt{\frac{e}{d}}}{4d^2e+4de^2x^2}-\frac{bd\operatorname{atan}\left(x\sqrt{\frac{e}{d}}\right)}{4d^2e+4de^2x^2}+\frac{bex^2\operatorname{atan}\left(x\sqrt{\frac{e}{d}}\right)}{4d^2e+4de^2x^2} & \text{for}\: c=\sqrt{\frac{e}{d}} \\ \frac{-\frac{a}{2x^2}-\frac{bc^2\operatorname{atan}(cx)}{2}-\frac{bc}{2x}-\frac{b\operatorname{atan}(cx)}{2x^2}}{e^2} & \text{for}\: d=0 \\ -\frac{2ac^2d\sqrt{-\frac{d}{e}}}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}+\frac{2ae\sqrt{-\frac{d}{e}}}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}+\frac{2bc^2ex^2\sqrt{-\frac{d}{e}}\operatorname{atan}(cx)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}-\frac{bcd\log\left(x-\sqrt{-\frac{d}{e}}\right)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}+\frac{bcd\log\left(x+\sqrt{-\frac{d}{e}}\right)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}-\frac{bcex^2\log\left(x-\sqrt{-\frac{d}{e}}\right)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}+\frac{bcex^2\log\left(x+\sqrt{-\frac{d}{e}}\right)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}}+\frac{2be\sqrt{-\frac{d}{e}}\operatorname{atan}(cx)}{4c^2d^2e\sqrt{-\frac{d}{e}}+4c^2de^2x^2\sqrt{-\frac{d}{e}}-4de^2\sqrt{-\frac{d}{e}}-4e^3x^2\sqrt{-\frac{d}{e}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Piecewise(((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2
))/d**2, Eq(e, 0)), (-2*a*d/(4*d**2*e + 4*d*e**2*x**2) - b*d*x*sqrt(e/d)/(4
*d**2*e + 4*d*e**2*x**2) + b*d*atan(x*sqrt(e/d))/(4*d**2*e + 4*d*e**2*x**2)
 - b*e*x**2*atan(x*sqrt(e/d))/(4*d**2*e + 4*d*e**2*x**2), Eq(c, -sqrt(e/d))
), (-2*a*d/(4*d**2*e + 4*d*e**2*x**2) + b*d*x*sqrt(e/d)/(4*d**2*e + 4*d*e**
2*x**2) - b*d*atan(x*sqrt(e/d))/(4*d**2*e + 4*d*e**2*x**2) + b*e*x**2*atan(
x*sqrt(e/d))/(4*d**2*e + 4*d*e**2*x**2), Eq(c, sqrt(e/d))), ((-a/(2*x**2) -
 b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2))/e**2, Eq(d, 0)), (-
2*a*c**2*d*sqrt(-d/e)/(4*c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt(-
d/e) - 4*d*e**2*sqrt(-d/e) - 4*e**3*x**2*sqrt(-d/e)) + 2*a*e*sqrt(-d/e)/(4*
c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt(-d/e) - 4*d*e**2*sqrt(-d/e
) - 4*e**3*x**2*sqrt(-d/e)) + 2*b*c**2*e*x**2*sqrt(-d/e)*atan(c*x)/(4*c**2*
d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt(-d/e) - 4*d*e**2*sqrt(-d/e) - 4
*e**3*x**2*sqrt(-d/e)) - b*c*d*log(x - sqrt(-d/e))/(4*c**2*d**2*e*sqrt(-d/e
) + 4*c**2*d*e**2*x**2*sqrt(-d/e) - 4*d*e**2*sqrt(-d/e) - 4*e**3*x**2*sqrt(
-d/e)) + b*c*d*log(x + sqrt(-d/e))/(4*c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**
2*x**2*sqrt(-d/e) - 4*d*e**2*sqrt(-d/e) - 4*e**3*x**2*sqrt(-d/e)) - b*c*e*x
**2*log(x - sqrt(-d/e))/(4*c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt
(-d/e) - 4*d*e**2*sqrt(-d/e) - 4*e**3*x**2*sqrt(-d/e)) + b*c*e*x**2*log(x +
 sqrt(-d/e))/(4*c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt(-d/e) - 4*
d*e**2*sqrt(-d/e) - 4*e**3*x**2*sqrt(-d/e)) + 2*b*e*sqrt(-d/e)*atan(c*x)/(4
*c**2*d**2*e*sqrt(-d/e) + 4*c**2*d*e**2*x**2*sqrt(-d/e) - 4*d*e**2*sqrt(-d/
e) - 4*e**3*x**2*sqrt(-d/e)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.85, size = 696, normalized size = 7.65

$$\frac{b\,c\,\ln\left(e\,x+\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,d\,e^2-4\,c^2\,d^2\,e}-\frac{2\,b\,c^2\,\mathrm{atan}\left(-\frac{\frac{c^2\,\left(c^8\,e\,x+\frac{c^2\,\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\,\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)}{4\,e^2-4\,c^2\,d\,e}-\frac{c^2\,\left(-c^8\,e\,x+\frac{c^2\,\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e-\frac{c^2\,x\,\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)}{4\,e^2-4\,c^2\,d\,e}}{\frac{c^2\,\left(c^8\,e\,x+\frac{c^2\,\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\,\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}+\frac{c^2\,\left(-c^8\,e\,x+\frac{c^2\,\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e-\frac{c^2\,x\,\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)\,1{}\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}}\right)}{4\,e^2-4\,c^2\,d\,e}-\frac{b\,\mathrm{atan}\left(c\,x\right)}{2\,e\,\left(e\,x^2+d\right)}-\frac{b\,c\,\ln\left(e\,x-\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,\left(d\,e^2-c^2\,d^2\,e\right)}-\frac{a}{2\,e^2\,x^2+2\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*atan(c*x)))/(d + e*x^2)^2,x)

[Out] (b*c*log(e*x + (-d*e)^(1/2))*(-d*e)^(1/2))/(4*d*e^2 - 4*c^2*d^2*e) - (2*b*c^2*atan(-((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x))/(4*e^2 - 4*c^2*d*e) - (c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) - c^8*e*x))/(4*e^2 - 4*c^2*d*e))/((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x)*1i)/(4*e^2 - 4*c^2*d*e) + (c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) - c^8*e*x)*1i)/(4*e^2 - 4*c^2*d*e))))/(4*e^2 - 4*c^2*d*e) - (b*atan(c*x))/(2*e*(d + e*x^2)) - (b*c*log(e*x - (-d*e)^(1/2))*(-d*e)^(1/2))/(4*(d*e^2 - c^2*d^2*e)) - a/(2*d*e + 2*e^2*x^2)

3.1160 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=443

$$-\frac{bc^2\text{ArcTan}(cx)}{2d\left(c^2d-e\right)}+\frac{a+b\text{ArcTan}(cx)}{2d\left(d+ex^2\right)}+\frac{bc\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)}+\frac{a\log(x)}{d^2}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}-\frac{(a$$

```
[Out] -1/2*b*c^2*arctan(c*x)/d/(c^2*d-e)+1/2*(a+b*arctan(c*x))/d/(e*x^2+d)+a*ln(x
)/d^2+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x))*ln(2*c*((
-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/2*(a+b*arcta
n(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d
^2+1/2*I*b*polylog(2,-I*c*x)/d^2-1/2*I*b*polylog(2,I*c*x)/d^2-1/2*I*b*polyl
og(2,1-2/(1-I*c*x))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e
^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2+1/2*b*c*arctan(x*e^(1/2)/d^
(1/2))*e^(1/2)/d^(3/2)/(c^2*d-e)
```

Rubi [A]
time = 0.36, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5100, 4940, 2438, 5094, 400, 209, 211, 4966, 2449, 2352, 2497}

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^2}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^2}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^2}+\frac{a+b\text{ArcTan}(cx)}{2d\left(d+ex^2\right)}+\frac{a\log(x)}{d^2}+\frac{bc\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)}-\frac{bc^2\text{ArcTan}(cx)}{2d\left(c^2d-e\right)}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\text{Li}_2(-icx)}{2d^2}-\frac{ib\text{Li}_2(icx)}{2d^2}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(b*c^2*ArcTan[c*x])/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])/(2*d*(d + e*
x^2)) + (b*c*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c^2*d - e)) +
 (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 - ((a + b*Ar
cTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(2*d^2) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + ((I/2)*b*PolyLog[2, (-I)*
c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 - ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*S
qrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{d\left(d+ex^2\right)^2} - \frac{ex(a+b\tan^{-1}(cx))}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^2} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{a\log(x)}{d^2} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^2} - \frac{(ib)\int \frac{\log(1+icx)}{x}\,dx}{2d^2} - \frac{(bc)\int \frac{}{(1+c^2x}}{2d}$$

$$= \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{a\log(x)}{d^2} + \frac{ib\text{Li}_2(-icx)}{2d^2} - \frac{ib\text{Li}_2(icx)}{2d^2} - \frac{(bc^3)\int \frac{1}{1+c^2x^2}\,dx}{2d\left(c^2d-e\right)} + \frac{\sqrt{e}\int}{}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{(a+b\tan}{}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{(a+b\tan}{}$$

$$= -\frac{bc^2\tan^{-1}(cx)}{2d\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{2d\left(d+ex^2\right)} + \frac{bc\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)} + \frac{a\log(x)}{d^2} + \frac{(a+b\tan}{}$$

Mathematica [A]
time = 3.71, size = 590, normalized size = 1.33

$$\frac{2a\left(\frac{d}{d+ex^2}+2\log(x)-\log\left(d+ex^2\right)\right)+b\left(-\frac{2c^2d\text{ArcTan}(cx)}{c^2d-e}+\frac{2d\text{ArcTan}(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\text{ ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{c^2d-e}+4\text{ArcTan}(cx)\log(x)-2\text{ArcTan}(cx)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-2\text{ArcTan}(cx)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-i\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\,(-1-icx)}{c\sqrt{d}-\sqrt{e}}\right)-2i\log(x)\log(1-icx)+i\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\,(1-icx)}{c\sqrt{d}+\sqrt{e}}\right)+i\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\,(-1+icx)}{c\sqrt{d}-\sqrt{e}}\right)+2i\log(x)\log(1+icx)-i\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\,(1+icx)}{c\sqrt{d}+\sqrt{e}}\right)+2i\text{PolyLog}(2,-icx)-2i\text{PolyLog}(2,icx)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}\,x\right)}{c\sqrt{d}-\sqrt{e}}\right)-i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}-i\sqrt{e}\,x\right)}{c\sqrt{d}+\sqrt{e}}\right)-i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}+i\sqrt{e}\,x\right)}{c\sqrt{d}-\sqrt{e}}\right)+i\text{PolyLog}\left(2,\frac{c\left(\sqrt{d}+i\sqrt{e}\,x\right)}{c\sqrt{d}+\sqrt{e}}\right)\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] (2*a*(d/(d + e*x^2) + 2*Log[x] - Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x]
)/(c^2*d - e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(c^2*d - e) + 4*ArcTan[c*x]*Log[x] - 2*ArcTan[c*x]*L
og[((-I)*Sqrt[d])/Sqrt[e] + x] - 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x]
 - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d]
- Sqrt[e])] - (2*I)*Log[x]*Log[1 - I*c*x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqr
t[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*Log[x]*
Log[1 + I*c*x] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(
c*Sqrt[d] + Sqrt[e])] + (2*I)*PolyLog[2, (-I)*c*x] - (2*I)*PolyLog[2, I*c*x
] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*Pol
yLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (
c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d
] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 847, normalized size = 1.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)+1
/2*b*c^2*arctan(c*x)/d/(c^2*e*x^2+c^2*d)-1/2*b*arctan(c*x)/d^2*ln(c^2*e*x^2
+c^2*d)+b*arctan(c*x)/d^2*ln(c*x)+1/2*b*c/d*e/(c^2*d-e)/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))-1/2*b*c^2*arctan(c*x)/d/(c^2*d-e)+1/2*I*b/d^2*ln(c*x)*ln(1
+I*c*x)-1/4*I*b/d^2*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c
*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))-1/4*I*b/d^2*ln(c*x-I)*ln(c^2
*e*x^2+c^2*d)+1/4*I*b/d^2*ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-1/4*I*b/d^2*ln(I+c*
x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_
Z+c^2*d-e,index=2))+1/4*I*b/d^2*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index
=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b/d^2*ln(c*x-I)*l
n((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^
2*d-e,index=2))-1/4*I*b/d^2*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-
c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/2*I*b/d^2*dilog(1-I*c*x)-
1/4*I*b/d^2*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*
_Z^2-2*I*e*_Z+c^2*d-e,index=1))-1/2*I*b/d^2*ln(c*x)*ln(1-I*c*x)+1/4*I*b/d^2
*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*
_Z+c^2*d-e,index=2))+1/4*I*b/d^2*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d
```

```
-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/2*I*b/d^2*dil
og(1+I*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(1/(d*x^2*e + d^2) - log(x^2*e + d)/d^2 + 2*log(x)/d^2) + 2*b*integra
te(1/2*arctan(c*x)/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)^2),x)
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)^2), x)
```

3.1161 $\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+ex^2)^2} dx$

Optimal. Leaf size=489

$$-\frac{bc}{2d^2x}-\frac{bc^2\text{ArcTan}(cx)}{2d^2}+\frac{bc^2e\text{ArcTan}(cx)}{2d^2(c^2d-e)}-\frac{a+b\text{ArcTan}(cx)}{2d^2x^2}-\frac{e(a+b\text{ArcTan}(cx))}{2d^2(d+ex^2)}-\frac{bce^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)}$$

```
[Out] -1/2*b*c/d^2/x-1/2*b*c^2*arctan(c*x)/d^2+1/2*b*c^2*e*arctan(c*x)/d^2/(c^2*d
-e)+1/2*(-a-b*arctan(c*x))/d^2/x^2-1/2*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)-1/
2*b*c*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(c^2*d-e)-2*a*e*ln(x)/d^3-2
*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^3+e*(a+b*arctan(c*x))*ln(2*c*((-d)^(
1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3+e*(a+b*arctan(c*x))
*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-I*b*
e*polylog(2,-I*c*x)/d^3+I*b*e*polylog(2,I*c*x)/d^3+I*b*e*polylog(2,1-2/(1-I
*c*x))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-
d)^(1/2)-I*e^(1/2)))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(
1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3
```

Rubi [A]
time = 0.39, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5100, 4946, 331, 209, 4940, 2438, 5094, 400, 211, 4966, 2449, 2352, 2497}

$$-\frac{2e\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{d^3}+\frac{e(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{d^3}-\frac{e(a+b\text{ArcTan}(cx))}{2d^2(d+ex^2)}-\frac{a+b\text{ArcTan}(cx)}{2d^2x^2}-\frac{2ae\log(x)}{d^3}-\frac{bce^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)}+\frac{bc^2e\text{ArcTan}(cx)}{2d^2(c^2d-e)}-\frac{bc^2\text{ArcTan}(cx)}{2d^2}-\frac{ibe\text{Li}_2(-icx)}{d^3}+\frac{ibe\text{Li}_2(icx)}{d^3}+\frac{ibe\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{d^3}-\frac{ibe\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^3}-\frac{ibe\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2d^3}-\frac{bc}{2d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(b*c)/(d^2*x) - (b*c^2*ArcTan[c*x])/(2*d^2) + (b*c^2*e*ArcTan[c*x])/(2
*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x]
))/(2*d^2*(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/
2)*(c^2*d - e)) - (2*a*e*Log[x])/d^3 - (2*e*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*
Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*
(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3 - (I*b
*e*PolyLog[2, (-I)*c*x])/d^3 + (I*b*e*PolyLog[2, I*c*x])/d^3 + (I*b*e*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^3 - ((I/2)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/2)*b*e*PolyL
og[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^3} - \frac{2e(a+b\tan^{-1}(cx))}{d^3x} + \frac{e^2x(a+b\tan^{-1}(cx))}{d^2\left(d+ex^2\right)^2} + \frac{2e^2x(a+b\tan}{d^3\left(d+\right.}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d^3} + \frac{e^2\int \frac{x(a+b\tan}{(d+}}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{2d^2\left(d+ex^2\right)} - \frac{2ae\log(x)}{d^3} + \frac{(bc)\int \frac{1}{x^2(1+c^2x^2)}\,dx}{2d^2} - \frac{(ibe)}{}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{2d^2\left(d+ex^2\right)} - \frac{2ae\log(x)}{d^3} - \frac{ibe\text{Li}_2(-icx)}{d^3} + \frac{ib}{}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2\tan^{-1}(cx)}{2d^2} + \frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2\tan^{-1}(cx)}{2d^2} + \frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bc}{2d^2x} - \frac{bc^2\tan^{-1}(cx)}{2d^2} + \frac{bc^2e\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} - \frac{a+b\tan^{-1}(cx)}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{2d^2\left(d+ex^2\right)}$$

Mathematica [A]
time = 8.78, size = 643, normalized size = 1.31

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(a*(d*(x^(-2) + e/(d + e*x^2)) + 4*e*Log[x] - 2*e*Log[d + e*x^2]) + b*
((c*d)/x + (c^2*d*(c^2*d - 2*e)*ArcTan[c*x])/(c^2*d - e) + d*(x^(-2) + e/(d
 + e*x^2))*ArcTan[c*x] + (c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c
^2*d - e) + 4*e*ArcTan[c*x]*Log[x] - 2*e*ArcTan[c*x]*Log[d + e*x^2] - (2*I)
*e*(Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - PolyLog[2, (-I)*c*x] + PolyL
og[2, I*c*x]) - e*(2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan
[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[
(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e
```

```
] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*
Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - 2*
ArcTan[c*x]*Log[d + e*x^2] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sq
rt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + S
qrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])]
- I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])])))/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 984, normalized size = 2.01

method	result
derivativedivides	Expression too large to display
default	Expression too large to display
risch	$\frac{ibe\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{2d^3}+\frac{ibe\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{2d^3}-\frac{ibe\ln(icx+1)\ln\left(\frac{c\sqrt{d}}{c}\right.}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*I*b/c^2/d^3*e*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+
I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/2*b*c^2*arctan(c*x)/d/(c^2*d-
e)+1/2*I*b/c^2/d^3*e*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/
RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/2*I*b/c^2/d^3*e*dilog((RootOf(e*
_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=
1))+1/2*I*b/c^2/d^3*e*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)
/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))-2*b/c^2*arctan(c*x)/d^3*e*ln(c*x)
+I*b/c^2/d^3*e*dilog(1-I*c*x)-I*b/c^2/d^3*e*dilog(1+I*c*x)+b/c^2*arctan(c*x
)*e/d^3*ln(c^2*e*x^2+c^2*d)-1/2*a*e/d^2/(c^2*e*x^2+c^2*d)-1/2*b/d^2/c/x-1/2
*a/d^2/c^2/x^2+1/2*I*b/c^2/d^3*e*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)+I*b/c^2/d^3*
e*ln(c*x)*ln(1-I*c*x)-1/2*b/c/d^2*e^2/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2))-1/2*I*b/c^2/d^3*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,ind
ex=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/2*I*b/c^2/d^3*e*ln(
c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I
*e*_Z+c^2*d-e,index=1))-1/2*I*b/c^2/d^3*e*ln(I+c*x)*ln(c^2*e*x^2+c^2*d)+1/2
*I*b/c^2/d^3*e*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)
/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))+1/2*I*b/c^2/d^3*e*ln(I+c*x)*ln((R
ootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-
e,index=1))-I*b/c^2/d^3*e*ln(c*x)*ln(1+I*c*x)+a/c^2*e/d^3*ln(c^2*e*x^2+c^2*
d)-2*a/c^2/d^3*e*ln(c*x)-1/2*b*arctan(c*x)/d^2/c^2/x^2+b/d^2/(c^2*d-e)*arct
an(c*x)*e-1/2*b*arctan(c*x)*e/d^2/(c^2*e*x^2+c^2*d))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*x^2*e + d)/(d^2*x^4*e + d^3*x^2) - 2*e*log(x^2*e + d)/d^3 + 4*e*
log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(x^7*e^2 + 2*d*x^5*e + d^2*x^3)
, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^7*e^2 + 2*d*x^5*e + d^2*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2), x)
```

3.1162 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=1335

$$-\frac{x(a+b\text{ArcTan}(cx))}{2e\,(d+ex^2)}+\frac{a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}}-\frac{(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}}-\frac{ib\log(1+icx)\log\left(\frac{c\left(\sqrt{-\right.}}{c\sqrt{-}}\right.}{4\sqrt{-d}\,e^{3/2}}$$

```
[Out] -1/2*x*(a+b*arctan(c*x))/e/(e*x^2+d)+1/4*b*c*ln(c^2*x^2+1)/(c^2*d-e)/e-1/4*
b*c*ln(e*x^2+d)/(c^2*d-e)/e-1/8*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^
2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/2)/d
^(1/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)
*d^(1/2)-I*e^(1/2)))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/8*I*b*c*polylog(2,(-c^2
)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/e^(3/2)/(-c
^2)^(1/2)/d^(1/2)+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)
))/e^(3/2)/(-d)^(1/2)+1/8*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1
/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/2)/d^(1/2
)+1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^
(1/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*
d^(1/2)-I*e^(1/2)))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/4*I*b*ln(1+I*c*x)*ln(c*(
(-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+a*arctan
(x*e^(1/2)/d^(1/2))/e^(3/2)/d^(1/2)-1/2*(a+b*arctan(c*x))*arctan(x*e^(1/2)/
d^(1/2))/e^(3/2)/d^(1/2)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c
*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(
1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*ln(1+I
*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d)^(1
/2)-1/8*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)
))*ln(1+I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/8*I*b*c*ln(-(1+
x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/
d^(1/2))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(
1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/e^(3/2)/(-c^2)^(1/2)/d^
(1/2)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))/e^(3/2)
/(-d)^(1/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))/e
^(3/2)/(-d)^(1/2)
```

Rubi [A]
time = 1.52, antiderivative size = 1335, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5100, 205, 211, 5032, 6857, 455, 36, 31, 5028, 2456, 2441, 2440, 2438, 5030}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(x*(a + b*ArcTan[c*x]))/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(3/2)) - ((a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*Sqrt[d]*e^(3/2)) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))
/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 - I*c*x]*Lo
g[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2))
- ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sq
rt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sq
rt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/8)*b*c*Log[(S
qrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*
(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sqrt[e]*(1 - S
qrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt
[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2
]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sq
rt[-c^2]*Sqrt[d]*e^(3/2)) + (b*c*Log[1 + c^2*x^2])/(4*(c^2*d - e)*e) - (b*c
*Log[d + e*x^2])/(4*(c^2*d - e)*e) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I - c*x)
)/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt
[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*P
olyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^(3/2
)) + ((I/4)*b*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sq
rt[-d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*
b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I
*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*S
qrt[d]*e^(3/2)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x)
)/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \left(-\frac{d(a+b\tan^{-1}(cx))}{e\,(d+ex^2)^2}+\frac{a+b\tan^{-1}(cx)}{e\,(d+ex^2)}\right)\,dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^2}\,dx}{e}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} + \frac{a\int \frac{1}{d+ex^2}\,dx}{e} + \frac{b\int \text{t}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} +$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} +$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} +$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} -$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} -$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} -$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{2e\,(d+ex^2)} + \frac{a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}} - \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}} -$$

Mathematica [**A**]
time = 8.85, size = 877, normalized size = 0.66

$$-\frac{ax}{2e\left(d+ex^2\right)}+\frac{a\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{3/2}}+\frac{bc\left(\ldots\right)}{8e}$$ [illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a*x)/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(
3/2)) + (b*c*((-2*Log[(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d +
 e)])/(c^2*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*A
rcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcC
os[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*L
og[(-2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d - c*S
qrt[-(c^2*d*e)]*x))] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[
(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[((2*I)*c^2*d*(e + I*Sqrt[-(c^2*d*e)])*(I + c
*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*d + e)/(-
(c^2*d) + e)] - (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + (2*I)*ArcTanh[(c*
e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-(c^2*d) + e]*
E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])]
- (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e
*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e
)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e + (-(c^2*d) + e
)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)
])*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]
*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d + c*Sqrt[-(
c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*
e)] - (4*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*Arc
Tan[c*x]])))/(8*e)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2343 vs. 2(991) = 1982.
time = 1.59, size = 2344, normalized size = 1.76

method	result	size
derivativedivides	Expression too large to display	2344
default	Expression too large to display	2344
risch	Expression too large to display	2391

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/2*a*c^5/e*x/(c^2*e*x^2+c^2*d)+1/4*b*c^7*(d*e)^(1/2)/e^2*d*arctanh
(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*
```

d-e)^2-1/8*b*c^2*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/4*b*c^3*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-1/4*b*c^2*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*arctan(c*x)^2+1/2*b*c^5*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x-1/4*I*b*c^8*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+3/4*I*b*c^6*d*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/4*I*b*c^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*e+1/2*a*c^3/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*I*b*c^6*arctan(c*x)/(c^2*d-e)/e/(c^2*e*x^2+c^2*d)*d-3/4*I*b*c^4*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/4*I*b*c^4*(c^2*d*e)^(1/2)/e^2/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/8*b*c^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*e-1/8*b*c^8*d^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/4*b*c^8*d^2*arctan(c*x)^2/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+3/4*b*c^6*d*arctan(c*x)^2/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/4*b*c^2*arctan(c*x)^2/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*e-1/2*I*b*c^6*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x^2+3/8*b*c^6*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*d/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/4*I*b*c^2*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/2*b*c^7*arctan(c*x)/(c^2*d-e)/e/(c^2*e*x^2+c^2*d)*d*x-1/4*b*c^3*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-1/4*b*c^6/(c^2*d-e)^2/e*d*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-3/4*b*c^4*arctan(c*x)^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-3/8*b*c^4*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/8*b*c^4*(c^2*d*e)^(1/2)/e^2/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/4*b*c^4/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-b*c^4/(c^2*d-e)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*b*c^5*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+1/4*b*c^4*(c^2*d*e)^(1/2)/e^2/(c^2*d-e)*arctan(c*x)^2+b*c^6/(c^2*d-e)^2/e*d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) - x/(x^2*e^2 + d*e))*a + 2*
b*integrate(1/2*x^2*arctan(c*x)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{atan}(cx))}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a + b\,\mathrm{atan}(c\,x))}{(e\,x^2 + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1163 $\int \frac{a+b\mathbf{ArcTan}(cx)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=819

$$\frac{x(a+b\text{ArcTan}(cx))}{2d\,(d+ex^2)}+\frac{(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right)}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\cdots$$

```
[Out] 1/2*x*(a+b*arctan(c*x))/d/(e*x^2+d)-1/4*b*c*ln(c^2*x^2+1)/d/(c^2*d-e)+1/4*b
*c*ln(e*x^2+d)/d/(c^2*d-e)+1/2*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(1/2))/
d^(3/2)/e^(1/2)-1/8*I*b*c*ln(-(1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^
(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)+1/8*
I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-
I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)-1/8*I*b*c*ln(-(1-x*(-c^2)
^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))
/d^(3/2)/(-c^2)^(1/2)/e^(1/2)+1/8*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-
c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c^2)^(1/2)
/e^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/
2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)-1/8*I*b*c*polylog(2,(-c
^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(3/2)/(
-c^2)^(1/2)/e^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/
((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)-1/8*I*b*c*po
lylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2))
)/d^(3/2)/(-c^2)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 819, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {205, 211, 5032, 6857, 455, 36, 31, 5028, 2456, 2441, 2440, 2438}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(a+b\text{ArcTan}(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\text{ArcTan}(cx))}{2d\,(ex^2+d)}+\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right)}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\frac{ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}\,x+1\right)}{i\sqrt{-c^2}\,\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right)}{i\sqrt{-c^2}\,\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}\,x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}+\frac{ibc\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}\,x+1\right)}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}\,x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\frac{bc\log\left(c^2x^2+1\right)}{4d\,(c^2d-e)}+\frac{bc\log\left(ex^2+d\right)}{4d\,(c^2d-e)}+\frac{ibc\text{Li}_2\left(\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{-c^2}\,\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\frac{ibc\text{Li}_2\left(\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{-c^2}\,\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}+\frac{ibc\text{Li}_2\left(\frac{\sqrt{-c^2}\left(i\sqrt{e}\,x+\sqrt{d}\right)}{\sqrt{-c^2}\,\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}-\frac{ibc\text{Li}_2\left(\frac{\sqrt{-c^2}\left(i\sqrt{e}\,x+\sqrt{d}\right)}{\sqrt{-c^2}\,\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

```
[Out] (x*(a + b*ArcTan[c*x]))/(2*d*(d + e*x^2)) + ((a + b*ArcTan[c*x])*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 - Sqrt[
-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x
))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*
Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^
```

```
2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*
Sqrt[e]) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)) + (b*c*Log[d + e*x^2])/
(4*d*(c^2*d - e)) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/8
)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] +
 I*Sqrt[e])]])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/8)*b*c*PolyLog[2, (Sqrt[-c
^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]
*d^(3/2)*Sqrt[e]) - ((I/8)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*
x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])]])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^2}\,dx = \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - (bc)\int \frac{\frac{x}{2d(d+ex^2)} + \frac{\tan^{-1}\left(\right.}{2d^{3/}}}{1+c^2x^2}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - (bc)\int \left(\frac{x}{2d\,(1+c^2x^2)\,(d}\right.$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{(bc)\int \frac{x}{(1+c^2x^2)(d+ex^2)}\,dx}{2d}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{(bc)\text{Subst}\left(\int \frac{1}{(1+c^2x)(d+ex}\right.}{4d}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x\right.}{4d\,(c^2d-e)}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} - \frac{bc\log(1+c^2x^2)}{4d\,(c^2d-e)} + \frac{bc\log}{4d}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right.}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right.}{8\sqrt{-c^2}\,d}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right.}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right.}{8\sqrt{-c^2}\,d}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{2d\,(d+ex^2)} + \frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}} + \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right.}{i\sqrt{-c^2}\,\sqrt{d}+\sqrt{e}}\right.}{8\sqrt{-c^2}\,d}$$

Mathematica [A]
time = 8.91, size = 861, normalized size = 1.05

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/(2*d*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e
]) + (b*c*((2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2*d
 - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2
*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d
+ e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*
((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*
d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d -
 e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))]
- (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d
*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*S
qrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt
[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog
[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/(
(c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*
I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d +
c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)] + (4*ArcTan[c*x]*Sin[2*ArcTan[c*
x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/(8*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2319 vs. 2(611) = 1222.
time = 0.91, size = 2320, normalized size = 2.83

method	result	size
risch	Expression too large to display	2173
derivativedivides	Expression too large to display	2320
default	Expression too large to display	2320

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*b*c^2*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^
2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/4*b*c^3*(d*e)^(1/2)/d/e*arcta
nh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^
```

```
2*d-e)+1/4*b*c^2*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*arctan(c*x)^2+1/2*b*c^5*arct
an(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x-1/4*b*c^5*(d*e)^(1/2)/e*arctanh(1/4*(
2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2
+1/4*b*c*(d*e)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2
*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-1/4*b*c^2/d/(c^2*d-e)^2*e*ln(c^2*d*(1+
I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x
^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+b*c^2/d/(c^2*d-e)^2*e*ln((1+I*
c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b*c^4*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)
+3/4*I*b*c^4*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/
2)-e))*arctan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/2*a*
c^3*x/d/(c^2*e*x^2+c^2*d)-3/8*b*c^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2
*d*e)^(1/2)*e-1/4*b*c^6*d*arctan(c*x)^2/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)
*(c^2*d*e)^(1/2)-3/4*b*c^2*arctan(c*x)^2/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2
)*(c^2*d*e)^(1/2)*e-1/8*b*c^6*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d+2*(c^2*d*e)^(1/2)-e))*d/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e
)^(1/2)+1/4*I*b*e^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d
*e)^(1/2)-e))*arctan(c*x)/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(
1/2)+1/4*I*b*c^2*(c^2*d*e)^(1/2)/(c^2*d-e)/d/e*arctan(c*x)*ln(1-(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/2*b*c^3*arctan(c*x)/
d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*e*x+1/4*b*c*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*
(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+1
/8*b*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1
/2)-e))/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/4*b*e^2*arc
tan(c*x)^2/d^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/4*I*b*(c
^2*d*e)^(1/2)/d^2/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2
+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/2*a*c/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2))-1/4*b*(c^2*d*e)^(1/2)/d^2/(c^2*d-e)*arctan(c*x)^2-1/8*b*(c^2*d*e)^(1/2
)/d^2/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*
d*e)^(1/2)-e))+3/4*b*c^4*arctan(c*x)^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c
^2*d*e)^(1/2)+3/8*b*c^4*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d
+2*(c^2*d*e)^(1/2)-e))/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/
4*b*c^4/(c^2*d-e)^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/
(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e
)-b*c^4/(c^2*d-e)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b*c^4*arctan(c*x)
/d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*e*x^2-3/4*I*b*c^2*ln(1-(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)*e/(c^2*d-e)/d/(c^4*d
^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/4*I*b*c^6*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c
^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/e/(c^2*d-e)/(c^4*d^2-2*
c^2*d*e+e^2)*(c^2*d*e)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) + x/(d*x^2*e + d^2)) + 2*
b*integrate(1/2*arctan(c*x)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{\left(d + ex^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x^2)^2, x)
```

3.1164 $\int \frac{a+b\text{ArcTan}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=1382

$$-\frac{a+b\text{ArcTan}(cx)}{d^2x}-\frac{ex(a+b\text{ArcTan}(cx))}{2d^2(d+ex^2)}-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}\,(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}}$$

```
[Out] (-a-b*arctan(c*x))/d^2/x-1/2*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)+b*c*ln(x)/
d^2-1/2*b*c*ln(c^2*x^2+1)/d^2+1/4*b*c*e*ln(c^2*x^2+1)/d^2/(c^2*d-e)-1/4*b*c
*e*ln(e*x^2+d)/d^2/(c^2*d-e)-a*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)-1/
2*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)-1/4*I*b*ln(1-
I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(
5/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d
^(1/2)-I*e^(1/2)))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)+1/8*I*b*c*ln(-(1-x*(-c^2)^(
1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))*e
^(1/2)/d^(5/2)/(-c^2)^(1/2)-1/8*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^
2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(-c^2)
^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)
*d^(1/2)+I*e^(1/2)))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)-1/4*I*b*ln(1+I*c*x)*ln(c*
((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/8*I*b
*c*ln(-(1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*
x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)+1/4*I*b*polylog(2,(1+I*c*x)
*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*I*b*polylog(2,(1-
I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/8*I*b*c*polyl
og(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))*e
^(1/2)/d^(5/2)/(-c^2)^(1/2)-1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2)
+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)-1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(
1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/
2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))*e^(1/2)/d^(5/2)/(-c^2)^(1
/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)
))*e^(1/2)/(-d)^(5/2)-1/8*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/
2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)
+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*
e^(1/2)/(-d)^(5/2)
```

Rubi [A]

time = 1.23, antiderivative size = 1382, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {5100, 4946, 272, 36, 29, 31, 205, 211, 5032, 6857, 455, 5028, 2456, 2441, 2440, 2438, 5030}

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]

[Out] -((a + b*ArcTan[c*x])/(d^2*x)) - (e*x*(a + b*ArcTan[c*x]))/(2*d^2*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(5/2) - (Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)) + (b*c*Log[x])/d^2 + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*e*Log[1 + c^2*x^2])/(4*d^2*(c^2*d - e)) - (b*c*e*Log[d + e*x^2])/(4*d^2*(c^2*d - e)) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2))

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^2x^2} - \frac{e(a+b\tan^{-1}(cx))}{d(d+ex^2)^2} - \frac{e(a+b\tan^{-1}(cx))}{d^2(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^2}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{\sqrt{e}\,(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}} +$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))}{2d^2(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{e}\,(a+b\tan^{-1}}{}$$

Mathematica [A]
time = 12.53, size = 992, normalized size = 0.72

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - (a*e*x)/(2*d^2*(d + e*x^2)) - (3*a*Sqrt[e]*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(2*d^(5/2)) + b*c^5*(-(ArcTan[c*x]/(c^5*d^2*x)) + Log[(c*x)/Sqr
t[1 + c^2*x^2]]/(c^4*d^2) - (e*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/
(c^2*d + e)])/(4*c^4*d^2*(c^2*d - e)) - (3*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(
Sqrt[-(c^2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c
*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c
^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)
]*x))] + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[
-(c^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*
Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (A
rcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*
x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(S
qrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[
c*x]]])] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcTanh[(c*d)/(Sqrt
[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(
c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*
Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])
*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2
*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2
*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))]))
)/(8*c^4*d^2*Sqrt[-(c^2*d*e)]) - (e*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(2*c^4*
d^2*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Cos[2*ArcTan[c*x]])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 3849, normalized size = 2.79

method	result	size
risch	Expression too large to display	2555
derivativedivides	Expression too large to display	3849
default	Expression too large to display	3849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-3/2*a/c/d^2*e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+3/8*b*c^3*(d*e)^(1/2)
/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^
```

(1/2))/(c^2*d-e)-3/4*b*c^5*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+5/16*b*c*(d*e)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+5/16*b*c^2/d/(c^2*d-e)^2*e*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+5*b*c^2/d/(c^2*d-e)^2*e*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*a/d^2*e*c*x/(c^2*e*x^2+c^2*d)-3/16*b/c^2/d^3/(c^2*d-e)^3*e^4*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-3/8*b/c^2/d^3/(c^2*d-e)*e^2*sum((_R1^2*c^2*d-_R1^2*e+3*c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))+3/8*b/c^2/d^3/(c^2*d-e)*e^2*sum((_R1^2*c^2*d-_R1^2*e-c^2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))+I*b*c^4*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)+17/16*b*c^2/d/(c^2*d-e)^3*e^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+3/8*b/c^2/d^3/(c^2*d-e)*e^2*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+3/16*b/c^2/d^3/(c^2*d-e)^2*e^3*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+I*b*c^4*arctan(c*x)/(c^2*d-e)/d/(c^2*e*x^2+c^2*d)*x^2*e-3/2*I*b*c^2*arctan(c*x)/(c^2*d-e)/d/(c^2*e*x^2+c^2*d)*e+3/2*b*arctan(c*x)/(c^2*d-e)/d^2/(c^2*e*x^2+c^2*d)*c*x*e^2+b*c*arctan(c*x)/(c^2*d-e)/d/(c^2*e*x^2+c^2*d)/x*e-3/2*I*b*arctan(c*x)/(c^2*d-e)/d^2/(c^2*e*x^2+c^2*d)*c^2*x^2*e^2-3/2*b*c^3*arctan(c*x)/d/(c^2*d-e)/(c^2*e*x^2+c^2*d)*e*x+2*b*c*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2-a/d^2/c/x-b*c^3*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)/x+b*c*(d*e)^(1/2)/d^2*e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-9/8*b/c*(d*e)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-7/4*b/c*(d*e)^(1/2)/d^3*e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^2+3/8*b*c^7*(d*e)^(1/2)*d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-1/4*b/c*(d*e)^(1/2)/d^3*e^3*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-1/8*b*c^3*(d*e)^(1/2)/d*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3-3/16*b/c^3*(d*e)^(1/2)/d^4*e^4*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)^3+3/16*b/c^3*(d*e)^(1/2)/d^4*e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)-1/16*b/d^2/(c^2*d-e)^3*e^3*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-b/d^2/(c^2*d-e)*e*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*b/d^2/(c^2*d-e)^2*ln(c^2*d*(1+I*

```
c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2
+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)*e^2-b/d^2/(c^2*d-e)*e*ln((1+I*c*
x)/(c^2*x^2+1)^(1/2)-1)+3/8*b/d^2/(c^2*d-e)*e*sum((_R1^2*c^2*d-_R1^2*e+3*c^
2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*
x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf
((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))-3*b/d^2/(c^2*d-e)^2*e^2*ln((1+
I*c*x)/(c^2*x^2+1)^(1/2))-3/8*b/d^2/(c^2*d-e)*e*sum((_R1^2*c^2*d-_R1^2*e-c^
2*d+e)/(_R1^2*c^2*d-_R1^2*e+c^2*d+e)*(I*arctan(c*x)*ln((_R1-(1+I*c*x)/(c^2*
x^2+1)^(1/2))/_R1)+dilog((_R1-(1+I*c*x)/(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf
((c^2*d-e)*_Z^4+(2*c^2*d+2*e)*_Z^2+c^2*d-e))+3/8*b*c^6*d/(c^2*d-e)^3*ln(c^2
*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/
(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-19/16*b*c^4/(c^2*d-e)^3*
ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c
*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)*e-13/16*b*c^5*(d*e
)^(1/2)*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*
e)^(1/2))/(c^2*d-e)^3-1/4*b*c^2/d/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1
)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(
1+I*c*x)^2/(c^2*x^2+1)-e)+b*c^2/d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-
1)+b*c^2/d/(c^2*d-e)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b*c^3*(d*e)^(1/2
)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*x^2*e + 2*d)/(d^2*x^3*e + d^3*x) + 3*arctan(x*e^(1/2)/sqrt(d))*e
^(1/2)/d^(5/2)) + 2*b*integrate(1/2*arctan(c*x)/(x^6*e^2 + 2*d*x^4*e + d^2*
x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^6*e^2 + 2*d*x^4*e + d^2*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2), x)
```

3.1165 $\int \frac{x^5(a+b\textbf{ArcTan}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=532

$$-\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)}+\frac{bc^4d^2\text{ArcTan}(cx)}{4(c^2d-e)^2e^3}-\frac{bc^2d\text{ArcTan}(cx)}{(c^2d-e)e^3}-\frac{d^2(a+b\text{ArcTan}(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\text{ArcTan}(cx))}{e^3(d+ex^2)}$$

```
[Out] -1/8*b*c*d*x/(c^2*d-e)/e^2/(e*x^2+d)+1/4*b*c^4*d^2*arctan(c*x)/(c^2*d-e)^2/
e^3-b*c^2*d*arctan(c*x)/(c^2*d-e)/e^3-1/4*d^2*(a+b*arctan(c*x))/e^3/(e*x^2+
d)^2+d*(a+b*arctan(c*x))/e^3/(e*x^2+d)-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^
3+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/
2)-I*e^(1/2)))/e^3+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^3+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e^3-1/
4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1
/2)))/e^3-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^
(1/2)+I*e^(1/2)))/e^3+b*c*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d-e)/e^(5/
2)-1/8*b*c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d-e)^2/e^(5/2
)
```

Rubi [A]
time = 0.47, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5100, 5094, 425, 536, 209, 211, 400, 4966, 2449, 2352, 2497}

$$-\frac{d^2(a+b\text{ArcTan}(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\text{ArcTan}(cx))}{e^3(d+ex^2)}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^3}+\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^3}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^3}-\frac{bc\sqrt{d}\,(3c^2d-e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8e^{5/2}(c^2d-e)^2}+\frac{bc\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(c^2d-e)}-\frac{bc^2d\text{ArcTan}(cx)}{e^3(c^2d-e)}+\frac{bc^4d^2\text{ArcTan}(cx)}{4e^3(c^2d-e)^2}-\frac{bcdx}{8e^2(c^2d-e)(d+ex^2)}-\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e^3}-\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4e^3}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*d*x)/((c^2*d - e)*e^2*(d + e*x^2)) + (b*c^4*d^2*ArcTan[c*x])/(4*(
c^2*d - e)^2*e^3) - (b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^3) - (d^2*(a + b*A
rcTan[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcTan[c*x]))/(e^3*(d + e*x^
2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*e^(5/2)) - (b*
c*Sqrt[d]*(3*c^2*d - e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*(c^2*d - e)^2*e^(5/
2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^3 + ((a + b*ArcTan[c*x])*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*e^3) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(2*e^3) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)
])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d]
- I*Sqrt[e])*(1 - I*c*x))])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] +
Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^5(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \left(\frac{d^2x(a+b\tan^{-1}(cx))}{e^2(d+ex^2)^3} - \frac{2dx(a+b\tan^{-1}(cx))}{e^2(d+ex^2)^2} + \frac{x(a+b\tan^{-1}(cx))}{e^2(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx}{e^2} + \frac{d^2\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx}{e^2}$$

$$= -\frac{d^2(a+b\tan^{-1}(cx))}{4e^3(d+ex^2)^2} + \frac{d(a+b\tan^{-1}(cx))}{e^3(d+ex^2)} - \frac{(bcd)\int \frac{1}{(1+c^2x^2)(d+ex^2)}\,dx}{e^3} + \frac{(bcd^2)}{}$$

$$= -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)} - \frac{d^2(a+b\tan^{-1}(cx))}{4e^3(d+ex^2)^2} + \frac{d(a+b\tan^{-1}(cx))}{e^3(d+ex^2)} + \frac{(bcd)\int}{8}$$

$$= -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)} - \frac{bc^2d\tan^{-1}(cx)}{(c^2d-e)e^3} - \frac{d^2(a+b\tan^{-1}(cx))}{4e^3(d+ex^2)^2} + \frac{d(a+b\tan}{e^3(d+}$$

$$= -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)} + \frac{bc^4d^2\tan^{-1}(cx)}{4(c^2d-e)^2e^3} - \frac{bc^2d\tan^{-1}(cx)}{(c^2d-e)e^3} - \frac{d^2(a+b\tan^{-1}}{4e^3(d+ex^2}$$

$$= -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)} + \frac{bc^4d^2\tan^{-1}(cx)}{4(c^2d-e)^2e^3} - \frac{bc^2d\tan^{-1}(cx)}{(c^2d-e)e^3} - \frac{d^2(a+b\tan^{-1}}{4e^3(d+ex^2}$$

Mathematica [A]
time = 9.31, size = 589, normalized size = 1.11

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*((d*(3*d + 4*e*x^2))/(d + e*x^2)^2 + 2*Log[d + e*x^2]) + b*(-1/2*(c*d*e*
x)/((c^2*d - e)*(d + e*x^2)) + (c^2*d*(-3*c^2*d + 4*e)*ArcTan[c*x])/(-(c^2*
d) + e)^2 + (d*(3*d + 4*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*Sqrt[d]*(5*c
^2*d - 7*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^2*d) + e)^2) + 2*A
rcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan[c*x]*Log[(I*Sqrt[d])/
Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))
/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1
 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqr
```

```
t[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqrt[e] + x]*
Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d]
 - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt
[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(
c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d]
 + Sqrt[e])]))/(4*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 1001, normalized size = 1.88

method	result	size
derivativedivides	Expression too large to display	1001
default	Expression too large to display	1001
risch	Expression too large to display	1666

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(a*c^8*d/e^3/(c^2*e*x^2+c^2*d)+1/2*a*c^6/e^3*ln(c^2*e*x^2+c^2*d)-1/4*
a*c^10*d^2/e^3/(c^2*e*x^2+c^2*d)^2+b*c^8*arctan(c*x)*d/e^3/(c^2*e*x^2+c^2*d
)+1/2*b*c^6*arctan(c*x)/e^3*ln(c^2*e*x^2+c^2*d)-1/4*b*c^10*arctan(c*x)*d^2/
e^3/(c^2*e*x^2+c^2*d)^2-1/8*b*c^11*d^2/e^2/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)+
1/8*b*c^9*d/e/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)+5/8*b*c^9*d^2/e^2/(c^2*d-e)^2
/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-7/8*b*c^7*d/e/(c^2*d-e)^2/(d*e)^(1/2)*
arctan(e*x/(d*e)^(1/2))-3/4*b*c^10*d^2/e^3/(c^2*d-e)^2*arctan(c*x)+b*c^8*d/
e^2/(c^2*d-e)^2*arctan(c*x)-1/4*I*b*c^6/e^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I
*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/4*
I*b*c^6/e^3*ln(c^2*e*x^2+c^2*d)*ln(c*x-I)-1/4*I*b*c^6/e^3*ln(c^2*e*x^2+c^2*
d)*ln(I+c*x)-1/4*I*b*c^6/e^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b*c^6/e^3*ln(
I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I
*e*_Z+c^2*d-e,index=1))+1/4*I*b*c^6/e^3*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d
-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-1/4*I*b*c^6/e^3
*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*
_Z+c^2*d-e,index=2))-1/4*I*b*c^6/e^3*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,
index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b*c^6/e^3*ln
(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*
I*e*_Z+c^2*d-e,index=2))+1/4*I*b*c^6/e^3*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*
d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*e^(-3)*log(x^2*e + d) + (4*d*x^2*e + 3*d^2)/(x^4*e^5 + 2*d*x^2*e^4 +
 d^2*e^3))*a + 2*b*integrate(1/2*x^5*arctan(c*x)/(x^6*e^3 + 3*d*x^4*e^2 + 3
*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arctan(c*x) + a*x^5)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e +
 d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

3.1166 $\int \frac{x^3(a+b\text{ArcTan}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{bcx}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{b\text{ArcTan}(cx)}{4d\left(c^2d-e\right)^2}+\frac{x^4(a+b\text{ArcTan}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc(c^2d-3e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{d}\left(c^2d-e\right)^2e^{3/2}}$$

```
[Out] 1/8*b*c*x/(c^2*d-e)/e/(e*x^2+d)-1/4*b*arctan(c*x)/d/(c^2*d-e)^2+1/4*x^4*(a+
b*arctan(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d-3*e)*arctan(x*e^(1/2)/d^(1/2))/
(c^2*d-e)^2/e^(3/2)/d^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5096, 12, 481, 536, 211}

$$\frac{x^4(a+b\text{ArcTan}(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc(c^2d-3e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{d}\,e^{3/2}\left(c^2d-e\right)^2}-\frac{b\text{ArcTan}(cx)}{4d\left(c^2d-e\right)^2}+\frac{bcx}{8e\left(c^2d-e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x)/(8*(c^2*d - e)*e*(d + e*x^2)) - (b*ArcTan[c*x])/(4*d*(c^2*d - e)^2)
 + (x^4*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(c^2*d - 3*e)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*(c^2*d - e)^2*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{x^4(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2} - (bc)\int \frac{x^4}{4\,(d+c^2dx^2)\,(d+ex^2)^2}\,dx$$

$$= \frac{x^4(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{1}{4}(bc)\int \frac{x^4}{(d+c^2dx^2)\,(d+ex^2)^2}\,dx$$

$$= \frac{bcx}{8\,(c^2d-e)\,e\,(d+ex^2)} + \frac{x^4(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{(bc)\int \frac{d^2+d(c^2d-2e)x^2}{(d+c^2dx^2)(d+ex^2)}\,dx}{8d\,(c^2d-e)\,e}$$

$$= \frac{bcx}{8\,(c^2d-e)\,e\,(d+ex^2)} + \frac{x^4(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{(bc)\int \frac{1}{d+c^2dx^2}\,dx}{4\,(c^2d-e)^2} - \frac{(bc(c^2d-3e}{8\,(c^2d}$$

$$= \frac{bcx}{8\,(c^2d-e)\,e\,(d+ex^2)} - \frac{b\tan^{-1}(cx)}{4d\,(c^2d-e)^2} + \frac{x^4(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{bc(c^2d-3e)\tan}{8\sqrt{d}\,(c^2d-}$$

Mathematica [A]

time = 2.33, size = 158, normalized size = 1.22

$$\frac{\frac{2ad}{(d+ex^2)^2}+\frac{-4ac^2d+4ae+bcex}{(c^2d-e)(d+ex^2)}+\frac{2bc^2(c^2d-2e)\mathrm{ArcTan}(cx)}{(-c^2d+e)^2}-\frac{2b(d+2ex^2)\mathrm{ArcTan}(cx)}{(d+ex^2)^2}-\frac{bc(c^2d-3e)\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(-c^2d+e)^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((2*a*d)/(d + e*x^2)^2 + (-4*a*c^2*d + 4*a*e + b*c*e*x)/((c^2*d - e)*(d + e
*x^2)) + (2*b*c^2*(c^2*d - 2*e)*ArcTan[c*x])/(-(c^2*d) + e)^2 - (2*b*(d + 2
*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 - (b*c*(c^2*d - 3*e)*Sqrt[e]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*(-(c^2*d) + e)^2))/(8*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(114) = 228.
time = 0.38, size = 304, normalized size = 2.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^6*(-1/2/e^2/(c^2*e*x^2+c^2*d)+1/4*d*c^2/e^2/(c^2*e*x^2+c^2*d)^2)
-1/2*b*c^6*arctan(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*b*c^8*arctan(c*x)*d/e^2/(c
^2*e*x^2+c^2*d)^2+1/8*b*c^9*d/e/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)-1/8*b*c^7/(
c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)-1/8*b*c^7*d/e/(c^2*d-e)^2/(d*e)^(1/2)*arctan
(e*x/(d*e)^(1/2))+3/8*b*c^5/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
+1/4*b*c^8*d/e^2/(c^2*d-e)^2*arctan(c*x)-1/2*b*c^6/e/(c^2*d-e)^2*arctan(c*x
))
```

Maxima [A]
time = 0.47, size = 207, normalized size = 1.59

$$-\frac{1}{8}\left(\left(\left(\frac{(c^2d-3e)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{(c^4d^2e-2c^2de^2+e^3)\sqrt{d}}-\frac{x}{c^2d^2e+(c^2de^2-e^3)x^2-de^2}-\frac{2(c^4d-2c^2e)\arctan(cx)}{(c^4d^2e^2-2c^2de^3+e^4)c}\right)c+\frac{2(2x^2e+d)\arctan(cx)}{x^4e^4+2dx^2e^3+d^2e^2}\right)b-\frac{(2x^2e+d)a}{4(x^4e^4+2dx^2e^3+d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((c^2*d - 3*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^4*d^2*e - 2*c^2
*d*e^2 + e^3)*sqrt(d)) - x/(c^2*d^2*e + (c^2*d*e^2 - e^3)*x^2 - d*e^2) - 2*
(c^4*d - 2*c^2*e)*arctan(c*x)/((c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*c))*c + 2*
(2*x^2*e + d)*arctan(c*x)/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2))*b - 1/4*(2*x^2
*e + d)*a/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(117) = 234.
time = 2.34, size = 680, normalized size = 5.23

$$\left[-\frac{4ac^4d^4-(bc^3d^3-3bcx^4e^3+(bc^3dx^4-6bcdx^2)e^2+(2bc^3d^2x^2-3bcd^2)e)\sqrt{-de}\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)+4(2(bc^2dx^4+bdx^2)e^3-(bc^4d^2x^4-bd^2)e^2)\arctan(cx)+2(bcdx^3+4adx^2)e^3-2(bc^3d^2x^3+8ac^2d^2x^2-bcd^2x-2ad^2)e^2+2(4ac^4d^3x^2-bc^3d^3x-4ac^2d^3)e}{16(c^4d^5e^2+dx^4e^6-2(c^2d^2x^4-d^2x^2)e^5+(c^4d^3x^4-4c^2d^3x^2+d^3)e^4+2(c^4d^4x^2-c^2d^4)e^3)},\ -\frac{2ac^4d^4+(bc^3d^3-3bcx^4e^3+(bc^3dx^4-6bcdx^2)e^2+(2bc^3d^2x^2-3bcd^2)e)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+2(2(bc^2dx^4+bdx^2)e^3-(bc^4d^2x^4-bd^2)e^2)\arctan(cx)+(bcdx^3+4adx^2)e^3-(bc^3d^2x^3+8ac^2d^2x^2-bcd^2x-2ad^2)e^2+(4ac^4d^3x^2-bc^3d^3x-4ac^2d^3)e}{8(c^4d^5e^2+dx^4e^6-2(c^2d^2x^4-d^2x^2)e^5+(c^4d^3x^4-4c^2d^3x^2+d^3)e^4+2(c^4d^4x^2-c^2d^4)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - (b*c^3*d^3 - 3*b*c*x^4*e^3 + (b*c^3*d*x^4 - 6*b*c*d*x
^2)*e^2 + (2*b*c^3*d^2*x^2 - 3*b*c*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-
d*e)*x - d)/(x^2*e + d)) + 4*(2*(b*c^2*d*x^4 + b*d*x^2)*e^3 - (b*c^4*d^2*x^
4 - b*d^2)*e^2)*arctan(c*x) + 2*(b*c*d*x^3 + 4*a*d*x^2)*e^3 - 2*(b*c^3*d^2*
x^3 + 8*a*c^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2)*e^2 + 2*(4*a*c^4*d^3*x^2 - b*c
^3*d^3*x - 4*a*c^2*d^3)*e)/(c^4*d^5*e^2 + d*x^4*e^6 - 2*(c^2*d^2*x^4 - d^2*
x^2)*e^5 + (c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*e^4 + 2*(c^4*d^4*x^2 - c^2*d
^4)*e^3), -1/8*(2*a*c^4*d^4 + (b*c^3*d^3 - 3*b*c*x^4*e^3 + (b*c^3*d*x^4 - 6
*b*c*d*x^2)*e^2 + (2*b*c^3*d^2*x^2 - 3*b*c*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)
/sqrt(d))*e^(1/2) + 2*(2*(b*c^2*d*x^4 + b*d*x^2)*e^3 - (b*c^4*d^2*x^4 - b*d
^2)*e^2)*arctan(c*x) + (b*c*d*x^3 + 4*a*d*x^2)*e^3 - (b*c^3*d^2*x^3 + 8*a*c
^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2)*e^2 + (4*a*c^4*d^3*x^2 - b*c^3*d^3*x - 4*
a*c^2*d^3)*e)/(c^4*d^5*e^2 + d*x^4*e^6 - 2*(c^2*d^2*x^4 - d^2*x^2)*e^5 + (c
^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*e^4 + 2*(c^4*d^4*x^2 - c^2*d^4)*e^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 3.30, size = 273, normalized size = 2.10

$$\frac{b\,c^4\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,{\left(e-c^2\,d\right)}^2}-\frac{a\,d}{4\,e^2\,{\left(e\,x^2+d\right)}^2}-\frac{b\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,{\left(e\,x^2+d\right)}^2}-\frac{b\,c\,x^3}{8\,\left(e-c^2\,d\right)\,{\left(e\,x^2+d\right)}^2}-\frac{b\,c^2\,\mathrm{atan}(c\,x)}{2\,e\,{\left(e-c^2\,d\right)}^2}-\frac{b\,x^2\,\mathrm{atan}(c\,x)}{2\,e\,{\left(e\,x^2+d\right)}^2}-\frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1{}\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,1{}\mathrm{i}}{8\,e^3\,{\left(e-c^2\,d\right)}^2}-\frac{a\,x^2}{2\,e\,{\left(e\,x^2+d\right)}^2}-\frac{b\,c\,d\,x}{8\,e\,\left(e-c^2\,d\right)\,{\left(e\,x^2+d\right)}^2}+\frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1{}\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,3{}\mathrm{i}}{8\,d\,e^2\,{\left(e-c^2\,d\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] (b*c^4*d*atan(c*x))/(4*e^2*(e - c^2*d)^2) - (a*d)/(4*e^2*(d + e*x^2)^2) - (
b*d*atan(c*x))/(4*e^2*(d + e*x^2)^2) - (b*c*x^3)/(8*(e - c^2*d)*(d + e*x^2)
^2) - (b*c^2*atan(c*x))/(2*e*(e - c^2*d)^2) - (b*x^2*atan(c*x))/(2*e*(d + e
*x^2)^2) - (b*c^3*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1/2)*1i)/(8*e
^3*(e - c^2*d)^2) - (a*x^2)/(2*e*(d + e*x^2)^2) - (b*c*d*x)/(8*e*(e - c^2*d
)*(d + e*x^2)^2) + (b*c*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1/2)*3i
)/(8*d*e^2*(e - c^2*d)^2)
```

3.1167 $\int \frac{x(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^3} dx$

Optimal. Leaf size=131

$$-\frac{bcx}{8d(c^2d-e)(d+ex^2)}+\frac{bc^4\text{ArcTan}(cx)}{4(c^2d-e)^2e}-\frac{a+b\text{ArcTan}(cx)}{4e(d+ex^2)^2}-\frac{bc(3c^2d-e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}(c^2d-e)^2\sqrt{e}}$$

```
[Out] -1/8*b*c*x/d/(c^2*d-e)/(e*x^2+d)+1/4*b*c^4*arctan(c*x)/(c^2*d-e)^2/e+1/4*(-
a-b*arctan(c*x))/e/(e*x^2+d)^2-1/8*b*c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)/(c^2*d-e)^2/e^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5094, 425, 536, 209, 211}

$$-\frac{a+b\text{ArcTan}(cx)}{4e(d+ex^2)^2}-\frac{bc(3c^2d-e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}\sqrt{e}\,(c^2d-e)^2}+\frac{bc^4\text{ArcTan}(cx)}{4e(c^2d-e)^2}-\frac{bcx}{8d(c^2d-e)(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x)/(d*(c^2*d - e)*(d + e*x^2)) + (b*c^4*ArcTan[c*x])/(4*(c^2*d -
e)^2*e) - (a + b*ArcTan[c*x])/(4*e*(d + e*x^2)^2) - (b*c*(3*c^2*d - e)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*(c^2*d - e)^2*Sqrt[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{a+b\tan^{-1}(cx)}{4e\,(d+ex^2)^2}+\frac{(bc)\int\frac{1}{(1+c^2x^2)(d+ex^2)^2}\,dx}{4e}$$
$$= -\frac{bcx}{8d\,(c^2d-e)\,(d+ex^2)}-\frac{a+b\tan^{-1}(cx)}{4e\,(d+ex^2)^2}+\frac{(bc)\int\frac{2c^2d-e-c^2ex^2}{(1+c^2x^2)(d+ex^2)}\,dx}{8d\,(c^2d-e)\,e}$$
$$= -\frac{bcx}{8d\,(c^2d-e)\,(d+ex^2)}-\frac{a+b\tan^{-1}(cx)}{4e\,(d+ex^2)^2}-\frac{(bc(3c^2d-e))\int\frac{1}{d+ex^2}\,dx}{8d\,(c^2d-e)^2}+\frac{(bc^5)}{4\,(c}$$
$$= -\frac{bcx}{8d\,(c^2d-e)\,(d+ex^2)}+\frac{bc^4\tan^{-1}(cx)}{4\,(c^2d-e)^2\,e}-\frac{a+b\tan^{-1}(cx)}{4e\,(d+ex^2)^2}-\frac{bc(3c^2d-e)\tan^{-}}{8d^{3/2}\,(c^2d-}$$

Mathematica [A]
time = 0.76, size = 131, normalized size = 1.00

$$\frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx(d+ex^2)}{d(c^2d-e)}}{(d+ex^2)^2}+\frac{2b\left(\frac{c^4}{(-c^2d+e)^2}-\frac{1}{(d+ex^2)^2}\right)\text{ArcTan}(cx)}{e}-\frac{bc(3c^2d-e)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}\,(-c^2d+e)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

[Out] (-(((2*a)/e + (b*c*x*(d + e*x^2))/(d*(c^2*d - e)))/(d + e*x^2)^2) + (2*b*(c
^4/(-(c^2*d) + e)^2 - (d + e*x^2)^(-2))*ArcTan[c*x])/e - (b*c*(3*c^2*d - e)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]*(-(c^2*d) + e)^2))/8

Maple [A]
time = 0.34, size = 222, normalized size = 1.69

method	result
derivativedivides	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}-\frac{b\,c^6\arctan(cx)}{4\left(e\,c^2x^2+c^2d\right)^2e}-\frac{b\,c^7x}{8\left(c^2d-e\right)^2\left(e\,c^2x^2+c^2d\right)}+\frac{b\,c^5ex}{8\left(c^2d-e\right)^2d\left(e\,c^2x^2+c^2d\right)}-\frac{3b\,c^5\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\left(c^2d-e\right)^2\sqrt{de}}+\frac{b\,c^3e\,\mathrm{ar}}{8\left(c^2\right.}}{c^2}$
default	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}-\frac{b\,c^6\arctan(cx)}{4\left(e\,c^2x^2+c^2d\right)^2e}-\frac{b\,c^7x}{8\left(c^2d-e\right)^2\left(e\,c^2x^2+c^2d\right)}+\frac{b\,c^5ex}{8\left(c^2d-e\right)^2d\left(e\,c^2x^2+c^2d\right)}-\frac{3b\,c^5\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\left(c^2d-e\right)^2\sqrt{de}}+\frac{b\,c^3e\,\mathrm{ar}}{8\left(c^2\right.}}{c^2}$
risch	$\frac{ic^8b\ln(-icx+1)e\,x^4}{8(-e\,c^2x^2-c^2d)^2(c^2d-e)^2}-\frac{ic^6bd}{16(c^2d-e)^2e(-e\,c^2x^2-c^2d)}+\frac{c^5bx}{16(c^2d-e)^2(-e\,c^2x^2-c^2d)}-\frac{ic^4b\ln(-icx+1)e}{8(-e\,c^2x^2-c^2d)^2(c^2d-e)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(-1/4*a*c^6/e/(c^2*e*x^2+c^2*d)^2-1/4*b*c^6/(c^2*e*x^2+c^2*d)^2*arcta
n(c*x)/e-1/8*b*c^7/(c^2*d-e)^2*x/(c^2*e*x^2+c^2*d)+1/8*b*c^5*e/(c^2*d-e)^2*
x/d/(c^2*e*x^2+c^2*d)-3/8*b*c^5/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2))+1/8*b*c^3*e/(c^2*d-e)^2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/4*b*c^
6/e/(c^2*d-e)^2*arctan(c*x))

Maxima [A]
time = 0.47, size = 183, normalized size = 1.40

$$\frac{1}{8}\left(\left(\left(\frac{2\,c^3\arctan\left(cx\right)}{c^4d^2e-2\,c^2de^2+e^3}-\frac{\left(3\,c^2d-e\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{\left(c^4d^3-2\,c^2d^2e+de^2\right)\sqrt{d}}-\frac{x}{c^2d^3+\left(c^2d^2e-de^2\right)x^2-d^2e}\right)c-\frac{2\arctan\left(cx\right)}{x^4e^3+2\,dx^2e^2+d^2e}\right)b-\frac{a}{4\left(x^4e^3+2\,dx^2e^2+d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8*((2*c^3*arctan(c*x)/(c^4*d^2*e - 2*c^2*d*e^2 + e^3) - (3*c^2*d - e)*arc
tan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^4*d^3 - 2*c^2*d^2*e + d*e^2)*sqrt(d)) -
 x/(c^2*d^3 + (c^2*d^2*e - d*e^2)*x^2 - d^2*e))*c - 2*arctan(c*x)/(x^4*e^3
+ 2*d*x^2*e^2 + d^2*e))*b - 1/4*a/(x^4*e^3 + 2*d*x^2*e^2 + d^2*e)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(117) = 234.
time = 2.50, size = 626, normalized size = 4.78

$$\left[-\frac{4\,ac^4d^4-2\,bcdx^3e^3-\left(3\,bc^3d^3-bcx^4e^3+\left(3\,bc^3dx^4-2\,bcdx^2\right)e^2+\left(6\,bc^3d^2x^2-bcd^2\right)e\right)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)-4\left(\left(bc^4d^2x^4-bd^2\right)e^2+2\left(bc^4d^3x^2+bc^2d^3\right)e\right)\arctan\left(cx\right)+2\left(bc^3d^2x^3-bcd^2x+2\,ad^2\right)e^2+2\left(bc^3d^3x-4\,ac^2d^3\right)e}{16\left(c^4d^6e+d^2x^4e^5-2\left(c^2d^3x^4-d^3x^2\right)e^4+\left(c^4d^4x^4-4\,c^2d^4x^2+d^4\right)e^3+2\left(c^4d^5x^2-c^2d^5\right)e^2\right)},-\frac{2\,ac^4d^4-bcdx^3e^3+\left(3\,bc^3d^3-bcx^4e^3+\left(3\,bc^3dx^4-2\,bcdx^2\right)e^2+\left(6\,bc^3d^2x^2-bcd^2\right)e\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-2\left(\left(bc^4d^2x^4-bd^2\right)e^2+2\left(bc^4d^3x^2+bc^2d^3\right)e\right)\arctan\left(cx\right)+\left(bc^3d^2x^3-bcd^2x+2\,ad^2\right)e^2+\left(bc^3d^3x-4\,ac^2d^3\right)e}{8\left(c^4d^6e+d^2x^4e^5-2\left(c^2d^3x^4-d^3x^2\right)e^4+\left(c^4d^4x^4-4\,c^2d^4x^2+d^4\right)e^3+2\left(c^4d^5x^2-c^2d^5\right)e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 2*b*c*d*x^3*e^3 - (3*b*c^3*d^3 - b*c*x^4*e^3 + (3*b*c
^3*d*x^4 - 2*b*c*d*x^2)*e^2 + (6*b*c^3*d^2*x^2 - b*c*d^2)*e)*sqrt(-d*e)*log
((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 4*((b*c^4*d^2*x^4 - b*d^2)*e^2
 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arctan(c*x) + 2*(b*c^3*d^2*x^3 - b*c*d^
2*x + 2*a*d^2)*e^2 + 2*(b*c^3*d^3*x - 4*a*c^2*d^3)*e)/(c^4*d^6*e + d^2*x^4*
e^5 - 2*(c^2*d^3*x^4 - d^3*x^2)*e^4 + (c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*e
^3 + 2*(c^4*d^5*x^2 - c^2*d^5)*e^2), -1/8*(2*a*c^4*d^4 - b*c*d*x^3*e^3 + (3
*b*c^3*d^3 - b*c*x^4*e^3 + (3*b*c^3*d*x^4 - 2*b*c*d*x^2)*e^2 + (6*b*c^3*d^2
*x^2 - b*c*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 2*((b*c^4*d^
2*x^4 - b*d^2)*e^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arctan(c*x) + (b*c^3*
d^2*x^3 - b*c*d^2*x + 2*a*d^2)*e^2 + (b*c^3*d^3*x - 4*a*c^2*d^3)*e)/(c^4*d^
6*e + d^2*x^4*e^5 - 2*(c^2*d^3*x^4 - d^3*x^2)*e^4 + (c^4*d^4*x^4 - 4*c^2*d^
4*x^2 + d^4)*e^3 + 2*(c^4*d^5*x^2 - c^2*d^5)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 2.61, size = 201, normalized size = 1.53

$$\frac{b\,c\,x}{8\,\left(e-c^2\,d\right)\,{\left(e\,x^2+d\right)}^2}-\frac{b\,\mathrm{atan}\left(c\,x\right)}{4\,e\,{\left(e\,x^2+d\right)}^2}-\frac{a}{4\,e\,{\left(e\,x^2+d\right)}^2}+\frac{b\,c^4\,\mathrm{atan}\left(c\,x\right)}{4\,e\,{\left(e-c^2\,d\right)}^2}+\frac{b\,c\,e\,x^3}{8\,d\,\left(e-c^2\,d\right)\,{\left(e\,x^2+d\right)}^2}+\frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1{}\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,1{}\mathrm{i}}{8\,d^3\,{\left(e-c^2\,d\right)}^2}-\frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1{}\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,3{}\mathrm{i}}{8\,d^2\,e\,{\left(e-c^2\,d\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] (b*c*x)/(8*(e - c^2*d)*(d + e*x^2)^2) - (b*atan(c*x))/(4*e*(d + e*x^2)^2) -
 a/(4*e*(d + e*x^2)^2) + (b*c^4*atan(c*x))/(4*e*(e - c^2*d)^2) + (b*c*atan(
(x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*1i)/(8*d^3*(e - c^2*d)^2) - (b*c^
3*atan((x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*3i)/(8*d^2*e*(e - c^2*d)^2
) + (b*c*e*x^3)/(8*d*(e - c^2*d)*(d + e*x^2)^2)
```

3.1168 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex^2)^3}\,dx$

Optimal. Leaf size=574

$$\frac{bcex}{8d^2(c^2d-e)(d+ex^2)}-\frac{bc^4\text{ArcTan}(cx)}{4d(c^2d-e)^2}-\frac{bc^2\text{ArcTan}(cx)}{2d^2(c^2d-e)}+\frac{a+b\text{ArcTan}(cx)}{4d(d+ex^2)^2}+\frac{a+b\text{ArcTan}(cx)}{2d^2(d+ex^2)}+\frac{bc\sqrt{e}\,\text{Ar}}{2d^{5/2}}$$

```
[Out] 1/8*b*c*e*x/d^2/(c^2*d-e)/(e*x^2+d)-1/4*b*c^4*arctan(c*x)/d/(c^2*d-e)^2-1/2
*b*c^2*arctan(c*x)/d^2/(c^2*d-e)+1/4*(a+b*arctan(c*x))/d/(e*x^2+d)^2+1/2*(a
+b*arctan(c*x))/d^2/(e*x^2+d)+a*ln(x)/d^3+(a+b*arctan(c*x))*ln(2/(1-I*c*x))
/d^3-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^
(1/2)-I*e^(1/2)))/d^3-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(
1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3+1/2*I*b*polylog(2,-I*c*x)/d^3-1/2*I*
b*polylog(2,I*c*x)/d^3-1/2*I*b*polylog(2,1-2/(1-I*c*x))/d^3+1/4*I*b*polylog
(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3+1/4
*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/
2)))/d^3+1/2*b*c*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(c^2*d-e)+1/8*b*
c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(c^2*d-e)^2
```

Rubi [A]
time = 0.45, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5100, 4940, 2438, 5094, 425, 536, 209, 211, 400, 4966, 2449, 2352, 2497}

$$-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^3}-\frac{(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^3}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^3}+\frac{a+b\text{ArcTan}(cx)}{2d^2(d+ex^2)}+\frac{a+b\text{ArcTan}(cx)}{4d(d+ex^2)^2}+\frac{a\log(x)}{d^3}+\frac{bc\sqrt{e}(3c^2d-e)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}(c^2d-e)^2}+\frac{bc\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)}-\frac{bc^2\text{ArcTan}(cx)}{2d^2(c^2d-e)}-\frac{bc^4\text{ArcTan}(cx)}{4d(c^2d-e)^2}+\frac{bcex}{8d^2(c^2d-e)(d+ex^2)}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^3}+\frac{ib\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4d^3}+\frac{ib\text{Li}_2(-icx)}{2d^3}-\frac{ib\text{Li}_2(icx)}{2d^3}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] (b*c*e*x)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (b*c^4*ArcTan[c*x])/(4*d*(c^2*d
 - e)^2) - (b*c^2*ArcTan[c*x])/(2*d^2*(c^2*d - e)) + (a + b*ArcTan[c*x])/(4
*d*(d + e*x^2)^2) + (a + b*ArcTan[c*x])/(2*d^2*(d + e*x^2)) + (b*c*Sqrt[e]*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*(c^2*d - e)) + (b*c*(3*c^2*d - e)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*(c^2*d - e)^2) + (a*Log[x])/
d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - ((a + b*ArcTan[c*x])*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*d^3) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(2*d^3) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 -
((I/2)*b*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt
[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{d\left(d+ex^2\right)^3} - \frac{ex(a+b\tan^{-1}(cx))}{d^2\left(d+ex^2\right)^2} - \frac{ex(a+b\tan^{-1}}{d^3\left(d+ex\right.}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^3} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx}{d^2} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))}{\left(d+ex^2\right)^3}}{d}$$

$$= \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{a\log(x)}{d^3} + \frac{(ib)\int \frac{\log(1-icx)}{x}\,dx}{2d^3} - \frac{(ib)\int \frac{\log(1+icx)}{x}}{2d^3}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{a\log(x)}{d^3} + \frac{ib\mathrm{Li}_2(-i}{2d^3}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\tan^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{bc\sqrt{}}{}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^4\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b}{2d^2}$$

$$= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{bc^4\tan^{-1}(cx)}{4d\left(c^2d-e\right)^2} - \frac{bc^2\tan^{-1}(cx)}{2d^2\left(c^2d-e\right)} + \frac{a+b\tan^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b}{2d^2}$$

Mathematica [A]
time = 9.47, size = 645, normalized size = 1.12

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] (2*a*((d*(3*d + 2*e*x^2))/(d + e*x^2)^2 + 4*Log[x] - 2*Log[d + e*x^2]) + b*
((c*d*e*x)/((c^2*d - e)*(d + e*x^2)) + (2*c^2*d*(-3*c^2*d + 2*e)*ArcTan[c*x
])/(-(c^2*d) + e)^2 + (2*d*(3*d + 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*
Sqrt[d]*(7*c^2*d - 5*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(-(c^2*d) + e)
^2 + 8*ArcTan[c*x]*Log[x] - 4*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] -
 4*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - (2*I)*Log[((-I)*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*Log[((-I
)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] +
```

```
(2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] -
Sqrt[e])] - (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c
*Sqrt[d] + Sqrt[e])] - (4*I)*(Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - Po
lyLog[2, (-I)*c*x] + PolyLog[2, I*c*x]) + (2*I)*PolyLog[2, (c*(Sqrt[d] - I*
Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[
e]*x))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x)
)/(c*Sqrt[d] - Sqrt[e])] + (2*I)*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*
Sqrt[d] + Sqrt[e])]))/(8*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 1041, normalized size = 1.81

method	result	size
derivativedivides	Expression too large to display	1041
default	Expression too large to display	1041
risch	Expression too large to display	2236

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*c^5*e/(c^2*d-e)^2*x/d/(c^2*e*x^2+c^2*d)+1/2*b*c^2/d^2/(c^2*d-e)^2*arc
tan(c*x)*e-1/2*a/d^3*ln(c^2*e*x^2+c^2*d)-5/8*b*c/d^2/(c^2*d-e)^2*e^2/(d*e)^
(1/2)*arctan(e*x/(d*e)^(1/2))+7/8*b*c^3*e/(c^2*d-e)^2/d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))-1/8*b*c^3/d^2/(c^2*d-e)^2*e^2*x/(c^2*e*x^2+c^2*d)+a/d^3*ln
(c*x)-3/4*b*c^4*arctan(c*x)/d/(c^2*d-e)^2+1/2*I*b/d^3*dilog(1+I*c*x)-1/4*I*
b/d^3*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2
*I*e*_Z+c^2*d-e,index=2))-1/2*I*b/d^3*dilog(1-I*c*x)+1/4*I*b/d^3*dilog((Roo
tOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,
index=1))-1/4*I*b/d^3*dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)
/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b/d^3*dilog((RootOf(e*_Z^2+
2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1
/2*b*arctan(c*x)/d^3*ln(c^2*e*x^2+c^2*d)+b*arctan(c*x)/d^3*ln(c*x)+1/2*a*c^
2/d^2/(c^2*e*x^2+c^2*d)+1/4*a*c^4/d/(c^2*e*x^2+c^2*d)^2+1/2*b*c^2*arctan(c*
x)/d^2/(c^2*e*x^2+c^2*d)+1/4*b*c^4*arctan(c*x)/d/(c^2*e*x^2+c^2*d)^2-1/4*I*
b/d^3*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e
*_Z^2-2*I*e*_Z+c^2*d-e,index=2))+1/4*I*b/d^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*
I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/4
*I*b/d^3*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+1/4*I*b/d^3*ln(I+c*x)*ln(c^2*e*x^2+c^2*
d)+1/4*I*b/d^3*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)
/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+1/2*I*b/d^3*ln(c*x)*ln(1+I*c*x)-1
/2*I*b/d^3*ln(c*x)*ln(1-I*c*x)-1/4*I*b/d^3*ln(c*x-I)*ln(c^2*e*x^2+c^2*d)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*x^2*e + 3*d)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) - 2*log(x^2*e + d)
/d^3 + 4*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(x^7*e^3 + 3*d*x^5*e^2
 + 3*d^2*x^3*e + d^3*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x),
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)^3), x)
```

3.1169 $\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+ex^2)^3}\,dx$

Optimal. Leaf size=629

$$-\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)}-\frac{bc^2\text{ArcTan}(cx)}{2d^3}+\frac{bc^4e\text{ArcTan}(cx)}{4d^2(c^2d-e)^2}+\frac{bc^2e\text{ArcTan}(cx)}{d^3(c^2d-e)}-\frac{a+b\text{ArcTan}(cx)}{2d^3x^2}-$$

```
[Out] -1/2*b*c/d^3/x-1/8*b*c*e^2*x/d^3/(c^2*d-e)/(e*x^2+d)-1/2*b*c^2*arctan(c*x)/
d^3+1/4*b*c^4*e*arctan(c*x)/d^2/(c^2*d-e)^2+b*c^2*e*arctan(c*x)/d^3/(c^2*d-
e)+1/2*(-a-b*arctan(c*x))/d^3/x^2-1/4*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)^2-e
*(a+b*arctan(c*x))/d^3/(e*x^2+d)-b*c*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d^(7
/2)/(c^2*d-e)-1/8*b*c*(3*c^2*d-e)*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)
/(c^2*d-e)^2-3*a*e*ln(x)/d^4-3*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^4+3/2*
e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I
*e^(1/2)))/d^4+3/2*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^4-3/2*I*b*e*polylog(2,-I*c*x)/d^4+3/2*I*b*e
*polylog(2,1-2/(1-I*c*x))/d^4-3/4*I*b*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/
2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^4-3/4*I*b*e*polylog(2,1-2*c*((-d)
^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^4+3/2*I*b*e*polylog
(2,I*c*x)/d^4
```

Rubi [A]
time = 0.50, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5100, 4946, 331, 209, 4940, 2438, 5094, 425, 536, 211, 400, 4966, 2449, 2352, 2497}

$$-\frac{3e\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d^4}+\frac{3e(a+b\text{ArcTan}(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d^4}+\frac{3e(a+b\text{ArcTan}(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d^4}-\frac{e(a+b\text{ArcTan}(cx))}{d^3(d+ex^2)}-\frac{a+b\text{ArcTan}(cx)}{2d^3x^2}-\frac{e(a+b\text{ArcTan}(cx))}{4d^2(d+ex^2)^2}-\frac{3ae\log(x)}{d^4}-\frac{bce^{3/2}(3c^2d-e)\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}(c^2d-e)^2}-\frac{bce^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}(c^2d-e)}+\frac{bc^2e\text{ArcTan}(cx)}{d^3(c^2d-e)}-\frac{bc^2\text{ArcTan}(cx)}{2d^3}+\frac{bc^4e\text{ArcTan}(cx)}{4d^2(c^2d-e)^2}-\frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)}-\frac{3ibe\text{Li}_2(-icx)}{2d^4}+\frac{3ibe\text{Li}_2(icx)}{2d^4}+\frac{3ibe\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2d^4}-\frac{3ibe\text{Li}_2\left(1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{3ibe\text{Li}_2\left(1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{bc}{2d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] -1/2*(b*c)/(d^3*x) - (b*c*e^2*x)/(8*d^3*(c^2*d - e)*(d + e*x^2)) - (b*c^2*A
rcTan[c*x])/(2*d^3) + (b*c^4*e*ArcTan[c*x])/(4*d^2*(c^2*d - e)^2) + (b*c^2*
e*ArcTan[c*x])/(d^3*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^3*x^2) - (e*(a
+ b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcTan[c*x]))/(d^3*(d +
 e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(7/2)*(c^2*d - e))
- (b*c*(3*c^2*d - e)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*(c^2*d
 - e)^2) - (3*a*e*Log[x])/d^4 - (3*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
)/d^4 + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[
-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) + (3*e*(a + b*ArcTan[c*x])*Log[(2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) -
```

```
(((3*I)/2)*b*e*PolyLog[2, (-I)*c*x])/d^4 + (((3*I)/2)*b*e*PolyLog[2, I*c*x
])/d^4 + (((3*I)/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^4 - (((3*I)/4)*b*e
*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
 I*c*x))]])/d^4 - (((3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))
/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/d^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^3}-\frac{3e(a+b\tan^{-1}(cx))}{d^4x}+\frac{e^2x(a+b\tan^{-1}(cx))}{d^2\left(d+ex^2\right)^3}+\frac{2e^2x(a+b\tan}{d^3\left(d+e\right.}\right.$$

$$=\frac{\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx}{d^3}-\frac{(3e)\int \frac{a+b\tan^{-1}(cx)}{x}\,dx}{d^4}+\frac{\left(3e^2\right)\int \frac{x(a+b\tan^{-1}(cx))}{d+ex^2}\,dx}{d^4}+\frac{\left(2e^2\right)\int \frac{x(a+b}{(d}}{d^3}$$

$$=-\frac{a+b\tan^{-1}(cx)}{2d^3x^2}-\frac{e(a+b\tan^{-1}(cx))}{4d^2\left(d+ex^2\right)^2}-\frac{e(a+b\tan^{-1}(cx))}{d^3\left(d+ex^2\right)}-\frac{3ae\log(x)}{d^4}+\frac{(bc)\int}{}$$

$$=-\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{a+b\tan^{-1}(cx)}{2d^3x^2}-\frac{e(a+b\tan^{-1}(cx))}{4d^2\left(d+ex^2\right)^2}-\frac{e(a+b}{d^3\left(\right.}$$

$$=-\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{bc^2\tan^{-1}(cx)}{2d^3}+\frac{bc^2e\tan^{-1}(cx)}{d^3\left(c^2d-e\right)}-\frac{a+b\tan^{-1}(c}{2d^3x^2}$$

$$=-\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{bc^2\tan^{-1}(cx)}{2d^3}+\frac{bc^4e\tan^{-1}(cx)}{4d^2\left(c^2d-e\right)^2}+\frac{bc^2e\tan^{-1}(cx}{d^3\left(c^2d-e\right)}$$

$$=-\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{bc^2\tan^{-1}(cx)}{2d^3}+\frac{bc^4e\tan^{-1}(cx)}{4d^2\left(c^2d-e\right)^2}+\frac{bc^2e\tan^{-1}(cx}{d^3\left(c^2d-e\right)}$$

Mathematica [A]
time = 13.08, size = 723, normalized size = 1.15

$$\frac{-a\left(\frac{d(2d^2+9dex^2+6e^2x^4)}{x^2(d+ex^2)^2}+12e\log(x)-6e\log(d+ex^2)\right)+b\left(-\frac{2cd}{x}-\frac{cde^2x}{2(c^2d-e)(d+ex^2)}+\frac{c^2d(-2c^4d^2+9c^2de-6e^2)\text{ArcTan}(cx)}{(-c^2d+e)^2}-\frac{d(2d^2+9dex^2+6e^2x^4)\text{ArcTan}(cx)}{x^2(d+ex^2)^2}+\frac{c\sqrt{d}e^{3/2}(-11c^2d+9e)\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2(-c^2d+e)^2}-12e\text{ArcTan}(cx)\log(x)+6e\text{ArcTan}(cx)\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-\log(d+ex^2)\right)+6e\text{ArcTan}(cx)\log(d+ex^2)-6ie(\log(x)\log(1+icx)+\text{PolyLog}(2,-icx))+6ie(\log(x)\log(1-icx)+\text{PolyLog}(2,icx))-3ie\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(-1+icx)}{c\sqrt{d}-\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{c(\sqrt{d}-i\sqrt{e}x)}{c\sqrt{d}-\sqrt{e}}\right)\right)+3ie\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(1+icx)}{c\sqrt{d}+\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{c(\sqrt{d}-i\sqrt{e}x)}{c\sqrt{d}+\sqrt{e}}\right)\right)+3ie\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(-1-icx)}{c\sqrt{d}-\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{c(\sqrt{d}+i\sqrt{e}x)}{c\sqrt{d}-\sqrt{e}}\right)\right)-3ie\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}(1-icx)}{c\sqrt{d}+\sqrt{e}}\right)+\text{PolyLog}\left(2,\frac{c(\sqrt{d}+i\sqrt{e}x)}{c\sqrt{d}+\sqrt{e}}\right)\right)\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (-(a*((d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4))/(x^2*(d + e*x^2)^2) + 12*e*Log[x]
 - 6*e*Log[d + e*x^2])) + b*((-2*c*d)/x - (c*d*e^2*x)/(2*(c^2*d - e)*(d + e
*x^2)) + (c^2*d*(-2*c^4*d^2 + 9*c^2*d*e - 6*e^2)*ArcTan[c*x])/(-(c^2*d) + e
)^2 - (d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4)*ArcTan[c*x])/(x^2*(d + e*x^2)^2) +
 (c*Sqrt[d]*e^(3/2)*(-11*c^2*d + 9*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^
2*d) + e)^2) - 12*e*ArcTan[c*x]*Log[x] + 6*e*ArcTan[c*x]*(Log[((-I)*Sqrt[d]
)/Sqrt[e] + x] + Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[d + e*x^2]) + 6*e*ArcTa
n[c*x]*Log[d + e*x^2] - (6*I)*e*(Log[x]*Log[1 + I*c*x] + PolyLog[2, (-I)*c*
x]) + (6*I)*e*(Log[x]*Log[1 - I*c*x] + PolyLog[2, I*c*x]) - (3*I)*e*(Log[(I
*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] +
PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])]) + (3*I)*e*(L
og[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])
] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]) + (3*I)*
e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] -
Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])])
- (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqr
t[d] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt
[e])])))/(4*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 1183, normalized size = 1.88

method	result	size
derivativedivides	Expression too large to display	1183
default	Expression too large to display	1183
risch	Expression too large to display	1835

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(3/2*I*b/c^2/d^4*e*dilog(1-I*c*x)+9/4*b*c^2/d^2/(c^2*d-e)^2*arctan(c*x)
*e-11/8*b*c/d^2/(c^2*d-e)^2*e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/8*b*c
^3/d^2/(c^2*d-e)^2*e^2*x/(c^2*e*x^2+c^2*d)+3/4*I*b/c^2/d^4*e*dilog((RootOf(
e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,inde
x=1))-1/4*b*c^2*arctan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)^2+3/4*I*b/c^2/d^4*e*dil
og((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c
```

```
^2*d-e,index=2))-3/4*I*b/c^2/d^4*e*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,in
dex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))-3/2*I*b/c^2/d^4*e*di
log(1+I*c*x)+3/2*b/c^2*arctan(c*x)*e/d^4*ln(c^2*e*x^2+c^2*d)-3*b/c^2*arctan
(c*x)/d^4*e*ln(c*x)-3/4*I*b/c^2/d^4*e*dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e
,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-1/2*b*c^4*arctan(
c*x)/d/(c^2*d-e)^2-a*e/d^3/(c^2*e*x^2+c^2*d)-1/2*b/d^3/c/x-1/2*a/d^3/c^2/x^
2+3/2*I*b/c^2/d^4*e*ln(c*x)*ln(1-I*c*x)+3/4*I*b/c^2/d^4*e*ln(c^2*e*x^2+c^2*
d)*ln(c*x-I)+9/8*b/c/d^3*e^3/(c^2*d-e)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)-3/2*b/d^3/(c^2*d-e)^2*arctan(c*x)*e^2-b*arctan(c*x)*e/d^3/(c^2*e*x^2+c^2*
d)-1/4*a*c^2*e/d^2/(c^2*e*x^2+c^2*d)^2+3/2*a/c^2*e/d^4*ln(c^2*e*x^2+c^2*d)-
3*a/c^2/d^4*e*ln(c*x)-1/2*b*arctan(c*x)/d^3/c^2/x^2-3/4*I*b/c^2/d^4*e*ln(c*
x-I)*ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e
*_Z+c^2*d-e,index=1))-3/4*I*b/c^2/d^4*e*ln(c*x-I)*ln((RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))-3/4*I*b/
c^2/d^4*e*ln(c^2*e*x^2+c^2*d)*ln(I+c*x)+3/4*I*b/c^2/d^4*e*ln(I+c*x)*ln((Roo
tOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,
index=1))+3/4*I*b/c^2/d^4*e*ln(I+c*x)*ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,in
dex=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))-3/2*I*b/c^2/d^4*e*ln
(c*x)*ln(1+I*c*x)+1/8*b/d^3*e^3/(c^2*d-e)^2*c*x/(c^2*e*x^2+c^2*d))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*x^4*e^2 + 9*d*x^2*e + 2*d^2)/(d^3*x^6*e^2 + 2*d^4*x^4*e + d^5*x^
2) - 6*e*log(x^2*e + d)/d^4 + 12*e*log(x)/d^4) + 2*b*integrate(1/2*arctan(c
*x)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2*x^5*e + d^3*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2*x^5*e + d^3*x^3
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3), x)
```

3.1170 $\int \frac{x^2(a+b\textbf{ArcTan}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=966

$$\frac{bc}{8\left(c^2d-e\right)e\left(d+ex^2\right)}-\frac{x(a+b\text{ArcTan}(cx))}{4e\left(d+ex^2\right)^2}+\frac{x(a+b\text{ArcTan}(cx))}{8de\left(d+ex^2\right)}+\frac{(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}+\cdots$$

```
[Out] 1/8*b*c/(c^2*d-e)/e/(e*x^2+d)-1/4*x*(a+b*arctan(c*x))/e/(e*x^2+d)^2+1/8*x*(
a+b*arctan(c*x))/d/e/(e*x^2+d)+1/8*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(1/
2))/d^(3/2)/e^(3/2)+1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d/(c^2*d-e)^2/e-1/
4*b*c*ln(c^2*x^2+1)/d/(c^2*d-e)/e-1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d/(c^2
*d-e)^2/e+1/4*b*c*ln(e*x^2+d)/d/(c^2*d-e)/e-1/32*I*b*c*ln(-(1-x*(-c^2)^(1/2
))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(3
/2)/e^(3/2)/(-c^2)^(1/2)+1/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(
1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)+1/32*I
*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I
*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)+1/32*I*b*c*ln((1+x*(-c^2)^
(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/
d^(3/2)/e^(3/2)/(-c^2)^(1/2)-1/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x
*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)+1/
32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)
-I*e^(1/2)))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)-1/32*I*b*c*polylog(2,(-c^2)^(1/2)
*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(3/2)/e^(3/2)/(-
c^2)^(1/2)-1/32*I*b*c*ln(-(1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2
)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)
```

Rubi [A]
time = 1.75, antiderivative size = 966, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5100, 205, 211, 5032, 6857, 585, 78, 5028, 2456, 2441, 2440, 2438, 455, 36, 31}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c)/(8*(c^2*d - e)*e*(d + e*x^2)) - (x*(a + b*ArcTan[c*x]))/(4*e*(d + e*x
^2)^2) + (x*(a + b*ArcTan[c*x]))/(8*d*e*(d + e*x^2)) + ((a + b*ArcTan[c*x])
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e
]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*
x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 +
```

```
 Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]
])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*
x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[-c^2]*d^(3/2)*e^(3/2)) + (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(16*d*(c^2
*d - e)^2*e) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)*e) - (b*c*(5*c^2*d -
 3*e)*Log[d + e*x^2])/(16*d*(c^2*d - e)^2*e) + (b*c*Log[d + e*x^2])/(4*d*(c
^2*d - e)*e) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/
(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b
*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*
Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2
]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d
^(3/2)*e^(3/2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x
))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2))
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 78</u>

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

<u>Rule 205</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
```

```
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \left(-\frac{d(a+b\tan^{-1}(cx))}{e\,(d+ex^2)^3}+\frac{a+b\tan^{-1}(cx)}{e\,(d+ex^2)^2}\right)\,dx$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^2}\,dx}{e} - \frac{d\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^3}\,dx}{e}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$= -\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$= \frac{bc}{8\,(c^2d-e)\,e\,(d+ex^2)}-\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan}{}$$

$$= \frac{bc}{8\,(c^2d-e)\,e\,(d+ex^2)}-\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan}{}$$

$$= \frac{bc}{8\,(c^2d-e)\,e\,(d+ex^2)}-\frac{x(a+b\tan^{-1}(cx))}{4e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))}{8de\,(d+ex^2)}+\frac{(a+b\tan}{}$$

Mathematica [A]
time = 12.21, size = 1744, normalized size = 1.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + (b*c^3*((-2*Log[1 + ((c^2*d - e)*Cos[
2*ArcTan[c*x]])/(c^2*d + e)])/(c^2*d) - (2*Log[1 + ((c^2*d - e)*Cos[2*ArcTa
n[c*x]])/(c^2*d + e)])/e + ((c^2*d - e)*e*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^
2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqr
t[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x
)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/(
(c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d
 - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt
[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (A
rcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*
x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(S
qrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[
c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt
[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(
c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*
Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])
*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x
))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^
2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))])))/(-(c^2*d*e))^(
3/2) + ((c^2*d - e)*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*A
rcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcC
os[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*L
og[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c
*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTan
h[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x
))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c
^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/
Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*Ar
cTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((
c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Arc
Tanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[
c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])]
+ I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2
*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2
*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d -
e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))])))/(e*Sqrt[-(c^2*d*e)]) + (16*(c^2*d -
```

```
e)*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*
x]])^2 + (-8*c^2*d*e - 4*(c^4*d^2 - e^2)*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c
^2*d*e*(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]))))/(32*(-(c^2*d) + e)^2
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3829 vs. 2(750) = 1500.
time = 0.74, size = 3830, normalized size = 3.96

method	result	size
derivativedivides	Expression too large to display	3830
default	Expression too large to display	3830
risch	Expression too large to display	6982

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/8*a*c^3/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/16*I*b*c^2*(c^2*
d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/d^2*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)
^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/8*I*b*c^10/(c^4*d^2-2*c^2*d*
e+e^2)/(c^2*e*x^2+c^2*d)^2*d^2/e*arctan(c*x)+1/16*I*b*c^6*(c^2*d*e)^(1/2)/(
c^4*d^2-2*c^2*d*e+e^2)/e^2*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+
1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/16*b*c^9*(d*e)^(1/2)/e^2*d*arctanh(1/4*(
2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*
c^2*d*e+e^2)/(c^2*d-e)-1/16*b*c^3*(d*e)^(1/2)*e/d^2*arctanh(1/4*(2*(c^2*d-e
)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^
2)/(c^2*d-e)-1/8*b*c^7/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan
(c*x)*x+1/4*b*c^9/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)
*x-1/4*b*c^9/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^3+
1/8*b*c^11/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)*x^3-3/
8*b*c^6*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-3/16*b*c^6*
polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(
c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/8*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/
(c^2*e*x^2+c^2*d)^2*d-1/4*b*c^4/(c^4*d^2-2*c^2*d*e+e^2)*e/d/(c^2*d-e)*ln((1
+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*b*c^10/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^
2*d)^2*d*x^2-1/8*b*c^10/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*x^4-1
/8*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*x^2-1/16*b*c^8/(c^4*
d^2-2*c^2*d*e+e^2)/e*d/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d
*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/
(c^2*x^2+1)-e)+1/4*b*c^8*arctan(c*x)^2*d/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d
*e)^(1/2)+1/8*b*c^4*e*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2
*(c^2*d*e)^(1/2)-e))/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/32*b*c^2
*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-
e))/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*b*c^5*(d*e)^(1/2)/d*
arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2)
```

```
)/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)+1/8*b*c^8*polylog(2,(c^2*d-e)*(1+I*c*x)
^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c
^2*d*e)^(1/2)*d+1/4*b*c^4*arctan(c*x)^2/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*
e)^(1/2)*e+1/4*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/e*d/(c^2*d-e)*ln((1+I*c*x)/(c^
2*x^2+1)^(1/2))-1/8*b*c^5*(d*e)^(1/2)/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)-1/8*b*c
^4*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2+1/16*b*c^4/(c^
4*d^2-2*c^2*d*e+e^2)*e/d/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2
*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^
2/(c^2*x^2+1)-e)+1/16*b*c^7*(d*e)^(1/2)/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-
e)-1/16*b*c^4*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/16*b*c^10*d^2*ar
ctan(c*x)^2/e^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/32*b*c^10*d^2*p
olylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e^
2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*b*c^2*e^2*arctan(c*x)^2/d^
2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-3/8*I*b*c^6*ln(1-(c^2*d-e)*(1+I
*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^
2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/8*I*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2
+c^2*d)^2*d*arctan(c*x)-1/8*a*c^7/(c^2*e*x^2+c^2*d)^2/e*x+1/8*a*c^7/(c^2*e*
x^2+c^2*d)^2/d*x^3+1/32*b*c^2*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/d^2*p
olylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/
32*b*c^6*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/e^2*polylog(2,(c^2*d-e)*(1
+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/16*b*c^6*(c^2*d*e)^(1
/2)/(c^4*d^2-2*c^2*d*e+e^2)/e^2*arctan(c*x)^2-1/16*b*c^3*(d*e)^(1/2)/d^2*ar
ctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/
(c^4*d^2-2*c^2*d*e+e^2)+1/16*b*c^2*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/
d^2*arctan(c*x)^2-1/16*b*c^7*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*
c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)-1/8*
I*b*c^4*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/4*I*b*c^4*e*ln(
1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*
x)/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*I*b*c^2*e^2*ln(1-(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/d^2/(
c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*I*b*c^10*d^2*ln(1-(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/e^2/(c^4*d
^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)+1/4*I*b*c^8*d*ln(1-(c^2*d-e)*(1+I*c*x)^
2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/e/(c^4*d^2-2*c^2*d*
e+e^2)^2*(c^2*d*e)^(1/2)+1/8*b*c^7/(c^4*d^2-2*c...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(3/2) + (x^3*e - d*x)/(d*x^4*e^
3 + 2*d^2*x^2*e^2 + d^3*e)) + 2*b*integrate(1/2*x^2*arctan(c*x)/(x^6*e^3 +
3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e +
 d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

3.1171 $\int \frac{a+b\mathbf{ArcTan}(cx)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=893

$$-\frac{bc}{8d\left(c^2d-e\right)\left(d+ex^2\right)}+\frac{x(a+b\text{ArcTan}(cx))}{4d\left(d+ex^2\right)^2}+\frac{3x(a+b\text{ArcTan}(cx))}{8d^2\left(d+ex^2\right)}+\frac{3(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}$$

```
[Out] -1/8*b*c/d/(c^2*d-e)/(e*x^2+d)+1/4*x*(a+b*arctan(c*x))/d/(e*x^2+d)^2+3/8*x*
(a+b*arctan(c*x))/d^2/(e*x^2+d)-1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d^2/(c
^2*d-e)^2+1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d^2/(c^2*d-e)^2+3/8*(a+b*arcta
n(c*x))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)-3/32*I*b*c*ln(-(1+x*(-c^2
)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2)
)/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*
(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/d^(5/2)/(-c^2)^(1/
2)/e^(1/2)-3/32*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2
)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b
*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x
*e^(1/2)/d^(1/2))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b*c*polylog(2,(-c^2)^
(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(5/2)/(-c^2
)^(1/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-
c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b*c*poly
log(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/
d^(5/2)/(-c^2)^(1/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x
*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 893, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {205, 211, 5032, 6857, 585, 78, 5028, 2456, 2441, 2440, 2438}

$$\frac{3ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}\,x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}\,x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}\,x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{3ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}\,x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}\,x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}+\frac{3ib\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}\,x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}\,x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{b(5c^2d-3e)\log\left(c^2x^2+1\right)c}{16d^2\left(c^2d-e\right)^2}+\frac{b(5c^2d-3e)\log\left(ex^2+d\right)c}{16d^2\left(c^2d-e\right)^2}+\frac{3ib\text{Li}_2\left(\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{3ib\text{Li}_2\left(\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}+\frac{3ib\text{Li}_2\left(\frac{\sqrt{-c^2}\left(i\sqrt{e}\,x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{3ib\text{Li}_2\left(\frac{\sqrt{-c^2}\left(i\sqrt{e}\,x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}\,d^{5/2}\sqrt{e}}-\frac{bc}{8d\left(c^2d-e\right)\left(ex^2+d\right)}+\frac{3x(a+b\text{ArcTan}(cx))}{8d^2\left(ex^2+d\right)}+\frac{x(a+b\text{ArcTan}(cx))}{4d\left(ex^2+d\right)^2}+\frac{3(a+b\text{ArcTan}(cx))\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c)/(d*(c^2*d - e)*(d + e*x^2)) + (x*(a + b*ArcTan[c*x]))/(4*d*(d +
e*x^2)^2) + (3*x*(a + b*ArcTan[c*x]))/(8*d^2*(d + e*x^2)) + (3*(a + b*ArcTa
n[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*
Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 -
(I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[
-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*Log[-
```

((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(16*d^2*(c^2*d - e)^2) + (b*c*(5*c^2*d - 3*e)*Log[d + e*x^2])/(16*d^2*(c^2*d - e)^2) + (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e])

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}-(b$$

$$=\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}-(b$$

$$=\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}-\frac{(b}{}$$

$$=\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}-\frac{(b}{}$$

$$=\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}(cx))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}-\frac{(b}{}$$

$$=-\frac{bc}{8d\,(c^2d-e)\,(d+ex^2)}+\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}}{}$$

$$=-\frac{bc}{8d\,(c^2d-e)\,(d+ex^2)}+\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}}{}$$

$$=-\frac{bc}{8d\,(c^2d-e)\,(d+ex^2)}+\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}}{}$$

$$=-\frac{bc}{8d\,(c^2d-e)\,(d+ex^2)}+\frac{x(a+b\tan^{-1}(cx))}{4d\,(d+ex^2)^2}+\frac{3x(a+b\tan^{-1}(cx))}{8d^2\,(d+ex^2)}+\frac{3(a+b\tan^{-1}}{}$$

Mathematica [A]
time = 10.33, size = 1745, normalized size = 1.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (b*c*(10*c^2*d*Log[1 + ((c^2*d - e)*
Cos[2*ArcTan[c*x]])/(c^2*d + e)] - 6*e*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*
x]])/(c^2*d + e)] + (3*c^2*d*(c^2*d - e)*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2
*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt
[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((
c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d
- e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[
-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (Ar
cCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sq
rt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c
*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c
^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*C
os[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*
(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x)
)] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2
*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))])))/Sqrt[-(c^2*d*e)
] - (3*(c^2*d - e)*e*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*
ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (Arc
Cos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*
Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d +
c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTa
nh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*
x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*A
rcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-(
(c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Ar
cTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan
[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])]
 + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^
2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^
2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d -
 e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))])))/Sqrt[-(c^2*d*e)] - (16*c^2*d*(c^2*d
```

```
- e)*e*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcT
an[c*x]])^2 + (8*c^2*d*e + 4*(5*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan[c*x]*Si
n[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/(32*d^2*(-
(c^2*d) + e)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4031 vs. $2(681) = 1362$.
time = 1.17, size = 4032, normalized size = 4.52

method	result	size
derivativedivides	Expression too large to display	4032
default	Expression too large to display	4032
risch	Expression too large to display	5059

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-5/4*b*c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^
(1/2))+5/16*b*c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4/(c
^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+c^2
*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+1/8*b*c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*
x^2+c^2*d)^2*e+3/8*a*c/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+5/16*b*c^3*(
d*e)^(1/2)*e/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2
*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-5/4*b*c^7/(c^4*d^2-2*c
^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x+5/8*b*c^9/(c^4*d^2-2*c^2*d*
e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)*x+3/8*b*c^9/(c^4*d^2-2*c^2*d*e+e^2
)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^3-3/16*b*c*(d*e)^(1/2)/d^3*e^2*arctan
h(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4
*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-3/8*I*b*c^2*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*
e+e^2)/d^2*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^
2*d*e)^(1/2)-e))-3/16*I*b*e^3*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*
d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)
^(1/2)+3/16*I*b*(c^2*d*e)^(1/2)/d^3*e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*l
n(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/8*b*c
^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*e^2*x^2+1/8*b*c^8/(c^4*d^2
-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*e^2*x^4+3/4*b*c^6*arctan(c*x)^2/(c^4*
d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)+3/8*b*c^6*polylog(2,(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^
2*d*e)^(1/2)+5/8*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arctan
(c*x)*e^2*x+3/8*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d^2*arcta
n(c*x)*e^3*x^3+5/4*I*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*arct
an(c*x)*e*x^2+3/16*I*b*c^4*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/d/e*arct
an(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e)
)+3/4*I*b*c^2*e^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e
)^(1/2)-e))*arctan(c*x)/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-9/8*I
```

```
*b*c^4*e*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e
))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)^2/d*(c^2*d*e)^(1/2)-3/16*I*b*c^8*ln(
1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*
x)/e/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)*d+2*b*c^4/(c^4*d^2-2*c^2*d*e
+e^2)*e/d/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8*b*c^8/(c^4*d^2-2*c^
2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*x^2-3/16*b*c^8*arctan(c*x)^2*d/e/(c^4*d^2-
2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-9/16*b*c^4*e*polylog(2,(c^2*d-e)*(1+I*c*x)
^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c
^2*d*e)^(1/2)+3/8*b*c^2*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c
^2*d+2*(c^2*d*e)^(1/2)-e))/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)+3/
16*b*c^5*(d*e)^(1/2)/d*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c
^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-3/32*b*c^8*polyl
og(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e/(c^4
*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)*d-9/8*b*c^4*arctan(c*x)^2/d/(c^4*d^2-
2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)*e+5/16*b*c^5*(d*e)^(1/2)/d/e*arctanh(1/4*(
2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*
c^2*d*e+e^2)+3/16*b*c^4*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(
c*x)^2-1/2*b*c^4/(c^4*d^2-2*c^2*d*e+e^2)*e/d/(c^2*d-e)*ln(c^2*d*(1+I*c*x)^4
/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^2+
c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-5/16*b*c^7*(d*e)^(1/2)/e*arctanh(1/4*(
2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*
c^2*d*e+e^2)/(c^2*d-e)+3/32*b*c^4*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^
2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e)
)+3/4*b*c^2*e^2*arctan(c*x)^2/d^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)
-3/16*b*c^2*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*e+e^2)/d^2*polylog(2,(c^2*d-e)
*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/8*b*c^3*(d*e)^(1/2
)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)
^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)-3/8*b*c^2*(c^2*d*e)^(1/2)/(c^4*d^2-2*c^2*d*
e+e^2)/d^2*arctan(c*x)^2-3/4*b*c^7/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d
)^2/d*e^2*arctan(c*x)*x^3+5/8*I*b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^
2*d)^2/d*arctan(c*x)*e^2*x^4-3/4*I*b*c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2
+c^2*d)^2/d*arctan(c*x)*e^2*x^2-3/8*I*b*c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*
x^2+c^2*d)^2/d^2*arctan(c*x)*e^3*x^4+1/4*a*c^5*x/d/(c^2*e*x^2+c^2*d)^2+3/8*
a*c^3/d^2*x/(c^2*e*x^2+c^2*d)-3/32*b*e^3*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c
^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*
d*e)^(1/2)+3/32*b*(c^2*d*e)^(1/2)/d^3*e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(
c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+3/16*b*(c^2*
d*e)^(1/2)/d^3*e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2-3/16*b*e^3*arctan(c*
x)^2/d^3/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*x^3*e + 5*d*x)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) + 3*arctan(x*e^(
1/2)/sqrt(d))*e^(-1/2)/d^(5/2)) + 2*b*integrate(1/2*arctan(c*x)/(x^6*e^3 +
3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x^2)^3, x)
```

3.1172 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^2(d+ex^2)^3}\,dx$

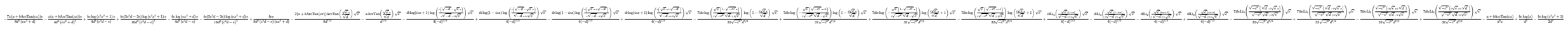
Optimal. Leaf size=1518

$$\frac{bce}{8d^2(c^2d-e)(d+ex^2)}-\frac{a+b\text{ArcTan}(cx)}{d^3x}-\frac{ex(a+b\text{ArcTan}(cx))}{4d^2(d+ex^2)^2}-\frac{7ex(a+b\text{ArcTan}(cx))}{8d^3(d+ex^2)}-\frac{a\sqrt{e}\,\text{ArcTan}\Big(}{d^{7/2}}$$

```
[Out] 7/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/
2)+I*e^(1/2)))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)+7/32*I*b*c*polylog(2,(-c^2)^(1/
2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))*e^(1/2)/d^(7/2)/
(-c^2)^(1/2)-a*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)-7/8*(a+b*arctan(c*
x))*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)+1/8*b*c*e/d^2/(c^2*d-e)/(e*x^
2+d)+b*c*ln(x)/d^3-1/2*b*c*ln(c^2*x^2+1)/d^3+(-a-b*arctan(c*x))/d^3/x+7/32*
I*b*c*ln(-(1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1
-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)+7/32*I*b*c*ln(-(1-x*(-c^
2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2
))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)-7/32*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I
*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)/(
-c^2)^(1/2)-7/32*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2
)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)+1/4*I*b*
polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(7/2)+1/4*
I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(7/2)-
1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^
(7/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)
/(-d)^(7/2)-1/4*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^2-7/8*e*x*(a+b*arctan(c
*x))/d^3/(e*x^2+d)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^
(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(7/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x
*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(7/2)+1/4*b*c*e*ln(c^2*x^2
+1)/d^3/(c^2*d-e)-1/4*b*c*e*ln(e*x^2+d)/d^3/(c^2*d-e)-1/4*I*b*ln(1+I*c*x)*l
n(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(7/2)-1/4
*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1
/2)/(-d)^(7/2)+1/16*b*c*(5*c^2*d-3*e)*e*ln(c^2*x^2+1)/d^3/(c^2*d-e)^2-1/16*
b*c*(5*c^2*d-3*e)*e*ln(e*x^2+d)/d^3/(c^2*d-e)^2-7/32*I*b*c*polylog(2,(-c^2)
^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))*e^(1/2)/d^(7
/2)/(-c^2)^(1/2)-7/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((
-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)
```

Rubi [A]

time = 2.05, antiderivative size = 1518, normalized size of antiderivative = 1.00, number of steps used = 73, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {5100, 4946, 272, 36, 29, 31, 205, 211, 5032, 6857, 585, 78, 5028, 2456, 2441, 2440, 2438, 455, 5030}



Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] (b*c*e)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (a + b*ArcTan[c*x])/(d^3*x) - (e*
x*(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (7*e*x*(a + b*ArcTan[c*x]))/
(8*d^3*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(7/2) - (7*
Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + (b*c
*Log[x])/d^3 - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x
))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*
Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - ((I/
4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*
Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] +
Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e]
*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 -
 (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log
[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 -
(I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*Log[
-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*Log[(
Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^3) +
(b*c*(5*c^2*d - 3*e)*e*Log[1 + c^2*x^2])/(16*d^3*(c^2*d - e)^2) + (b*c*e*Lo
g[1 + c^2*x^2])/(4*d^3*(c^2*d - e)) - (b*c*(5*c^2*d - 3*e)*e*Log[d + e*x^2]
)/(16*d^3*(c^2*d - e)^2) - (b*c*e*Log[d + e*x^2])/(4*d^3*(c^2*d - e)) + ((I
/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d
)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] +
 Sqrt[e])])/(-d)^(7/2) - ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/
(I*c*Sqrt[-d] + Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b*c*Sqrt[e
]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*S
qrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-
c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2
]*d^(7/2)) - (((7*I)/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sq
rt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)
/32)*b*c*Sqrt[e]*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2
]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(7/2))
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2(d+ex^2)^3}\,dx = \int \left(\frac{a+b\tan^{-1}(cx)}{d^3x^2} - \frac{e(a+b\tan^{-1}(cx))}{d(d+ex^2)^3} - \frac{e(a+b\tan^{-1}(cx))}{d^2(d+ex^2)^2} - \frac{e(a+b\tan^{-1}(cx}{d^3(d+ex^2)}\right.$$

$$= \frac{\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^2}\,dx}{d^2} - \frac{e\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^3}\,dx}{d}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}(cx))}{8d^3(d+ex^2)} - \frac{7\sqrt{e}\,(a+b\tan}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}(cx))}{8d^3(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{d^{7/2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}(cx))}{8d^3(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{d^{7/2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}(cx))}{8d^3(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{d^{7/2}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}(cx))}{8d^3(d+ex^2)} - \frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{d^{7/2}}$$

$$= \frac{bce}{8d^2(c^2d-e)(d+ex^2)} - \frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}}{8d^3(d+ex^2}$$

$$= \frac{bce}{8d^2(c^2d-e)(d+ex^2)} - \frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}}{8d^3(d+ex^2}$$

$$= \frac{bce}{8d^2(c^2d-e)(d+ex^2)} - \frac{a+b\tan^{-1}(cx)}{d^3x} - \frac{ex(a+b\tan^{-1}(cx))}{4d^2(d+ex^2)^2} - \frac{7ex(a+b\tan^{-1}}{8d^3(d+ex^2}$$

Mathematica [A]
time = 12.78, size = 2005, normalized size = 1.32

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - (a*e*x)/(4*d^2*(d + e*x^2)^2) - (7*a*e*x)/(8*d^3*(d + e*x^2)
) - (15*a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + b*c^7*(-(ArcTa
n[c*x]/(c^7*d^3*x)) + Log[(c*x)/Sqrt[1 + c^2*x^2]]/(c^6*d^3) - (9*e*Log[1 -
 ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c^4*d^2*(c^2*d - e)^
2) + (7*e^2*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c
^6*d^3*(c^2*d - e)^2) - (15*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)
]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e
)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^
2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt
[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (-ArcC
os[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*L
og[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)
]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[(-(c^2*d)
 - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(
c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*
E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (Arc
Cos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)
] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*
ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*
x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c
*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] - P
olyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d
*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))])))/(32*c^4*d^2*(
c^2*d - e)*Sqrt[-(c^2*d*e)]) + (15*e^2*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-
(c^2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c*e*x)/Sqrt[
-(c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/
Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d -
 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))]
 + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*
d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-
(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[
(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] +
ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^
2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]]
)] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2
*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*
e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*
```

```
ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^
2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]
*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))])))/(32*
c^6*d^3*(c^2*d - e)*Sqrt[-(c^2*d*e)]) + (e^2*ArcTan[c*x]*Sin[2*ArcTan[c*x]]
)/(2*c^4*d^2*(c^2*d - e)*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]] - e*Cos[2*Ar
cTan[c*x]])^2) + (-2*c^2*d*e^2 - 9*c^4*d^2*e*ArcTan[c*x]*Sin[2*ArcTan[c*x]]
 + 16*c^2*d*e^2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 7*e^3*ArcTan[c*x]*Sin[2*Ar
cTan[c*x]])/(8*c^6*d^3*(c^2*d - e)^2*(c^2*d + e + c^2*d*Cos[2*ArcTan[c*x]]
- e*Cos[2*ArcTan[c*x]])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 6658, normalized size = 4.39

method	result	size
derivativedivides	Expression too large to display	6658
default	Expression too large to display	6658
risch	Expression too large to display	7490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((15*x^4*e^2 + 25*d*x^2*e + 8*d^2)/(d^3*x^5*e^2 + 2*d^4*x^3*e + d^5*
x) + 15*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/d^(7/2)) + 2*b*integrate(1/2*arct
an(c*x)/(x^8*e^3 + 3*d*x^6*e^2 + 3*d^2*x^4*e + d^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(x^8*e^3 + 3*d*x^6*e^2 + 3*d^2*x^4*e + d^3*x^2
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3), x)
```

3.1173 $\int x^3\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=223

$$-\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3e^2}+\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(c$$

```
[Out] -1/20*b*x*(e*x^2+d)^(3/2)/c/e-1/3*d*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+1
/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e^2+1/15*b*(c^2*d-e)^(3/2)*(2*c^2*d+3*
e)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^5/e^2+1/120*b*(15*c^4*d^2+20
*c^2*d*e-24*e^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^5/e^(3/2)-1/120*b*(c^
2*d-12*e)*x*(e*x^2+d)^(1/2)/c^3/e
```

Rubi [A]
time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {272, 45, 5096, 12, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3e^2}+\frac{b(c^2d-e)^{3/2}\,(2c^2d+3e)\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{15c^5e^2}-\frac{bx(c^2d-12e)\,\sqrt{d+ex^2}}{120c^3e}+\frac{b(15c^4d^2+20c^2de-24e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}-\frac{bx(d+ex^2)^{3/2}}{20ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/120*(b*(c^2*d - 12*e)*x*Sqrt[d + e*x^2])/(c^3*e) - (b*x*(d + e*x^2)^(3/2
))/(20*c*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + ((d + e*x
^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + (b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*
e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(15*c^5*e^2) + (b*(15*c^4*d
^2 + 20*c^2*d*e - 24*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(120*c^5*e^
(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx = -\frac{d(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-$$

$$= -\frac{d(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}-$$

$$= -\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}(a}{5e}$$

$$= -\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}(a+b\,t}{3e^2}$$

$$= -\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}(a+b\,t}{3e^2}$$

$$= -\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}(a+b\,t}{3e^2}$$

$$= -\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}-\frac{d(d+ex^2)^{3/2}(a+b\,t}{3e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 391, normalized size = 1.75

$$\frac{-c^2\sqrt{d+ex^2}\left(8ac^3(2d^2-dex^2-3e^2x^4)+bex(-12e+c^2(7d+6ex^2))\right)-8bc^5\sqrt{d+ex^2}\,(2d^2-dex^2-3e^2x^4)\operatorname{ArcTan}(cx)-4ib(c^2d-e)^{3/2}(2c^2d+3e)\log\left(-\frac{60ic^6e^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(2c^2d+3e)(i+cx)}\right)+4ib(c^2d-e)^{3/2}(2c^2d+3e)\log\left(\frac{60ic^6e^2\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(2c^2d+3e)(-i+cx)}\right)+b\sqrt{e}\,(15c^4d^2+20c^2de-24e^2)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{120c^5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (-(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*x*(-12*
e + c^2*(7*d + 6*e*x^2)))) - 8*b*c^5*Sqrt[d + e*x^2]*(2*d^2 - d*e*x^2 - 3*e
^2*x^4)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)*Log[((-60*I
)*c^6*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(
5/2)*(2*c^2*d + 3*e)*(I + c*x))] + (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e
)*Log[((60*I)*c^6*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(
```

```
c^2*d - e)^(5/2)*(2*c^2*d + 3*e)*(-I + c*x))] + b*Sqrt[e]*(15*c^4*d^2 + 20*
c^2*d*e - 24*e^2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(120*c^5*e^2)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
[Out] int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/15*(3*(x^2*e + d)^(3/2)*x^2*e^(-1) - 2*(x^2*e + d)^(3/2)*d*e^(-2))*a + b*
integrate(sqrt(x^2*e + d)*x^3*arctan(c*x), x)
```

Fricas [A]
time = 7.93, size = 617, normalized size = 2.77

$$\left[-\frac{\left(\left(15\,bc^{4}d^{2}+20\,bc^{2}de-24\,be^{2}\right)e^{\frac{1}{2}}\log\left(-2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)+4\,\left(2\,bc^{4}d^{2}+bc^{2}de-3\,be^{2}\right)\sqrt{-c^{2}d+e}\,\log\left(\frac{c^{4}d^{2}x^{4}-6\,c^{2}d^{2}x^{2}+8\,x^{4}e^{2}-4\,\left(c^{2}dx^{3}-2\,x^{3}e-dx\right)\sqrt{-c^{2}d+e}\,\sqrt{x^{2}e+d}+d^{2}-8\,\left(c^{2}dx^{4}-dx^{2}\right)e}{c^{4}x^{4}+2\,c^{2}x^{2}+1}\right)+2\,\left(16\,ac^{5}d^{2}-8\,\left(3\,bc^{5}x^{4}e^{2}+bc^{5}dx^{2}e-2\,bc^{5}d^{2}\right)\arctan\left(cx\right)-6\,\left(4\,ac^{5}x^{4}-bc^{4}x^{3}+2\,bc^{2}x\right)e^{2}-\left(8\,ac^{5}dx^{2}-7\,bc^{4}dx\right)e\right)\sqrt{x^{2}e+d}\right)e^{\left(-2\right)}}{240\,c^{5}},\ -\frac{\left(\left(15\,bc^{4}d^{2}+20\,bc^{2}de-24\,be^{2}\right)e^{\frac{1}{2}}\log\left(-2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)-8\,\left(2\,bc^{4}d^{2}+bc^{2}de-3\,be^{2}\right)\sqrt{c^{2}d-e}\,\arctan\left(\frac{\left(c^{2}dx^{2}-2\,x^{2}e-d\right)\sqrt{c^{2}d-e}\,\sqrt{x^{2}e+d}}{2\,\left(c^{2}d^{2}x-x^{3}e^{2}+\left(c^{2}dx^{3}-dx\right)e\right)}\right)+2\,\left(16\,ac^{5}d^{2}-8\,\left(3\,bc^{5}x^{4}e^{2}+bc^{5}dx^{2}e-2\,bc^{5}d^{2}\right)\arctan\left(cx\right)-6\,\left(4\,ac^{5}x^{4}-bc^{4}x^{3}+2\,bc^{2}x\right)e^{2}-\left(8\,ac^{5}dx^{2}-7\,bc^{4}dx\right)e\right)\sqrt{x^{2}e+d}\right)e^{\left(-2\right)}}{240\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] [-1/240*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*e^(1/2)*log(-2*x^2*e + 2*
sqrt(x^2*e + d)*x*e^(1/2) - d) + 4*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt
(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e^2 - 4*(c^2*d*x^3 -
2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^
2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(16*a*c^5*d^2 - 8*(3*b*c^5*x^4*e^2 + b
*c^5*d*x^2*e - 2*b*c^5*d^2)*arctan(c*x) - 6*(4*a*c^5*x^4 - b*c^4*x^3 + 2*b*
c^2*x)*e^2 - (8*a*c^5*d*x^2 - 7*b*c^4*d*x)*e)*sqrt(x^2*e + d))*e^(-2)/c^5,
-1/240*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*e^(1/2)*log(-2*x^2*e + 2*s
qrt(x^2*e + d)*x*e^(1/2) - d) - 8*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt(
c^2*d - e)*arctan(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e
+ d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) + 2*(16*a*c^5*d^2 - 8*(3*
b*c^5*x^4*e^2 + b*c^5*d*x^2*e - 2*b*c^5*d^2)*arctan(c*x) - 6*(4*a*c^5*x^4 -
 b*c^4*x^3 + 2*b*c^2*x)*e^2 - (8*a*c^5*d*x^2 - 7*b*c^4*d*x)*e)*sqrt(x^2*e +
 d))*e^(-2)/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1174 $\int x^2\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=97

$$\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}-\frac{ad^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}}+b\text{Int}\left(x^2\sqrt{d+ex^2}\,\text{ArcTan}(cx),x\right)$$

```
[Out] -1/8*a*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+1/8*a*d*x*(e*x^2+d)^(
1/2)/e+1/4*a*x^3*(e*x^2+d)^(1/2)+b*Unintegrable(x^2*arctan(c*x)*(e*x^2+d)^(
1/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d*x*Sqrt[d + e*x^2])/(8*e) + (a*x^3*Sqrt[d + e*x^2])/4 - (a*d^2*ArcTanh[
(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(3/2)) + b*Defer[Int][x^2*Sqrt[d + e*x^2
]*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{d+ex^2}\,\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^2\sqrt{d+ex^2}\,dx+b\int x^2\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx\\ &=\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx+\frac{1}{4}(ad)\int\frac{x^2}{\sqrt{d+}}\\ &=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx\\ &=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}+b\int x^2\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx\\ &=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}-\frac{ad^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}}+b\end{aligned}$$

Mathematica [A]
time = 11.18, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^2\sqrt{e\,x^2+d}\,\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
[Out] Integral(x**2*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,(a+b\operatorname{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
[Out] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1175 $\int x\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=140

$$-\frac{bx\sqrt{d+ex^2}}{6c}+\frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{3e}-\frac{b(c^2d-e)^{3/2}\text{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{3c^3e}-\frac{b(3c^2d-2e)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e-1/3*b*(c^2*d-e)^(3/2)*arctan(x*(c^2
*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^3/e-1/6*b*(3*c^2*d-2*e)*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))/c^3/e^(1/2)-1/6*b*x*(e*x^2+d)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {5094, 427, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{3e}-\frac{b(c^2d-e)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e}-\frac{b(3c^2d-2e)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}-\frac{bx\sqrt{d+ex^2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x*Sqrt[d + e*x^2])/c + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e
) - (b*(c^2*d - e)^(3/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^
3*e) - (b*(3*c^2*d - 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*Sqrt
[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\tan^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{\left(d+ex^2\right)^{3/2}}{1+c^2x^2}\,dx}{3e}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\int \frac{d(2c^2d-e)+(3c^2d-}{(1+c^2x^2)\sqrt{d+e}}}{6ce}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(b(3c^2d-2e))\int -}{6c^3}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(b(3c^2d-2e))\,\text{Sub}}{}$$

$$= -\frac{bx\sqrt{d+ex^2}}{6c} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\left(c^2d-e\right)^{3/2}\tan^{-1}}{3c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 279, normalized size = 1.99

$$\frac{c^2\sqrt{d+ex^2}\left(-bex+2ac\left(d+ex^2\right)\right)+2bc^3\left(d+ex^2\right)^{3/2}\text{ArcTan}(cx)-ib\left(c^2d-e\right)^{3/2}\log\left(\frac{12c^4e\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{5/2}(-i+cx)}\right)+ib\left(c^2d-e\right)^{3/2}\log\left(\frac{12c^4e\left(icd+ex+i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{5/2}(i+cx)}\right)+b\sqrt{e}\left(-3c^2d+2e\right)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{6c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(-(b*e*x) + 2*a*c*(d + e*x^2)) + 2*b*c^3*(d + e*x^2)^(
3/2)*ArcTan[c*x] - I*b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*((-I)*c*d + e*x - I*
Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))] + I*b*(
c^2*d - e)^(3/2)*Log[(12*c^4*e*(I*c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*
x^2]))/(b*(c^2*d - e)^(5/2)*(I + c*x))] + b*Sqrt[e]*(-3*c^2*d + 2*e)*Log[e*
x + Sqrt[e]*Sqrt[d + e*x^2]])/(6*c^3*e)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x\sqrt{e\,x^2+d}\,\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [**A**]
time = 5.72, size = 472, normalized size = 3.37

$$\left[-\frac{\left((3\,bc^2d-2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+(bc^2d-be)\sqrt{-c^2d+e}\,\log\left(\frac{c^4d^2x^4-6\,c^2d^2x^2+8\,x^4e^2+4\,(c^2dx^3-2\,x^3e-dx)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}+d^2-8\,(c^2dx^4-dx^2)e}{c^4x^4+2\,c^2x^2+1}\right)-2\,(2\,ac^3d+2\,(bc^3x^2e+bc^3d)\arctan(cx)+(2\,ac^3x^2-bc^2x)e)\sqrt{x^2e+d}\right)e^{(-1)}}{12\,c^3},-\frac{\left((3\,bc^2d-2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+2\,(bc^2d-be)\sqrt{c^2d-e}\,\arctan\left(\frac{(c^2dx^2-2\,x^2e-d)\sqrt{c^2d-e}\,\sqrt{x^2e+d}}{2\,(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)-2\,(2\,ac^3d+2\,(bc^3x^2e+bc^3d)\arctan(cx)+(2\,ac^3x^2-bc^2x)e)\sqrt{x^2e+d}\right)e^{(-1)}}{12\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/12*((3*b*c^2*d - 2*b*e)*e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1
/2) - d) + (b*c^2*d - b*e)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^
2 + 8*x^4*e^2 + 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e +
 d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^
3*d + 2*(b*c^3*x^2*e + b*c^3*d)*arctan(c*x) + (2*a*c^3*x^2 - b*c^2*x)*e)*sq
rt(x^2*e + d))*e^(-1)/c^3, -1/12*((3*b*c^2*d - 2*b*e)*e^(1/2)*log(-2*x^2*e
- 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 2*(b*c^2*d - b*e)*sqrt(c^2*d - e)*arct
an(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x
 - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) - 2*(2*a*c^3*d + 2*(b*c^3*x^2*e + b*c^3*
d)*arctan(c*x) + (2*a*c^3*x^2 - b*c^2*x)*e)*sqrt(x^2*e + d))*e^(-1)/c^3]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1176 $\int \sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx)),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\,\left(a+b\tan^{-1}(cx)\right)\,dx = \int \sqrt{d+ex^2}\,\left(a+b\tan^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 5.00, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \sqrt{e\,x^2+d}\,\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atan}(cx)) \sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
[Out] int((a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1177 $\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=64

$$a\sqrt{d+ex^2}-a\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{\sqrt{d+ex^2}\,\text{ArcTan}(cx)}{x},x\right)$$

```
[Out] -a*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)+a*(e*x^2+d)^(1/2)+b*Unintegrabl
e(arctan(c*x)*(e*x^2+d)^(1/2)/x,x)
```

Rubi [A]

time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + e*x^2] - a*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Defer[In
t][(Sqrt[d + e*x^2]*ArcTan[c*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x}\,dx &= a\int \frac{\sqrt{d+ex^2}}{x}\,dx+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x}\,dx,x,x^2\right)+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x}\,dx\\
&= a\sqrt{d+ex^2}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x}\,dx+\frac{1}{2}(ad)\text{Subst}\left(\int \frac{1}{x\sqrt{d+e}}\right.\\
&= a\sqrt{d+ex^2}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x}\,dx+\frac{(ad)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x\right.}{e}\\
&= a\sqrt{d+ex^2}-a\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x}
\end{aligned}$$

Mathematica [A]

time = 6.85, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(c x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x)) - sqrt(x^2*e + d))*a + b*integra
te(sqrt(x^2*e + d)*arctan(c*x)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atan}\left(c x\right)\right)\sqrt{d+e x^{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x, x)
```

3.1178 $\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{\sqrt{d+ex^2}\,\text{ArcTan}(cx)}{x^2},x\right)$$

```
[Out] a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)-a*(e*x^2+d)^(1/2)/x+b*Unintegr
able(arctan(c*x)*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
[Out] -((a*Sqrt[d + e*x^2])/x) + a*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] +
 b*Defer[Int][(Sqrt[d + e*x^2]*ArcTan[c*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x^2}\,dx &= a\int \frac{\sqrt{d+ex^2}}{x^2}\,dx+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^2}\,dx\\ &= -\frac{a\sqrt{d+ex^2}}{x}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^2}\,dx+(ae)\int \frac{1}{\sqrt{d+ex^2}}\,dx\\ &= -\frac{a\sqrt{d+ex^2}}{x}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^2}\,dx+(ae)\text{Subst}\left(\int \frac{1}{1-ex^2}\right.\\ &= -\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(c}{x^2}\end{aligned}$$

Mathematica [A]
time = 7.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(c x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (arcsinh(x*e^(1/2)/sqrt(d))*e^(1/2) - sqrt(x^2*e + d)/x)*a + b*integrate(sq
rt(x^2*e + d)*arctan(c*x)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atan}\left(c x\right)\right)\sqrt{d+e x^{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2, x)
```

3.1179 $\int \frac{\sqrt{d+ex^2}\,(a+b\,\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=73

$$-\frac{a\sqrt{d+ex^2}}{2x^2}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}}+b\text{Int}\left(\frac{\sqrt{d+ex^2}\,\text{ArcTan}(cx)}{x^3},x\right)$$

```
[Out] -1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)-1/2*a*(e*x^2+d)^(1/2)/x^2
+b*Unintegrable(arctan(c*x)*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[d + e*x^2])/x^2 - (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*Sq
rt[d]) + b*Defer[Int][(Sqrt[d + e*x^2]*ArcTan[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x^3}\,dx = a\int \frac{\sqrt{d+ex^2}}{x^3}\,dx+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^3}\,dx$$
$$=\frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2}\,dx,x,x^2\right)+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^3}\,dx$$
$$=-\frac{a\sqrt{d+ex^2}}{2x^2}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^3}\,dx+\frac{1}{4}(ae)\text{Subst}\left(\int \frac{1}{x\sqrt{d+\cdots}}\right.$$
$$=-\frac{a\sqrt{d+ex^2}}{2x^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)+b\int \frac{\sqrt{d+\cdots}}{}$$
$$=-\frac{a\sqrt{d+ex^2}}{2x^2}-\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}}+b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^3}\,d\cdots$$

Mathematica [A]
time = 8.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\arctan\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/sqrt(d) - sqrt(x^2*e + d)*e/d + (x
^2*e + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(x^2*e + d)*arctan(c*x)/x^3, x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{atan}\left(cx\right)\right)\sqrt{d+ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3, x)
```

3.1180 $\int \frac{\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=137

$$-\frac{bc\sqrt{d+ex^2}}{6x^2}-\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3dx^3}+\frac{bc(2c^2d-3e)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}}-\frac{b(c^2d-e)^{3/2}\tanh^{-}}{3d}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^3-1/3*b*(c^2*d-e)^(3/2)*arctanh(
c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d+1/6*b*c*(2*c^2*d-3*e)*arctanh((e*x^2+d
)^(1/2)/d^(1/2))/d^(1/2)-1/6*b*c*(e*x^2+d)^(1/2)/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {270, 5096, 12, 457, 100, 162, 65, 214}

$$-\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3dx^3}-\frac{b(c^2d-e)^{3/2}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d}+\frac{bc(2c^2d-3e)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}}-\frac{bc\sqrt{d+ex^2}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[d + e*x^2])/x^2 - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3
*d*x^3) + (b*c*(2*c^2*d - 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*Sqrt[d]
) - (b*(c^2*d - e)^(3/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x^4}\,dx = -\frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - (bc)\int \frac{(d+ex^2)^{3/2}}{3x^3(-d-c^2dx^2)}\,dx$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - \frac{1}{3}(bc)\int \frac{(d+ex^2)^{3/2}}{x^3(-d-c^2dx^2)}\,dx$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - \frac{1}{6}(bc)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2(-d-c^2dx)}\,dx,\right.$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - \frac{(bc)\text{Subst}\left(\int \frac{-\frac{1}{2}d^2(}{x(-}\right.}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - \frac{1}{12}\left(bc(2c^2d-3e)\right)$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} - \frac{(bc(2c^2d-3e))\,\text{Sub}}{}$$

$$= -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{3dx^3} + \frac{bc(2c^2d-3e)\tanh}{6\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 288, normalized size = 2.10

$$-\frac{\sqrt{d+ex^2}\,(bcdx+2a(d+ex^2))+2b(d+ex^2)^{3/2}\text{ArcTan}(cx)+bc\sqrt{d}\,(2c^2d-3e)\,x^3\log(x)-bc\sqrt{d}\,(2c^2d-3e)\,x^3\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)+b(c^2d-e)^{3/2}x^3\log\left(\frac{12cd\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(i+cx)}\right)+b(c^2d-e)^{3/2}x^3\log\left(\frac{12cd\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(-i+cx)}\right)}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d + e*x^2)) + 2*b*(d + e*x^2)^(3/2)*A
rcTan[c*x] + b*c*Sqrt[d]*(2*c^2*d - 3*e)*x^3*Log[x] - b*c*Sqrt[d]*(2*c^2*d
- 3*e)*x^3*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*(c^2*d - e)^(3/2)*x^3*Log[(
12*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2
)*(I + c*x))] + b*(c^2*d - e)^(3/2)*x^3*Log[(12*c*d*(c*d + I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))])/(d*x^3)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\arctan(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 3.83, size = 926, normalized size = 6.76

$$\left[-\frac{\left(bc^2dx^3-bx^3e\right)\sqrt{c^2d-e}\log\left(\frac{8c^4d^2+4\left(2c^3d+\left(c^3x^2-c\right)e\right)\sqrt{c^2d-e}\sqrt{x^2e+d}+\left(c^4x^4-6c^2x^2+1\right)e^2+8\left(c^4dx^2-c^2d\right)e}{c^4x^4+2c^2x^2+1}\right)+\left(2bc^3dx^3-3bcx^3e\right)\sqrt{d}\log\left(-\frac{x^2e-2\sqrt{x^2e+d}\sqrt{d}+2d}{x^2}\right)+2\left(bcdx+2ax^2e+2ad+2\left(bx^2e+bd\right)\arctan\left(cx\right)\right)\sqrt{x^2e+d}}{12\,dx^3},\ -\frac{2\left(bc^2dx^3-bx^3e\right)\sqrt{-c^2d+e}\arctan\left(-\frac{\left(2c^2d+\left(c^2x^2-1\right)e\right)\sqrt{-c^2d+e}\sqrt{x^2e+d}}{2\left(c^3d^2-cx^2e^2+\left(c^3dx^2-cd\right)e\right)}\right)+\left(2bc^3dx^3-3bcx^3e\right)\sqrt{d}\log\left(-\frac{x^2e-2\sqrt{x^2e+d}\sqrt{d}+2d}{x^2}\right)+2\left(bcdx+2ax^2e+2ad+2\left(bx^2e+bd\right)\arctan\left(cx\right)\right)\sqrt{x^2e+d}}{12\,dx^3},\ -\frac{2\left(2bc^3dx^3-3bcx^3e\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{x^2e+d}}\right)+\left(bc^2dx^3-bx^3e\right)\sqrt{c^2d-e}\log\left(\frac{8c^4d^2+4\left(2c^3d+\left(c^3x^2-c\right)e\right)\sqrt{c^2d-e}\sqrt{x^2e+d}+\left(c^4x^4-6c^2x^2+1\right)e^2+8\left(c^4dx^2-c^2d\right)e}{c^4x^4+2c^2x^2+1}\right)+2\left(bcdx+2ax^2e+2ad+2\left(bx^2e+bd\right)\arctan\left(cx\right)\right)\sqrt{x^2e+d}}{12\,dx^3},\ -\frac{\left(bc^2dx^3-bx^3e\right)\sqrt{-c^2d+e}\arctan\left(-\frac{\left(2c^2d+\left(c^2x^2-1\right)e\right)\sqrt{-c^2d+e}\sqrt{x^2e+d}}{2\left(c^3d^2-cx^2e^2+\left(c^3dx^2-cd\right)e\right)}\right)+\left(2bc^3dx^3-3bcx^3e\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{x^2e+d}}\right)+\left(bcdx+2ax^2e+2ad+2\left(bx^2e+bd\right)\arctan\left(cx\right)\right)\sqrt{x^2e+d}}{6\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d*x^3 - b*x^3*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d
 + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2
+ 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*c^3*d
*x^3 - 3*b*c*x^3*e)*sqrt(d)*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/
x^2) + 2*(b*c*d*x + 2*a*x^2*e + 2*a*d + 2*(b*x^2*e + b*d)*arctan(c*x))*sqrt
(x^2*e + d))/(d*x^3), -1/12*(2*(b*c^2*d*x^3 - b*x^3*e)*sqrt(-c^2*d + e)*arc
tan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*
d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + (2*b*c^3*d*x^3 - 3*b*c*x^3*e)*sqr
t(d)*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x + 2*a
*x^2*e + 2*a*d + 2*(b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d))/(d*x^3), -
1/12*(2*(2*b*c^3*d*x^3 - 3*b*c*x^3*e)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e +
 d)) + (b*c^2*d*x^3 - b*x^3*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d
+ (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 +
 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c*d*x
+ 2*a*x^2*e + 2*a*d + 2*(b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d))/(d*x^
3), -1/6*((b*c^2*d*x^3 - b*x^3*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (
c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^
3*d*x^2 - c*d)*e)) + (2*b*c^3*d*x^3 - 3*b*c*x^3*e)*sqrt(-d)*arctan(sqrt(-d)
/sqrt(x^2*e + d)) + (b*c*d*x + 2*a*x^2*e + 2*a*d + 2*(b*x^2*e + b*d)*arctan
(c*x))*sqrt(x^2*e + d))/(d*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,\sqrt{d + ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2 + d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4, x)
```

3.1181 $\int \frac{\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=98

$$-\frac{a\sqrt{d+ex^2}}{4x^4}-\frac{ae\sqrt{d+ex^2}}{8dx^2}+\frac{ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}}+b\text{Int}\left(\frac{\sqrt{d+ex^2}\,\text{ArcTan}(cx)}{x^5},x\right)$$

```
[Out] 1/8*a*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/4*a*(e*x^2+d)^(1/2)/x^
4-1/8*a*e*(e*x^2+d)^(1/2)/d/x^2+b*Unintegrable(arctan(c*x)*(e*x^2+d)^(1/2)/
x^5,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] -1/4*(a*Sqrt[d + e*x^2])/x^4 - (a*e*Sqrt[d + e*x^2])/(8*d*x^2) + (a*e^2*Arc
Tanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*d^(3/2)) + b*Defer[Int][(Sqrt[d + e*x^2]*
ArcTan[c*x])/x^5, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x^5}\,dx = a\int \frac{\sqrt{d+ex^2}}{x^5}\,dx + b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3}\,dx, x, x^2\right) + b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} + b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^5}\,dx + \frac{1}{8}(ae)\text{Subst}\left(\int \frac{}{x^2\sqrt{a}}\right.$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^5}\,dx - \frac{(ae^2)}{}$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + b\int \frac{\sqrt{d+ex^2}\,\tan^{-1}(cx)}{x^5}\,dx - \frac{(ae)\text{S}}{}$$
$$= -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + \frac{ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}} + b\int \frac{\sqrt{d}}{}$$

Mathematica [A]
time = 10.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\arctan\left(cx\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e^2/d^(3/2) - sqrt(x^2*e + d)*e^2/d
^2 + (x^2*e + d)^(3/2)*e/(d^2*x^2) - 2*(x^2*e + d)^(3/2)/(d*x^4)) + b*integ
rate(sqrt(x^2*e + d)*arctan(c*x)/x^5, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/x^5, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx)) \sqrt{d + ex^2}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**5, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atan}(c\,x))\,\sqrt{e\,x^2 + d}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5, x)
```

3.1182 $\int \frac{\sqrt{d+ex^2}\,(a+b\mathbf{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=224

$$\frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2}-\frac{bc(d+ex^2)^{3/2}}{20dx^4}-\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{5dx^5}+\frac{2e(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(c$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ar
ctan(c*x))/d^2/x^3+1/30*b*c*(3*c^2*d-e)*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/
d^(3/2)+1/40*b*c*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/15*b*c*(c^2
*d-e)*(3*c^2*d+2*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+1/15*b*(c^2*d-
e)^(3/2)*(3*c^2*d+2*e)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2-1/20*
b*c*(e*x^2+d)^(1/2)/x^4+1/30*b*c*(3*c^2*d-e)*(e*x^2+d)^(1/2)/d/x^2-1/40*b*c
*e*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.24, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {277, 270, 5096, 12, 587, 154, 162, 65, 214}

$$\frac{2e(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{5dx^5}+\frac{b(3c^2d+2e)\,(c^2d-e)^{3/2}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{15d^2}+\frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2}-\frac{bc(24c^4d^2-20c^2de-15e^2)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{120d^{3/2}}-\frac{bc(d+ex^2)^{3/2}}{20dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (b*c*(12*c^2*d - e)*Sqrt[d + e*x^2])/(120*d*x^2) - (b*c*(d + e*x^2)^(3/2))/
(20*d*x^4) - ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(15*d^2*x^3) - (b*c*(24*c^4*d^2 - 20*c^2*
d*e - 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(120*d^(3/2)) + (b*(c^2*d -
 e)^(3/2)*(3*c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(15
*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
```

```
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{x^6}\,dx = -\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{15d^2x^3} -$$

$$= -\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{15d^2x^3} -$$

$$= -\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{15d^2x^3} -$$

$$= -\frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\,(a}{15d^2}$$

$$= \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2} - \frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}}{5dx^5}$$

$$= \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2} - \frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}}{5dx^5}$$

$$= \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2} - \frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}}{5dx^5}$$

$$= \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2} - \frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}}{5dx^5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 413, normalized size = 1.84

$$\frac{-\sqrt{d+ex^2}\,(8a(3d^2+dex^2-2e^2x^4)+bcdx(7ex^2+d(6-12c^2x^2)))-8b\sqrt{d+ex^2}\,(3d^2+dex^2-2e^2x^4)\,\mathrm{ArcTan}(cx)+bc\sqrt{d}\,(24c^4d^2-20c^2de-15e^2)\,x^5\log(x)-bc\sqrt{d}\,(24c^4d^2-20c^2de-15e^2)\,x^5\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)+4b(c^2d-e)^{3/2}\,(3c^2d+2e)\,x^5\log\left(-\frac{60cd^2\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(3c^2d+2e)(i+cx)}\right)+4b(c^2d-e)^{3/2}\,(3c^2d+2e)\,x^5\log\left(-\frac{60cd^2\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{5/2}(3c^2d+2e)(-i+cx)}\right)}{120d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (-(Sqrt[d + e*x^2]*(8*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*d*x*(7*e*x^2 +
d*(6 - 12*c^2*x^2)))) - 8*b*Sqrt[d + e*x^2]*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*A
```

```
rcTan[c*x] + b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[x] - b*
c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[d + Sqrt[d]*Sqrt[d + e
*x^2]] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c*d - I*
e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*d + 2*e
)*(I + c*x))] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c
*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*
d + 2*e)*(-I + c*x))])/(120*d^2*x^5)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(cx\right)\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
[Out] int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
[Out] 1/15*a*(2*(x^2*e + d)^(3/2)*e/(d^2*x^3) - 3*(x^2*e + d)^(3/2)/(d*x^5)) + b*
integrate(sqrt(x^2*e + d)*arctan(c*x)/x^6, x)
```

Fricas [A]
time = 2.82, size = 1244, normalized size = 5.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
[Out] [-1/240*(4*(3*b*c^4*d^2*x^5 - b*c^2*d*x^5*e - 2*b*x^5*e^2)*sqrt(c^2*d - e)*
log((8*c^4*d^2 - 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e +
 d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2
*c^2*x^2 + 1)) + (24*b*c^5*d^2*x^5 - 20*b*c^3*d*x^5*e - 15*b*c*x^5*e^2)*sqr
t(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) - 2*(12*b*c^3*d^2*
x^3 + 16*a*x^4*e^2 - 6*b*c*d^2*x - 24*a*d^2 + 8*(2*b*x^4*e^2 - b*d*x^2*e -
3*b*d^2)*arctan(c*x) - (7*b*c*d*x^3 + 8*a*d*x^2)*e)*sqrt(x^2*e + d))/(d^2*x
^5), 1/240*(8*(3*b*c^4*d^2*x^5 - b*c^2*d*x^5*e - 2*b*x^5*e^2)*sqrt(-c^2*d +
```

```
 e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d
)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - (24*b*c^5*d^2*x^5 - 20*b*c
^3*d*x^5*e - 15*b*c*x^5*e^2)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d
) + 2*d)/x^2) + 2*(12*b*c^3*d^2*x^3 + 16*a*x^4*e^2 - 6*b*c*d^2*x - 24*a*d^2
 + 8*(2*b*x^4*e^2 - b*d*x^2*e - 3*b*d^2)*arctan(c*x) - (7*b*c*d*x^3 + 8*a*d
*x^2)*e)*sqrt(x^2*e + d))/(d^2*x^5), 1/120*((24*b*c^5*d^2*x^5 - 20*b*c^3*d*
x^5*e - 15*b*c*x^5*e^2)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) - 2*(3*b*
c^4*d^2*x^5 - b*c^2*d*x^5*e - 2*b*x^5*e^2)*sqrt(c^2*d - e)*log((8*c^4*d^2 -
 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 -
 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) +
 (12*b*c^3*d^2*x^3 + 16*a*x^4*e^2 - 6*b*c*d^2*x - 24*a*d^2 + 8*(2*b*x^4*e^2
 - b*d*x^2*e - 3*b*d^2)*arctan(c*x) - (7*b*c*d*x^3 + 8*a*d*x^2)*e)*sqrt(x^2
*e + d))/(d^2*x^5), 1/120*(4*(3*b*c^4*d^2*x^5 - b*c^2*d*x^5*e - 2*b*x^5*e^2
)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)
*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + (24*b*c^5*d
^2*x^5 - 20*b*c^3*d*x^5*e - 15*b*c*x^5*e^2)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x
^2*e + d)) + (12*b*c^3*d^2*x^3 + 16*a*x^4*e^2 - 6*b*c*d^2*x - 24*a*d^2 + 8*
(2*b*x^4*e^2 - b*d*x^2*e - 3*b*d^2)*arctan(c*x) - (7*b*c*d*x^3 + 8*a*d*x^2)
*e)*sqrt(x^2*e + d))/(d^2*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,\sqrt{d + ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**6,x)
[Out] Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2 + d}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6, x)
```

3.1183 $\int x^3(d+ex^2)^{3/2}(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=279

$$\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex^2)^{3/2}}{840c^3e}-\frac{bx(d+ex^2)^{5/2}}{42ce}-\frac{d(d+ex^2)^{5/2}\,(a+b}{5e^2}$$

```
[Out] -1/840*b*(13*c^2*d-30*e)*x*(e*x^2+d)^(3/2)/c^3/e-1/42*b*x*(e*x^2+d)^(5/2)/c
/e-1/5*d*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arc
tan(c*x))/e^2+1/35*b*(c^2*d-e)^(5/2)*(2*c^2*d+5*e)*arctan(x*(c^2*d-e)^(1/2)
/(e*x^2+d)^(1/2))/c^7/e^2+1/560*b*(35*c^6*d^3+70*c^4*d^2*e-168*c^2*d*e^2+80
*e^3)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^7/e^(3/2)+1/560*b*(3*c^4*d^2+54*
c^2*d*e-40*e^2)*x*(e*x^2+d)^(1/2)/c^5/e
```

Rubi [A]
time = 0.33, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {272, 45, 5096, 12, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{7/2}\,(a+b\text{ArcTan}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5e^2}+\frac{b(c^2d-e)^{5/2}\,(2c^2d+5e)\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{35c^7e^2}-\frac{bx(13c^2d-30e)\,(d+ex^2)^{3/2}}{840c^3e}+\frac{bx(3c^4d^2+54c^2de-40e^2)\,\sqrt{d+ex^2}}{560c^5e}+\frac{b(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}-\frac{bx(d+ex^2)^{5/2}}{42ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 54*c^2*d*e - 40*e^2)*x*Sqrt[d + e*x^2])/(560*c^5*e) - (b*(1
3*c^2*d - 30*e)*x*(d + e*x^2)^(3/2))/(840*c^3*e) - (b*x*(d + e*x^2)^(5/2))/
(42*c*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e^2) + ((d + e*x^2)
^(7/2)*(a + b*ArcTan[c*x]))/(7*e^2) + (b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*
ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(35*c^7*e^2) + (b*(35*c^6*d^3
+ 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2
]])/(560*c^7*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
```

```
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx=-\frac{d(d+ex^2)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}$$

$$=-\frac{d(d+ex^2)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}$$

$$=-\frac{bx(d+ex^2)^{5/2}}{42ce}-\frac{d(d+ex^2)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/}}{}$$

$$=-\frac{b(13c^2d-30e)\,x(d+ex^2)^{3/2}}{840c^3e}-\frac{bx(d+ex^2)^{5/2}}{42ce}-\frac{d(d+ex^2)^{5/2}}{}$$

$$=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex}{840c^3e}$$

$$=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex}{840c^3e}$$

$$=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex}{840c^3e}$$

$$=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex}{840c^3e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 418, normalized size = 1.50

$$-\frac{c^2\sqrt{d+ex^2}\left(48ac^5(2d-5ex^2)(d+ex^2)^2+bex(120e^2-6c^2e(37d+10ex^2)+c^4(57d^2+106dex^2+40e^2x^4))\right)+48bc^7(2d-5ex^2)(d+ex^2)^{5/2}\operatorname{ArcTan}(cx)+24ib(c^2d-e)^{5/2}(2c^2d+5e)\log\left(-\frac{140ic^8e^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(2c^2d+5e)(i+cx)}\right)-24ib(c^2d-e)^{5/2}(2c^2d+5e)\log\left(\frac{140ic^8e^2\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(2c^2d+5e)(-i+cx)}\right)-3b\sqrt{e}\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{1680c^7e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/1680*(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*
x*(120*e^2 - 6*c^2*e*(37*d + 10*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40*e^2
*x^4))) + 48*b*c^7*(2*d - 5*e*x^2)*(d + e*x^2)^(5/2)*ArcTan[c*x] + (24*I)*b
```

```
*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((-140*I)*c^8*e^2*(c*d - I*e*x + Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d + 5*e)*(I + c*
x))] - (24*I)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((140*I)*c^8*e^2*(c*d
 + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d
+ 5*e)*(-I + c*x))] - 3*b*Sqrt[e]*(35*c^6*d^3 + 70*c^4*d^2*e - 168*c^2*d*e^
2 + 80*e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(c^7*e^2)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \arctan \left(c x \right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(x^2*e + d)^(5/2)*x^2*e^(-1) - 2*(x^2*e + d)^(5/2)*d*e^(-2))*a + 1/
2*b*integrate(2*(x^5*e + d*x^3)*sqrt(x^2*e + d)*arctan(c*x), x)
```

Fricas [A]
time = 17.15, size = 783, normalized size = 2.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [1/3360*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*e^(
1/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 24*(2*b*c^6*d^3 + b*
c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*
c^2*d^2*x^2 + 8*x^4*e^2 + 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sq
rt(x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) -
 2*(96*a*c^7*d^3 - 48*(5*b*c^7*x^6*e^3 + 8*b*c^7*d*x^4*e^2 + b*c^7*d^2*x^2*
e - 2*b*c^7*d^3)*arctan(c*x) - 20*(12*a*c^7*x^6 - 2*b*c^6*x^5 + 3*b*c^4*x^3
 - 6*b*c^2*x)*e^3 - 2*(192*a*c^7*d*x^4 - 53*b*c^6*d*x^3 + 111*b*c^4*d*x)*e^
2 - 3*(16*a*c^7*d^2*x^2 - 19*b*c^6*d^2*x)*e)*sqrt(x^2*e + d))*e^(-2)/c^7, 1
```

```
/3360*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*e^(1/
2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 48*(2*b*c^6*d^3 + b*c^
4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(c^2*d - e)*arctan(1/2*(c^2*d*x^2 -
2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*
x^3 - d*x)*e)) - 2*(96*a*c^7*d^3 - 48*(5*b*c^7*x^6*e^3 + 8*b*c^7*d*x^4*e^2
+ b*c^7*d^2*x^2*e - 2*b*c^7*d^3)*arctan(c*x) - 20*(12*a*c^7*x^6 - 2*b*c^6*x
^5 + 3*b*c^4*x^3 - 6*b*c^2*x)*e^3 - 2*(192*a*c^7*d*x^4 - 53*b*c^6*d*x^3 + 1
11*b*c^4*d*x)*e^2 - 3*(16*a*c^7*d^2*x^2 - 19*b*c^6*d^2*x)*e)*sqrt(x^2*e + d
))*e^(-2)/c^7]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\operatorname{atan}(cx))\left(d + ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
[Out] Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,{\left(e\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
[Out] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1184 $\int x^2(d+ex^2)^{3/2}(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=119

$$\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}-\frac{ad^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{16e^{3/2}}+b\text{Int}\left(x^2(d+ex^2)^{3/2}\,\text{ArcT}\right.$$

```
[Out] 1/6*a*x^3*(e*x^2+d)^(3/2)-1/16*a*d^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(
3/2)+1/16*a*d^2*x*(e*x^2+d)^(1/2)/e+1/8*a*d*x^3*(e*x^2+d)^(1/2)+b*Unintegra
ble(x^2*(e*x^2+d)^(3/2)*arctan(c*x),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d^2*x*Sqrt[d + e*x^2])/(16*e) + (a*d*x^3*Sqrt[d + e*x^2])/8 + (a*x^3*(d
+ e*x^2)^(3/2))/6 - (a*d^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(3/2
)) + b*Defer[Int][x^2*(d + e*x^2)^(3/2)*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^2(d+ex^2)^{3/2}\,dx+b\int x^2(d+ex^2)^{3/2}\tan^{-1}(cx)\,dx\\
&=\frac{1}{6}ax^3(d+ex^2)^{3/2}+b\int x^2(d+ex^2)^{3/2}\tan^{-1}(cx)\,dx+\frac{1}{2}(ad)\int x^2\\
&=\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}+b\int x^2(d+ex^2)^{3/2}\tan^{-1}(c\\
&=\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}+b\int x^2(\\
&=\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}+b\int x^2(\\
&=\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}-\frac{ad^3\tanh}{}\end{aligned}$$

Mathematica [A]
time = 10.70, size = 0, normalized size = 0.00

$$\int x^2(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^2\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e + a*d*x^2 + (b*x^4*e + b*d*x^2)*arctan(c*x))*sqrt(x^2*e +
 d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,{\left(e\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1185 $\int x(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=181

$$-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx(d+ex^2)^{3/2}}{20c}+\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5e}-\frac{b(c^2d-e)^{5/2}\,\text{ArcTan}\left(\frac{\sqrt{c^2d}}{\sqrt{d-}}\right.}{5c^5e}$$

```
[Out] -1/20*b*x*(e*x^2+d)^(3/2)/c+1/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e-1/5*b*(
c^2*d-e)^(5/2)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^5/e-1/40*b*(15*c
^4*d^2-20*c^2*d*e+8*e^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^5/e^(1/2)-1/4
0*b*(7*c^2*d-4*e)*x*(e*x^2+d)^(1/2)/c^3
```

Rubi [A]

time = 0.16, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5094, 427, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5e}-\frac{b(c^2d-e)^{5/2}\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{5c^5e}-\frac{bx(7c^2d-4e)\,\sqrt{d+ex^2}}{40c^3}-\frac{b(15c^4d^2-20c^2de+8e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bx(d+ex^2)^{3/2}}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/40*(b*(7*c^2*d - 4*e)*x*Sqrt[d + e*x^2])/c^3 - (b*x*(d + e*x^2)^(3/2))/(
20*c) + ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e) - (b*(c^2*d - e)^(5/2
)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(5*c^5*e) - (b*(15*c^4*d^2 -
 20*c^2*d*e + 8*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(40*c^5*Sqrt[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^{5/2}}{1+c^2x^2}\,dx}{5e}$$

$$=-\frac{bx\left(d+ex^2\right)^{3/2}}{20c}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{b\int\sqrt{d+ex^2}}{}$$

$$=-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx\left(d+ex^2\right)^{3/2}}{20c}+\frac{\left(d+ex^2\right)^{5/2}(a+b}{5e}$$

$$=-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx\left(d+ex^2\right)^{3/2}}{20c}+\frac{\left(d+ex^2\right)^{5/2}(a+b}{5e}$$

$$=-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx\left(d+ex^2\right)^{3/2}}{20c}+\frac{\left(d+ex^2\right)^{5/2}(a+b}{5e}$$

$$=-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx\left(d+ex^2\right)^{3/2}}{20c}+\frac{\left(d+ex^2\right)^{5/2}(a+b}{5e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 313, normalized size = 1.73

$$\frac{c^2\sqrt{d+ex^2}\left(8ac^3(d+ex^2)^2+bex(4e-c^2(9d+2ex^2))\right)+8bc^5(d+ex^2)^{5/2}\operatorname{ArcTan}(cx)-4ib(c^2d-e)^{5/2}\log\left(\frac{20c^6e\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(-i+cx)}\right)+4ib(c^2d-e)^{5/2}\log\left(\frac{20c^6e\left(icd+ex+i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(i+cx)}\right)-b\sqrt{e}\,(15c^4d^2-20c^2de+8e^2)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{40c^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*x*(4*e - c^2*(9*d + 2*e*x
^2))) + 8*b*c^5*(d + e*x^2)^(5/2)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(5/2)*L
og[(20*c^6*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*
d - e)^(7/2)*(-I + c*x))] + (4*I)*b*(c^2*d - e)^(5/2)*Log[(20*c^6*e*(I*c*d
+ e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I + c*x))
] - b*Sqrt[e]*(15*c^4*d^2 - 20*c^2*d*e + 8*e^2)*Log[e*x + Sqrt[e]*Sqrt[d +
e*x^2]])/(40*c^5*e)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 5.71, size = 611, normalized size = 3.38

$$\left[\frac{\left((15\,bc^4d^2-20\,bc^2de+8\,be^2)e^{\frac{1}{2}}\log\left(-2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+4\,(bc^4d^2-2\,bc^2de+be^2)\sqrt{-c^2d+e}\,\log\left(\frac{c^4d^2x^4-6\,c^2d^2x^2+8\,x^4e^2-4\,(c^2dx^3-2\,x^3e-dx)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}+d^2-8\,(c^2dx^4-dx^2)e}{c^4x^4+2\,c^2x^2+1}\right)+2\,(8\,ac^5d^2+8\,(bc^5x^4e^2+2\,bc^5dx^2e+bc^5d^2)\arctan(cx)+2\,(4\,ac^5x^4-bc^4x^3+2\,bc^2x)e^2+(16\,ac^5dx^2-9\,bc^4dx)e)\sqrt{x^2e+d}\right)e^{(-1)}}{80\,c^5}, \frac{\left((15\,bc^4d^2-20\,bc^2de+8\,be^2)e^{\frac{1}{2}}\log\left(-2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)-8\,(bc^4d^2-2\,bc^2de+be^2)\sqrt{c^2d-e}\,\arctan\left(\frac{(c^2dx^2-2\,x^2e-d)\sqrt{c^2d-e}\,\sqrt{x^2e+d}}{2\,(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)+2\,(8\,ac^5d^2+8\,(bc^5x^4e^2+2\,bc^5dx^2e+bc^5d^2)\arctan(cx)+2\,(4\,ac^5x^4-bc^4x^3+2\,bc^2x)e^2+(16\,ac^5dx^2-9\,bc^4dx)e)\sqrt{x^2e+d}\right)e^{(-1)}}{80\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [1/80*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*e^(1/2)*log(-2*x^2*e + 2*sqr
t(x^2*e + d)*x*e^(1/2) - d) + 4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2
*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e^2 - 4*(c^2*d*x^3 - 2*x^3
*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)
/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(8*a*c^5*d^2 + 8*(b*c^5*x^4*e^2 + 2*b*c^5*d
*x^2*e + b*c^5*d^2)*arctan(c*x) + 2*(4*a*c^5*x^4 - b*c^4*x^3 + 2*b*c^2*x)*e
^2 + (16*a*c^5*d*x^2 - 9*b*c^4*d*x)*e)*sqrt(x^2*e + d))*e^(-1)/c^5, 1/80*((
15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e
+ d)*x*e^(1/2) - d) - 8*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*a
rctan(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^
2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) + 2*(8*a*c^5*d^2 + 8*(b*c^5*x^4*e^2 +
 2*b*c^5*d*x^2*e + b*c^5*d^2)*arctan(c*x) + 2*(4*a*c^5*x^4 - b*c^4*x^3 + 2*
b*c^2*x)*e^2 + (16*a*c^5*d*x^2 - 9*b*c^4*d*x)*e)*sqrt(x^2*e + d))*e^(-1)/c^
5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
[Out] int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1186 $\int (d+ex^2)^{3/2} (a+b\mathbf{ArcTan}(cx)) \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(d+ex^2\right)^{3/2}(a+b\text{ArcTan}(cx)),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx=\int \left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 5.27, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(cx)) \left(d + ex^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
[Out] int((a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1187 $\int \frac{\left(d+ex^2\right)^{3/2}(a+b\mathbf{ArcTan}(cx))}{x}\,dx$

Optimal. Leaf size=81

$$ad\sqrt{d+ex^2}+\frac{1}{3}a\left(d+ex^2\right)^{3/2}-ad^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\text{ArcTan}(cx)}{x},x\right)$$

```
[Out] 1/3*a*(e*x^2+d)^(3/2)-a*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))+a*d*(e*x^2
+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*d*Sqrt[d + e*x^2] + (a*(d + e*x^2)^(3/2))/3 - a*d^(3/2)*ArcTanh[Sqrt[d +
e*x^2]/Sqrt[d]] + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTan[c*x])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\tan^{-1}(cx)\right)}{x}\,dx &= a\int \frac{\left(d+ex^2\right)^{3/2}}{x}\,dx+b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x}\,dx,x,x^2\right)+b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x}\,dx\\
&= \frac{1}{3}a\left(d+ex^2\right)^{3/2}+b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x}\,dx+\frac{1}{2}(ad)\text{Subst}\left(\int \right.\\
&= ad\sqrt{d+ex^2}+\frac{1}{3}a\left(d+ex^2\right)^{3/2}+b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x}\,dx+\frac{1}{2}\\
&= ad\sqrt{d+ex^2}+\frac{1}{3}a\left(d+ex^2\right)^{3/2}+b\int \frac{\left(d+ex^2\right)^{3/2}\tan^{-1}(cx)}{x}\,dx+\\
&= ad\sqrt{d+ex^2}+\frac{1}{3}a\left(d+ex^2\right)^{3/2}-ad^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\int
\end{aligned}$$

Mathematica [A]
time = 7.06, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\arctan(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)/x, x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atan}\,(cx))\,(d + ex^2)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atan}(c\,x))\,\left(e\,x^2 + d\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x, x)
```

3.1188 $\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x^2}\,dx$

Optimal. Leaf size=90

$$\frac{3}{2}aex\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{x}+\frac{3}{2}ad\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\,\text{ArcTan}(cx)}{x^2},x\right)$$

```
[Out] -a*(e*x^2+d)^(3/2)/x+3/2*a*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)+3/2
*a*e*x*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^2,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (3*a*e*x*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/x + (3*a*d*Sqrt[e]*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2 + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTa
n[c*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{x^2}\,dx &= a\int \frac{(d+ex^2)^{3/2}}{x^2}\,dx+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx\\ &= -\frac{a(d+ex^2)^{3/2}}{x}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx+(3ae)\int \sqrt{d+ex^2}\\ &= \frac{3}{2}aex\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{x}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx+\frac{1}{2}\\ &= \frac{3}{2}aex\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{x}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^2}\,dx+\frac{1}{2}\\ &= \frac{3}{2}aex\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{x}+\frac{3}{2}ad\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\end{aligned}$$

Mathematica [A]
time = 9.60, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan \left(c x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)/x^2,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2, x)
```

3.1189 $\int \frac{(d+ex^2)^{3/2}(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=90

$$\frac{3}{2}ae\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{2x^2}-\frac{3}{2}a\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\,\mathrm{ArcTan}(cx)}{x^3},x\right)$$

```
[Out] -1/2*a*(e*x^2+d)^(3/2)/x^2-3/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)
+3/2*a*e*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^3,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{ArcTan}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (3*a*e*Sqrt[d + e*x^2])/2 - (a*(d + e*x^2)^(3/2))/(2*x^2) - (3*a*Sqrt[d]*e*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTa
n[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{x^3}\,dx = a\int \frac{(d+ex^2)^{3/2}}{x^3}\,dx + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^3}\,dx$$
$$= \frac{1}{2}a\mathrm{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2}\,dx,x,x^2\right) + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^3}\,dx$$
$$= -\frac{a(d+ex^2)^{3/2}}{2x^2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}(3ae)\mathrm{Subst}\Bigg(\int$$
$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{2x^2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{4}$$
$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{2x^2} + b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^3}\,dx + \frac{1}{2}$$
$$= \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{2x^2} - \frac{3}{2}a\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b$$

Mathematica [A]
time = 8.47, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\arctan(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e - 3*sqrt(x^2*e + d)*e -
(x^2*e + d)^(3/2)*e/d + (x^2*e + d)^(5/2)/(d*x^2))*a + 1/2*b*integrate(2*(x
^2*e + d)^(3/2)*arctan(c*x)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)/x^3,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**3,x)
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\left(e\,x^2 + d\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3, x)
```

3.1190 $\int \frac{(d+ex^2)^{3/2}(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=88

$$-\frac{ae\sqrt{d+ex^2}}{x}-\frac{a(d+ex^2)^{3/2}}{3x^3}+ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\,\text{ArcTan}(cx)}{x^4},x\right)$$

```
[Out] -1/3*a*(e*x^2+d)^(3/2)/x^3+a*e^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-a*e
*(e*x^2+d)^(1/2)/x+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^4,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] -((a*e*Sqrt[d + e*x^2])/x) - (a*(d + e*x^2)^(3/2))/(3*x^3) + a*e^(3/2)*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + b*Defer[Int][((d + e*x^2)^(3/2)*ArcTan[c
*x])/x^4, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= a\int \frac{(d+ex^2)^{3/2}}{x^4}\,dx+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^4}\,dx\\
&= -\frac{a(d+ex^2)^{3/2}}{3x^3}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^4}\,dx+(ae)\int \frac{\sqrt{d+ex^2}}{x^2}\\
&= -\frac{ae\sqrt{d+ex^2}}{x}-\frac{a(d+ex^2)^{3/2}}{3x^3}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^4}\,dx+(a\\
&= -\frac{ae\sqrt{d+ex^2}}{x}-\frac{a(d+ex^2)^{3/2}}{3x^3}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^4}\,dx+(a\\
&= -\frac{ae\sqrt{d+ex^2}}{x}-\frac{a(d+ex^2)^{3/2}}{3x^3}+ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\int
\end{aligned}$$

Mathematica [A]
time = 30.19, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan \left(c x\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)/x^4,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4, x)
```

3.1191 $\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x^5}\,dx$

Optimal. Leaf size=95

$$-\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a(d+ex^2)^{3/2}}{4x^4}-\frac{3ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}}+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\text{ArcTan}(cx)}{x^5},x\right)$$

```
[Out] -1/4*a*(e*x^2+d)^(3/2)/x^4-3/8*a*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/
2)-3/8*a*e*(e*x^2+d)^(1/2)/x^2+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x
^5,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] (-3*a*e*Sqrt[d + e*x^2])/(8*x^2) - (a*(d + e*x^2)^(3/2))/(4*x^4) - (3*a*e^2
*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*Sqrt[d]) + b*Defer[Int][((d + e*x^2)^
(3/2)*ArcTan[c*x])/x^5, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^5}\,dx &= a\int \frac{(d+ex^2)^{3/2}}{x^5}\,dx+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^5}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^3}\,dx,x,x^2\right)+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^5}\,dx\\
&= -\frac{a(d+ex^2)^{3/2}}{4x^4}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^5}\,dx+\frac{1}{8}(3ae)\text{Subst}\Bigg(\int\\
&= -\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a(d+ex^2)^{3/2}}{4x^4}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^5}\,dx+\\
&= -\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a(d+ex^2)^{3/2}}{4x^4}+b\int \frac{(d+ex^2)^{3/2}\tan^{-1}(cx)}{x^5}\,dx+\\
&= -\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a(d+ex^2)^{3/2}}{4x^4}-\frac{3ae^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}}+b
\end{aligned}$$

Mathematica [A]
time = 9.68, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\arctan(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e^2/sqrt(d) - (x^2*e + d)^(3/2)*e^
2/d^2 - 3*sqrt(x^2*e + d)*e^2/d + (x^2*e + d)^(5/2)*e/(d^2*x^2) + 2*(x^2*e
+ d)^(5/2)/(d*x^4))*a + 1/2*b*integrate(2*(x^2*e + d)^(3/2)*arctan(c*x)/x^5
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)/x^5,
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**5,x)
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**5, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atan}(c\,x))\,\left(e\,x^2 + d\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5, x)
```

3.1192 $\int \frac{(d+ex^2)^{3/2}(a+b\mathbf{ArcTan}(cx))}{x^6}\,dx$

Optimal. Leaf size=178

$$\frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}-\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5dx^5}-\frac{bc(8c^4d^2-20c^2de+15e^2)\tanh}{40\sqrt{d}}$$

```
[Out] -1/20*b*c*(e*x^2+d)^(3/2)/x^4-1/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/d/x^5+1
/5*b*(c^2*d-e)^(5/2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d-1/40*b*c*
(8*c^4*d^2-20*c^2*d*e+15*e^2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+1/40
*b*c*(4*c^2*d-7*e)*(e*x^2+d)^(1/2)/x^2
```

Rubi [A]
time = 0.24, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$,
Rules used = {270, 5096, 12, 457, 100, 154, 162, 65, 214}

$$-\frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{5dx^5}+\frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}+\frac{b(c^2d-e)^{5/2}\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{5d}-\frac{bc(8c^4d^2-20c^2de+15e^2)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40\sqrt{d}}-\frac{bc(d+ex^2)^{3/2}}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (b*c*(4*c^2*d - 7*e)*Sqrt[d + e*x^2])/(40*x^2) - (b*c*(d + e*x^2)^(3/2))/(2
0*x^4) - ((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*d*x^5) - (b*c*(8*c^4*d^
2 - 20*c^2*d*e + 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*Sqrt[d]) + (
b*(c^2*d - e)^(5/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
```

```
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)^{3/2}(a+b\tan^{-1}(cx))}{x^6}\,dx &= -\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-(bc)\int \frac{(d+ex^2)^{5/2}}{5x^5(-d-c^2dx^2)}\,dx\\
&= -\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-\frac{1}{5}(bc)\int \frac{(d+ex^2)^{5/2}}{x^5(-d-c^2dx^2)}\,dx\\
&= -\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-\frac{1}{10}(bc)\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x^3(-d-c^2dx)}\,dx\right.\\
&= -\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{5dx^5}-\frac{(bc)\text{Subst}\left(\int \sqrt{d}\right.}{}\\
&= \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}-\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}}{5dx^5}\\
&= \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}-\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}}{5dx^5}\\
&= \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}-\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}}{5dx^5}\\
&= \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2}-\frac{bc(d+ex^2)^{3/2}}{20x^4}-\frac{(d+ex^2)^{5/2}(a+b\tan^{-1}}{5dx^5}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 334, normalized size = 1.88

$$\frac{-\sqrt{d+ex^2}\left(8a(d+ex^2)^2+bcdx(9ex^2+d(2-4c^2x^2))\right)-8b(d+ex^2)^{5/2}\text{ArcTan}(cx)+bc\sqrt{d}\left(8c^4d^2-20c^2de+15e^2\right)x^5\log(x)-bc\sqrt{d}\left(8c^4d^2-20c^2de+15e^2\right)x^5\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)+4b(c^2d-e)^{5/2}x^5\log\left(-\frac{20cd\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(i+cx)}\right)+4b(c^2d-e)^{5/2}x^5\log\left(-\frac{20cd\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{7/2}(-i+cx)}\right)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

```
[Out] (-(Sqrt[d + e*x^2]*(8*a*(d + e*x^2)^2 + b*c*d*x*(9*e*x^2 + d*(2 - 4*c^2*x^2
)))) - 8*b*(d + e*x^2)^(5/2)*ArcTan[c*x] + b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*
```

```
d*e + 15*e^2)*x^5*Log[x] - b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)*x^
5*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d
*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I +
 c*x))] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d*(c*d + I*e*x + Sqrt[c^2*d
- e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(-I + c*x))])/(40*d*x^5)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan\left(cx\right)\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.58, size = 1227, normalized size = 6.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(4*(b*c^4*d^2*x^5 - 2*b*c^2*d*x^5*e + b*x^5*e^2)*sqrt(c^2*d - e)*log(
(8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d)
+ (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2
*x^2 + 1)) + (8*b*c^5*d^2*x^5 - 20*b*c^3*d*x^5*e + 15*b*c*x^5*e^2)*sqrt(d)*
log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 2*(4*b*c^3*d^2*x^3 -
8*a*x^4*e^2 - 2*b*c*d^2*x - 8*a*d^2 - 8*(b*x^4*e^2 + 2*b*d*x^2*e + b*d^2)*a
rctan(c*x) - (9*b*c*d*x^3 + 16*a*d*x^2)*e)*sqrt(x^2*e + d))/(d*x^5), 1/80*(
8*(b*c^4*d^2*x^5 - 2*b*c^2*d*x^5*e + b*x^5*e^2)*sqrt(-c^2*d + e)*arctan(-1/
```

```
2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c
*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + (8*b*c^5*d^2*x^5 - 20*b*c^3*d*x^5*e + 15
*b*c*x^5*e^2)*sqrt(d)*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) +
 2*(4*b*c^3*d^2*x^3 - 8*a*x^4*e^2 - 2*b*c*d^2*x - 8*a*d^2 - 8*(b*x^4*e^2 +
2*b*d*x^2*e + b*d^2)*arctan(c*x) - (9*b*c*d*x^3 + 16*a*d*x^2)*e)*sqrt(x^2*e
 + d))/(d*x^5), 1/40*((8*b*c^5*d^2*x^5 - 20*b*c^3*d*x^5*e + 15*b*c*x^5*e^2)
*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + 2*(b*c^4*d^2*x^5 - 2*b*c^2*d*x
^5*e + b*x^5*e^2)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 -
c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(
c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (4*b*c^3*d^2*x^3 - 8*a*x
^4*e^2 - 2*b*c*d^2*x - 8*a*d^2 - 8*(b*x^4*e^2 + 2*b*d*x^2*e + b*d^2)*arctan
(c*x) - (9*b*c*d*x^3 + 16*a*d*x^2)*e)*sqrt(x^2*e + d))/(d*x^5), 1/40*(4*(b*
c^4*d^2*x^5 - 2*b*c^2*d*x^5*e + b*x^5*e^2)*sqrt(-c^2*d + e)*arctan(-1/2*(2*
c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*
e^2 + (c^3*d*x^2 - c*d)*e)) + (8*b*c^5*d^2*x^5 - 20*b*c^3*d*x^5*e + 15*b*c*
x^5*e^2)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + (4*b*c^3*d^2*x^3 - 8*a
*x^4*e^2 - 2*b*c*d^2*x - 8*a*d^2 - 8*(b*x^4*e^2 + 2*b*d*x^2*e + b*d^2)*arct
an(c*x) - (9*b*c*d*x^3 + 16*a*d*x^2)*e)*sqrt(x^2*e + d))/(d*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**6,x)
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,{(e\,x^2 + d)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6, x)
```

3.1193 $\int x^3(d+ex^2)^{5/2}(a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=345

$$\frac{b(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3)\,x\sqrt{d+ex^2}}{8064c^7e}-\frac{b(69c^4d^2-520c^2de+336e^2)\,x(d+ex^2)^{3/2}}{12096c^5e}-\frac{b(33c$$

```
[Out] -1/12096*b*(69*c^4*d^2-520*c^2*d*e+336*e^2)*x*(e*x^2+d)^(3/2)/c^5/e-1/3024*
b*(33*c^2*d-56*e)*x*(e*x^2+d)^(5/2)/c^3/e-1/72*b*x*(e*x^2+d)^(7/2)/c/e-1/7*
d*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e^2+1/9*(e*x^2+d)^(9/2)*(a+b*arctan(c*x
))/e^2+1/63*b*(c^2*d-e)^(7/2)*(2*c^2*d+7*e)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2
+d)^(1/2))/c^9/e^2+1/8064*b*(315*c^8*d^4+840*c^6*d^3*e-3024*c^4*d^2*e^2+288
0*c^2*d*e^3-896*e^4)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^9/e^(3/2)+1/8064*
b*(59*c^6*d^3+712*c^4*d^2*e-1104*c^2*d*e^2+448*e^3)*x*(e*x^2+d)^(1/2)/c^7/e
```

Rubi [A]
time = 0.43, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {272, 45, 5096, 12, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{9/2}(a+b\text{ArcTan}(cx))}{9e^2}-\frac{d(d+ex^2)^{7/2}(a+b\text{ArcTan}(cx))}{7e^2}+\frac{b(c^2d-e)^{7/2}(2c^2d+7e)\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{63c^9e^2}-\frac{bx(33c^2d-56e)(d+ex^2)^{5/2}}{3024c^3e}-\frac{bx(69c^4d^2-520c^2de+336e^2)(d+ex^2)^{3/2}}{12096c^5e}+\frac{bx(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3)\sqrt{d+ex^2}}{8064c^7e}+\frac{b(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8064c^9e^{3/2}}-\frac{bx(d+ex^2)^{7/2}}{72ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*(59*c^6*d^3 + 712*c^4*d^2*e - 1104*c^2*d*e^2 + 448*e^3)*x*Sqrt[d + e*x^2
])/(8064*c^7*e) - (b*(69*c^4*d^2 - 520*c^2*d*e + 336*e^2)*x*(d + e*x^2)^(3/
2))/(12096*c^5*e) - (b*(33*c^2*d - 56*e)*x*(d + e*x^2)^(5/2))/(3024*c^3*e)
- (b*x*(d + e*x^2)^(7/2))/(72*c*e) - (d*(d + e*x^2)^(7/2)*(a + b*ArcTan[c*x
]))/(7*e^2) + ((d + e*x^2)^(9/2)*(a + b*ArcTan[c*x]))/(9*e^2) + (b*(c^2*d -
 e)^(7/2)*(2*c^2*d + 7*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(63*
c^9*e^2) + (b*(315*c^8*d^4 + 840*c^6*d^3*e - 3024*c^4*d^2*e^2 + 2880*c^2*d*
e^3 - 896*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8064*c^9*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)\,dx = -\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}$$

$$= -\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/2}\left(a+b\tan^{-1}(cx)\right)}{9e^2}$$

$$= -\frac{bx\left(d+ex^2\right)^{7/2}}{72ce}-\frac{d\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e^2}+\frac{\left(d+ex^2\right)^{9/}}{}$$

$$= -\frac{b\left(33c^2d-56e\right)x\left(d+ex^2\right)^{5/2}}{3024c^3e}-\frac{bx\left(d+ex^2\right)^{7/2}}{72ce}-\frac{d\left(d+ex^2\right)^{7/2}}{}$$

$$= -\frac{b\left(69c^4d^2-520c^2de+336e^2\right)x\left(d+ex^2\right)^{3/2}}{12096c^5e}-\frac{b\left(33c^2d-56e\right)x(}{3024c^3}$$

$$= \frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b(69c^4}{}$$

$$= \frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b(69c^4}{}$$

$$= \frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b(69c^4}{}$$

$$= \frac{b\left(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3\right)x\sqrt{d+ex^2}}{8064c^7e}-\frac{b(69c^4}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 470, normalized size = 1.36

$$-\frac{c^2\sqrt{d+ex^2}\left(384ac^7\left(2d-7ex^2\right)\left(d+ex^2\right)^3+bex\left(-1344e^3+48c^2e^2\left(83d+14ex^2\right)-8c^4e\left(453d^2+242dex^2+56e^2x^4\right)+3c^6\left(187d^3+558d^2ex^2+424de^2x^4+112e^3x^6\right)\right)\right)+384bc^9\left(2d-7ex^2\right)\left(d+ex^2\right)^{7/2}\text{ArcTan}(cx)+192ib\left(c^2d-e\right)^{7/2}\left(2c^2d+7e\right)\log\left(-\frac{252ic^{10}e^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{9/2}\left(2c^2d+7e\right)(i+cx)}\right)-192ib\left(c^2d-e\right)^{7/2}\left(2c^2d+7e\right)\log\left(\frac{252ic^{10}e^2\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{9/2}\left(2c^2d+7e\right)(-i+cx)}\right)+3b\sqrt{e}\left(-315c^8d^4-840c^6d^3e+3024c^4d^2e^2-2880c^2de^3+896e^4\right)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{24192c^9e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/24192*(c^2*Sqrt[d + e*x^2]*(384*a*c^7*(2*d - 7*e*x^2)*(d + e*x^2)^3 + b*
e*x*(-1344*e^3 + 48*c^2*e^2*(83*d + 14*e*x^2) - 8*c^4*e*(453*d^2 + 242*d*e*
x^2 + 56*e^2*x^4) + 3*c^6*(187*d^3 + 558*d^2*e*x^2 + 424*d*e^2*x^4 + 112*e^
3*x^6))) + 384*b*c^9*(2*d - 7*e*x^2)*(d + e*x^2)^(7/2)*ArcTan[c*x] + (192*I
)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((-252*I)*c^10*e^2*(c*d - I*e*x +
 Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*c^2*d + 7*e)*(I
+ c*x))] - (192*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((252*I)*c^10*e^
2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*
c^2*d + 7*e)*(-I + c*x))] + 3*b*Sqrt[e]*(-315*c^8*d^4 - 840*c^6*d^3*e + 302
4*c^4*d^2*e^2 - 2880*c^2*d*e^3 + 896*e^4)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]
])/(c^9*e^2)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/63*(7*(x^2*e + d)^(7/2)*x^2*e^(-1) - 2*(x^2*e + d)^(7/2)*d*e^(-2))*a + 1/
2*b*integrate(2*(x^7*e^2 + 2*d*x^5*e + d^2*x^3)*sqrt(x^2*e + d)*arctan(c*x)
, x)
```

Fricas [A]
time = 90.01, size = 971, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/48384*(3*(315*b*c^8*d^4 + 840*b*c^6*d^3*e - 3024*b*c^4*d^2*e^2 + 2880*b
*c^2*d*e^3 - 896*b*e^4)*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2)
```

```
- d) + 192*(2*b*c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d^2*e^2 + 19*b*c^2*d*e^3 -
 7*b*e^4)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e^2 - 4
*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 - 8*(c^
2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(768*a*c^9*d^4 - 384*(7*
b*c^9*x^8*e^4 + 19*b*c^9*d*x^6*e^3 + 15*b*c^9*d^2*x^4*e^2 + b*c^9*d^3*x^2*e
 - 2*b*c^9*d^4)*arctan(c*x) - 112*(24*a*c^9*x^8 - 3*b*c^8*x^7 + 4*b*c^6*x^5
 - 6*b*c^4*x^3 + 12*b*c^2*x)*e^4 - 8*(912*a*c^9*d*x^6 - 159*b*c^8*d*x^5 + 2
42*b*c^6*d*x^3 - 498*b*c^4*d*x)*e^3 - 6*(960*a*c^9*d^2*x^4 - 279*b*c^8*d^2*
x^3 + 604*b*c^6*d^2*x)*e^2 - 3*(128*a*c^9*d^3*x^2 - 187*b*c^8*d^3*x)*e)*sqr
t(x^2*e + d))*e^(-2)/c^9, -1/48384*(3*(315*b*c^8*d^4 + 840*b*c^6*d^3*e - 30
24*b*c^4*d^2*e^2 + 2880*b*c^2*d*e^3 - 896*b*e^4)*e^(1/2)*log(-2*x^2*e + 2*s
qrt(x^2*e + d)*x*e^(1/2) - d) - 384*(2*b*c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d
^2*e^2 + 19*b*c^2*d*e^3 - 7*b*e^4)*sqrt(c^2*d - e)*arctan(1/2*(c^2*d*x^2 -
2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*
x^3 - d*x)*e)) + 2*(768*a*c^9*d^4 - 384*(7*b*c^9*x^8*e^4 + 19*b*c^9*d*x^6*e
^3 + 15*b*c^9*d^2*x^4*e^2 + b*c^9*d^3*x^2*e - 2*b*c^9*d^4)*arctan(c*x) - 11
2*(24*a*c^9*x^8 - 3*b*c^8*x^7 + 4*b*c^6*x^5 - 6*b*c^4*x^3 + 12*b*c^2*x)*e^4
 - 8*(912*a*c^9*d*x^6 - 159*b*c^8*d*x^5 + 242*b*c^6*d*x^3 - 498*b*c^4*d*x)*
e^3 - 6*(960*a*c^9*d^2*x^4 - 279*b*c^8*d^2*x^3 + 604*b*c^6*d^2*x)*e^2 - 3*(
128*a*c^9*d^3*x^2 - 187*b*c^8*d^3*x)*e)*sqrt(x^2*e + d))*e^(-2)/c^9]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
[Out] Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,{\left(e\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
[Out] int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1194 $\int x^2(d+ex^2)^{5/2}(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=141

$$\frac{5ad^3x\sqrt{d+ex^2}}{128e}+\frac{5}{64}ad^2x^3\sqrt{d+ex^2}+\frac{5}{48}adx^3\left(d+ex^2\right)^{3/2}+\frac{1}{8}ax^3\left(d+ex^2\right)^{5/2}-\frac{5ad^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{128e^{3/2}}$$

```
[Out] 5/48*a*d*x^3*(e*x^2+d)^(3/2)+1/8*a*x^3*(e*x^2+d)^(5/2)-5/128*a*d^4*arctanh(
x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+5/128*a*d^3*x*(e*x^2+d)^(1/2)/e+5/64*a*d
^2*x^3*(e*x^2+d)^(1/2)+b*Unintegrable(x^2*(e*x^2+d)^(5/2)*arctan(c*x),x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (5*a*d^3*x*Sqrt[d + e*x^2])/(128*e) + (5*a*d^2*x^3*Sqrt[d + e*x^2])/64 + (5
*a*d*x^3*(d + e*x^2)^(3/2))/48 + (a*x^3*(d + e*x^2)^(5/2))/8 - (5*a*d^4*Arc
Tanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(3/2)) + b*Defer[Int][x^2*(d + e*
x^2)^(5/2)*ArcTan[c*x], x]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^{5/2} \left(a+b\tan^{-1}(cx)\right) \, dx = a \int x^2 \left(d+ex^2\right)^{5/2} \, dx + b \int x^2 \left(d+ex^2\right)^{5/2} \tan^{-1}(cx) \, dx$$

$$= \frac{1}{8}ax^3\left(d+ex^2\right)^{5/2} + b\int x^2\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)\,dx + \frac{1}{8}(5ad)\int x$$

$$= \frac{5}{48}adx^3\left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3\left(d+ex^2\right)^{5/2} + b\int x^2\left(d+ex^2\right)^{5/2}\tan^{-}$$

$$= \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3\left(d+ex^2\right)^{3/2} + \frac{1}{8}ax^3\left(d+ex^2\right)^{5/2} + b$$

$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3\left(d+ex^2\right)^{3/2} + \frac{1}{8}$$

$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3\left(d+ex^2\right)^{3/2} + \frac{1}{8}$$

$$= \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3\left(d+ex^2\right)^{3/2} + \frac{1}{8}$$

Mathematica [**A**]
time = 11.56, size = 0, normalized size = 0.00

$$\int x^2\left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [**A**]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^2\left(e\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
[Out] int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^6*e^2 + 2*a*d*x^4*e + a*d^2*x^2 + (b*x^6*e^2 + 2*b*d*x^4*e +
b*d^2*x^2)*arctan(c*x))*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{atan}(cx))\left(d + ex^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,{\left(e\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1195 $\int x(d+ex^2)^{5/2}(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=233

$$-\frac{b(19c^4d^2-22c^2de+8e^2)x\sqrt{d+ex^2}}{112c^5}-\frac{b(11c^2d-6e)x(d+ex^2)^{3/2}}{168c^3}-\frac{bx(d+ex^2)^{5/2}}{42c}+\frac{(d+ex^2)^{7/2}(a+b\mathrm{Ar}}{7e}$$

```
[Out] -1/168*b*(11*c^2*d-6*e)*x*(e*x^2+d)^(3/2)/c^3-1/42*b*x*(e*x^2+d)^(5/2)/c+1/
7*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e-1/7*b*(c^2*d-e)^(7/2)*arctan(x*(c^2*d
-e)^(1/2)/(e*x^2+d)^(1/2))/c^7/e-1/112*b*(35*c^6*d^3-70*c^4*d^2*e+56*c^2*d*
e^2-16*e^3)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^7/e^(1/2)-1/112*b*(19*c^4*
d^2-22*c^2*d*e+8*e^2)*x*(e*x^2+d)^(1/2)/c^5
```

Rubi [A]
time = 0.23, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5094, 427, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{7/2}(a+b\mathrm{ArcTan}(cx))}{7e}-\frac{b(c^2d-e)^{7/2}\mathrm{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{7c^7e}-\frac{bx(11c^2d-6e)(d+ex^2)^{3/2}}{168c^3}-\frac{bx(19c^4d^2-22c^2de+8e^2)\sqrt{d+ex^2}}{112c^5}-\frac{b(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{112c^7\sqrt{e}}-\frac{bx(d+ex^2)^{5/2}}{42c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/112*(b*(19*c^4*d^2 - 22*c^2*d*e + 8*e^2)*x*Sqrt[d + e*x^2])/c^5 - (b*(11
*c^2*d - 6*e)*x*(d + e*x^2)^(3/2))/(168*c^3) - (b*x*(d + e*x^2)^(5/2))/(42*
c) + ((d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/(7*e) - (b*(c^2*d - e)^(7/2)*A
rcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(7*c^7*e) - (b*(35*c^6*d^3 - 70
*c^4*d^2*e + 56*c^2*d*e^2 - 16*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
112*c^7*Sqrt[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^{7/2}}{1+c^2x^2}\,dx}{7e}$$

$$=-\frac{bx\left(d+ex^2\right)^{5/2}}{42c}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\tan^{-1}(cx)\right)}{7e}-\frac{b\int\frac{\left(d+ex^2\right)^{3/2}(d(}{}}{}$$

$$=-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)^{3/2}}{168c^3}-\frac{bx\left(d+ex^2\right)^{5/2}}{42c}+\frac{\left(d+ex^2\right)^{7/2}(a+}{7e}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)}{168c^3}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)}{168c^3}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)}{168c^3}$$

$$=-\frac{b\left(19c^4d^2-22c^2de+8e^2\right)x\sqrt{d+ex^2}}{112c^5}-\frac{b\left(11c^2d-6e\right)x\left(d+ex^2\right)}{168c^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 353, normalized size = 1.52

$$\frac{c^2\sqrt{d+ex^2}\left(48ac^5\left(d+ex^2\right)^3-bex\left(24e^2-6c^2e\left(13d+2ex^2\right)+c^4\left(87d^2+38dex^2+8e^2x^4\right)\right)\right)+48bc^7\left(d+ex^2\right)^{7/2}\text{ArcTan}(cx)-24ib\left(c^2d-e\right)^{7/2}\log\left(\frac{28c^8e\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{9/2}(-i+cx)}\right)+24ib\left(c^2d-e\right)^{7/2}\log\left(\frac{28c^8e\left(icd+ex+i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\left(c^2d-e\right)^{9/2}(i+cx)}\right)+3b\sqrt{e}\left(-35c^6d^3+70c^4d^2e-56c^2de^2+16e^3\right)\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{336c^7e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (c^2*Sqrt[d + e*x^2]*(48*a*c^5*(d + e*x^2)^3 - b*e*x*(24*e^2 - 6*c^2*e*(13*
d + 2*e*x^2) + c^4*(87*d^2 + 38*d*e*x^2 + 8*e^2*x^4))) + 48*b*c^7*(d + e*x^
2)^(7/2)*ArcTan[c*x] - (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*((-I)*c*d +
 e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(-I + c*x))
] + (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*(I*c*d + e*x + I*Sqrt[c^2*d -
e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(I + c*x))] + 3*b*Sqrt[e]*(-35*c^
6*d^3 + 70*c^4*d^2*e - 56*c^2*d*e^2 + 16*e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*
x^2]])/(336*c^7*e)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x\left(e\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
[Out] int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 25.07, size = 781, normalized size = 3.35

$$\left[-\frac{3\,(35\,bc^6d^3-70\,bc^4d^2e+56\,bc^2de^2-16\,be^3)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+24\,(bc^6d^3-3\,bc^4d^2e+3\,bc^2de^2-be^3)\sqrt{-c^2d+e}\,\log\left(\frac{c^4d^2x^4-6\,c^2d^2x^2+8\,x^4e^2+4\,(c^2dx^3-2\,x^3e-dx)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}+d^2-8\,(c^2dx^4-dx^2)e}{c^4x^4+2\,c^2x^2+1}\right)-2\,(48\,ac^7d^3+48\,(bc^7x^6e^3+3\,bc^7dx^4e^2+3\,bc^7d^2x^2e+bc^7d^3)\arctan(cx)+4\,(12\,ac^7x^6-2\,bc^6x^5+3\,bc^4x^3-6\,bc^2x)e^3+2\,(72\,ac^7dx^4-19\,bc^6dx^3+39\,bc^4dx)e^2+3\,(48\,ac^7d^2x^2-29\,bc^6d^2x)e)\sqrt{x^2e+d}\right)e^{(-1)}}{672\,c^7},\ -\frac{3\,(35\,bc^6d^3-70\,bc^4d^2e+56\,bc^2de^2-16\,be^3)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+48\,(bc^6d^3-3\,bc^4d^2e+3\,bc^2de^2-be^3)\sqrt{c^2d-e}\,\arctan\left(\frac{(c^2dx^2-2\,x^2e-d)\sqrt{c^2d-e}\,\sqrt{x^2e+d}}{2\,(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)-2\,(48\,ac^7d^3+48\,(bc^7x^6e^3+3\,bc^7dx^4e^2+3\,bc^7d^2x^2e+bc^7d^3)\arctan(cx)+4\,(12\,ac^7x^6-2\,bc^6x^5+3\,bc^4x^3-6\,bc^2x)e^3+2\,(72\,ac^7dx^4-19\,bc^6dx^3+39\,bc^4dx)e^2+3\,(48\,ac^7d^2x^2-29\,bc^6d^2x)e)\sqrt{x^2e+d}\,e^{(-1)}}{672\,c^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] [-1/672*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*e^(1
/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 24*(b*c^6*d^3 - 3*b*c
^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2
*d^2*x^2 + 8*x^4*e^2 + 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(
x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*
(48*a*c^7*d^3 + 48*(b*c^7*x^6*e^3 + 3*b*c^7*d*x^4*e^2 + 3*b*c^7*d^2*x^2*e +
 b*c^7*d^3)*arctan(c*x) + 4*(12*a*c^7*x^6 - 2*b*c^6*x^5 + 3*b*c^4*x^3 - 6*b
*c^2*x)*e^3 + 2*(72*a*c^7*d*x^4 - 19*b*c^6*d*x^3 + 39*b*c^4*d*x)*e^2 + 3*(4
8*a*c^7*d^2*x^2 - 29*b*c^6*d^2*x)*e)*sqrt(x^2*e + d))*e^(-1)/c^7, -1/672*(3
*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*e^(1/2)*log(-2
*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 48*(b*c^6*d^3 - 3*b*c^4*d^2*e +
 3*b*c^2*d*e^2 - b*e^3)*sqrt(c^2*d - e)*arctan(1/2*(c^2*d*x^2 - 2*x^2*e - d
)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*
e)) - 2*(48*a*c^7*d^3 + 48*(b*c^7*x^6*e^3 + 3*b*c^7*d*x^4*e^2 + 3*b*c^7*d^2
*x^2*e + b*c^7*d^3)*arctan(c*x) + 4*(12*a*c^7*x^6 - 2*b*c^6*x^5 + 3*b*c^4*x
^3 - 6*b*c^2*x)*e^3 + 2*(72*a*c^7*d*x^4 - 19*b*c^6*d*x^3 + 39*b*c^4*d*x)*e^
2 + 3*(48*a*c^7*d^2*x^2 - 29*b*c^6*d^2*x)*e)*sqrt(x^2*e + d))*e^(-1)/c^7]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1196 $\int (d+ex^2)^{5/2} (a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx)),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)\,dx = \int \left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 5.18, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
[Out] Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atan}(cx)) \left(d + ex^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b\,\mathrm{atan}(c\,x))\ \left(e\,x^2 + d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
[Out] int((a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1197 $\int \frac{(d+ex^2)^{5/2}(a+b\mathbf{ArcTan}(cx))}{x} dx$

Optimal. Leaf size=100

$$ad^2\sqrt{d+ex^2}+\frac{1}{3}ad(d+ex^2)^{3/2}+\frac{1}{5}a(d+ex^2)^{5/2}-ad^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\text{ArcTan}(c$$

```
[Out] 1/3*a*d*(e*x^2+d)^(3/2)+1/5*a*(e*x^2+d)^(5/2)-a*d^(5/2)*arctanh((e*x^2+d)^(
1/2)/d^(1/2))+a*d^2*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(5/2)*arctan(c
*x)/x,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{5/2}(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

```
[Out] a*d^2*Sqrt[d + e*x^2] + (a*d*(d + e*x^2)^(3/2))/3 + (a*(d + e*x^2)^(5/2))/5
 - a*d^(5/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + b*Defer[Int][((d + e*x^2)^(
5/2)*ArcTan[c*x])/x, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{x}\,dx = a\int \frac{(d+ex^2)^{5/2}}{x}\,dx + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x}\,dx$$

$$= \frac{1}{2}a\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x}\,dx, x, x^2\right) + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x}\,dx$$

$$= \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x}\,dx + \frac{1}{2}(ad)\text{Subst}\Bigg(\int$$

$$= \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x}\,dx$$

$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{(d+ex^2)}{}$$

$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} + b\int \frac{(d+ex^2)}{}$$

$$= ad^2\sqrt{d+ex^2} + \frac{1}{3}ad\left(d+ex^2\right)^{3/2} + \frac{1}{5}a\left(d+ex^2\right)^{5/2} - ad^{5/2}\tanh^{-1}\Bigg($$

Mathematica [A]
time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\text{ArcTan}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{5}{2}}(a+b\arctan(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d)/x, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))(d + ex^2)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x, x)
```

3.1198 $\int \frac{(d+ex^2)^{5/2}(a+b\text{ArcTan}(cx))}{x^2} dx$

Optimal. Leaf size=111

$$\frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex\left(d+ex^2\right)^{3/2}-\frac{a\left(d+ex^2\right)^{5/2}}{x}+\frac{15}{8}ad^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right.$$

```
[Out] 5/4*a*e*x*(e*x^2+d)^(3/2)-a*(e*x^2+d)^(5/2)/x+15/8*a*d^2*arctanh(x*e^(1/2)/
(e*x^2+d)^(1/2))*e^(1/2)+15/8*a*d*e*x*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2
+d)^(5/2)*arctan(c*x)/x^2,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] (15*a*d*e*x*Sqrt[d + e*x^2])/8 + (5*a*e*x*(d + e*x^2)^(3/2))/4 - (a*(d + e*
x^2)^(5/2))/x + (15*a*d^2*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/8 +
 b*Defer[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^{5/2}\left(a+b\tan^{-1}(cx)\right)}{x^2}\,dx &= a\int \frac{\left(d+ex^2\right)^{5/2}}{x^2}\,dx+b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^2}\,dx\\
&= -\frac{a\left(d+ex^2\right)^{5/2}}{x}+b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^2}\,dx+(5ae)\int \left(d+ex^2\right)\\
&= \frac{5}{4}aex\left(d+ex^2\right)^{3/2}-\frac{a\left(d+ex^2\right)^{5/2}}{x}+b\int \frac{\left(d+ex^2\right)^{5/2}\tan^{-1}(cx)}{x^2}\,dx+\\
&= \frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex\left(d+ex^2\right)^{3/2}-\frac{a\left(d+ex^2\right)^{5/2}}{x}+b\int \frac{(d+e}{}\\
&= \frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex\left(d+ex^2\right)^{3/2}-\frac{a\left(d+ex^2\right)^{5/2}}{x}+b\int \frac{(d+e}{}\\
&= \frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex\left(d+ex^2\right)^{3/2}-\frac{a\left(d+ex^2\right)^{5/2}}{x}+\frac{15}{8}ad^2\sqrt{e}
\end{aligned}$$

Mathematica [A]
time = 9.38, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

```
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\arctan(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d)/x^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atan}\,(cx))\,(d + ex^2)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**2,x)
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atan}(c\,x))\,\left(e\,x^2 + d\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2,x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2, x)
```

3.1199 $\int \frac{(d+ex^2)^{5/2}(a+b\mathbf{ArcTan}(cx))}{x^3}\,dx$

Optimal. Leaf size=108

$$\frac{5}{2}ade\sqrt{d+ex^2}+\frac{5}{6}ae(d+ex^2)^{3/2}-\frac{a(d+ex^2)^{5/2}}{2x^2}-\frac{5}{2}ad^{3/2}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\mathrm{Int}\left(\frac{(d+ex^2)^{5/2}\,\mathrm{ArcT}}{x^3}\right.$$

```
[Out] 5/6*a*e*(e*x^2+d)^(3/2)-1/2*a*(e*x^2+d)^(5/2)/x^2-5/2*a*d^(3/2)*e*arctanh((
e*x^2+d)^(1/2)/d^(1/2))+5/2*a*d*e*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^
(5/2)*arctan(c*x)/x^3,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{5/2}(a+b\mathrm{ArcTan}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

```
[Out] (5*a*d*e*Sqrt[d + e*x^2])/2 + (5*a*e*(d + e*x^2)^(3/2))/6 - (a*(d + e*x^2)^
(5/2))/(2*x^2) - (5*a*d^(3/2)*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + b*Def
er[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^3, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{5/2}(a+b\tan^{-1}(cx))}{x^3}dx = a\int \frac{(d+ex^2)^{5/2}}{x^3}dx + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^3}dx$$
$$= \frac{1}{2}a\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x^2}dx, x, x^2\right) + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^3}dx$$
$$= -\frac{a(d+ex^2)^{5/2}}{2x^2} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^3}dx + \frac{1}{4}(5ae)\text{Subst}\left(\int \right.$$
$$= \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2} + b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^3}dx + $$
$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2} + b\int$$
$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2} + b\int$$
$$= \frac{5}{2}ade\sqrt{d+ex^2} + \frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2} - \frac{5}{2}ad^{3/2}e\tanh^{-1}$$

Mathematica [A]
time = 8.63, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\text{ArcTan}(cx))}{x^3}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{5}{2}}(a+b\arctan(cx))}{x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))\,(d + ex^2)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\left(e\,x^2 + d\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3, x)
```

3.1200 $\int \frac{(d+ex^2)^{5/2}(a+b\mathbf{ArcTan}(cx))}{x^4}\,dx$

Optimal. Leaf size=114

$$\frac{5}{2}ae^2x\sqrt{d+ex^2}-\frac{5ae(d+ex^2)^{3/2}}{3x}-\frac{a(d+ex^2)^{5/2}}{3x^3}+\frac{5}{2}ade^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\,\text{ArcTa}}{x^4}\right.$$

```
[Out] -5/3*a*e*(e*x^2+d)^(3/2)/x-1/3*a*(e*x^2+d)^(5/2)/x^3+5/2*a*d*e^(3/2)*arctan
h(x*e^(1/2)/(e*x^2+d)^(1/2))+5/2*a*e^2*x*(e*x^2+d)^(1/2)+b*Unintegrable((e*
x^2+d)^(5/2)*arctan(c*x)/x^4,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\text{ArcTan}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

```
[Out] (5*a*e^2*x*Sqrt[d + e*x^2])/2 - (5*a*e*(d + e*x^2)^(3/2))/(3*x) - (a*(d + e
*x^2)^(5/2))/(3*x^3) + (5*a*d*e^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/2 + b*Defer[Int][((d + e*x^2)^(5/2)*ArcTan[c*x])/x^4, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{5/2}\,(a+b\tan^{-1}(cx))}{x^4}\,dx &= a\int \frac{(d+ex^2)^{5/2}}{x^4}\,dx+b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^4}\,dx\\
&= -\frac{a(d+ex^2)^{5/2}}{3x^3}+b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^4}\,dx+\frac{1}{3}(5ae)\int \frac{(d+ex^2}{x^2}\\
&= -\frac{5ae(d+ex^2)^{3/2}}{3x}-\frac{a(d+ex^2)^{5/2}}{3x^3}+b\int \frac{(d+ex^2)^{5/2}\tan^{-1}(cx)}{x^4}\,dx+\\
&= \frac{5}{2}ae^2x\sqrt{d+ex^2}-\frac{5ae(d+ex^2)^{3/2}}{3x}-\frac{a(d+ex^2)^{5/2}}{3x^3}+b\int \frac{(d+ex^2)^5}{}\\
&= \frac{5}{2}ae^2x\sqrt{d+ex^2}-\frac{5ae(d+ex^2)^{3/2}}{3x}-\frac{a(d+ex^2)^{5/2}}{3x^3}+b\int \frac{(d+ex^2)^5}{}\\
&= \frac{5}{2}ae^2x\sqrt{d+ex^2}-\frac{5ae(d+ex^2)^{3/2}}{3x}-\frac{a(d+ex^2)^{5/2}}{3x^3}+\frac{5}{2}ade^{3/2}\tanh^-
\end{aligned}$$

Mathematica [A]
time = 9.47, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{5/2}(a+b\text{ArcTan}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^{\frac{5}{2}}(a+b\arctan(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
[Out] int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d)/x^4, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atan}\,(cx))\,(d + ex^2)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atan}(c\,x))\,\left(e\,x^2 + d\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4, x)
```

3.1201 $\int \frac{x^3(a+b\textbf{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=176

$$-\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3e^2}+\frac{b\sqrt{c^2d-e}\,(2c^2d+e)\,\text{A}}{3c^3e}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+1/6*b*(3*c^2*d+2*e)*arctanh(x*e^(
1/2)/(e*x^2+d)^(1/2))/c^3/e^(3/2)+1/3*b*(2*c^2*d+e)*arctan(x*(c^2*d-e)^(1/2
)/(e*x^2+d)^(1/2))*(c^2*d-e)^(1/2)/c^3/e^2-1/6*b*x*(e*x^2+d)^(1/2)/c/e-d*(a
+b*arctan(c*x))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.18, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {272, 45, 5096, 12, 542, 537, 223, 212, 385, 209}

$$\frac{(d+ex^2)^{3/2}\,(a+b\text{ArcTan}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{e^2}+\frac{b\sqrt{c^2d-e}\,(2c^2d+e)\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3c^3e^2}+\frac{b(3c^2d+2e)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}}-\frac{bx\sqrt{d+ex^2}}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/6*(b*x*Sqrt[d + e*x^2])/(c*e) - (d*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/
e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e^2) + (b*Sqrt[c^2*d - e]*
(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*c^3*e^2) + (b
*(3*c^2*d + 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(6*c^3*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= -\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{3e^2}-(bc)\int \frac{(-2\\
&= -\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx))}{3e^2}-\frac{(bc)\int \frac{(-2d\\
&= -\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx\\
&= -\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx\\
&= -\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx\\
&= -\frac{bx\sqrt{d+ex^2}}{6ce}-\frac{d\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2}+\frac{(d+ex^2)^{3/2}\,(a+b\tan^{-1}(cx\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 377, normalized size = 2.14

$$\frac{-\frac{\sqrt{d+ex^2}\,(bex+ac(4d-2ex^2))}{c}+2b(-2d+ex^2)\sqrt{d+ex^2}\,\mathrm{ArcTan}(cx)-\frac{ib(2c^4d^2-c^2de-e^2)\log\left(\frac{12ic^4e^2\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(-2c^4d^2+c^2de+e^2)(i+cx)}\right)}{c^3\sqrt{c^2d-e}}+\frac{ib(2c^4d^2-c^2de-e^2)\log\left(-\frac{12ic^4e^2\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(-2c^4d^2+c^2de+e^2)(-i+cx)}\right)}{c^3\sqrt{c^2d-e}}+\frac{b\sqrt{e}\,(3c^2d+2e)\log\left(ex+\sqrt{e}\,\sqrt{d+ex^2}\right)}{c^3}}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (-((Sqrt[d + e*x^2]*(b*e*x + a*c*(4*d - 2*e*x^2)))/c) + 2*b*(-2*d + e*x^2)*
Sqrt[d + e*x^2]*ArcTan[c*x] - (I*b*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((12*I)*
c^4*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]
*(-2*c^4*d^2 + c^2*d*e + e^2)*(I + c*x))])/(c^3*Sqrt[c^2*d - e]) + (I*b*(2*
c^4*d^2 - c^2*d*e - e^2)*Log[((-12*I)*c^4*e^2*(c*d + I*e*x + Sqrt[c^2*d - e
]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(-I + c
*x))])/(c^3*Sqrt[c^2*d - e]) + (b*Sqrt[e]*(3*c^2*d + 2*e)*Log[e*x + Sqrt[e]
*Sqrt[d + e*x^2]])/c^3)/(6*e^2)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(x^2*e + d)*x^2*e^(-1) - 2*sqrt(x^2*e + d)*d*e^(-2))*a + b*integra
te(x^3*arctan(c*x)/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 2.64, size = 476, normalized size = 2.70

$$\left[\frac{\left((3\,bc^2d+2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+(2\,bc^2d+be)\sqrt{-c^2d+e}\,\log\left(\frac{c^4d^2x^4-6\,c^2d^2x^2+8\,x^4e^2+4\,(c^2dx^3-2\,x^3e-dx)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}+d^2-8\,(c^2dx^4-dx^2)e}{c^4x^4+2\,c^2x^2+1}\right)-2\,(4\,ac^3d-2\,(bc^3x^2e-2\,bc^3d)\arctan(cx)-(2\,ac^3x^2-bc^2x)e)\sqrt{x^2e+d}\right)e^{(-2)}}{12\,c^3}, \frac{\left((3\,bc^2d+2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+2\,(2\,bc^2d+be)\sqrt{c^2d-e}\,\arctan\left(\frac{(c^2dx^2-2\,x^2e-d)\sqrt{c^2d-e}\,\sqrt{x^2e+d}}{2\,(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)-2\,(4\,ac^3d-2\,(bc^3x^2e-2\,bc^3d)\arctan(cx)-(2\,ac^3x^2-bc^2x)e)\sqrt{x^2e+d}\right)e^{(-2)}}{12\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*b*c^2*d + 2*b*e)*e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/
2) - d) + (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x
^2 + 8*x^4*e^2 + 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e
+ d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(4*a*c
^3*d - 2*(b*c^3*x^2*e - 2*b*c^3*d)*arctan(c*x) - (2*a*c^3*x^2 - b*c^2*x)*e)
*sqrt(x^2*e + d))*e^(-2)/c^3, 1/12*((3*b*c^2*d + 2*b*e)*e^(1/2)*log(-2*x^2*
e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 2*(2*b*c^2*d + b*e)*sqrt(c^2*d - e)*
arctan(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d
^2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) - 2*(4*a*c^3*d - 2*(b*c^3*x^2*e - 2*
b*c^3*d)*arctan(c*x) - (2*a*c^3*x^2 - b*c^2*x)*e)*sqrt(x^2*e + d))*e^(-2)/c
^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{atan}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1202 $\int \frac{x^2(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=75

$$\frac{ax\sqrt{d+ex^2}}{2e}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}+b\text{Int}\left(\frac{x^2\text{ArcTan}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] -1/2*a*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+1/2*a*x*(e*x^2+d)^(1/2)
/e+b*Unintegrable(x^2*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x^2(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (a*x*Sqrt[d + e*x^2])/(2*e) - (a*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*e^(3/2)) + b*Defer[Int][(x^2*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= a\int \frac{x^2}{\sqrt{d+ex^2}}\,dx+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx\\
&= \frac{ax\sqrt{d+ex^2}}{2e}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx-\frac{(ad)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e}\\
&= \frac{ax\sqrt{d+ex^2}}{2e}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx-\frac{(ad)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2e}\\
&= \frac{ax\sqrt{d+ex^2}}{2e}-\frac{ad\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}+b\int \frac{x^2\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 10.39, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{2} (a +b \arctan \left(c x \right))}{\sqrt{e \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{atan}{\left(c x \right)}\right)}{\sqrt{d + e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1203 $\int \frac{x(a+b\mathbf{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{d+ex^2}\,(a+b\mathrm{ArcTan}(cx))}{e}-\frac{b\sqrt{c^2d-e}\,\mathrm{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] -b*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))*(c^2*d-e)^(1/2)/c/e-b*arctanh(
x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)+(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5094, 399, 223, 212, 385, 209}

$$\frac{\sqrt{d+ex^2}\,(a+b\mathrm{ArcTan}(cx))}{e}-\frac{b\sqrt{c^2d-e}\,\mathrm{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e - (b*Sqrt[c^2*d - e]*ArcTan[(Sqrt[c
^2*d - e]*x)/Sqrt[d + e*x^2]])/(c*e) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e} - \frac{(bc)\int \frac{\sqrt{d+ex^2}}{1+c^2x^2}\,dx}{e}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} + \frac{(b(-c^2d+e))\int \frac{1}{(1+c^2x^2)\sqrt{d+}}}{ce}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e} - \frac{b\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} + \frac{(b(-c^2d+}{}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e} - \frac{b\sqrt{c^2d-e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{ce} - \frac{b\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{c\sqrt{e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 251, normalized size = 2.44

$$\frac{2ac\sqrt{d+ex^2}+2bc\sqrt{d+ex^2}\,\text{ArcTan}(cx)-ib\sqrt{c^2d-e}\,\log\left(\frac{4c^2e\left(-icd+ex-i\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(-i+cx)}\right)+ib\sqrt{c^2d-e}\,\log\left(\frac{4c^2e\left(icd+ex+i\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(i+cx)}\right)-2b\sqrt{e}\,\log\left(ex+\sqrt{e}\,\sqrt{d+ex^2}\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (2*a*c*Sqrt[d + e*x^2] + 2*b*c*Sqrt[d + e*x^2]*ArcTan[c*x] - I*b*Sqrt[c^2*d
 - e]*Log[(4*c^2*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b
```

```
*(c^2*d - e)^(3/2)*(-I + c*x))] + I*b*Sqrt[c^2*d - e]*Log[(4*c^2*e*(I*c*d +
 e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(I + c*x))]
 - 2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2*c*e)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 3.70, size = 347, normalized size = 3.37

$$\left[\frac{\left(2\,be^{\frac{1}{2}}\log\left(-2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+\sqrt{-c^2d+e}\,b\log\left(\frac{c^4d^2x^4-6\,c^2d^2x^2+8\,x^4e^2-4\,(c^2dx^3-2\,x^3e-dx)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}+d^2-8\,(c^2dx^4-dx^2)e}{c^4x^4+2\,c^2x^2+1}\right)+4\,(bc\arctan(cx)+ac)\sqrt{x^2e+d}\right)e^{(-1)}}{4\,c},\ \frac{\left(be^{\frac{1}{2}}\log\left(-2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)-\sqrt{c^2d-e}\,b\arctan\left(\frac{(c^2dx^2-2\,x^2e-d)\sqrt{c^2d-e}\,\sqrt{x^2e+d}}{2\,(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)+2\,(bc\arctan(cx)+ac)\sqrt{x^2e+d}\right)e^{(-1)}}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + sqrt(-c
^2*d + e)*b*log((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e^2 - 4*(c^2*d*x^3 - 2
*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2
)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(b*c*arctan(c*x) + a*c)*sqrt(x^2*e + d)
)*e^(-1)/c, 1/2*(b*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d)
- sqrt(c^2*d - e)*b*arctan(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sq
rt(x^2*e + d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) + 2*(b*c*arctan(
c*x) + a*c)*sqrt(x^2*e + d))*e^(-1)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1204 $\int \frac{a+b\text{ArcTan}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a+b\text{ArcTan}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{ArcTan}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan(cx)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + b*integrate(arctan(c*x)/sqrt(x^2*e
+ d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{atan}(c\,x)}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*atan(c*x))/(d + e*x^2)^(1/2), x)
```

3.1205 $\int \frac{a+b\text{ArcTan}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+b\text{Int}\left(\frac{\text{ArcTan}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] -a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*Unintegrable(arctan(c*x)/x/(e
*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{ArcTan}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] -((a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d]) + b*Defer[Int][ArcTan[c*x]/
(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx &= a\int \frac{1}{x\sqrt{d+ex^2}}\,dx+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx\\
&= b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+b\int \frac{\tan^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 4.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)/(sqrt(x^2*e + d)*x), x) - a*arcsinh(sqrt(d)*e^(-1/2
)/abs(x))/sqrt(d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^3*e + d*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atan}\left(cx\right)}{x\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)), x)
```

3.1206 $\int \frac{a+b\text{ArcTan}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{dx}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{b\sqrt{c^2d-e}\,\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}$$

```
[Out] -b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*arctanh(c*(e*x^2+d)^(1/2)/(
c^2*d-e)^(1/2))*(c^2*d-e)^(1/2)/d-(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {270, 5096, 457, 85, 65, 214}

$$-\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{dx}+\frac{b\sqrt{c^2d-e}\,\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x)) - (b*c*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d] + (b*Sqrt[c^2*d - e]*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[
c^2*d - e]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{dx}-(bc)\int \frac{\sqrt{d+ex^2}}{x\,(-d-c^2dx^2)}\,dx$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{dx}-\frac{1}{2}(bc)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\,(-d-c^2dx)}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{dx}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)+\frac{1}{2}\big(bc(c^2d -$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{dx}+\frac{(bc)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{e}+\frac{(bc(c^2d -}{}$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{dx}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{b\sqrt{c^2d-e}\,\tanh^{-1}\left(\frac{c\sqrt{}}{\sqrt{}}\right.}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 247, normalized size = 2.47

$$\frac{-2a\sqrt{d+ex^2}-2b\sqrt{d+ex^2}\,\text{ArcTan}(cx)+2bc\sqrt{d}\,x\log(x)-2bc\sqrt{d}\,x\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)+b\sqrt{c^2d-e}\,x\log\left(-\frac{4cd\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(i+cx)}\right)+b\sqrt{c^2d-e}\,x\log\left(-\frac{4cd\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(-i+cx)}\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (-2*a*Sqrt[d + e*x^2] - 2*b*Sqrt[d + e*x^2]*ArcTan[c*x] + 2*b*c*Sqrt[d]*x*L
og[x] - 2*b*c*Sqrt[d]*x*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*Sqrt[c^2*d - e
]*x*Log[(-4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d
- e)^(3/2)*(I + c*x))] + b*Sqrt[c^2*d - e]*x*Log[(-4*c*d*(c*d + I*e*x + Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(-I + c*x))])/(2*d*x)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x^{2}\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.75, size = 688, normalized size = 6.88

$$\left[\frac{2\,bc\sqrt{d}\,x\log\left(-\frac{x^{2}e-2\,\sqrt{x^{2}e+d}\,\sqrt{d}+2\,d}{x^{2}}\right)+\sqrt{c^{2}d-e}\,bx\log\left(\frac{8\,c^{4}d^{2}+4\,\left(2\,c^{3}d+\left(c^{3}x^{2}-c\right)e\right)\sqrt{c^{2}d-e}\,\sqrt{x^{2}e+d}+\left(c^{4}x^{4}-6\,c^{2}x^{2}+1\right)e^{2}+8\,\left(c^{4}dx^{2}-c^{2}d\right)e}{c^{4}x^{4}+2\,c^{2}x^{2}+1}\right)-4\,\sqrt{x^{2}e+d}\,\left(b\arctan\left(cx\right)+a\right)}{4\,dx},\ \frac{bc\sqrt{d}\,x\log\left(-\frac{x^{2}e-2\,\sqrt{x^{2}e+d}\,\sqrt{d}+2\,d}{x^{2}}\right)+\sqrt{-c^{2}d+e}\,bx\arctan\left(-\frac{\left(2\,c^{2}d+\left(c^{2}x^{2}-1\right)e\right)\sqrt{-c^{2}d+e}\,\sqrt{x^{2}e+d}}{2\,\left(c^{3}d^{2}-cx^{2}e^{2}+\left(c^{3}dx^{2}-cd\right)e\right)}\right)-2\,\sqrt{x^{2}e+d}\,\left(b\arctan\left(cx\right)+a\right)}{2\,dx},\ \frac{4\,bc\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}}{\sqrt{x^{2}e+d}}\right)+\sqrt{c^{2}d-e}\,bx\log\left(\frac{8\,c^{4}d^{2}+4\,\left(2\,c^{3}d+\left(c^{3}x^{2}-c\right)e\right)\sqrt{c^{2}d-e}\,\sqrt{x^{2}e+d}+\left(c^{4}x^{4}-6\,c^{2}x^{2}+1\right)e^{2}+8\,\left(c^{4}dx^{2}-c^{2}d\right)e}{c^{4}x^{4}+2\,c^{2}x^{2}+1}\right)-4\,\sqrt{x^{2}e+d}\,\left(b\arctan\left(cx\right)+a\right)}{4\,dx},\ \frac{2\,bc\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}}{\sqrt{x^{2}e+d}}\right)+\sqrt{-c^{2}d+e}\,bx\arctan\left(-\frac{\left(2\,c^{2}d+\left(c^{2}x^{2}-1\right)e\right)\sqrt{-c^{2}d+e}\,\sqrt{x^{2}e+d}}{2\,\left(c^{3}d^{2}-cx^{2}e^{2}+\left(c^{3}dx^{2}-cd\right)e\right)}\right)-2\,\sqrt{x^{2}e+d}\,\left(b\arctan\left(cx\right)+a\right)}{2\,dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) +
 sqrt(c^2*d - e)*b*x*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^
2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c
^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sqrt(x^2*e + d)*(b*arctan(c*x) + a)
)/(d*x), 1/2*(b*c*sqrt(d)*x*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/
```

```
x^2) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c
^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - 2*
sqrt(x^2*e + d)*(b*arctan(c*x) + a))/(d*x), 1/4*(4*b*c*sqrt(-d)*x*arctan(sq
rt(-d)/sqrt(x^2*e + d)) + sqrt(c^2*d - e)*b*x*log((8*c^4*d^2 + 4*(2*c^3*d +
 (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 +
1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sqrt(x^2*e
 + d)*(b*arctan(c*x) + a))/(d*x), 1/2*(2*b*c*sqrt(-d)*x*arctan(sqrt(-d)/sqr
t(x^2*e + d)) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e
)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)
*e)) - 2*sqrt(x^2*e + d)*(b*arctan(c*x) + a))/(d*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atan}{(cx)}}{x^2\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{atan}(c\,x)}{x^2\,\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.1207 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=76

$$-\frac{a\sqrt{d+ex^2}}{2dx^2}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}}+b\mathrm{Int}\left(\frac{\mathrm{ArcTan}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] 1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/2*a*(e*x^2+d)^(1/2)/d/x^
2+b*Unintegrable(arctan(c*x)/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\mathrm{ArcTan}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/2*(a*Sqrt[d + e*x^2])/(d*x^2) + (a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(
2*d^(3/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx &= a\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx+b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{2}a\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)+b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx\\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2}+b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx-\frac{(ae)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{4d}\\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2}+b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx-\frac{a\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{2d}\\
&= -\frac{a\sqrt{d+ex^2}}{2dx^2}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}}+b\int \frac{\tan^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 9.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\text{ArcTan}(cx)}{x^3\sqrt{d + ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a + b\arctan(cx)}{x^3\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(3/2) - sqrt(x^2*e + d)/(d*x^2)
) + b*integrate(arctan(c*x)/(sqrt(x^2*e + d)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^5*e + d*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atan}(cx)}{x^3\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.1208 $\int \frac{a+b\text{ArcTan}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{3d^2x}+\frac{bc(2c^2d+3e)\tanh^{-1}\left(\frac{\sqrt{d}\ldots}{}\right)}{6d^{3/2}}$$

[Out] 1/6*b*c*(2*c^2*d+3*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/3*b*(c^2*d+2*e)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))*(c^2*d-e)^(1/2)/d^2-1/6*b*c*(e*x^2+d)^(1/2)/d/x^2-1/3*(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d^2/x

Rubi [A]
time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {277, 270, 5096, 12, 587, 154, 162, 65, 214}

$$\frac{2e\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{3dx^3}+\frac{bc(2c^2d+3e)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{b\sqrt{c^2d-e}\,(c^2d+2e)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2}-\frac{bc\sqrt{d+ex^2}}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + e*x^2])/(d*x^2) - (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/
(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(3*d^2*x) + (b*c*(2*c
^2*d + 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2)) - (b*Sqrt[c^2*d -
 e]*(c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
```

```
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx &= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}-(bc)\int\frac{\sqrt{d+ex^2}}{3d^2x^3\,(} \\
&= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}-\frac{(bc)\int\frac{\sqrt{d+ex^2}}{x^3(1+c}}{3d^2} \\
&= -\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}-\frac{(bc)\mathrm{Subst}\Big(\int\frac{\sqrt{d}}{}}{} \\
&= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}-\frac{(}{} \\
&= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}+\frac{(}{} \\
&= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}+\frac{(}{} \\
&= -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{3d^2x}+\frac{b}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 372, normalized size = 2.08

$$-\frac{\frac{\sqrt{d+ex^2}\,(bcdx+2a(d-2ex^2))}{x^3}+\frac{2b(d-2ex^2)\sqrt{d+ex^2}\,\mathrm{ArcTan}(cx)}{x^3}+bc\sqrt{d}\,(2c^2d+3e)\log(x)-bc\sqrt{d}\,(2c^2d+3e)\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)+\frac{b(c^4d^2+c^2de-2e^2)\log\left(\frac{12cd^2\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(c^4d^2+c^2de-2e^2)(i+cx)}\right)}{\sqrt{c^2d-e}}+\frac{b(c^4d^2+c^2de-2e^2)\log\left(\frac{12cd^2\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(c^4d^2+c^2de-2e^2)(-i+cx)}\right)}{\sqrt{c^2d-e}}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/6*((Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d - 2*e*x^2)))/x^3 + (2*b*(d - 2*e*x
^2)*Sqrt[d + e*x^2]*ArcTan[c*x])/x^3 + b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[x] -
 b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2
 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e
*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2)*(I + c*x))])/Sqrt[c^
2*d - e] + (b*(c^4*d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d + I*e*x + Sqrt
```

```
[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2
)*(-I + c*x))])/Sqrt[c^2*d - e])/d^2
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a + b \arctan(cx)}{x^4 \sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(x^2*e + d)*e/(d^2*x) - sqrt(x^2*e + d)/(d*x^3)) + b*integrate
(arctan(c*x)/(sqrt(x^2*e + d)*x^4), x)
```

Fricas [**A**]
time = 2.72, size = 936, normalized size = 5.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*((b*c^2*d*x^3 + 2*b*x^3*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 - 4*(2*c^3*
d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2
 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*c^3*
d*x^3 + 3*b*c*x^3*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)
/x^2) - 2*(b*c*d*x - 4*a*x^2*e + 2*a*d - 2*(2*b*x^2*e - b*d)*arctan(c*x))*s
qrt(x^2*e + d))/(d^2*x^3), -1/12*(2*(b*c^2*d*x^3 + 2*b*x^3*e)*sqrt(-c^2*d +
 e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d
)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - (2*b*c^3*d*x^3 + 3*b*c*x^3
*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*
x - 4*a*x^2*e + 2*a*d - 2*(2*b*x^2*e - b*d)*arctan(c*x))*sqrt(x^2*e + d))/(
d^2*x^3), -1/12*(2*(2*b*c^3*d*x^3 + 3*b*c*x^3*e)*sqrt(-d)*arctan(sqrt(-d)/s
qrt(x^2*e + d)) - (b*c^2*d*x^3 + 2*b*x^3*e)*sqrt(c^2*d - e)*log((8*c^4*d^2
- 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4
```

```
- 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1))
+ 2*(b*c*d*x - 4*a*x^2*e + 2*a*d - 2*(2*b*x^2*e - b*d)*arctan(c*x))*sqrt(x^
2*e + d))/(d^2*x^3), -1/6*((b*c^2*d*x^3 + 2*b*x^3*e)*sqrt(-c^2*d + e)*arcta
n(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^
2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + (2*b*c^3*d*x^3 + 3*b*c*x^3*e)*sqrt(
-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + (b*c*d*x - 4*a*x^2*e + 2*a*d - 2*(2*
b*x^2*e - b*d)*arctan(c*x))*sqrt(x^2*e + d))/(d^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atan}{(cx)}}{x^4\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*atan(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,\sqrt{e\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.1209 $\int \frac{x^3(a+b\,\mathbf{ArcTan}(cx))}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{d(a+b\text{ArcTan}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{e^2}-\frac{b(2c^2d-e)\,\text{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2d-e}\,e^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\cdot}}{\sqrt{d+\cdot}}\right.}{ce^{3/2}}$$

[Out] -b*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*(2*c^2*d-e)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c/e^2/(c^2*d-e)^(1/2)+d*(a+b*arctan(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/e^2

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {272, 45, 5096, 12, 537, 223, 212, 385, 209}

$$\frac{\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))}{e^2}+\frac{d(a+b\text{ArcTan}(cx))}{e^2\sqrt{d+ex^2}}-\frac{b(2c^2d-e)\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{c^2d-e}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]

[Out] (d*(a + b*ArcTan[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e^2 - (b*(2*c^2*d - e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(c*Sqrt[c^2*d - e]*e^2) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*e^(3/2))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\tan^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2} - (bc)\int \frac{2d+ex^2}{e^2\,(1+c^2x^2)\,\sqrt{d+}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2} - \frac{(bc)\int \frac{2d+ex^2}{(1+c^2x^2)\sqrt{d+ex^2}}\,d}{e^2}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{ce} - \frac{(bc(}{}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{}{\sqrt{}}\right.}{ce}$$

$$= \frac{d(a+b\tan^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\tan^{-1}(cx))}{e^2} - \frac{bc\left(2d-\frac{e}{c^2}\right)\tan^{-1}\left(\frac{\sqrt{c^2d}}{\sqrt{d-}}\right.}{\sqrt{c^2d-e}\,e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 321, normalized size = 2.34

$$\frac{\frac{2a(2d+ex^2)}{\sqrt{d+ex^2}} + \frac{2b(2d+ex^2)\text{ArcTan}(cx)}{\sqrt{d+ex^2}} - \frac{ib(2c^2d-e)\log\left(\frac{4c^2e^2\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(2c^2d-e)(-i+cx)}\right)}{c\sqrt{c^2d-e}} + \frac{ib(2c^2d-e)\log\left(\frac{4c^2e^2\left(icd+ex+i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(2c^2d-e)(i+cx)}\right)}{c\sqrt{c^2d-e}} - \frac{2b\sqrt{e}\log\left(ex+\sqrt{e}\sqrt{d+ex^2}\right)}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*a*(2*d + e*x^2))/Sqrt[d + e*x^2] + (2*b*(2*d + e*x^2)*ArcTan[c*x])/Sqrt
[d + e*x^2] - (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*((-I)*c*d + e*x - I*Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(-I + c*x))])/(
c*Sqrt[c^2*d - e]) + (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*(I*c*d + e*x + I*Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(I + c*x))]
)/(c*Sqrt[c^2*d - e]) - (2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/c)
/(2*e^2)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] (x^2*e^(-1)/sqrt(x^2*e + d) + 2*d*e^(-2)/sqrt(x^2*e + d))*a + 2*b*integrate
(1/2*x^3*arctan(c*x)/(x^2*e + d)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(123) = 246$.
time = 3.28, size = 681, normalized size = 4.97

$$\left[\frac{2\left(bc^{2}d^{2}-bx^{2}e^{2}+\left(bc^{2}dx^{2}-bd\right)e\right)e^{\frac{1}{2}}\log\left(-2x^{2}e+2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)+\left(2bc^{2}d^{2}-bx^{2}e^{2}+\left(2bc^{2}dx^{2}-bd\right)e\right)\sqrt{-c^{2}d+e}\log\left(\frac{c^{4}d^{2}x^{4}-6c^{2}d^{2}x^{2}+8x^{4}e^{2}-4\left(c^{2}dx^{3}-2x^{3}e-dx\right)\sqrt{-c^{2}d+e}\sqrt{x^{2}e+d}+d^{2}-8\left(c^{2}dx^{4}-dx^{2}\right)e}{c^{4}x^{4}+2c^{2}x^{2}+1}\right)+4\left(2ac^{3}d^{2}-acx^{2}e^{2}+\left(2bc^{3}d^{2}-bcx^{2}e^{2}+\left(bc^{3}dx^{2}-2bcd\right)e\right)\arctan\left(cx\right)+\left(ac^{3}dx^{2}-2acd\right)e\right)\sqrt{x^{2}e+d}}{4\left(c^{3}d^{2}e^{2}-cx^{2}e^{4}+\left(c^{3}dx^{2}-cd\right)e^{3}\right)},\ \frac{\left(bc^{2}d^{2}-bx^{2}e^{2}+\left(bc^{2}dx^{2}-bd\right)e\right)e^{\frac{1}{2}}\log\left(-2x^{2}e+2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)-\left(2bc^{2}d^{2}-bx^{2}e^{2}+\left(2bc^{2}dx^{2}-bd\right)e\right)\sqrt{c^{2}d-e}\arctan\left(\frac{\left(c^{2}dx^{2}-2x^{2}e-d\right)\sqrt{c^{2}d-e}\sqrt{x^{2}e+d}}{2\left(c^{2}d^{2}x-x^{3}e^{2}+\left(c^{2}dx^{3}-dx\right)e\right)}\right)+2\left(2ac^{3}d^{2}-acx^{2}e^{2}+\left(2bc^{3}d^{2}-bcx^{2}e^{2}+\left(bc^{3}dx^{2}-2bcd\right)e\right)\arctan\left(cx\right)+\left(ac^{3}dx^{2}-2acd\right)e\right)\sqrt{x^{2}e+d}}{2\left(c^{3}d^{2}e^{2}-cx^{2}e^{4}+\left(c^{3}dx^{2}-cd\right)e^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c^2*d^2 - b*x^2*e^2 + (b*c^2*d*x^2 - b*d)*e)*e^(1/2)*log(-2*x^2*
e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + (2*b*c^2*d^2 - b*x^2*e^2 + (2*b*c^2*
d*x^2 - b*d)*e)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e
^2 - 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 -
 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - a*c
*x^2*e^2 + (2*b*c^3*d^2 - b*c*x^2*e^2 + (b*c^3*d*x^2 - 2*b*c*d)*e)*arctan(c
*x) + (a*c^3*d*x^2 - 2*a*c*d)*e)*sqrt(x^2*e + d))/(c^3*d^2*e^2 - c*x^2*e^4
+ (c^3*d*x^2 - c*d)*e^3), 1/2*((b*c^2*d^2 - b*x^2*e^2 + (b*c^2*d*x^2 - b*d)
*e)*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d) - (2*b*c^2*d^2
- b*x^2*e^2 + (2*b*c^2*d*x^2 - b*d)*e)*sqrt(c^2*d - e)*arctan(1/2*(c^2*d*x^
2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x^3*e^2 + (c^
2*d*x^3 - d*x)*e)) + 2*(2*a*c^3*d^2 - a*c*x^2*e^2 + (2*b*c^3*d^2 - b*c*x^2*
e^2 + (b*c^3*d*x^2 - 2*b*c*d)*e)*arctan(c*x) + (a*c^3*d*x^2 - 2*a*c*d)*e)*s
qrt(x^2*e + d))/(c^3*d^2*e^2 - c*x^2*e^4 + (c^3*d*x^2 - c*d)*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(a+b\operatorname{atan}\left(cx\right)\right)}{\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1210 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{ax}{e\sqrt{d+ex^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+b\text{Int}\left(\frac{x^2\text{ArcTan}(cx)}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)-a*x/e/(e*x^2+d)^(1/2)+b*Uninte
grable(x^2*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
[Out] -((a*x)/(e*Sqrt[d + e*x^2])) + (a*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(
3/2) + b*Defer[Int][(x^2*ArcTan[c*x])/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= a\int \frac{x^2}{(d+ex^2)^{3/2}}\,dx+b\int \frac{x^2\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx\\
&= -\frac{ax}{e\sqrt{d+ex^2}}+b\int \frac{x^2\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx+\frac{a\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e}\\
&= -\frac{ax}{e\sqrt{d+ex^2}}+b\int \frac{x^2\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{e}\\
&= -\frac{ax}{e\sqrt{d+ex^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+b\int \frac{x^2\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 16.51, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b \arctan(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(x^2*e + d)/(x^4*e^2 + 2*d*x^2*e +
 d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b \operatorname{atan}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1211 $\int \frac{x(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{a+b\text{ArcTan}(cx)}{e\sqrt{d+ex^2}}+\frac{bc\text{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}\,e}$$

```
[Out] b*c*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/e/(c^2*d-e)^(1/2)+(-a-b*arcta
n(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {5094, 385, 209}

$$\frac{bc\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{e\sqrt{c^2d-e}}-\frac{a+b\text{ArcTan}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*ArcTan[(Sqrt[c^2*d - e]*x
)/Sqrt[d + e*x^2]])/(Sqrt[c^2*d - e]*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= -\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bc)\int\frac{1}{(1+c^2x^2)\sqrt{d+ex^2}}\,dx}{e}\\
&= -\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bc)\text{Subst}\left(\int\frac{1}{1-(-c^2d+e)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{e}\\
&= -\frac{a+b\tan^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bc\tan^{-1}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}\,e}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 210, normalized size = 2.96

$$
-\frac{\frac{2a}{\sqrt{d+ex^2}}+\frac{2b\text{ArcTan}(cx)}{\sqrt{d+ex^2}}+\frac{ibc\log\left(-\frac{4ie\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(i+cx)}\right)}{\sqrt{c^2d-e}}-\frac{ibc\log\left(\frac{4ie\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -1/2*((2*a)/Sqrt[d + e*x^2] + (2*b*ArcTan[c*x])/Sqrt[d + e*x^2] + (I*b*c*Lo
g[((-4*I)*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d
- e]*(I + c*x))])/Sqrt[c^2*d - e] - (I*b*c*Log[((4*I)*e*(c*d + I*e*x + Sqrt
[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d -
 e])/e
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$
\int \frac{x(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(65) = 130$.
time = 2.39, size = 401, normalized size = 5.65

$$\left[-\frac{(bcx^2e+bcd)\sqrt{-c^2d+e}\log\left(\frac{c^4d^2x^4-6c^2d^2x^2+8x^4e^2-4(c^2dx^3-2x^3e-dx)\sqrt{-c^2d+e}\sqrt{x^2e+d}+d^2-8(c^2dx^4-dx^2)e}{c^4x^4+2c^2x^2+1}\right)+4(ac^2d+(bc^2d-be)\arctan(cx)-ae)\sqrt{x^2e+d}}{4(c^2d^2e-x^2e^3+(c^2dx^2-d)e^2)}, \frac{(bcx^2e+bcd)\sqrt{c^2d-e}\arctan\left(\frac{(c^2dx^2-2x^2e-d)\sqrt{c^2d-e}\sqrt{x^2e+d}}{2(c^2d^2x-x^3e^2+(c^2dx^3-dx)e)}\right)-2(ac^2d+(bc^2d-be)\arctan(cx)-ae)\sqrt{x^2e+d}}{2(c^2d^2e-x^2e^3+(c^2dx^2-d)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*x^2*e + b*c*d)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*d^2*x^
2 + 8*x^4*e^2 - 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x^2*e +
 d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(a*c^2*
d + (b*c^2*d - b*e)*arctan(c*x) - a*e)*sqrt(x^2*e + d))/(c^2*d^2*e - x^2*e^
3 + (c^2*d*x^2 - d)*e^2), 1/2*((b*c*x^2*e + b*c*d)*sqrt(c^2*d - e)*arctan(1
/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x
^3*e^2 + (c^2*d*x^3 - d*x)*e)) - 2*(a*c^2*d + (b*c^2*d - b*e)*arctan(c*x) -
 a*e)*sqrt(x^2*e + d))/(c^2*d^2*e - x^2*e^3 + (c^2*d*x^2 - d)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1212 $\int \frac{a+b\text{ArcTan}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x(a+b\text{ArcTan}(cx))}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

```
[Out] b*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(1/2)+x*(a+b*arcta
n(c*x))/d/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {197, 5032, 12, 455, 65, 214}

$$\frac{x(a+b\text{ArcTan}(cx))}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcTan[c*x]))/(d*Sqrt[d + e*x^2]) + (b*ArcTanh[(c*Sqrt[d + e*x^2]
)/Sqrt[c^2*d - e]])/(d*Sqrt[c^2*d - e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx &= \frac{x(a+b\tan^{-1}(cx))}{d\sqrt{d+ex^2}}-(bc)\int \frac{x}{d\,(1+c^2x^2)\,\sqrt{d+ex^2}}\,dx\\
&= \frac{x(a+b\tan^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x}{(1+c^2x^2)\sqrt{d+ex^2}}\,dx}{d}\\
&= \frac{x(a+b\tan^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{(bc)\mathrm{Subst}\left(\int \frac{1}{(1+c^2x)\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}\\
&= \frac{x(a+b\tan^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{(bc)\mathrm{Subst}\left(\int \frac{1}{1-\frac{c^2d}{e}+\frac{c^2x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de}\\
&= \frac{x(a+b\tan^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 202, normalized size = 2.89

$$
\frac{\frac{2ax}{\sqrt{d+ex^2}}+\frac{2bx\mathrm{ArcTan}(cx)}{\sqrt{d+ex^2}}+\frac{b\log\left(-\frac{4cd\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(i+cx)}\right)}{\sqrt{c^2d-e}}+\frac{b\log\left(-\frac{4cd\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*a*x)/Sqrt[d + e*x^2] + (2*b*x*ArcTan[c*x])/Sqrt[d + e*x^2] + (b*Log[(-4
*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(I
 + c*x))])/Sqrt[c^2*d - e] + (b*Log[(-4*c*d*(c*d + I*e*x + Sqrt[c^2*d - e]*
Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d - e])/(2*d)
```

Maple [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(66) = 132$.
time = 2.62, size = 409, normalized size = 5.84

$$\left[\frac{\left(bx^{2}e+bd\right)\sqrt{c^{2}d-e}\,\log\left(\frac{8\,c^{4}d^{2}+4\left(2\,c^{3}d+\left(c^{3}x^{2}-c\right)e\right)\sqrt{c^{2}d-e}\,\sqrt{x^{2}e+d}+\left(c^{4}x^{4}-6\,c^{2}x^{2}+1\right)e^{2}+8\left(c^{4}dx^{2}-c^{2}d\right)e}{c^{4}x^{4}+2\,c^{2}x^{2}+1}\right)+4\left(ac^{2}dx-axe+\left(bc^{2}dx-bxe\right)\arctan\left(cx\right)\right)\sqrt{x^{2}e+d}}{4\left(c^{2}d^{3}-dx^{2}e^{2}+\left(c^{2}d^{2}x^{2}-d^{2}\right)e\right)},\ \frac{\left(bx^{2}e+bd\right)\sqrt{-c^{2}d+e}\,\arctan\left(-\frac{\left(2\,c^{2}d+\left(c^{2}x^{2}-1\right)e\right)\sqrt{-c^{2}d+e}\,\sqrt{x^{2}e+d}}{2\left(c^{3}d^{2}-cx^{2}e^{2}+\left(c^{3}dx^{2}-cd\right)e\right)}\right)+2\left(ac^{2}dx-axe+\left(bc^{2}dx-bxe\right)\arctan\left(cx\right)\right)\sqrt{x^{2}e+d}}{2\left(c^{2}d^{3}-dx^{2}e^{2}+\left(c^{2}d^{2}x^{2}-d^{2}\right)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^2*e + b*d)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^
2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 +
 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(a*c^2*d*x - a*x*e
 + (b*c^2*d*x - b*x*e)*arctan(c*x))*sqrt(x^2*e + d))/(c^2*d^3 - d*x^2*e^2 +
```

```
(c^2*d^2*x^2 - d^2)*e), 1/2*((b*x^2*e + b*d)*sqrt(-c^2*d + e)*arctan(-1/2*
(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x
^2*e^2 + (c^3*d*x^2 - c*d)*e)) + 2*(a*c^2*d*x - a*x*e + (b*c^2*d*x - b*x*e)
*arctan(c*x))*sqrt(x^2*e + d))/(c^2*d^3 - d*x^2*e^2 + (c^2*d^2*x^2 - d^2)*e
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{\left(d + ex^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x^2)^(3/2), x)
```

3.1213 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{a}{d\sqrt{d+ex^2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+b\text{Int}\left(\frac{\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] -a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+a/d/(e*x^2+d)^(1/2)+b*Unintegra
ble(arctan(c*x)/x/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] a/(d*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + b*De
fer[Int][ArcTan[c*x]/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx &= a\int \frac{1}{x\left(d+ex^2\right)^{3/2}}\,dx+b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(d+ex)^{3/2}}\,dx,x,x^2\right)+b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx\\
&= \frac{a}{d\sqrt{d+ex^2}}+b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}\\
&= \frac{a}{d\sqrt{d+ex^2}}+b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de}\\
&= \frac{a}{d\sqrt{d+ex^2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 8.42, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(3/2) - 1/(sqrt(x^2*e + d)*d)) + 2*b
*integrate(1/2*arctan(c*x)/((x^3*e + d*x)*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atan}\left(cx\right)}{x\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*atan(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)), x)
```

3.1214 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a+b\text{ArcTan}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\text{ArcTan}(cx))}{d^2\sqrt{d+ex^2}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b(c^2d-2e)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}$$

```
[Out] -b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+b*(c^2*d-2*e)*arctanh(c*(e*x^
2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(1/2)+(-a-b*arctan(c*x))/d/x/(e*x
^2+d)^(1/2)-2*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {277, 197, 5096, 12, 587, 162, 65, 214}

$$-\frac{2ex(a+b\text{ArcTan}(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{ArcTan}(cx)}{dx\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcTan[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcTan[c*x]))/
(d^2*Sqrt[d + e*x^2]) - (b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2) + (b
*(c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(d^2*Sqrt[c^2*
d - e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx &= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}-(bc)\int\frac{-d-2ex^2}{d^2x\left(1+c^2x^2\right)\sqrt{d+ex^2}}\,dx \\
&= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{(bc)\int\frac{-d-2ex^2}{x(1+c^2x^2)\sqrt{d+ex^2}}\,dx}{d^2} \\
&= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{(bc)\text{Subst}\left(\int\frac{-d-2ex}{x(1+c^2x)\sqrt{d+ex}}\,dx,x,x^2\right.}{2d^2} \\
&= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}+\frac{(bc)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{2d}- \\
&= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}+\frac{(bc)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{de} \\
&= -\frac{a+b\tan^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\tan^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b(c^2d-2e)}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 306, normalized size = 2.27

$$
\frac{-\frac{2a(d+2ex^2)}{x\sqrt{d+ex^2}}-\frac{2b(d+2ex^2)\text{ArcTan}(cx)}{x\sqrt{d+ex^2}}+2bc\sqrt{d}\log(x)-2bc\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)+\frac{b(c^2d-2e)\log\left(-\frac{4cd^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-2e)\sqrt{c^2d-e}(i+cx)}\right)}{\sqrt{c^2d-e}}+\frac{b(c^2d-2e)\log\left(-\frac{4cd^2\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-2e)\sqrt{c^2d-e}(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] ((-2*a*(d + 2*e*x^2))/(x*Sqrt[d + e*x^2]) - (2*b*(d + 2*e*x^2)*ArcTan[c*x])
/(x*Sqrt[d + e*x^2]) + 2*b*c*Sqrt[d]*Log[x] - 2*b*c*Sqrt[d]*Log[d + Sqrt[d]
*Sqrt[d + e*x^2]] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d - I*e*x + Sqrt[c^2*
d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(I + c*x))])/Sqrt
[c^2*d - e] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d + I*e*x + Sqrt[c^2*d - e]
*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*
d - e])/(2*d^2)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$
\int \frac{a+b\arctan\left(cx\right)}{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*x*e/(sqrt(x^2*e + d)*d^2) + 1/(sqrt(x^2*e + d)*d*x)) + 2*b*integrate(
1/2*arctan(c*x)/((x^4*e + d*x^2)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(127) = 254$.
time = 3.04, size = 1373, normalized size = 10.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2*d^2*x - 2*b*x^3*e^2 + (b*c^2*d*x^3 - 2*b*d*x)*e)*sqrt(c^2*d -
 e)*log((8*c^4*d^2 - 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2
*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4
 + 2*c^2*x^2 + 1)) - 2*(b*c^3*d^2*x - b*c*x^3*e^2 + (b*c^3*d*x^3 - b*c*d*x)
*e)*sqrt(d)*log(-(x^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 4*(a*c^2*
d^2 - 2*a*x^2*e^2 + (b*c^2*d^2 - 2*b*x^2*e^2 + (2*b*c^2*d*x^2 - b*d)*e)*arc
tan(c*x) + (2*a*c^2*d*x^2 - a*d)*e)*sqrt(x^2*e + d))/(c^2*d^4*x - d^2*x^3*e
^2 + (c^2*d^3*x^3 - d^3*x)*e), 1/2*((b*c^2*d^2*x - 2*b*x^3*e^2 + (b*c^2*d*x
^3 - 2*b*d*x)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*s
qrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)
) + (b*c^3*d^2*x - b*c*x^3*e^2 + (b*c^3*d*x^3 - b*c*d*x)*e)*sqrt(d)*log(-(x
^2*e - 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) - 2*(a*c^2*d^2 - 2*a*x^2*e^2 +
 (b*c^2*d^2 - 2*b*x^2*e^2 + (2*b*c^2*d*x^2 - b*d)*e)*arctan(c*x) + (2*a*c^2
*d*x^2 - a*d)*e)*sqrt(x^2*e + d))/(c^2*d^4*x - d^2*x^3*e^2 + (c^2*d^3*x^3 -
 d^3*x)*e), 1/4*(4*(b*c^3*d^2*x - b*c*x^3*e^2 + (b*c^3*d*x^3 - b*c*d*x)*e)*
sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) - (b*c^2*d^2*x - 2*b*x^3*e^2 + (b
*c^2*d*x^3 - 2*b*d*x)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 - 4*(2*c^3*d + (c^3
*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^
2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(a*c^2*d^2 - 2*
a*x^2*e^2 + (b*c^2*d^2 - 2*b*x^2*e^2 + (2*b*c^2*d*x^2 - b*d)*e)*arctan(c*x)
```

```
 + (2*a*c^2*d*x^2 - a*d)*e)*sqrt(x^2*e + d))/(c^2*d^4*x - d^2*x^3*e^2 + (c^
2*d^3*x^3 - d^3*x)*e), 1/2*((b*c^2*d^2*x - 2*b*x^3*e^2 + (b*c^2*d*x^3 - 2*b
*d*x)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2
*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + 2*(b
*c^3*d^2*x - b*c*x^3*e^2 + (b*c^3*d*x^3 - b*c*d*x)*e)*sqrt(-d)*arctan(sqrt(
-d)/sqrt(x^2*e + d)) - 2*(a*c^2*d^2 - 2*a*x^2*e^2 + (b*c^2*d^2 - 2*b*x^2*e^
2 + (2*b*c^2*d*x^2 - b*d)*e)*arctan(c*x) + (2*a*c^2*d*x^2 - a*d)*e)*sqrt(x^
2*e + d))/(c^2*d^4*x - d^2*x^3*e^2 + (c^2*d^3*x^3 - d^3*x)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{(cx)}}{x^2 (d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atan}(c\,x)}{x^2\,(e\,x^2 + d)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.1215 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{3ae}{2d^2\sqrt{d+ex^2}}-\frac{a}{2dx^2\sqrt{d+ex^2}}+\frac{3ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}}+b\mathrm{Int}\left(\frac{\mathrm{ArcTan}(cx)}{x^3\,(d+ex^2)^{3/2}},x\right)$$

```
[Out] 3/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)-3/2*a*e/d^2/(e*x^2+d)^(1/2
)-1/2*a/d/x^2/(e*x^2+d)^(1/2)+b*Unintegrable(arctan(c*x)/x^3/(e*x^2+d)^(3/2
),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\mathrm{ArcTan}(cx)}{x^3\,(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-3*a*e)/(2*d^2*Sqrt[d + e*x^2]) - a/(2*d*x^2*Sqrt[d + e*x^2]) + (3*a*e*Arc
Tanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(5/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*
(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = a\int \frac{1}{x^3\left(d+ex^2\right)^{3/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{3/2}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{a}{dx^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx + \frac{(3a)\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx - \frac{(3ae)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\right.}{4d^2}$$

$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx - \frac{(3a)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,\right.}{2d^2}$$

$$= \frac{a}{dx^2\sqrt{d+ex^2}} - \frac{3a\sqrt{d+ex^2}}{2d^2x^2} + \frac{3ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,d$$

Mathematica [A]
time = 10.93, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(5/2) - 3*e/(sqrt(x^2*e + d)*
d^2) - 1/(sqrt(x^2*e + d)*d*x^2)) + 2*b*integrate(1/2*arctan(c*x)/((x^5*e +
 d*x^3)*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^7*e^2 + 2*d*x^5*e + d^2*x^3
), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}{\left(cx \right)}}{x^{3} \left(d + ex^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{x^3\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.1216 $\int \frac{a+b\text{ArcTan}(cx)}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{bc\sqrt{d+ex^2}}{6d^2x^2}-\frac{a+b\text{ArcTan}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\text{ArcTan}(cx))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\text{ArcTan}(cx))}{3d^3\sqrt{d+ex^2}}+\frac{bce\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right.}{6d^{5/2}}$$

```
[Out] 1/6*b*c*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/3*b*c*(c^2*d+4*e)*arct
anh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)-1/3*b*(c^4*d^2+4*c^2*d*e-8*e^2)*arctan
h(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^3/(c^2*d-e)^(1/2)+1/3*(-a-b*arctan(c
*x))/d/x^3/(e*x^2+d)^(1/2)+4/3*e*(a+b*arctan(c*x))/d^2/x/(e*x^2+d)^(1/2)+8/
3*e^2*x*(a+b*arctan(c*x))/d^3/(e*x^2+d)^(1/2)-1/6*b*c*(e*x^2+d)^(1/2)/d^2/x
^2
```

Rubi [A]
time = 0.62, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 197, 5096, 12, 6857, 272, 44, 65, 214, 455}

$$\frac{8e^2x(a+b\text{ArcTan}(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\text{ArcTan}(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\text{ArcTan}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{bc(c^2d+4e)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{b(c^4d^2+4c^2de-8e^2)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3\sqrt{c^2d-e}}+\frac{bce\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{5/2}}-\frac{bc\sqrt{d+ex^2}}{6d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] -1/6*(b*c*Sqrt[d + e*x^2])/(d^2*x^2) - (a + b*ArcTan[c*x])/(3*d*x^3*Sqrt[d
+ e*x^2]) + (4*e*(a + b*ArcTan[c*x]))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*
(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/(6*d^(5/2)) + (b*c*(c^2*d + 4*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[
d]])/(3*d^(5/2)) - (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x
^2])/Sqrt[c^2*d - e]])/(3*d^3*Sqrt[c^2*d - e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
```

```
 - 1)/2, 0]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - (bc)\int \ldots$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \ldots$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \ldots$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \ldots$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \ldots$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \ldots$$

$$= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2} - \frac{a+b\tan^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\tan^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 405, normalized size = 1.63

$$-\frac{\frac{bcdx(d+ex^2)+2a(d^2-4dex^2-8e^2x^4)}{x^3\sqrt{d+ex^2}} + \frac{2b(d^2-4dex^2-8e^2x^4)\text{ArcTan}(cx)}{x^3\sqrt{d+ex^2}} + bc\sqrt{d}\,(2c^2d+9e)\log(x) - bc\sqrt{d}\,(2c^2d+9e)\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right) + \frac{b(c^4d^2+4c^2de-8e^2)\log\left(\frac{12cd^3\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(c^4d^2+4c^2de-8e^2)(i+cx)}\right)}{\sqrt{c^2d-e}} + \frac{b(c^4d^2+4c^2de-8e^2)\log\left(\frac{12cd^3\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\,(c^4d^2+4c^2de-8e^2)(-i+cx)}\right)}{\sqrt{c^2d-e}}}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] -1/6*((b*c*d*x*(d + e*x^2) + 2*a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4))/(x^3*Sqrt[d
 + e*x^2]) + (2*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcTan[c*x])/(x^3*Sqrt[d +
e*x^2]) + b*c*Sqrt[d]*(2*c^2*d + 9*e)*Log[x] - b*c*Sqrt[d]*(2*c^2*d + 9*e)*
Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*Log[(12
*c*d^3*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*
(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(I + c*x))])/Sqrt[c^2*d - e] + (b*(c^4*d^2 +
4*c^2*d*e - 8*e^2)*Log[(12*c*d^3*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*
x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(-I + c*x))])/Sqrt[
c^2*d - e])/d^3
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(8*x*e^2/(sqrt(x^2*e + d)*d^3) + 4*e/(sqrt(x^2*e + d)*d^2*x) - 1/(sqr
t(x^2*e + d)*d*x^3)) + 2*b*integrate(1/2*arctan(c*x)/((x^6*e + d*x^4)*sqrt(
x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(220) = 440.
time = 4.20, size = 2008, normalized size = 8.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^4*d^3*x^3 - 8*b*x^5*e^3 + 4*(b*c^2*d*x^5 - 2*b*d*x^3)*e^2 + (b
*c^4*d^2*x^5 + 4*b*c^2*d^2*x^3)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^
```

```
3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x
^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - (2*b*c^
5*d^3*x^3 - 9*b*c*x^5*e^3 + (7*b*c^3*d*x^5 - 9*b*c*d*x^3)*e^2 + (2*b*c^5*d^
2*x^5 + 7*b*c^3*d^2*x^3)*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d)
 + 2*d)/x^2) + 2*(b*c^3*d^3*x + 2*a*c^2*d^3 + 16*a*x^4*e^3 + 2*(b*c^2*d^3 +
 8*b*x^4*e^3 - 4*(2*b*c^2*d*x^4 - b*d*x^2)*e^2 - (4*b*c^2*d^2*x^2 + b*d^2)*
e)*arctan(c*x) - (16*a*c^2*d*x^4 + b*c*d*x^3 - 8*a*d*x^2)*e^2 + (b*c^3*d^2*
x^3 - 8*a*c^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2)*e)*sqrt(x^2*e + d))/(c^2*d^5*x
^3 - d^3*x^5*e^2 + (c^2*d^4*x^5 - d^4*x^3)*e), -1/12*(2*(b*c^4*d^3*x^3 - 8*
b*x^5*e^3 + 4*(b*c^2*d*x^5 - 2*b*d*x^3)*e^2 + (b*c^4*d^2*x^5 + 4*b*c^2*d^2*
x^3)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*
d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - (2*b*
c^5*d^3*x^3 - 9*b*c*x^5*e^3 + (7*b*c^3*d*x^5 - 9*b*c*d*x^3)*e^2 + (2*b*c^5*
d^2*x^5 + 7*b*c^3*d^2*x^3)*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(
d) + 2*d)/x^2) + 2*(b*c^3*d^3*x + 2*a*c^2*d^3 + 16*a*x^4*e^3 + 2*(b*c^2*d^3
 + 8*b*x^4*e^3 - 4*(2*b*c^2*d*x^4 - b*d*x^2)*e^2 - (4*b*c^2*d^2*x^2 + b*d^2
)*e)*arctan(c*x) - (16*a*c^2*d*x^4 + b*c*d*x^3 - 8*a*d*x^2)*e^2 + (b*c^3*d^
2*x^3 - 8*a*c^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2)*e)*sqrt(x^2*e + d))/(c^2*d^5
*x^3 - d^3*x^5*e^2 + (c^2*d^4*x^5 - d^4*x^3)*e), -1/12*(2*(2*b*c^5*d^3*x^3
- 9*b*c*x^5*e^3 + (7*b*c^3*d*x^5 - 9*b*c*d*x^3)*e^2 + (2*b*c^5*d^2*x^5 + 7*
b*c^3*d^2*x^3)*e)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + (b*c^4*d^3*x^
3 - 8*b*x^5*e^3 + 4*(b*c^2*d*x^5 - 2*b*d*x^3)*e^2 + (b*c^4*d^2*x^5 + 4*b*c^
2*d^2*x^3)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 - c)*e
)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*
d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c^3*d^3*x + 2*a*c^2*d^3
 + 16*a*x^4*e^3 + 2*(b*c^2*d^3 + 8*b*x^4*e^3 - 4*(2*b*c^2*d*x^4 - b*d*x^2)*
e^2 - (4*b*c^2*d^2*x^2 + b*d^2)*e)*arctan(c*x) - (16*a*c^2*d*x^4 + b*c*d*x^
3 - 8*a*d*x^2)*e^2 + (b*c^3*d^2*x^3 - 8*a*c^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2
)*e)*sqrt(x^2*e + d))/(c^2*d^5*x^3 - d^3*x^5*e^2 + (c^2*d^4*x^5 - d^4*x^3)*
e), -1/6*((b*c^4*d^3*x^3 - 8*b*x^5*e^3 + 4*(b*c^2*d*x^5 - 2*b*d*x^3)*e^2 +
(b*c^4*d^2*x^5 + 4*b*c^2*d^2*x^3)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d
+ (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 +
(c^3*d*x^2 - c*d)*e)) + (2*b*c^5*d^3*x^3 - 9*b*c*x^5*e^3 + (7*b*c^3*d*x^5 -
 9*b*c*d*x^3)*e^2 + (2*b*c^5*d^2*x^5 + 7*b*c^3*d^2*x^3)*e)*sqrt(-d)*arctan(
sqrt(-d)/sqrt(x^2*e + d)) + (b*c^3*d^3*x + 2*a*c^2*d^3 + 16*a*x^4*e^3 + 2*(
b*c^2*d^3 + 8*b*x^4*e^3 - 4*(2*b*c^2*d*x^4 - b*d*x^2)*e^2 - (4*b*c^2*d^2*x^
2 + b*d^2)*e)*arctan(c*x) - (16*a*c^2*d*x^4 + b*c*d*x^3 - 8*a*d*x^2)*e^2 +
(b*c^3*d^2*x^3 - 8*a*c^2*d^2*x^2 - b*c*d^2*x - 2*a*d^2)*e)*sqrt(x^2*e + d))
/(c^2*d^5*x^3 - d^3*x^5*e^2 + (c^2*d^4*x^5 - d^4*x^3)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atan}\,(cx)}{x^4\,(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(3/2),x)

[Out] Integral((a + b*atan(c*x))/(x**4*(d + e*x**2)**(3/2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")

[Out] sage0*x

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)),x)

[Out] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)), x)

3.1217 $\int \frac{x^4(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{ax^3}{3e(d+ex^2)^{3/2}}-\frac{ax}{e^2\sqrt{d+ex^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}+b\text{Int}\left(\frac{x^4\text{ArcTan}(cx)}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] -1/3*a*x^3/e/(e*x^2+d)^(3/2)+a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-a
*x/e^2/(e*x^2+d)^(1/2)+b*Unintegrable(x^4*arctan(c*x)/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*x^3)/(e*(d + e*x^2)^(3/2)) - (a*x)/(e^2*Sqrt[d + e*x^2]) + (a*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(5/2) + b*Defer[Int][(x^4*ArcTan[c*x])/(
d + e*x^2)^(5/2), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx &= a\int \frac{x^4}{(d+ex^2)^{5/2}}\,dx+b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx\\
&= -\frac{ax^3}{3e(d+ex^2)^{3/2}}+b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx+\frac{a\int \frac{x^2}{(d+ex^2)^{3/2}}\,dx}{e}\\
&= -\frac{ax^3}{3e(d+ex^2)^{3/2}}-\frac{ax}{e^2\sqrt{d+ex^2}}+b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx+\frac{a\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}\\
&= -\frac{ax^3}{3e(d+ex^2)^{3/2}}-\frac{ax}{e^2\sqrt{d+ex^2}}+b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx\right.}{e^2}\\
&= -\frac{ax^3}{3e(d+ex^2)^{3/2}}-\frac{ax}{e^2\sqrt{d+ex^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}+b\int \frac{x^4\tan^{-1}(cx)}{(d+ex^2)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 15.68, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arctan(c*x) + a*x^4)*sqrt(x^2*e + d)/(x^6*e^3 + 3*d*x^4*e^2
 + 3*d^2*x^2*e + d^3), x)
```

<u>**Sympy**</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

<u>**Mupad**</u> [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1218 $\int \frac{x^3(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}}+\frac{d(a+b\text{ArcTan}(cx))}{3e^2(d+ex^2)^{3/2}}-\frac{a+b\text{ArcTan}(cx)}{e^2\sqrt{d+ex^2}}+\frac{bc(2c^2d-3e)\,\text{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{3(c^2d-e)^{3/2}e^2}$$

```
[Out] 1/3*d*(a+b*arctan(c*x))/e^2/(e*x^2+d)^(3/2)+1/3*b*c*(2*c^2*d-3*e)*arctan(x*
(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e^2+1/3*b*c*x/(c^2*d-e)/e/
(e*x^2+d)^(1/2)+(-a-b*arctan(c*x))/e^2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {272, 45, 5096, 12, 541, 385, 209}

$$-\frac{a+b\text{ArcTan}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\text{ArcTan}(cx))}{3e^2(d+ex^2)^{3/2}}+\frac{bc(2c^2d-3e)\,\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e^2(c^2d-e)^{3/2}}+\frac{bcx}{3e(c^2d-e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (d*(a + b*ArcTan[c*x]))/(3*e^2*
(d + e*x^2)^(3/2)) - (a + b*ArcTan[c*x])/(e^2*Sqrt[d + e*x^2]) + (b*c*(2*c^
2*d - 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c^2*d - e)^(3/2
)*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx &= \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} - (bc)\int \frac{-2d-3ex^2}{3e^2(1+c^2x^2)(d+ex^2)^{3/2}}\,dx \\ &= \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{-2d-3ex^2}{(1+c^2x^2)(d+ex^2)^{3/2}}\,dx}{3e^2} \\ &= \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}} + \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bc)\int \frac{d(2}{(1+c^2x^2}}{3d(c^2c} \\ &= \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}} + \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bc(2c^2d-3e}{} \\ &= \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}} + \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bc(2c^2d-3e}{} \\ &= \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}} + \frac{d(a+b\tan^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\tan^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{bc(2c^2d-3e)}{3(c}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.41, size = 326, normalized size = 2.28

$$\frac{2\sqrt{c^2d-e}\,(bcex(d+ex^2)-a(c^2d-e)(2d+3ex^2)) - 2b(c^2d-e)^{3/2}(2d+3ex^2)\,\mathrm{ArcTan}(cx) - ibc(2c^2d-3e)(d+ex^2)^{3/2}\log\left(-\frac{12i\sqrt{c^2d-e}\,e^2\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(2c^2d-3e)(i+cx)}\right) + ibc(2c^2d-3e)(d+ex^2)^{3/2}\log\left(\frac{12i\sqrt{c^2d-e}\,e^2\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(2c^2d-3e)(-i+cx)}\right)}{6(c^2d-e)^{3/2}e^2(d+ex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[c^2*d - e]*(b*c*e*x*(d + e*x^2) - a*(c^2*d - e)*(2*d + 3*e*x^2)) -
2*b*(c^2*d - e)^(3/2)*(2*d + 3*e*x^2)*ArcTan[c*x] - I*b*c*(2*c^2*d - 3*e)*(
d + e*x^2)^(3/2)*Log[((-12*I)*Sqrt[c^2*d - e]*e^2*(c*d - I*e*x + Sqrt[c^2*d
 - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(I + c*x))] + I*b*c*(2*c^2*d - 3
*e)*(d + e*x^2)^(3/2)*Log[((12*I)*Sqrt[c^2*d - e]*e^2*(c*d + I*e*x + Sqrt[c
^2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(-I + c*x))])/(6*(c^2*d - e)
^(3/2)*e^2*(d + e*x^2)^(3/2))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(129) = 258.
time = 3.61, size = 877, normalized size = 6.13

$$\left[-\frac{\left(2 b c^3 d^3 - 3 b c x^4 e^3 + 2 \left(b c^3 d x^4 - 3 b c d x^2\right) e^2 + \left(4 b c^3 d^2 x^2 - 3 b c d^2\right) e\right) \sqrt{-c^2 d + e} \log\left(\frac{c^4 d^2 x^4 - 6 c^2 d^2 x^2 + 8 x^4 e^2 - 4 \left(c^2 d x^3 - 2 x^3 e - d x\right) \sqrt{-c^2 d + e} \sqrt{x^2 e + d} + d^2 - 8 \left(c^2 d x^4 - d x^2\right) e}{c^4 x^4 + 2 c^2 x^2 + 1}\right) + 4 \left(2 a c^4 d^3 + \left(2 b c^4 d^3 + 3 b x^2 e^3 - 2 \left(3 b c^2 d x^2 - b d\right) e^2 + \left(3 b c^4 d^2 x^2 - 4 b c^2 d^2\right) e\right) \arctan\left(c x\right) + \left(b c x^3 + 3 a x^2\right) e^3 - \left(b c^3 d x^3 + 6 a c^2 d x^2 - b c d x - 2 a d\right) e^2 + \left(3 a c^4 d^2 x^2 - b c^3 d^2 x - 4 a c^2 d^2\right) e\right) \sqrt{x^2 e + d}}{12 \left(c^4 d^4 e^2 + x^4 e^6 - 2 \left(c^2 d x^4 - d x^2\right) e^5 + \left(c^4 d^2 x^4 - 4 c^2 d^2 x^2 + d^2\right) e^4 + 2 \left(c^4 d^3 x^2 - c^2 d^3\right) e^3\right)}, \frac{\left(2 b c^3 d^3 - 3 b c x^4 e^3 + 2 \left(b c^3 d x^4 - 3 b c d x^2\right) e^2 + \left(4 b c^3 d^2 x^2 - 3 b c d^2\right) e\right) \sqrt{c^2 d - e} \arctan\left(\frac{\left(c^2 d x^2 - 2 x^2 e - d\right) \sqrt{c^2 d - e} \sqrt{x^2 e + d}}{2 \left(c^2 d^2 x - x^3 e^2 + \left(c^2 d x^3 - d x\right) e\right)}\right) - 2 \left(2 a c^4 d^3 + \left(2 b c^4 d^3 + 3 b x^2 e^3 - 2 \left(3 b c^2 d x^2 - b d\right) e^2 + \left(3 b c^4 d^2 x^2 - 4 b c^2 d^2\right) e\right) \arctan\left(c x\right) + \left(b c x^3 + 3 a x^2\right) e^3 - \left(b c^3 d x^3 + 6 a c^2 d x^2 - b c d x - 2 a d\right) e^2 + \left(3 a c^4 d^2 x^2 - b c^3 d^2 x - 4 a c^2 d^2\right) e\right) \sqrt{x^2 e + d}}{6 \left(c^4 d^4 e^2 + x^4 e^6 - 2 \left(c^2 d x^4 - d x^2\right) e^5 + \left(c^4 d^2 x^4 - 4 c^2 d^2 x^2 + d^2\right) e^4 + 2 \left(c^4 d^3 x^2 - c^2 d^3\right) e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [-1/12*((2*b*c^3*d^3 - 3*b*c*x^4*e^3 + 2*(b*c^3*d*x^4 - 3*b*c*d*x^2)*e^2 +
(4*b*c^3*d^2*x^2 - 3*b*c*d^2)*e)*sqrt(-c^2*d + e)*log((c^4*d^2*x^4 - 6*c^2*
d^2*x^2 + 8*x^4*e^2 - 4*(c^2*d*x^3 - 2*x^3*e - d*x)*sqrt(-c^2*d + e)*sqrt(x
^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(
2*a*c^4*d^3 + (2*b*c^4*d^3 + 3*b*x^2*e^3 - 2*(3*b*c^2*d*x^2 - b*d)*e^2 + (3
*b*c^4*d^2*x^2 - 4*b*c^2*d^2)*e)*arctan(c*x) + (b*c*x^3 + 3*a*x^2)*e^3 - (b
*c^3*d*x^3 + 6*a*c^2*d*x^2 - b*c*d*x - 2*a*d)*e^2 + (3*a*c^4*d^2*x^2 - b*c^
3*d^2*x - 4*a*c^2*d^2)*e)*sqrt(x^2*e + d))/(c^4*d^4*e^2 + x^4*e^6 - 2*(c^2*
d*x^4 - d*x^2)*e^5 + (c^4*d^2*x^4 - 4*c^2*d^2*x^2 + d^2)*e^4 + 2*(c^4*d^3*x
^2 - c^2*d^3)*e^3), 1/6*((2*b*c^3*d^3 - 3*b*c*x^4*e^3 + 2*(b*c^3*d*x^4 - 3*
b*c*d*x^2)*e^2 + (4*b*c^3*d^2*x^2 - 3*b*c*d^2)*e)*sqrt(c^2*d - e)*arctan(1/
2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*e + d)/(c^2*d^2*x - x^
3*e^2 + (c^2*d*x^3 - d*x)*e)) - 2*(2*a*c^4*d^3 + (2*b*c^4*d^3 + 3*b*x^2*e^3
 - 2*(3*b*c^2*d*x^2 - b*d)*e^2 + (3*b*c^4*d^2*x^2 - 4*b*c^2*d^2)*e)*arctan(
c*x) + (b*c*x^3 + 3*a*x^2)*e^3 - (b*c^3*d*x^3 + 6*a*c^2*d*x^2 - b*c*d*x - 2
*a*d)*e^2 + (3*a*c^4*d^2*x^2 - b*c^3*d^2*x - 4*a*c^2*d^2)*e)*sqrt(x^2*e + d
))/(c^4*d^4*e^2 + x^4*e^6 - 2*(c^2*d*x^4 - d*x^2)*e^5 + (c^4*d^2*x^4 - 4*c^
2*d^2*x^2 + d^2)*e^4 + 2*(c^4*d^3*x^2 - c^2*d^3)*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b \operatorname{atan}(cx))}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1219 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{bc}{3\left(c^2d-e\right)e\sqrt{d+ex^2}}+\frac{x^3(a+b\text{ArcTan}(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

```
[Out] 1/3*x^3*(a+b*arctan(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*arctanh(c*(e*x^2+d)^(1/2)
/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)+1/3*b*c/(c^2*d-e)/e/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {270, 5096, 457, 79, 65, 214}

$$\frac{x^3(a+b\text{ArcTan}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{bc}{3e\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c)/(3*(c^2*d - e)*e*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcTan[c*x]))/(3*d*(d
 + e*x^2)^(3/2)) - (b*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d*(c
^2*d - e)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx &= \frac{x^3(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}}-(bc)\int\frac{x^3}{(3d+3c^2dx^2)\,(d+ex^2)^{3/2}}\,dx\\
&= \frac{x^3(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{1}{2}(bc)\text{Subst}\left(\int\frac{x}{(3d+3c^2dx)\,(d+ex)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{bc}{3\,(c^2d-e)\,e\sqrt{d+ex^2}}+\frac{x^3(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{(bc)\text{Subst}\left(\int\frac{1}{(3d+3c^2dx)\sqrt{d+ex}}\right.}{2\,(c^2d-e)}\\
&= \frac{bc}{3\,(c^2d-e)\,e\sqrt{d+ex^2}}+\frac{x^3(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{(bc)\text{Subst}\left(\int\frac{1}{3d-\frac{3c^2d^2}{e}+\frac{3c^2dx^2}{e}}\,dx\right.}{(c^2d-e)\,e}\\
&= \frac{bc}{3\,(c^2d-e)\,e\sqrt{d+ex^2}}+\frac{x^3(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\,(c^2d-e)^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 252, normalized size = 2.31

$$-\frac{\frac{2adx}{e(d+ex^2)^{3/2}}-\frac{2(bcd+a(c^2d-e)x)}{(c^2d-e)e\sqrt{d+ex^2}}-\frac{2bx^3\text{ArcTan}(cx)}{(d+ex^2)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(i+cx)}\right)}{(c^2d-e)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(-i+cx)}\right)}{(c^2d-e)^{3/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/6*((2*a*d*x)/(e*(d + e*x^2)^(3/2)) - (2*(b*c*d + a*(c^2*d - e)*x))/((c^2
*d - e)*e*Sqrt[d + e*x^2]) - (2*b*x^3*ArcTan[c*x])/(d + e*x^2)^(3/2) + (b*L
og[(12*c*d*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))
/(b*(I + c*x))])/(c^2*d - e)^(3/2) + (b*Log[(12*c*d*Sqrt[c^2*d - e]*(c*d +
I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(-I + c*x))])/(c^2*d - e)^(3/2
))/d
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x*e^(-1)/(x^2*e + d)^(3/2) - x*e^(-1)/(sqrt(x^2*e + d)*d)) + 2*b*in
tegrate(1/2*x^2*arctan(c*x)/((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^2*e + d)),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(98) = 196.
time = 2.57, size = 699, normalized size = 6.41

$$\left[-\frac{(bx^4e^3+2\,bdx^2e^2+bd^2e)\sqrt{c^2d-e}\,\log\left(\frac{8\,c^4d^2+4\,(2\,c^3d+(c^3x^2-c)e)\sqrt{c^2d-e}\,\sqrt{x^2e+d}+(c^4x^4-6\,c^2x^2+1)e^2+8\,(c^4dx^2-c^2d)e}{c^4x^4+2\,c^2x^2+1}\right)-4\,(bc^3d^3+ax^3e^3+(bc^4d^2x^3e-2\,bc^2dx^3e^2+bx^3e^3)\arctan(cx)-(2\,ac^2dx^3+bcdx^2)e^2+(ac^4d^2x^3+bc^3d^2x^2-bcd^2)e)\sqrt{x^2e+d}}{12\,(c^4d^5e+dx^4e^5-2\,(c^2d^2x^4-d^2x^2)e^4+(c^4d^3x^4-4\,c^2d^3x^2+d^3)e^3+2\,(c^4d^4x^2-c^2d^4)e^2)},-\frac{(bx^4e^3+2\,bdx^2e^2+bd^2e)\sqrt{-c^2d+e}\,\arctan\left(-\frac{(2\,c^2d+(c^2x^2-1)e)\sqrt{-c^2d+e}\,\sqrt{x^2e+d}}{2\,(c^3d^2-cx^2e^2+(c^3dx^2-cd)e)}\right)-2\,(bc^3d^3+ax^3e^3+(bc^4d^2x^3e-2\,bc^2dx^3e^2+bx^3e^3)\arctan(cx)-(2\,ac^2dx^3+bcdx^2)e^2+(ac^4d^2x^3+bc^3d^2x^2-bcd^2)e)\sqrt{x^2e+d}}{6\,(c^4d^5e+dx^4e^5-2\,(c^2d^2x^4-d^2x^2)e^4+(c^4d^3x^4-4\,c^2d^3x^2+d^3)e^3+2\,(c^4d^4x^2-c^2d^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*x^4*e^3 + 2*b*d*x^2*e^2 + b*d^2*e)*sqrt(c^2*d - e)*log((8*c^4*d^
2 + 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^
4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)
) - 4*(b*c^3*d^3 + a*x^3*e^3 + (b*c^4*d^2*x^3*e - 2*b*c^2*d*x^3*e^2 + b*x^3
*e^3)*arctan(c*x) - (2*a*c^2*d*x^3 + b*c*d*x^2)*e^2 + (a*c^4*d^2*x^3 + b*c^
3*d^2*x^2 - b*c*d^2)*e)*sqrt(x^2*e + d))/(c^4*d^5*e + d*x^4*e^5 - 2*(c^2*d^
2*x^4 - d^2*x^2)*e^4 + (c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*e^3 + 2*(c^4*d^4
*x^2 - c^2*d^4)*e^2), -1/6*((b*x^4*e^3 + 2*b*d*x^2*e^2 + b*d^2*e)*sqrt(-c^2
*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e
 + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - 2*(b*c^3*d^3 + a*x^3*e
^3 + (b*c^4*d^2*x^3*e - 2*b*c^2*d*x^3*e^2 + b*x^3*e^3)*arctan(c*x) - (2*a*c
^2*d*x^3 + b*c*d*x^2)*e^2 + (a*c^4*d^2*x^3 + b*c^3*d^2*x^2 - b*c*d^2)*e)*sq
rt(x^2*e + d))/(c^4*d^5*e + d*x^4*e^5 - 2*(c^2*d^2*x^4 - d^2*x^2)*e^4 + (c^
4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*e^3 + 2*(c^4*d^4*x^2 - c^2*d^4)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1220 $\int \frac{x(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{a+b\text{ArcTan}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bc^3\text{ArcTan}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{3\left(c^2d-e\right)^{3/2}e}$$

```
[Out] 1/3*(-a-b*arctan(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c^3*arctan(x*(c^2*d-e)^(1/2)
/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e-1/3*b*c*x/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5094, 390, 385, 209}

$$-\frac{a+b\text{ArcTan}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bc^3\text{ArcTan}\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{3e\left(c^2d-e\right)^{3/2}}-\frac{bcx}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x)/(d*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*ArcTan[c*x])/(3*e*(d
+ e*x^2)^(3/2)) + (b*c^3*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(3*(c
^2*d - e)^(3/2)*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{(bc)\int \frac{1}{(1+c^2x^2)(d+ex^2)^{3/2}}\,dx}{3e}$$

$$= -\frac{bcx}{3d\,(c^2d-e)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{(bc^3)\int \frac{1}{(1+c^2x^2)\sqrt{d+ex^2}}\,dx}{3\,(c^2d-e)\,e}$$

$$= -\frac{bcx}{3d\,(c^2d-e)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{(bc^3)\,\text{Subst}\left(\int \frac{1}{1-(-c^2d+e)x^2}\,dx, x, \right.}{3\,(c^2d-e)\,e}$$

$$= -\frac{bcx}{3d\,(c^2d-e)\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{bc^3\tan^{-1}\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{3\,(c^2d-e)^{3/2}\,e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 259, normalized size = 2.35

$$\frac{1}{6}\left(-\frac{2a}{e\,(d+ex^2)^{3/2}}-\frac{2bcx}{(c^2d^2-de)\sqrt{d+ex^2}}-\frac{2b\text{ArcTan}(cx)}{e\,(d+ex^2)^{3/2}}-\frac{ibc^3\log\left(-\frac{12i\sqrt{c^2d-e}\,e\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{bc^2(i+cx)}\right)}{(c^2d-e)^{3/2}\,e}+\frac{ibc^3\log\left(\frac{12i\sqrt{c^2d-e}\,e\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{bc^2(-i+cx)}\right)}{(c^2d-e)^{3/2}\,e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((-2*a)/(e*(d + e*x^2)^(3/2)) - (2*b*c*x)/((c^2*d^2 - d*e)*Sqrt[d + e*x^2])
 - (2*b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) - (I*b*c^3*Log[((-12*I)*Sqrt[c^2
*d - e]*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(I + c*x)
)])/((c^2*d - e)^(3/2)*e) + (I*b*c^3*Log[((12*I)*Sqrt[c^2*d - e]*e*(c*d + I
*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(-I + c*x))])/((c^2*d - e)^
(3/2)*e))/6
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\arctan(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-%e>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(98) = 196.
time = 2.94, size = 687, normalized size = 6.25

$$\left[\frac{(bc^3dx^4e^2 + 2\,bc^3d^2x^2e + bc^3d^3)\sqrt{-c^2d + e}\,\log\left(\frac{c^4d^2x^4 - 6\,c^2d^2x^2 + 8\,x^4e^2 + 4\,(c^2dx^3 - 2\,x^3e - dx)\sqrt{-c^2d + e}\,\sqrt{x^2e + d} + d^2 - 8\,(c^2dx^4 - dx^2)e}{c^4x^4 + 2\,c^2x^2 + 1}\right) - 4\,(ac^4d^3 - bcx^3e^3 + (bc^4d^3 - 2\,bc^2d^2e + bde^2)\arctan(cx) + (bc^3dx^3 - bcdx + ad)e^2 + (bc^3d^2x - 2\,ac^2d^2)e)\sqrt{x^2e + d}}{12\,(c^4d^5e + dx^4e^5 - 2\,(c^2d^2x^4 - d^2x^2)e^4 + (c^4d^3x^4 - 4\,c^2d^3x^2 + d^3)e^3 + 2\,(c^4d^4x^2 - c^2d^4)e^2)}, \frac{(bc^3dx^4e^2 + 2\,bc^3d^2x^2e + bc^3d^3)\sqrt{c^2d - e}\,\arctan\left(\frac{(c^2dx^2 - 2\,x^2e - d)\sqrt{c^2d - e}\,\sqrt{x^2e + d}}{2\,(c^2d^2x - x^3e^2 + (c^2dx^3 - dx)e)}\right) - 2\,(ac^4d^3 - bcx^3e^3 + (bc^4d^3 - 2\,bc^2d^2e + bde^2)\arctan(cx) + (bc^3dx^3 - bcdx + ad)e^2 + (bc^3d^2x - 2\,ac^2d^2)e)\sqrt{x^2e + d}}{6\,(c^4d^5e + dx^4e^5 - 2\,(c^2d^2x^4 - d^2x^2)e^4 + (c^4d^3x^4 - 4\,c^2d^3x^2 + d^3)e^3 + 2\,(c^4d^4x^2 - c^2d^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((b*c^3*d*x^4*e^2 + 2*b*c^3*d^2*x^2*e + b*c^3*d^3)*sqrt(-c^2*d + e)*l
og((c^4*d^2*x^4 - 6*c^2*d^2*x^2 + 8*x^4*e^2 + 4*(c^2*d*x^3 - 2*x^3*e - d*x)
*sqrt(-c^2*d + e)*sqrt(x^2*e + d) + d^2 - 8*(c^2*d*x^4 - d*x^2)*e)/(c^4*x^4
 + 2*c^2*x^2 + 1)) - 4*(a*c^4*d^3 - b*c*x^3*e^3 + (b*c^4*d^3 - 2*b*c^2*d^2*
e + b*d*e^2)*arctan(c*x) + (b*c^3*d*x^3 - b*c*d*x + a*d)*e^2 + (b*c^3*d^2*x
 - 2*a*c^2*d^2)*e)*sqrt(x^2*e + d))/(c^4*d^5*e + d*x^4*e^5 - 2*(c^2*d^2*x^4
 - d^2*x^2)*e^4 + (c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*e^3 + 2*(c^4*d^4*x^2
- c^2*d^4)*e^2), 1/6*((b*c^3*d*x^4*e^2 + 2*b*c^3*d^2*x^2*e + b*c^3*d^3)*sqr
t(c^2*d - e)*arctan(1/2*(c^2*d*x^2 - 2*x^2*e - d)*sqrt(c^2*d - e)*sqrt(x^2*
e + d)/(c^2*d^2*x - x^3*e^2 + (c^2*d*x^3 - d*x)*e)) - 2*(a*c^4*d^3 - b*c*x^
3*e^3 + (b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*arctan(c*x) + (b*c^3*d*x^3 -
b*c*d*x + a*d)*e^2 + (b*c^3*d^2*x - 2*a*c^2*d^2)*e)*sqrt(x^2*e + d))/(c^4*d
```

```
^5*e + d*x^4*e^5 - 2*(c^2*d^2*x^4 - d^2*x^2)*e^4 + (c^4*d^3*x^4 - 4*c^2*d^3
*x^2 + d^3)*e^3 + 2*(c^4*d^4*x^2 - c^2*d^4)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{atan}(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
[Out] Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\operatorname{atan}(c\,x))}{(e\,x^2 + d)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1221 $\int \frac{a+b\text{ArcTan}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{bc}{3d(c^2d-e)\sqrt{d+ex^2}}+\frac{x(a+b\text{ArcTan}(cx))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\text{ArcTan}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{b(3c^2d-2e)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2(c^2d-e)^{3/2}}$$

```
[Out] 1/3*x*(a+b*arctan(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*(3*c^2*d-2*e)*arctanh(c*(e*
x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)-1/3*b*c/d/(c^2*d-e)/(e*x^
2+d)^(1/2)+2/3*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {198, 197, 5032, 6820, 12, 585, 79, 65, 214}

$$\frac{2x(a+b\text{ArcTan}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\text{ArcTan}(cx))}{3d(d+ex^2)^{3/2}}+\frac{b(3c^2d-2e)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2(c^2d-e)^{3/2}}-\frac{bc}{3d(c^2d-e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c)/(d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcTan[c*x]))/(3*d*(
d + e*x^2)^(3/2)) + (2*x*(a + b*ArcTan[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*
(3*c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^2*(c^2*d
 - e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{\frac{x}{3d(d+ex^2)^{3/2}} + \frac{2x}{3d^2\sqrt{d+ex^2}}}{1+c^2x^2}\,dx$$

$$= \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x(3d+2ex^2)}{3d^2\,(1+c^2x^2)\,(d+ex^2)^{3/2}}\,dx$$

$$= \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x(3d+2ex^2)}{(1+c^2x^2)(d+ex^2)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\text{Subst}\left(\int \frac{3d+2ex}{(1+c^2x)(d+ex)^{3/2}}\,dx, x, x^2\right)}{6d^2}$$

$$= -\frac{bc}{3d\,(c^2d-e)\,\sqrt{d+ex^2}} + \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc(3c^2d}{}$$

$$= -\frac{bc}{3d\,(c^2d-e)\,\sqrt{d+ex^2}} + \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{(bc(3c^2d}{}$$

$$= -\frac{bc}{3d\,(c^2d-e)\,\sqrt{d+ex^2}} + \frac{x(a+b\tan^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\tan^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{b(3c^2d-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 317, normalized size = 2.20

$$\frac{2\sqrt{c^2d-e}\,(-bcd(d+ex^2)+a(c^2d-e)\,x(3d+2ex^2))+2b(c^2d-e)^{3/2}\,x(3d+2ex^2)\,\text{ArcTan}(cx)+b(3c^2d-2e)\,(d+ex^2)^{3/2}\log\left(-\frac{12cd^2\sqrt{c^2d-e}\,\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(3c^2d-2e)(i+cx)}\right)+b(3c^2d-2e)\,(d+ex^2)^{3/2}\log\left(-\frac{12cd^2\sqrt{c^2d-e}\,\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b(3c^2d-2e)(-i+cx)}\right)}{6d^2\,(c^2d-e)^{3/2}\,(d+ex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[c^2*d - e]*(-(b*c*d*(d + e*x^2)) + a*(c^2*d - e)*x*(3*d + 2*e*x^2))
 + 2*b*(c^2*d - e)^(3/2)*x*(3*d + 2*e*x^2)*ArcTan[c*x] + b*(3*c^2*d - 2*e)*
(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d
- e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(I + c*x))] + b*(3*c^2*d - 2*e)*(
d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt[c^2*d -
 e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(-I + c*x))])/(6*d^2*(c^2*d - e)^(
3/2)*(d + e*x^2)^(3/2))
```

Maple [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan(cx)}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + 2*b*integrate
(1/2*arctan(c*x)/((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(132) = 264.
time = 2.41, size = 897, normalized size = 6.23

$$\left[\frac{{\left(3 \, b c^{2} d^{3} - 2 \, b x^{4} e^{3} + {\left(3 \, b c^{2} d x^{4} - 4 \, b d x^{2}\right)} e^{2} + 2 \, {\left(3 \, b c^{2} d^{2} x^{2} - b d^{2}\right)} e\right)} \sqrt{c^{2} d - e} \log\left(\frac{8 \, c^{4} d^{2} + 4 \, {\left(2 \, c^{3} d + {\left(c^{3} x^{2} - c\right)} e\right)} \sqrt{c^{2} d - e} \sqrt{x^{2} e + d} + {\left(c^{4} x^{4} - 6 \, c^{2} x^{2} + 1\right)} e^{2} + 8 \, {\left(c^{4} d x^{2} - c^{2} d\right)} e}{c^{4} x^{4} + 2 \, c^{2} x^{2} + 1}\right) + 4 \, {\left(3 \, a c^{4} d^{3} x - b c^{3} d^{3} + 2 \, a x^{3} e^{3} + {\left(3 \, b c^{4} d^{3} x + 2 \, b x^{3} e^{3} - {\left(4 \, b c^{2} d x^{3} - 3 \, b d x\right)} e^{2} + 2 \, {\left(b c^{4} d^{2} x^{3} - 3 \, b c^{2} d^{2} x\right)} e\right)} \arctan\left(c x\right) - {\left(4 \, a c^{2} d x^{3} - b c d x^{2} - 3 \, a d x\right)} e^{2} + {\left(2 \, a c^{4} d^{2} x^{3} - b c^{3} d^{2} x^{2} - 6 \, a c^{2} d^{2} x + b c d^{2}\right)} e\right)} \sqrt{x^{2} e + d}}{12 \, {\left(c^{4} d^{6} + d^{2} x^{4} e^{4} - 2 \, {\left(c^{2} d^{3} x^{4} - d^{3} x^{2}\right)} e^{3} + {\left(c^{4} d^{4} x^{4} - 4 \, c^{2} d^{4} x^{2} + d^{4}\right)} e^{2} + 2 \, {\left(c^{4} d^{5} x^{2} - c^{2} d^{5}\right)} e\right)}}, \frac{{\left(3 \, b c^{2} d^{3} - 2 \, b x^{4} e^{3} + {\left(3 \, b c^{2} d x^{4} - 4 \, b d x^{2}\right)} e^{2} + 2 \, {\left(3 \, b c^{2} d^{2} x^{2} - b d^{2}\right)} e\right)} \sqrt{-c^{2} d + e} \arctan\left(-\frac{{\left(2 \, c^{2} d + {\left(c^{2} x^{2} - 1\right)} e\right)} \sqrt{-c^{2} d + e} \sqrt{x^{2} e + d}}{2 \, {\left(c^{3} d^{2} - c x^{2} e^{2} + {\left(c^{3} d x^{2} - c d\right)} e\right)}}\right) + 2 \, {\left(3 \, a c^{4} d^{3} x - b c^{3} d^{3} + 2 \, a x^{3} e^{3} + {\left(3 \, b c^{4} d^{3} x + 2 \, b x^{3} e^{3} - {\left(4 \, b c^{2} d x^{3} - 3 \, b d x\right)} e^{2} + 2 \, {\left(b c^{4} d^{2} x^{3} - 3 \, b c^{2} d^{2} x\right)} e\right)} \arctan\left(c x\right) - {\left(4 \, a c^{2} d x^{3} - b c d x^{2} - 3 \, a d x\right)} e^{2} + {\left(2 \, a c^{4} d^{2} x^{3} - b c^{3} d^{2} x^{2} - 6 \, a c^{2} d^{2} x + b c d^{2}\right)} e\right)} \sqrt{x^{2} e + d}}{6 \, {\left(c^{4} d^{6} + d^{2} x^{4} e^{4} - 2 \, {\left(c^{2} d^{3} x^{4} - d^{3} x^{2}\right)} e^{3} + {\left(c^{4} d^{4} x^{4} - 4 \, c^{2} d^{4} x^{2} + d^{4}\right)} e^{2} + 2 \, {\left(c^{4} d^{5} x^{2} - c^{2} d^{5}\right)} e\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [1/12*((3*b*c^2*d^3 - 2*b*x^4*e^3 + (3*b*c^2*d*x^4 - 4*b*d*x^2)*e^2 + 2*(3*
b*c^2*d^2*x^2 - b*d^2)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^
3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e
^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(3*a*c^4*d^3*x
 - b*c^3*d^3 + 2*a*x^3*e^3 + (3*b*c^4*d^3*x + 2*b*x^3*e^3 - (4*b*c^2*d*x^3
- 3*b*d*x)*e^2 + 2*(b*c^4*d^2*x^3 - 3*b*c^2*d^2*x)*e)*arctan(c*x) - (4*a*c^
2*d*x^3 - b*c*d*x^2 - 3*a*d*x)*e^2 + (2*a*c^4*d^2*x^3 - b*c^3*d^2*x^2 - 6*a
*c^2*d^2*x + b*c*d^2)*e)*sqrt(x^2*e + d))/(c^4*d^6 + d^2*x^4*e^4 - 2*(c^2*d
^3*x^4 - d^3*x^2)*e^3 + (c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*e^2 + 2*(c^4*d^
5*x^2 - c^2*d^5)*e), 1/6*((3*b*c^2*d^3 - 2*b*x^4*e^3 + (3*b*c^2*d*x^4 - 4*b
*d*x^2)*e^2 + 2*(3*b*c^2*d^2*x^2 - b*d^2)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(
2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^
2*e^2 + (c^3*d*x^2 - c*d)*e)) + 2*(3*a*c^4*d^3*x - b*c^3*d^3 + 2*a*x^3*e^3
+ (3*b*c^4*d^3*x + 2*b*x^3*e^3 - (4*b*c^2*d*x^3 - 3*b*d*x)*e^2 + 2*(b*c^4*d
^2*x^3 - 3*b*c^2*d^2*x)*e)*arctan(c*x) - (4*a*c^2*d*x^3 - b*c*d*x^2 - 3*a*d
*x)*e^2 + (2*a*c^4*d^2*x^3 - b*c^3*d^2*x^2 - 6*a*c^2*d^2*x + b*c*d^2)*e)*sq
rt(x^2*e + d))/(c^4*d^6 + d^2*x^4*e^4 - 2*(c^2*d^3*x^4 - d^3*x^2)*e^3 + (c^
4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*e^2 + 2*(c^4*d^5*x^2 - c^2*d^5)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*atan(c*x))/(d + e*x^2)^(5/2), x)
```

3.1222 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{a}{3d\left(d+ex^2\right)^{3/2}}+\frac{a}{d^2\sqrt{d+ex^2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+b\text{Int}\left(\frac{\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] 1/3*a/d/(e*x^2+d)^(3/2)-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+a/d^2/(e
*x^2+d)^(1/2)+b*Unintegrable(arctan(c*x)/x/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] a/(3*d*(d + e*x^2)^(3/2)) + a/(d^2*Sqrt[d + e*x^2]) - (a*ArcTanh[Sqrt[d + e
*x^2]/Sqrt[d]])/d^(5/2) + b*Defer[Int][ArcTan[c*x]/(x*(d + e*x^2)^(5/2)), x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx &= a\int \frac{1}{x\left(d+ex^2\right)^{5/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx \\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(d+ex)^{5/2}}\,dx, x, x^2\right) + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx \\
&= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\text{Subst}\left(\int \frac{1}{x(d+ex)^{3/2}}\,dx, x, x^2\right)}{2d} \\
&= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,d\right.}{2d^2} \\
&= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x,\right.}{d^2e} \\
&= \frac{a}{3d\left(d+ex^2\right)^{3/2}} + \frac{a}{d^2\sqrt{d+ex^2}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}} + b\int \frac{\tan^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 11.05, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(5/2) - 3/(sqrt(x^2*e + d)*d^2
) - 1/((x^2*e + d)^(3/2)*d)) + 2*b*integrate(1/2*arctan(c*x)/((x^5*e^2 + 2*
d*x^3*e + d^2*x)*sqrt(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2
*x^3*e + d^3*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)),x)
```

```
[Out] int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)), x)
```

3.1223 $\int \frac{a+b\text{ArcTan}(cx)}{x^2(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{bc}{d^2\sqrt{d+ex^2}}-\frac{8be}{3cd^3\sqrt{d+ex^2}}-\frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3(c^2d-e)\sqrt{d+ex^2}}-\frac{a+b\text{ArcTan}(cx)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\text{ArcTan}(cx))}{3d^2(d+ex^2)^{3/2}}-\frac{8ex}{}$$

```
[Out] (-a-b*arctan(c*x))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*arctan(c*x))/d^2/(e*x^2
+d)^(3/2)-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/3*b*(3*c^4*d^2-12*
c^2*d*e+8*e^2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^3/(c^2*d-e)^(3/
2)+b*c/d^2/(e*x^2+d)^(1/2)-8/3*b*e/c/d^3/(e*x^2+d)^(1/2)-1/3*b*(3*c^4*d^2-1
2*c^2*d*e+8*e^2)/c/d^3/(c^2*d-e)/(e*x^2+d)^(1/2)-8/3*e*x*(a+b*arctan(c*x))/
d^3/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 198, 197, 5096, 12, 6857, 272, 53, 65, 214, 267, 455}

$$-\frac{8ex(a+b\text{ArcTan}(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\text{ArcTan}(cx))}{3d^2(d+ex^2)^{3/2}}-\frac{a+b\text{ArcTan}(cx)}{dx(d+ex^2)^{3/2}}-\frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3(c^2d-e)\sqrt{d+ex^2}}+\frac{b(3c^4d^2-12c^2de+8e^2)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3(c^2d-e)^{3/2}}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}-\frac{8be}{3cd^3\sqrt{d+ex^2}}+\frac{bc}{d^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(d^2*Sqrt[d + e*x^2]) - (8*b*e)/(3*c*d^3*Sqrt[d + e*x^2]) - (b*(3*c^4
*d^2 - 12*c^2*d*e + 8*e^2))/(3*c*d^3*(c^2*d - e)*Sqrt[d + e*x^2]) - (a + b*
ArcTan[c*x])/(d*x*(d + e*x^2)^(3/2)) - (4*e*x*(a + b*ArcTan[c*x]))/(3*d^2*(
d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d + e*x^2]) - (
b*c*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(5/2) + (b*(3*c^4*d^2 - 12*c^2*d*e
+ 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(3*d^3*(c^2*d - e)^(
3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\tan^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - (bc)\int \frac{-3d^2}{3d^3x(1}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\tan^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{-3d^2-12d}{x(1+c^2x^2)}}{3d^3}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\tan^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bc)\int\left(-\frac{3}{x(d+e}\right.}{}$$

$$= -\frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\tan^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\tan^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{(bc)\int \frac{1}{x(d+ex^2)^3}}{d}$$

$$= -\frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\tan^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\tan^{-1}(cx)}{3d^3\sqrt{d+ex^2}}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4e}{}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4e}{}$$

$$= \frac{bc}{d^2\sqrt{d+ex^2}} - \frac{8be}{3cd^3\sqrt{d+ex^2}} - \frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3\left(c^2d-e\right)\sqrt{d+ex^2}} - \frac{a+b\tan^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4e}{}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 0.79, size = 418, normalized size = 1.53

$$\frac{-\frac{2adex}{(d+ex^2)^{3/2}} + \frac{2e\left(bcd+5a\left(-c^2d+e\right)x\right)}{(c^2d-e)\sqrt{d+ex^2}} - \frac{6a\sqrt{d+ex^2}}{x} - \frac{2b\left(3d^2+12dex^2+8e^2x^4\right)\mathrm{ArcTan}(cx)}{x(d+ex^2)^{3/2}} + 6bc\sqrt{d}\,\log(x) - 6bc\sqrt{d}\,\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right) + \frac{b(3c^4d^2-12c^2de+8e^2)\log\left(-\frac{12cd^3\sqrt{c^2d-e}\left(cd-iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\left(3c^4d^2-12c^2de+8e^2\right)(i+cx)}\right)}{(c^2d-e)^{3/2}} + \frac{b(3c^4d^2-12c^2de+8e^2)\log\left(-\frac{12cd^3\sqrt{c^2d-e}\left(cd+iex+\sqrt{c^2d-e}\,\sqrt{d+ex^2}\right)}{b\left(3c^4d^2-12c^2de+8e^2\right)(-i+cx)}\right)}{(c^2d-e)^{3/2}}}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] ((-2*a*d*e*x)/(d + e*x^2)^(3/2) + (2*e*(b*c*d + 5*a*(-(c^2*d) + e)*x))/((c^
2*d - e)*Sqrt[d + e*x^2]) - (6*a*Sqrt[d + e*x^2])/x - (2*b*(3*d^2 + 12*d*e*
x^2 + 8*e^2*x^4)*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)) + 6*b*c*Sqrt[d]*Log[x]
- 6*b*c*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(3*c^4*d^2 - 12*c^2*d
*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*S
qrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*(I + c*x))])/(c^2*d -
e)^(3/2) + (b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d -
```

```
e]*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*
d*e + 8*e^2)*(-I + c*x))])/(c^2*d - e)^(3/2))/(6*d^3)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(8*x*e/(sqrt(x^2*e + d)*d^3) + 4*x*e/((x^2*e + d)^(3/2)*d^2) + 3/((x
^2*e + d)^(3/2)*d*x)) + 2*b*integrate(1/2*arctan(c*x)/((x^6*e^2 + 2*d*x^4*e
 + d^2*x^2)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(254) = 508.
time = 4.39, size = 2774, normalized size = 10.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((3*b*c^4*d^4*x + 8*b*x^5*e^4 - 4*(3*b*c^2*d*x^5 - 4*b*d*x^3)*e^3 +
(3*b*c^4*d^2*x^5 - 24*b*c^2*d^2*x^3 + 8*b*d^2*x)*e^2 + 6*(b*c^4*d^3*x^3 - 2
*b*c^2*d^3*x)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 - 4*(2*c^3*d + (c^3*x^2 - c
)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c
^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 6*(b*c^5*d^4*x + b*c*x^5*
e^4 - 2*(b*c^3*d*x^5 - b*c*d*x^3)*e^3 + (b*c^5*d^2*x^5 - 4*b*c^3*d^2*x^3 +
b*c*d^2*x)*e^2 + 2*(b*c^5*d^3*x^3 - b*c^3*d^3*x)*e)*sqrt(d)*log(-(x^2*e - 2
*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 4*(3*a*c^4*d^4 + 8*a*x^4*e^4 + (3*b*
c^4*d^4 + 8*b*x^4*e^4 - 4*(4*b*c^2*d*x^4 - 3*b*d*x^2)*e^3 + (8*b*c^4*d^2*x^
4 - 24*b*c^2*d^2*x^2 + 3*b*d^2)*e^2 + 6*(2*b*c^4*d^3*x^2 - b*c^2*d^3)*e)*ar
ctan(c*x) - (16*a*c^2*d*x^4 - b*c*d*x^3 - 12*a*d*x^2)*e^3 + (8*a*c^4*d^2*x^
```

4 - b*c^3*d^2*x^3 - 24*a*c^2*d^2*x^2 + b*c*d^2*x + 3*a*d^2)*e^2 + (12*a*c^4
*d^3*x^2 - b*c^3*d^3*x - 6*a*c^2*d^3)*e)*sqrt(x^2*e + d))/(c^4*d^7*x + d^3*
x^5*e^4 - 2*(c^2*d^4*x^5 - d^4*x^3)*e^3 + (c^4*d^5*x^5 - 4*c^2*d^5*x^3 + d^
5*x)*e^2 + 2*(c^4*d^6*x^3 - c^2*d^6*x)*e), 1/6*((3*b*c^4*d^4*x + 8*b*x^5*e^
4 - 4*(3*b*c^2*d*x^5 - 4*b*d*x^3)*e^3 + (3*b*c^4*d^2*x^5 - 24*b*c^2*d^2*x^3
 + 8*b*d^2*x)*e^2 + 6*(b*c^4*d^3*x^3 - 2*b*c^2*d^3*x)*e)*sqrt(-c^2*d + e)*a
rctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^
3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + 3*(b*c^5*d^4*x + b*c*x^5*e^4 -
2*(b*c^3*d*x^5 - b*c*d*x^3)*e^3 + (b*c^5*d^2*x^5 - 4*b*c^3*d^2*x^3 + b*c*d^
2*x)*e^2 + 2*(b*c^5*d^3*x^3 - b*c^3*d^3*x)*e)*sqrt(d)*log(-(x^2*e - 2*sqrt(
x^2*e + d)*sqrt(d) + 2*d)/x^2) - 2*(3*a*c^4*d^4 + 8*a*x^4*e^4 + (3*b*c^4*d^
4 + 8*b*x^4*e^4 - 4*(4*b*c^2*d*x^4 - 3*b*d*x^2)*e^3 + (8*b*c^4*d^2*x^4 - 24
*b*c^2*d^2*x^2 + 3*b*d^2)*e^2 + 6*(2*b*c^4*d^3*x^2 - b*c^2*d^3)*e)*arctan(c
*x) - (16*a*c^2*d*x^4 - b*c*d*x^3 - 12*a*d*x^2)*e^3 + (8*a*c^4*d^2*x^4 - b*
c^3*d^2*x^3 - 24*a*c^2*d^2*x^2 + b*c*d^2*x + 3*a*d^2)*e^2 + (12*a*c^4*d^3*x
^2 - b*c^3*d^3*x - 6*a*c^2*d^3)*e)*sqrt(x^2*e + d))/(c^4*d^7*x + d^3*x^5*e^
4 - 2*(c^2*d^4*x^5 - d^4*x^3)*e^3 + (c^4*d^5*x^5 - 4*c^2*d^5*x^3 + d^5*x)*e
^2 + 2*(c^4*d^6*x^3 - c^2*d^6*x)*e), 1/12*(12*(b*c^5*d^4*x + b*c*x^5*e^4 -
2*(b*c^3*d*x^5 - b*c*d*x^3)*e^3 + (b*c^5*d^2*x^5 - 4*b*c^3*d^2*x^3 + b*c*d^
2*x)*e^2 + 2*(b*c^5*d^3*x^3 - b*c^3*d^3*x)*e)*sqrt(-d)*arctan(sqrt(-d)/sqrt
(x^2*e + d)) - (3*b*c^4*d^4*x + 8*b*x^5*e^4 - 4*(3*b*c^2*d*x^5 - 4*b*d*x^3)
*e^3 + (3*b*c^4*d^2*x^5 - 24*b*c^2*d^2*x^3 + 8*b*d^2*x)*e^2 + 6*(b*c^4*d^3*
x^3 - 2*b*c^2*d^3*x)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 - 4*(2*c^3*d + (c^3*
x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2
 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(3*a*c^4*d^4 + 8
*a*x^4*e^4 + (3*b*c^4*d^4 + 8*b*x^4*e^4 - 4*(4*b*c^2*d*x^4 - 3*b*d*x^2)*e^3
 + (8*b*c^4*d^2*x^4 - 24*b*c^2*d^2*x^2 + 3*b*d^2)*e^2 + 6*(2*b*c^4*d^3*x^2
- b*c^2*d^3)*e)*arctan(c*x) - (16*a*c^2*d*x^4 - b*c*d*x^3 - 12*a*d*x^2)*e^3
 + (8*a*c^4*d^2*x^4 - b*c^3*d^2*x^3 - 24*a*c^2*d^2*x^2 + b*c*d^2*x + 3*a*d^
2)*e^2 + (12*a*c^4*d^3*x^2 - b*c^3*d^3*x - 6*a*c^2*d^3)*e)*sqrt(x^2*e + d))
/(c^4*d^7*x + d^3*x^5*e^4 - 2*(c^2*d^4*x^5 - d^4*x^3)*e^3 + (c^4*d^5*x^5 -
4*c^2*d^5*x^3 + d^5*x)*e^2 + 2*(c^4*d^6*x^3 - c^2*d^6*x)*e), 1/6*((3*b*c^4*
d^4*x + 8*b*x^5*e^4 - 4*(3*b*c^2*d*x^5 - 4*b*d*x^3)*e^3 + (3*b*c^4*d^2*x^5
- 24*b*c^2*d^2*x^3 + 8*b*d^2*x)*e^2 + 6*(b*c^4*d^3*x^3 - 2*b*c^2*d^3*x)*e)*
sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*s
qrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + 6*(b*c^5*d^4*
x + b*c*x^5*e^4 - 2*(b*c^3*d*x^5 - b*c*d*x^3)*e^3 + (b*c^5*d^2*x^5 - 4*b*c^
3*d^2*x^3 + b*c*d^2*x)*e^2 + 2*(b*c^5*d^3*x^3 - b*c^3*d^3*x)*e)*sqrt(-d)*ar
ctan(sqrt(-d)/sqrt(x^2*e + d)) - 2*(3*a*c^4*d^4 + 8*a*x^4*e^4 + (3*b*c^4*d^
4 + 8*b*x^4*e^4 - 4*(4*b*c^2*d*x^4 - 3*b*d*x^2)*e^3 + (8*b*c^4*d^2*x^4 - 24
*b*c^2*d^2*x^2 + 3*b*d^2)*e^2 + 6*(2*b*c^4*d^3*x^2 - b*c^2*d^3)*e)*arctan(c
*x) - (16*a*c^2*d*x^4 - b*c*d*x^3 - 12*a*d*x^2)*e^3 + (8*a*c^4*d^2*x^4 - b*
c^3*d^2*x^3 - 24*a*c^2*d^2*x^2 + b*c*d^2*x + 3*a*d^2)*e^2 + (12*a*c^4*d^3*x
^2 - b*c^3*d^3*x - 6*a*c^2*d^3)*e)*sqrt(x^2*e + d))/(c^4*d^7*x + d^3*x^5*e^
4 - 2*(c^2*d^4*x^5 - d^4*x^3)*e^3 + (c^4*d^5*x^5 - 4*c^2*d^5*x^3 + d^5*x)*e

```
^2 + 2*(c^4*d^6*x^3 - c^2*d^6*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)), x)
```

3.1224 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{5ae}{6d^2(d+ex^2)^{3/2}}-\frac{a}{2dx^2(d+ex^2)^{3/2}}-\frac{5ae}{2d^3\sqrt{d+ex^2}}+\frac{5ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}+b\text{Int}\left(\frac{\text{ArcTan}(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

```
[Out] -5/6*a*e/d^2/(e*x^2+d)^(3/2)-1/2*a/d/x^2/(e*x^2+d)^(3/2)+5/2*a*e*arctanh((e
*x^2+d)^(1/2)/d^(1/2))/d^(7/2)-5/2*a*e/d^3/(e*x^2+d)^(1/2)+b*Unintegrable(a
rctan(c*x)/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{ArcTan}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] (-5*a*e)/(6*d^2*(d + e*x^2)^(3/2)) - a/(2*d*x^2*(d + e*x^2)^(3/2)) - (5*a*e
)/(2*d^3*Sqrt[d + e*x^2]) + (5*a*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(
7/2)) + b*Defer[Int][ArcTan[c*x]/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = a\int \frac{1}{x^3\left(d+ex^2\right)^{5/2}}\,dx + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

$$= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{5/2}}\,dx,x,x^2\right) + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

$$= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx + \frac{(5a)\text{Subst}\left(\int \frac{1}{x^2(d+ex)^{3/2}}\,dx,x,x^2\right)}{6d}$$

$$= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx + \frac{(5a)\text{Subst}\left(\int \frac{}{x^2\sqrt{}}\right.}{2}$$

$$= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx -$$

$$= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + b\int \frac{\tan^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx -$$

$$= \frac{a}{3dx^2\left(d+ex^2\right)^{3/2}} + \frac{5a}{3d^2x^2\sqrt{d+ex^2}} - \frac{5a\sqrt{d+ex^2}}{2d^3x^2} + \frac{5ae\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}$$

Mathematica [A]
time = 13.58, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcTan}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(7/2) - 15*e/(sqrt(x^2*e + d
)*d^3) - 5*e/((x^2*e + d)^(3/2)*d^2) - 3/((x^2*e + d)^(3/2)*d*x^2)) + 2*b*i
ntegrate(1/2*arctan(c*x)/((x^7*e^2 + 2*d*x^5*e + d^2*x^3)*sqrt(x^2*e + d)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2
*x^5*e + d^3*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.1225 $\int \frac{a+b\mathbf{ArcTan}(cx)}{x^4(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{bce}{2d^3\sqrt{d+ex^2}}+\frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{bc}{6d^2x^2\sqrt{d+ex^2}}-\frac{a+}{3dx}$$

```
[Out] 1/3*(-a-b*arctan(c*x))/d/x^3/(e*x^2+d)^(3/2)+2*e*(a+b*arctan(c*x))/d^2/x/(e
*x^2+d)^(3/2)+8/3*e^2*x*(a+b*arctan(c*x))/d^3/(e*x^2+d)^(3/2)+1/2*b*c*e*arc
tanh((e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)+1/3*b*c*(c^2*d+6*e)*arctanh((e*x^2+d)
^(1/2)/d^(1/2))/d^(7/2)-1/3*b*(c^2*d-2*e)*(c^4*d^2+8*c^2*d*e-8*e^2)*arctanh
(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^4/(c^2*d-e)^(3/2)-1/2*b*c*e/d^3/(e*x^
2+d)^(1/2)+16/3*b*e^2/c/d^4/(e*x^2+d)^(1/2)-1/3*b*c*(c^2*d+6*e)/d^3/(e*x^2+
d)^(1/2)+1/3*b*(c^2*d-2*e)*(c^4*d^2+8*c^2*d*e-8*e^2)/c/d^4/(c^2*d-e)/(e*x^2
+d)^(1/2)-1/6*b*c/d^2/x^2/(e*x^2+d)^(1/2)+16/3*e^2*x*(a+b*arctan(c*x))/d^4/
(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {277, 198, 197, 5096, 12, 6857, 272, 44, 53, 65, 214, 267, 455}

$$\frac{16e^2x(a+b\text{ArcTan}(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\text{ArcTan}(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\text{ArcTan}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{ArcTan}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{bc(c^2d+6e)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{7/2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)\tanh^{-1}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^4\left(c^2d-e\right)^{3/2}}+\frac{bce\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}+\frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bce}{2d^3\sqrt{d+ex^2}}-\frac{bc}{6d^2x^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] -1/2*(b*c*e)/(d^3*Sqrt[d + e*x^2]) + (16*b*e^2)/(3*c*d^4*Sqrt[d + e*x^2]) -
 (b*c*(c^2*d + 6*e))/(3*d^3*Sqrt[d + e*x^2]) + (b*(c^2*d - 2*e)*(c^4*d^2 +
8*c^2*d*e - 8*e^2))/(3*c*d^4*(c^2*d - e)*Sqrt[d + e*x^2]) - (b*c)/(6*d^2*x^
2*Sqrt[d + e*x^2]) - (a + b*ArcTan[c*x])/(3*d*x^3*(d + e*x^2)^(3/2)) + (2*e
*(a + b*ArcTan[c*x]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*ArcTan[c*
x]))/(3*d^3*(d + e*x^2)^(3/2)) + (16*e^2*x*(a + b*ArcTan[c*x]))/(3*d^4*Sqrt
[d + e*x^2]) + (b*c*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*d^(7/2)) + (b*c*
(c^2*d + 6*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(3*d^(7/2)) - (b*(c^2*d - 2
*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d -
e]])/(3*d^4*(c^2*d - e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\tan^{-1}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}+\frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{16e^2x(a+b}{3d^4\sqrt{d}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\tan^{-1}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}+\frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{16e^2x(a+b}{3d^4\sqrt{d}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\tan^{-1}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}+\frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{16e^2x(a+b}{3d^4\sqrt{d}}$$

$$= -\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\tan^{-1}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}+\frac{8e^2x(a+b\tan^{-1}(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{16e^2x(a+b}{3d^4\sqrt{d}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{a+b\tan^{-1}(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\tan^{-1}(cx))}{d^2x\left(d+ex^2\right)^{3/2}}+\frac{8e^2x(a+b\tan^{-1}(cx)}{3d^3\left(d+ex^2\right)^{3/2}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc}{3d^2x^2\sqrt{d+}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc}{3d^2x^2\sqrt{d+}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc}{3d^2x^2\sqrt{d+}}$$

$$= \frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)\left(c^4d^2+8c^2de-8e^2\right)}{3cd^4\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc}{3d^2x^2\sqrt{d+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.32, size = 510, normalized size = 1.21

$$-\frac{\frac{2a(d^3-6d^2ex^2-24de^2x^4-16e^3x^6)}{x^3(d+ex^2)^{3/2}}+\frac{bcd(e(-d+ex^2)+c^2d(d+ex^2))}{(c^2d-e)x^2\sqrt{d+ex^2}}+\frac{2b(d^3-6d^2ex^2-24de^2x^4-16e^3x^6)\text{ArcTan}(cx)}{x^3(d+ex^2)^{3/2}}+bc\sqrt{d}\,(2c^2d+15e)\log(x)-bc\sqrt{d}\,(2c^2d+15e)\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)+\frac{b(c^6d^3+6c^4d^2e-24c^2de^2+16e^3)\log\left(\frac{12cd^4\sqrt{c^2d-e}\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^6d^3+6c^4d^2e-24c^2de^2+16e^3)(i+cx)}\right)}{(c^2d-e)^{3/2}}+\frac{b(c^6d^3+6c^4d^2e-24c^2de^2+16e^3)\log\left(\frac{12cd^4\sqrt{c^2d-e}\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^6d^3+6c^4d^2e-24c^2de^2+16e^3)(-i+cx)}\right)}{(c^2d-e)^{3/2}}}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

```
[Out] -1/6*((2*a*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6))/(x^3*(d + e*x^2
)^(3/2)) + (b*c*d*(e*(-d + e*x^2) + c^2*d*(d + e*x^2)))/((c^2*d - e)*x^2*Sq
rt[d + e*x^2]) + (2*b*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6)*ArcTa
n[c*x])/(x^3*(d + e*x^2)^(3/2)) + b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[x] - b*c
```

```
*Sqrt[d]*(2*c^2*d + 15*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^6*d^3 +
6*c^4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d - I
*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2
*d*e^2 + 16*e^3)*(I + c*x))])/(c^2*d - e)^(3/2) + (b*(c^6*d^3 + 6*c^4*d^2*e
 - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt
[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2*d*e^2 + 16
*e^3)*(-I + c*x))])/(c^2*d - e)^(3/2))/d^4
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\left(cx\right)}{x^{4}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(16*x*e^2/(sqrt(x^2*e + d)*d^4) + 8*x*e^2/((x^2*e + d)^(3/2)*d^3) + 6
*e/((x^2*e + d)^(3/2)*d^2*x) - 1/((x^2*e + d)^(3/2)*d*x^3)) + 2*b*integrate
(1/2*arctan(c*x)/((x^8*e^2 + 2*d*x^6*e + d^2*x^4)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(384) = 768.
time = 5.79, size = 3608, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^6*d^5*x^3 + 16*b*x^7*e^5 - 8*(3*b*c^2*d*x^7 - 4*b*d*x^5)*e^4 +
 2*(3*b*c^4*d^2*x^7 - 24*b*c^2*d^2*x^5 + 8*b*d^2*x^3)*e^3 + (b*c^6*d^3*x^7
+ 12*b*c^4*d^3*x^5 - 24*b*c^2*d^3*x^3)*e^2 + 2*(b*c^6*d^4*x^5 + 3*b*c^4*d^4
*x^3)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqr
t(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2
 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) - (2*b*c^7*d^5*x^3 + 15*b*c*x^7*e^5
```

- 2*(14*b*c^3*d*x^7 - 15*b*c*d*x^5)*e^4 + (11*b*c^5*d^2*x^7 - 56*b*c^3*d^2*x^5 + 15*b*c*d^2*x^3)*e^3 + 2*(b*c^7*d^3*x^7 + 11*b*c^5*d^3*x^5 - 14*b*c^3*d^3*x^3)*e^2 + (4*b*c^7*d^4*x^5 + 11*b*c^5*d^4*x^3)*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c^5*d^5*x + 2*a*c^4*d^5 - 32*a*x^6*e^5 + 2*(b*c^4*d^5 - 16*b*x^6*e^5 + 8*(4*b*c^2*d*x^6 - 3*b*d*x^4)*e^4 - 2*(8*b*c^4*d^2*x^6 - 24*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*e^3 - (24*b*c^4*d^3*x^4 - 12*b*c^2*d^3*x^2 - b*d^3)*e^2 - 2*(3*b*c^4*d^4*x^2 + b*c^2*d^4)*e)*arctan(c*x) + (64*a*c^2*d*x^6 - b*c*d*x^5 - 48*a*d*x^4)*e^4 - 4*(8*a*c^4*d^2*x^6 - 24*a*c^2*d^2*x^4 + 3*a*d^2*x^2)*e^3 + (b*c^5*d^3*x^5 - 48*a*c^4*d^3*x^4 - 2*b*c^3*d^3*x^3 + 24*a*c^2*d^3*x^2 + b*c*d^3*x + 2*a*d^3)*e^2 + 2*(b*c^5*d^4*x^3 - 6*a*c^4*d^4*x^2 - b*c^3*d^4*x - 2*a*c^2*d^4)*e)*sqrt(x^2*e + d))/(c^4*d^8*x^3 + d^4*x^7*e^4 - 2*(c^2*d^5*x^7 - d^5*x^5)*e^3 + (c^4*d^6*x^7 - 4*c^2*d^6*x^5 + d^6*x^3)*e^2 + 2*(c^4*d^7*x^5 - c^2*d^7*x^3)*e), -1/12*(2*(b*c^6*d^5*x^3 + 16*b*x^7*e^5 - 8*(3*b*c^2*d*x^7 - 4*b*d*x^5)*e^4 + 2*(3*b*c^4*d^2*x^7 - 24*b*c^2*d^2*x^5 + 8*b*d^2*x^3)*e^3 + (b*c^6*d^3*x^7 + 12*b*c^4*d^3*x^5 - 24*b*c^2*d^3*x^3)*e^2 + 2*(b*c^6*d^4*x^5 + 3*b*c^4*d^4*x^3)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) - (2*b*c^7*d^5*x^3 + 15*b*c*x^7*e^5 - 2*(14*b*c^3*d*x^7 - 15*b*c*d*x^5)*e^4 + (11*b*c^5*d^2*x^7 - 56*b*c^3*d^2*x^5 + 15*b*c*d^2*x^3)*e^3 + 2*(b*c^7*d^3*x^7 + 11*b*c^5*d^3*x^5 - 14*b*c^3*d^3*x^3)*e^2 + (4*b*c^7*d^4*x^5 + 11*b*c^5*d^4*x^3)*e)*sqrt(d)*log(-(x^2*e + 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c^5*d^5*x + 2*a*c^4*d^5 - 32*a*x^6*e^5 + 2*(b*c^4*d^5 - 16*b*x^6*e^5 + 8*(4*b*c^2*d*x^6 - 3*b*d*x^4)*e^4 - 2*(8*b*c^4*d^2*x^6 - 24*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*e^3 - (24*b*c^4*d^3*x^4 - 12*b*c^2*d^3*x^2 - b*d^3)*e^2 - 2*(3*b*c^4*d^4*x^2 + b*c^2*d^4)*e)*arctan(c*x) + (64*a*c^2*d*x^6 - b*c*d*x^5 - 48*a*d*x^4)*e^4 - 4*(8*a*c^4*d^2*x^6 - 24*a*c^2*d^2*x^4 + 3*a*d^2*x^2)*e^3 + (b*c^5*d^3*x^5 - 48*a*c^4*d^3*x^4 - 2*b*c^3*d^3*x^3 + 24*a*c^2*d^3*x^2 + b*c*d^3*x + 2*a*d^3)*e^2 + 2*(b*c^5*d^4*x^3 - 6*a*c^4*d^4*x^2 - b*c^3*d^4*x - 2*a*c^2*d^4)*e)*sqrt(x^2*e + d))/(c^4*d^8*x^3 + d^4*x^7*e^4 - 2*(c^2*d^5*x^7 - d^5*x^5)*e^3 + (c^4*d^6*x^7 - 4*c^2*d^6*x^5 + d^6*x^3)*e^2 + 2*(c^4*d^7*x^5 - c^2*d^7*x^3)*e), -1/12*(2*(2*b*c^7*d^5*x^3 + 15*b*c*x^7*e^5 - 2*(14*b*c^3*d*x^7 - 15*b*c*d*x^5)*e^4 + (11*b*c^5*d^2*x^7 - 56*b*c^3*d^2*x^5 + 15*b*c*d^2*x^3)*e^3 + 2*(b*c^7*d^3*x^7 + 11*b*c^5*d^3*x^5 - 14*b*c^3*d^3*x^3)*e^2 + (4*b*c^7*d^4*x^5 + 11*b*c^5*d^4*x^3)*e)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + (b*c^6*d^5*x^3 + 16*b*x^7*e^5 - 8*(3*b*c^2*d*x^7 - 4*b*d*x^5)*e^4 + 2*(3*b*c^4*d^2*x^7 - 24*b*c^2*d^2*x^5 + 8*b*d^2*x^3)*e^3 + (b*c^6*d^3*x^7 + 12*b*c^4*d^3*x^5 - 24*b*c^2*d^3*x^3)*e^2 + 2*(b*c^6*d^4*x^5 + 3*b*c^4*d^4*x^3)*e)*sqrt(c^2*d - e)*log((8*c^4*d^2 + 4*(2*c^3*d + (c^3*x^2 - c)*e)*sqrt(c^2*d - e)*sqrt(x^2*e + d) + (c^4*x^4 - 6*c^2*x^2 + 1)*e^2 + 8*(c^4*d*x^2 - c^2*d)*e)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c^5*d^5*x + 2*a*c^4*d^5 - 32*a*x^6*e^5 + 2*(b*c^4*d^5 - 16*b*x^6*e^5 + 8*(4*b*c^2*d*x^6 - 3*b*d*x^4)*e^4 - 2*(8*b*c^4*d^2*x^6 - 24*b*c^2*d^2*x^4 + 3*b*d^2*x^2)*e^3 - (24*b*c^4*d^3*x^4 - 12*b*c^2*d^3*x^2 - b*d^3)*e^2 - 2*(3*b*c^4*d^4*x^2 + b*c^2*d^4)*e)*arctan(c*x) + (64*a*c^2*d*x^6 - b*c*d*x^5 - 48*a*d*x^4)*e^4 - 4*(8*

```
a*c^4*d^2*x^6 - 24*a*c^2*d^2*x^4 + 3*a*d^2*x^2)*e^3 + (b*c^5*d^3*x^5 - 48*a
*c^4*d^3*x^4 - 2*b*c^3*d^3*x^3 + 24*a*c^2*d^3*x^2 + b*c*d^3*x + 2*a*d^3)*e^
2 + 2*(b*c^5*d^4*x^3 - 6*a*c^4*d^4*x^2 - b*c^3*d^4*x - 2*a*c^2*d^4)*e)*sqrt
(x^2*e + d))/(c^4*d^8*x^3 + d^4*x^7*e^4 - 2*(c^2*d^5*x^7 - d^5*x^5)*e^3 + (
c^4*d^6*x^7 - 4*c^2*d^6*x^5 + d^6*x^3)*e^2 + 2*(c^4*d^7*x^5 - c^2*d^7*x^3)*
e), -1/6*((b*c^6*d^5*x^3 + 16*b*x^7*e^5 - 8*(3*b*c^2*d*x^7 - 4*b*d*x^5)*e^4
 + 2*(3*b*c^4*d^2*x^7 - 24*b*c^2*d^2*x^5 + 8*b*d^2*x^3)*e^3 + (b*c^6*d^3*x^
7 + 12*b*c^4*d^3*x^5 - 24*b*c^2*d^3*x^3)*e^2 + 2*(b*c^6*d^4*x^5 + 3*b*c^4*d
^4*x^3)*e)*sqrt(-c^2*d + e)*arctan(-1/2*(2*c^2*d + (c^2*x^2 - 1)*e)*sqrt(-c
^2*d + e)*sqrt(x^2*e + d)/(c^3*d^2 - c*x^2*e^2 + (c^3*d*x^2 - c*d)*e)) + (2
*b*c^7*d^5*x^3 + 15*b*c*x^7*e^5 - 2*(14*b*c^3*d*x^7 - 15*b*c*d*x^5)*e^4 + (
11*b*c^5*d^2*x^7 - 56*b*c^3*d^2*x^5 + 15*b*c*d^2*x^3)*e^3 + 2*(b*c^7*d^3*x^
7 + 11*b*c^5*d^3*x^5 - 14*b*c^3*d^3*x^3)*e^2 + (4*b*c^7*d^4*x^5 + 11*b*c^5*
d^4*x^3)*e)*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)) + (b*c^5*d^5*x + 2*a*
c^4*d^5 - 32*a*x^6*e^5 + 2*(b*c^4*d^5 - 16*b*x^6*e^5 + 8*(4*b*c^2*d*x^6 - 3
*b*d*x^4)*e^4 - 2*(8*b*c^4*d^2*x^6 - 24*b*c^2*d...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)), x)
```

3.1226 $\int \frac{\text{ArcTan}(ax)}{(c+dx^2)^{7/2}} \, dx$

Optimal. Leaf size=208

$$-\frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}}-\frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\,\sqrt{c+dx^2}}+\frac{x\text{ArcTan}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\text{ArcTan}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\text{ArcTan}(ax}{15c^3\sqrt{c+dx^2}}$$

```
[Out] -1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)+1/5*x*arctan(a*x)/c/(d*x^2+c)^(5/2)+4/1
5*x*arctan(a*x)/c^2/(d*x^2+c)^(3/2)+1/15*(15*a^4*c^2-20*a^2*c*d+8*d^2)*arct
anh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(5/2)-1/15*a*(7*a^2*c-
4*d)/c^2/(a^2*c-d)^2/(d*x^2+c)^(1/2)+8/15*x*arctan(a*x)/c^3/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 5032, 6820, 12, 6847, 911, 1275, 214}

$$-\frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\,\sqrt{c+dx^2}}-\frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}}+\frac{(15a^4c^2-20a^2cd+8d^2)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\,(a^2c-d)^{5/2}}+\frac{8x\text{ArcTan}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\text{ArcTan}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{x\text{ArcTan}(ax)}{5c\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/15*a/(c*(a^2*c - d)*(c + d*x^2)^(3/2)) - (a*(7*a^2*c - 4*d))/(15*c^2*(a^
2*c - d)^2*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*
x*ArcTan[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTan[a*x])/(15*c^3*Sqrt[
c + d*x^2]) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c - d]])/(15*c^3*(a^2*c - d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

0] && NeQ[p, -1]

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx &= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}}+\frac{4x}{15c^2(c+dx^2)^{3/2}}}{1+a^2x^2} \\
&= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-a\int \frac{x(15c^2+20cdx^2+8d^2x^4}{15c^3\,(1+a^2x^2)\,(c+dx^2)} \\
&= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-\frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1+a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3} \\
&= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-\frac{a\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1+a^2x)(c+dx)^{5/2}}\,dx,\right.}{30c^3} \\
&= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-\frac{a\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{-a^2c+d}{d}+\frac{a^2x^2}{d}\right)}\,dx,\right.}{15c^3d} \\
&= \frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{8x\tan^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-\frac{a\text{Subst}\left(\int \left(\frac{3c^2d}{(-a^2c+d)x^4}-\frac{c(7a^2c}{(-a^2c}\right.\right.}{} \\
&= -\frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}}-\frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}}+\frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan}{15c^2\,(c+} \\
&= -\frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}}-\frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}}+\frac{x\tan^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}+\frac{4x\tan}{15c^2\,(c+}
\end{aligned}
$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 0.66, size = 345, normalized size = 1.66

$$
\frac{-\frac{2ac(-d(5c+4dx^2)+a^2c(8c+7dx^2))}{(-a^2c+d)^2(c+dx^2)^{3/2}}+\frac{2x(15c^2+20cdx^2+8d^2x^4)\text{ArcTan}(ax)}{(c+dx^2)^{5/2}}+\frac{(15a^4c^2-20a^2cd+8d^2)\log\left(-\frac{60ac^3(a^2c-d)^{3/2}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(15a^4c^2-20a^2cd+8d^2)(i+ax)}\right)}{(a^2c-d)^{5/2}}+\frac{(15a^4c^2-20a^2cd+8d^2)\log\left(-\frac{60ac^3(a^2c-d)^{3/2}\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(15a^4c^2-20a^2cd+8d^2)(-i+ax)}\right)}{(a^2c-d)^{5/2}}}{30c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] ((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) + d)^2*
(c + d*x^2)^(3/2)) + (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTan[a*x])/(c
 + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c
- d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 -
20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20*a^2*
c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2*c - d
]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])/(a^2*c
 - d)^(5/2))/(30*c^3)
```

Maple [F]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\arctan(ax)}{(d\,x^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(180) = 360.
time = 2.74, size = 1280, normalized size = 6.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*x^2
 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x
^2 + 1)) - 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*
a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3
)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 + 20*(a^6*
c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 - 3*a^4*c^
4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(a^6*c^9 -
3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5*d^4 + 3*a^
2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 -
```

```
 c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 - c^5*d^4)*x^2
), 1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^
5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*c + d)*arctan(-1
/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*
c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 +
 (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*
c^3*d^2 + 3*a*c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4
- d^5)*x^5 + 20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 1
5*(a^6*c^5 - 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*
x^2 + c))/(a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 -
 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*
d^3 + 3*a^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c
^6*d^3 - c^5*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(d*x**2+c)**(7/2),x)
[Out] Integral(atan(a*x)/(c + d*x**2)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{{(d\,x^2+c)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + d*x^2)^(7/2),x)
[Out] int(atan(a*x)/(c + d*x^2)^(7/2), x)
```

3.1227 $\int \frac{\text{ArcTan}(ax)}{(c+dx^2)^{9/2}} \, dx$

Optimal. Leaf size=293

$$-\frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}}-\frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}}-\frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}}+\frac{x\text{ArcTan}(ax)}{7c\left(c+dx^2\right)^{7/2}}+\frac{6x}{35c}$$

```
[Out] -1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)-1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2/
(d*x^2+c)^(3/2)+1/7*x*arctan(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctan(a*x)/c^2/
(d*x^2+c)^(5/2)+8/35*x*arctan(a*x)/c^3/(d*x^2+c)^(3/2)+1/35*(35*a^6*c^3-70*
a^4*c^2*d+56*a^2*c*d^2-16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c
^4/(a^2*c-d)^(7/2)-1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^3/(d*
x^2+c)^(1/2)+16/35*x*arctan(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 5032, 6820, 12, 6847, 1633, 65, 214}

$$-\frac{a\left(11a^2c-6d\right)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}}-\frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}}-\frac{a\left(19a^4c^2-22a^2cd+8d^2\right)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}}+\frac{\left(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4\left(a^2c-d\right)^{7/2}}+\frac{16x\text{ArcTan}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\text{ArcTan}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\text{ArcTan}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\text{ArcTan}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] -1/35*a/(c*(a^2*c - d)*(c + d*x^2)^(5/2)) - (a*(11*a^2*c - 6*d))/(105*c^2*(
a^2*c - d)^2*(c + d*x^2)^(3/2)) - (a*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/(35
*c^3*(a^2*c - d)^3*Sqrt[c + d*x^2]) + (x*ArcTan[a*x])/(7*c*(c + d*x^2)^(7/2
)) + (6*x*ArcTan[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTan[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcTan[a*x])/(35*c^4*Sqrt[c + d*x^2]) + ((35*
a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2])
/Sqrt[a^2*c - d]])/(35*c^4*(a^2*c - d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\tan^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x}{7c(c+dx}$$

$$= \frac{x\tan^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x(35c^3}{3}$$

$$= \frac{x\tan^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x(35c^3+7}{(1}}{}$$

$$= \frac{x\tan^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Big(\int}{}$$

$$= \frac{x\tan^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\tan^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\tan^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\tan^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Big(\int}{}$$

$$= -\frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} - \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} - \frac{a(19a^4c^2-22a^2cd+8d^2}{35c^3\,(a^2c-d)^3\sqrt{c+dx^2}}$$

$$= -\frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} - \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} - \frac{a(19a^4c^2-22a^2cd+8d^2}{35c^3\,(a^2c-d)^3\sqrt{c+dx^2}}$$

$$= -\frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} - \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} - \frac{a(19a^4c^2-22a^2cd+8d^2}{35c^3\,(a^2c-d)^3\sqrt{c+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.00, size = 450, normalized size = 1.54

$$\frac{-\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}} + \frac{6x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\text{ArcTan}(ax)}{(c+dx^2)^{7/2}} + \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\log\left(-\frac{140ac^4(a^2c-d)^{5/2}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(i+ax)}\right)}{(a^2c-d)^{7/2}} + \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\log\left(-\frac{140ac^4(a^2c-d)^{5/2}\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(-i+ax)}\right)}{(a^2c-d)^{7/2}}}{210c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] ((-2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x^
2) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(c
+ d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*
ArcTan[a*x])/(c + d*x^2)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d
^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2*c -
d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(
I + a*x))])/(a^2*c - d)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^
```

```
2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a^2*c - d
]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(-
I + a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)}{\left(d\,x^{2}+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(257) = 514$.
time = 3.00, size = 1986, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*x^2 +
2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2
+ 1)) - 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*
(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a
```

```
^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a
^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 - 3*(16*(a
^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a
^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 7
0*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3
 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*
arctan(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4
*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a
^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5
- 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^
8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a
^4*c^9*d^3 - 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35*a^6*c^7 - 70*a^4*c
^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 70*a^4*c^2*d^5 + 56*a^2*c*d^6 - 1
6*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d
^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 -
16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^
3*d^4)*x^2)*sqrt(-a^2*c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^
2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(71
*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*(19*a^7*c^4*d
^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a^7*c^5*d^2 -
397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a^7*c^6*d - 43
4*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 - 3*(16*(a^8*c^4*d^3 -
4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a^8*c^5*d^2 -
4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^8*c^6*d
- 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^8*c^
7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*arctan(a*x))*
sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4*a^2*c^9*d^3
+ c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a^2*c^5*d^7 +
c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*d
^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^2
*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 -
4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(atan(a*x)/(c + d*x**2)**(9/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a\,x)}{{\left(d\,x^2+c\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(atan(a*x)/(c + d*x^2)^(9/2), x)
```

3.1228 $\int x^m(d+ex^2)^3(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=378

$$-\frac{be(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12m+m^2))\,x^{2+m}}{c^5(2+m)(3+m)(5+m)(7+m)}+\frac{be^2(e(5+m)-3c^2d(7+}{c^3(4+m)(5+m)(7}$$

```
[Out] -b*e*(e^2*(m^2+8*m+15)-3*c^2*d*e*(m^2+10*m+21)+3*c^4*d^2*(m^2+12*m+35))*x^(
2+m)/c^5/(2+m)/(7+m)/(m^2+8*m+15)+b*e^2*(e*(5+m)-3*c^2*d*(7+m))*x^(4+m)/c^3
/(4+m)/(5+m)/(7+m)-b*e^3*x^(6+m)/c/(6+m)/(7+m)+d^3*x^(1+m)*(a+b*arctan(c*x)
)/(1+m)+3*d^2*e*x^(3+m)*(a+b*arctan(c*x))/(3+m)+3*d*e^2*x^(5+m)*(a+b*arctan
(c*x))/(5+m)+e^3*x^(7+m)*(a+b*arctan(c*x))/(7+m)+b*(e^3*(m^3+9*m^2+23*m+15)
-3*c^2*d*e^2*(m^3+11*m^2+31*m+21)+3*c^4*d^2*e*(m^3+13*m^2+47*m+35)-c^6*d^3*
(m^3+15*m^2+71*m+105))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^2)/c
^5/(m^2+12*m+35)/(m^3+6*m^2+11*m+6)
```

Rubi [A]
time = 1.34, antiderivative size = 374, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {276, 5096, 1816, 371}

$$\frac{d^3x^{m+1}(a+b\text{ArcTan}(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\text{ArcTan}(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\text{ArcTan}(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\text{ArcTan}(cx))}{m+7}-\frac{be^2x^{m+4}\left(\frac{3c^2d}{m+5}-\frac{e}{m+7}\right)}{c^3(m+4)}-\frac{bex^{m+2}(3c^4d^2(m^2+12m+35)-3c^2de(m^2+10m+21)+e^2(m^2+8m+15))}{c^5(m+2)(m+3)(m+5)(m+7)}+\frac{bx^{m+2}(c^6(-d^3)\,(m^3+15m^2+71m+105)+3c^4d^2e(m^3+13m^2+47m+35)-3c^2de^2(m^3+11m^2+31m+21)+e^3(m^3+9m^2+23m+15))\;{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{c^5(m+1)(m+2)(m+3)(m+5)(m+7)}-\frac{be^3x^{m+6}}{c(m+6)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2) - 3*c^2*d*e*(21 + 10*m + m^2) + 3*c^4*d^2*(35
+ 12*m + m^2))*x^(2 + m))/(c^5*(2 + m)*(3 + m)*(5 + m)*(7 + m))) - (b*e^2*(
(3*c^2*d)/(5 + m) - e/(7 + m))*x^(4 + m))/(c^3*(4 + m)) - (b*e^3*x^(6 + m))
/(c*(6 + m)*(7 + m)) + (d^3*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2
*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (3*d*e^2*x^(5 + m)*(a + b*ArcTa
n[c*x]))/(5 + m) + (e^3*x^(7 + m)*(a + b*ArcTan[c*x]))/(7 + m) + (b*(e^3*(1
5 + 23*m + 9*m^2 + m^3) - 3*c^2*d*e^2*(21 + 31*m + 11*m^2 + m^3) + 3*c^4*d^
2*e*(35 + 47*m + 13*m^2 + m^3) - c^6*d^3*(105 + 71*m + 15*m^2 + m^3))*x^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(c^5*(1 + m)*(
2 + m)*(3 + m)*(5 + m)*(7 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^m\left(d+ex^2\right)^3\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{d^3x^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{3d^2ex^{3+m}(a+b\tan^{-1}(cx))}{3+m}+\frac{3de^2x^{5+}}{}$$

$$=\frac{d^3x^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{3d^2ex^{3+m}(a+b\tan^{-1}(cx))}{3+m}+\frac{3de^2x^{5+}}{}$$

$$=-\frac{be(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12}{c^5(2+m)(3+m)(5+m)(7+m)}$$

$$=-\frac{be(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12}{c^5(2+m)(3+m)(5+m)(7+m)}$$

Mathematica [F]
time = 5.41, size = 0, normalized size = 0.00

$$\int x^m\left(d+ex^2\right)^3\left(a+b\text{ArcTan}(cx)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

[Out] Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]), x]

Maple [F]
time = 1.81, size = 0, normalized size = 0.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{3} \left(a +b \arctan \left(c x \right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)

[Out] int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")

[Out] a*d^3*x^(m + 1)/(m + 1) + 3*a*d^2*x^(m + 3)*e/(m + 3) + 3*a*d*x^(m + 5)*e^2 /(m + 5) + a*x^(m + 7)*e^3/(m + 7) + (((b*m^3*e^3 + 9*b*m^2*e^3 + 23*b*m*e^ 3 + 15*b*e^3)*x^7 + 3*(b*d*m^3*e^2 + 11*b*d*m^2*e^2 + 31*b*d*m*e^2 + 21*b*d *e^2)*x^5 + 3*(b*d^2*m^3*e + 13*b*d^2*m^2*e + 47*b*d^2*m*e + 35*b*d^2*e)*x^ 3 + (b*d^3*m^3 + 15*b*d^3*m^2 + 71*b*d^3*m + 105*b*d^3)*x)*x^m*arctan(c*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(((b*c*m^3*e^3 + 9*b*c*m^2 *e^3 + 23*b*c*m*e^3 + 15*b*c*e^3)*x^7 + 3*(b*c*d*m^3*e^2 + 11*b*c*d*m^2*e^2 + 31*b*c*d*m*e^2 + 21*b*c*d*e^2)*x^5 + 3*(b*c*d^2*m^3*e + 13*b*c*d^2*m^2*e + 47*b*c*d^2*m*e + 35*b*c*d^2*e)*x^3 + (b*c*d^3*m^3 + 15*b*c*d^3*m^2 + 71* b*c*d^3*m + 105*b*c*d^3)*x)*x^m/(m^4 + 16*m^3 + (c^2*m^4 + 16*c^2*m^3 + 86* c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m ^3 + 86*m^2 + 176*m + 105)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 + 3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arctan(c*x))*x^m, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(a + b \operatorname{atan}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3, x)
```

3.1229 $\int x^m(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=230

$$\frac{be(e(3+m)-2c^2d(5+m))x^{2+m}}{c^3(2+m)(3+m)(5+m)}-\frac{be^2x^{4+m}}{c(4+m)(5+m)}+\frac{d^2x^{1+m}(a+b\text{ArcTan}(cx))}{1+m}+\frac{2dex^{3+m}(a+b\text{ArcTan}(c$$

```
[Out] b*e*(e*(3+m)-2*c^2*d*(5+m))*x^(2+m)/c^3/(5+m)/(m^2+5*m+6)-b*e^2*x^(4+m)/c/(
4+m)/(5+m)+d^2*x^(1+m)*(a+b*arctan(c*x))/(1+m)+2*d*e*x^(3+m)*(a+b*arctan(c*
x))/(3+m)+e^2*x^(5+m)*(a+b*arctan(c*x))/(5+m)-b*(e^2*(m^2+4*m+3)-2*c^2*d*e*
(m^2+6*m+5)+c^4*d^2*(m^2+8*m+15))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],
-c^2*x^2)/c^3/(m^2+3*m+2)/(m^2+8*m+15)
```

Rubi [A]
time = 0.22, antiderivative size = 226, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {276, 5096, 1275, 371}

$$\frac{d^2x^{m+1}(a+b\text{ArcTan}(cx))}{m+1}+\frac{2dex^{m+3}(a+b\text{ArcTan}(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\text{ArcTan}(cx))}{m+5}-\frac{bex^{m+2}\left(\frac{2c^2d}{m+3}-\frac{e}{m+5}\right)}{c^3(m+2)}-\frac{bx^{m+2}(c^4d^2(m^2+8m+15)-2c^2de(m^2+6m+5)+e^2(m^2+4m+3))\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{c^3(m+1)(m+2)(m+3)(m+5)}-\frac{be^2x^{m+4}}{c(m+4)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*e*((2*c^2*d)/(3 + m) - e/(5 + m))*x^(2 + m))/(c^3*(2 + m))) - (b*e^2*x
^(4 + m))/(c*(4 + m)*(5 + m)) + (d^2*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m)
 + (2*d*e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) + (e^2*x^(5 + m)*(a + b*Ar
cTan[c*x]))/(5 + m) - (b*(e^2*(3 + 4*m + m^2) - 2*c^2*d*e*(5 + 6*m + m^2) +
 c^4*d^2*(15 + 8*m + m^2))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m
)/2, -(c^2*x^2)])/(c^3*(1 + m)*(2 + m)*(3 + m)*(5 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^m\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{d^2x^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{2dex^{3+m}(a+b\tan^{-1}(cx))}{3+m}+\frac{e^2x^{5+m}(a$$

$$=\frac{d^2x^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{2dex^{3+m}(a+b\tan^{-1}(cx))}{3+m}+\frac{e^2x^{5+m}(a$$

$$=-\frac{be\left(\frac{2c^2d}{3+m}-\frac{e}{5+m}\right)x^{2+m}}{c^3(2+m)}-\frac{be^2x^{4+m}}{c(4+m)(5+m)}+\frac{d^2x^{1+m}(a+b\tan^{-}}{1+m}$$

$$=-\frac{be\left(\frac{2c^2d}{3+m}-\frac{e}{5+m}\right)x^{2+m}}{c^3(2+m)}-\frac{be^2x^{4+m}}{c(4+m)(5+m)}+\frac{d^2x^{1+m}(a+b\tan^{-}}{1+m}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 18.93, size = 8408, normalized size = 36.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int x^m\left(e\,x^2+d\right)^2\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] a*d^2*x^(m + 1)/(m + 1) + 2*a*d*x^(m + 3)*e/(m + 3) + a*x^(m + 5)*e^2/(m +
5) + (((b*m^2*e^2 + 4*b*m*e^2 + 3*b*e^2)*x^5 + 2*(b*d*m^2*e + 6*b*d*m*e + 5
*b*d*e)*x^3 + (b*d^2*m^2 + 8*b*d^2*m + 15*b*d^2)*x)*x^m*arctan(c*x) - (m^3
+ 9*m^2 + 23*m + 15)*integrate(((b*c*m^2*e^2 + 4*b*c*m*e^2 + 3*b*c*e^2)*x^5
 + 2*(b*c*d*m^2*e + 6*b*c*d*m*e + 5*b*c*d*e)*x^3 + (b*c*d^2*m^2 + 8*b*c*d^2
*m + 15*b*c*d^2)*x)*x^m/(m^3 + (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^
2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2, x)
```

3.1230 $\int x^m(d+ex^2)(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=122

$$-\frac{bex^{2+m}}{c(6+5m+m^2)}+\frac{dx^{1+m}(a+b\text{ArcTan}(cx))}{1+m}+\frac{ex^{3+m}(a+b\text{ArcTan}(cx))}{3+m}-\frac{b\left(\frac{c^2d}{1+m}-\frac{e}{3+m}\right)x^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\right.}{c(2+m)}$$

```
[Out] -b*e*x^(2+m)/c/(m^2+5*m+6)+d*x^(1+m)*(a+b*arctan(c*x))/(1+m)+e*x^(3+m)*(a+b
*arctan(c*x))/(3+m)-b*(c^2*d/(1+m)-e/(3+m))*x^(2+m)*hypergeom([1, 1+1/2*m],
[2+1/2*m],-c^2*x^2)/c/(2+m)
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {14, 5096, 470, 371}

$$\frac{dx^{m+1}(a+b\text{ArcTan}(cx))}{m+1}+\frac{ex^{m+3}(a+b\text{ArcTan}(cx))}{m+3}-\frac{bx^{m+2}\left(\frac{c^2d}{m+1}-\frac{e}{m+3}\right)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{c(m+2)}-\frac{bex^{m+2}}{c(m^2+5m+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*e*x^(2 + m))/(c*(6 + 5*m + m^2))) + (d*x^(1 + m)*(a + b*ArcTan[c*x]))/
(1 + m) + (e*x^(3 + m)*(a + b*ArcTan[c*x]))/(3 + m) - (b*((c^2*d)/(1 + m) -
 e/(3 + m))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)
])/(c*(2 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] && !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^m\left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{dx^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{ex^{3+m}(a+b\tan^{-1}(cx))}{3+m}-(bc)\int\frac{x^{1+m}(}{1}$$
$$=-\frac{bex^{2+m}}{c\left(6+5m+m^2\right)}+\frac{dx^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{ex^{3+m}(a+b\tan^{-1}}{3+m}$$
$$=-\frac{bex^{2+m}}{c\left(6+5m+m^2\right)}+\frac{dx^{1+m}(a+b\tan^{-1}(cx))}{1+m}+\frac{ex^{3+m}(a+b\tan^{-1}}{3+m}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.27, size = 733, normalized size = 6.01

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (x*((1 + m)*x^m*(d + e*x^2)*((2 + m)*(a + b*ArcTan[c*x]) - b*c*x*Hypergeome
tric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)]) - 2*e*(1 + m)*((x^(1 + m)*(-b + a
*c*x + a*c*m*x + b*c*(1 + m)*x*ArcTan[c*x] + b*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -(c^2*x^2)]))/(c*(1 + m)) - b*c*x^(3 + m)*Gamma[2 + m/2]*Hyp
ergeometricPFQRegularized[{1, 1 + m/2}, {2 + m/2}, -(c^2*x^2)] + (b*c*x^(3
+ m)*Gamma[2 + m/2]*Gamma[(3 + m)/2]*HypergeometricPFQRegularized[{1, (3 +
m)/2}, {(5 + m)/2}, -(c^2*x^2)])/Gamma[1 + m/2]) + e*x^(1 + m)*((-2*b*Gamma
[1 + m/2]*Gamma[(3 + m)/2]*HypergeometricPFQRegularized[{1, 1 + m/2}, {2 +
m/2}, -(c^2*x^2)])/(c*Gamma[(1 + m)/2]) - b*c*(1 + m)*x^2*Gamma[2 + m/2]*Hy
pergeometricPFQRegularized[{1, 1 + m/2}, {3 + m/2}, -(c^2*x^2)] + 2*(-(b/(c
```

```
*(2 + m))) - b/(c*(6 + 5*m + m^2)) - (b*m)/(c*(6 + 5*m + m^2)) + (a*x)/(3 +
 m) + (a*m*x)/(3 + m) + (b*x*ArcTan[c*x])/(3 + m) + (b*m*x*ArcTan[c*x])/(3
+ m) + (b*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(c*(6 + 5*m +
 m^2)) + (b*m*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(c*(6 + 5
*m + m^2)) - (b*c*(1 + m)*x^2*Gamma[2 + m/2]^2*HypergeometricPFQRegularized
[{1, 2 + m/2}, {3 + m/2}, -(c^2*x^2)])/Gamma[1 + m/2] + (b*Gamma[(3 + m)/2]
*HypergeometricPFQRegularized[{1, (1 + m)/2}, {(3 + m)/2}, -(c^2*x^2)])/c +
 (b*c*x^2*Gamma[2 + m/2]*Gamma[(3 + m)/2]*HypergeometricPFQRegularized[{1,
(3 + m)/2}, {(5 + m)/2}, -(c^2*x^2)])/Gamma[1 + m/2] + (b*c*m*x^2*Gamma[2 +
 m/2]*Gamma[(3 + m)/2]*HypergeometricPFQRegularized[{1, (3 + m)/2}, {(5 + m
)/2}, -(c^2*x^2)])/Gamma[1 + m/2]))))/((1 + m)^2*(2 + m))
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int x^{m} \left(e\,x^{2}+d\right) \left(a +b \arctan \left(c x \right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] a*d*x^(m + 1)/(m + 1) + a*x^(m + 3)*e/(m + 3) + (((b*m*e + b*e)*x^3 + (b*d*
m + 3*b*d)*x)*x^m*arctan(c*x) - (m^2 + 4*m + 3)*integrate(((b*c*m*e + b*c*e
)*x^3 + (b*c*d*m + 3*b*c*d)*x)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 + m^2 +
 4*m + 3), x))/(m^2 + 4*m + 3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+b\operatorname{atan}(cx))\left(d+ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)*(a+b*atan(c*x)),x)
[Out] Integral(x**m*(a + b*atan(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,(a+b\operatorname{atan}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2),x)
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2), x)
```

3.1231 $\int \frac{x^m(a+b\mathbf{ArcTan}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=63

$$\frac{ax^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\mathrm{Int}\left(\frac{x^m\mathrm{ArcTan}(cx)}{d+ex^2},x\right)$$

```
[Out] a*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)+b*Unintegr
able(x^m*arctan(c*x)/(e*x^2+d),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x^m(a+b\mathrm{ArcTan}(cx))}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d*(
1 + m)) + b*Defer[Int][(x^m*ArcTan[c*x])/(d + e*x^2), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\tan^{-1}(cx))}{d+ex^2}\,dx &= a\int \frac{x^m}{d+ex^2}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{d+ex^2}\,dx\\ &= \frac{ax^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\int \frac{x^m\tan^{-1}(cx)}{d+ex^2}\,dx\end{aligned}$$

Mathematica [A]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\mathrm{ArcTan}(cx))}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2), x]
```

Maple [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+b\arctan \left(c x\right)\right)}{e\,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*x^m/(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+b\operatorname{atan}\left(cx\right)\right)}{d+ex^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*atan(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^m*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1232 $\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^2} dx$

Optimal. Leaf size=63

$$\frac{ax^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d^2(1+m)}+b\text{Int}\left(\frac{x^m\text{ArcTan}(cx)}{(d+ex^2)^2},x\right)$$

```
[Out] a*x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d^2/(1+m)+b*Uninte
grable(x^m*arctan(c*x)/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d^2
*(1 + m)) + b*Defer[Int][(x^m*ArcTan[c*x])/(d + e*x^2)^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\tan^{-1}(cx))}{(d+ex^2)^2}\,dx &= a\int \frac{x^m}{(d+ex^2)^2}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^2}\,dx\\ &= \frac{ax^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d^2(1+m)}+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 3.87, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2, x]
```

Maple [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+b\arctan\left(c x\right)\right)}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*x^m/(x^2*e + d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*x^m/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1233 $\int x^m (d+ex^2)^{5/2} (a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=76

$$\frac{ax^{1+m}(d+ex^2)^{7/2}\,{}_2F_1\left(1,\frac{8+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(x^m(d+ex^2)^{5/2}\,\text{ArcTan}(cx),x\right)$$

```
[Out] a*x^(1+m)*(e*x^2+d)^(7/2)*hypergeom([1, 4+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1
+m)+b*Unintegrable(x^m*(e*x^2+d)^(5/2)*arctan(c*x),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m (d+ex^2)^{5/2} (a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d^2*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)
/2, -((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*(d + e*
x^2)^(5/2)*ArcTan[c*x], x]
```

Rubi steps

$$\int x^m (d+ex^2)^{5/2} \left(a+b\tan^{-1}(cx)\right)\,dx = a\int x^m (d+ex^2)^{5/2}\,dx + b\int x^m (d+ex^2)^{5/2}\tan^{-1}(cx)\,dx$$

$$= b\int x^m(d+ex^2)^{5/2}\tan^{-1}(cx)\,dx + \frac{\left(ad^2\sqrt{d+ex^2}\right)\int x^m\left(1+\frac{ex^2}{d}\right.}{\sqrt{1+\frac{ex^2}{d}}}$$

$$= \frac{ad^2x^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}} + b\int x^m(d+ex^2)^{5}$$

Mathematica [A]
time = 2.56, size = 0, normalized size = 0.00

$$\int x^m (d+ex^2)^{5/2} (a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \arctan \left(c x \right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
[Out] int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arctan(c*x))*sqrt(x^2*e + d)*x^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

```
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1234 $\int x^m(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=76

$$\frac{ax^{1+m}(d+ex^2)^{5/2}\,{}_2F_1\left(1,\frac{6+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(x^m(d+ex^2)^{3/2}\text{ArcTan}(cx),x\right)$$

```
[Out] a*x^(1+m)*(e*x^2+d)^(5/2)*hypergeom([1, 3+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1
+m)+b*Unintegrable(x^m*(e*x^2+d)^(3/2)*arctan(c*x),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*d*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2
, -((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*(d + e*x^
2)^(3/2)*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m(d+ex^2)^{3/2}\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^m(d+ex^2)^{3/2}\,dx+b\int x^m(d+ex^2)^{3/2}\tan^{-1}(cx)\,dx\\ &= b\int x^m(d+ex^2)^{3/2}\tan^{-1}(cx)\,dx+\frac{\left(ad\sqrt{d+ex^2}\right)\int x^m\left(1+\frac{ex^2}{d}\right.}{\sqrt{1+\frac{ex^2}{d}}}\\ &= \frac{adx^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\int x^m(d+ex^2)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^m(d+ex^2)^{3/2}(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b\arctan \left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arctan(c*x))*sqrt(x^2*e + d)*x^m,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1235 $\int x^m\sqrt{d+ex^2}\,(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=76

$$\frac{ax^{1+m}(d+ex^2)^{3/2}\,{}_2F_1\left(1,\frac{4+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(x^m\sqrt{d+ex^2}\,\text{ArcTan}(cx),x\right)$$

```
[Out] a*x^(1+m)*(e*x^2+d)^(3/2)*hypergeom([1, 2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1
+m)+b*Unintegrable(x^m*arctan(c*x)*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^m\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[d + e*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
-((e*x^2)/d)])/((1 + m)*Sqrt[1 + (e*x^2)/d]) + b*Defer[Int][x^m*Sqrt[d + e*
x^2]*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{d+ex^2}\,\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^m\sqrt{d+ex^2}\,dx+b\int x^m\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx\\ &= b\int x^m\sqrt{d+ex^2}\,\tan^{-1}(cx)\,dx+\frac{\left(a\sqrt{d+ex^2}\right)\int x^m\sqrt{1+\frac{ex^2}{d}}\,dx}{\sqrt{1+\frac{ex^2}{d}}}\\ &= \frac{ax^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\int x^m\sqrt{d+ex^2}\,\tan\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^m\sqrt{d+ex^2}\,(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int x^{m} \sqrt{e\,x^{2}+d}\,\left(a +b \arctan \left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*(b*arctan(c*x) + a)*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)*x^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(a + b \operatorname{atan}{\left(c x \right)}\right) \sqrt{d + e x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

```
[Out] Integral(x**m*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1236 $\int \frac{x^m(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=76

$$\frac{ax^{1+m}\sqrt{d+ex^2}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(\frac{x^m\text{ArcTan}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] a*x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-e*x^2/d)*(e*x^2+d)^(1/2)/d/(1
+m)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -((e*x^2)/d)])/((1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[c*x]
)/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m(a+b\tan^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= a\int \frac{x^m}{\sqrt{d+ex^2}}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx\\
&= b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx+\frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}\\
&= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{(1+m)\sqrt{d+ex^2}}+b\int \frac{x^m\tan^{-1}(cx)}{\sqrt{d+ex^2}}\,dx
\end{aligned}$$

Mathematica [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+b\arctan\left(cx\right)\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*x^m/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*x^m/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(a+b\operatorname{atan}\left(cx\right)\right)}{\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**m*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
[Out] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1237 $\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{ax^{1+m}\,{}_2F_1\left(1,\frac{m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)\sqrt{d+ex^2}}+b\text{Int}\left(\frac{x^m\text{ArcTan}(cx)}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] a*x^(1+m)*hypergeom([1, 1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)/(e*x^2+d)^(1/2
)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/
2, -((e*x^2)/d)])/(d*(1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[c*
x])/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\tan^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= a\int \frac{x^m}{(d+ex^2)^{3/2}}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx\\ &= b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx+\frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\left(1+\frac{ex^2}{d}\right)^{3/2}}\,dx}{d\sqrt{d+ex^2}}\\ &= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)\sqrt{d+ex^2}}+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx\end{aligned}$$

Mathematica [A]
time = 3.65, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(a+b\arctan (cx))}{(e\,x^{2}+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(x^2*e + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)*x^m/(x^4*e^2 + 2*d*x^2*e + d^2
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \left(a + b\,\mathrm{atan}(c\,x)\right)}{\left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1238 $\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{ax^{1+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)\,(d+ex^2)^{3/2}}+b\text{Int}\left(\frac{x^m\text{ArcTan}(cx)}{(d+ex^2)^{5/2}},x\right)$$

[Out] a*x^(1+m)*hypergeom([1, -1+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)/(e*x^2+d)^(3/2)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(5/2),x)

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]

[Out] (a*x^(1 + m)*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, -((e*x^2)/d)])/(d^2*(1 + m)*Sqrt[d + e*x^2]) + b*Defer[Int][(x^m*ArcTan[c*x])/(d + e*x^2)^(5/2), x]

Rubi steps

$$\begin{aligned}\int \frac{x^m(a+b\tan^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx &= a\int \frac{x^m}{(d+ex^2)^{5/2}}\,dx+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx\\ &= b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx+\frac{\left(a\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{x^m}{\left(1+\frac{ex^2}{d}\right)^{5/2}}\,dx}{d^2\sqrt{d+ex^2}}\\ &= \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d^2(1+m)\sqrt{d+ex^2}}+b\int \frac{x^m\tan^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx\end{aligned}$$

Mathematica [A]
time = 4.89, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+b\text{ArcTan}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(a+b\arctan\left(cx\right))}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)*x^m/(x^2*e + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arctan(c*x) + a)*x^m/(x^6*e^3 + 3*d*x^4*e^2 + 3
*d^2*x^2*e + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1239 $\int x^m(d+ex^2)^p(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=77

$$\frac{ax^{1+m}(d+ex^2)^{1+p}\,{}_2F_1\left(1,\frac{1}{2}(3+m+2p);\frac{3+m}{2};-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(x^m\left(d+ex^2\right)^p\text{ArcTan}(cx),x\right)$$

```
[Out] a*x^(1+m)*(e*x^2+d)^(1+p)*hypergeom([1, 3/2+1/2*m+p],[3/2+1/2*m],-e*x^2/d)/
d/(1+m)+b*Unintegrable(x^m*(e*x^2+d)^p*arctan(c*x),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^m\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] (a*x^(1 + m)*(d + e*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((e
*x^2)/d)])/((1 + m)*(1 + (e*x^2)/d)^p) + b*Defer[Int][x^m*(d + e*x^2)^p*Arc
Tan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx&=a\int x^m\left(d+ex^2\right)^p\,dx+b\int x^m\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\&=b\int x^m\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\right)\int x\\&=\frac{ax^{1+m}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{ex^2}{d}\right)}{1+m}+b\int x^m(\end{aligned}$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int x^m\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 1.47, size = 0, normalized size = 0.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{p} \left(a +b \arctan \left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*(x^2*e + d)^p*x^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p,x)
```

```
[Out] int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p, x)
```

3.1240 $\int x^{-2-2p}(d+ex^2)^p(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=81

$$-\frac{ax^{-1-2p}(d+ex^2)^{1+p}\,{}_2F_1\left(\frac{1}{2},1;\frac{1}{2}(1-2p);-\frac{ex^2}{d}\right)}{d(1+2p)}+b\text{Int}\left(x^{-2-2p}\left(d+ex^2\right)^p\text{ArcTan}(cx),x\right)$$

```
[Out] -a*x^(-1-2*p)*(e*x^2+d)^(1+p)*hypergeom([1/2, 1],[1/2-p],-e*x^2/d)/d/(1+2*p
)+b*Unintegrable(x^(-2-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^{-2-2p}\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-1 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*
p)/2, -((e*x^2)/d)])/((1 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-2 -
2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-2-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-2-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^{-2-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right.\right.\\ &= -\frac{ax^{-1-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-1-2p),-p;\frac{1}{2}(1-2\right.}{1+2p}\end{aligned}$$

Mathematica [A]
time = 2.30, size = 0, normalized size = 0.00

$$\int x^{-2-2p}\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int x^{-2-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(c x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4373 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2),x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2), x)
```

3.1241 $\int x^{-3-2p}(d+ex^2)^p\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=129

$$-\frac{bcx^{-1-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-1-2p);1,-1-p;\frac{1}{2}(1-2p);-c^2x^2,-\frac{ex^2}{d}\right)}{2\left(1+3p+2p^2\right)}-\frac{x^{-2(1+p)}(d+ex^2)^{1+p}}{2d(1-$$

```
[Out] -1/2*b*c*x^(-1-2*p)*(e*x^2+d)^p*AppellF1(-1/2-p,1,-1-p,1/2-p,-c^2*x^2,-e*x^
2/d)/(2*p^2+3*p+1)/((1+e*x^2/d)^p)-1/2*(e*x^2+d)^(1+p)*(a+b*arctan(c*x))/d/
(1+p)/(x^(2+2*p))
```

Rubi [A]

time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {270, 5096, 12, 525, 524}

$$-\frac{x^{-2(p+1)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{2d(p+1)}-\frac{bcx^{-2p-1}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}F_1\left(\frac{1}{2}(-2p-1);1,-p-1;\frac{1}{2}(1-2p);-c^2x^2,-\frac{ex^2}{d}\right)}{2\left(2p^2+3p+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*c*x^(-1 - 2*p)*(d + e*x^2)^p*AppellF1[(-1 - 2*p)/2, 1, -1 - p, (1 -
 2*p)/2, -(c^2*x^2), -((e*x^2)/d)])/((1 + 3*p + 2*p^2)*(1 + (e*x^2)/d)^p) -
 ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(1 + p)*x^(2*(1 + p)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}-(bc)\int -\frac{x^{-2(1+p)}(d}{2d(1+p)}$$

$$=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}+\frac{(bc)\int\frac{x^{-2(1+p)}(d+ex^2}{1+c^2x^2}}{2d(1+p)}$$

$$=-\frac{x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d(1+p)}+\frac{\left(bc\left(d+ex^2\right)^p\left(1+\right.\right.}{}$$

$$=-\frac{bcx^{-1-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-1-2p);1,-1-p;\frac{1}{2}\right.}{2\left(1+3p+2p^2\right)}$$

Mathematica [A]
time = 0.45, size = 166, normalized size = 1.29

$$-\frac{x^{-2(1+p)}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\left(b(c^2d-e)\,xF_1\left(-\frac{1}{2}-p;-p,1;\frac{1}{2}-p;-\frac{ex^2}{d},-c^2x^2\right)+c(1+2p)\left(d+ex^2\right)\left(1+\frac{ex^2}{d}\right)^p\left(a+b\text{ArcTan}(cx)\right)+bex\,{}_2F_1\left(-\frac{1}{2}-p,-p;\frac{1}{2}-p;-\frac{ex^2}{d}\right)\right)}{2cd(1+p)(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*((d + e*x^2)^p*(b*(c^2*d - e)*x*AppellF1[-1/2 - p, -p, 1, 1/2 - p, -((
e*x^2)/d), -(c^2*x^2)] + c*(1 + 2*p)*(d + e*x^2)*(1 + (e*x^2)/d)^p*(a + b*A
rcTan[c*x]) + b*e*x*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((e*x^2)/d)])
)/(c*d*(1 + p)*(1 + 2*p)*x^(2*(1 + p))*(1 + (e*x^2)/d)^p)
```

Maple [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int x^{-3-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan \left(c x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] b*integrate(arctan(c*x)*e^(p*log(x^2*e + d) - 2*p*log(x))/x^3, x) - 1/2*(x^
2*e + d)*a*e^(p*log(x^2*e + d) - 2*p*log(x))/(d*(p + 1)*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6193 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3),x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3), x)
```

3.1242 $\int x^{-4-2p}(d+ex^2)^p\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=81

$$-\frac{ax^{-3-2p}(d+ex^2)^{1+p}\ {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}(-1-2p);-\frac{ex^2}{d}\right)}{d(3+2p)}+b\text{Int}\left(x^{-4-2p}\left(d+ex^2\right)^p\text{ArcTan}(cx),x\right)$$

```
[Out] -a*x^(-3-2*p)*(e*x^2+d)^(1+p)*hypergeom([-1/2, 1],[-1/2-p],-e*x^2/d)/d/(3+2
*p)+b*Unintegrable(x^(-4-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2
*p)/2, -((e*x^2)/d)])/((3 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-4 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx=a\int x^{-4-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-4-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx$$

$$=b\int x^{-4-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)\right.$$

$$=-\frac{ax^{-3-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(-3-2p),-p;\frac{1}{2}(-1-\right.}{3+2p}$$

Mathematica [A]
time = 2.53, size = 0, normalized size = 0.00

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int x^{-4-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b \arctan \left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4),x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4), x)
```

3.1243 $\int x^{-5-2p}(d+ex^2)^p\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=285

$$-\frac{b(e+c^2d(1+p))\,x^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-3-2p);1,-1-p;\frac{1}{2}(-1-2p);-c^2x^2,-\frac{ex^2}{d}\right)}{2cd(1+p)(2+p)(3+2p)}+\frac{ex}{}$$

```
[Out] -1/2*b*(e+c^2*d*(1+p))*x^(-3-2*p)*(e*x^2+d)^p*AppellF1(-3/2-p,1,-1-p,-1/2-p
,-c^2*x^2,-e*x^2/d)/c/d/(3+2*p)/(p^2+3*p+2)/((1+e*x^2/d)^p)+1/2*e*(e*x^2+d)
^(1+p)*(a+b*arctan(c*x))/d^2/(1+p)/(2+p)/(x^(2+2*p))-1/2*(e*x^2+d)^(1+p)*(a
+b*arctan(c*x))/d/(2+p)/(x^(4+2*p))+1/2*b*e*x^(-3-2*p)*(e*x^2+d)^p*hypergeo
m([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d/(2*p^3+9*p^2+13*p+6)/((1+e*x^2/d)^p
)
```

Rubi [A]
time = 0.27, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {277, 270, 5096, 12, 598, 372, 371, 525, 524}

$$\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{2d(p+2)}-\frac{bx^{-2p-3}(c^2d(p+1)+e)(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}F_1\left(\frac{1}{2}(-2p-3);1,-p-1;\frac{1}{2}(-2p-1);-c^2x^2,-\frac{ex^2}{d}\right)}{2cd(p+1)(p+2)(2p+3)}+\frac{bex^{-2p-3}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}(-2p-3),-p-1;\frac{1}{2}(-2p-1);-\frac{ex^2}{d}\right)}{2cd\left(2p^3+9p^2+13p+6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*(e + c^2*d*(1 + p))*x^(-3 - 2*p)*(d + e*x^2)^p*AppellF1[(-3 - 2*p)/
2, 1, -1 - p, (-1 - 2*p)/2, -(c^2*x^2), -((e*x^2)/d)])/(c*d*(1 + p)*(2 + p)
*(3 + 2*p)*(1 + (e*x^2)/d)^p) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))
/(2*d^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan
[c*x]))/(2*d*(2 + p)*x^(2*(2 + p))) + (b*e*x^(-3 - 2*p)*(d + e*x^2)^p*Hyper
geometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(2*c*d*(6 +
 13*p + 9*p^2 + 2*p^3)*(1 + (e*x^2)/d)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
```

```
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx=\frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+}}{2d(2}$$

$$=\frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+}}{2d(2}$$

$$=\frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+}}{2d(2}$$

$$=\frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+}}{2d(2}$$

$$=\frac{ex^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(d+ex^2\right)^{1+}}{2d(2}$$

$$=-\frac{b\left(e+c^2d(1+p)\right)x^{-3-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-3-2\right.}{2cd(1+p)(2+p)(3+2}$$

Mathematica [F]
time = 3.33, size = 0, normalized size = 0.00

$$\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\text{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int x^{-5-2p}\left(e\,x^2+d\right)^p\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

[Out] int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")

[Out] b*integrate(arctan(c*x)*e^(p*log(x^2*e + d) - 2*p*log(x))/x^5, x) + 1/2*(x^4*e^2 - d*p*x^2*e - d^2*(p + 1))*a*e^(p*log(x^2*e + d) - 2*p*log(x))/((p^2 + 3*p + 2)*d^2*x^4)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 5), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-5-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")

[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5),x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5), x)
```

3.1244 $\int x^{-6-2p}(d+ex^2)^p\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=81

$$-\frac{ax^{-5-2p}(d+ex^2)^{1+p}\ {}_2F_1\left(-\frac{3}{2},1;\frac{1}{2}(-3-2p);-\frac{ex^2}{d}\right)}{d(5+2p)}+b\mathrm{Int}\left(x^{-6-2p}(d+ex^2)^p\,\mathrm{ArcTan}(cx),x\right)$$

```
[Out] -a*x^(-5-2*p)*(e*x^2+d)^(1+p)*hypergeom([-3/2, 1],[-3/2-p],-e*x^2/d)/d/(5+2
*p)+b*Unintegrable(x^(-6-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^{-6-2p}\left(d+ex^2\right)^p(a+b\mathrm{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-5 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2
*p)/2, -((e*x^2)/d)])/((5 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-6 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-6-2p}\left(d+ex^2\right)^p\,dx+b\int x^{-6-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx\\ &= b\int x^{-6-2p}\left(d+ex^2\right)^p\tan^{-1}(cx)\,dx+\left(a\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)\right.\\ &= -\frac{ax^{-5-2p}\left(d+ex^2\right)^p\left(1+\frac{ex^2}{d}\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(-5-2p),-p;\frac{1}{2}(-3-\right.}{5+2p}\end{aligned}$$

Mathematica [A]
time = 2.68, size = 0, normalized size = 0.00

$$\int x^{-6-2p}\left(d+ex^2\right)^p(a+b\mathrm{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int x^{-6-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 6), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 6), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-6-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6),x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6), x)
```

3.1245 $\int x^{-7-2p}(d+ex^2)^p\,(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=466

$$-\frac{b(2e^2+2c^2de(1+p)+c^4d^2(2+3p+p^2))\,x^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}F_1\left(\frac{1}{2}(-5-2p);1,-1-p;\frac{1}{2}(-3\right.}{2c^3d^2(1+p)(2+p)(3+p)(5+2p)}$$

```
[Out] -1/2*b*(2*e^2+2*c^2*d*e*(1+p)+c^4*d^2*(p^2+3*p+2))*x^(-5-2*p)*(e*x^2+d)^p*A
ppellF1(-5/2-p,1,-1-p,-3/2-p,-c^2*x^2,-e*x^2/d)/c^3/d^2/(3+p)/(5+2*p)/(p^2+
3*p+2)/((1+e*x^2/d)^p)-e^2*(e*x^2+d)^(1+p)*(a+b*arctan(c*x))/d^3/(2+p)/(p^2
+4*p+3)/(x^(2+2*p))+e*(e*x^2+d)^(1+p)*(a+b*arctan(c*x))/d^2/(2+p)/(3+p)/(x^
(4+2*p))-1/2*(e*x^2+d)^(1+p)*(a+b*arctan(c*x))/d/(3+p)/(x^(6+2*p))+b*e*(e+c
^2*d*(1+p))*x^(-5-2*p)*(e*x^2+d)^p*hypergeom([-1-p, -5/2-p],[-3/2-p],-e*x^2
/d)/c^3/d^2/(3+p)/(5+2*p)/(p^2+3*p+2)/((1+e*x^2/d)^p)-b*e^2*x^(-3-2*p)*(e*x
^2+d)^p*hypergeom([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d^2/(p^2+3*p+2)/(2*p^
2+9*p+9)/((1+e*x^2/d)^p)
```

Rubi [A]

time = 1.03, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {277, 270, 5096, 12, 6857, 372, 371, 525, 524}

$$-\frac{e^2x^{-2(p+1)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}(d+ex^2)^{p+1}(a+b\text{ArcTan}(cx))}{2d(p+3)}-\frac{bx^{-2p-5}(c^4d^2(p^2+3p+2)+2c^2de(p+1)+2e^2)(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}F_1\left(\frac{1}{2}(-2p-5);1,-p-1;\frac{1}{2}(-2p-3);-c^2x^2,-\frac{ex^2}{d}\right)}{2c^3d^2(p+1)(p+2)(p+3)(2p+5)}+\frac{bex^{-2p-5}(c^2d(p+1)+e)(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}(-2p-5),-p-1;\frac{1}{2}(-2p-3);-\frac{ex^2}{d}\right)}{c^3d^2(p+1)(p+2)(p+3)(2p+5)}-\frac{be^2x^{-2p-3}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}(-2p-3),-p-1;\frac{1}{2}(-2p-1);-\frac{ex^2}{d}\right)}{cd^2(p+1)(p+2)(p+3)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*(2*e^2 + 2*c^2*d*e*(1 + p) + c^4*d^2*(2 + 3*p + p^2))*x^(-5 - 2*p)*
(d + e*x^2)^p*AppellF1[(-5 - 2*p)/2, 1, -1 - p, (-3 - 2*p)/2, -(c^2*x^2), -
((e*x^2)/d)])/(c^3*d^2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p)
 - (e^2*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^3*(1 + p)*(2 + p)*(3 +
p)*x^(2*(1 + p))) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^2*(2 + p
)*(3 + p)*x^(2*(2 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(
3 + p)*x^(2*(3 + p))) + (b*e*(e + c^2*d*(1 + p))*x^(-5 - 2*p)*(d + e*x^2)^p
*Hypergeometric2F1[(-5 - 2*p)/2, -1 - p, (-3 - 2*p)/2, -((e*x^2)/d)])/(c^3*
d^2*(1 + p)*(2 + p)*(3 + p)*(5 + 2*p)*(1 + (e*x^2)/d)^p) - (b*e^2*x^(-3 - 2
*p)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((
e*x^2)/d)])/(c*d^2*(1 + p)*(2 + p)*(3 + p)*(3 + 2*p)*(1 + (e*x^2)/d)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
```

```
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\tan^{-1}(cx)\right)\,dx=-\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right.}{d^2(2}$$

$$=-\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right.}{d^2(2}$$

$$=-\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right.}{d^2(2}$$

$$=-\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right.}{d^2(2}$$

$$=-\frac{e^2x^{-2(1+p)}\left(d+ex^2\right)^{1+p}\left(a+b\tan^{-1}(cx)\right)}{d^3(1+p)(2+p)(3+p)}+\frac{ex^{-2(2+p)}\left(d+ex^2\right.}{d^2(2}$$

$$=-\frac{b\left(2e^2+2c^2de(1+p)+c^4d^2(2+3p+p^2)\right)x^{-5-2p}\left(d+ex^2\right)^p\,(}{2c^3d^2(1+p)(2}$$

Mathematica [F]
time = 3.72, size = 0, normalized size = 0.00

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\text{ArcTan}(cx)\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [F]
time = 1.25, size = 0, normalized size = 0.00

$$\int x^{-7-2p}(e\,x^2+d)^p\,(a+b\arctan(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

```
[Out] int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)*e^(p*log(x^2*e + d) - 2*p*log(x))/x^7, x) - 1/2*(2*
x^6*e^3 - 2*d*p*x^4*e^2 + (p^2 + p)*d^2*x^2*e + (p^2 + 3*p + 2)*d^3)*a*e^(p
*log(x^2*e + d) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*d^3*x^6)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 7), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-7-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7),x)
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7), x)
```

3.1246 $\int x^{-8-2p}(d+ex^2)^p (a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=81

$$-\frac{ax^{-7-2p}(d+ex^2)^{1+p}\ {}_2F_1\left(-\frac{5}{2},1;\frac{1}{2}(-5-2p);-\frac{ex^2}{d}\right)}{d(7+2p)}+b\mathrm{Int}\left(x^{-8-2p}(d+ex^2)^p \operatorname{ArcTan}(cx),x\right)$$

```
[Out] -a*x^(-7-2*p)*(e*x^2+d)^(1+p)*hypergeom([-5/2, 1],[-5/2-p],-e*x^2/d)/d/(7+2
*p)+b*Unintegrable(x^(-8-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^{-8-2p}(d+ex^2)^p (a+b\operatorname{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((a*x^(-7 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-7 - 2*p)/2, -p, (-5 - 2
*p)/2, -((e*x^2)/d)])/((7 + 2*p)*(1 + (e*x^2)/d)^p)) + b*Defer[Int][x^(-8 -
 2*p)*(d + e*x^2)^p*ArcTan[c*x], x]
```

Rubi steps

$$\begin{aligned}
\int x^{-8-2p}(d+ex^2)^p \left(a+b\tan^{-1}(cx)\right)\,dx &= a\int x^{-8-2p}(d+ex^2)^p\,dx+b\int x^{-8-2p}(d+ex^2)^p\tan^{-1}(cx)\,dx\\
&= b\int x^{-8-2p}(d+ex^2)^p\tan^{-1}(cx)\,dx+\left(a(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-}\right.\\
&= -\frac{ax^{-7-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(-7-2p),-p;\frac{1}{2}(-5-\right.}{7+2p}
\end{aligned}$$

Mathematica [A]
time = 2.16, size = 0, normalized size = 0.00

$$\int x^{-8-2p}(d+ex^2)^p (a+b\operatorname{ArcTan}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
[Out] Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

Maple [**A**]
time = 1.25, size = 0, normalized size = 0.00

$$\int x^{-8-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
[Out] int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 8), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(x^2*e + d)^p*x^(-2*p - 8), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-8-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8),x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8), x)
```

3.1247 $\int x^3(d+ex^2)\left(a+b\mathbf{ArcTan}(cx)\right)^2 dx$

Optimal. Leaf size=271

$$\frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{4b^2ex^2}{45c^4}+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\text{ArcTan}(cx)}{2c^3}-\frac{b^2ex\text{ArcTan}(cx)}{3c^5}-\frac{bdx^3(a+b\text{ArcTan}(cx))}{6c}+\frac{bex^3(}{}$$

```
[Out] 1/2*a*b*d*x/c^3-1/3*a*b*e*x/c^5+1/12*b^2*d*x^2/c^2-4/45*b^2*e*x^2/c^4+1/60*
b^2*e*x^4/c^2+1/2*b^2*d*x*arctan(c*x)/c^3-1/3*b^2*e*x*arctan(c*x)/c^5-1/6*b
*d*x^3*(a+b*arctan(c*x))/c+1/9*b*e*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e*x^5*(
a+b*arctan(c*x))/c-1/4*d*(a+b*arctan(c*x))^2/c^4+1/6*e*(a+b*arctan(c*x))^2/
c^6+1/4*d*x^4*(a+b*arctan(c*x))^2+1/6*e*x^6*(a+b*arctan(c*x))^2-1/3*b^2*d*l
n(c^2*x^2+1)/c^4+23/90*b^2*e*ln(c^2*x^2+1)/c^6
```

Rubi [A]
time = 0.45, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5100, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{e(a+b\text{ArcTan}(cx))^2}{6c^6}-\frac{d(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{bex^3(a+b\text{ArcTan}(cx))}{9c^3}+\frac{1}{4}dx^4(a+b\text{ArcTan}(cx))^2-\frac{bdx^3(a+b\text{ArcTan}(cx))}{6c}+\frac{1}{6}ex^6(a+b\text{ArcTan}(cx))^2-\frac{bex^5(a+b\text{ArcTan}(cx))}{15c}-\frac{abex}{3c^5}+\frac{abdx}{2c^3}-\frac{b^2ex\text{ArcTan}(cx)}{3c^5}+\frac{b^2dx\text{ArcTan}(cx)}{2c^3}-\frac{4b^2ex^2}{45c^4}+\frac{b^2dx^2}{12c^2}+\frac{b^2ex^4}{60c^2}+\frac{23b^2e\log(c^2x^2+1)}{90c^6}-\frac{b^2d\log(c^2x^2+1)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*d*x)/(2*c^3) - (a*b*e*x)/(3*c^5) + (b^2*d*x^2)/(12*c^2) - (4*b^2*e*x^2
)/(45*c^4) + (b^2*e*x^4)/(60*c^2) + (b^2*d*x*ArcTan[c*x])/(2*c^3) - (b^2*e*
x*ArcTan[c*x])/(3*c^5) - (b*d*x^3*(a + b*ArcTan[c*x]))/(6*c) + (b*e*x^3*(a
+ b*ArcTan[c*x]))/(9*c^3) - (b*e*x^5*(a + b*ArcTan[c*x]))/(15*c) - (d*(a +
b*ArcTan[c*x])^2)/(4*c^4) + (e*(a + b*ArcTan[c*x])^2)/(6*c^6) + (d*x^4*(a +
 b*ArcTan[c*x])^2)/4 + (e*x^6*(a + b*ArcTan[c*x])^2)/6 - (b^2*d*Log[1 + c^2
*x^2])/(3*c^4) + (23*b^2*e*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int x^3(d+ex^2)\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(dx^3\left(a+b\tan^{-1}(cx)\right)^2+ex^5\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx \\
&= d\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^5\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int \frac{x^4(a}{} \\
&= \frac{1}{4}dx^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}ex^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{(bd)\int x^2(a+}{} \\
&= -\frac{bdx^3(a+b\tan^{-1}(cx))}{6c}-\frac{bex^5(a+b\tan^{-1}(cx))}{15c}+\frac{1}{4}dx^4(a+b\tan \\
&= \frac{abdx}{2c^3}-\frac{bdx^3(a+b\tan^{-1}(cx))}{6c}+\frac{bex^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{bex^5(a+}{} \\
&= \frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{bdx^3(a+b\tan^{-1}(cx))}{6c}+\frac{bex^3(a}{} \\
&= \frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{b^2ex^2}{30c^4}+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{b^2ex\,t}{} \\
&= \frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{4b^2ex^2}{45c^4}+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\tan^{-1}(cx)}{2c^3}-\frac{b^2ex}{}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 240, normalized size = 0.89

$$
\frac{cx(15a^2c^5x^3(3d+2ex^2)+b^2cx(-16e+3c^2(5d+ex^2))-2ab(30e-5c^2(9d+2ex^2)+3c^4(5dx^2+2ex^4)))+2b(bcx(-30e+5c^2(9d+2ex^2)-3c^4(5dx^2+2ex^4))+15a(-3c^2d+2e+c^6(3dx^4+2ex^6)))\operatorname{ArcTan}(cx)+15b^2(-3c^2d+2e+c^6(3dx^4+2ex^6))\operatorname{ArcTan}(cx)^2+2b^2(-30c^2d+23e)\log(1+c^2x^2)}{180c^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(15*a^2*c^5*x^3*(3*d + 2*e*x^2) + b^2*c*x*(-16*e + 3*c^2*(5*d + e*x^2)
) - 2*a*b*(30*e - 5*c^2*(9*d + 2*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4))) + 2*b
*(b*c*x*(-30*e + 5*c^2*(9*d + 2*e*x^2) - 3*c^4*(5*d*x^2 + 2*e*x^4)) + 15*a*
(-3*c^2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6)))*ArcTan[c*x] + 15*b^2*(-3*c^2*d
+ 2*e + c^6*(3*d*x^4 + 2*e*x^6))*ArcTan[c*x]^2 + 2*b^2*(-30*c^2*d + 23*e)*L
og[1 + c^2*x^2])/(180*c^6)
```

Maple [A]
time = 0.63, size = 333, normalized size = 1.23

method	result

derivativedivides	$\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\arctan(cx)^2 d\,c^4x^4}{4}+\frac{b^2c^4\arctan(cx)^2 e\,x^6}{6}-\frac{b^2\arctan(cx)d\,c^3x^3}{6}-\frac{b^2c^3\arctan(cx)e\,x^5}{15}+\frac{b^2\arctan(cx)dcx}{2}$
default	$\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\arctan(cx)^2 d\,c^4x^4}{4}+\frac{b^2c^4\arctan(cx)^2 e\,x^6}{6}-\frac{b^2\arctan(cx)d\,c^3x^3}{6}-\frac{b^2c^3\arctan(cx)e\,x^5}{15}+\frac{b^2\arctan(cx)dcx}{2}$
risch	$-\frac{ib^2e\,x^5\ln(-icx+1)}{30c}-\frac{ib^2d\,x^3\ln(-icx+1)}{12c}+\frac{abdx}{2c^3}+\frac{x^6e\,a^2}{6}+\frac{x^4d\,a^2}{4}-\frac{b^2\left(2e\,c^6x^6+3d\,c^6x^4-3c^2d+2e\right)\ln(icx+1)}{48c^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a^2/c^2*(1/4*d*c^6*x^4+1/6*e*c^6*x^6)+1/4*b^2*arctan(c*x)^2*d*c^4*x^
4+1/6*b^2*c^4*arctan(c*x)^2*e*x^6-1/6*b^2*arctan(c*x)*d*c^3*x^3-1/15*b^2*c^
3*arctan(c*x)*e*x^5+1/2*b^2*arctan(c*x)*d*c*x+1/9*b^2*c*arctan(c*x)*e*x^3-1
/3*b^2*e*x*arctan(c*x)/c-1/4*b^2*arctan(c*x)^2*d+1/6*b^2/c^2*arctan(c*x)^2*
e+1/12*b^2*d*c^2*x^2+1/60*b^2*c^2*e*x^4-4/45*b^2*e*x^2-1/3*b^2*ln(c^2*x^2+1
)*d+23/90*b^2*e*ln(c^2*x^2+1)/c^2+1/2*a*b*arctan(c*x)*d*c^4*x^4+1/3*a*b*c^4
*arctan(c*x)*e*x^6-1/6*a*b*d*c^3*x^3-1/15*a*b*c^3*e*x^5+1/2*a*b*d*c*x+1/9*a
*b*c*e*x^3-1/3*a*b*e*x/c-1/2*a*b*arctan(c*x)*d+1/3*a*b/c^2*arctan(c*x)*e)
```

Maxima [A]
time = 0.50, size = 310, normalized size = 1.14

$$\frac{1}{6}b^2x^6\arctan(cx)^2e+\frac{1}{4}b^2dx^4\arctan(cx)^2+\frac{1}{6}a^2x^6e+\frac{1}{4}a^2dx^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abd-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abe-\frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+46\log(c^2x^2+1)}{c^6}\right)b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x)^2*e + 1/4*b^2*d*x^4*arctan(c*x)^2 + 1/6*a^2*x^6*e +
 1/4*a^2*d*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan
(c*x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arc
tan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d + 1/
45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(
c*x)/c^7))*a*b*e - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arct
an(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*
log(c^2*x^2 + 1))/c^6)*b^2*e
```

Fricas [A]
time = 1.99, size = 286, normalized size = 1.06

$$\frac{45a^2c^6dx^4-30abc^5dx^3+15b^2c^4dx^2+90abc^3dx+15\left(3b^2c^6dx^4-3b^2c^2d+2\left(b^2c^6x^6+b^2\right)e\right)\arctan(cx)^2+2\left(45abc^6dx^4-15b^2c^5dx^3+45b^2c^3dx-45abc^2d+2\left(15abc^6x^6-3b^2c^5x^5+5b^2c^3x^3-15b^2cx+15ab\right)e\right)\arctan(cx)+\left(30a^2c^6x^6-12abc^5x^5+3b^2c^4x^4+20abc^3x^3-16b^2c^2x^2-60abcx\right)e-2\left(30b^2c^2d-23b^2e\right)\log(c^2x^2+1)}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(45*a^2*c^6*d*x^4 - 30*a*b*c^5*d*x^3 + 15*b^2*c^4*d*x^2 + 90*a*b*c^3*
d*x + 15*(3*b^2*c^6*d*x^4 - 3*b^2*c^2*d + 2*(b^2*c^6*x^6 + b^2)*e)*arctan(c
*x)^2 + 2*(45*a*b*c^6*d*x^4 - 15*b^2*c^5*d*x^3 + 45*b^2*c^3*d*x - 45*a*b*c^
2*d + 2*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b^2*c*x + 15*a
*b)*e)*arctan(c*x) + (30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*
a*b*c^3*x^3 - 16*b^2*c^2*x^2 - 60*a*b*c*x)*e - 2*(30*b^2*c^2*d - 23*b^2*e)*
log(c^2*x^2 + 1))/c^6
```

Sympy [A]
time = 0.65, size = 398, normalized size = 1.47

$$\begin{cases} \frac{a^2dx^4}{4} + \frac{a^2ex^6}{6} + \frac{abdx^4\operatorname{atan}(cx)}{2} + \frac{abex^6\operatorname{atan}(cx)}{3} - \frac{abdx^3}{6c} - \frac{abex^5}{15c} + \frac{abdx}{2c^3} + \frac{abex^3}{9c^3} - \frac{abd\operatorname{atan}(cx)}{2c^4} - \frac{abex}{3c^5} + \frac{abe\operatorname{atan}(cx)}{3c^6} + \frac{b^2dx^4\operatorname{atan}^2(cx)}{4} + \frac{b^2ex^6\operatorname{atan}^2(cx)}{6} - \frac{b^2dx^3\operatorname{atan}(cx)}{6c} - \frac{b^2ex^5\operatorname{atan}(cx)}{15c} + \frac{b^2dx^2}{12c^2} + \frac{b^2ex^4}{60c^2} + \frac{b^2dx\operatorname{atan}(cx)}{2c^3} + \frac{b^2ex^3\operatorname{atan}(cx)}{9c^3} - \frac{b^2d\log\left(x^2+\frac{1}{c^2}\right)}{3c^4} - \frac{b^2d\operatorname{atan}^2(cx)}{4c^4} - \frac{4b^2ex^2}{45c^4} - \frac{b^2ex\operatorname{atan}(cx)}{3c^5} + \frac{23b^2e\log\left(x^2+\frac{1}{c^2}\right)}{90c^6} + \frac{b^2e\operatorname{atan}^2(cx)}{6c^6} & \text{for}\: c \neq 0 \\ a^2\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x**4/4 + a**2*e*x**6/6 + a*b*d*x**4*atan(c*x)/2 + a*b*e*x
**6*atan(c*x)/3 - a*b*d*x**3/(6*c) - a*b*e*x**5/(15*c) + a*b*d*x/(2*c**3) +
 a*b*e*x**3/(9*c**3) - a*b*d*atan(c*x)/(2*c**4) - a*b*e*x/(3*c**5) + a*b*e*
atan(c*x)/(3*c**6) + b**2*d*x**4*atan(c*x)**2/4 + b**2*e*x**6*atan(c*x)**2/
6 - b**2*d*x**3*atan(c*x)/(6*c) - b**2*e*x**5*atan(c*x)/(15*c) + b**2*d*x**
2/(12*c**2) + b**2*e*x**4/(60*c**2) + b**2*d*x*atan(c*x)/(2*c**3) + b**2*e*
x**3*atan(c*x)/(9*c**3) - b**2*d*log(x**2 + c**(-2))/(3*c**4) - b**2*d*atan
(c*x)**2/(4*c**4) - 4*b**2*e*x**2/(45*c**4) - b**2*e*x*atan(c*x)/(3*c**5) +
 23*b**2*e*log(x**2 + c**(-2))/(90*c**6) + b**2*e*atan(c*x)**2/(6*c**6), Ne
(c, 0)), (a**2*(d*x**4/4 + e*x**6/6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.56, size = 338, normalized size = 1.25

$$\frac{46\,b^2\,e\,\ln\left(c^2\,x^2+1\right)+30\,b^2\,e\,\mathrm{atan}(c\,x)^2-60\,b^2\,c^2\,d\,\ln\left(c^2\,x^2+1\right)+45\,a^2\,c^6\,d\,x^4+15\,b^2\,c^4\,d\,x^2+30\,a^2\,c^6\,e\,x^6-16\,b^2\,c^2\,e\,x^2+3\,b^2\,c^4\,e\,x^4+60\,a\,b\,e\,\mathrm{atan}(c\,x)-45\,b^2\,c^2\,d\,\mathrm{atan}(c\,x)^2+45\,b^2\,c^6\,d\,x^4\,\mathrm{atan}(c\,x)^2+30\,b^2\,c^6\,e\,x^6\,\mathrm{atan}(c\,x)^2-30\,a\,b\,c^5\,d\,x^3+20\,a\,b\,c^3\,e\,x^3-12\,a\,b\,c^5\,e\,x^5+90\,b^2\,c^3\,d\,x\,\mathrm{atan}(c\,x)-60\,a\,b\,c\,e\,x-30\,b^2\,c^5\,d\,x^3\,\mathrm{atan}(c\,x)+20\,b^2\,c^3\,e\,x^3\,\mathrm{atan}(c\,x)-12\,b^2\,c^5\,e\,x^5\,\mathrm{atan}(c\,x)+90\,a\,b\,c^3\,d\,x-90\,a\,b\,c^2\,d\,\mathrm{atan}(c\,x)-60\,b^2\,c\,e\,x\,\mathrm{atan}(c\,x)+90\,a\,b\,c^6\,d\,x^4\,\mathrm{atan}(c\,x)+60\,a\,b\,c^6\,e\,x^6\,\mathrm{atan}(c\,x)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

```
[Out] (46*b^2*e*log(c^2*x^2 + 1) + 30*b^2*e*atan(c*x)^2 - 60*b^2*c^2*d*log(c^2*x^
2 + 1) + 45*a^2*c^6*d*x^4 + 15*b^2*c^4*d*x^2 + 30*a^2*c^6*e*x^6 - 16*b^2*c^
2*e*x^2 + 3*b^2*c^4*e*x^4 + 60*a*b*e*atan(c*x) - 45*b^2*c^2*d*atan(c*x)^2 +
 45*b^2*c^6*d*x^4*atan(c*x)^2 + 30*b^2*c^6*e*x^6*atan(c*x)^2 - 30*a*b*c^5*d
*x^3 + 20*a*b*c^3*e*x^3 - 12*a*b*c^5*e*x^5 + 90*b^2*c^3*d*x*atan(c*x) - 60*
a*b*c*e*x - 30*b^2*c^5*d*x^3*atan(c*x) + 20*b^2*c^3*e*x^3*atan(c*x) - 12*b^
2*c^5*e*x^5*atan(c*x) + 90*a*b*c^3*d*x - 90*a*b*c^2*d*atan(c*x) - 60*b^2*c*
e*x*atan(c*x) + 90*a*b*c^6*d*x^4*atan(c*x) + 60*a*b*c^6*e*x^6*atan(c*x))/(1
80*c^6)
```

3.1248 $\int x^2(d+ex^2)\left(a+b\mathbf{ArcTan}(cx)\right)^2\,dx$

Optimal. Leaf size=323

$$\frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\text{ArcTan}(cx)}{3c^3}+\frac{3b^2e\text{ArcTan}(cx)}{10c^5}-\frac{bdx^2(a+b\text{ArcTan}(cx))}{3c}+\frac{bex^2(a+b\text{ArcTan}(cx))}{5c^3}$$

```
[Out] 1/3*b^2*d*x/c^2-3/10*b^2*e*x/c^4+1/30*b^2*e*x^3/c^2-1/3*b^2*d*arctan(c*x)/c
^3+3/10*b^2*e*arctan(c*x)/c^5-1/3*b*d*x^2*(a+b*arctan(c*x))/c+1/5*b*e*x^2*(
a+b*arctan(c*x))/c^3-1/10*b*e*x^4*(a+b*arctan(c*x))/c+1/5*I*b^2*e*polylog(2
,1-2/(1+I*c*x))/c^5-1/3*I*d*(a+b*arctan(c*x))^2/c^3+1/3*d*x^3*(a+b*arctan(c
*x))^2+1/5*e*x^5*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c^3+2/5*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+1/5*I*e*(a+b*arctan(c
*x))^2/c^5-1/3*I*b^2*d*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A]
time = 0.43, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5100, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 308}

$$\frac{ie(a+b\text{ArcTan}(cx))^2}{5c^5}+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{5c^5}-\frac{id(a+b\text{ArcTan}(cx))^2}{3c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}+\frac{bex^2(a+b\text{ArcTan}(cx))}{5c^3}+\frac{1}{3}dx^3(a+b\text{ArcTan}(cx))^2-\frac{bdx^2(a+b\text{ArcTan}(cx))}{3c}+\frac{1}{5}ex^5(a+b\text{ArcTan}(cx))^2-\frac{bex^4(a+b\text{ArcTan}(cx))}{10c}+\frac{3b^2e\text{ArcTan}(cx)}{10c^5}-\frac{b^2d\text{ArcTan}(cx)}{3c^3}+\frac{ib^2e\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2ex}{10c^4}-\frac{ib^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}+\frac{b^2ex^3}{30c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*d*x)/(3*c^2) - (3*b^2*e*x)/(10*c^4) + (b^2*e*x^3)/(30*c^2) - (b^2*d*Ar
cTan[c*x])/(3*c^3) + (3*b^2*e*ArcTan[c*x])/(10*c^5) - (b*d*x^2*(a + b*ArcTa
n[c*x]))/(3*c) + (b*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e*x^4*(a + b*Ar
cTan[c*x]))/(10*c) - ((I/3)*d*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e*(a + b*
ArcTan[c*x])^2)/c^5 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (e*x^5*(a + b*ArcTa
n[c*x])^2)/5 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (2*
b*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - ((I/3)*b^2*d*PolyLog[
2, 1 - 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^
5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int x^2(d+ex^2)\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(dx^2\left(a+b\tan^{-1}(cx)\right)^2+ex^4\left(a+b\tan^{-1}(cx)\right)^2\right)dx \\
&= d\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx \\
&= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bcd)\int \frac{x^3}{} \\
&= \frac{1}{3}dx^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}ex^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2bd)\int x(a+}{} \\
&= -\frac{bdx^2(a+b\tan^{-1}(cx))}{3c}-\frac{bex^4(a+b\tan^{-1}(cx))}{10c}-\frac{id(a+b\tan^{-1}(}{3c^3} \\
&= \frac{b^2dx}{3c^2}-\frac{bdx^2(a+b\tan^{-1}(cx))}{3c}+\frac{bex^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bex^4(a+}{} \\
&= \frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\tan^{-1}(cx)}{3c^3}-\frac{bdx^2(a+b\tan^{-1}(cx))}{3c}+ \\
&= \frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{3b^2e\tan^{-1}(cx)}{10c^5}-\frac{bdx^2(a}{} \\
&= \frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\tan^{-1}(cx)}{3c^3}+\frac{3b^2e\tan^{-1}(cx)}{10c^5}-\frac{bdx^2(a}{}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 287, normalized size = 0.89

$$\frac{9abe+10b^2c^3dx-9b^2cex-10abc^4dx^2+6abc^2ex^2+10a^2c^5dx^3+b^2c^3ex^3-3abc^4ex^4+6a^2c^5ex^5+2b^2(5ic^2d-3ie+c^5(5dx^3+3ex^5))\,\mathrm{ArcTan}(cx)^2-b\,\mathrm{ArcTan}(cx)\left(-4ac^5x^3(5d+3ex^2)+b(1+c^2x^2)\left(-9e+c^2(10d+3ex^2)\right)+4b(5c^2d-3e)\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)+10abc^2d\log\left(1+c^2x^2\right)-6abe\log\left(1+c^2x^2\right)+2ib^2(5c^2d-3e)\,\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)}{30c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

[Out] (9*a*b*e + 10*b^2*c^3*d*x - 9*b^2*c*e*x - 10*a*b*c^4*d*x^2 + 6*a*b*c^2*e*x^2 + 10*a^2*c^5*d*x^3 + b^2*c^3*e*x^3 - 3*a*b*c^4*e*x^4 + 6*a^2*c^5*e*x^5 + 2*b^2*((5*I)*c^2*d - (3*I)*e + c^5*(5*d*x^3 + 3*e*x^5))*ArcTan[c*x]^2 - b*ArcTan[c*x]*(-4*a*c^5*x^3*(5*d + 3*e*x^2) + b*(1 + c^2*x^2)*(-9*e + c^2*(10*d + 3*e*x^2)) + 4*b*(5*c^2*d - 3*e)*Log[1 + E^((2*I)*ArcTan[c*x])]) + 10*a*b*c^2*d*Log[1 + c^2*x^2] - 6*a*b*e*Log[1 + c^2*x^2] + (2*I)*b^2*(5*c^2*d - 3*e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(283) = 566.
time = 0.86, size = 648, normalized size = 2.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^3*(a^2/c^2*(1/3*d*c^5*x^3+1/5*e*c^5*x^5)+2/3*a*b*arctan(c*x)*d*c^3*x^3-1/10*a*b*c^2*e*x^4-1/10*I*b^2/c^2*ln(c*x-I)*ln(c^2*x^2+1)*e+1/10*I*b^2/c^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*e+1/10*I*b^2/c^2*ln(I+c*x)*ln(c^2*x^2+1)*e-1/10*I*b^2/c^2*ln(1/2*I*(c*x-I))*ln(I+c*x)*e+1/5*b^2*c^3*arctan(c*x)^2*e*x^5-1/10*b^2*c^2*arctan(c*x)*e*x^4+1/5*b^2*arctan(c*x)*e*x^2+1/5*a*b*e*x^2-1/5*b^2/c^2*arctan(c*x)*ln(c^2*x^2+1)*e-1/3*b^2*arctan(c*x)*d+1/10*I*b^2/c^2*dilog(-1/2*I*(I+c*x))*e-1/20*I*b^2/c^2*ln(I+c*x)^2*e-1/10*I*b^2/c^2*dilog(1/2*I*(c*x-I))*e-1/3*b^2*arctan(c*x)*d*c^2*x^2+1/3*b^2*arctan(c*x)^2*d*c^3*x^3+1/30*b^2*c*e*x^3-3/10*b^2/c*e*x+2/5*a*b*c^3*arctan(c*x)*e*x^5+1/6*I*b^2*dilog(1/2*I*(c*x-I))*d+1/3*b^2*d*c*x+3/10*b^2/c^2*arctan(c*x)*e+1/3*b^2*arctan(c*x)*ln(c^2*x^2+1)*d+1/3*a*b*ln(c^2*x^2+1)*d-1/12*I*b^2*ln(c*x-I)^2*d-1/6*I*b^2*dilog(-1/2*I*(I+c*x))*d+1/12*I*b^2*ln(I+c*x)^2*d-1/5*a*b/c^2*ln(c^2*x^2+1)*e-1/6*I*b^2*ln(I+c*x)*ln(c^2*x^2+1)*d+1/6*I*b^2*ln(1/2*I*(c*x-I))*ln(I+c*x)*d+1/6*I*b^2*ln(c*x-I)*ln(c^2*x^2+1)*d-1/6*I*b^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*d+1/20*I*b^2/c^2*ln(c*x-I)^2*e-1/3*a*b*d*c^2*x^2)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] 1/5*a^2*x^5*e + 1/3*a^2*d*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*e + 1/60*(3*b^2*x^5*e + 5*b^2*d*x^3)*arctan(c*x)^2 - 1/240*(3*b^2*x^5*e + 5*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + integrate(1/240*(180*(b^2*c^2*x^6*e + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*arctan(c*x)^2 + 15*(b^2*c^2*x^6*e + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*log(c^2*x^2 + 1)^2 - 8*(3*b^2*c*x^5*e + 5*b^2*c*d*x^3)*arctan(c*x) + 4*(3*b^2*c^2*x^6*e + 5*b^2*c^2*d*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^4*e + a^2*d*x^2 + (b^2*x^4*e + b^2*d*x^2)*arctan(c*x)^2 + 2*
(a*b*x^4*e + a*b*d*x^2)*arctan(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{atan}(cx))^2 \left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a + b\operatorname{atan}(c\,x))^2\,\left(e\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2*(d + e*x^2), x)
```

3.1249 $\int x(d+ex^2)\,(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=199

$$-\frac{abdx}{c}+\frac{abex}{2c^3}+\frac{b^2ex^2}{12c^2}-\frac{b^2dx\text{ArcTan}(cx)}{c}+\frac{b^2ex\text{ArcTan}(cx)}{2c^3}-\frac{bex^3(a+b\text{ArcTan}(cx))}{6c}+\frac{d(a+b\text{ArcTan}(cx))^2}{2c^2}$$

```
[Out] -a*b*d*x/c+1/2*a*b*e*x/c^3+1/12*b^2*e*x^2/c^2-b^2*d*x*arctan(c*x)/c+1/2*b^2
*e*x*arctan(c*x)/c^3-1/6*b*e*x^3*(a+b*arctan(c*x))/c+1/2*d*(a+b*arctan(c*x)
)^2/c^2-1/4*e*(a+b*arctan(c*x))^2/c^4+1/2*d*x^2*(a+b*arctan(c*x))^2+1/4*e*x
^4*(a+b*arctan(c*x))^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2+1)/
c^4
```

Rubi [A]
time = 0.29, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5100, 4946, 5036, 4930, 266, 5004, 272, 45}

$$-\frac{e(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{d(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))^2+\frac{1}{4}ex^4(a+b\text{ArcTan}(cx))^2-\frac{bex^3(a+b\text{ArcTan}(cx))}{6c}+\frac{abex}{2c^3}-\frac{abdx}{c}+\frac{b^2ex\text{ArcTan}(cx)}{2c^3}-\frac{b^2dx\text{ArcTan}(cx)}{c}+\frac{b^2d\log(c^2x^2+1)}{2c^2}+\frac{b^2ex^2}{12c^2}-\frac{b^2e\log(c^2x^2+1)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*d*x)/c) + (a*b*e*x)/(2*c^3) + (b^2*e*x^2)/(12*c^2) - (b^2*d*x*ArcTan
[c*x])/c + (b^2*e*x*ArcTan[c*x])/(2*c^3) - (b*e*x^3*(a + b*ArcTan[c*x]))/(6
*c) + (d*(a + b*ArcTan[c*x])^2)/(2*c^2) - (e*(a + b*ArcTan[c*x])^2)/(4*c^4)
 + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (e*x^4*(a + b*ArcTan[c*x])^2)/4 + (b^2
*d*Log[1 + c^2*x^2])/(2*c^2) - (b^2*e*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int x(d+ex^2)\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(dx\left(a+b\tan^{-1}(cx)\right)^2+ex^3\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx$$
$$=d\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx$$
$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-(bcd)\int\frac{x^2(a+}{1}$$
$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{4}ex^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{(bd)\int(a+b\tan}{c}$$
$$=-\frac{abdx}{c}-\frac{bex^3(a+b\tan^{-1}(cx))}{6c}+\frac{d(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{1}{2}dx^2\left(a+b\right.$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}-\frac{b^2dx\tan^{-1}(cx)}{c}-\frac{bex^3(a+b\tan^{-1}(cx))}{6c}+\frac{d(a+b}{}$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}-\frac{b^2dx\tan^{-1}(cx)}{c}+\frac{b^2ex\tan^{-1}(cx)}{2c^3}-\frac{bex^3(a+b\tan}{6c}$$
$$=-\frac{abdx}{c}+\frac{abex}{2c^3}+\frac{b^2ex^2}{12c^2}-\frac{b^2dx\tan^{-1}(cx)}{c}+\frac{b^2ex\tan^{-1}(cx)}{2c^3}-\frac{bex^3(a}{}$$

Mathematica [A]

time = 0.08, size = 179, normalized size = 0.90

$$\frac{cx(6abe+b^2cex+3a^2c^3x(2d+ex^2)-2abc^2(6d+ex^2))+2b(6ac^2d-3ae+3bcex-bc^3x(6d+ex^2)+3ac^4(2dx^2+ex^4))\,\mathrm{ArcTan}(cx)+3b^2(2c^2d-e+c^4(2dx^2+ex^4))\,\mathrm{ArcTan}(cx)^2+2b^2(3c^2d-2e)\log(1+c^2x^2)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b*e + b^2*c*e*x + 3*a^2*c^3*x*(2*d + e*x^2) - 2*a*b*c^2*(6*d + e*
x^2)) + 2*b*(6*a*c^2*d - 3*a*e + 3*b*c*e*x - b*c^3*x*(6*d + e*x^2) + 3*a*c^
4*(2*d*x^2 + e*x^4))*ArcTan[c*x] + 3*b^2*(2*c^2*d - e + c^4*(2*d*x^2 + e*x^
4))*ArcTan[c*x]^2 + 2*b^2*(3*c^2*d - 2*e)*Log[1 + c^2*x^2])/(12*c^4)
```

Maple [A]

time = 0.42, size = 254, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{(c^2ex^2+c^2d)^2a^2}{4c^2e}+\frac{b^2\arctan(cx)^2dc^2x^2}{2}+\frac{b^2c^2e\arctan(cx)^2x^4}{4}-b^2\arctan(cx)dcx-\frac{b^2c\arctan(cx)ex^3}{6}+\frac{b^2ex\arctan(cx)}{2c}+\frac{b^2\,\mathrm{arct}}{}}{}$
default	$\frac{\frac{(c^2ex^2+c^2d)^2a^2}{4c^2e}+\frac{b^2\arctan(cx)^2dc^2x^2}{2}+\frac{b^2c^2e\arctan(cx)^2x^4}{4}-b^2\arctan(cx)dcx-\frac{b^2c\arctan(cx)ex^3}{6}+\frac{b^2ex\arctan(cx)}{2c}+\frac{b^2\,\mathrm{arct}}{}}{}$

risch	$-\frac{b^2(c^4 e\,x^4+2c^4 d\,x^2+2c^2 d-e)\ln(icx+1)^2}{16c^4}+\frac{iabe\,x^4\ln(-icx+1)}{4}-\frac{b^2 e\,x^4\ln(-icx+1)^2}{16}-\frac{ib(6a\,c^4 e\,x^4+3ib\,c^4 e\,x^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*(c^2*e*x^2+c^2*d)^2*a^2/c^2/e+1/2*b^2*arctan(c*x)^2*d*c^2*x^2+1/
4*b^2*c^2*e*arctan(c*x)^2*x^4-b^2*arctan(c*x)*d*c*x-1/6*b^2*c*arctan(c*x)*e
*x^3+1/2*b^2*e*x*arctan(c*x)/c+1/2*b^2*arctan(c*x)^2*d-1/4*b^2/c^2*arctan(c
*x)^2*e+1/12*b^2*e*x^2+1/2*b^2*ln(c^2*x^2+1)*d-1/3*b^2*e*ln(c^2*x^2+1)/c^2+
a*b*arctan(c*x)*d*c^2*x^2+1/2*a*b*c^2*e*arctan(c*x)*x^4-a*b*d*c*x-1/6*a*b*c
*e*x^3+1/2*a*b*e*x/c+a*b*arctan(c*x)*d-1/2*a*b/c^2*arctan(c*x)*e)
```

Maxima [A]
time = 0.50, size = 251, normalized size = 1.26

$$\frac{1}{4}b^2x^4\arctan(cx)^2e+\frac{1}{2}b^2dx^2\arctan(cx)^2+\frac{1}{4}a^2x^4e+\frac{1}{2}a^2dx^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)abd-\frac{1}{2}\left(2c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\arctan(cx)+\frac{\arctan(cx)^2-\log(c^2x^2+1)}{c^2}\right)b^2d+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abe-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x)^2*e + 1/2*b^2*d*x^2*arctan(c*x)^2 + 1/4*a^2*x^4*e +
 1/2*a^2*d*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/
2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2
 + 1))/c^2)*b^2*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arc
tan(c*x)/c^5))*a*b*e - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*
arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*e
```

Fricas [A]
time = 2.46, size = 218, normalized size = 1.10

$$\frac{6a^2c^4dx^2-12abc^3dx+3\left(2b^2c^4dx^2+2b^2c^2d+(b^2c^4x^4-b^2)e\right)\arctan(cx)^2+2\left(6abc^4dx^2-6b^2c^3dx+6abc^2d+(3abc^4x^4-b^2c^3x^3+3b^2cx-3ab)e\right)\arctan(cx)+(3a^2c^4x^4-2abc^3x^3+b^2c^2x^2+6abcx)e+2(3b^2c^2d-2b^2e)\log(c^2x^2+1)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*c^4*d*x^2 - 12*a*b*c^3*d*x + 3*(2*b^2*c^4*d*x^2 + 2*b^2*c^2*d +
 (b^2*c^4*x^4 - b^2)*e)*arctan(c*x)^2 + 2*(6*a*b*c^4*d*x^2 - 6*b^2*c^3*d*x
+ 6*a*b*c^2*d + (3*a*b*c^4*x^4 - b^2*c^3*x^3 + 3*b^2*c*x - 3*a*b)*e)*arctan
(c*x) + (3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x)*e + 2*(3*
b^2*c^2*d - 2*b^2*e)*log(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 0.42, size = 296, normalized size = 1.49

$$\begin{cases}\frac{a^2dx^2}{2}+\frac{a^2ex^4}{4}+abdx^2\operatorname{atan}(cx)+\frac{abex^4\operatorname{atan}(cx)}{2}-\frac{abdx}{c}-\frac{abex^3}{6c}+\frac{abd\operatorname{atan}(cx)}{c^2}+\frac{abex}{2c^3}-\frac{abe\operatorname{atan}(cx)}{2c^4}+\frac{b^2dx^2\operatorname{atan}^2(cx)}{2}+\frac{b^2ex^4\operatorname{atan}^2(cx)}{4}-\frac{b^2dx\operatorname{atan}(cx)}{c}-\frac{b^2ex^3\operatorname{atan}(cx)}{6c}+\frac{b^2d\log\left(x^2+\frac{1}{c^2}\right)}{2c^2}+\frac{b^2d\operatorname{atan}^2(cx)}{2c^2}+\frac{b^2ex^2}{12c^2}+\frac{b^2ex\operatorname{atan}(cx)}{2c^3}-\frac{b^2e\log\left(x^2+\frac{1}{c^2}\right)}{3c^4}-\frac{b^2e\operatorname{atan}^2(cx)}{4c^4} & \text{for } c\neq 0\\ a^2\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x**2/2 + a**2*e*x**4/4 + a*b*d*x**2*atan(c*x) + a*b*e*x**
4*atan(c*x)/2 - a*b*d*x/c - a*b*e*x**3/(6*c) + a*b*d*atan(c*x)/c**2 + a*b*e
*x/(2*c**3) - a*b*e*atan(c*x)/(2*c**4) + b**2*d*x**2*atan(c*x)**2/2 + b**2*
e*x**4*atan(c*x)**2/4 - b**2*d*x*atan(c*x)/c - b**2*e*x**3*atan(c*x)/(6*c)
+ b**2*d*log(x**2 + c**(-2))/(2*c**2) + b**2*d*atan(c*x)**2/(2*c**2) + b**2
*e*x**2/(12*c**2) + b**2*e*x*atan(c*x)/(2*c**3) - b**2*e*log(x**2 + c**(-2)
)/(3*c**4) - b**2*e*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*(d*x**2/2 + e*x
**4/4), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 1.00, size = 248, normalized size = 1.25

$$\frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}-\frac{b^2\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^4}+\frac{b^2\,e\,x^2}{12\,c^2}+\frac{b^2\,d\,\mathrm{atan}(c\,x)^2}{2\,c^2}-\frac{b^2\,e\,\mathrm{atan}(c\,x)^2}{4\,c^4}+\frac{b^2\,d\,x^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,e\,x^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{a\,b\,e\,x^3}{6\,c}-\frac{b^2\,d\,x\,\mathrm{atan}(c\,x)}{c}+\frac{b^2\,e\,x\,\mathrm{atan}(c\,x)}{2\,c^3}-\frac{b^2\,e\,x^3\,\mathrm{atan}(c\,x)}{6\,c}-\frac{a\,b\,d\,x}{c}+\frac{a\,b\,e\,x}{2\,c^3}+\frac{a\,b\,d\,\mathrm{atan}(c\,x)}{c^2}-\frac{a\,b\,e\,\mathrm{atan}(c\,x)}{2\,c^4}+a\,b\,d\,x^2\,\mathrm{atan}(c\,x)+\frac{a\,b\,e\,x^4\,\mathrm{atan}(c\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

```
[Out] (a^2*d*x^2)/2 + (a^2*e*x^4)/4 + (b^2*d*log(c^2*x^2 + 1))/(2*c^2) - (b^2*e*l
og(c^2*x^2 + 1))/(3*c^4) + (b^2*e*x^2)/(12*c^2) + (b^2*d*atan(c*x)^2)/(2*c^
2) - (b^2*e*atan(c*x)^2)/(4*c^4) + (b^2*d*x^2*atan(c*x)^2)/2 + (b^2*e*x^4*a
tan(c*x)^2)/4 - (a*b*e*x^3)/(6*c) - (b^2*d*x*atan(c*x))/c + (b^2*e*x*atan(c
*x))/(2*c^3) - (b^2*e*x^3*atan(c*x))/(6*c) - (a*b*d*x)/c + (a*b*e*x)/(2*c^3
) + (a*b*d*atan(c*x))/c^2 - (a*b*e*atan(c*x))/(2*c^4) + a*b*d*x^2*atan(c*x)
 + (a*b*e*x^4*atan(c*x))/2
```

3.1250 $\int (d+ex^2) (a+b\mathbf{ArcTan}(cx))^2 \, dx$

Optimal. Leaf size=231

$$\frac{b^2ex}{3c^2}-\frac{b^2e\text{ArcTan}(cx)}{3c^3}-\frac{bex^2(a+b\text{ArcTan}(cx))}{3c}+\frac{id(a+b\text{ArcTan}(cx))^2}{c}-\frac{ie(a+b\text{ArcTan}(cx))^2}{3c^3}+dx(a+b\text{A}$$

```
[Out] 1/3*b^2*e*x/c^2-1/3*b^2*e*arctan(c*x)/c^3-1/3*b*e*x^2*(a+b*arctan(c*x))/c+I
*d*(a+b*arctan(c*x))^2/c-1/3*I*e*(a+b*arctan(c*x))^2/c^3+d*x*(a+b*arctan(c*
x))^2+1/3*e*x^3*(a+b*arctan(c*x))^2+2*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))
/c-2/3*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+I*b^2*d*polylog(2,1-2/(1+I
*c*x))/c-1/3*I*b^2*e*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A]
time = 0.25, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5034, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 327, 209}

$$-\frac{ie(a+b\text{ArcTan}(cx))^2}{3c^3}-\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}+dx(a+b\text{ArcTan}(cx))^2+\frac{id(a+b\text{ArcTan}(cx))^2}{c}+\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{1}{3}ex^3(a+b\text{ArcTan}(cx))^2-\frac{bex^2(a+b\text{ArcTan}(cx))}{3c}-\frac{b^2e\text{ArcTan}(cx)}{3c^3}-\frac{ib^2e\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2ex}{3c^2}+\frac{ib^2d\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*e*x)/(3*c^2) - (b^2*e*ArcTan[c*x])/(3*c^3) - (b*e*x^2*(a + b*ArcTan[c*
x]))/(3*c) + (I*d*(a + b*ArcTan[c*x])^2)/c - ((I/3)*e*(a + b*ArcTan[c*x])^2
)/c^3 + d*x*(a + b*ArcTan[c*x])^2 + (e*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*b*
d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/(3*c^3) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I
/3)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int \left(d\left(a+b\tan^{-1}(cx)\right)^2+ex^2\left(a+b\tan^{-1}(cx)\right)^2\right)\,dx$$
$$= d\int \left(a+b\tan^{-1}(cx)\right)^2\,dx+e\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx$$
$$= dx\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right)^2-(2bcd)\int \frac{x(a+b\,\text{t}}{1+}$$
$$= \frac{id(a+b\tan^{-1}(cx))^2}{c}+dx\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\tan^{-1}(cx)\right.$$
$$= -\frac{bex^2(a+b\tan^{-1}(cx))}{3c}+\frac{id(a+b\tan^{-1}(cx))^2}{c}-\frac{ie(a+b\tan^{-1}(cx)}{3c^3}$$
$$= \frac{b^2ex}{3c^2}-\frac{bex^2(a+b\tan^{-1}(cx))}{3c}+\frac{id(a+b\tan^{-1}(cx))^2}{c}-\frac{ie(a+b\tan^{-}}{3c^3}$$
$$= \frac{b^2ex}{3c^2}-\frac{b^2e\tan^{-1}(cx)}{3c^3}-\frac{bex^2(a+b\tan^{-1}(cx))}{3c}+\frac{id(a+b\tan^{-1}(cx))}{c}$$
$$= \frac{b^2ex}{3c^2}-\frac{b^2e\tan^{-1}(cx)}{3c^3}-\frac{bex^2(a+b\tan^{-1}(cx))}{3c}+\frac{id(a+b\tan^{-1}(cx))}{c}$$

Mathematica [A]
time = 0.29, size = 208, normalized size = 0.90

$$\frac{3a^2c^3dx+b^2cex-abc^2ex^2+a^2c^3ex^3+b^2(-3ic^2d+ie+c^3(3dx+ex^3))\,\text{ArcTan}(cx)^2-b\text{ArcTan}(cx)\left(-2ac^3x(3d+ex^2)+b(e+c^2ex^2)+2b(-3c^2d+e)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)-3abc^2d\log\left(1+c^2x^2\right)+abe\log\left(1+c^2x^2\right)-ib^2(3c^2d-e)\,\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (3*a^2*c^3*d*x + b^2*c*e*x - a*b*c^2*e*x^2 + a^2*c^3*e*x^3 + b^2*((-3*I)*c^
2*d + I*e + c^3*(3*d*x + e*x^3))*ArcTan[c*x]^2 - b*ArcTan[c*x]*(-2*a*c^3*x*
(3*d + e*x^2) + b*(e + c^2*e*x^2) + 2*b*(-3*c^2*d + e)*Log[1 + E^((2*I)*Arc
Tan[c*x])]) - 3*a*b*c^2*d*Log[1 + c^2*x^2] + a*b*e*Log[1 + c^2*x^2] - I*b^2
*(3*c^2*d - e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(209) = 418.
time = 0.31, size = 550, normalized size = 2.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c*(-1/6*I*b^2/c^2*ln(c^2*x^2+1)*ln(I+c*x)*e+a^2/c^2*(d*c^3*x+1/3*e*c^3*x^3)+2/3*a*b*c*arctan(c*x)*e*x^3-1/3*b^2*arctan(c*x)*e*x^2-1/3*a*b*e*x^2+1/3*b^2*c*arctan(c*x)^2*e*x^3+2*a*b*arctan(c*x)*d*c*x+1/6*I*b^2/c^2*ln(c^2*x^2+1)*ln(c*x-I)*e-1/6*I*b^2/c^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*e+1/6*I*b^2/c^2*ln(1/2*I*(c*x-I))*ln(I+c*x)*e+1/3*b^2/c^2*arctan(c*x)*ln(c^2*x^2+1)*e+1/6*I*b^2/c^2*dilog(1/2*I*(c*x-I))*e+1/2*I*b^2*ln(c^2*x^2+1)*ln(I+c*x)*d+1/12*I*b^2/c^2*ln(I+c*x)^2*e+1/2*I*b^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*d-1/6*I*b^2/c^2*dilog(-1/2*I*(I+c*x))*e-1/2*I*b^2*ln(1/2*I*(c*x-I))*ln(I+c*x)*d+b^2*arctan(c*x)^2*d*c*x-1/12*I*b^2/c^2*ln(c*x-I)^2*e-1/2*I*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d+1/3*b^2/c*e*x-1/3*b^2/c^2*arctan(c*x)*e-b^2*arctan(c*x)*ln(c^2*x^2+1)*d-a*b*ln(c^2*x^2+1)*d+1/3*a*b/c^2*ln(c^2*x^2+1)*e+1/4*I*b^2*ln(c*x-I)^2*d-1/4*I*b^2*ln(I+c*x)^2*d-1/2*I*b^2*dilog(1/2*I*(c*x-I))*d+1/2*I*b^2*dilog(-1/2*I*(I+c*x))*d)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] 1/3*a^2*x^3*e + 36*b^2*c^2*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 36*b^2*c^2*d*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*c^2*d*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 1/4*b^2*d*arctan(c*x)^3/c - 8*b^2*c*e*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b^2*c*d*integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d*x + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e + 36*b^2*e*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*b^2*d*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c + 1/12*(b^2*x^3*e + 3*b^2*d*x)*arctan(c*x)^2 - 1/48*(b^2*x^3*e + 3*b^2*d*x)*log(c^2*x^2 + 1)^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arctan(c*x)^2 + 2*(a*b*x^2
*e + a*b*d)*arctan(c*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}\,(cx))^2\,(d + ex^2)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{atan}(c\,x))^2\,(e\,x^2 + d)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d + e*x^2),x)
```

```
[Out] int((a + b*atan(c*x))^2*(d + e*x^2), x)
```

3.1251 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))^2}{x}\,dx$

Optimal. Leaf size=217

$$-\frac{abex}{c}-\frac{b^2ex\text{ArcTan}(cx)}{c}+\frac{e(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))^2+2d(a+b\text{ArcTan}(cx))^2\tanh^{-1}\Big(1$$

```
[Out] -a*b*e*x/c-b^2*e*x*arctan(c*x)/c+1/2*e*(a+b*arctan(c*x))^2/c^2+1/2*e*x^2*(a
+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+1/2*b^2*e
*ln(c^2*x^2+1)/c^2-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d*(
a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(1+I*c*x
))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]

time = 0.32, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5100, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 4930, 266}

$$\frac{e(a+b\text{ArcTan}(cx))^2}{2c^2}-ibd\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibd\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+2d\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))^2-\frac{abex}{c}-\frac{b^2ex\text{ArcTan}(cx)}{c}+\frac{b^2e\log\left(c^2x^2+1\right)}{2c^2}-\frac{1}{2}b^2d\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (e*(a + b*ArcTan[c*x])^2)/(2*c^2
) + (e*x^2*(a + b*ArcTan[c*x])^2)/2 + 2*d*(a + b*ArcTan[c*x])^2*ArcTanh[1 -
 2/(1 + I*c*x)] + (b^2*e*Log[1 + c^2*x^2])/(2*c^2) - I*b*d*(a + b*ArcTan[c*
x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, -1
 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
```

```
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\tan^{-1}(cx))^2}{x}\,dx &= \int \left(\frac{d(a+b\tan^{-1}(cx))^2}{x}+ex\big(a+b\tan^{-1}(cx)\big)^2\right)\,dx \\
&= d\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx+e\int x\big(a+b\tan^{-1}(cx)\big)^2\,dx \\
&= \frac{1}{2}ex^2\big(a+b\tan^{-1}(cx)\big)^2+2d\big(a+b\tan^{-1}(cx)\big)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) \\
&= \frac{1}{2}ex^2\big(a+b\tan^{-1}(cx)\big)^2+2d\big(a+b\tan^{-1}(cx)\big)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right) \\
&= -\frac{abex}{c}+\frac{e(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{1}{2}ex^2\big(a+b\tan^{-1}(cx)\big)^2+2d\big(a+b\tan \\
&= -\frac{abex}{c}-\frac{b^2ex\tan^{-1}(cx)}{c}+\frac{e(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{1}{2}ex^2\big(a+b\tan^{-1}(cx \\
&= -\frac{abex}{c}-\frac{b^2ex\tan^{-1}(cx)}{c}+\frac{e(a+b\tan^{-1}(cx))^2}{2c^2}+\frac{1}{2}ex^2\big(a+b\tan^{-1}(cx
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 263, normalized size = 1.21

$$
\frac{1}{2}a^2ex^2+\frac{abe(-cx+(1+c^2x^2)\,\mathrm{ArcTan}(cx))}{c^2}+a^2d\log(x)+\frac{b^2e(-2cx\mathrm{ArcTan}(cx)+(1+c^2x^2)\,\mathrm{ArcTan}(cx)^2+\log(1+c^2x^2))}{2c^2}+iabd(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+b^2d\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\mathrm{ArcTan}(cx)^3+\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\mathrm{ArcTan}(cx)}\right)-\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)+i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,e^{-2i\mathrm{ArcTan}(cx)}\right)+i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\mathrm{ArcTan}(cx)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (a^2*e*x^2)/2 + (a*b*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^2 + a^2*d*Lo
g[x] + (b^2*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c
^2*x^2]))/(2*c^2) + I*a*b*d*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^
2*d*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-
2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTa
n[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2
```

```
*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((
2*I)*ArcTan[c*x])]/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.96, size = 1284, normalized size = 5.92

method	result	size
derivativedivides	Expression too large to display	1284
default	Expression too large to display	1284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*arctan(c*x)^2*d*ln(c*x)-b^2/c^2*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-b^2*d*a
rctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*d*arctan(c*x)^2*ln(1+(1+I*c*
x)/(c^2*x^2+1)^(1/2))+b^2*d*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))
+a*b/c^2*arctan(c*x)*e+1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-a*b*e*x/c-b^2*e*x*arctan(c*x)/c+1/2*b
^2/c^2*arctan(c*x)^2*e+1/2*I*b^2*d*Pi*arctan(c*x)^2-2*I*b^2*d*arctan(c*x)*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*d*arctan(c*x)*polylog(2,-(1+I
*c*x)/(c^2*x^2+1)^(1/2))+I*a*b*d*ln(c*x)*ln(1+I*c*x)+2*a*b*arctan(c*x)*d*ln
(c*x)+I*a*b*d*dilog(1+I*c*x)-I*a*b*d*dilog(1-I*c*x)+I*b^2/c^2*arctan(c*x)*e
+I*b^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*b^2*d*Pi*csg
n(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*a*b*d*ln(c*x)*ln(1-I*c*x)+1/2*b^2*a
rctan(c*x)^2*e*x^2-1/2*I*b^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2*d*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+
a^2*d*ln(c*x)+2*b^2*d*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*d*polyl
og(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b^2*d*polylog(3,-(1+I*c*x)^2/(c^2*x^2
+1))+1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arct
an(c*x)^2-1/2*I*b^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2
*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+a*
b*arctan(c*x)*e*x^2+1/2*a^2*e*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^2*arctan(c*x)^2*e - 1/32*b^2*x^2*e*log(c^2*x^2 + 1)^2 + 12*b^2*c^
2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*c^2*e*integrat
e(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*e*integrate(1/
16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 2*b^2*c^2*e*integrate(1/16*x^4*log(c
^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3
 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + 1/2*a^2*x^2*e - 4*b^2*c*e*integ
rate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e*integrate(1/16*x^2*a
rctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*e*integrate(1/16*x^2*arctan(c*x)/(c
^2*x^3 + x), x) + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) +
 b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integ
rate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*e*log(c^2*x^2 + 1)^3/c^2
 + a^2*d*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arctan(c*x)^2 + 2*(a*b*x^
2*e + a*b*d)*arctan(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x \right)}\right)^{2} \left(d + e x^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2))/x,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2))/x, x)
```

3.1252 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=172

$$-icd(a+b\text{ArcTan}(cx))^2+\frac{ie(a+b\text{ArcTan}(cx))^2}{c}-\frac{d(a+b\text{ArcTan}(cx))^2}{x}+ex(a+b\text{ArcTan}(cx))^2+\frac{2be(a+b\text{Arc}}{}$$

```
[Out] -I*c*d*(a+b*arctan(c*x))^2+I*e*(a+b*arctan(c*x))^2/c-d*(a+b*arctan(c*x))^2/
x+e*x*(a+b*arctan(c*x))^2+2*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+2*b*c*d
*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d*polylog(2,-1+2/(1-I*c*x))+I*
b^2*e*polylog(2,1-2/(1+I*c*x))/c
```

Rubi [A]
time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5100, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497}

$$-icd(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+\frac{ie(a+b\text{ArcTan}(cx))^2}{c}+ex(a+b\text{ArcTan}(cx))^2+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}-ib^2cd\text{Li}_2\left(\frac{2}{1-icx}-1\right)+\frac{ib^2e\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-I)*c*d*(a + b*ArcTan[c*x])^2 + (I*e*(a + b*ArcTan[c*x])^2)/c - (d*(a + b*
ArcTan[c*x])^2)/x + e*x*(a + b*ArcTan[c*x])^2 + (2*b*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/c + 2*b*c*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)]
- I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + (I*b^2*e*PolyLog[2, 1 - 2/(1 +
 I*c*x)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\tan^{-1}(cx))^2}{x^2}\,dx &= \int \left(e\big(a+b\tan^{-1}(cx)\big)^2+\frac{d(a+b\tan^{-1}(cx))^2}{x^2}\right)dx \\
&= d\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx+e\int \big(a+b\tan^{-1}(cx)\big)^2\,dx \\
&= -\frac{d(a+b\tan^{-1}(cx))^2}{x}+ex\big(a+b\tan^{-1}(cx)\big)^2+(2bcd)\int \frac{a+b\tan^{-1}(c}{x\,(1+c^2x^2} \\
&= -icd\big(a+b\tan^{-1}(cx)\big)^2+\frac{ie(a+b\tan^{-1}(cx))^2}{c}-\frac{d(a+b\tan^{-1}(cx))^2}{x}+ \\
&= -icd\big(a+b\tan^{-1}(cx)\big)^2+\frac{ie(a+b\tan^{-1}(cx))^2}{c}-\frac{d(a+b\tan^{-1}(cx))^2}{x}+ \\
&= -icd\big(a+b\tan^{-1}(cx)\big)^2+\frac{ie(a+b\tan^{-1}(cx))^2}{c}-\frac{d(a+b\tan^{-1}(cx))^2}{x}+ \\
&= -icd\big(a+b\tan^{-1}(cx)\big)^2+\frac{ie(a+b\tan^{-1}(cx))^2}{c}-\frac{d(a+b\tan^{-1}(cx))^2}{x}+
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 204, normalized size = 1.19

$$
\frac{-a^2cd+a^2cex^2+abcd(-2\text{ArcTan}(cx)+cx(2\log(cx)-\log(1+c^2x^2)))+abex(2cx\text{ArcTan}(cx)-\log(1+c^2x^2))+b^2ex\left(\text{ArcTan}(cx)\left((-i+cx)\text{ArcTan}(cx)+2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)-b^2cd\left(\text{ArcTan}(cx)^2-2cx\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)+icx\left(\text{ArcTan}(cx)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)\right)}{cx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (-(a^2*c*d) + a^2*c*e*x^2 + a*b*c*d*(-2*ArcTan[c*x] + c*x*(2*Log[c*x] - Log
[1 + c^2*x^2])) + a*b*e*x*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) + b^2*e*x*
(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) -
I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) - b^2*c*d*(ArcTan[c*x]^2 - 2*c*x*ArcT
an[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2,
E^((2*I)*ArcTan[c*x])])))/(c*x)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(164) = 328$.
time = 0.92, size = 599, normalized size = 3.48

method	result
derivativedivides	$c\left(-\frac{ib^2\ln(cx-i)\ln(c^2x^2+1)e}{2c^2}+\frac{ib^2\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)e}{2c^2}+\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{2ab\arctan(cx)ex}{c}+\frac{b^2\arctan(c}{c}\right.$
default	$c\left(-\frac{ib^2\ln(cx-i)\ln(c^2x^2+1)e}{2c^2}+\frac{ib^2\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)e}{2c^2}+\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{2ab\arctan(cx)ex}{c}+\frac{b^2\arctan(c}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*I*b^2/c^2*ln(c^2*x^2+1)*ln(c*x-I)*e+a^2/c^2*(e*c*x-d*c/x)+2*a*b/c*a
rctan(c*x)*e*x+b^2/c*arctan(c*x)^2*e*x+1/2*I*b^2/c^2*ln(-1/2*I*(I+c*x))*ln(
c*x-I)*e+1/2*I*b^2/c^2*ln(c^2*x^2+1)*ln(I+c*x)*e-1/2*I*b^2/c^2*ln(1/2*I*(c*
x-I))*ln(I+c*x)*e-2*a*b*arctan(c*x)*d/c/x+I*b^2*d*ln(c*x)*ln(1+I*c*x)-I*b^2
*d*ln(c*x)*ln(1-I*c*x)-1/4*I*b^2/c^2*ln(I+c*x)^2*e-b^2*arctan(c*x)^2*d/c/x-
1/2*I*b^2/c^2*dilog(1/2*I*(c*x-I))*e+1/4*I*b^2/c^2*ln(c*x-I)^2*e+1/2*I*b^2/
c^2*dilog(-1/2*I*(I+c*x))*e-b^2/c^2*arctan(c*x)*ln(c^2*x^2+1)*e+1/2*I*b^2*l
n(c^2*x^2+1)*ln(I+c*x)*d+1/2*I*b^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*d-1/2*I*b^2
*ln(1/2*I*(c*x-I))*ln(I+c*x)*d-1/2*I*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d-b^2*arct
an(c*x)*ln(c^2*x^2+1)*d-a*b*ln(c^2*x^2+1)*d-a*b/c^2*ln(c^2*x^2+1)*e+2*a*b*d
*ln(c*x)+I*b^2*d*dilog(1+I*c*x)+2*b^2*arctan(c*x)*d*ln(c*x)-I*b^2*d*dilog(1
-I*c*x)+1/4*I*b^2*ln(c*x-I)^2*d-1/4*I*b^2*ln(I+c*x)^2*d-1/2*I*b^2*dilog(1/2
*I*(c*x-I))*d+1/2*I*b^2*dilog(-1/2*I*(I+c*x))*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d + a^2*x*e + (2*c
*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*e/c - a^2*d/x + 1/16*(4*(b^2*x^2*e -
 b^2*d)*arctan(c*x)^2 - (b^2*x^2*e - b^2*d)*log(c^2*x^2 + 1)^2 + 4*(b^2*c*d
*arctan(c*x)^3 + 48*b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^4 + x
^2), x) + 4*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2)
, x) + 16*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x)
 + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) -
16*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + b^2*
arctan(c*x)^3*e/c - 32*b^2*c*e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^4 + x^
2), x) + 32*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 4*b^
2*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 48*b^2*d*in
```

```
tegrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integrate(1/16*log
(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
[Out] integral((a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arctan(c*x)^2 + 2*(a*b*x^
2*e + a*b*d)*arctan(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))^2 (d + ex^2)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**2,x)
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{atan}(c\,x))^2\,(e\,x^2 + d)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2,x)
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2, x)
```

3.1253 $\int \frac{(d+ex^2)(a+b\text{ArcTan}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=220

$$-\frac{bcd(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}c^2d(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{2x^2}+2e(a+b\text{ArcTan}(cx))^2\tanh^{-1}\Big(1$$

```
[Out] -b*c*d*(a+b*arctan(c*x))/x-1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan(
c*x))^2/x^2-2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d*ln(x)
-1/2*b^2*c^2*d*ln(c^2*x^2+1)-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x
))+I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*e*polylog(3,1-
2/(1+I*c*x))+1/2*b^2*e*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.32, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5100, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745}

$$-\frac{1}{2}c^2d(a+b\text{ArcTan}(cx))^2-\frac{d(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{bcd(a+b\text{ArcTan}(cx))}{x}-ibe\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibe\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+2e\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)-\frac{1}{2}b^2e\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2e\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*(a + b*ArcTan[c*x]))/x) - (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
+ b*ArcTan[c*x])^2)/(2*x^2) + 2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 +
I*c*x)] + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 - I*b*e*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, -1 + 2/(1 + I*c*x)] - (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2
*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
```

```
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\tan^{-1}(cx))^2}{x^3}\,dx &= \int \left(\frac{d(a+b\tan^{-1}(cx))^2}{x^3}+\frac{e(a+b\tan^{-1}(cx))^2}{x}\right)dx\\
&= d\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx+e\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx\\
&= -\frac{d(a+b\tan^{-1}(cx))^2}{2x^2}+2e\big(a+b\tan^{-1}(cx)\big)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)\\
&= -\frac{d(a+b\tan^{-1}(cx))^2}{2x^2}+2e\big(a+b\tan^{-1}(cx)\big)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)\\
&= -\frac{bcd(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d\big(a+b\tan^{-1}(cx)\big)^2-\frac{d(a+b\tan^{-1}(cx)}{2x^2}\\
&= -\frac{bcd(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d\big(a+b\tan^{-1}(cx)\big)^2-\frac{d(a+b\tan^{-1}(cx)}{2x^2}\\
&= -\frac{bcd(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d\big(a+b\tan^{-1}(cx)\big)^2-\frac{d(a+b\tan^{-1}(cx)}{2x^2}\\
&= -\frac{bcd(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d\big(a+b\tan^{-1}(cx)\big)^2-\frac{d(a+b\tan^{-1}(cx)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 273, normalized size = 1.24

$$-\frac{a^2d}{2x^2}-\frac{abd(\text{ArcTan}(cx)+cx(1+cx\text{ArcTan}(cx)))}{x^2}+a^2e\log(x)-\frac{b^2d\left(2cx\text{ArcTan}(cx)+(1+c^2x^2)\,\text{ArcTan}(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{2x^2}+iabe(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx))+\frac{1}{24}b^2e\left(-i\pi^3+16i\text{ArcTan}(cx)^3+24\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)-24\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+24i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+24i\text{ArcTan}(cx)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)-12\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -1/2*(a^2*d)/x^2 - (a*b*d*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
a^2*e*Log[x] - (b^2*d*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*
c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/(2*x^2) + I*a*b*e*(PolyLog[2, (-I)*c
*x] - PolyLog[2, I*c*x]) + (b^2*e*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 24*Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log[1 + E^((
2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])]
+ (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyLog[3, E^(
(-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.84, size = 1390, normalized size = 6.32

method	result	size
derivativedivides	Expression too large to display	1390
default	Expression too large to display	1390

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(I*b^2/c^2*e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*b^2*ar
ctan(c*x)^2*d-a*b*arctan(c*x)*d+1/2*I*b^2/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-a*b*arctan(c*x)*d/c^2/x^
2+2*a*b/c^2*arctan(c*x)*e*ln(c*x)+I*a*b/c^2*e*dilog(1+I*c*x)-I*a*b/c^2*e*di
log(1-I*c*x)-2*I*b^2/c^2*e*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2
))+1/2*I*b^2/c^2*e*Pi*arctan(c*x)^2-2*I*b^2/c^2*e*arctan(c*x)*polylog(2,-(1
+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/c^2*e*Pi*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^2-1/2*I*b^2/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c
*x)^2-1/2*I*b^2/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*arctan(c*x)^2+b^2*d*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*d*ln((1+I*c*x)
/(c^2*x^2+1)^(1/2)-1)-1/2*a^2*d/c^2/x^2+a^2/c^2*e*ln(c*x)+b^2/c^2*e*arctan(
c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/c^2*arctan(c*x)^2*e*ln(c*x)+b^
2/c^2*e*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-a*b*d/c/x-1/2*I*b^2
/c^2*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*a
rctan(c*x)^2+1/2*I*b^2/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^2/c^2*e*Pi*csgn(((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*a*b/c^2*e*ln(c
*x)*ln(1+I*c*x)-I*a*b/c^2*e*ln(c*x)*ln(1-I*c*x)-1/2*b^2*arctan(c*x)^2*d/c^2
```

```
/x^2-b^2*d*arctan(c*x)/c/x-b^2/c^2*e*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+
1)-1)-1/2*b^2/c^2*e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2/c^2*e*polylog
(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2/c^2*e*polylog(3,(1+I*c*x)/(c^2*x^2+1
)^(1/2))-I*b^2*d*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + a^2*e*log(x) - 1/2*a^2
*d/x^2 - 1/96*(12*b^2*d*arctan(c*x)^2 - 3*b^2*d*log(c^2*x^2 + 1)^2 - (1152*
b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c
^2*e*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d*in
tegrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d*integrate
(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d*integrate(
1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + b^2*e*log(c^2*x^2 + 1)^3 +
384*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*e*i
ntegrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*e*integrate(1/
16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e*integrate(1/16*x
^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*d*integrate(1/16*arctan(c*x)^
2/(c^2*x^5 + x^3), x) + 96*b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5
 + x^3), x))*x^2)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2*e + a^2*d + (b^2*x^2*e + b^2*d)*arctan(c*x)^2 + 2*(a*b*x^
2*e + a*b*d)*arctan(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x \right)}\right)^{2} \left(d + e x^{2}\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3, x)
```

3.1254 $\int x^3(d+ex^2)^2\,(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=502

$$\frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{71b^2e^2x^2}{840c^6}+\frac{b^2dex^4}{30c^2}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2e^2x^6}{168c^2}+\frac{b^2d^2x\text{ArcTan}(cx)}{2c^3}-\frac{2b^2de}{}$$

```
[Out] 71/840*b^2*e^2*x^2/c^6-3/140*b^2*e^2*x^4/c^4+1/168*b^2*e^2*x^6/c^2+1/3*d*e*
(a+b*arctan(c*x))^2/c^6+1/3*d*e*x^6*(a+b*arctan(c*x))^2-1/3*b^2*d^2*ln(c^2*
x^2+1)/c^4-2/3*a*b*d*e*x/c^5-2/3*b^2*d*e*x*arctan(c*x)/c^5+2/9*b*d*e*x^3*(a
+b*arctan(c*x))/c^3-2/15*b*d*e*x^5*(a+b*arctan(c*x))/c+1/2*a*b*d^2*x/c^3+1/
2*b^2*d^2*x*arctan(c*x)/c^3-1/6*b*d^2*x^3*(a+b*arctan(c*x))/c+1/4*a*b*e^2*x
/c^7-8/45*b^2*d*e*x^2/c^4+1/30*b^2*d*e*x^4/c^2+1/4*b^2*e^2*x*arctan(c*x)/c^
7-1/12*b*e^2*x^3*(a+b*arctan(c*x))/c^5+1/20*b*e^2*x^5*(a+b*arctan(c*x))/c^3
-1/28*b*e^2*x^7*(a+b*arctan(c*x))/c-1/4*d^2*(a+b*arctan(c*x))^2/c^4+1/4*d^2
*x^4*(a+b*arctan(c*x))^2+23/45*b^2*d*e*ln(c^2*x^2+1)/c^6-1/8*e^2*(a+b*arcta
n(c*x))^2/c^8+1/8*e^2*x^8*(a+b*arctan(c*x))^2-22/105*b^2*e^2*ln(c^2*x^2+1)/
c^8+1/12*b^2*d^2*x^2/c^2
```

Rubi [A]
time = 0.78, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5100, 4946, 5036, 272, 45, 4930, 266, 5004}

$$-\frac{e^2(a+b\text{ArcTan}(cx))^2}{8c^8}+\frac{de(a+b\text{ArcTan}(cx))^2}{3c^6}-\frac{be^2x^3(a+b\text{ArcTan}(cx))}{12c^5}-\frac{d^2(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{2bdex^3(a+b\text{ArcTan}(cx))}{9c^3}+\frac{be^2x^5(a+b\text{ArcTan}(cx))}{20c^3}+\frac{1}{4}d^2x^4(a+b\text{ArcTan}(cx))^2-\frac{bd^2x^3(a+b\text{ArcTan}(cx))}{6c}+\frac{1}{3}dex^6(a+b\text{ArcTan}(cx))^2-\frac{2bdex^5(a+b\text{ArcTan}(cx))}{15c}+\frac{1}{8}e^2x^8(a+b\text{ArcTan}(cx))^2-\frac{be^2x^7(a+b\text{ArcTan}(cx))}{28c}+\frac{abe^2x}{4c^7}-\frac{2abdex}{3c^5}+\frac{abd^2x}{2c^3}+\frac{b^2e^2x\text{ArcTan}(cx)}{4c^7}-\frac{2b^2dex\text{ArcTan}(cx)}{3c^5}+\frac{b^2d^2x\text{ArcTan}(cx)}{2c^3}+\frac{71b^2e^2x^2}{840c^6}-\frac{8b^2dex^2}{45c^4}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2d^2x^2}{12c^2}+\frac{b^2dex^4}{30c^2}+\frac{b^2e^2x^6}{168c^2}-\frac{22b^2e^2\log(c^2x^2+1)}{105c^8}+\frac{23b^2de\log(c^2x^2+1)}{45c^6}-\frac{b^2d^2\log(c^2x^2+1)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*d^2*x)/(2*c^3) - (2*a*b*d*e*x)/(3*c^5) + (a*b*e^2*x)/(4*c^7) + (b^2*d^
2*x^2)/(12*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (71*b^2*e^2*x^2)/(840*c^6) + (
b^2*d*e*x^4)/(30*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (b^2*e^2*x^6)/(168*c^2)
 + (b^2*d^2*x*ArcTan[c*x])/(2*c^3) - (2*b^2*d*e*x*ArcTan[c*x])/(3*c^5) + (b
^2*e^2*x*ArcTan[c*x])/(4*c^7) - (b*d^2*x^3*(a + b*ArcTan[c*x]))/(6*c) + (2*
b*d*e*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*e^2*x^3*(a + b*ArcTan[c*x]))/(1
2*c^5) - (2*b*d*e*x^5*(a + b*ArcTan[c*x]))/(15*c) + (b*e^2*x^5*(a + b*ArcTa
n[c*x]))/(20*c^3) - (b*e^2*x^7*(a + b*ArcTan[c*x]))/(28*c) - (d^2*(a + b*Ar
cTan[c*x])^2)/(4*c^4) + (d*e*(a + b*ArcTan[c*x])^2)/(3*c^6) - (e^2*(a + b*A
rcTan[c*x])^2)/(8*c^8) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + (d*e*x^6*(a +
b*ArcTan[c*x])^2)/3 + (e^2*x^8*(a + b*ArcTan[c*x])^2)/8 - (b^2*d^2*Log[1 +
c^2*x^2])/(3*c^4) + (23*b^2*d*e*Log[1 + c^2*x^2])/(45*c^6) - (22*b^2*e^2*Lo
g[1 + c^2*x^2])/(105*c^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
```

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \int \left(d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+2dex^5\left(a+b\tan^{-1}(cx)\right)^2+e^2x^7\left(a+\right.\right.\\
&= d^2\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^5\left(a+b\tan^{-1}(cx)\right)^2\,dx\\
&= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{8}e^2x^8\left(a+\right.\\
&= \frac{1}{4}d^2x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}dex^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{8}e^2x^8\left(a+\right.\\
&= -\frac{bd^2x^3(a+b\tan^{-1}(cx))}{6c}-\frac{2bdex^5(a+b\tan^{-1}(cx))}{15c}-\frac{be^2x^7(a+}{2}\\
&= \frac{abd^2x}{2c^3}-\frac{bd^2x^3(a+b\tan^{-1}(cx))}{6c}+\frac{2bdex^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{2ba}{}\\
&= \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{b^2d^2x\tan^{-1}(cx)}{2c^3}-\frac{bd^2x^3(a+b\tan^{-1}(cx))}{6c}+\frac{2}{}\\
&= \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{b^2dex^2}{15c^4}+\frac{b^2e^2x^2}{56c^6}+\frac{b^2dex^4}{30c^2}\\
&= \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{3b^2e^2x^2}{70c^6}+\frac{b^2dex}{30c^2}\\
&= \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{71b^2e^2x^2}{840c^6}+\frac{b^2de}{30c}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 414, normalized size = 0.82

$$\frac{cx(105a^2c^7x^3(6d^2+8dex^2+3e^2x^4)+b^2cx(213e^2-2c^2e(224d+27ex^2)+3c^4(70d^2+28dex^2+5e^2x^4))-2ab(-315e^2+105c^2e(8d+ex^2)-7c^4(90d^2+40dex^2+9e^2x^4)+3c^6(70d^2x^2+56dex^4+15e^2x^6)))+2b(bcx(315e^2-105c^2e(8d+ex^2)+7c^4(90d^2+40dex^2+9e^2x^4)-3c^6(70d^2x^2+56dex^4+15e^2x^6))+105a(-6c^4d^2+8c^2de-3e^2+c^8(6d^2x^4+8dex^6+3e^2x^8)))\,\mathrm{ArcTan}(cx)+105b^2(-6c^4d^2+8c^2de-3e^2+c^8(6d^2x^4+8dex^6+3e^2x^8))\,\mathrm{ArcTan}(cx)^2-8b^2(105c^4d^2-161c^2de+66e^2)\log(1+c^2x^2)}{2520c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(105*a^2*c^7*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b^2*c*x*(213*e^2 -
2*c^2*e*(224*d + 27*e*x^2) + 3*c^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)) - 2*a
*b*(-315*e^2 + 105*c^2*e*(8*d + e*x^2) - 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2
```

*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6))) + 2*b*(b*c*x*(315*e^
2 - 105*c^2*e*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4) - 3*c
^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*a*(-6*c^4*d^2 + 8*c^2*d*e
- 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8)))*ArcTan[c*x] + 105*b^2*(
-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8))*A
rcTan[c*x]^2 - 8*b^2*(105*c^4*d^2 - 161*c^2*d*e + 66*e^2)*Log[1 + c^2*x^2])
/(2520*c^8)

Maple [A]
time = 0.70, size = 624, normalized size = 1.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(1/2*a*b*c*d^2*x+1/2*b^2*c*d^2*x*arctan(c*x)-1/12*a*b/c*e^2*x^3+2/3*a
*b*c^4*arctan(c*x)*d*e*x^6-1/4*b^2*arctan(c*x)^2*d^2+1/168*b^2*c^2*e^2*x^6-
3/140*b^2*e^2*x^4+1/4*b^2*arctan(c*x)^2*d^2*c^4*x^4-1/6*b^2*arctan(c*x)*d^2
*c^3*x^3-1/4*a*b/c^4*arctan(c*x)*e^2-1/6*a*b*d^2*c^3*x^3+1/3*b^2/c^2*arctan
(c*x)^2*d*e+1/30*b^2*c^2*d*e*x^4-8/45*b^2*d*e*x^2+1/8*b^2*c^4*arctan(c*x)^2
*e^2*x^8+2/3*a*b/c^2*arctan(c*x)*d*e+1/2*a*b*arctan(c*x)*d^2*c^4*x^4-1/28*a
*b*c^3*e^2*x^7+1/20*a*b*c*e^2*x^5-1/28*b^2*c^3*arctan(c*x)*e^2*x^7+1/20*b^2
*c*arctan(c*x)*e^2*x^5-1/12*b^2/c*arctan(c*x)*e^2*x^3-22/105*b^2*e^2*ln(c^2
*x^2+1)/c^4+23/45*b^2*d*e*ln(c^2*x^2+1)/c^2+2/9*a*b*c*d*e*x^3+2/9*b^2*c*arc
tan(c*x)*d*e*x^3+1/3*b^2*c^4*arctan(c*x)^2*d*e*x^6-2/15*b^2*c^3*arctan(c*x)
*d*e*x^5+1/4*a*b*c^4*arctan(c*x)*e^2*x^8-2/15*a*b*c^3*d*e*x^5+71/840*b^2*e^
2*x^2/c^2-1/3*b^2*d^2*ln(c^2*x^2+1)+1/4*a*b*e^2*x/c^3+1/4*b^2*e^2*x*arctan(
c*x)/c^3-2/3*a*b*d*e*x/c-2/3*b^2*d*e*x*arctan(c*x)/c+1/12*b^2*d^2*c^2*x^2-1
/2*a*b*arctan(c*x)*d^2-1/8*b^2/c^4*arctan(c*x)^2*e^2+a^2/c^4*(1/4*d^2*c^8*x
^4+1/3*d*c^8*e*x^6+1/8*e^2*c^8*x^8))

Maxima [A]
time = 0.51, size = 516, normalized size = 1.03

$$\frac{1}{8}b^2x^8\arctan(cx)^2e^2+\frac{1}{3}b^2dx^6\arctan(cx)^2e+\frac{1}{8}a^2x^8e^2+\frac{1}{4}b^2d^2x^4\arctan(cx)^2+\frac{1}{3}a^2dx^6e+\frac{1}{4}a^2d^2x^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abd^2-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d^2+\frac{2}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abde-\frac{1}{90}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+46\log(c^2x^2+1)}{c^6}\right)b^2de+\frac{1}{420}\left(105x^8\arctan(cx)-c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\right)abe^2-\frac{1}{840}\left(2c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\arctan(cx)-\frac{5c^6x^6-18c^4x^4+71c^2x^2+105\arctan(cx)^2-176\log(c^2x^2+1)}{c^8}\right)b^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] 1/8*b^2*x^8*arctan(c*x)^2*e^2 + 1/3*b^2*d*x^6*arctan(c*x)^2*e + 1/8*a^2*x^8
*e^2 + 1/4*b^2*d^2*x^4*arctan(c*x)^2 + 1/3*a^2*d*x^6*e + 1/4*a^2*d^2*x^4 +
1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d
^2 - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2
*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 + 2/45*(15*x^6*ar
ctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*
b*d*e - 1/90*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)
*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2

```
+ 1))/c^6)*b^2*d*e + 1/420*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x
^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*a*b*e^2 - 1/840*(2*c*(
(15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9)*a
rctan(c*x) - (5*c^6*x^6 - 18*c^4*x^4 + 71*c^2*x^2 + 105*arctan(c*x)^2 - 176
*log(c^2*x^2 + 1))/c^8)*b^2*e^2
```

Fricas [A]
time = 2.28, size = 508, normalized size = 1.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(630*a^2*c^8*d^2*x^4 - 420*a*b*c^7*d^2*x^3 + 210*b^2*c^6*d^2*x^2 + 1
260*a*b*c^5*d^2*x + 105*(6*b^2*c^8*d^2*x^4 - 6*b^2*c^4*d^2 + 3*(b^2*c^8*x^8
 - b^2)*e^2 + 8*(b^2*c^8*d*x^6 + b^2*c^2*d)*e)*arctan(c*x)^2 + 2*(630*a*b*c
^8*d^2*x^4 - 210*b^2*c^7*d^2*x^3 + 630*b^2*c^5*d^2*x - 630*a*b*c^4*d^2 + 3*
(105*a*b*c^8*x^8 - 15*b^2*c^7*x^7 + 21*b^2*c^5*x^5 - 35*b^2*c^3*x^3 + 105*b
^2*c*x - 105*a*b)*e^2 + 56*(15*a*b*c^8*d*x^6 - 3*b^2*c^7*d*x^5 + 5*b^2*c^5*
d*x^3 - 15*b^2*c^3*d*x + 15*a*b*c^2*d)*e)*arctan(c*x) + 3*(105*a^2*c^8*x^8
- 30*a*b*c^7*x^7 + 5*b^2*c^6*x^6 + 42*a*b*c^5*x^5 - 18*b^2*c^4*x^4 - 70*a*b
*c^3*x^3 + 71*b^2*c^2*x^2 + 210*a*b*c*x)*e^2 + 28*(30*a^2*c^8*d*x^6 - 12*a*
b*c^7*d*x^5 + 3*b^2*c^6*d*x^4 + 20*a*b*c^5*d*x^3 - 16*b^2*c^4*d*x^2 - 60*a*
b*c^3*d*x)*e - 8*(105*b^2*c^4*d^2 - 161*b^2*c^2*d*e + 66*b^2*e^2)*log(c^2*x
^2 + 1))/c^8
```

Sympy [A]
time = 0.89, size = 758, normalized size = 1.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x**4/4 + a**2*d*e*x**6/3 + a**2*e**2*x**8/8 + a*b*d**2
*x**4*atan(c*x)/2 + 2*a*b*d*e*x**6*atan(c*x)/3 + a*b*e**2*x**8*atan(c*x)/4
- a*b*d**2*x**3/(6*c) - 2*a*b*d*e*x**5/(15*c) - a*b*e**2*x**7/(28*c) + a*b*
d**2*x/(2*c**3) + 2*a*b*d*e*x**3/(9*c**3) + a*b*e**2*x**5/(20*c**3) - a*b*d
**2*atan(c*x)/(2*c**4) - 2*a*b*d*e*x/(3*c**5) - a*b*e**2*x**3/(12*c**5) + 2
*a*b*d*e*atan(c*x)/(3*c**6) + a*b*e**2*x/(4*c**7) - a*b*e**2*atan(c*x)/(4*c
**8) + b**2*d**2*x**4*atan(c*x)**2/4 + b**2*d*e*x**6*atan(c*x)**2/3 + b**2*
e**2*x**8*atan(c*x)**2/8 - b**2*d**2*x**3*atan(c*x)/(6*c) - 2*b**2*d*e*x**5
*atan(c*x)/(15*c) - b**2*e**2*x**7*atan(c*x)/(28*c) + b**2*d**2*x**2/(12*c*
*2) + b**2*d*e*x**4/(30*c**2) + b**2*e**2*x**6/(168*c**2) + b**2*d**2*x*ata
n(c*x)/(2*c**3) + 2*b**2*d*e*x**3*atan(c*x)/(9*c**3) + b**2*e**2*x**5*atan(
```

```
c*x)/(20*c**3) - b**2*d**2*log(x**2 + c**(-2))/(3*c**4) - b**2*d**2*atan(c*
x)**2/(4*c**4) - 8*b**2*d*e*x**2/(45*c**4) - 3*b**2*e**2*x**4/(140*c**4) -
2*b**2*d*e*x*atan(c*x)/(3*c**5) - b**2*e**2*x**3*atan(c*x)/(12*c**5) + 23*b
**2*d*e*log(x**2 + c**(-2))/(45*c**6) + b**2*d*e*atan(c*x)**2/(3*c**6) + 71
*b**2*e**2*x**2/(840*c**6) + b**2*e**2*x*atan(c*x)/(4*c**7) - 22*b**2*e**2*
log(x**2 + c**(-2))/(105*c**8) - b**2*e**2*atan(c*x)**2/(8*c**8), Ne(c, 0))
, (a**2*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 6.90, size = 929, normalized size = 1.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] (a^2*d^2*x^4)/4 + (a^2*e^2*x^8)/8 - (b^2*d^2*log(c^2*x^2 + 1))/(3*c^4) - (2
2*b^2*e^2*log(c^2*x^2 + 1))/(105*c^8) + (b^2*d^2*x^2)/(12*c^2) + (b^2*e^2*x
^6)/(168*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (71*b^2*e^2*x^2)/(840*c^6) - (b
^2*d^2*atan(c*x)^2)/(4*c^4) - (b^2*e^2*atan(c*x)^2)/(8*c^8) + (b^2*d^2*x^4*
atan(c*x)^2)/4 + (b^2*e^2*x^8*atan(c*x)^2)/8 + (a^2*d*e*x^6)/3 - (b^2*d^2*x
^3*atan(c*x))/(6*c) - (b^2*e^2*x^7*atan(c*x))/(28*c) + (b^2*e^2*x^5*atan(c*
x))/(20*c^3) - (b^2*e^2*x^3*atan(c*x))/(12*c^5) + (a*b*d^2*x)/(2*c^3) + (a*
b*e^2*x)/(4*c^7) + (a*b*d^2*x^4*atan(c*x))/2 + (a*b*e^2*x^8*atan(c*x))/4 +
(23*b^2*d*e*log(c^2*x^2 + 1))/(45*c^6) - (a*b*d^2*x^3)/(6*c) - (a*b*e^2*x^7
)/(28*c) + (a*b*e^2*x^5)/(20*c^3) - (a*b*e^2*x^3)/(12*c^5) + (b^2*d*e*x^4)/
(30*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (b^2*d*e*atan(c*x)^2)/(3*c^6) + (b^2*
d^2*x*atan(c*x))/(2*c^3) + (b^2*e^2*x*atan(c*x))/(4*c^7) + (b^2*d*e*x^6*ata
n(c*x)^2)/3 - (a*b*d^2*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*
d*e) + (6*b*c^5*d^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) - (8*b*c^3*d*e
*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)))/(2*c^4) - (a*b*e^2*atan((3*b*c*
e^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) + (6*b*c^5*d^2*x)/(3*b*e^2 + 6
*b*c^4*d^2 - 8*b*c^2*d*e) - (8*b*c^3*d*e*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^
2*d*e)))/(4*c^8) - (2*b^2*d*e*x^5*atan(c*x))/(15*c) + (2*b^2*d*e*x^3*atan(c
*x))/(9*c^3) - (2*a*b*d*e*x)/(3*c^5) + (2*a*b*d*e*x^6*atan(c*x))/3 - (2*a*b
*d*e*x^5)/(15*c) + (2*a*b*d*e*x^3)/(9*c^3) - (2*b^2*d*e*x*atan(c*x))/(3*c^5
```

```
) + (2*a*b*d*e*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) + (
6*b*c^5*d^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) - (8*b*c^3*d*e*x)/(3*b
*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)))/(3*c^6)
```

3.1255 $\int x^2(d+ex^2)^2\,(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=580

$$\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\mathrm{ArcTan}(cx)}{3c^3}+\frac{3b^2de\mathrm{ArcTan}(cx)}{5c^5}-\frac{11b^2e^2\mathrm{ArcTan}(c}{42c^7}$$

```
[Out] -2/3*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2/5*I*d*e*(a+b*arctan(c*x)
)^2/c^5-1/3*b*d^2*x^2*(a+b*arctan(c*x))/c+1/3*b^2*d^2*x/c^2-1/3*b^2*d^2*arc
tan(c*x)/c^3+2/5*b*d*e*x^2*(a+b*arctan(c*x))/c^3-1/5*b*d*e*x^4*(a+b*arctan(
c*x))/c-3/5*b^2*d*e*x/c^4+1/15*b^2*d*e*x^3/c^2+3/5*b^2*d*e*arctan(c*x)/c^5-
1/7*b*e^2*x^2*(a+b*arctan(c*x))/c^5+1/14*b*e^2*x^4*(a+b*arctan(c*x))/c^3-1/
21*b*e^2*x^6*(a+b*arctan(c*x))/c-1/3*I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^3
-1/7*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^7+4/5*b*d*e*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c^5+1/3*d^2*x^3*(a+b*arctan(c*x))^2-2/7*b*e^2*(a+b*arctan(c*x)
)*ln(2/(1+I*c*x))/c^7+11/42*b^2*e^2*x/c^6-5/126*b^2*e^2*x^3/c^4+1/105*b^2*e
^2*x^5/c^2-11/42*b^2*e^2*arctan(c*x)/c^7+2/5*d*e*x^5*(a+b*arctan(c*x))^2-1/
3*I*d^2*(a+b*arctan(c*x))^2/c^3-1/7*I*e^2*(a+b*arctan(c*x))^2/c^7+1/7*e^2*x
^7*(a+b*arctan(c*x))^2+2/5*I*b^2*d*e*polylog(2,1-2/(1+I*c*x))/c^5
```

Rubi [A]
time = 0.76, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5100, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 308}

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*d^2*x)/(3*c^2) - (3*b^2*d*e*x)/(5*c^4) + (11*b^2*e^2*x)/(42*c^6) + (b^
2*d*e*x^3)/(15*c^2) - (5*b^2*e^2*x^3)/(126*c^4) + (b^2*e^2*x^5)/(105*c^2) -
 (b^2*d^2*ArcTan[c*x])/(3*c^3) + (3*b^2*d*e*ArcTan[c*x])/(5*c^5) - (11*b^2*
e^2*ArcTan[c*x])/(42*c^7) - (b*d^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(7*c^5
) - (b*d*e*x^4*(a + b*ArcTan[c*x]))/(5*c) + (b*e^2*x^4*(a + b*ArcTan[c*x]))
/(14*c^3) - (b*e^2*x^6*(a + b*ArcTan[c*x]))/(21*c) - ((I/3)*d^2*(a + b*ArcT
an[c*x])^2)/c^3 + (((2*I)/5)*d*e*(a + b*ArcTan[c*x])^2)/c^5 - ((I/7)*e^2*(a
 + b*ArcTan[c*x])^2)/c^7 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*d*e*x^5*(
a + b*ArcTan[c*x])^2)/5 + (e^2*x^7*(a + b*ArcTan[c*x])^2)/7 - (2*b*d^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (4*b*d*e*(a + b*ArcTan[c*x])*
Log[2/(1 + I*c*x)])/(5*c^5) - (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/(7*c^7) - ((I/3)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (((2*I)/5
)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5 - ((I/7)*b^2*e^2*PolyLog[2, 1
- 2/(1 + I*c*x)])/c^7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int\left(d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+2dex^4\left(a+b\tan^{-1}(cx)\right)^2+e^2x^6(a+\right.$$

$$=d^2\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{7}e^2x^7(a+$$

$$=\frac{1}{3}d^2x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{5}dex^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{7}e^2x^7(a+$$

$$=-\frac{bd^2x^2(a+b\tan^{-1}(cx))}{3c}-\frac{bdex^4(a+b\tan^{-1}(cx))}{5c}-\frac{be^2x^6(a+b}{21}$$

$$=\frac{b^2d^2x}{3c^2}-\frac{bd^2x^2(a+b\tan^{-1}(cx))}{3c}+\frac{2bdex^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bde}{}$$

$$=\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{b^2e^2x}{21c^6}+\frac{b^2dex^3}{15c^2}-\frac{b^2e^2x^3}{63c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\tan}{3c^3}$$

$$=\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2}{}$$

$$=\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2}{}$$

$$=\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2}{}$$

Mathematica [A]
time = 1.18, size = 513, normalized size = 0.88

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (378*a*b*c^2*d*e - 165*a*b*e^2 + 210*b^2*c^5*d^2*x - 378*b^2*c^3*d*e*x + 16
5*b^2*c*e^2*x - 210*a*b*c^6*d^2*x^2 + 252*a*b*c^4*d*e*x^2 - 90*a*b*c^2*e^2*
x^2 + 210*a^2*c^7*d^2*x^3 + 42*b^2*c^5*d*e*x^3 - 25*b^2*c^3*e^2*x^3 - 126*a
*b*c^6*d*e*x^4 + 45*a*b*c^4*e^2*x^4 + 252*a^2*c^7*d*e*x^5 + 6*b^2*c^5*e^2*x
^5 - 30*a*b*c^6*e^2*x^6 + 90*a^2*c^7*e^2*x^7 + 6*b^2*((35*I)*c^4*d^2 - (42*
I)*c^2*d*e + (15*I)*e^2 + c^7*(35*d^2*x^3 + 42*d*e*x^5 + 15*e^2*x^7))*ArcTa
n[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)
 + b*(1 + c^2*x^2)*(55*e^2 - c^2*e*(126*d + 25*e*x^2) + 2*c^4*(35*d^2 + 21*
```

```
d*e*x^2 + 5*e^2*x^4)) + 4*b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log[1 + E^((
2*I)*ArcTan[c*x])]) + 210*a*b*c^4*d^2*Log[1 + c^2*x^2] - 252*a*b*c^2*d*e*Lo
g[1 + c^2*x^2] + 90*a*b*e^2*Log[1 + c^2*x^2] + (6*I)*b^2*(35*c^4*d^2 - 42*c
^2*d*e + 15*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(630*c^7)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1136 vs. 2(514) = 1028.
time = 0.98, size = 1137, normalized size = 1.96

method	result	size
derivativedivides	Expression too large to display	1137
default	Expression too large to display	1137
risch	Expression too large to display	1414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/5*I*b^2/c^2*ln(c^2*x^2+1)*ln(I+c*x)*d*e-1/5*I*b^2/c^2*ln(I+c*x)*ln
(1/2*I*(c*x-I))*d*e-1/5*a*b*c^2*d*e*x^4+2/5*a*b*d*e*x^2+2/7*a*b*c^3*arctan(
c*x)*e^2*x^7+2/5*b^2*c^3*arctan(c*x)^2*d*e*x^5-1/5*b^2*c^2*arctan(c*x)*d*e*
x^4+2/5*b^2*arctan(c*x)*d*e*x^2+4/5*a*b*c^3*arctan(c*x)*d*e*x^5-1/5*I*b^2/c
^2*ln(c^2*x^2+1)*ln(c*x-I)*d*e+1/5*I*b^2/c^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))*d
*e+1/6*I*b^2*dilog(1/2*I*(c*x-I))*d^2-1/12*I*b^2*ln(c*x-I)^2*d^2-1/6*I*b^2*
dilog(-1/2*I*(I+c*x))*d^2+1/3*b^2*d^2*c*x-11/42*b^2/c^4*arctan(c*x)*e^2+1/3
*a*b*ln(c^2*x^2+1)*d^2+1/12*I*b^2*ln(I+c*x)^2*d^2+1/3*b^2*arctan(c*x)*ln(c^
2*x^2+1)*d^2+1/15*b^2*c*d*e*x^3-3/5*b^2/c*d*e*x+1/7*b^2*c^3*arctan(c*x)^2*e
^2*x^7-1/21*b^2*c^2*arctan(c*x)*e^2*x^6+1/14*b^2*arctan(c*x)*e^2*x^4-1/21*a
*b*c^2*e^2*x^6+1/14*a*b*e^2*x^4-1/7*b^2/c^2*arctan(c*x)*e^2*x^2-1/5*I*b^2/c
^2*dilog(1/2*I*(c*x-I))*d*e+1/14*I*b^2/c^4*ln(I+c*x)*ln(1/2*I*(c*x-I))*e^2-
2/5*b^2/c^2*arctan(c*x)*ln(c^2*x^2+1)*d*e-1/7*a*b/c^2*e^2*x^2+2/3*a*b*arcta
n(c*x)*d^2*c^3*x^3+1/5*I*b^2/c^2*dilog(-1/2*I*(I+c*x))*d*e-1/14*I*b^2/c^4*l
n(c^2*x^2+1)*ln(I+c*x)*e^2-1/10*I*b^2/c^2*ln(I+c*x)^2*d*e-2/5*a*b/c^2*ln(c^
2*x^2+1)*d*e+1/14*I*b^2/c^4*ln(c^2*x^2+1)*ln(c*x-I)*e^2-1/14*I*b^2/c^4*ln(c
*x-I)*ln(-1/2*I*(I+c*x))*e^2+1/10*I*b^2/c^2*ln(c*x-I)^2*d*e-1/3*a*b*d^2*c^2
*x^2+1/28*I*b^2/c^4*ln(I+c*x)^2*e^2+1/14*I*b^2/c^4*dilog(1/2*I*(c*x-I))*e^2
+3/5*b^2/c^2*arctan(c*x)*d*e-1/6*I*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))*d^2-1/6
*I*b^2*ln(c^2*x^2+1)*ln(I+c*x)*d^2+1/6*I*b^2*ln(I+c*x)*ln(1/2*I*(c*x-I))*d^
2+1/6*I*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d^2-1/28*I*b^2/c^4*ln(c*x-I)^2*e^2-1/14
*I*b^2/c^4*dilog(-1/2*I*(I+c*x))*e^2+1/7*b^2/c^4*arctan(c*x)*ln(c^2*x^2+1)*
e^2-1/3*b^2*arctan(c*x)*d^2*c^2*x^2+1/105*b^2*c*e^2*x^5-5/126*b^2/c*e^2*x^3
+11/42*b^2/c^3*e^2*x+1/3*b^2*arctan(c*x)^2*d^2*c^3*x^3+1/7*a*b/c^4*ln(c^2*x
^2+1)*e^2+a^2/c^4*(1/3*d^2*c^7*x^3+2/5*d*c^7*e*x^5+1/7*e^2*c^7*x^7)-1/3*b^2
*arctan(c*x)*d^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*a^2*x^7*e^2 + 2/5*a^2*d*x^5*e + 1/3*a^2*d^2*x^3 + 1/3*(2*x^3*arctan(c*x
) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^2 + 1/5*(4*x^5*arctan(c*x) -
c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*d*e + 1/42*(12*x^7*
arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c
^8))*a*b*e^2 + 1/420*(15*b^2*x^7*e^2 + 42*b^2*d*x^5*e + 35*b^2*d^2*x^3)*arc
tan(c*x)^2 - 1/1680*(15*b^2*x^7*e^2 + 42*b^2*d*x^5*e + 35*b^2*d^2*x^3)*log(
c^2*x^2 + 1)^2 + integrate(1/1680*(1260*(b^2*c^2*x^8*e^2 + (2*b^2*c^2*d*e +
 b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*arctan(c*x)^2
+ 105*(b^2*c^2*x^8*e^2 + (2*b^2*c^2*d*e + b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2
*c^2*d^2 + 2*b^2*d*e)*x^4)*log(c^2*x^2 + 1)^2 - 8*(15*b^2*c*x^7*e^2 + 42*b^
2*c*d*x^5*e + 35*b^2*c*d^2*x^3)*arctan(c*x) + 4*(15*b^2*c^2*x^8*e^2 + 42*b^
2*c^2*d*x^6*e + 35*b^2*c^2*d^2*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^6*e^2 + 2*a^2*d*x^4*e + a^2*d^2*x^2 + (b^2*x^6*e^2 + 2*b^2*d
*x^4*e + b^2*d^2*x^2)*arctan(c*x)^2 + 2*(a*b*x^6*e^2 + 2*a*b*d*x^4*e + a*b*
d^2*x^2)*arctan(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{atan}(cx))^2 \left(d + ex^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

3.1256 $\int x(d+ex^2)^2 (a+b\text{ArcTan}(cx))^2 \, dx$

Optimal. Leaf size=380

$$-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\text{ArcTan}(cx)}{c}+\frac{b^2dex\text{ArcTan}(cx)}{c^3}-\frac{b^2e^2x\text{ArcTan}(}{3c^5}$$

```
[Out] -a*b*d^2*x/c+a*b*d*e*x/c^3-1/3*a*b*e^2*x/c^5+1/6*b^2*d*e*x^2/c^2-4/45*b^2*e
^2*x^2/c^4+1/60*b^2*e^2*x^4/c^2-b^2*d^2*x*arctan(c*x)/c+b^2*d*e*x*arctan(c*
x)/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^5-1/3*b*d*e*x^3*(a+b*arctan(c*x))/c+1/9*
b*e^2*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e^2*x^5*(a+b*arctan(c*x))/c+1/2*d^2*
(a+b*arctan(c*x))^2/c^2-1/2*d*e*(a+b*arctan(c*x))^2/c^4+1/6*e^2*(a+b*arctan
(c*x))^2/c^6+1/2*d^2*x^2*(a+b*arctan(c*x))^2+1/2*d*e*x^4*(a+b*arctan(c*x))^
2+1/6*e^2*x^6*(a+b*arctan(c*x))^2+1/2*b^2*d^2*ln(c^2*x^2+1)/c^2-2/3*b^2*d*e
*ln(c^2*x^2+1)/c^4+23/90*b^2*e^2*ln(c^2*x^2+1)/c^6
```

Rubi [A]
time = 0.55, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5100, 4946, 5036, 4930, 266, 5004, 272, 45}

$$\frac{e^2(a+b\text{ArcTan}(cx))^2}{6c^6}-\frac{de(a+b\text{ArcTan}(cx))^2}{2c^4}+\frac{be^2x^3(a+b\text{ArcTan}(cx))}{9c^3}+\frac{d^2(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{1}{2}d^2x^2(a+b\text{ArcTan}(cx))^2+\frac{1}{2}dex^4(a+b\text{ArcTan}(cx))^2-\frac{bdex^3(a+b\text{ArcTan}(cx))}{3c}+\frac{1}{6}e^2x^6(a+b\text{ArcTan}(cx))^2-\frac{be^2x^5(a+b\text{ArcTan}(cx))}{15c}-\frac{abe^2x}{3c^5}+\frac{abdex}{c^3}-\frac{abd^2x}{c}-\frac{b^2e^2x\text{ArcTan}(cx)}{3c^5}+\frac{b^2dex\text{ArcTan}(cx)}{c^3}-\frac{b^2d^2x\text{ArcTan}(cx)}{c}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2d^2\log(c^2x^2+1)}{2c^2}+\frac{b^2dex^2}{6c^2}+\frac{b^2e^2x^4}{60c^2}+\frac{23b^2e^2\log(c^2x^2+1)}{90c^6}-\frac{2b^2de\log(c^2x^2+1)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*d^2*x)/c) + (a*b*d*e*x)/c^3 - (a*b*e^2*x)/(3*c^5) + (b^2*d*e*x^2)/(6
*c^2) - (4*b^2*e^2*x^2)/(45*c^4) + (b^2*e^2*x^4)/(60*c^2) - (b^2*d^2*x*ArcT
an[c*x])/c + (b^2*d*e*x*ArcTan[c*x])/c^3 - (b^2*e^2*x*ArcTan[c*x])/(3*c^5)
- (b*d*e*x^3*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^3*(a + b*ArcTan[c*x]))/(
9*c^3) - (b*e^2*x^5*(a + b*ArcTan[c*x]))/(15*c) + (d^2*(a + b*ArcTan[c*x])^
2)/(2*c^2) - (d*e*(a + b*ArcTan[c*x])^2)/(2*c^4) + (e^2*(a + b*ArcTan[c*x])
^2)/(6*c^6) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + (d*e*x^4*(a + b*ArcTan[c*
x])^2)/2 + (e^2*x^6*(a + b*ArcTan[c*x])^2)/6 + (b^2*d^2*Log[1 + c^2*x^2])/(
2*c^2) - (2*b^2*d*e*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*e^2*Log[1 + c^2*x^2
])/(90*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2\,dx = \int\left(d^2x\left(a+b\tan^{-1}(cx)\right)^2+2dex^3\left(a+b\tan^{-1}(cx)\right)^2+e^2x^5\left(a+b\right.\right.$$

$$= d^2\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx+(2de)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx+$$

$$= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}dex^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}e^2x^6\left(a+\right.$$

$$= \frac{1}{2}d^2x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{2}dex^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}e^2x^6\left(a+\right.$$

$$= -\frac{abd^2x}{c}-\frac{bdex^3(a+b\tan^{-1}(cx))}{3c}-\frac{be^2x^5(a+b\tan^{-1}(cx))}{15c}+\frac{d^2(}{}$$

$$= -\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{b^2d^2x\tan^{-1}(cx)}{c}-\frac{bdex^3(a+b\tan^{-1}(cx))}{3c}+\frac{b}{}$$

$$= -\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}-\frac{b^2d^2x\tan^{-1}(cx)}{c}+\frac{b^2dex\tan^{-1}(cx)}{c^3}-$$

$$= -\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{b^2e^2x^2}{30c^4}+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\,\mathrm{ta}}{c}$$

$$= -\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.17, size = 317, normalized size = 0.83

$$\frac{cx(30a^2c^5x(3d^2+3dex^2+e^2x^4)+b^2cex(-16e+3c^2(10d+ex^2))-4ab(15e^2-5c^2e(9d+ex^2)+3c^4(15d^2+5dex^2+e^2x^4)))+4b(-bcx(15e^2-5c^2e(9d+ex^2)+3c^4(15d^2+5dex^2+e^2x^4))+15a(3c^4d^2-3c^2de+e^2+c^6(3d^2x^2+3dex^4+e^2x^6)))\operatorname{ArcTan}(cx)+30b^2(3c^4d^2-3c^2de+e^2+c^6(3d^2x^2+3dex^4+e^2x^6))\operatorname{ArcTan}(cx)^2+2b^2(45c^4d^2-60c^2de+23e^2)\log(1+c^2x^2)}{180c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(30*a^2*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + b^2*c*e*x*(-16*e + 3*c^2
*(10*d + e*x^2)) - 4*a*b*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c^4*(15*d^2 +
5*d*e*x^2 + e^2*x^4))) + 4*b*(-(b*c*x*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c
^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4))) + 15*a*(3*c^4*d^2 - 3*c^2*d*e + e^2 + c
^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6)))*ArcTan[c*x] + 30*b^2*(3*c^4*d^2 - 3*
c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x]^2 + 2*b^
2*(45*c^4*d^2 - 60*c^2*d*e + 23*e^2)*Log[1 + c^2*x^2])/(180*c^6)
```

Maple [A]
time = 0.48, size = 474, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*b^2*arctan(c*x)^2*d^2*c^2*x^2-a*b*c*d^2*x-b^2*c*d^2*x*arctan(c*x
)+a*b*arctan(c*x)*d^2*c^2*x^2+1/9*a*b/c*e^2*x^3+a*b*c^2*e*arctan(c*x)*d*x^4
+1/2*b^2*arctan(c*x)^2*d^2+1/60*b^2*e^2*x^4+1/3*a*b/c^4*arctan(c*x)*e^2-1/2
*b^2/c^2*arctan(c*x)^2*d*e+1/6*b^2*c^2*e^2*arctan(c*x)^2*x^6+1/6*b^2*d*e*x^
2-a*b/c^2*arctan(c*x)*d*e-1/15*a*b*c*e^2*x^5-1/15*b^2*c*arctan(c*x)*e^2*x^5
+1/9*b^2/c*arctan(c*x)*e^2*x^3+23/90*b^2*e^2*ln(c^2*x^2+1)/c^4-2/3*b^2*d*e*
ln(c^2*x^2+1)/c^2+1/3*a*b*c^2*e^2*arctan(c*x)*x^6+1/2*b^2*c^2*e*arctan(c*x)
^2*d*x^4-1/3*a*b*c*d*e*x^3-1/3*b^2*c*arctan(c*x)*d*e*x^3-4/45*b^2*e^2*x^2/c
^2+1/2*b^2*d^2*ln(c^2*x^2+1)-1/3*a*b*e^2*x/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^
3+a*b*d*e*x/c+b^2*d*e*x*arctan(c*x)/c+a*b*arctan(c*x)*d^2+1/6*b^2/c^4*arcta
n(c*x)^2*e^2+1/6*(c^2*e*x^2+c^2*d)^3*a^2/c^4/e)
```

Maxima [**A**]
time = 0.50, size = 433, normalized size = 1.14

$$\frac{1}{6}b^2x^6\arctan(cx)^2e^2+\frac{1}{2}b^2dx^4\arctan(cx)^2e+\frac{1}{6}a^2x^6e^2+\frac{1}{2}b^2d^2x^2\arctan(cx)^2+\frac{1}{2}a^2dx^4e+\frac{1}{2}a^2d^2x^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)abd^2-\frac{1}{2}\left(2c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\arctan(cx)+\frac{\arctan(cx)^2-\log(c^2x^2+1)}{c^2}\right)b^2d^2+\frac{1}{3}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abde-\frac{1}{6}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2de+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abe^2-\frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+46\log(c^2x^2+1)}{c^6}\right)b^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x)^2*e^2 + 1/2*b^2*d*x^4*arctan(c*x)^2*e + 1/6*a^2*x^6
*e^2 + 1/2*b^2*d^2*x^2*arctan(c*x)^2 + 1/2*a^2*d*x^4*e + 1/2*a^2*d^2*x^2 +
(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2 - 1/2*(2*c*(x/c^2 -
 arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
*d^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)
)*a*b*d*e - 1/6*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x)
- (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d*e + 1/45*(15*
x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^
7))*a*b*e^2 - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*
x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c
^2*x^2 + 1))/c^6)*b^2*e^2
```

Fricas [**A**]
time = 2.44, size = 395, normalized size = 1.04

$$\frac{90a^2c^6d^2x^2-180abc^5d^2x+30\left(3b^2c^6d^2x^2+3b^2c^4d^2+(b^2c^6x^6+b^2)e^2+3(b^2c^6dx^4-b^2c^2d)e\right)\arctan(cx)^2+4\left(45abc^6d^2x^2-45b^2c^5d^2x+45abc^4d^2+(15abc^6x^6-3b^2c^5x^5+5b^2c^3x^3-15b^2cx+15ab)e^2+15(3abc^6dx^4-b^2c^5dx^3+3b^2c^3dx-3abc^2d)e\right)\arctan(cx)+(30a^2c^6x^6-12abc^5x^5+3b^2c^4x^4+20abc^3x^3-16b^2c^2x^2-60abcx)e^2+30(3a^2c^6dx^4-2abc^5dx^3+b^2c^4dx^2+6abc^3dx)e+2(45b^2c^4d^2-60b^2c^2de+23b^2e^2)\log(c^2x^2+1)}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(90*a^2*c^6*d^2*x^2 - 180*a*b*c^5*d^2*x + 30*(3*b^2*c^6*d^2*x^2 + 3*b
^2*c^4*d^2 + (b^2*c^6*x^6 + b^2)*e^2 + 3*(b^2*c^6*d*x^4 - b^2*c^2*d)*e)*arc
tan(c*x)^2 + 4*(45*a*b*c^6*d^2*x^2 - 45*b^2*c^5*d^2*x + 45*a*b*c^4*d^2 + (1
5*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b^2*c*x + 15*a*b)*e^2 +
15*(3*a*b*c^6*d*x^4 - b^2*c^5*d*x^3 + 3*b^2*c^3*d*x - 3*a*b*c^2*d)*e)*arcta
n(c*x) + (30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3
```

```
- 16*b^2*c^2*x^2 - 60*a*b*c*x)*e^2 + 30*(3*a^2*c^6*d*x^4 - 2*a*b*c^5*d*x^3
+ b^2*c^4*d*x^2 + 6*a*b*c^3*d*x)*e + 2*(45*b^2*c^4*d^2 - 60*b^2*c^2*d*e + 2
3*b^2*e^2)*log(c^2*x^2 + 1))/c^6
```

Sympy [A]
time = 1.68, size = 575, normalized size = 1.51

$$\begin{cases} \frac{a^2d^2x^2}{2} + \frac{a^2dex^4}{2} + \frac{a^2e^2x^6}{6} + abd^2x^2\operatorname{atan}(cx) + abdex^4\operatorname{atan}(cx) + \frac{abe^2x^6\operatorname{atan}(cx)}{3} - \frac{abd^2x}{c} - \frac{abdex^3}{3c} - \frac{abe^2x^5}{15c} + \frac{abd^2\operatorname{atan}(cx)}{c^2} + \frac{abdex}{c^3} + \frac{abe^2x^3}{9c^3} - \frac{abde\operatorname{atan}(cx)}{c^4} - \frac{abe^2x}{3c^5} + \frac{abe^2\operatorname{atan}(cx)}{3c^6} + \frac{b^2d^2x^2\operatorname{atan}^2(cx)}{2} + \frac{b^2dex^4\operatorname{atan}^2(cx)}{2} + \frac{b^2e^2x^6\operatorname{atan}^2(cx)}{6} - \frac{b^2d^2x\operatorname{atan}(cx)}{c} - \frac{b^2dex^3\operatorname{atan}(cx)}{3c} - \frac{b^2e^2x^5\operatorname{atan}(cx)}{15c} + \frac{b^2d^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^2} + \frac{b^2d^2\operatorname{atan}^2(cx)}{2c^2} + \frac{b^2dex^2}{6c^2} + \frac{b^2e^2x^4}{60c^2} + \frac{b^2dex\operatorname{atan}(cx)}{c^3} + \frac{b^2e^2x^3\operatorname{atan}(cx)}{9c^3} - \frac{2b^2de\log\left(x^2+\frac{1}{c^2}\right)}{3c^4} - \frac{b^2de\operatorname{atan}^2(cx)}{2c^4} - \frac{4b^2e^2x^2}{45c^4} - \frac{b^2e^2x\operatorname{atan}(cx)}{3c^5} + \frac{23b^2e^2\log\left(x^2+\frac{1}{c^2}\right)}{90c^6} + \frac{b^2e^2\operatorname{atan}^2(cx)}{6c^6} & \text{for}\: c \neq 0 \\ a^2\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x**2/2 + a**2*d*e*x**4/2 + a**2*e**2*x**6/6 + a*b*d**2
*x**2*atan(c*x) + a*b*d*e*x**4*atan(c*x) + a*b*e**2*x**6*atan(c*x)/3 - a*b*
d**2*x/c - a*b*d*e*x**3/(3*c) - a*b*e**2*x**5/(15*c) + a*b*d**2*atan(c*x)/c
**2 + a*b*d*e*x/c**3 + a*b*e**2*x**3/(9*c**3) - a*b*d*e*atan(c*x)/c**4 - a*
b*e**2*x/(3*c**5) + a*b*e**2*atan(c*x)/(3*c**6) + b**2*d**2*x**2*atan(c*x)*
*2/2 + b**2*d*e*x**4*atan(c*x)**2/2 + b**2*e**2*x**6*atan(c*x)**2/6 - b**2*
d**2*x*atan(c*x)/c - b**2*d*e*x**3*atan(c*x)/(3*c) - b**2*e**2*x**5*atan(c*
x)/(15*c) + b**2*d**2*log(x**2 + c**(-2))/(2*c**2) + b**2*d**2*atan(c*x)**2
/(2*c**2) + b**2*d*e*x**2/(6*c**2) + b**2*e**2*x**4/(60*c**2) + b**2*d*e*x*
atan(c*x)/c**3 + b**2*e**2*x**3*atan(c*x)/(9*c**3) - 2*b**2*d*e*log(x**2 +
c**(-2))/(3*c**4) - b**2*d*e*atan(c*x)**2/(2*c**4) - 4*b**2*e**2*x**2/(45*c
**4) - b**2*e**2*x*atan(c*x)/(3*c**5) + 23*b**2*e**2*log(x**2 + c**(-2))/(9
0*c**6) + b**2*e**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*(d**2*x**2/2 +
d*e*x**4/2 + e**2*x**6/6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 5.28, size = 780, normalized size = 2.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] (a^2*d^2*x^2)/2 + (a^2*e^2*x^6)/6 + (b^2*d^2*log(c^2*x^2 + 1))/(2*c^2) + (2
3*b^2*e^2*log(c^2*x^2 + 1))/(90*c^6) + (b^2*e^2*x^4)/(60*c^2) - (4*b^2*e^2*
```

x^2)/(45*c^4) + (b^2*d^2*atan(c*x)^2)/(2*c^2) + (b^2*e^2*atan(c*x)^2)/(6*c^6) + (b^2*d^2*x^2*atan(c*x)^2)/2 + (b^2*e^2*x^6*atan(c*x)^2)/6 + (a^2*d*e*x^4)/2 - (b^2*e^2*x^5*atan(c*x))/(15*c) + (b^2*e^2*x^3*atan(c*x))/(9*c^3) - (a*b*d^2*x)/c - (a*b*e^2*x)/(3*c^5) + a*b*d^2*x^2*atan(c*x) + (a*b*e^2*x^6*atan(c*x))/3 - (2*b^2*d*e*log(c^2*x^2 + 1))/(3*c^4) - (a*b*e^2*x^5)/(15*c) + (a*b*e^2*x^3)/(9*c^3) + (b^2*d*e*x^2)/(6*c^2) - (b^2*d*e*atan(c*x)^2)/(2*c^4) - (b^2*d^2*x*atan(c*x))/c - (b^2*e^2*x*atan(c*x))/(3*c^5) + (b^2*d*e*x^4*atan(c*x)^2)/2 + (a*b*d^2*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e)))/c^2 + (a*b*e^2*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e)))/(3*c^6) - (b^2*d*e*x^3*atan(c*x))/(3*c) + (a*b*d*e*x)/c^3 + a*b*d*e*x^4*atan(c*x) - (a*b*d*e*x^3)/(3*c) + (b^2*d*e*x*atan(c*x))/c^3 - (a*b*d*e*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e)))/c^4

3.1257 $\int (d+ex^2)^2 (a+b\mathbf{ArcTan}(cx))^2 \, dx$

Optimal. Leaf size=442

$$\frac{2b^2dex}{3c^2}-\frac{3b^2e^2x}{10c^4}+\frac{b^2e^2x^3}{30c^2}-\frac{2b^2de\text{ArcTan}(cx)}{3c^3}+\frac{3b^2e^2\text{ArcTan}(cx)}{10c^5}-\frac{2bdex^2(a+b\text{ArcTan}(cx))}{3c}+\frac{be^2x^2(a+b\text{A}}{5c^3}$$

```
[Out] 2/3*b^2*d*e*x/c^2-3/10*b^2*e^2*x/c^4+1/30*b^2*e^2*x^3/c^2-2/3*b^2*d*e*arcta
n(c*x)/c^3+3/10*b^2*e^2*arctan(c*x)/c^5-2/3*b*d*e*x^2*(a+b*arctan(c*x))/c+1
/5*b*e^2*x^2*(a+b*arctan(c*x))/c^3-1/10*b*e^2*x^4*(a+b*arctan(c*x))/c+I*b^2
*d^2*polylog(2,1-2/(1+I*c*x))/c-2/3*I*b^2*d*e*polylog(2,1-2/(1+I*c*x))/c^3+
1/5*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^5+d^2*x*(a+b*arctan(c*x))^2+2/3*d*
e*x^3*(a+b*arctan(c*x))^2+1/5*e^2*x^5*(a+b*arctan(c*x))^2+2*b*d^2*(a+b*arct
an(c*x))*ln(2/(1+I*c*x))/c-4/3*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+
2/5*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5-2/3*I*d*e*(a+b*arctan(c*x))
^2/c^3+I*d^2*(a+b*arctan(c*x))^2/c+1/5*I*e^2*(a+b*arctan(c*x))^2/c^5
```

Rubi [A]

time = 0.53, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5034, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 327, 209, 308}

$$\frac{ie^2(a+b\text{ArcTan}(cx))^2}{5c^5}+\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{5c^5}-\frac{2ide(a+b\text{ArcTan}(cx))^2}{3c^3}-\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}+\frac{be^2x^2(a+b\text{ArcTan}(cx))}{5c^3}+d^2x(a+b\text{ArcTan}(cx))^2+\frac{id^2(a+b\text{ArcTan}(cx))^2}{c}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{2}{3}dex^3(a+b\text{ArcTan}(cx))^2-\frac{2bdex^2(a+b\text{ArcTan}(cx))}{3c}+\frac{1}{5}e^2x^5(a+b\text{ArcTan}(cx))^2-\frac{be^2x^4(a+b\text{ArcTan}(cx))}{10c}+\frac{3b^2e^2\text{ArcTan}(cx)}{10c^5}-\frac{2b^2de\text{ArcTan}(cx)}{3c^3}+\frac{ib^2e^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2e^2x}{10c^4}-\frac{2ib^2de\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{2b^2dex}{3c^2}+\frac{b^2e^2x^3}{30c^2}+\frac{ib^2d^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (2*b^2*d*e*x)/(3*c^2) - (3*b^2*e^2*x)/(10*c^4) + (b^2*e^2*x^3)/(30*c^2) - (
2*b^2*d*e*ArcTan[c*x])/(3*c^3) + (3*b^2*e^2*ArcTan[c*x])/(10*c^5) - (2*b*d*
e*x^2*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^2*(a + b*ArcTan[c*x]))/(5*c^3)
- (b*e^2*x^4*(a + b*ArcTan[c*x]))/(10*c) + (I*d^2*(a + b*ArcTan[c*x])^2)/c
- (((2*I)/3)*d*e*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e^2*(a + b*ArcTan[c*x]
)^2)/c^5 + d^2*x*(a + b*ArcTan[c*x])^2 + (2*d*e*x^3*(a + b*ArcTan[c*x])^2)/
3 + (e^2*x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*d^2*(a + b*ArcTan[c*x])*Log[2/
(1 + I*c*x)])/c - (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3)
+ (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + (I*b^2*d^2*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c - (((2*I)/3)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*
c*x)])/c^3 + ((I/5)*b^2*e^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex^2)^2 \left(a+b\tan^{-1}(cx)\right)^2 \, dx = \int \left(d^2\left(a+b\tan^{-1}(cx)\right)^2+2dex^2\left(a+b\tan^{-1}(cx)\right)^2+e^2x^4\left(a+b\tan\right.\right.$$

$$= d^2\int \left(a+b\tan^{-1}(cx)\right)^2 \, dx+(2de)\int x^2\left(a+b\tan^{-1}(cx)\right)^2 \, dx+e^2$$

$$= d^2x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}dex^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{5}e^2x^5\left(a+b\tan\right.$$

$$= \frac{id^2\left(a+b\tan^{-1}(cx)\right)^2}{c}+d^2x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2}{3}dex^3\left(a+b\tan^{-1}\right.$$

$$= -\frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{be^2x^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{id^2\left(a+b\tan\right.}{c}$$

$$= \frac{2b^2dex}{3c^2}-\frac{2bdex^2\left(a+b\tan^{-1}(cx)\right)}{3c}+\frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{be^2x}{}$$

$$= \frac{2b^2dex}{3c^2}-\frac{3b^2e^2x}{10c^4}+\frac{b^2e^2x^3}{30c^2}-\frac{2b^2de\tan^{-1}(cx)}{3c^3}-\frac{2bdex^2\left(a+b\tan^{-1}\right.}{3c}$$

$$= \frac{2b^2dex}{3c^2}-\frac{3b^2e^2x}{10c^4}+\frac{b^2e^2x^3}{30c^2}-\frac{2b^2de\tan^{-1}(cx)}{3c^3}+\frac{3b^2e^2\tan^{-1}(cx)}{10c^5}-$$

$$= \frac{2b^2dex}{3c^2}-\frac{3b^2e^2x}{10c^4}+\frac{b^2e^2x^3}{30c^2}-\frac{2b^2de\tan^{-1}(cx)}{3c^3}+\frac{3b^2e^2\tan^{-1}(cx)}{10c^5}-$$

Mathematica [A]
time = 0.73, size = 391, normalized size = 0.88

$$\frac{9abe^2+30a^2c^5d^2x+20b^2c^3dex-9b^2ce^2x-20abc^4dex^2+6abc^2e^2x^2+20a^2c^5dex^3+b^2c^3e^2x^3-3abc^4e^2x^4+6a^2c^5e^2x^5+2b^2(-15ic^4d^2+10ic^2de-3ie^2+c^5(15d^2x+10dex^3+3e^2x^5))\operatorname{ArcTan}(cx)^2+b\operatorname{ArcTan}(cx)\left(4ac^5x(15d^2+10dex^2+3e^2x^4)-be(1+c^2x^2)\left(-9e+c^2(20d+3ex^2)\right)+4b(15c^4d^2-10c^2de+3e^2)\log\left(1+e^{2i\operatorname{ArcTan}(cx)}\right)\right)-30abc^4d^2\log\left(1+c^2x^2\right)+20abc^2de\log\left(1+c^2x^2\right)-6abe^2\log\left(1+c^2x^2\right)-2ib^2(15c^4d^2-10c^2de+3e^2)\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx)}\right)}{30c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b*e^2 + 30*a^2*c^5*d^2*x + 20*b^2*c^3*d*e*x - 9*b^2*c*e^2*x - 20*a*b*c
^4*d*e*x^2 + 6*a*b*c^2*e^2*x^2 + 20*a^2*c^5*d*e*x^3 + b^2*c^3*e^2*x^3 - 3*a
*b*c^4*e^2*x^4 + 6*a^2*c^5*e^2*x^5 + 2*b^2*((-15*I)*c^4*d^2 + (10*I)*c^2*d*
e - (3*I)*e^2 + c^5*(15*d^2*x + 10*d*e*x^3 + 3*e^2*x^5))*ArcTan[c*x]^2 + b*
ArcTan[c*x]*(4*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*(1 + c^2*x^2
)*(-9*e + c^2*(20*d + 3*e*x^2)) + 4*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log
[1 + E^((2*I)*ArcTan[c*x])]) - 30*a*b*c^4*d^2*Log[1 + c^2*x^2] + 20*a*b*c^2
*d*e*Log[1 + c^2*x^2] - 6*a*b*e^2*Log[1 + c^2*x^2] - (2*I)*b^2*(15*c^4*d^2
- 10*c^2*d*e + 3*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(398) = 796.
time = 0.41, size = 981, normalized size = 2.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(2*a*b*arctan(c*x)*d^2*c*x-2/3*a*b*d*e*x^2-2/3*b^2*arctan(c*x)*d*e*x^2+
4/3*a*b*c*arctan(c*x)*d*e*x^3-1/3*I*b^2/c^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))*d*
e-1/3*I*b^2/c^2*ln(c^2*x^2+1)*ln(I+c*x)*d*e+1/3*I*b^2/c^2*ln(I+c*x)*ln(1/2*
I*(c*x-I))*d*e+1/3*I*b^2/c^2*ln(c^2*x^2+1)*ln(c*x-I)*d*e+2/5*a*b*c*arctan(c
*x)*e^2*x^5+2/3*b^2*c*arctan(c*x)^2*d*e*x^3+1/20*I*b^2/c^4*ln(c*x-I)^2*e^2+
1/10*I*b^2/c^4*dilog(-1/2*I*(I+c*x))*e^2-1/20*I*b^2/c^4*ln(I+c*x)^2*e^2-1/1
0*I*b^2/c^4*dilog(1/2*I*(c*x-I))*e^2+1/2*I*b^2*ln(c^2*x^2+1)*ln(I+c*x)*d^2-
1/2*I*b^2*ln(I+c*x)*ln(1/2*I*(c*x-I))*d^2+1/2*I*b^2*ln(c*x-I)*ln(-1/2*I*(I+
c*x))*d^2-1/2*I*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d^2+b^2*arctan(c*x)^2*d^2*c*x+3
/10*b^2/c^4*arctan(c*x)*e^2-a*b*ln(c^2*x^2+1)*d^2-b^2*arctan(c*x)*ln(c^2*x^
2+1)*d^2+1/2*I*b^2*dilog(-1/2*I*(I+c*x))*d^2-1/4*I*b^2*ln(I+c*x)^2*d^2-1/2*
I*b^2*dilog(1/2*I*(c*x-I))*d^2+1/4*I*b^2*ln(c*x-I)^2*d^2-1/6*I*b^2/c^2*ln(c
*x-I)^2*d*e-1/3*I*b^2/c^2*dilog(-1/2*I*(I+c*x))*d*e-1/10*I*b^2/c^4*ln(c^2*x
^2+1)*ln(c*x-I)*e^2+1/10*I*b^2/c^4*ln(c*x-I)*ln(-1/2*I*(I+c*x))*e^2+1/10*I*
b^2/c^4*ln(c^2*x^2+1)*ln(I+c*x)*e^2-1/10*I*b^2/c^4*ln(I+c*x)*ln(1/2*I*(c*x-
I))*e^2+1/6*I*b^2/c^2*ln(I+c*x)^2*d*e+1/5*b^2*c*arctan(c*x)^2*e^2*x^5+1/3*I
*b^2/c^2*dilog(1/2*I*(c*x-I))*d*e+2/3*b^2/c*d*e*x-1/10*b^2*arctan(c*x)*e^2*
x^4-1/10*a*b*e^2*x^4+1/5*b^2/c^2*arctan(c*x)*e^2*x^2+2/3*b^2/c^2*arctan(c*x
)*ln(c^2*x^2+1)*d*e+1/5*a*b/c^2*e^2*x^2+2/3*a*b/c^2*ln(c^2*x^2+1)*d*e-2/3*b
^2/c^2*arctan(c*x)*d*e-1/5*b^2/c^4*arctan(c*x)*ln(c^2*x^2+1)*e^2+1/30*b^2/c
```

```
*e^2*x^3-3/10*b^2/c^3*e^2*x-1/5*a*b/c^4*ln(c^2*x^2+1)*e^2+a^2/c^4*(d^2*c^5*
x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5*e^2 + 2/3*a^2*d*x^3*e + 360*b^2*c^2*d*e*integrate(1/240*x^4*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 30*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) + 40*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^
2 + 1)/(c^2*x^2 + 1), x) + 180*b^2*c^2*d^2*integrate(1/240*x^2*arctan(c*x)^
2/(c^2*x^2 + 1), x) + 15*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)^2
/(c^2*x^2 + 1), x) + 60*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 1/4*b^2*d^2*arctan(c*x)^3/c + 180*b^2*c^2*e^2*integrate(1
/240*x^6*arctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*c^2*e^2*integrate(1/240*x
^6*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*e^2*integrate(1/240*x^
6*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 80*b^2*c*d*e*integrate(1/240*x^3*arc
tan(c*x)/(c^2*x^2 + 1), x) - 120*b^2*c*d^2*integrate(1/240*x*arctan(c*x)/(c
^2*x^2 + 1), x) + a^2*d^2*x + 2/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2
*x^2 + 1)/c^4))*a*b*d*e - 24*b^2*c*e^2*integrate(1/240*x^5*arctan(c*x)/(c^2
*x^2 + 1), x) + 360*b^2*d*e*integrate(1/240*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + 30*b^2*d*e*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 15*b^2*d^2*integrate(1/240*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*
arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/10*(4*x^5*arctan(c*x) - c*((c
^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*e^2 + 180*b^2*e^2*integr
ate(1/240*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*e^2*integrate(1/240*
x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1/60*(3*b^2*x^5*e^2 + 10*b^2*d*x
^3*e + 15*b^2*d^2*x)*arctan(c*x)^2 - 1/240*(3*b^2*x^5*e^2 + 10*b^2*d*x^3*e
+ 15*b^2*d^2*x)*log(c^2*x^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*x^2
*e + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)*arc
tan(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}\left(cx\right)\right)^{2} \left(d + ex^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2,x)
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d + e*x^2)^2,x)
[Out] int((a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

3.1258 $\int \frac{(d+ex^2)^2(a+b\mathbf{ArcTan}(cx))^2}{x}\,dx$

Optimal. Leaf size=355

$$-\frac{2abdex}{c}+\frac{abe^2x}{2c^3}+\frac{b^2e^2x^2}{12c^2}-\frac{2b^2dex\text{ArcTan}(cx)}{c}+\frac{b^2e^2x\text{ArcTan}(cx)}{2c^3}-\frac{be^2x^3(a+b\text{ArcTan}(cx))}{6c}+\frac{de(a+b\text{Arc}}{c^2}$$

```
[Out] -2*a*b*d*e*x/c+1/2*a*b*e^2*x/c^3+1/12*b^2*e^2*x^2/c^2-2*b^2*d*e*x*arctan(c*
x)/c+1/2*b^2*e^2*x*arctan(c*x)/c^3-1/6*b*e^2*x^3*(a+b*arctan(c*x))/c+d*e*(a
+b*arctan(c*x))^2/c^2-1/4*e^2*(a+b*arctan(c*x))^2/c^4+d*e*x^2*(a+b*arctan(c
*x))^2+1/4*e^2*x^4*(a+b*arctan(c*x))^2-2*d^2*(a+b*arctan(c*x))^2*arctanh(-1
+2/(1+I*c*x))+b^2*d*e*ln(c^2*x^2+1)/c^2-1/3*b^2*e^2*ln(c^2*x^2+1)/c^4-I*b*d
^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d^2*(a+b*arctan(c*x))*pol
ylog(2,-1+2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^2*pol
ylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.52, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5100, 4942, 5108, 5004, 5114, 6745, 4946, 5036, 4930, 266, 272, 45}

$$-\frac{e^2(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{de(a+b\text{ArcTan}(cx))^2}{c^2}-ibd^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ibd^2\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+2d^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+dex^2(a+b\text{ArcTan}(cx))^2+\frac{1}{4}e^2x^4(a+b\text{ArcTan}(cx))^2-\frac{be^2x^3(a+b\text{ArcTan}(cx))}{6c}+\frac{abe^2x}{2c^3}-\frac{2abdex}{c}+\frac{b^2e^2x\text{ArcTan}(cx)}{2c^3}-\frac{2b^2dex\text{ArcTan}(cx)}{c}+\frac{b^2de\log(c^2x^2+1)}{c^2}+\frac{b^2e^2x^2}{12c^2}-\frac{b^2e^2\log(c^2x^2+1)}{3c^4}-\frac{1}{2}b^2d^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] (-2*a*b*d*e*x)/c + (a*b*e^2*x)/(2*c^3) + (b^2*e^2*x^2)/(12*c^2) - (2*b^2*d*
e*x*ArcTan[c*x])/c + (b^2*e^2*x*ArcTan[c*x])/(2*c^3) - (b*e^2*x^3*(a + b*Ar
cTan[c*x]))/(6*c) + (d*e*(a + b*ArcTan[c*x])^2)/c^2 - (e^2*(a + b*ArcTan[c*
x])^2)/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x])^2 + (e^2*x^4*(a + b*ArcTan[c*x
])^2)/4 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (b^2*d*e
*Log[1 + c^2*x^2])/c^2 - (b^2*e^2*Log[1 + c^2*x^2])/(3*c^4) - I*b*d^2*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*
PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2
+ (b^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\tan^{-1}(cx))^2}{x} dx = \int \left(\frac{d^2(a+b\tan^{-1}(cx))^2}{x} + 2dex\left(a+b\tan^{-1}(cx)\right)^2 + e^2x^3\left(a+b\tan^{-1}\right.\right.$$

$$= d^2 \int \frac{(a+b\tan^{-1}(cx))^2}{x} dx + (2de)\int x\left(a+b\tan^{-1}(cx)\right)^2 dx + e^2 \int$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)^2 + 2d^2\left(a+b\tan^{-1}(\right.$$

$$= dex^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{4}e^2x^4\left(a+b\tan^{-1}(cx)\right)^2 + 2d^2\left(a+b\tan^{-1}(\right.$$

$$= -\frac{2abdex}{c} - \frac{be^2x^3(a+b\tan^{-1}(cx))}{6c} + \frac{de(a+b\tan^{-1}(cx))^2}{c^2} + dex^2(a$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^3(a+b\tan^{-1}(cx))}{6c} +$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} + \frac{b^2e^2x\tan^{-1}(cx)}{2c^3} - \frac{be^2x^3(a+}{}$$

$$= -\frac{2abdex}{c} + \frac{abe^2x}{2c^3} + \frac{b^2e^2x^2}{12c^2} - \frac{2b^2dex\tan^{-1}(cx)}{c} + \frac{b^2e^2x\tan^{-1}(cx)}{2c^3} -$$

Mathematica [A]
time = 0.43, size = 389, normalized size = 1.10

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

```
[Out] a^2*d*e*x^2 + (a^2*e^2*x^4)/4 + (2*a*b*d*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c
*x]))/c^2 + (a*b*e^2*(3*c*x - c^3*x^3 + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(6*c
^4) + a^2*d^2*Log[x] + (b^2*e^2*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*ArcTan[c
*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]))/(12*c^4) + (b^2
*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))
/c^2 + I*a*b*d^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*d^2*((-1/
24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcT
an[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*Po
lyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTa
n[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcT
an[c*x])]/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.57, size = 1549, normalized size = 4.36

method	result	size
derivativedivides	Expression too large to display	1549
default	Expression too large to display	1549

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a*b/c*e^2*x^3-I*a*b*d^2*ln(c*x)*ln(1-I*c*x)+a^2*d*e*x^2+1/4*b^2*arctan
(c*x)^2*e^2*x^4+1/2*I*b^2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x
)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2
*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d^2*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)
^2+2*I*b^2/c^2*arctan(c*x)*d*e+I*a*b*d^2*ln(c*x)*ln(1+I*c*x)+I*a*b*d^2*dilo
g(1+I*c*x)-I*a*b*d^2*dilog(1-I*c*x)+1/4*a^2*e^2*x^4-1/2*a*b/c^4*arctan(c*x)
*e^2+b^2/c^2*arctan(c*x)^2*d*e+2*a*b/c^2*arctan(c*x)*d*e-1/6*b^2/c*arctan(c
*x)*e^2*x^3+2/3*b^2/c^4*e^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-b^2*d^2*arctan(c*
x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*d^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^
2*x^2+1)^(1/2))+b^2*d^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2
*arctan(c*x)^2*d^2*ln(c*x)+1/12*b^2*e^2*x^2/c^2+1/2*a*b*e^2*x/c^3+1/2*b^2*e
^2*x*arctan(c*x)/c^3-2*a*b*d*e*x/c-2*b^2*d*e*x*arctan(c*x)/c-1/4*b^2/c^4*ar
ctan(c*x)^2*e^2-2*b^2/c^2*d*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+I*b^2*d^2*arcta
n(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*b^2*d^2*arctan(c*x)*polylog(
2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*d^2*arctan(c*x)*polylog(2,(1+I*c*x)
/(c^2*x^2+1)^(1/2))+1/2*I*b^2*d^2*Pi*arctan(c*x)^2-2/3*I*b^2/c^4*arctan(c*x
)*e^2+2*a*b*arctan(c*x)*d^2*ln(c*x)-1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d^2*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(I
/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
)^2*arctan(c*x)^2+b^2*arctan(c*x)^2*d*e*x^2+1/2*a*b*arctan(c*x)*e^2*x^4+1/1
2*b^2/c^4*e^2-1/2*b^2*d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*d^2*pol
ylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*d^2*polylog(3,(1+I*c*x)/(c^2*x^2
+1)^(1/2))+1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*arctan(c*x)^2+2*a*b*arctan(c*x)*d*e*x^2+a^2*d^2*ln(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")

[Out] 1/4*a^2*x^4*e^2 + 24*b^2*c^2*d*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*b^2*c^2*d*e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 64*a*b*c^2*d*e*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 4*b^2*c^2*d*e*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d^2*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d^2*log(c^2*x^2 + 1)^3 + a^2*d*x^2*e + 12*b^2*c^2*e^2*integrate(1/16*x^6*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*e^2*integrate(1/16*x^6*arctan(c*x)/(c^2*x^3 + x), x) + b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 8*b^2*c*d*e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) - 2*b^2*c*e^2*integrate(1/16*x^5*arctan(c*x)/(c^2*x^3 + x), x) + 24*b^2*d*e*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 64*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d^2*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/48*b^2*d*e*log(c^2*x^2 + 1)^3/c^2 + a^2*d^2*log(x) + 12*b^2*e^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*e^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*e^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 1/16*(b^2*x^4*e^2 + 4*b^2*d*x^2*e)*arctan(c*x)^2 - 1/64*(b^2*x^4*e^2 + 4*b^2*d*x^2*e)*log(c^2*x^2 + 1)^2

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")

[Out] integral((a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*x^2*e + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)*arctan(c*x))/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\operatorname{atan}(cx))^2\,(d + ex^2)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x,x)

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x, x)
```

3.1259 $\int \frac{(d+ex^2)^2(a+b\text{ArcTan}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=343

$$\frac{b^2e^2x}{3c^2}-\frac{b^2e^2\text{ArcTan}(cx)}{3c^3}-\frac{be^2x^2(a+b\text{ArcTan}(cx))}{3c}-icd^2(a+b\text{ArcTan}(cx))^2+\frac{2ide(a+b\text{ArcTan}(cx))^2}{c}-\frac{ie^2(a$$

```
[Out] 1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctan(c*x)/c^3-1/3*b*e^2*x^2*(a+b*arctan(c*x
))/c-I*c*d^2*(a+b*arctan(c*x))^2+2*I*d*e*(a+b*arctan(c*x))^2/c-1/3*I*e^2*(a
+b*arctan(c*x))^2/c^3-d^2*(a+b*arctan(c*x))^2/x+2*d*e*x*(a+b*arctan(c*x))^2
+1/3*e^2*x^3*(a+b*arctan(c*x))^2+4*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/
c-2/3*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2*b*c*d^2*(a+b*arctan(c*x
))*ln(2-2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))+2*I*b^2*d*e*poly
log(2,1-2/(1+I*c*x))/c-1/3*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A]
time = 0.43, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5100, 4930, 5040, 4964, 2449, 2352, 4946, 5044, 4988, 2497, 5036, 327, 209}

$$-\frac{ie^2(a+b\text{ArcTan}(cx))^2}{3c^3}-\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}-icd^2(a+b\text{ArcTan}(cx))^2-\frac{d^2(a+b\text{ArcTan}(cx))^2}{x}+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))+2dex(a+b\text{ArcTan}(cx))^2+\frac{2ide(a+b\text{ArcTan}(cx))^2}{c}+\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{1}{3}e^2x^3(a+b\text{ArcTan}(cx))^2-\frac{be^2x^2(a+b\text{ArcTan}(cx))}{3c}-\frac{b^2e^2\text{ArcTan}(cx)}{3c^3}-\frac{ib^2e^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2e^2x}{3c^2}-ib^2cd^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)+\frac{2ib^2de\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (b*e^2*x^2*(a + b*Arc
Tan[c*x]))/(3*c) - I*c*d^2*(a + b*ArcTan[c*x])^2 + ((2*I)*d*e*(a + b*ArcTan
[c*x])^2)/c - ((I/3)*e^2*(a + b*ArcTan[c*x])^2)/c^3 - (d^2*(a + b*ArcTan[c*
x])^2)/x + 2*d*e*x*(a + b*ArcTan[c*x])^2 + (e^2*x^3*(a + b*ArcTan[c*x])^2)/
3 + (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e^2*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[
2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)] + ((2*I)*b^
2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
```

```
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(2de\left(a+b\tan^{-1}(cx)\right)^2 + \frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x^2} + e^2x^2\left(a+b\tan^{-}\right.\right.$$

$$= d^2\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx + (2de)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx + e^2\int$$

$$= -\frac{d^2\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2dex\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{3}e^2x^3\left(a+b\tan^{-}\right.$$

$$= -icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{d^2\left(a+b\tan^{-1}(\right.}{x}$$

$$= -\frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide\left(a+b\tan^{-}\right.}{c}$$

$$= \frac{b^2e^2x}{3c^2} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}(cx)\right)^2 + \frac{2ide(a+}{}$$

$$= \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}\right.$$

$$= \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} - icd^2\left(a+b\tan^{-1}\right.$$

Mathematica [**A**]
time = 0.50, size = 349, normalized size = 1.02

$$\frac{1}{3}\left(-\frac{3a^2d^2}{x} + 6a^2dex + a^2e^2x^3 + \frac{6abde\left(2cx\mathrm{ArcTan}(cx) - \log\left(1+c^2x^2\right)\right)}{c} + \frac{abe^2\left(-c^2x^2 + 2c^3x^3\mathrm{ArcTan}(cx) + \log\left(1+c^2x^2\right)\right)}{c^3} - \frac{3abd^2\left(2\mathrm{ArcTan}(cx) + cx\left(-2\log(cx) + \log\left(1+c^2x^2\right)\right)\right)}{x} + \frac{6b^2de\left(\mathrm{ArcTan}(cx)\left((-i+cx)\mathrm{ArcTan}(cx) + 2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right) - i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{c} + \frac{b^2e^2\left(cx + \left(i+c^3x^3\right)\mathrm{ArcTan}(cx)^2 - \mathrm{ArcTan}(cx)\left(1+c^2x^2+2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right) + i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{c^3} + 3b^2cd^2\left(\mathrm{ArcTan}(cx)\left(\left(-i-\frac{1}{cx}\right)\mathrm{ArcTan}(cx) + 2\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)\right) - i\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

```
[Out] ((-3*a^2*d^2)/x + 6*a^2*d*e*x + a^2*e^2*x^3 + (6*a*b*d*e*(2*c*x*ArcTan[c*x]
 - Log[1 + c^2*x^2]))/c + (a*b*e^2*(-(c^2*x^2) + 2*c^3*x^3*ArcTan[c*x] + Lo
g[1 + c^2*x^2]))/c^3 - (3*a*b*d^2*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1
 + c^2*x^2])))/x + (6*b^2*d*e*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[
1 + E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b
^2*e^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Lo
g[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
+ 3*b^2*c*d^2*(ArcTan[c*x]*((-I - 1/(c*x))*ArcTan[c*x] + 2*Log[1 - E^((2*I)
*ArcTan[c*x])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(317) = 634.
time = 0.79, size = 1009, normalized size = 2.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/6*I*b^2/c^4*dilog(-1/2*I*(I+c*x))*e^2+1/6*I*b^2/c^4*dilog(1/2*I*(c*x-
I))*e^2-1/12*I*b^2/c^4*ln(c*x-I)^2*e^2+1/12*I*b^2/c^4*ln(I+c*x)^2*e^2-I*b^2
*d^2*ln(c*x)*ln(1-I*c*x)+I*b^2*d^2*ln(c*x)*ln(1+I*c*x)-b^2*arctan(c*x)^2*d^
2/c/x-2*a*b*arctan(c*x)*d^2/c/x+4*a*b/c*arctan(c*x)*d*e*x+a^2/c^4*(2*c^3*d*
e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+I*b^2/c^2*ln(-1/2*I*(I+c*x))*ln(c*x-I)*d*e+I
*b^2/c^2*ln(c^2*x^2+1)*ln(I+c*x)*d*e-I*b^2/c^2*ln(c^2*x^2+1)*ln(c*x-I)*d*e-
I*b^2/c^2*ln(1/2*I*(c*x-I))*ln(I+c*x)*d*e+2/3*a*b/c*arctan(c*x)*e^2*x^3+2*b
^2/c*arctan(c*x)^2*d*e*x+1/2*I*b^2*ln(c^2*x^2+1)*ln(I+c*x)*d^2-1/2*I*b^2*ln
(I+c*x)*ln(1/2*I*(c*x-I))*d^2+1/2*I*b^2*ln(c*x-I)*ln(-1/2*I*(I+c*x))*d^2-1/
2*I*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d^2-1/3*b^2/c^4*arctan(c*x)*e^2-a*b*ln(c^2*
x^2+1)*d^2-b^2*arctan(c*x)*ln(c^2*x^2+1)*d^2+2*a*b*d^2*ln(c*x)+I*b^2*d^2*di
log(1+I*c*x)+2*b^2*arctan(c*x)*d^2*ln(c*x)-I*b^2*d^2*dilog(1-I*c*x)+1/2*I*b
^2*dilog(-1/2*I*(I+c*x))*d^2-1/4*I*b^2*ln(I+c*x)^2*d^2-1/2*I*b^2*dilog(1/2*
I*(c*x-I))*d^2+1/4*I*b^2*ln(c*x-I)^2*d^2+1/6*I*b^2/c^4*ln(c^2*x^2+1)*ln(c*x
-I)*e^2-I*b^2/c^2*dilog(1/2*I*(c*x-I))*d*e-1/2*I*b^2/c^2*ln(I+c*x)^2*d*e+1/
2*I*b^2/c^2*ln(c*x-I)^2*d*e+1/3*b^2/c*arctan(c*x)^2*e^2*x^3+I*b^2/c^2*dilog
(-1/2*I*(I+c*x))*d*e-1/6*I*b^2/c^4*ln(-1/2*I*(I+c*x))*ln(c*x-I)*e^2-1/6*I*b
^2/c^4*ln(c^2*x^2+1)*ln(I+c*x)*e^2+1/6*I*b^2/c^4*ln(1/2*I*(c*x-I))*ln(I+c*x
)*e^2-1/3*b^2/c^2*arctan(c*x)*e^2*x^2-2*b^2/c^2*arctan(c*x)*ln(c^2*x^2+1)*d
*e-1/3*a*b/c^2*e^2*x^2-2*a*b/c^2*ln(c^2*x^2+1)*d*e+1/3*b^2/c^4*arctan(c*x)*
ln(c^2*x^2+1)*e^2+1/3*b^2/c^3*e^2*x+1/3*a*b/c^4*ln(c^2*x^2+1)*e^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3*e^2 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d
^2 + 2*a^2*d*x*e + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c
^4))*a*b*e^2 + 2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d*e/c - a^2*d^2
/x + 1/48*(4*(b^2*x^4*e^2 + 6*b^2*d*x^2*e - 3*b^2*d^2)*arctan(c*x)^2 - (b^2
*x^4*e^2 + 6*b^2*d*x^2*e - 3*b^2*d^2)*log(c^2*x^2 + 1)^2 + 12*(b^2*c*d^2*ar
ctan(c*x)^3 + 288*b^2*c^2*d*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^4 + x
^2), x) + 24*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 + x
^2), x) + 96*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^4 + x^2
), x) + 12*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2
), x) - 48*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 2*b^2*d*arctan(c*x)^3*e/c + 144*b^2*c^2*e^2*integrate(1/48*x^6*arctan
(c*x)^2/(c^2*x^4 + x^2), x) + 12*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2
```

```
 + 1)^2/(c^2*x^4 + x^2), x) + 16*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2
 + 1)/(c^2*x^4 + x^2), x) - 192*b^2*c*d*e*integrate(1/48*x^3*arctan(c*x)/(c
^2*x^4 + x^2), x) + 96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^4 + x^
2), x) - 32*b^2*c*e^2*integrate(1/48*x^5*arctan(c*x)/(c^2*x^4 + x^2), x) +
24*b^2*d*e*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 144*
b^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 12*b^2*d^2*integ
rate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 144*b^2*e^2*integrate(1/
48*x^4*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 12*b^2*e^2*integrate(1/48*x^4*lo
g(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*x^
2*e + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)*ar
ctan(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(cx \right)}\right)^{2} \left(d + ex^{2}\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2,x)
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2, x)
```

3.1260 $\int \frac{(d+ex^2)^2(a+b\text{ArcTan}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=320

$$-\frac{abe^2x}{c}-\frac{b^2e^2x\text{ArcTan}(cx)}{c}-\frac{bcd^2(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}c^2d^2(a+b\text{ArcTan}(cx))^2+\frac{e^2(a+b\text{ArcTan}(cx))^2}{2c^2}-\frac{d^2}{}$$

```
[Out] -a*b*e^2*x/c-b^2*e^2*x*arctan(c*x)/c-b*c*d^2*(a+b*arctan(c*x))/x-1/2*c^2*d^
2*(a+b*arctan(c*x))^2+1/2*e^2*(a+b*arctan(c*x))^2/c^2-1/2*d^2*(a+b*arctan(c
*x))^2/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))^2-4*d*e*(a+b*arctan(c*x))^2*arctan
h(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^2+1)+1/2*b^2*e
^2*ln(c^2*x^2+1)/c^2-2*I*b*d*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+2
*I*b*d*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-b^2*d*e*polylog(3,1-2/
(1+I*c*x))+b^2*d*e*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.44, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5100, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745, 5036, 4930, 266}

$$-\frac{1}{2}c^2d^2(a+b\text{ArcTan}(cx))^2+\frac{e^2(a+b\text{ArcTan}(cx))^2}{2c^2}-\frac{d^2(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{bcd^2(a+b\text{ArcTan}(cx))}{x}-2ibde\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+2ibde\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+4de\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2+\frac{1}{2}e^2x^2(a+b\text{ArcTan}(cx))^2-\frac{abe^2x}{c}-\frac{b^2e^2x\text{ArcTan}(cx)}{c}-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)+\frac{b^2e^2\log\left(c^2x^2+1\right)}{2c^2}-b^2de\text{Li}_3\left(1-\frac{2}{icx+1}\right)+b^2de\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] -((a*b*e^2*x)/c) - (b^2*e^2*x*ArcTan[c*x])/c - (b*c*d^2*(a + b*ArcTan[c*x])
)/x - (c^2*d^2*(a + b*ArcTan[c*x])^2)/2 + (e^2*(a + b*ArcTan[c*x])^2)/(2*c^
2) - (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) + (e^2*x^2*(a + b*ArcTan[c*x])^2)/
2 + 4*d*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^2*Lo
g[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (b^2*e^2*Log[1 + c^2*x^2])/(2*c^2
) - (2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + (2*I)*b
*d*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - b^2*d*e*PolyLog[3
, 1 - 2/(1 + I*c*x)] + b^2*d*e*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2 \left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx &= \int \left(\frac{d^2(a+b\tan^{-1}(cx))^2}{x^3}+\frac{2de(a+b\tan^{-1}(cx))^2}{x}+e^2x\big(a+b\tan^{-1}(\right. \\
&= d^2\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx+(2de)\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx+e^2\int x(\\
&= -\frac{d^2(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{2}e^2x^2\big(a+b\tan^{-1}(cx)\big)^2+4de\big(a+b\tan^{-1}(\\
&= -\frac{d^2(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{2}e^2x^2\big(a+b\tan^{-1}(cx)\big)^2+4de\big(a+b\tan^{-1}(\\
&= -\frac{abe^2x}{c}-\frac{bcd^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d^2\big(a+b\tan^{-1}(cx)\big)^2+\frac{e^2(a+}{} \\
&= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d^2\big(a+b\tan \\
&= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d^2\big(a+b\tan \\
&= -\frac{abe^2x}{c}-\frac{b^2e^2x\tan^{-1}(cx)}{c}-\frac{bcd^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}c^2d^2\big(a+b\tan
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 367, normalized size = 1.15

$$
\frac{1}{2}\left(-\frac{a^2d^2}{x^2}+a^2e^2x^2+\frac{2abe^2(-cx+(1+c^2x^2)\,\mathrm{ArcTan}(cx))}{c^2}-\frac{2abd^2(\mathrm{ArcTan}(cx)+cx(1+cx\,\mathrm{ArcTan}(cx)))}{x^2}+4a^2de\log(x)-\frac{b^2d^2\left(2cx\,\mathrm{ArcTan}(cx)+(1+c^2x^2)\,\mathrm{ArcTan}(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{x^2}+\frac{b^2e^2(-2cx\,\mathrm{ArcTan}(cx)+(1+c^2x^2)\,\mathrm{ArcTan}(cx)^2+\log(1+c^2x^2))}{c^2}+4iabde(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+\frac{1}{6}b^2de\left(-i\pi^3+16i\,\mathrm{ArcTan}(cx)^3+24\,\mathrm{ArcTan}(cx)^2\log\left(1-e^{-2i\,\mathrm{ArcTan}(cx)}\right)-24\,\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\,\mathrm{ArcTan}(cx)}\right)+24i\,\mathrm{ArcTan}(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\,\mathrm{ArcTan}(cx)}\right)+24i\,\mathrm{ArcTan}(cx)\,\mathrm{PolyLog}\left(2,-e^{2i\,\mathrm{ArcTan}(cx)}\right)+12\,\mathrm{PolyLog}\left(3,e^{-2i\,\mathrm{ArcTan}(cx)}\right)-12\,\mathrm{PolyLog}\left(3,-e^{2i\,\mathrm{ArcTan}(cx)}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

```
[Out] (-((a^2*d^2)/x^2) + a^2*e^2*x^2 + (2*a*b*e^2*(-(c*x) + (1 + c^2*x^2)*ArcTan
[c*x]))/c^2 - (2*a*b*d^2*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 + 4
*a^2*d*e*Log[x] - (b^2*d^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2
 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/x^2 + (b^2*e^2*(-2*c*x*ArcTan[c
*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))/c^2 + (4*I)*a*b*d*e*
(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*d*e*((-I)*Pi^3 + (16*I)*A
rcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan
[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
 + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c
*x])]))/6)/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.38, size = 1597, normalized size = 4.99

method	result	size
derivativedivides	Expression too large to display	1597
default	Expression too large to display	1597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*b^2*arctan(c*x)^2*d^2+a*b/c^4*arctan(c*x)*e^2-b^2/c^4*e^2*ln((1+I
*c*x)^2/(c^2*x^2+1)+1)-a*b*e^2*x/c^3-b^2*e^2*x*arctan(c*x)/c^3-a*b*arctan(c
*x)*d^2/c^2/x^2+2*b^2/c^2*d*e*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2
))+2*b^2/c^2*arctan(c*x)^2*d*e*ln(c*x)+2*b^2/c^2*d*e*arctan(c*x)^2*ln(1-(1+
I*c*x)/(c^2*x^2+1)^(1/2))-2*b^2/c^2*d*e*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x
^2+1)-1)+1/2*b^2/c^2*arctan(c*x)^2*e^2*x^2-a*b*arctan(c*x)*d^2+1/2*b^2/c^4*
arctan(c*x)^2*e^2+I*b^2/c^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*arctan(c*x)^2-I*b^2/c^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan
(c*x)^2-I*b^2/c^2*d*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*b^2/c^2
*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+
I*b^2/c^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))*arctan(c*x)^2+4*a*b/c^2*arctan(c*x)*d*e*ln(c*x)+2*I*a*b/c^2*d*e*dilog(1+
I*c*x)-2*I*a*b/c^2*d*e*dilog(1-I*c*x)+a*b/c^2*arctan(c*x)*e^2*x^2+I*b^2/c^2
*d*e*Pi*arctan(c*x)^2+2*I*b^2/c^2*d*e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c
^2*x^2+1))-4*I*b^2/c^2*d*e*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/
2))-4*I*b^2/c^2*d*e*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*b^
2/c^2*d*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^2+2*I*a*b/c^2*d*e*ln(c*x)*ln(1+I*c*x)-2*I*a*b/c^2*d*e*ln(c*x)
*ln(1-I*c*x)-I*b^2/c^2*d*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b^2/c^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-a*b*d^2/c/x+2*a^2/c^
2*d*e*ln(c*x)-1/2*b^2*arctan(c*x)^2*d^2/c^2/x^2-b^2*d^2*arctan(c*x)/c/x+1/2
*a^2/c^2*e^2*x^2+I*b^2/c^4*arctan(c*x)*e^2-b^2/c^2*d*e*polylog(3,-(1+I*c*x)
^2/(c^2*x^2+1))+4*b^2/c^2*d*e*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*b^2
/c^2*d*e*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*d^2*ln((1+I*c*x)/(c^2*x
^2+1)^(1/2)-1)+b^2*d^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*b^2*d^2*arctan(c
*x)-1/2*a^2*d^2/c^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 + 1/2*a^2*x^2*e^2 + 2*
a^2*d*e*log(x) - 1/2*a^2*d^2/x^2 + 1/96*((2304*b^2*c^2*d*e*integrate(1/16*x
^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 6144*a*b*c^2*d*e*integrate(1/16*x^4*
arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d^2*integrate(1/16*x^2*arcta
n(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^
2 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x
^2 + 1)/(c^2*x^5 + x^3), x) + 2*b^2*d*e*log(c^2*x^2 + 1)^3 + 1152*b^2*c^2*e
^2*integrate(1/16*x^6*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*e^2*in
tegrate(1/16*x^6*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*e^2*
integrate(1/16*x^6*arctan(c*x)/(c^2*x^5 + x^3), x) + 192*b^2*c^2*e^2*integr
ate(1/16*x^6*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 384*b^2*c*d^2*integrate
(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) - 384*b^2*c*e^2*integrate(1/16*x^5*
arctan(c*x)/(c^2*x^5 + x^3), x) + 2304*b^2*d*e*integrate(1/16*x^2*arctan(c*
x)^2/(c^2*x^5 + x^3), x) + 192*b^2*d*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^
2/(c^2*x^5 + x^3), x) + 6144*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^
5 + x^3), x) + 1152*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3), x
) + 96*b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 1152
*b^2*e^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e^
2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + b^2*e^2*log(c^2*x^2
+ 1)^3/c^2)*x^2 + 12*(b^2*x^4*e^2 - b^2*d^2)*arctan(c*x)^2 - 3*(b^2*x^4*e^2
 - b^2*d^2)*log(c^2*x^2 + 1)^2)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^4*e^2 + 2*a^2*d*x^2*e + a^2*d^2 + (b^2*x^4*e^2 + 2*b^2*d*x^
2*e + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*x^4*e^2 + 2*a*b*d*x^2*e + a*b*d^2)*ar
ctan(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))^2 (d + ex^2)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3, x)
```

3.1261 $\int \frac{x^3(a+b\text{ArcTan}(cx))^2}{d+ex^2}\,dx$

Optimal. Leaf size=590

$$-\frac{abx}{ce}-\frac{b^2x\text{ArcTan}(cx)}{ce}+\frac{(a+b\text{ArcTan}(cx))^2}{2c^2e}+\frac{x^2(a+b\text{ArcTan}(cx))^2}{2e}+\frac{d(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{d(}{}$$

```
[Out] -a*b*x/c/e-b^2*x*arctan(c*x)/c/e+1/2*(a+b*arctan(c*x))^2/c^2/e+1/2*x^2*(a+b
*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^2+1/2*b^2*ln(c^2*
x^2+1)/c^2/e-1/2*d*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/2*d*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1
/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-I*b*d*(a+b*arctan(c*
x))*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*(a+b*arctan(c*x))*polylog(2,1-2*
c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2*I*b*d*
(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^
(1/2)+I*e^(1/2)))/e^2+1/2*b^2*d*polylog(3,1-2/(1-I*c*x))/e^2-1/4*b^2*d*poly
log(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-
1/4*b^2*d*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*
e^(1/2)))/e^2
```

Rubi [A]
time = 0.36, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5036, 4946, 4930, 266, 5004, 5100, 4968}

$$\frac{(a+b\text{ArcTan}(cx))^2}{2c^2e}-\frac{ibd\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e^2}+\frac{ibd(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e^2}+\frac{ibd(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e^2}-\frac{d(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^2}-\frac{d(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^2}+\frac{x^2(a+b\text{ArcTan}(cx))^2}{2e}-\frac{abx}{ce}-\frac{b^2x\text{ArcTan}(cx)}{ce}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2e}+\frac{b^2d\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e^2}-\frac{b^2d\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e^2}-\frac{b^2d\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] -((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) + (a + b*ArcTan[c*x])^2/(2*c^2
*e) + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) + (d*(a + b*ArcTan[c*x])^2*Log[2/(1
 - I*c*x)])/e^2 - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) - (d*(a + b*ArcTan[c*x])^
2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*e^2) + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) - (I*b*d*(a + b*ArcTan[c*x])*Po
lyLog[2, 1 - 2/(1 - I*c*x)])/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^2 + ((I/2)*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 + (b^2*d*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) - (b^2*d*PolyLog[3, 1 -
```

```
(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e
^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx = \frac{\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx}{e} - \frac{d\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{e}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{e} - \frac{d\int \left(-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}\right.}{}$$

$$= \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}} - \frac{d\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2e^{3/2}} - \frac{b\int (a+}{}$$

$$= -\frac{abx}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d\left(a+b\tan^{-1}(cx)\right)^2\log(}{e^2}$$

$$= -\frac{abx}{ce} - \frac{b^2x\tan^{-1}(cx)}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d(a+b}{}$$

$$= -\frac{abx}{ce} - \frac{b^2x\tan^{-1}(cx)}{ce} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2e} + \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{d(a+b}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(590) = 1180$.
time = 6.89, size = 1569, normalized size = 2.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] (2*a^2*e*x^2 - 2*a^2*d*Log[d + e*x^2] + 4*a*b*(-((e*x)/c) - I*d*ArcTan[c*x]
^2 + ArcTan[c*x]*(e*(c^(-2) + x^2) + 2*d*Log[1 + E^((2*I)*ArcTan[c*x])]) -
```

```
I*d*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (2*d*(-(c^2*d) + e)*((-I)*ArcTan[c
*x]^2 + (2*I)*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e
]] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e
 + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2
*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]
) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^
2*d - e)] - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^((2*I)*Ar
cTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((
2*I)*ArcTan[c*x]))/(c^2*d - e))])))/(2*c^2*d - 2*e)) + (b^2*(-4*c*e*x*ArcTa
n[c*x] + 2*e*ArcTan[c*x]^2 + 2*c^2*e*x^2*ArcTan[c*x]^2 + 4*c^2*d*ArcTan[c*x
]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c*Sqrt
[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - 2*c^2*d*ArcT
an[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
- Sqrt[e])] + 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*Sqrt[c^2*d*e])*
E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d -
e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x
]))/(c^2*d - e)] - 2*c^2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d
*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2
*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 +
 E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] -
 4*c^2*d*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1
+ E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)]
+ 4*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d -
(2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))]
+ 2*c^2*d*ArcTan[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e +
Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 2*e*Log[1 + c^2*x^2] - 4*c^2*d
*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt
[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + 2*c^
2*d*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]
] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] - (4*I)*c^2*d*ArcTan[c*x]*PolyLog[2
, -E^((2*I)*ArcTan[c*x])] + (2*I)*c^2*d*ArcTan[c*x]*PolyLog[2, ((-(c*Sqrt[d
]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] + (2*I)*c^2*d*A
rcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sq
rt[d] - Sqrt[e]))] + 2*c^2*d*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - c^2*d*Pol
yLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[
e])] - c^2*d*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*
Sqrt[d] - Sqrt[e]))]))/c^2)/(4*e^2)
```

Maple [F]

time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\arctan(cx))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

[Out] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e^(-1) - d*e^(-2)*log(x^2*e + d))*a^2 + integrate((b^2*x^3*arctan(
c*x)^2 + 2*a*b*x^3*arctan(c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(x^2*e +
 d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{atan}(cx))^2}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2),x)
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1262 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))^2}{d+ex^2}\,dx$

Optimal. Leaf size=554

$$\frac{i(a+b\text{ArcTan}(cx))^2}{ce}+\frac{x(a+b\text{ArcTan}(cx))^2}{e}+\frac{2b(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{ce}+\frac{\sqrt{-d}\,(a+b\text{ArcTan}(cx))^2\,\text{lo}}{2e^3}$$

```
[Out] I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+2*b*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c/e+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e+1/2*(a+b*arctan(c*x))^2
*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/
2)/e^(3/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*(a+b*arctan(c*x))*poly
log(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)
^(1/2)/e^(3/2)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1
/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*b^2*polylog
(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1
/2)/e^(3/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d
)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5036, 4930, 5040, 4964, 2449, 2352, 5034, 4968}

$$-\frac{ib\sqrt{-d}\,(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e^{3/2}}+\frac{x(a+b\text{ArcTan}(cx))^2}{e}+\frac{i(a+b\text{ArcTan}(cx))^2}{ce}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{ce}+\frac{b^2\sqrt{-d}\,\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e^{3/2}}-\frac{b^2\sqrt{-d}\,\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4e^{3/2}}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (2*b*(a + b
*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e) + (Sqrt[-d]*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2
*e^(3/2)) - (Sqrt[-d]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x)
)/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^(3/2)) + (I*b^2*PolyLog[2,
1 - 2/(1 + I*c*x)])/(c*e) - ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2
, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])
/e^(3/2) + ((I/2)*b*Sqrt[-d]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^(3/2) + (b^2*S
qrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e
])*(1 - I*c*x))])/(4*e^(3/2)) - (b^2*Sqrt[-d]*PolyLog[3, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e^(3/2))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx = \frac{\int (a+b\tan^{-1}(cx))^2\,dx}{e} - \frac{d\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{e}$$

$$= \frac{x(a+b\tan^{-1}(cx))^2}{e} - \frac{(2bc)\int \frac{x(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{e} - \frac{d\int \left(\frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \right.}{e}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{(2b)\int \frac{a+b\tan^{-1}(cx)}{i-cx}\,dx}{e} - \frac{\sqrt{-d}\int}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{ce} +$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{ce} +$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{ce} + \frac{x(a+b\tan^{-1}(cx))^2}{e} + \frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{ce} +$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
[Out] $Aborted
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 73.26, size = 94172, normalized size = 169.99

method	result	size
derivativedivides	Expression too large to display	94172
default	Expression too large to display	94172

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1))*a^2 + 1/16*(4*b^2*
x*arctan(c*x)^2 - b^2*x*log(c^2*x^2 + 1)^2 + 16*e*integrate(1/16*(12*(b^2*c
^2*x^4*e + b^2*x^2*e)*arctan(c*x)^2 + (b^2*c^2*x^4*e + b^2*x^2*e)*log(c^2*x
^2 + 1)^2 + 8*(4*a*b*c^2*x^4*e - b^2*c*x^3*e - b^2*c*d*x + 4*a*b*x^2*e)*arc
tan(c*x) + 4*(b^2*c^2*x^4*e + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*x^4*e^2
 + (c^2*d*e + e^2)*x^2 + d*e), x))*e^(-1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(x^2*e +
 d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{atan}(cx))^2}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{e\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2),x)
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1263 $\int \frac{x(a+b\mathbf{ArcTan}(cx))^2}{d+ex^2}\,dx$

Optimal. Leaf size=492

$$-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{(a+b\text{ArcTan}(cx))^2\,\mathrm{l}}{}$$

```
[Out] -(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)
^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e+1/2*(a+b*arctan(c*x
))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e+I*
b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*(a+b*arctan(c*x))*po
lylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e-
1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))/e-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e+1/4*b^2*poly
log(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e+1/
4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1
/2)))/e
```

Rubi [A]

time = 0.19, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5100, 4968}

$$-\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}-\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2e}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2e}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{e}+\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4e}+\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4e}-\frac{b^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/(2*
e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))]])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[
-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/e - ((I/2)*b*(a
+ b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))]])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) +
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*
(1 - I*c*x))]])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/(4*e)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
```

```
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx = \int \left(-\frac{(a+b\tan^{-1}(cx))^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{(a+b\tan^{-1}(cx))^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{e}}+\frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{e}}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e}+\frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right.}{2e}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1529 vs. 2(492) = 984.
time = 5.76, size = 1529, normalized size = 3.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

```
[Out] ((8*I)*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]]
- 8*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 4*b^2*ArcTan[c*x]^2*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + 2*b^2*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] - Sq
rt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] + 2*b^2*ArcTan[c*x]^2*
Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e])
```

```
] - 2*b^2*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*Sqrt[c^2*d*e])*E^((2*I)*Arc
Tan[c*x]))/(c^2*d - e)] - 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[1 + (
(c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*a*b*A
rcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c
^2*d - e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((
c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 2*b^2*Ar
cTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(
c^2*d - e)] + 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2*Sqrt[c^2*d*e]
*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*
I)*ArcTan[c*x])))/(c^2*d - e)] + 4*a*b*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^
((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*
ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTa
n[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTa
n[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcTan[c
*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan
[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*b^2*ArcSin[Sq
rt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e]
 + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] - 2*b^2*ArcTan[c*x]
^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c
^2*d - e)*(I + c*x))] + 2*a^2*Log[d + e*x^2] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(c
^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTa
n[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] - 2*b^2*ArcTan[c*x]^2*Log[1 +
 ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]])
)/(c^2*d - e)] + (4*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] - (2*I)*b^2*ArcTan[c*x]*PolyLog[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*
ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*b^2*ArcTan[c*x]*PolyLog[2, -((
(c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))] - (2*I
)*a*b*PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(
c^2*d - e)] - (2*I)*a*b*PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I
)*ArcTan[c*x]))/(c^2*d - e))] - 2*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])] +
b^2*PolyLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
+ Sqrt[e])] + b^2*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x])
)/(c*Sqrt[d] - Sqrt[e]))])/(4*e)
```

Maple [F]

time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\arctan(cx))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

```
[Out] int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(-1)*log(x^2*e + d) + integrate((b^2*x*arctan(c*x)^2 + 2*a*b*x*ar
ctan(c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(x^2*e + d), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{atan}(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a + b\,\mathrm{atan}(c\,x))^2}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1264 $\int \frac{(a+b\text{ArcTan}(cx))^2}{d+ex^2}\,dx$

Optimal. Leaf size=460

$$\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{ib(a$$

```
[Out] 1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/
2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2
)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b
*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)
^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1
-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/
e^(1/2)+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1
/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^
(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5034, 4968}

$$-\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4\sqrt{-d}\,\sqrt{e}}-\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[
-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) +
((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e])
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
```

```
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right) dx$$

$$= -\frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2c\left(\sqrt{-\right.}\right.}{\left(c\sqrt{-\right.}\right.}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
[Out] $Aborted
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.08, size = 5665, normalized size = 12.32

method	result	size
derivativedivides	Expression too large to display	5665
default	Expression too large to display	5665

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + integrate(1/16*(12*b^2*arc
tan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + e*x^2),x)
[Out] int((a + b*atan(c*x))^2/(d + e*x^2), x)
```

3.1265 $\int \frac{(a+b\text{ArcTan}(cx))^2}{x(d+ex^2)}\,dx$

Optimal. Leaf size=637

$$\frac{2(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d}+\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c(\cdots}{(c\sqrt{-\cdots}}\right.}{2d}$$

```
[Out] -2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/(
1-I*c*x))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)
/(c*(-d)^(1/2)-I*e^(1/2)))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e
^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d-I*b*(a+b*arctan(c*x))*polylog
(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d+I*b*(a
+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+1/2*I*b*(a+b*arctan(c*x))*polyl
og(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d+1/2
*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*
(-d)^(1/2)+I*e^(1/2)))/d+1/2*b^2*polylog(3,1-2/(1-I*c*x))/d-1/2*b^2*polylog
(3,1-2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d-1/4*b^2*polylog(3,1
-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d-1/4*b^2*p
olylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d
```

Rubi [A]
time = 0.50, antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5100, 4942, 5108, 5004, 5114, 6745, 4968}

$$\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d}+\frac{ib(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d}-\frac{(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}-\frac{ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2}{d}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{d}-\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d}-\frac{b^2\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4d}+\frac{b^2\text{Li}_3\left(1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

```
[Out] (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*x
])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sq
rt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTan[c
*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*
x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d - (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a + b*ArcT
an[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*PolyLog[3, 1 -
 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d) + (b^2*P
olyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) - (b^2*PolyLog[
```

```
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*d)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+ex^2)}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{ex(a+b\tan^{-1}(cx))^2}{d(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{d}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} - \frac{(4bc)\int \frac{(a+b\tan^{-1}(cx))\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(2bc)\int \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d} - (2$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - (a+$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - (a+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1412 vs. 2(637) = 1274.
time = 4.87, size = 1412, normalized size = 2.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
[Out] (24*a^2*Log[x] - 12*a^2*Log[d + e*x^2] - 24*a*b*((-I)*ArcTan[c*x]^2 + (2*I)
*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] - 2*ArcTan
```

```
[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]
+ ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x])
)/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[(-2*
Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*
d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + I*(ArcTan[c*x]^2 + PolyLog[2,
 E^((2*I)*ArcTan[c*x])]) - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*
e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt
[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])) + b^2*((-I)*Pi^3 + (16*I)
*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 12*ArcT
an[c*x]^2*Log[1 + ((c*Sqrt[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
+ Sqrt[e])] - 12*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcT
an[c*x]))/(c*Sqrt[d] - Sqrt[e])] + 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2
*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 24*ArcSin[Sqrt[(c^2*d
)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)
*ArcTan[c*x]))/(c^2*d - e)] - 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt
[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 24*ArcSin[Sqrt[(c^2*d)/(c^
2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1
+ E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)]
- 24*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^
((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 24
*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqr
t[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 12*ArcT
an[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])
*x)/((c^2*d - e)*(I + c*x))] - 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[
c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*A
rcTan[c*x]]))/(c^2*d - e)] + 12*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[
c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + (24*I
)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (12*I)*ArcTan[c*x]*PolyL
og[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e]
)] + (12*I)*ArcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan
[c*x]))/(c*Sqrt[d] - Sqrt[e]))] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 6
*PolyLog[3, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + S
qrt[e])] - 6*PolyLog[3, -(((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*
Sqrt[d] - Sqrt[e]))]))/(24*d)
```

Maple [F]

time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arctan (cx))^2}{x (e x^2 + d)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

```
[Out] int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a^2*(log(x^2*e + d)/d - 2*log(x)/d) + integrate((b^2*arctan(c*x)^2 + 2
*a*b*arctan(c*x))/(x^3*e + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^3*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x**2+d),x)
[Out] Integral((a + b*atan(c*x))**2/(x*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atan}(c\,x))^2}{x\,(e\,x^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + e*x^2)),x)
[Out] int((a + b*atan(c*x))^2/(x*(d + e*x^2)), x)
```

3.1266 $\int \frac{(a+b\text{ArcTan}(cx))^2}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=553

$$-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}+\frac{\sqrt{e}\,(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\,(\cdots}{}$$

```
[Out] -I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*b*c*(a+b*arctan(c*x))*
ln(2-2/(1-I*c*x))/d-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d+1/2*(a+b*arctan(c*x
))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1
/2)/(-d)^(3/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*(a+b*arctan(c*x))*
polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*
e^(1/2)/(-d)^(3/2)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*b^2*pol
ylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(
1/2)/(-d)^(3/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c
*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)
```

Rubi [A]
time = 0.39, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5038, 4946, 5044, 4988, 2497, 5034, 4968}

$$-\frac{ib\sqrt{e}\,(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\,(a+b\text{ArcTan}(cx))\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\,(a+b\text{ArcTan}(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2(-d)^{3/2}}-\frac{ic(a+b\text{ArcTan}(cx))^2}{d}-\frac{(a+b\text{ArcTan}(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))}{d}+\frac{b^2\sqrt{e}\,\text{Li}_3\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{3/2}}-\frac{b^2\sqrt{e}\,\text{Li}_3\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{3/2}}-\frac{ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) + (Sqrt[e]*(
a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^
(3/2)) + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d - (I*b^2*c*Po
lyLog[2, -1 + 2/(1 - I*c*x)])/d - ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*
x))])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(3/2
) + (b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/2)) - (b^2*Sqrt[e]*PolyLog[3, 1 - (2*
c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^
(3/2))
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2(d+ex^2)}\,dx = \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx}{d} - \frac{e\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x(1+c^2x^2)}\,dx}{d} - \frac{e\int \left(\frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-\ldots}}{2d(\ldots}\right.}{d}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{(2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx}{d} - \frac{e\int \frac{(a+b\ldots}{\sqrt{-\ldots}}}{2(\ldots}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{\sqrt{e}\,(a+b\tan^{-1}(cx))^2\log\left(\frac{2c(\ldots}{(c\sqrt{-\ldots}}\right.}{2(-d)^{3/2}}$$

$$= -\frac{ic(a+b\tan^{-1}(cx))^2}{d} - \frac{(a+b\tan^{-1}(cx))^2}{dx} + \frac{\sqrt{e}\,(a+b\tan^{-1}(cx))^2\log\left(\frac{2c(\ldots}{(c\sqrt{-\ldots}}\right.}{2(-d)^{3/2}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

```
[Out] $Aborted
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 39.35, size = 102882, normalized size = 186.04

method	result	size

derivativedivides	Expression too large to display	102882
default	Expression too large to display	102882

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^4*e + d*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))^2}{x^2\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(x**2*(d + e*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)), x)
```

3.1267 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=745

$$-\frac{bc(a+b\text{ArcTan}(cx))}{dx}-\frac{c^2(a+b\text{ArcTan}(cx))^2}{2d}-\frac{(a+b\text{ArcTan}(cx))^2}{2dx^2}-\frac{2e(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\right.}{d^2}$$

```
[Out] -b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c*
x))^2/d/x^2+2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+b^2*c^2*ln(
x)/d-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d+
1/2*e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(
1/2)-I*e^(1/2)))/d^2+1/2*e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2)
)/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2+I*b*e*(a+b*arctan(c*x))*polylog(2
,1-2/(1-I*c*x))/d^2+I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^2-1/
2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))/d^2-1/2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*(
(-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-I*b*e*(a+b*ar
ctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-1/2*b^2*e*polylog(3,1-2/(1-I*c*x))
/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x))/d^2-1/2*b^2*e*polylog(3,-1+2/(1+I*c
*x))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)
^(1/2)-I*e^(1/2)))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-
I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2
```

Rubi [A]
time = 0.67, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5038, 4946, 272, 36, 29, 31, 5004, 5100, 4942, 5108, 5114, 6745, 4968}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) - (a
 + b*ArcTan[c*x])^2/(2*d*x^2) - (2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
 + I*c*x)])/d^2 + (b^2*c^2*Log[x])/d - (e*(a + b*ArcTan[c*x])^2*Log[2/(1 -
I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((
c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (e*(a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2
*d^2) - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) + (I*b*e*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 - I*c*x)])/d^2 + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*
x)])/d^2 - ((I/2)*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - S
```

```
qrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 - ((I/2)*b*e*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2)
 + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^2*e*PolyLog[3, -1 + 2
/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) + (b^2*e*PolyLog[3, 1 - (
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+ex^2)}\,dx = \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx}{d} - \frac{e\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+ex^2)}\,dx}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2dx^2} + \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2(1+c^2x^2)}\,dx}{d} - \frac{e\int \left(\frac{(a+b\tan^{-1}(cx))^2}{dx} - \frac{ex(a+b\tan^{-1}(c}{d(d+ex^2)}\right.}{d}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2dx^2} + \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx}{d} - \frac{(bc^3)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{d} - \frac{e\int \frac{(a+b\tan}{}}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} - \frac{(a+b\tan^{-1}(cx))^2}{2dx^2} - \frac{2e(a+b\tan}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} - \frac{(a+b\tan^{-1}(cx))^2}{2dx^2} - \frac{2e(a+b\tan}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} - \frac{(a+b\tan^{-1}(cx))^2}{2dx^2} - \frac{2e(a+b\tan}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{dx} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d} - \frac{(a+b\tan^{-1}(cx))^2}{2dx^2} - \frac{2e(a+b\tan}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1557 vs. 2(745) = 1490.
time = 7.53, size = 1557, normalized size = 2.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/24*((12*a^2*d)/x^2 + (24*a*b*c*d)/x + (24*a*b*d*(1 + c^2*x^2)*ArcTan[c*x
])/x^2 + 24*a^2*e*Log[x] - 12*a^2*e*Log[d + e*x^2] - (24*I)*a*b*e*(ArcTan[c
*x]*(ArcTan[c*x] + (2*I)*Log[1 - E^((2*I)*ArcTan[c*x])]) + PolyLog[2, E^((2
```

```
*I)*ArcTan[c*x])]) - (48*a*b*(c^2*d - e)*e*((-I)*ArcTan[c*x]^2 + (2*I)*ArcS
in[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin[Sqrt
[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e]
)*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]
+ ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*
I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - (I/2)*
(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d
 - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))
/(c^2*d - e))])))/(2*c^2*d - 2*e) + b^2*((-I)*e*Pi^3 + (24*c*d*ArcTan[c*x])
/x + (12*d*(1 + c^2*x^2)*ArcTan[c*x]^2)/x^2 + (8*I)*e*ArcTan[c*x]^3 + 24*e*
ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*c^2*d*Log[(c*x)/Sqrt[1 +
 c^2*x^2]] + (24*I)*e*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 12*e
*PolyLog[3, E^((-2*I)*ArcTan[c*x])]) + 2*b^2*e*((4*I)*ArcTan[c*x]^3 - 6*Arc
Tan[c*x]^2*Log[1 + ((c*Sqrt[d] - Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d]
 + Sqrt[e])] - 6*ArcTan[c*x]^2*Log[1 + ((c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcT
an[c*x]))/(c*Sqrt[d] - Sqrt[e])] + 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e - 2*
Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 12*ArcSin[Sqrt[(c^2*d)
/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*
ArcTan[c*x]))/(c^2*d - e)] - 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c
^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*
d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 +
E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] -
12*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((
2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 12*A
rcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[
c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*x))] + 6*ArcTan[
c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)
/((c^2*d - e)*(I + c*x))] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x
]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcT
an[c*x]]))/(c^2*d - e)] + 6*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*
d*e])*(Cos[2*ArcTan[c*x]] + I*Sin[2*ArcTan[c*x]]))/(c^2*d - e)] + (6*I)*Arc
Tan[c*x]*PolyLog[2, ((-(c*Sqrt[d]) + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqr
t[d] + Sqrt[e])] + (6*I)*ArcTan[c*x]*PolyLog[2, -(((c*Sqrt[d] + Sqrt[e])*E^
((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))] - 3*PolyLog[3, ((-(c*Sqrt[d])
+ Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] + Sqrt[e])] - 3*PolyLog[3, -((
(c*Sqrt[d] + Sqrt[e])*E^((2*I)*ArcTan[c*x]))/(c*Sqrt[d] - Sqrt[e]))]))/d^2
```

Maple [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))^2}{x^3\,(e\,x^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(e*log(x^2*e + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + integrate((b^
2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(x^5*e + d*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^5*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x^3 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(x**3*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atan}(c\,x))^2}{x^3\,(e\,x^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)),x)
[Out] int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)), x)
```

3.1268 $\int \frac{x^3(a+b\mathbf{ArcTan}(cx))^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=943

$$-\frac{c^2d(a+b\mathrm{ArcTan}(cx))^2}{2\left(c^2d-e\right)e^2}+\frac{(a+b\mathrm{ArcTan}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}+\frac{(a+b\mathrm{ArcTan}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}-\frac{(a+b\mathrm{ArcTan}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}-\frac{bc\sqrt{}}{}$$

[Out] -1/2*c^2*d*(a+b*arctan(c*x))^2/(c^2*d-e)/e^2-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^2+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e^2+1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)/(c^2*d-e)/e^(3/2)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/(c^2*d-e)/e^(3/2)-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e^2+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)/(c^2*d-e)/e^(3/2)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/(c^2*d-e)/e^(3/2)-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*(a+b*arctan(c*x))^2/e^2/(1-x*e^(1/2)/(-d)^(1/2))+1/4*(a+b*arctan(c*x))^2/e^2/(1+x*e^(1/2)/(-d)^(1/2))

Rubi [A]
time = 1.30, antiderivative size = 943, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5100, 5098, 4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968}

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]

[Out] -1/2*(c^2*d*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e^2) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 + (Sqrt[e]*x)/Sqrt[-d])) - ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 - (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a + b*ArcTan[c*x]

```
)^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))
])/(2*e^2) + (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a
 + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[
e])*(1 - I*c*x))])/(2*e^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/e^2 + ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[
e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e^(3/2)) - ((I
/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*S
qrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2,
1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(
(c^2*d - e)*e^(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sq
rt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2 - (b^2*Po
lyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] -
 Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*e^2) + (b^2*PolyLo
g[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)])/(4*e^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
```

```
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5098

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x])^p/(1 - R
t[-e/d, 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x
])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p,
0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx = \int \left(-\frac{dx(a+b\tan^{-1}(cx))^2}{e\,(d+ex^2)^2}+\frac{x(a+b\tan^{-1}(cx))^2}{e\,(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{e} - \frac{d\int \frac{x(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx}{e}$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)^2}\,dx}{4\sqrt{-d}\,e^{3/2}} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)^2}\,dx}{4\sqrt{-d}\,e^{3/2}} + \frac{\int \left(-\frac{(a+b\tan^{-1}(cx))^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{(a+}{2\sqrt{e}\,(}\right.}{e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(bc)\int \left(\frac{\sqrt{-d}\,e(a+b\tan^{-1}(cx))}{(c^2d-e)\left(-\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{c^2}{}\right.}{2e^2}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} + \frac{(a}{}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{bc}{}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{bc}{}$$

$$= -\frac{c^2d(a+b\tan^{-1}(cx))^2}{2\,(c^2d-e)\,e^2} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4e^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\,\mathrm{ta}}{}$$

Mathematica [F]
time = 11.93, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\mathrm{ArcTan}(cx))^2}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

Maple [F]
time = 2.43, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a+b \arctan \left(c x\right)\right)^{2}}{\left(e \,x^{2}+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

```
[Out] int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2)*log(x^2*e + d) + d/(x^2*e^3 + d*e^2))*a^2 + integrate((b^2*x^3*
arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x))/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(x^4*e^2
 + 2*d*x^2*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1269 $\int \frac{x^2(a+b\mathbf{ArcTan}(cx))^2}{(d+ex^2)^2} \, dx$

Optimal. Leaf size=1033

$$-\frac{ic(a+b\text{ArcTan}(cx))^2}{2(c^2d-e)e}+\frac{(a+b\text{ArcTan}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{(a+b\text{ArcTan}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{bc(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d-e)e}$$

```
[Out] 1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+
I*e^(1/2)))/(c^2*d-e)/e+b*c*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/(c^2*d-e)/e-b
*c*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/(c^2*d-e)/e-1/2*b*c*(a+b*arctan(c*x))*
ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*d-e)
/e-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d
)^(1/2)+I*e^(1/2)))/(c^2*d-e)/e-1/2*I*b^2*c*polylog(2,1-2/(1-I*c*x))/(c^2*d
-e)/e-1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b^2*c*polylog(2,1-
2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*d-e)/e-
1/2*I*b^2*c*polylog(2,1-2/(1+I*c*x))/(c^2*d-e)/e+1/4*(a+b*arctan(c*x))^2*ln
(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d
)^(1/2)-1/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*
(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*(a+b*arctan(c*x))*polylog
(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)
/(-d)^(1/2)-1/2*I*c*(a+b*arctan(c*x))^2/(c^2*d-e)/e+1/8*b^2*polylog(3,1-2*c
*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d)^(1
/2)-1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+
I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arctan(c*x))^2/e^(3/2)/((-d)^(1/2)-
x*e^(1/2))-1/4*(a+b*arctan(c*x))^2/e^(3/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A]
time = 1.39, antiderivative size = 1033, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5100, 5034, 4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] ((-1/2*I)*c*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e) + (a + b*ArcTan[c*x])^2/
(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcTan[c*x])^2/(4*e^(3/2)*(Sqrt
[-d] + Sqrt[e]*x)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/((c^2*d -
 e)*e) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/((c^2*d - e)*e) - (b*
```

```
c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqr
t[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(S
qrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*
e^(3/2)) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sq
rt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) - ((a + b*ArcTan[c*x])
^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]
)/(4*Sqrt[-d]*e^(3/2)) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/((c^2*
d - e)*e) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/((c^2*d - e)*e) + (
(I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/((c^2*d - e)*e) - ((I/4)*b*(a + b*ArcTan[c*x])*PolyLo
g[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e) + ((I/4)*b*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d
] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + (b^2*PolyLog[3, 1 - (2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-
d]*e^(3/2)) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
 + I*Sqrt[e])*(1 - I*c*x))])/(8*Sqrt[-d]*e^(3/2))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]

Rule 4968

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(PolyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]

Rule 4974

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5004

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5034

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]

Rule 5040

Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]

Rule 5100

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]

```
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx = \int \left(-\frac{d(a+b\tan^{-1}(cx))^2}{e\,(d+ex^2)^2}+\frac{(a+b\tan^{-1}(cx))^2}{e\,(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{e} - \frac{d\int \frac{(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx}{e}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,(a+b\tan^{-1}(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e(a+b\tan^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\right.}\right.}{}$$

$$= \frac{1}{4}\int \frac{(a+b\tan^{-1}(cx))^2}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx + \frac{1}{4}\int \frac{(a+b\tan^{-1}(cx))^2}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx + \frac{1}{2}\int \frac{(a+b\tan^{-}}{-de-e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\right.}{2\sqrt{-d}}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{(a+b\tan^{-1}(cx))^2\log\left(\right.}{2\sqrt{-d}}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc(a+b\tan^{-1}(cx))\log\left(\right.}{(c^2d-e)\,e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc(a+b\tan^{-1}(cx))\log\left(\right.}{(c^2d-e)\,e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc(a+b\tan^{-1}(cx))\log\left(\right.}{(c^2d-e)\,e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc(a+b\tan^{-1}(cx))\log\left(\right.}{(c^2d-e)\,e}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc(a+b\tan^{-1}(cx))\log\left(\right.}{(c^2d-e)\,e}$$

Mathematica [F]
time = 30.20, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\text{ArcTan}(cx))^2}{(d + ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
[Out] Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 179.74, size = 6636, normalized size = 6.42

method	result	size
derivativedivides	Expression too large to display	6636
default	Expression too large to display	6636

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(x^4*e^2
 + 2*d*x^2*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1270 $\int \frac{x(a+b\text{ArcTan}(cx))^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=457

$$\frac{c^2(a+b\text{ArcTan}(cx))^2}{2(c^2d-e)e}-\frac{(a+b\text{ArcTan}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}-\frac{(a+b\text{ArcTan}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}-\frac{bc(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\cdots\right.}{\left(c\sqrt{-d}-i\sqrt{e}\cdots\right.}\right.}{2\sqrt{-d}\,(c^2d-e)\sqrt{e}}$$

```
[Out] 1/2*c^2*(a+b*arctan(c*x))^2/(c^2*d-e)/e-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((
-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*d-e)/(-d)^(1/
2)/e^(1/2)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x
)/(c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/(-d)^(1/2)/e^(1/2)+1/4*I*b^2*c*polylo
g(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*d
-e)/(-d)^(1/2)/e^(1/2)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(
1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/(-d)^(1/2)/e^(1/2)-1/4*(a+b*ar
ctan(c*x))^2/d/e/(1-x*e^(1/2)/(-d)^(1/2))-1/4*(a+b*arctan(c*x))^2/d/e/(1+x*
e^(1/2)/(-d)^(1/2))
```

Rubi [A]

time = 0.80, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5098, 4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964}

$$\frac{c^2(a+b\text{ArcTan}(cx))^2}{2e(c^2d-e)}-\frac{bc(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}\,(c^2d-e)}+\frac{bc(a+b\text{ArcTan}(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}\,(c^2d-e)}-\frac{(a+b\text{ArcTan}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}-\frac{(a+b\text{ArcTan}(cx))^2}{4de\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}+1\right)}+\frac{ib^2c\,\text{Li}_2\left(1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4\sqrt{-d}\,\sqrt{e}\,(c^2d-e)}-\frac{ib^2c\,\text{Li}_2\left(1-\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}\,c+i\sqrt{e}\right)(1-icx)}\right)}{4\sqrt{-d}\,\sqrt{e}\,(c^2d-e)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] (c^2*(a + b*ArcTan[c*x])^2)/(2*(c^2*d - e)*e) - (a + b*ArcTan[c*x])^2/(4*d*
e*(1 - (Sqrt[e]*x)/Sqrt[-d])) - (a + b*ArcTan[c*x])^2/(4*d*e*(1 + (Sqrt[e]*
x)/Sqrt[-d])) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) +
(b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*(c^2*d - e)*Sqrt[e]) + ((I/4)*b^2*c*Pol
yLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(S
qrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*(c
^2*d - e)*Sqrt[e])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5098

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x])^p/(1 - R
t[-e/d, 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x
])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p,
0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx = \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)^2}\,dx}{4(-d)^{3/2}\sqrt{e}} - \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)^2}\,dx}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(bc)\int \left(\frac{\sqrt{-d}\,e(a+b\tan^{-1}(cx))}{(c^2d-e)\left(-\sqrt{-d}+\sqrt{e}\,x\right)} + \cdots\right.}{2\cdots}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(bc)\int \frac{a+b\tan^{-1}(cx)}{-\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}\,(c^2d-e)} + \frac{(bc)\int \cdots}{2\sqrt{\cdots}}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{bc(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{\cdots}\right.}{\left(c\sqrt{-d}\cdots\right.}\right.}{2\sqrt{-d}\,(c^2d-e)\sqrt{\cdots}}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{bc(a+b\tan^{-1}(cx))\log\left(\frac{2c\left(\sqrt{\cdots}\right.}{\left(c\sqrt{-d}\cdots\right.}\right.}{2\sqrt{-d}\,(c^2d-e)\sqrt{\cdots}}$$

$$= \frac{c^2(a+b\tan^{-1}(cx))^2}{2(c^2d-e)e} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{(a+b\tan^{-1}(cx))^2}{4de\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{bc(a+b\tan\cdots}{\cdots}$$

Mathematica [A]
time = 6.50, size = 836, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

```
[Out] ((-2*a^2)/(e*(d + e*x^2)) + (4*a*b*(-(((1 + c^2*x^2)*ArcTan[c*x])/(d + e*x^
2)) + (c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])))/(-(c^2*d) + e) +
```

```
(b^2*c^2*((4*ArcTan[c*x]^2)/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]) +
(-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] - 2*ArcCos[-((c^2*d + e
)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcCos[-((c^2*d + e)/(
c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((-I)*e
+ Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*e)]*x))] +
 (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e
)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c*d + Sqr
t[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(
c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[
2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c
^2*d - e)*Cos[2*ArcTan[c*x]]])] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*
I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]
))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c
^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - I*(PolyLog[2, ((c^2*d + e -
(2*I)*Sqrt[-(c^2*d*e)])*(c*d - Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d + Sqr
t[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c*d
- Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c
^2*d*e)]))/(c^2*d - e))/4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(377) = 754$.
time = 5.20, size = 1209, normalized size = 2.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a^2*c^4/e/(c^2*e*x^2+c^2*d)-1/2*b^2*c^4/e/(c^2*e*x^2+c^2*d)*arc
tan(c*x)^2+b^2*c^4/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*d*e
)^(1/2)+1/2*b^2*c^4/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)-1/4*b^
2*c^2*e/(c^2*d-e)/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)-1/4*b^2*c^6/e/(c
^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)*d+1/2*b^2*c^2/e*(c^2*d*e)^(1
/2)/(c^2*d-e)/d*arctan(c*x)^2+1/2*b^2*c^4/e*arctan(c*x)^2/(c^2*d-e)+I*b^2*c
^4*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arc
tan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/2*I*b^2*c^2/e*
(c^2*d*e)^(1/2)/(c^2*d-e)/d*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2
+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))-1/2*I*b^2*c^6/e*ln(1-(c^2*d-e)*(1+I*c*x)^
2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/(c^4*d^2-
2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*d-1/2*I*b^2*c^2*e*ln(1-(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/d/(c^4*d^2
-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/2*b^2*c^2*e/(c^2*d-e)/d/(c^4*d^2-2*c^2*d*
e+e^2)*arctan(c*x)^2*(c^2*d*e)^(1/2)-1/2*b^2*c^6/e/(c^2*d-e)/(c^4*d^2-2*c^2
*d*e+e^2)*arctan(c*x)^2*(c^2*d*e)^(1/2)*d+1/4*b^2*c^2/e*(c^2*d*e)^(1/2)/(c^
2*d-e)/d*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1
```

```
/2)-e))-a*b*c^4/e/(c^2*e*x^2+c^2*d)*arctan(c*x)-a*b*c^3/(c^2*d-e)/(d*e)^(1/
2)*arctan(e*x/(d*e)^(1/2))+a*b*c^4/e/(c^2*d-e)*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] (c*(c*arctan(c*x)/(c^2*d*e - e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^
2*d - e)*sqrt(d))) - arctan(c*x)/(x^2*e^2 + d*e))*a*b - 1/32*(4*arctan(c*x)
^2 - 32*(x^2*e^2 + d*e)*integrate(1/16*(12*(c^2*x^3*e + x*e)*arctan(c*x)^2
+ (c^2*x^3*e + x*e)*log(c^2*x^2 + 1)^2 + 4*(c*x^2*e + c*d)*arctan(c*x) - 2*
(c^2*x^3*e + c^2*d*x)*log(c^2*x^2 + 1))/(c^2*x^6*e^3 + (2*c^2*d*e^2 + e^3)*
x^4 + (c^2*d^2*e + 2*d*e^2)*x^2 + d^2*e), x) - log(c^2*x^2 + 1)^2)*b^2/(x^2
*e^2 + d*e) - 1/2*a^2/(x^2*e^2 + d*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(x^4*e^2 + 2*d
*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{atan}(cx))^2}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**2/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1271 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=1039

$$\frac{ic(a+b\text{ArcTan}(cx))^2}{2d\,(c^2d-e)}-\frac{(a+b\text{ArcTan}(cx))^2}{4d\sqrt{e}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{(a+b\text{ArcTan}(cx))^2}{4d\sqrt{e}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{bc(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1-ic}\right.}{d\,(c^2d-e)}$$

```
[Out] 1/2*I*b^2*c*polylog(2,1-2/(1+I*c*x))/d/(c^2*d-e)-b*c*(a+b*arctan(c*x))*ln(2
/(1-I*c*x))/d/(c^2*d-e)+b*c*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d/(c^2*d-e)+1
/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1
/2)-I*e^(1/2)))/d/(c^2*d-e)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d/(c^2*d-e)-1/4*I*b^2*c*polylo
g(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d/(c^2
*d-e)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^
(1/2)-I*e^(1/2)))/d/(c^2*d-e)+1/2*I*c*(a+b*arctan(c*x))^2/d/(c^2*d-e)+1/4*I
*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-
d)^(1/2)-I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arctan(c*x))^2*ln(2*c*((-d
)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(3/2)/e^(1/2)+1
/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2
)+I*e^(1/2)))/(-d)^(3/2)/e^(1/2)+1/2*I*b^2*c*polylog(2,1-2/(1-I*c*x))/d/(c^
2*d-e)-1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-
I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/8*b^2*polylog(3,1-2*c
*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+
I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arctan(c*x))^2/d/e^(1/2)/((-d)^(1/2
)-x*e^(1/2))+1/4*(a+b*arctan(c*x))^2/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A]

time = 0.99, antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5034, 4974, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

```
[Out] ((I/2)*c*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(4*
d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcTan[c*x])^2/(4*d*Sqrt[e]*(Sqr
t[-d] + Sqrt[e]*x)) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(d*(c^2*
d - e)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d*(c^2*d - e)) + (b
```

```
*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(3/
2)*Sqrt[e]) + (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) + ((a + b*ArcTan[c*
x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x
))])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*x)])/(
d*(c^2*d - e)) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d*(c^2*d - e)
) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) + ((I/4)*b*(a + b*ArcTan[c*x])*P
olyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d]
+ Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) - ((
I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*
Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]) - (b^2*PolyLog[3,
 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
(8*(-d)^(3/2)*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/(
(c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(3/2)*Sqrt[e])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
```

```
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{4d\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(-de-e^2x^2\right)}\right)$$

$$= -\frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\,\sqrt{e}\,-ex\right)^2}\,dx}{4d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(\sqrt{-d}\,\sqrt{e}\,+ex\right)^2}\,dx}{4d} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{-de-e^2x^2}\,dx}{2d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{(bc)\int \left(\frac{\sqrt{e}\,(a+b\tan}{(-c^2d+e)\left(\sqrt{-d}\right.}\right.}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} + \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\sqrt{-d}\,-\sqrt{e}\,x}\,dx}{4(-d)^{3/2}} +$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{4d\sqrt{e}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} - \frac{bc\left(a+b\tan^{-1}(cx)\right)\log}{d\left(c^2d-e\right)}$$

Mathematica [F]
time = 17.18, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(cx))^2}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 179.98, size = 6570, normalized size = 6.32

method	result	size
derivativedivides	Expression too large to display	6570
default	Expression too large to display	6570

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) + x/(d*x^2*e + d^2)) +
1/32*(4*b^2*x*arctan(c*x)^2 - b^2*x*log(c^2*x^2 + 1)^2 + 32*(d*x^2*e + d^2)
*integrate(1/16*(12*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x)^2 + (b^2*c^2*d*x^2
+ b^2*d)*log(c^2*x^2 + 1)^2 + 4*(8*a*b*c^2*d*x^2 - b^2*c*x^3*e - b^2*c*d*x
+ 8*a*b*d)*arctan(c*x) + 2*(b^2*c^2*x^4*e + b^2*c^2*d*x^2)*log(c^2*x^2 + 1)
)/(c^2*d*x^6*e^2 + (2*c^2*d^2*e + d*e^2)*x^4 + d^3 + (c^2*d^3 + 2*d^2*e)*x^
2), x))/(d*x^2*e + d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^4*e^2 + 2*d*x^2*e
 + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^2}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^2}{(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*atan(c*x))^2/(d + e*x^2)^2, x)
```

3.1272 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{x\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=1087

$$-\frac{c^2(a+b\text{ArcTan}(cx))^2}{2d\left(c^2d-e\right)}+\frac{(a+b\text{ArcTan}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}+\frac{(a+b\text{ArcTan}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)}+\frac{2(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right.}{d^2}$$

```
[Out] -1/2*c^2*(a+b*arctan(c*x))^2/d/(c^2*d-e)-2*(a+b*arctan(c*x))^2*arctanh(-1+2
/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x
))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-
1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/
2)+I*e^(1/2)))/d^2+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2+1/2*I*b*(a+b*arctan(c*x))*
polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/
d^2-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^2+I*b*(a+b*arctan(c*x)
)*polylog(2,-1+2/(1+I*c*x))/d^2+1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-x*e
^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(3/2)/(c^2*d-e)+1/
2*b^2*polylog(3,1-2/(1-I*c*x))/d^2-1/2*b^2*polylog(3,1-2/(1+I*c*x))/d^2+1/2
*b^2*polylog(3,-1+2/(1+I*c*x))/d^2-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^
(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/4*b^2*polylog(3,1-2*c*((-d
)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2-1/2*b*c*(a+b*arc
tan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))
*e^(1/2)/(-d)^(3/2)/(c^2*d-e)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)/(c^2*d-e)
-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d^2-1/4*I*b^2*c*polylog(2,1
-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d
)^(3/2)/(c^2*d-e)+1/4*(a+b*arctan(c*x))^2/d^2/(1-x*e^(1/2)/(-d)^(1/2))+1/4*
(a+b*arctan(c*x))^2/d^2/(1+x*e^(1/2)/(-d)^(1/2))
```

Rubi [A]
time = 1.37, antiderivative size = 1087, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {5100, 4942, 5108, 5004, 5114, 6745, 5098, 4974, 4966, 2449, 2352, 2497, 5104, 5040, 4964, 4968}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(c^2*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])^2/(4
*d^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*d^2*(1 + (Sqrt[
```

```
e]*x)/Sqrt[-d])) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2
 + ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^2 - (b*c*Sqrt[e]*(a + b*Arc
Tan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt
[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + (b*c*
Sqrt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
 I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x]
)^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))
])/(2*d^2) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 -
(I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 + (I*b*(a + b*A
rcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + ((I/4)*b^2*c*Sqrt[e]*Poly
Log[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*
x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1
- (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2
 - ((I/4)*b^2*c*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqr
t[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I
*Sqrt[e])*(1 - I*c*x))])/d^2 + (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^2)
- (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) + (b^2*PolyLog[3, -1 + 2/(1 +
 I*c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d^2) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
```

```
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5098

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x])^p/(1 - R
t[-e/d, 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x
])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p,
0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x(d+ex^2)^2}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^2x} - \frac{ex(a+b\tan^{-1}(cx))^2}{d(d+ex^2)^2} - \frac{ex(a+b\tan^{-1}(cx))^2}{d^2(d+ex^2)}\right)dx$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^2} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{x(a+b\tan^{-1}(cx))^2}{(d+ex^2)^2}\,dx}{d}$$

$$= \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{(4bc)\int \frac{(a+b\tan^{-1}(cx))\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{d^2}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2}$$

$$= \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{2(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^2}$$

$$= -\frac{c^2(a+b\tan^{-1}(cx))^2}{2d(c^2d-e)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{(a+b\tan^{-1}(cx))^2}{4d^2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} + \frac{2(a+b\tan^{-1}}{}$$

Mathematica [**F**]

time = 10.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\mathrm{ArcTan}(cx))^2}{x(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2), x]
```

Maple [**F**]
time = 3.46, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan \left(c x\right)\right)^{2}}{x \left(e \,x^{2}+d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(d*x^2*e + d^2) - log(x^2*e + d)/d^2 + 2*log(x)/d^2) + integrate
((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^5*e^2 + 2*d*x^3*e
 + d^2*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2),x)
[Out] int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2), x)
```

3.1273 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=1141

$$-\frac{ic(a+b\text{ArcTan}(cx))^2}{d^2}-\frac{ice(a+b\text{ArcTan}(cx))^2}{2d^2(c^2d-e)}-\frac{(a+b\text{ArcTan}(cx))^2}{d^2x}+\frac{\sqrt{e}\,(a+b\text{ArcTan}(cx))^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{\sqrt{e}\,(a-}{4d^2\left(\sqrt{}\right.}$$

```
[Out] -1/2*I*b^2*c*e*polylog(2,1-2/(1-I*c*x))/d^2/(c^2*d-e)-3/4*I*b*(a+b*arctan(c
*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/
2)))*e^(1/2)/(-d)^(5/2)-(a+b*arctan(c*x))^2/d^2/x+b*c*e*(a+b*arctan(c*x))*l
n(2/(1-I*c*x))/d^2/(c^2*d-e)-b*c*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2/(c
^2*d-e)+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^2-1/2*b*c*e*(a+b*arctan
(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^
2/(c^2*d-e)-1/2*b*c*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*
c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2/(c^2*d-e)+1/4*I*b^2*c*e*polylog(2,1-2*c*
((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2/(c^2*d-e)-1/
2*I*c*e*(a+b*arctan(c*x))^2/d^2/(c^2*d-e)+3/4*I*b*(a+b*arctan(c*x))*polylog
(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)
/(-d)^(5/2)-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d^2-1/2*I*b^2*c*e*polylog(2,1
-2/(1+I*c*x))/d^2/(c^2*d-e)-3/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^
(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arct
an(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2))
)*e^(1/2)/(-d)^(5/2)+1/4*I*b^2*c*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(
1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2/(c^2*d-e)-I*c*(a+b*arctan(c*x))^2/d^
2-3/8*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*
e^(1/2)))*e^(1/2)/(-d)^(5/2)+3/8*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))
/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*(a+b*arctan(c*x
))^2*e^(1/2)/d^2/((-d)^(1/2)-x*e^(1/2))-1/4*(a+b*arctan(c*x))^2*e^(1/2)/d^2
/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A]
time = 1.49, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5100, 4946, 5044, 4988, 2497, 5034, 4974, 4966, 2449, 2352, 5104, 5004, 5040, 4964, 4968}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-I)*c*(a + b*ArcTan[c*x])^2)/d^2 - ((I/2)*c*e*(a + b*ArcTan[c*x])^2)/(d^2
*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(d^2*x) + (Sqrt[e]*(a + b*ArcTan[c*x]
)^2)/(4*d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2)/(4*d^
2*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/
(d^2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d^2*(c^
2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d - e)) - (3*Sqrt[e]*(a +
 b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(4*(-d)^(5/2)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt
[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d -
 e)) + (3*Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((
c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) + (2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(d^2*(c^2*d - e)) - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2 -
 ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d^2*(c^2*d - e)) + ((I/4)*b
^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(d^2*(c^2*d - e)) + (((3*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1
 - I*c*x))])/(-d)^(5/2) + ((I/4)*b^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d^2*(c^2*d - e)) - (((3
*I)/4)*b*Sqrt[e]*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e
]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(-d)^(5/2) - (3*b^2*Sqrt[e]*
PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(8*(-d)^(5/2)) + (3*b^2*Sqrt[e]*PolyLog[3, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(8*(-d)^(5/2))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
```

```
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5034

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(d+ex^2\right)^2}\,dx = \int \left(\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d\left(d+ex^2\right)^2} - \frac{e\left(a+b\tan^{-1}(cx)\right)^2}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{(2bc)\int \frac{a+b\tan^{-1}(cx)}{x(1+c^2x^2)}\,dx}{d^2} - \frac{e\int \left(\frac{\sqrt{-d}\left(a+b\tan^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-\cdots}}{2d\cdots}\right.}{d^2}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{(2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx}{d^2} + \frac{e\int \frac{(a+\cdots}{\sqrt{\cdots}}}{2\cdots}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

$$= -\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d^2x} + \frac{\sqrt{e}\left(a+b\tan^{-1}(cx)\right)^2}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\,(a\cdots}{4d^2\left(\cdots\right.}\cdots$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

```
[Out] $Aborted
```

Maple [F]
time = 12.30, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arctan (cx))^2}{x^2 (e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^6*e^2 + 2*d*x^4*e
 + d^2*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2), x)
```

3.1274 $\int \frac{(a+b\textbf{ArcTan}(cx))^2}{x^3(d+ex^2)^2}\,dx$

Optimal. Leaf size=1181

$$-\frac{bc(a+b\text{ArcTan}(cx))}{d^2x}-\frac{c^2(a+b\text{ArcTan}(cx))^2}{2d^2}+\frac{c^2e(a+b\text{ArcTan}(cx))^2}{2d^2\left(c^2d-e\right)}-\frac{(a+b\text{ArcTan}(cx))^2}{2d^2x^2}-\frac{e(a+b\text{ArcT}}{4d^3\left(1-\right.}$$

```
[Out] b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+4*e*(a+b*arctan(c*x))^2*arc
tanh(-1+2/(1+I*c*x))/d^3+e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2)
)/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3+e*(a+b*arctan(c*x))^2*ln(2*c*((-d
)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-2*e*(a+b*arctan(
c*x))^2*ln(2/(1-I*c*x))/d^3+1/2*b^2*e*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2)
)/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3+1/2*b^2*e*polylog(3,1-2*c*((-d)^(
1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-1/4*e*(a+b*arctan(c
*x))^2/d^3/(1-x*e^(1/2)/(-d)^(1/2))-1/4*e*(a+b*arctan(c*x))^2/d^3/(1+x*e^(1
/2)/(-d)^(1/2))+2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d^3+2*I*
b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^3-b*c*(a+b*arctan(c*x))/d^
2/x-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x
)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-
d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-b^2*e*polylog(3
,1-2/(1-I*c*x))/d^3+b^2*e*polylog(3,1-2/(1+I*c*x))/d^3-b^2*e*polylog(3,-1+2
/(1+I*c*x))/d^3-1/2*c^2*(a+b*arctan(c*x))^2/d^2-1/2*(a+b*arctan(c*x))^2/d^2
/x^2-1/2*b*c*e^(3/2)*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(5/2)/(c^2*d-e)+1/2*b*c*e^(3/2)*(a+b*arc
tan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))
/(-d)^(5/2)/(c^2*d-e)-1/4*I*b^2*c*e^(3/2)*polylog(2,1-2*c*((-d)^(1/2)+x*e^(
1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(5/2)/(c^2*d-e)+1/4*I*b^2*c*
e^(3/2)*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^
(1/2)))/(-d)^(5/2)/(c^2*d-e)-2*I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*
c*x))/d^3+1/2*c^2*e*(a+b*arctan(c*x))^2/d^2/(c^2*d-e)
```

Rubi [A]

time = 1.49, antiderivative size = 1181, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 22, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.956$, Rules used = {5100, 4946, 5038, 272, 36, 29, 31, 5004, 4942, 5108, 5114, 6745, 5098, 4974, 4966, 2449, 2352, 2497, 5104, 5040, 4964, 4968}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/(d^2*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d^2)
+ (c^2*e*(a + b*ArcTan[c*x])^2)/(2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])^2
/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 - (Sqrt[e]*x)/Sqrt[-d]))
 - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 + (Sqrt[e]*x)/Sqrt[-d])) - (4*e*(a +
 b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d^2 -
(2*e*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^3 - (b*c*e^(3/2)*(a + b*Ar
cTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(S
qrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (b*c*e
^(3/2)*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +
I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x
])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + ((2*I)*b*e*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((2*I)*b*e*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - 2/(1 + I*c*x)])/d^3 - ((2*I)*b*e*(a + b*ArcTan[c*x])*PolyLog[2,
-1 + 2/(1 + I*c*x)])/d^3 + ((I/4)*b^2*c*e^(3/2)*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((-d)^(5/2)*(c^2*
d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e
]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/4)*b^2*c*e^(3/2)*P
olyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I
*c*x))])/((-d)^(5/2)*(c^2*d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^
3 - (b^2*e*PolyLog[3, 1 - 2/(1 - I*c*x)])/d^3 + (b^2*e*PolyLog[3, 1 - 2/(1
+ I*c*x)])/d^3 - (b^2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*e*PolyLo
g[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x)
)])/(2*d^3) + (b^2*e*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-
d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5098

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x])^p/(1 - R
t[-e/d, 2]*x)^2, x], x] - Dist[1/(4*d^2*Rt[-e/d, 2]), Int[(a + b*ArcTan[c*x
])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p,
0]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(d+ex^2)^2}\,dx = \int \left(\frac{(a+b\tan^{-1}(cx))^2}{d^2x^3} - \frac{2e(a+b\tan^{-1}(cx))^2}{d^3x} + \frac{e^2x(a+b\tan^{-1}(cx))^2}{d^2(d+ex^2)^2} + \frac{2e^2x\right.$$

$$= \frac{\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{x(a+b\tan^{-1}(cx))^2}{d+ex^2}\,dx}{d^3} + \frac{e^2}{}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} - \frac{4e(a+b\tan^{-1}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{(bc)\int \frac{a+b\tan^{-1}}{x^2(1+c}}{d^2}$$

$$= -\frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} - \frac{e(a+b\tan^{-1}(cx))^2}{4d^3\left(1-\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{e(a+b\tan^{-1}(cx))^2}{4d^3\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)} - \frac{4e(a+b\,\mathrm{ta}}{}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} - \frac{e(a+b\,\mathrm{ta}}{4d^3\left(1-\right.}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} - \frac{e(a+b\,\mathrm{ta}}{4d^3\left(1-\right.}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d^2} - \frac{(a+b\tan^{-1}(cx))^2}{2d^2x^2} - \frac{e(a+b\,\mathrm{ta}}{4d^3\left(1-\right.}$$

$$= -\frac{bc(a+b\tan^{-1}(cx))}{d^2x} - \frac{c^2(a+b\tan^{-1}(cx))^2}{2d^2} + \frac{c^2e(a+b\tan^{-1}(cx))^2}{2d^2(c^2d-e)} - \frac{(a+b}{2}$$

Mathematica [F]
time = 20.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\mathrm{ArcTan}(cx))^2}{x^3(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2), x]
```

Maple [F]
time = 75.36, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((2*x^2*e + d)/(d^2*x^4*e + d^3*x^2) - 2*e*log(x^2*e + d)/d^3 + 4*
e*log(x)/d^3) + integrate((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(x^7*e^2
+ 2*d*x^5*e + d^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(x^7*e^2 + 2*d*x^5*e
 + d^2*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2),x)
[Out] int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2), x)
```

3.1275 $\int x^4 \mathbf{ArcTan}(x) \log (1 + x^2) \, dx$

Optimal. Leaf size=111

$$-\frac{77x^2}{300}+\frac{9x^4}{200}-\frac{2}{5}x\text{ArcTan}(x)+\frac{2}{15}x^3\text{ArcTan}(x)-\frac{2}{25}x^5\text{ArcTan}(x)+\frac{\text{ArcTan}(x)^2}{5}+\frac{137}{300}\log\left(1+x^2\right)+\frac{1}{10}x^2\log\left(\right.$$

```
[Out] -77/300*x^2+9/200*x^4-2/5*x*arctan(x)+2/15*x^3*arctan(x)-2/25*x^5*arctan(x)
+1/5*arctan(x)^2+137/300*ln(x^2+1)+1/10*x^2*ln(x^2+1)-1/20*x^4*ln(x^2+1)+1/
5*x^5*arctan(x)*ln(x^2+1)-1/20*ln(x^2+1)^2
```

Rubi [A]
time = 0.30, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {4946, 272, 45, 5141, 6857, 457, 78, 5036, 4930, 266, 5004, 2525, 2437, 2338}

$$-\frac{2}{25}x^5\text{ArcTan}(x)+\frac{2}{15}x^3\text{ArcTan}(x)+\frac{1}{5}x^5\text{ArcTan}(x)\log\left(x^2+1\right)-\frac{2}{5}x\text{ArcTan}(x)+\frac{\text{ArcTan}(x)^2}{5}+\frac{9x^4}{200}-\frac{77x^2}{300}-\frac{1}{20}\log^2\left(x^2+1\right)+\frac{1}{10}x^2\log\left(x^2+1\right)+\frac{137}{300}\log\left(x^2+1\right)-\frac{1}{20}x^4\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (-77*x^2)/300 + (9*x^4)/200 - (2*x*ArcTan[x])/5 + (2*x^3*ArcTan[x])/15 - (2
*x^5*ArcTan[x])/25 + ArcTan[x]^2/5 + (137*Log[1 + x^2])/300 + (x^2*Log[1 +
x^2])/10 - (x^4*Log[1 + x^2])/20 + (x^5*ArcTan[x]*Log[1 + x^2])/5 - Log[1 +
 x^2]^2/20
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^4 \tan^{-1}(x) \log(1+x^2)\, dx = \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x) \log(1+x^2) - \frac{1}{10} \mathrm{lo}$$
$$= \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x) \log(1+x^2) - \frac{1}{10} \mathrm{lo}$$
$$= \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x) \log(1+x^2) - \frac{1}{10} \mathrm{lo}$$
$$= \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x) \log(1+x^2) - \frac{1}{10} \mathrm{lo}$$
$$= \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x) \log(1+x^2) - \frac{1}{20} \mathrm{lo}$$
$$= -\frac{2}{25}x^5 \tan^{-1}(x) + \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \tan^{-1}(x)$$
$$= -\frac{3x^2}{20} + \frac{x^4}{40} + \frac{2}{15}x^3 \tan^{-1}(x) - \frac{2}{25}x^5 \tan^{-1}(x) + \frac{3}{20}\log(1+x^2) + \frac{1}{10}x^2 \mathrm{lo}$$
$$= -\frac{3x^2}{20} + \frac{x^4}{40} - \frac{2}{5}x \tan^{-1}(x) + \frac{2}{15}x^3 \tan^{-1}(x) - \frac{2}{25}x^5 \tan^{-1}(x) + \frac{1}{5}\tan^{-1}(x$$
$$= -\frac{19x^2}{100} + \frac{9x^4}{200} - \frac{2}{5}x \tan^{-1}(x) + \frac{2}{15}x^3 \tan^{-1}(x) - \frac{2}{25}x^5 \tan^{-1}(x) + \frac{1}{5}\tan^{-1}$$
$$= -\frac{77x^2}{300} + \frac{9x^4}{200} - \frac{2}{5}x \tan^{-1}(x) + \frac{2}{15}x^3 \tan^{-1}(x) - \frac{2}{25}x^5 \tan^{-1}(x) + \frac{1}{5}\tan^{-1}$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 0.71

$$\frac{1}{600}\left(x^2(-154+27x^2) + 120\mathrm{ArcTan}(x)^2 + (274+60x^2-30x^4)\log(1+x^2) - 30\log^2(1+x^2) + 8x\mathrm{ArcTan}(x)(-30+10x^2-6x^4+15x^4\log(1+x^2))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (x^2*(-154 + 27*x^2) + 120*ArcTan[x]^2 + (274 + 60*x^2 - 30*x^4)*Log[1 + x^
2] - 30*Log[1 + x^2]^2 + 8*x*ArcTan[x]*(-30 + 10*x^2 - 6*x^4 + 15*x^4*Log[1
 + x^2]))/600
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.90, size = 1286, normalized size = 11.59

method	result	size
default	Expression too large to display	1286
risch	Expression too large to display	5733

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -77/300*x^2+9/200*x^4+1/5*ln(2)*x^2-1/10*ln(2)*x^4-1/5*Pi*csgn(I*((1+I*x)^2
/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)+1/5*Pi*csgn(I/(x^2+1)^(1/2))
^3*arctan(x)+2/5*ln(2)*arctan(x)*x^5+3/20*I*Pi*csgn(I/(x^2+1)^(1/2))^3-2/5*
I*ln(2)*arctan(x)-2/5*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x^5-2/5*x*arctan(x)
+2/15*x^3*arctan(x)-2/25*x^5*arctan(x)-1/5*ln((1+I*x)^2/(x^2+1)+1)*x^2+1/10
*ln((1+I*x)^2/(x^2+1)+1)*x^4+46/75*I*arctan(x)+3/10*ln(2)-137/150*ln((1+I*x
)^2/(x^2+1)+1)+1/5*I*csgn(I/(x^2+1)^(1/2))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*cs
gn(I*((1+I*x)^2/(x^2+1)+1))*ln((1+I*x)^2/(x^2+1)+1)*Pi+1/10*I*Pi*csgn(I*((1
+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))*x^2
-1/5*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)*x
^5-1/5*I*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)
*x^5-1/20*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*
csgn(I/(x^2+1)^(1/2))*x^4-1/5*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+
1)^(1/2))^2*arctan(x)-1/20*I*Pi*csgn(I/(x^2+1)^(1/2))^3*x^4+1/10*I*Pi*csgn(
I/(x^2+1)^(1/2))^3*x^2-3/20*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+
1)^(1/2))^2-3/20*I*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2
+1/5*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I/(x^2+1)^(1/2))^3-1/5*ln((1+I*x)^2/
(x^2+1)+1)^2+1/5*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^
(1/2))*csgn(I/(x^2+1)^(1/2))*arctan(x)*x^5+1/10*(-4*I*arctan(x)+4*x^5*arcta
n(x)+4*ln((1+I*x)^2/(x^2+1)+1)+3+2*x^2-x^4)*ln((1+I*x)/(x^2+1)^(1/2))+2/5*l
n((1+I*x)^2/(x^2+1)+1)*ln(2)-1/10*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I
/(x^2+1)^(1/2))^2*x^2-1/10*I*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1
)^(1/2))^2*x^2-1/5*I*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*
ln((1+I*x)^2/(x^2+1)+1)*Pi-1/5*I*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^
2+1)^(1/2))*ln((1+I*x)^2/(x^2+1)+1)*Pi+3/20*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+
1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))+1/5*I*Pi*csgn(I/(x^
2+1)^(1/2))^3*arctan(x)*x^5+1/5*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+
I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))*arctan(x)+1/20*I*Pi*csgn(I*((1+I*
x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*x^4+1/20*I*Pi*csgn(I/(1+I*x)*(x^2+
1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*x^4-181/600
```

Maxima [A]
time = 0.48, size = 80, normalized size = 0.72

$$\frac{9}{200}\,x^{4}-\frac{77}{300}\,x^{2}+\frac{1}{75}\,\left(15\,x^{5}\log\left(x^{2}+1\right)-6\,x^{5}+10\,x^{3}-30\,x+30\,\arctan\left(x\right)\right)\arctan\left(x\right)-\frac{1}{5}\,\arctan\left(x\right)^{2}-\frac{1}{300}\,\left(15\,x^{4}-30\,x^{2}-137\right)\log\left(x^{2}+1\right)-\frac{1}{20}\,\log\left(x^{2}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 9/200*x^4 - 77/300*x^2 + 1/75*(15*x^5*log(x^2 + 1) - 6*x^5 + 10*x^3 - 30*x
+ 30*arctan(x))*arctan(x) - 1/5*arctan(x)^2 - 1/300*(15*x^4 - 30*x^2 - 137)
*log(x^2 + 1) - 1/20*log(x^2 + 1)^2
```

Fricas [A]
time = 3.12, size = 72, normalized size = 0.65

$$\frac{9}{200}\,x^4-\frac{77}{300}\,x^2-\frac{2}{75}\,\left(3\,x^5-5\,x^3+15\,x\right)\arctan\left(x\right)+\frac{1}{5}\,\arctan\left(x\right)^2+\frac{1}{300}\,\left(60\,x^5\arctan\left(x\right)-15\,x^4+30\,x^2+137\right)\log\left(x^2+1\right)-\frac{1}{20}\,\log\left(x^2+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 9/200*x^4 - 77/300*x^2 - 2/75*(3*x^5 - 5*x^3 + 15*x)*arctan(x) + 1/5*arctan
(x)^2 + 1/300*(60*x^5*arctan(x) - 15*x^4 + 30*x^2 + 137)*log(x^2 + 1) - 1/2
0*log(x^2 + 1)^2
```

Sympy [A]
time = 0.91, size = 107, normalized size = 0.96

$$\frac{x^5\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{5}-\frac{2x^5\operatorname{atan}\left(x\right)}{25}-\frac{x^4\log\left(x^2+1\right)}{20}+\frac{9x^4}{200}+\frac{2x^3\operatorname{atan}\left(x\right)}{15}+\frac{x^2\log\left(x^2+1\right)}{10}-\frac{77x^2}{300}-\frac{2x\operatorname{atan}\left(x\right)}{5}-\frac{\log\left(x^2+1\right)^2}{20}+\frac{137\log\left(x^2+1\right)}{300}+\frac{\operatorname{atan}^2\left(x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(x)*ln(x**2+1),x)
```

```
[Out] x**5*log(x**2 + 1)*atan(x)/5 - 2*x**5*atan(x)/25 - x**4*log(x**2 + 1)/20 +
9*x**4/200 + 2*x**3*atan(x)/15 + x**2*log(x**2 + 1)/10 - 77*x**2/300 - 2*x*
atan(x)/5 - log(x**2 + 1)**2/20 + 137*log(x**2 + 1)/300 + atan(x)**2/5
```

Giac [A]
time = 0.39, size = 168, normalized size = 1.51

$$\frac{1}{10}\,\pi x^5\log\left(x^2+1\right)\operatorname{sgn}(x)-\frac{1}{5}\,x^5\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)-\frac{1}{25}\,\pi x^5\operatorname{sgn}(x)+\frac{2}{25}\,x^5\arctan\left(\frac{1}{x}\right)-\frac{1}{20}\,x^4\log\left(x^2+1\right)+\frac{1}{15}\,\pi x^3\operatorname{sgn}(x)+\frac{9}{200}\,x^4-\frac{2}{15}\,x^3\arctan\left(\frac{1}{x}\right)+\frac{1}{10}\,x^2\log\left(x^2+1\right)-\frac{3}{10}\,\pi^2\operatorname{sgn}(x)-\frac{1}{5}\,\pi x\operatorname{sgn}(x)-\frac{1}{5}\,\pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x)+\frac{1}{10}\,\pi^2-\frac{77}{300}\,x^2+\frac{1}{5}\,\pi\arctan\left(x\right)+\frac{1}{5}\,\pi\arctan\left(\frac{1}{x}\right)+\frac{2}{5}\,x\arctan\left(\frac{1}{x}\right)+\frac{1}{5}\,\arctan\left(\frac{1}{x}\right)^2-\frac{1}{20}\,\log\left(x^2+1\right)^2+\frac{137}{300}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*pi*x^5*log(x^2 + 1)*sgn(x) - 1/5*x^5*arctan(1/x)*log(x^2 + 1) - 1/25*p
i*x^5*sgn(x) + 2/25*x^5*arctan(1/x) - 1/20*x^4*log(x^2 + 1) + 1/15*pi*x^3*s
gn(x) + 9/200*x^4 - 2/15*x^3*arctan(1/x) + 1/10*x^2*log(x^2 + 1) - 3/10*pi^
2*sgn(x) - 1/5*pi*x*sgn(x) - 1/5*pi*arctan(1/x)*sgn(x) + 1/10*pi^2 - 77/300
*x^2 + 1/5*pi*arctan(x) + 1/5*pi*arctan(1/x) + 2/5*x*arctan(1/x) + 1/5*arct
an(1/x)^2 - 1/20*log(x^2 + 1)^2 + 137/300*log(x^2 + 1)
```

Mupad [B]
time = 0.48, size = 82, normalized size = 0.74

$$\frac{137\,\ln\left(x^2+1\right)}{300}-\frac{{\ln\left(x^2+1\right)}^2}{20}+\frac{{\mathrm{atan}\left(x\right)}^2}{5}-\mathrm{atan}\left(x\right)\,\left(\frac{2\,x}{5}-\frac{2\,x^3}{15}+\frac{2\,x^5}{25}-\frac{x^5\,\ln\left(x^2+1\right)}{5}\right)+\ln\left(x^2+1\right)\,\left(\frac{x^2}{10}-\frac{x^4}{20}\right)-\frac{77\,x^2}{300}+\frac{9\,x^4}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(x^2 + 1)*atan(x),x)
```

```
[Out] (137*log(x^2 + 1))/300 - log(x^2 + 1)^2/20 + atan(x)^2/5 - atan(x)*((2*x)/5
 - (2*x^3)/15 + (2*x^5)/25 - (x^5*log(x^2 + 1))/5) + log(x^2 + 1)*(x^2/10 -
 x^4/20) - (77*x^2)/300 + (9*x^4)/200
```

3.1276 $\int x^3 \mathbf{ArcTan}(x) \log(1+x^2)\,dx$

Optimal. Leaf size=88

$$-\frac{25x}{24}+\frac{7x^3}{72}+\frac{25\text{ArcTan}(x)}{24}+\frac{1}{4}x^2\text{ArcTan}(x)-\frac{1}{8}x^4\text{ArcTan}(x)+\frac{1}{4}x\log(1+x^2)-\frac{1}{12}x^3\log(1+x^2)-\frac{1}{4}\text{ArcTa}$$

```
[Out] -25/24*x+7/72*x^3+25/24*arctan(x)+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/4*x
*ln(x^2+1)-1/12*x^3*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)+1/4*x^4*arctan(x)*ln(
x^2+1)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4946, 308, 209, 2504, 2442, 45, 5139, 470, 327, 2521, 2498, 2505}

$$-\frac{1}{8}x^4\text{ArcTan}(x)+\frac{1}{4}x^2\text{ArcTan}(x)-\frac{1}{4}\text{ArcTan}(x)\log(x^2+1)+\frac{1}{4}x^4\text{ArcTan}(x)\log(x^2+1)+\frac{25\text{ArcTan}(x)}{24}+\frac{7x^3}{72}+\frac{1}{4}x\log(x^2+1)-\frac{1}{12}x^3\log(x^2+1)-\frac{25x}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (-25*x)/24 + (7*x^3)/72 + (25*ArcTan[x])/24 + (x^2*ArcTan[x])/4 - (x^4*ArcT
an[x])/8 + (x*Log[1 + x^2])/4 - (x^3*Log[1 + x^2])/12 - (ArcTan[x]*Log[1 +
x^2])/4 + (x^4*ArcTan[x]*Log[1 + x^2])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
```

```
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5139

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tan^{-1}(x) \log\left(1+x^2\right)\,dx &= \frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{4}x^4\tan^{-1}(x)\,\text{lo}\\
&= \frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{4}x^4\tan^{-1}(x)\,\text{lo}\\
&= \frac{x^3}{24}+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{4}x^4\tan^{-1}\\
&= -\frac{3x}{8}+\frac{x^3}{24}+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{4}\\
&= -\frac{3x}{8}+\frac{x^3}{24}+\frac{3}{8}\tan^{-1}(x)+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)+\frac{1}{4}x\log\left(1+x^2\right.\\
&= -\frac{7x}{8}+\frac{x^3}{24}+\frac{3}{8}\tan^{-1}(x)+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)+\frac{1}{4}x\log\left(1+x^2\right.\\
&= -\frac{25x}{24}+\frac{7x^3}{72}+\frac{7}{8}\tan^{-1}(x)+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)+\frac{1}{4}x\log\left(1+\right.\\
&= -\frac{25x}{24}+\frac{7x^3}{72}+\frac{25}{24}\tan^{-1}(x)+\frac{1}{4}x^2\tan^{-1}(x)-\frac{1}{8}x^4\tan^{-1}(x)+\frac{1}{4}x\log\left(1+\right.
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 56, normalized size = 0.64

$$\frac{1}{72}\left(x\left(-75+7x^2-6\left(-3+x^2\right)\log\left(1+x^2\right)\right)+3\text{ArcTan}(x)\left(25+6x^2-3x^4+6\left(-1+x^4\right)\log\left(1+x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (x*(-75 + 7*x^2 - 6*(-3 + x^2)*Log[1 + x^2]) + 3*ArcTan[x]*(25 + 6*x^2 - 3*
x^4 + 6*(-1 + x^4)*Log[1 + x^2]))/72
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.35, size = 997, normalized size = 11.33

method	result	size
default	Expression too large to display	997
risch	Expression too large to display	16521

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(4+3*I*arctan(x)+I*x-3*x*arctan(x)-x^2-3*I*arctan(x)*x^2+3*x^3*arctan(x
))*(x+I)*ln((1+I*x)/(x^2+1)^(1/2))+1/72*I*(-82+18*Pi*csgn(I/(x^2+1)^(1/2))^
3*x-36*I*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)-18*I*arctan(x)*x^2+9*I*arctan(x)
*x^4+36*I*ln((1+I*x)^2/(x^2+1)+1)*x-12*I*ln((1+I*x)^2/(x^2+1)+1)*x^3+36*I*l
n(2)*arctan(x)+24*I*Pi*csgn(I/(x^2+1)^(1/2))^3-6*Pi*csgn(I/(x^2+1)^(1/2))^3
*x^3-24*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2-24*I*Pi*
csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2-36*I*ln(2)*arctan(x)*
x^4+36*I*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x^4+6*Pi*csgn(I*((1+I*x)^2/(x^2+
1)+1))*csgn(I/(x^2+1)^(1/2))^2*x^3+6*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(
I/(x^2+1)^(1/2))^2*x^3+18*Pi*arctan(x)*csgn(I/(x^2+1)^(1/2))^3*x^4-18*Pi*cs
gn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*x-18*Pi*csgn(I/(1+I*x)*
(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*x+18*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1)
)*csgn(I/(x^2+1)^(1/2))^2*arctan(x)-18*Pi*csgn(I/(x^2+1)^(1/2))^3*arctan(x)
+18*Pi*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2)
)*csgn(I/(x^2+1)^(1/2))*x^4-123*I*arctan(x)+48*ln(2)-7*I*x^3+18*Pi*csgn(I/(
1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)+75*I*x-36*I*ln(2)*x
+12*I*ln(2)*x^3-18*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^
(1/2))*csgn(I/(x^2+1)^(1/2))*arctan(x)+18*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*
csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))*x+24*I*Pi*csgn(I*((1+I*
x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))-6*Pi*c
sgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(
1/2))*x^3-18*Pi*arctan(x)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2
))^2*x^4-18*Pi*arctan(x)*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2)
)^2*x^4)
```

Maxima [A]
time = 0.48, size = 62, normalized size = 0.70

$$\frac{7}{72}\,x^{3}+\frac{1}{8}\,\left(2\,x^{4}\log\left(x^{2}+1\right)-x^{4}+2\,x^{2}-2\,\log\left(x^{2}+1\right)\right)\arctan\left(x\right)-\frac{1}{12}\,\left(x^{3}-3\,x\right)\log\left(x^{2}+1\right)-\frac{25}{24}\,x+\frac{25}{24}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 7/72*x^3 + 1/8*(2*x^4*log(x^2 + 1) - x^4 + 2*x^2 - 2*log(x^2 + 1))*arctan(x
) - 1/12*(x^3 - 3*x)*log(x^2 + 1) - 25/24*x + 25/24*arctan(x)
```

Fricas [A]
time = 1.62, size = 49, normalized size = 0.56

$$\frac{7}{72}\,x^{3}-\frac{1}{24}\,\left(3\,x^{4}-6\,x^{2}-25\right)\arctan\left(x\right)-\frac{1}{12}\,\left(x^{3}-3\,\left(x^{4}-1\right)\arctan\left(x\right)-3\,x\right)\log\left(x^{2}+1\right)-\frac{25}{24}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 7/72*x^3 - 1/24*(3*x^4 - 6*x^2 - 25)*arctan(x) - 1/12*(x^3 - 3*(x^4 - 1)*ar
ctan(x) - 3*x)*log(x^2 + 1) - 25/24*x
```

Sympy [A]
time = 0.61, size = 83, normalized size = 0.94

$$\frac{x^{4}\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{4}-\frac{x^{4}\operatorname{atan}\left(x\right)}{8}-\frac{x^{3}\log\left(x^{2}+1\right)}{12}+\frac{7x^{3}}{72}+\frac{x^{2}\operatorname{atan}\left(x\right)}{4}+\frac{x\log\left(x^{2}+1\right)}{4}-\frac{25x}{24}-\frac{\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{4}+\frac{25\operatorname{atan}\left(x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)*ln(x**2+1),x)
```

```
[Out] x**4*log(x**2 + 1)*atan(x)/4 - x**4*atan(x)/8 - x**3*log(x**2 + 1)/12 + 7*x
**3/72 + x**2*atan(x)/4 + x*log(x**2 + 1)/4 - 25*x/24 - log(x**2 + 1)*atan(
x)/4 + 25*atan(x)/24
```

Giac [A]
time = 0.40, size = 124, normalized size = 1.41

$$\frac{1}{8}\,\pi x^{4}\log\left(x^{2}+1\right)\operatorname{sgn}(x)-\frac{1}{4}\,x^{4}\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)-\frac{1}{16}\,\pi x^{4}\operatorname{sgn}(x)+\frac{1}{8}\,x^{4}\arctan\left(\frac{1}{x}\right)-\frac{1}{12}\,x^{3}\log\left(x^{2}+1\right)+\frac{1}{8}\,\pi x^{2}\operatorname{sgn}(x)+\frac{7}{72}\,x^{3}-\frac{1}{4}\,x^{2}\arctan\left(\frac{1}{x}\right)-\frac{1}{8}\,\pi\log\left(x^{2}+1\right)\operatorname{sgn}(x)+\frac{1}{4}\,x\log\left(x^{2}+1\right)+\frac{1}{4}\,\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)-\frac{25}{24}\,\pi\operatorname{sgn}(x)-\frac{25}{24}\,x+\frac{25}{24}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*pi*x^4*log(x^2 + 1)*sgn(x) - 1/4*x^4*arctan(1/x)*log(x^2 + 1) - 1/16*pi
*x^4*sgn(x) + 1/8*x^4*arctan(1/x) - 1/12*x^3*log(x^2 + 1) + 1/8*pi*x^2*sgn(
x) + 7/72*x^3 - 1/4*x^2*arctan(1/x) - 1/8*pi*log(x^2 + 1)*sgn(x) + 1/4*x*lo
```

```
g(x^2 + 1) + 1/4*arctan(1/x)*log(x^2 + 1) - 25/24*pi*sgn(x) - 25/24*x + 25/
24*arctan(x)
```

Mupad [B]
time = 0.53, size = 69, normalized size = 0.78

$$\frac{25\,\mathrm{atan}(x)}{24}+\frac{x^2\,\mathrm{atan}(x)}{4}+x\left(\frac{\ln\left(x^2+1\right)}{4}-\frac{25}{24}\right)-x^3\left(\frac{\ln\left(x^2+1\right)}{12}-\frac{7}{72}\right)-x^4\left(\frac{\mathrm{atan}(x)}{8}-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}\right)-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x^2 + 1)*atan(x),x)
```

```
[Out] (25*atan(x))/24 + (x^2*atan(x))/4 + x*(log(x^2 + 1)/4 - 25/24) - x^3*(log(x
^2 + 1)/12 - 7/72) - x^4*(atan(x)/8 - (log(x^2 + 1)*atan(x))/4) - (log(x^2
+ 1)*atan(x))/4
```

3.1277 $\int x^2 \textbf{ArcTan}(x) \log(1+x^2)\,dx$

Optimal. Leaf size=82

$$\frac{5x^2}{18}+\frac{2}{3}x\text{ArcTan}(x)-\frac{2}{9}x^3\text{ArcTan}(x)-\frac{\text{ArcTan}(x)^2}{3}-\frac{11}{18}\log(1+x^2)-\frac{1}{6}x^2\log(1+x^2)+\frac{1}{3}x^3\text{ArcTan}(x)\log($$

[Out] 5/18*x^2+2/3*x*arctan(x)-2/9*x^3*arctan(x)-1/3*arctan(x)^2-11/18*ln(x^2+1)-1/6*x^2*ln(x^2+1)+1/3*x^3*arctan(x)*ln(x^2+1)+1/12*ln(x^2+1)^2

Rubi [A]
time = 0.23, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4946, 272, 45, 5141, 6857, 5036, 4930, 266, 5004, 2525, 2437, 2338}

$$-\frac{2}{9}x^3\text{ArcTan}(x)+\frac{1}{3}x^3\text{ArcTan}(x)\log(x^2+1)+\frac{2}{3}x\text{ArcTan}(x)-\frac{\text{ArcTan}(x)^2}{3}+\frac{5x^2}{18}+\frac{1}{12}\log^2(x^2+1)-\frac{1}{6}x^2\log(x^2+1)-\frac{11}{18}\log(x^2+1)$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcTan[x]*Log[1 + x^2],x]

[Out] (5*x^2)/18 + (2*x*ArcTan[x])/3 - (2*x^3*ArcTan[x])/9 - ArcTan[x]^2/3 - (11*Log[1 + x^2])/18 - (x^2*Log[1 + x^2])/6 + (x^3*ArcTan[x]*Log[1 + x^2])/3 + Log[1 + x^2]^2/12

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(x)\log\left(1+x^2\right)\,dx &= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-2\int\Big(\\ &= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{3}\int\frac{x}{}\\ &= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{6}\text{Subs}\\ &= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{6}\log^2\left(1+x^2\right)-\frac{1}{6}\text{Subs}\\ &= -\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{1}{6}\text{Sub}\\ &= -\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{6}x^2\log\left(1+x^2\right)+\frac{1}{3}x^3\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{12}\log^2\Big(\\ &= \frac{x^2}{6}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}x^2\text{l}\\ &= \frac{x^2}{6}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{1}{2}\log\left(1+x^2\right)-\frac{1}{6}x^2\text{l}\\ &= \frac{5x^2}{18}+\frac{2}{3}x\tan^{-1}(x)-\frac{2}{9}x^3\tan^{-1}(x)-\frac{1}{3}\tan^{-1}(x)^2-\frac{11}{18}\log\left(1+x^2\right)-\frac{1}{6}x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.78

$$\frac{1}{36}\left(10x^2-12\text{ArcTan}(x)^2-2\left(11+3x^2\right)\log\left(1+x^2\right)+3\log^2\left(1+x^2\right)+4x\text{ArcTan}(x)\left(6-2x^2+3x^2\log\left(1+x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (10*x^2 - 12*ArcTan[x]^2 - 2*(11 + 3*x^2)*Log[1 + x^2] + 3*Log[1 + x^2]^2 +
 4*x*ArcTan[x]*(6 - 2*x^2 + 3*x^2*Log[1 + x^2]))/36
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.25, size = 1078, normalized size = 13.15

method	result	size
default	Expression too large to display	1078
risch	Expression too large to display	5252

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*csgn(I/(x^2+1)^(1/2))^3*ln((1+I*x)^2/(x^2+1)+1)*Pi-1/6*I*csgn(I/(x^2
+1)^(1/2))^3*Pi*x^2+1/6*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(
1/2))^2+1/6*I*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2+5/18
*x^2-1/3*ln(2)*x^2+5/18+1/3*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)
^(1/2))^2*arctan(x)-1/3*Pi*csgn(I/(x^2+1)^(1/2))^3*arctan(x)+2/3*x*arctan(x
)-2/9*x^3*arctan(x)+1/3*ln((1+I*x)^2/(x^2+1)+1)*x^2-1/3*ln(2)-1/6*I*csgn(I*
((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*
Pi*x^2-1/3*I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(
I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))+11/9*ln((1+I*x)^2/(x^2+1)+1)
+1/3*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)+1/3
*I*csgn(I/(x^2+1)^(1/2))^3*arctan(x)*Pi*x^3+1/6*I*csgn(I*((1+I*x)^2/(x^2+1)
+1))*csgn(I/(x^2+1)^(1/2))^2*Pi*x^2+1/6*I*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1
+I*x)*(x^2+1)^(1/2))*Pi*x^2-1/6*I*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(
1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))+1/3*I*Pi*ln((1+I*x)^2/(x^2+1)+1
)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2+1/3*I*Pi*ln((1+I*x)
^2/(x^2+1)+1)*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2+1/3*I*c
sgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))*arctan(x)*csgn(I/(1+I*x)
*(x^2+1)^(1/2))*Pi*x^3+1/3*ln((1+I*x)^2/(x^2+1)+1)^2-2/3*arctan(x)*ln((1+I*
x)^2/(x^2+1)+1)*x^3+2/3*arctan(x)*ln(2)*x^3-1/6*I*csgn(I/(x^2+1)^(1/2))^3*P
i+2/3*I*ln(2)*arctan(x)-1/3*I*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^
(1/2))^2*arctan(x)*Pi*x^3-1/3*I*csgn(I/(x^2+1)^(1/2))^2*arctan(x)*csgn(I/(1
+I*x)*(x^2+1)^(1/2))*Pi*x^3-2/3*ln((1+I*x)^2/(x^2+1)+1)*ln(2)-8/9*I*arctan(
x)-1/3*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(
I/(x^2+1)^(1/2))*arctan(x)+1/3*(2*x^3*arctan(x)+2*I*arctan(x)-x^2-2*ln((1+I
*x)^2/(x^2+1)+1)-1)*ln((1+I*x)/(x^2+1)^(1/2))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 0.79

$$\frac{5}{18}\,x^2+\frac{1}{9}\left(3\,x^3\log\left(x^2+1\right)-2\,x^3+6\,x-6\,\arctan\left(x\right)\right)\arctan\left(x\right)+\frac{1}{3}\,\arctan\left(x\right)^2-\frac{1}{18}\left(3\,x^2+11\right)\log\left(x^2+1\right)+\frac{1}{12}\,\log\left(x^2+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] 5/18*x^2 + 1/9*(3*x^3*log(x^2 + 1) - 2*x^3 + 6*x - 6*arctan(x))*arctan(x) +
 1/3*arctan(x)^2 - 1/18*(3*x^2 + 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

Fricas [A]
time = 1.74, size = 55, normalized size = 0.67

$$\frac{5}{18}\,x^2-\frac{2}{9}\,(x^3-3\,x)\arctan\left(x\right)-\frac{1}{3}\arctan\left(x\right)^2+\frac{1}{18}\,(6\,x^3\arctan\left(x\right)-3\,x^2-11)\log\left(x^2+1\right)+\frac{1}{12}\log\left(x^2+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] 5/18*x^2 - 2/9*(x^3 - 3*x)*arctan(x) - 1/3*arctan(x)^2 + 1/18*(6*x^3*arctan
(x) - 3*x^2 - 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

Sympy [A]
time = 0.40, size = 78, normalized size = 0.95

$$\frac{x^3\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{3}-\frac{2x^3\operatorname{atan}\left(x\right)}{9}-\frac{x^2\log\left(x^2+1\right)}{6}+\frac{5x^2}{18}+\frac{2x\operatorname{atan}\left(x\right)}{3}+\frac{\log\left(x^2+1\right)^2}{12}-\frac{11\log\left(x^2+1\right)}{18}-\frac{\operatorname{atan}^2\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)*ln(x**2+1),x)
```

```
[Out] x**3*log(x**2 + 1)*atan(x)/3 - 2*x**3*atan(x)/9 - x**2*log(x**2 + 1)/6 + 5*
x**2/18 + 2*x*atan(x)/3 + log(x**2 + 1)**2/12 - 11*log(x**2 + 1)/18 - atan(
x)**2/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(66) = 132.
time = 0.40, size = 135, normalized size = 1.65

$$\frac{1}{6}\,\pi x^3\log\left(x^2+1\right)\operatorname{sgn}(x)-\frac{1}{3}\,x^3\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)-\frac{1}{9}\,\pi x^3\operatorname{sgn}(x)+\frac{2}{9}\,x^3\arctan\left(\frac{1}{x}\right)-\frac{1}{6}\,x^2\log\left(x^2+1\right)+\frac{1}{6}\,\pi^2\operatorname{sgn}(x)+\frac{1}{3}\,\pi x\operatorname{sgn}(x)+\frac{1}{3}\,\pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x)-\frac{1}{6}\,\pi^2+\frac{5}{18}\,x^2-\frac{1}{3}\,\pi\arctan\left(x\right)-\frac{1}{3}\,\pi\arctan\left(\frac{1}{x}\right)-\frac{2}{3}\,x\arctan\left(\frac{1}{x}\right)-\frac{1}{3}\arctan\left(\frac{1}{x}\right)^2+\frac{1}{12}\log\left(x^2+1\right)^2-\frac{11}{18}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*pi*x^3*log(x^2 + 1)*sgn(x) - 1/3*x^3*arctan(1/x)*log(x^2 + 1) - 1/9*pi*
x^3*sgn(x) + 2/9*x^3*arctan(1/x) - 1/6*x^2*log(x^2 + 1) + 1/6*pi^2*sgn(x) +
 1/3*pi*x*sgn(x) + 1/3*pi*arctan(1/x)*sgn(x) - 1/6*pi^2 + 5/18*x^2 - 1/3*pi
*arctan(x) - 1/3*pi*arctan(1/x) - 2/3*x*arctan(1/x) - 1/3*arctan(1/x)^2 + 1
/12*log(x^2 + 1)^2 - 11/18*log(x^2 + 1)
```

Mupad [B]
time = 0.46, size = 65, normalized size = 0.79

$$\frac{{\ln\left(x^2+1\right)}^2}{12}-\frac{11\,\ln\left(x^2+1\right)}{18}-\frac{{\mathrm{atan}\left(x\right)}^2}{3}-x^2\,\left(\frac{\ln\left(x^2+1\right)}{6}-\frac{5}{18}\right)-x^3\,\left(\frac{2\,\mathrm{atan}\left(x\right)}{9}-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}\left(x\right)}{3}\right)+\frac{2\,x\,\mathrm{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x^2 + 1)*atan(x),x)
```

```
[Out] log(x^2 + 1)^2/12 - (11*log(x^2 + 1))/18 - atan(x)^2/3 - x^2*(log(x^2 + 1)/
6 - 5/18) - x^3*((2*atan(x))/9 - (log(x^2 + 1)*atan(x))/3) + (2*x*atan(x))/
3
```

3.1278 $\int x\mathbf{ArcTan}(x)\log(1+x^2)\,dx$

Optimal. Leaf size=49

$$\frac{3x}{2}-\frac{3\text{ArcTan}(x)}{2}-\frac{1}{2}x^2\text{ArcTan}(x)-\frac{1}{2}x\log(1+x^2)+\frac{1}{2}(1+x^2)\text{ArcTan}(x)\log(1+x^2)$$

```
[Out] 3/2*x-3/2*arctan(x)-1/2*x^2*arctan(x)-1/2*x*ln(x^2+1)+1/2*(x^2+1)*arctan(x)
*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.800$, Rules used = {4946, 327, 209, 2504, 2436, 2332, 5139, 2498}

$$-\frac{1}{2}x^2\text{ArcTan}(x)+\frac{1}{2}(x^2+1)\text{ArcTan}(x)\log(x^2+1)-\frac{3\text{ArcTan}(x)}{2}-\frac{1}{2}x\log(x^2+1)+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] (3*x)/2 - (3*ArcTan[x])/2 - (x^2*ArcTan[x])/2 - (x*Log[1 + x^2])/2 + ((1 +
x^2)*ArcTan[x]*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5139

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(x)\log\left(1+x^2\right)\,dx &= -\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)-\int\left(-\frac{x^2}{2\left(1+x^2\right)}+\frac{1}{2}\text{lo}\right.\\
&= -\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx-\frac{1}{2}\int\text{lo}\\
&= \frac{x}{2}-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{2}\\
&= \frac{3x}{2}-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\text{log}\\
&= \frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)\text{log}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.78

$$\frac{1}{2}\left(3x - 3\text{ArcTan}(x) - x^2\text{ArcTan}(x) + \left(-x + \left(1 + x^2\right)\text{ArcTan}(x)\right)\log\left(1 + x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]*Log[1 + x^2],x]
[Out] (3*x - 3*ArcTan[x] - x^2*ArcTan[x] + (-x + (1 + x^2)*ArcTan[x])*Log[1 + x^2
])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.10, size = 762, normalized size = 15.55

method	result
default	$(-i\arctan(x) + x\arctan(x) - 1)(x+i)\ln\left(\frac{ix+1}{\sqrt{x^2+1}}\right) + \frac{i\left(3-2i\arctan(x)\ln(2)x^2+2i\arctan(x)\ln\left(\frac{(ix+1)}{x^2+}\right.\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
[Out] (-I*arctan(x)+x*arctan(x)-1)*(x+I)*ln((1+I*x)/(x^2+1)^(1/2))+1/2*I*(3-3*I*x
-Pi*csgn(I/(x^2+1)^(1/2))^3*x+Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+
1)^(1/2))^2*x+Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2*x-Pi
*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*arctan(x)+Pi*csgn(I/
(x^2+1)^(1/2))^3*arctan(x)+2*I*ln(2)*x-2*ln(2)-arctan(x)*Pi*csgn(I/(x^2+1)^
(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*x^2-arctan(x)*Pi*csgn(I/(x^2+1)^(1/2
))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))*x^2-I*Pi*csgn(I/(x^2+1)^(1/2))*csgn(I*((
1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))-I*Pi*csgn(I/(x^2+1)^(1/2
))^3+I*arctan(x)*x^2+2*I*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)-2*I*arctan(x)*ln
(2)-2*I*ln((1+I*x)^2/(x^2+1)+1)*x-Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(
x^2+1)^(1/2))^2*arctan(x)+5*I*arctan(x)+I*Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*
csgn(I/(x^2+1)^(1/2))^2+arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^3*x^2+I*Pi*csgn(
I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2-2*I*arctan(x)*ln(2)*x^2+2*
I*arctan(x)*ln((1+I*x)^2/(x^2+1)+1)*x^2+arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))*
csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*x^2+Pi*csgn(I*(
(1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))*a
rctan(x)-Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csg
n(I/(x^2+1)^(1/2))*x)
```

Maxima [A]
time = 0.55, size = 39, normalized size = 0.80

$$-\frac{1}{2}\left(x^2 - \left(x^2+1\right)\log\left(x^2+1\right) + 1\right)\arctan(x) - \frac{1}{2}x\log\left(x^2+1\right) + \frac{3}{2}x - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x) - 1/2*x*log(x^2 + 1) + 3/
2*x - arctan(x)
```

Fricas [A]
time = 2.27, size = 33, normalized size = 0.67

$$-\frac{1}{2}\left(x^{2}+3\right)\arctan\left(x\right)+\frac{1}{2}\left(\left(x^{2}+1\right)\arctan\left(x\right)-x\right)\log\left(x^{2}+1\right)+\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + 3)*arctan(x) + 1/2*((x^2 + 1)*arctan(x) - x)*log(x^2 + 1) + 3/2
*x
```

Sympy [A]
time = 0.26, size = 56, normalized size = 1.14

$$\frac{x^{2}\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{2}-\frac{x^{2}\operatorname{atan}\left(x\right)}{2}-\frac{x\log\left(x^{2}+1\right)}{2}+\frac{3x}{2}+\frac{\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{2}-\frac{3\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)*ln(x**2+1),x)
```

```
[Out] x**2*log(x**2 + 1)*atan(x)/2 - x**2*atan(x)/2 - x*log(x**2 + 1)/2 + 3*x/2 +
 log(x**2 + 1)*atan(x)/2 - 3*atan(x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(39) = 78$.
time = 0.38, size = 86, normalized size = 1.76

$$\frac{1}{4}\pi x^{2}\log\left(x^{2}+1\right)\operatorname{sgn}(x)-\frac{1}{2}x^{2}\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)-\frac{1}{4}\pi x^{2}\operatorname{sgn}(x)+\frac{1}{2}x^{2}\arctan\left(\frac{1}{x}\right)+\frac{1}{4}\pi\log\left(x^{2}+1\right)\operatorname{sgn}(x)-\frac{1}{2}x\log\left(x^{2}+1\right)-\frac{1}{2}\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)+\frac{3}{2}x-\frac{3}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*x^2*log(x^2 + 1)*sgn(x) - 1/2*x^2*arctan(1/x)*log(x^2 + 1) - 1/4*pi*
x^2*sgn(x) + 1/2*x^2*arctan(1/x) + 1/4*pi*log(x^2 + 1)*sgn(x) - 1/2*x*log(x
^2 + 1) - 1/2*arctan(1/x)*log(x^2 + 1) + 3/2*x - 3/2*arctan(x)
```

Mupad [B]
time = 0.47, size = 48, normalized size = 0.98

$$\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2}-x\,\left(\frac{\ln\left(x^2+1\right)}{2}-\frac{3}{2}\right)-x^2\,\left(\frac{\mathrm{atan}(x)}{2}-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2}\right)-\frac{3\,\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x^2 + 1)*atan(x),x)
```

```
[Out] (log(x^2 + 1)*atan(x))/2 - x*(log(x^2 + 1)/2 - 3/2) - x^2*(atan(x)/2 - (log
(x^2 + 1)*atan(x))/2) - (3*atan(x))/2
```

3.1279 $\int \mathbf{ArcTan}(x)\log(1+x^2)\,dx$

Optimal. Leaf size=38

$$-2x\text{ArcTan}(x)+\text{ArcTan}(x)^2+\log(1+x^2)+x\text{ArcTan}(x)\log(1+x^2)-\frac{1}{4}\log^2(1+x^2)$$

```
[Out] -2*x*arctan(x)+arctan(x)^2+ln(x^2+1)+x*arctan(x)*ln(x^2+1)-1/4*ln(x^2+1)^2
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.889$, Rules used = {4930, 266, 5129, 2525, 2437, 2338, 5036, 5004}

$$x\text{ArcTan}(x)\log(x^2+1)+\text{ArcTan}(x)^2-2x\text{ArcTan}(x)-\frac{1}{4}\log^2(x^2+1)+\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] -2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - Lo
g[1 + x^2]^2/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5129

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*
e*g, Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(x)\log\left(1+x^2\right)\,dx &= x\tan^{-1}(x)\log\left(1+x^2\right)-2\int\frac{x^2\tan^{-1}(x)}{1+x^2}\,dx-\int\frac{x\log\left(1+x^2\right)}{1+x^2}\,dx\\ &= x\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\log(1+x)}{1+x}\,dx,x,x^2\right)-2\int\tan^{-1}(x)\,dx\\ &= -2x\tan^{-1}(x)+\tan^{-1}(x)^2+x\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,\right.\\ &= -2x\tan^{-1}(x)+\tan^{-1}(x)^2+\log\left(1+x^2\right)+x\tan^{-1}(x)\log\left(1+x^2\right)-\frac{1}{4}\log^2\left(\right.\end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.00

$$-2x\text{ArcTan}(x)+\text{ArcTan}(x)^2+\log\left(1+x^2\right)+x\text{ArcTan}(x)\log\left(1+x^2\right)-\frac{1}{4}\log^2\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]*Log[1 + x^2],x]
```

```
[Out] -2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - Lo
g[1 + x^2]^2/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.76, size = 697, normalized size = 18.34

method	result
default	$2\left(-i\arctan\left(x\right)+x\arctan\left(x\right)+\ln\left(\frac{(ix+1)^2}{x^2+1}+1\right)\right)\ln\left(\frac{ix+1}{\sqrt{x^2+1}}\right)+i\text{csgn}\left(\frac{i}{\sqrt{x^2+1}}\right)\text{csgn}\left(\frac{i}{\ldots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-I*arctan(x)+x*arctan(x)+ln((1+I*x)^2/(x^2+1)+1))*ln((1+I*x)/(x^2+1)^(1/
2))+I*csgn(I/(x^2+1)^(1/2))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I*((1+I*x)^2
/(x^2+1)+1))*Pi*ln((1+I*x)^2/(x^2+1)+1)-I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I
/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))^2+I*csgn(I/(x^2+1)^(1/2))*csg
n(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I*((1+I*x)^2/(x^2+1)+1))*Pi*arctan(x)*x+2*I
*arctan(x)+I*ln((1+I*x)^2/(x^2+1)+1)*Pi*csgn(I/(x^2+1)^(1/2))^3-I*csgn(I/(x
^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))*Pi*arctan(x)*x-I*csgn(I/(x^2+1
)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*Pi*arctan(x)*x-I*ln((1+I*x)^2/(x^2
+1)+1)*Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2+Pi*csgn(I/(
x^2+1)^(1/2))^3*arctan(x)-Pi*csgn(I/(1+I*x)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(
1/2))^2*arctan(x)-Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(x^2+1)^(1/2))^2*
arctan(x)+Pi*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*cs
gn(I/(x^2+1)^(1/2))*arctan(x)-2*I*arctan(x)*ln(2)+2*arctan(x)*ln(2)*x-2*arc
tan(x)*ln((1+I*x)^2/(x^2+1)+1)*x+I*csgn(I/(x^2+1)^(1/2))^3*Pi*arctan(x)*x+2
*ln((1+I*x)^2/(x^2+1)+1)*ln(2)-2*x*arctan(x)-ln((1+I*x)^2/(x^2+1)+1)^2-2*ln
((1+I*x)^2/(x^2+1)+1)
```

Maxima [A]
time = 0.48, size = 42, normalized size = 1.11

$$\left(x\log\left(x^2+1\right)-2\,x+2\,\arctan\left(x\right)\right)\arctan\left(x\right)-\arctan\left(x\right)^2-\frac{1}{4}\,\log\left(x^2+1\right)^2+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="maxima")
```

```
[Out] (x*log(x^2 + 1) - 2*x + 2*arctan(x))*arctan(x) - arctan(x)^2 - 1/4*log(x^2
+ 1)^2 + log(x^2 + 1)
```

Fricas [A]
time = 1.66, size = 33, normalized size = 0.87

$$-2\,x\arctan\left(x\right)+\arctan\left(x\right)^{2}+\left(x\arctan\left(x\right)+1\right)\log\left(x^{2}+1\right)-\frac{1}{4}\log\left(x^{2}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*x*arctan(x) + arctan(x)^2 + (x*arctan(x) + 1)*log(x^2 + 1) - 1/4*log(x^2
 + 1)^2
```

Sympy [A]
time = 0.18, size = 39, normalized size = 1.03

$$x\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)-2x\operatorname{atan}\left(x\right)-\frac{\log\left(x^{2}+1\right)^{2}}{4}+\log\left(x^{2}+1\right)+\operatorname{atan}^{2}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1)*atan(x) - 2*x*atan(x) - log(x**2 + 1)**2/4 + log(x**2 + 1)
+ atan(x)**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(36) = 72.
time = 0.40, size = 92, normalized size = 2.42

$$\frac{1}{2}\pi x\log\left(x^{2}+1\right)\operatorname{sgn}(x)-x\arctan\left(\frac{1}{x}\right)\log\left(x^{2}+1\right)-\frac{3}{2}\pi^{2}\operatorname{sgn}(x)-\pi x\operatorname{sgn}(x)-\pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x)+\frac{1}{2}\pi^{2}+\pi\arctan\left(x\right)+\pi\arctan\left(\frac{1}{x}\right)+2\,x\arctan\left(\frac{1}{x}\right)+\arctan\left(\frac{1}{x}\right)^{2}-\frac{1}{4}\log\left(x^{2}+1\right)^{2}+\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*pi*x*log(x^2 + 1)*sgn(x) - x*arctan(1/x)*log(x^2 + 1) - 3/2*pi^2*sgn(x)
 - pi*x*sgn(x) - pi*arctan(1/x)*sgn(x) + 1/2*pi^2 + pi*arctan(x) + pi*arcta
n(1/x) + 2*x*arctan(1/x) + arctan(1/x)^2 - 1/4*log(x^2 + 1)^2 + log(x^2 + 1
)
```

Mupad [B]
time = 0.46, size = 39, normalized size = 1.03

$$\ln\left(x^{2}+1\right)-\frac{\ln\left(x^{2}+1\right)^{2}}{4}+\operatorname{atan}(x)^{2}-x\left(2\operatorname{atan}(x)-\ln\left(x^{2}+1\right)\operatorname{atan}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2 + 1)*atan(x),x)
```

```
[Out] log(x^2 + 1) - log(x^2 + 1)^2/4 + atan(x)^2 - x*(2*atan(x) - log(x^2 + 1)*a
tan(x))
```

3.1280 $\int \frac{\textbf{ArcTan}(x)\log(1+x^2)}{x}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{2}i\log^2(1+ix)\log(-ix)+\frac{1}{2}i\log^2(1-ix)\log(ix)+i\log(1-ix)\text{PolyLog}(2,1-ix)-i\log(1+ix)\text{PolyLog}(2,1+i$$

```
[Out] -1/2*I*ln(1+I*x)^2*ln(-I*x)+1/2*I*ln(1-I*x)^2*ln(I*x)+I*ln(1-I*x)*polylog(2
,1-I*x)-I*ln(1+I*x)*polylog(2,1+I*x)-1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1))*
polylog(2,-I*x)+1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1))*polylog(2,I*x)-I*poly
log(3,1-I*x)+I*polylog(3,1+I*x)
```

Rubi [A]
time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4940, 2438, 5131, 2443, 2481, 2421, 6724}

$$-\frac{1}{2}i\text{Li}_2(-ix)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)+\frac{1}{2}i\text{Li}_2(ix)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)-i\text{Li}_3(1-ix)+i\text{Li}_3(ix+1)+i\text{Li}_2(1-ix)\log(1-ix)-i\text{Li}_2(ix+1)\log(1+ix)+\frac{1}{2}i\log(ix)\log^2(1-ix)-\frac{1}{2}i\log^2(1+ix)\log(-ix)$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x,x]
```

```
[Out] (-1/2*I)*Log[1 + I*x]^2*Log[(-I)*x] + (I/2)*Log[1 - I*x]^2*Log[I*x] + I*Log
[1 - I*x]*PolyLog[2, 1 - I*x] - I*Log[1 + I*x]*PolyLog[2, 1 + I*x] - (I/2)*
(Log[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, (-I)*x] + (I/2)*(Lo
g[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*PolyLog[2, I*x] - I*PolyLog[3, 1
- I*x] + I*PolyLog[3, 1 + I*x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5131

```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist[
Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x], Int[ArcTan[c*x]/x, x], x]
 + (Dist[I/2, Int[Log[1 - I*c*x]^2/x, x], x] - Dist[I/2, Int[Log[1 + I*c*x]
^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}\,dx = \frac{1}{2}i\int \frac{\log^2(1-ix)}{x}\,dx - \frac{1}{2}i\int \frac{\log^2(1+ix)}{x}\,dx + \left(-\log(1-ix)-\log(1+ix)+\right.$$

$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + \frac{1}{2}\left(i\left(\log(1-ix)+\log(1\right.\right.$$

$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) - \frac{1}{2}i\left(\log(1-ix)+\log(1-\right.$$

$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + i\log(1-ix)\mathrm{Li}_2(1-ix)$$

$$= -\frac{1}{2}i\log^2(1+ix)\log(-ix) + \frac{1}{2}i\log^2(1-ix)\log(ix) + i\log(1-ix)\mathrm{Li}_2(1-ix)$$

Mathematica [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(x)\log(1+x^2)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x,x]
```

```
[Out] Integrate[(ArcTan[x]*Log[1 + x^2])/x, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.26, size = 5237, normalized size = 27.71

method	result	size
risch	Expression too large to display	5237

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x^2+1)\,\text{atan}(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x,x)
```

```
[Out] int((log(x^2 + 1)*atan(x))/x, x)
```

3.1281 $\int \frac{\text{ArcTan}(x)\log(1+x^2)}{x^2}\,dx$

Optimal. Leaf size=41

$$\text{ArcTan}(x)^2-\frac{\text{ArcTan}(x)\log\left(1+x^2\right)}{x}-\frac{1}{4}\log^2\left(1+x^2\right)-\frac{1}{2}\text{PolyLog}\left(2,-x^2\right)$$

```
[Out] arctan(x)^2-arctan(x)*ln(x^2+1)/x-1/4*ln(x^2+1)^2-1/2*polylog(2,-x^2)
```

Rubi [A]
time = 0.09, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4946, 272, 36, 29, 31, 5137, 2525, 2457, 2437, 2338, 2438, 5004}

$$-\frac{\text{ArcTan}(x)\log\left(x^2+1\right)}{x}+\text{ArcTan}(x)^2-\frac{\text{Li}_2\left(-x^2\right)}{2}-\frac{1}{4}\log^2\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

```
[Out] ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -x
^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^2}\,dx &= -\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+2\int \frac{\tan^{-1}(x)}{1+x^2}\,dx+\int \frac{\log\left(1+x^2\right)}{x\left(1+x^2\right)}\,dx\\ &= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x(1+x)}\,dx,x,x^2\right)\\ &= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{\log(1+x)}{-1-x}+\frac{\log(1+x)}{x}\right)\right.\\ &= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{-1-x}\,dx,x,x^2\right)+\frac{1}{2}\text{Su}\\ &= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}-\frac{\text{Li}_2\left(-x^2\right)}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{\log(x)}{x}\,dx,x,1\right.\\ &= \tan^{-1}(x)^2-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x}-\frac{1}{4}\log^2\left(1+x^2\right)-\frac{\text{Li}_2\left(-x^2\right)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\text{ArcTan}(x)^2-\frac{\text{ArcTan}(x)\log\left(1+x^2\right)}{x}-\frac{1}{4}\log^2\left(1+x^2\right)-\frac{1}{2}\text{PolyLog}\left(2,-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

```
[Out] ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -x
^2]/2
```

Maple [F]
time = 7.56, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(x\right)\ln\left(x^2+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^2,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^2,x)
```

Maxima [A]
time = 0.48, size = 58, normalized size = 1.41

$$-\left(\frac{\log\left(x^2+1\right)}{x}-2\arctan\left(x\right)\right)\arctan\left(x\right)-\arctan\left(x\right)^2+\frac{1}{2}\log\left(-x^2\right)\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2+1\right)^2+\frac{1}{2}\mathrm{Li}_2(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(x^2 + 1)/x - 2*arctan(x))*arctan(x) - arctan(x)^2 + 1/2*log(-x^2)*log
(x^2 + 1) - 1/4*log(x^2 + 1)^2 + 1/2*dilog(x^2 + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 55.39, size = 37, normalized size = 0.90

$$-\frac{\log\left(x^2+1\right)^2}{4}+\operatorname{atan}^2\left(x\right)-\frac{\mathrm{Li}_2\left(x^2e^{i\pi}\right)}{2}-\frac{\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**2,x)
```

```
[Out] -log(x**2 + 1)**2/4 + atan(x)**2 - polylog(2, x**2*exp_polar(I*pi))/2 - log
(x**2 + 1)*atan(x)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^2, x)
```

Mupad [B]
time = 0.11, size = 36, normalized size = 0.88

$$\mathrm{atan}(x)^2-\frac{\ln\left(x^2+1\right)^2}{4}-\frac{\mathrm{Li}_2\left(x^2+1\right)}{2}-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x^2,x)
[Out] atan(x)^2 - log(x^2 + 1)^2/4 - dilog(x^2 + 1)/2 - (log(x^2 + 1)*atan(x))/x
```

3.1282 $\int \frac{\mathbf{ArcTan}(x)\log(1+x^2)}{x^3}\,dx$

Optimal. Leaf size=69

$$\text{ArcTan}(x)-\frac{\log\left(1+x^2\right)}{2x}-\frac{1}{2}\text{ArcTan}(x)\log\left(1+x^2\right)-\frac{\text{ArcTan}(x)\log\left(1+x^2\right)}{2x^2}+\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLo}$$

```
[Out] arctan(x)-1/2*ln(x^2+1)/x-1/2*arctan(x)*ln(x^2+1)-1/2*arctan(x)*ln(x^2+1)/x
^2+1/2*I*polylog(2,-I*x)-1/2*I*polylog(2,I*x)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 331, 209, 5141, 4940, 2438}

$$-\frac{\text{ArcTan}(x)\log\left(x^2+1\right)}{2x^2}-\frac{1}{2}\text{ArcTan}(x)\log\left(x^2+1\right)+\text{ArcTan}(x)+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)-\frac{\log\left(x^2+1\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

```
[Out] ArcTan[x] - Log[1 + x^2]/(2*x) - (ArcTan[x]*Log[1 + x^2])/2 - (ArcTan[x]*Lo
g[1 + x^2])/(2*x^2) + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
```

```
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(x)\log\left(1+x^2\right)}{x^3}\,dx &= -\frac{\log\left(1+x^2\right)}{2x}-\frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2}-2\int\Big(-\frac{}{2}\\ &= -\frac{\log\left(1+x^2\right)}{2x}-\frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2}+\int\frac{1}{1+x^2}\\ &= \tan^{-1}(x)-\frac{\log\left(1+x^2\right)}{2x}-\frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2}+\\ &= \tan^{-1}(x)-\frac{\log\left(1+x^2\right)}{2x}-\frac{1}{2}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log\left(1+x^2\right)}{2x^2}+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.71

$$\text{ArcTan}(x)-\frac{(x+\text{ArcTan}(x)+x^2\text{ArcTan}(x))\log\left(1+x^2\right)}{2x^2}+\frac{1}{2}i(\text{PolyLog}(2,-ix)-\text{PolyLog}(2,ix))$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

```
[Out] ArcTan[x] - ((x + ArcTan[x] + x^2*ArcTan[x])*Log[1 + x^2])/(2*x^2) + (I/2)*
(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

Maple [F]
time = 20.40, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(x\right)\ln\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^3,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^3,x)
```

Maxima [A]
time = 0.52, size = 70, normalized size = 1.01

$$\frac{4\,x^2\arctan\left(x\right)\log\left(x\right)+4\,x^2\arctan\left(x\right)-2i\,x^2\mathrm{Li}_2(i\,x+1)+2i\,x^2\mathrm{Li}_2(-i\,x+1)-\left(\pi x^2+2\left(x^2+1\right)\arctan\left(x\right)+2\,x\right)\log\left(x^2+1\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2*arctan(x)*log(x) + 4*x^2*arctan(x) - 2*I*x^2*dilog(I*x + 1) + 2*
I*x^2*dilog(-I*x + 1) - (pi*x^2 + 2*(x^2 + 1)*arctan(x) + 2*x)*log(x^2 + 1)
)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**3,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x^3,x)
```

```
[Out] int((log(x^2 + 1)*atan(x))/x^3, x)
```

3.1283 $\int \frac{\text{ArcTan}(x)\log(1+x^2)}{x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{2\text{ArcTan}(x)}{3x}-\frac{\text{ArcTan}(x)^2}{3}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)-\frac{\log\left(1+x^2\right)}{6x^2}-\frac{\text{ArcTan}(x)\log\left(1+x^2\right)}{3x^3}+\frac{1}{12}\log^2\left(1+x^2\right.$$

[Out] -2/3*arctan(x)/x-1/3*arctan(x)^2+ln(x)-1/2*ln(x^2+1)-1/6*ln(x^2+1)/x^2-1/3*arctan(x)*ln(x^2+1)/x^3+1/12*ln(x^2+1)^2+1/6*polylog(2,-x^2)

Rubi [A]
time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {4946, 272, 46, 5137, 2525, 2457, 2442, 36, 29, 31, 2438, 2437, 2338, 5038, 5004}

$$-\frac{\text{ArcTan}(x)\log\left(x^2+1\right)}{3x^3}-\frac{1}{3}\text{ArcTan}(x)^2-\frac{2\text{ArcTan}(x)}{3x}+\frac{\text{Li}_2(-x^2)}{6}+\frac{1}{12}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{6x^2}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(ArcTan[x]*Log[1 + x^2])/x^4,x]

[Out] (-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x^2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyLog[2, -x^2]/6

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^4}\,dx = -\frac{\tan^{-1}(x)\log(1+x^2)}{3x^3} + \frac{1}{3}\int \frac{\log(1+x^2)}{x^3(1+x^2)}\,dx + \frac{2}{3}\int \frac{\tan^{-1}(x)}{x^2(1+x^2)}\,dx$$

$$= -\frac{\tan^{-1}(x)\log(1+x^2)}{3x^3} + \frac{1}{6}\text{Subst}\left(\int \frac{\log(1+x)}{x^2(1+x)}\,dx, x, x^2\right) + \frac{2}{3}\int \frac{\tan^{-1}(x)}{x^2}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)\log(1+x^2)}{3x^3} + \frac{1}{6}\text{Subst}\left(\int \left(\frac{\log(1+}{x^2}\right.\right.$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)\log(1+x^2)}{3x^3} + \frac{1}{6}\text{Subst}\left(\int \frac{\log(1+x)}{x^2}\right.$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 - \frac{\log(1+x^2)}{6x^2} - \frac{\tan^{-1}(x)\log(1+x^2)}{3x^3} + \frac{\text{Li}_2(-x}{6}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 + \frac{2\log(x)}{3} - \frac{1}{3}\log(1+x^2) - \frac{\log(1+x^2)}{6x^2} - \frac{\tan}{}$$

$$= -\frac{2\tan^{-1}(x)}{3x} - \frac{1}{3}\tan^{-1}(x)^2 + \log(x) - \frac{1}{2}\log(1+x^2) - \frac{\log(1+x^2)}{6x^2} - \frac{\tan^{-}}{}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.00

$$-\frac{2\text{ArcTan}(x)}{3x} - \frac{\text{ArcTan}(x)^2}{3} + \log(x) - \frac{1}{2}\log(1+x^2) - \frac{\log(1+x^2)}{6x^2} - \frac{\text{ArcTan}(x)\log(1+x^2)}{3x^3} + \frac{1}{12}\log^2(1+x^2) + \frac{1}{6}\text{PolyLog}(2,-x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

```
[Out] (-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x^
2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyLog
[2, -x^2]/6
```

Maple [F]
time = 6.70, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^4,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^4,x)
```

Maxima [A]
time = 0.49, size = 95, normalized size = 1.17

$$-\frac{1}{3}\left(\frac{2}{x} + \frac{\log(x^2+1)}{x^3} + 2\arctan(x)\right)\arctan(x) + \frac{4x^2\arctan(x)^2 + x^2\log(x^2+1)^2 - 2x^2\text{Li}_2(x^2+1) + 12x^2\log(x) - 2(x^2\log(-x^2) + 3x^2 + 1)\log(x^2+1)}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(2/x + log(x^2 + 1)/x^3 + 2*arctan(x))*arctan(x) + 1/12*(4*x^2*arctan(
x)^2 + x^2*log(x^2 + 1)^2 - 2*x^2*dilog(x^2 + 1) + 12*x^2*log(x) - 2*(x^2*l
og(-x^2) + 3*x^2 + 1)*log(x^2 + 1))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.52, size = 97, normalized size = 1.20

$$\frac{2\log(x)}{3}+\frac{\log(2x^2)}{6}+\frac{\log(x^2+1)^2}{12}-\frac{\log(x^2+1)}{3}-\frac{\log(2x^2+2)}{6}-\frac{\operatorname{atan}^2(x)}{3}+\frac{\operatorname{Li}_2(x^2e^{i\pi})}{6}-\frac{2\operatorname{atan}(x)}{3x}-\frac{\log(x^2+1)}{6x^2}-\frac{\log(x^2+1)\operatorname{atan}(x)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**4,x)
```

```
[Out] 2*log(x)/3 + log(2*x**2)/6 + log(x**2 + 1)**2/12 - log(x**2 + 1)/3 - log(2*
x**2 + 2)/6 - atan(x)**2/3 + polylog(2, x**2*exp_polar(I*pi))/6 - 2*atan(x)
/(3*x) - log(x**2 + 1)/(6*x**2) - log(x**2 + 1)*atan(x)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(x^2+1)\operatorname{atan}(x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x^4,x)
```

```
[Out] int((log(x^2 + 1)*atan(x))/x^4, x)
```

3.1284 $\int \frac{\text{ArcTan}(x)\log(1+x^2)}{x^5}\,dx$

Optimal. Leaf size=102

$$-\frac{5}{12x}-\frac{11\text{ArcTan}(x)}{12}-\frac{\text{ArcTan}(x)}{4x^2}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\text{ArcTan}(x)\log(1+x^2)-\frac{\text{ArcTan}(x)\log(}{4x^4}$$

```
[Out] -5/12/x-11/12*arctan(x)-1/4*arctan(x)/x^2-1/12*ln(x^2+1)/x^3+1/4*ln(x^2+1)/
x+1/4*arctan(x)*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)/x^4-1/4*I*polylog(2,-I*x)
+1/4*I*polylog(2,I*x)
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$,
Rules used = {4946, 331, 209, 5141, 464, 5100, 4940, 2438}

$$-\frac{\text{ArcTan}(x)}{4x^2}+\frac{1}{4}\text{ArcTan}(x)\log(x^2+1)-\frac{\text{ArcTan}(x)\log(x^2+1)}{4x^4}-\frac{11\text{ArcTan}(x)}{12}-\frac{1}{4}i\text{Li}_2(-ix)+\frac{1}{4}i\text{Li}_2(ix)+\frac{\log(x^2+1)}{4x}-\frac{\log(x^2+1)}{12x^3}-\frac{5}{12x}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

```
[Out] -5/(12*x) - (11*ArcTan[x])/12 - ArcTan[x]/(4*x^2) - Log[1 + x^2]/(12*x^3) +
 Log[1 + x^2]/(4*x) + (ArcTan[x]*Log[1 + x^2])/4 - (ArcTan[x]*Log[1 + x^2])
/(4*x^4) - (I/4)*PolyLog[2, (-I)*x] + (I/4)*PolyLog[2, I*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^5}\,dx &= -\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log(1+x}{4x^4} \\
&= -\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log(1+x}{4x^4} \\
&= -\frac{1}{6x}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)-\frac{\tan^{-1}(x)\log}{4x^4} \\
&= -\frac{1}{6x}-\frac{2}{3}\tan^{-1}(x)-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\log\left(1+x^2\right)- \\
&= -\frac{1}{6x}-\frac{2}{3}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\,\mathrm{lo} \\
&= -\frac{5}{12x}-\frac{2}{3}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x)\,\mathrm{l} \\
&= -\frac{5}{12x}-\frac{11}{12}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^2}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}+\frac{1}{4}\tan^{-1}(x
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.96

$$-\frac{1}{6x}-\frac{2\mathrm{ArcTan}(x)}{3}+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{x}-\mathrm{ArcTan}(x)\right)-\frac{\mathrm{ArcTan}(x)}{2x^2}\right)+\frac{(-x+3x^3-3\mathrm{ArcTan}(x)+3x^4\mathrm{ArcTan}(x))\log(1+x^2)}{12x^4}-\frac{1}{4}i(\mathrm{PolyLog}(2,-ix)-\mathrm{PolyLog}(2,ix))$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

```
[Out] -1/6*1/x - (2*ArcTan[x])/3 + ((-x^(-1) - ArcTan[x])/2 - ArcTan[x]/(2*x^2))/
2 + ((-x + 3*x^3 - 3*ArcTan[x] + 3*x^4*ArcTan[x])*Log[1 + x^2])/(12*x^4) -
(I/4)*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

Maple [F]
time = 7.87, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^5,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^5,x)
```

Maxima [A]
time = 0.52, size = 89, normalized size = 0.87

$$-\frac{12\,x^4\arctan(x)\log(x)-6i\,x^4\mathrm{Li}_2(i\,x+1)+6i\,x^4\mathrm{Li}_2(-i\,x+1)+10\,x^3+2\,(11\,x^4+3\,x^2)\arctan(x)-(3\,\pi x^4+6\,x^3+6\,(x^4-1)\arctan(x)-2\,x)\log(x^2+1)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*x^4*arctan(x)*log(x) - 6*I*x^4*dilog(I*x + 1) + 6*I*x^4*dilog(-I*
x + 1) + 10*x^3 + 2*(11*x^4 + 3*x^2)*arctan(x) - (3*pi*x^4 + 6*x^3 + 6*(x^4
 - 1)*arctan(x) - 2*x)*log(x^2 + 1))/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x^2+1)\operatorname{atan}(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**5,x)
```

```
[Out] Integral(log(x**2 + 1)*atan(x)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x^5,x)
```

```
[Out] int((log(x^2 + 1)*atan(x))/x^5, x)
```

3.1285 $\int \frac{\operatorname{ArcTan}(x) \log(1+x^2)}{x^6} \, dx$

Optimal. Leaf size=114

$$-\frac{7}{60x^2}-\frac{2\operatorname{ArcTan}(x)}{15x^3}+\frac{2\operatorname{ArcTan}(x)}{5x}+\frac{\operatorname{ArcTan}(x)^2}{5}-\frac{5\log(x)}{6}+\frac{5}{12}\log\left(1+x^2\right)-\frac{\log\left(1+x^2\right)}{20x^4}+\frac{\log\left(1+x^2\right)}{10x^2}-$$

```
[Out] -7/60/x^2-2/15*arctan(x)/x^3+2/5*arctan(x)/x+1/5*arctan(x)^2-5/6*ln(x)+5/12
*ln(x^2+1)-1/20*ln(x^2+1)/x^4+1/10*ln(x^2+1)/x^2-1/5*arctan(x)*ln(x^2+1)/x^
5-1/20*ln(x^2+1)^2-1/10*polylog(2,-x^2)
```

Rubi [A]
time = 0.20, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {4946, 272, 46, 5137, 2525, 2457, 2437, 2338, 2442, 36, 29, 31, 2438, 5038, 5004}

$$-\frac{2\operatorname{ArcTan}(x)}{15x^3}-\frac{\operatorname{ArcTan}(x)\log\left(x^2+1\right)}{5x^5}+\frac{\operatorname{ArcTan}(x)^2}{5}+\frac{2\operatorname{ArcTan}(x)}{5x}-\frac{\operatorname{Li}_2\left(-x^2\right)}{10}-\frac{7}{60x^2}-\frac{1}{20}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{10x^2}+\frac{5}{12}\log\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{20x^4}-\frac{5\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

```
[Out] -7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
- (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^2]
/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyLog[
2, -x^2]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
```

```
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log(1+x^2)}{x^6}\,dx = -\frac{\tan^{-1}(x)\log(1+x^2)}{5x^5} + \frac{1}{5}\int \frac{\log(1+x^2)}{x^5(1+x^2)}\,dx + \frac{2}{5}\int \frac{\tan^{-1}(x)}{x^4(1+x^2)}\,dx$$
$$= -\frac{\tan^{-1}(x)\log(1+x^2)}{5x^5} + \frac{1}{10}\text{Subst}\left(\int \frac{\log(1+x)}{x^3(1+x)}\,dx, x, x^2\right) + \frac{2}{5}\int \frac{\tan^{-1}(x)}{x^4}$$
$$= -\frac{2\tan^{-1}(x)}{15x^3} - \frac{\tan^{-1}(x)\log(1+x^2)}{5x^5} + \frac{1}{10}\text{Subst}\left(\int \left(\frac{\log(1+x)}{-1-x} + \frac{\log(1+x}{x^3}\right.\right.$$
$$= -\frac{2\tan^{-1}(x)}{15x^3} + \frac{2\tan^{-1}(x)}{5x} + \frac{1}{5}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)\log(1+x^2)}{5x^5} + \frac{1}{15}\text{Subst}\Big($$
$$= -\frac{2\tan^{-1}(x)}{15x^3} + \frac{2\tan^{-1}(x)}{5x} + \frac{1}{5}\tan^{-1}(x)^2 - \frac{\log(1+x^2)}{20x^4} + \frac{\log(1+x^2)}{10x^2} - \tan$$
$$= -\frac{1}{15x^2} - \frac{2\tan^{-1}(x)}{15x^3} + \frac{2\tan^{-1}(x)}{5x} + \frac{1}{5}\tan^{-1}(x)^2 - \frac{2\log(x)}{15} + \frac{1}{15}\log(1+x^2)$$
$$= -\frac{7}{60x^2} - \frac{2\tan^{-1}(x)}{15x^3} + \frac{2\tan^{-1}(x)}{5x} + \frac{1}{5}\tan^{-1}(x)^2 - \frac{5\log(x)}{6} + \frac{5}{12}\log(1+x^2)$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 1.00

$$-\frac{7}{60x^2} - \frac{2\text{ArcTan}(x)}{15x^3} + \frac{2\text{ArcTan}(x)}{5x} + \frac{\text{ArcTan}(x)^2}{5} - \frac{5\log(x)}{6} + \frac{5}{12}\log(1+x^2) - \frac{\log(1+x^2)}{20x^4} + \frac{\log(1+x^2)}{10x^2} - \frac{\text{ArcTan}(x)\log(1+x^2)}{5x^5} - \frac{1}{20}\log^2(1+x^2) - \frac{1}{10}\text{PolyLog}(2,-x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

```
[Out] -7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
- (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^2]
/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyLog[
2, -x^2]/10
```

Maple [F]
time = 6.36, size = 0, normalized size = 0.00

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x^2+1)/x^6,x)
```

```
[Out] int(arctan(x)*ln(x^2+1)/x^6,x)
```

Maxima [A]
time = 0.47, size = 115, normalized size = 1.01

$$\frac{1}{15}\left(\frac{2(3x^2-1)}{x^3} - \frac{3\log(x^2+1)}{x^5} + 6\arctan(x)\right)\arctan(x) - \frac{12x^4\arctan(x)^2 + 3x^4\log(x^2+1)^2 - 6x^4\text{Li}_2(x^2+1) + 50x^4\log(x) + 7x^2 - (6x^4\log(-x^2) + 25x^4 + 6x^2 - 3)\log(x^2+1)}{60x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*(2*(3*x^2 - 1)/x^3 - 3*log(x^2 + 1)/x^5 + 6*arctan(x))*arctan(x) - 1/6
0*(12*x^4*arctan(x)^2 + 3*x^4*log(x^2 + 1)^2 - 6*x^4*dilog(x^2 + 1) + 50*x^
4*log(x) + 7*x^2 - (6*x^4*log(-x^2) + 25*x^4 + 6*x^2 - 3)*log(x^2 + 1))/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x^2 + 1)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.39, size = 134, normalized size = 1.18

$$-\frac{8\log(x)}{15}-\frac{\log(x^2)}{20}-\frac{\log(2x^2)}{10}-\frac{\log(x^2+1)^2}{20}+\frac{19\log(x^2+1)}{60}+\frac{\log(2x^2+2)}{10}+\frac{\operatorname{atan}^2(x)}{5}-\frac{\operatorname{Li}_2(x^2e^{i\pi})}{10}+\frac{2\operatorname{atan}(x)}{5x}+\frac{\log(x^2+1)}{10x^2}-\frac{7}{60x^2}-\frac{2\operatorname{atan}(x)}{15x^3}-\frac{\log(x^2+1)}{20x^4}-\frac{\log(x^2+1)\operatorname{atan}(x)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x**2+1)/x**6,x)
```

```
[Out] -8*log(x)/15 - log(x**2)/20 - log(2*x**2)/10 - log(x**2 + 1)**2/20 + 19*log
(x**2 + 1)/60 + log(2*x**2 + 2)/10 + atan(x)**2/5 - polylog(2, x**2*exp_pol
ar(I*pi))/10 + 2*atan(x)/(5*x) + log(x**2 + 1)/(10*x**2) - 7/(60*x**2) - 2*
atan(x)/(15*x**3) - log(x**2 + 1)/(20*x**4) - log(x**2 + 1)*atan(x)/(5*x**5
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x^2 + 1)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(x^2+1)\operatorname{atan}(x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x^2 + 1)*atan(x))/x^6,x)
```

```
[Out] int((log(x^2 + 1)*atan(x))/x^6, x)
```

3.1286 $\int x^4(a+b\text{ArcTan}(cx)) \left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=278

$$-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\text{ArcTan}(cx)}{5c^5}-\frac{2bex\text{ArcTan}(cx)}{5c^4}+\frac{2bex^3\text{ArcTan}(cx)}{15c^2}-\frac{2}{25}bex^5\text{A}$$

```
[Out] -2/5*a*e*x/c^4-77/300*b*e*x^2/c^3+2/15*a*e*x^3/c^2+9/200*b*e*x^4/c-2/25*a*e
*x^5+2/5*a*e*arctan(c*x)/c^5-2/5*b*e*x*arctan(c*x)/c^4+2/15*b*e*x^3*arctan(
c*x)/c^2-2/25*b*e*x^5*arctan(c*x)+1/5*b*e*arctan(c*x)^2/c^5+137/300*b*e*ln(
c^2*x^2+1)/c^5+1/20*b*e*ln(c^2*x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(c^2*x^2+1))/
c^3-1/20*b*x^4*(d+e*ln(c^2*x^2+1))/c+1/5*x^5*(a+b*arctan(c*x))*(d+e*ln(c^2*
x^2+1))-1/10*b*ln(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/c^5
```

Rubi [A]
time = 0.47, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {4946, 272, 45, 5141, 6857, 1816, 649, 209, 266, 5036, 4930, 5004, 2525, 2437, 2338}

$$\frac{1}{5}x^5(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{2ae\text{ArcTan}(cx)}{5c^5}-\frac{2aex}{5c^4}+\frac{2aex^3}{15c^2}-\frac{2}{25}aex^5+\frac{be\text{ArcTan}(cx)^2}{5c^5}-\frac{2bex\text{ArcTan}(cx)}{5c^4}+\frac{2bex^3\text{ArcTan}(cx)}{15c^2}-\frac{2}{25}bex^5\text{ArcTan}(cx)-\frac{77bex^2}{300c^3}-\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}-\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^5}+\frac{be\log^2\left(c^2x^2+1\right)}{20c^5}+\frac{137be\log\left(c^2x^2+1\right)}{300c^5}+\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^3}+\frac{9bex^4}{200c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) + (2*a*e*x^3)/(15*c^2) + (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 + (2*a*e*ArcTan[c*x])/(5*c^5) - (2*b*e*x*Arc
Tan[c*x])/(5*c^4) + (2*b*e*x^3*ArcTan[c*x])/(15*c^2) - (2*b*e*x^5*ArcTan[c*
x])/25 + (b*e*ArcTan[c*x]^2)/(5*c^5) + (137*b*e*Log[1 + c^2*x^2])/(300*c^5)
 + (b*e*Log[1 + c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 + c^2*x^2]))/(10
*c^3) - (b*x^4*(d + e*Log[1 + c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTan[c*x])*
(d + e*Log[1 + c^2*x^2]))/5 - (b*Log[1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))
/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\frac{bx^2(d+e\log(1+c^2x^2))}{10c^3}-\frac{bx^4(d+e\log(1+c^2x^2))}{20c}+$$

$$=\frac{bx^2(d+e\log(1+c^2x^2))}{10c^3}-\frac{bx^4(d+e\log(1+c^2x^2))}{20c}+$$

$$=\frac{bx^2(d+e\log(1+c^2x^2))}{10c^3}-\frac{bx^4(d+e\log(1+c^2x^2))}{20c}+$$

$$=\frac{bx^2(d+e\log(1+c^2x^2))}{10c^3}-\frac{bx^4(d+e\log(1+c^2x^2))}{20c}+$$

$$=\frac{be\log^2(1+c^2x^2)}{20c^5}+\frac{bx^2(d+e\log(1+c^2x^2))}{10c^3}-\frac{bx^4(d+$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\ \mathrm{ta}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5+\frac{2bex^3\tan}{15c}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}+\frac{2aex^3}{15c^2}+\frac{bex^4}{40c}-\frac{2}{25}aex^5+\frac{2ae\tan^{-1}}{5c^5}$$

$$=-\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\tan}{5c}$$

$$=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\tan}{5c}$$

Mathematica [A]
time = 0.10, size = 214, normalized size = 0.77

$$\frac{cx(bcx(-30d(-2+c^2x^2)+e(-154+27c^2x^2))+8a(15c^4dx^4-2e(15-5c^2x^2+3c^4x^4)))+120be\mathrm{ArcTan}(cx)^2+(-60bd+120ac^5ex^5+2be(137+30c^2x^2-15c^4x^4))\log(1+c^2x^2)-30be\log^2(1+c^2x^2)+8\mathrm{ArcTan}(cx)(30ae+15bc^5dx^5-2bcex(15-5c^2x^2+3c^4x^4)+15bc^5ex^5\log(1+c^2x^2))}{600c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (c*x*(b*c*x*(-30*d*(-2 + c^2*x^2) + e*(-154 + 27*c^2*x^2)) + 8*a*(15*c^4*d*
x^4 - 2*e*(15 - 5*c^2*x^2 + 3*c^4*x^4))) + 120*b*e*ArcTan[c*x]^2 + (-60*b*d
 + 120*a*c^5*e*x^5 + 2*b*e*(137 + 30*c^2*x^2 - 15*c^4*x^4))*Log[1 + c^2*x^2
] - 30*b*e*Log[1 + c^2*x^2]^2 + 8*ArcTan[c*x]*(30*a*e + 15*b*c^5*d*x^5 - 2*
b*c*e*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 15*b*c^5*e*x^5*Log[1 + c^2*x^2]))/(6
00*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.46, size = 4941, normalized size = 17.77

method	result	size
default	Expression too large to display	4941
risch	Expression too large to display	23634

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e*a*x^5*ln(c^2*x^2+1)-181/600*b/c^5*e-2/5*b*e*x*arctan(c*x)/c^4+2/15*b*
e*x^3*arctan(c*x)/c^2-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3+2/15*a*e*x^3/c^2+2/5
*a*e*arctan(c*x)/c^5-2/25*b*e*x^5*arctan(c*x)-2/25*a*e*x^5+9/200*b*e*x^4/c-
1/20*b/c*x^4*d+1/10*b/c^3*x^2*d+1/5*b/c^5*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*d-1
/5*b/c^5*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)^2-137/150*b/c^5*e*ln((1+I*c*x)^2/(
c^2*x^2+1)+1)+1/5*b*arctan(c*x)*x^5*d-1/10*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan
(c*x)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^5*e+3/10*b/c^5*ln(2)*e+1/5*b/
c^5*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1))^2+1/10*b/c^5*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/5*
b/c^5*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)^2+1/10*b/c^5*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/
10*b/c^5*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)+1)^2)+3/40*I*b/c^5*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/10*I*b/
c^5*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-3/
40*I*b/c^5*e*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1))+1/10*I*b/c^5*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)^3+1/40*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x^4
*e-1/10*I*b/c^5*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I/((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/40*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((
1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^4*e-1/20*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(
I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+2/5*b*ln(2)*arctan(c*x)*x^5*e-2/5*b*
ln((1+I*c*x)^2/(c^2*x^2+1)+1)*arctan(c*x)*x^5*e+2/5*b/c^5*e*ln((1+I*c*x)^2/
(c^2*x^2+1)+1)*ln(2)-1/10*b/c*ln(2)*x^4*e+1/5*b/c^3*ln(2)*x^2*e-1/5*b/c^3*l
n((1+I*c*x)^2/(c^2*x^2+1)+1)*x^2*e+1/10*b/c*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x
^4*e+46/75*I*b/c^5*e*arctan(c*x)-1/5*I*b/c^5*arctan(c*x)*d+1/10*I*b/c^3*Pi*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^2*e
+1/20*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)
^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/10*I*b*Pi*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^2*
```

```
x^2+1))*arctan(c*x)*x^5*e-1/10*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arcta
n(c*x)*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*x^5*e-1/10*I*b/c^5*Pi*ln((1+I*
c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1))+1/5*x^5*d*a+1/10*b/c^5*e*(4*arctan(c*x)*x^5*c^5-x^4*c^4-
4*I*arctan(c*x)+2*c^2*x^2+4*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+3)*ln((1+I*c*x)/(
c^2*x^2+1)^(1/2))+1/10*b/c^5*arctan(c*x)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)^3-1/10*b/c^5*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)*Pi*e-1/
10*b/c^5*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*ar
ctan(c*x)*Pi*e+3/40*I*b/c^5*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-3/
40*I*b/c^5*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*e*Pi-2/5*I*b/c^5*arctan(c*x)*l
n(2)*e-3/40*I*b/c^5*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)^3*e*Pi-1/40*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^4*e+1
/40*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^
3*x^4*e-1/20*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x^2*e+1/20*I*b/c^
3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e-1/20*I*b/c^3*Pi*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e+3/40*I*b/c^5*
csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+3/20*I*b/c^5*e*Pi*csgn(I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+3/40*I*b/c^5*e*Pi*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/10*I*b/c^
5*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)^3-3/20*I*b/c^5*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/10*I*b*Pi*arctan(c*x)*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^5*e-1/10*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1))^3*arctan(c*x)*x^5*e-1/10*b/c^5*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^
2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/10*I*b*Pi*arctan(c*x)*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*x^5*e-1/5*I
*b*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*((1+I*c*x)
^2/(c^2*x^2+1)+1))*x^5*e+1/5*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arcta
n(c*x)*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^5*...
```

Maxima [A]
time = 0.48, size = 259, normalized size = 0.93

$$\frac{1}{5}adx^5+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)b\arctan\left(cx\right)e+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bd+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)ae+\frac{\left(27\,c^4x^4-154\,c^2x^2-120\,\arctan\left(cx\right)^2-2\left(15\,c^4x^4-30\,c^2x^2-137\right)\log\left(c^2x^2+1\right)-30\,\log\left(c^2x^2+1\right)^2\right)be}{600\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3
 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*arctan(c*x)*e + 1/20*(4*x^5*arctan(c*
x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d + 1/75*(15*x^5
*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c
*x)/c^7))*a*e + 1/600*(27*c^4*x^4 - 154*c^2*x^2 - 120*arctan(c*x)^2 - 2*(15
```

```
*c^4*x^4 - 30*c^2*x^2 - 137)*log(c^2*x^2 + 1) - 30*log(c^2*x^2 + 1)^2)*b*e/
c^5
```

Fricas [A]
time = 2.09, size = 224, normalized size = 0.81

$$\frac{120\,ac^5dx^5 - 30\,bc^4dx^4 + 60\,bc^2dx^2 + 120\,b\arctan(cx)^2\,e - 30\,be\log(c^2x^2+1)^2 + 8\,(15\,bc^5dx^5 - 2\,(3\,bc^5x^5 - 5\,bc^3x^3 + 15\,bcx - 15\,a)e)\arctan(cx) - (48\,ac^5x^5 - 27\,bc^4x^4 - 80\,ac^3x^3 + 154\,bc^2x^2 + 240\,acx)e + 2\,(60\,bc^5x^5\arctan(cx)\,e - 30\,bd + (60\,ac^5x^5 - 15\,bc^4x^4 + 30\,bc^2x^2 + 137\,b)e)\log(c^2x^2+1)}{600\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/600*(120*a*c^5*d*x^5 - 30*b*c^4*d*x^4 + 60*b*c^2*d*x^2 + 120*b*arctan(c*x
)^2*e - 30*b*e*log(c^2*x^2 + 1)^2 + 8*(15*b*c^5*d*x^5 - 2*(3*b*c^5*x^5 - 5*
b*c^3*x^3 + 15*b*c*x - 15*a)*e)*arctan(c*x) - (48*a*c^5*x^5 - 27*b*c^4*x^4
- 80*a*c^3*x^3 + 154*b*c^2*x^2 + 240*a*c*x)*e + 2*(60*b*c^5*x^5*arctan(c*x)
*e - 30*b*d + (60*a*c^5*x^5 - 15*b*c^4*x^4 + 30*b*c^2*x^2 + 137*b)*e)*log(c
^2*x^2 + 1))/c^5
```

Sympy [A]
time = 1.96, size = 338, normalized size = 1.22

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^5\log(c^2x^2+1)}{5} - \frac{2aex^5}{25} + \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae\,\mathrm{atan}(cx)}{5c^5} + \frac{bdx^5\,\mathrm{atan}(cx)}{5} + \frac{bex^5\log(c^2x^2+1)\,\mathrm{atan}(cx)}{5} - \frac{2bex^5\,\mathrm{atan}(cx)}{25} - \frac{bdx^4}{20c} - \frac{bex^4\log(c^2x^2+1)}{20c} + \frac{9bex^4}{200c} + \frac{2bex^3\,\mathrm{atan}(cx)}{15c^2} + \frac{bdx^2}{10c^3} + \frac{bex^2\log(c^2x^2+1)}{10c^3} - \frac{77bex^2}{300c^3} - \frac{2bex\,\mathrm{atan}(cx)}{5c^4} - \frac{bd\log(c^2x^2+1)}{10c^5} - \frac{be\log(c^2x^2+1)^2}{20c^5} + \frac{137be\log(c^2x^2+1)}{300c^5} + \frac{be\,\mathrm{atan}^2(cx)}{5c^5} & \text{for } c \neq 0 \\ \frac{adx^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(c**2*x**2 + 1)/5 - 2*a*e*x**5/25 + 2*a
*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atan(c*x)/(5*c**5) + b*d*x**5*
atan(c*x)/5 + b*e*x**5*log(c**2*x**2 + 1)*atan(c*x)/5 - 2*b*e*x**5*atan(c*x
)/25 - b*d*x**4/(20*c) - b*e*x**4*log(c**2*x**2 + 1)/(20*c) + 9*b*e*x**4/(2
00*c) + 2*b*e*x**3*atan(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x**2*log(
c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atan(c*x)/(5*c*
*4) - b*d*log(c**2*x**2 + 1)/(10*c**5) - b*e*log(c**2*x**2 + 1)**2/(20*c**5
) + 137*b*e*log(c**2*x**2 + 1)/(300*c**5) + b*e*atan(c*x)**2/(5*c**5), Ne(c
, 0)), (a*d*x**5/5, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 3.35, size = 276, normalized size = 0.99

$$\frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}-\frac{b\,e\,\ln\left(c^2\,x^2+1\right)^2}{20\,c^5}-\frac{2\,a\,e\,x}{5\,c^4}+\frac{2\,a\,e\,\mathrm{atan}\left(c\,x\right)}{5\,c^5}+\frac{b\,d\,x^5\,\mathrm{atan}\left(c\,x\right)}{5}-\frac{2\,b\,e\,x^5\,\mathrm{atan}\left(c\,x\right)}{25}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}+\frac{137\,b\,e\,\ln\left(c^2\,x^2+1\right)}{300\,c^5}+\frac{2\,a\,e\,x^3}{15\,c^2}-\frac{b\,d\,x^4}{20\,c}+\frac{b\,d\,x^2}{10\,c^3}+\frac{9\,b\,e\,x^4}{200\,c}-\frac{77\,b\,e\,x^2}{300\,c^3}+\frac{a\,e\,x^5\,\ln\left(c^2\,x^2+1\right)}{5}+\frac{b\,e\,{\mathrm{atan}\left(c\,x\right)}^2}{5\,c^5}+\frac{2\,b\,e\,x^3\,\mathrm{atan}\left(c\,x\right)}{15\,c^2}+\frac{b\,e\,x^5\,\mathrm{atan}\left(c\,x\right)\,\ln\left(c^2\,x^2+1\right)}{5}-\frac{b\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{20\,c}+\frac{b\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^3}-\frac{2\,b\,e\,x\,\mathrm{atan}\left(c\,x\right)}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

```
[Out] (a*d*x^5)/5 - (2*a*e*x^5)/25 - (b*e*log(c^2*x^2 + 1)^2)/(20*c^5) - (2*a*e*x
)/(5*c^4) + (2*a*e*atan(c*x))/(5*c^5) + (b*d*x^5*atan(c*x))/5 - (2*b*e*x^5*
atan(c*x))/25 - (b*d*log(c^2*x^2 + 1))/(10*c^5) + (137*b*e*log(c^2*x^2 + 1)
)/(300*c^5) + (2*a*e*x^3)/(15*c^2) - (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c^3)
+ (9*b*e*x^4)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(c^2*x^2 + 1))
/5 + (b*e*atan(c*x)^2)/(5*c^5) + (2*b*e*x^3*atan(c*x))/(15*c^2) + (b*e*x^5*
atan(c*x)*log(c^2*x^2 + 1))/5 - (b*e*x^4*log(c^2*x^2 + 1))/(20*c) + (b*e*x^
2*log(c^2*x^2 + 1))/(10*c^3) - (2*b*e*x*atan(c*x))/(5*c^4)
```

3.1287 $\int x^3(a+b\textbf{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=221

$$\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)\text{ArcTan}(cx)}{8c^4}+\frac{2be\text{ArcTan}(cx)}{3c^4}+\frac{ex^2(a+b\text{ArcTan}(cx))}{4c^2}-$$

```
[Out] 1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3-1/24*b*(2*d-e)*x^3/c+1/18*b*e*x^3/c-1/8
*b*(2*d-3*e)*arctan(c*x)/c^4+2/3*b*e*arctan(c*x)/c^4+1/4*e*x^2*(a+b*arctan(
c*x))/c^2-1/8*e*x^4*(a+b*arctan(c*x))+1/4*b*e*x*ln(c^2*x^2+1)/c^3-1/12*b*e*
x^3*ln(c^2*x^2+1)/c-1/4*e*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^4+1/4*x^4*(a+b*
arctan(c*x))*(d+e*ln(c^2*x^2+1))
```

Rubi [A]
time = 0.18, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2504, 2442, 45, 5139, 470, 327, 209, 2521, 2498, 2505, 308}

$$\frac{1}{4}x^4(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2(a+b\text{ArcTan}(cx))}{4c^2}-\frac{e\log\left(c^2x^2+1\right)(a+b\text{ArcTan}(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\text{ArcTan}(cx))-\frac{b(2d-3e)\text{ArcTan}(cx)}{8c^4}+\frac{2be\text{ArcTan}(cx)}{3c^4}+\frac{bx(2d-3e)}{8c^3}-\frac{2bex}{3c^3}-\frac{bex^3\log\left(c^2x^2+1\right)}{12c}+\frac{bex\log\left(c^2x^2+1\right)}{4c^3}-\frac{bx^3(2d-e)}{24c}+\frac{bex^3}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) - (b*(2*d - e)*x^3)/(24*c) +
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTan[c*x])/(8*c^4) + (2*b*e*ArcTan[c*x]
)/(3*c^4) + (e*x^2*(a + b*ArcTan[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTan[c*x]
))/8 + (b*e*x*Log[1 + c^2*x^2])/(4*c^3) - (b*e*x^3*Log[1 + c^2*x^2])/(12*c)
 - (e*(a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTan[c*
x])*(d + e*Log[1 + c^2*x^2]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

Q[m, 2*n - 1]

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 5139

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\frac{ex^2(a+b\tan^{-1}(cx))}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-1}(cx)\right)-\frac{e(a+b$$

$$=\frac{ex^2(a+b\tan^{-1}(cx))}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-1}(cx)\right)-\frac{e(a+b$$

$$=-\frac{b(2d-e)x^3}{24c}+\frac{ex^2(a+b\tan^{-1}(cx))}{4c^2}-\frac{1}{8}ex^4\left(a+b\tan^{-}\right.$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{b(2d-e)x^3}{24c}+\frac{ex^2(a+b\tan^{-1}(cx))}{4c^2}-\frac{1}{8}e$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tan^{-1}(cx)}{8c^4}+ex$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{bex}{2c^3}-\frac{b(2d-e)x^3}{24c}-\frac{b(2d-3e)\tan^{-1}(cx}{8c^4}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)}{8c}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)}{8c}$$

Mathematica [A]
time = 0.11, size = 164, normalized size = 0.74

$$\frac{cx(18ac^3dx^3-6bd(-3+c^2x^2)-9acex(-2+c^2x^2)+be(-75+7c^2x^2))-6e(bcx(-3+c^2x^2)+a(3-3c^4x^4))\log\left(1+c^2x^2\right)+3b\text{ArcTan}(cx)\left(e(25+6c^2x^2-3c^4x^4)+6d(-1+c^4x^4)+6e(-1+c^4x^4)\log\left(1+c^2x^2\right)\right)}{72c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (c*x*(18*a*c^3*d*x^3 - 6*b*d*(-3 + c^2*x^2) - 9*a*c*e*x*(-2 + c^2*x^2) + b*
e*(-75 + 7*c^2*x^2)) - 6*e*(b*c*x*(-3 + c^2*x^2) + a*(3 - 3*c^4*x^4))*Log[1
 + c^2*x^2] + 3*b*ArcTan[c*x]*(e*(25 + 6*c^2*x^2 - 3*c^4*x^4) + 6*d*(-1 + c
^4*x^4) + 6*e*(-1 + c^4*x^4)*Log[1 + c^2*x^2]))/(72*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.30, size = 3897, normalized size = 17.63

method	result	size
default	Expression too large to display	3897
risch	Expression too large to display	22188

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*d*a+1/8*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x*e-1/4*I*b/c^3*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x
*e+1/8*I*b/c^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)*x*e-1/12*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn
(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^3*e-1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^
3*e+1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))^2*x^3*e-1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^3*e-25/24*b*e
*x/c^3+41/24*b*e*arctan(c*x)/c^4+7/72*b*e*x^3/c+1/4*b*d*x/c^3-1/12*b*d*x^3/
c-1/4*b*d*arctan(c*x)/c^4+1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)*x^3*e-1/8*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1)^2)*x*e+1/8*I*b/c^4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)*
csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*e+1/4*e*a*x^4*ln(c^2*x^2+1)+1/4*e*a/c
^2*x^2-1/4*e*a/c^4*ln(c^2*x^2+1)-1/8*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1))^3*x*e-1/8*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1)^2)^3*x*e+1/8*I*b/c^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x
*e+1/8*I*b/c^4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)*e+1/8*I*b/c
^4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arcta
n(c*x)*e-1/8*I*b/c^4*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3
*e-1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)*x^4*e+1/3*I*b/c^4*d-41/36*I*b/c^4*e+1/4*b/c^2*arctan(c*x)*x^2*e+1/6
*b/c^4*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+1/6*b/c^4*e*Pi*csgn(I*(1+I*c*
```

```
x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/6*b/c^4*e*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+1/2*b/c^3*ln(2)*x*e-1/2*b/c^3*ln((1+I*c*x)^2/
(c^2*x^2+1)+1)*x*e-1/2*b/c^4*ln(2)*arctan(c*x)*e+1/2*b/c^4*arctan(c*x)*ln((
1+I*c*x)^2/(c^2*x^2+1)+1)*e+1/2*b*ln(2)*arctan(c*x)*x^4*e-1/2*b*arctan(c*x)
*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^4*e-1/6*b/c*ln(2)*x^3*e+1/6*b/c*ln((1+I*c*
x)^2/(c^2*x^2+1)+1)*x^3*e+1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^4*e
-1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))^2*x^4*e-1/4*I*b/c^4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*a
rctan(c*x)*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*e-1/8*I*b/c^4*Pi*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1))*arctan(c*x)*e+1/8*I*b/c^4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arct
an(c*x)*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*e-1/8*I*b/c^4*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1)^2)*e+1/4*I*b/c^4*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*e-1/8*I*b/c^4*Pi*a
rctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)+1))^2*e+1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^4*e-1/4*I
*b*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)^2*x^4*e+1/8*I*b*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^4*e+1/4*I*b*Pi*arctan(c*
x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^
4*e+1/12*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)^2*x^3*e-1/24*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^
2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^3*e+1/4*I*b/c^3*Pi*csgn(I*(1+I*c*
x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x*e+1/8*I*b/c^3*Pi*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2
/(c^2*x^2+1)+1)^2)^2*x*e-1/8*I*b/c^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csg
n(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*x*e+1/4*b*arctan(c*x)*x^4*d-1/8*b*arctan
(c*x)*x^4*e+2/3*I*b/c^4*ln(2)*e-1/8*e*a*x^4+1/6*b/c^4*e*Pi*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))^3*x^4*e-1/8*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^4*e+1/8*I*b*Pi*arctan(c*x)*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^4*e+1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))^3*x^3*e+1/24*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)
^2/(c^2*x^2+1)+1)^2)^3*x^3*e-1/24*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)^3*x^3*e-1/3*b/c^4*e*Pi*csgn(I*(1+I*c*x)/(...
```

Maxima [A]

time = 0.49, size = 227, normalized size = 1.03

$$\frac{1}{4}adx^4+\frac{1}{72}bc\left(\frac{7c^2x^3-6(c^2x^3-3x)\log(c^2x^2+1)-75x}{c^4}+\frac{75\arctan(cx)}{c^5}\right)e+\frac{1}{8}\left(2x^4\log(c^2x^2+1)-c^2\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)b\arctan(cx)e+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd+\frac{1}{8}\left(2x^4\log(c^2x^2+1)-c^2\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log(c^2x^2+1)}{c^6}\right)\right)ae$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d*x^4 + 1/72*b*c*((7*c^2*x^3 - 6*(c^2*x^3 - 3*x)*log(c^2*x^2 + 1) - 7
5*x)/c^4 + 75*arctan(c*x)/c^5)*e + 1/8*(2*x^4*log(c^2*x^2 + 1) - c^2*((c^2*
x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*arctan(c*x)*e + 1/12*(3*x^4*a
rctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d + 1/8*(2*x^4*
log(c^2*x^2 + 1) - c^2*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*
e
```

Fricas [A]
time = 2.37, size = 179, normalized size = 0.81

$$\frac{18\,ac^{4}dx^{4} - 6\,bc^{3}dx^{3} + 18\,bcdx + 3\,(6\,bc^{4}dx^{4} - 6\,bd - (3\,bc^{4}x^{4} - 6\,bc^{2}x^{2} - 25\,b)e)\arctan\left(cx\right) - (9\,ac^{4}x^{4} - 7\,bc^{3}x^{3} - 18\,ac^{2}x^{2} + 75\,bcx)e + 6\,(3\,(bc^{4}x^{4} - b)\arctan\left(cx\right)e + (3\,ac^{4}x^{4} - bc^{3}x^{3} + 3\,bcx - 3\,a)e)\log\left(c^{2}x^{2} + 1\right)}{72\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/72*(18*a*c^4*d*x^4 - 6*b*c^3*d*x^3 + 18*b*c*d*x + 3*(6*b*c^4*d*x^4 - 6*b*
d - (3*b*c^4*x^4 - 6*b*c^2*x^2 - 25*b)*e)*arctan(c*x) - (9*a*c^4*x^4 - 7*b*
c^3*x^3 - 18*a*c^2*x^2 + 75*b*c*x)*e + 6*(3*(b*c^4*x^4 - b)*arctan(c*x)*e +
 (3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x - 3*a)*e)*log(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 1.32, size = 279, normalized size = 1.26

$$\begin{cases} \frac{adx^{4}}{4} + \frac{aex^{4}\log\left(c^{2}x^{2}+1\right)}{4} - \frac{aex^{4}}{8} + \frac{aex^{2}}{4c^{2}} - \frac{ae\log\left(c^{2}x^{2}+1\right)}{4c^{4}} + \frac{bdx^{4}\operatorname{atan}\left(cx\right)}{4} + \frac{bex^{4}\log\left(c^{2}x^{2}+1\right)\operatorname{atan}\left(cx\right)}{4} - \frac{bex^{4}\operatorname{atan}\left(cx\right)}{8} - \frac{bdx^{3}}{12c} - \frac{bex^{3}\log\left(c^{2}x^{2}+1\right)}{12c} + \frac{7bex^{3}}{72c} + \frac{bex^{2}\operatorname{atan}\left(cx\right)}{4c^{2}} + \frac{bdx}{4c^{3}} + \frac{bex\log\left(c^{2}x^{2}+1\right)}{4c^{3}} - \frac{25bex}{24c^{3}} - \frac{bd\operatorname{atan}\left(cx\right)}{4c^{4}} - \frac{be\log\left(c^{2}x^{2}+1\right)\operatorname{atan}\left(cx\right)}{4c^{4}} + \frac{25be\operatorname{atan}\left(cx\right)}{24c^{4}} & \text{for}\: c \neq 0 \\ \frac{adx^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(c**2*x**2 + 1)/4 - a*e*x**4/8 + a*e*x*
*2/(4*c**2) - a*e*log(c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atan(c*x)/4 + b*e*
x**4*log(c**2*x**2 + 1)*atan(c*x)/4 - b*e*x**4*atan(c*x)/8 - b*d*x**3/(12*c
) - b*e*x**3*log(c**2*x**2 + 1)/(12*c) + 7*b*e*x**3/(72*c) + b*e*x**2*atan(
c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(c**2*x**2 + 1)/(4*c**3) - 25*b*e
*x/(24*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*log(c**2*x**2 + 1)*atan(c*x)/(4
*c**4) + 25*b*e*atan(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.78, size = 297, normalized size = 1.34

$$\frac{a\,d\,x^4}{4}-\frac{a\,e\,x^4}{8}+\frac{b\,d\,x}{4\,c^3}-\frac{25\,b\,e\,x}{24\,c^3}+\frac{b\,d\,x^4\,\mathrm{atan}\left(c\,x\right)}{4}-\frac{b\,e\,x^4\,\mathrm{atan}\left(c\,x\right)}{8}-\frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{a\,e\,x^2}{4\,c^2}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,d\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{4\,c^4}+\frac{7\,b\,e\,x^3}{72\,c}+\frac{25\,b\,e\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{24\,c^4}+\frac{a\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{4}+\frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{4\,c^3}-\frac{b\,e\,\mathrm{atan}\left(c\,x\right)\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{b\,e\,x^2\,\mathrm{atan}\left(c\,x\right)}{4\,c^2}+\frac{b\,e\,x^4\,\mathrm{atan}\left(c\,x\right)\,\ln\left(c^2\,x^2+1\right)}{4}-\frac{b\,e\,x^3\,\ln\left(c^2\,x^2+1\right)}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

```
[Out] (a*d*x^4)/4 - (a*e*x^4)/8 + (b*d*x)/(4*c^3) - (25*b*e*x)/(24*c^3) + (b*d*x^
4*atan(c*x))/4 - (b*e*x^4*atan(c*x))/8 - (a*e*log(c^2*x^2 + 1))/(4*c^4) + (
a*e*x^2)/(4*c^2) - (b*d*x^3)/(12*c) - (b*d*atan((6*b*c*d*x)/(6*b*d - 25*b*e
) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(4*c^4) + (7*b*e*x^3)/(72*c) + (25*b*e*
atan((6*b*c*d*x)/(6*b*d - 25*b*e) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(24*c^4
) + (a*e*x^4*log(c^2*x^2 + 1))/4 + (b*e*x*log(c^2*x^2 + 1))/(4*c^3) - (b*e*
atan(c*x)*log(c^2*x^2 + 1))/(4*c^4) + (b*e*x^2*atan(c*x))/(4*c^2) + (b*e*x^
4*atan(c*x)*log(c^2*x^2 + 1))/4 - (b*e*x^3*log(c^2*x^2 + 1))/(12*c)
```

3.1288 $\int x^2(a+b\textbf{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=213

$$\frac{2aex}{3c^2}+\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2ae\text{ArcTan}(cx)}{3c^3}+\frac{2bex\text{ArcTan}(cx)}{3c^2}-\frac{2}{9}bex^3\text{ArcTan}(cx)-\frac{be\text{ArcTan}(cx)^2}{3c^3}-\frac{11be\log(1}{18c}$$

```
[Out] 2/3*a*e*x/c^2+5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*a*e*arctan(c*x)/c^3+2/3*b*e*x*
arctan(c*x)/c^2-2/9*b*e*x^3*arctan(c*x)-1/3*b*e*arctan(c*x)^2/c^3-11/18*b*e
*ln(c^2*x^2+1)/c^3-1/12*b*e*ln(c^2*x^2+1)^2/c^3-1/6*b*x^2*(d+e*ln(c^2*x^2+1
))/c+1/3*x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))+1/6*b*ln(c^2*x^2+1)*(d+e
*ln(c^2*x^2+1))/c^3
```

Rubi [A]

time = 0.39, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {4946, 272, 45, 5141, 6857, 815, 649, 209, 266, 5036, 4930, 5004, 2525, 2437, 2338}

$$\frac{1}{3}x^3(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{2ae\text{ArcTan}(cx)}{3c^3}+\frac{2aex}{3c^2}-\frac{2}{9}aex^3-\frac{be\text{ArcTan}(cx)^2}{3c^3}+\frac{2bex\text{ArcTan}(cx)}{3c^2}-\frac{2}{9}bex^3\text{ArcTan}(cx)-\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{6c^3}-\frac{be\log^2\left(c^2x^2+1\right)}{12c^3}-\frac{11be\log\left(c^2x^2+1\right)}{18c^3}+\frac{5bex^2}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (2*a*e*x)/(3*c^2) + (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*a*e*ArcTan[c*x]
)/(3*c^3) + (2*b*e*x*ArcTan[c*x])/(3*c^2) - (2*b*e*x^3*ArcTan[c*x])/9 - (b*
e*ArcTan[c*x]^2)/(3*c^3) - (11*b*e*Log[1 + c^2*x^2])/(18*c^3) - (b*e*Log[1
+ c^2*x^2]^2)/(12*c^3) - (b*x^2*(d + e*Log[1 + c^2*x^2]))/(6*c) + (x^3*(a +
 b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/3 + (b*Log[1 + c^2*x^2]*(d + e*Lo
g[1 + c^2*x^2]))/(6*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^2(a+b\tan^{-1}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = -\frac{bx^2(d+e\log(1+c^2x^2))}{6c}+\frac{1}{3}x^3(a+b\tan^{-1}(cx))\,(d+e$$

$$= -\frac{bx^2(d+e\log(1+c^2x^2))}{6c}+\frac{1}{3}x^3(a+b\tan^{-1}(cx))\,(d+e$$

$$= -\frac{bx^2(d+e\log(1+c^2x^2))}{6c}+\frac{1}{3}x^3(a+b\tan^{-1}(cx))\,(d+e$$

$$= -\frac{bx^2(d+e\log(1+c^2x^2))}{6c}+\frac{1}{3}x^3(a+b\tan^{-1}(cx))\,(d+e$$

$$= -\frac{be\log^2(1+c^2x^2)}{12c^3}-\frac{bx^2(d+e\log(1+c^2x^2))}{6c}+\frac{1}{3}x^3(a+$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2}{9}bex^3\tan^{-1}(cx)-\frac{be\log^2(1+c$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3+\frac{2bex\tan^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tan^{-1}(c$$

$$= \frac{2aex}{3c^2}+\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2ae\tan^{-1}(cx)}{3c^3}+\frac{2bex\tan^{-1}(cx)}{3c^2}$$

$$= \frac{2aex}{3c^2}+\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2ae\tan^{-1}(cx)}{3c^3}+\frac{2bex\tan^{-1}(cx}{3c^2}$$

Mathematica [A]
time = 0.07, size = 171, normalized size = 0.80

$$\frac{2cx(bc(-3d+5e)x+6ac^2dx^2-4ae(-3+c^2x^2))-12be\text{ArcTan}(cx)^2+2(3bd+6ac^3ex^3-be(11+3c^2x^2))\log(1+c^2x^2)+3be\log^2(1+c^2x^2)-4\text{ArcTan}(cx)\left(6ae+bcx(-6e-3c^2dx^2+2c^2ex^2)-3bc^3ex^3\log(1+c^2x^2)\right)}{36c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] (2*c*x*(b*c*(-3*d + 5*e)*x + 6*a*c^2*d*x^2 - 4*a*e*(-3 + c^2*x^2)) - 12*b*e
*ArcTan[c*x]^2 + 2*(3*b*d + 6*a*c^3*e*x^3 - b*e*(11 + 3*c^2*x^2))*Log[1 + c
^2*x^2] + 3*b*e*Log[1 + c^2*x^2]^2 - 4*ArcTan[c*x]*(6*a*e + b*c*x*(-6*e - 3
*c^2*d*x^2 + 2*c^2*e*x^2) - 3*b*c^3*e*x^3*Log[1 + c^2*x^2]))/(36*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.49, size = 4146, normalized size = 19.46

method	result	size
default	Expression too large to display	4146

risch	Expression too large to display	22991

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b/c^3*arctan(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)+1/6*I*b*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*x^3*e+1/3*I*b*Pi
*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*arct
an(c*x)*x^3*e+1/6*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*x^3*e-1/6*I*b*Pi*
csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*arcta
n(c*x)*x^3*e+1/6*I*b*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)*x^3*e-1/3*I*b*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)*x^3*e-1/
12*I*b/c*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e-1/6*I*b/c*Pi*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^2*e-1/12*I*b/c*Pi*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)^2*x^2*e+1/12*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*x^2*e-1/12*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/6*I*b/c*Pi*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*
x^2*e-1/6*I*b/c^3*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I/((1+I*c*x)^2/(c
^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^2-1/3*I*b/c^3*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+1/6*b/c^3*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1))^3*arctan(c*x)*Pi*e+1/6*b/c^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)*Pi*e-1/6*b/c^3*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)*Pi*e+1/12*I*b/c^3*Pi*e*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1))^3+1/12*I*b/c^3*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)^3-1/12*I*b/c^3*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+
1)^2)^3+2/3*I*b/c^3*arctan(c*x)*ln(2)*e+1/3*I*b/c^3*arctan(c*x)*d-8/9*I*b/c
^3*arctan(c*x)*e-1/3*b/c^3*ln(2)*e-1/6*I*b*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)*x^3*e+1/12*I*b/c*Pi*csgn(I/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/6*I*b/c^3*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1
)+5/18*e*b/c^3-2/3*b/c^3*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*ln(2)+2/3*b*e*x*ar
ctan(c*x)/c^2+2/3*a*e*x/c^2-2/3*a*e*arctan(c*x)/c^3-2/9*b*e*x^3*arctan(c*x)
-2/9*a*e*x^3+5/18*b*e*x^2/c-1/6*b*d*x^2/c-1/12*I*b/c^3*csgn(I*(1+I*c*x)^2/(
```

```
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x
^2+1)+1)^2)-1/6*I*b/c^3*Pi*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))^2-1/12*I*b/c^3*Pi*e*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+2/3*b*ln(2)*
arctan(c*x)*x^3*e-2/3*b*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^3*e-1/3
*b/c*ln(2)*x^2*e+1/3*b/c*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^2*e+1/3*b/c^3*e*(2
*arctan(c*x)*x^3*c^3-c^2*x^2+2*I*arctan(c*x)-2*ln((1+I*c*x)^2/(c^2*x^2+1)+1
)-1)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/3*b/c^3*e*ln((1+I*c*x)^2/(c^2*x^2+1)
+1)^2-1/3*b/c^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*d+11/9*b/c^3*e*ln((1+I*c*x)^2
/(c^2*x^2+1)+1)+1/3*b*arctan(c*x)*x^3*d+1/3*e*a*x^3*ln(c^2*x^2+1)-1/6*I*b/c
^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1))*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e+1/6*I*b/c^3*Pi*l
n((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))-1/6*I*b/c^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e+1
/3*I*b/c^3*Pi*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/12*I*b/c^3*Pi*e*csgn(I/((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)-1/6*b/c^3*d-1/6*b/c^3*arctan
(c*x)*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/3*b/c^3*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)*Pi*e-1/6*b/c^3*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1))*arctan(c*x)*Pi*e+1/6*b/c^3*arctan(c*x)*Pi*e*csgn(I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-1/6*b/c^3*arctan(c*x
)*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)-1/6*I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)*x^3*e-1/6*
I*b*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x...
```

Maxima [A]
time = 0.48, size = 215, normalized size = 1.01

$$\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)b\arctan\left(cx\right)e+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd+\frac{1}{9}\left(3\,x^3\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)ae+\frac{\left(10\,c^2x^2+12\,\arctan\left(cx\right)^2-2\left(3\,c^2x^2+11\right)\log\left(c^2x^2+1\right)+3\,\log\left(c^2x^2+1\right)^2\right)be}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 - 3*x)/c^4 + 3*
arctan(c*x)/c^5))*b*arctan(c*x)*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - l
og(c^2*x^2 + 1)/c^4))*b*d + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 -
 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*e + 1/36*(10*c^2*x^2 + 12*arctan(c*x)^2 -
 2*(3*c^2*x^2 + 11)*log(c^2*x^2 + 1) + 3*log(c^2*x^2 + 1)^2)*b*e/c^3
```

Fricas [A]
time = 2.53, size = 177, normalized size = 0.83

$$\frac{12\,ac^3dx^3-6\,bc^2dx^2-12\,b\arctan\left(cx\right)^2e+3\,be\log\left(c^2x^2+1\right)^2+4\left(3\,bc^3dx^3-2\left(bc^3x^3-3\,bcx+3\,a\right)e\right)\arctan\left(cx\right)-2\left(4\,ac^3x^3-5\,bc^2x^2-12\,acx\right)e+2\left(6\,bc^3x^3\arctan\left(cx\right)e+3\,bd+\left(6\,ac^3x^3-3\,bc^2x^2-11\,b\right)e\right)\log\left(c^2x^2+1\right)}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*c^3*d*x^3 - 6*b*c^2*d*x^2 - 12*b*arctan(c*x)^2*e + 3*b*e*log(c^2
*x^2 + 1)^2 + 4*(3*b*c^3*d*x^3 - 2*(b*c^3*x^3 - 3*b*c*x + 3*a)*e)*arctan(c*
x) - 2*(4*a*c^3*x^3 - 5*b*c^2*x^2 - 12*a*c*x)*e + 2*(6*b*c^3*x^3*arctan(c*x
)*e + 3*b*d + (6*a*c^3*x^3 - 3*b*c^2*x^2 - 11*b)*e)*log(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.92, size = 258, normalized size = 1.21

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3 \log(c^2x^2+1)}{3} - \frac{2aex^3}{9} + \frac{2aex}{3c^2} - \frac{2ae \operatorname{atan}(cx)}{3c^3} + \frac{bdx^3 \operatorname{atan}(cx)}{3} + \frac{bex^3 \log(c^2x^2+1) \operatorname{atan}(cx)}{3} - \frac{2bex^3 \operatorname{atan}(cx)}{9} - \frac{bdx^2}{6c} - \frac{bex^2 \log(c^2x^2+1)}{6c} + \frac{5bex^2}{18c} + \frac{2bex \operatorname{atan}(cx)}{3c^2} + \frac{bd \log(c^2x^2+1)}{6c^3} + \frac{be \log(c^2x^2+1)^2}{12c^3} - \frac{11be \log(c^2x^2+1)}{18c^3} - \frac{be \operatorname{atan}^2(cx)}{3c^3} & \text{for}\ c \neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(c**2*x**2 + 1)/3 - 2*a*e*x**3/9 + 2*a*
e*x/(3*c**2) - 2*a*e*atan(c*x)/(3*c**3) + b*d*x**3*atan(c*x)/3 + b*e*x**3*l
og(c**2*x**2 + 1)*atan(c*x)/3 - 2*b*e*x**3*atan(c*x)/9 - b*d*x**2/(6*c) - b
*e*x**2*log(c**2*x**2 + 1)/(6*c) + 5*b*e*x**2/(18*c) + 2*b*e*x*atan(c*x)/(3
*c**2) + b*d*log(c**2*x**2 + 1)/(6*c**3) + b*e*log(c**2*x**2 + 1)**2/(12*c*
*3) - 11*b*e*log(c**2*x**2 + 1)/(18*c**3) - b*e*atan(c*x)**2/(3*c**3), Ne(c
, 0)), (a*d*x**3/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.53, size = 212, normalized size = 1.00

$$\frac{a\,d\,x^3}{3} - \frac{2\,a\,e\,x^3}{9} + \frac{b\,e\ln(c^2\,x^2+1)^2}{12\,c^3} + \frac{2\,a\,e\,x}{3\,c^2} - \frac{2\,a\,e\operatorname{atan}(c\,x)}{3\,c^3} + \frac{b\,d\,x^3\operatorname{atan}(c\,x)}{3} - \frac{2\,b\,e\,x^3\operatorname{atan}(c\,x)}{9} + \frac{b\,d\,\ln(c^2\,x^2+1)}{6\,c^3} - \frac{11\,b\,e\,\ln(c^2\,x^2+1)}{18\,c^3} - \frac{b\,d\,x^2}{6\,c} + \frac{5\,b\,e\,x^2}{18\,c} + \frac{a\,e\,x^3\,\ln(c^2\,x^2+1)}{3} - \frac{b\,e\operatorname{atan}(c\,x)^2}{3\,c^3} + \frac{b\,e\,x^3\operatorname{atan}(c\,x)\,\ln(c^2\,x^2+1)}{3} - \frac{b\,e\,x^2\,\ln(c^2\,x^2+1)}{6\,c} + \frac{2\,b\,e\,x\operatorname{atan}(c\,x)}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

```
[Out] (a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*e*log(c^2*x^2 + 1)^2)/(12*c^3) + (2*a*e*x)
/(3*c^2) - (2*a*e*atan(c*x))/(3*c^3) + (b*d*x^3*atan(c*x))/3 - (2*b*e*x^3*a
tan(c*x))/9 + (b*d*log(c^2*x^2 + 1))/(6*c^3) - (11*b*e*log(c^2*x^2 + 1))/(1
8*c^3) - (b*d*x^2)/(6*c) + (5*b*e*x^2)/(18*c) + (a*e*x^3*log(c^2*x^2 + 1))/
3 - (b*e*atan(c*x)^2)/(3*c^3) + (b*e*x^3*atan(c*x)*log(c^2*x^2 + 1))/3 - (b
*e*x^2*log(c^2*x^2 + 1))/(6*c) + (2*b*e*x*atan(c*x))/(3*c^2)
```

3.1289 $\int x(a+b\mathbf{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

Optimal. Leaf size=137

$$-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\text{ArcTan}(cx)}{2c^2}-\frac{be\text{ArcTan}(cx)}{c^2}+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))-\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))-\frac{b}{}$$

```
[Out] -1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2-b*e*arctan(c*x)/c^2+1/
2*d*x^2*(a+b*arctan(c*x))-1/2*e*x^2*(a+b*arctan(c*x))-1/2*b*e*x*ln(c^2*x^2+
1)/c+1/2*e*(c^2*x^2+1)*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^2
```

Rubi [A]

time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.292$, Rules used = {2504, 2436, 2332, 5139, 327, 209, 2498}

$$\frac{e(c^2x^2+1)\log(c^2x^2+1)(a+b\text{ArcTan}(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))-\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))+\frac{b(d-e)\text{ArcTan}(cx)}{2c^2}-\frac{be\text{ArcTan}(cx)}{c^2}-\frac{bex\log(c^2x^2+1)}{2c}-\frac{bx(d-e)}{2c}+\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] -1/2*(b*(d - e)*x)/c + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) - (b*e*A
rcTan[c*x])/c^2 + (d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]
))/2 - (b*e*x*Log[1 + c^2*x^2])/(2*c) + (e*(1 + c^2*x^2)*(a + b*ArcTan[c*x]
)*Log[1 + c^2*x^2])/(2*c^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 5139

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rule steps

$$\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e(1+$$

$$=\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e(1+$$

$$=-\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-}\right.$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}-\frac{be\tan^{-1}(cx)}{c^2}$$

Mathematica [A]
time = 0.06, size = 105, normalized size = 0.77

$$\frac{cx(-b(d-3e)+ac(d-e)x)+e(a-bcx+ac^2x^2)\log\left(1+c^2x^2\right)+b\text{ArcTan}(cx)\left(d+c^2dx^2-e(3+c^2x^2)+(e+c^2ex^2)\log\left(1+c^2x^2\right)\right)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]

[Out] (c*x*(-(b*(d - 3*e)) + a*c*(d - e)*x) + e*(a - b*c*x + a*c^2*x^2)*Log[1 + c^2*x^2] + b*ArcTan[c*x]*(d + c^2*d*x^2 - e*(3 + c^2*x^2) + (e + c^2*e*x^2)*Log[1 + c^2*x^2]))/(2*c^2)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.05, size = 3074, normalized size = 22.44

method	result	size
default	Expression too large to display	3074
risch	Expression too large to display	21445

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)

[Out] 1/4*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x*e-1/4*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*e-1/4*b/c^2*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-1/4*b/c^2*e*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+1/4*b/c^2*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-b/c*ln(2)*x*e+b/c*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x*e+b/c^2*e*arctan(c*x)*ln(2)-b/c^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*arctan(c*x)*e+b*arctan(c*x)*ln(2)*x^2*e-b*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*x^2*e-1/2*a*e*x^2-1/2*arctan(c*x)*b*e*x^2-1/2*a/c^2*e+3/2*b*e*x/c-5/2*b*e*arctan(c*x)/c^2-1/2*b*d*x/c+1/2*b*d*arctan(c*x)/c^2+1/4*b/c^2*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+1/2*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*Pi*e+1/4*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*e*Pi-1/4*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*e+1/4*b/c^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*e*Pi-1/2*b/c^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi*e+b/c^2*e*(arctan(c*x)*x*c-1-I*arctan(c*x))*(I+c*x)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x^2*e-1/2*I*b*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e+1/4*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e-1/4*I*b*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*x^2*e+1/4*I*b*Pi*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/4*I*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*arctan(c

```
*x)*e+1/2*I*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))*Pi*arctan(c*x)*e+1/2*a/c^2*e*ln(c^2*x^2+1)-1/2*I*b/c^2*d+1/4*I
*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x*e
-1/4*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x*e-1/4*I*b*Pi*arctan
(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*x^2*e-1/4*I*b*Pi*arctan(c*x)*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e+1/4*I*b*Pi*
arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^2*e-1/4*I*b/c^2*csgn(
I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi*arctan(c*x)*e-1/4*I*b/c^2*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*arctan(c*x)*e+1/4*I*b/c^2
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*Pi*arctan(c*x)*e-1/4*I*b*Pi*arctan
(c*x)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))
*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^2*e+1/4*I*
b/c*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x*e-1/4*I*b
/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*Pi*arctan(c*x
)*e+1/4*I*b/c^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*Pi*arctan(c*x)*e-1/4*I*b/c^2*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*Pi*arctan(c*x)
*e+1/4*I*b/c^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*Pi*arctan(c*x)*e-1/2*I*b/c^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*Pi*arctan(c*x)*e-1/4*I*b/c*Pi
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)^2*x*e-1/2*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)
)^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*x*e-1/4*I*b/c*Pi*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)
^2*x*e+1/4*I*b/c*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))^2*x*e-1/4*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x*e+1/2*I*b/c*Pi*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x*e+1/4*I*b*Pi*arctan(c*
x)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2*e-I*b/c^2*e*ln(2)+1/2*a*e*ln(c^2*x^2+1)*x
^2+1/2*b*arctan(c*x)*x^2*d+3/2*I*b/c^2*e+1/2*a*d*x^2
```

Maxima [A]
time = 0.49, size = 152, normalized size = 1.11

$$\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd-\frac{\left(x\log(c^2x^2+1)-3x+\frac{2\arctan(cx)}{c}\right)be}{2c}-\frac{(c^2x^2-(c^2x^2+1)\log(c^2x^2+1)+1)b\arctan(cx)e}{2c^2}-\frac{(c^2x^2-(c^2x^2+1)\log(c^2x^2+1)+1)ae}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/2
*(x*log(c^2*x^2 + 1) - 3*x + 2*arctan(c*x)/c)*b*e/c - 1/2*(c^2*x^2 - (c^2*x
^2 + 1)*log(c^2*x^2 + 1) + 1)*b*arctan(c*x)*e/c^2 - 1/2*(c^2*x^2 - (c^2*x^2
 + 1)*log(c^2*x^2 + 1) + 1)*a*e/c^2
```

Fricas [A]
time = 2.10, size = 120, normalized size = 0.88

$$\frac{ac^2dx^2 - bcdx + (bc^2dx^2 + bd - (bc^2x^2 + 3b)e)\arctan(cx) - (ac^2x^2 - 3bcx)e + ((bc^2x^2 + b)\arctan(cx)e + (ac^2x^2 - bcx + a)e)\log(c^2x^2 + 1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*d*x^2 - b*c*d*x + (b*c^2*d*x^2 + b*d - (b*c^2*x^2 + 3*b)*e)*arct
an(c*x) - (a*c^2*x^2 - 3*b*c*x)*e + ((b*c^2*x^2 + b)*arctan(c*x)*e + (a*c^2
*x^2 - b*c*x + a)*e)*log(c^2*x^2 + 1))/c^2
```

Sympy [A]
time = 0.60, size = 202, normalized size = 1.47

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^2\log(c^2x^2+1)}{2} - \frac{aex^2}{2} + \frac{ae\log(c^2x^2+1)}{2c^2} + \frac{bdx^2\operatorname{atan}(cx)}{2} + \frac{bex^2\log(c^2x^2+1)\operatorname{atan}(cx)}{2} - \frac{bex^2\operatorname{atan}(cx)}{2} - \frac{bdx}{2c} - \frac{bex\log(c^2x^2+1)}{2c} + \frac{3bex}{2c} + \frac{bd\operatorname{atan}(cx)}{2c^2} + \frac{be\log(c^2x^2+1)\operatorname{atan}(cx)}{2c^2} - \frac{3be\operatorname{atan}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(c**2*x**2 + 1)/2 - a*e*x**2/2 + a*e*lo
g(c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atan(c*x)/2 + b*e*x**2*log(c**2*x**2 +
 1)*atan(c*x)/2 - b*e*x**2*atan(c*x)/2 - b*d*x/(2*c) - b*e*x*log(c**2*x**2
+ 1)/(2*c) + 3*b*e*x/(2*c) + b*d*atan(c*x)/(2*c**2) + b*e*log(c**2*x**2 + 1
)*atan(c*x)/(2*c**2) - 3*b*e*atan(c*x)/(2*c**2), Ne(c, 0)), (a*d*x**2/2, Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.26, size = 227, normalized size = 1.66

$$\frac{a\,d\,x^2}{2} - \frac{a\,e\,x^2}{2} - \frac{b\,d\,x}{2\,c} + \frac{3\,b\,e\,x}{2\,c} + \frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2} - \frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{2} + \frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c^2} + \frac{b\,d\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e} - \frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2} - \frac{3\,b\,e\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e} - \frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2} + \frac{a\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{2} - \frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{2\,c} + \frac{b\,e\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2\,c^2} + \frac{b\,e\,x^2\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

```
[Out] (a*d*x^2)/2 - (a*e*x^2)/2 - (b*d*x)/(2*c) + (3*b*e*x)/(2*c) + (b*d*x^2*atan
(c*x))/2 - (b*e*x^2*atan(c*x))/2 + (a*e*log(c^2*x^2 + 1))/(2*c^2) + (b*d*at
an((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) - (3*b*e*a
tan((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) + (a*e*x^
2*log(c^2*x^2 + 1))/2 - (b*e*x*log(c^2*x^2 + 1))/(2*c) + (b*e*atan(c*x)*log
(c^2*x^2 + 1))/(2*c^2) + (b*e*x^2*atan(c*x)*log(c^2*x^2 + 1))/2
```

3.1290 $\int (a+b\mathbf{ArcTan}(cx)) \left(d+e\log\left(1+c^2x^2\right)\right) dx$

Optimal. Leaf size=100

$$-2aex-2bex\text{ArcTan}(cx)+\frac{e(a+b\text{ArcTan}(cx))^2}{bc}+\frac{be\log\left(1+c^2x^2\right)}{c}+x(a+b\text{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)$$

[Out] -2*a*e*x-2*b*e*x*arctan(c*x)+e*(a+b*arctan(c*x))^2/b/c+b*e*ln(c^2*x^2+1)/c+x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))-1/4*b*(d+e*ln(c^2*x^2+1))^2/c/e

Rubi [A]
time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5129, 2525, 2437, 2338, 5036, 4930, 266, 5004}

$$x(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{e(a+b\text{ArcTan}(cx))^2}{bc}-2aex-2bex\text{ArcTan}(cx)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}+\frac{be\log\left(c^2x^2+1\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]

[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (e*(a + b*ArcTan[c*x])^2)/(b*c) + (b*e*Log[1 + c^2*x^2])/c + x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]) - (b*(d + e*Log[1 + c^2*x^2])^2)/(4*c*e)

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
```

```
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5129

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*
e*g, Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)-(bc)\int \frac{x(d+\ldots}{}$$

$$= x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{1}{2}(bc)\text{Subst}\Bigg(\ldots$$

$$= -2aex+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+x\left(a+b\tan^{-1}(cx)\right)\left(d+e\ldots\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+x\left(a+b\ldots\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{e\left(a+b\tan^{-1}(cx)\right)^2}{bc}+\frac{be\log\left(1\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.02, size = 138, normalized size = 1.38

$$adx - 2aex + \frac{2ae\text{ArcTan}(cx)}{c} + bdx\text{ArcTan}(cx) - 2bex\text{ArcTan}(cx) + \frac{be\text{ArcTan}(cx)^2}{c} - \frac{bd\log(1+c^2x^2)}{2c} + \frac{be\log(1+c^2x^2)}{c} + aex\log(1+c^2x^2) + bex\text{ArcTan}(cx)\log(1+c^2x^2) - \frac{be\log^2(1+c^2x^2)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*ArcTan[c*x])/c + b*d*x*ArcTan[c*x] - 2*b*e*x*ArcTa
n[c*x] + (b*e*ArcTan[c*x]^2)/c - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1
+ c^2*x^2])/c + a*e*x*Log[1 + c^2*x^2] + b*e*x*ArcTan[c*x]*Log[1 + c^2*x^2]
 - (b*e*Log[1 + c^2*x^2]^2)/(4*c)
```

Maple [A]
time = 1.99, size = 173, normalized size = 1.73

method	result
default	$adx + bd\arctan(cx)x - \frac{bd\ln(c^2x^2+1)}{2c} + \frac{b\arctan(cx)^2e}{c} - \frac{be\ln\left(\frac{2}{\frac{-c^2x^2+1}{c^2x^2+1}+1}\right)^2}{4c} - 2bex\arctan(cx) + be\,\text{arc}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+b*d*arctan(c*x)*x-1/2*b/c*d*ln(c^2*x^2+1)+b/c*arctan(c*x)^2*e-1/4*b/c
*e*ln(2/((-c^2*x^2+1)/(c^2*x^2+1)+1))^2-2*b*e*x*arctan(c*x)+b*e*arctan(c*x)
*x*ln(2/((-c^2*x^2+1)/(c^2*x^2+1)+1))+b*e*ln(c^2*x^2+1)/c+e*a*x*ln(c^2*x^2+
1)-2*a*e*x+2*e*a/c*arctan(c*x)
```

Maxima [A]
time = 0.49, size = 156, normalized size = 1.56

$$-\left(2\,c^2\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right) - x\log(c^2x^2+1)\right)b\arctan(cx)\,e + adx - \left(2\,c^2\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right) - x\log(c^2x^2+1)\right)ae + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd}{2\,c} - \frac{\left(4\arctan(cx)^2 + \log(c^2x^2+1)^2 - 4\log(c^2x^2+1)\right)be}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*b*arctan(c*x)*e + a
*d*x - (2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*a*e + 1/2*(2*
c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c - 1/4*(4*arctan(c*x)^2 + log(c^2*
x^2 + 1)^2 - 4*log(c^2*x^2 + 1))*b*e/c
```

Fricas [A]
time = 2.50, size = 109, normalized size = 1.09

$$\frac{4\,acdx - 8\,acxe + 4\,b\arctan(cx)^2\,e - be\log(c^2x^2+1)^2 + 4\,(bcdx - 2\,(bcx-a)e)\arctan(cx) + 2\,(2\,bcx\arctan(cx)\,e - bd + 2\,(acx+b)e)\log(c^2x^2+1)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*c*d*x - 8*a*c*x*e + 4*b*arctan(c*x)^2*e - b*e*log(c^2*x^2 + 1)^2 +
 4*(b*c*d*x - 2*(b*c*x - a)*e)*arctan(c*x) + 2*(2*b*c*x*arctan(c*x)*e - b*d
 + 2*(a*c*x + b)*e)*log(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.39, size = 148, normalized size = 1.48

$$\begin{cases} adx + aex\log\left(c^2x^2+1\right) - 2aex + \frac{2ae\operatorname{atan}(cx)}{c} + bdx\operatorname{atan}(cx) + bex\log\left(c^2x^2+1\right)\operatorname{atan}(cx) - 2bex\operatorname{atan}(cx) - \frac{bd\log\left(c^2x^2+1\right)}{2c} - \frac{be\log\left(c^2x^2+1\right)^2}{4c} + \frac{be\log\left(c^2x^2+1\right)}{c} + \frac{be\operatorname{atan}^2(cx)}{c} & \text{for } c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atan(c*x)/c +
 b*d*x*atan(c*x) + b*e*x*log(c**2*x**2 + 1)*atan(c*x) - 2*b*e*x*atan(c*x) -
 b*d*log(c**2*x**2 + 1)/(2*c) - b*e*log(c**2*x**2 + 1)**2/(4*c) + b*e*log(c
**2*x**2 + 1)/c + b*e*atan(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.99, size = 134, normalized size = 1.34

$$a\,d\,x - 2\,a\,e\,x - \frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{4\,c} + b\,d\,x\,\mathrm{atan}(c\,x) - 2\,b\,e\,x\,\mathrm{atan}(c\,x) + a\,e\,x\,\ln\left(c^2\,x^2+1\right) + \frac{2\,a\,e\,\mathrm{atan}(c\,x)}{c} - \frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c} + \frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{c} + \frac{b\,e\,\mathrm{atan}(c\,x)^2}{c} + b\,e\,x\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

```
[Out] a*d*x - 2*a*e*x - (b*e*log(c^2*x^2 + 1)^2)/(4*c) + b*d*x*atan(c*x) - 2*b*e*
x*atan(c*x) + a*e*x*log(c^2*x^2 + 1) + (2*a*e*atan(c*x))/c - (b*d*log(c^2*x
^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/c + (b*e*atan(c*x)^2)/c + b*e*x*ata
n(c*x)*log(c^2*x^2 + 1)
```

3.1291 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x}\,dx$

Optimal. Leaf size=282

$$ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibe(\log(1-icx)$$

```
[Out] a*d*ln(x)+1/2*I*b*e*ln(I*c*x)*ln(1-I*c*x)^2-1/2*I*b*e*ln(-I*c*x)*ln(1+I*c*x
)^2+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*e*(ln(1-I*c*x)+ln(1+I*c*x)-ln(c^2*x
^2+1))*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)+1/2*I*b*e*(ln(1-I*c*x)+
ln(1+I*c*x)-ln(c^2*x^2+1))*polylog(2,I*c*x)-1/2*a*e*polylog(2,-c^2*x^2)+I*b
*e*ln(1-I*c*x)*polylog(2,1-I*c*x)-I*b*e*ln(1+I*c*x)*polylog(2,1+I*c*x)-I*b*
e*polylog(3,1-I*c*x)+I*b*e*polylog(3,1+I*c*x)
```

Rubi [A]
time = 0.24, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5135, 4940, 2438, 5133, 5131, 2443, 2481, 2421, 6724}

$$-\frac{1}{2}ae\text{Li}_2(-c^2x^2)+ad\log(x)-\frac{1}{2}ibe\text{Li}_2(-icx)\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)+\frac{1}{2}ibe\text{Li}_2(icx)\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)+\frac{1}{2}ibd\text{Li}_2(-icx)-\frac{1}{2}ibd\text{Li}_2(icx)-ibe\text{Li}_3(1-icx)+ibe\text{Li}_3(icx+1)+ibe\text{Li}_2(1-icx)\log(1-icx)-ibe\text{Li}_2(icx+1)\log(1+icx)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (I/2)*b*e*Log[I*c*x]*Log[1 - I*c*x]^2 - (I/2)*b*e*Log[(-I)*c*x
]*Log[1 + I*c*x]^2 + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*e*(Log[1 - I*
c*x] + Log[1 + I*c*x] - Log[1 + c^2*x^2])*PolyLog[2, (-I)*c*x] - (I/2)*b*d*
PolyLog[2, I*c*x] + (I/2)*b*e*(Log[1 - I*c*x] + Log[1 + I*c*x] - Log[1 + c^
2*x^2])*PolyLog[2, I*c*x] - (a*e*PolyLog[2, -(c^2*x^2)])/2 + I*b*e*Log[1 -
I*c*x]*PolyLog[2, 1 - I*c*x] - I*b*e*Log[1 + I*c*x]*PolyLog[2, 1 + I*c*x] -
 I*b*e*PolyLog[3, 1 - I*c*x] + I*b*e*PolyLog[3, 1 + I*c*x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5131

```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist[
Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x], Int[ArcTan[c*x]/x, x], x]
 + (Dist[I/2, Int[Log[1 - I*c*x]^2/x, x], x] - Dist[I/2, Int[Log[1 + I*c*x]
^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

Rule 5133

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTan[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_S
ymbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[Log[f + g*x^2]
*(ArcTan[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 5135

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcTan[c*x])/x, x], x] + Dist[
e, Int[Log[f + g*x^2]*((a + b*ArcTan[c*x])/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, g}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x}\,dx = d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx + e\int \frac{(a+b\tan^{-1}(cx))\log(1+c^2x^2)}{x}$$
$$= ad\log(x) + \frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx - \frac{1}{2}(ibd)\int \frac{\log(1+icx)}{x}$$
$$= ad\log(x) + \frac{1}{2}ibd\mathrm{Li}_2(-icx) - \frac{1}{2}ibd\mathrm{Li}_2(icx) - \frac{1}{2}ae\mathrm{Li}_2(-c^2x^2)$$
$$= ad\log(x) + \frac{1}{2}ibe\log(icx)\log^2(1-icx) - \frac{1}{2}ibe\log(-icx)\log^2$$
$$= ad\log(x) + \frac{1}{2}ibe\log(icx)\log^2(1-icx) - \frac{1}{2}ibe\log(-icx)\log^2$$
$$= ad\log(x) + \frac{1}{2}ibe\log(icx)\log^2(1-icx) - \frac{1}{2}ibe\log(-icx)\log^2$$
$$= ad\log(x) + \frac{1}{2}ibe\log(icx)\log^2(1-icx) - \frac{1}{2}ibe\log(-icx)\log^2$$

Mathematica [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\mathrm{ArcTan}(cx))\,(d+e\log(1+c^2x^2))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
[Out] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 51.53, size = 6931, normalized size = 24.58

method	result	size
risch	Expression too large to display	6931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*arctan(c*x)*e + a*e)*log
(c^2*x^2 + 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x, x)
```

3.1292 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x^2}\,dx$

Optimal. Leaf size=100

$$\frac{ce(a+b\text{ArcTan}(cx))^2}{b}-\frac{(a+b\text{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}+\frac{1}{2}bc\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)$$

```
[Out] c*e*(a+b*arctan(c*x))^2/b-(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x+1/2*b*c*(
d+e*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(c^2*x^2+1))
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5137, 2525, 2458, 2379, 2438, 5004}

$$-\frac{(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}+\frac{ce(a+b\text{ArcTan}(cx))^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{c^2x^2+1}\right)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{1}{2}bce\text{Li}_2\left(\frac{1}{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

```
[Out] (c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2
]))/x + (b*c*(d + e*Log[1 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/2 - (b*c
*e*PolyLog[2, (1 + c^2*x^2)^(-1)])/2
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x^2}\,dx = -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x} + (bc)\int \frac{d+e\log}{x\,(1}$$

$$= \frac{ce(a+b\tan^{-1}(cx))^2}{b} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2}{x}$$

$$= \frac{ce(a+b\tan^{-1}(cx))^2}{b} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2}{x}$$

$$= \frac{ce(a+b\tan^{-1}(cx))^2}{b} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2}{x}$$

$$= \frac{ce(a+b\tan^{-1}(cx))^2}{b} + bcd\log(x) - \frac{(a+b\tan^{-1}(cx))\,(d+}{x}$$

$$= \frac{ce(a+b\tan^{-1}(cx))^2}{b} + bcd\log(x) - \frac{(a+b\tan^{-1}(cx))\,(d+}{x}$$

Mathematica [A]
time = 0.07, size = 111, normalized size = 1.11

$$\frac{ce(a+b\text{ArcTan}(cx))^2}{b} - \frac{(a+b\text{ArcTan}(cx))\,(d+e\log(1+c^2x^2))}{x} + bc\left(-\frac{(d+e\log(1+c^2x^2))\,(d-2e\log(-c^2x^2)+e\log(1+c^2x^2))}{4e} + \frac{1}{2}e\text{PolyLog}(2,1+c^2x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

```
[Out] (c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2
]))/x + b*c*(-1/4*((d + e*Log[1 + c^2*x^2])*(d - 2*e*Log[-(c^2*x^2)] + e*Lo
g[1 + c^2*x^2]))/e + (e*PolyLog[2, 1 + c^2*x^2])/2)
```

Maple [F]
time = 15.90, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*c*arctan(
c*x) - log(c^2*x^2 + 1)/x)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 + 1)
/x^2, x) - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x^2, x)
```

Sympy [A]
time = 95.40, size = 160, normalized size = 1.60

$$-\frac{ad}{x}+\frac{2ae\operatorname{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{\sqrt{\frac{1}{c^2}}}-\frac{ae\log\left(c^2x^2+1\right)}{x}-bc^3e\left(\begin{cases}0 & \text{for } c^2=0\\ \frac{\log\left(c^2x^2+1\right)^2}{4c^2} & \text{otherwise}\end{cases}\right)+4bc^2e\left(\begin{cases}0 & \text{for } c=0\\ \frac{\operatorname{atan}^2(cx)}{4c} & \text{otherwise}\end{cases}\right)-\frac{bcd\log\left(c^2+\frac{1}{x^2}\right)}{2}-\frac{bce\operatorname{Li}_2\left(c^2x^2e^{i\pi}\right)}{2}-\frac{bd\operatorname{atan}(cx)}{x}-\frac{be\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**2,x)
```

```
[Out] -a*d/x + 2*a*e*atan(x/sqrt(c**(-2)))/sqrt(c**(-2)) - a*e*log(c**2*x**2 + 1)
/x - b*c**3*e*Piecewise((0, Eq(c**2, 0)), (log(c**2*x**2 + 1)**2/(4*c**2),
True)) + 4*b*c**2*e*Piecewise((0, Eq(c, 0)), (atan(c*x)**2/(4*c), True)) -
b*c*d*log(c**2 + x**(-2))/2 - b*c*e*polylog(2, c**2*x**2*exp_polar(I*pi))/2
 - b*d*atan(c*x)/x - b*e*log(c**2*x**2 + 1)*atan(c*x)/x
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2, x)
```

3.1293 $\int \frac{(a+b\text{ArcTan}(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=154

$$bc^2e\text{ArcTan}(cx)+ac^2e\log(x)-\frac{1}{2}ac^2e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log\left(1+c^2x^2\right))}{2x}-\frac{1}{2}bc^2\text{ArcTan}(cx)\left(d+e\log\left(1-\right.\right.$$

```
[Out] b*c^2*e*arctan(c*x)+a*c^2*e*ln(x)-1/2*a*c^2*e*ln(c^2*x^2+1)-1/2*b*c*(d+e*ln
(c^2*x^2+1))/x-1/2*b*c^2*arctan(c*x)*(d+e*ln(c^2*x^2+1))-1/2*(a+b*arctan(c*
x))*(d+e*ln(c^2*x^2+1))/x^2+1/2*I*b*c^2*e*polylog(2,-I*c*x)-1/2*I*b*c^2*e*p
olylog(2,I*c*x)
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4946, 331, 209, 5141, 815, 649, 266, 4940, 2438}

$$-\frac{(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-\frac{1}{2}ac^2e\log\left(c^2x^2+1\right)+ac^2e\log(x)-\frac{1}{2}bc^2\text{ArcTan}(cx)\left(e\log\left(c^2x^2+1\right)+d\right)+bc^2e\text{ArcTan}(cx)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{2x}+\frac{1}{2}ibc^2e\text{Li}_2(-icx)-\frac{1}{2}ibc^2e\text{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

```
[Out] b*c^2*e*ArcTan[c*x] + a*c^2*e*Log[x] - (a*c^2*e*Log[1 + c^2*x^2])/2 - (b*c*
(d + e*Log[1 + c^2*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2
]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(2*x^2) + (I/2)*b*c^
2*e*PolyLog[2, (-I)*c*x] - (I/2)*b*c^2*e*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x^3}\,dx &= -\frac{bc(d+e\log(1+c^2x^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\,\bigl(d+e\log(1+c^2 \\
&= -\frac{bc(d+e\log(1+c^2x^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\,\bigl(d+e\log(1+c^2 \\
&= -\frac{bc(d+e\log(1+c^2x^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\,\bigl(d+e\log(1+c^2 \\
&= ac^2e\log(x) - \frac{bc(d+e\log(1+c^2x^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\,\bigl(d+ \\
&= ac^2e\log(x) - \frac{bc(d+e\log(1+c^2x^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\,\bigl(d+ \\
&= bc^2e\tan^{-1}(cx) + ac^2e\log(x) - \frac{1}{2}ac^2e\log\left(1+c^2x^2\right) - \frac{bc(d+
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 189, normalized size = 1.23

$$-\frac{ad+bcdx+bd\text{ArcTan}(cx)+bc^2dx^2\text{ArcTan}(cx)-2bc^2ex^2\text{ArcTan}(cx)-2ac^2ex^2\log(x)+ae\log\left(1+c^2x^2\right)+bcex\log\left(1+c^2x^2\right)+ac^2ex^2\log\left(1+c^2x^2\right)+be\text{ArcTan}(cx)\log\left(1+c^2x^2\right)+bc^2ex^2\text{ArcTan}(cx)\log\left(1+c^2x^2\right)-ibc^2ex^2\text{PolyLog}(2,-icx)+ibc^2ex^2\text{PolyLog}(2,icx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

```
[Out] -1/2*(a*d + b*c*d*x + b*d*ArcTan[c*x] + b*c^2*d*x^2*ArcTan[c*x] - 2*b*c^2*e
*x^2*ArcTan[c*x] - 2*a*c^2*e*x^2*Log[x] + a*e*Log[1 + c^2*x^2] + b*c*e*x*Lo
g[1 + c^2*x^2] + a*c^2*e*x^2*Log[1 + c^2*x^2] + b*e*ArcTan[c*x]*Log[1 + c^2
*x^2] + b*c^2*e*x^2*ArcTan[c*x]*Log[1 + c^2*x^2] - I*b*c^2*e*x^2*PolyLog[2,
 (-I)*c*x] + I*b*c^2*e*x^2*PolyLog[2, I*c*x])/x^2
```

Maple [F]
time = 74.66, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(c^2*(log(c^2*x^
2 + 1) - log(x^2)) + log(c^2*x^2 + 1)/x^2)*a*e + 1/2*(4*c^4*x^2*integrate(1
/2*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*c^2*x^2*arctan(c*x) + 4*c^2*x^2*inte
grate(1/2*arctan(c*x)/(c^2*x^3 + x), x) - (c*x + (c^2*x^2 + 1)*arctan(c*x))
*log(c^2*x^2 + 1))*b*e/x^2 - 1/2*a*d/x^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx)) (d + e \log(c^2x^2 + 1))}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atan}(c\,x))\,(d + e\,\ln(c^2\,x^2 + 1))}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3, x)
```

3.1294 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{2c^2e(a+b\text{ArcTan}(cx))}{3x}-\frac{c^3e(a+b\text{ArcTan}(cx))^2}{3b}+bc^3e\log(x)-\frac{1}{3}bc^3e\log\left(1+c^2x^2\right)-\frac{bc(1+c^2x^2)\left(d+e\log\right.}{6x^2}$$

```
[Out] -2/3*c^2*e*(a+b*arctan(c*x))/x-1/3*c^3*e*(a+b*arctan(c*x))^2/b+b*c^3*e*ln(x
)-1/3*b*c^3*e*ln(c^2*x^2+1)-1/6*b*c*(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/x^2-1/3
*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3-1/6*b*c^3*(d+e*ln(c^2*x^2+1))*ln
(1-1/(c^2*x^2+1))+1/6*b*c^3*e*polylog(2,1/(c^2*x^2+1))
```

Rubi [A]
time = 0.27, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5137, 2525, 2458, 2389, 2379, 2438, 2351, 31, 5038, 4946, 272, 36, 29, 5004}

$$-\frac{c^3e(a+b\text{ArcTan}(cx))^2}{3b}-\frac{(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}-\frac{2c^2e(a+b\text{ArcTan}(cx))}{3x}+bc^3e\log(x)-\frac{bc(c^2x^2+1)\left(e\log\left(c^2x^2+1\right)+d\right)}{6x^2}-\frac{1}{6}bc^3\log\left(1-\frac{1}{c^2x^2+1}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{1}{6}bc^3e\text{Li}_2\left(\frac{1}{c^2x^2+1}\right)-\frac{1}{3}bc^3e\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

```
[Out] (-2*c^2*e*(a + b*ArcTan[c*x]))/(3*x) - (c^3*e*(a + b*ArcTan[c*x])^2)/(3*b)
+ b*c^3*e*Log[x] - (b*c^3*e*Log[1 + c^2*x^2])/3 - (b*c*(1 + c^2*x^2)*(d + e
*Log[1 + c^2*x^2]))/(6*x^2) - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2])
)/(3*x^3) - (b*c^3*(d + e*Log[1 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/6
+ (b*c^3*e*PolyLog[2, (1 + c^2*x^2)^(-1)])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x^4}\,dx = -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{3x^3} + \frac{1}{3}(bc)\int \frac{d+e\,\mathrm{l}}{x^3\,(}$$

$$= -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{3x^3} + \frac{1}{6}(bc)\mathrm{Subst}\Bigl(\int$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{3x} - \frac{c^3e(a+b\tan^{-1}(cx))^2}{3b} - \frac{(a+b\,\mathrm{t}}{}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{3x} - \frac{c^3e(a+b\tan^{-1}(cx))^2}{3b} - \frac{(a+b\,\mathrm{t}}{}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{3x} - \frac{c^3e(a+b\tan^{-1}(cx))^2}{3b} - \frac{bc(1+}{}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{3x} - \frac{c^3e(a+b\tan^{-1}(cx))^2}{3b} - \frac{1}{3}bc^3d\,\mathrm{lo}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{3x} - \frac{c^3e(a+b\tan^{-1}(cx))^2}{3b} - \frac{1}{3}bc^3d\,\mathrm{lo}$$

Mathematica [A]
time = 0.12, size = 181, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{8c^2e(a+b\text{ArcTan}(cx))}{x} - \frac{4c^3e(a+b\text{ArcTan}(cx))^2}{b} + 6bc^3e(2\log(x)-\log(1+c^2x^2)) - \frac{2bc(d+e\log(1+c^2x^2))}{x^2} - \frac{4(a+b\text{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x^3} + \frac{bc^3(d+e\log(1+c^2x^2))^2}{e} - 2bc^3(\log(-c^2x^2)(d+e\log(1+c^2x^2)) + e\text{PolyLog}(2,1+c^2x^2))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

```
[Out] ((-8*c^2*e*(a + b*ArcTan[c*x]))/x - (4*c^3*e*(a + b*ArcTan[c*x])^2)/b + 6*b
*c^3*e*(2*Log[x] - Log[1 + c^2*x^2]) - (2*b*c*(d + e*Log[1 + c^2*x^2]))/x^2
 - (4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*Log
[1 + c^2*x^2])^2)/e - 2*b*c^3*(Log[-(c^2*x^2)]*(d + e*Log[1 + c^2*x^2]) + e
*PolyLog[2, 1 + c^2*x^2]))/12
```

Maple [F]
time = 20.84, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
*d - 1/3*(2*(c*arctan(c*x) + 1/x)*c^2 + log(c^2*x^2 + 1)/x^3)*a*e + b*e*int
egrate(arctan(c*x)*log(c^2*x^2 + 1)/x^4, x) - 1/3*a*d/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x^4, x)
```

Sympy [A]
time = 47.91, size = 428, normalized size = 2.26

$$-\frac{2ac^2e\,\mathrm{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{3\sqrt{\frac{1}{c^2}}}-\frac{2ac^2e}{3x}-\frac{ad}{3x^3}-\frac{ae\log\left(c^2x^2+1\right)}{3x^3}-2bc^7e\left(\begin{cases}\frac{x^2}{12c^2}-\frac{\log\left(c^2x^2+1\right)}{12c^4} & \text{for } c=0\\ \frac{\log\left(c^2x^2+1\right)^2}{24c^4} & \text{otherwise}\end{cases}\right)+\frac{bc^5d\left(\begin{cases}x^2 & \text{for } c^2=0\\ \frac{\log\left(c^2x^2+1\right)}{c^2} & \text{otherwise}\end{cases}\right)}{6}+\frac{bc^5e\left(\begin{cases}x^2 & \text{for } c^2=0\\ \frac{\log\left(c^2x^2+1\right)}{c^2} & \text{otherwise}\end{cases}\right)\log\left(c^2x^2+1\right)}{6}-\frac{bc^3d\log\left(x^2\right)}{6}+\frac{bc^3e\log\left(x\right)}{3}-\frac{bc^3e\log\left(c^2x^2+1\right)}{6}-\frac{bc^3e\log\left(6c^2\sqrt{\frac{1}{c^2}}+\frac{6\sqrt{\frac{1}{c^2}}}{x^2}\right)}{3}+\frac{bc^3e\,\mathrm{atan}^2\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{3}+\frac{bc^3e\,\mathrm{Li}_2\left(c^2x^2e^{i\pi}\right)}{6}-\frac{2bc^2e\,\mathrm{atan}\left(cx\right)\mathrm{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{3\sqrt{\frac{1}{c^2}}}-\frac{2bc^2e\,\mathrm{atan}\left(cx\right)}{3x}-\frac{bcd}{6x^2}-\frac{bce\log\left(c^2x^2+1\right)}{6x^2}-\frac{bd\,\mathrm{atan}\left(cx\right)}{3x^3}-\frac{be\log\left(c^2x^2+1\right)\mathrm{atan}\left(cx\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**4,x)
```

```
[Out] -2*a*c**2*e*atan(x/sqrt(c**(-2)))/(3*sqrt(c**(-2))) - 2*a*c**2*e/(3*x) - a*
d/(3*x**3) - a*e*log(c**2*x**2 + 1)/(3*x**3) - 2*b*c**7*e*Piecewise((x**2/(
12*c**2) - log(c**2*x**2 + 1)/(12*c**4), Eq(c, 0)), (log(c**2*x**2 + 1)**2/
(24*c**4), True)) + b*c**5*d*Piecewise((x**2, Eq(c**2, 0)), (log(c**2*x**2
+ 1)/c**2, True))/6 + b*c**5*e*Piecewise((x**2, Eq(c**2, 0)), (log(c**2*x**
2 + 1)/c**2, True))*log(c**2*x**2 + 1)/6 - b*c**3*d*log(x**2)/6 + b*c**3*e*
log(x)/3 - b*c**3*e*log(c**2*x**2 + 1)/6 - b*c**3*e*log(6*c**2*sqrt(c**(-2)
) + 6*sqrt(c**(-2))/x**2)/3 + b*c**3*e*atan(x/sqrt(c**(-2)))**2/3 + b*c**3*
e*polylog(2, c**2*x**2*exp_polar(I*pi))/6 - 2*b*c**2*e*atan(c*x)*atan(x/sqr
t(c**(-2)))/(3*sqrt(c**(-2))) - 2*b*c**2*e*atan(c*x)/(3*x) - b*c*d/(6*x**2)
 - b*c*e*log(c**2*x**2 + 1)/(6*x**2) - b*d*atan(c*x)/(3*x**3) - b*e*log(c**
2*x**2 + 1)*atan(c*x)/(3*x**3)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4,x)
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4, x)
```

3.1295 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x^5}\,dx$

Optimal. Leaf size=225

$$-\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{11}{12}bc^4e\text{ArcTan}(cx)-\frac{bc^2e\text{ArcTan}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)+\frac{1}{4}ac^4e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log(1+}{12x^3}$$

```
[Out] -1/4*a*c^2*e/x^2-5/12*b*c^3*e/x-11/12*b*c^4*e*arctan(c*x)-1/4*b*c^2*e*arcta
n(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/4*a*c^4*e*ln(c^2*x^2+1)-1/12*b*c*(d+e*ln(c^2
*x^2+1))/x^3+1/4*b*c^3*(d+e*ln(c^2*x^2+1))/x+1/4*b*c^4*arctan(c*x)*(d+e*ln(
c^2*x^2+1))-1/4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4-1/4*I*b*c^4*e*pol
ylog(2,-I*c*x)+1/4*I*b*c^4*e*polylog(2,I*c*x)
```

Rubi [A]
time = 0.18, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4946, 331, 209, 5141, 1816, 649, 266, 5100, 4940, 2438}

$$-\frac{(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}-\frac{1}{2}ac^4e\log(x)-\frac{ac^2e}{4x^2}+\frac{1}{4}ac^4e\log\left(c^2x^2+1\right)-\frac{11}{12}bc^4e\text{ArcTan}(cx)-\frac{bc^2e\text{ArcTan}(cx)}{4x^2}+\frac{1}{4}bc^4\text{ArcTan}(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{1}{4}ibc^4e\text{Li}_2(-icx)+\frac{1}{4}ibc^4e\text{Li}_2(icx)-\frac{5bc^3e}{12x}-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{12x^3}+\frac{bc^3\left(e\log\left(c^2x^2+1\right)+d\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

```
[Out] -1/4*(a*c^2*e)/x^2 - (5*b*c^3*e)/(12*x) - (11*b*c^4*e*ArcTan[c*x])/12 - (b*
c^2*e*ArcTan[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + (a*c^4*e*Log[1 + c^2*x^2]
)/4 - (b*c*(d + e*Log[1 + c^2*x^2]))/(12*x^3) + (b*c^3*(d + e*Log[1 + c^2*x
^2]))/(4*x) + (b*c^4*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2]))/4 - ((a + b*ArcT
an[c*x])*(d + e*Log[1 + c^2*x^2]))/(4*x^4) - (I/4)*b*c^4*e*PolyLog[2, (-I)*
c*x] + (I/4)*b*c^4*e*PolyLog[2, I*c*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5100

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
```

```
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx &= -\frac{bc(d+e\log(1+c^2x^2))}{12x^3}+\frac{bc^3(d+e\log(1+c^2x^2))}{4x}+\frac{1}{4}bc^4\\
&= -\frac{bc(d+e\log(1+c^2x^2))}{12x^3}+\frac{bc^3(d+e\log(1+c^2x^2))}{4x}+\frac{1}{4}bc^4\\
&= -\frac{bc(d+e\log(1+c^2x^2))}{12x^3}+\frac{bc^3(d+e\log(1+c^2x^2))}{4x}+\frac{1}{4}bc^4\\
&= -\frac{ac^2e}{4x^2}-\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)-\frac{bc(d+e\log(1+c^2x^2))}{12x^3}+\frac{bc}{}\\
&= -\frac{ac^2e}{4x^2}-\frac{bc^3e}{6x}-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)-\frac{bc(d+e\log}{1}\\
&= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{2}{3}bc^4e\tan^{-1}(cx)-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4e\\
&= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{11}{12}bc^4e\tan^{-1}(cx)-\frac{bc^2e\tan^{-1}(cx)}{4x^2}-\frac{1}{2}ac^4
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 260, normalized size = 1.16

$$-\frac{3ad+bcdx+3ac^2ex^2-3bc^3dx^3+5bc^3ex^3+3bd\mathrm{ArcTan}(cx)+3bc^2ex^2\mathrm{ArcTan}(cx)-3bc^4dx^4\mathrm{ArcTan}(cx)+11bc^4ex^4\mathrm{ArcTan}(cx)+6ac^4ex^4\log(x)+3ae\log(1+c^2x^2)+bcex\log(1+c^2x^2)-3bc^3ex^3\log(1+c^2x^2)-3ac^4ex^4\log(1+c^2x^2)+3be\mathrm{ArcTan}(cx)\log(1+c^2x^2)-3bc^4ex^4\mathrm{ArcTan}(cx)\log(1+c^2x^2)+3ibc^4ex^4\mathrm{PolyLog}(2,-icx)-3ibc^4ex^4\mathrm{PolyLog}(2,icx)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

```
[Out] -1/12*(3*a*d + b*c*d*x + 3*a*c^2*e*x^2 - 3*b*c^3*d*x^3 + 5*b*c^3*e*x^3 + 3*
b*d*ArcTan[c*x] + 3*b*c^2*e*x^2*ArcTan[c*x] - 3*b*c^4*d*x^4*ArcTan[c*x] + 1
1*b*c^4*e*x^4*ArcTan[c*x] + 6*a*c^4*e*x^4*Log[x] + 3*a*e*Log[1 + c^2*x^2] +
 b*c*e*x*Log[1 + c^2*x^2] - 3*b*c^3*e*x^3*Log[1 + c^2*x^2] - 3*a*c^4*e*x^4*
Log[1 + c^2*x^2] + 3*b*e*ArcTan[c*x]*Log[1 + c^2*x^2] - 3*b*c^4*e*x^4*ArcTa
n[c*x]*Log[1 + c^2*x^2] + (3*I)*b*c^4*e*x^4*PolyLog[2, (-I)*c*x] - (3*I)*b*
c^4*e*x^4*PolyLog[2, I*c*x])/x^4
```

Maple [F]
time = 24.50, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
+ 1/4*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c^2 - log(c^2*x^2 + 1)
/x^4)*a*e - 1/12*(72*c^6*x^4*integrate(1/12*x*arctan(c*x)/(c^2*x^2 + 1), x)
 + 8*c^4*x^4*arctan(c*x) - 72*c^2*x^4*integrate(1/12*arctan(c*x)/(c^2*x^5 +
 x^3), x) + 2*c^3*x^3 - (3*c^3*x^3 - c*x + 3*(c^4*x^4 - 1)*arctan(c*x))*log
(c^2*x^2 + 1))*b*e/x^4 - 1/4*a*d/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}{(cx)})\,(d + e \log{(c^2x^2 + 1)})}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**5, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\left(d + e\,\ln\left(c^2\,x^2 + 1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5, x)
```

3.1296 $\int \frac{(a+b\mathbf{ArcTan}(cx))(d+e\log(1+c^2x^2))}{x^6}\,dx$

Optimal. Leaf size=248

$$-\frac{7bc^3e}{60x^2}-\frac{2c^2e(a+b\text{ArcTan}(cx))}{15x^3}+\frac{2c^4e(a+b\text{ArcTan}(cx))}{5x}+\frac{c^5e(a+b\text{ArcTan}(cx))^2}{5b}-\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e$$

```
[Out] -7/60*b*c^3*e/x^2-2/15*c^2*e*(a+b*arctan(c*x))/x^3+2/5*c^4*e*(a+b*arctan(c*
x))/x+1/5*c^5*e*(a+b*arctan(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(c^
2*x^2+1)-1/20*b*c*(d+e*ln(c^2*x^2+1))/x^4+1/10*b*c^3*(c^2*x^2+1)*(d+e*ln(c^
2*x^2+1))/x^2-1/5*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5+1/10*b*c^5*(d+e
*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(c^2*x^2+1))
```

Rubi [A]

time = 0.40, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5137, 2525, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 5038, 4946, 272, 36, 29, 5004}

$$\frac{c^5e(a+b\text{ArcTan}(cx))^2}{5b}+\frac{2c^4e(a+b\text{ArcTan}(cx))}{5x}-\frac{(a+b\text{ArcTan}(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{2c^2e(a+b\text{ArcTan}(cx))}{15x^3}-\frac{5}{6}bc^5e\log(x)-\frac{7bc^3e}{60x^2}-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{20x^4}+\frac{1}{10}bc^5\log\left(1-\frac{1}{c^2x^2+1}\right)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{1}{10}bc^5e\text{Li}_2\left(\frac{1}{c^2x^2+1}\right)+\frac{19}{60}bc^5e\log\left(c^2x^2+1\right)+\frac{bc^3\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

```
[Out] (-7*b*c^3*e)/(60*x^2) - (2*c^2*e*(a + b*ArcTan[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTan[c*x]))/(5*x) + (c^5*e*(a + b*ArcTan[c*x])^2)/(5*b) - (5*b*c^5*
e*Log[x])/6 + (19*b*c^5*e*Log[1 + c^2*x^2])/60 - (b*c*(d + e*Log[1 + c^2*x^
2]))/(20*x^4) + (b*c^3*(1 + c^2*x^2)*(d + e*Log[1 + c^2*x^2]))/(10*x^2) - (
(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Log[1
 + c^2*x^2])*Log[1 - (1 + c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 + c^2
*x^2)^(-1)])/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{x^6}\,dx = -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{5x^5} + \frac{1}{5}(bc)\int \frac{d+e\log}{x^5\,(1}$$

$$= -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c^2x^2))}{5x^5} + \frac{1}{10}(bc)\text{Subst}\Big(\int$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(1+c}{5x^5}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} + \frac{2c^4e(a+b\tan^{-1}(cx))}{5x} + \frac{c^5e(a+b}{}$$

$$= -\frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} + \frac{2c^4e(a+b\tan^{-1}(cx))}{5x} + \frac{c^5e(a+b}{}$$

$$= -\frac{bc^3e}{15x^2} - \frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} + \frac{2c^4e(a+b\tan^{-1}(cx))}{5x} + $$

$$= -\frac{7bc^3e}{60x^2} - \frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} + \frac{2c^4e(a+b\tan^{-1}(cx))}{5x} + $$

$$= -\frac{7bc^3e}{60x^2} - \frac{2c^2e(a+b\tan^{-1}(cx))}{15x^3} + \frac{2c^4e(a+b\tan^{-1}(cx))}{5x} + $$

Mathematica [A]
time = 0.23, size = 259, normalized size = 1.04

$$\frac{1}{60}\left(-\frac{8c^2e(a+b\text{ArcTan}(cx))}{x^3}+\frac{24c^4e(a+b\text{ArcTan}(cx))}{x}+\frac{12c^5e(a+b\text{ArcTan}(cx))^2}{b}-18bc^5e(2\log(x)-\log(1+c^2x^2))+7bc^3e\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log(1+c^2x^2)\right)-\frac{3bc(d+e\log(1+c^2x^2))}{x^4}+\frac{6bc^3(d+e\log(1+c^2x^2))}{x^2}-\frac{12(a+b\text{ArcTan}(cx))\,(d+e\log(1+c^2x^2))}{x^5}+6bc^5\log(-c^2x^2)\,(d+e\log(1+c^2x^2))-\frac{3bc^5(d+e\log(1+c^2x^2))^2}{e}+6bc^5e\text{PolyLog}(2,1+c^2x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

```
[Out] ((-8*c^2*e*(a + b*ArcTan[c*x]))/x^3 + (24*c^4*e*(a + b*ArcTan[c*x]))/x + (1
2*c^5*e*(a + b*ArcTan[c*x])^2)/b - 18*b*c^5*e*(2*Log[x] - Log[1 + c^2*x^2])
 + 7*b*c^3*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log[1 + c^2*x^2]) - (3*b*c*(d +
e*Log[1 + c^2*x^2]))/x^4 + (6*b*c^3*(d + e*Log[1 + c^2*x^2]))/x^2 - (12*(a
+ b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5 + 6*b*c^5*Log[-(c^2*x^2)]*(d
 + e*Log[1 + c^2*x^2]) - (3*b*c^5*(d + e*Log[1 + c^2*x^2])^2)/e + 6*b*c^5*e
*PolyLog[2, 1 + c^2*x^2])/60
```

Maple [F]
time = 25.48, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b*d + 1/15*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*
c^2 - 3*log(c^2*x^2 + 1)/x^5)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 +
 1)/x^6, x) - 1/5*a*d/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(c^2*x^2 + 1))
/x^6, x)
```

Sympy [A]
time = 43.59, size = 474, normalized size = 1.91

$$\frac{2ac^4e\operatorname{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{5\sqrt{\frac{1}{c^2}}}+\frac{2ac^4e}{5x}-\frac{2ac^2e}{15x^3}-\frac{ad}{5x^5}-\frac{ae\log\left(c^2x^2+1\right)}{5x^5}+4bc^9e\left(\begin{cases}\frac{x^2}{40c^2}-\frac{\log\left(c^2x^2+1\right)}{40c^4} & \text{for } c=0\\ \frac{\log\left(c^2x^2+1\right)^2}{80c^4} & \text{otherwise}\end{cases}\right)-\frac{bc^7d\left(\begin{cases}x^2 & \text{for } c^2=0\\ \frac{\log\left(c^2x^2+1\right)}{c^2} & \text{otherwise}\end{cases}\right)}{10}-\frac{bc^7e\left(\begin{cases}x^2 & \text{for } c^2=0\\ \frac{\log\left(c^2x^2+1\right)}{c^2} & \text{otherwise}\end{cases}\right)\log\left(c^2x^2+1\right)}{10}+\frac{bc^5d\log\left(x^2\right)}{10}-\frac{5bc^5e\log\left(x\right)}{6}+\frac{5bc^5e\log\left(c^2x^2+1\right)}{12}-\frac{bc^5e\operatorname{atan}^2\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{5}-\frac{bc^5e\operatorname{Li}_2\left(c^2x^2e^{i\pi}\right)}{10}+\frac{2bc^4e\operatorname{atan}\left(cx\right)\operatorname{atan}\left(\frac{x}{\sqrt{\frac{1}{c^2}}}\right)}{5\sqrt{\frac{1}{c^2}}}+\frac{2bc^4e\operatorname{atan}\left(cx\right)}{5x}+\frac{bc^3d}{10x^2}+\frac{bc^3e\log\left(c^2x^2+1\right)}{10x^2}-\frac{7bc^3e}{60x^2}-\frac{2bc^2e\operatorname{atan}\left(cx\right)}{15x^3}-\frac{bcd}{20x^4}-\frac{bce\log\left(c^2x^2+1\right)}{20x^4}-\frac{bd\operatorname{atan}\left(cx\right)}{5x^5}-\frac{be\log\left(c^2x^2+1\right)\operatorname{atan}\left(cx\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**6,x)
```

```
[Out] 2*a*c**4*e*atan(x/sqrt(c**(-2)))/(5*sqrt(c**(-2))) + 2*a*c**4*e/(5*x) - 2*a
*c**2*e/(15*x**3) - a*d/(5*x**5) - a*e*log(c**2*x**2 + 1)/(5*x**5) + 4*b*c*
*9*e*Piecewise((x**2/(40*c**2) - log(c**2*x**2 + 1)/(40*c**4), Eq(c, 0)), (
log(c**2*x**2 + 1)**2/(80*c**4), True)) - b*c**7*d*Piecewise((x**2, Eq(c**2
, 0)), (log(c**2*x**2 + 1)/c**2, True))/10 - b*c**7*e*Piecewise((x**2, Eq(c
**2, 0)), (log(c**2*x**2 + 1)/c**2, True))*log(c**2*x**2 + 1)/10 + b*c**5*d
*log(x**2)/10 - 5*b*c**5*e*log(x)/6 + 5*b*c**5*e*log(c**2*x**2 + 1)/12 - b*
c**5*e*atan(x/sqrt(c**(-2)))**2/5 - b*c**5*e*polylog(2, c**2*x**2*exp_polar
(I*pi))/10 + 2*b*c**4*e*atan(c*x)*atan(x/sqrt(c**(-2)))/(5*sqrt(c**(-2))) +
```

```
 2*b*c**4*e*atan(c*x)/(5*x) + b*c**3*d/(10*x**2) + b*c**3*e*log(c**2*x**2 +
 1)/(10*x**2) - 7*b*c**3*e/(60*x**2) - 2*b*c**2*e*atan(c*x)/(15*x**3) - b*c
*d/(20*x**4) - b*c*e*log(c**2*x**2 + 1)/(20*x**4) - b*d*atan(c*x)/(5*x**5)
- b*e*log(c**2*x**2 + 1)*atan(c*x)/(5*x**5)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6, x)
```

3.1297 $\int x(a+b\text{ArcTan}(cx)) \left(d+e\log \left(f+gx^2\right)\right) \, dx$

Optimal. Leaf size=562

$$-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))-\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))-\frac{be\sqrt{f}\,\text{ArcTan}\left(\right.}{c\sqrt{g}}$$

```
[Out] -1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan(
c*x))-1/2*e*x^2*(a+b*arctan(c*x))-b*e*(c^2*f-g)*arctan(c*x)*ln(2/(1-I*c*x))
/c^2/g-1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(g*x^2+f)/c^
2/g+1/2*e*(g*x^2+f)*(a+b*arctan(c*x))*ln(g*x^2+f)/g+1/2*b*e*(c^2*f-g)*arcta
n(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)))/c^
2/g+1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/(
c*(-f)^(1/2)+I*g^(1/2)))/c^2/g+1/2*I*b*e*(c^2*f-g)*polylog(2,1-2/(1-I*c*x))
/c^2/g-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)
/(c*(-f)^(1/2)-I*g^(1/2)))/c^2/g-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^
(1/2)+x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/c^2/g-b*e*arctan(x*g^(
1/2)/f^(1/2))*f^(1/2)/c/g^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2504, 2436, 2332, 5139, 327, 209, 2608, 2498, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{1}{2}dx^2(a+b\text{ArcTan}(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\text{ArcTan}(cx))}{2g}-\frac{1}{2}ex^2(a+b\text{ArcTan}(cx))+\frac{b(d-e)\text{ArcTan}(cx)}{2c^2}-\frac{be\text{ArcTan}(cx)(c^2f-g)\log(f+gx^2)}{2c^2g}-\frac{be\text{ArcTan}(cx)(c^2f-g)\log\left(\frac{2}{1-icx}\right)}{c^2g}+\frac{be\text{ArcTan}(cx)(c^2f-g)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{2c^2g}+\frac{be\text{ArcTan}(cx)(c^2f-g)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{(1-icx)\left(c\sqrt{-f}+i\sqrt{g}\right)}\right)}{2c^2g}-\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}+\frac{ibe(c^2f-g)\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2c^2g}-\frac{ibe(c^2f-g)\text{Li}_2\left(1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-i\sqrt{g}\right)(1-icx)}\right)}{4c^2g}-\frac{ibe(c^2f-g)\text{Li}_2\left(1-\frac{2c\left(\sqrt{g}x+\sqrt{-f}\right)}{\left(\sqrt{-f}c+i\sqrt{g}\right)(1-icx)}\right)}{4c^2g}-\frac{bx(d-e)}{2c}-\frac{bex\log(f+gx^2)}{2c}+\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -1/2*(b*(d - e)*x)/c + (b*e*x)/c + (b*(d - e)*ArcTan[c*x])/(2*c^2) + (d*x^2
*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 - (b*e*Sqrt[f]*ArcT
an[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[g]) - (b*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(
1 - I*c*x)])/(c^2*g) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sq
rt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(2*c^2*g) + (b*e*(c^2*f
- g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])
*(1 - I*c*x))])/(2*c^2*g) - (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f - g)
*ArcTan[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTan[c*x])
*Log[f + g*x^2])/(2*g) + ((I/2)*b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x
)])/(c^2*g) - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g
]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(c^2*g) - ((I/4)*b*e*(c^2*f
- g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*
(1 - I*c*x))])/(c^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5139

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Dist[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 +
 c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2
, 0]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e(f+}{}$$

$$= \frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}(cx)\right)+\frac{e(f+}{}$$

$$= -\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tan^{-1}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tan^{-1}(cx\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

$$= -\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\tan^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\,\mathrm{ta}\right.$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1140 vs. $2(562) = 1124$.
time = 5.78, size = 1140, normalized size = 2.03

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (-2*b*c*d*g*x + 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 + 2*b*d*g*A
rcTan[c*x] - 2*b*e*g*ArcTan[c*x] + 2*b*c^2*d*g*x^2*ArcTan[c*x] - 2*b*c^2*e*
g*x^2*ArcTan[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + (
4*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*
g]] - (4*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2
*f*g]] - 4*b*c^2*e*f*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b*e*g*A
rcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)
/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcT
an[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] - 2*b*e*g
*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f +
 (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c
^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f
 + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/
(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f +
 (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c
^2*f - g)] - 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I
)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*e*g*ArcSin
[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*S
qrt[c^2*f*g]))/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[1 + (E^((2*I)*Arc
Tan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]
*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)]
 + 2*a*c^2*e*f*Log[f + g*x^2] - 2*b*c*e*g*x*Log[f + g*x^2] + 2*a*c^2*e*g*x^
2*Log[f + g*x^2] + 2*b*e*g*ArcTan[c*x]*Log[f + g*x^2] + 2*b*c^2*e*g*x^2*Arc
Tan[c*x]*Log[f + g*x^2] + (2*I)*b*e*(c^2*f - g)*PolyLog[2, -E^((2*I)*ArcTan
[c*x])] - I*b*e*(c^2*f - g)*PolyLog[2, (E^((2*I)*ArcTan[c*x])*(-(c^2*f) - g
 + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - I*b*c^2*e*f*PolyLog[2, -((E^((2*I)*ArcT
an[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))] + I*b*e*g*PolyLog[2,
-((E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))])/(4*c
^2*g)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 17.07, size = 21442, normalized size = 38.15

method	result	size
default	Expression too large to display	21442

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/2
*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g - 1/2*(2*c*f*arctan(g*x/sqr
t(f*g)) + (4*c^4*g*integrate(1/2*x^3*arctan(c*x)/(c^2*g*x^2 + c^2*f), x) +
4*c^2*g*integrate(1/2*x*arctan(c*x)/(c^2*g*x^2 + c^2*f), x) - 2*c*x + (c*x
- (c^2*x^2 + 1)*arctan(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)*c^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arctan(c*x) + a*d*x + (b*x*arctan(c*x)*e + a*x*e)*log(g*x^2
+ f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
[Out] int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

3.1298 $\int (a+b\mathbf{ArcTan}(cx)) (d+e\log (f+gx^2)) \, dx$

Optimal. Leaf size=656

$$-2aex-2bex\text{ArcTan}(cx)+\frac{2ae\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{-f}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe}{}$$

[Out] -2*a*e*x-2*b*e*x*arctan(c*x)+b*e*ln(c^2*x^2+1)/c+x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))-1/2*b*ln(-g*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))/c-1/2*b*e*polylog(2,c^2*(g*x^2+f)/(c^2*f-g))/c+1/2*I*b*e*ln(1+I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*ln(1+I*c*x)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*polylog(2,(I-c*x)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*polylog(2,(c*x+I)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*I*b*e*polylog(2,(1-I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*I*b*e*polylog(2,(1+I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/2)/g^(1/2)

Rubi [A]
time = 0.63, antiderivative size = 656, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5129, 2525, 2441, 2440, 2438, 5036, 4930, 266, 5030, 211, 5028, 2456}

$$x(a+b\text{ArcTan}(cx))\left(d+e\log\left(f+gx^2\right)\right)+\frac{2ae\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-2aex-2bex\text{ArcTan}(cx)-\frac{b\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}-\frac{be\text{Li}_2\left(\frac{c^2(gx^2+f)}{c^2f-g}\right)}{2c}+\frac{be\log\left(c^2x^2+1\right)}{c}-\frac{ibe\sqrt{-f}\,\text{Li}_2\left(\frac{\sqrt{g}(i-cx)}{\sqrt{-f}\,c+i\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\,\text{Li}_2\left(\frac{\sqrt{g}(1-icx)}{i\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\,\text{Li}_2\left(\frac{\sqrt{g}(icx+1)}{i\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\,\text{Li}_2\left(\frac{\sqrt{g}(cx+i)}{\sqrt{-f}\,c+i\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]

[Out] -2*a*e*x - 2*b*e*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + (b*e*Log[1 + c^2*x^2])/c + x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]) - (b*Log[-((g*(1 + c^2*x^2))/(c^2*f - g))]*(d + e*Log[f + g*x^2]))/(2*c) - ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[g] + ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g]

```
)])/Sqrt[g] - ((I/2)*b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f
] + I*Sqrt[g])])/Sqrt[g] - (b*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/
(2*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5129

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*
e*g, Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x(d+e\,\mathrm{lo}}{1-}$$

$$= x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\mathrm{Subst}\left(\int\right.$$

$$= -2aex+x\left(a+b\tan^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{b\log\left(\right.}{}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+x\left(a+\right.$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\log}{}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\log}{}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{}}{}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{}}{}$$

$$= -2aex-2bex\tan^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{ibe\sqrt{}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1352 vs. $2(656) = 1312$.

time = 2.89, size = 1352, normalized size = 2.06

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + b*d*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqr
t[f]])/Sqrt[g] - (b*d*Log[1 + c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e*
(x*ArcTan[c*x] - Log[1 + c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*g*(((-Log[(-
I)/c + x] - Log[I/c + x] + Log[1 + c^2*x^2])*Log[f + g*x^2])/(2*g) + (Log[(
-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)] +
 PolyLog[2, (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)])/(2*g) +
 (Log[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c
)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c)])/(2*g)
+ (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)]
 + PolyLog[2, (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g) +
(Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)] + Po
lyLog[2, (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g)))/c - (b*e
*(4*c*x*ArcTan[c*x] + 4*Log[1/Sqrt[1 + c^2*x^2]] + (c^2*f*(4*ArcTan[c*x]*Ar
cTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - 2*ArcCos[(c^2*f + g)/(-(c^2*f) + g)]*ArcT
anh[(c*g*x)/Sqrt[-(c^2*f*g)]] - (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] - (2*I)
*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(-2*c^2*f*(I*g + Sqrt[-(c^2*f*g)])*
(-I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))] - (ArcCos[(c^2*f
+ g)/(-(c^2*f) + g)] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[((2*I)*
c^2*f*(g + I*Sqrt[-(c^2*f*g)])*(I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^
2*f*g)]*x))] + (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] - (2*I)*ArcTanh[Sqrt[-(c
^2*f*g)]/(c*g*x)] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(Sqrt[2]*S
qrt[-(c^2*f*g)])/(E^(I*ArcTan[c*x])*Sqrt[-(c^2*f) + g]*Sqrt[-(c^2*f) - g +
(-(c^2*f) + g)*Cos[2*ArcTan[c*x]]])] + (ArcCos[(c^2*f + g)/(-(c^2*f) + g)]
+ (2*I)*ArcTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^
2*f*g)]])*Log[(Sqrt[2]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*f*g)])/(Sqrt[-(c^2*f) +
 g]*Sqrt[-(c^2*f) - g + (-(c^2*f) + g)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[
2, ((c^2*f + g - (2*I)*Sqrt[-(c^2*f*g)])*(c^2*f + c*Sqrt[-(c^2*f*g)]*x))/((
c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))] + PolyLog[2, ((c^2*f + g + (2*I
)*Sqrt[-(c^2*f*g)])*(c^2*f + c*Sqrt[-(c^2*f*g)]*x))/((c^2*f - g)*(c^2*f - c
*Sqrt[-(c^2*f*g)]*x))])))/Sqrt[-(c^2*f*g)]))/(2*c)
```

Maple [**F**]
time = 14.23, size = 0, normalized size = 0.00

$$\int \left(a+b\arctan\left(cx\right)\right)\left(d+e\ln\left(x^{2}g+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] a*d*x + (2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 +
f))*a*e + b*e*integrate(arctan(c*x)*log(g*x^2 + f), x) + 1/2*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(g*x^2 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int((a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

3.1299 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=102

$$ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}ibd\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\text{PolyLog}(2,icx)+\frac{1}{2}ae\text{PolyLog}\Big(2,1$$

```
[Out] b*e*CannotIntegrate(arctan(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*x^
2/f)*ln(g*x^2+f)+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)+1/2
*a*e*polylog(2,1+g*x^2/f)
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\text{ArcTan}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (I/2)*b*d*PolyLog[2
, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x] + (a*e*PolyLog[2, 1 + (g*x^2)/f])
/2 + b*e*Defer[Int][(ArcTan[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\tan^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx &= d\int \frac{a+b\tan^{-1}(cx)}{x}\,dx+e\int \frac{(a+b\tan^{-1}(cx))\log\left(f+gx\right.}{x}\\
&= ad\log(x)+\frac{1}{2}(ibd)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ibd)\int \frac{\log(1+ic}{x}\\
&= ad\log(x)+\frac{1}{2}ibd\text{Li}_2(-icx)-\frac{1}{2}ibd\text{Li}_2(icx)+\frac{1}{2}(ae)\text{Subst}\Big(\int\\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}ibd\text{Li}_2(-icx\\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}ibd\text{Li}_2(-icx\end{aligned}$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A]
time = 4.72, size = 0, normalized size = 0.00

$$\int \frac{(a + b\arctan(cx))\,(d + e\ln(x^2 g + f))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*arctan(c*x)*e + a*e)*log
(g*x^2 + f))/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(g*x^2 + f))/x
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}(cx))\,(d + e\log(f + gx^2))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))*(d + e*log(f + g*x**2))/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

3.1300 $\int \frac{(a+b\text{ArcTan}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal. Leaf size=672

$$\frac{2ae\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{ibe\sqrt{g}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}\right.}{c\sqrt{-f}}\right.}{2\sqrt{-f}}$$

```
[Out] -(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f
))-1/2*b*c*ln(-g*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polylog
(2,c^2*(g*x^2+f)/(c^2*f-g))+1/2*b*c*e*polylog(2,1+g*x^2/f)-1/2*I*b*e*ln(1+I
*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-I*g^(1/2)))*g^(1/2)/(-f)^(1
/2)+1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/
2)))*g^(1/2)/(-f)^(1/2)-1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)+x*g^(1/2))/(
c*(-f)^(1/2)-I*g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*I*b*e*ln(1+I*c*x)*ln(c*((-f
)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*I*b*e*p
olylog(2,(I-c*x)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*I
*b*e*polylog(2,(c*x+I)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/(-f)^(1/2)
-1/2*I*b*e*polylog(2,(1-I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*g^(1/2)/(-
f)^(1/2)-1/2*I*b*e*polylog(2,(1+I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*g^
(1/2)/(-f)^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5137, 2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438, 5030, 211, 5028, 2456}

$$-\frac{(a+b\text{ArcTan}(cx))(d+e\log(f+gx^2))}{x}+\frac{2ae\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{1}{2}bc\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))-\frac{1}{2}bce\text{Li}_2\left(\frac{c^2(gx^2+f)}{c^2f-g}\right)+\frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+gx^2))+\frac{1}{2}bce\text{Li}_2\left(\frac{gx^2}{f}+1\right)+\frac{ibe\sqrt{g}\,\text{Li}_2\left(\frac{\sqrt{g}\,(i-cx)}{\sqrt{-f}\,c+i\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\,\text{Li}_2\left(\frac{\sqrt{g}\,(1-icx)}{i\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\,\text{Li}_2\left(\frac{\sqrt{g}\,(icx+1)}{i\sqrt{-f}\,c+\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{ibe\sqrt{g}\,\text{Li}_2\left(\frac{\sqrt{g}\,(cx+i)}{\sqrt{-f}\,c+i\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{ibe\sqrt{g}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}}-\frac{ibe\sqrt{g}\,\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - ((I/2)*b*e*Sqrt[g]*Lo
g[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt
[-f] + ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*
Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*Log[1 - I*c*x]*Log[(c
*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/Sqrt[-f] + ((I/2)*b*e*S
qrt[g]*Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g
])])/Sqrt[-f] - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-
((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[-((g*(1 + c^2*x^2))/(c^2*
f - g))]*(d + e*Log[f + g*x^2]))/2 + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]
*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e*Sqrt[g]*PolyLo
g[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-f] - ((I/2)*b*e
```

```
*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])])/Sqrt[-
f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqrt
[g])])/Sqrt[-f] - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/2 + (b*
c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5030

```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTan[c*x]/(d + e*x^2), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 5137

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e*L
og[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m + 2)
*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g},
 x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+gx^2))}{x^2}\,dx = -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+gx^2))}{x} + (bc)\int \frac{d+e\log}{x\,(1+}$$

$$= -\frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+gx^2))}{x} + \frac{1}{2}(bc)\text{Subst}\Bigg(\int$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\,\log(1+icx)\log\left(\frac{c\left(\sqrt{-}\right.}{c\sqrt{-}}\right.}{2\sqrt{-f}}$$

Mathematica [A]
time = 0.72, size = 552, normalized size = 0.82

$$\frac{1}{2}\left(-\frac{2(a+b\text{ArcTan}(cx))\,(d+e\log(f+gx^2))}{x}+\frac{e\sqrt{g}\left(4a\sqrt{-f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)+ib\sqrt{f}\left(\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{g}\,(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)\right)-ib\sqrt{f}\left(\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{g}\,(1-icx)}{ic\sqrt{-f}+\sqrt{g}}\right)\right)-ib\sqrt{f}\left(\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{g}\,(1+icx)}{ic\sqrt{-f}+\sqrt{g}}\right)\right)+ib\sqrt{f}\left(\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{g}\,(i+cx)}{c\sqrt{-f}+i\sqrt{g}}\right)\right)\right)}{\sqrt{-f^2}}+bc\left(\left(\log\left(-\frac{gx^2}{f}\right)-\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)\right)(d+e\log(f+gx^2))-e\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)+e\text{PolyLog}\left(2,1+\frac{gx^2}{f}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] ((-2*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (e*Sqrt[g]*(4*a*Sqrt[-
f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt[-
f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I - c*x))
/(c*Sqrt[-f] + I*Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[-f]
+ Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 - I*c*x))/
(I*c*Sqrt[-f] + Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt[-f] -
 Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(
I*c*Sqrt[-f] + Sqrt[g])]) + I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[-f] -
Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*S
qrt[-f] + I*Sqrt[g])])))/Sqrt[-f^2] + b*c*((Log[-((g*x^2)/f)] - Log[-((g*(1
 + c^2*x^2))/(c^2*f - g))])*(d + e*Log[f + g*x^2]) - e*PolyLog[2, (c^2*(f +
 g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

Maple [F]
time = 7.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(cx\right)\right)\left(d+e\ln\left(x^{2}g+f\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*g*arctan(
g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + b*e*integrate(arctan(c*x
)*log(g*x^2 + f)/x^2, x) - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(g*x^2 + f))/x
^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))\,\left(d + e\,\ln\left(g\,x^2 + f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

3.1301 $\int \frac{(a+b\mathbf{ArcTan}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal. Leaf size=528

$$\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{be(c^2f-g)\,\text{ArcTan}(cx)\log\left(\frac{2}{1-icx}\right)}{f}+\frac{be(c^2f-g)\,\text{ArcTan}(cx)\log\left(\cdots\right)}{2f}$$

```
[Out] a*e*g*ln(x)/f-b*e*(c^2*f-g)*arctan(c*x)*ln(2/(1-I*c*x))/f-1/2*a*e*g*ln(g*x^
2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x-1/2*b*c^2*arctan(c*x)*(d+e*ln(g*x^2+f))-
1/2*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*e*(c^2*f-g)*arctan(c*x)*l
n(2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)))/f+1/2*b*e*
(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)
+I*g^(1/2)))/f+1/2*I*b*e*g*polylog(2,-I*c*x)/f-1/2*I*b*e*g*polylog(2,I*c*x)
/f+1/2*I*b*e*(c^2*f-g)*polylog(2,1-2/(1-I*c*x))/f-1/4*I*b*e*(c^2*f-g)*polyl
og(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)))/f-1/4
*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/(c*(-f)^(
1/2)+I*g^(1/2)))/f+b*c*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 331, 209, 5141, 815, 649, 211, 266, 457, 78, 6857, 4940, 2438, 5048, 4966, 2449, 2352, 2497}

$$-\frac{(a+b\text{ArcTan}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{aeg\log\left(f+gx^2\right)}{2f}+\frac{aeg\log(x)}{f}-\frac{1}{2}bc^2\text{ArcTan}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{be\text{ArcTan}(cx)\left(c^2f-g\right)\log\left(\frac{2}{1-icx}\right)}{f}+\frac{be\text{ArcTan}(cx)\left(c^2f-g\right)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{(1-icx)\left(c\sqrt{-f}-i\sqrt{g}\right)}\right)}{2f}+\frac{be\text{ArcTan}(cx)\left(c^2f-g\right)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{(1-icx)\left(c\sqrt{-f}+i\sqrt{g}\right)}\right)}{2f}+\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{ibe\left(c^2f-g\right)\text{Li}_2\left(1-\frac{2}{1-icx}\right)}{2f}-\frac{ibe\left(c^2f-g\right)\text{Li}_2\left(1-\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\left(c\sqrt{-f}-i\sqrt{g}\right)(1-icx)}\right)}{4f}-\frac{ibe\left(c^2f-g\right)\text{Li}_2\left(1-\frac{2c\left(\sqrt{g}\,x+\sqrt{-f}\right)}{\left(\sqrt{-f}\,c+i\sqrt{g}\right)(1-icx)}\right)}{4f}-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{ibeg\text{Li}_2(-icx)}{2f}-\frac{ibeg\text{Li}_2(icx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f - (b
*e*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])/f + (b*e*(c^2*f - g)*ArcTan[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x)
)])/(2*f) + (b*e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/(
(c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f
) - (b*c*(d + e*Log[f + g*x^2]))/(2*x) - (b*c^2*ArcTan[c*x]*(d + e*Log[f +
g*x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + ((I/2)*
b*e*g*PolyLog[2, (-I)*c*x])/f - ((I/2)*b*e*g*PolyLog[2, I*c*x])/f + ((I/2)*
b*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x)])/f - ((I/4)*b*e*(c^2*f - g)*P
olyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I
*c*x))])/f - ((I/4)*b*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]
*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/f
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
```

```
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5141

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*(u
/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m
] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = -\frac{bc(d+e\log(f+gx^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

$$= -\frac{bc(d+e\log(f+gx^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

$$= -\frac{bc(d+e\log(f+gx^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\log\left(f+gx\right.\right.$$

$$= \frac{aeg\log(x)}{f} - \frac{bc(d+e\log(f+gx^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\right.$$

$$= \frac{aeg\log(x)}{f} - \frac{bc(d+e\log(f+gx^2))}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx)\left(d+e\right.$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{aeg\log(f+gx^2)}{2f} - \cdots$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be(c^2f-g)\tan^{-1}(cx\cdots}{f}$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be(c^2f-g)\tan^{-1}(cx\cdots}{f}$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} - \frac{be(c^2f-g)\tan^{-1}(cx\cdots}{f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1217 vs. 2(528) = 1056.
time = 4.73, size = 1217, normalized size = 2.30

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -1/4*(2*a*d*f + 2*b*c*d*f*x + 2*b*d*f*ArcTan[c*x] + 2*b*c^2*d*f*x^2*ArcTan[
c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - (4*I)*b*c^
2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] +
 (4*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*
f*g]] - 4*b*e*g*x^2*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + 4*b*c^2*e*
f*x^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*b*c^2*e*f*x^2*ArcSin[S
qrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^
((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)
] + 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*Ar
cTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sq
rt[c^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[(c^2*(1 + E^((2
*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*
x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[(c^2*(1 + E^(
(2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[
c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^
2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c
^2*f - g)] - 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I
)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 2*b*c^2*e*f*x^
2*ArcTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))
/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2
*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - 4*a*e*g*x^2*Log[x] + 2*a*e*f*Log[
f + g*x^2] + 2*b*c*e*f*x*Log[f + g*x^2] + 2*a*e*g*x^2*Log[f + g*x^2] + 2*b*
e*f*ArcTan[c*x]*Log[f + g*x^2] + 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[f + g*x^2]
 - (2*I)*b*c^2*e*f*x^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (2*I)*b*e*g*x^2
*PolyLog[2, E^((2*I)*ArcTan[c*x])] + I*b*c^2*e*f*x^2*PolyLog[2, (E^((2*I)*A
rcTan[c*x])*(-(c^2*f) - g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] - I*b*e*g*x^2*Po
lyLog[2, (E^((2*I)*ArcTan[c*x])*(-(c^2*f) - g + 2*Sqrt[c^2*f*g]))/(c^2*f -
g)] + I*b*c^2*e*f*x^2*PolyLog[2, -((E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sq
rt[c^2*f*g]))/(c^2*f - g))] - I*b*e*g*x^2*PolyLog[2, -((E^((2*I)*ArcTan[c*x
])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g))])/(f*x^2)
```

Maple [F]
time = 12.44, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(x^2g+f))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

```
[Out] int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(g*(log(g*x^2 +
f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e + 1/2*(2*c*g*x^2*arctan(g*x/sq
rt(f*g)) + (4*c^2*g*x^2*integrate(1/2*x*arctan(c*x)/(g*x^2 + f), x) + 4*g*x
^2*integrate(1/2*arctan(c*x)/(g*x^3 + f*x), x) - (c*x + (c^2*x^2 + 1)*arcta
n(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)*x^2) - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctan(c*x) + a*d + (b*arctan(c*x)*e + a*e)*log(g*x^2 + f))/x
^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

```
[Out] int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```